

# Lecture 10

## Planning - II - Bugs

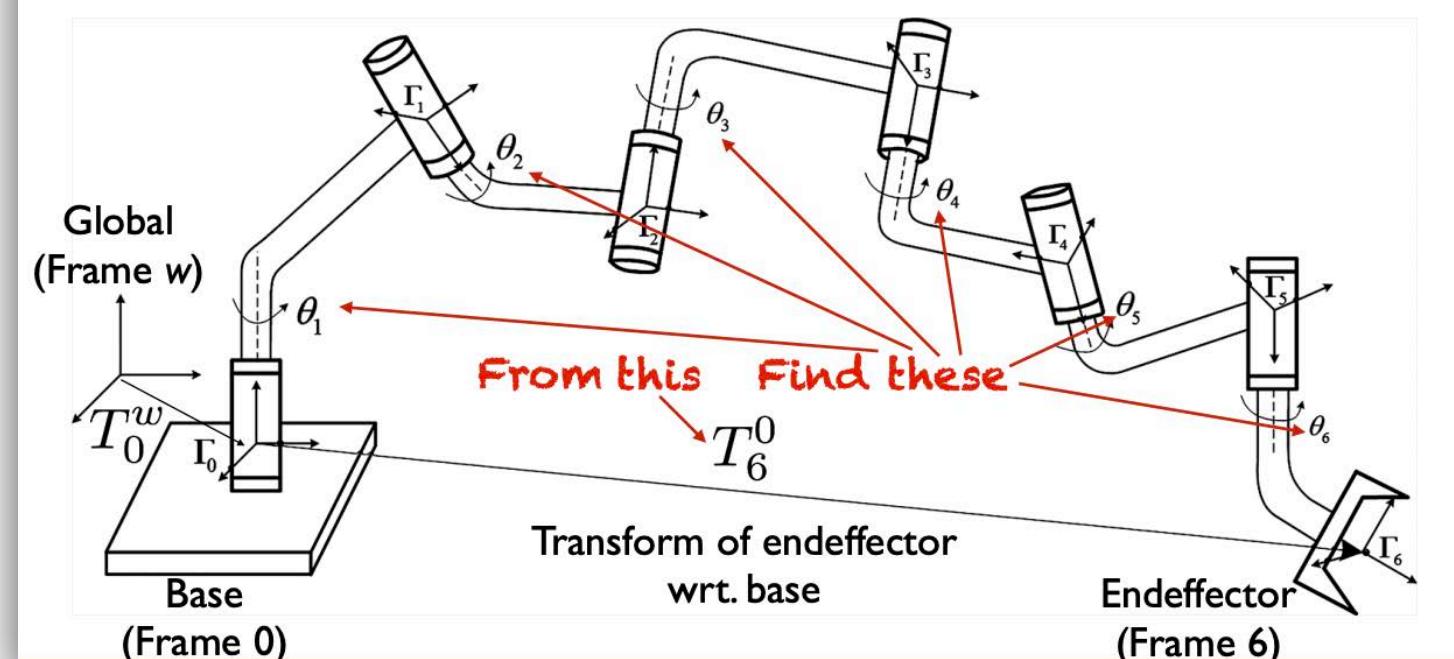


# Course Logistics

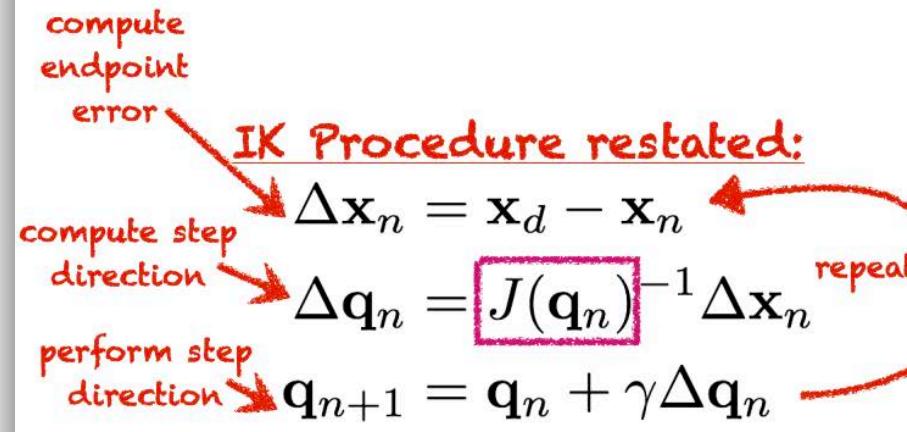
- Project 4 was posted on 02/19 and will be due on 03/05.
  - Start early!
- Quiz 5 will be posted tomorrow at noon and will be due at noon on Wed.

# Previously in Manipulation Lectures

Inverse kinematics: how to solve for  $q = \{\theta_1, \dots, \theta_N\}$  from  $T^w_n$ ?



How to use this Jacobian for IK as optimization?

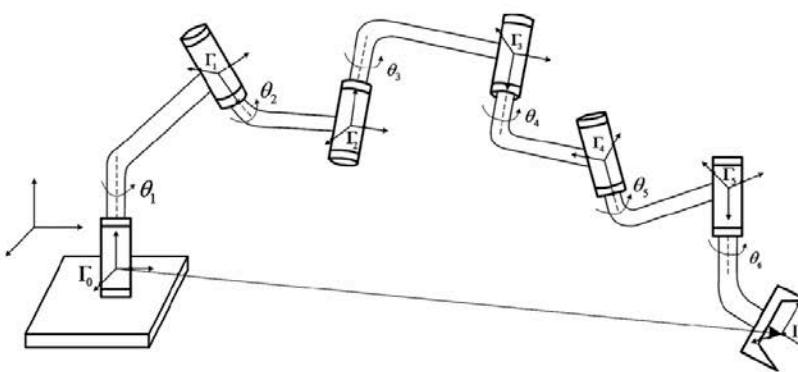


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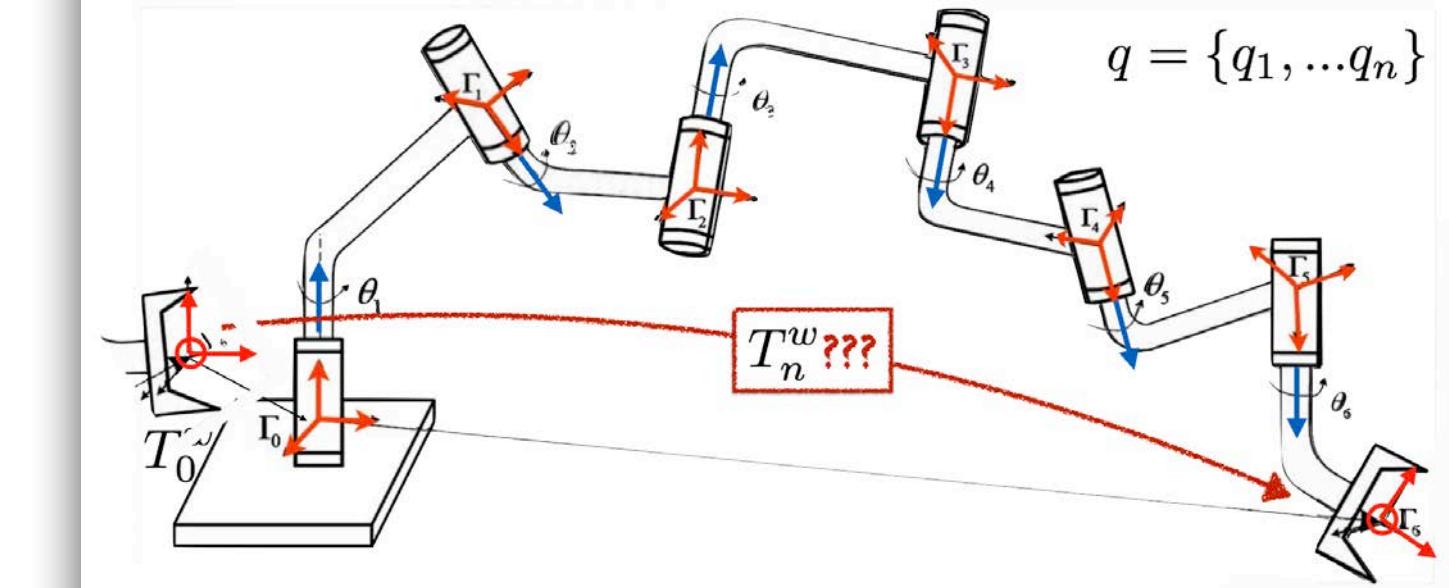
## Robot Kinematics

**Goal:** Given the structure of a robot arm, compute

- **Forward kinematics:** infer the pose of the end-effector, given the state of each joint.
- **Inverse kinematics:** infer the joint states to reach a desired end-effector pose.



**Forward kinematics restated:** Given  $q$ , find  $T^w_n$ ;  $T^w_n$  transforms endeffector into workspace



## Inverse Kinematics: 2 possibilites

- **Closed-form solution:** geometrically infer satisfying configuration
  - *Speed:* solution often computed in constant time
  - *Predictability:* solution is selected in a consistent manner
- **Solve by optimization:** minimize error of endeffector to desired pose
  - often some form of Gradient Descent (a la Jacobian Transpose)
  - *Generality:* same solver can be used for many different robots

## Robot arm and its Jacobian

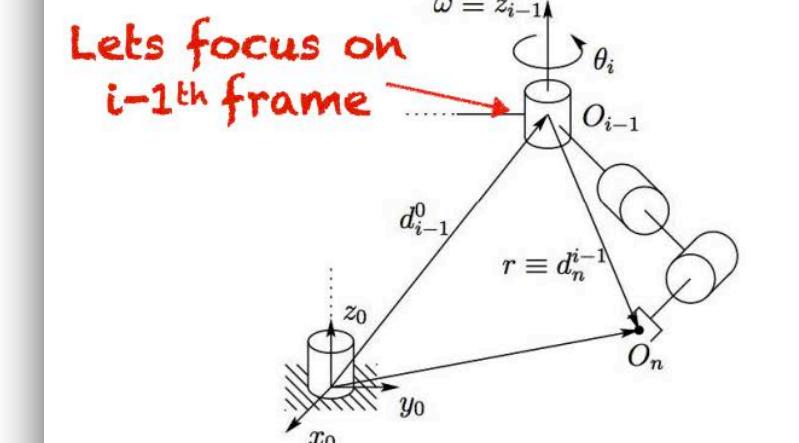


Figure 5.1: Motion of the end-effector due to link  $i$ .

$J_i$  for a prismatic joint

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} \quad J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix}$$

*i-1<sup>th</sup> frame maps to i<sup>th</sup> column in*

## The Jacobian

A  $6 \times N$  matrix

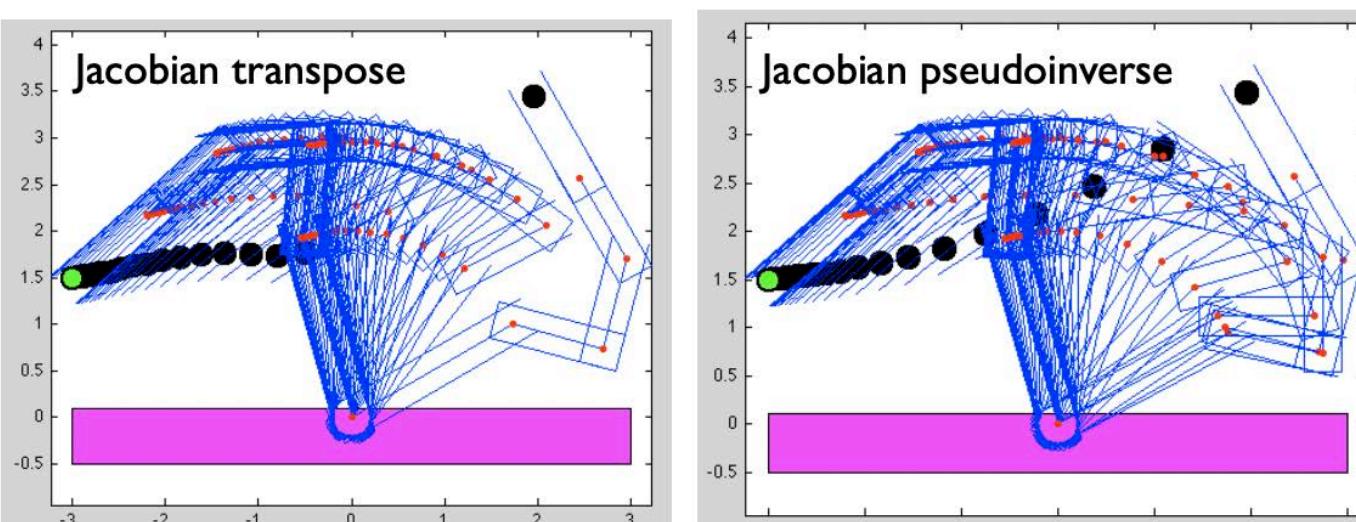
$$J = [J_1 \ J_2 \ \dots \ J_n]$$

consisting of two  $3 \times N$  matrices

$$J = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$$

$J_i$  for a rotational joint

## Matlab 5-link arm example: Jacobian transpose



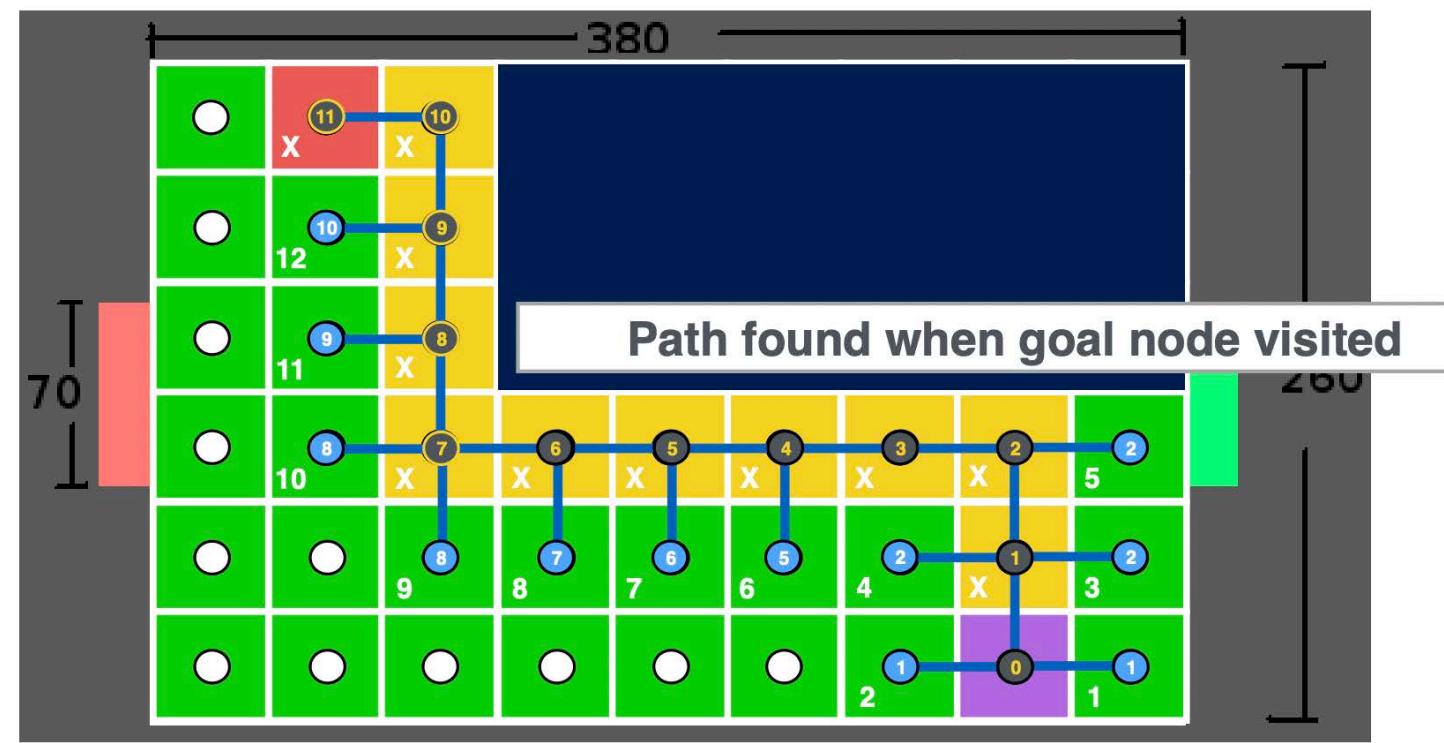
## Definition of Manipulation

Mason, Matthew T. "Toward robotic manipulation." *Annual Review of Control, Robotics, and Autonomous Systems* 1 (2018): 1-28.

This lecture uses the structure and material from this review paper!

# Previously in Planning, Decision Making, Control

## Depth-first search



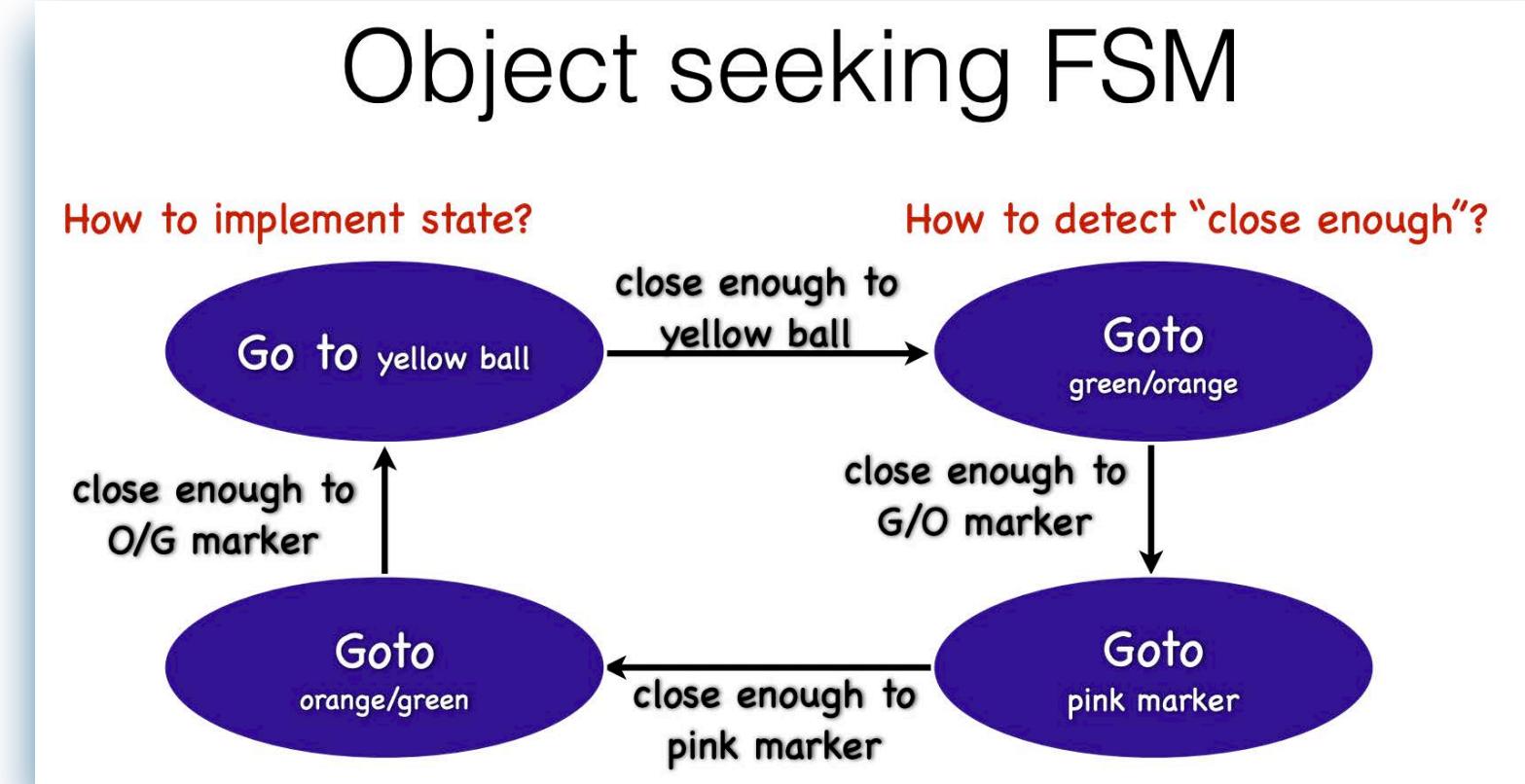
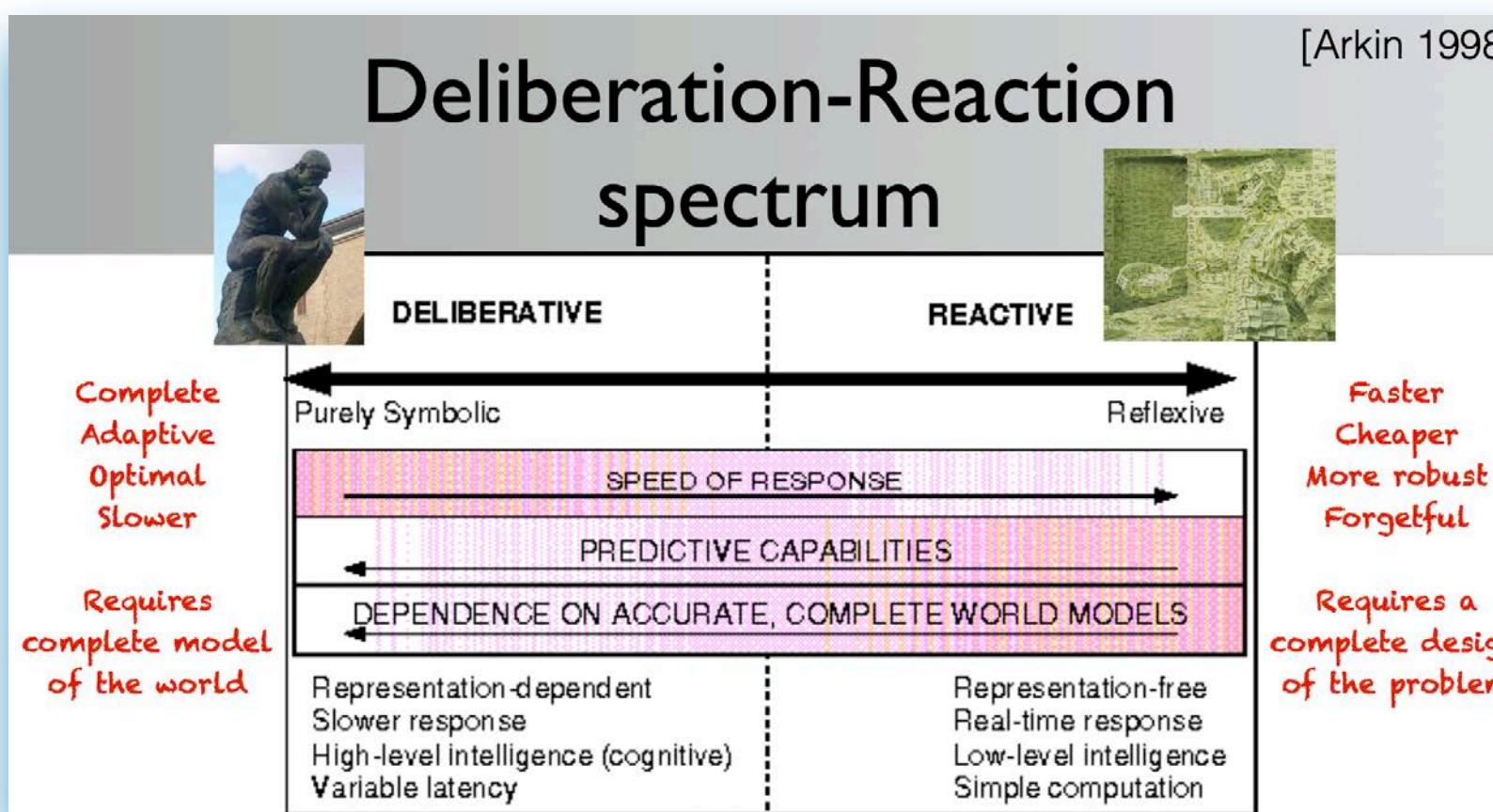
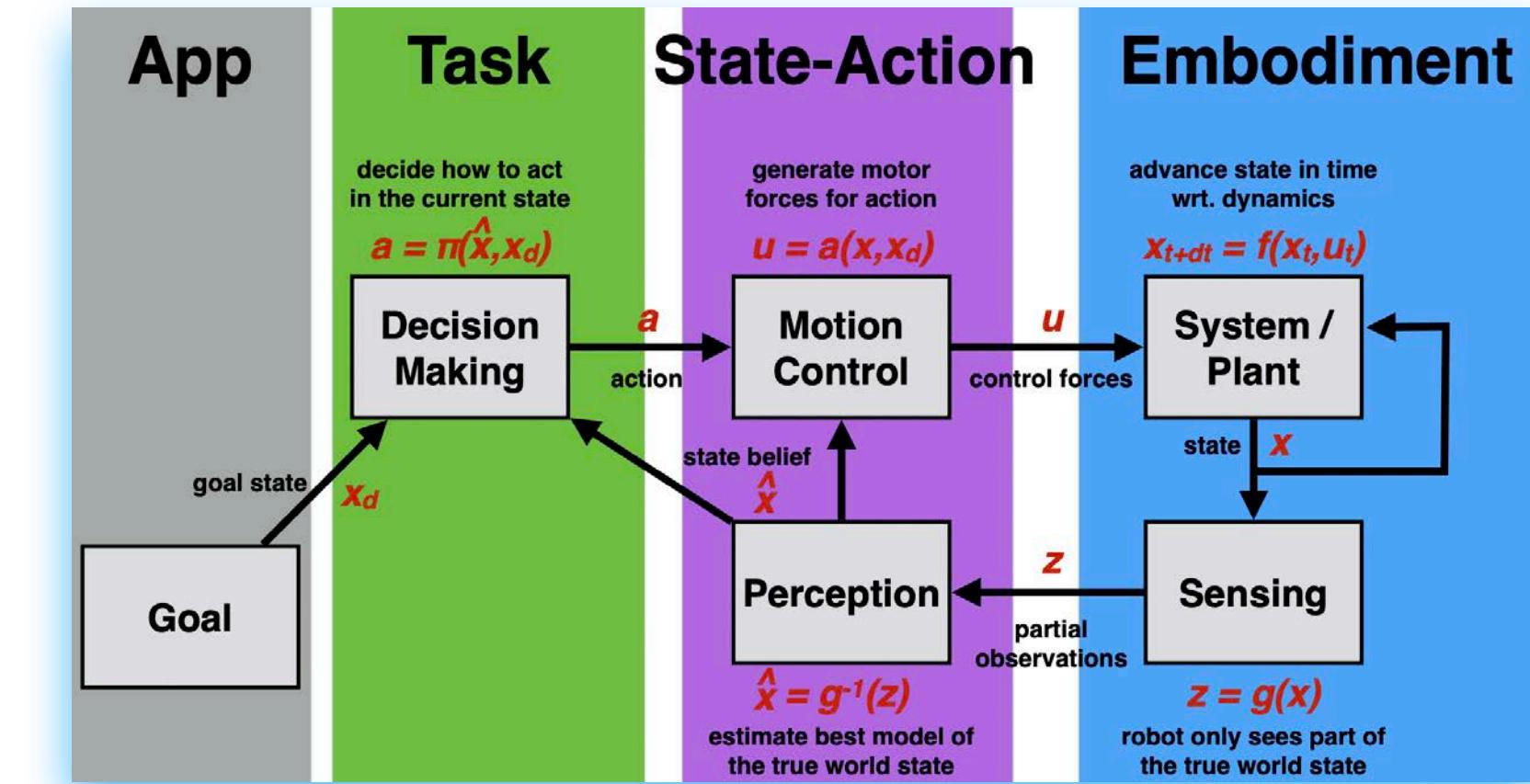
### Search algorithm template

```

all nodes ← {diststart ← infinity, parentstart ← none, visitedstart ← false}
start_node ← {diststart ← 0, parentstart ← none, visitedstart ← true}
visit_list ← start_node

while visit_list != empty && current_node != goal
    cur_node ← highestPriority(visit_list)
    visitedcur_node ← true
    for each nbr in not_visited(adjacent(cur_node))
        add(nbr to visit_list)
        if distnbr > distcur_node + distStraightLine(nbr,cur_node)
            parentnbr ← current_node
            distnbr ← distcur_node + distStraightLine(nbr,cur_node)
        end if
    end for loop
end while loop
output ← parent, distance

```



## PID Control

Error signal:

$$e(t) = x_{desired}(t) - x(t)$$

Control signal:

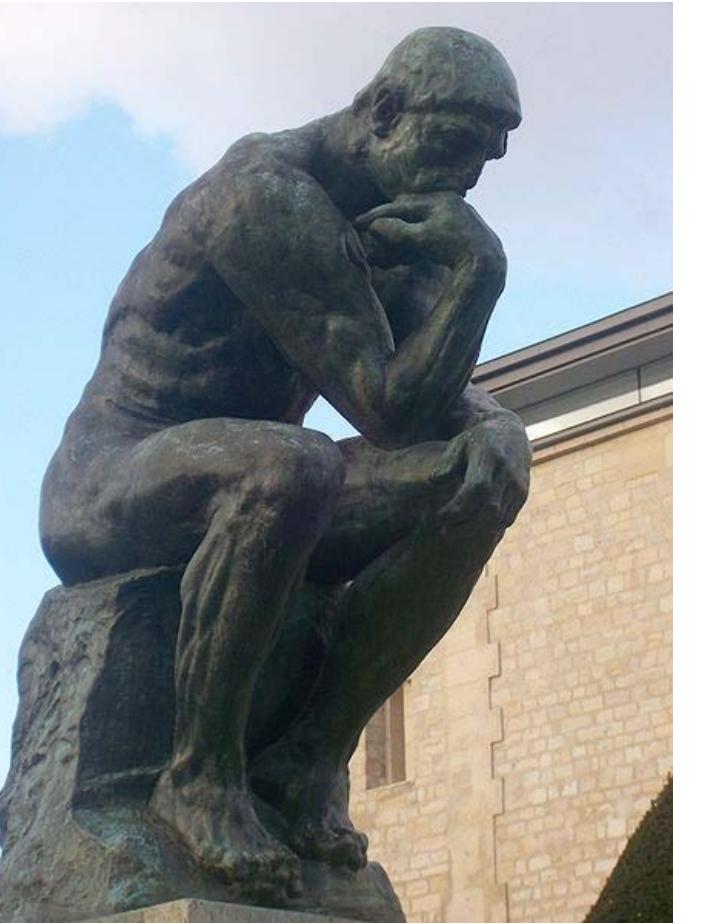
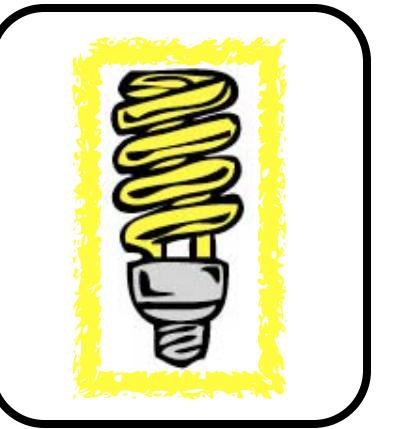
$$u(t) = K_p e(t) + K_i \int_0^t e(\alpha) d\alpha + K_d \frac{d}{dt} e(t)$$

P  $K_p e(t)$   
I  $K_i \int_0^t e(\alpha) d\alpha$   
D  $K_d \frac{d}{dt} e(t)$   
**Current**      **Past**      **Future**

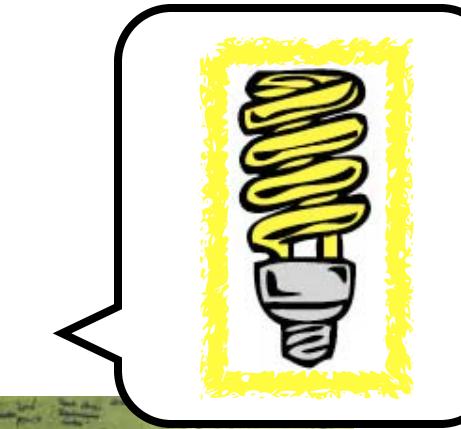
# Approaches to motion planning

- **Bug algorithms: Bug[0-2], Tangent Bug**
- Graph Search (fixed graph)
  - Depth-first, Breadth-first, Dijkstra, A-star
- Sampling-based Search (build graph):
  - Probabilistic Road Maps, Rapidly-exploring Random Trees
- Optimization (local search):
  - Gradient descent, potential fields, Wavefront

# Should your robot's decision making



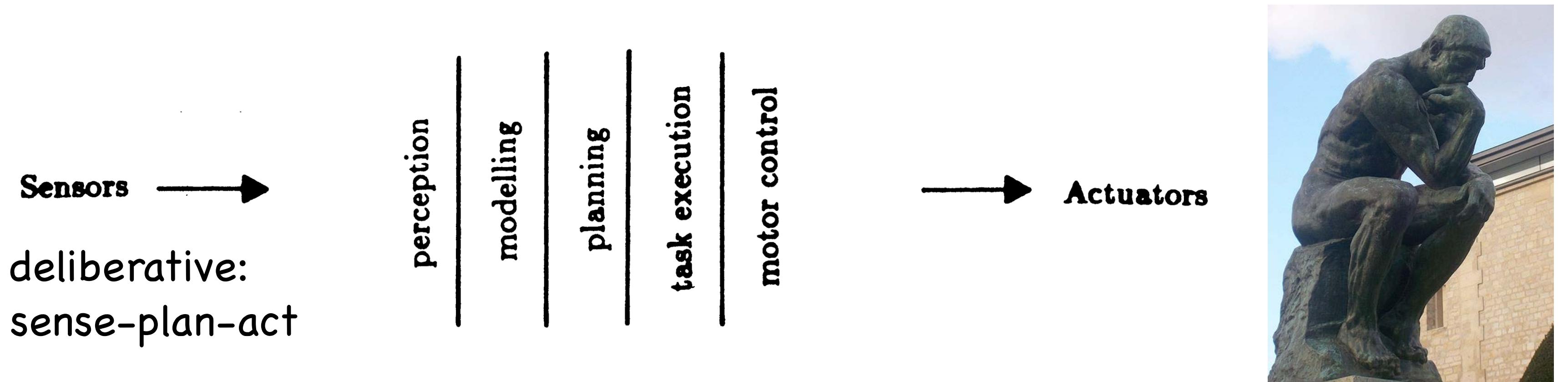
OR



fully think through  
solving a problem?

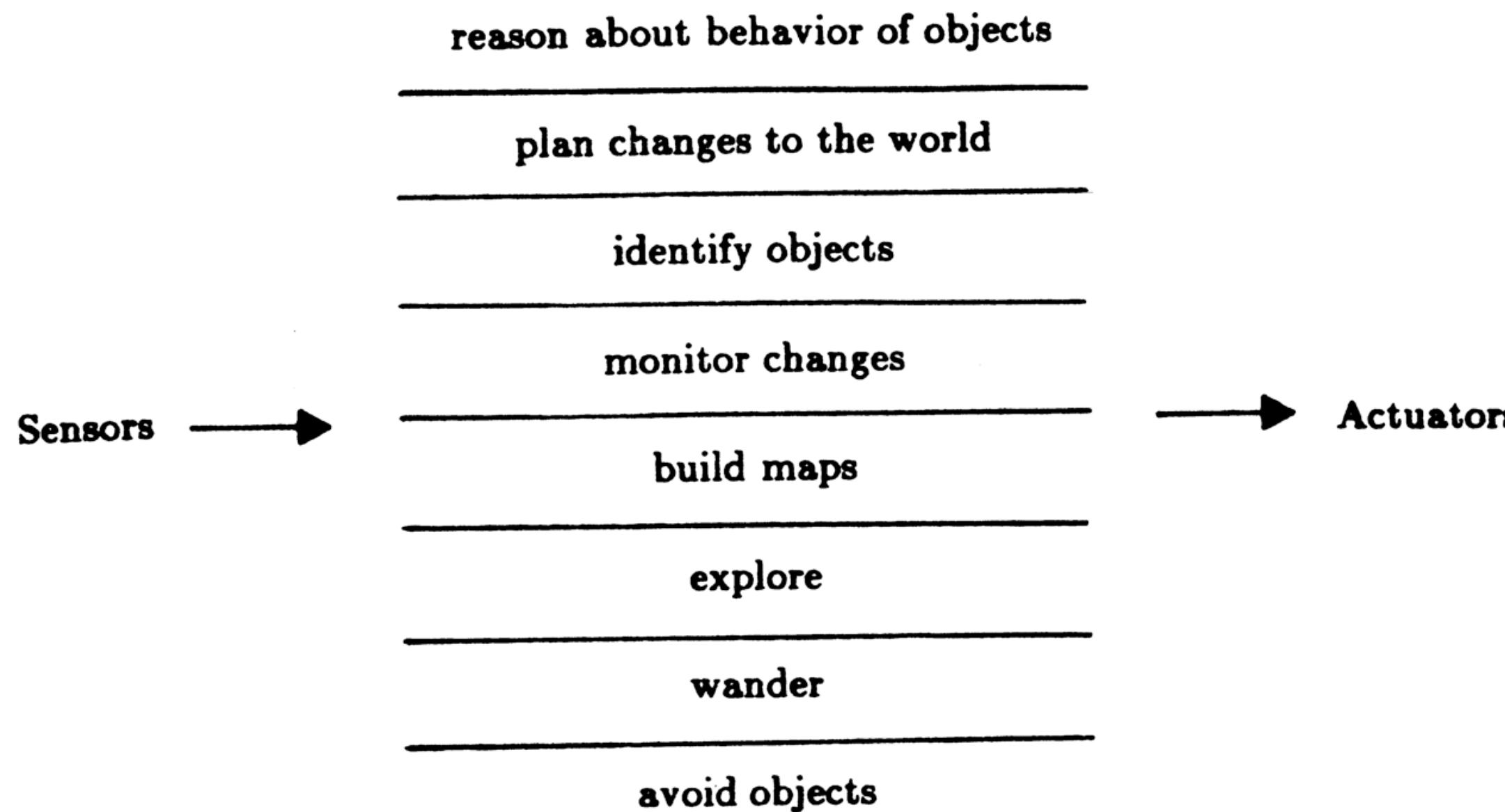
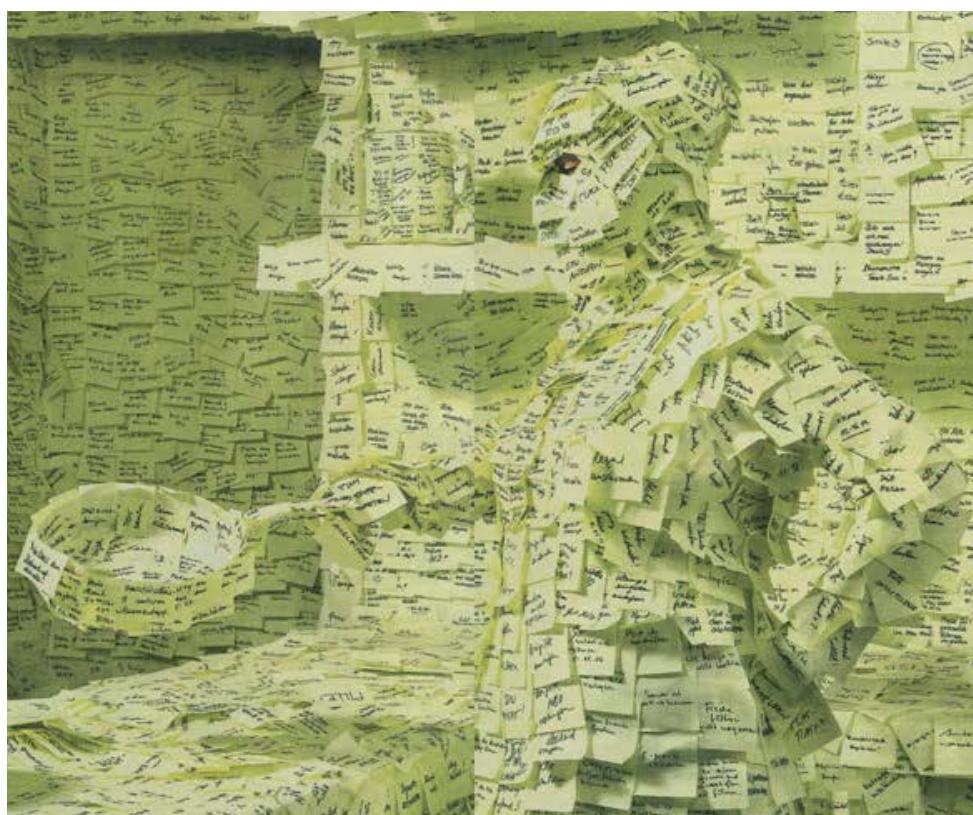
react quickly to  
changes in its world?

