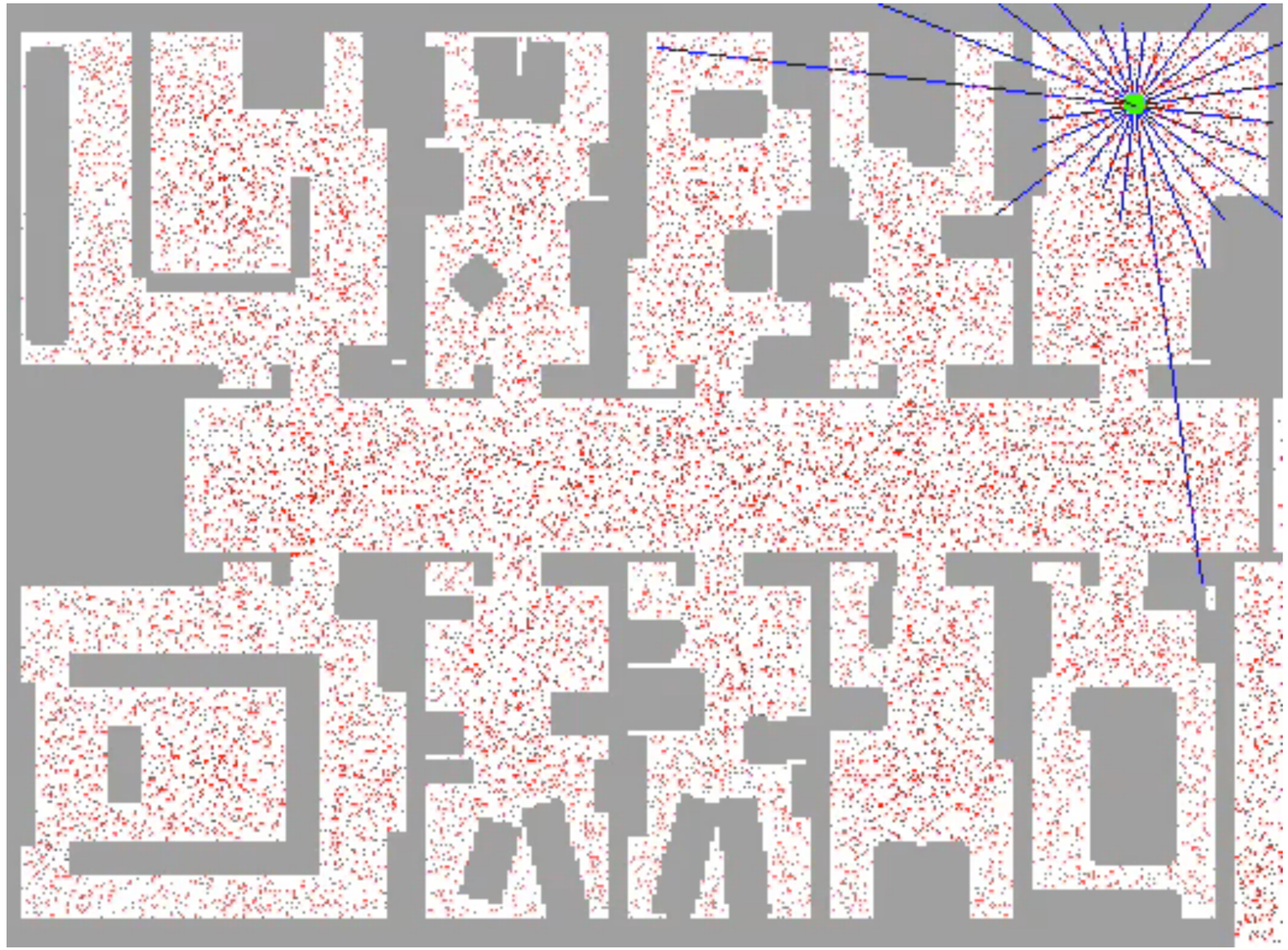
Lecture 20
Mobile Robotics - IV - Particle Filter

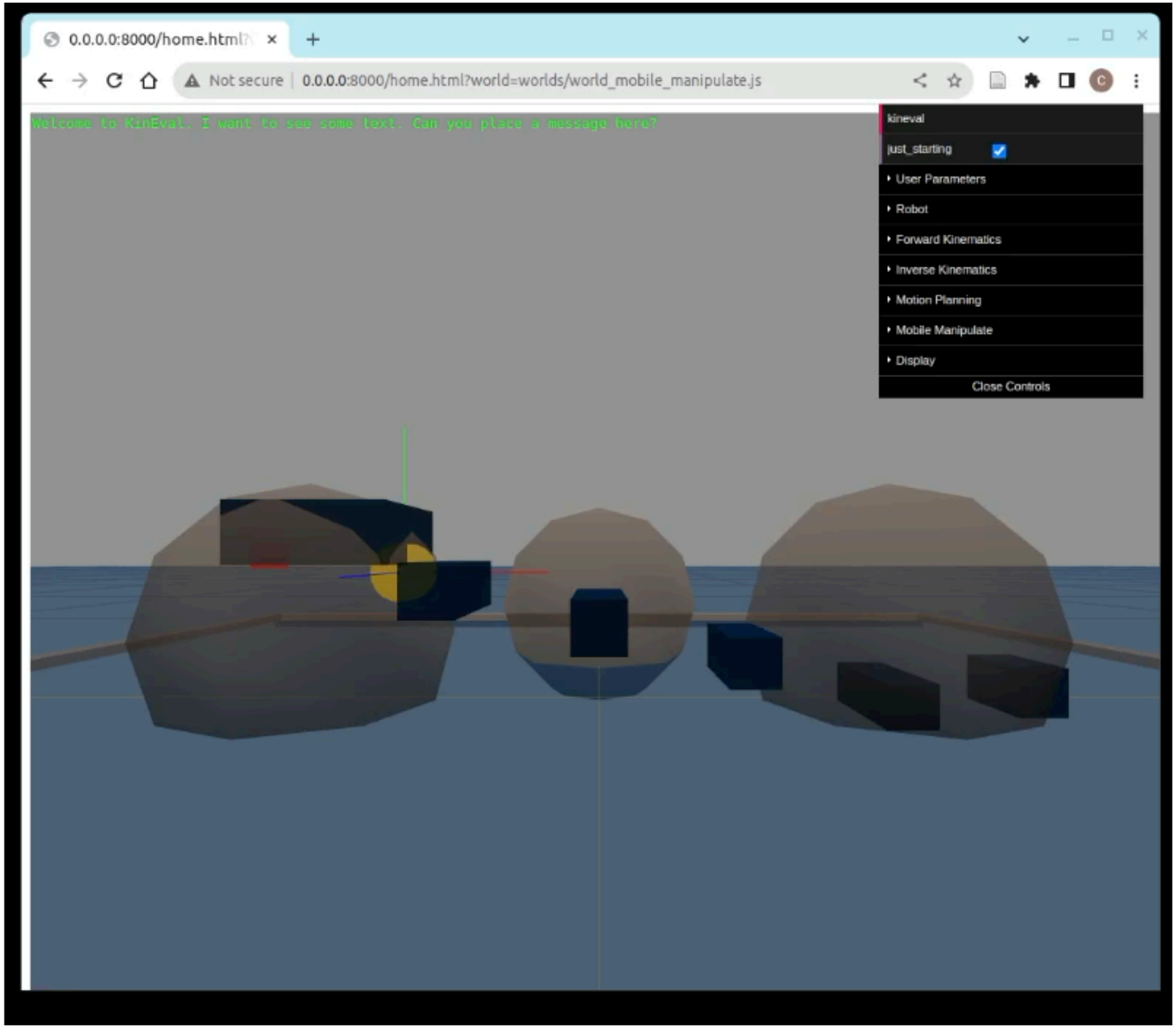


Course logistics

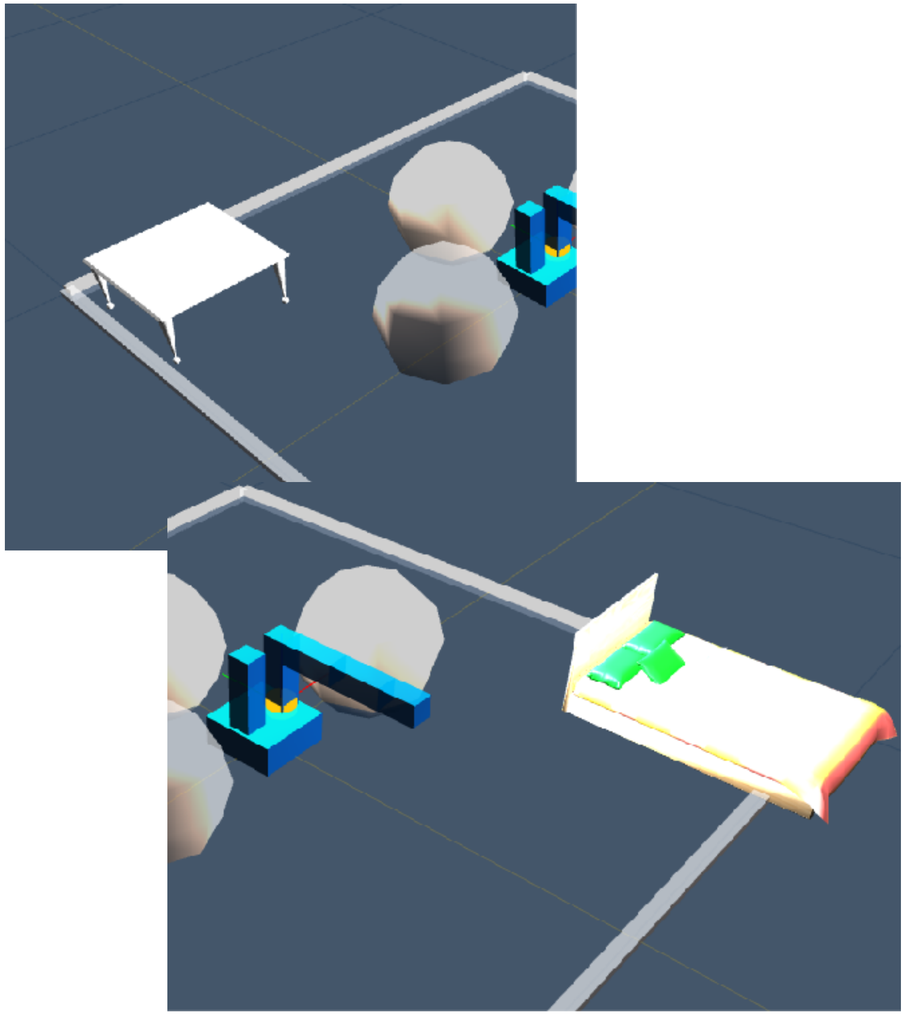
- **Quiz 18-19 was posted yesterday and was due before today's lecture**
- Project 4 is posted on 10/30 and will be due **11/15 (today)**
- Project 5 will be posted today
  - Start early!

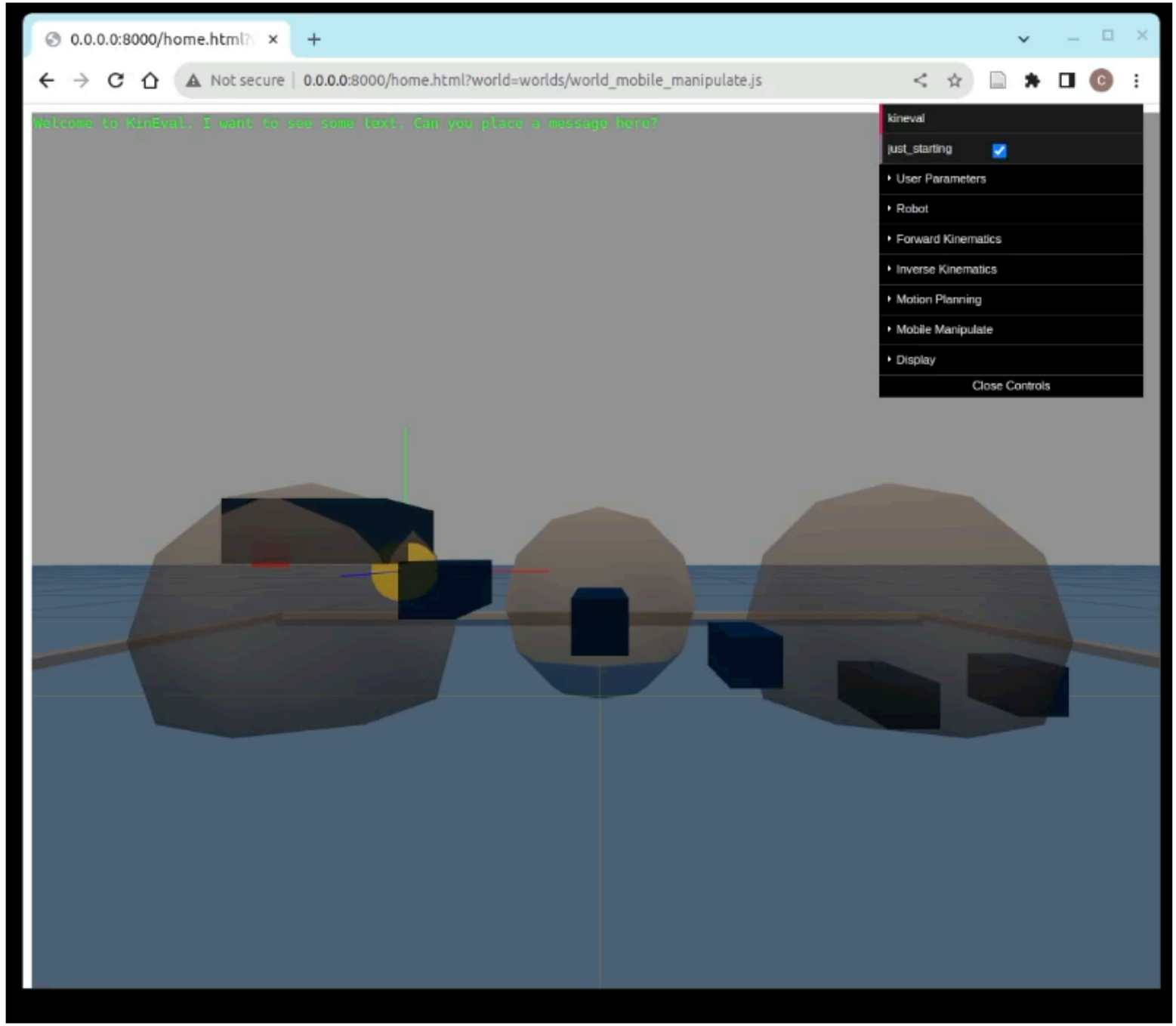
Project-5

- RRT-Connect
- FSM
- IK

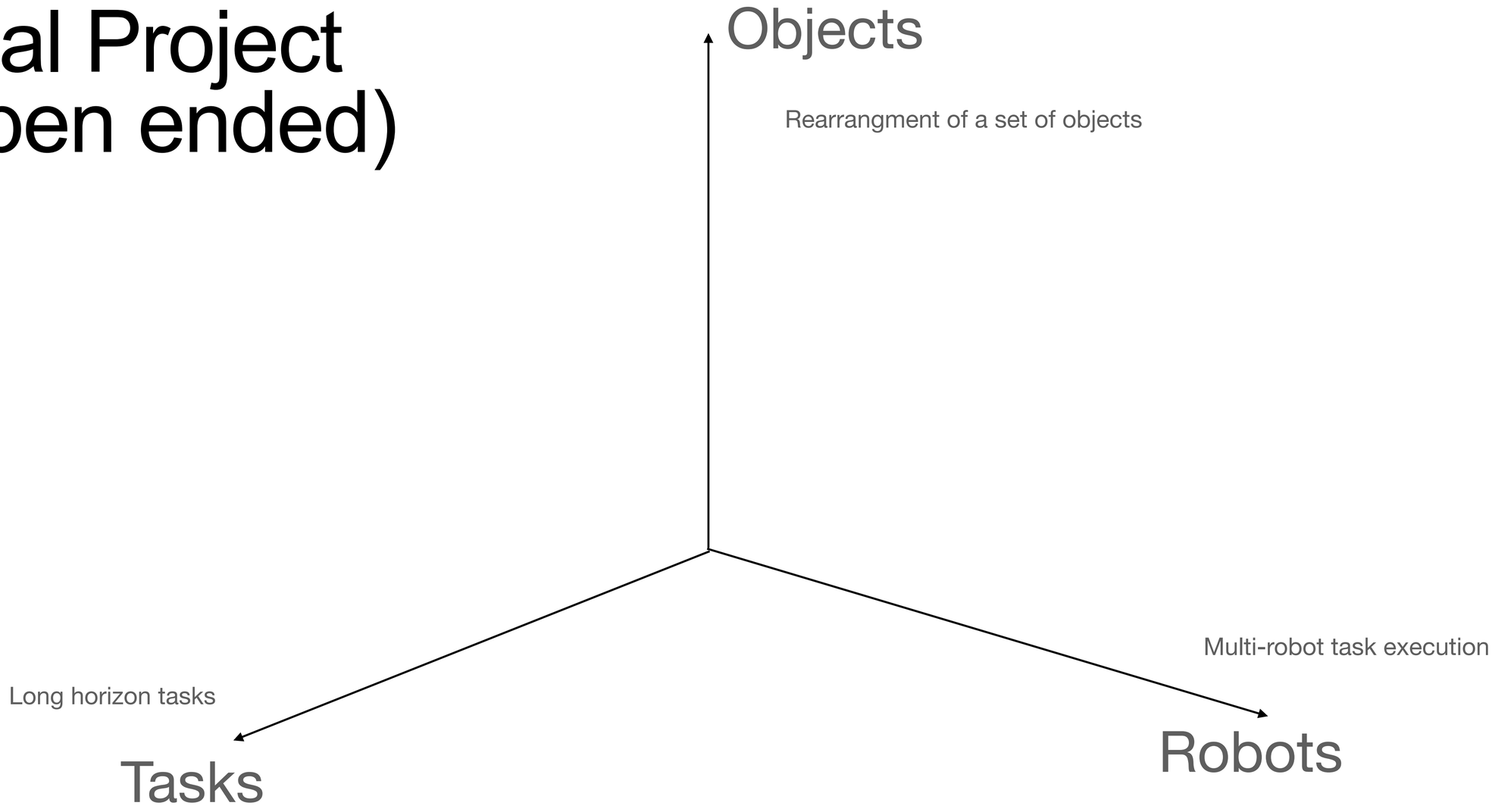


Final Project (Open ended)





