Lecture 14
Planning - VI - Collision Detection

Fishman, Adam, Adithyavairavan Murali, Clemens Eppner, Bryan Peele, Byron Boots, and Dieter Fox. "Motion policy networks." In *Conference on Robot Learning*, pp. 967-977.PMLR, 2023.

Course Logistics

- **No quiz today!**
- Project 3 was posted on 10/11 and is due on **10/30 (new)**.
- Project 4 will be posted on 10/30.

Some updates based on the feedback

- Projects:
  - Number of submissions to autogravder will be increased to 10 per day.
  - Test case failure error will be available for the students.
- Quiz:
  - *We encourage you to discuss this in Ed after the deadline.*

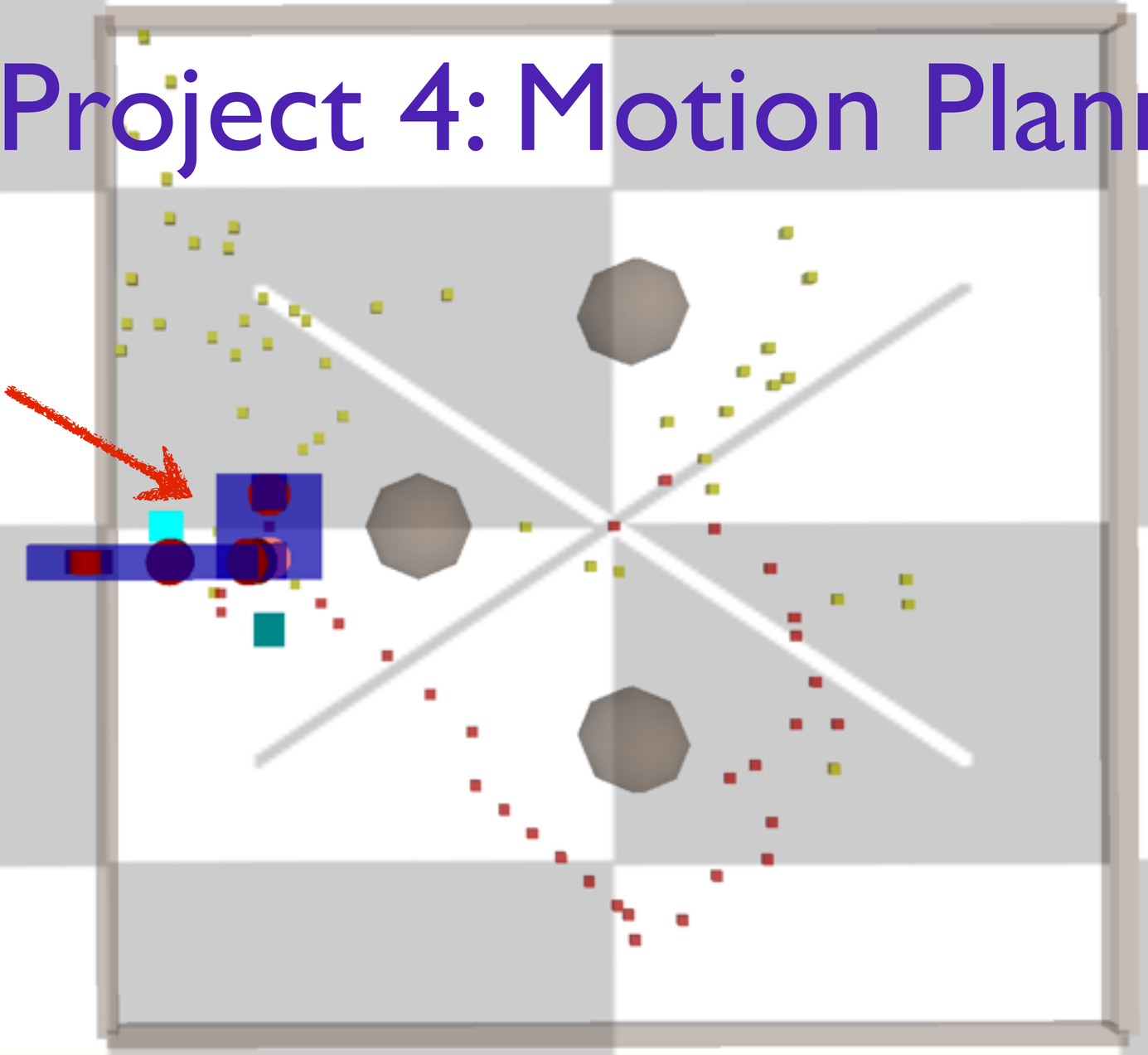
Where are we in the course?

Project 4: Motion Planning

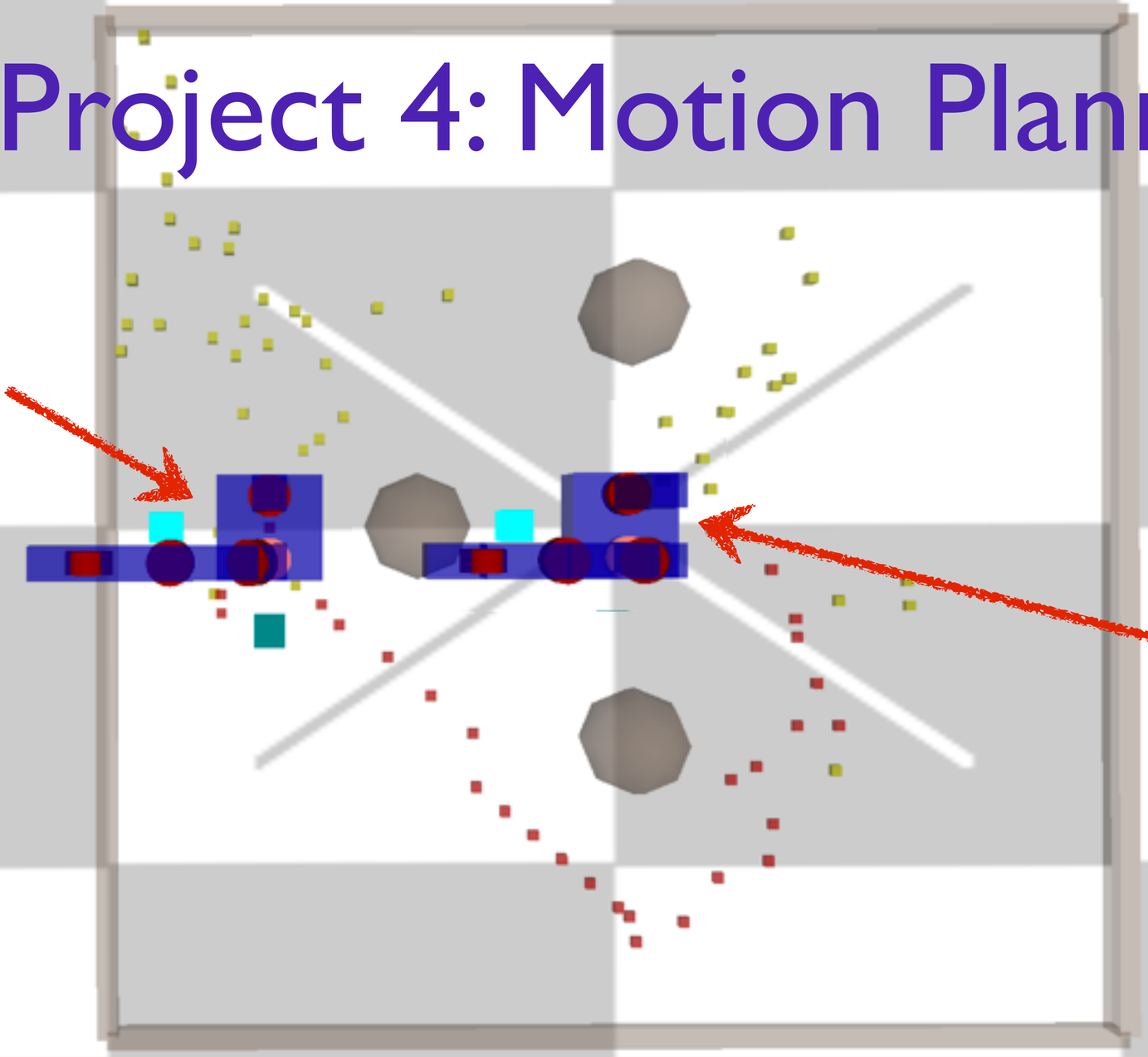
- Generate a collision free motion plan to the world origin and zero joint angle configuration

Project 4: Motion Planning

Start: random non-colliding configuration



Project 4: Motion Planning



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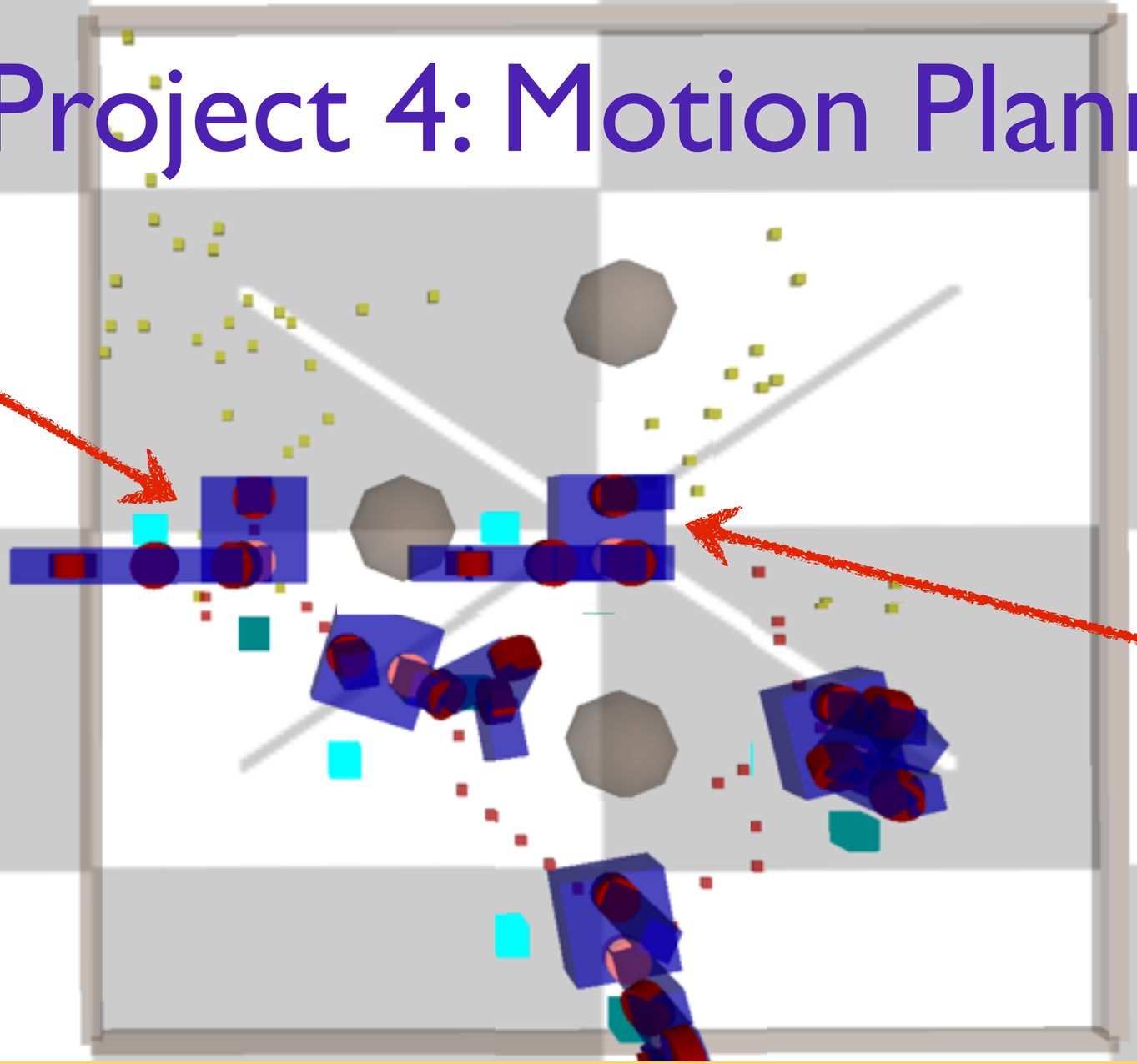
Project 4: Motion Planning



Start: random non-colliding configuration

Goal: zero configuration at world origin

Generate collision-free motion plan

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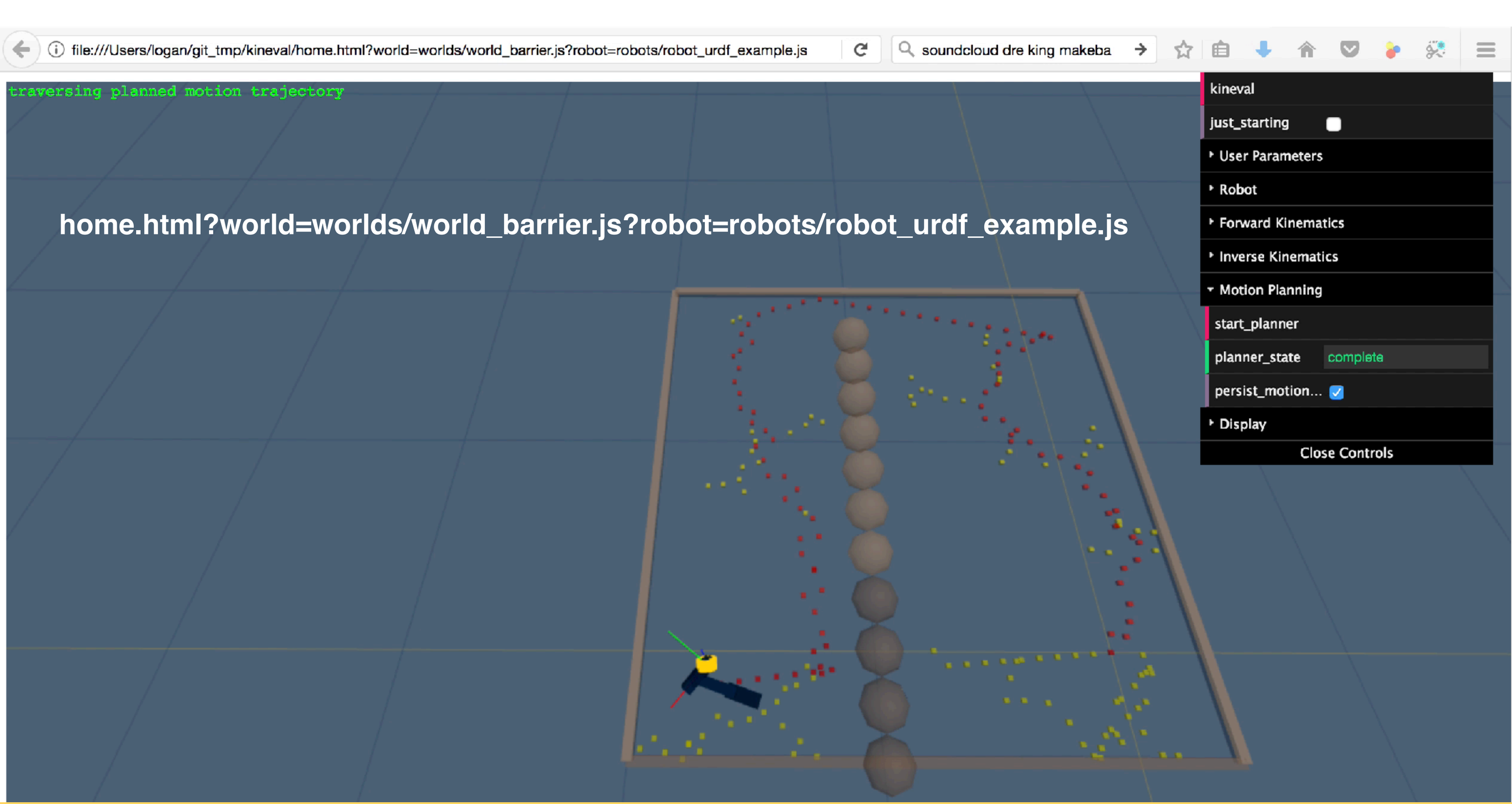

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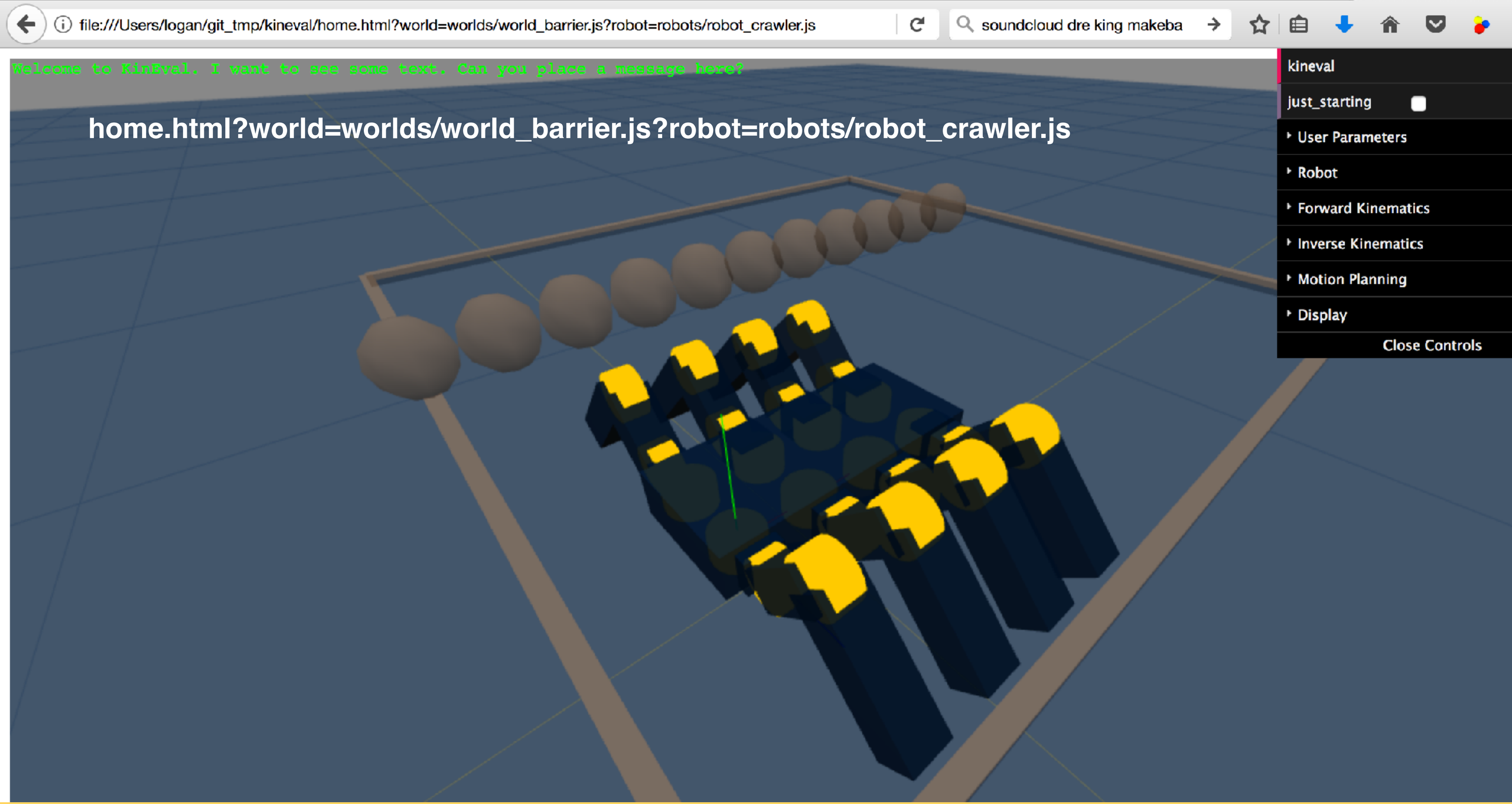

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GitHub, Inc. (US) https://github.com/ohseejay/kineval-stencil-fall16 rrt connect

ohseejay / kineval-stencil-fall16

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Stencil code for KinEval (Kinematic Evaluato

2 commits

Branch: master New pull request

odestcj initial commit

js		
kineval		
project_pathplan		
project_pendularm		
robots		
tutorial_heapsort		
tutorial_js	initial commit	26 days ago
worlds	initial commit	26 days ago
README.md	initial commit	26 days ago

home.html

```
<script src="worlds/world_basic.js"></script>
...

function my_animate() {
    ...
    // detect robot collisions
    kineval.robotIsCollision();
    ...
    // if requested, perform configuration space
motion planning to home pose
    kineval.planMotionRRTConnect();
}
```


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home.html

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<script src="worlds/world_basic.js"></script>
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function my_animate() {
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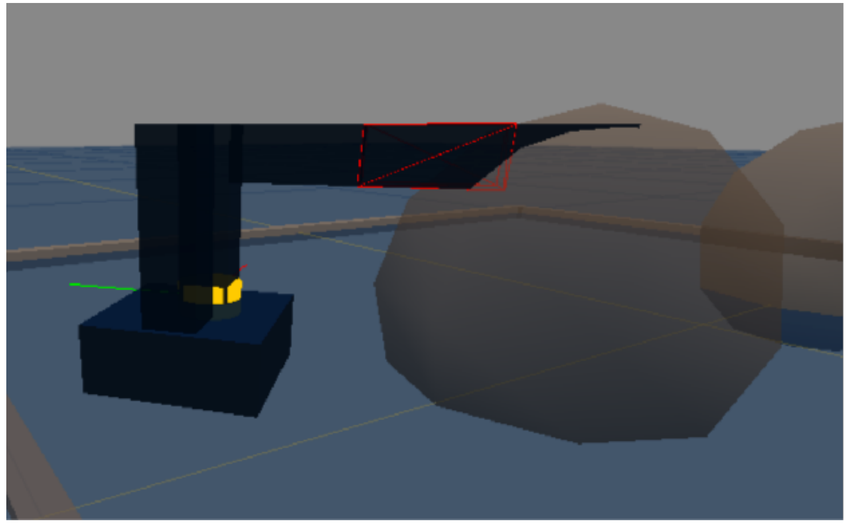
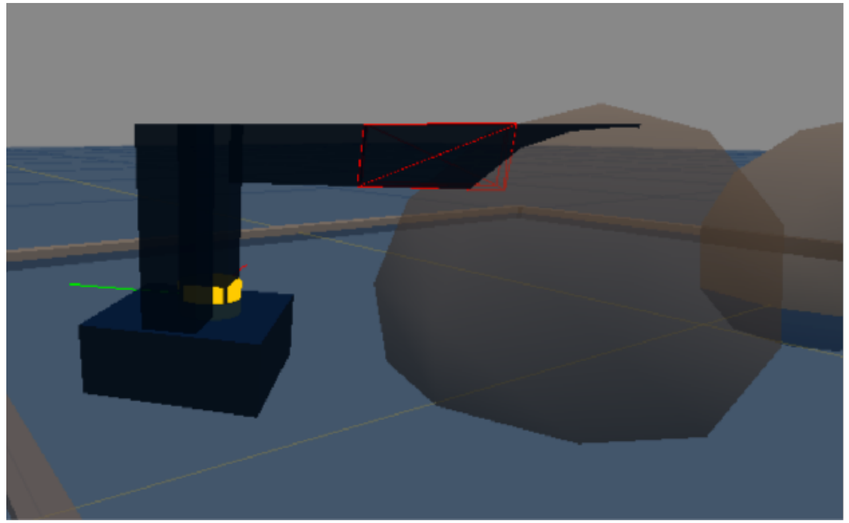


world file can be alternatively loaded by a script tag (avoid doing this)



home.html

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    ...
    // if requested, perform configuration space
motion planning to home pose
    kineval.planMotionRRTConnect();

}
```

detect if current configuration is in collision (colliding link turns red)

iterate motion planner

https://github.com/ohseejay/kineval-stencil/tree/master/kineval

Branch: **master** ▾ **kineval-stencil** / **kineval** / | New file | Upload files | Find file | History

odestcj initial commit — Latest commit 2a1bd6e on Jan 11

..		
kineval.js		
kineval_collision.js		
kineval_controls.js		
kineval_forward_kinematics.js		
kineval_inverse_kinematics.js	initial commit	2 months ago
kineval_matrix.js	initial commit	2 months ago
kineval_quaternion.js	initial commit	2 months ago
kineval_robot_init.js	initial commit	2 months ago
kineval_rosbridge.js	initial commit	2 months ago
kineval_rrt_connect.js	initial commit	2 months ago
kineval_servo_control.js	initial commit	2 months ago
kineval_startingpoint.js	initial commit	2 months ago
kineval_threejs.js	initial commit	2 months ago
kineval_userinput.js	initial commit	2 months ago

`kineval.robotIsCollision();`

Update collision detection with your forward kinematics

`kineval.planMotionRRTConnect();`

Implement RRT-Connect planner

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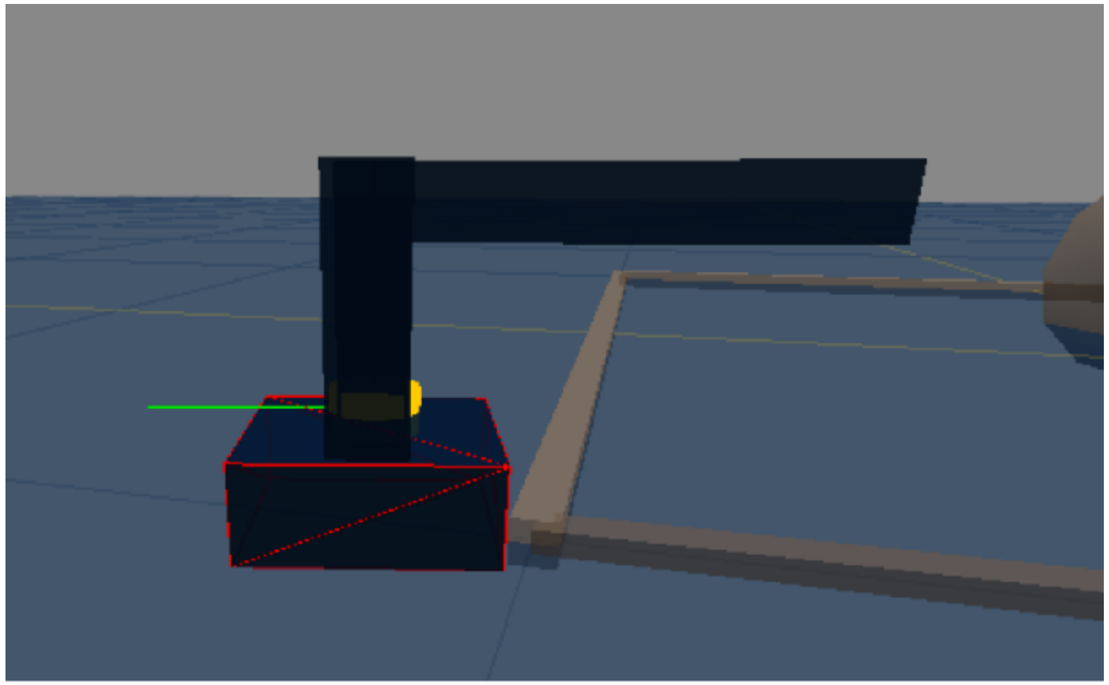
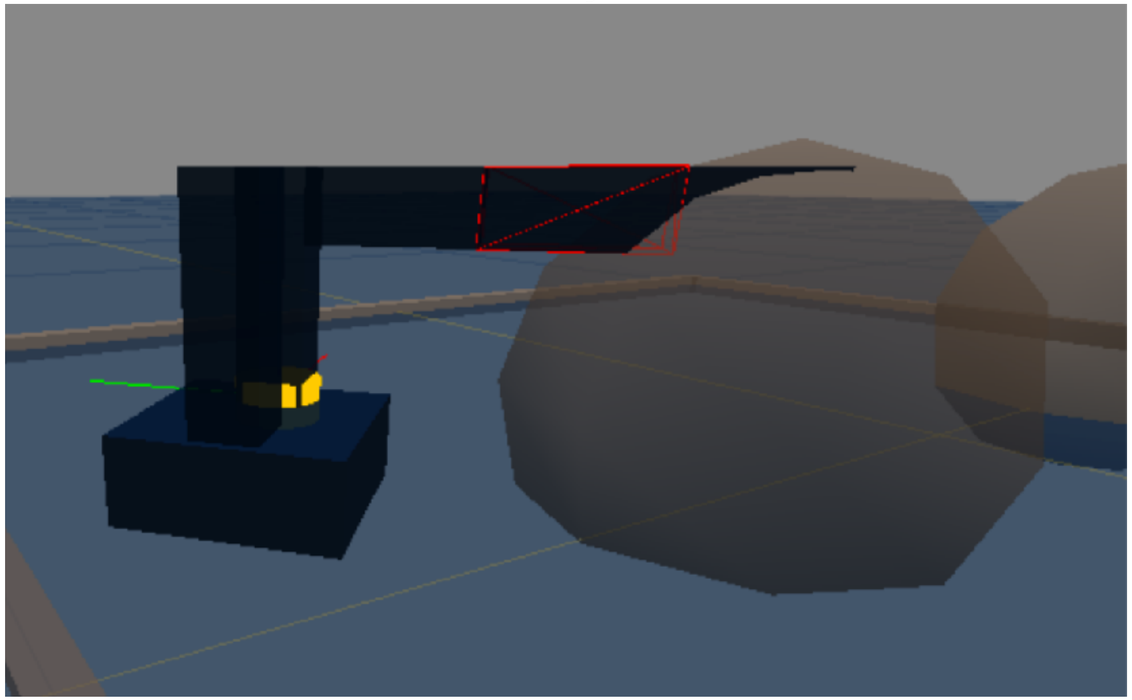
```
kineval.robotIsCollision();
```

Project 4 collision detection

Boundary Collision
(provided by default)



Link Collision
(requires your FK)



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input: q (robot configuration)
output: false (for no collision) or name of link in collision

kineval_collision.js

```
kineval.poseIsCollision = function robot_collision_test(q) {
    // perform collision test of robot geometry against planning world

    // test base origin (not extents) against world boundary extents
    if ((q[0]<robot_boundary[0][0])||(q[0]>robot_boundary[1][0])||
        (q[2]<robot_boundary[0][2])||(q[2]>robot_boundary[1][2]))
        return robot.base;

    // traverse robot kinematics to test each body for collision
    // STENCIL: implement forward kinematics for collision detection
    return robot_collision_forward_kinematics(q);