# Deliberation v. Reaction



deliberative:  
sense-plan-act

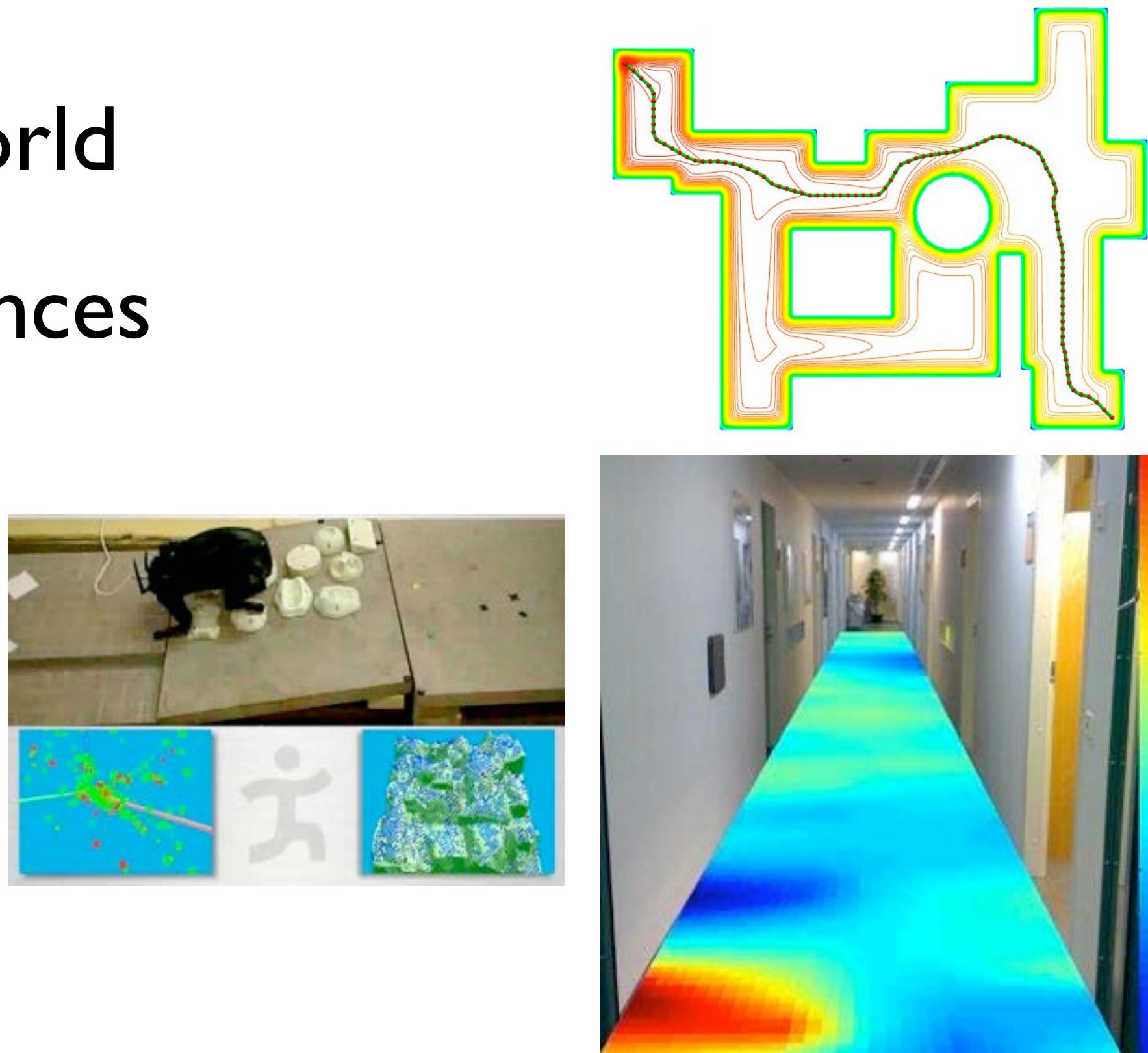
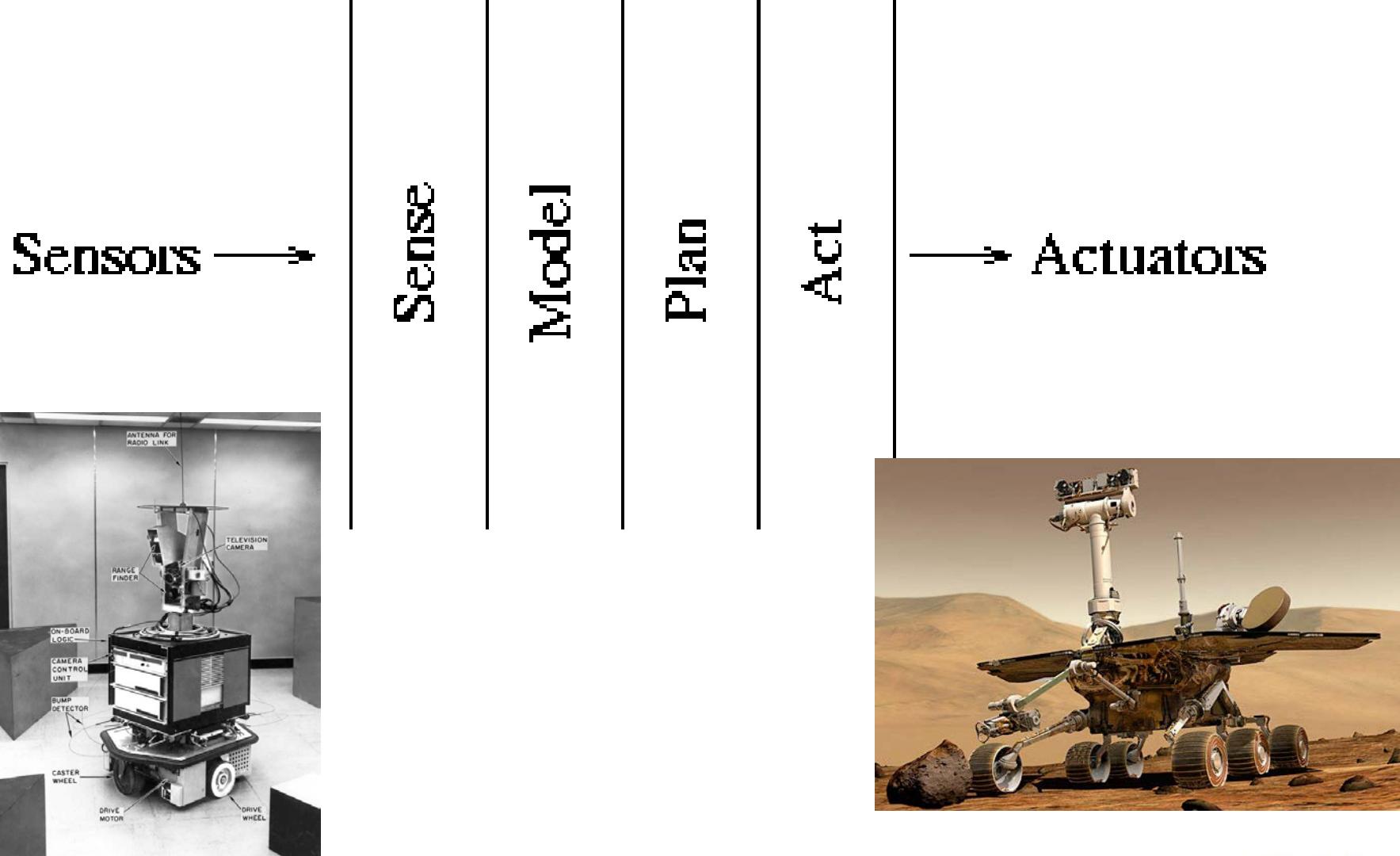
reaction: subsumption,  
Finite State Machine  
controllers act in parallel



# Deliberation

## “Sense-Plan-Act” paradigm

- sense: build most complete model of world
- GPS, SLAM, 3D reconstruction, affordances
- plan: search over all possible outcomes
- BFS, DFS, Dijkstra, A\*, RRT
- act: execute plan through motor forces



# Reaction

- No representation of state
- Typically, fast hardcoded rules
- Embodied intelligence
  - behavior := control + embodiment
  - ant analogy, stigmergy
- Subsumption architecture
- prioritized reactive policies
- Ghengis hexpod video

Sensors → → Actuators

Avoid Obstacles

Avoid Collision



Explore

Wander Around

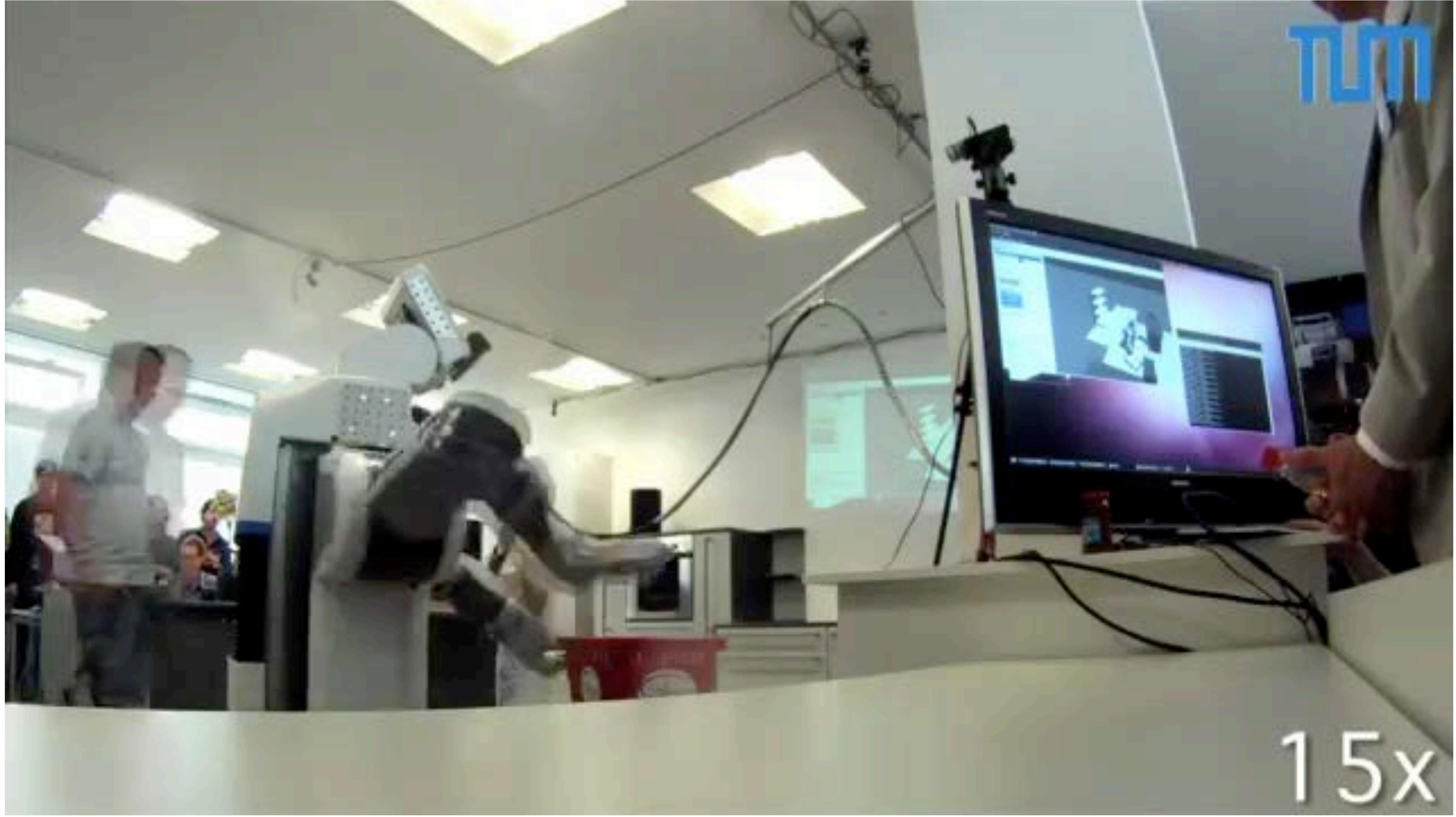
Avoid Collision



# MIT Genghis

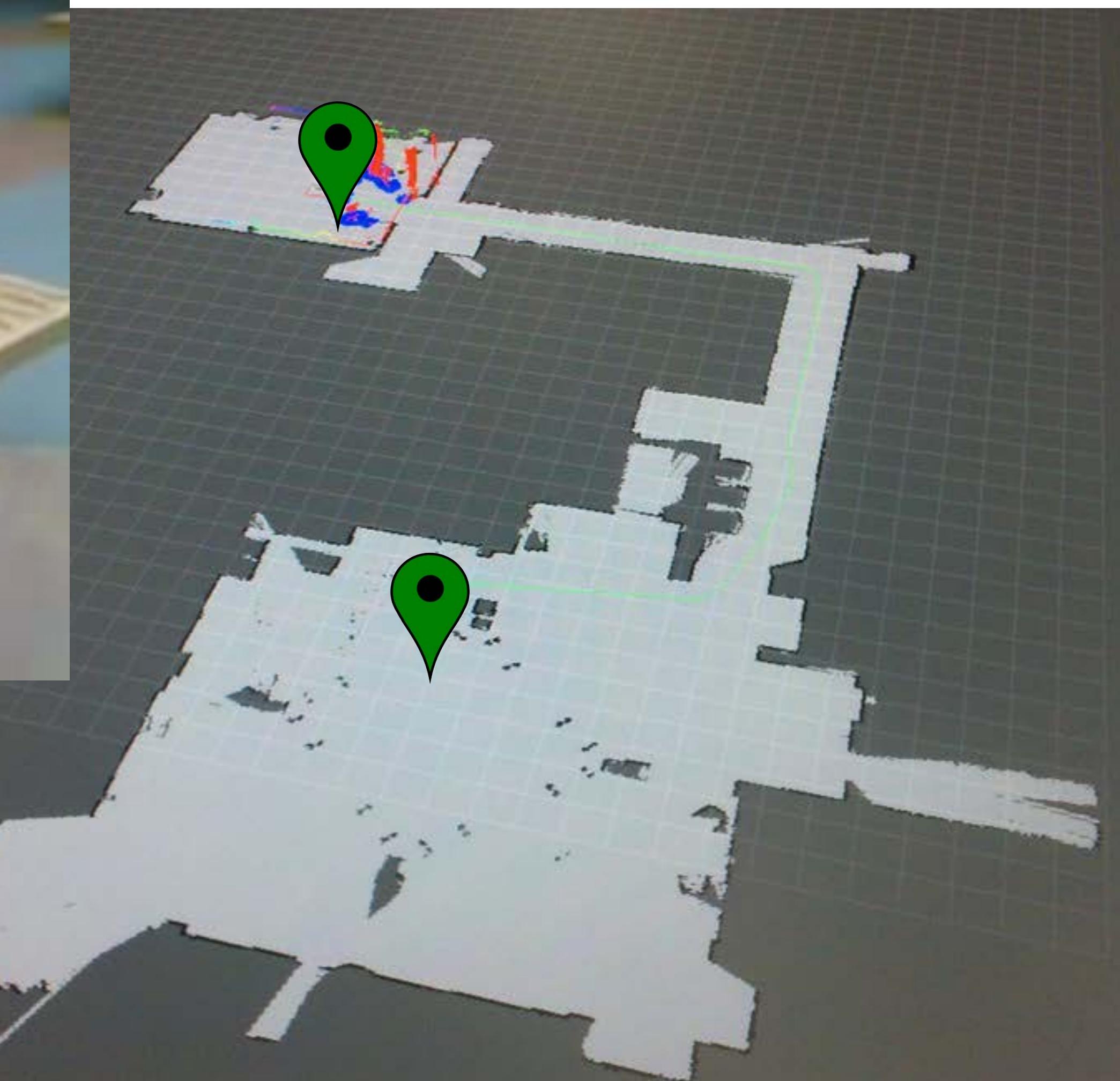
<https://www.youtube.com/watch?v=1j6CliOwRng>





15x

Robots have to make lots of decisions



Remember your path planner?

How to get from Location A to  
Location B?

# Base Navigation

- How get from point A to point B
- **What is the simplest policy to perform navigation?**
  - Remember: simplest reactive policy?

# Random Walk: Goal Seeking

- Move in a random direction until you hit something
- Then go in a new direction
- Stop when you get to the goal, assuming it can be recognized



Lisa Miller, <http://www.youtube.com/watch?v=VBzXDrz8rMI>

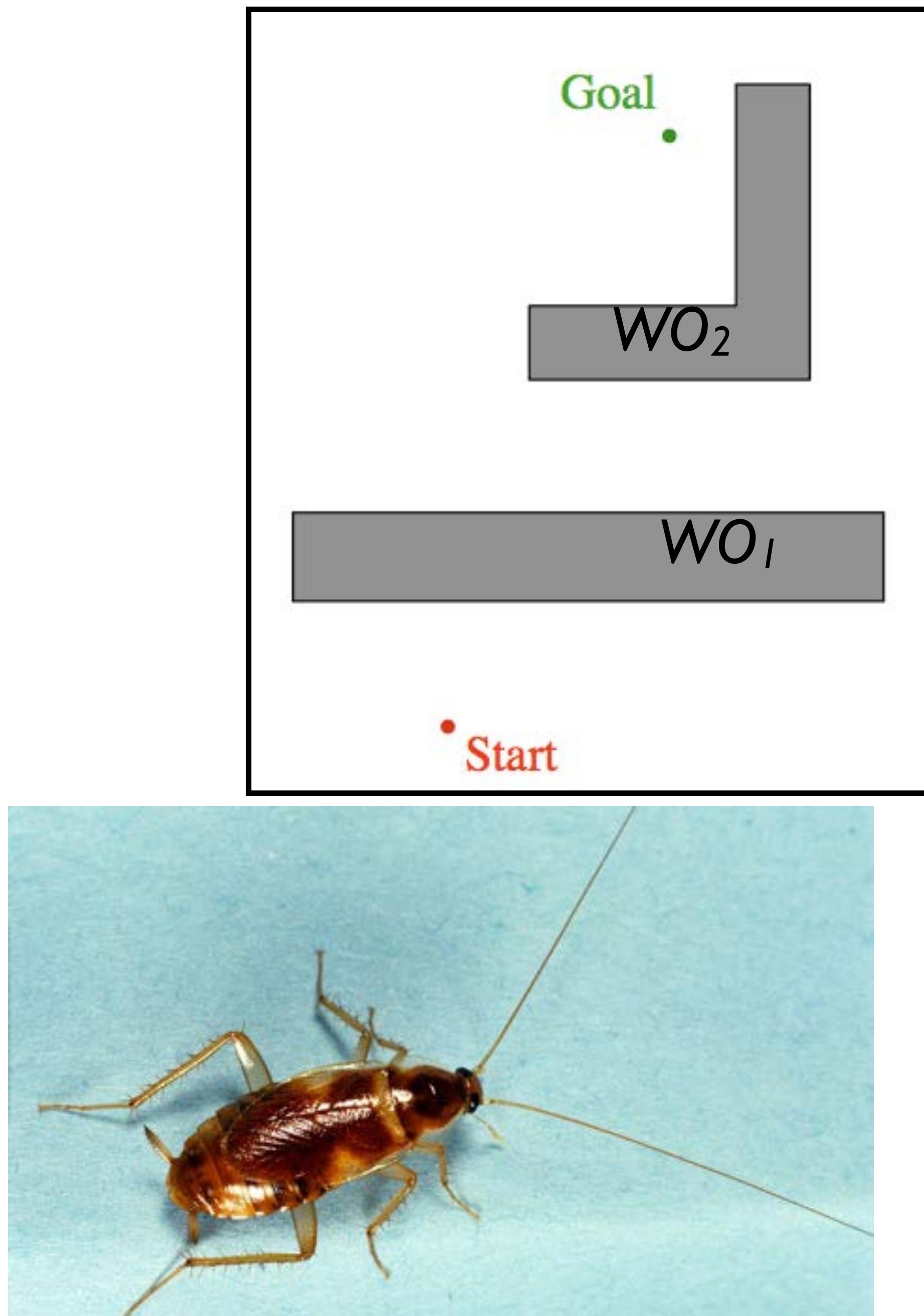


# Base Navigation

- How get from point A to point B
- What is the simplest policy to perform navigation?
  - random walk
  - reactive: embodied intelligence
- **What is a “simple” deliberative policy?**

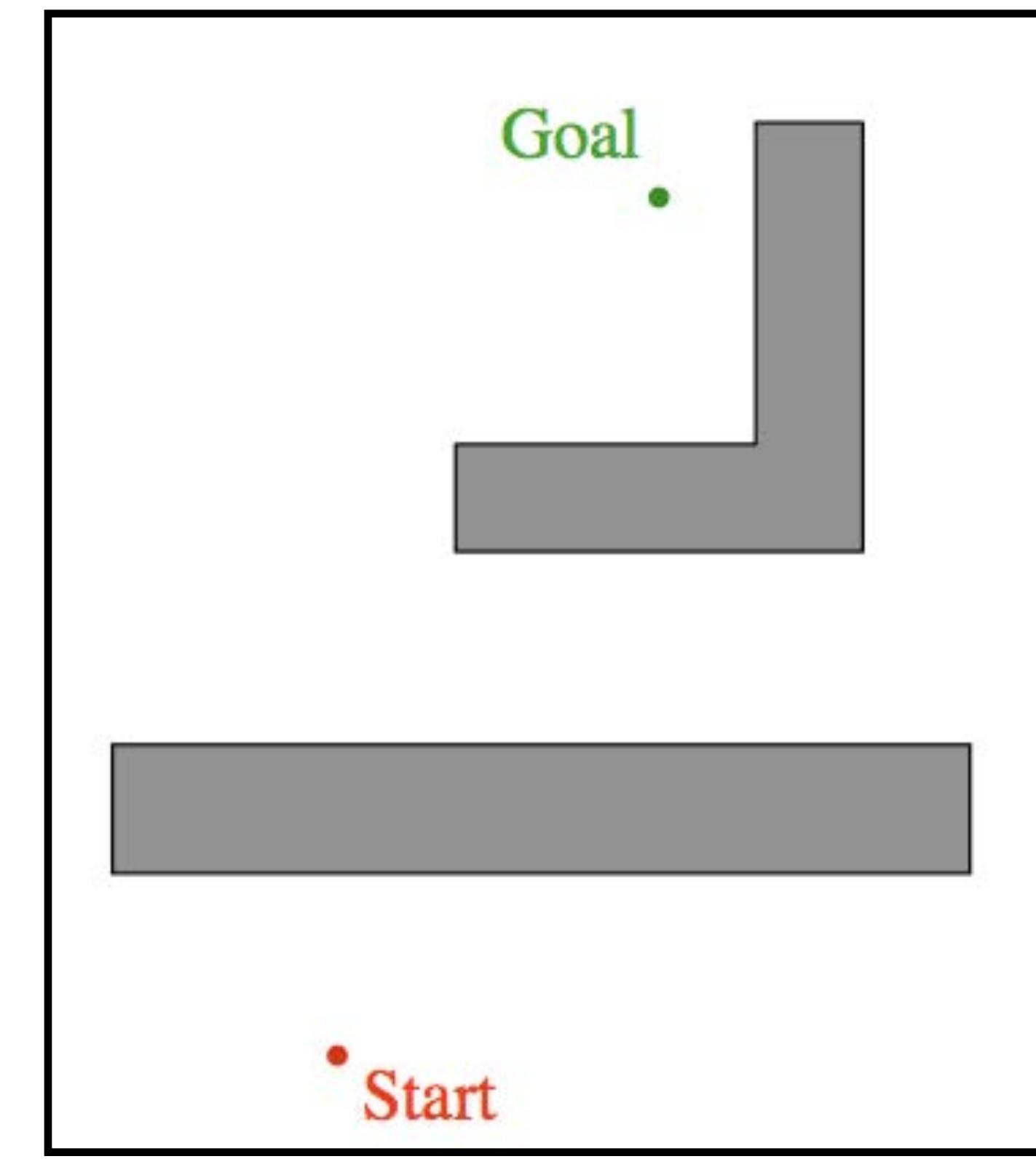
# Bug Algorithms

- Assume bounded world  $W$
- Known: global goal
  - measurable distance  $d(x,y)$
- Unknown: obstacles  $WO_i$
- Local sensing
  - tactile
  - distance traveled



# Bug Algorithms

- Assume bounded world  $W$
- Known: global goal
  - measurable distance  $d(x,y)$
- Unknown: obstacles  $WO_i$
- Local sensing
  - **bump sensor**
  - distance traveled



$\approx$

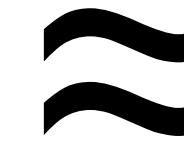
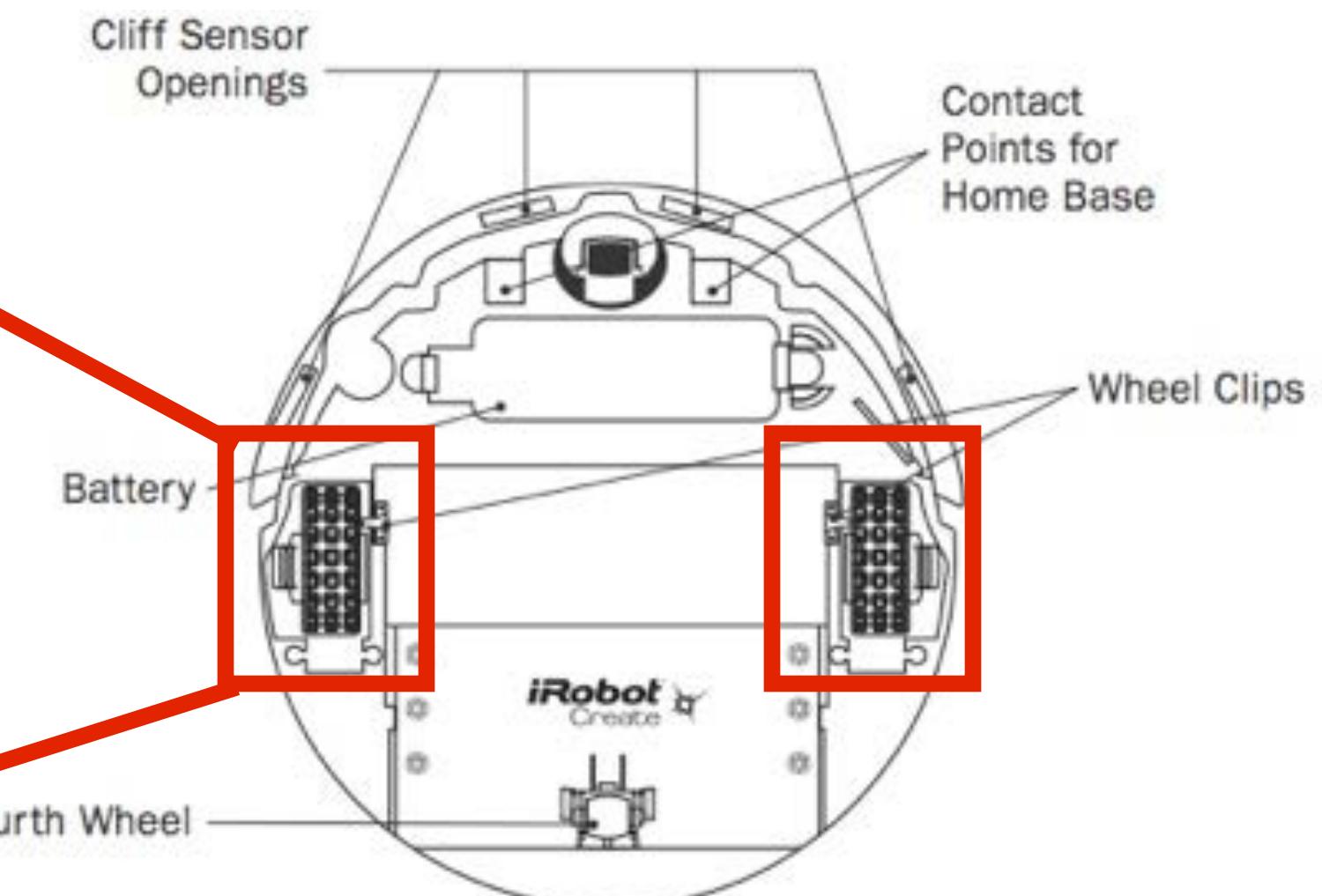
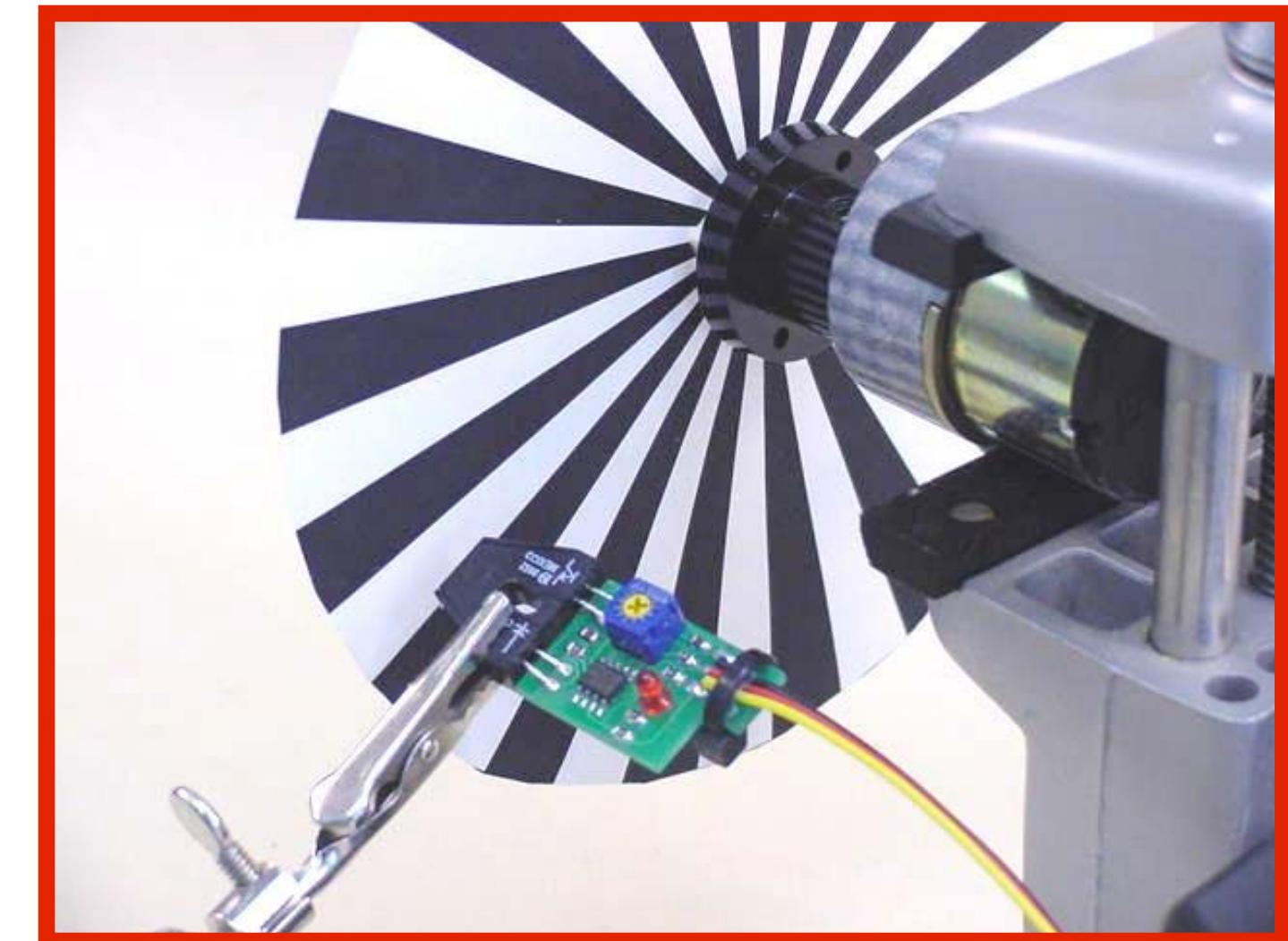


bumper is essentially an on/off button

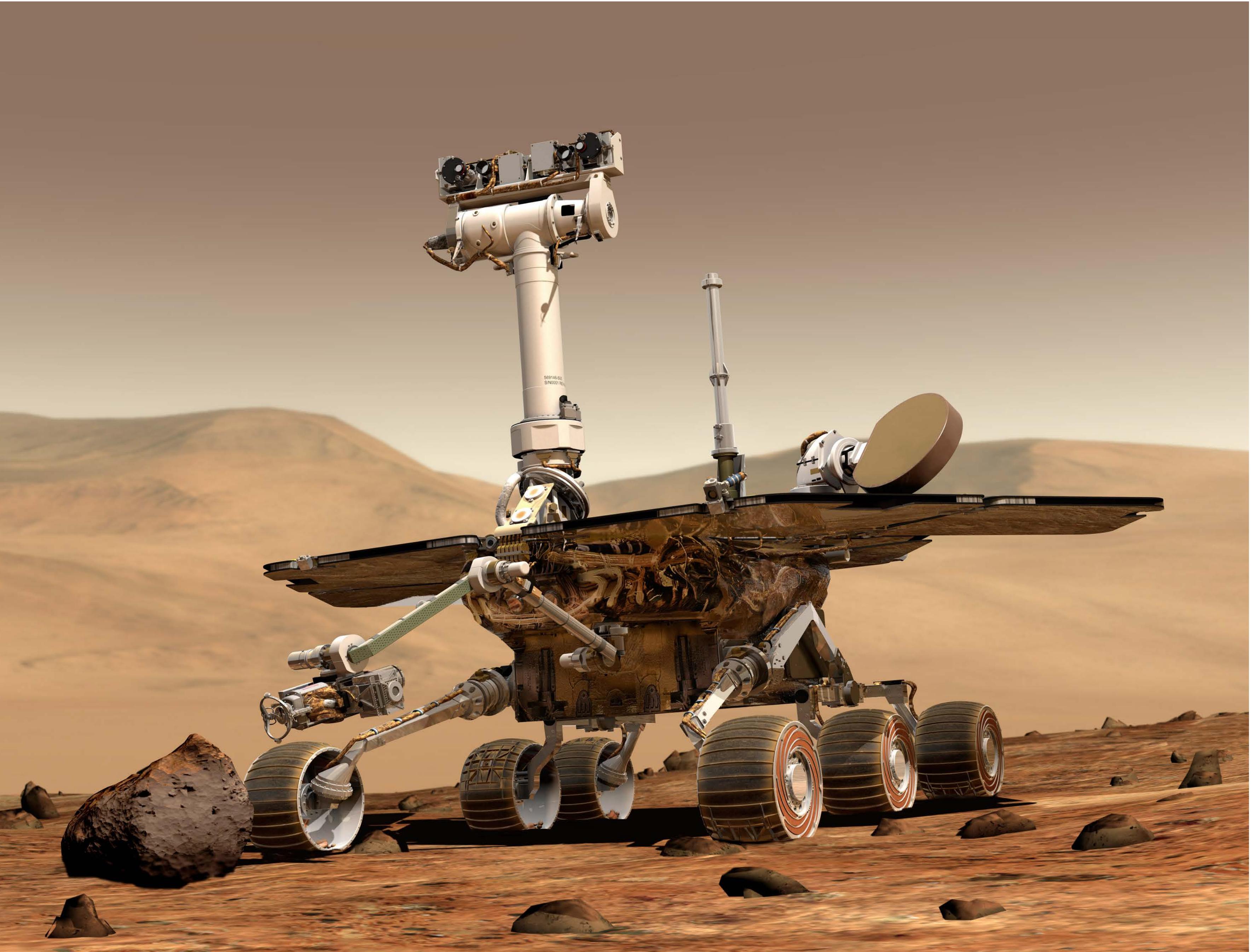
# Bug

## Optical encoders

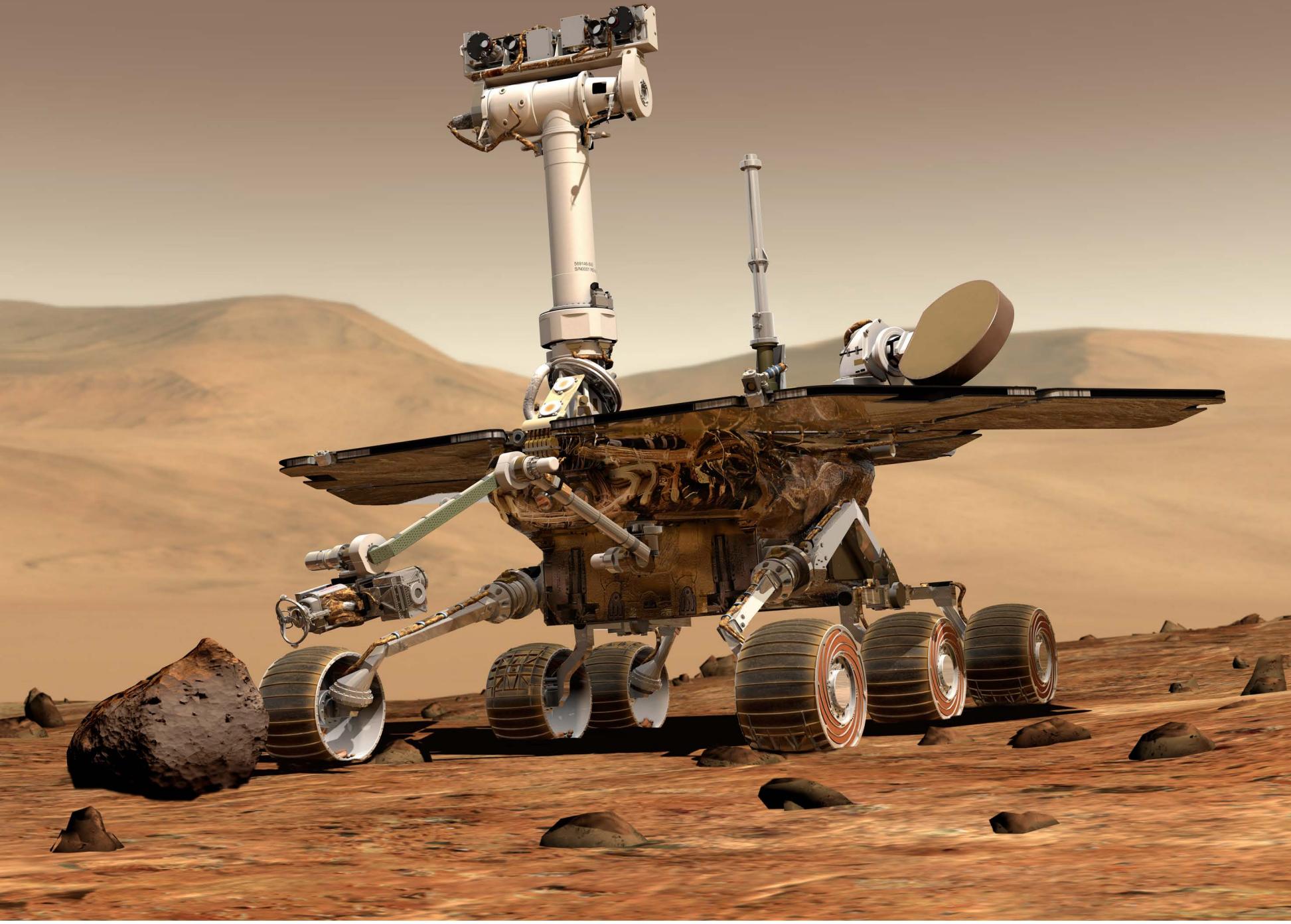
- Assume
- Known:
  - meas
- Unknown
- Local sensing
  - bump
  - odometry



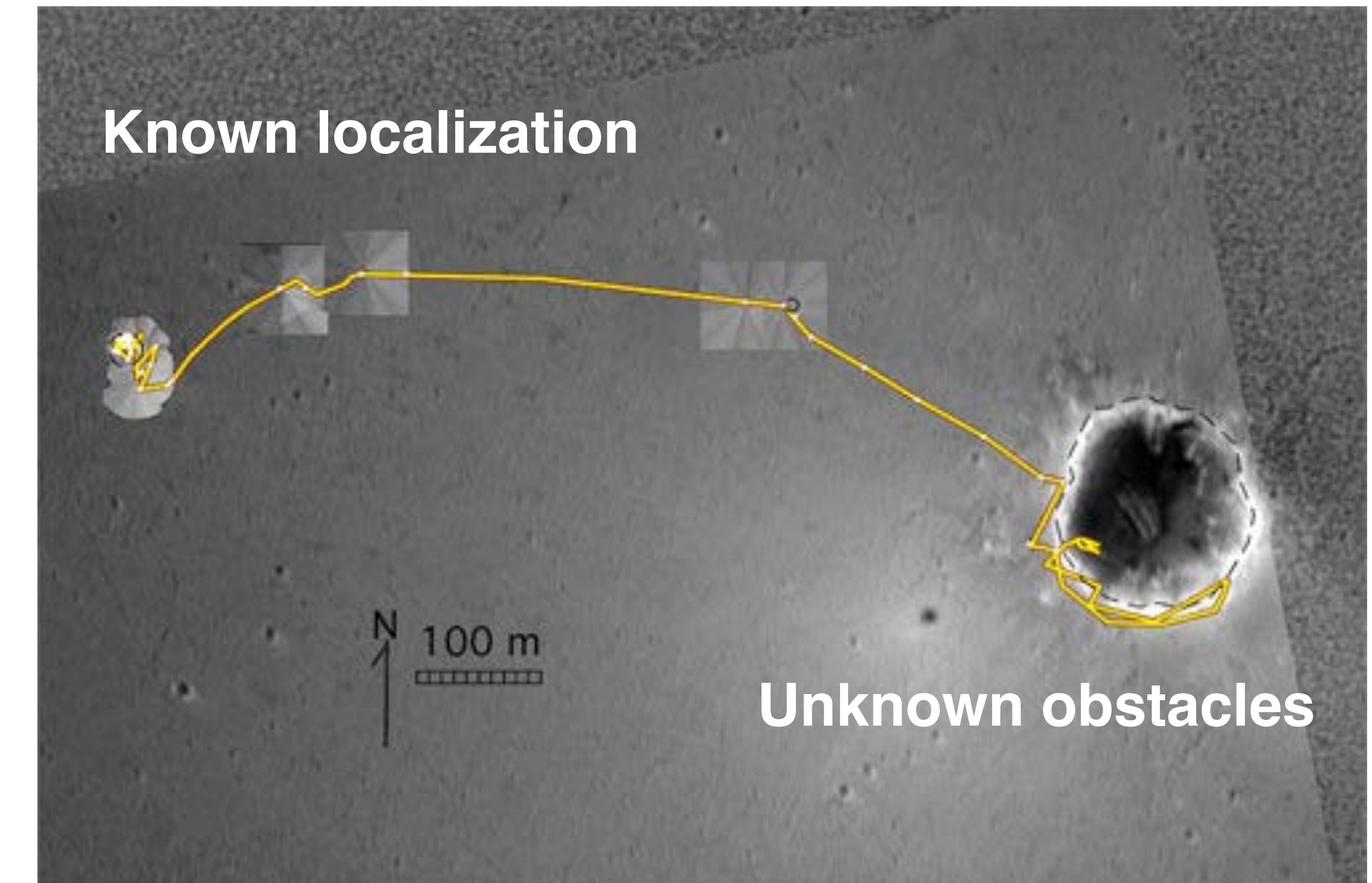
# Interesting application of Bug algorithms ?