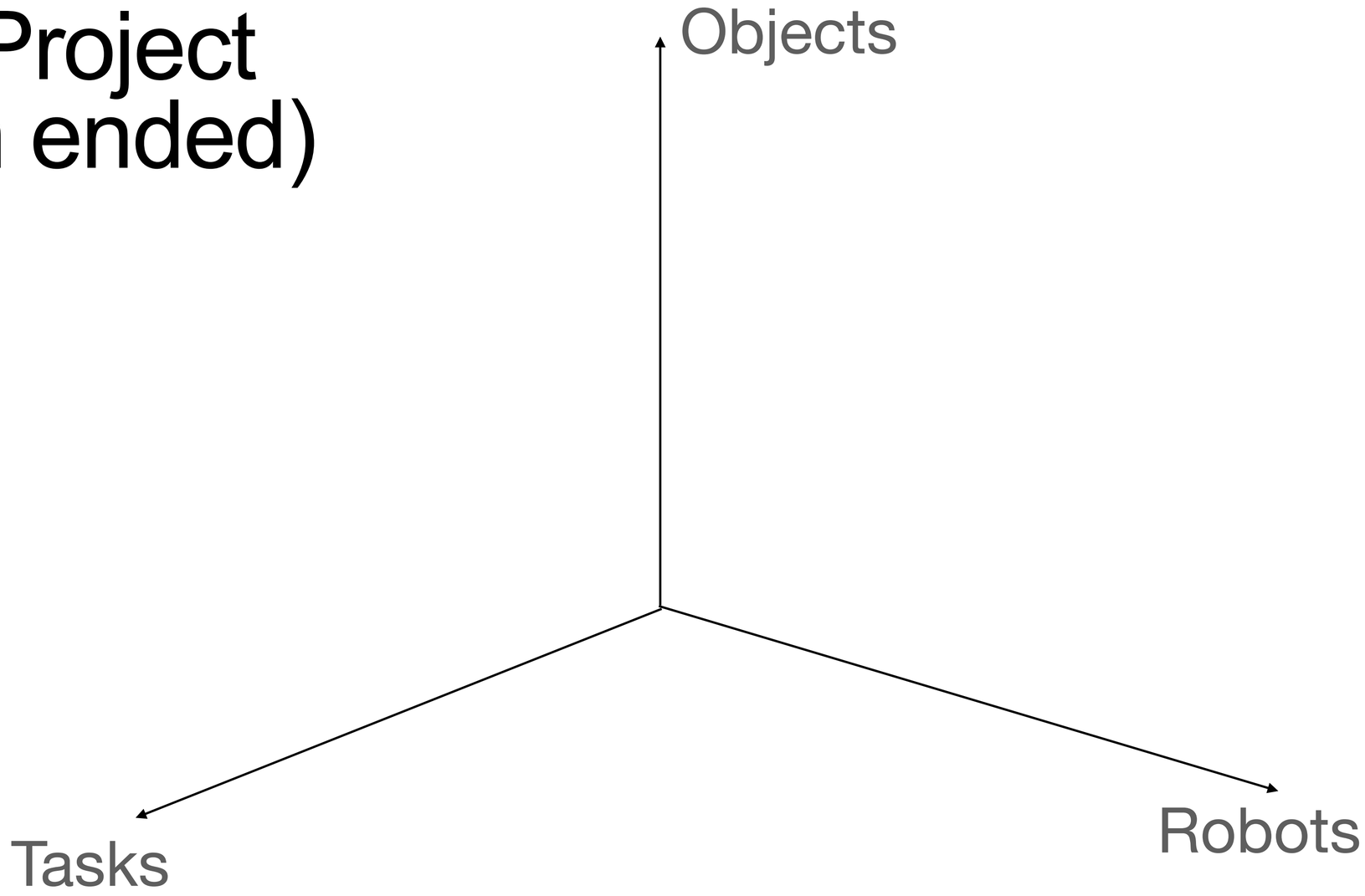
Final Project (Open ended)



Final Project (Open ended)
For inspiration!

Yang, Zhutian, Caelan Reed Garrett, Tomás Lozano-Pérez, Leslie Kaelbling, and Dieter Fox. "Sequence-based plan feasibility prediction for efficient task and motion planning." *arXiv preprint arXiv:2211.01576* (2022).

Final Project (Open ended)



Continuing previous Lecture
KF and EKF

Discrete Kalman Filter

Estimates the state x of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$

with a measurement

$$z_t = C_t x_t + \delta_t$$

Components of a Kalman Filter

A_t Matrix (nxn) that describes how the state evolves from t-1 to t without controls or noise.

B_t Matrix (nxl) that describes how the control u_t changes the state from t-1 to t.

C_t Matrix (kxn) that describes how to map the state x_t to an observation z_t.

ε_t δ_t Random variables representing the process and measurement noise that are assumed to be independent and normally distributed with covariance R_t and Q_t respectively.

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

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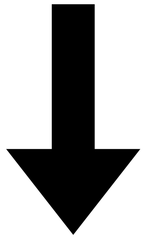
Non-linear Dynamic Systems

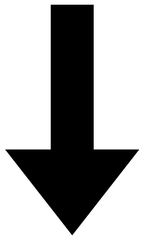
- Most realistic problems involve nonlinear functions

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$$



$$x_t = g(u_t, x_{t-1}) + \epsilon_t$$

$$z_t = C_t x_t + \delta_t$$



$$z_t = h(x_t) + \delta_t$$

Linearization

$$x_t = g(u_t, x_{t-1}) + \varepsilon_t$$

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \underbrace{g'(u_t, \mu_{t-1})}_{=: G_t} (x_{t-1} - \mu_{t-1})$$

$$= g(u_t, \mu_{t-1}) + G_t (x_{t-1} - \mu_{t-1})$$

$$z_t = h(x_t) + \delta_t$$

$$h(x_t) \approx h(\bar{\mu}_t) + \underbrace{\frac{\partial h(\bar{\mu}_t)}{\partial x_t}}_{=: H_t} (x_t - \bar{\mu}_t)$$

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EKF Algorithm

1. **Extended_Kalman_filter**$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

2. Prediction:

3. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ $\longleftarrow$ $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$

4. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$ $\longleftarrow$ $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:

6. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$ $\longleftarrow$ $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$

7. $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$ $\longleftarrow$ $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

8. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ $\longleftarrow$ $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t} \qquad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

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Localization

> "Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities." [Cox '91]

- **Given**
  - Map of the environment.
  - Sequence of sensor measurements.
- **Wanted**
  - Estimate of the robot's position.
- **Problem classes**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)

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EKF Summary

- Highly efficient: Polynomial in measurement dimensionality k and state dimensionality n:

$$O(k^{2.376} + n^2)$$

- Not optimal!
- Can diverge if nonlinearities are large!
- Works surprisingly well even when all assumptions are violated!

Particle Filter

A Bayesian Filter Implementation

Motivation

- So far, we discussed the
  - Kalman filter: Gaussian, linearization problems, multi-modal beliefs

- Particle filters are a way to **efficiently** represent **non-Gaussian distributions**

- Basic principle
  - Set of state hypotheses (“particles”)
  - Survival-of-the-fittest

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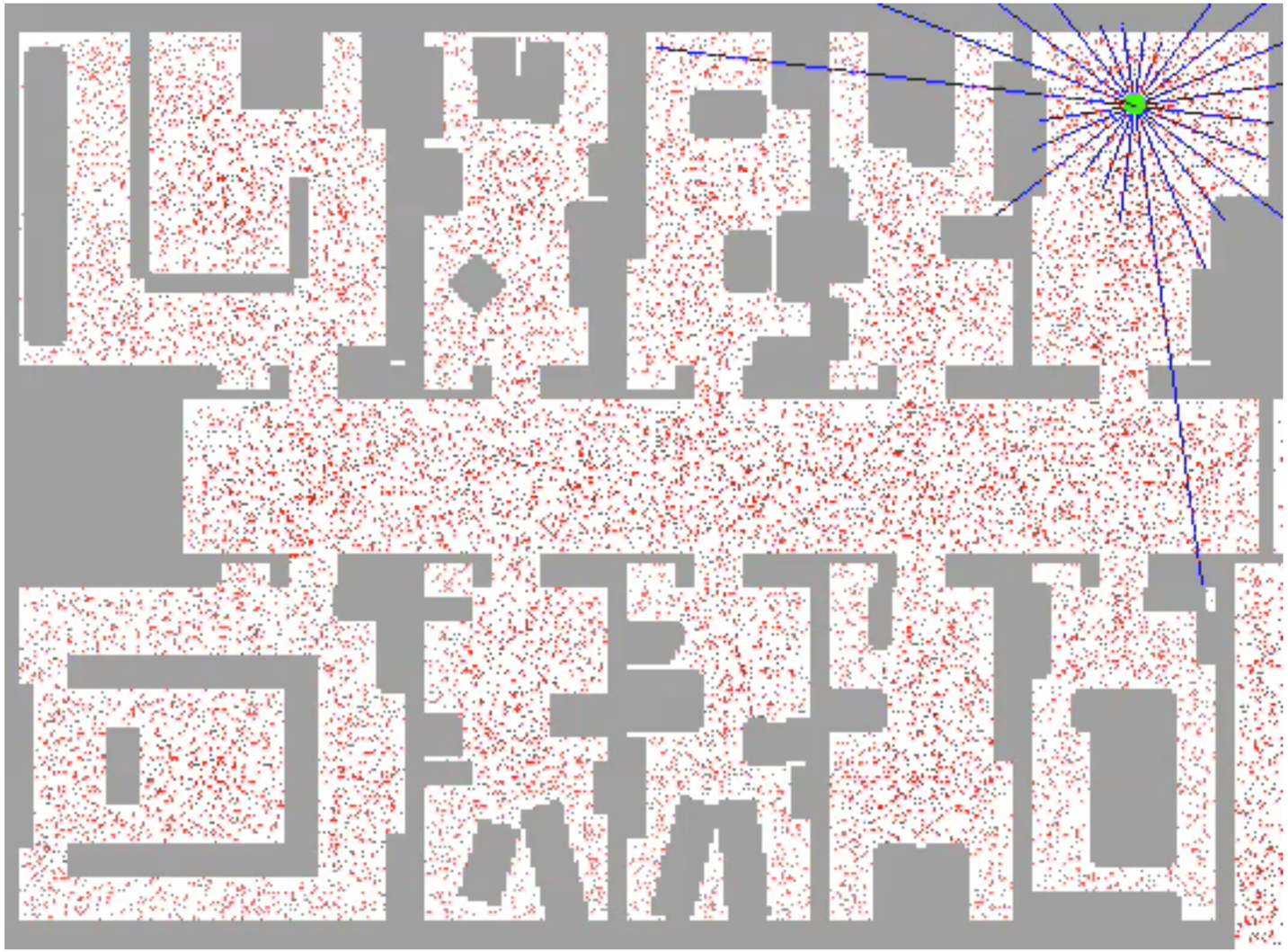
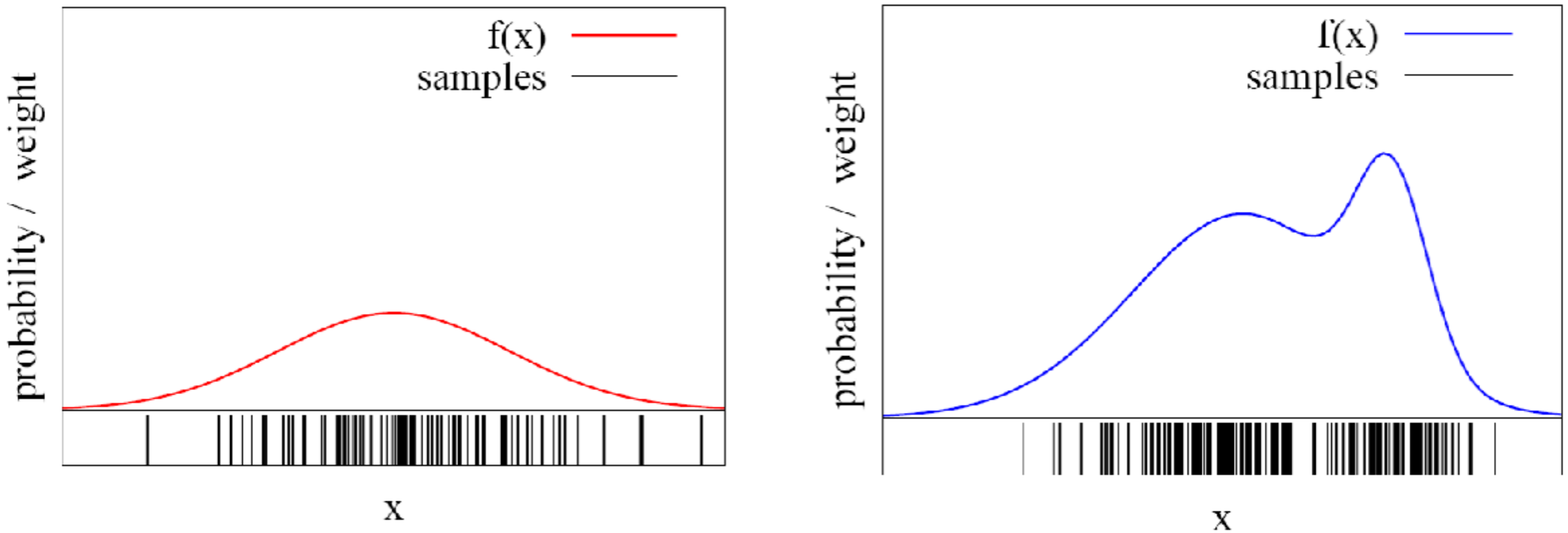
Sample-based Localization (sonar)



Density Approximation

- Particle sets can be used to approximate densities

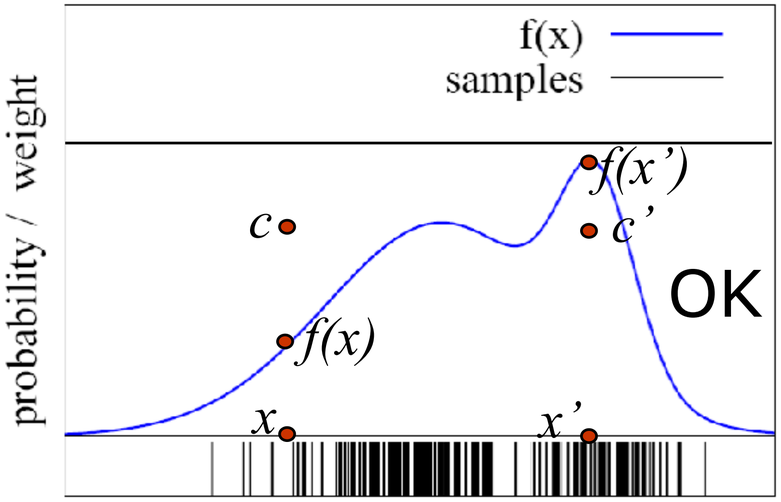


- The more particles fall into an interval, the higher the probability of that interval

- How to draw samples form a function/distribution?