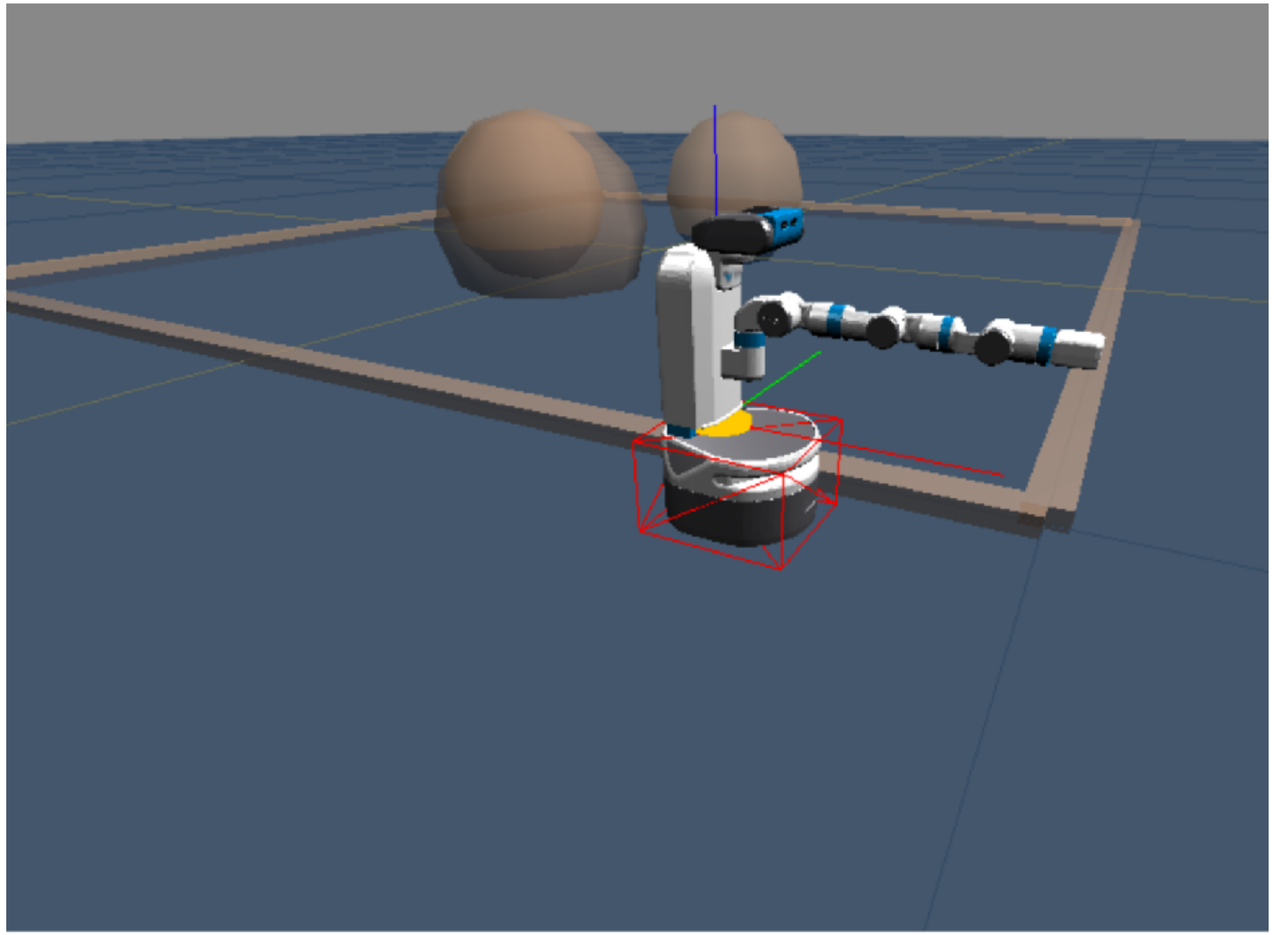
}
```

world boundary detection is provided

Uncomment this call;

Implement this function with FK transforms to test for collisions;

Use provided link collision function to test bounding box of each link



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if ((q[0]<robot_boundary[0][0])||(q[0]>robot_boundary[1][0])||
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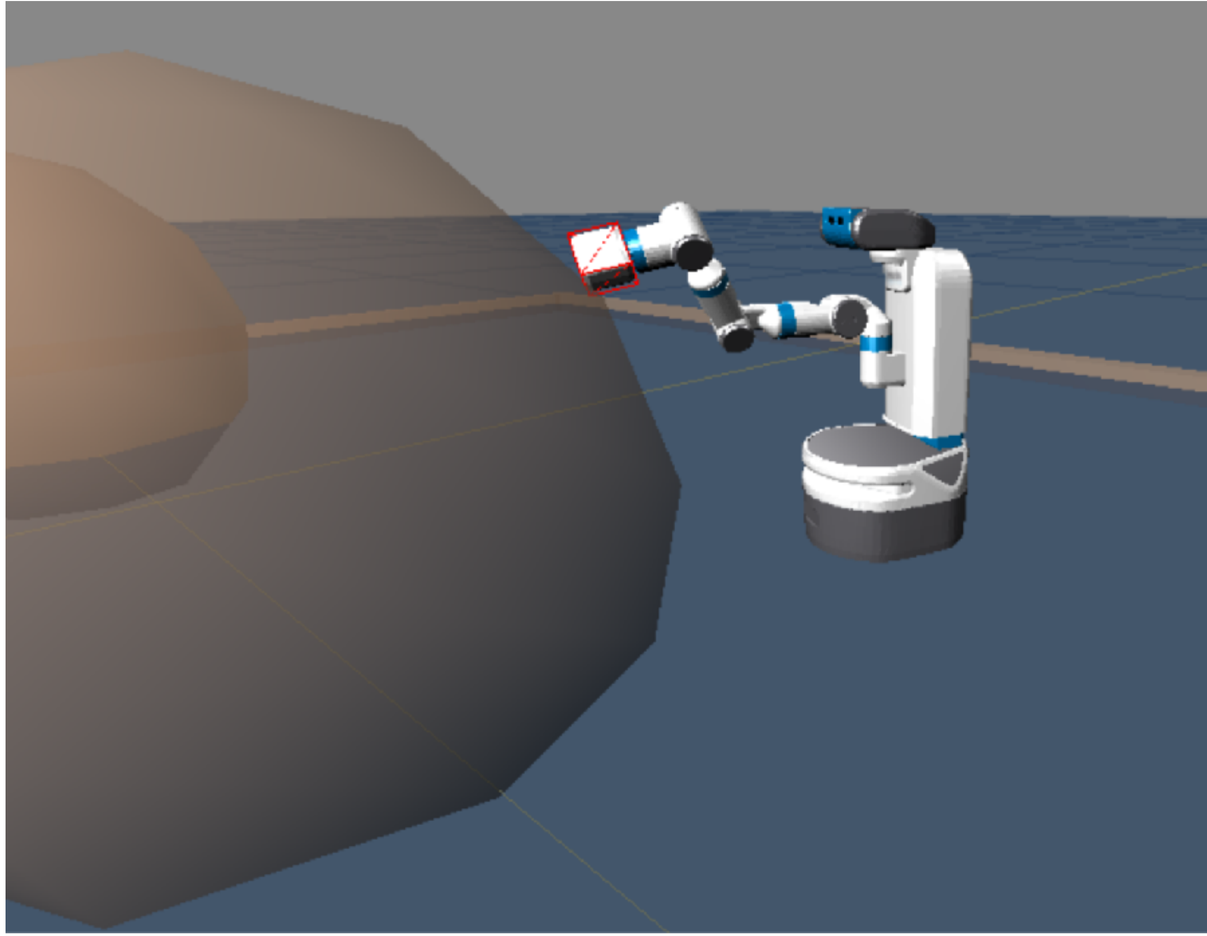