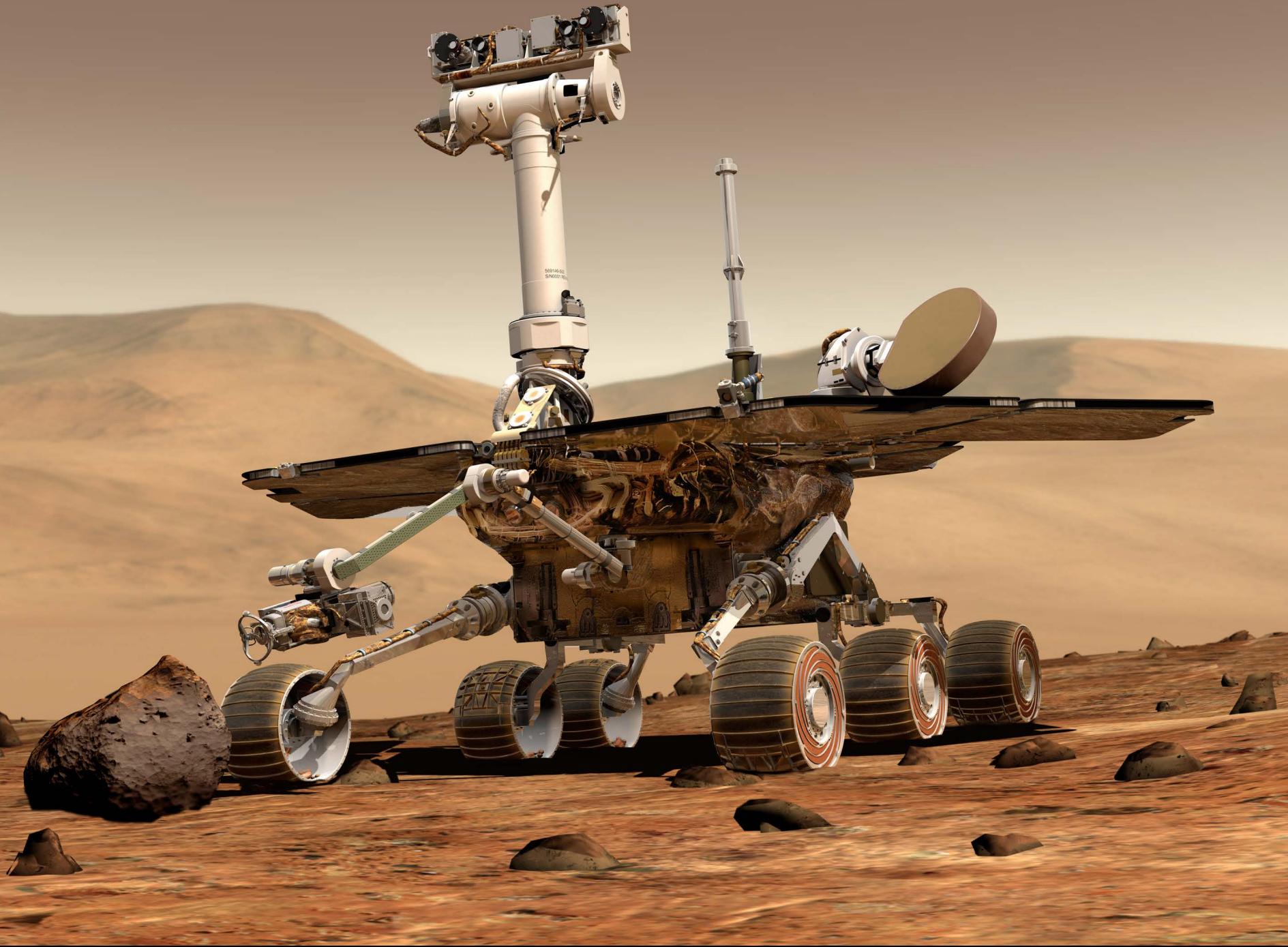
# Mars Exploration Rover



<http://mars.nasa.gov/mer/gallery/press/opportunity/20040921a.html>

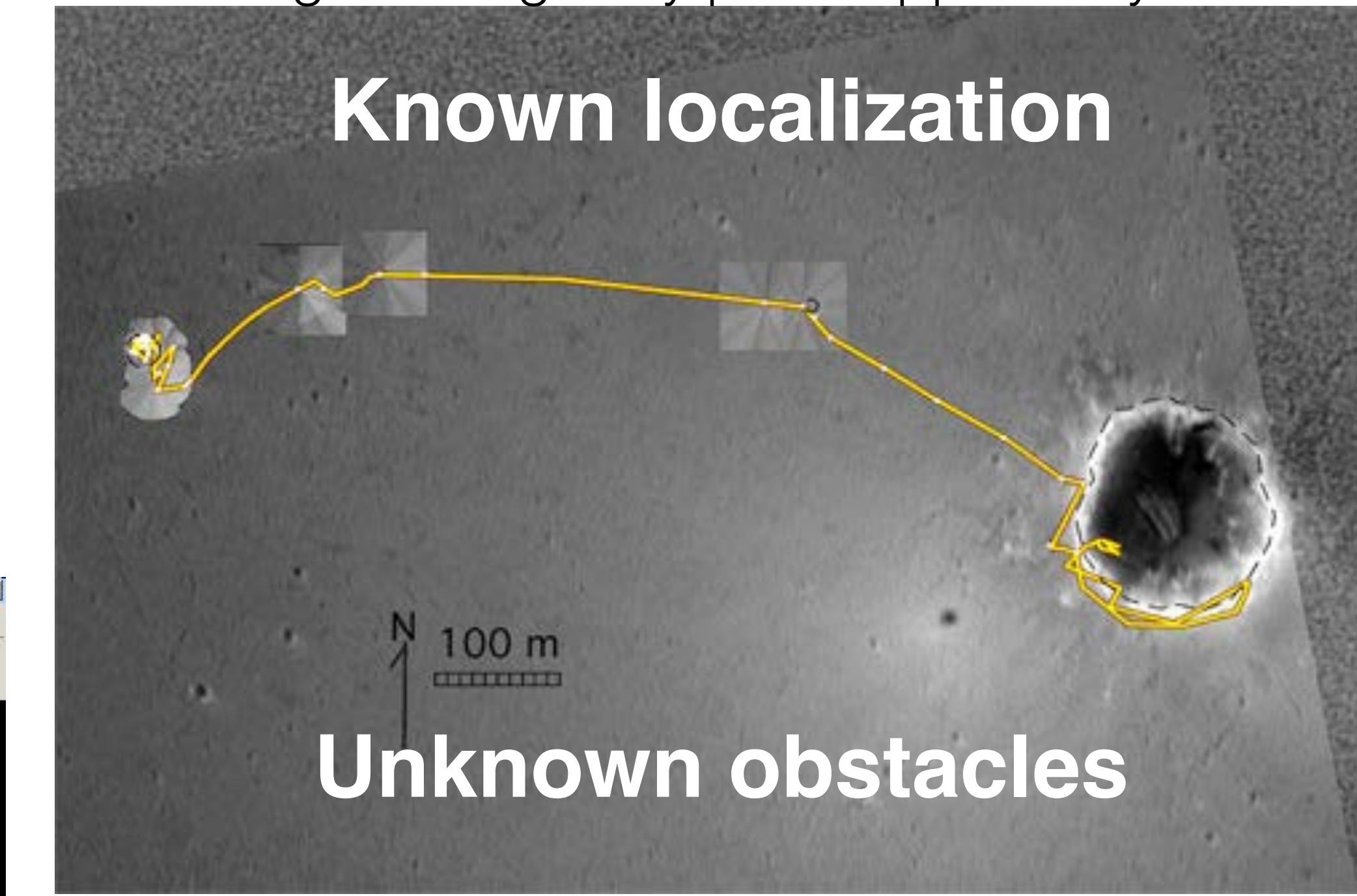


# Mars Exploration Rover

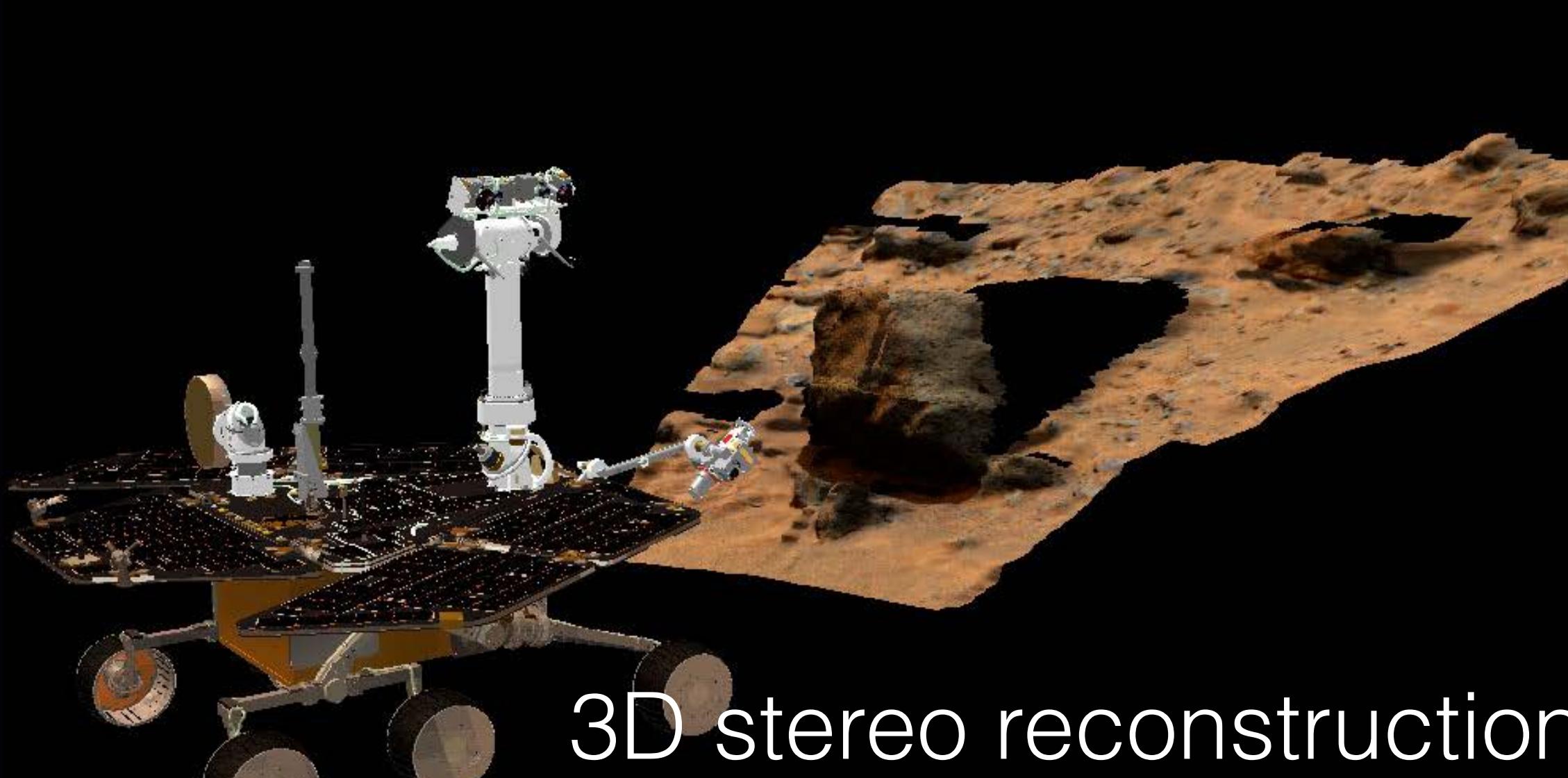


<http://mars.nasa.gov/mer/gallery/press/opportunity/20040921a.html>

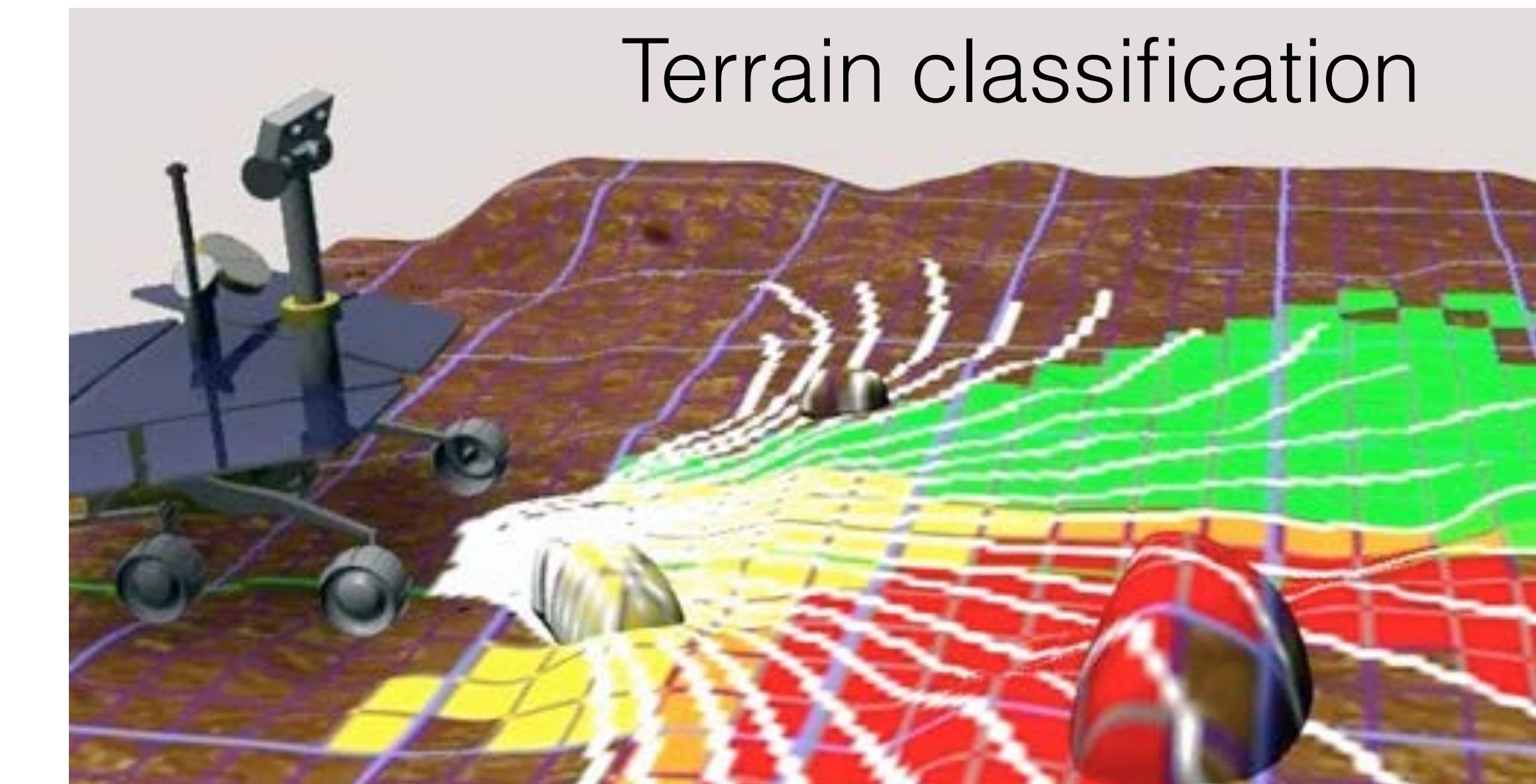
Known localization



Unknown obstacles

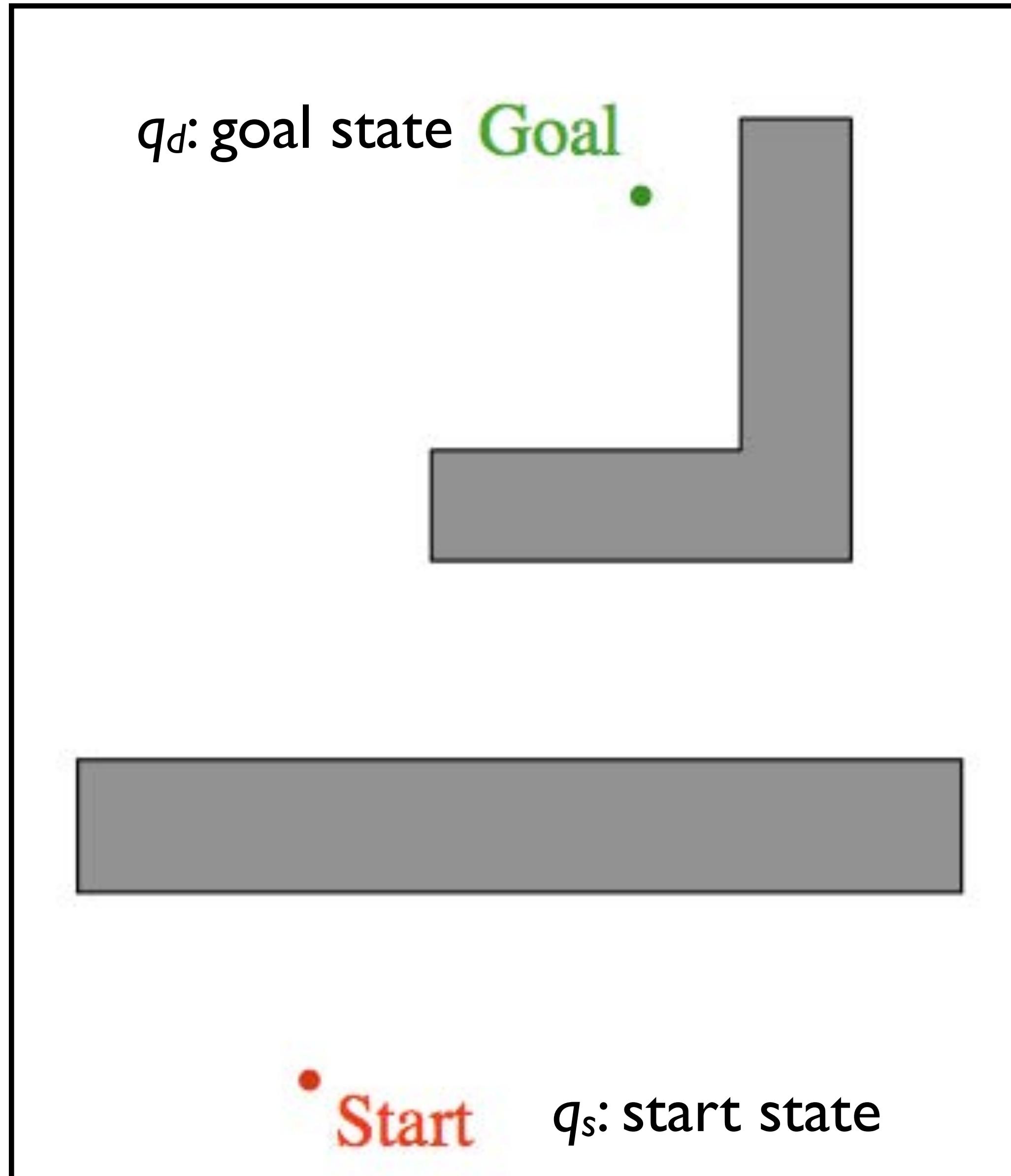


3D stereo reconstruction



Terrain classification

# Bug Navigation

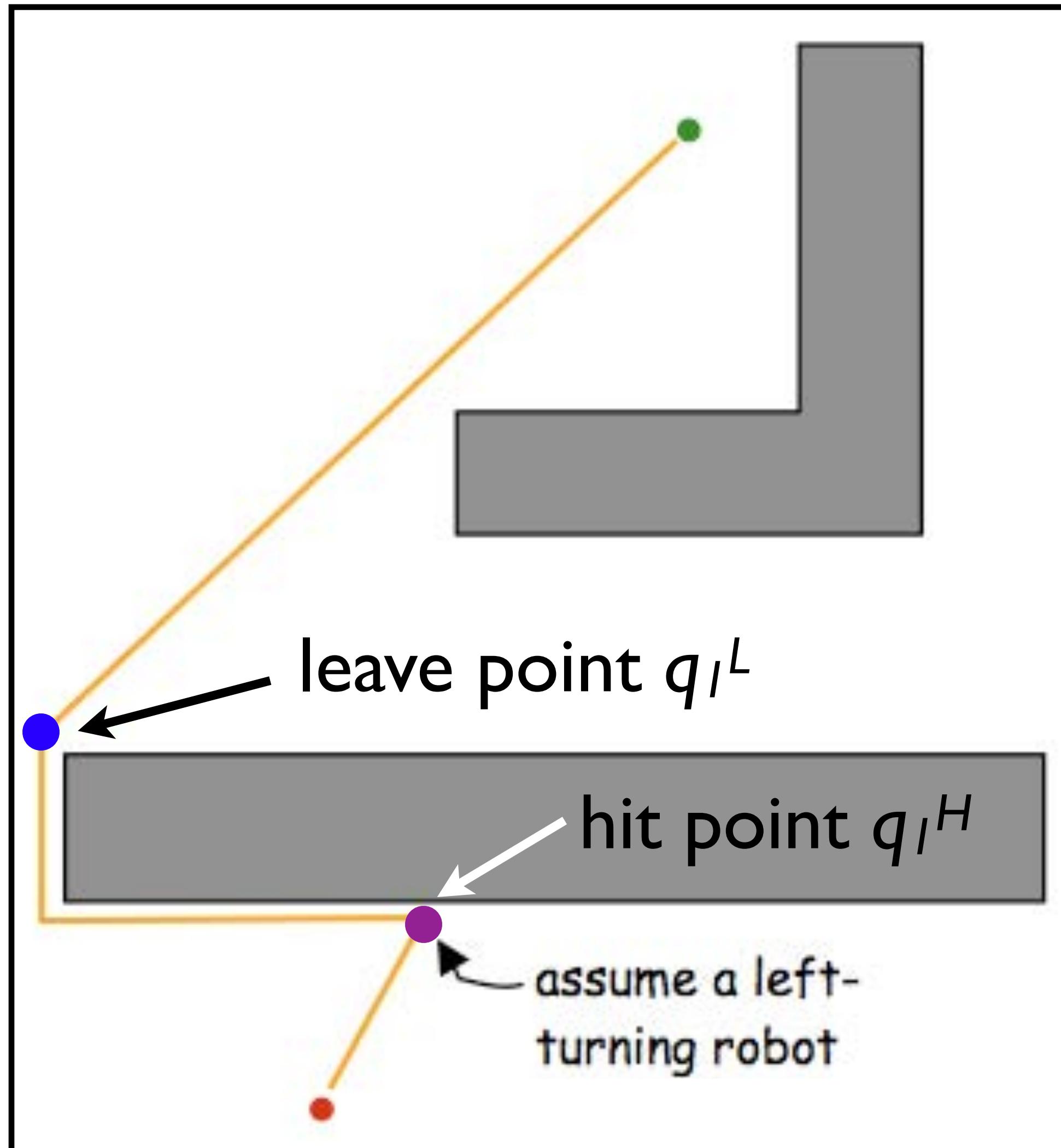


Plan navigation path from start  $q_s$  to goal  $q_d$

as a sequence of hit/leave point pairs on obstacles

Hit point:  $q_i^H$   
Leave point:  $q_i^L$

# Bug 0



- 1) Head towards goal
- 2) When hit point set, **follow wall**, until you can move towards goal again (leave point)
- 3) continue from (1)

# Wall following

follow wall



One approach:

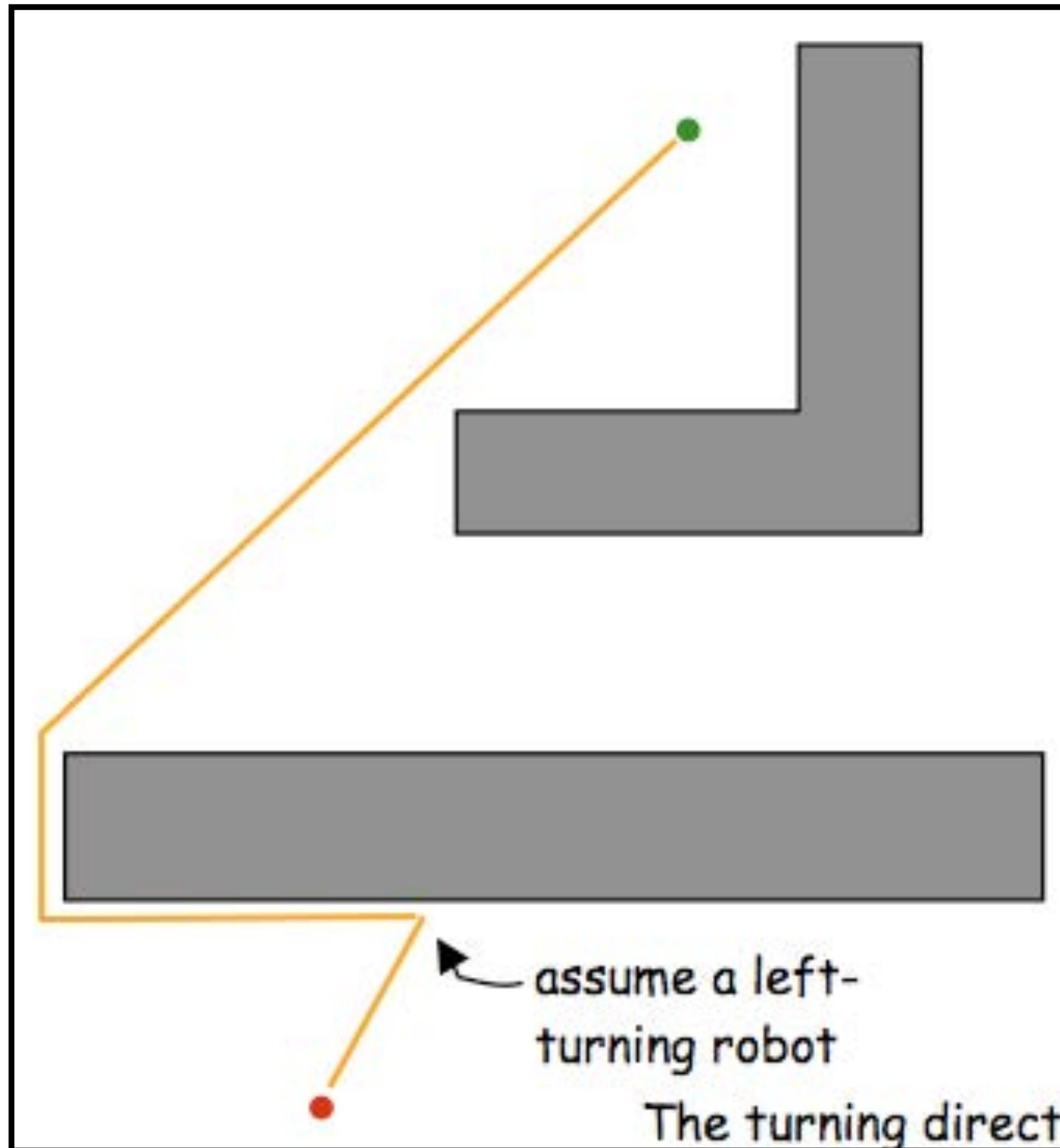
- a) move forward with slight turn
- b) when bumped, turn opposite direction
- c) goto (a)

Trevor Jay



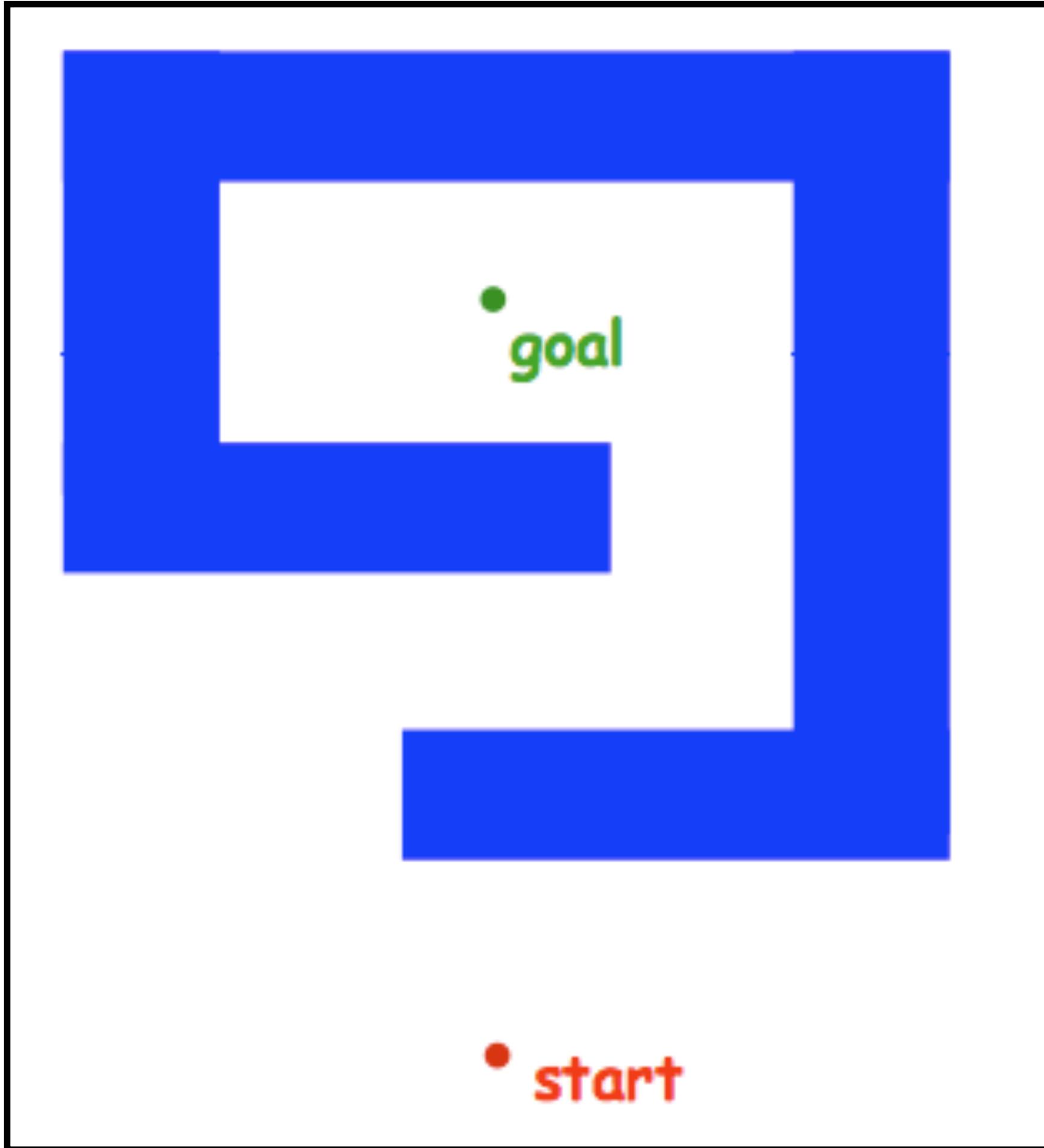
# What map would foil Bug 0?

## Bug 0



- 1) Head towards goal
- 2) When hit point set, follow wall, until you can move towards goal again (leave point)
- 3) continue from (1)

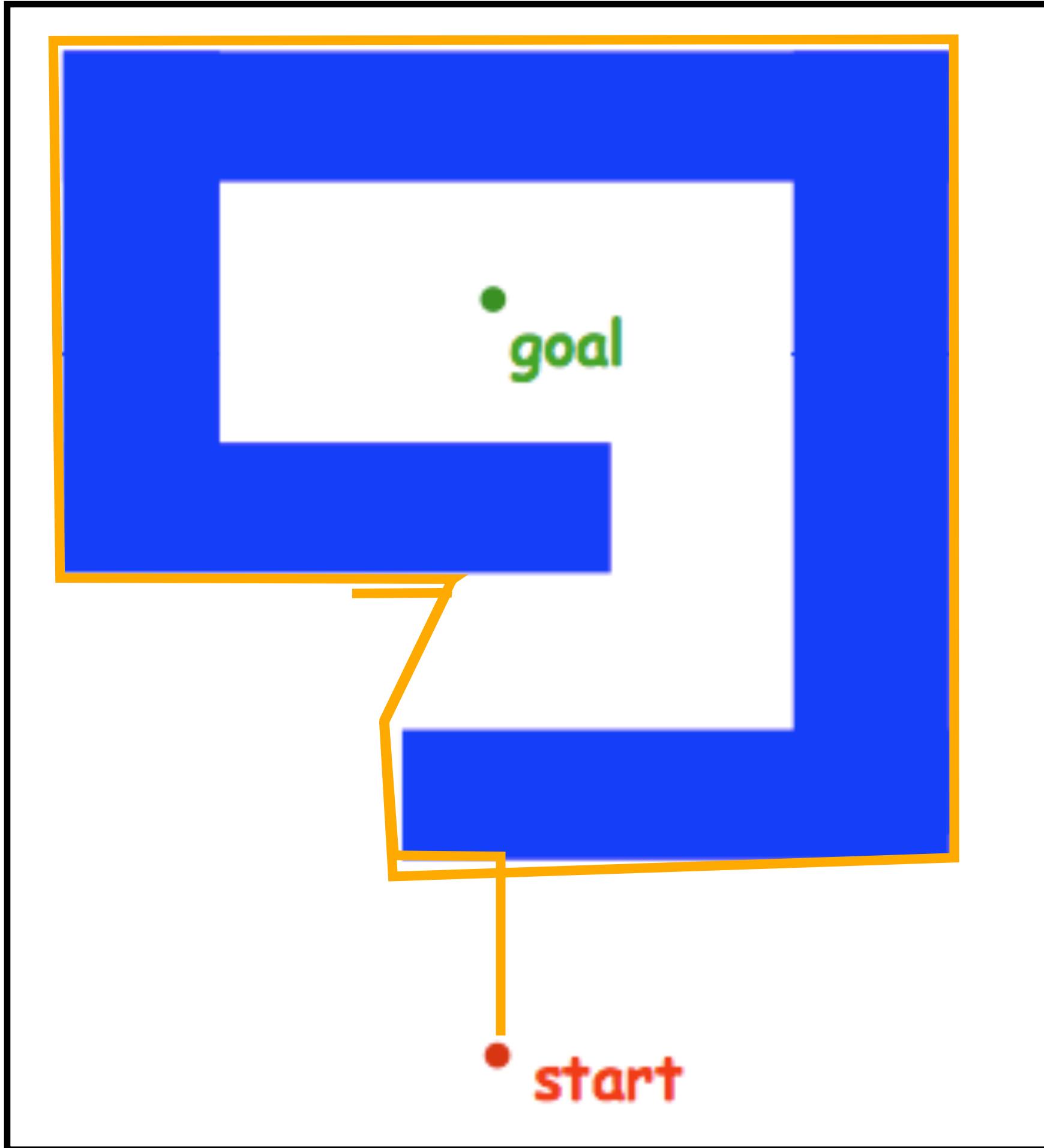
# Bug 0



- 1) Head towards goal
- 2) When hit point set, follow wall, until you can move towards goal again (leave point)
- 3) continue from (1)

Can you trace the Bug 0 path?