Rejection Sampling

- Let us assume that $f(x)<=1$ for all x
- Sample x from a uniform distribution
- Sample c from [0,1]
- if $f(x) > c$ keep the sample
  otherwise reject the sampe



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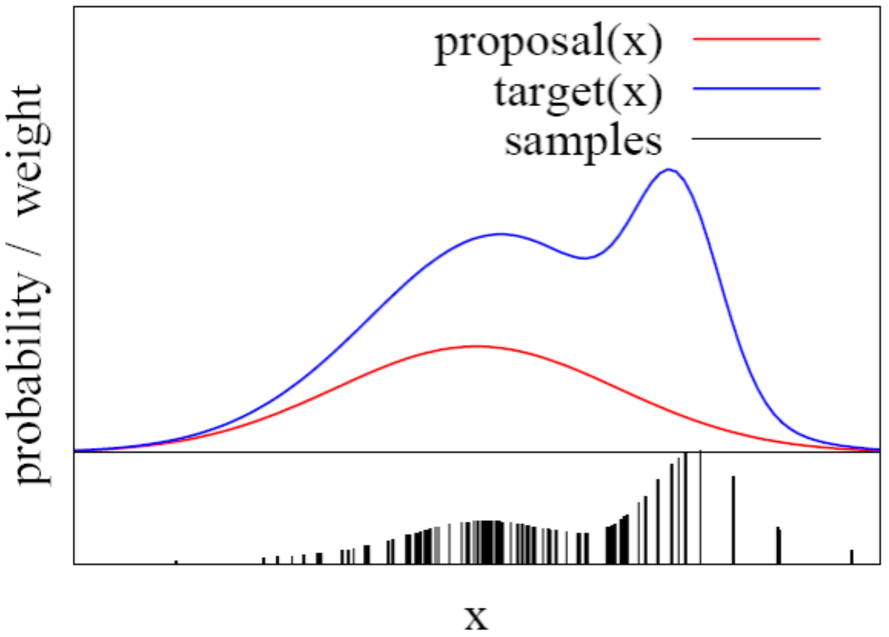
Importance Sampling Principle

- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the "differences between g and f"
- $w = f / g$
- f is often called target
- g is often called proposal



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Particle Filter for State estimation

- Non-parametric approach
- Recursive Bayes Filter
- Models the distribution by samples
- **Prediction:** draw from the proposal g
- **Correction:** weighting by the ratio of the target f and the proposal g

The more samples we use, the better is the estimate

Particle Filter Algorithm

1. Sample the particles using the proposal distribution.

$$x_t^{[j]} \sim \text{proposal}(x_t | \dots)$$

2. Compute the importance weights

$$w_t^{[j]} = \frac{\text{target}(x_t^{[j]})}{\text{proposal}(x_t^{[j]})}$$

3. Resampling: Draw samples i with propobability $w_t^{[i]}$ and repeat J times

Particle Filter Algorithm

Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**

1: $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

2: *for* $j = 1$ *to* J *do*

3: *sample* $x_t^{[j]} \sim \pi(x_t)$

4: $w_t^{[j]} = \frac{p(x_t^{[j]})}{\pi(x_t^{[j]})}$

5: $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[j]}, w_t^{[j]} \rangle$

6: *endfor*

7: *for* $j = 1$ *to* J *do*

8: *draw* $i \in 1, \ldots, J$ *with probability* $\propto w_t^{[i]}$

9: *add* $x_t^{[i]}$ *to* $\mathcal{X}_t$

10: *endfor*

11: *return* $\mathcal{X}_t$

Particle Filter Algorithm

$$Bel(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})\, dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$\begin{aligned} w^i_t &= \frac{\text{target distribution}}{\text{proposal distribution}} \\ &= \frac{\eta\; p(z_t \mid x_t)\; p(x_t \mid x_{t-1}, u_{t-1})\; Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\; Bel(x_{t-1})} \\ &\propto p(z_t \mid x_t) \end{aligned}$$

Particle Filter

Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**

1: $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
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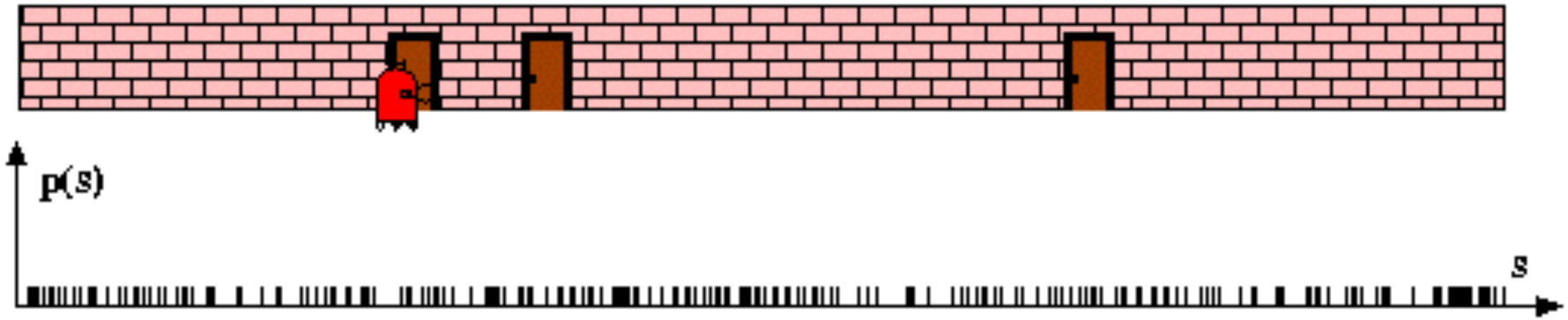
6: *endfor*
7: *for* $j = 1$ *to* J *do*
8: *draw* $i \in 1, \ldots, J$ *with probability* $\propto w_t^{[i]}$
9: *add* $x_t^{[i]}$ *to* $\mathcal{X}_t$
10: *endfor*
11: *return* $\mathcal{X}_t$

Particle Filter for Localization

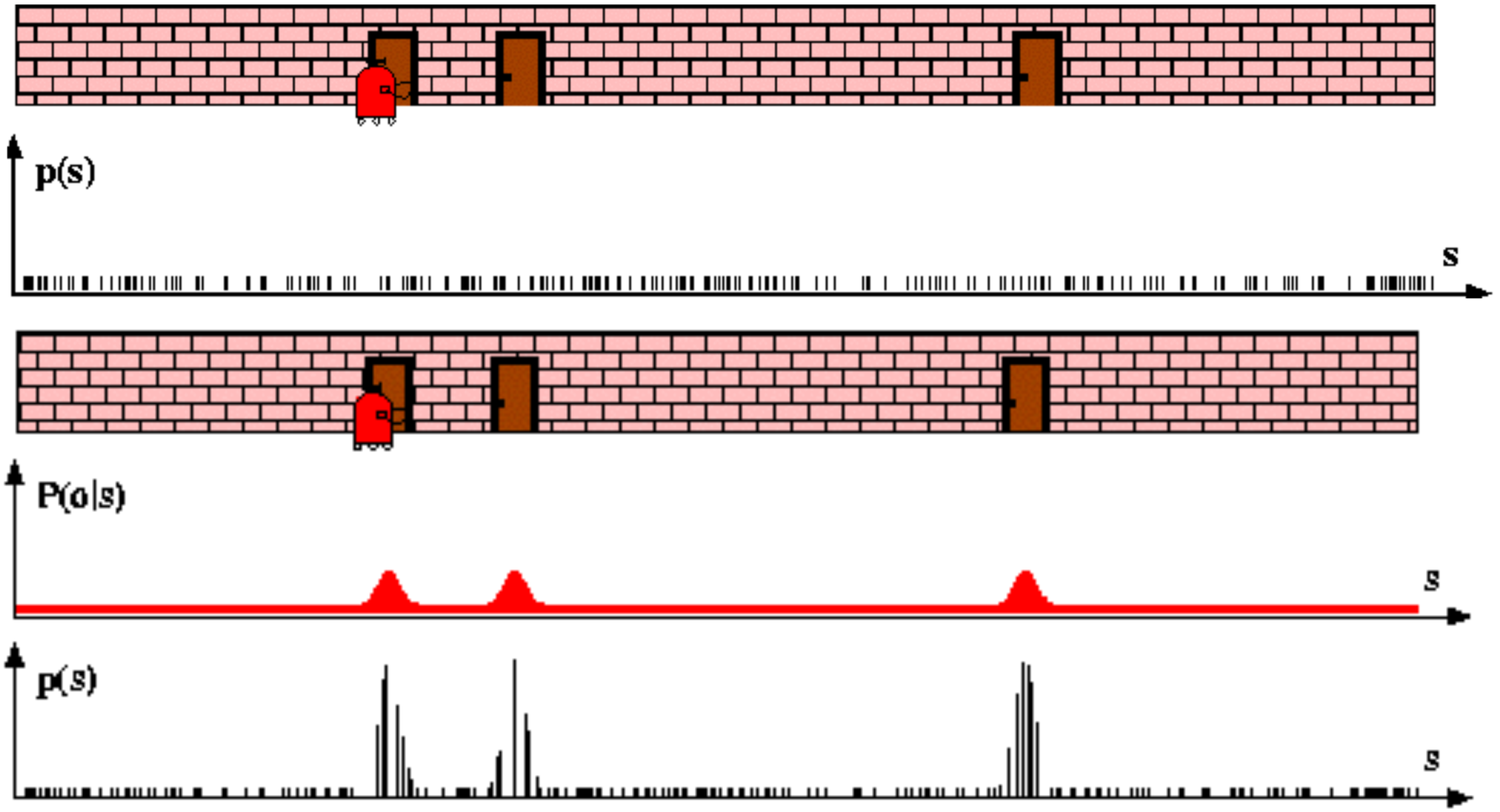
Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**

1: $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
2: *for* $j = 1$ *to* J *do*
3: *sample* $x_t^{[j]} \sim p(x_t \mid u_t, x_{t-1}^{[j]})$
4: $w_t^{[j]} = p(z_t \mid x_t^{[j]})$
5: $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[j]}, w_t^{[j]} \rangle$
6: *endfor*
7: *for* $j = 1$ *to* J *do*
8: *draw* $i \in 1, \ldots, J$ *with probability* $\propto w_t^{[i]}$
9: *add* $x_t^{[i]}$ *to* $\mathcal{X}_t$
10: *endfor*
11: *return* $\mathcal{X}_t$

Particle Filters



Sensor Information: Importance Sampling



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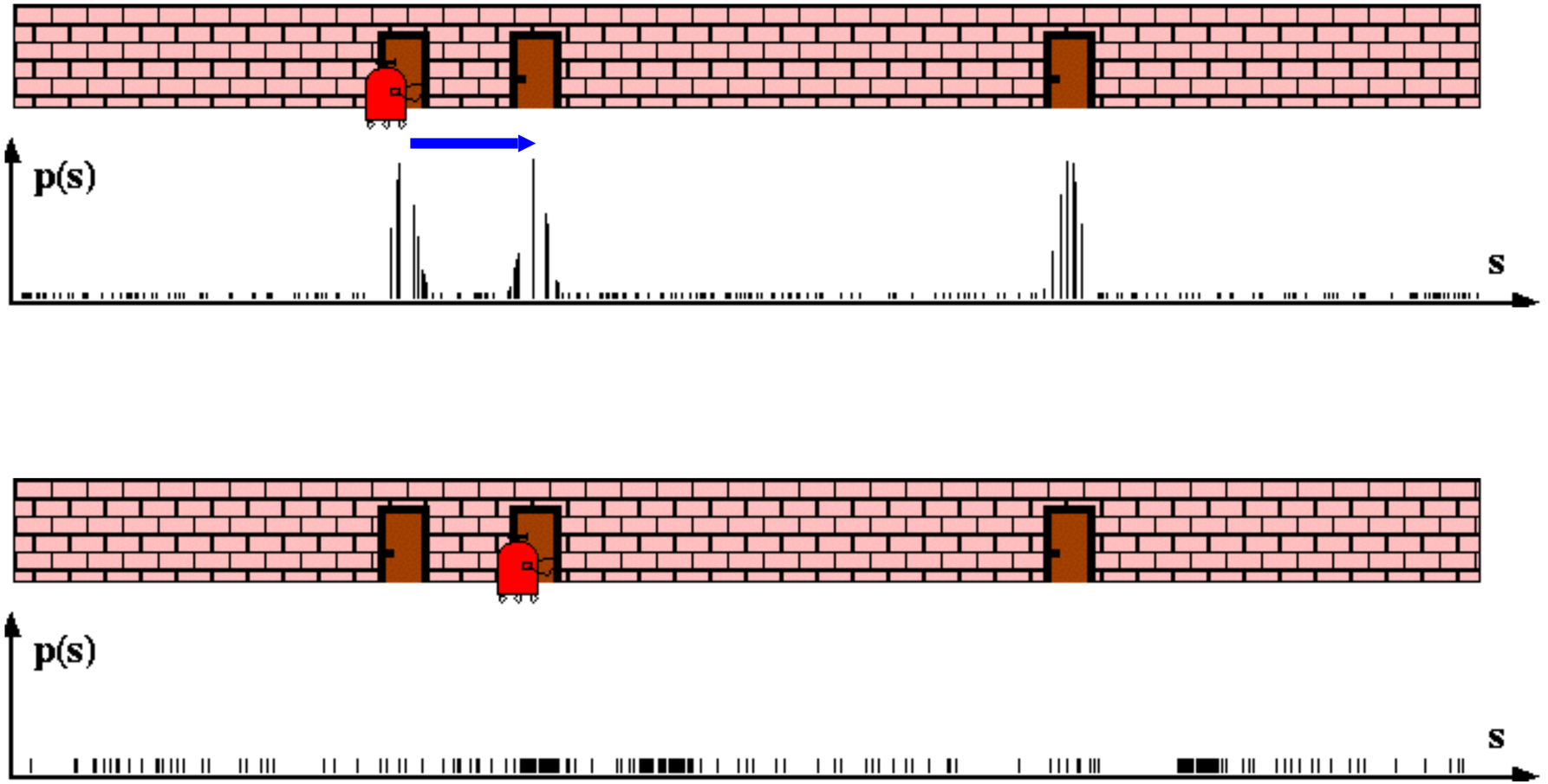
Robot Motion



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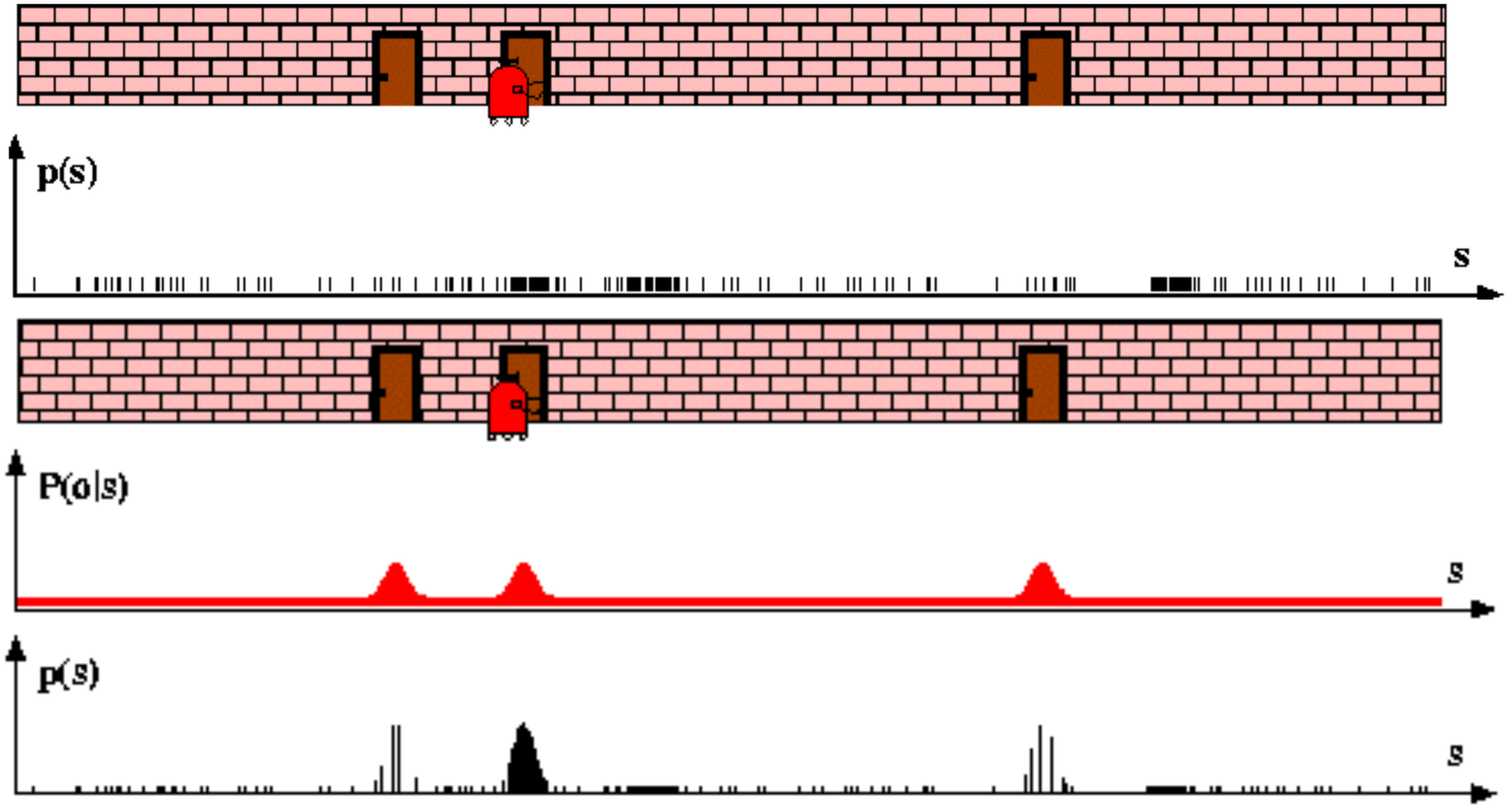
Sensor Information: Importance Sampling