```
// traverse robot kinematics to test each body for collision
// STENCIL: implement forward kinematics for collision detection
return robot_collision_forward_kinematics(q);
```

What item is not real in this picture?

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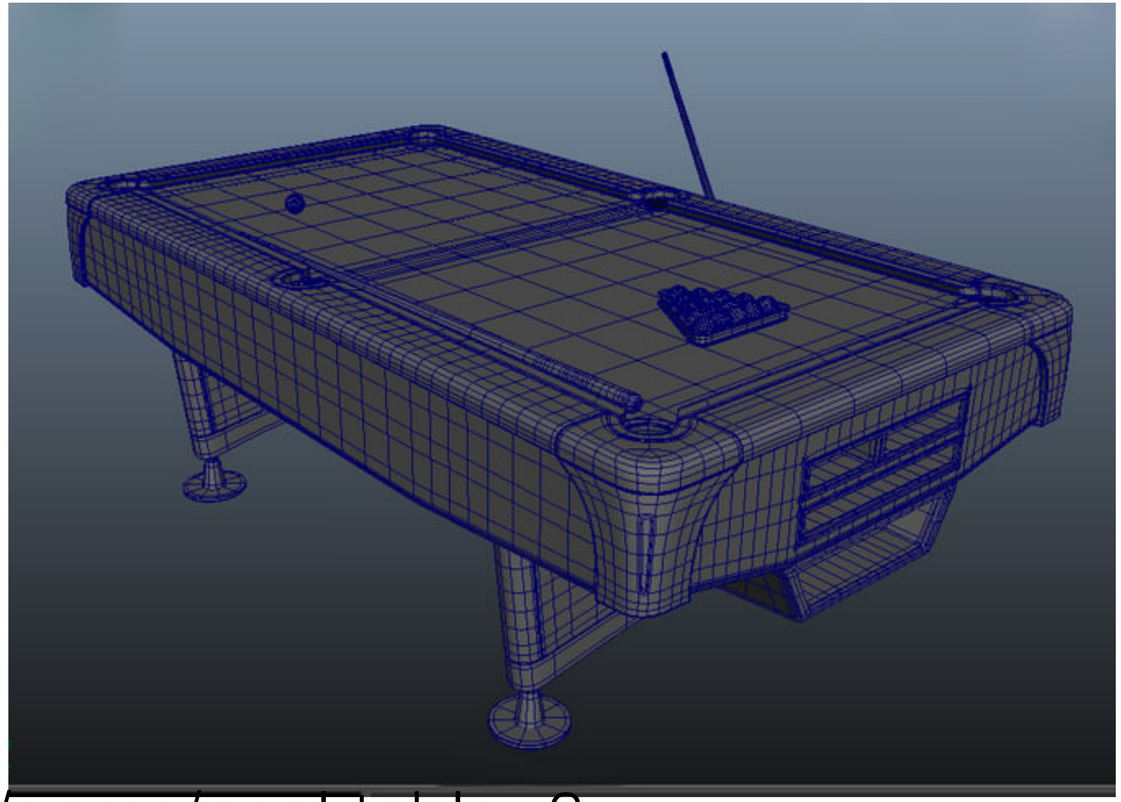
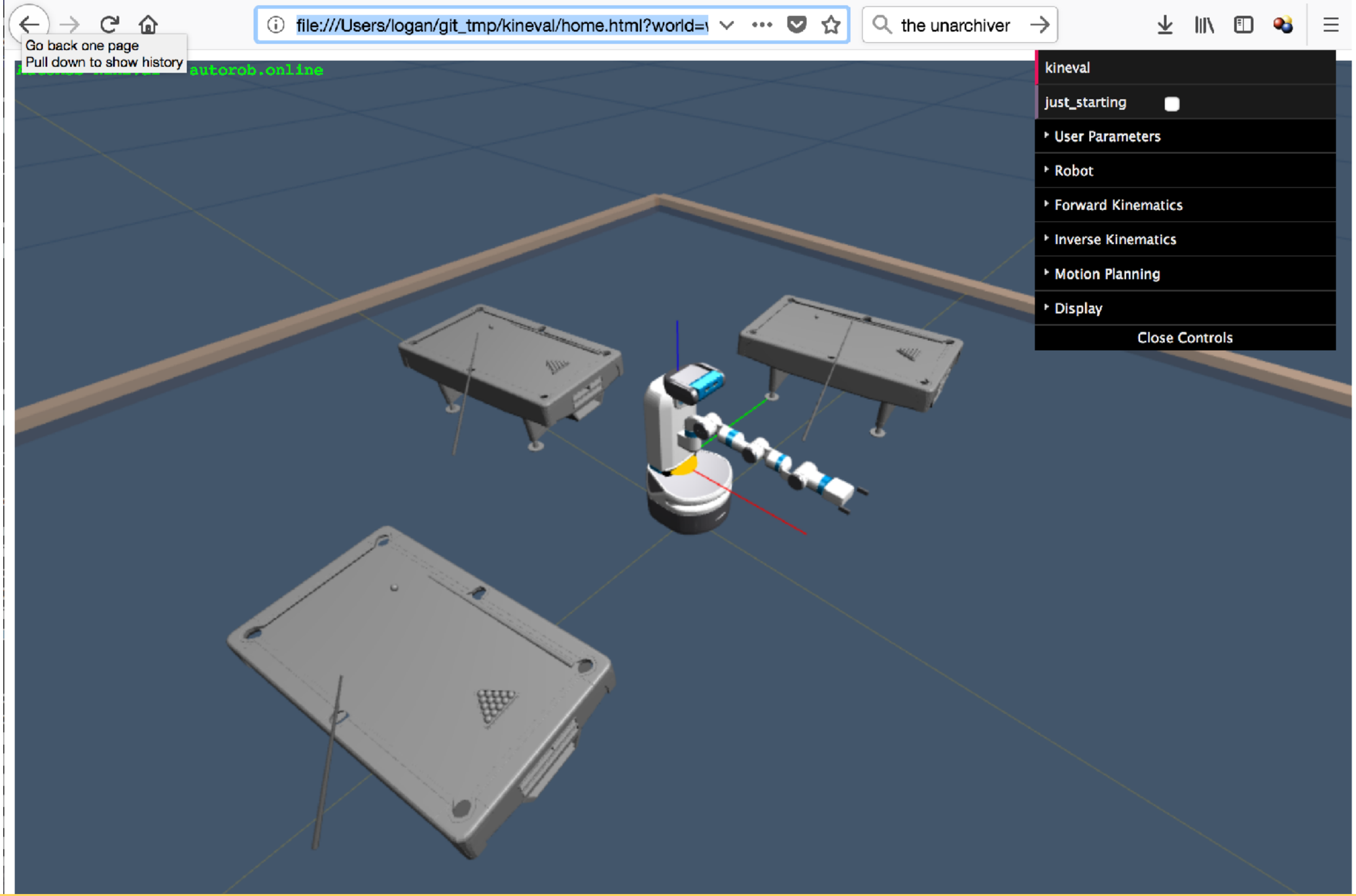
Link geometries are represented as triangular geometric meshes



https://www.cgtrader.com/3d-models/sports/game/pool-table--3



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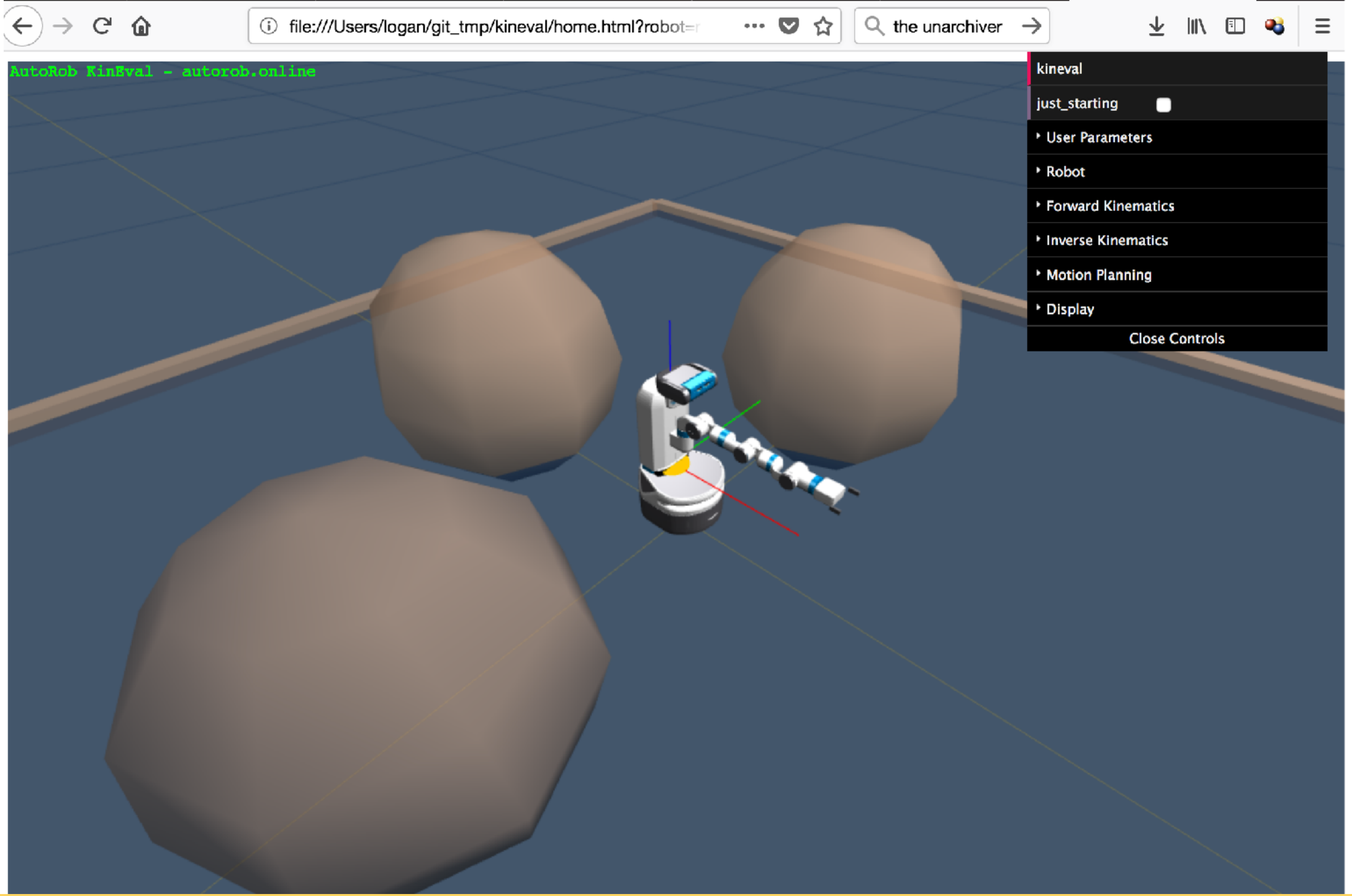

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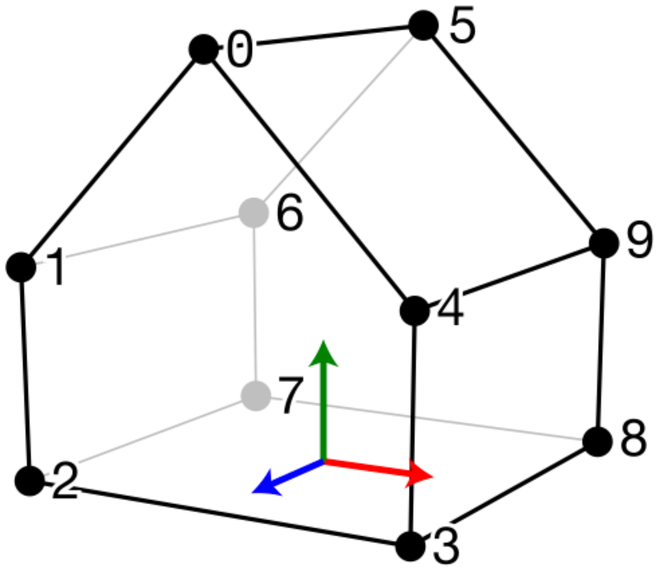
Remember:

Link Geometry



i	x	y	z
0	0.0	1.0	0.5
1	–0.5	0.5	0.5
2	–0.5	0.0	0.5
3	0.5	0.0	0.5
4	0.5	0.5	0.5
5	0.0	1.0	–0.5
6	–0.5	0.5	–0.5
7	–0.5	0.0	–0.5
8	0.5	0.0	–0.5
9	0.5	0.5	–0.5

Each link has a geometry specified as 3D vertices in the frame of the link connected into faces of its surface

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Individual link geometries…

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Individual link geometries…

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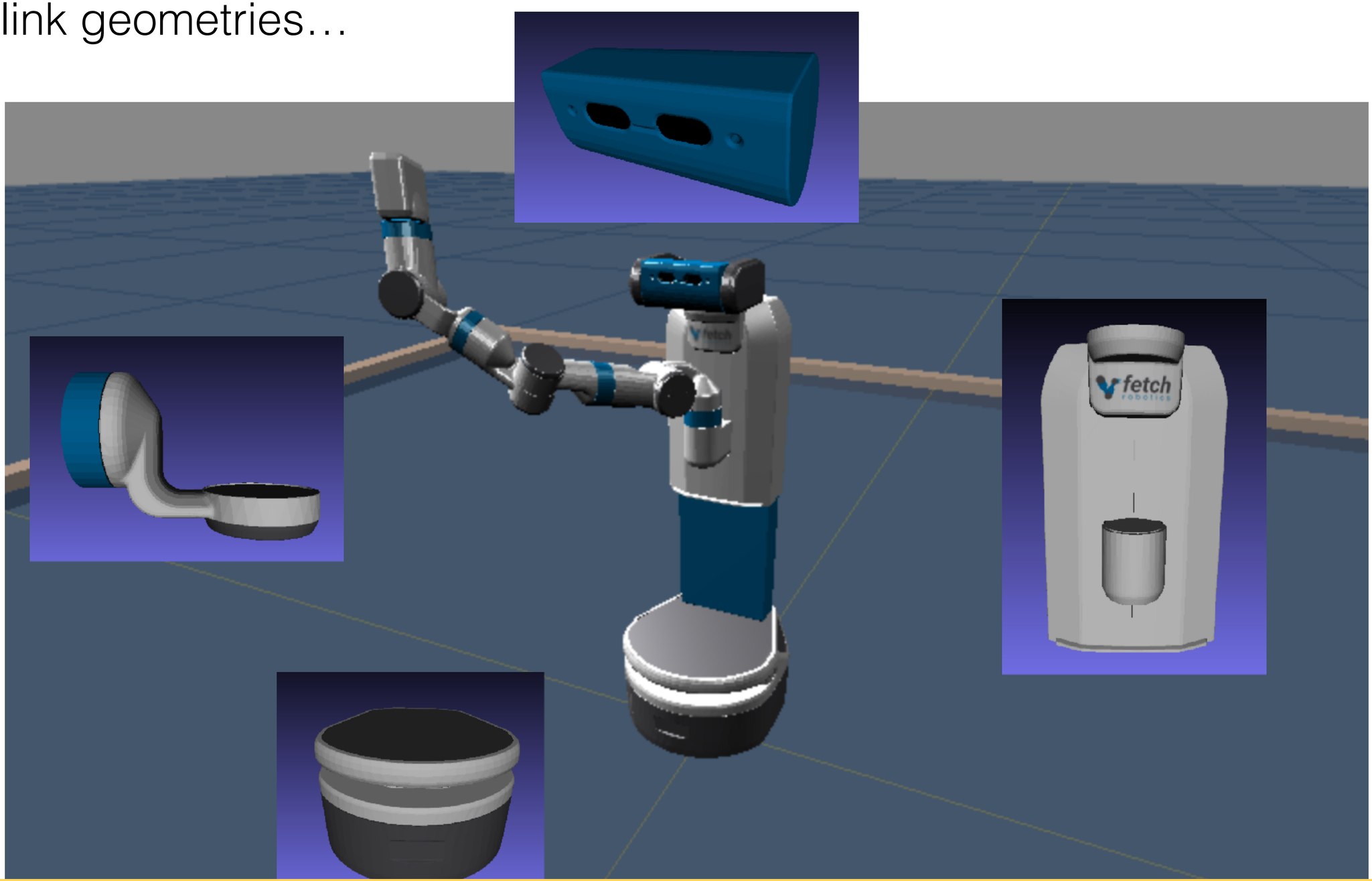
Individual link geometries…



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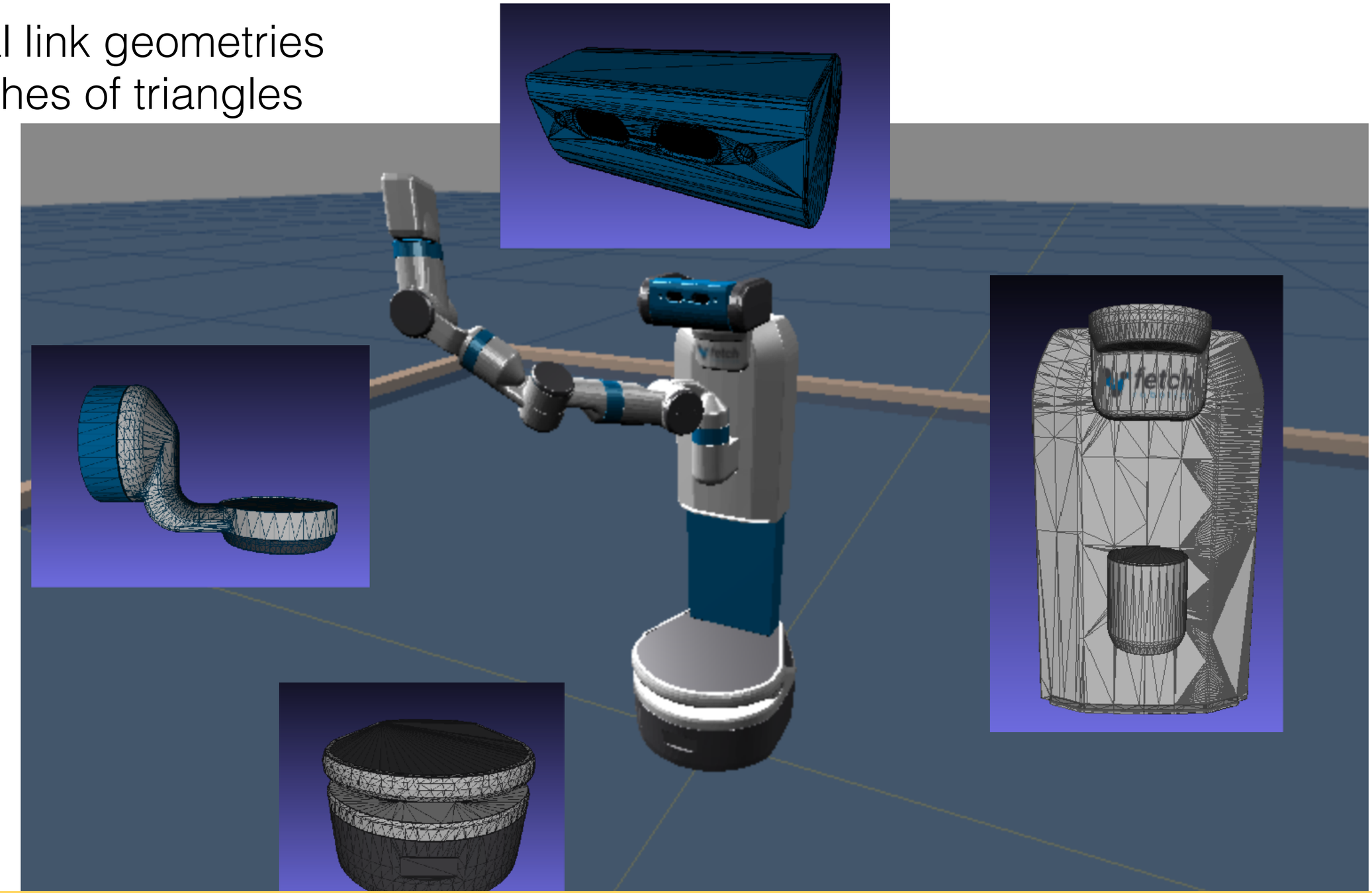
Individual link geometries are meshes of triangles



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GitHub, Inc. (US) https://github.com/fetchrobotics/fetch_ros/tree/indigo-devel/fetch_description/meshes

marketing vs accomplishment

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fetchrobotics / fetch_ros

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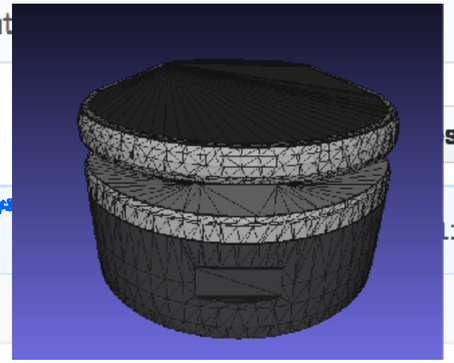

Branch: indigo-devel fetch_ros / fetch_description / meshes / Find file History

mikeferguson update gripper model 119857 on Oct 10, 2015

..

base_link.dae	add fetch description package	3 years ago
base_link_collision.STL	remove laser opening from collision mesh	3 years ago
base_link_uv.png	add fetch description package	3 years ago
bellows_link.STL	add fetch description package	3 years ago
bellows_link_collision.STL	add fetch description package	3 years ago
elbow_flex_link.dae	add fetch description package	3 years ago
elbow_flex_link_collision.STL	add fetch description package	3 years ago
elbow_flex_uv.png	add fetch description package	3 years ago
estop_link.STL	add fetch description package	3 years ago
forearm_roll_link.dae	add fetch description package	3 years ago

GitHub, Inc. (US) https://github.com/fetchrobotics/fetch_ros/tree/indigo-devel/fetch_description/meshes

marketing vs accomplishment

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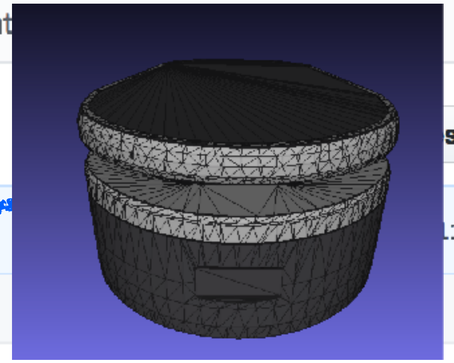

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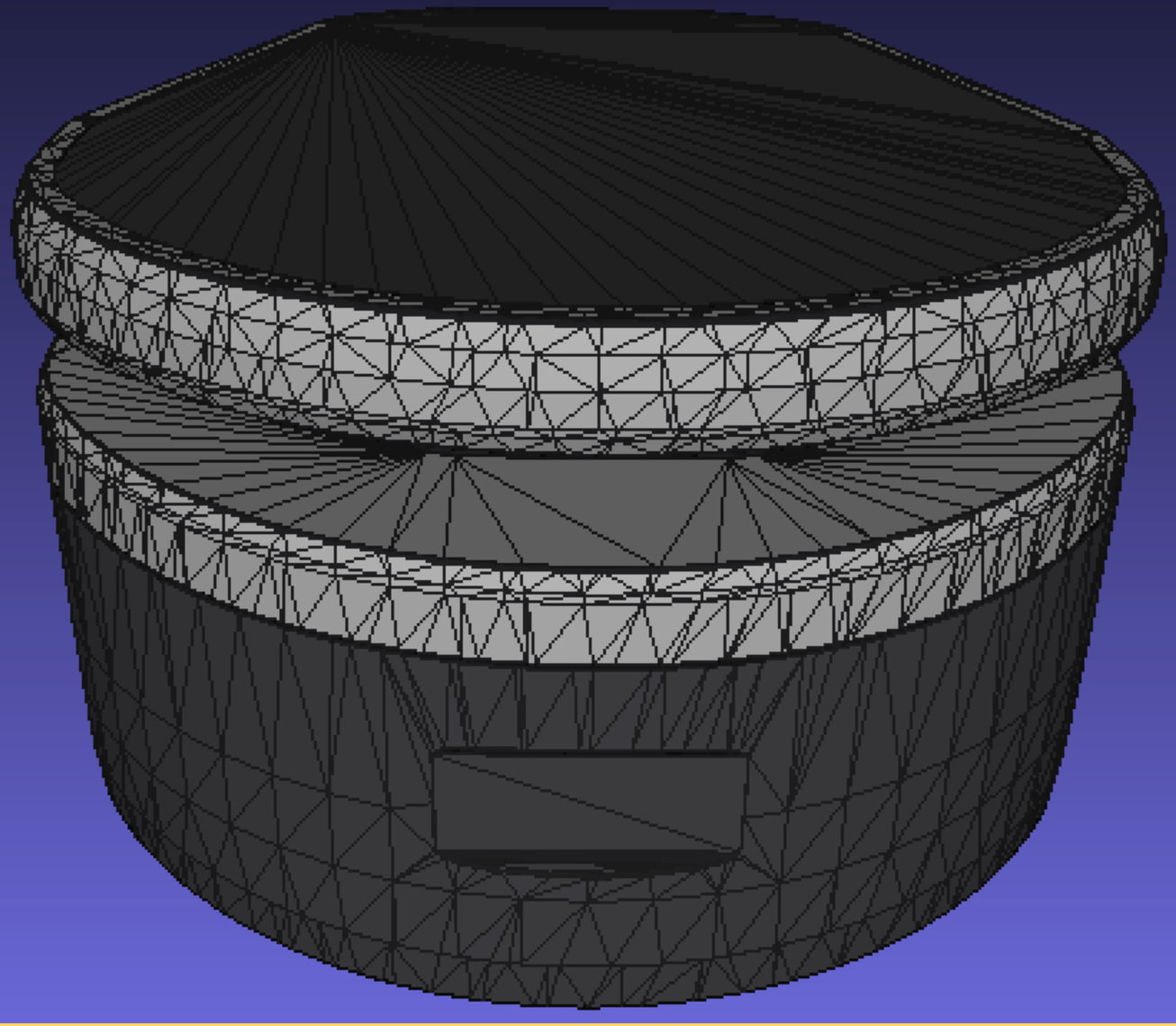
base_li		
bellows		
bellows		
elbow_		
elbow_		
elbow_		
estop_l		
forearm		

COLLADA

COLLADA (***COLLA**borative **D**esign **A**ctivity*) is an interchange file format for interactive 3D applications. It is managed by the nonprofit technology consortium, the Khronos Group, and has been adopted by ISO as a publicly available specification, ISO/PAS 17506.[1]