# Bug 0

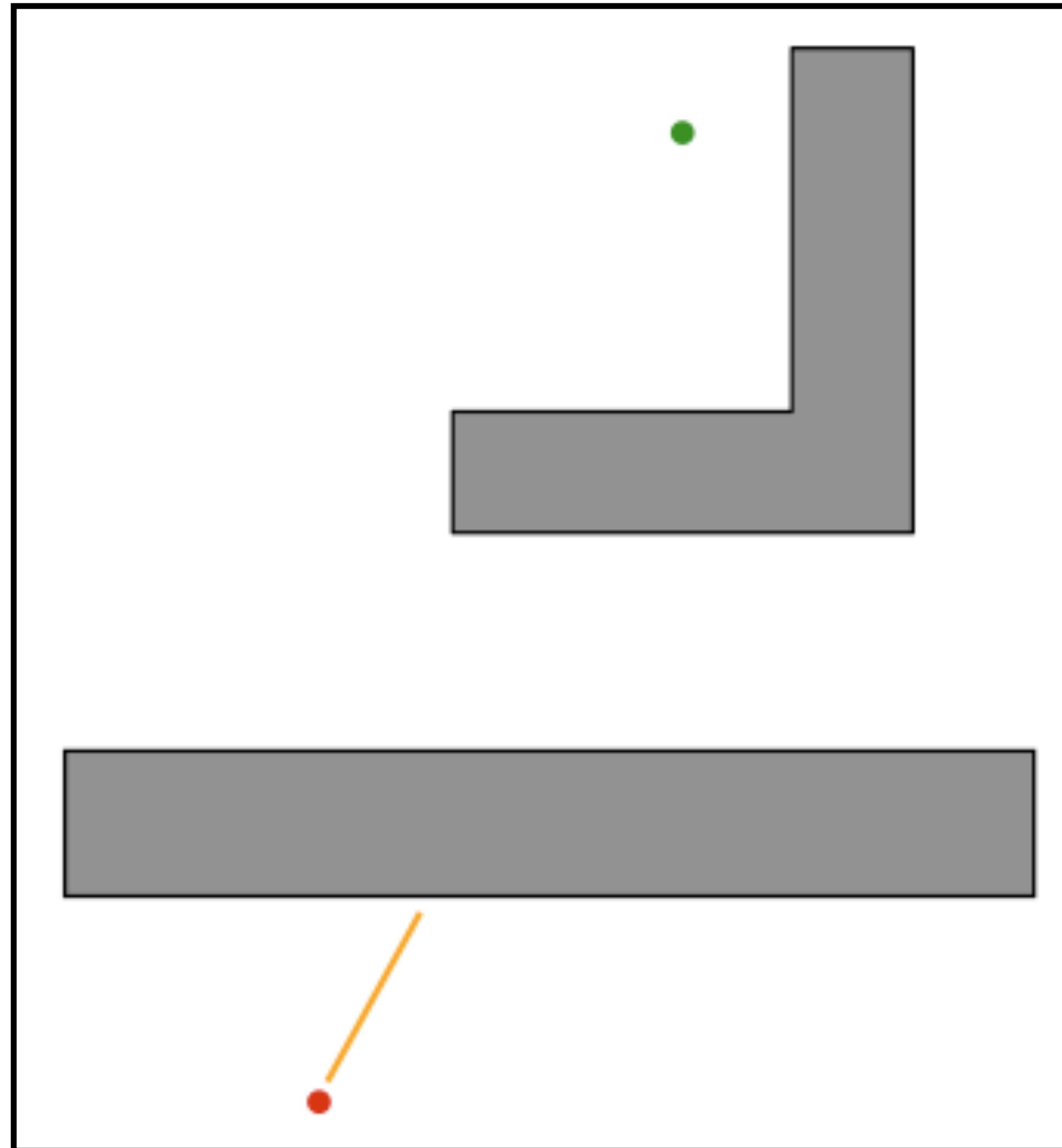


- 1) Head towards goal
- 2) When hit point set, follow wall,  
until you can move towards goal  
again (leave point)
- 3) continue from (1)

Can you trace the Bug 0 path?

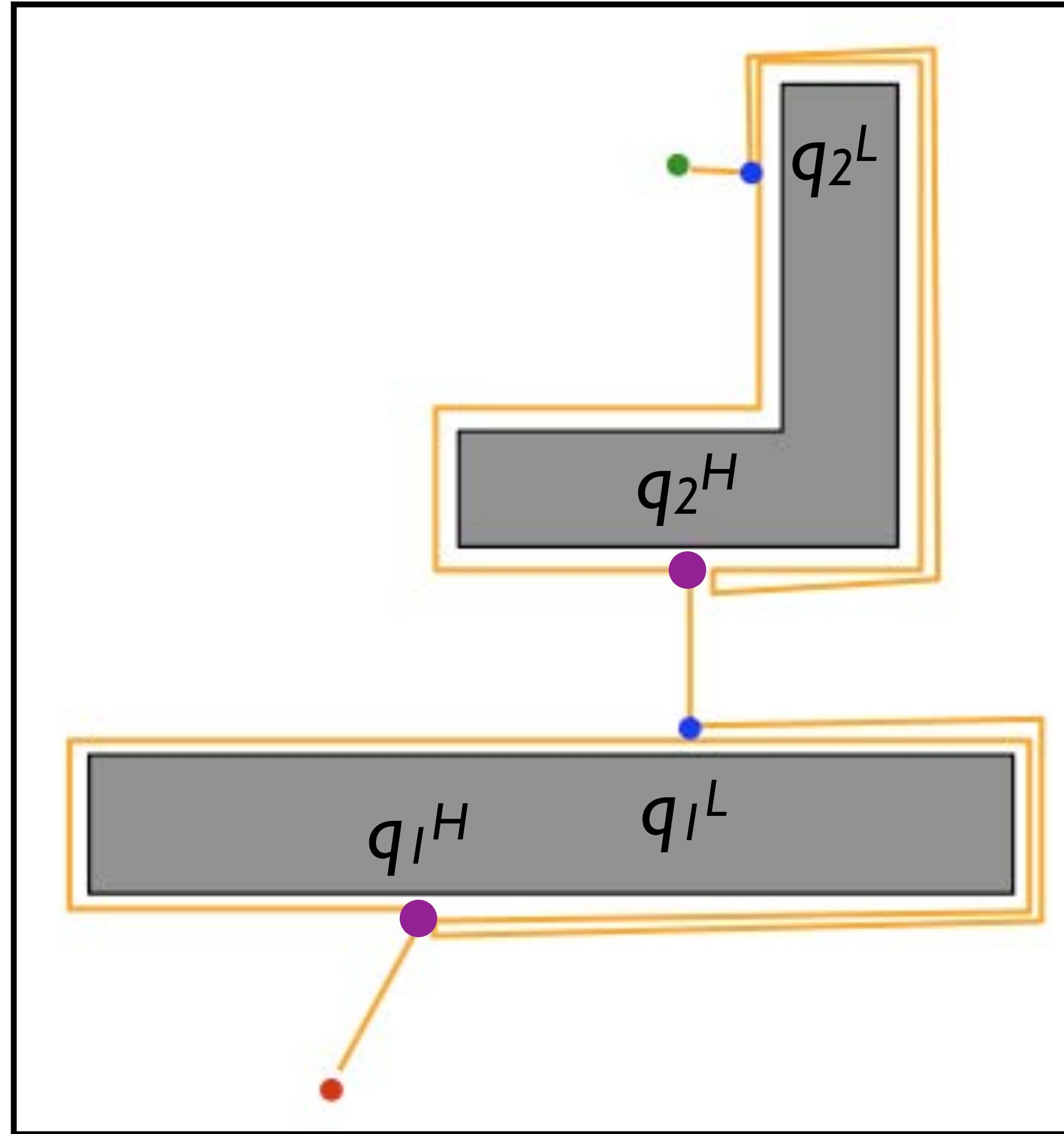
Can we make a better bug? How?

# Bug I



- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) continue from (1)

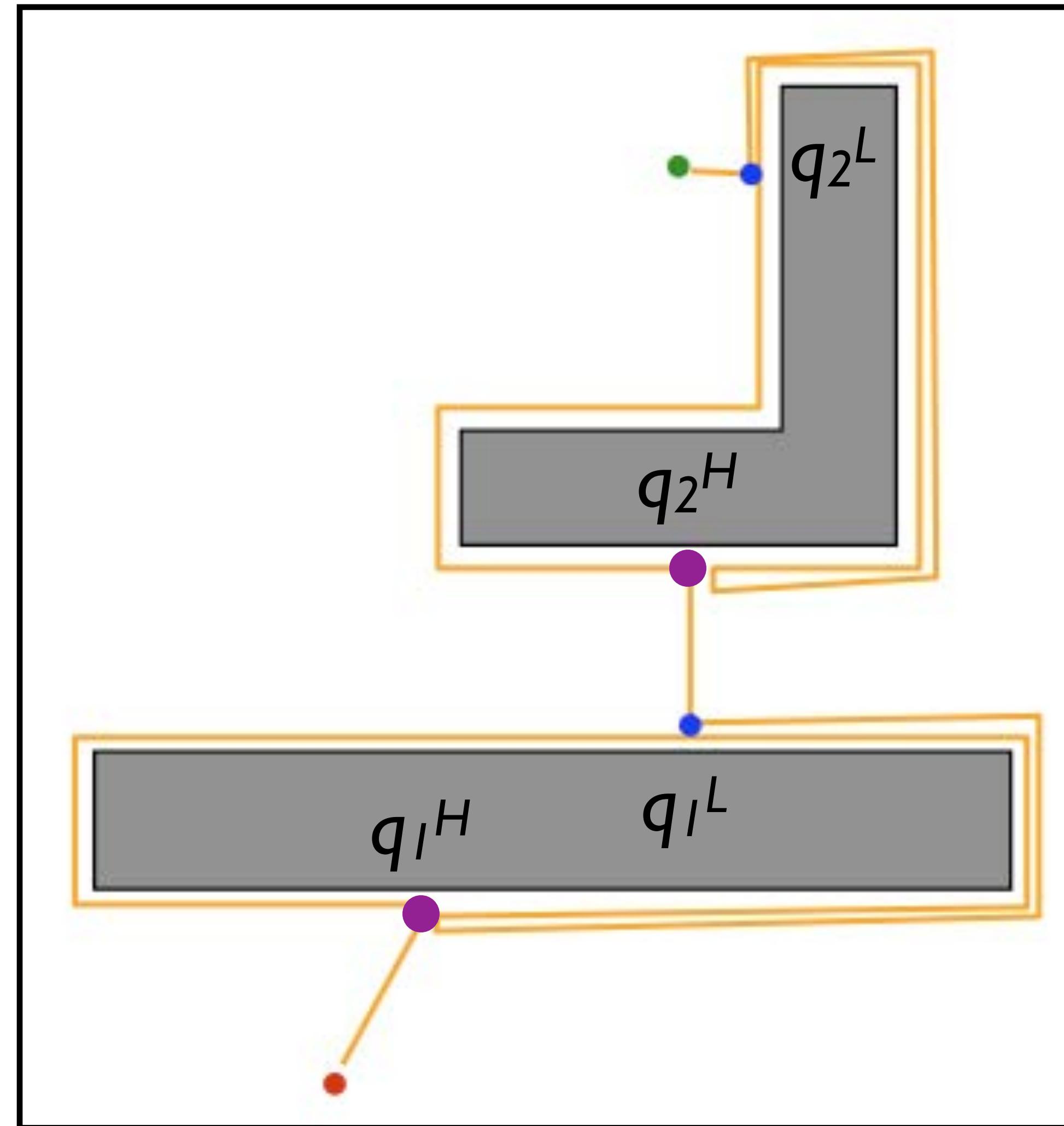
# Bug I



- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) continue from (1)

# What map would foil Bug 1?

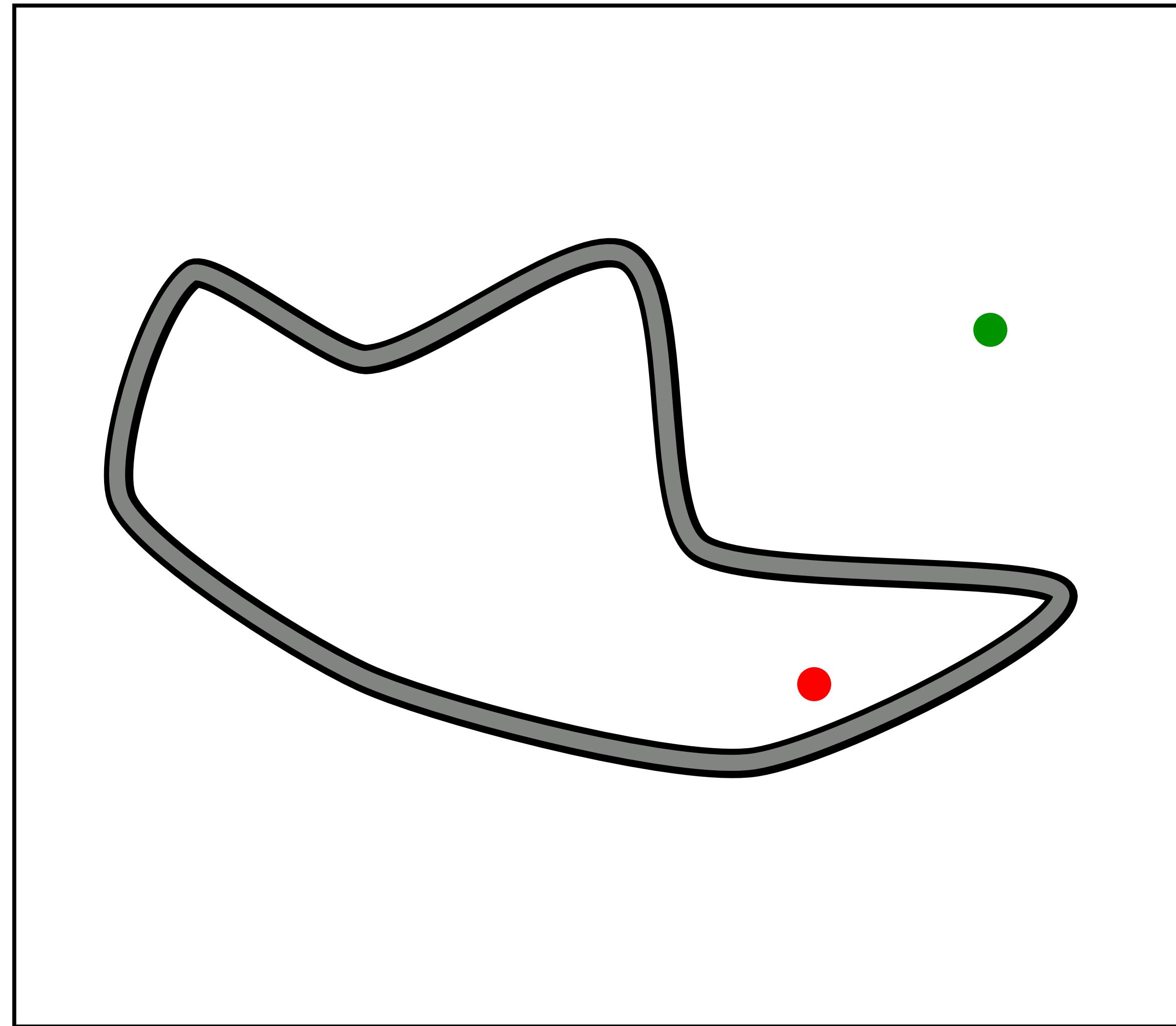
## Bug 1



- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) continue from (1)

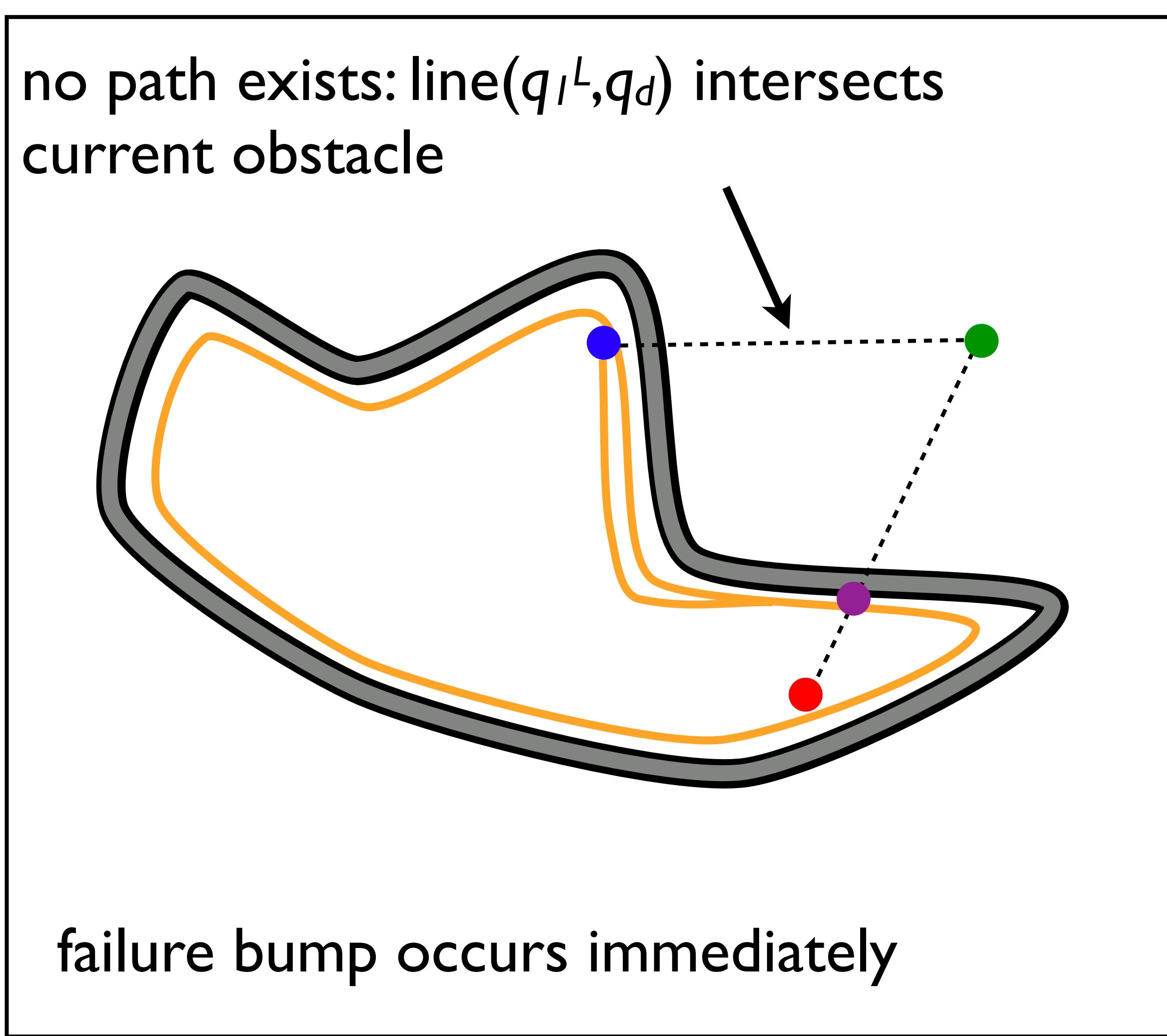
# What map would foil Bug 1?

## Bug 1



- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) if bump current obstacle,  
    return fail;  
else, continue from (1)

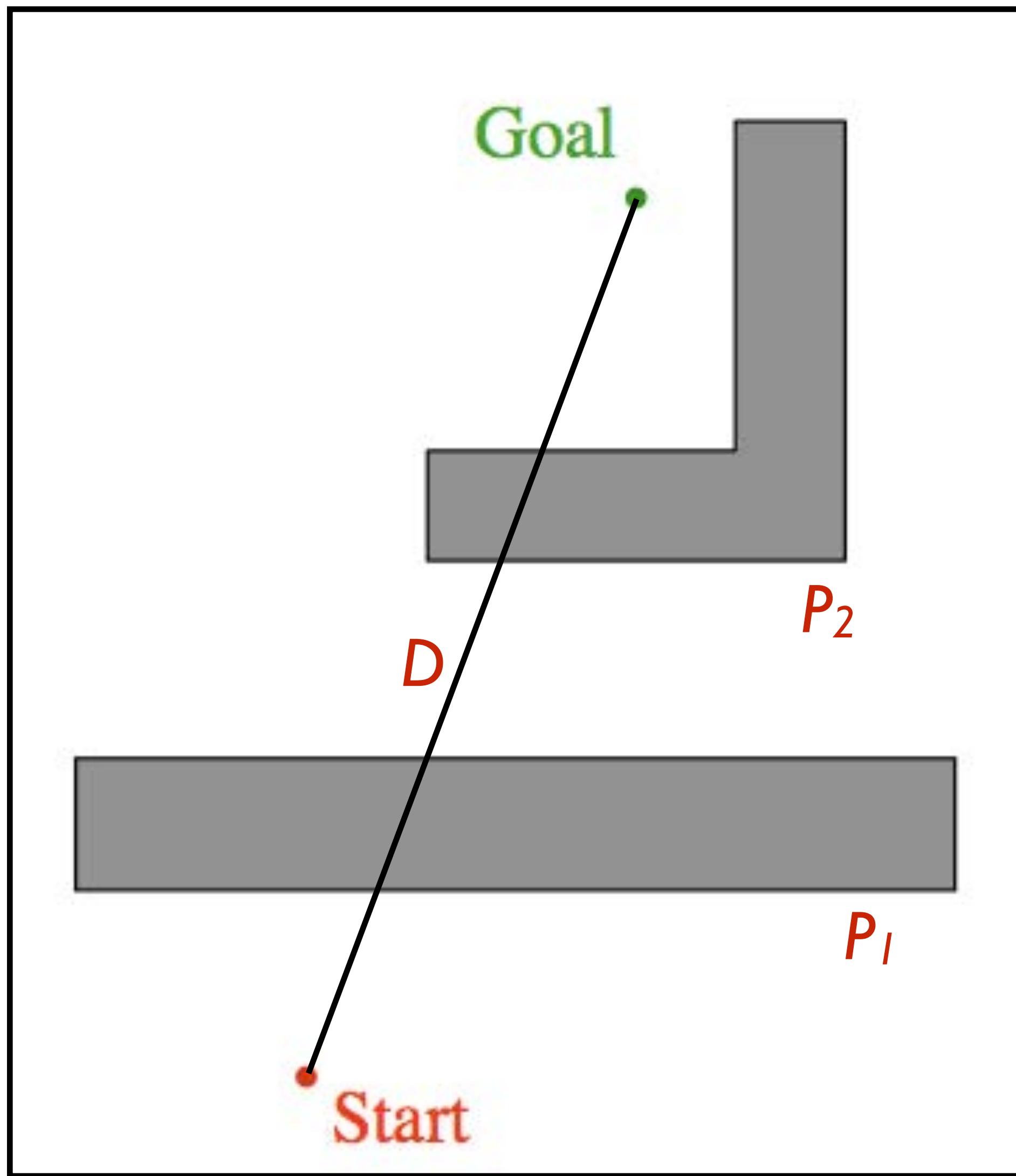
What map would foil Bug 1?



## Bug 1: Detecting Failure

- 1) Head towards goal
- 2) When hit point set, circumnavigate obstacle, setting leave point as closest to goal
- 3) return to leave point
- 4) if bump current obstacle,  
    return **fail**;  
else, continue from (1)

# Bug I: Search Bounds

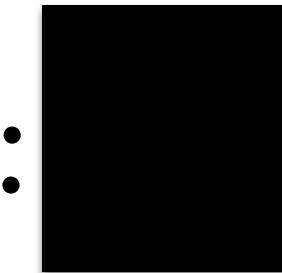


Bounds on path distance, assuming

$D$ : distance start-to-goal

$P_i$ : obstacle perimeter

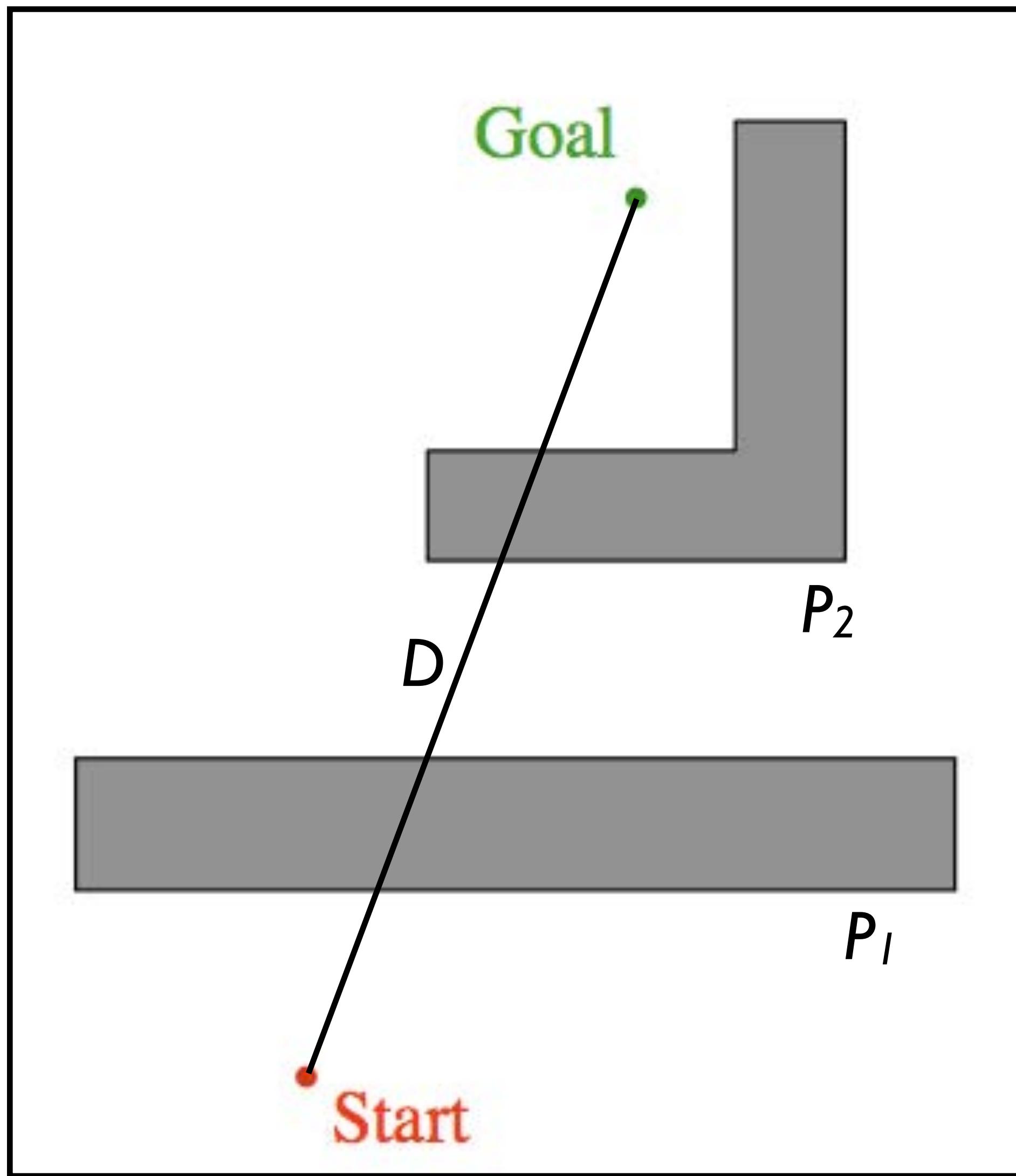
Best case:



Worst case:



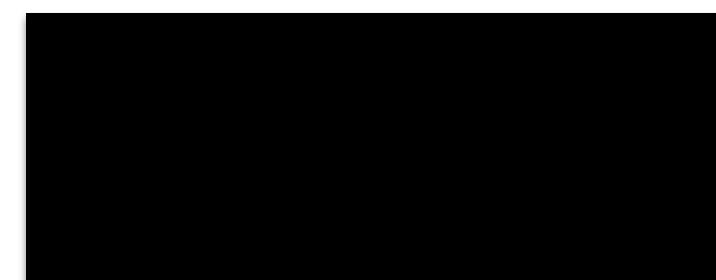
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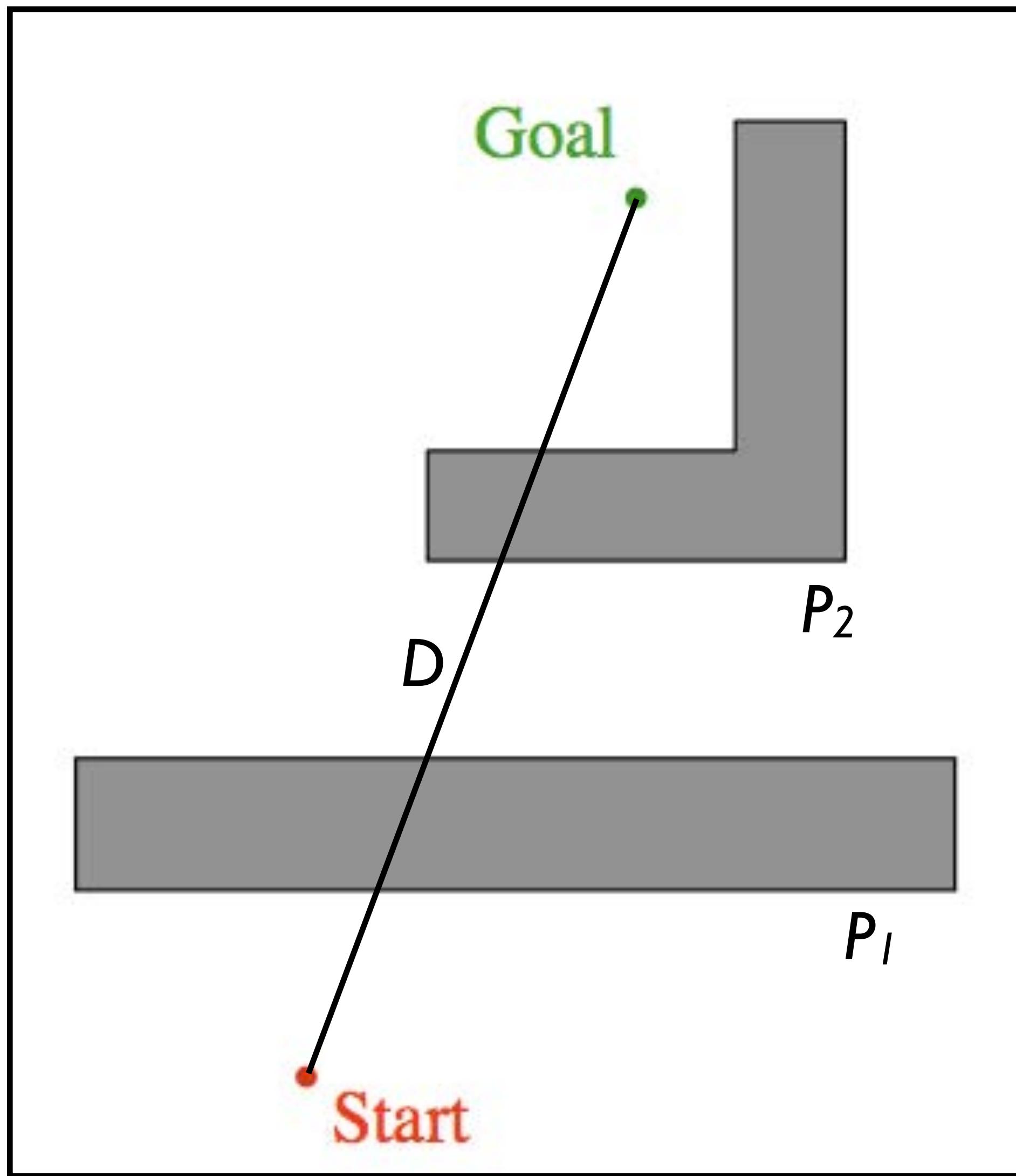
Bounds on path distance, assuming  
 $D$ : distance start-to-goal  
 $P_i$ : obstacle perimeter

Best case:  $D$

Worst case:



# Bug I: Search Bounds



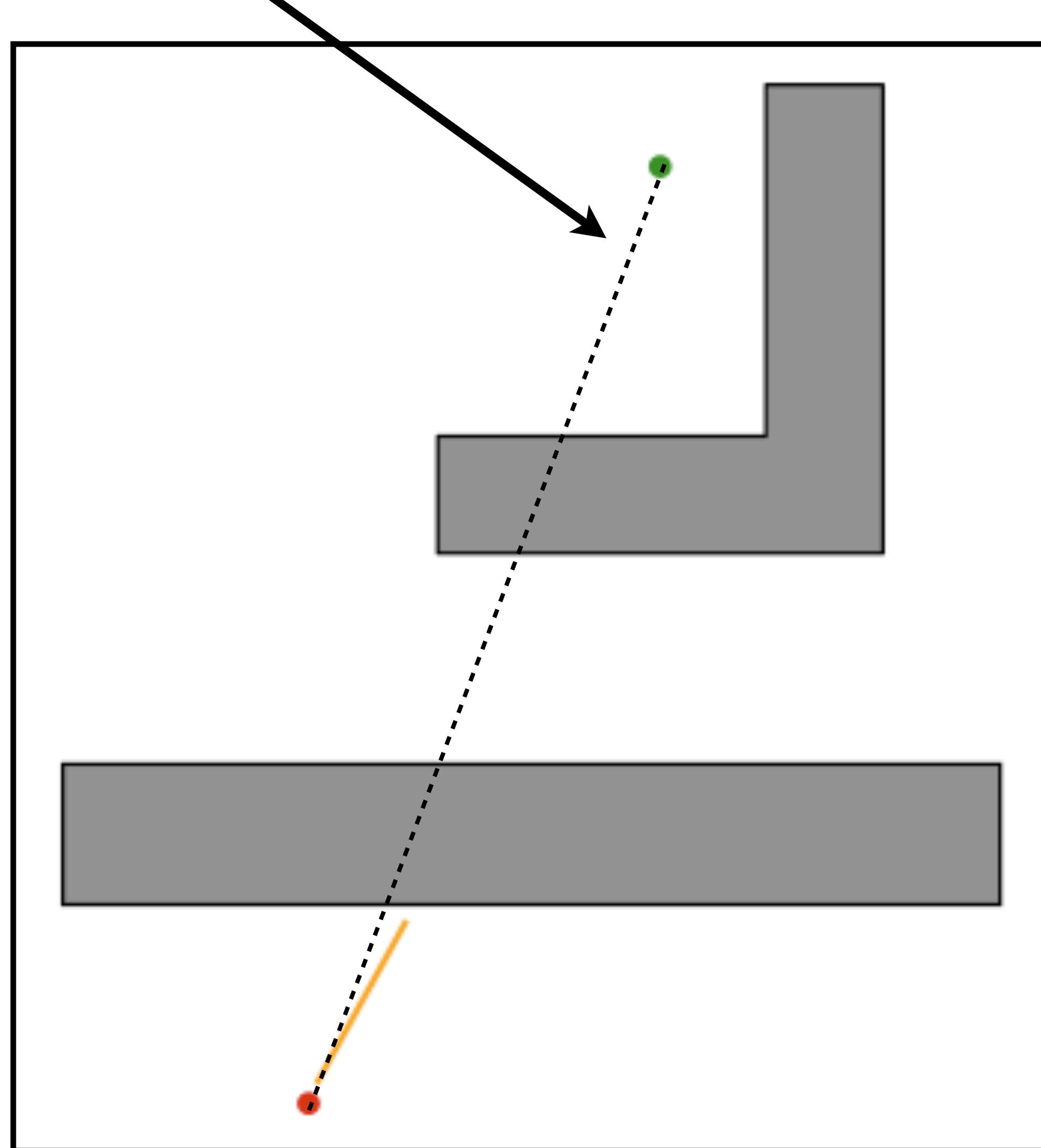
Bounds on path distance, assuming  
 $D$ : distance start-to-goal  
 $P_i$ : obstacle perimeter

Best case:  $D$

Worst case:  $D + 1.5 \sum_i P_i$

Is there a faster bug?