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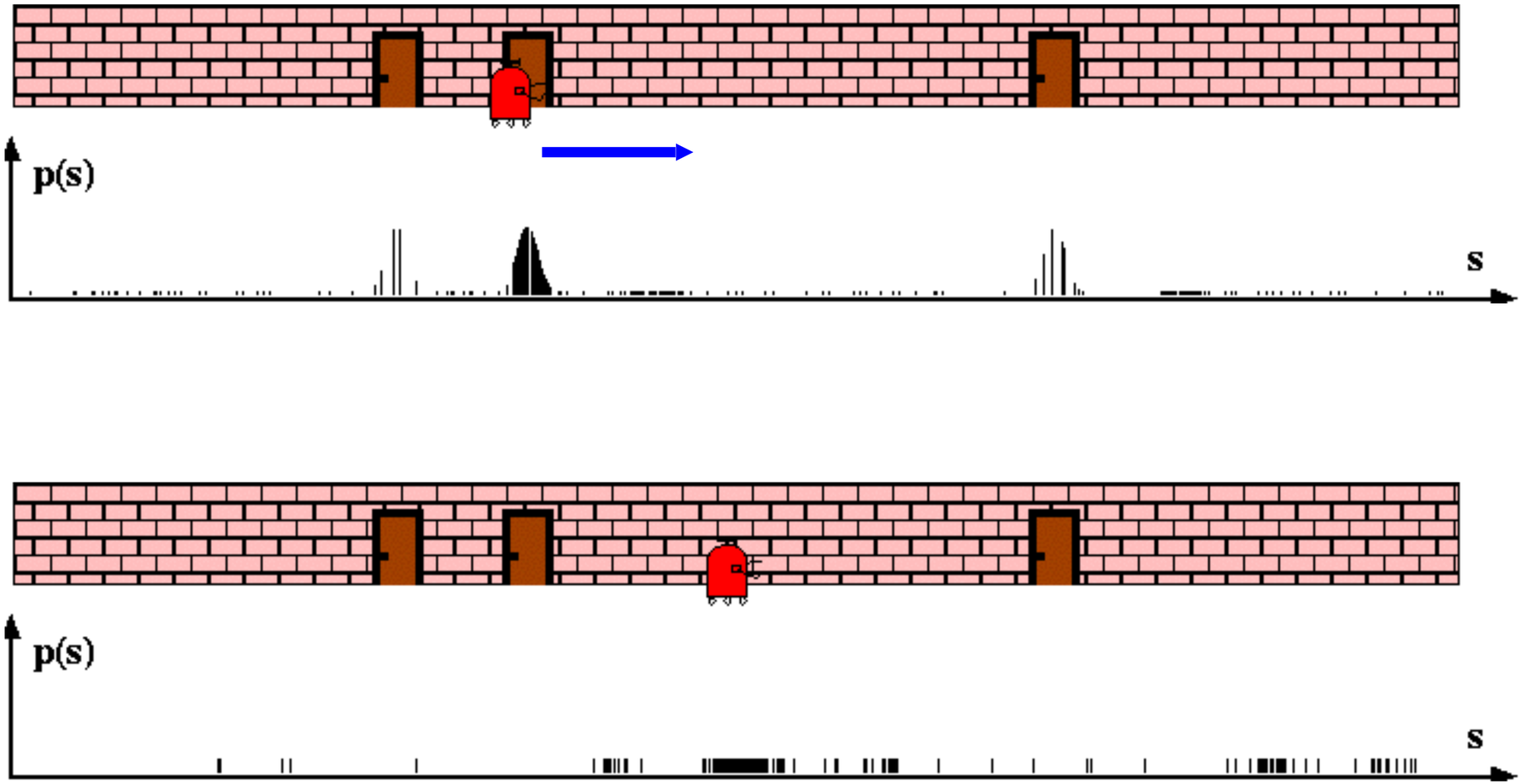
Robot Motion



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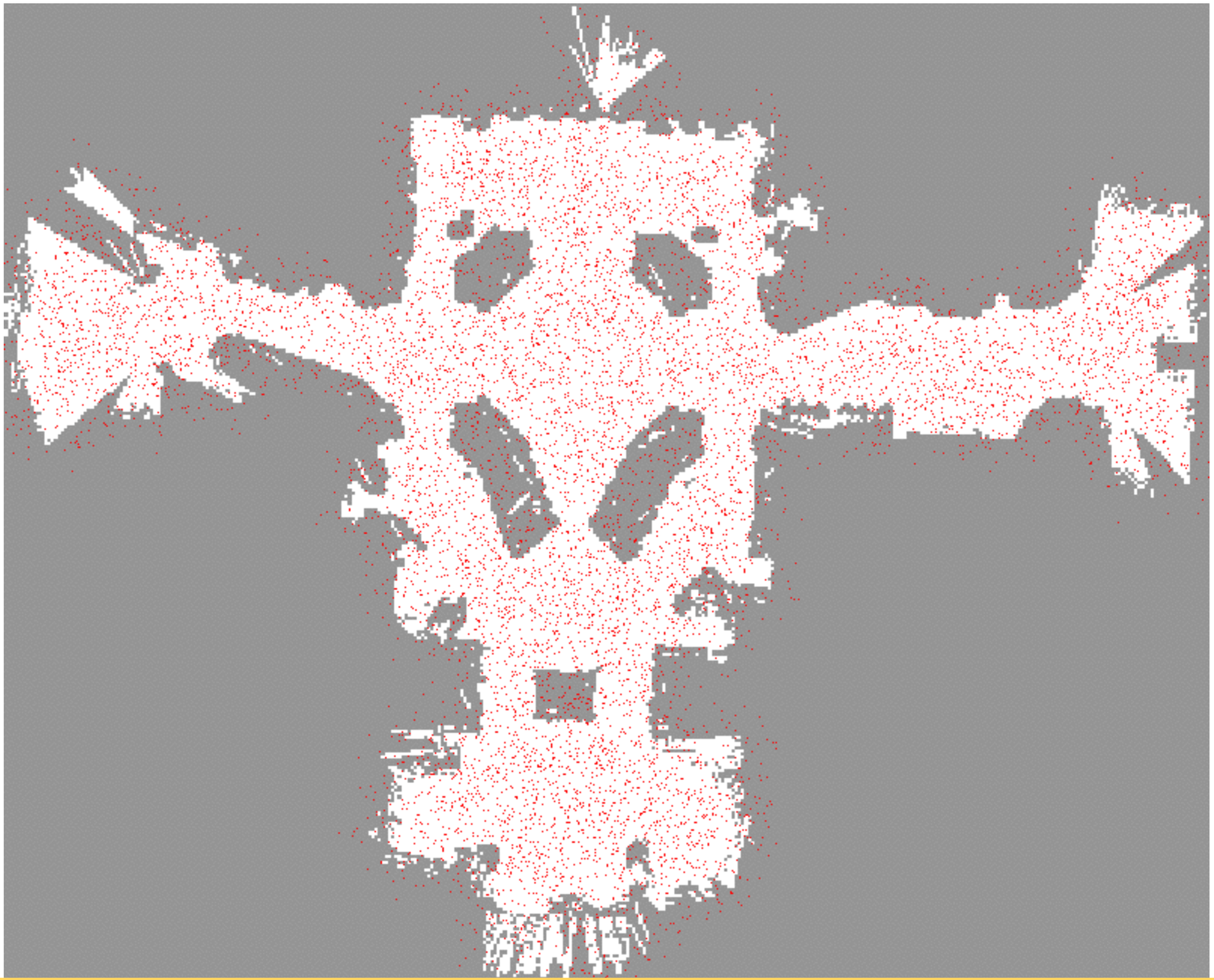

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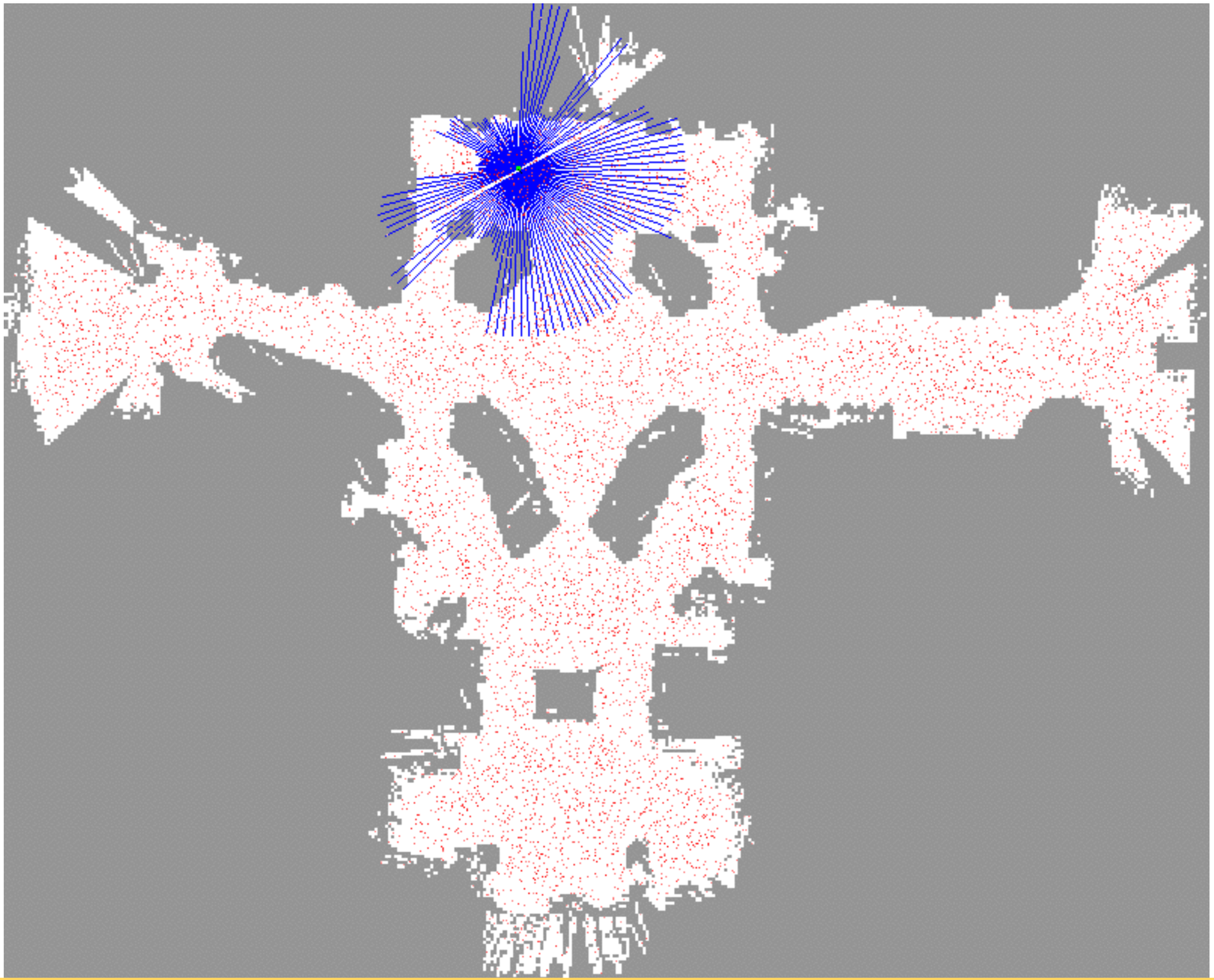

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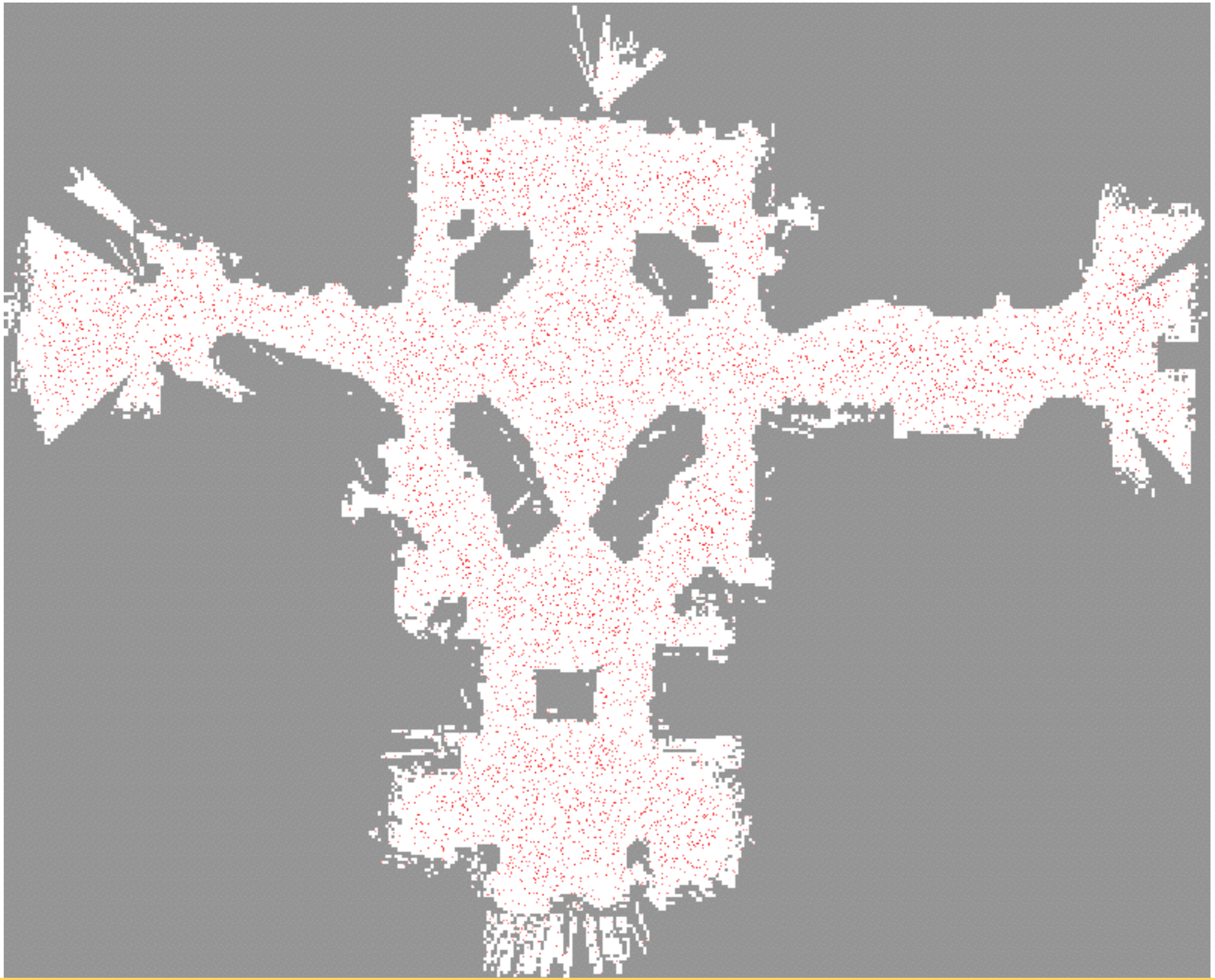

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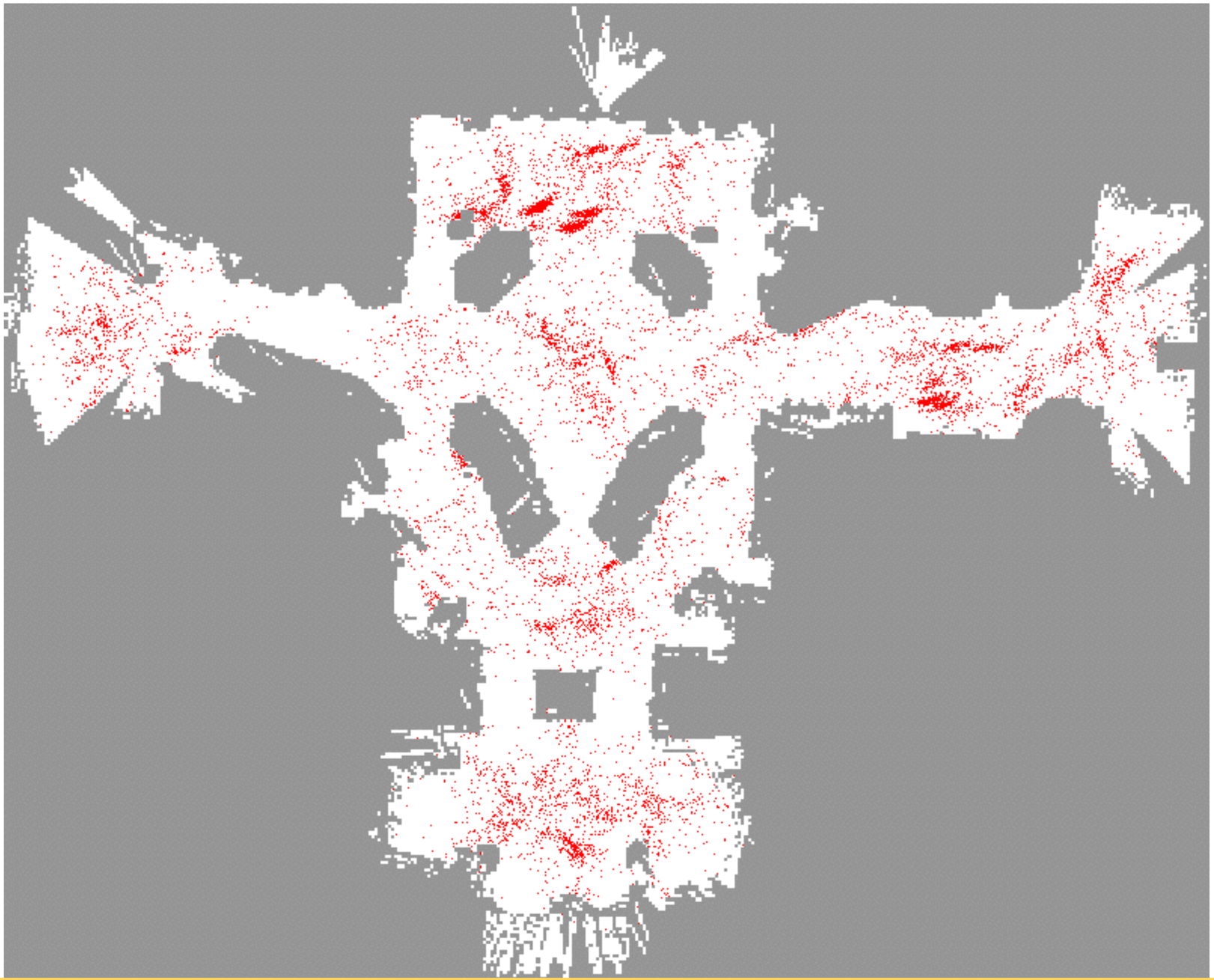