COLLADA defines an open standard XML schema for exchanging digital assets among various graphics software applications that might otherwise store their assets in incompatible file formats. COLLADA documents that describe digital assets are XML files, usually identified with a **.dae** (digital asset exchange) filename extension.

Vertices: robot.links[robot.base].geom.children[1].children[0].geometry.vertices
Faces: robot.links[robot.base].geom.children[1].children[0].geometry.faces



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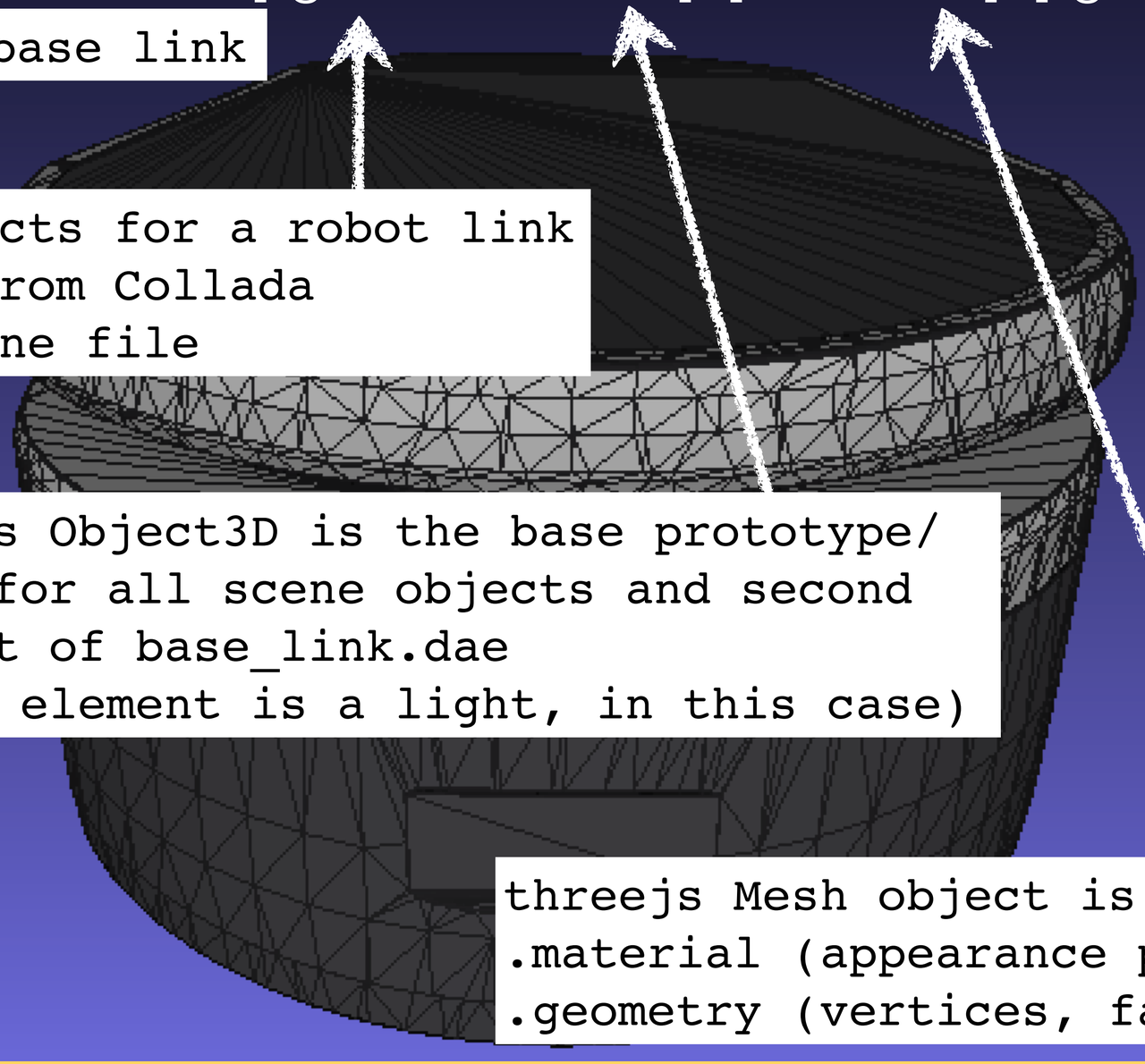

KinEval robot base link

.geom: threejs objects for a robot link (or joint) loaded from Collada (base_link.dae) scene file

threejs Object3D is the base prototype/ class for all scene objects and second element of base_link.dae (first element is a light, in this case)

threejs Mesh object is consists of:
.material (appearance properties)
.geometry (vertices, faces, normals)

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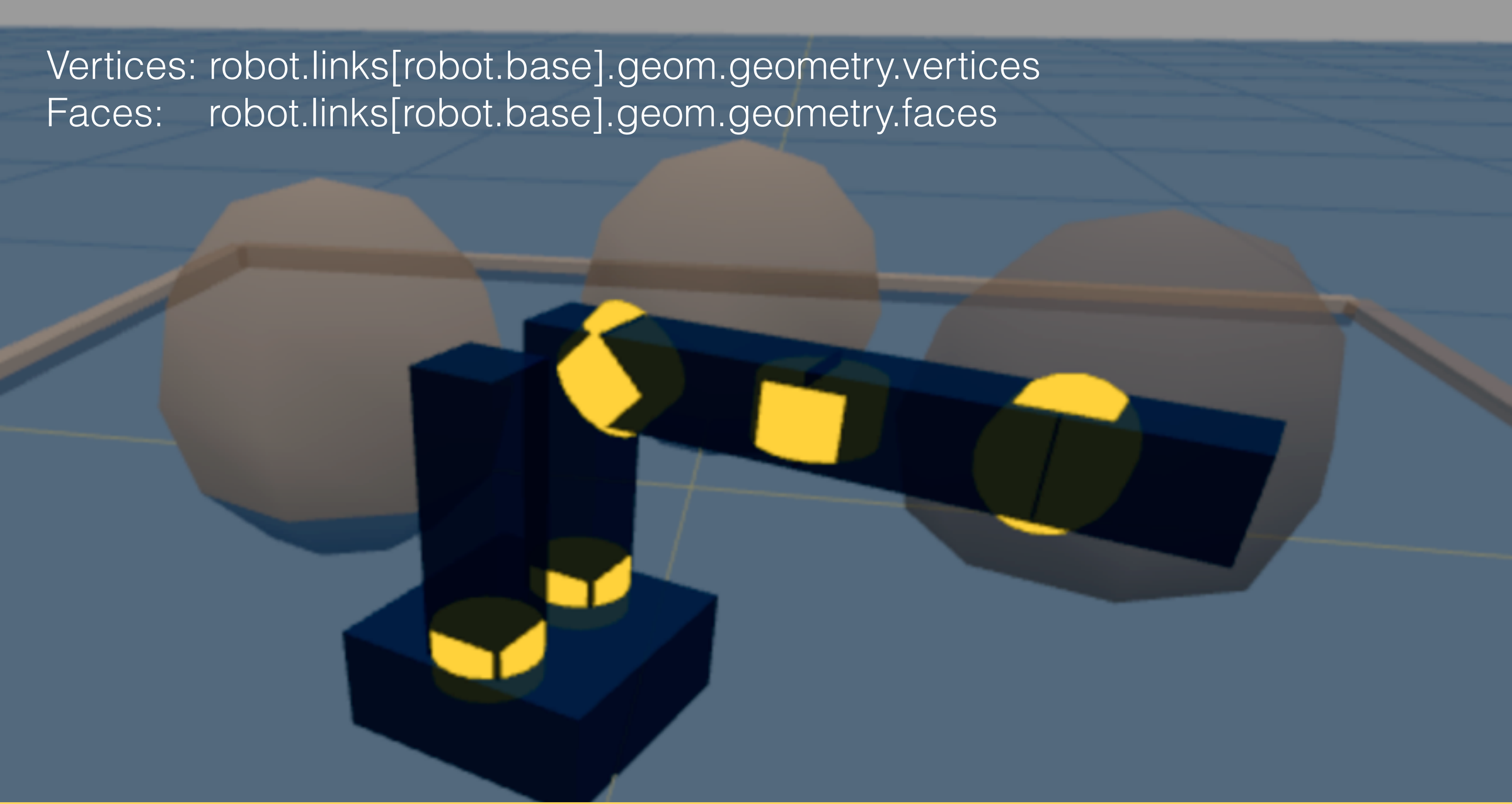
Vertices: robot.links[robot.base].geom.geometry.vertices
Faces: robot.links[robot.base].geom.geometry.faces



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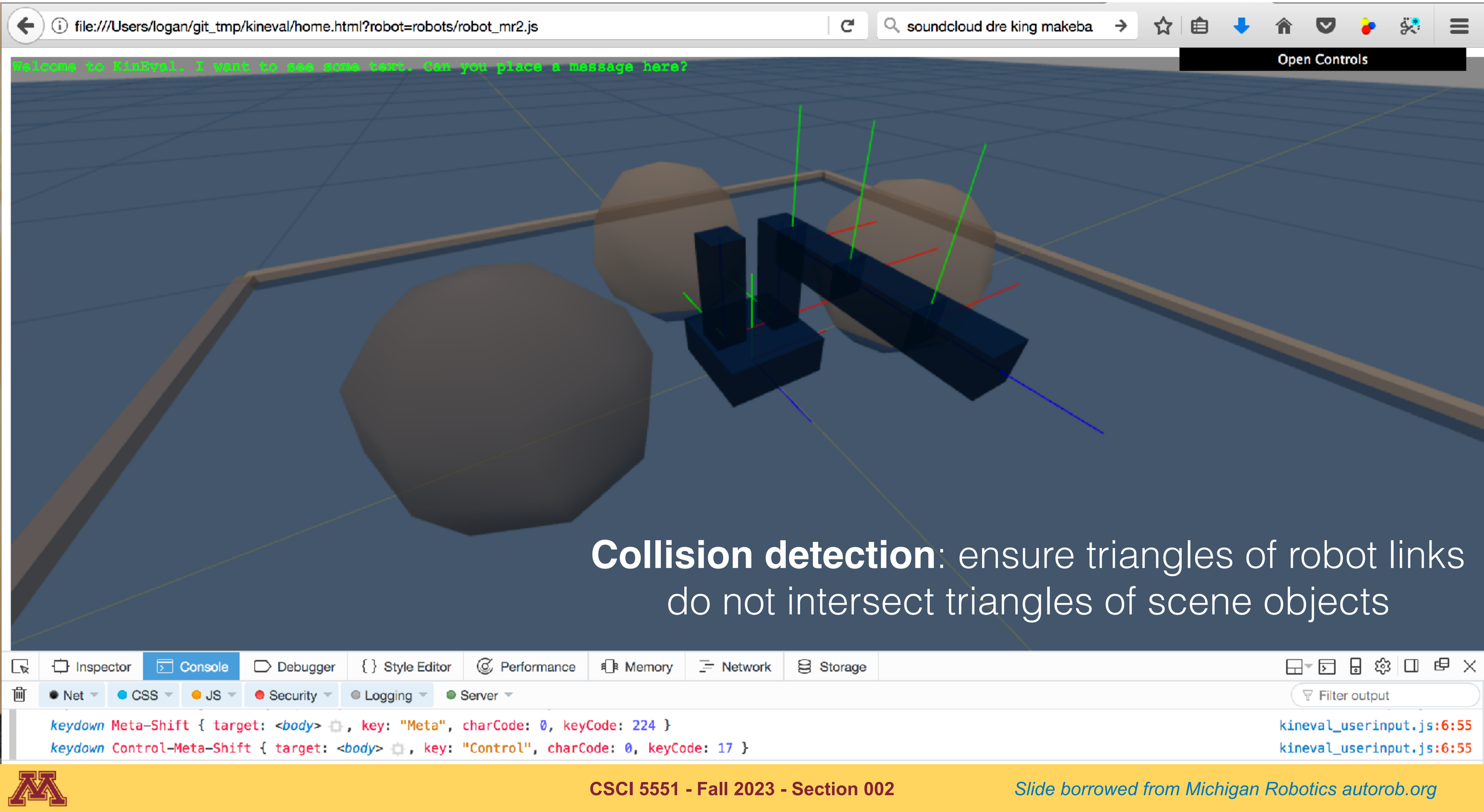
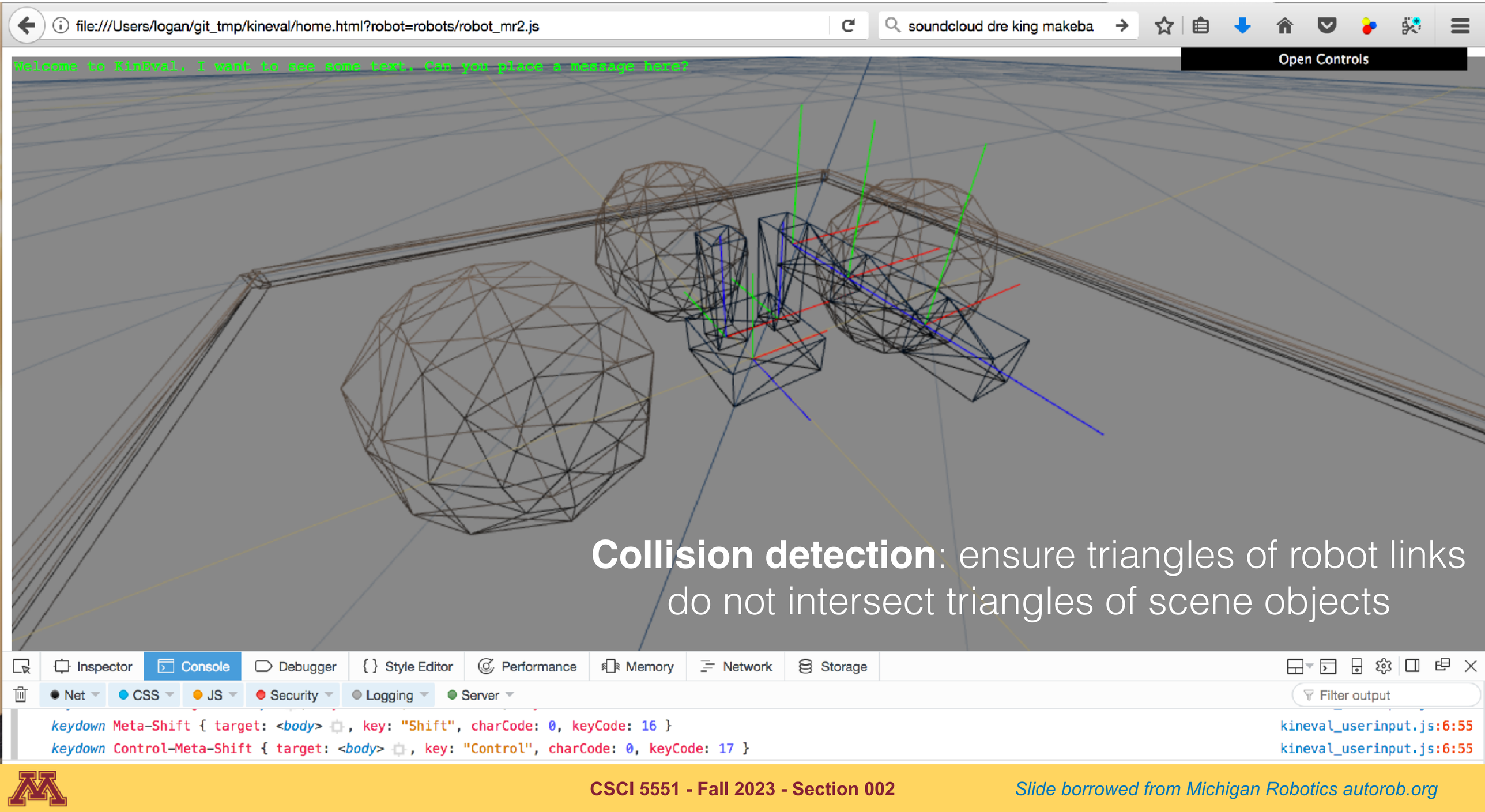

file:///Users/logan/git_tmp/kineval/home.html?robot=robots/robot_mr2.js
soundcloud dre king makeba
Welcome to KinEval. I want to see some text. Can you place a message here?
Open Controls
Collision detection: ensure triangles of robot links do not intersect triangles of scene objects
Inspector
Console
Debugger
Style Editor
Performance
Memory
Network
Storage
Net
CSS
JS
Security
Logging
Server
Filter output
keydown Meta-Shift { target: <body> , key: "Meta", charCode: 0, keyCode: 224 }
kineval_userinput.js:6:55
keydown Control-Meta-Shift { target: <body> , key: "Control", charCode: 0, keyCode: 17 }
kineval_userinput.js:6:55



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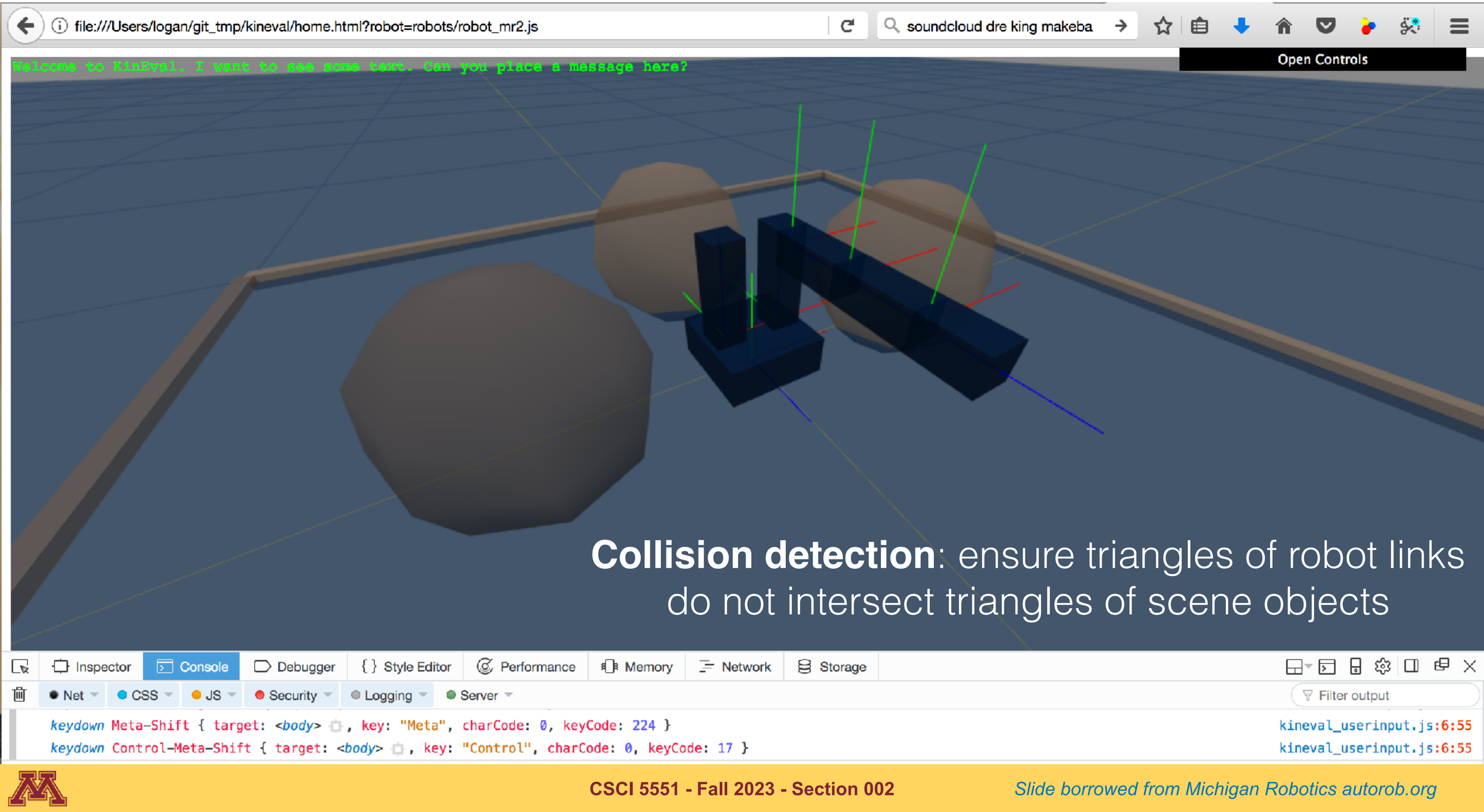
file:///Users/logan/git_tmp/kineval/home.html?robot=robots/robot_mr2.js
soundcloud dre king makeba
Welcome to KinEval. I want to see some text. Can you place a message here?
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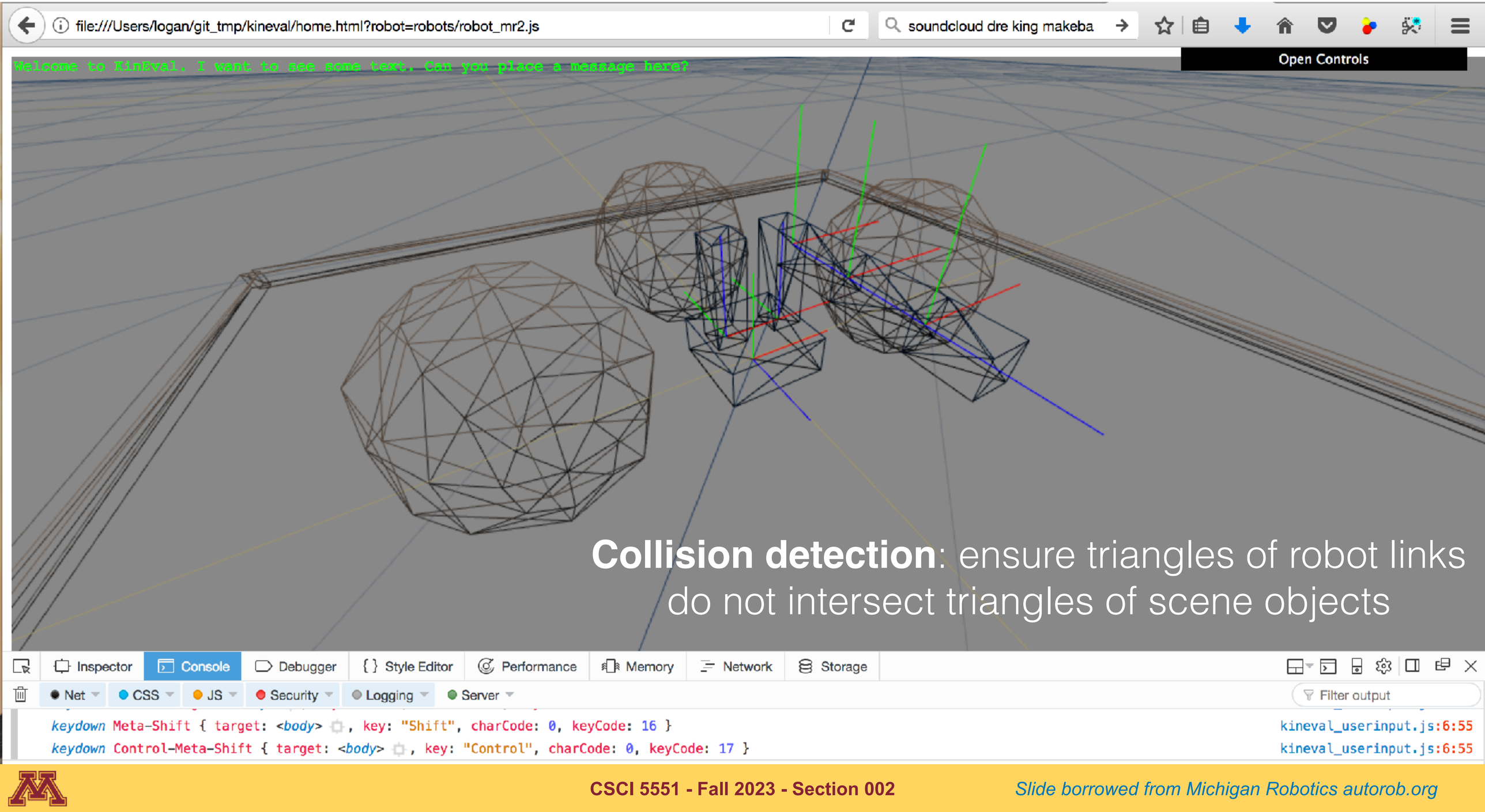
Inspector
Console
Debugger
Style Editor
Performance
Memory
Network
Storage
Net
CSS
JS
Security
Logging
Server
Filter output
keydown Meta-Shift { target: <body> , key: "Meta", charCode: 0, keyCode: 224 }
kineval_userinput.js:6:55
keydown Control-Meta-Shift { target: <body> , key: "Control", charCode: 0, keyCode: 17 }
kineval_userinput.js:6:55



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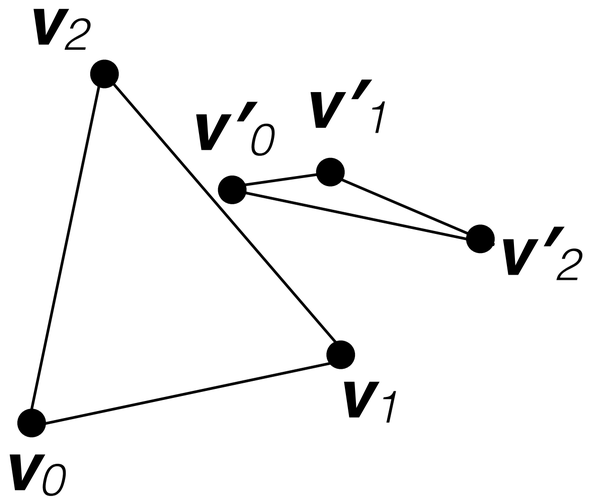
How do we test whether two triangles intersect?

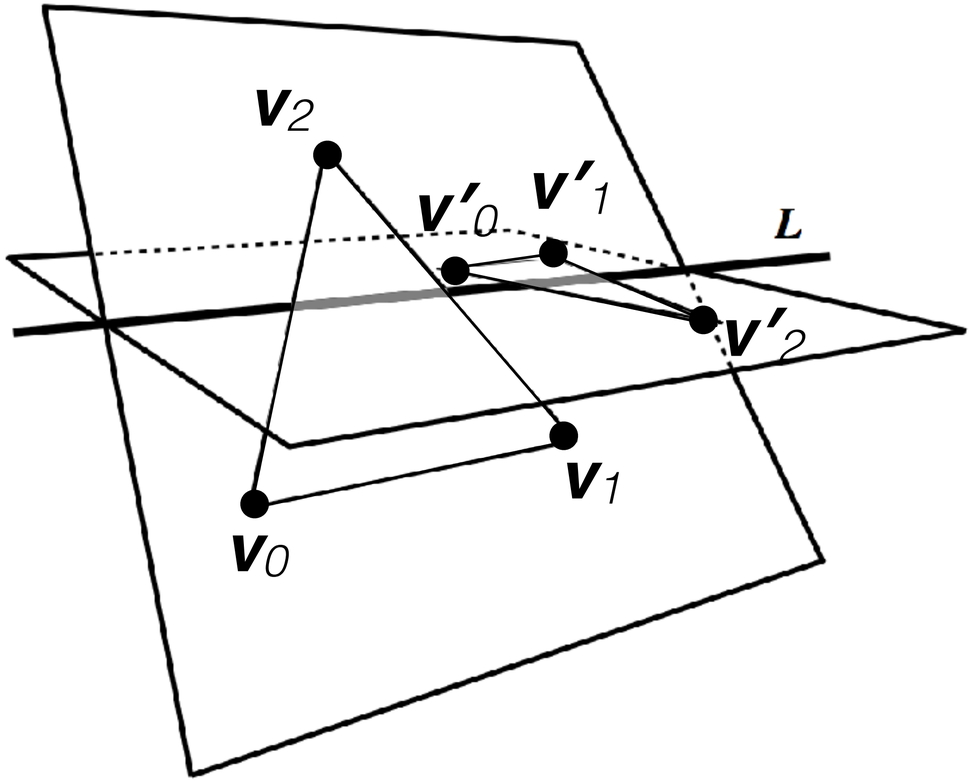
3D Triangle-Triangle Test

- Given two triangles each with three vertices
  - $T = \{\boldsymbol{v}_0, \boldsymbol{v}_1, \boldsymbol{v}_2\}$
  - $T' = \{\boldsymbol{v}'_0, \boldsymbol{v}'_1, \boldsymbol{v}'_2\}$
- Return true if T and T' intersect



3D Triangle-Triangle Test

1. Compute plane equation of triangle 2.
2. Reject as trivial if all points of triangle 1 are on same side.