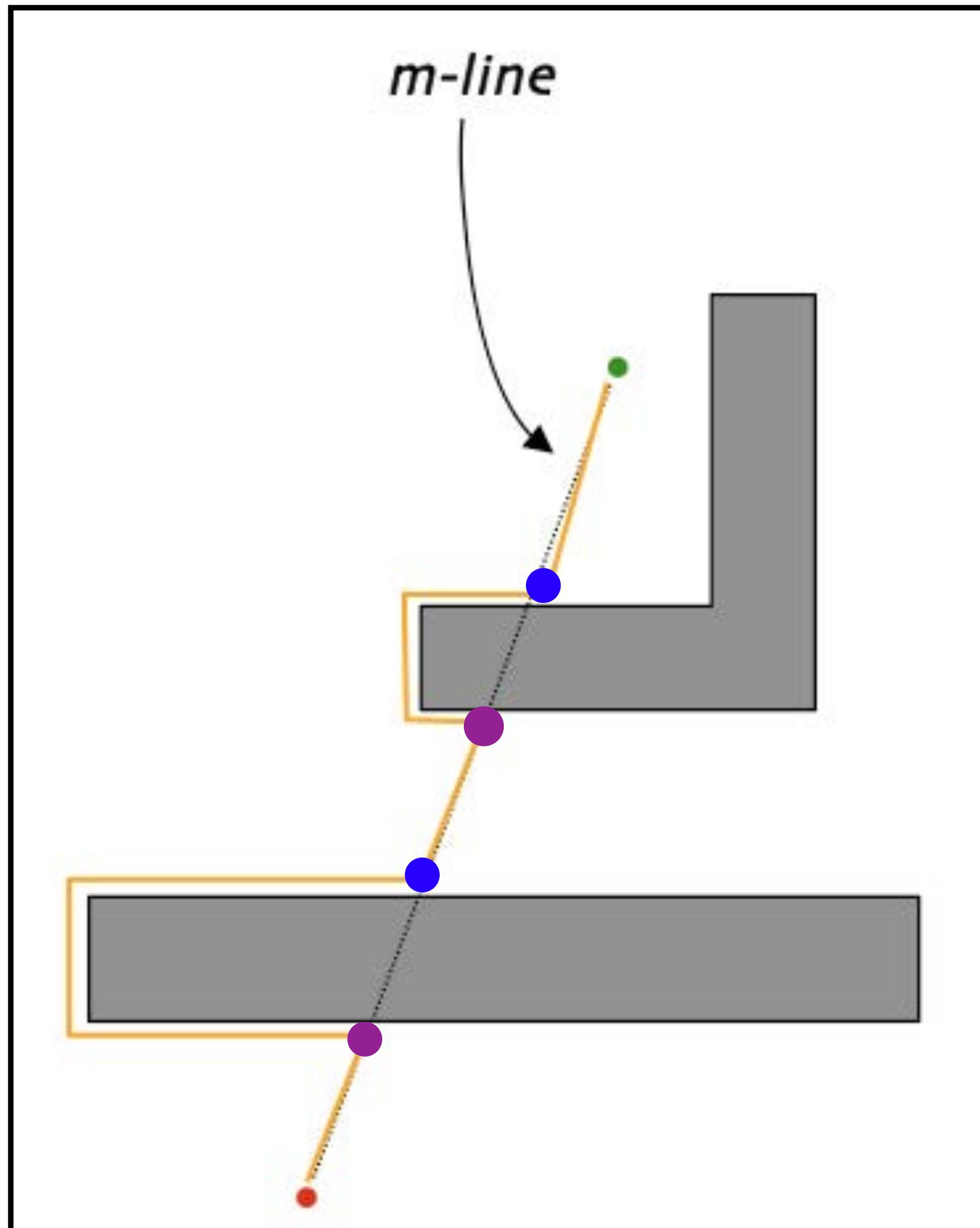
**m-line:** straight line path to goal



# Bug 2

- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered
- 3) set leave point and exit obstacle
- 4) continue from (1)

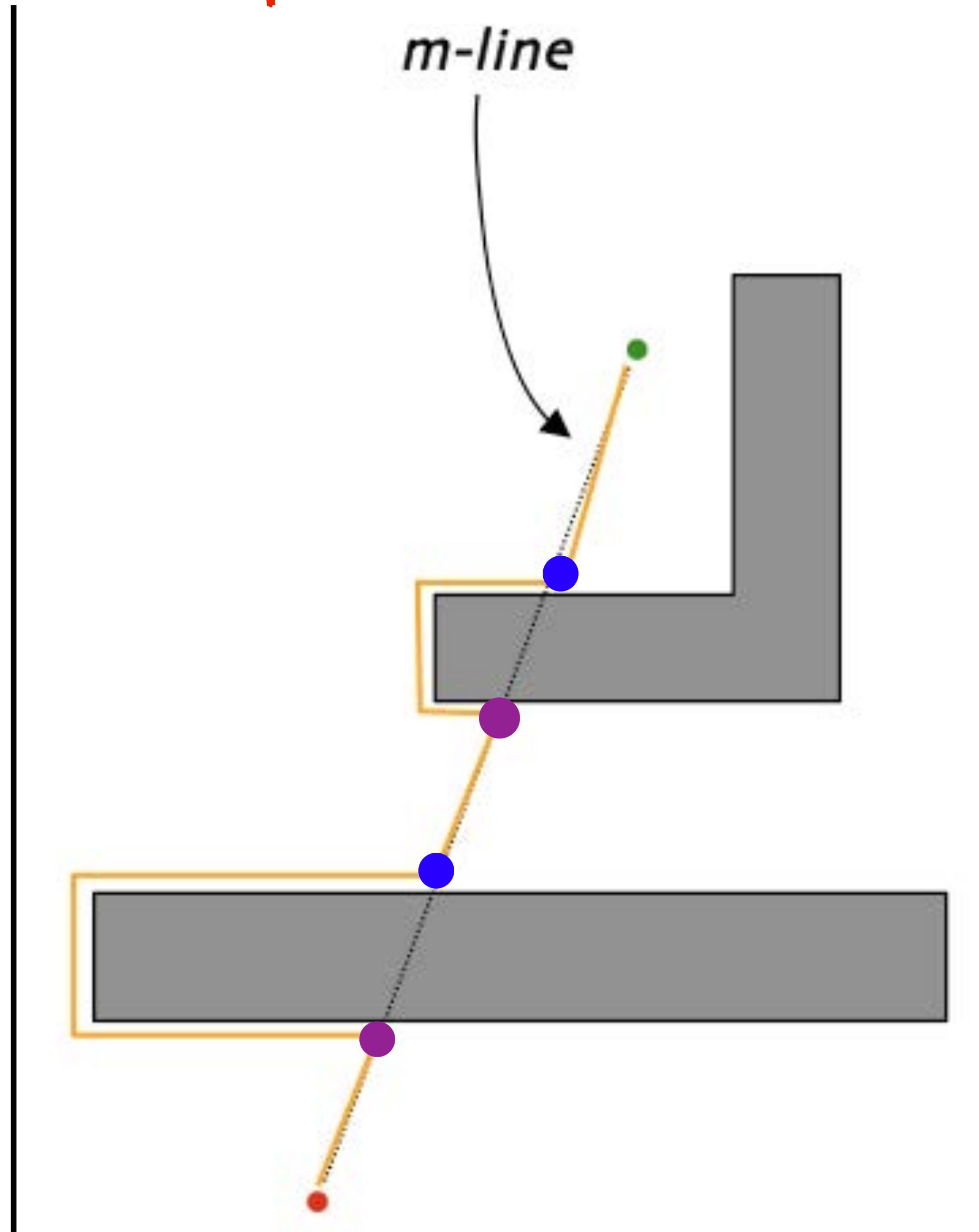
# Bug 2



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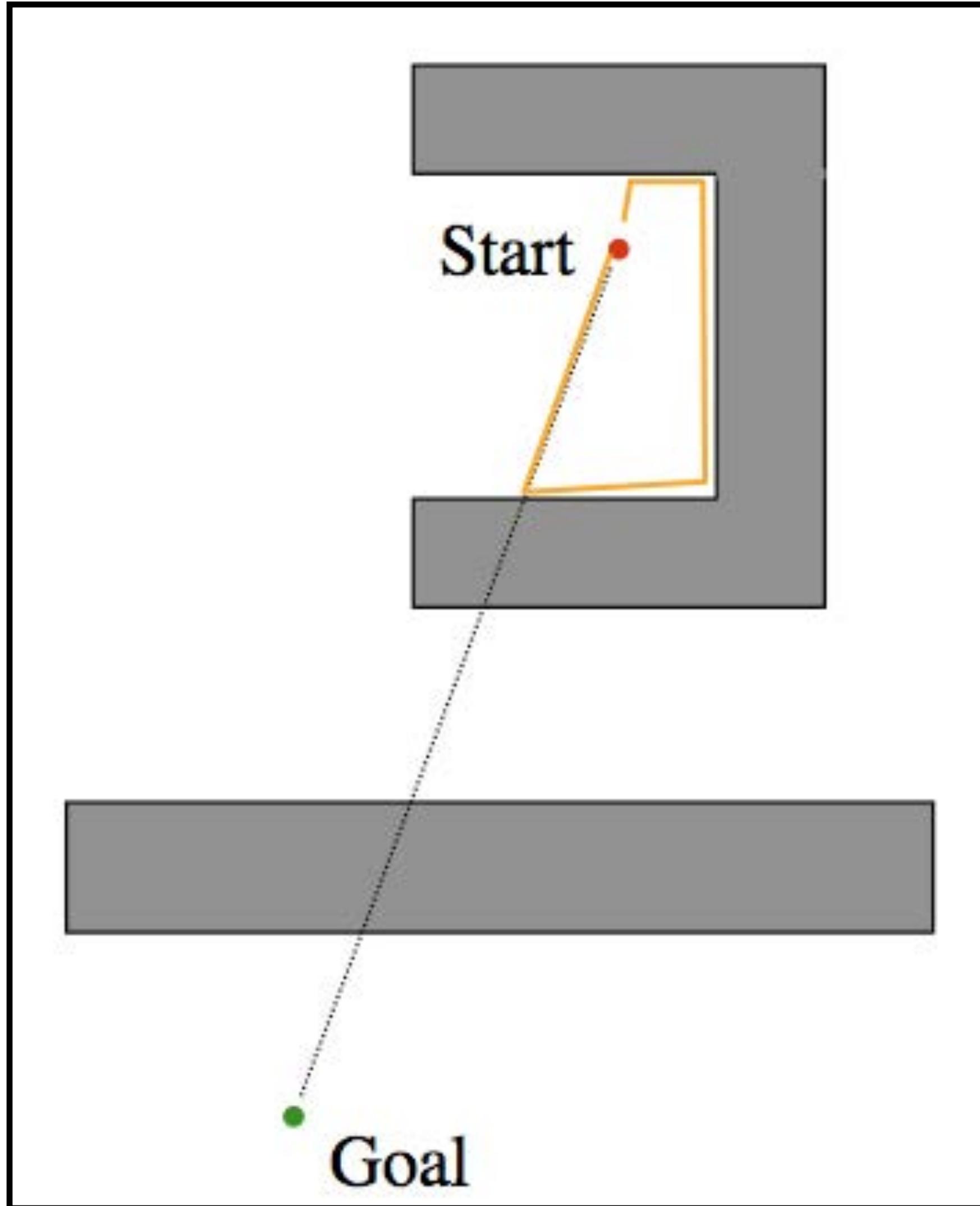
# What map would foil Bug 2?

Bug 2



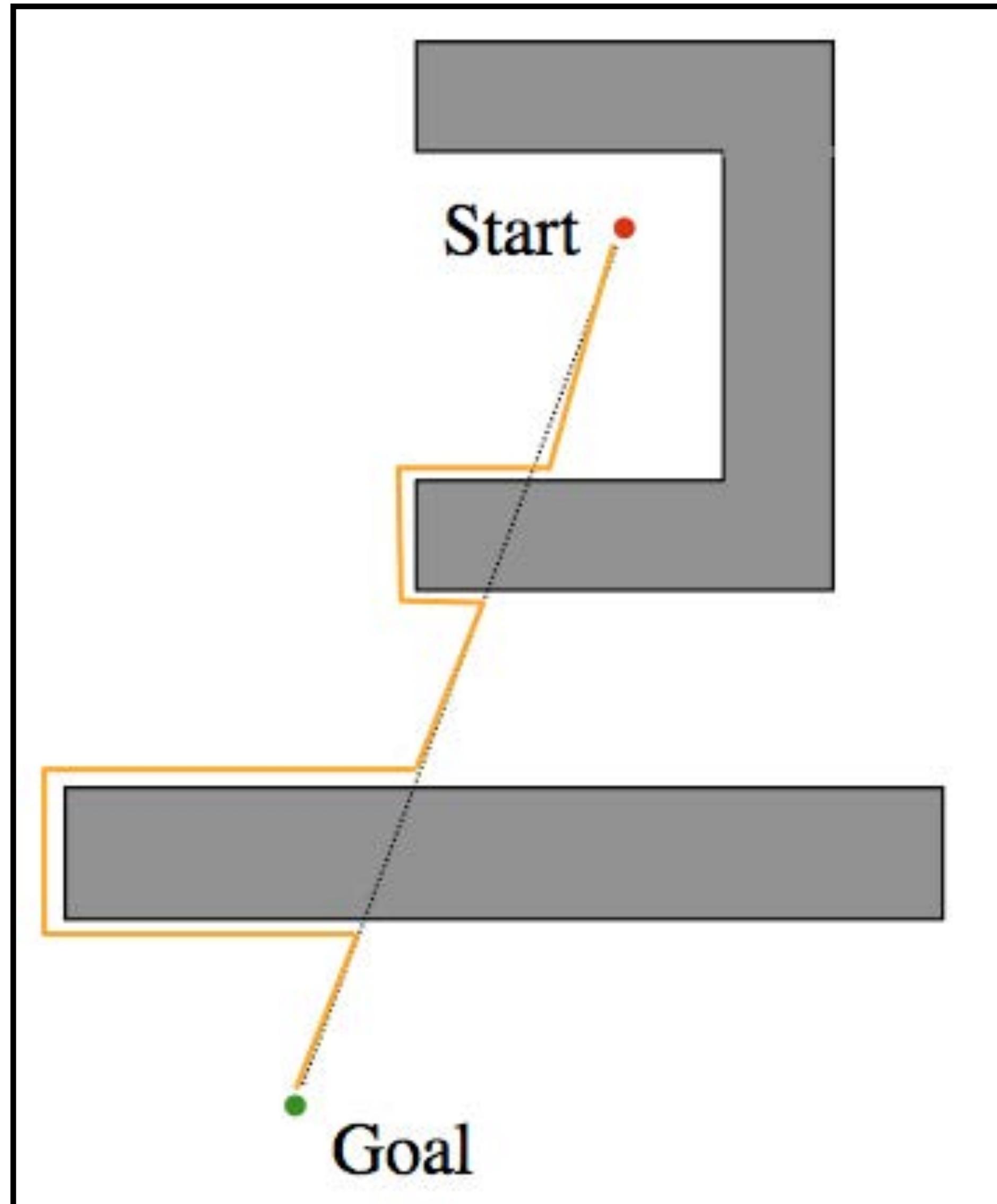
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# Bug 2



- 1) Head towards goal on m-line
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- 3) set leave point and exit obstacle
- 4) continue from (1)

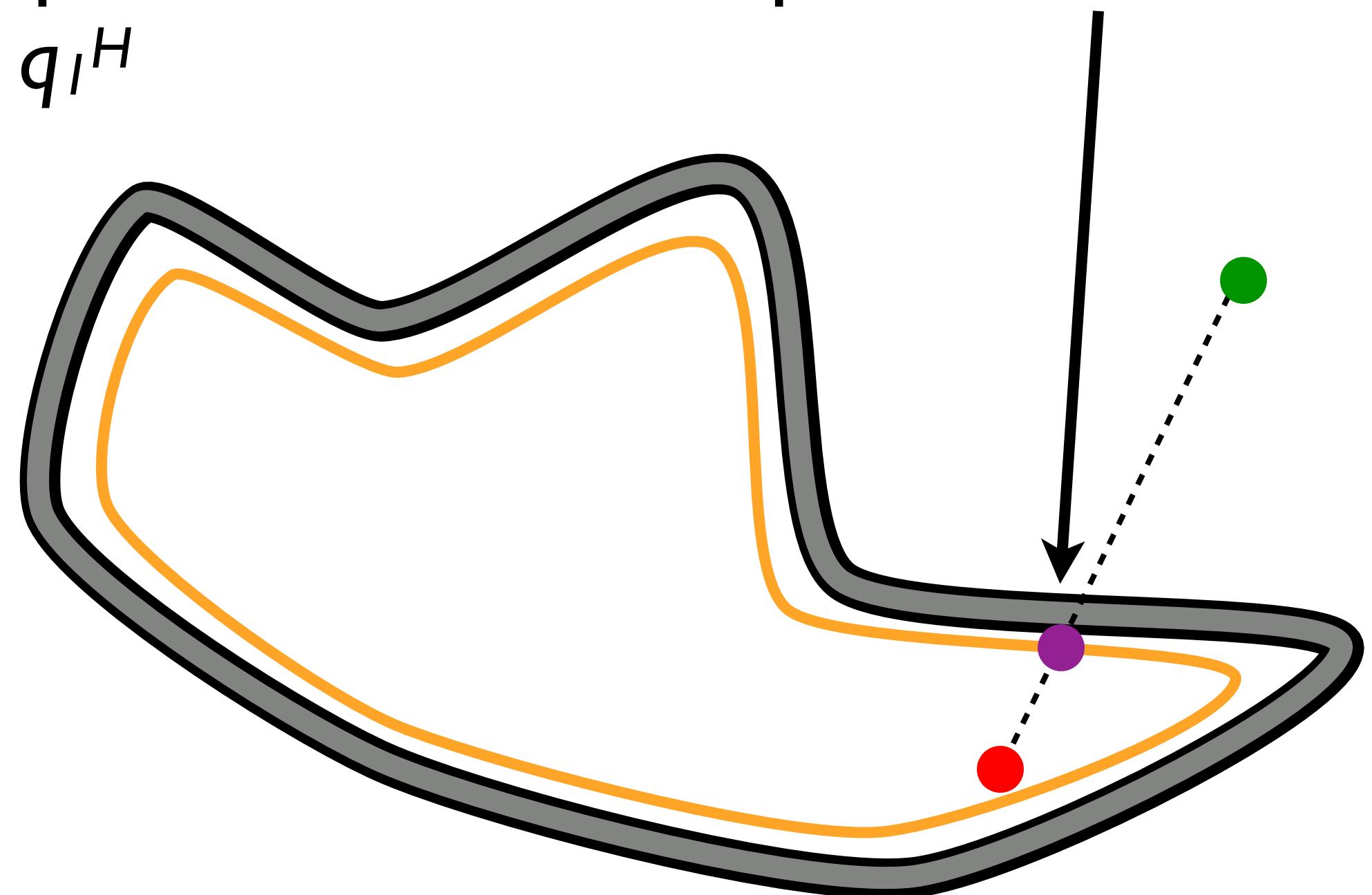
# Bug 2



- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered  
**& closer to the goal**
- 3) set leave point and exit obstacle
- 4) continue from (1)

# Bug 2: Detecting Failure

no path exists: no leave point before returning  
to  $q_1^H$

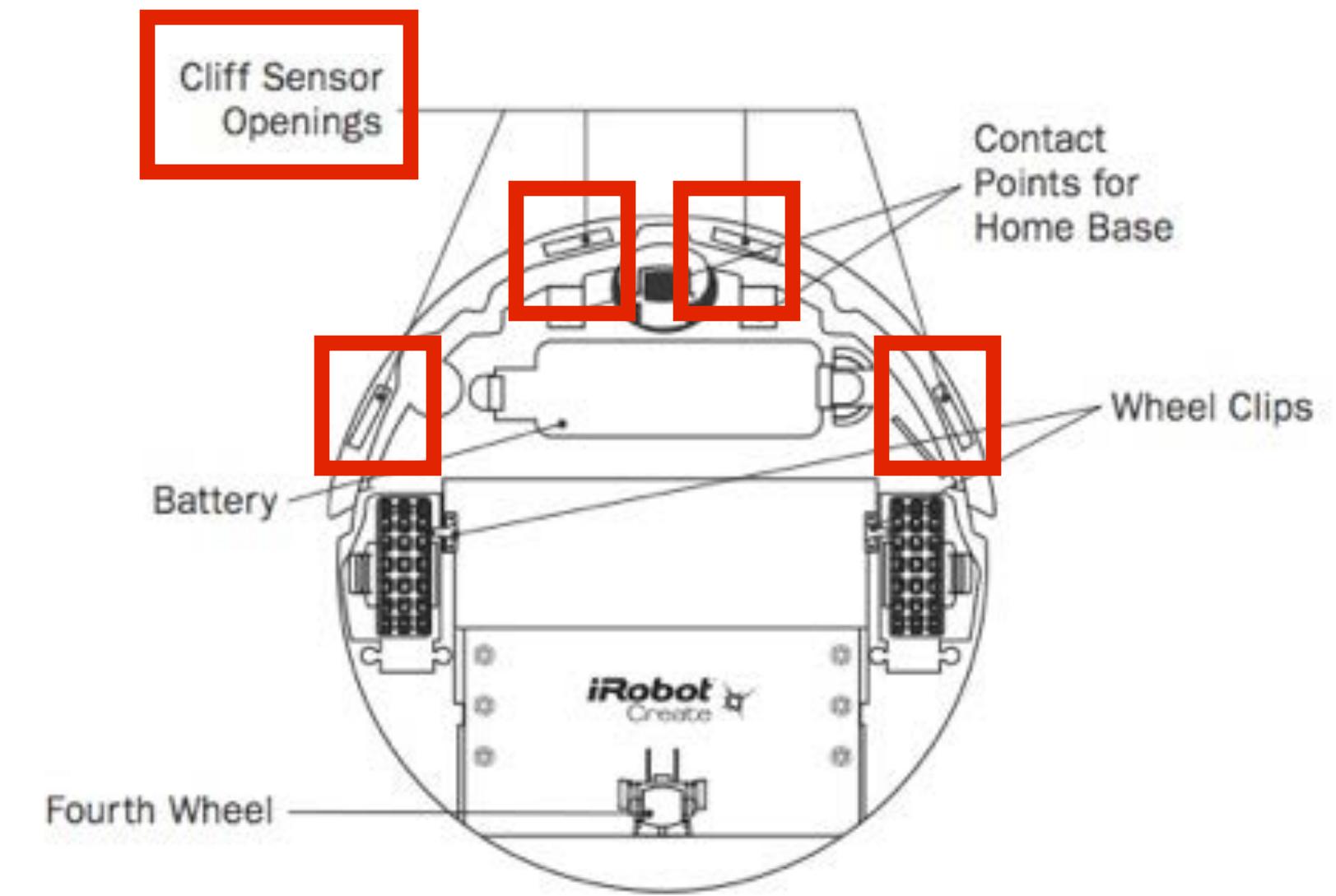


- 1) Head towards goal on m-line
- 2) When hit point set, traverse obstacle until m-line is encountered & closer to the goal  
**or hit point reached**
- 3) **if not  $j^{th}$  hit point**, set leave pt. and exit
- 4) continue from (1)

# Bug 2 in action



m-line drawn on floor  
with tape recognizable by  
Create cliff sensor



Kayle Gishen



# Is Bug2 better than Bug1?



# Bug 1 v. Bug 2:

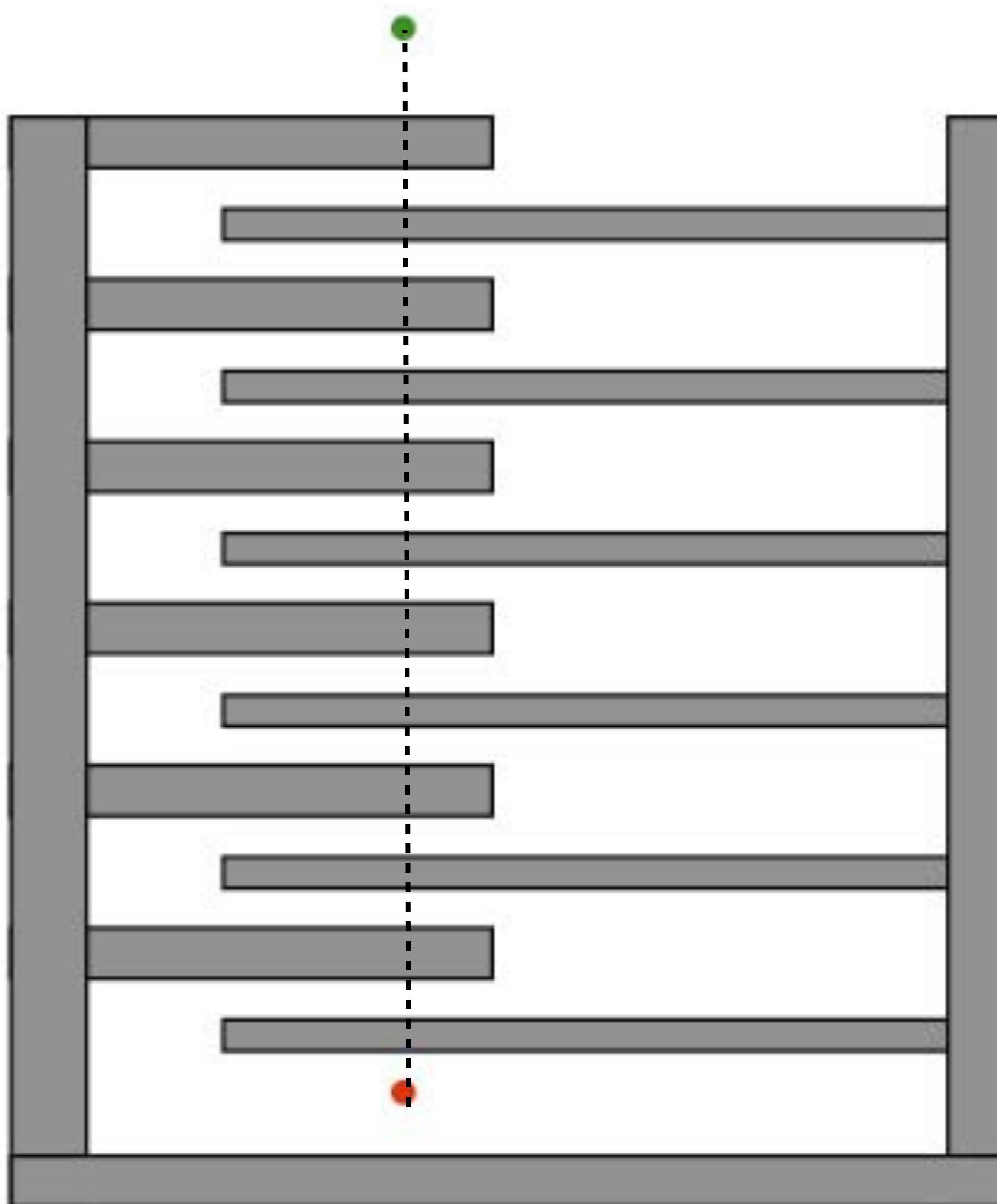
Draw worlds where Bug 2 performs better than Bug 1 (and vice versa)

**Bug 2 beats Bug 1**

**Bug 1 beats Bug 2**

Home work!

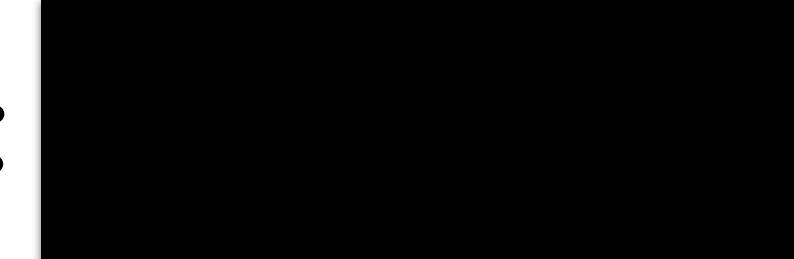
# Bug 2: Search Bounds



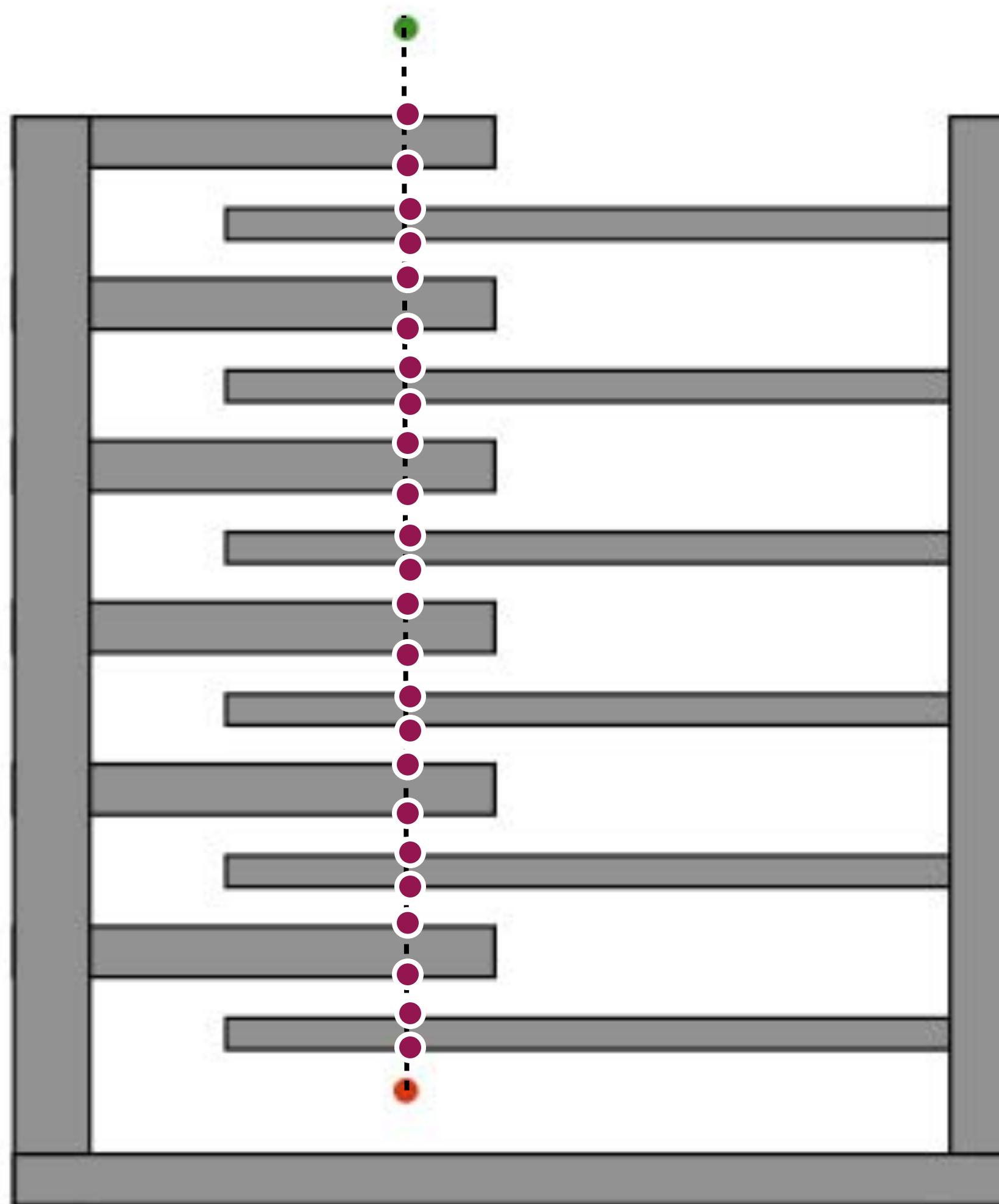
Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case: 

Worst case: 

# Bug 2: Search Bounds

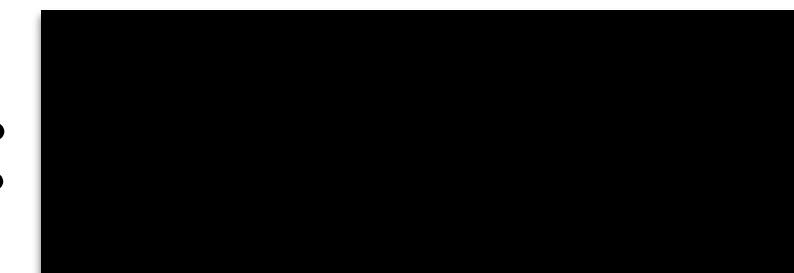


Bounds on path distance, assuming

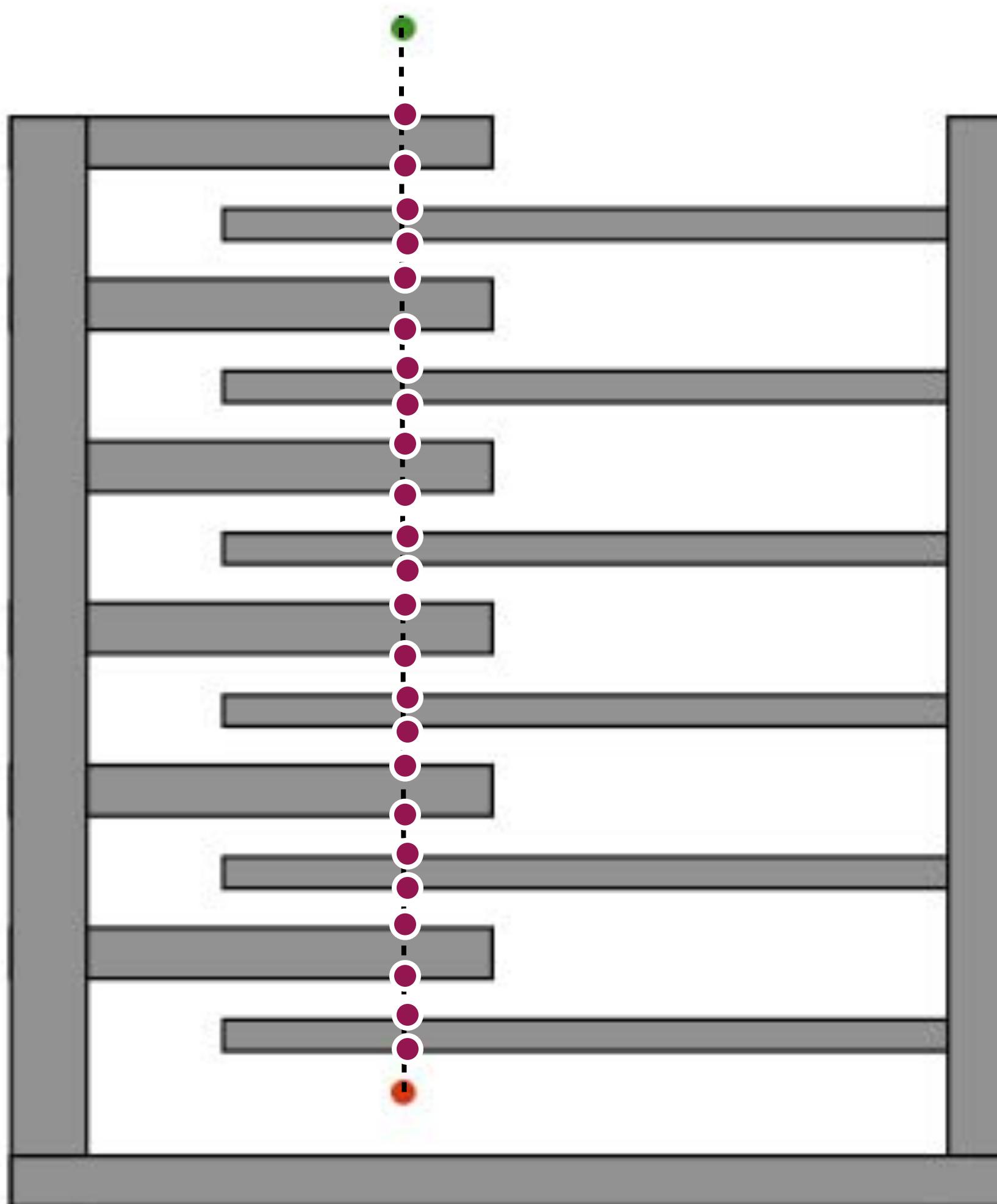
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# Bug 2: Search Bounds



Bounds on path distance, assuming

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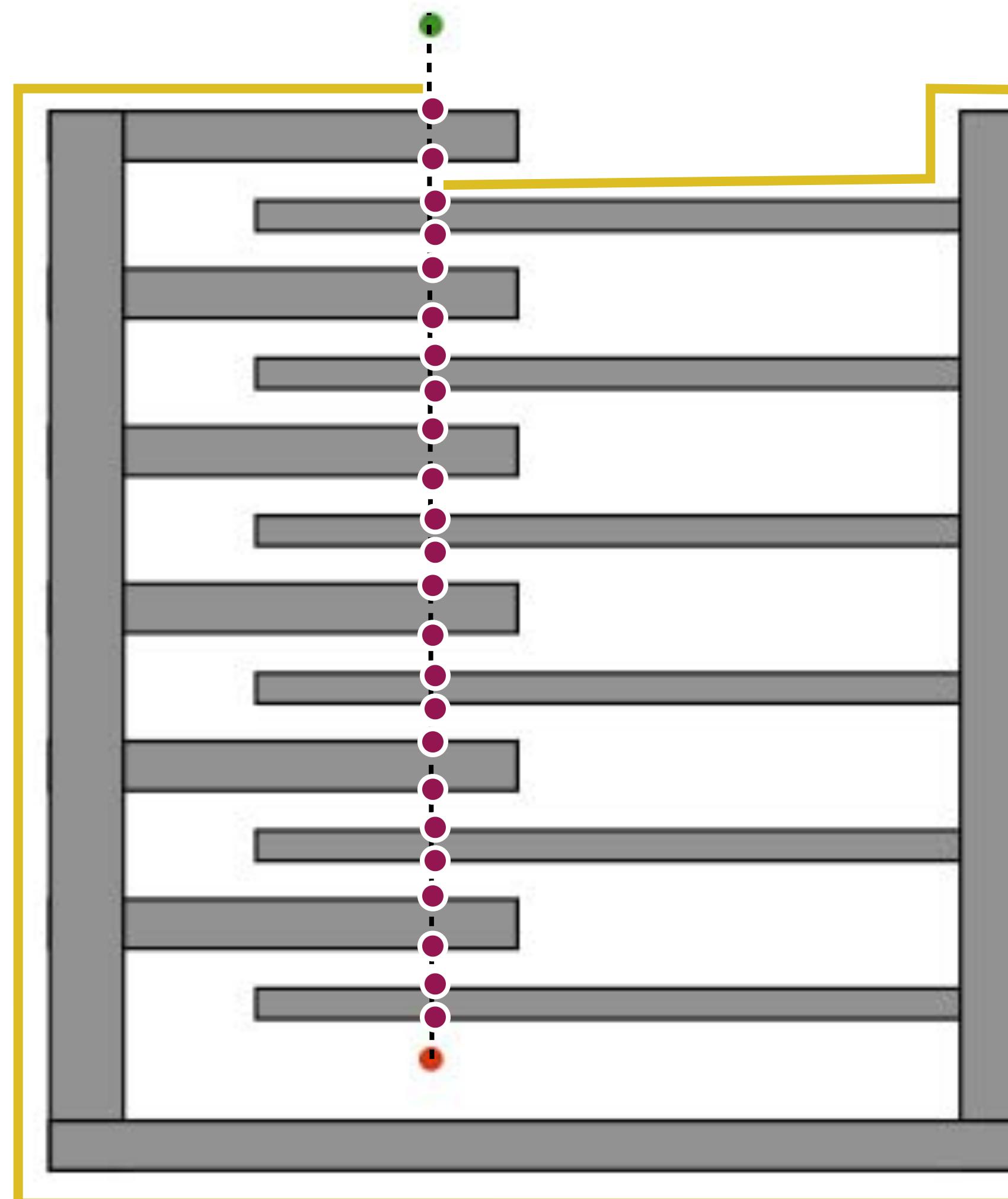
Best case:  $D$

Worst case:  $D + \sum_i (n_i/2)P_i$

Why?

Why?

Consider all leave points on m-line;  
only half are valid



Each leave pt might require traversing entire  
obstacle perimeter, including the outside

## Bug 2: Search Bounds

Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

Worst case:  $D + \sum_i (n_i/2)P_i$

# Search Bounds:

## Bug 1

Bounds on path distance, assuming

$D$ : distance start-to-goal

$P_i$ : obstacle perimeter

Best case:  $D$

Worst case:  $D + 1.5 \sum_i P_i$

## Bug 2

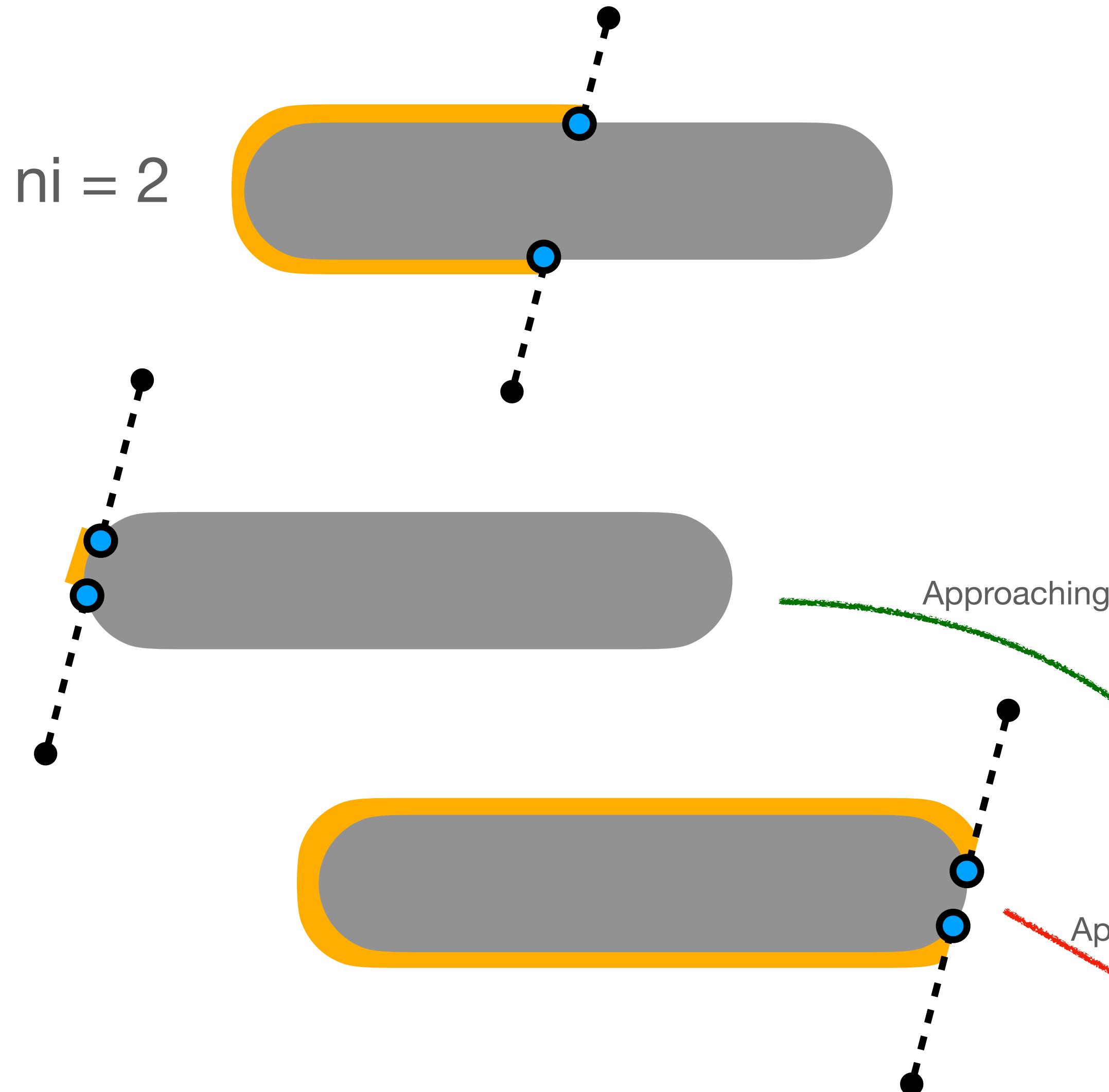
Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

Worst case:  $D + \sum_i (n_i/2)P_i$

# Search Bounds:



## Bug 2

Bounds on path distance, assuming

- $D$ : distance start-to-goal
- $P_i$ : obstacle perimeter
- $n_i$ : number of m-line intersections for  $WO_i$

Best case:  $D$

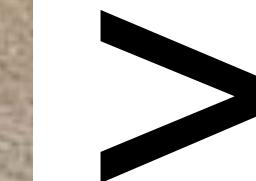
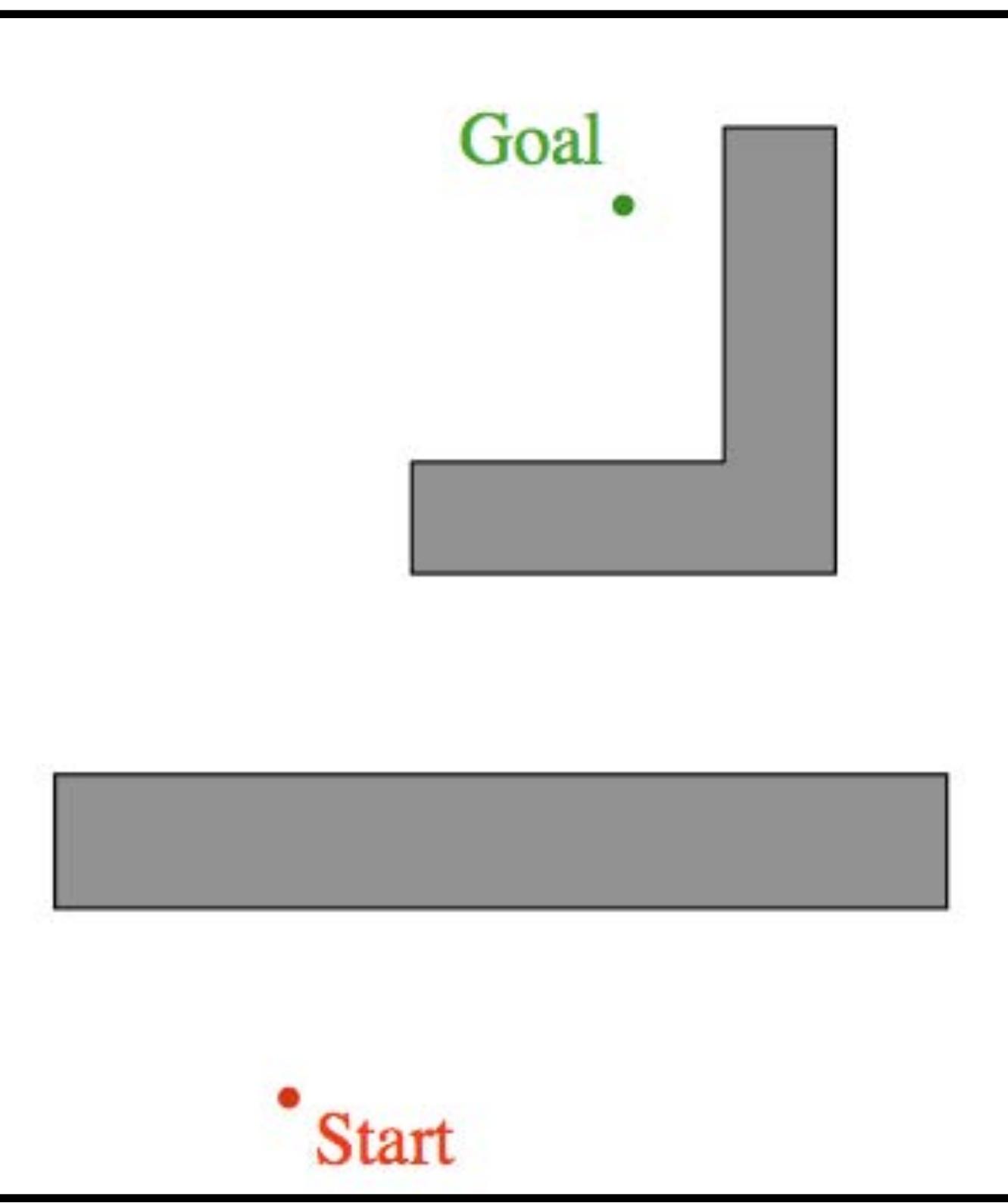
Worst case:  $D + \sum_i (n_i/2) P_i$

Suppose robot has a range sensor.