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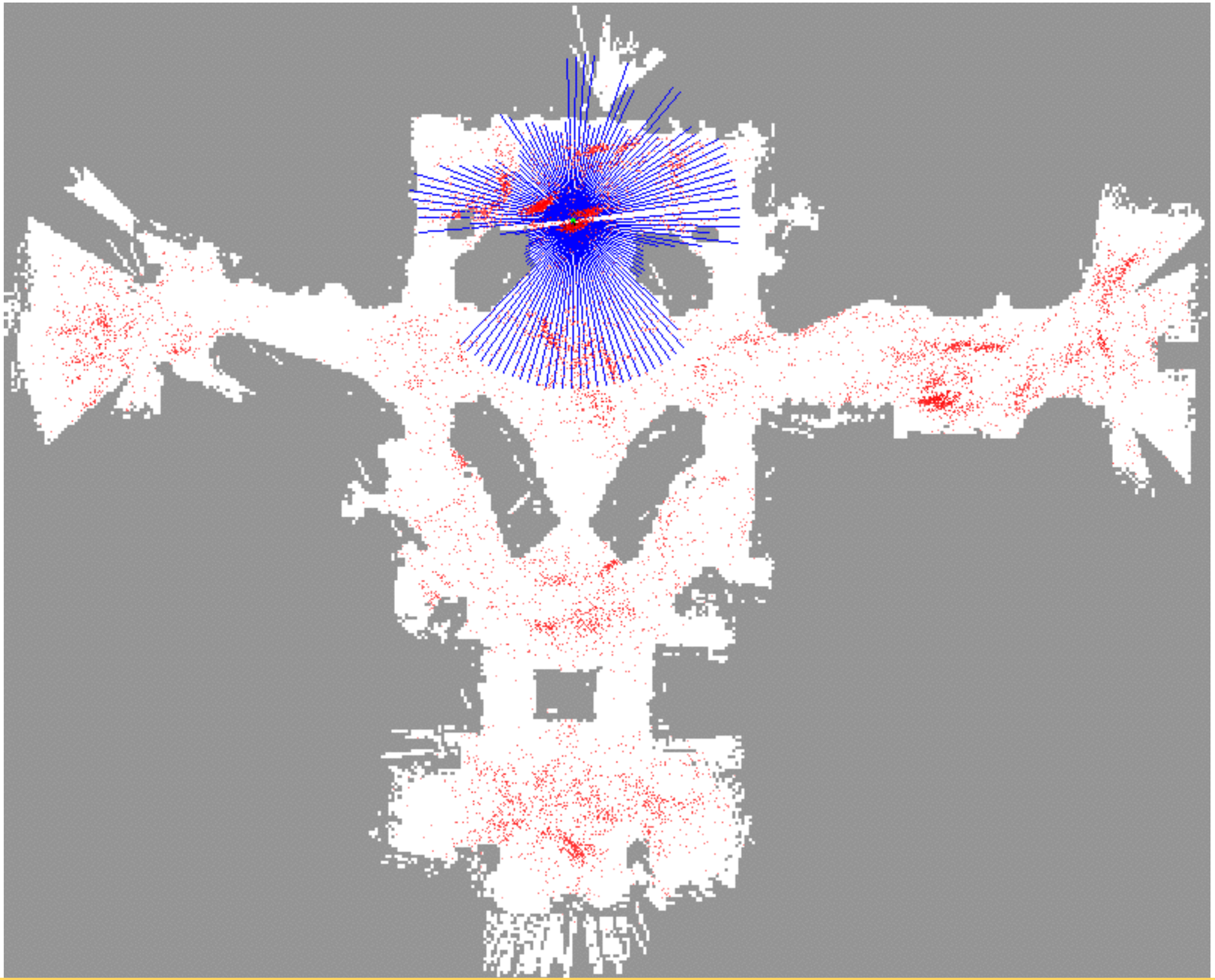

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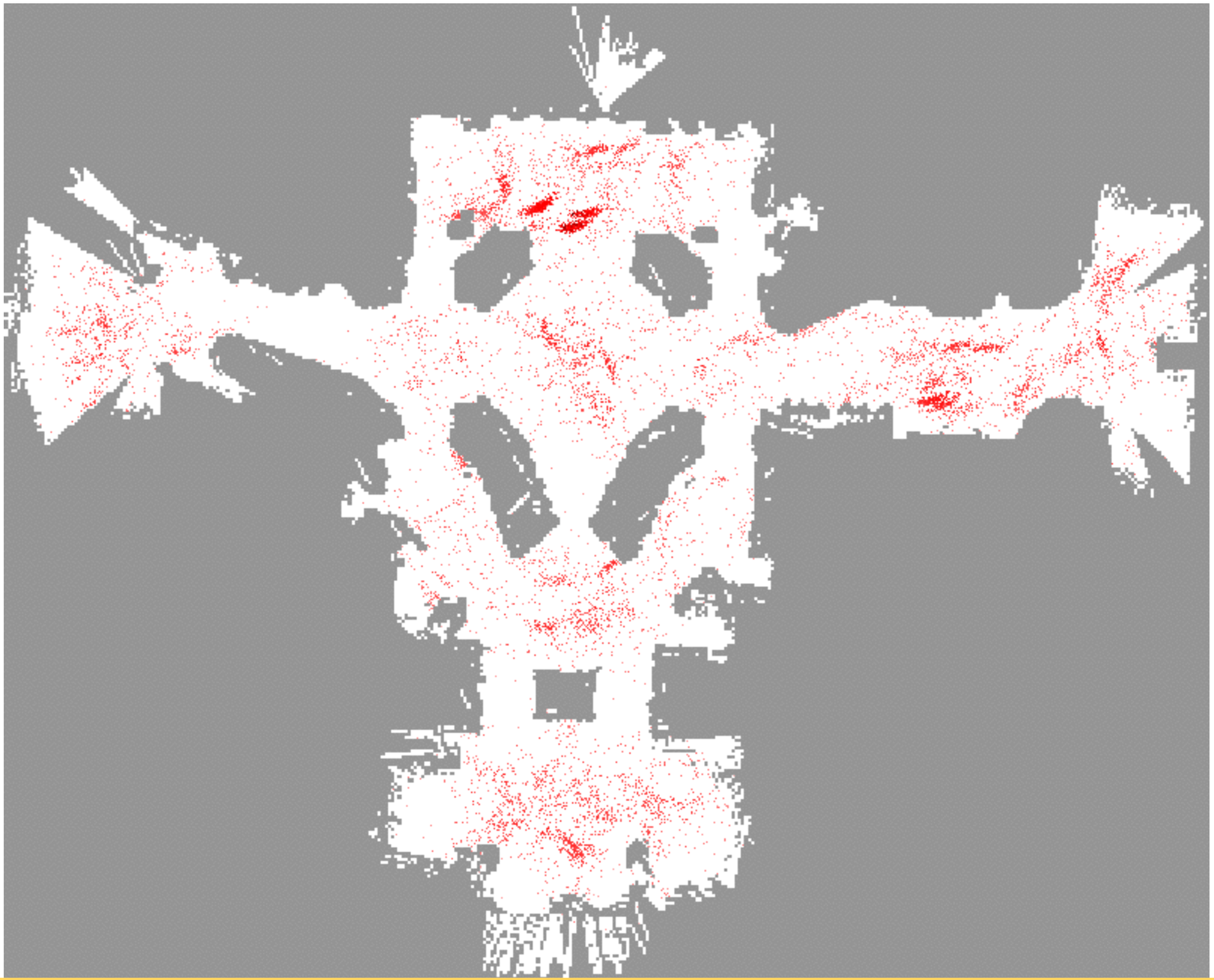

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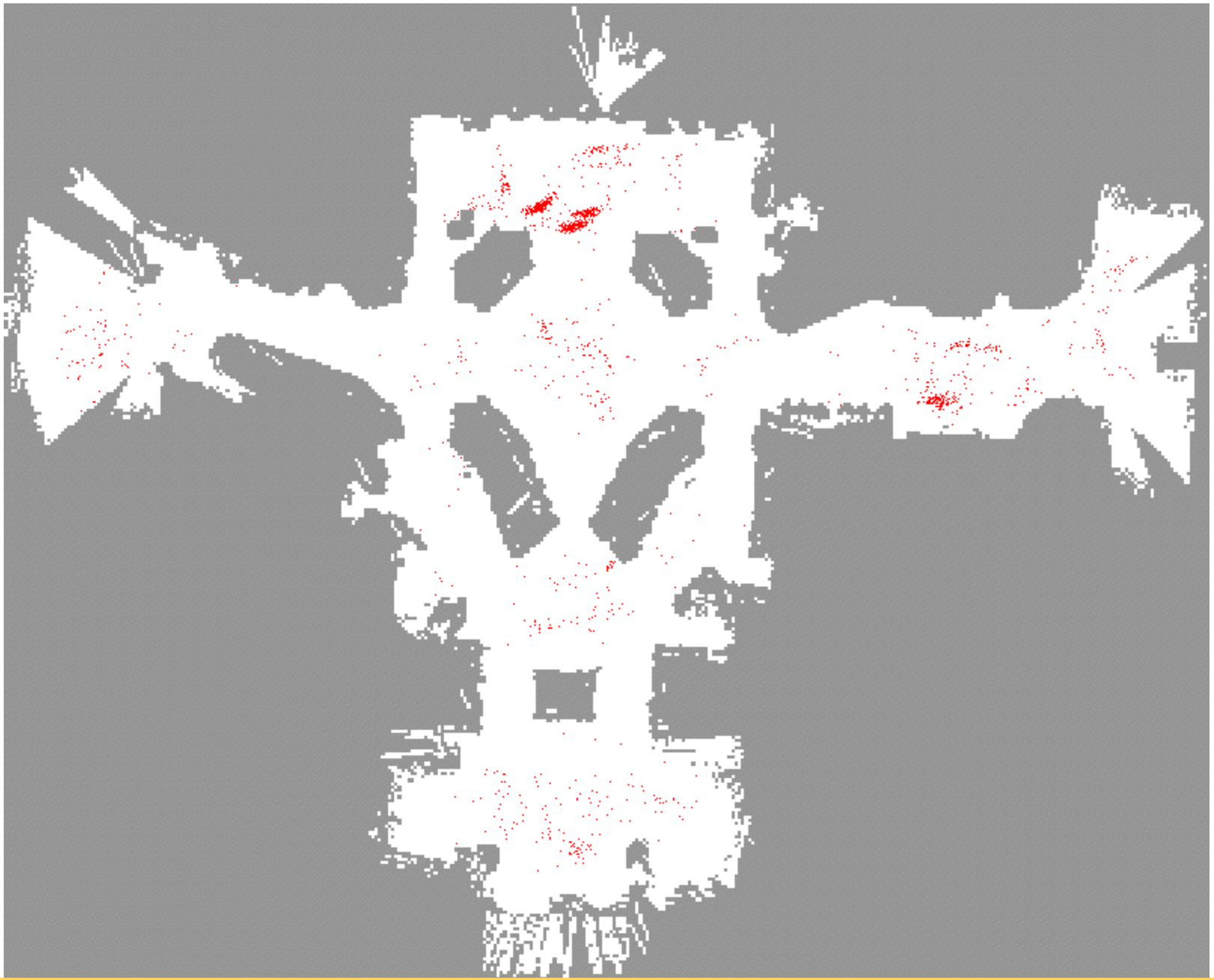

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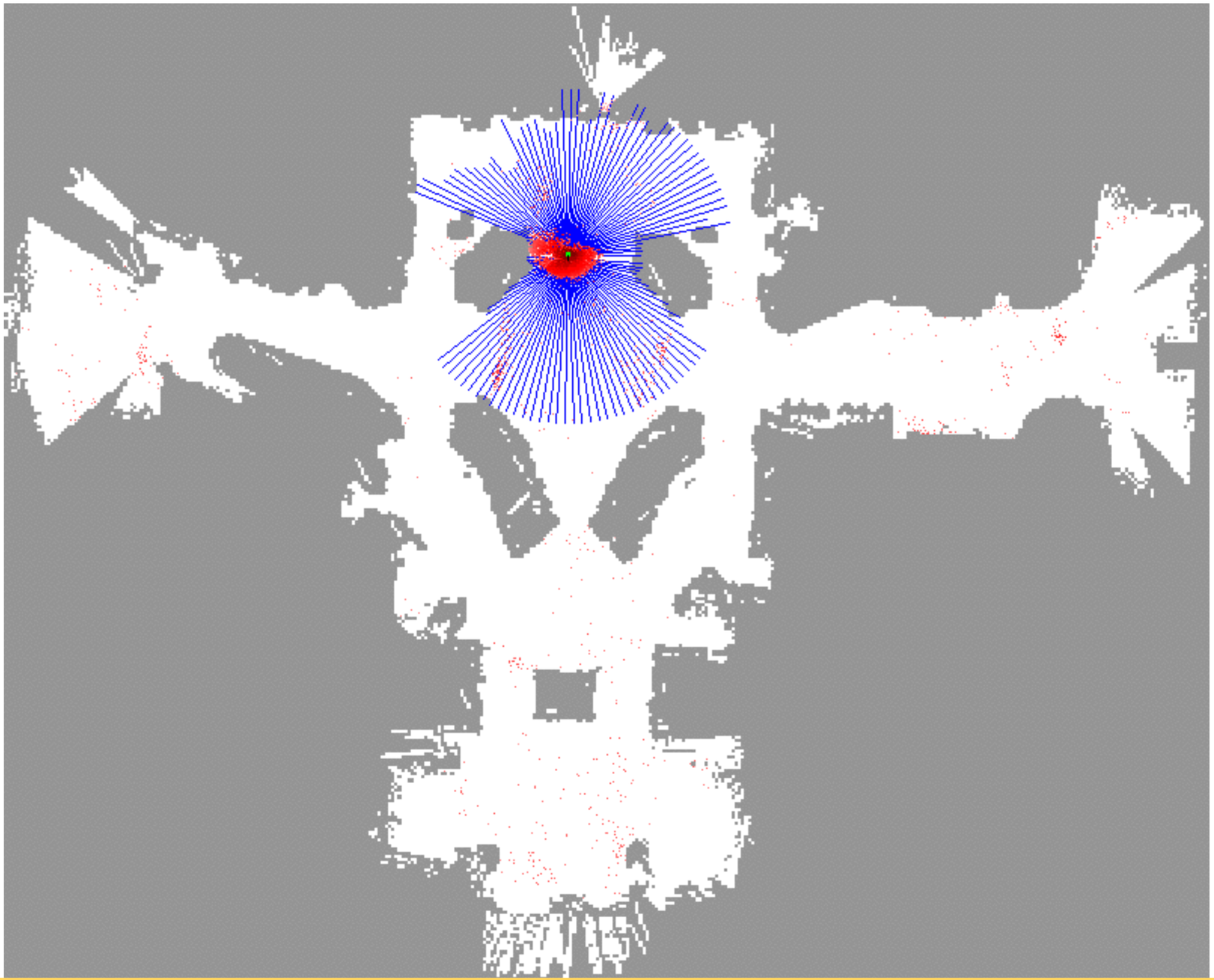