3. Compute plane equation of triangle 1.
4. Reject as trivial if all points of triangle 2 are on same side.
5. Compute intersection line and project onto largest axis.
6. Compute the intervals for each triangle.
7. Intersect the intervals.



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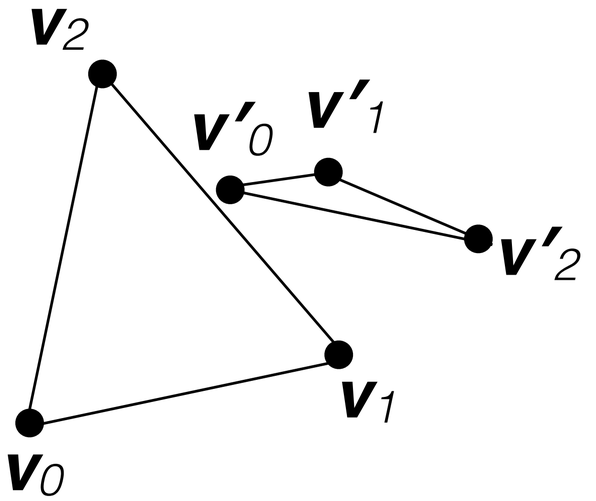

3D Triangle-Triangle Test

1. Compute plane equation of triangle 2.
2. Reject as trivial if all points of triangle 1 are on same side.
3. Compute plane equation of triangle 1.
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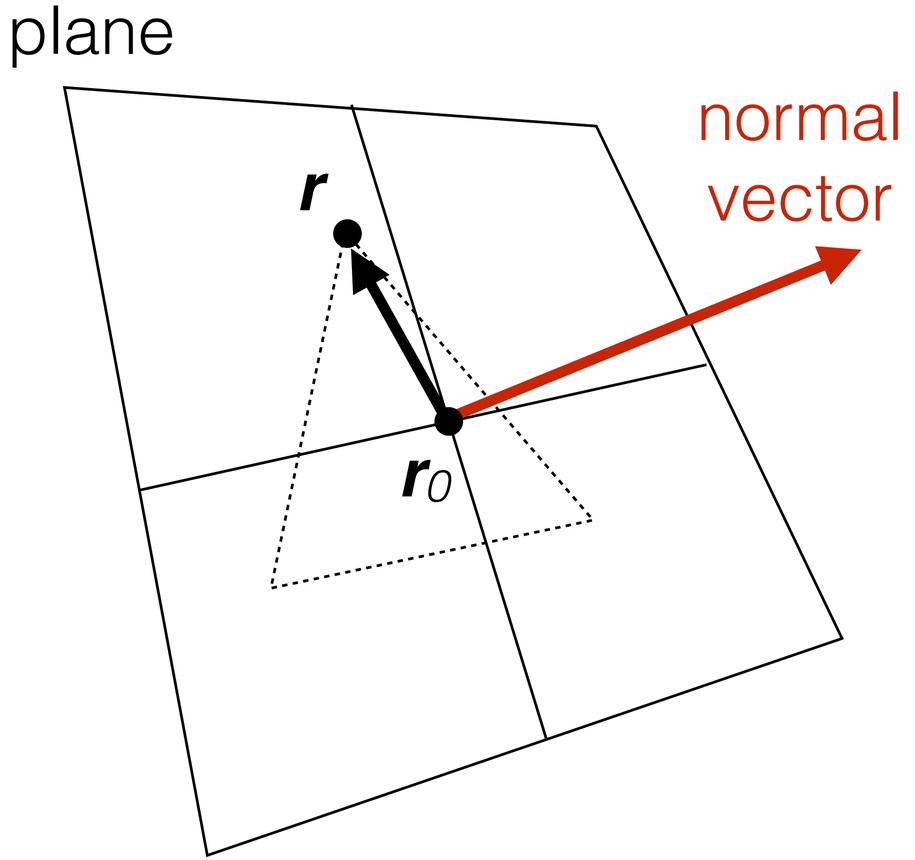

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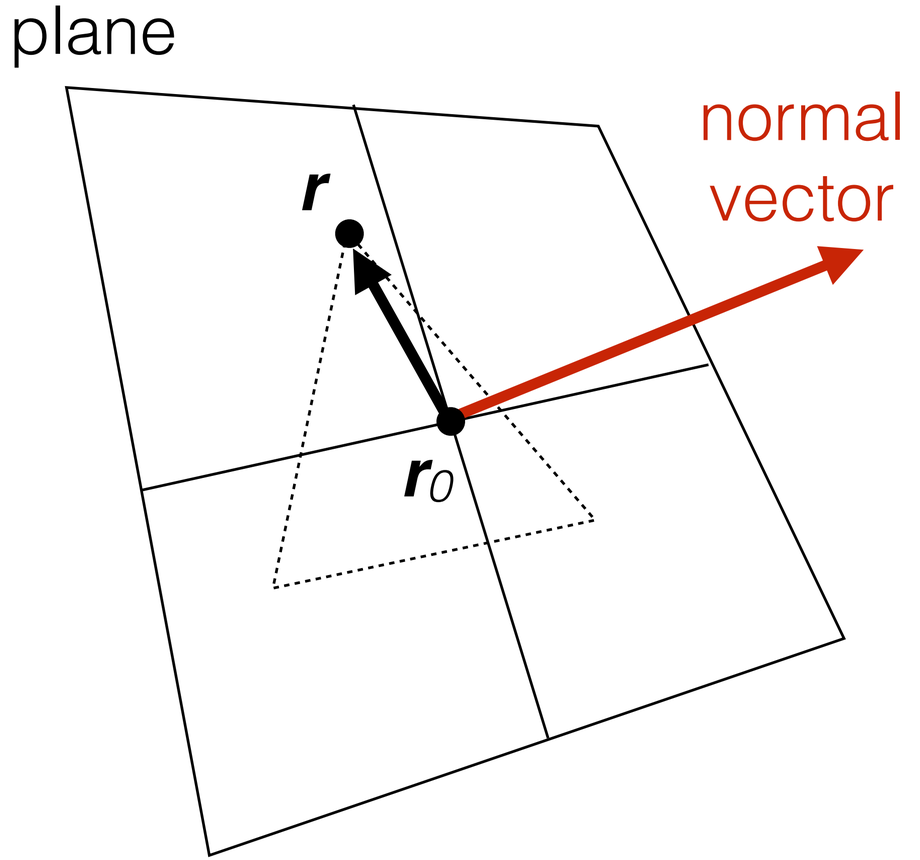
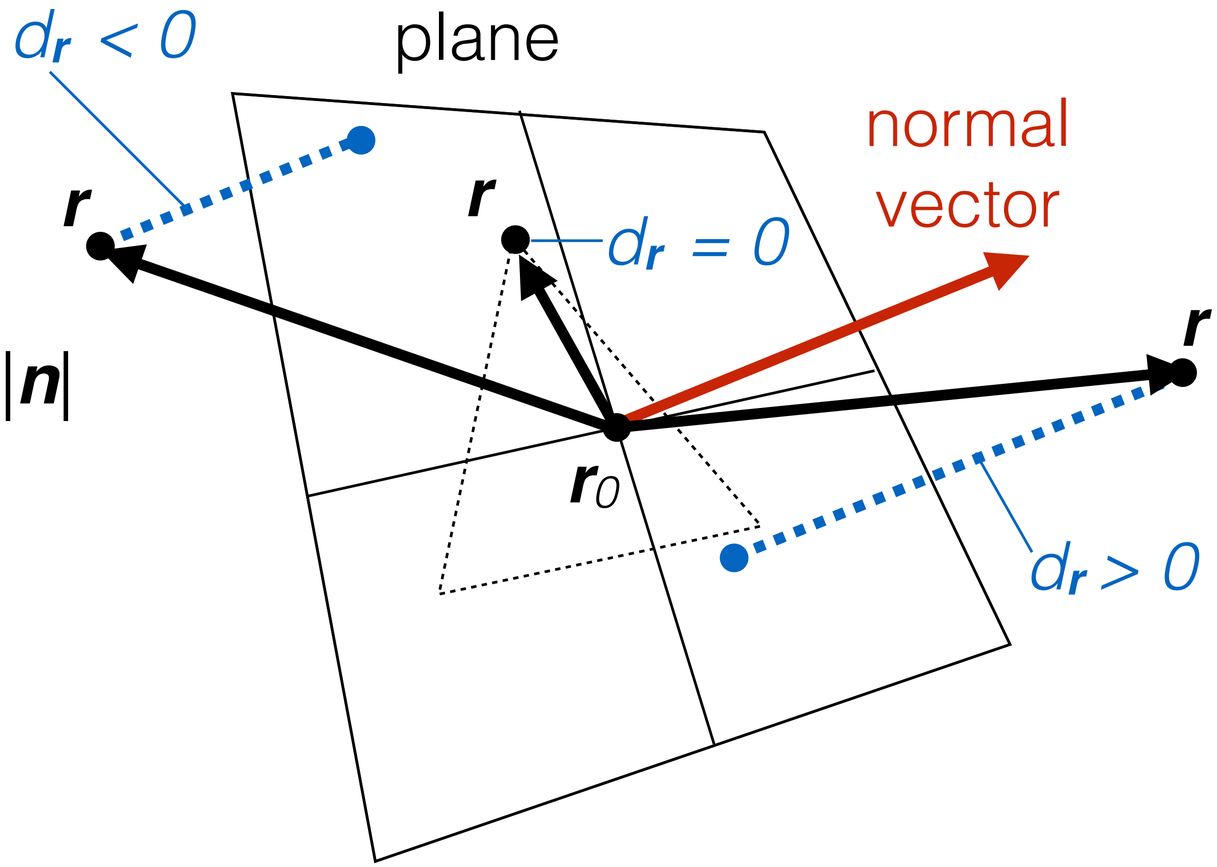
3D Plane Definition

- General form
  - $ax + by + cz + d = 0$
  - $d = -(ax_0 + by_0 + cz_0)$
- Point-normal form (equivalently)
  - $\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) = 0$
- Normal vector $\boldsymbol{n} = [a,b,c]$ orthogonal to plane rooted at location $\boldsymbol{r}_0 = [x_0,y_0,z_0]$
- Any point $\boldsymbol{r} = [x,y,z]$ lying within this plane will evaluate to zero



3D Plane Definition

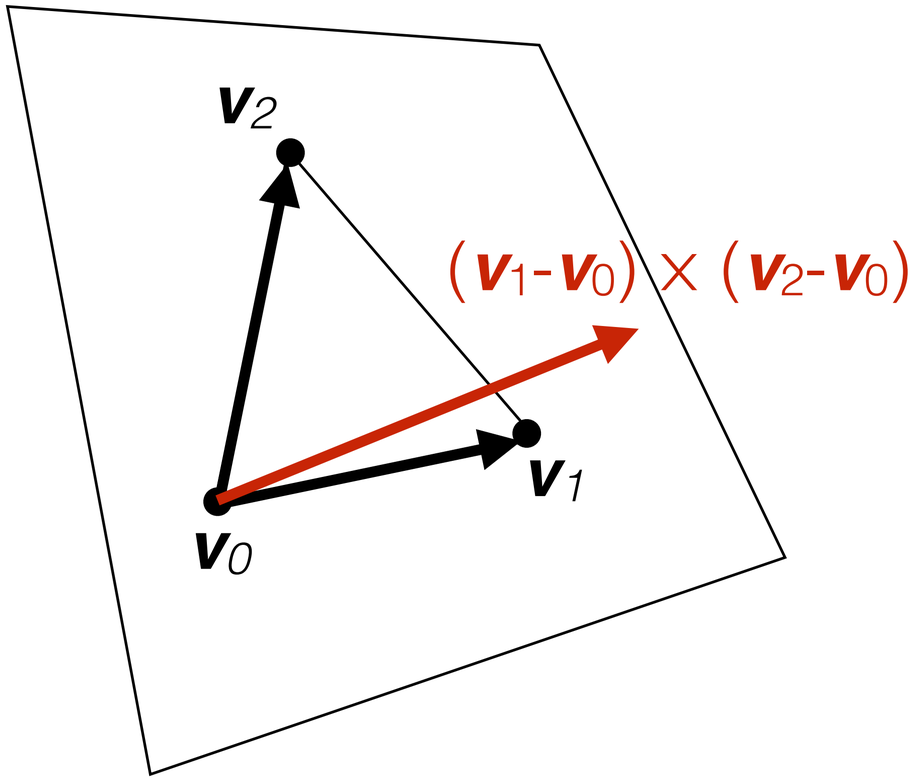
- Scalar projection with normal gives signed distance of point from plane
  - $d_{\boldsymbol{r}} = (\boldsymbol{r} - \boldsymbol{r}_0) \cdot \boldsymbol{n} \,/\, |\boldsymbol{n}| = (\boldsymbol{n} \cdot \boldsymbol{r} - d) \,/\, |\boldsymbol{n}|$
- Any point $\boldsymbol{r} = [x,y,z]$ lying within this plane will have distance $d_{\boldsymbol{r}} = 0$
- Any point below plane: $d_{\boldsymbol{r}} < 0$
- Any point above plane: $d_{\boldsymbol{r}} > 0$



3D Plane Definition

$ax + by + cz + d = 0$

- Plane coefficients can be computed from points of triangle
  - $\boldsymbol{n} = [a,b,c] = (\boldsymbol{v}_1 - \boldsymbol{v}_0) \times (\boldsymbol{v}_2 - \boldsymbol{v}_0)$
  - $d = - \boldsymbol{v}_2 \cdot \boldsymbol{n}$



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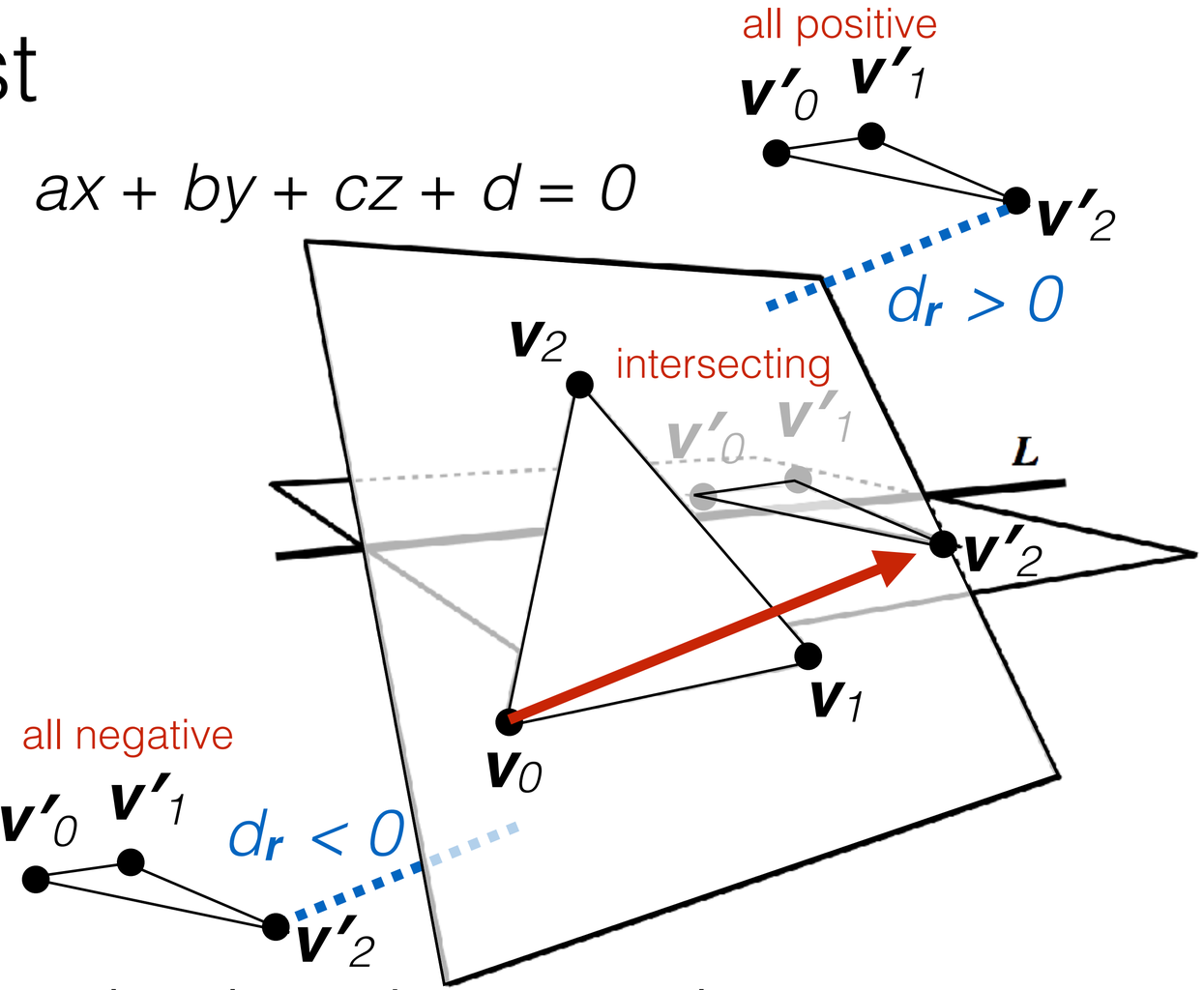

Input points into plane equation.

If all have the same sign, planes of triangles do not intersect

Möller 1997

3D Triangle-Triangle Test

$$a'x + b'y + c'z + d' = 0$$

1. Compute plane equation of triangle 2.
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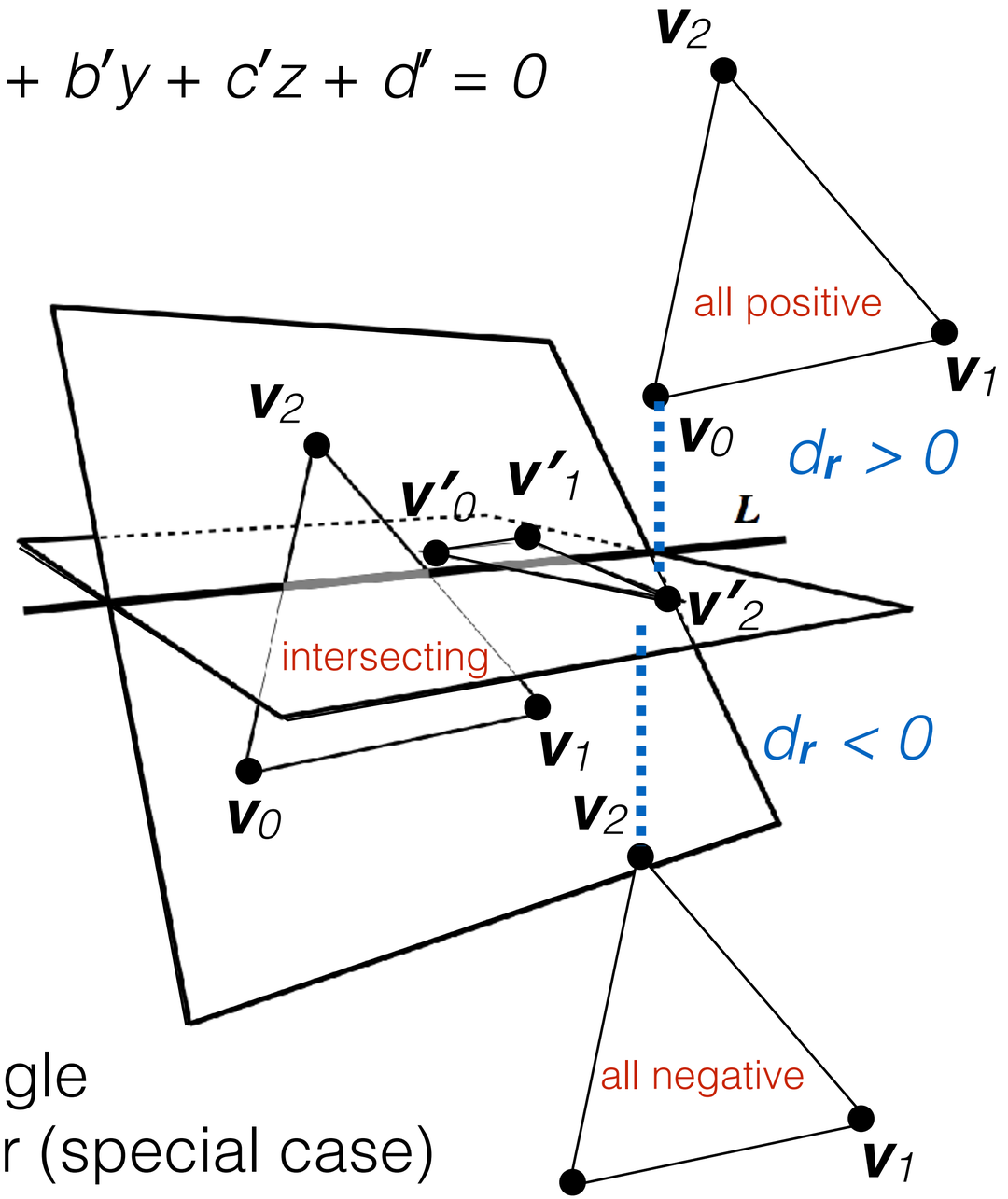


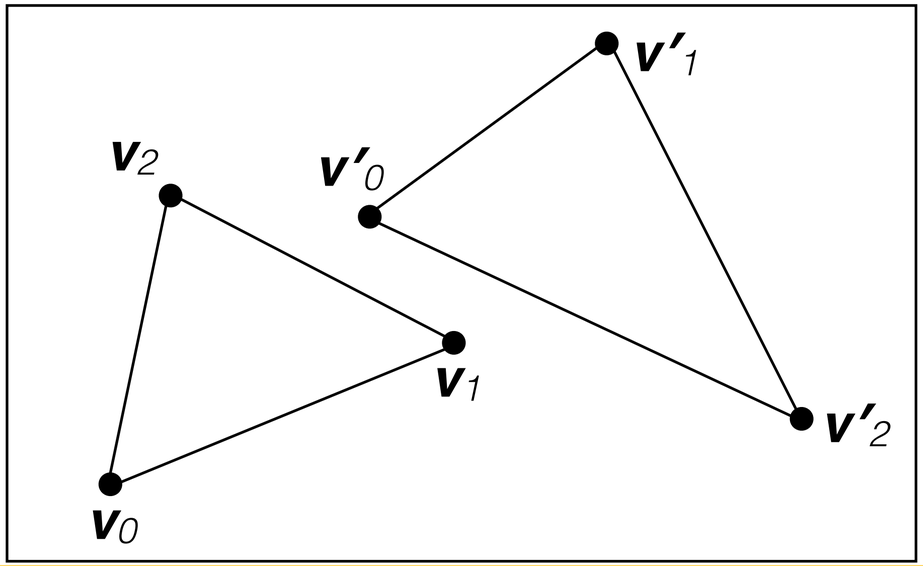
Repeat for other plane of other triangle

If all evaluations are zero, triangles are co-planar (special case)

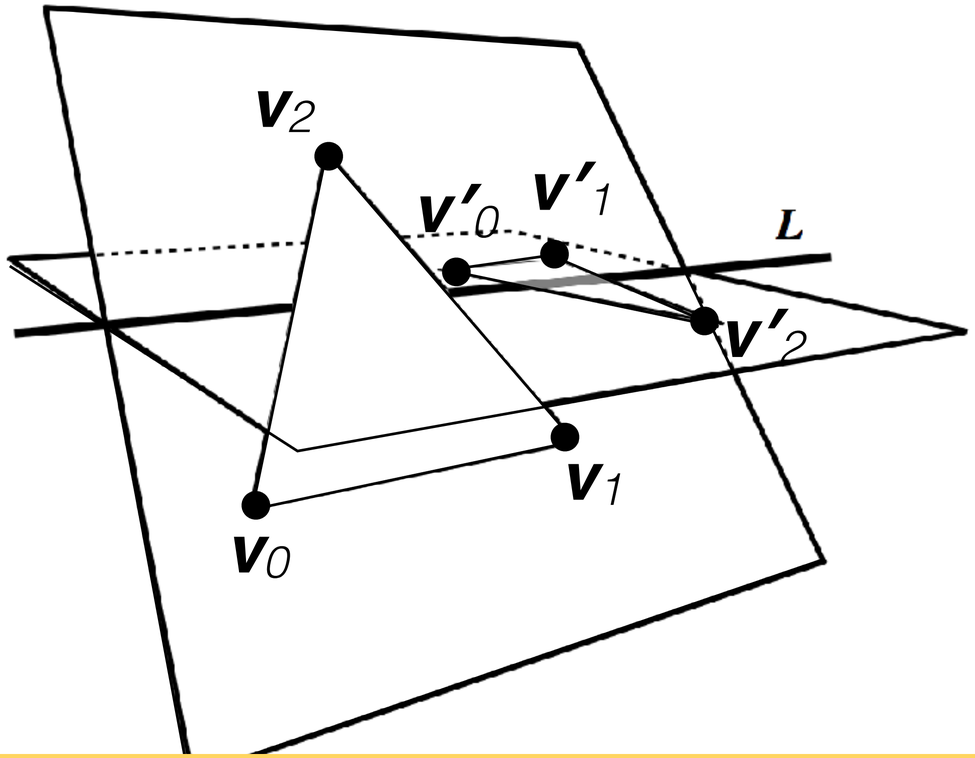
Three possible cases can occur based on
evaluation of vertices of one triangle against the plane of the other triangle

1. Triangle does not intersect plane
(all positive or all negative evaluations)
return non-collision

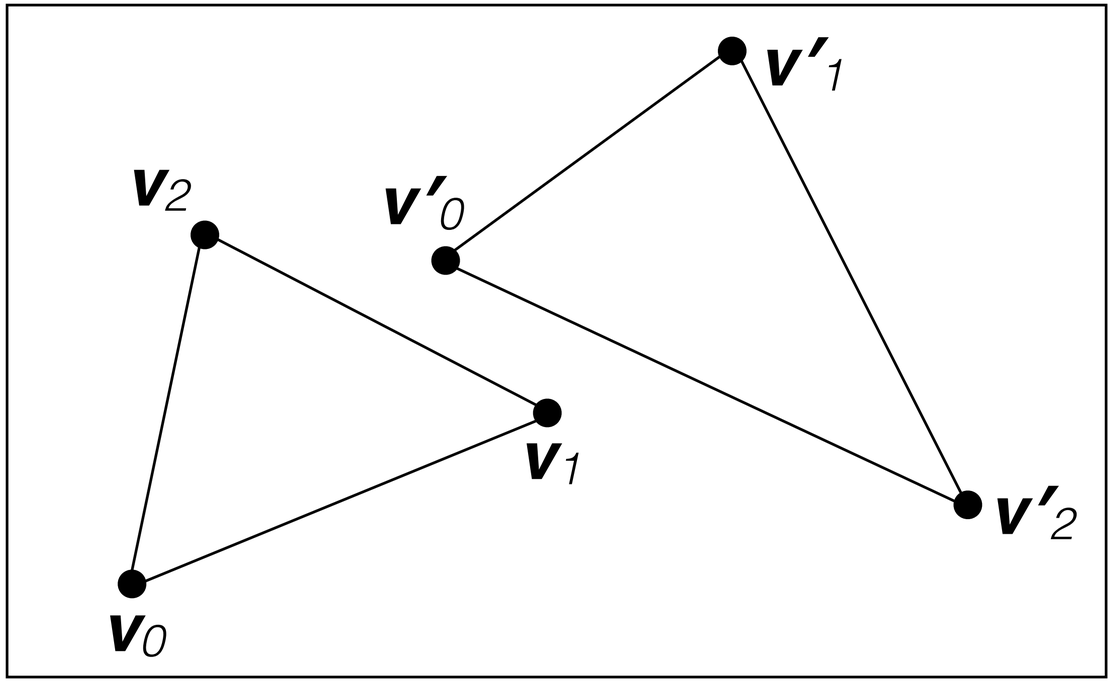
2. Triangles are coplanar
(all evaluations are zero)

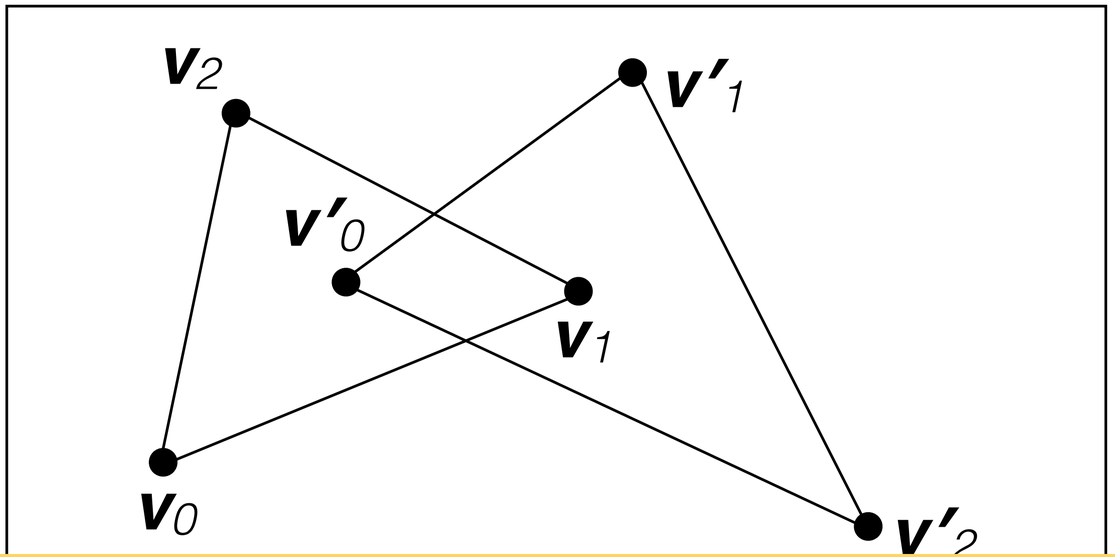


3. Triangles are not coplanar
(positive and negative evaluations)

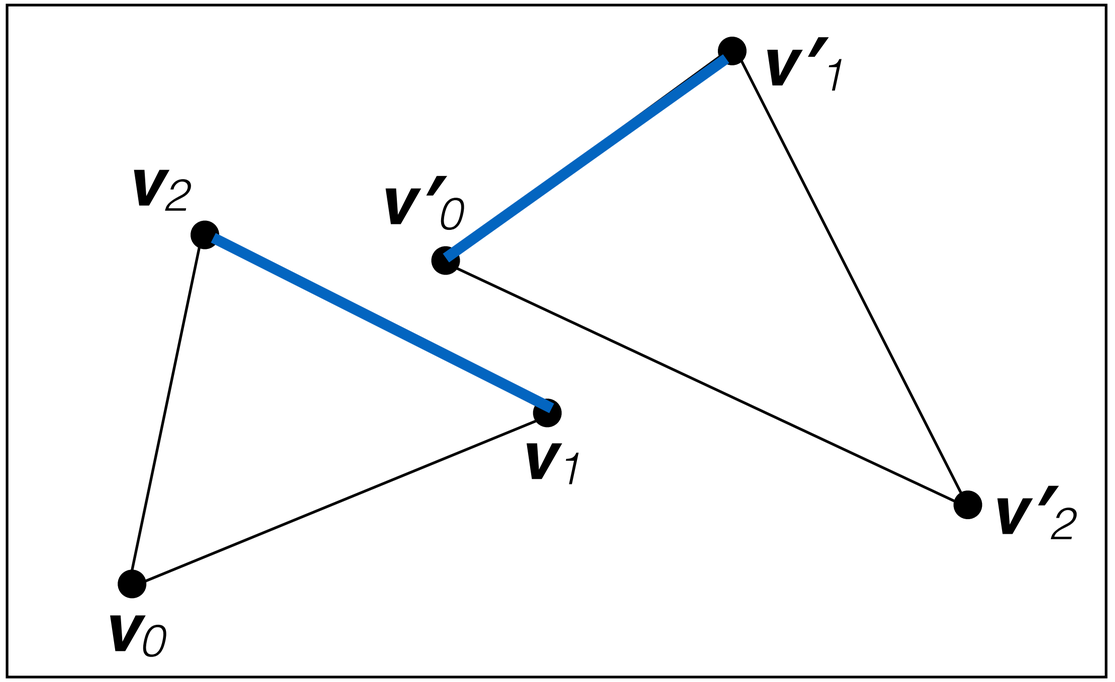


Suppose triangles are coplanar

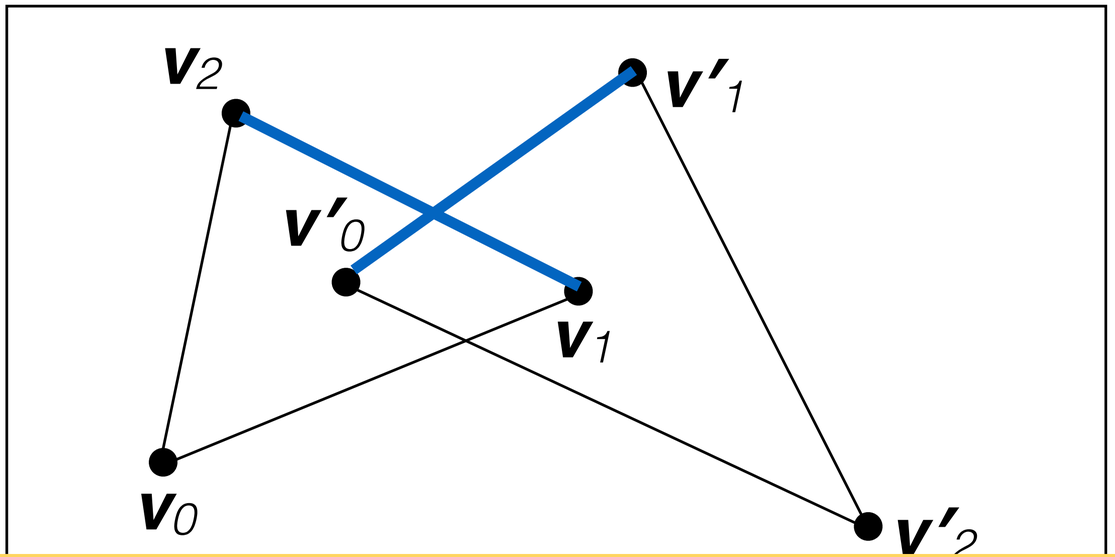




Suppose triangles are coplanar



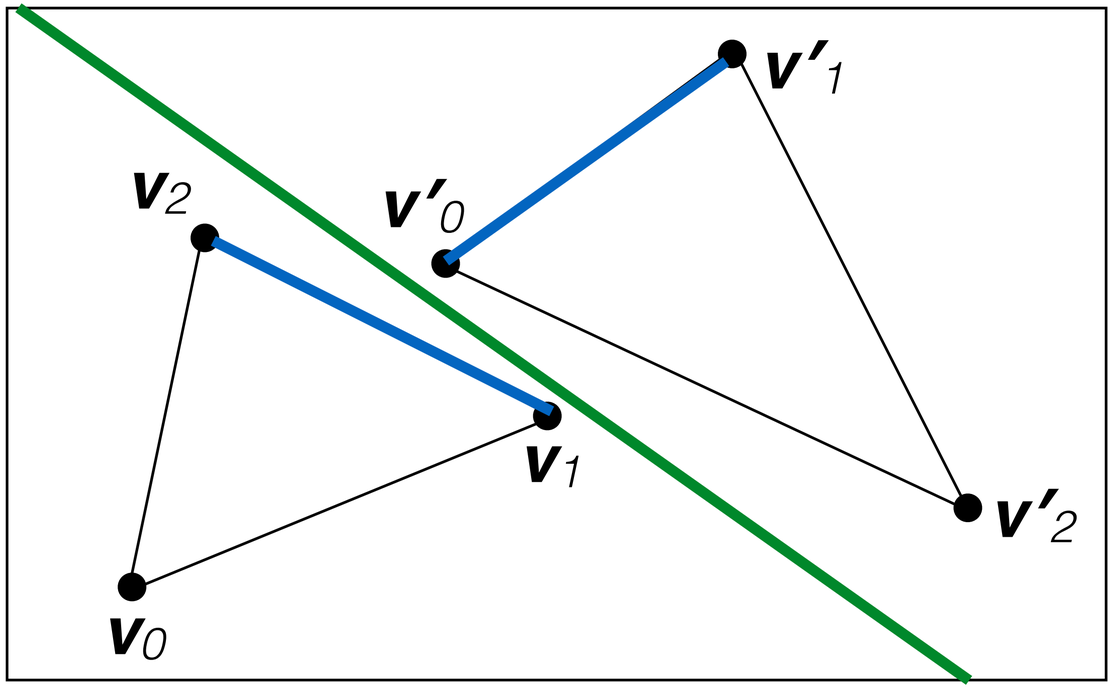
Compare each pair of line segments