Is there a better Bug algorithm?

# Tangent Bug

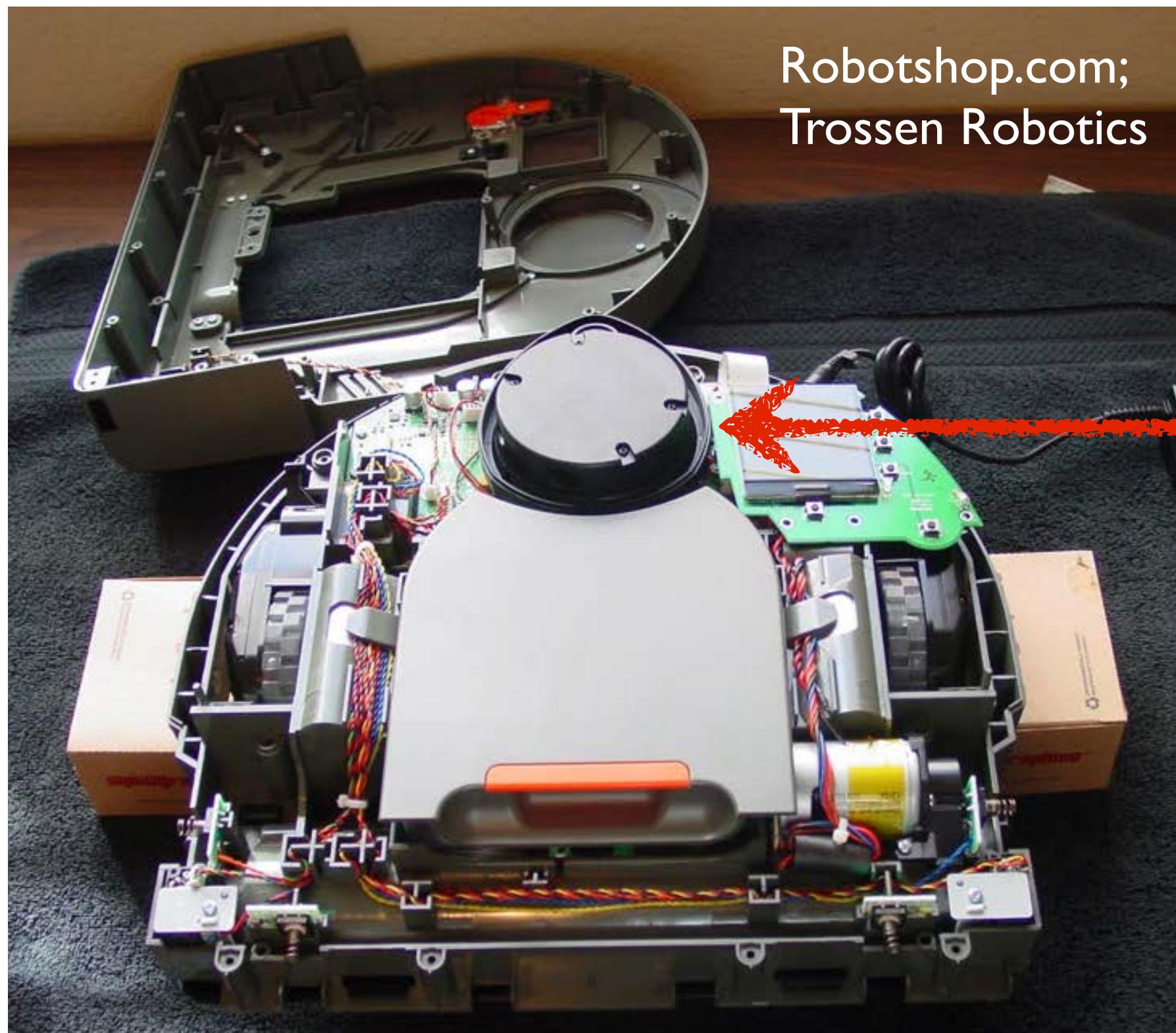
- Assume bounded world
- Known: global goal
  - measurable distance  $d(x,y)$
- Local sensing
  - **range finding**
  - odometry



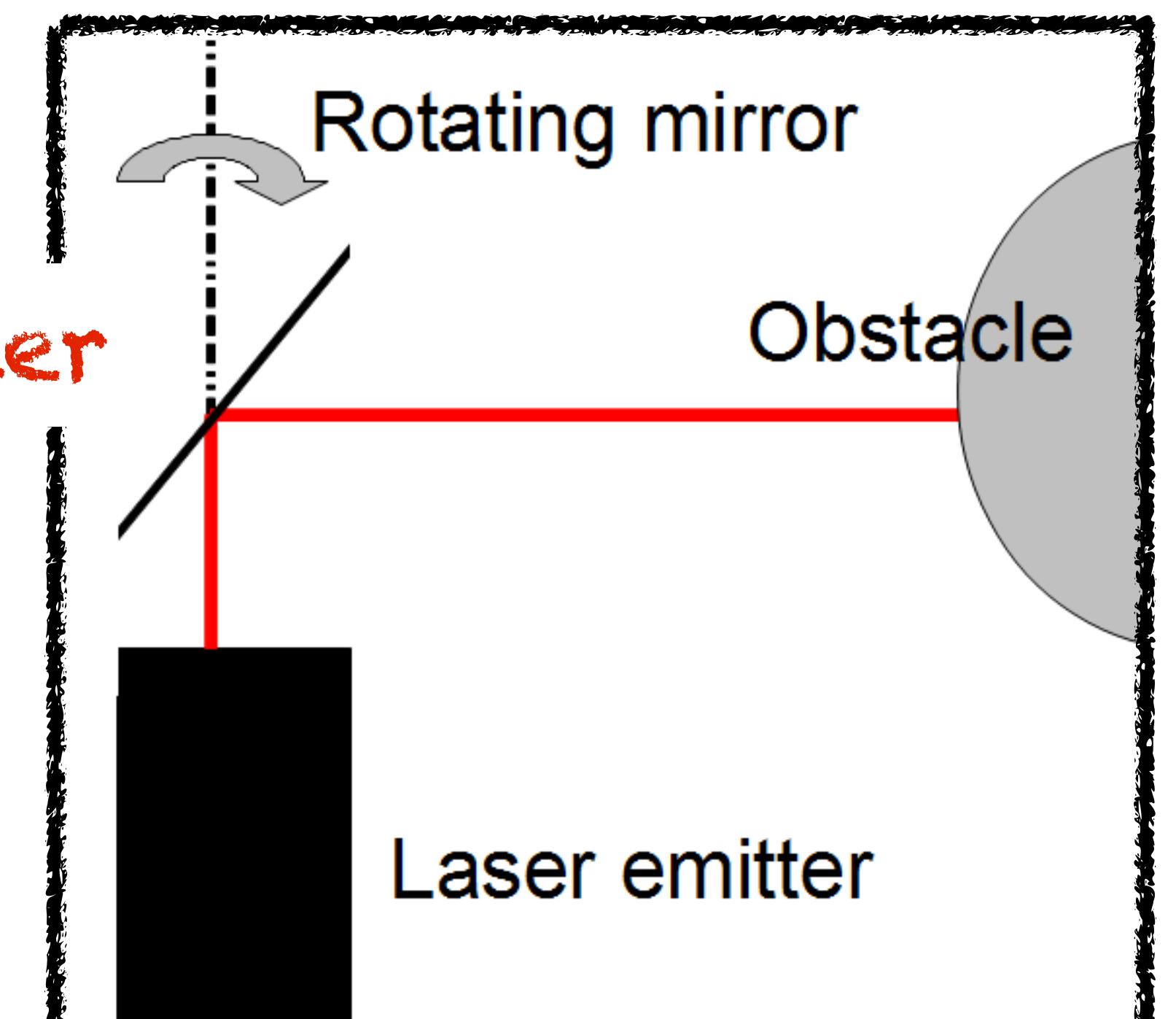
# Laser Rangefinding (briefly)

Emit laser beam in a direction

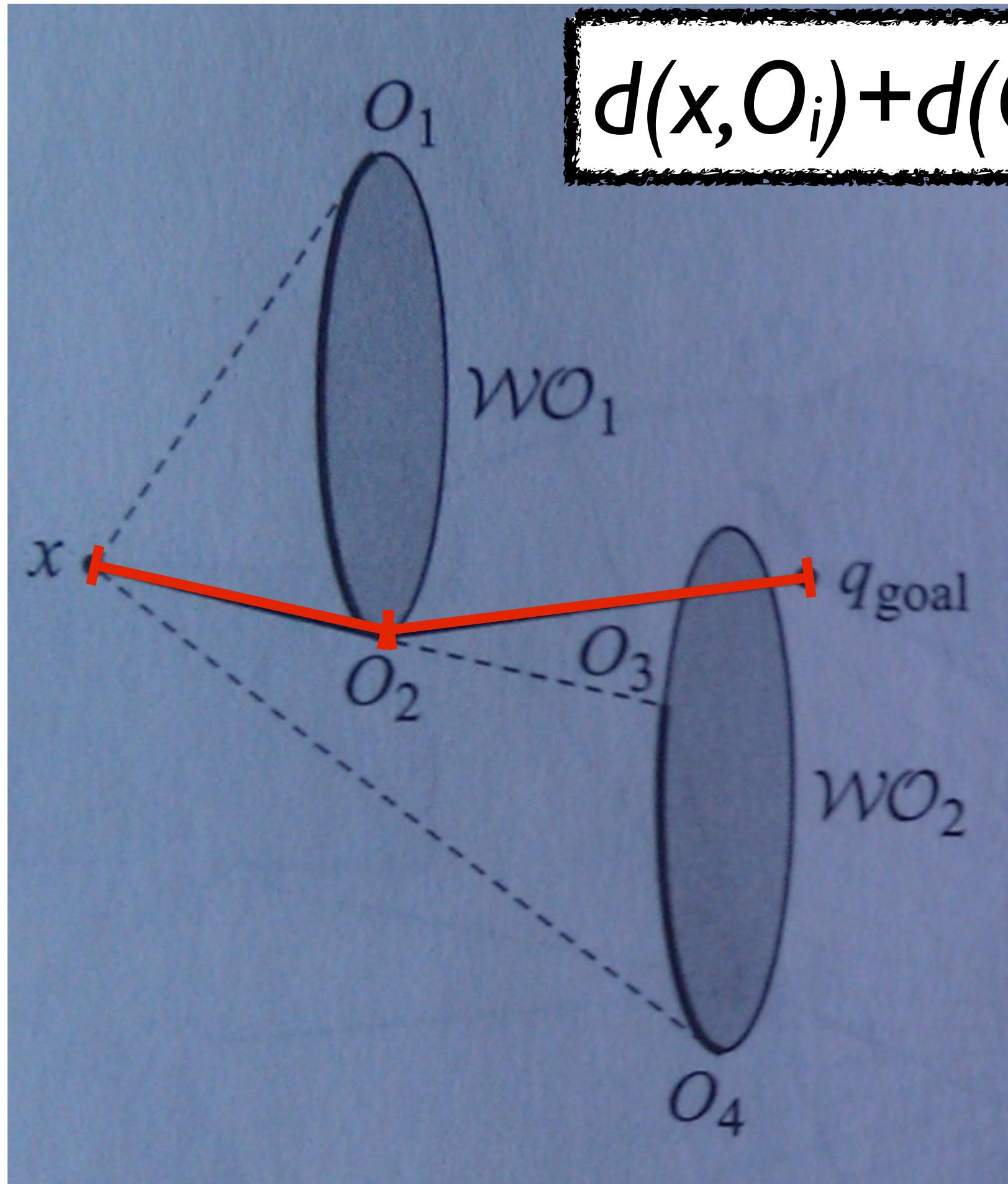
Distance to nearest object related to time from emission to sensing of beam  
(assumes speed of light is known)



Planar range finding : reflect laser on spinning mirror (typically at 10Hz)



# Tangent Bug: Heuristic Distance-to-Goal



$O_i$  are visible obstacle extents

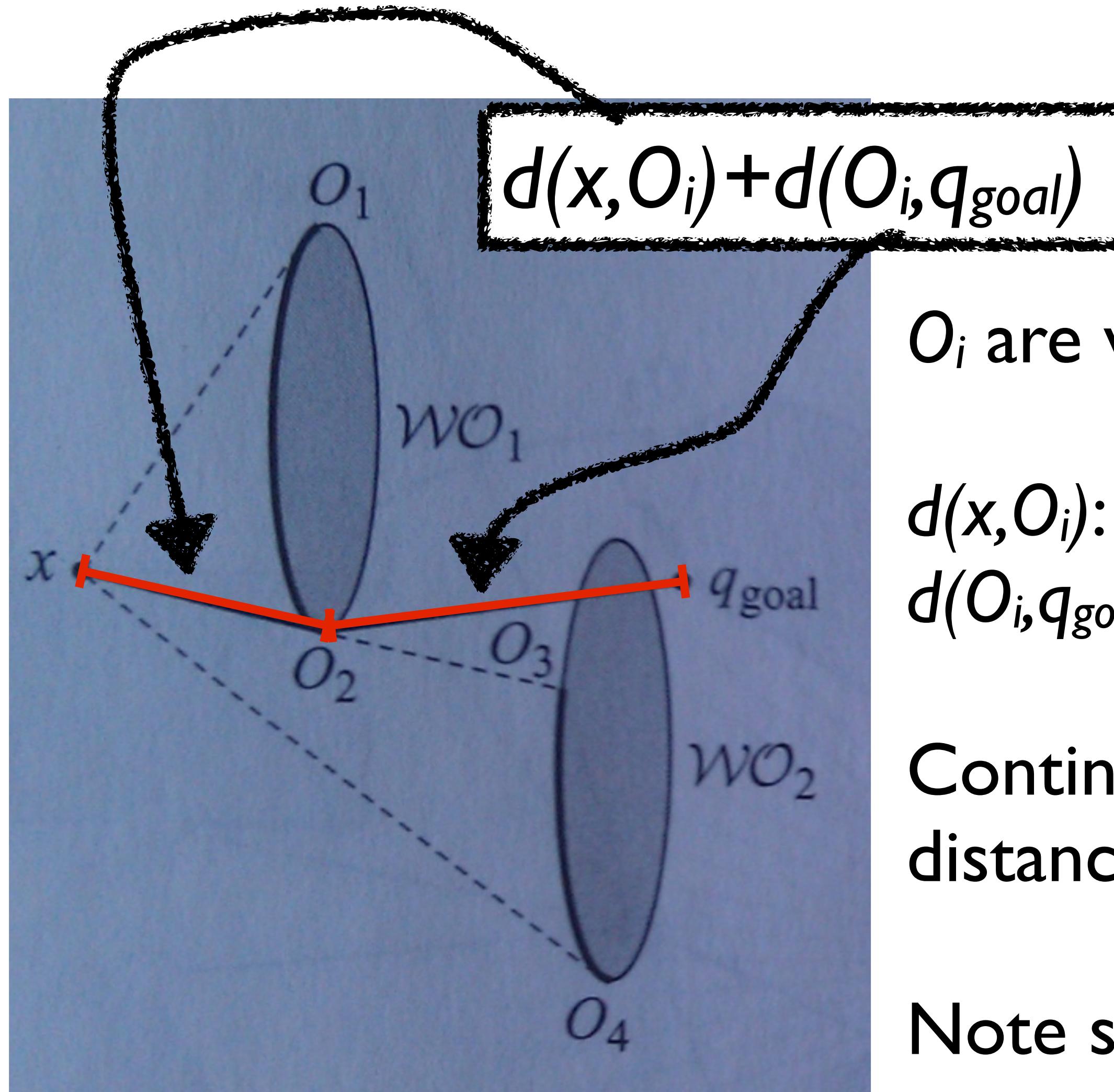
$d(x, O_i)$ : robot can see

$d(O_i, q_{goal})$ : best path robot cannot see

Continually move robot such that distance to goal is decreased

Note similarity to A\* search heuristic

# Tangent Bug: Heuristic Distance-to-Goal



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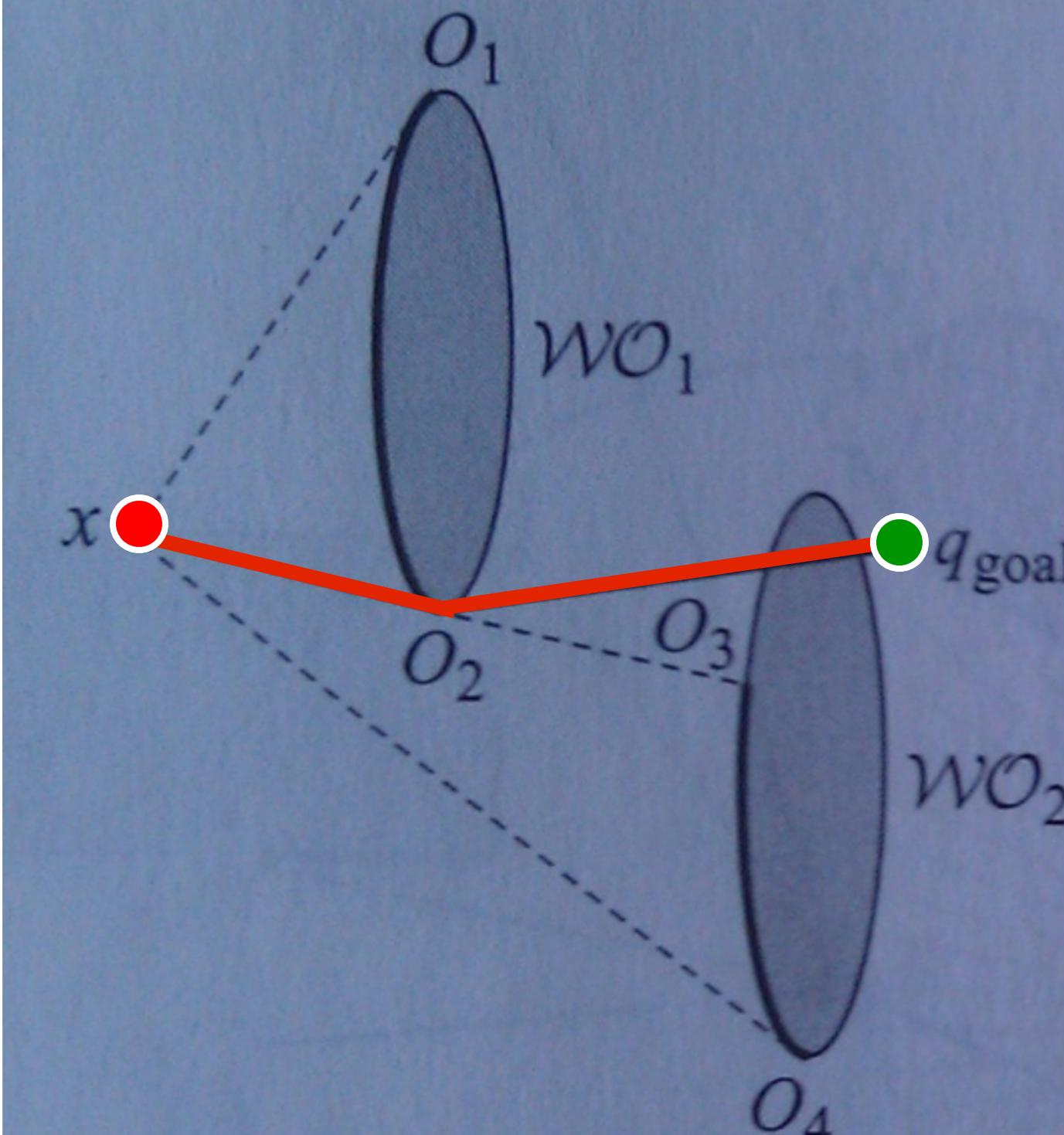
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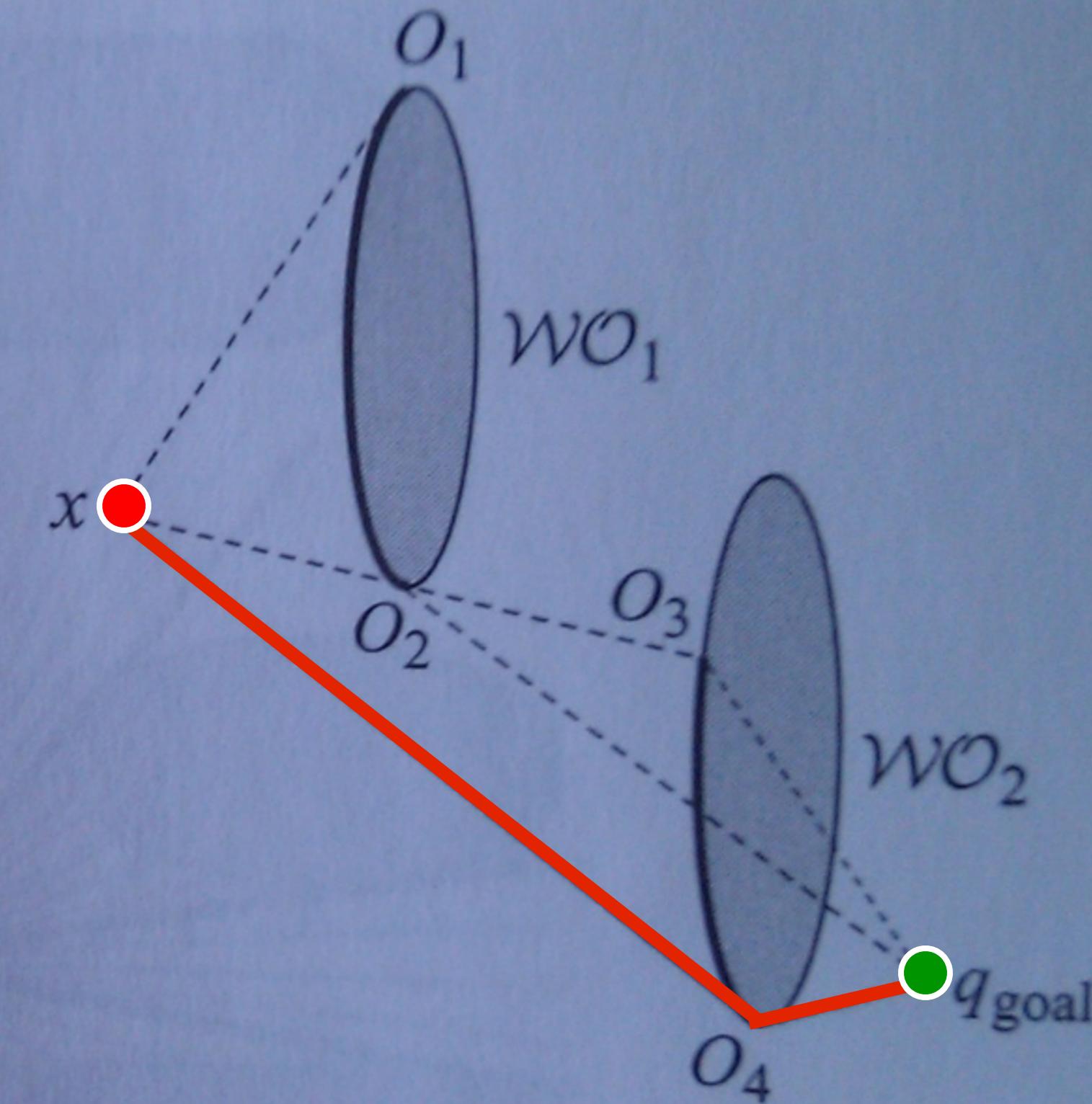
Continually move robot such that  
distance to goal is decreased

Note similarity to A\* search heuristic

$$d(x, O_2) + d(O_2, q_{goal})$$



$$d(x, O_4) + d(O_4, q_{goal})$$

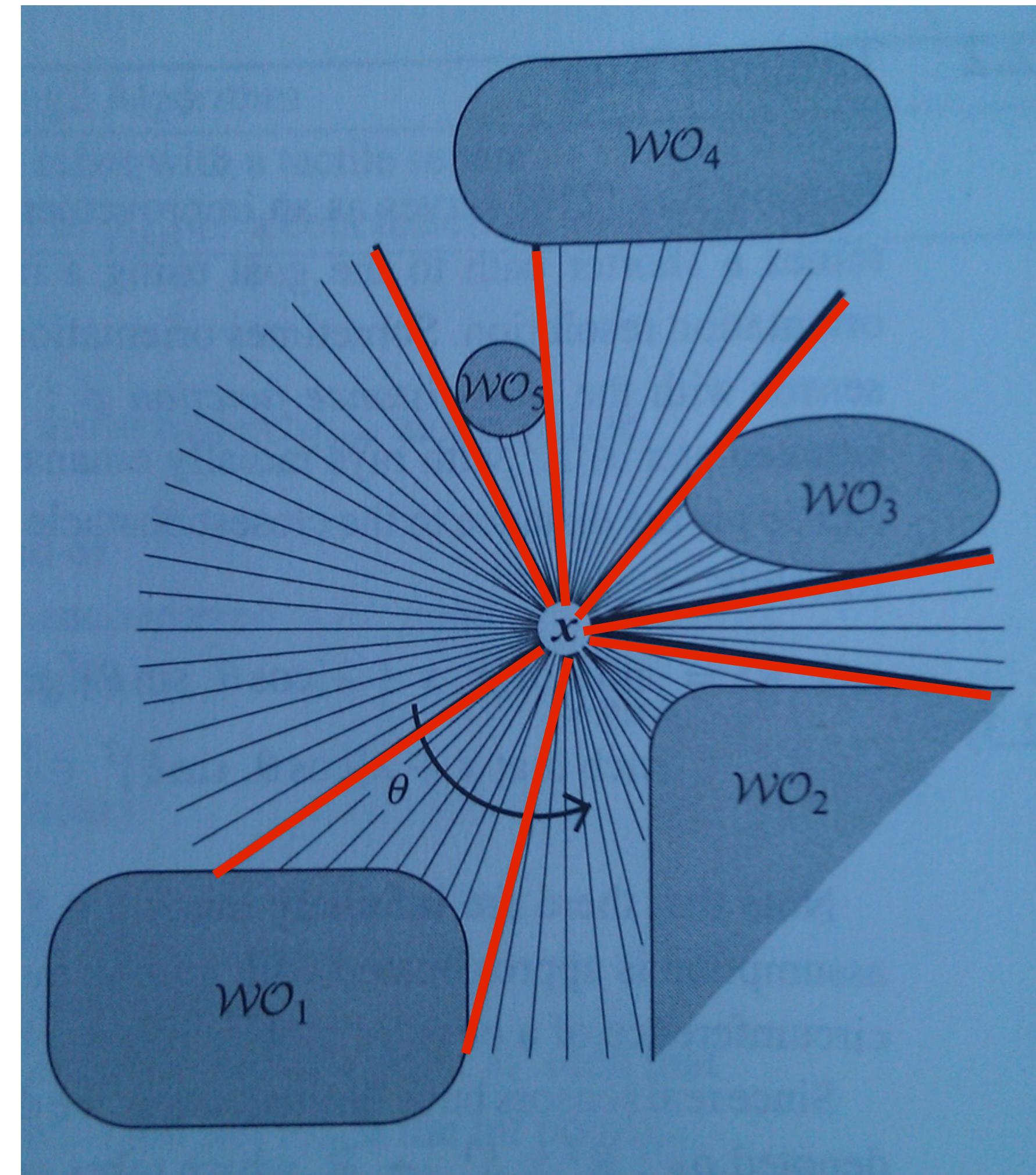


# Range Segmentation

range scan  $\rho(x, \Theta)$ : sensed distance along ray at angle  $\Theta$  within limit  $R$

discontinuities  $\{O_i\}$  in scan result from obstacles

$\{O_i\}$  segments scan into intervals  
continuity, with obstacles and free space

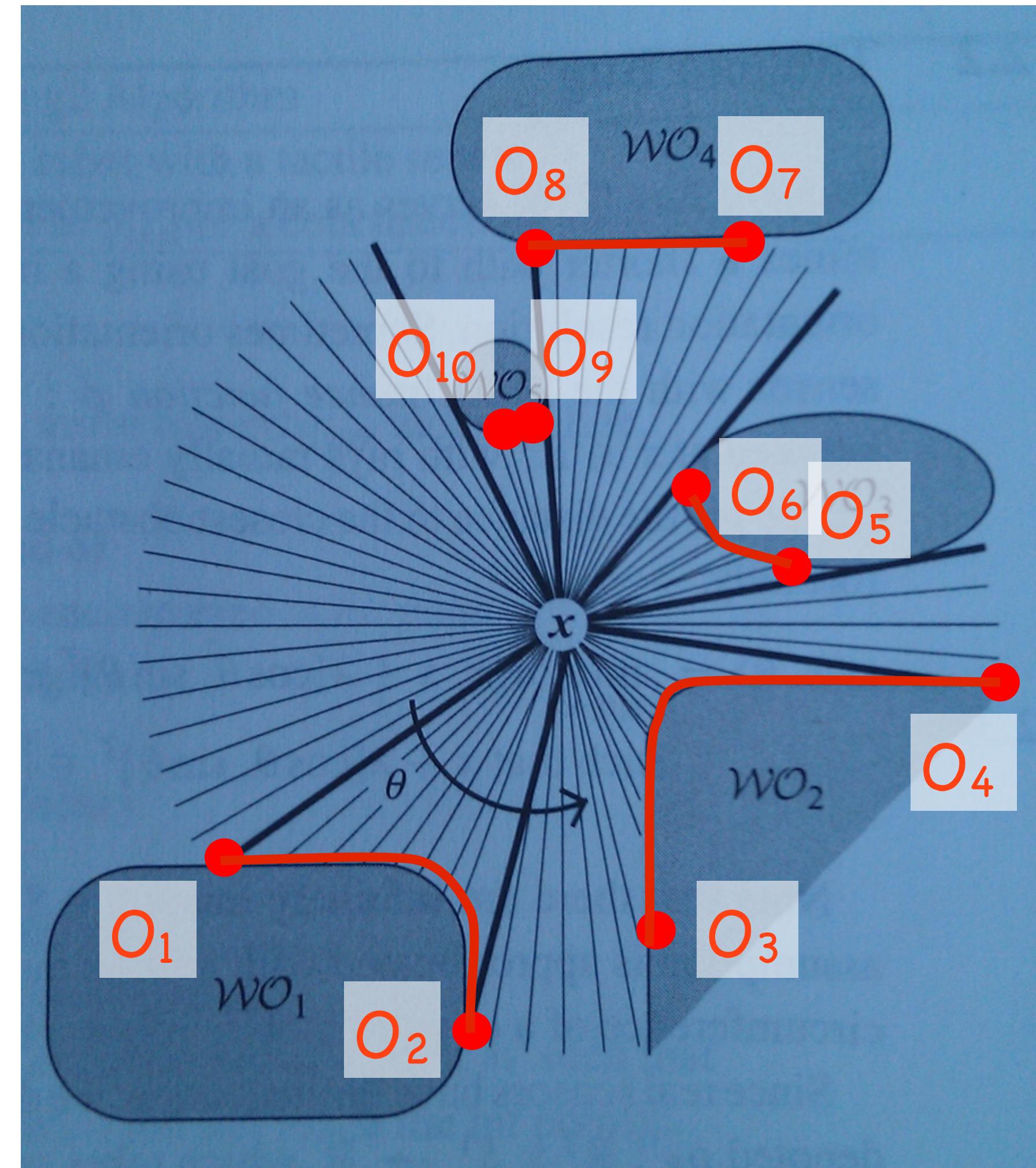


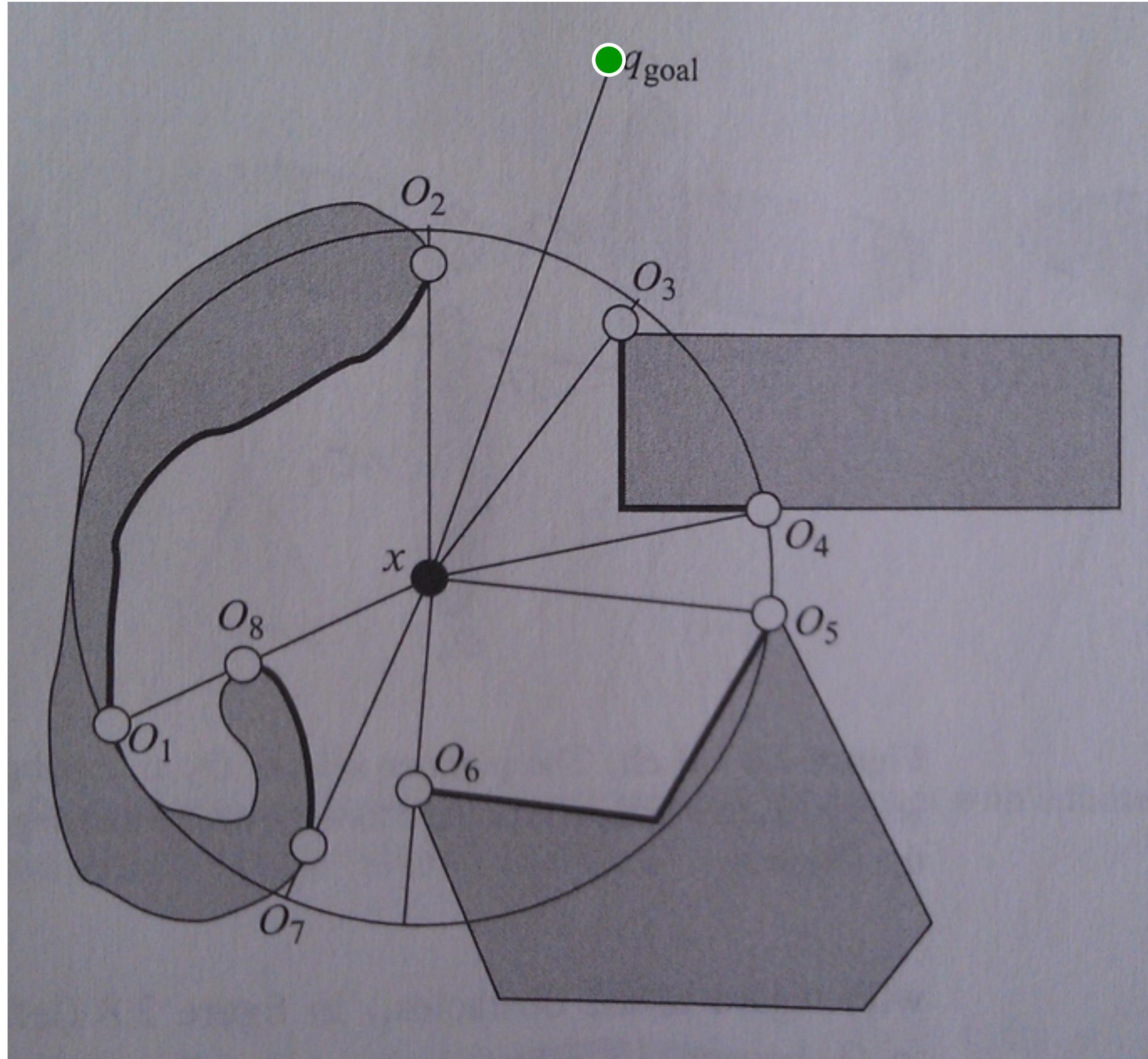
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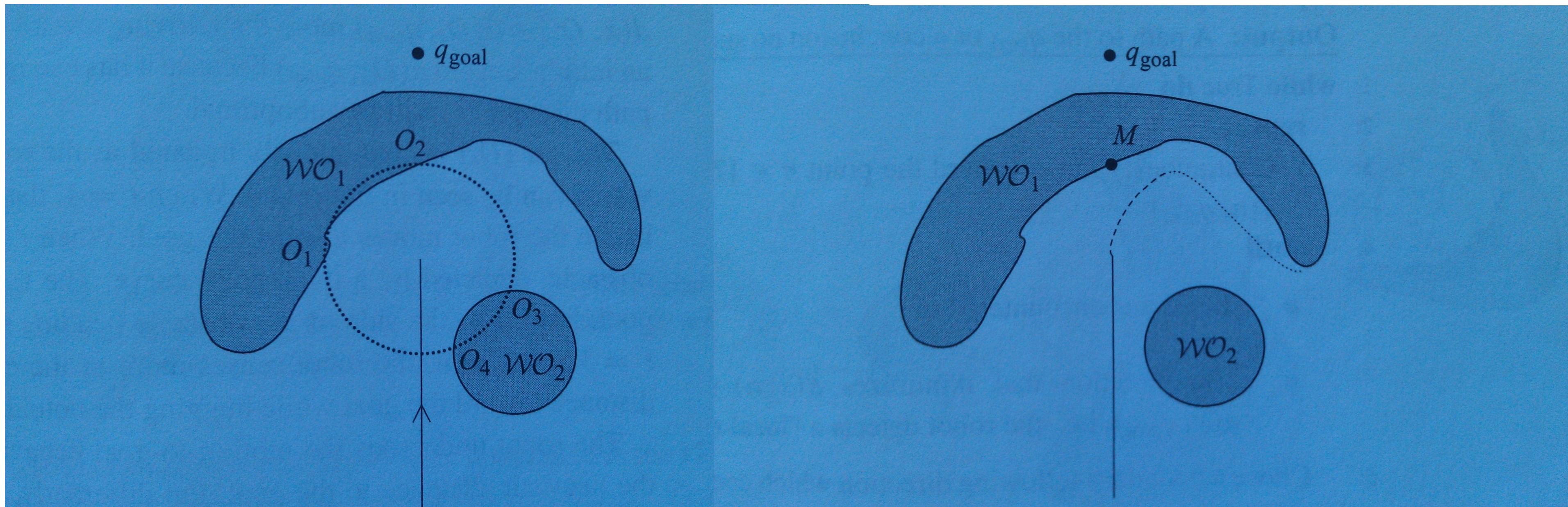
$\{O_i\}$  segments scan into intervals  
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# Tangent Bug Behaviors