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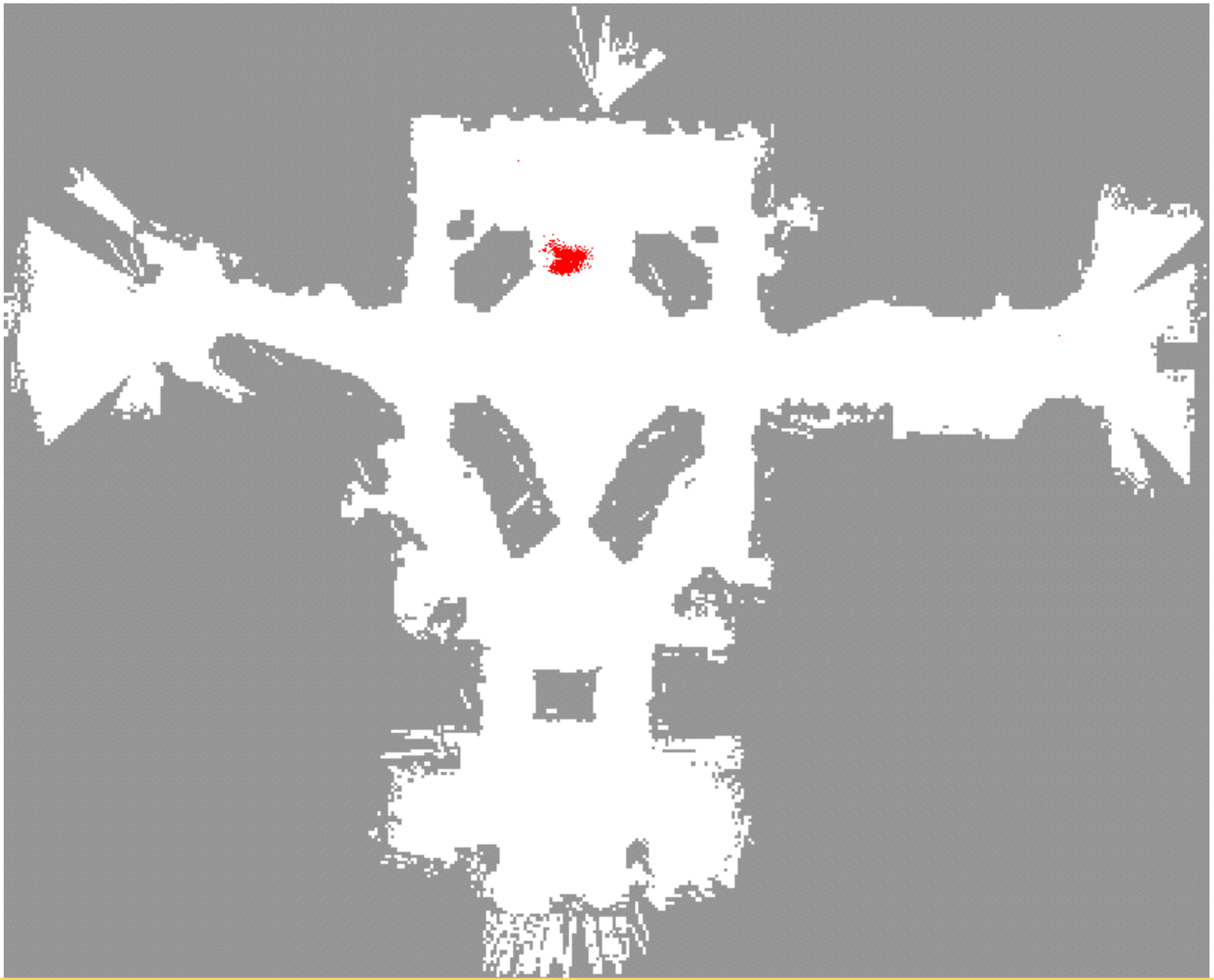

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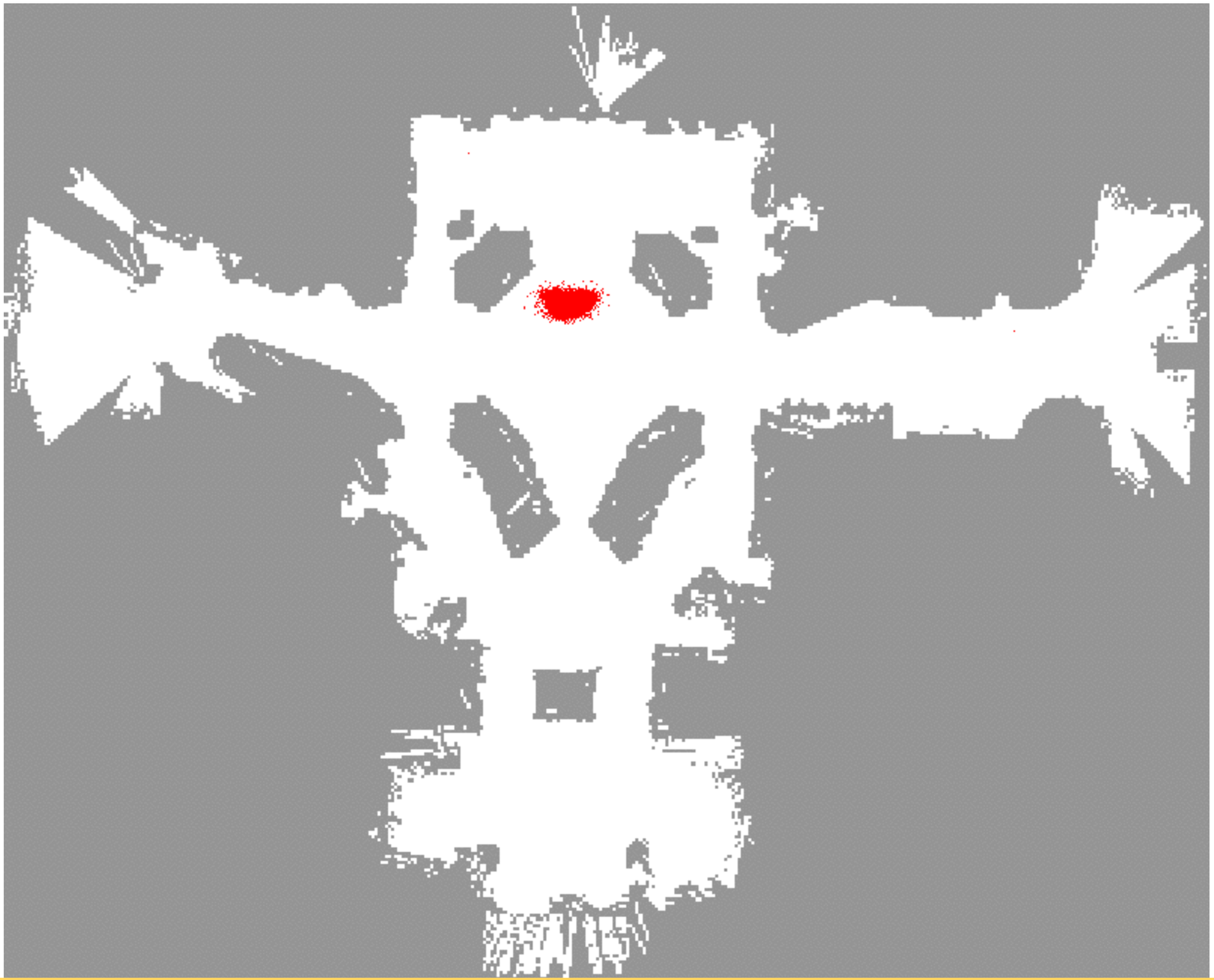

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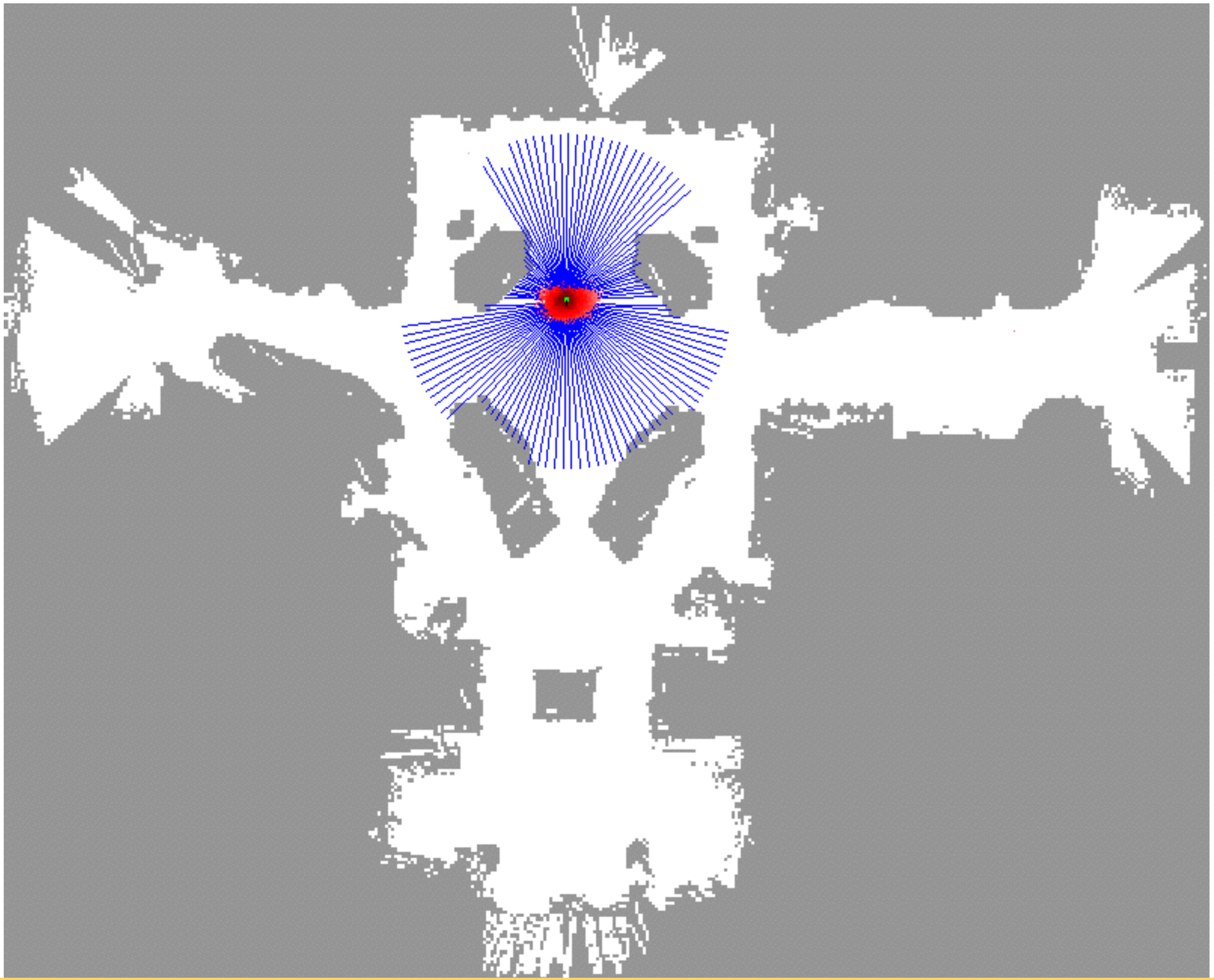

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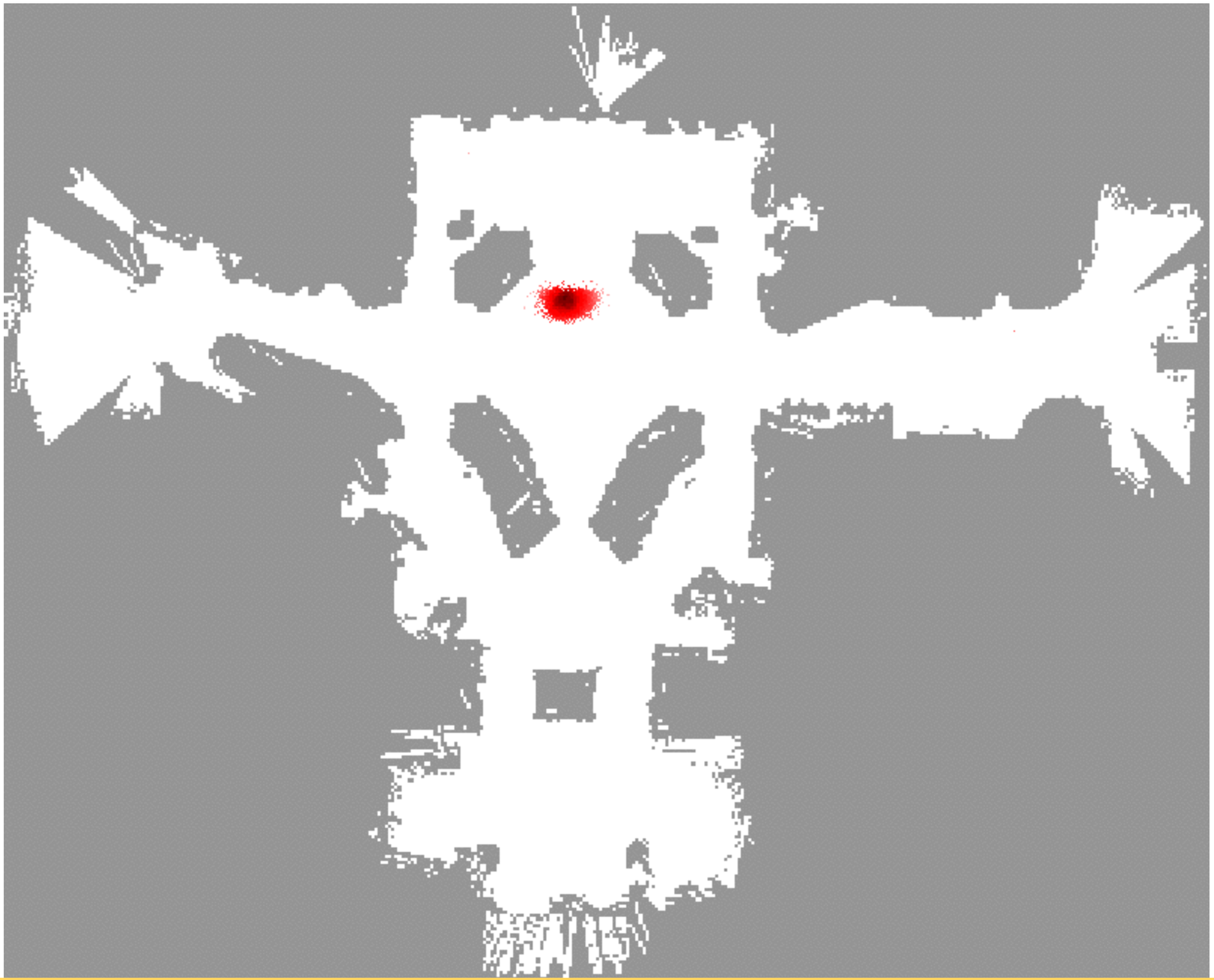

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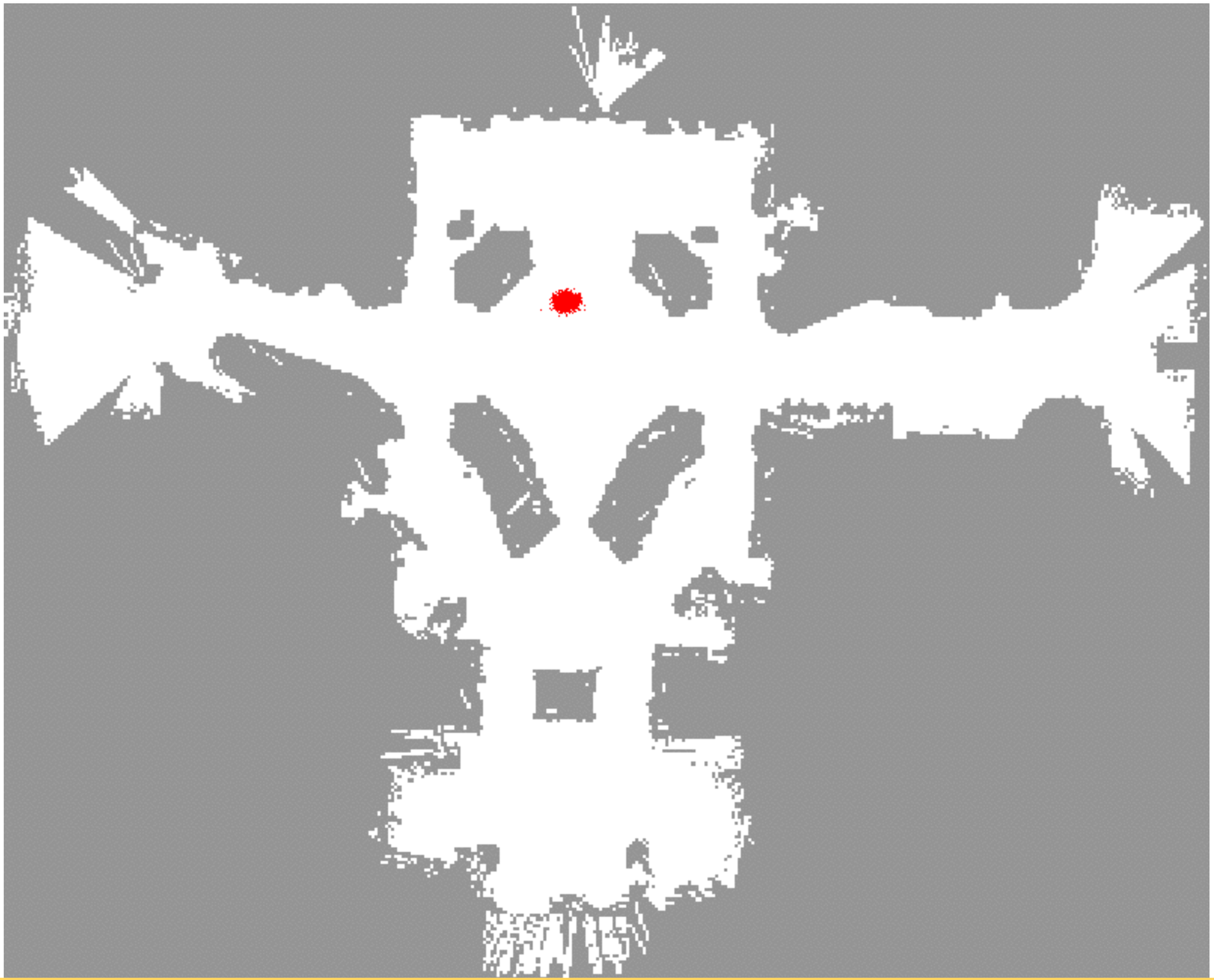