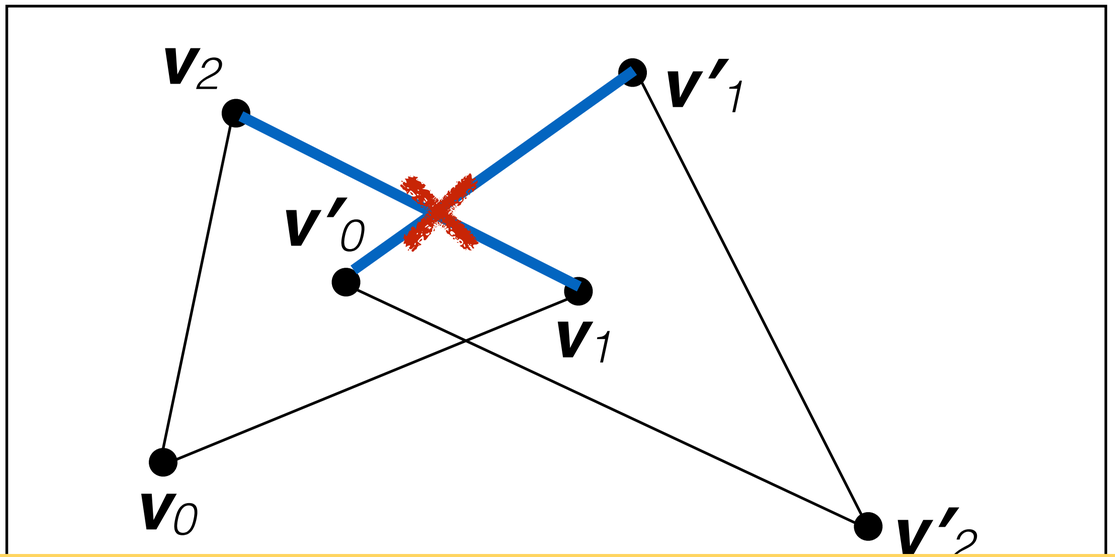


Suppose triangles are coplanar

Compare each pair of line segments

Find intersection point as solution to linear system





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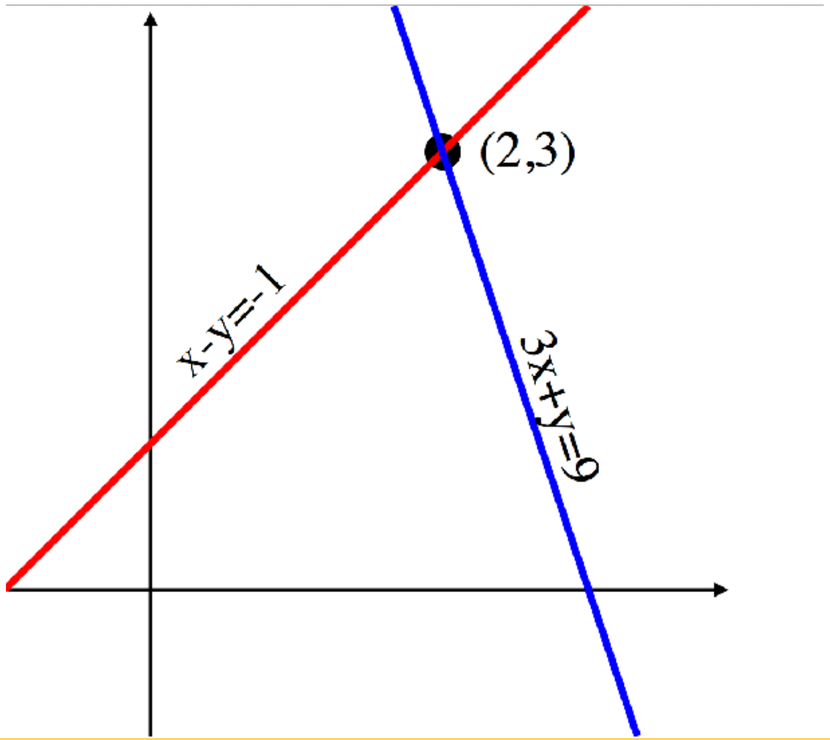
Remember:

2D Example

only single point satisfies both lines

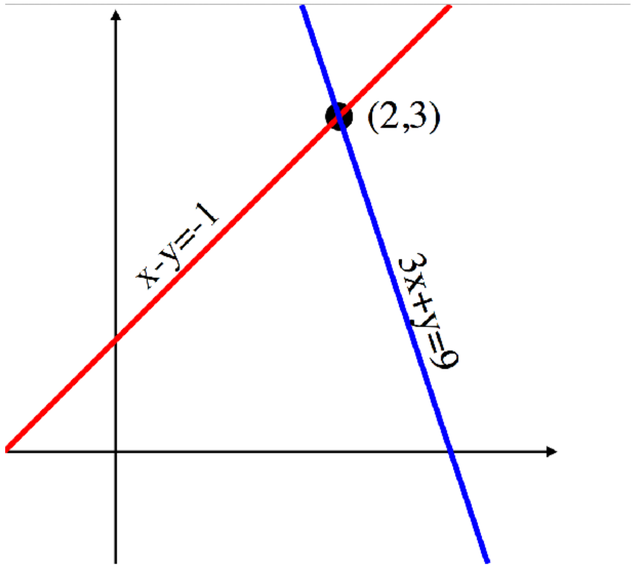


Cramer's rule

From Wikipedia, the free encyclopedia

In linear algebra, **Cramer's rule** is an explicit formula for the solution of a system of linear equations with as many equations as unknowns, valid whenever the system has a unique solution. It expresses the solution in terms of the determinants of the (square) coefficient matrix and of matrices obtained from it by replacing one column by the vector of right-hand-sides of the equations. It is named after Gabriel Cramer (1704–1752), who published the rule for an arbitrary number of unknowns in 1750,[1][2] although Colin Maclaurin also published special cases of the rule in 1748[3] (and possibly knew of it as early as 1729).[4][5][6]

Cramer's rule is computationally very inefficient for systems of more than two or three equations;[7] its asymptotic complexity is O(n·n!) compared to elimination methods that have polynomial time complexity.[8][9] Cramer's rule is also numerically unstable even for 2×2 systems.[10]



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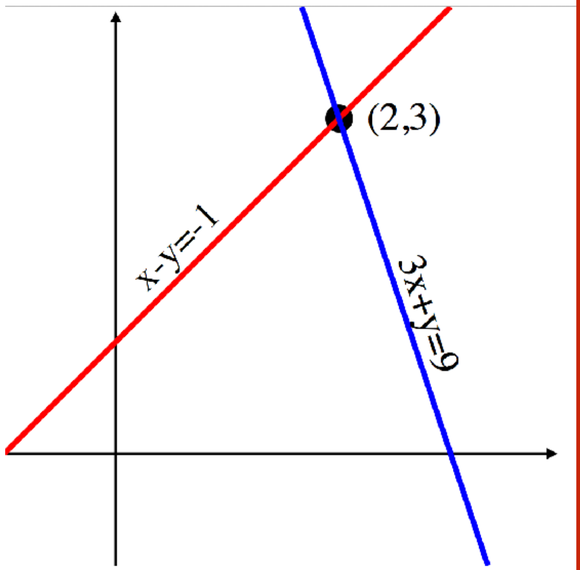


Explicit formulas for small systems [edit]

Consider the linear system

$$\begin{cases} a_1 x + b_1 y &= c_1 \\ a_2 x + b_2 y &= c_2 \end{cases}$$

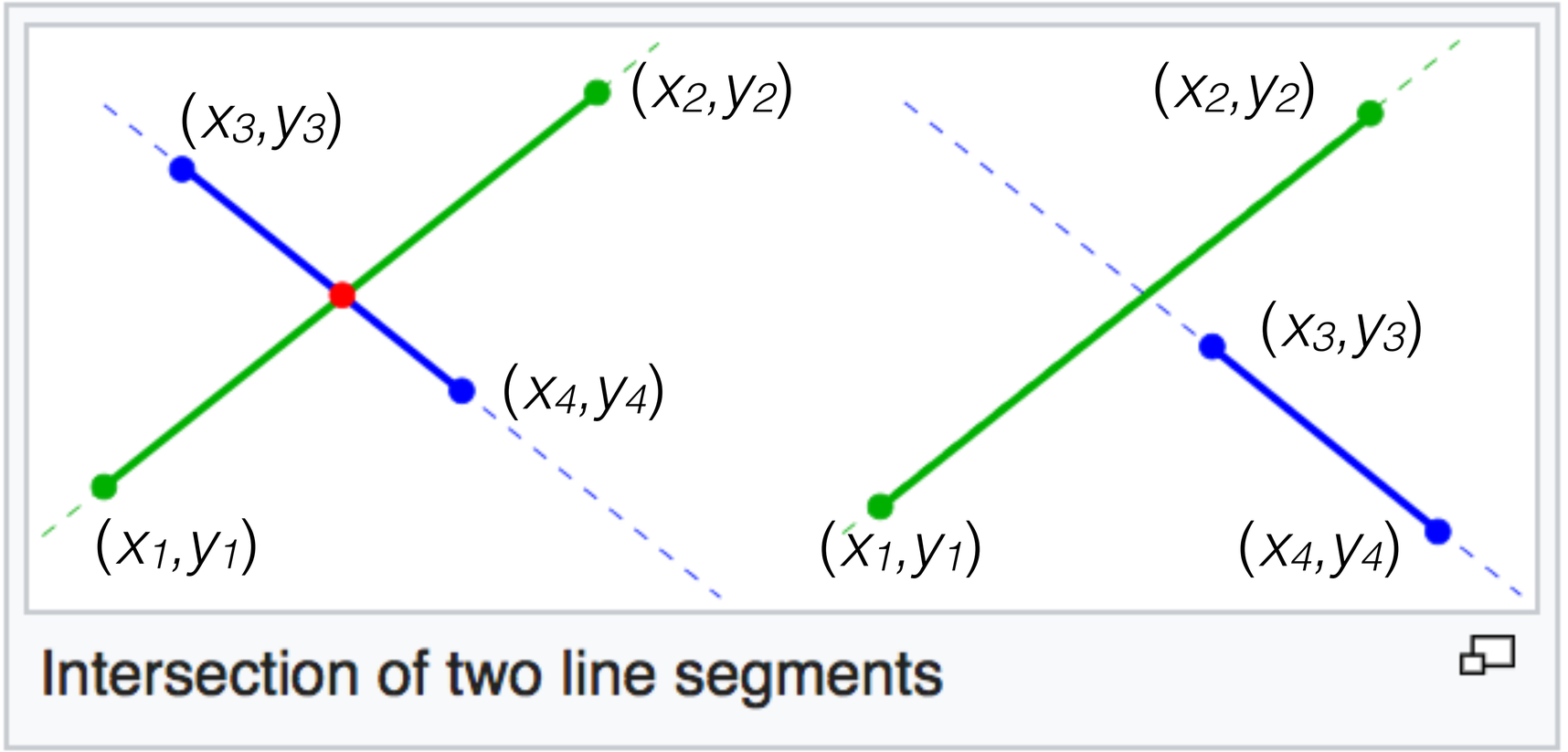
which in matrix format is

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

Assume $a_1 b_2 - b_1 a_2$ nonzero. Then, with help of determinants, x and y can be found with Cramer's rule as

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1 b_2 - b_1 c_2}{a_1 b_2 - b_1 a_2}, \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1 c_2 - c_1 a_2}{a_1 b_2 - b_1 a_2}.$$

Line Segment Intersection Test



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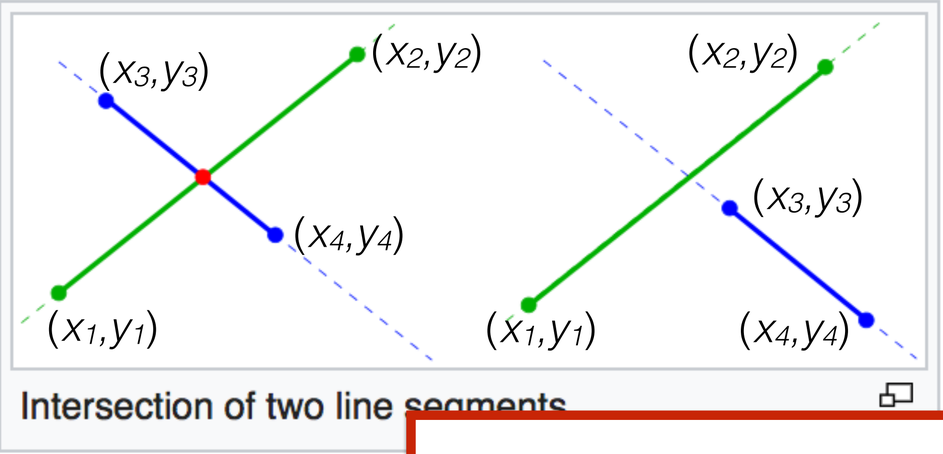
Line Segment Intersection Test



Intersection of two line segments

Parametric expression of line segments

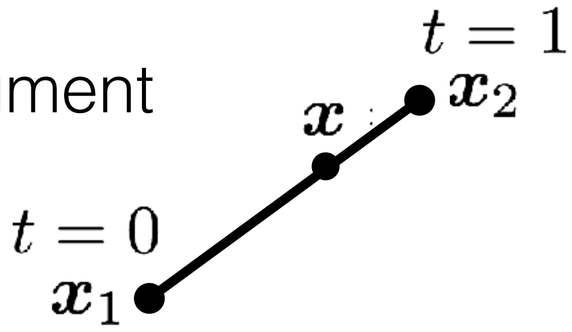
$$(x(s), y(s)) = (x_1 + s(x_2 - x_1), y_1 + s(y_2 - y_1))$$

$$(x(t), y(t)) = (x_3 + t(x_4 - x_3), y_3 + t(y_4 - y_3))$$

Segments span parameter interval

$$0 \leq s, t \leq 1$$

Parametric expression of a line segment



$$\boldsymbol{x} = \boldsymbol{x}_1 + t\boldsymbol{x}_2 = (x_1 + t(x_2 - x_1), y_1 + t(y_2 - y_1)) \text{ , for } 0 \leq t \leq 1$$

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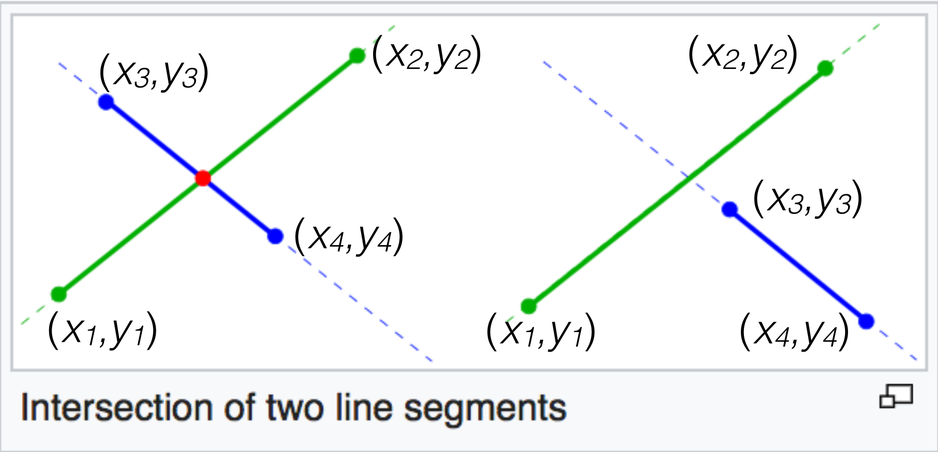
Line Segment Intersection Test



Intersection of two line segments

Form linear system s.t. $(\boldsymbol{x}(s), \boldsymbol{y}(s)) = (\boldsymbol{x}(t), \boldsymbol{y}(t))$

$$\begin{bmatrix} (x_2 - x_1) & -(x_4 - x_3) \\ (y_2 - y_1) & -(y_4 - y_3) \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} x_3 - x_1 \\ y_3 - y_1 \end{bmatrix}$$

Solve for parameters at intersection point using Cramer's rule

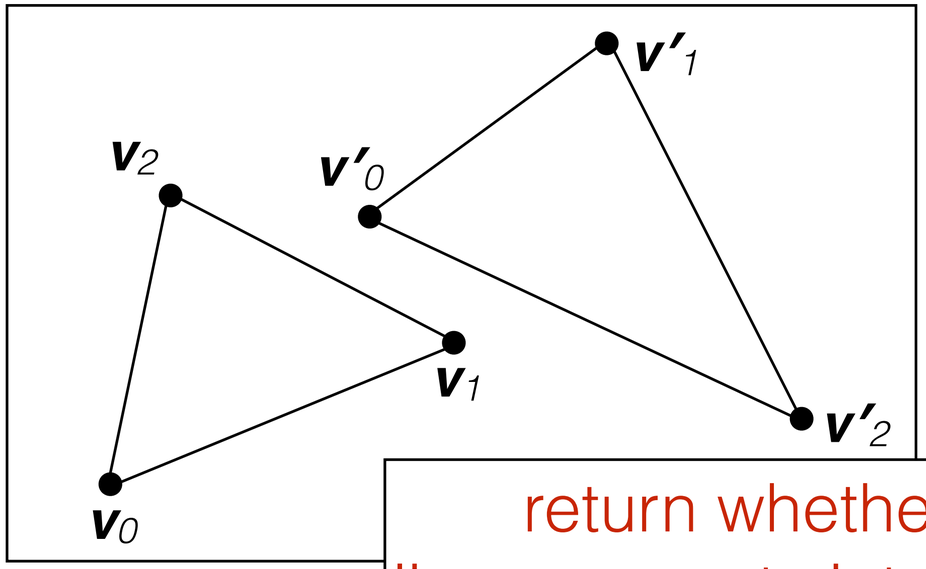