Similar to other bug algorithms, Tangent Bug uses two behaviors:

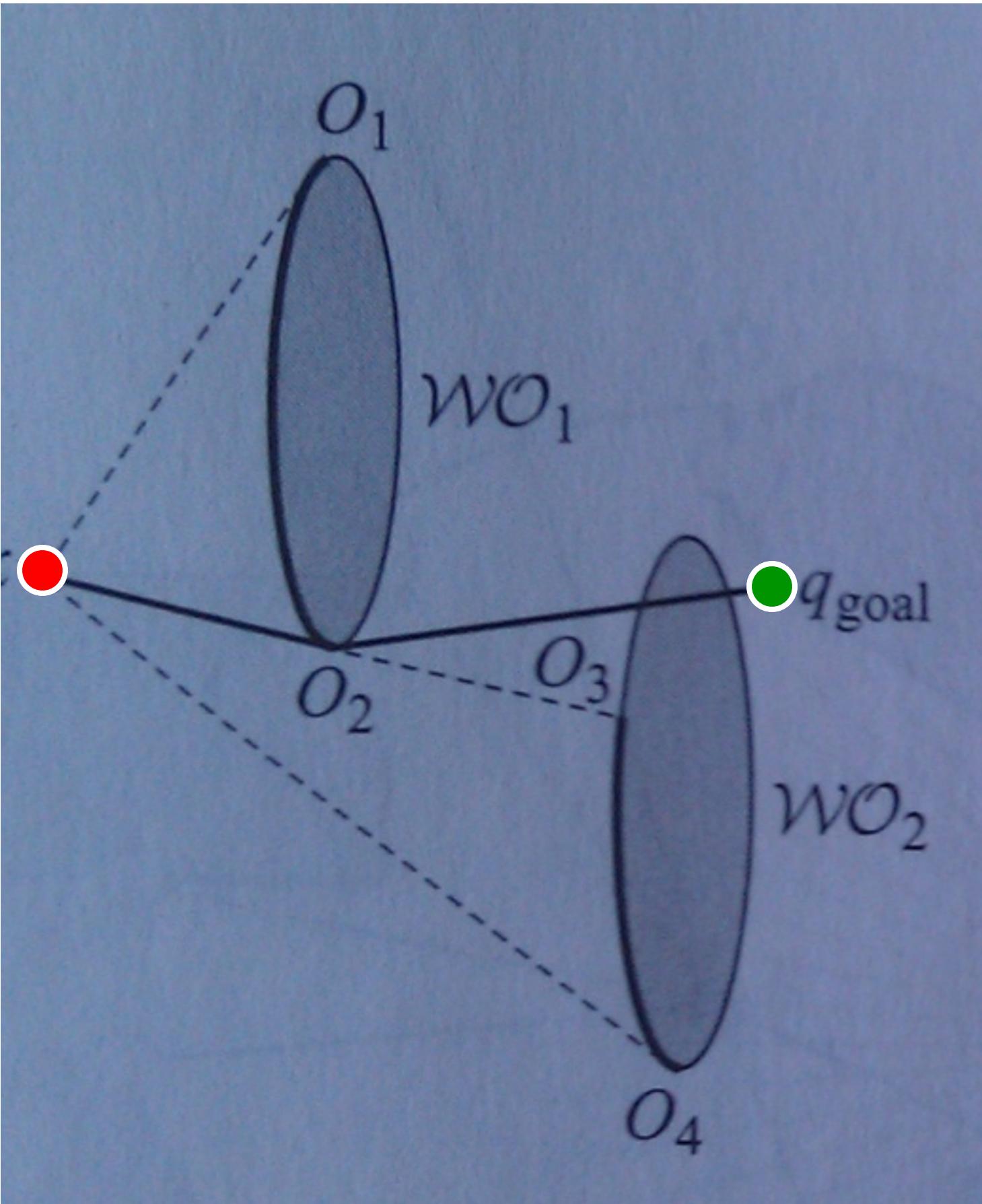


motion-to-goal

boundary-follow

# Tangent Bug

$$G(x) = d(x, O_i) + d(O_i, q_{goal})$$



I) motion-to-goal: Move to current  $O_i$  to minimize  $G(x)$ , until goal (success) or  $G(x)$  increases (local minima)

2) boundary-follow: move in while loop:

a) repeat updates

$$d_{reach} = \min d(q_{goal}, \{\text{visible } O_i\})$$

$$d_{follow} = \min d(q_{goal}, \text{sensed}(WO_j))$$

$$O_i = \operatorname{argmin}_i d(x, O_i) + d(O_i, q_{goal})$$

b) until

goal reached, (**success**)

robot cycles around obstacle, (**fail**)

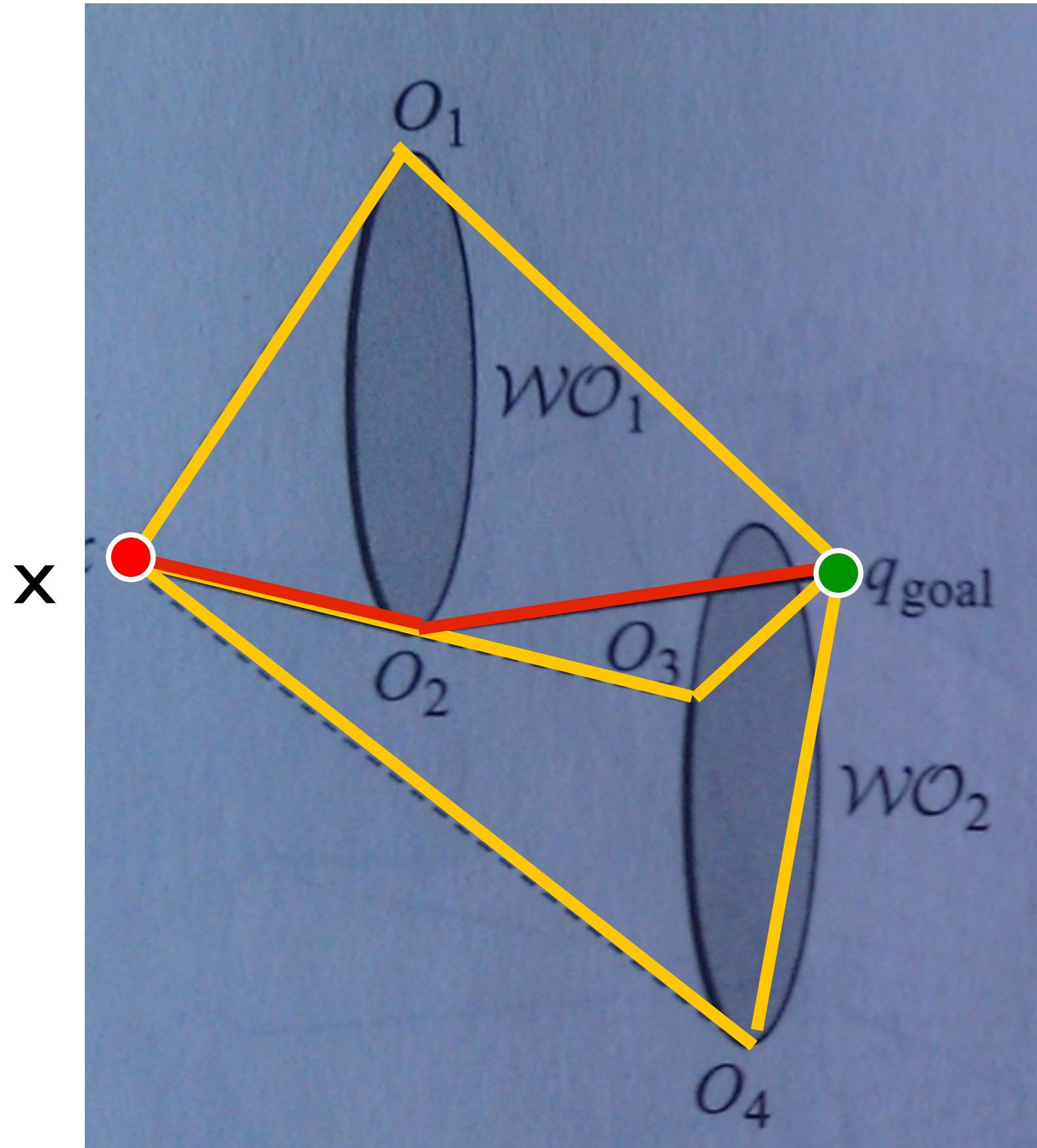
$$d_{reach} < d_{follow},$$

(**cleared obstacle or local minima**)

3) continue from (I)

# Tangent Bug

$$G(x) = d(x, O_2) + d(O_2, q_{goal})$$



min  $G(x)$  in red, others in yellow

I) motion-to-goal: Move to current  $O_i$  to minimize  $G(x)$ , until goal (success) or  $G(x)$  increases (local minima)

2) boundary-follow: move in while loop:

a) repeat updates

$$d_{reach} = \min d(q_{goal}, \{\text{visible } O_i\})$$

$$d_{follow} = \min d(q_{goal}, \text{sensed}(W O_j))$$

$$O_i = \operatorname{argmin}_i d(x, O_i) + d(O_i, q_{goal})$$

b) until

goal reached, (**success**)

robot cycles around obstacle, (**fail**)

$$d_{reach} < d_{follow},$$

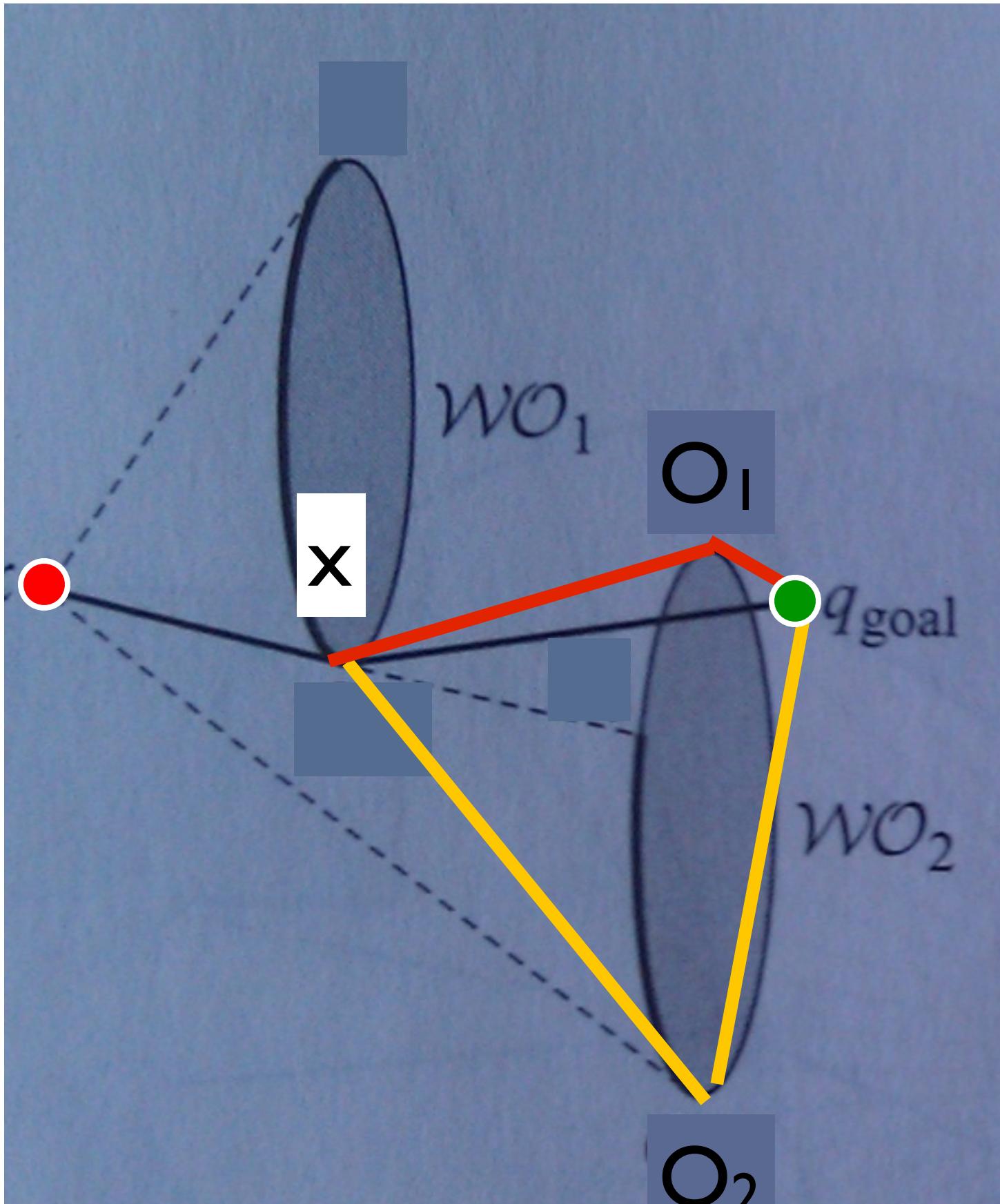
**(cleared obstacle or local minima)**

3) continue from (I)



# Tangent Bug

$$G(x) = d(x, O_1) + d(O_1, q_{goal})$$



min  $G(x)$  in red, others in yellow

I) motion-to-goal: Move to current  $O_i$  to minimize  $G(x)$ , until goal (success) or  $G(x)$  increases (local minima)

2) boundary-follow: move in while loop:

a) repeat updates

$$d_{reach} = \min d(q_{goal}, \{\text{visible } O_i\})$$

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$$O_i = \operatorname{argmin}_i d(x, O_i) + d(O_i, q_{goal})$$

b) until

goal reached, (**success**)

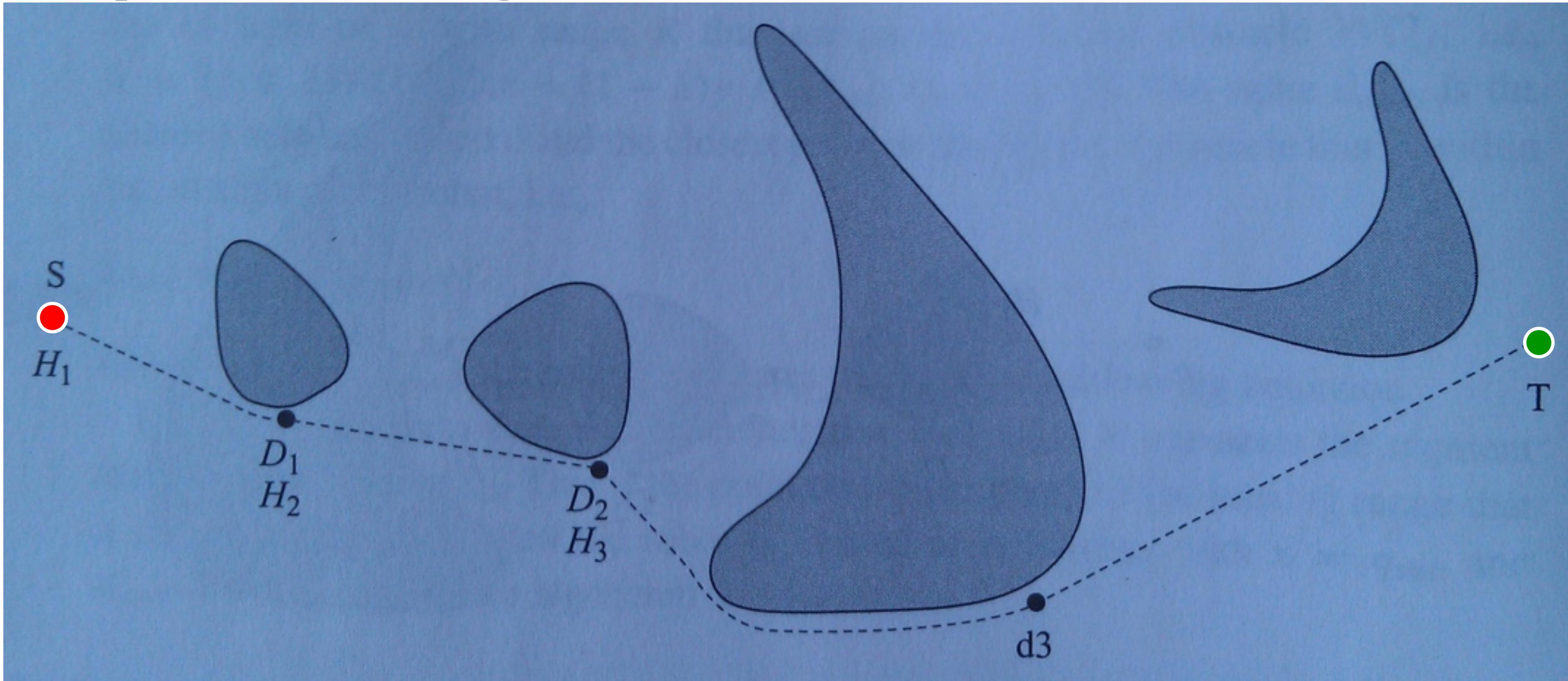
robot cycles around obstacle, (**fail**)

$$d_{reach} < d_{follow},$$

**(cleared obstacle or local minima)**

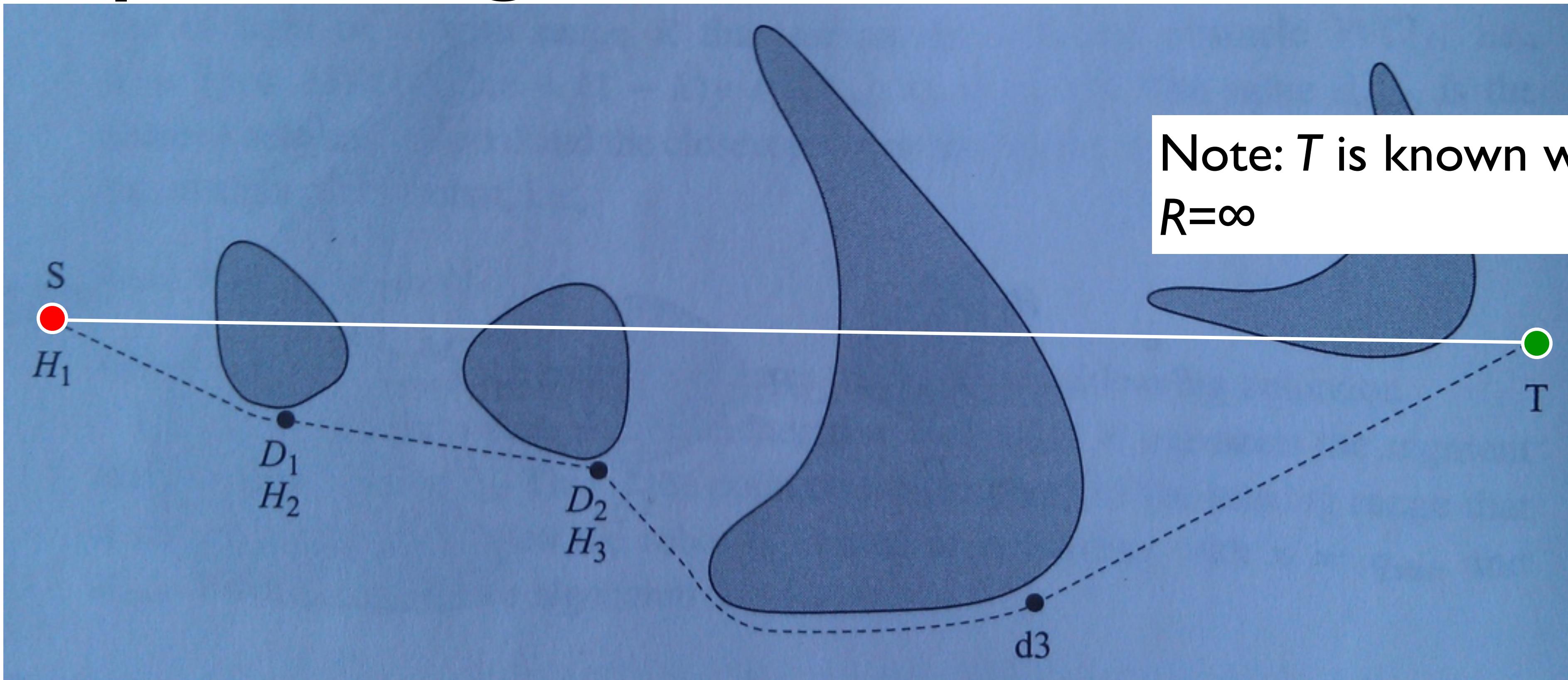
3) continue from (I)

# Example: range $R=\infty$



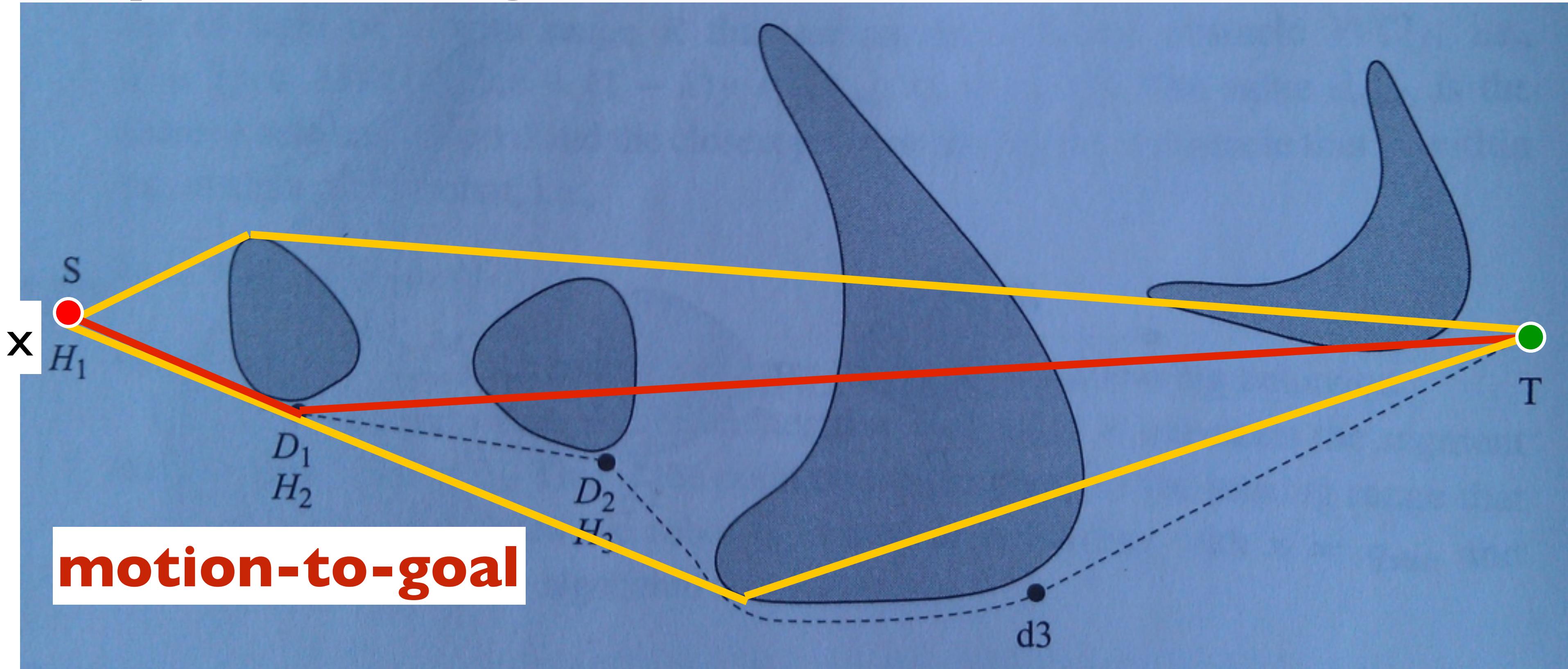
$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima

# Example: range $R=\infty$



$H_i$ : hit point  
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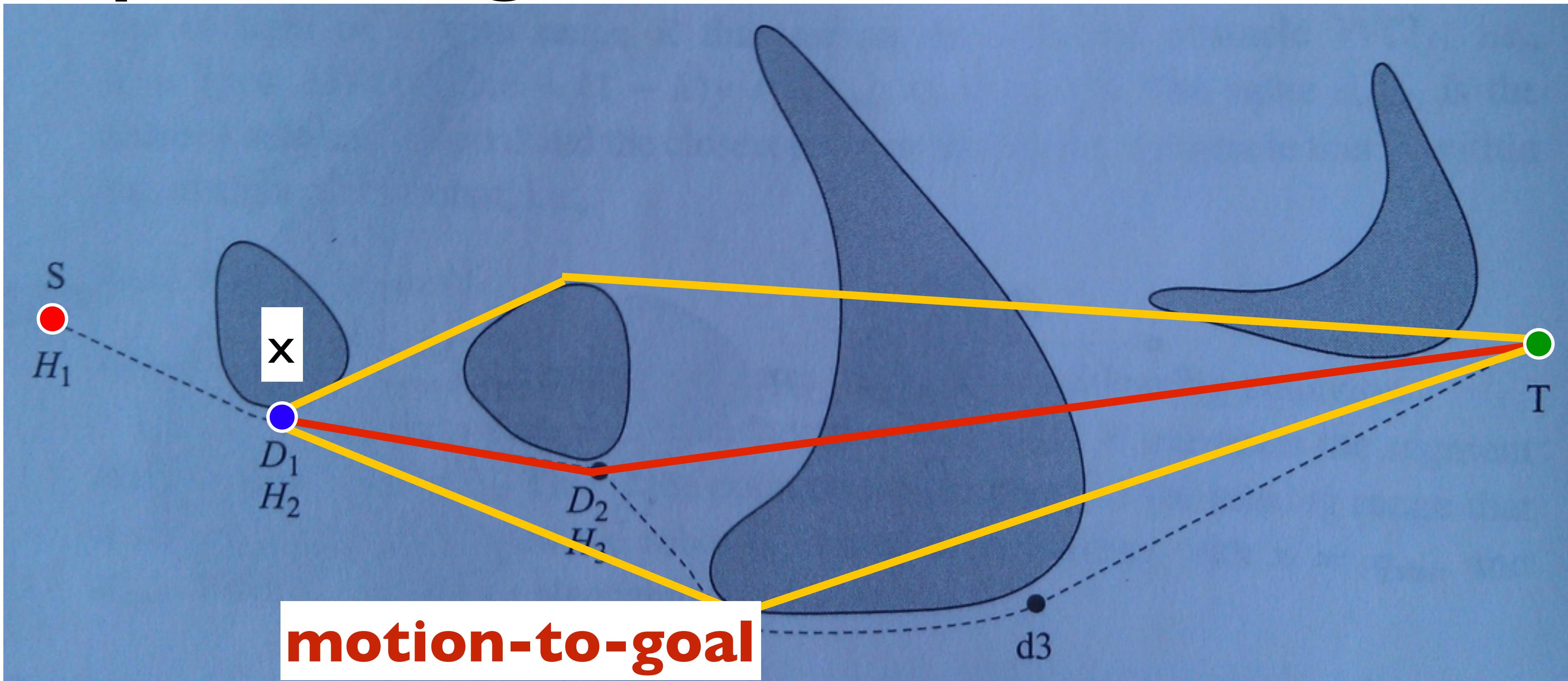
# Example: range $R=\infty$



$\min G(x)$  in red, others in yellow

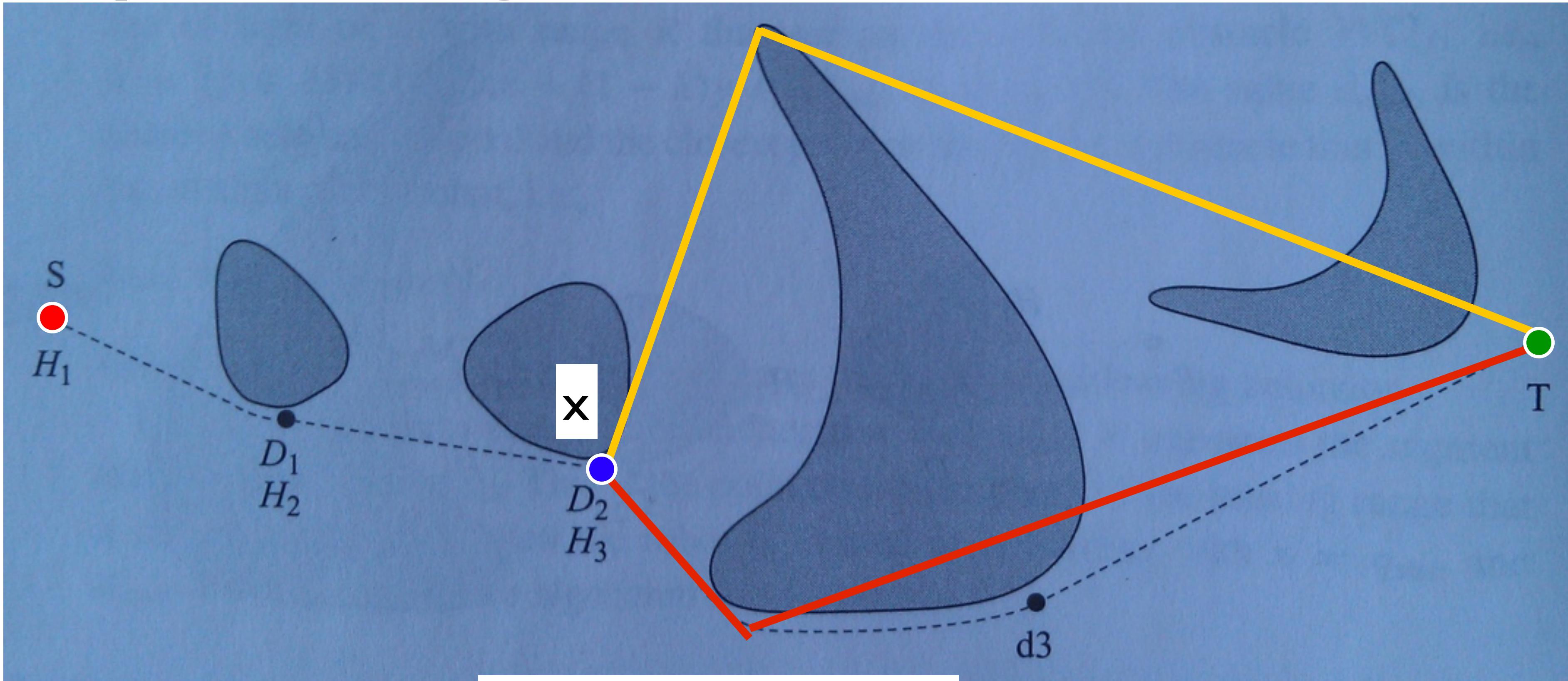
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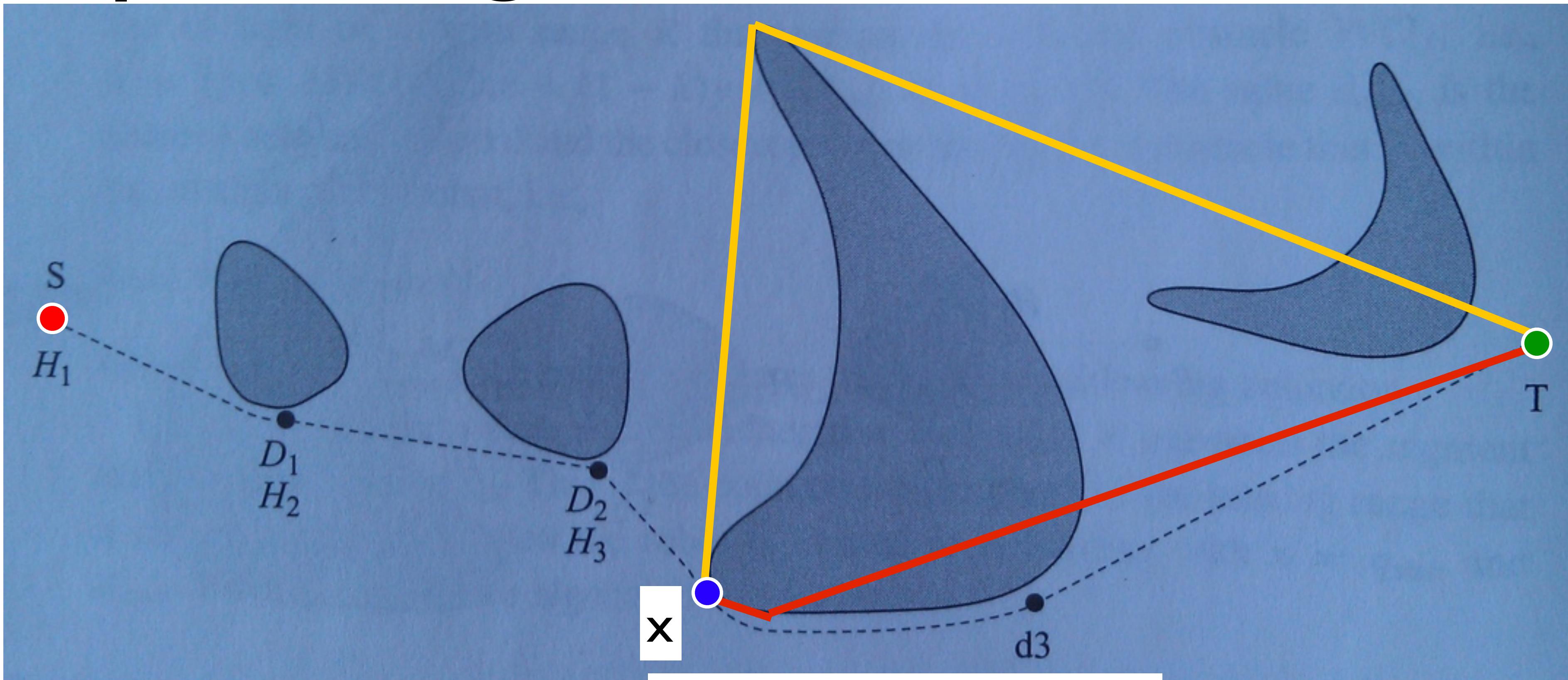
# Example: range $R=\infty$



**motion-to-goal**

$H_i$ : hit point  
 $D_i$ : Depart point  
 $L_i$ : Leave point  
 $M_i$ : local minima

# Example: range $R=\infty$



start following:

$$\min d(q_{goal}, \{\text{visible } O_i\}) < \min d(q_{goal}, \text{sensed}(WO_j))$$

**follow-boundary**

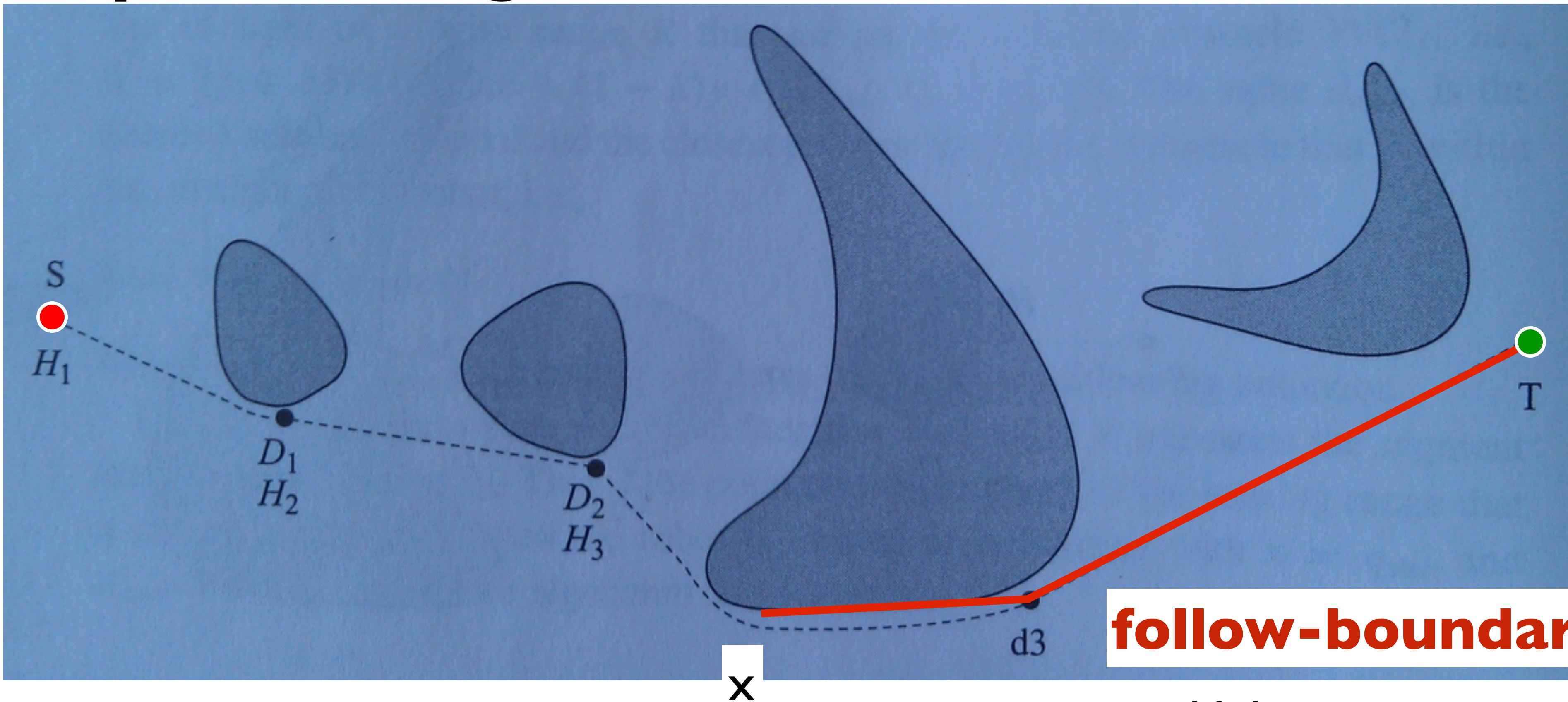
$H_i$ : hit point

$D_i$ : Depart point

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# Example: range $R=\infty$

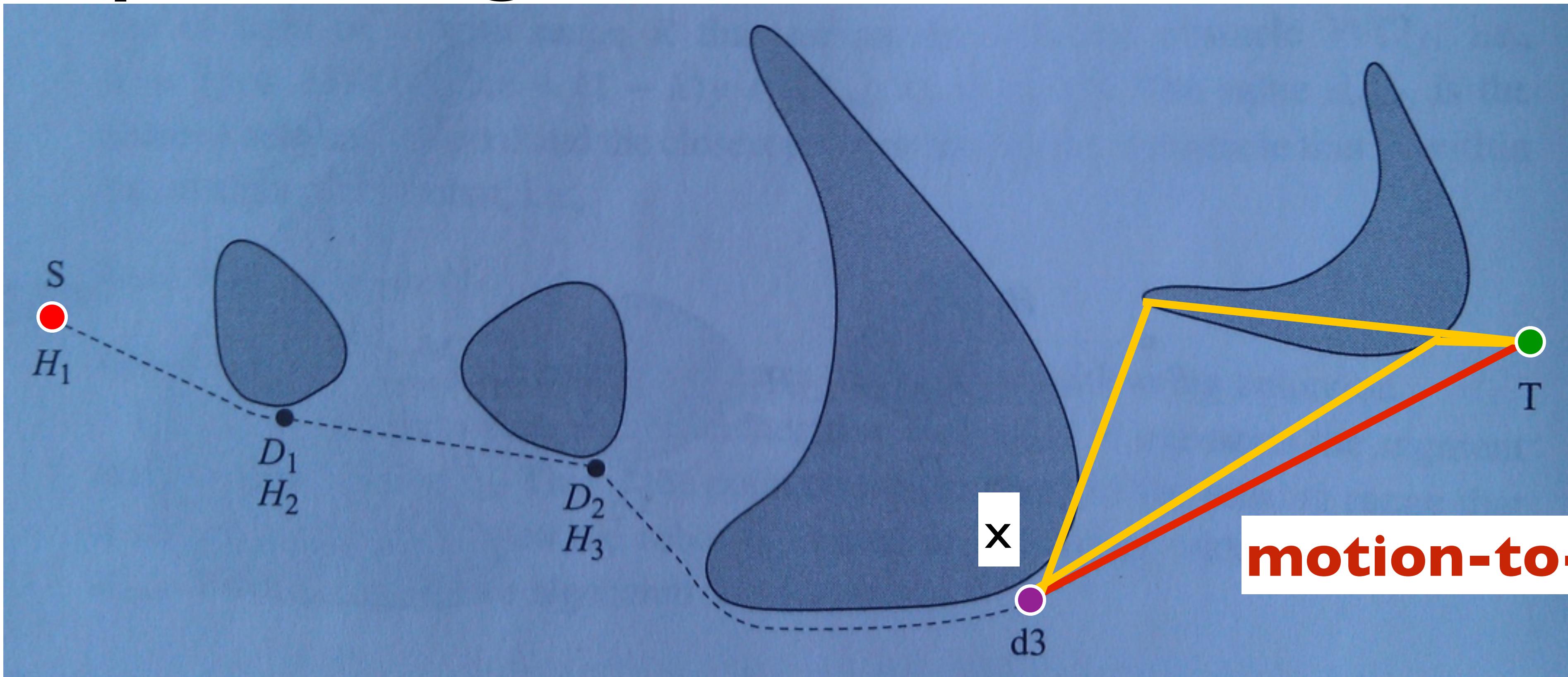


end following:

$$\min d(q_{goal}, \{\text{visible } O_i\}) < \min d(q_{goal}, \text{sensed}(WO_j))$$