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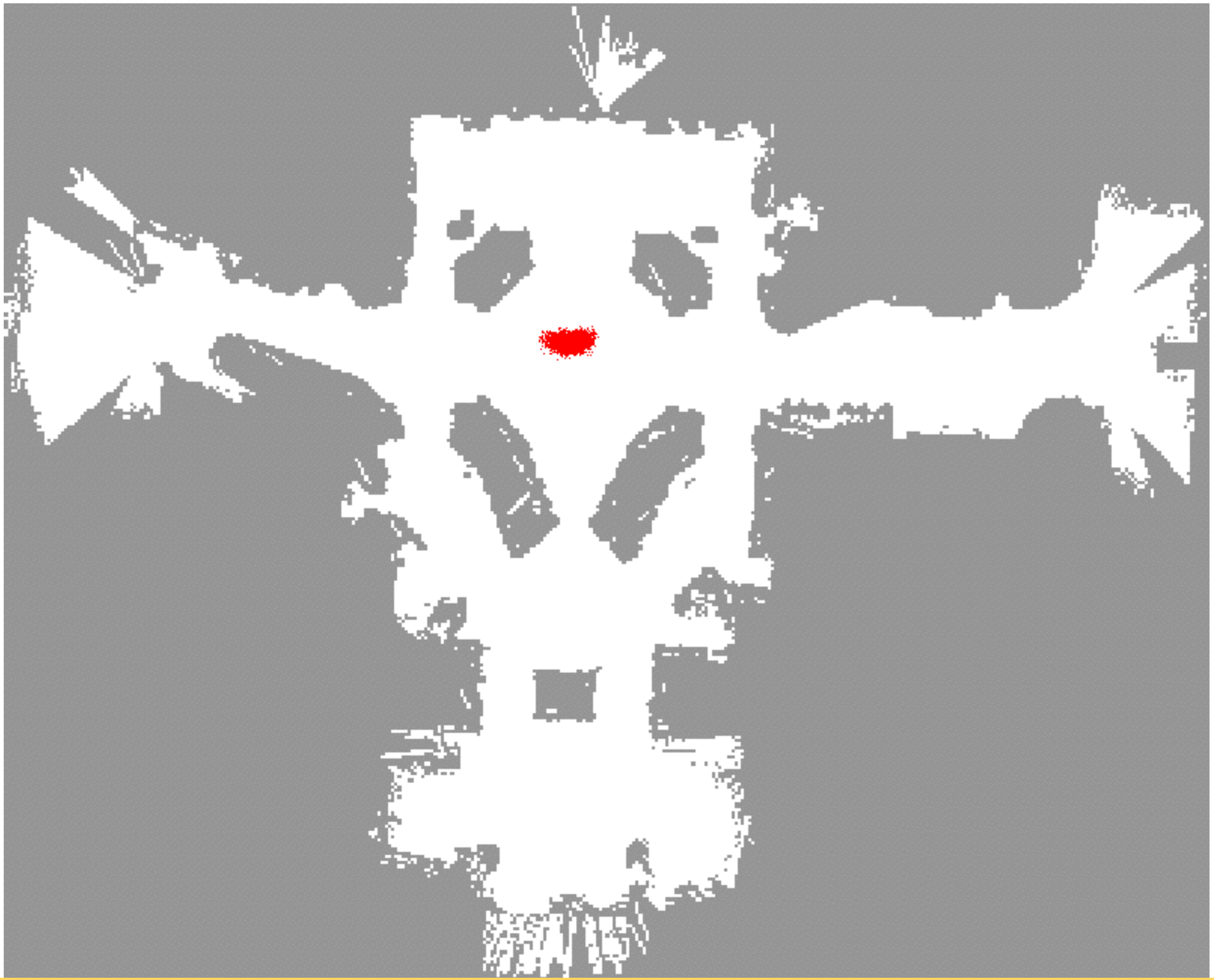

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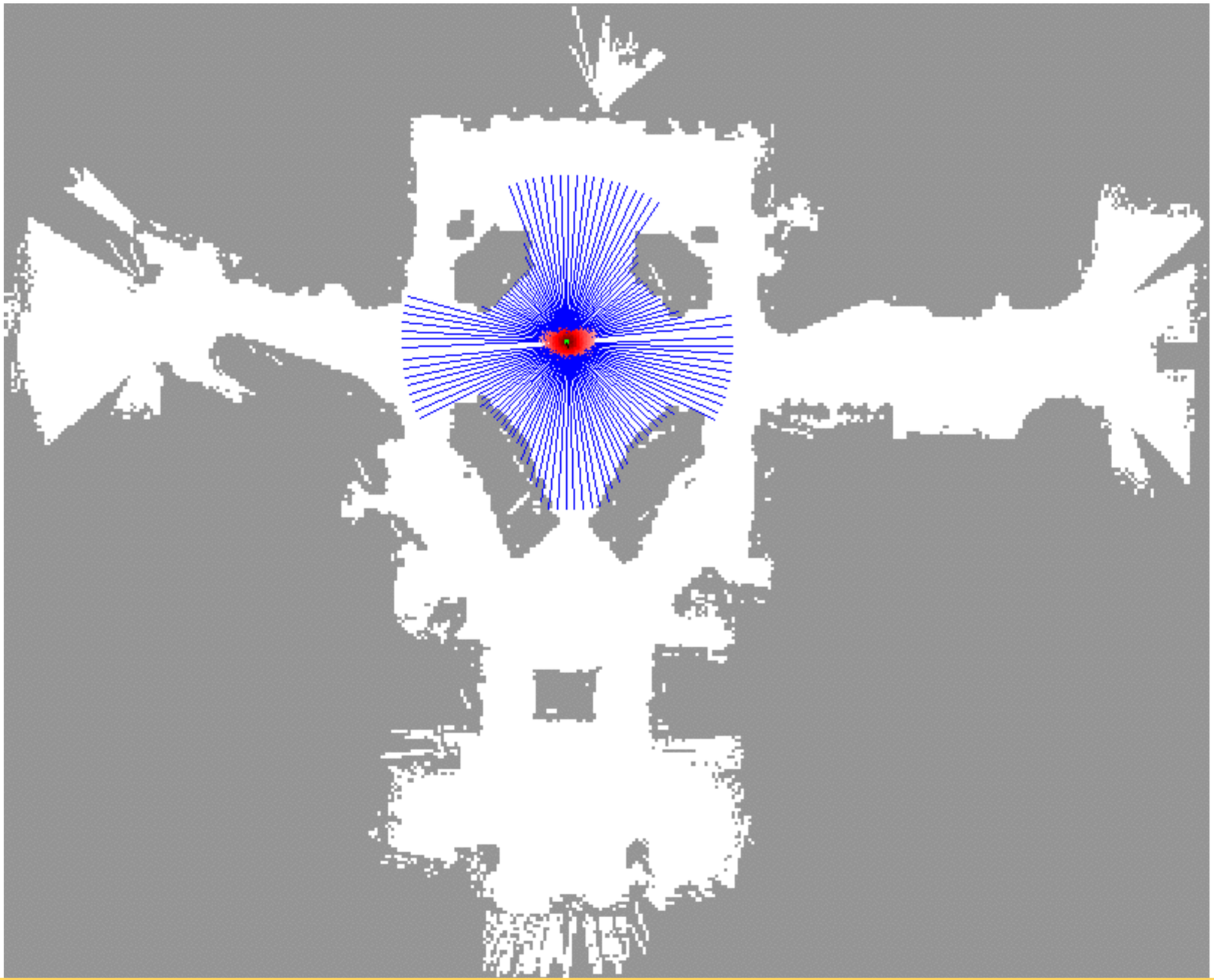

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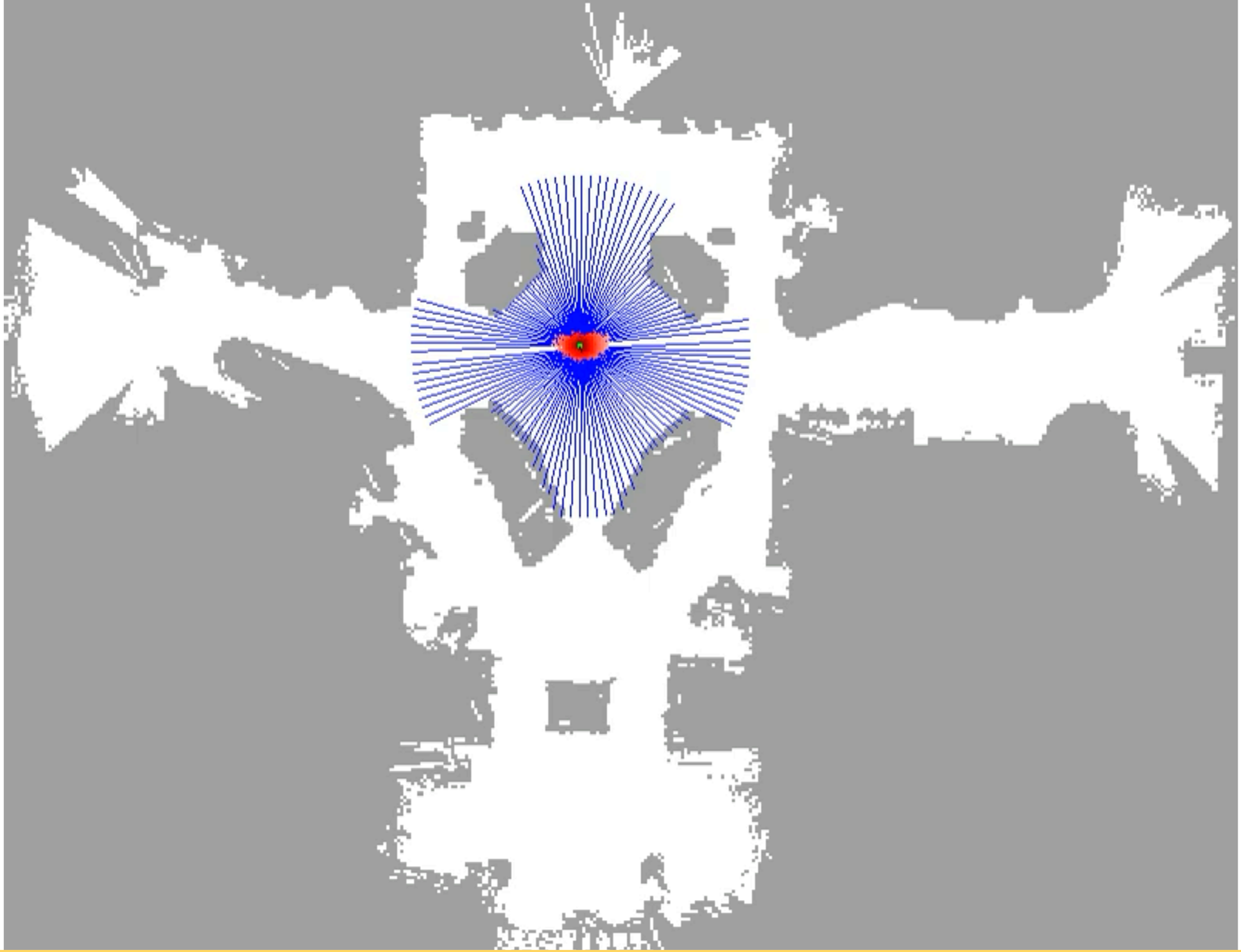

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Resampling

- **Given**: Set S of weighted samples.
- **Wanted** : Random sample, where the probability of drawing x_i is given by w_i.
- Typically done n times with replacement to generate new sample set S'.

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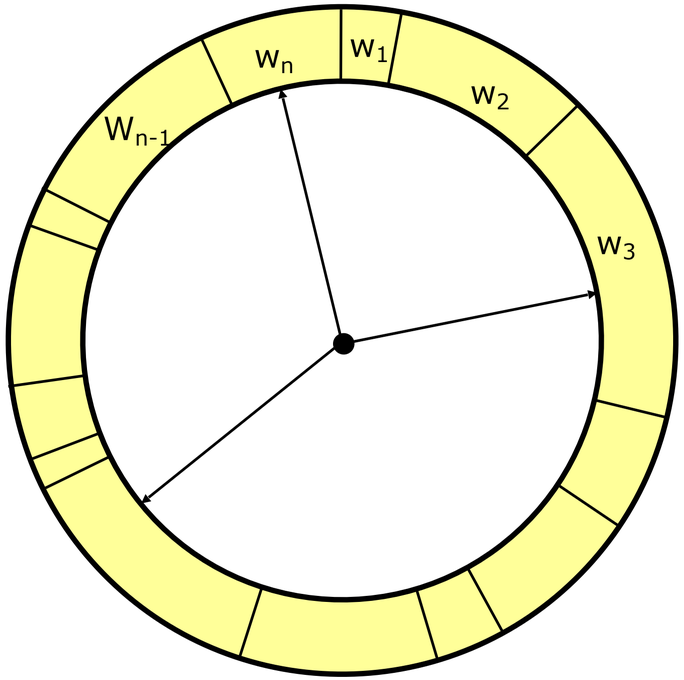
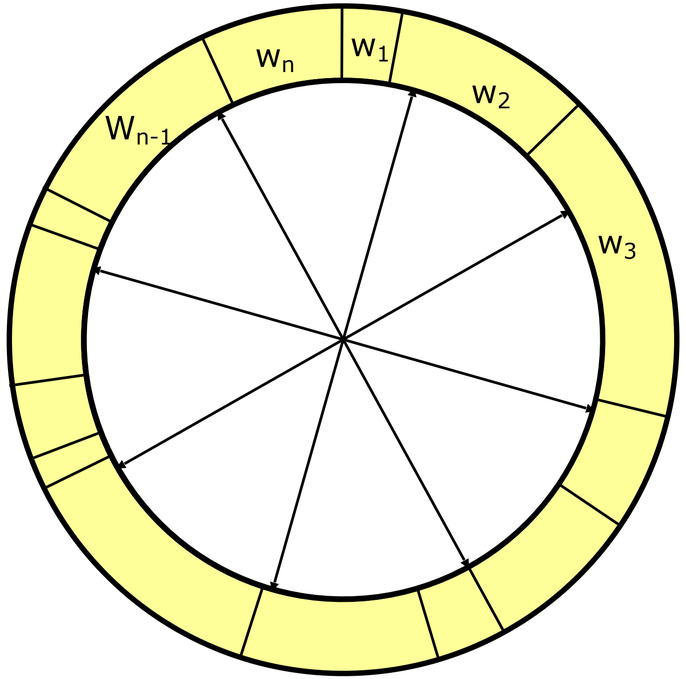
Resampling





- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

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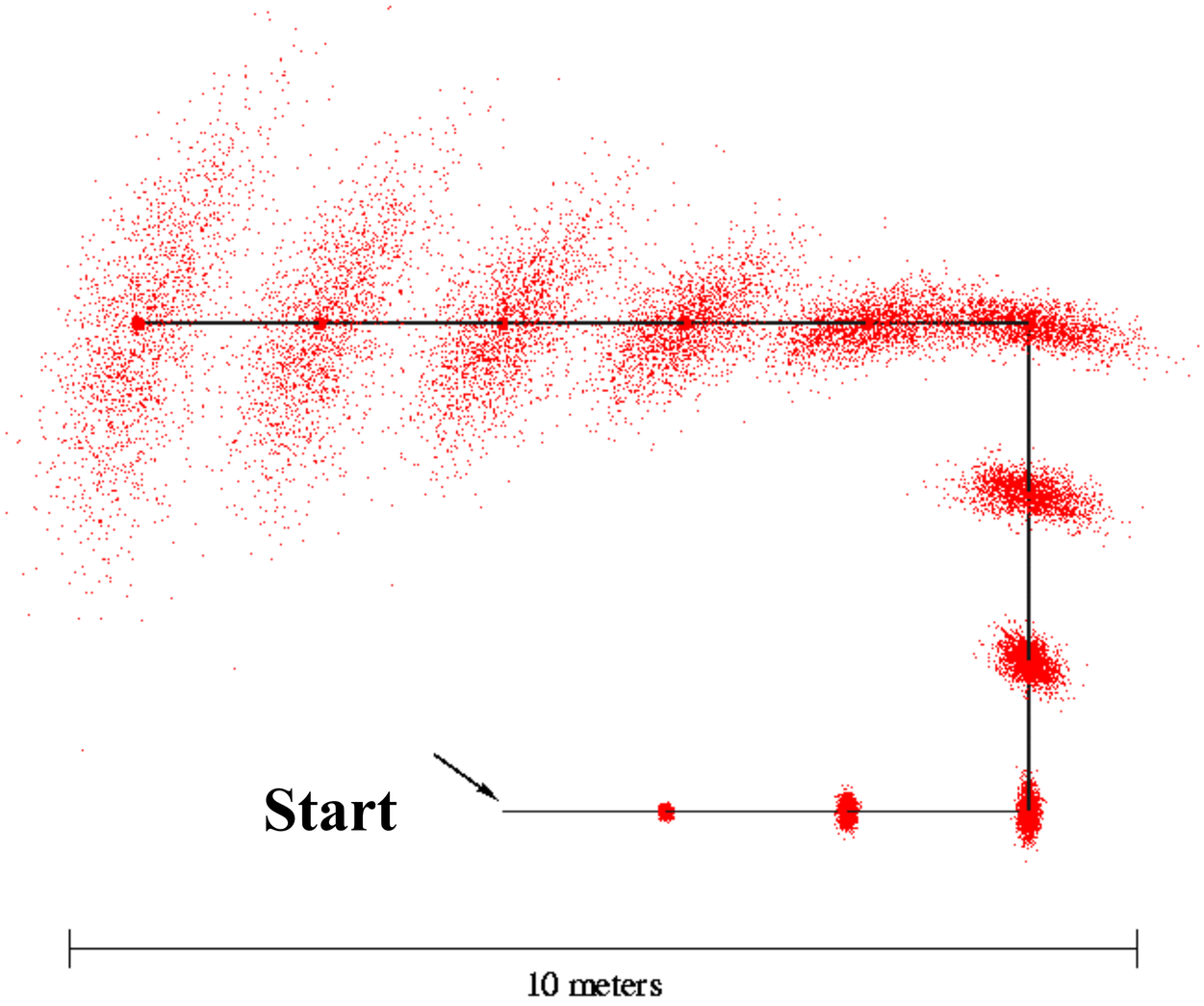
Motion Model Reminder