Return intersection, if parameters lie within segments, as $0 \leq s, t \leq 1$

Return non-intersection, otherwise

Three possible cases can occur based on
evaluation of vertices of one triangle against the plane of the other triangle

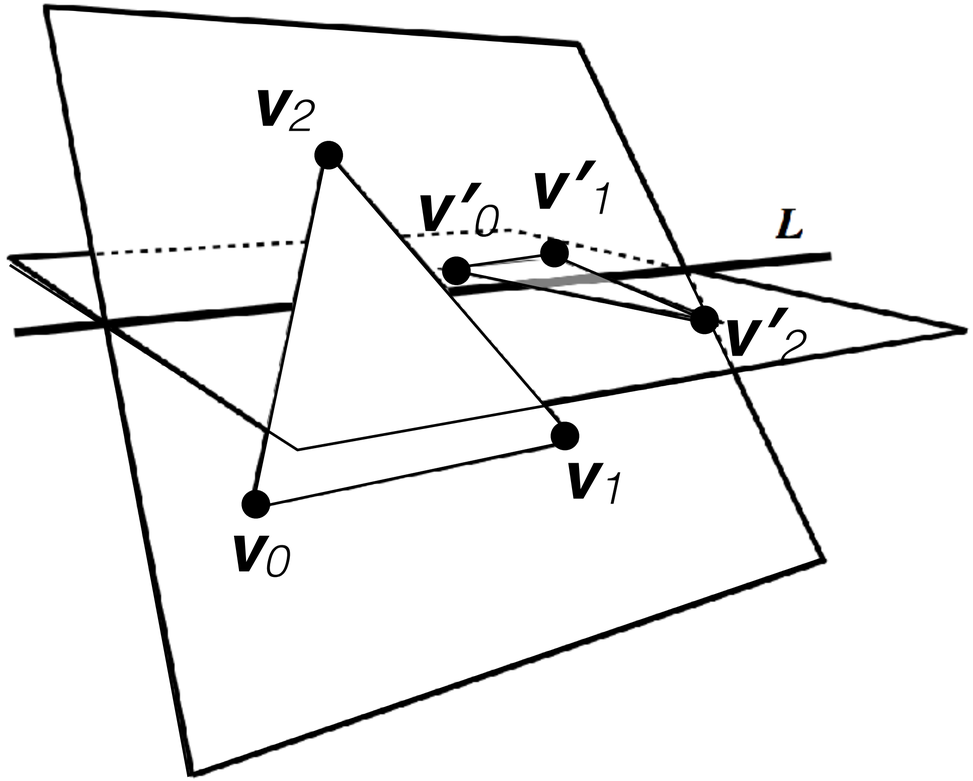
1. Triangle does not intersect plane
(all positive or all negative evaluations)
return non-collision

2. Triangles are coplanar
(all evaluations are zero)



return whether
line segments intersect

3. Triangles are not coplanar
(positive and negative evaluations)



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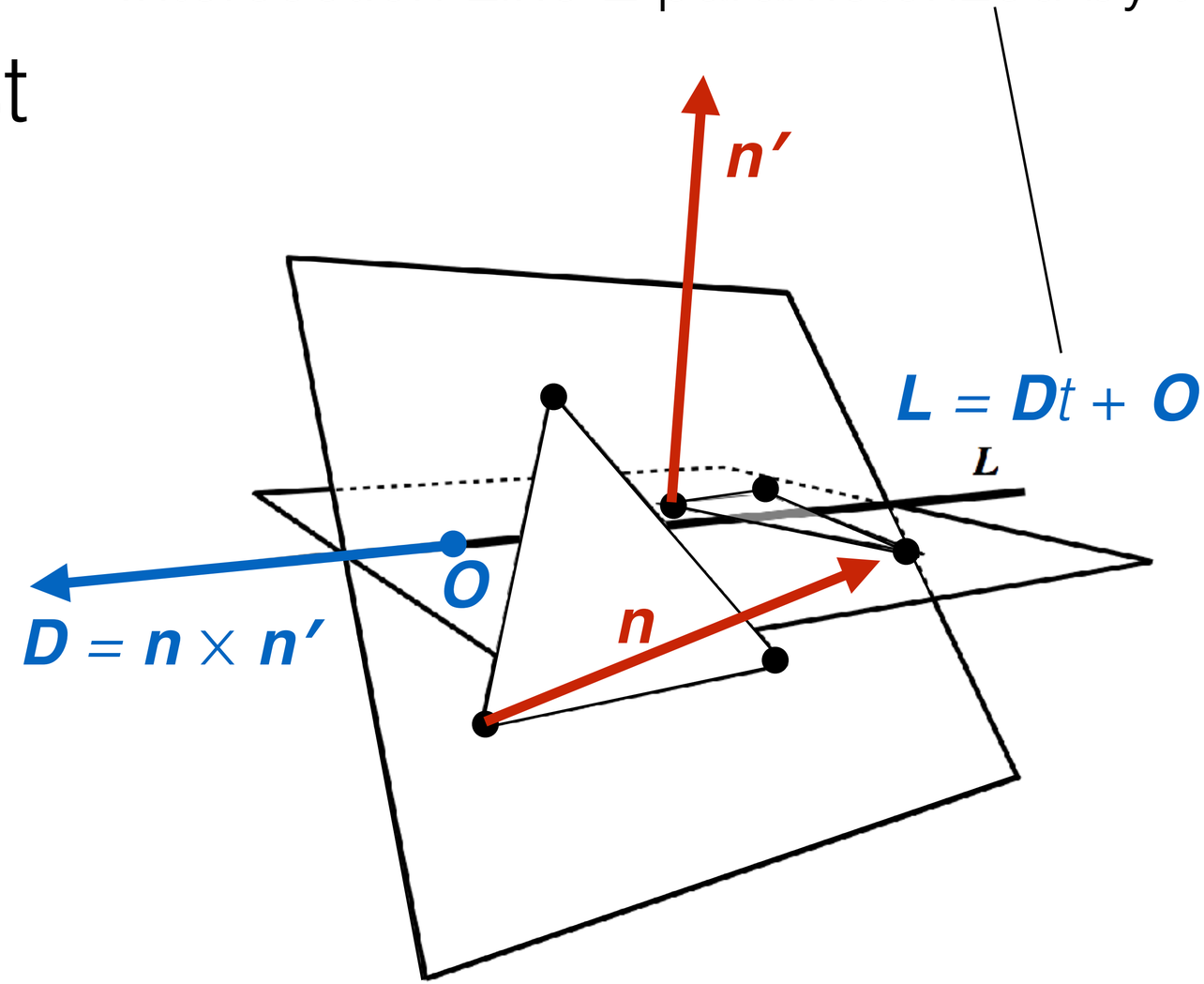
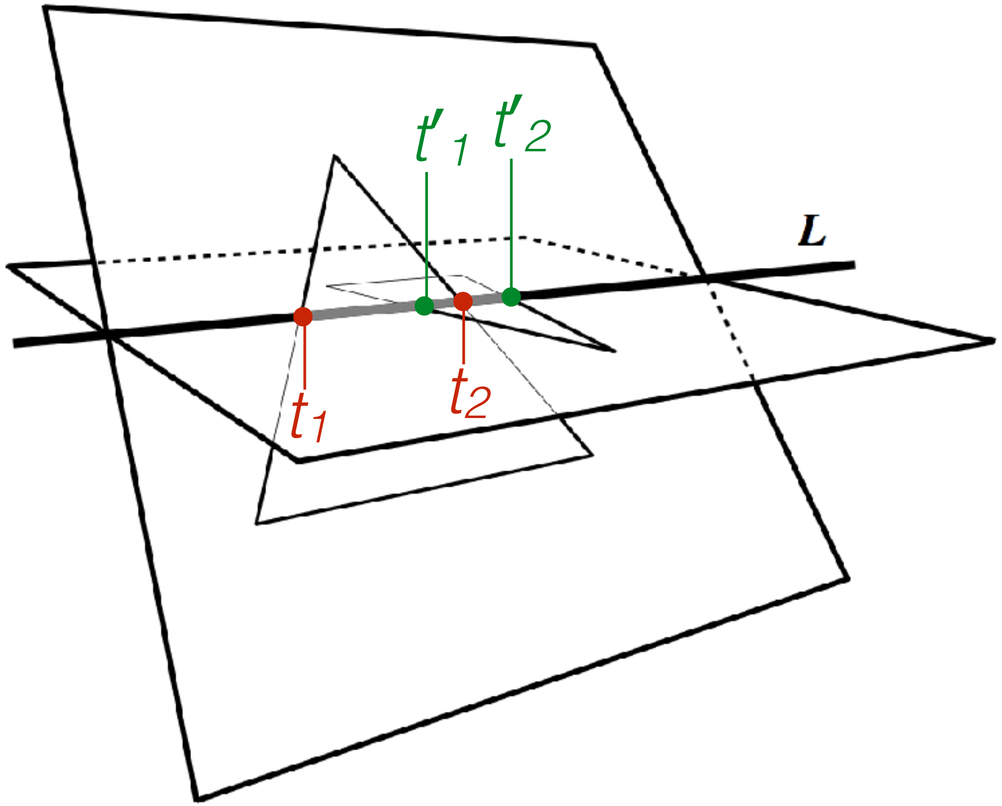
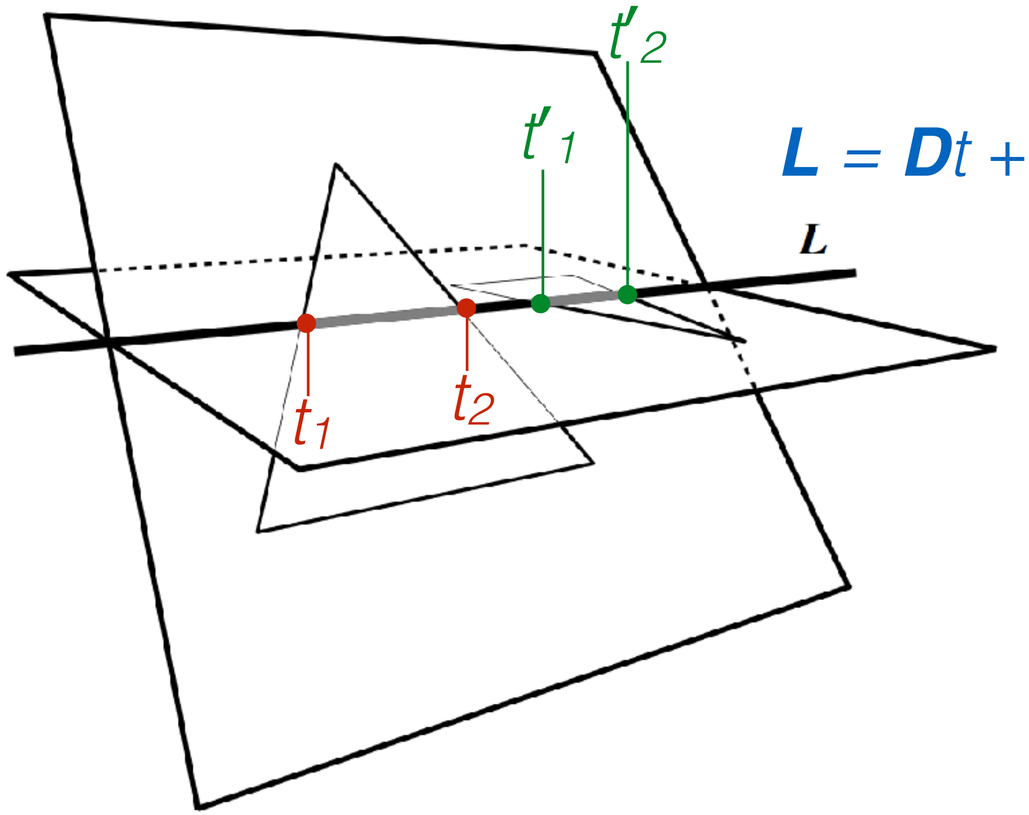


Two possible cases can occur based on interval spans ($[t_1,t_2]$, $[t'_1,t'_2]$) of triangle intersections with ***L***

Collision, if intervals overlap



No Collision, if intervals do not overlap



$\boldsymbol{L} = \boldsymbol{D}t + \boldsymbol{O}$

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3D Triangle-Triangle Test

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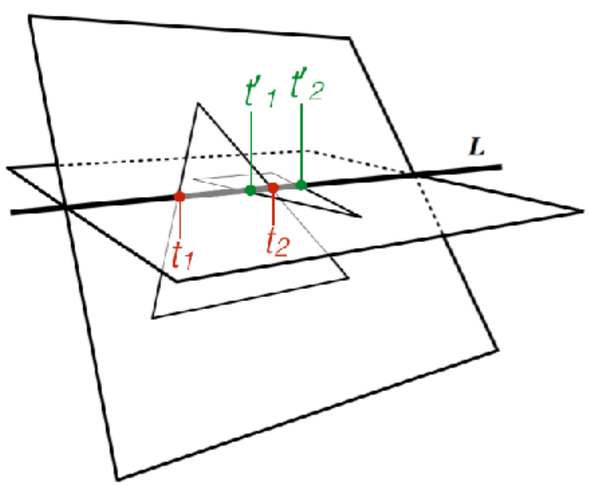
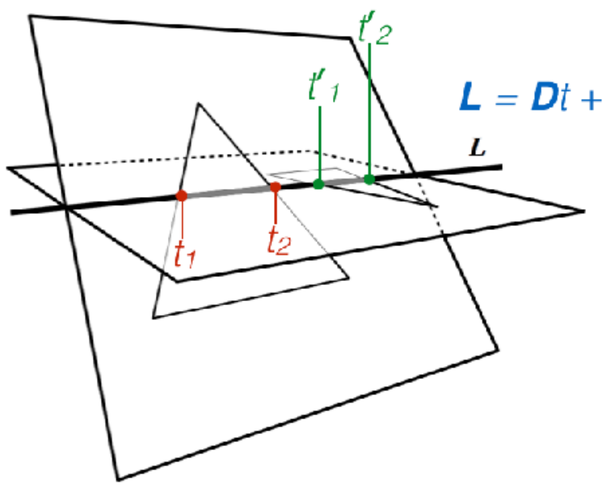
Collision



Non-collision



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How many triangle tests must be performed?

How many triangle tests must be performed?

Can we reduce the number of tests to evaluate?

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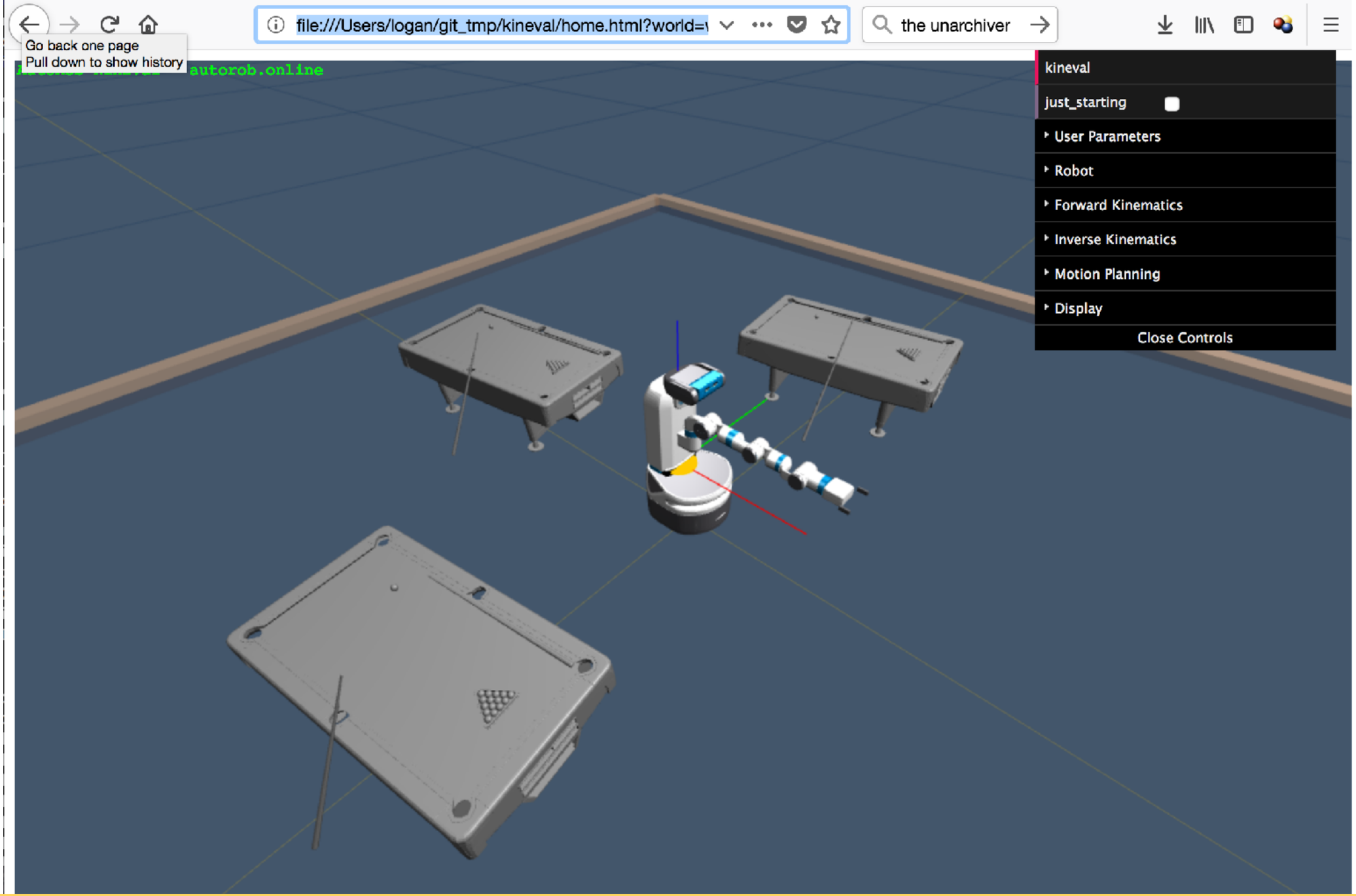

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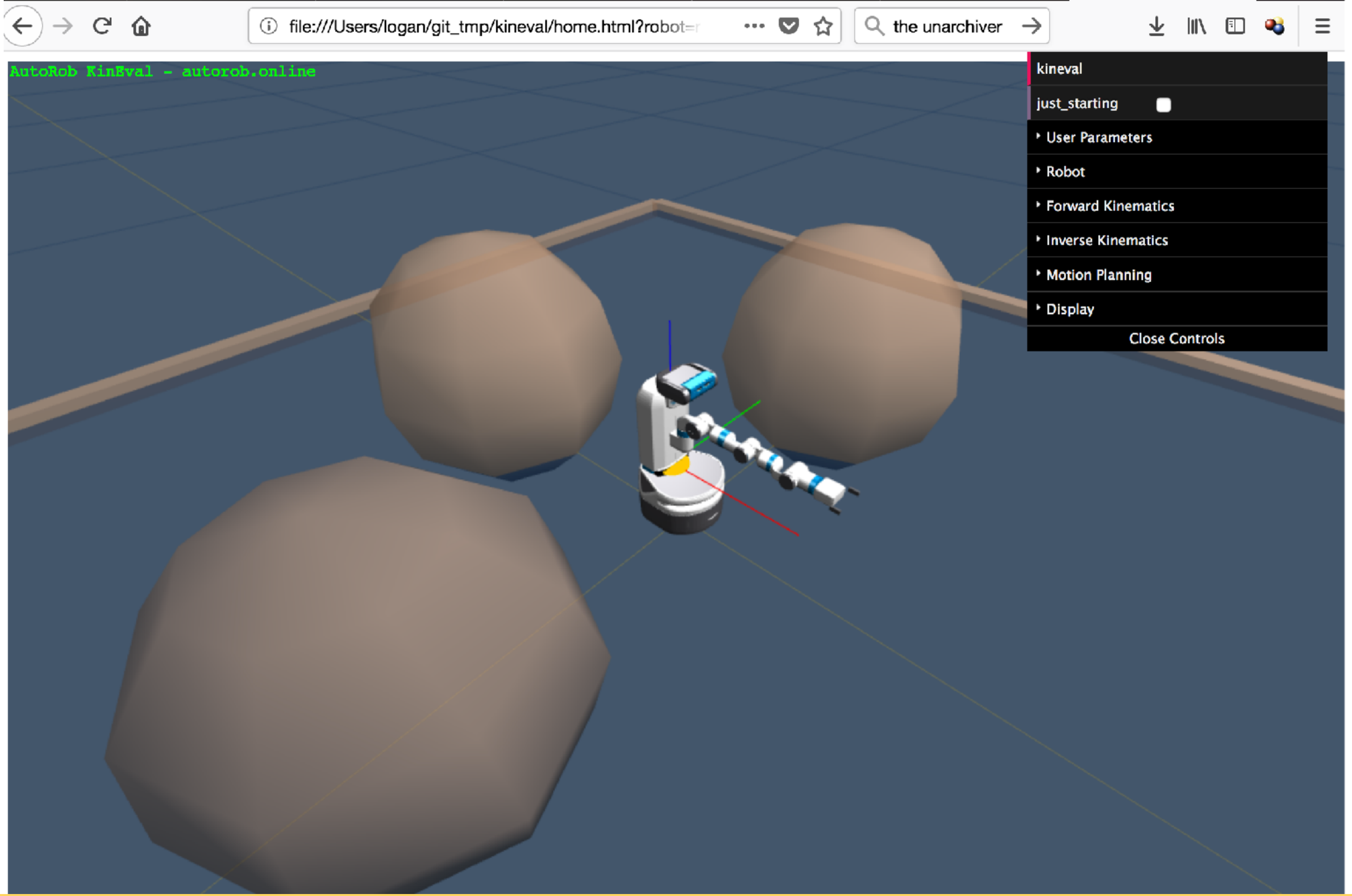

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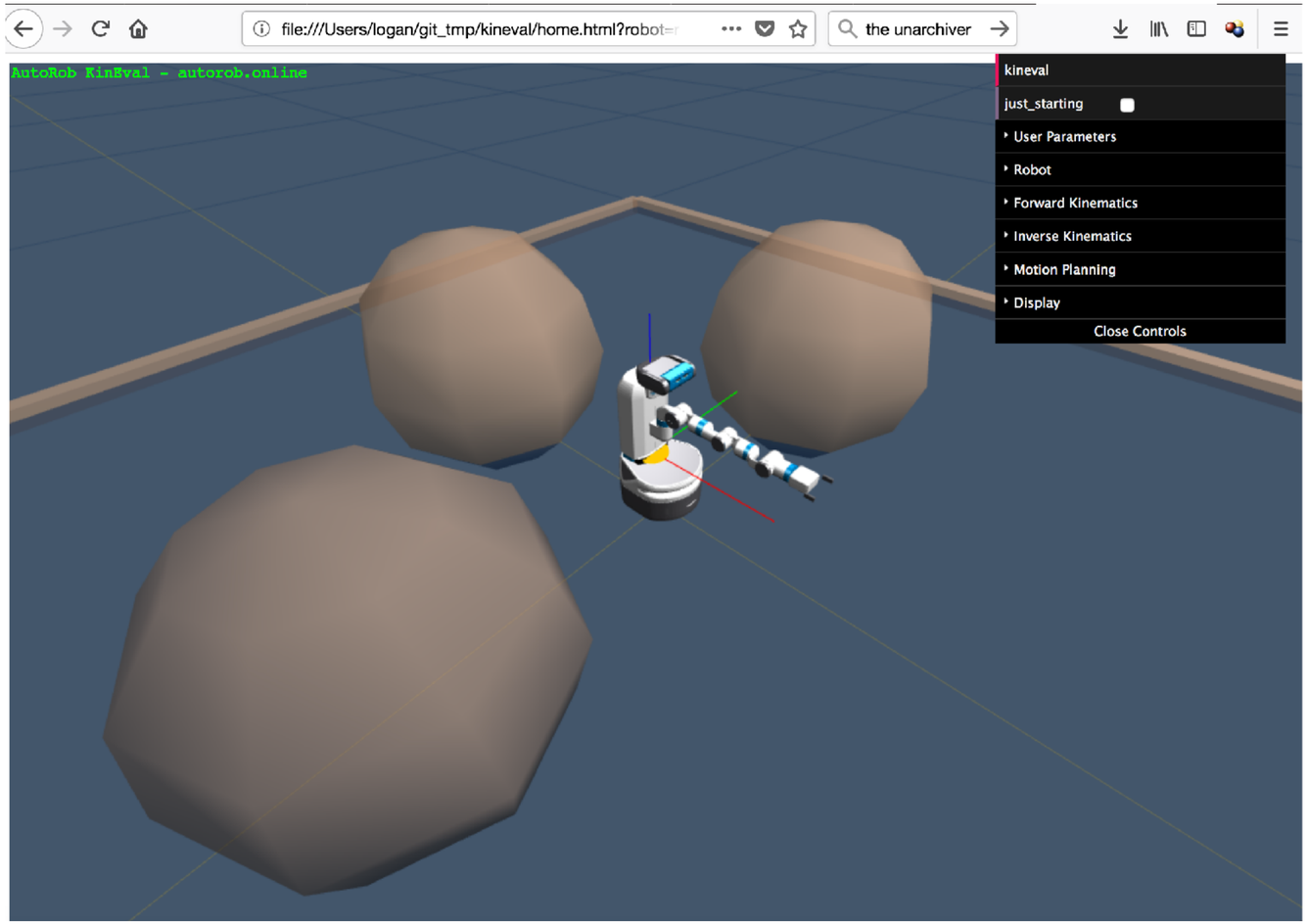
KinEval approximates obstacles with bounding spheres



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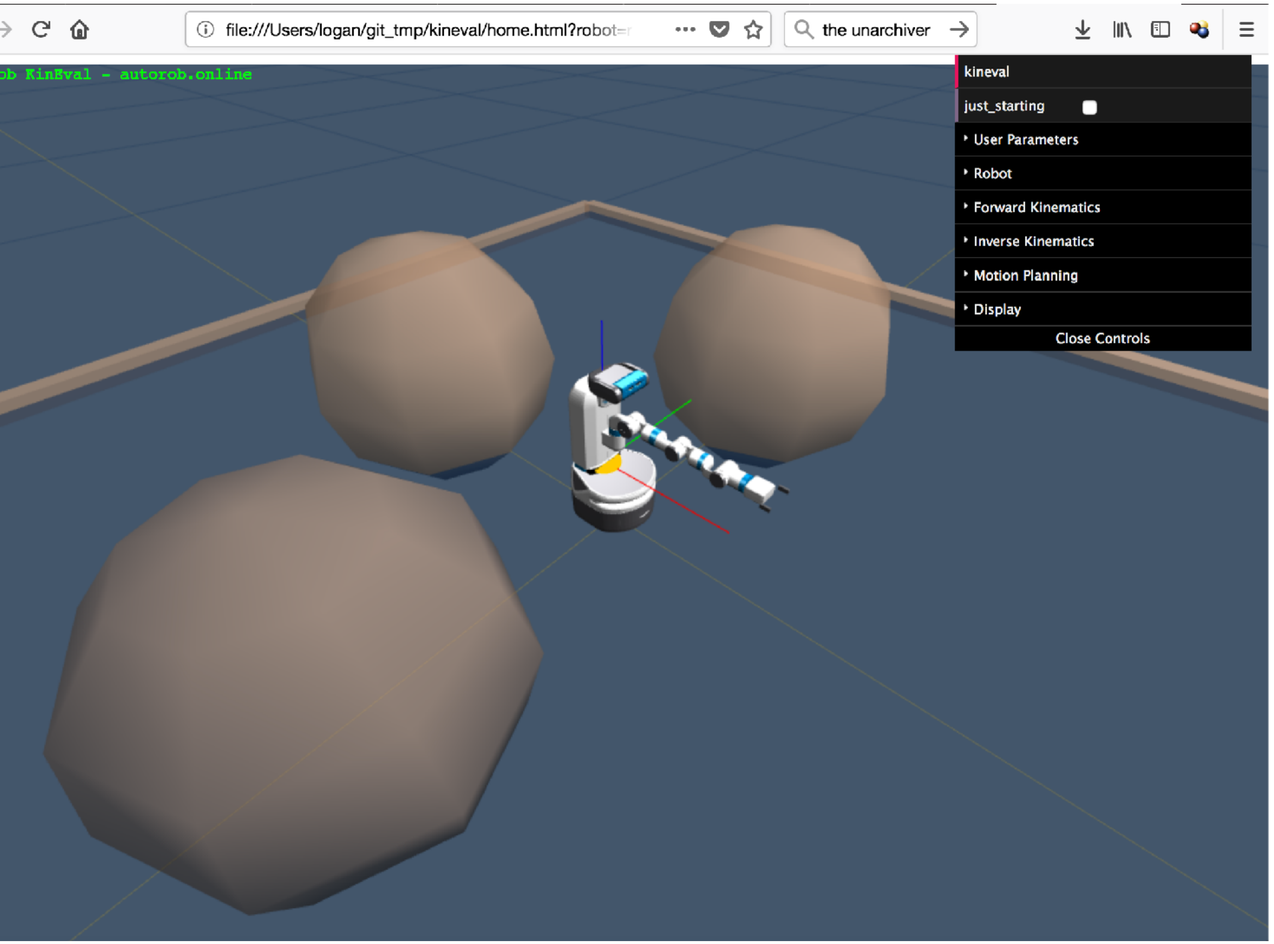
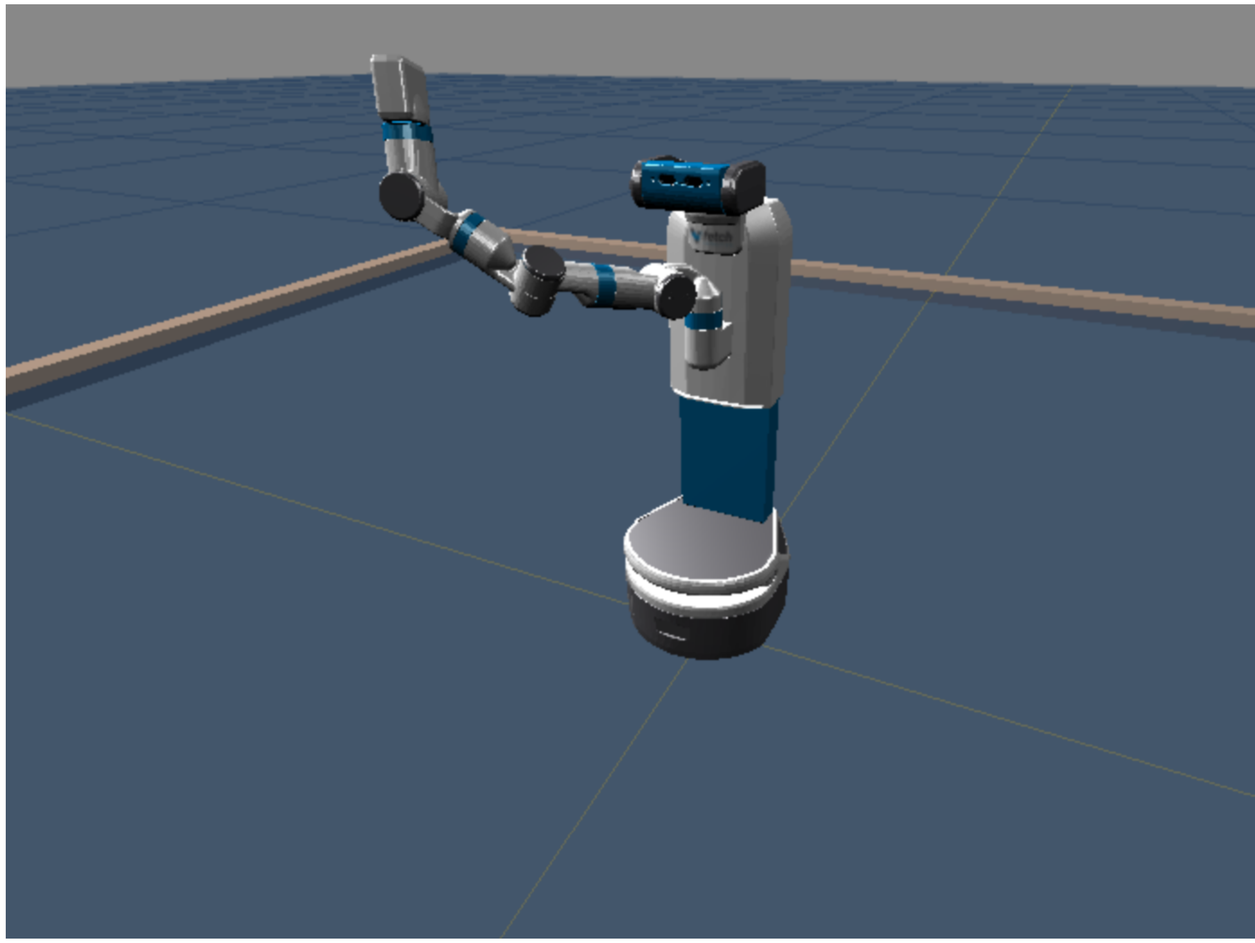
KinEval approximates

obstacles with bounding spheres



and the robot?



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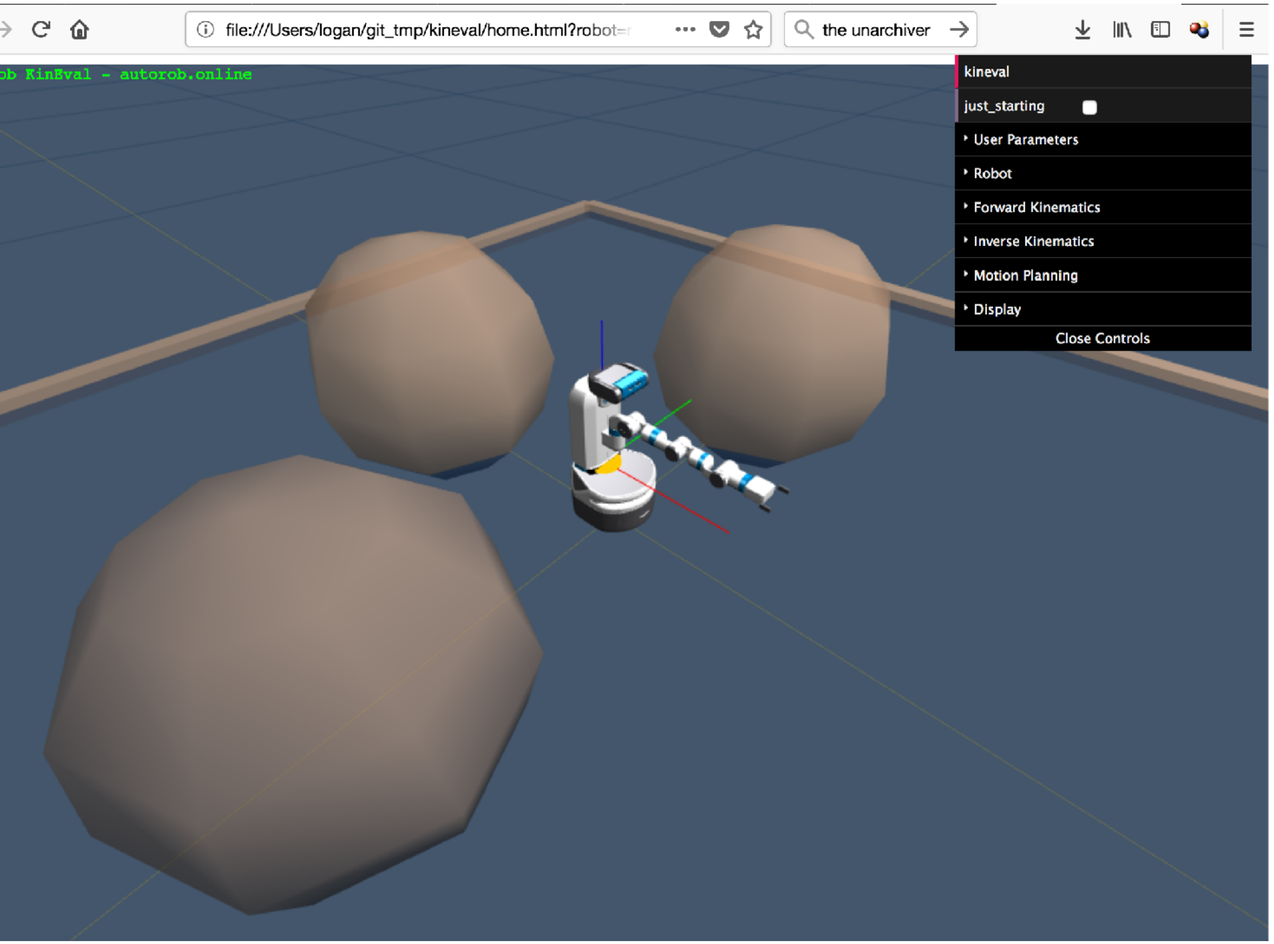
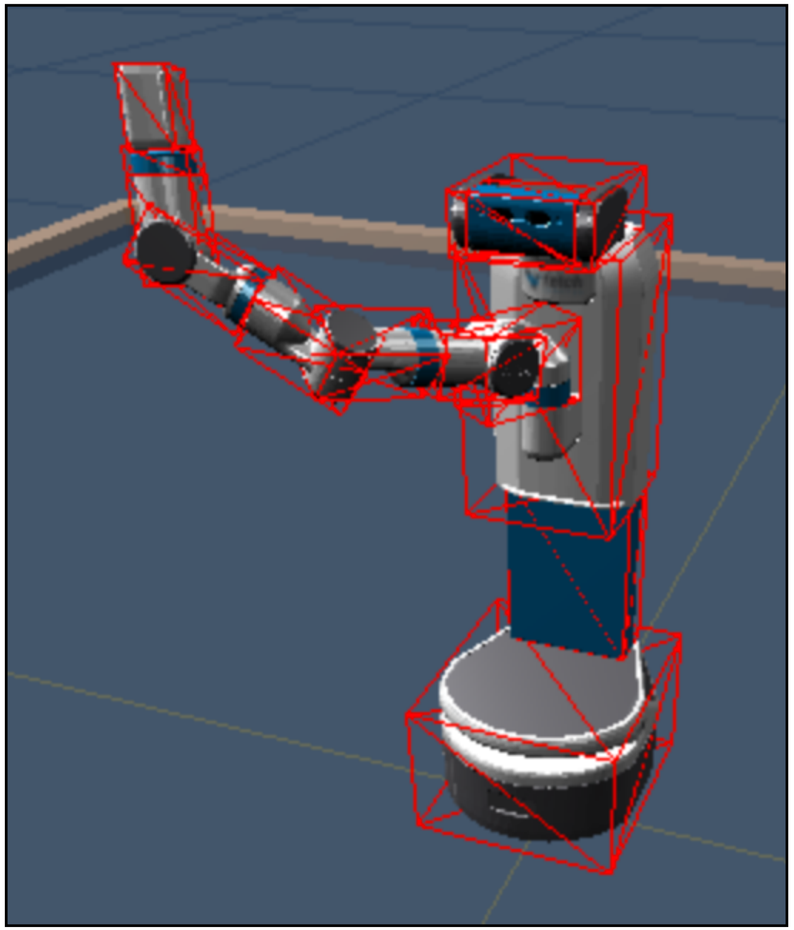
KinEval approximates

obstacles with bounding spheres



robot as bounding boxes



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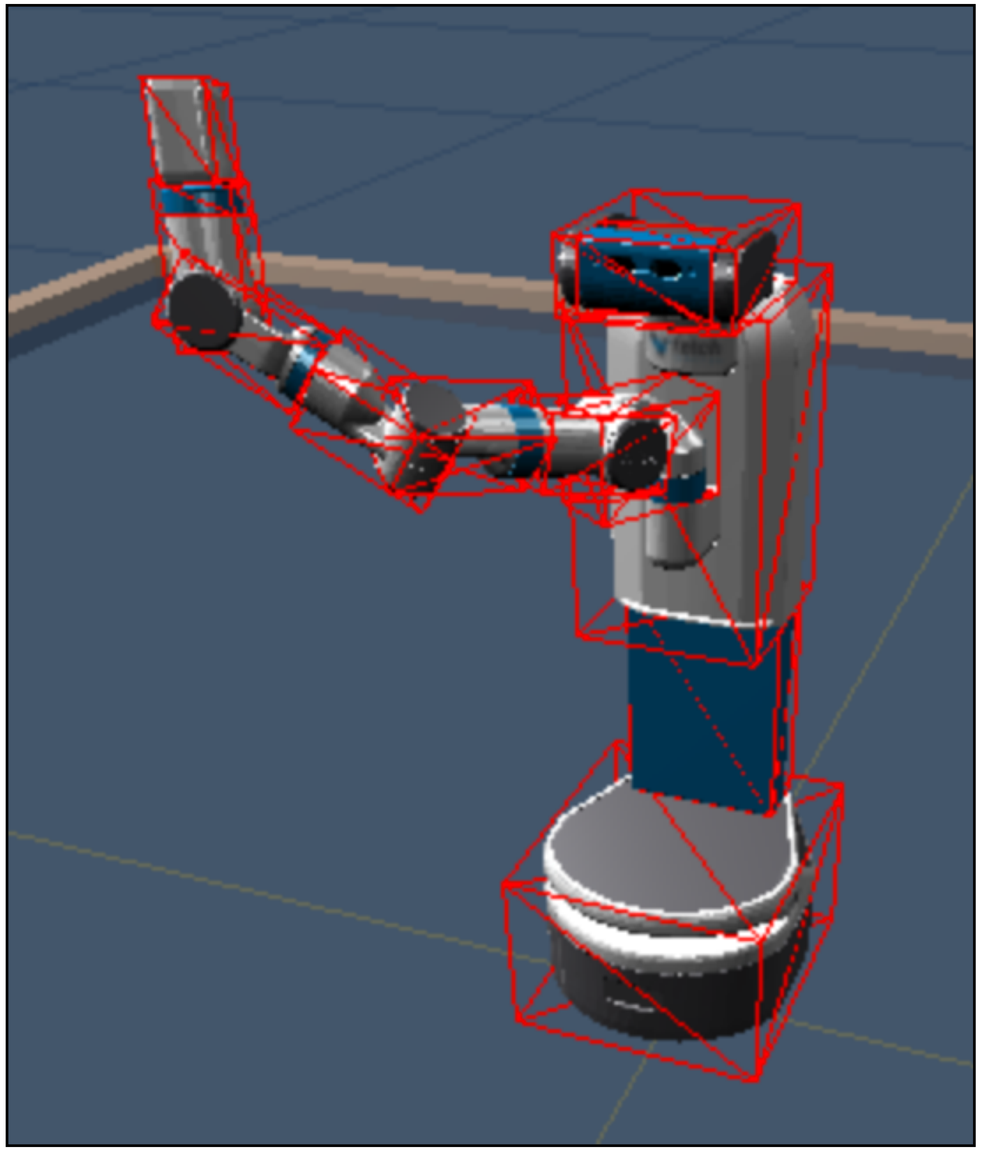
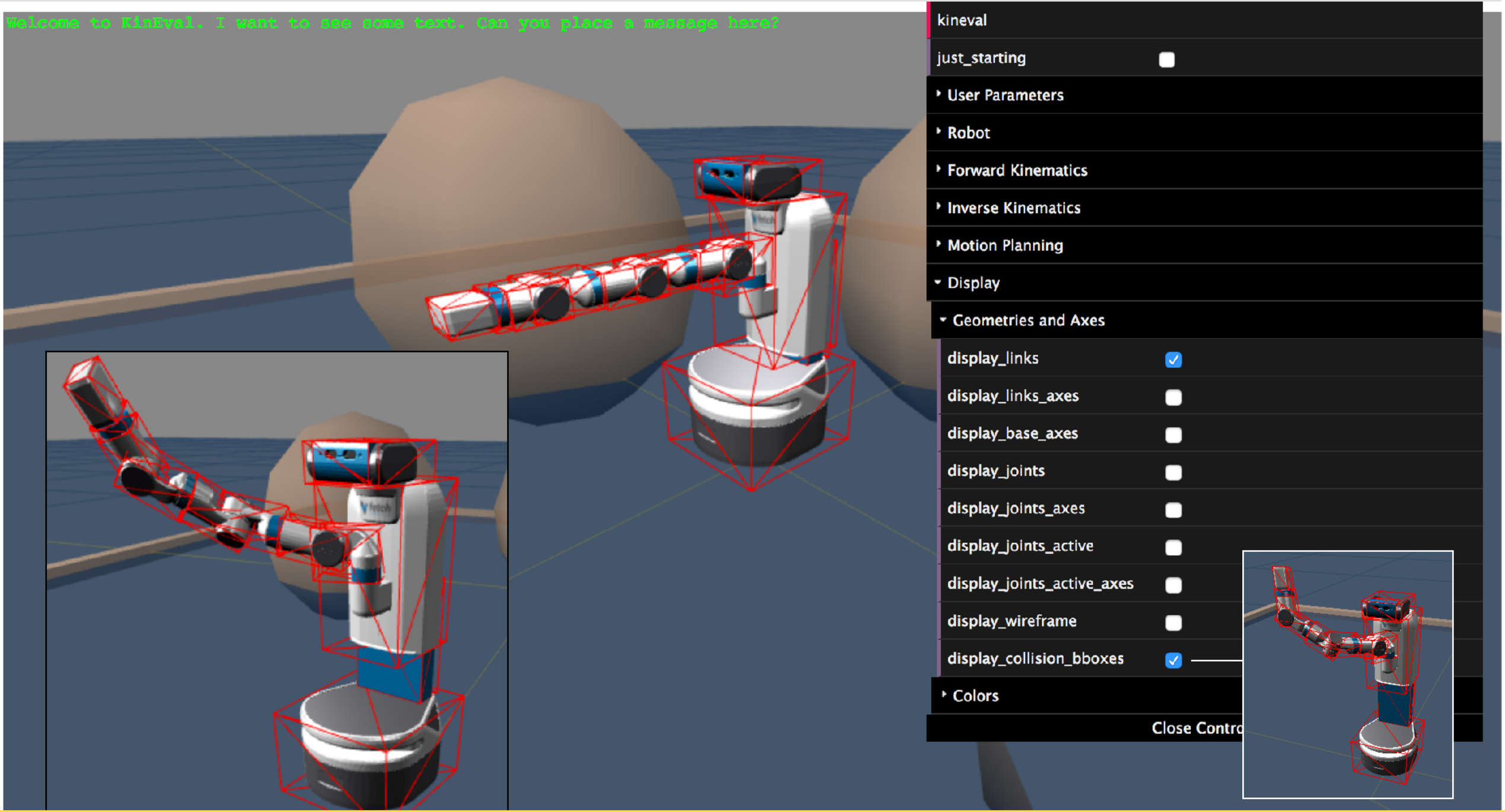
KinEval approximates link geometries with bounding boxes





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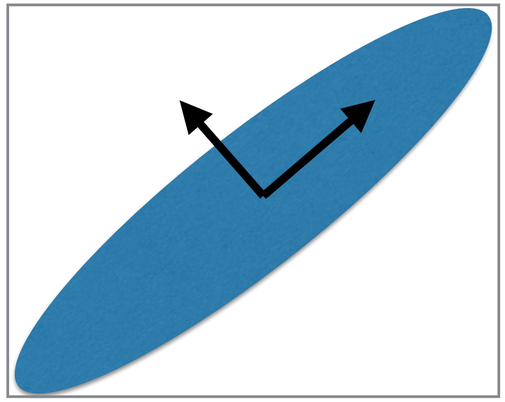
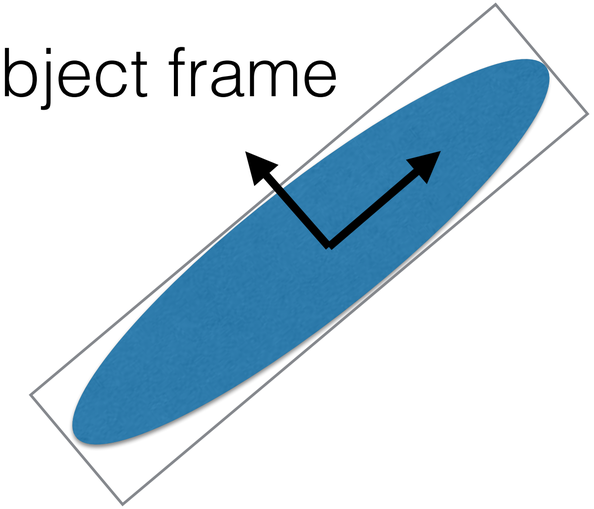
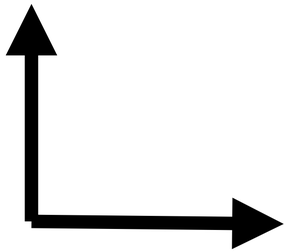
Bounding Boxes



Axis-aligned Bounding Box
(AABB)



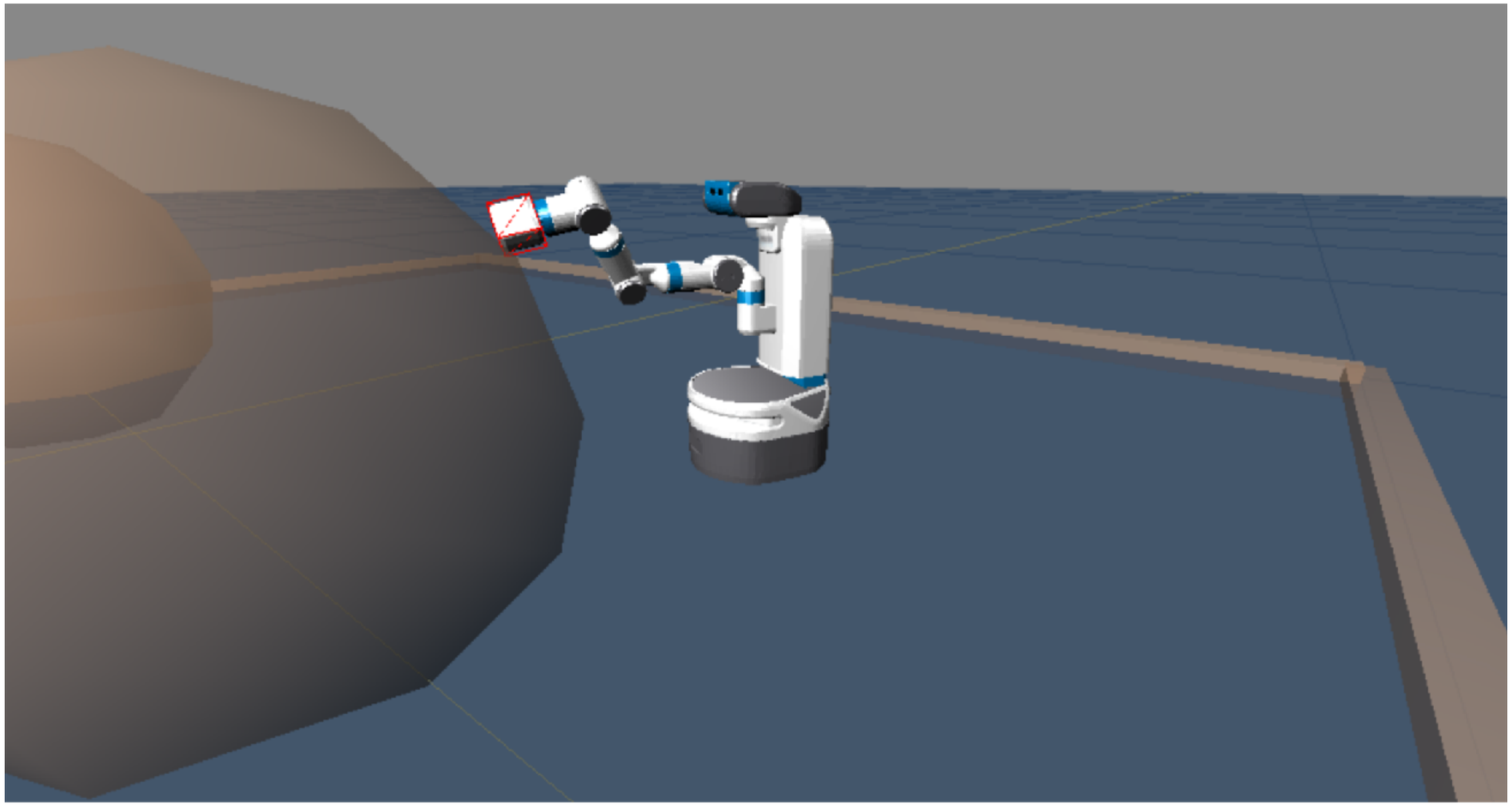
Oriented Bounding Box
(OBB)



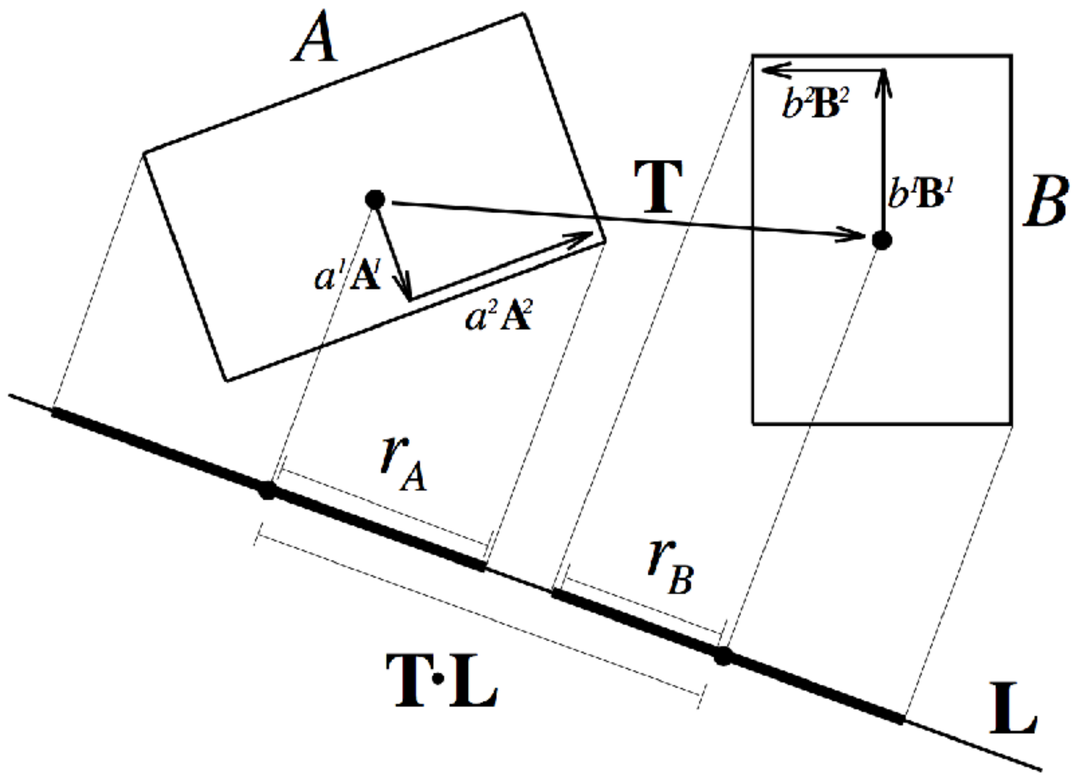
world frame

Gottschalk et al. 1996

Only a “separating axis” needs to be found



Separating Axis Theorem



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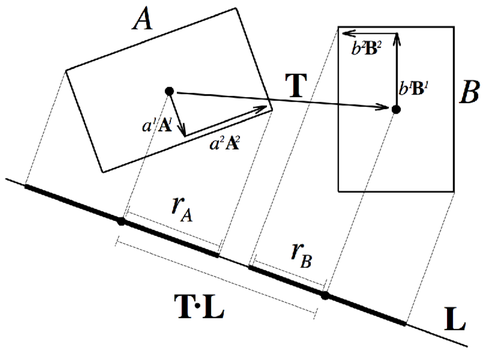
Hyperplane separation theorem



From Wikipedia, the free encyclopedia

(Redirected from Separating axis theorem)