- $H_i$ : hit point
- $D_i$ : Depart point
- $L_i$ : Leave point
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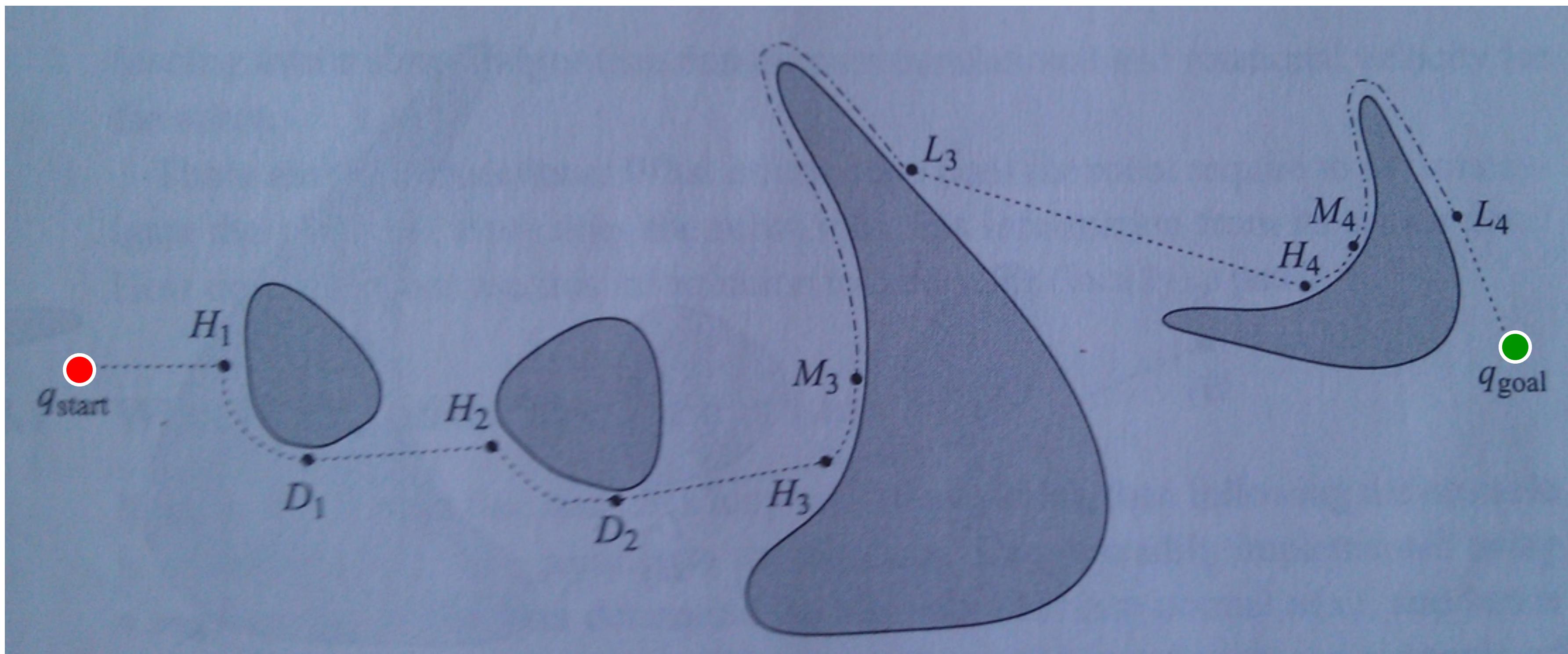
# Example: range $R=\infty$



$H_i$ : hit point  
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# Example: range $R=0$

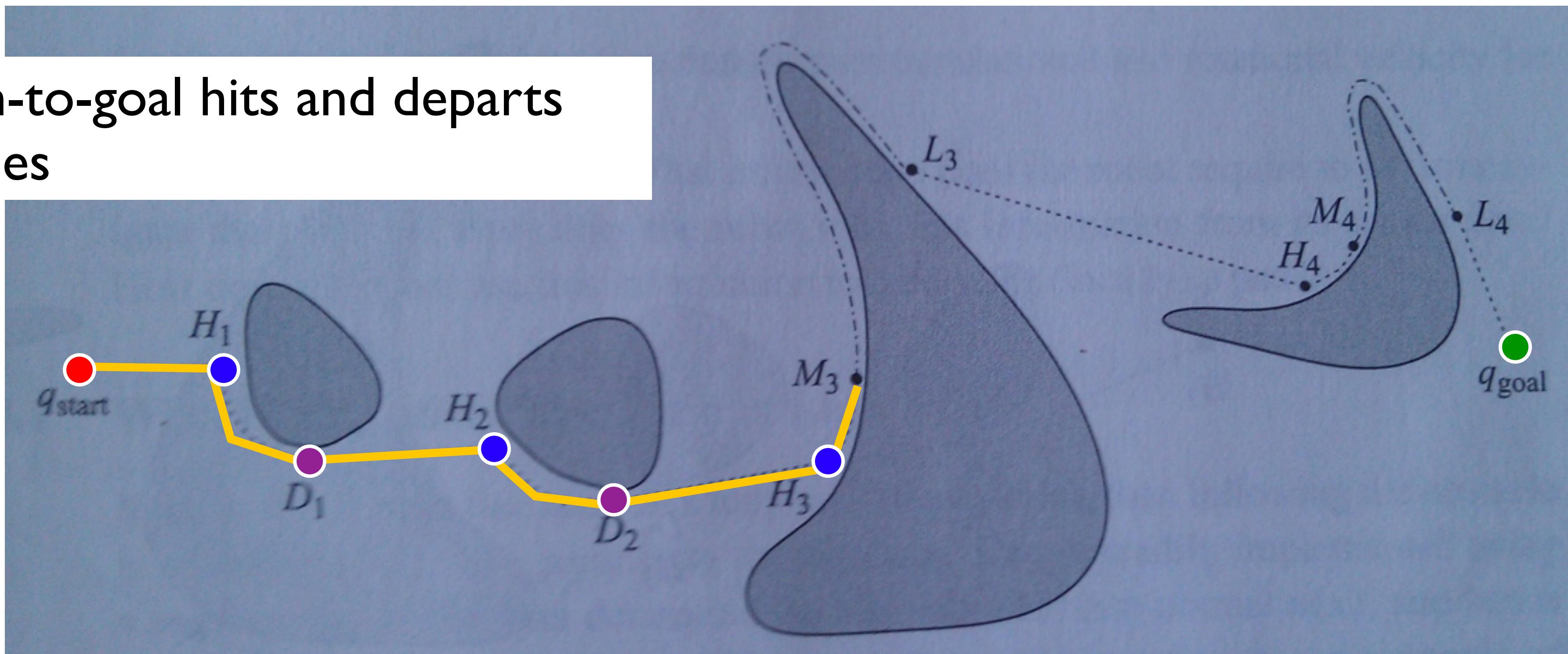
$H_i$ : hit point  
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# Example: range $R=0$

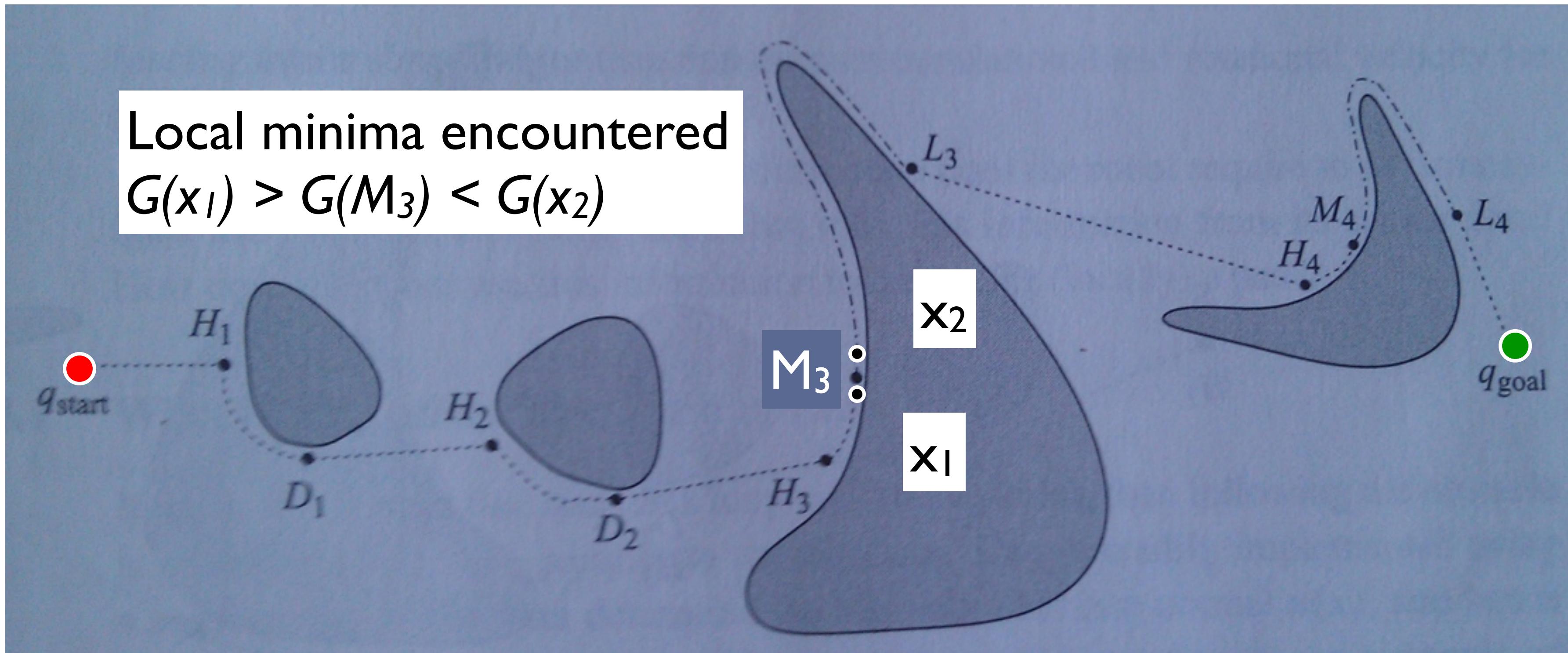
$H_i$ : hit point  
 $D_i$ : Depart point  
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 $M_i$ : local minima

Motion-to-goal hits and departs obstacles



# Example: range $R=0$

$H_i$ : hit point  
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 $L_i$ : Leave point  
 $M_i$ : local minima

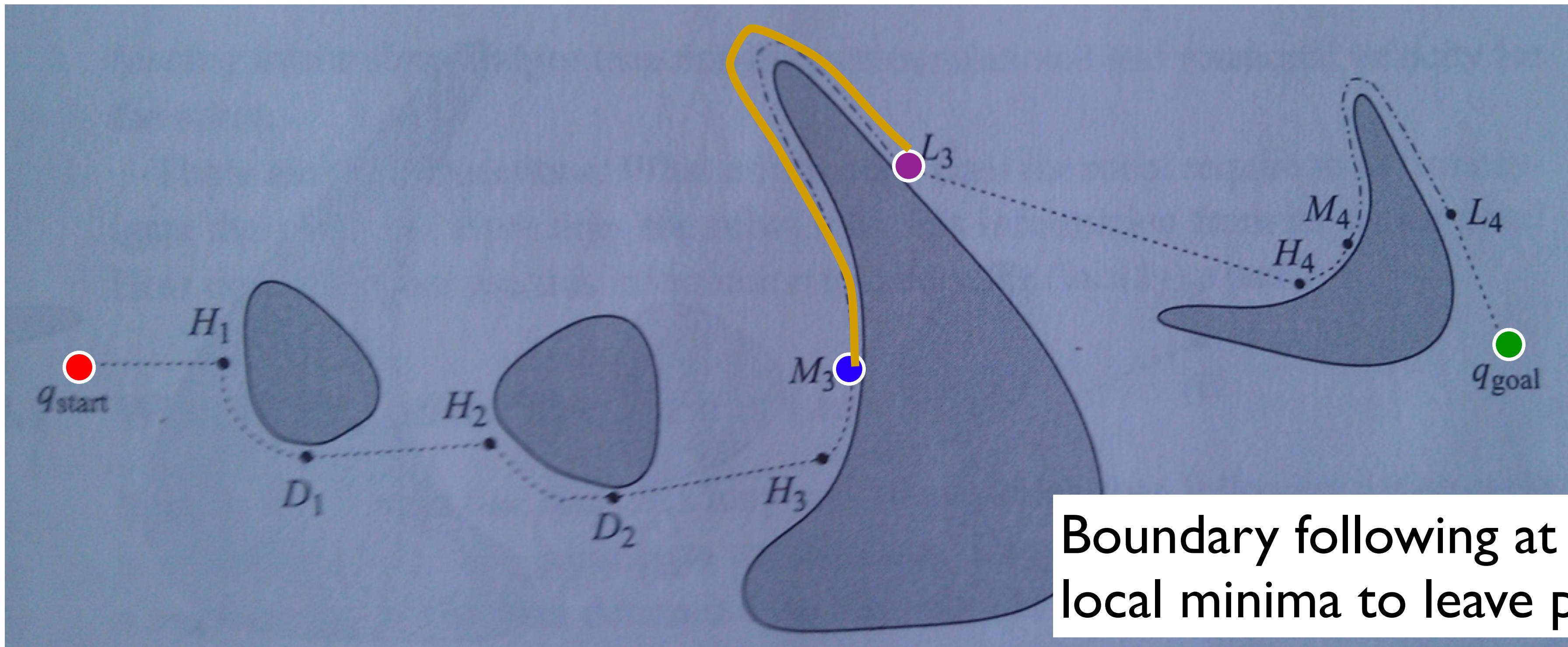


Local minima at increase of  $G(x) = d(x, O_i) + d(O_i, q_{goal})$



# Example: range $R=0$

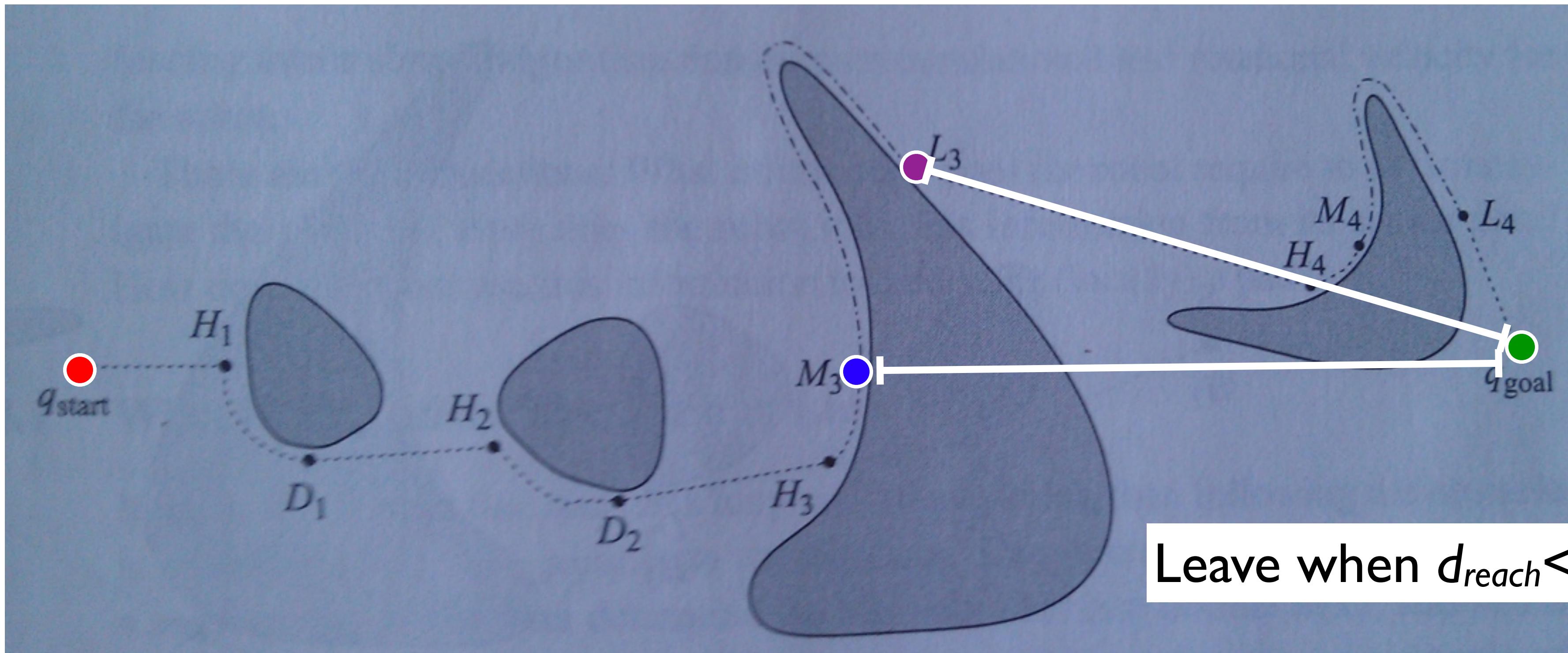
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Local minima at increase of  $G(x) = d(x, O_i) + d(O_i, q_{goal})$

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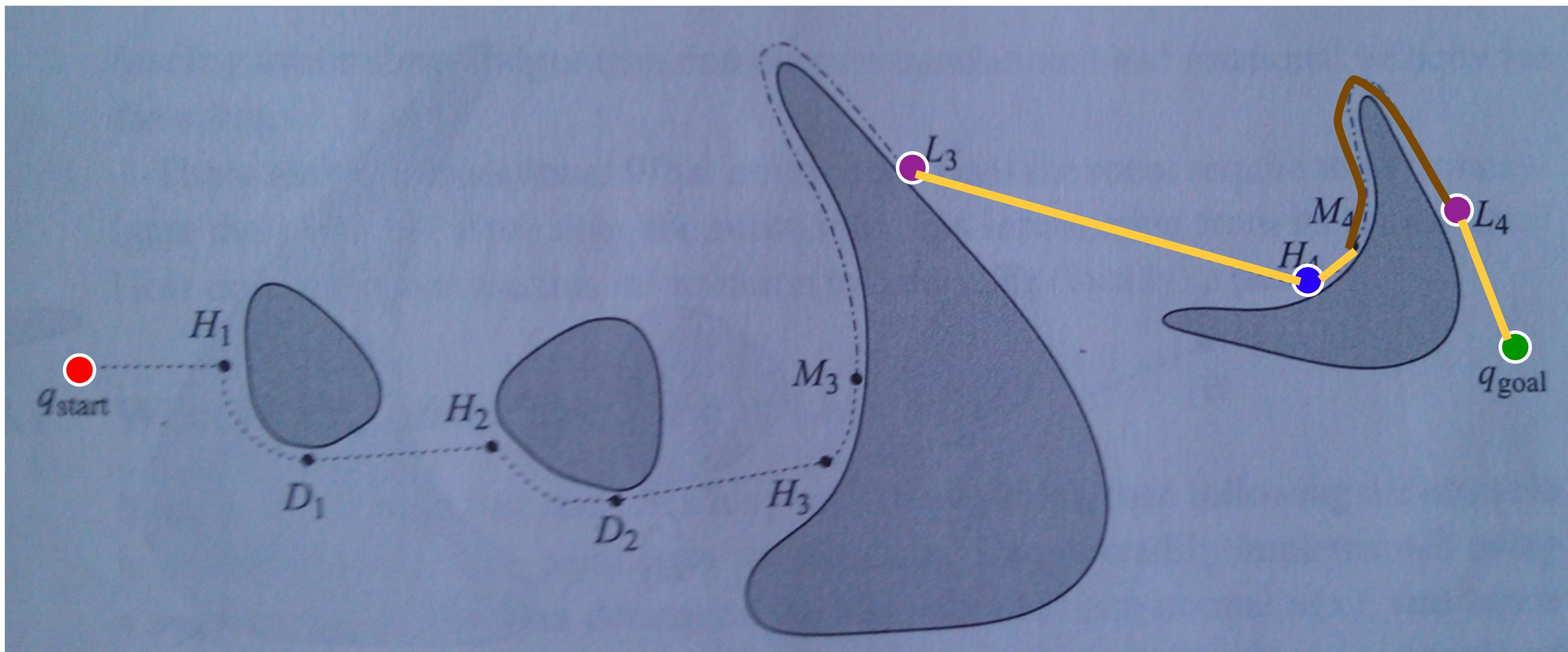
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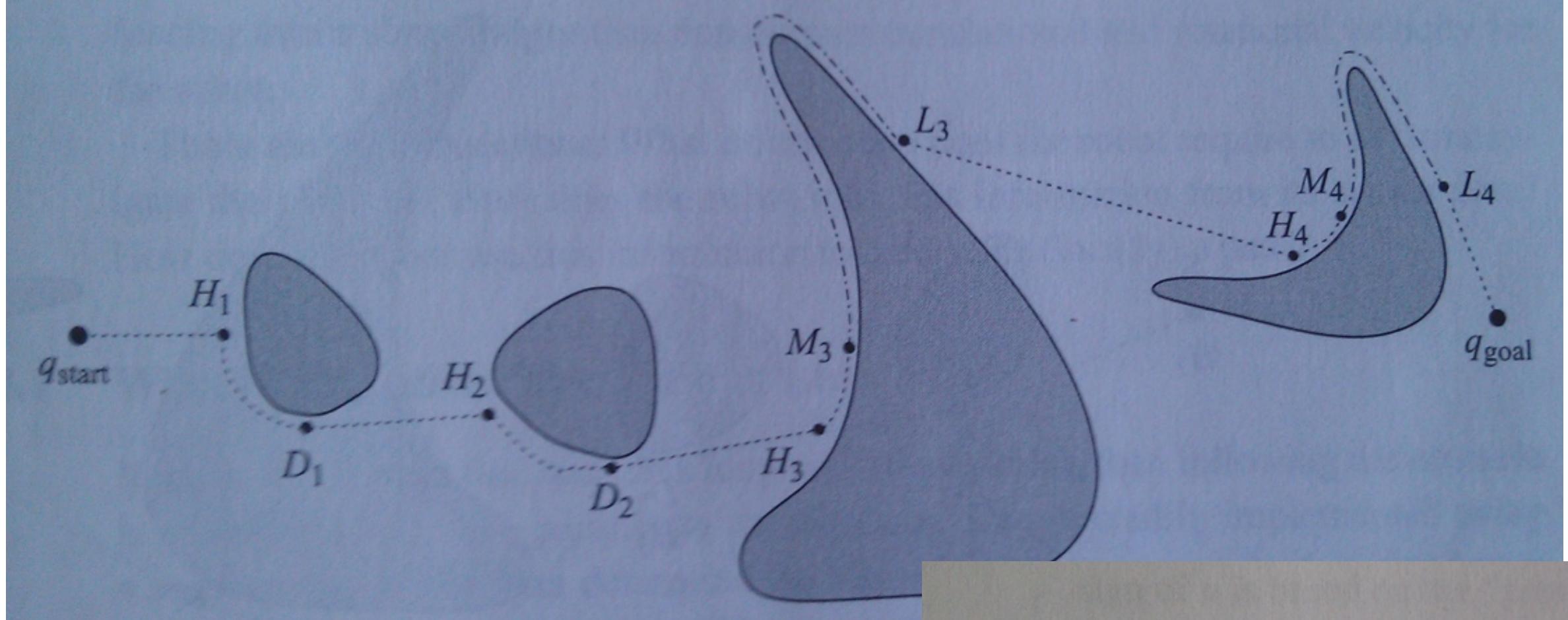


Local minima at increase of  $G(x) = d(x, O_i) + d(O_i, q_{goal})$

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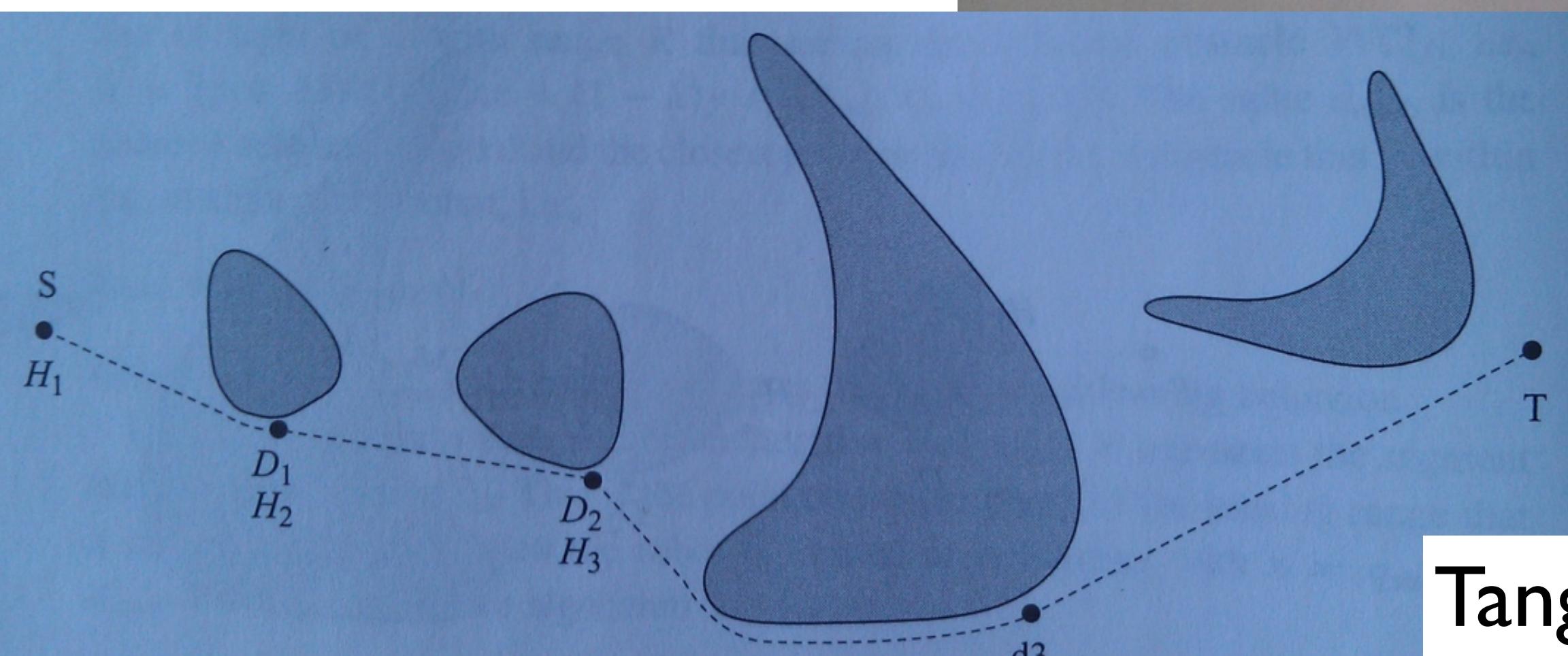
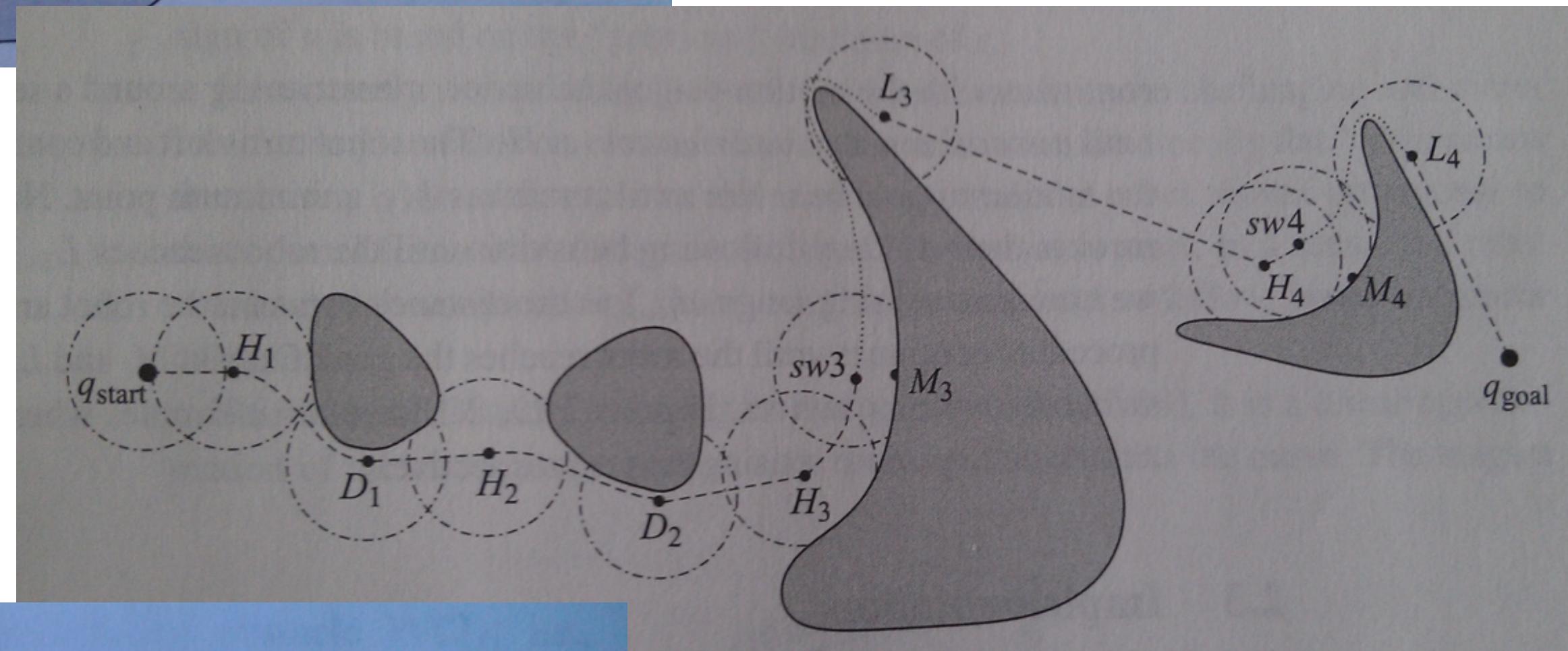
$H_i$ : hit point  
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Tangent bug  $R=0$

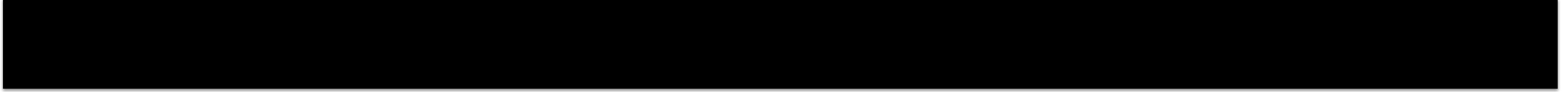
Tangent bug with limited radius



Tangent bug  $R=\infty$



What does BugX assume that Random Walk does not?



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Localization: knowing the robot's location, at least wrt. distance to goal

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What do graph search algorithms assume that BugX does not?



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A graph of valid locations that can be traversed

What does BugX assume that Random Walk does not?

Localization: knowing the robot's location, at least wrt. distance to goal

What do graph search algorithms assume that BugX does not?

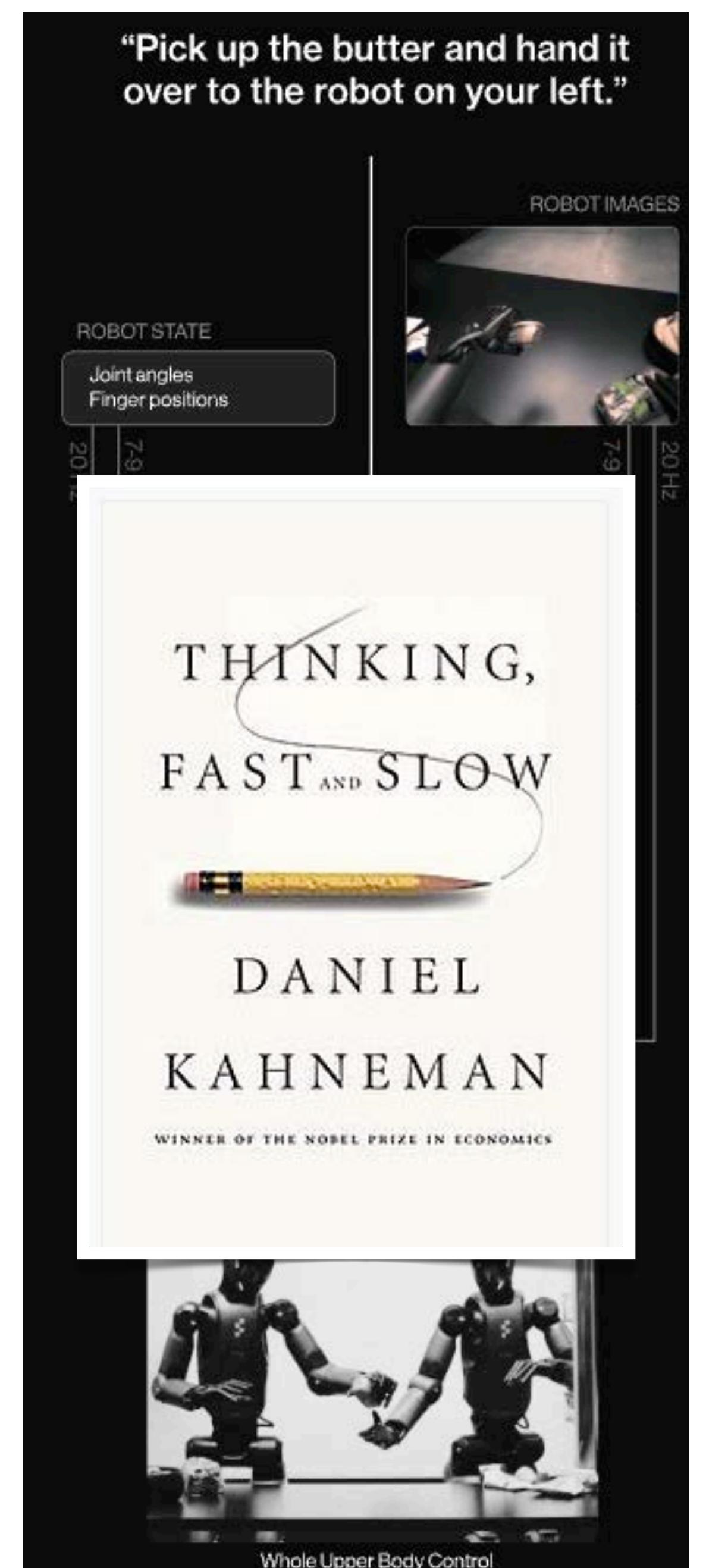
A graph of valid locations that can be traversed

Suppose we have or can build such a graph...

# Next Lecture

## Planning - III - Configuration Space

"Pick up the butter and hand it over to the robot on your left."



# HELIx

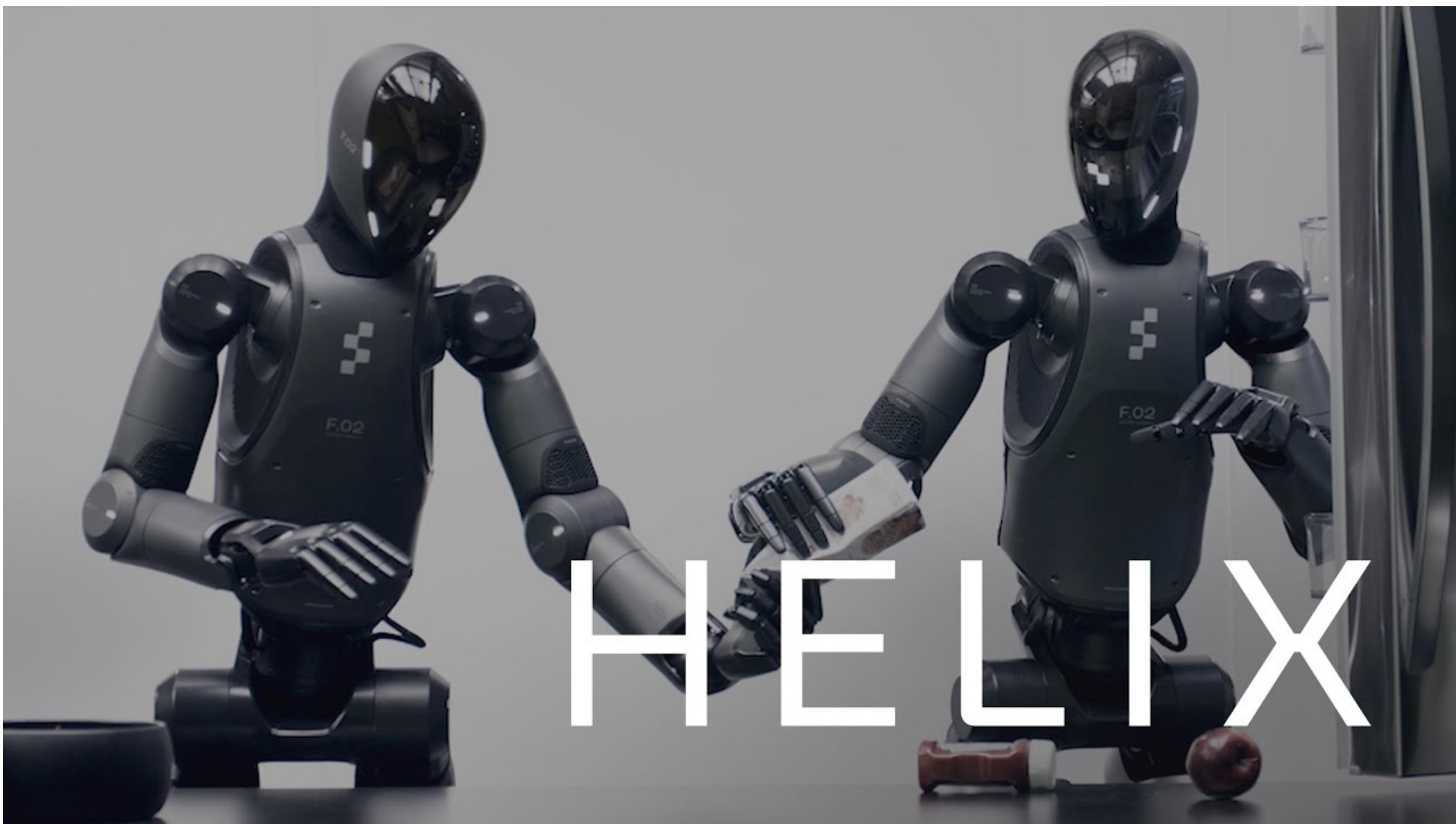


Figure - <https://www.youtube.com/watch?v=Z3yQHYNXPws>