In geometry, the **hyperplane separation theorem** is a theorem about disjoint convex sets in *n*-dimensional Euclidean space. There are several rather similar versions. In one version of the theorem, if both these sets are closed and at least one of them is compact, then there is a hyperplane in between them and even two parallel hyperplanes in between them separated by a gap. In another version, if both disjoint convex sets are open, then there is a hyperplane in between them, but not necessarily any gap. An axis which is orthogonal to a separating hyperplane is a **separating axis**, because the orthogonal projections of the convex bodies onto the axis are disjoint.

The hyperplane separation theorem is due to Hermann Minkowski. The Hahn–Banach separation theorem generalizes the result to topological vector spaces.

A related result is the supporting hyperplane theorem.

In geometry, a **maximum-margin hyperplane** is a hyperplane which separates two 'clouds' of points and is at equal distance from the two. The margin between the hyperplane and the clouds is maximal. See the article on Support Vector Machines for more details.

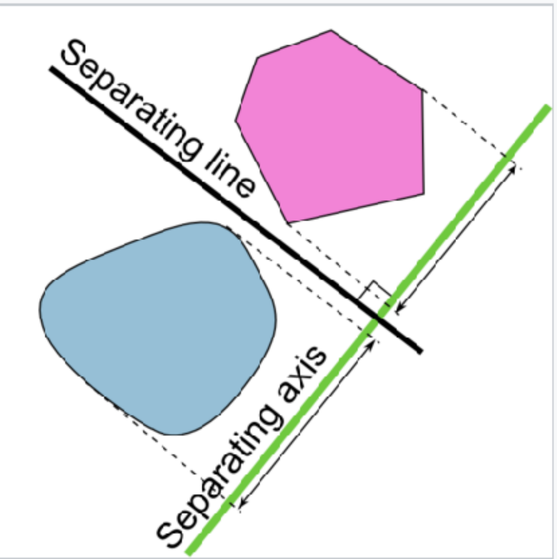
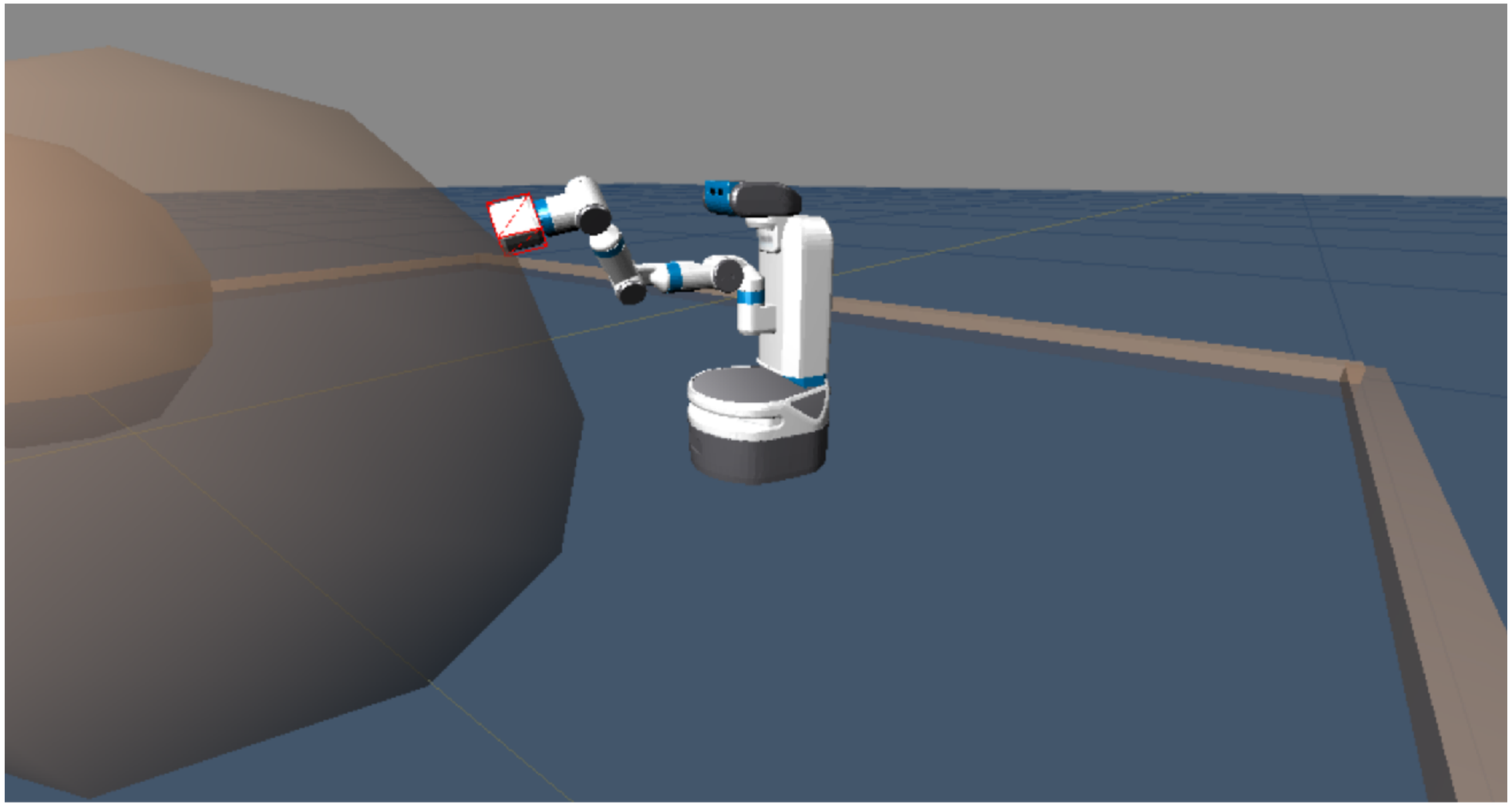


Illustration of the hyperplane separation theorem.

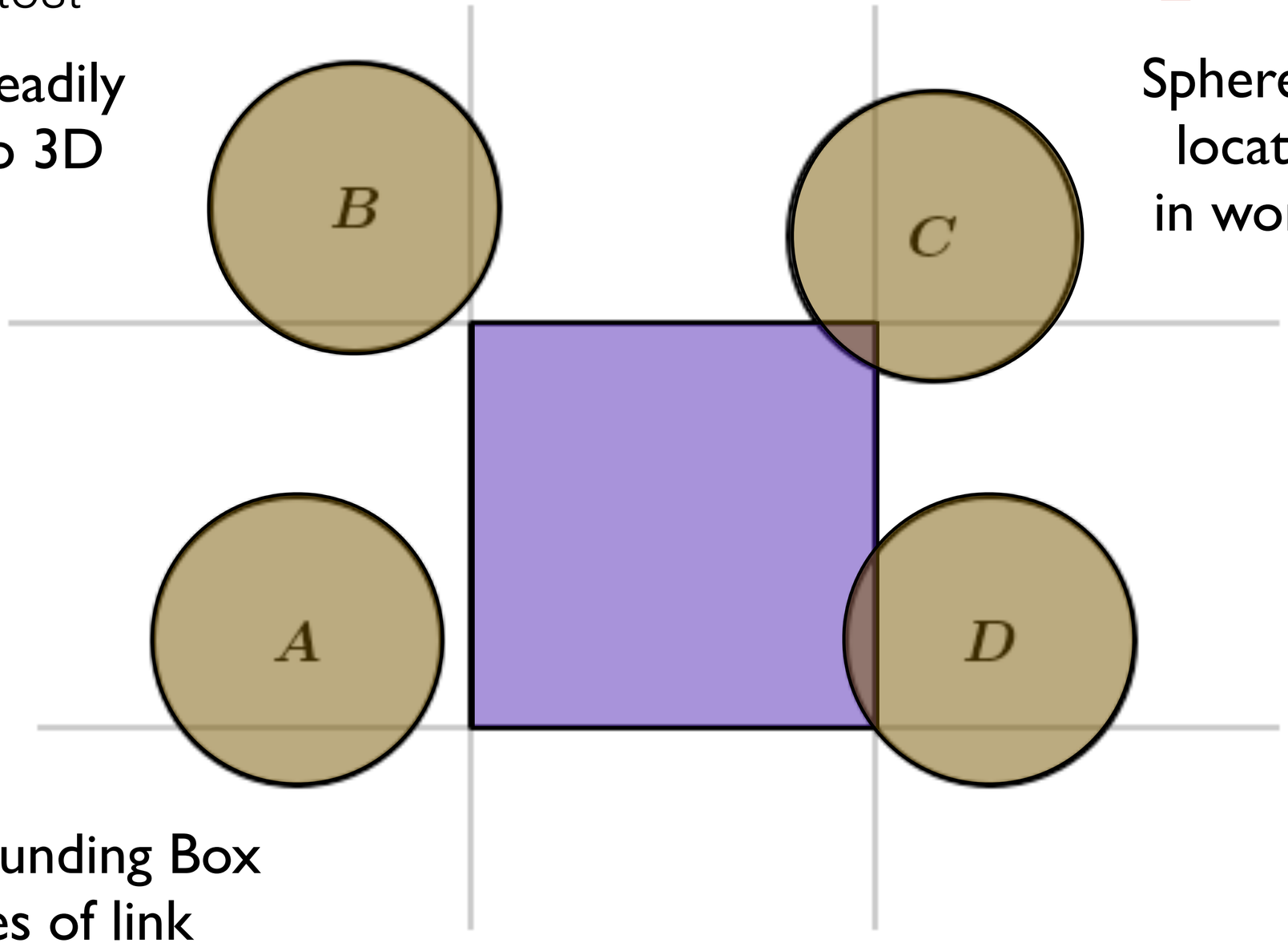
Consider AABB link tested against spherical obstacles in link frame



Sphere-bbox test

`robot_obstacles[i]`

2D example readily generalizes to 3D



Sphere obstacles with location and radius in world coordinates

Axis Aligned Bounding Box in coordinates of link

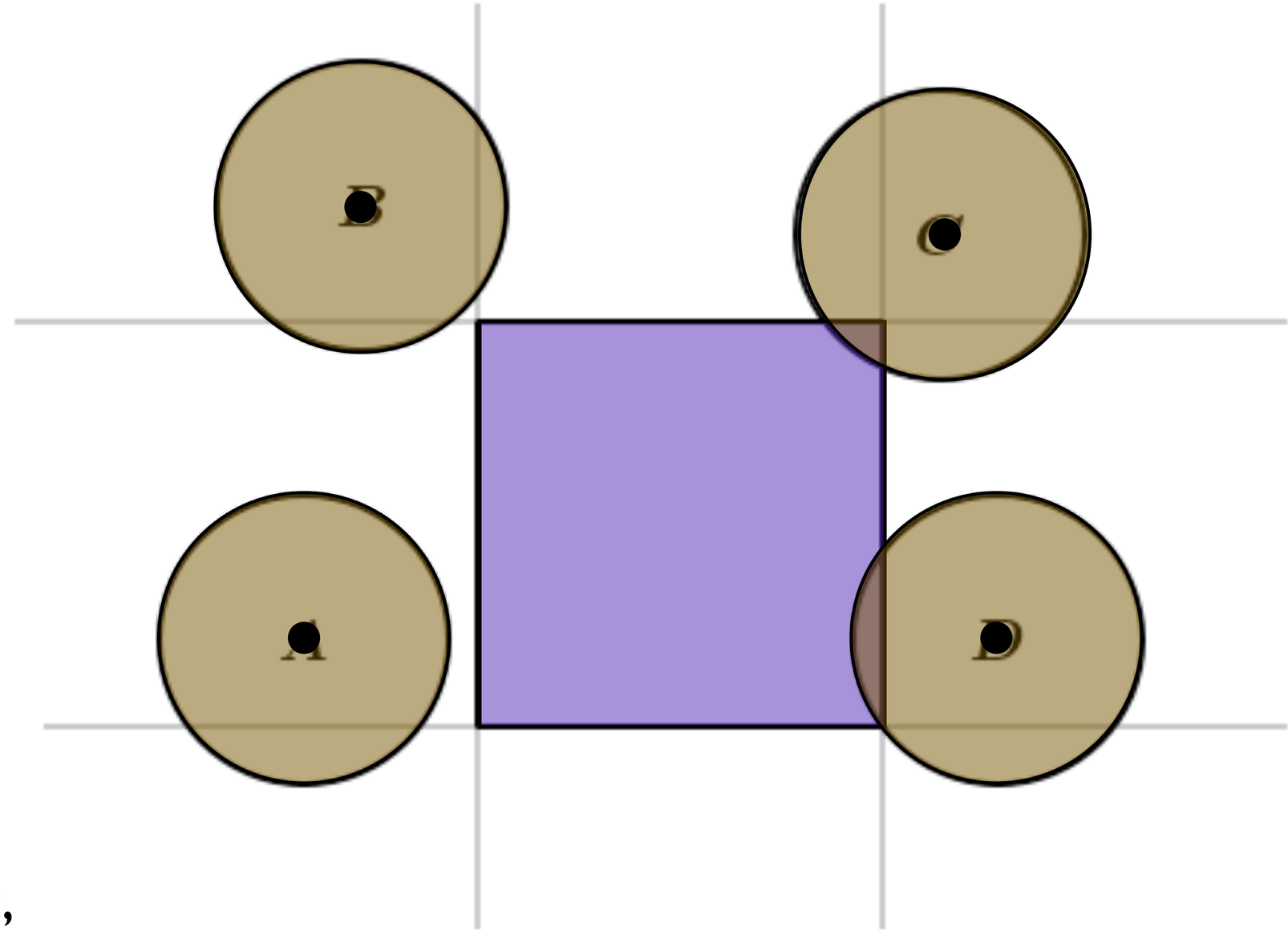
```
robot.links[x].bbox = [[x_min,y_min,z_min], [x_max,y_max,z_max]]
```

Sphere-bbox test

If sphere separable from AABB in any dimension, return no collision

`loc_y-radius>y_max?`

`loc_y+radius<y_min?`

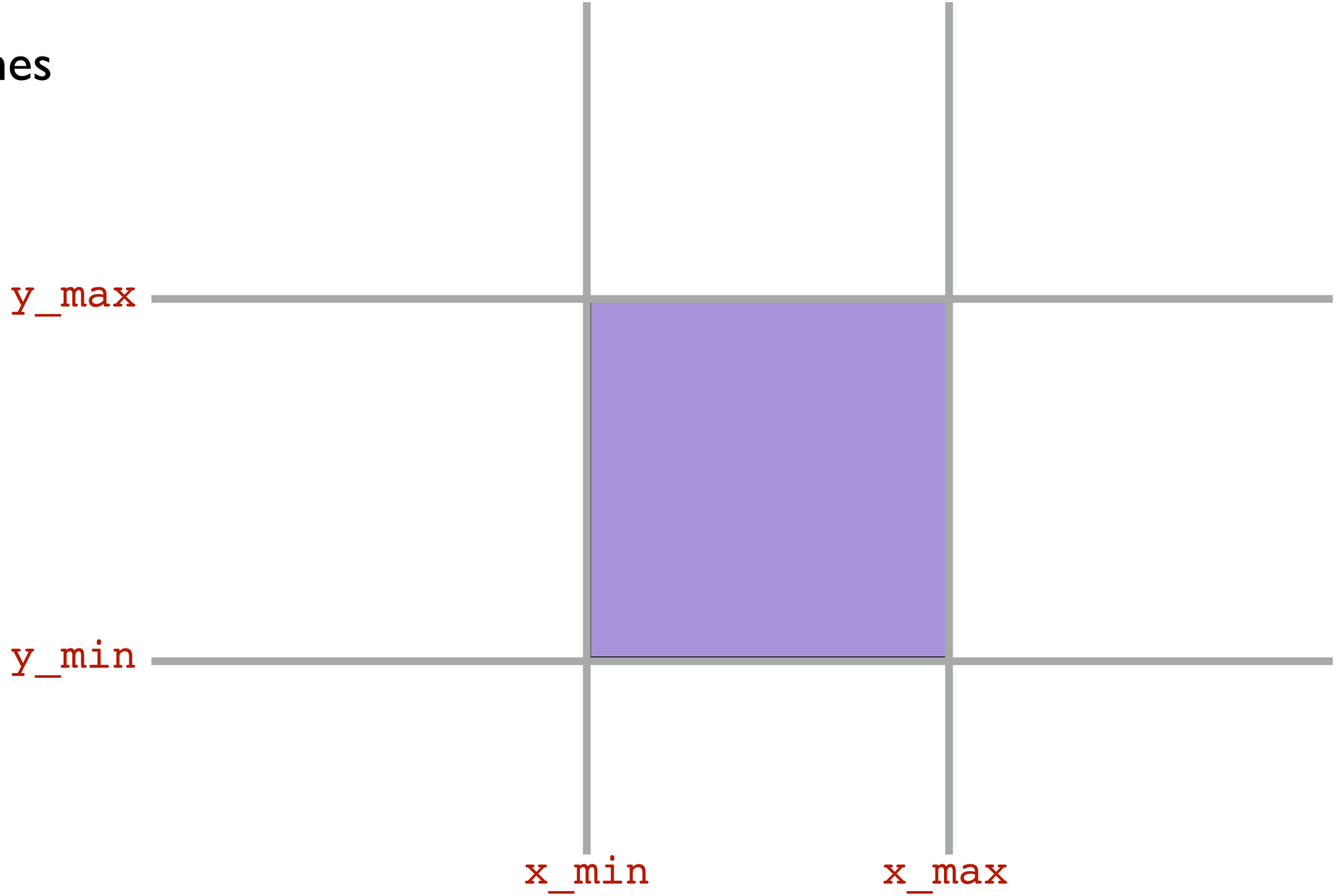


`loc_x+radius<x_min?`

`loc_x-radius>x_max?`

If sphere collides on all tests, return collision

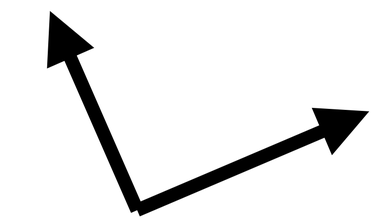
Separating planes



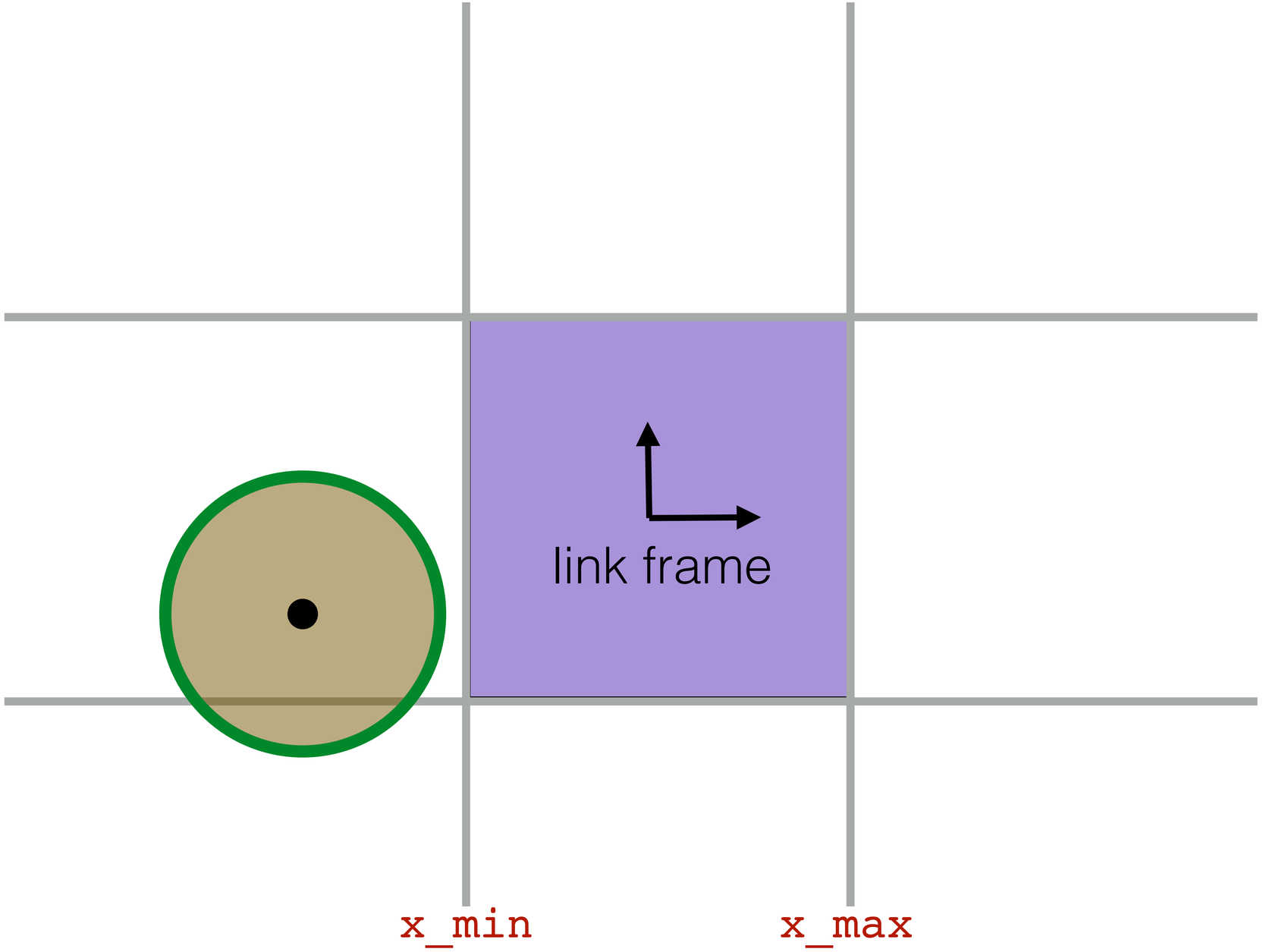
Transform centers of sphere obstacles into link coordinates

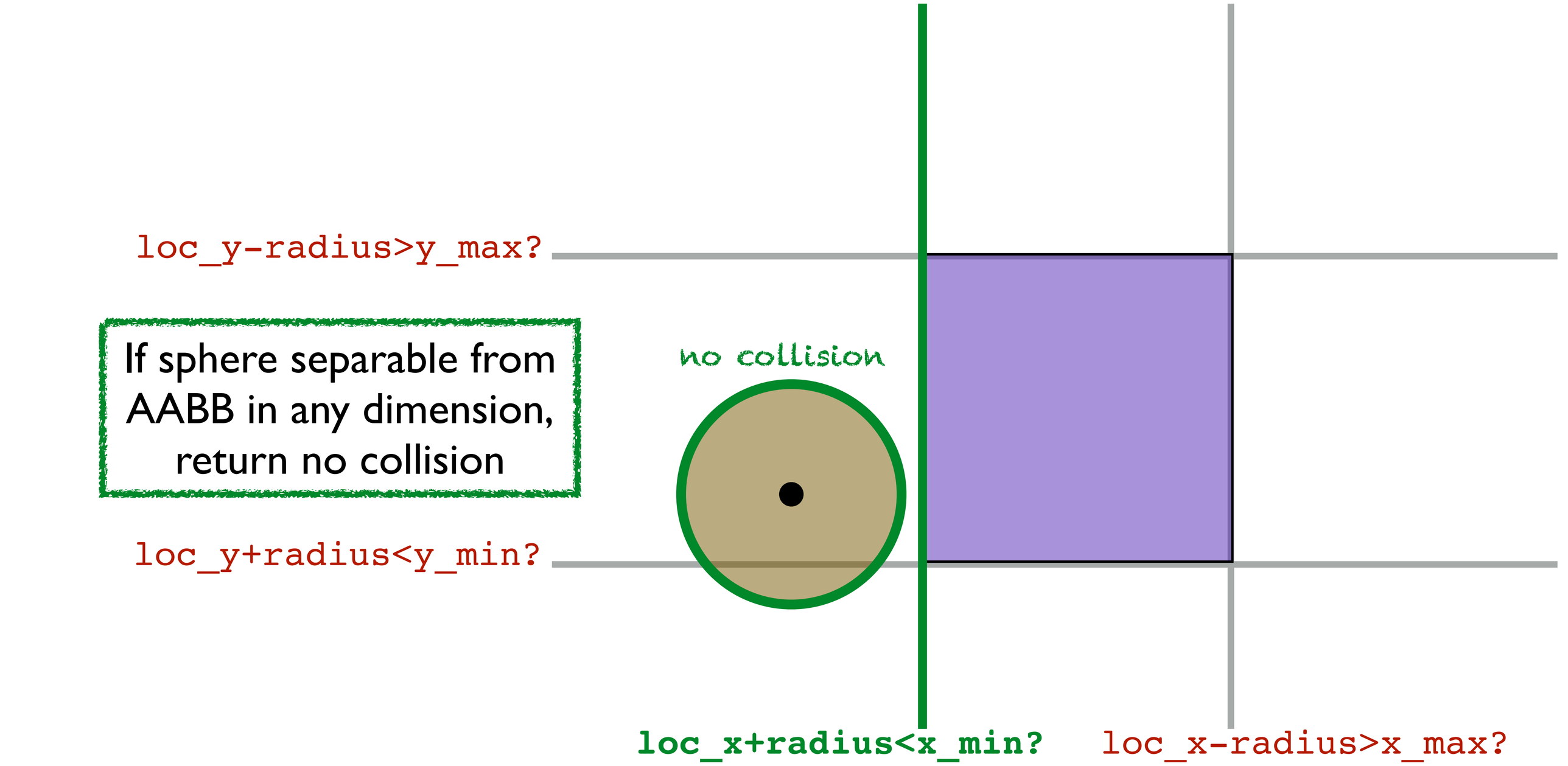
(Remember inverse of homogeneous transform?)

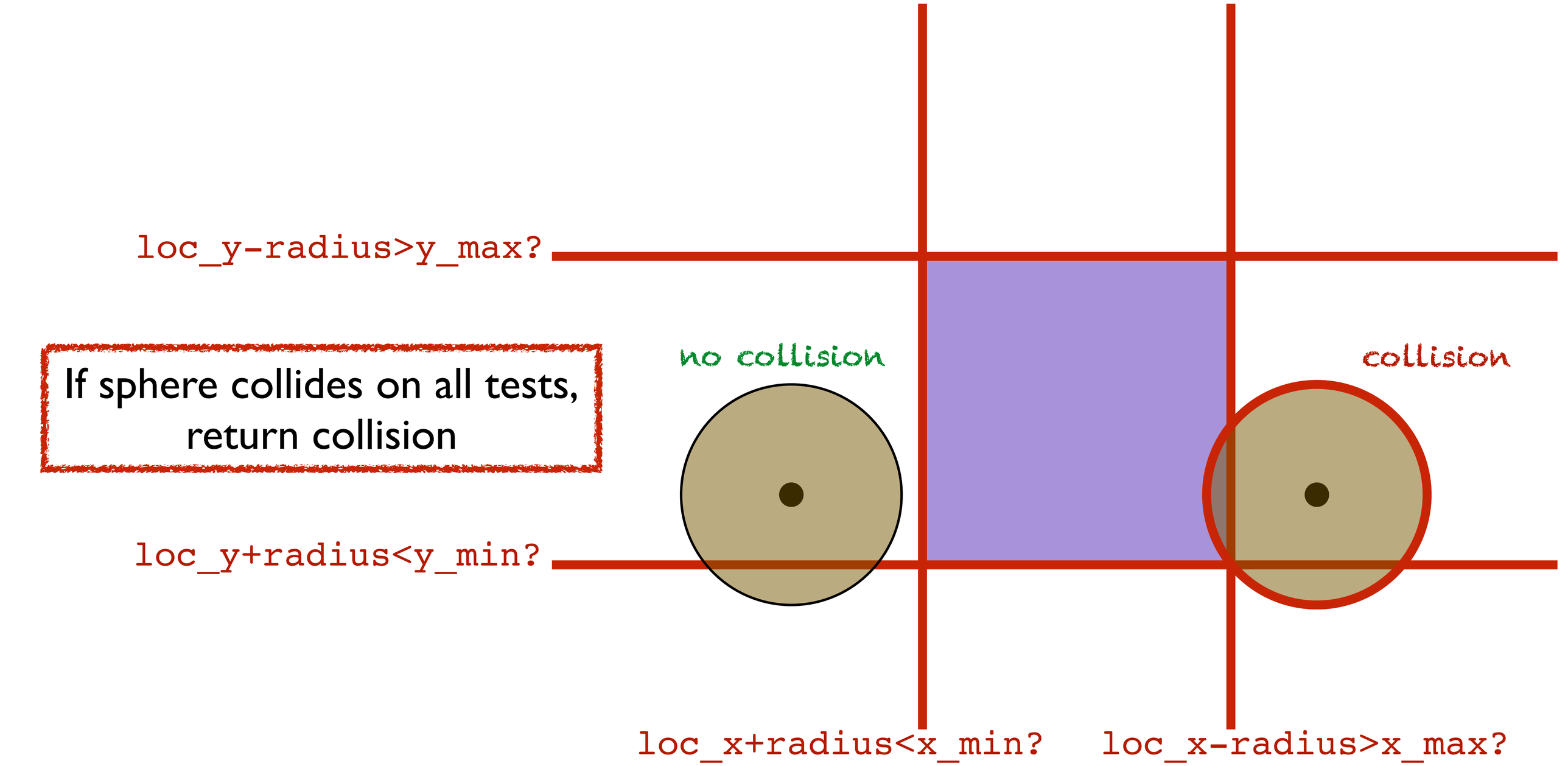
$$p^{link} = (T_{link}^{world})^{-1} p^{world}$$

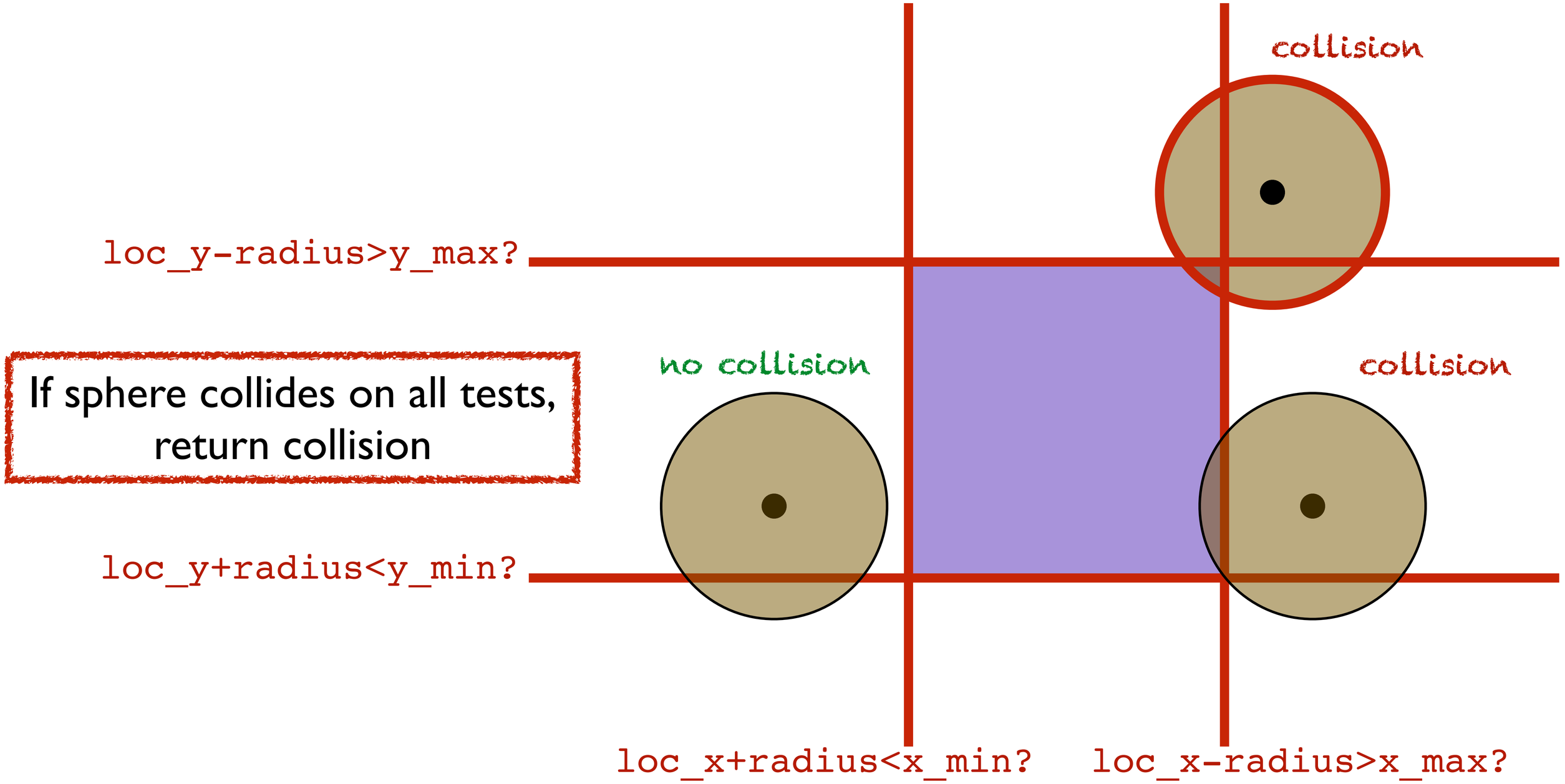


world frame

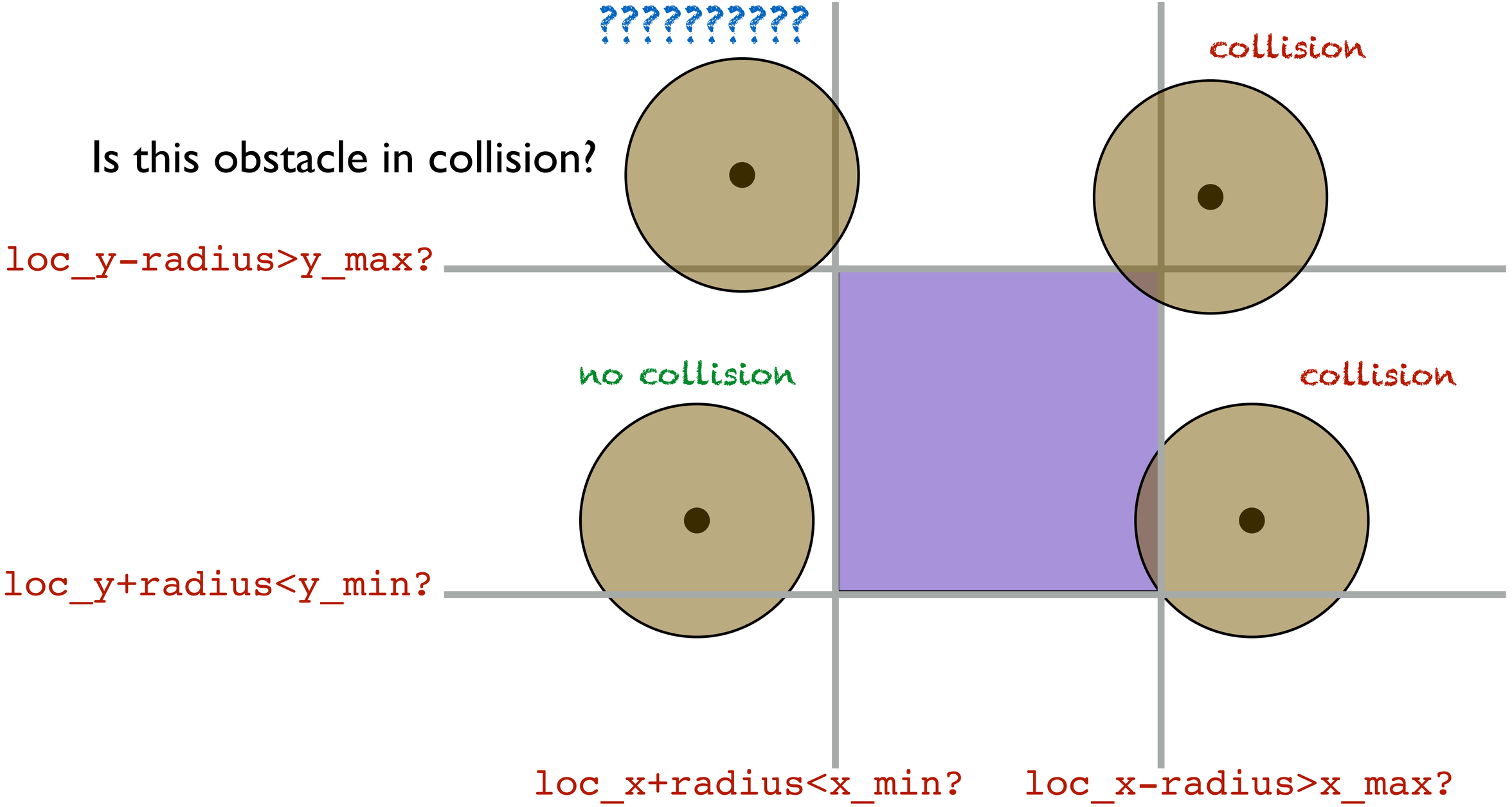



loc_y-radius>y_max?
If sphere separable from AABB in any dimension, return no collision
no collision
loc_y+radius<y_min?
loc_x+radius<x_min?
loc_x-radius>x_max?

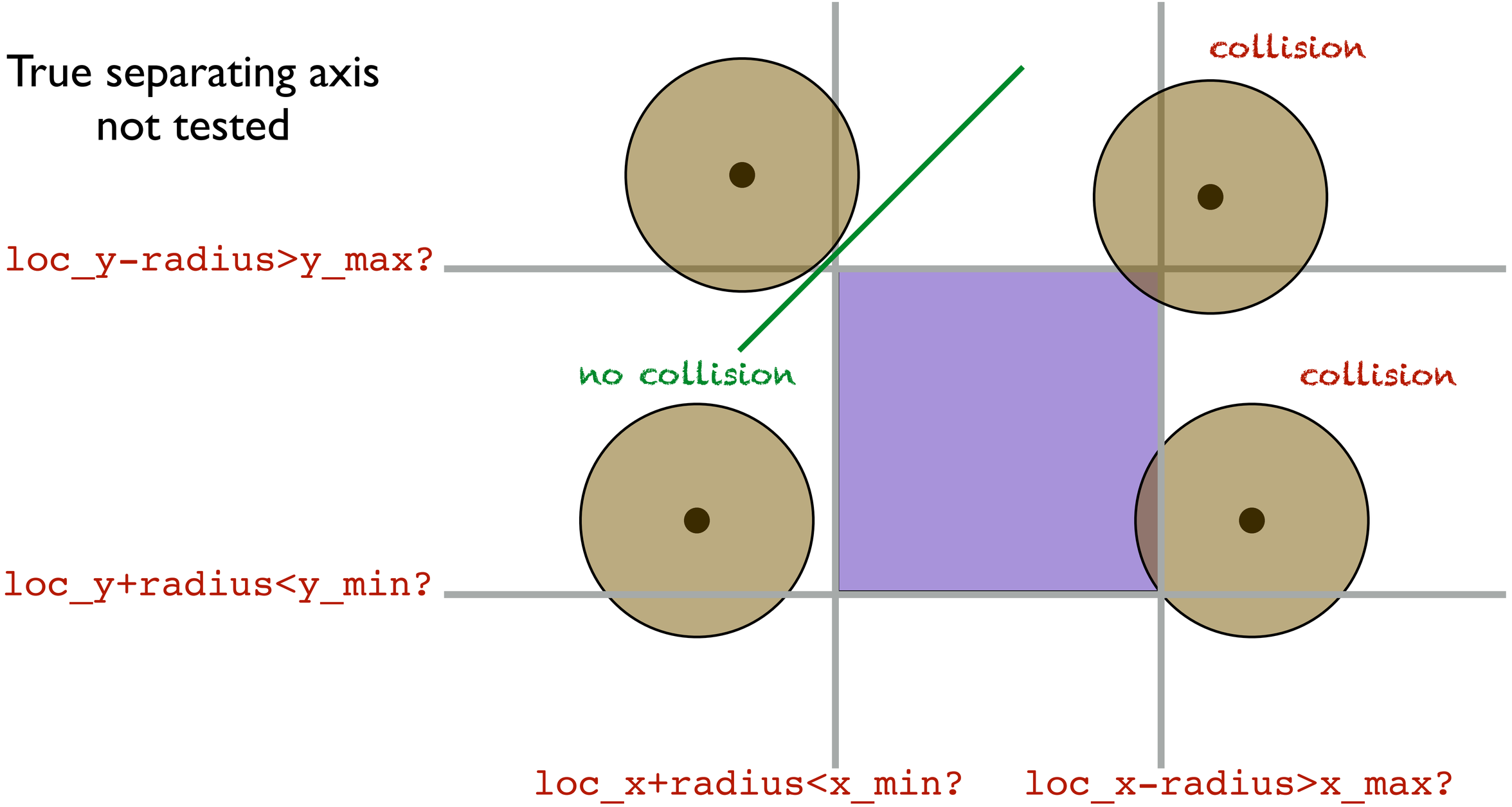

loc_y-radius>y_max?
If sphere collides on all tests, return collision
no collision
collision
loc_y+radius<y_min?
loc_x+radius<x_min?
loc_x-radius>x_max?

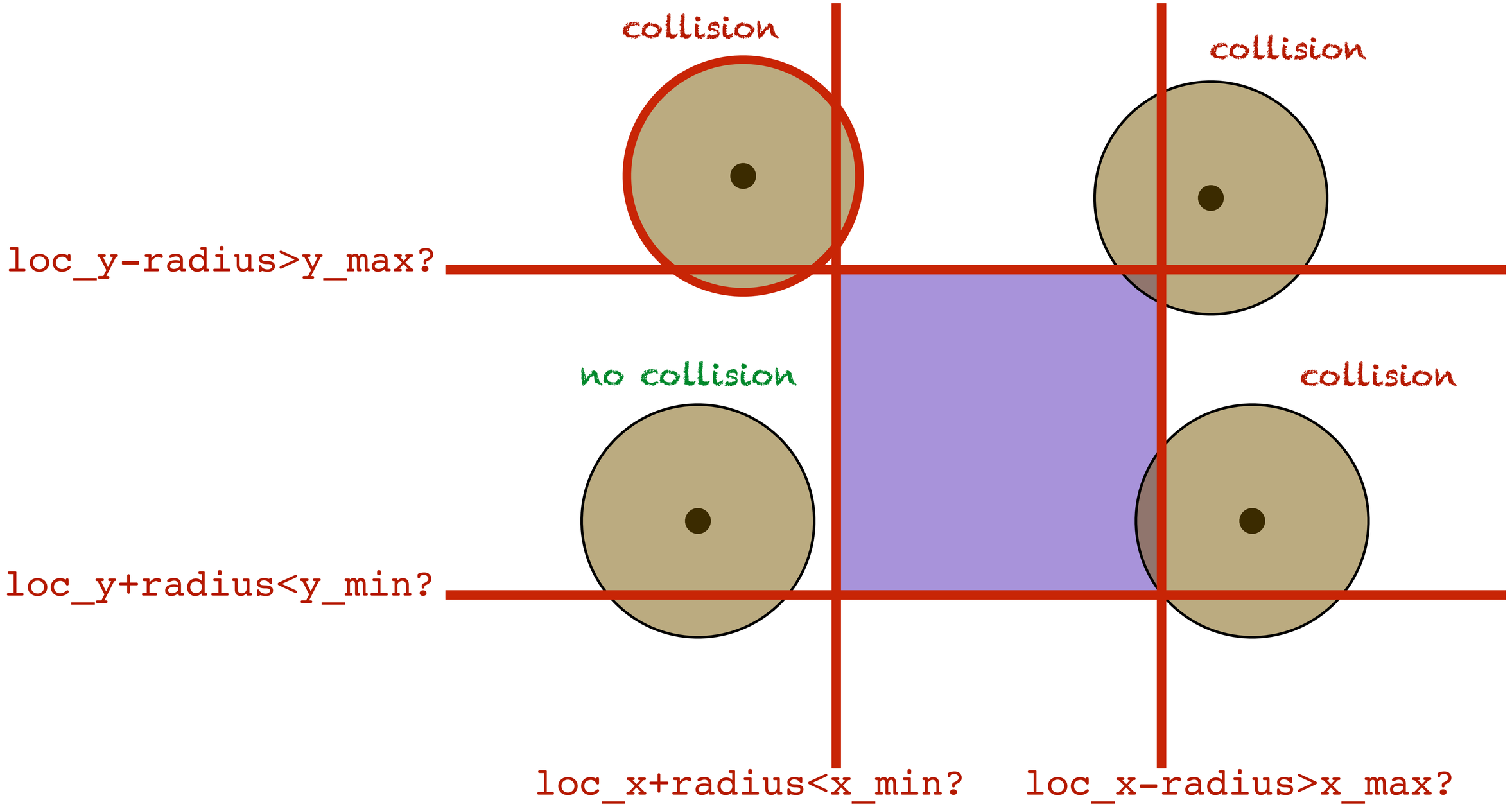

collision
loc_y-radius>y_max?
If sphere collides on all tests, return collision
no collision
collision
loc_y+radius<y_min?
loc_x+radius<x_min?
loc_x-radius>x_max?

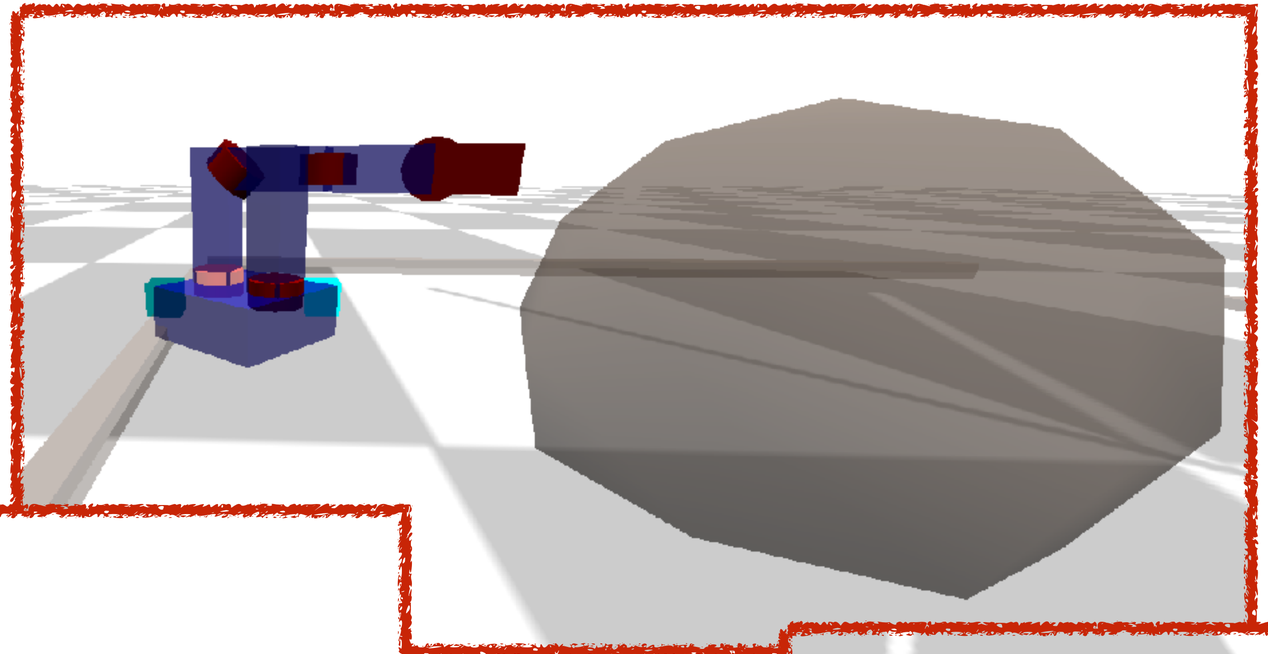
Is this obstacle in collision?

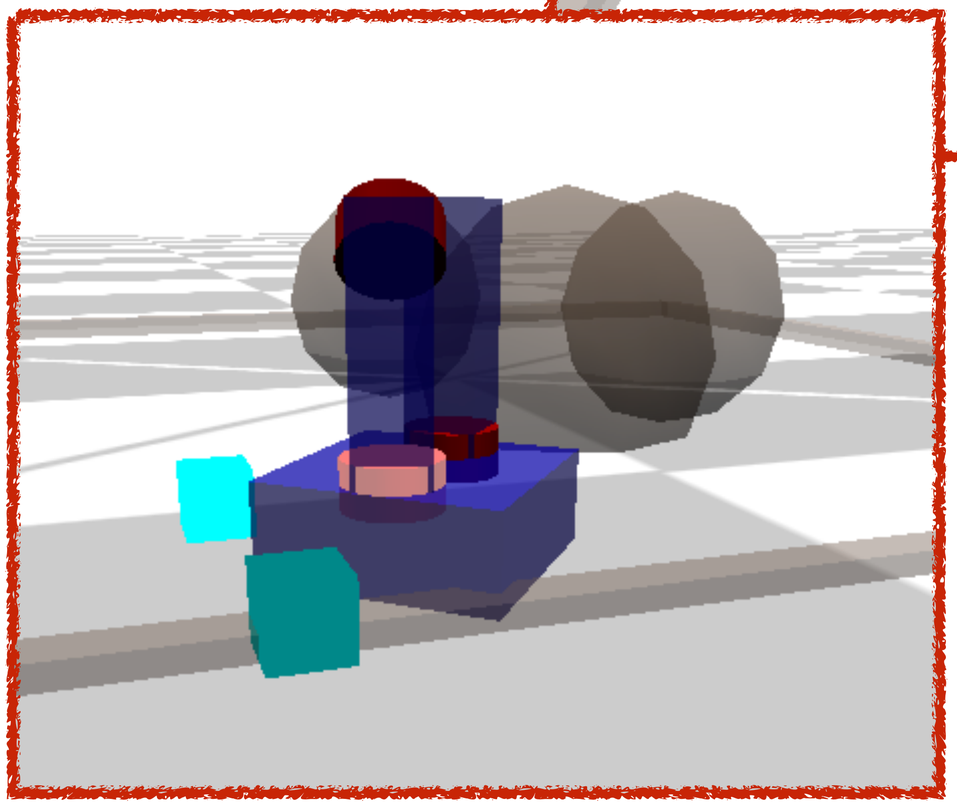


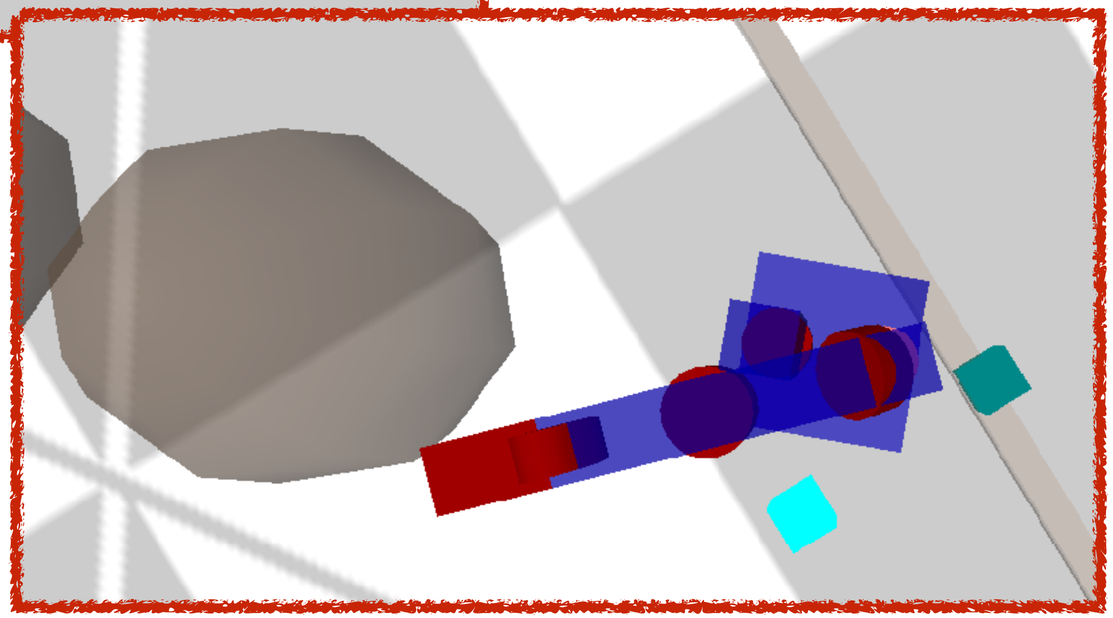
True separating axis not tested




collision
collision
loc_y-radius>y_max?
no collision
collision
loc_y+radius<y_min?
loc_x+radius<x_min?
loc_x-radius>x_max?







Last notes about planning visualization

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Next Lecture
Planning - New Frontiers