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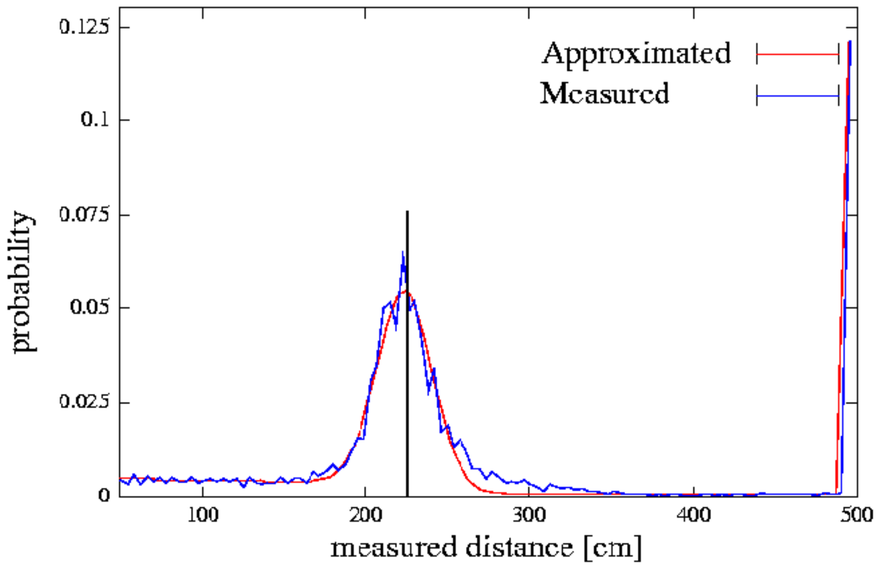
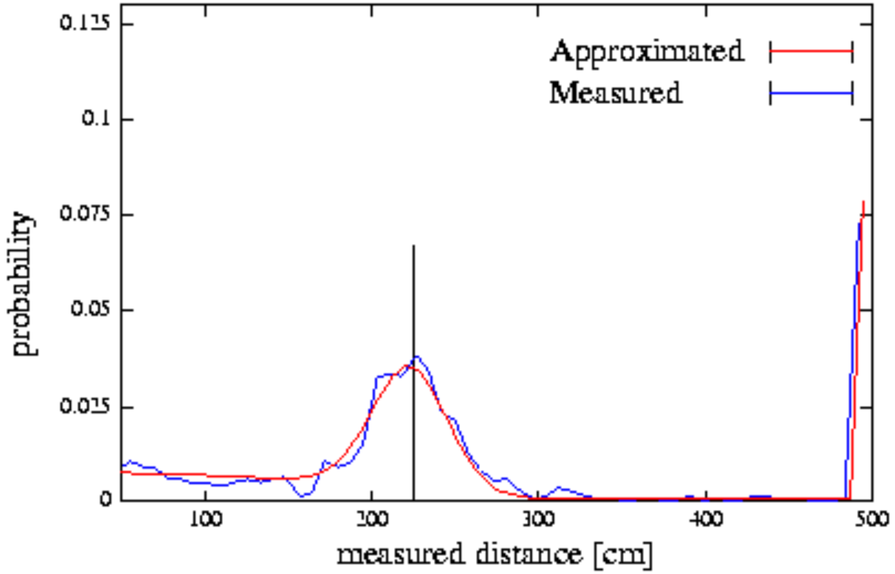
Proximity Sensor Model Reminder

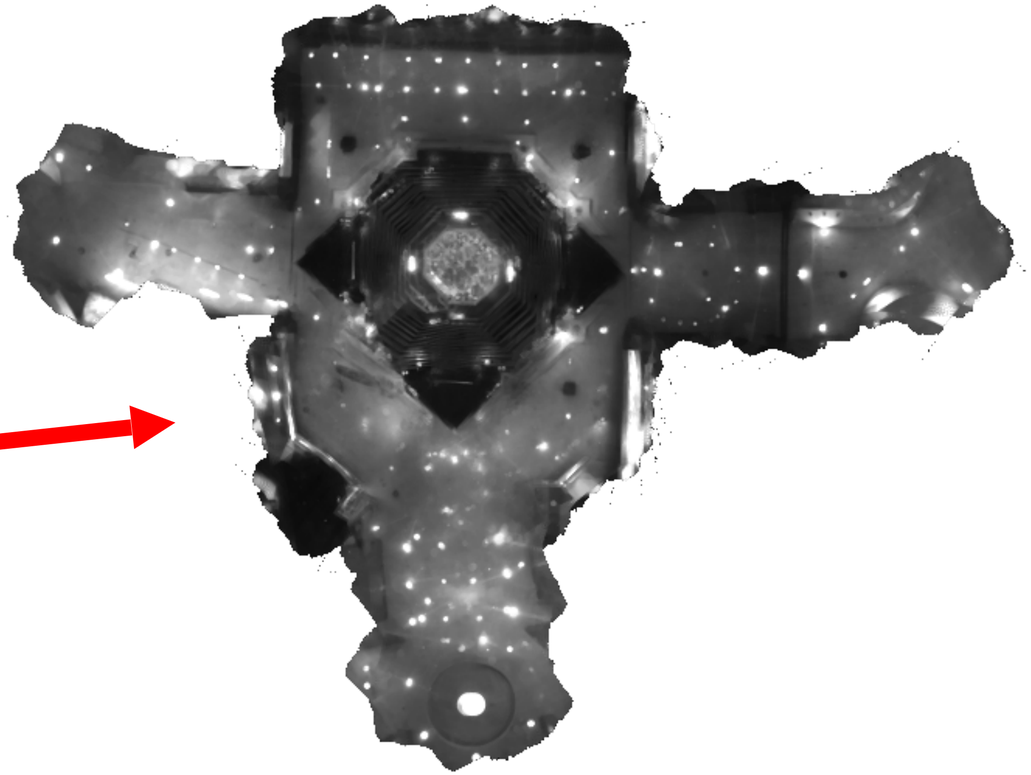


Laser sensor



Sonar sensor

Using Ceiling Maps for Localization



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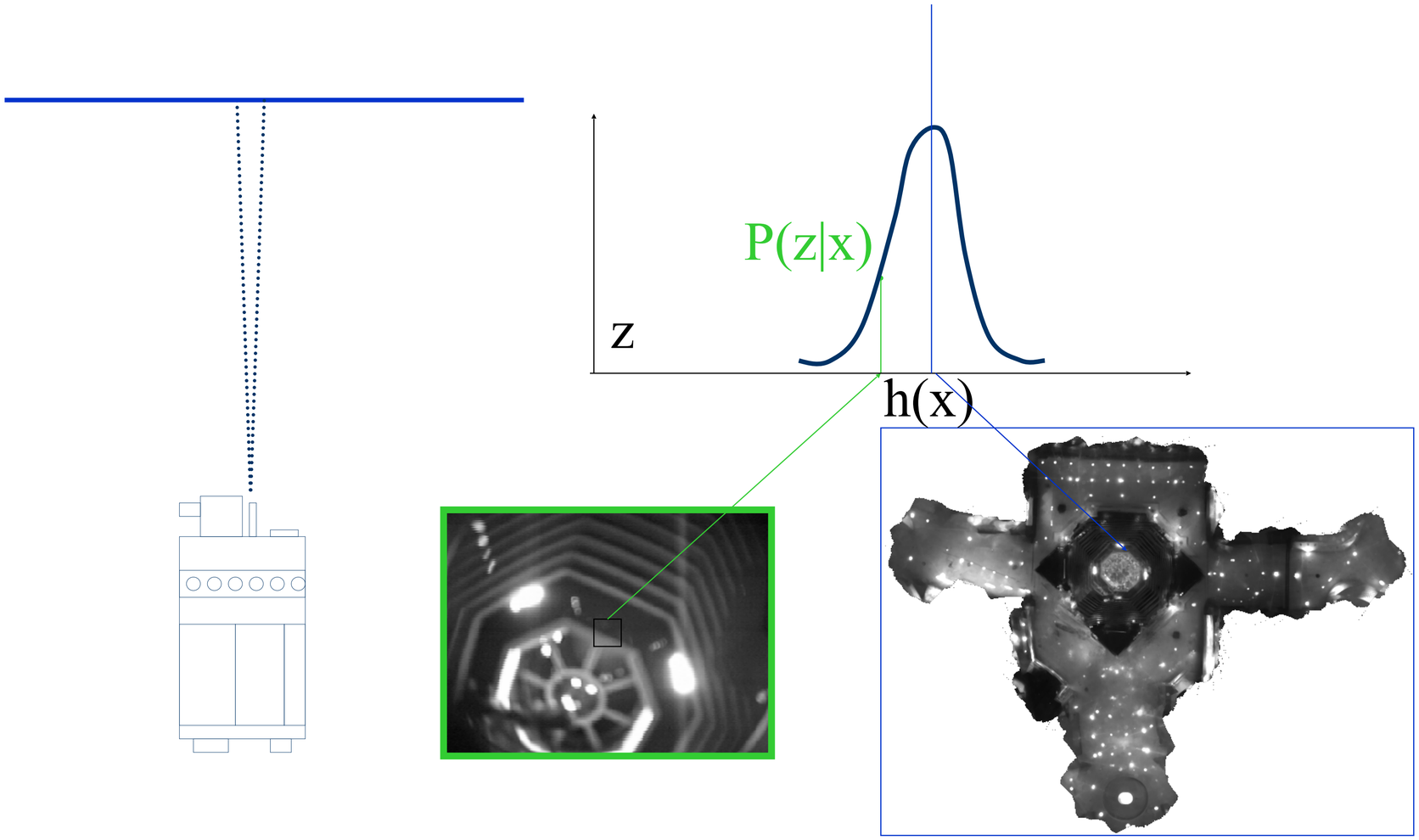
Vision-based Localization



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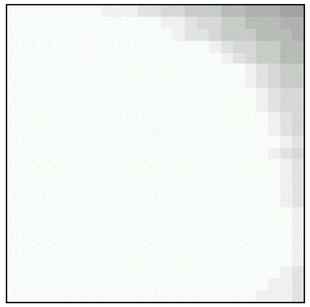
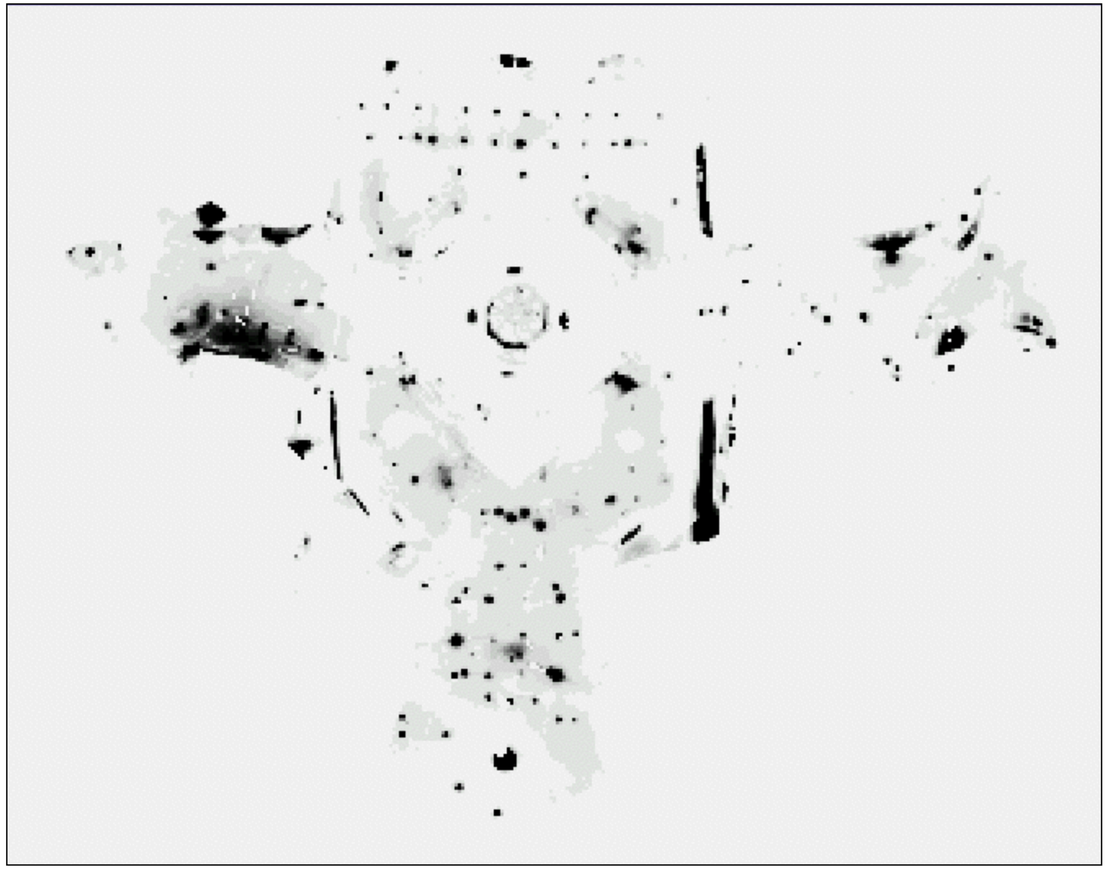
Under a Light

Measurement z:



***P(z|x)*:**



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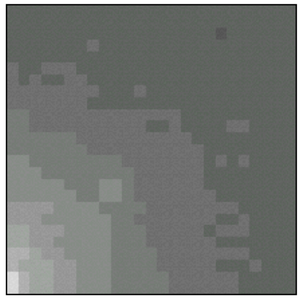
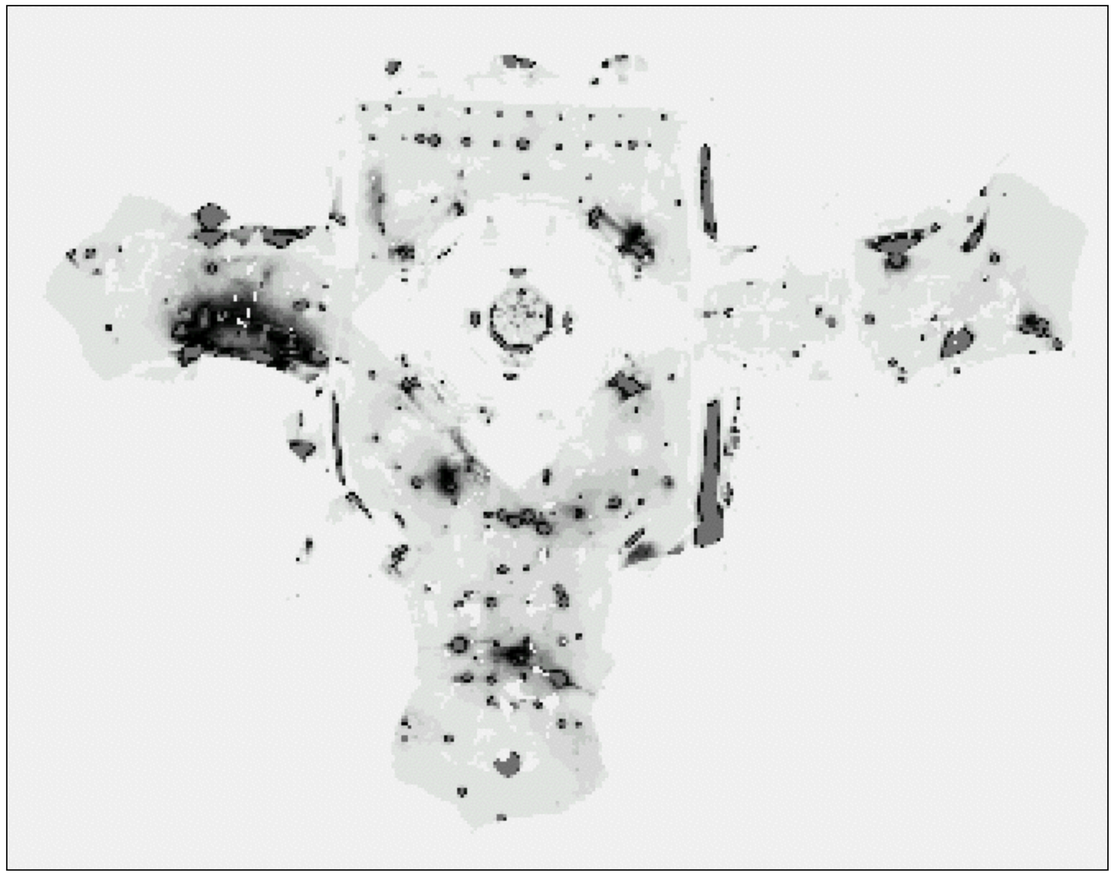
Next to a Light

Measurement z:



***P(z|x)*:**



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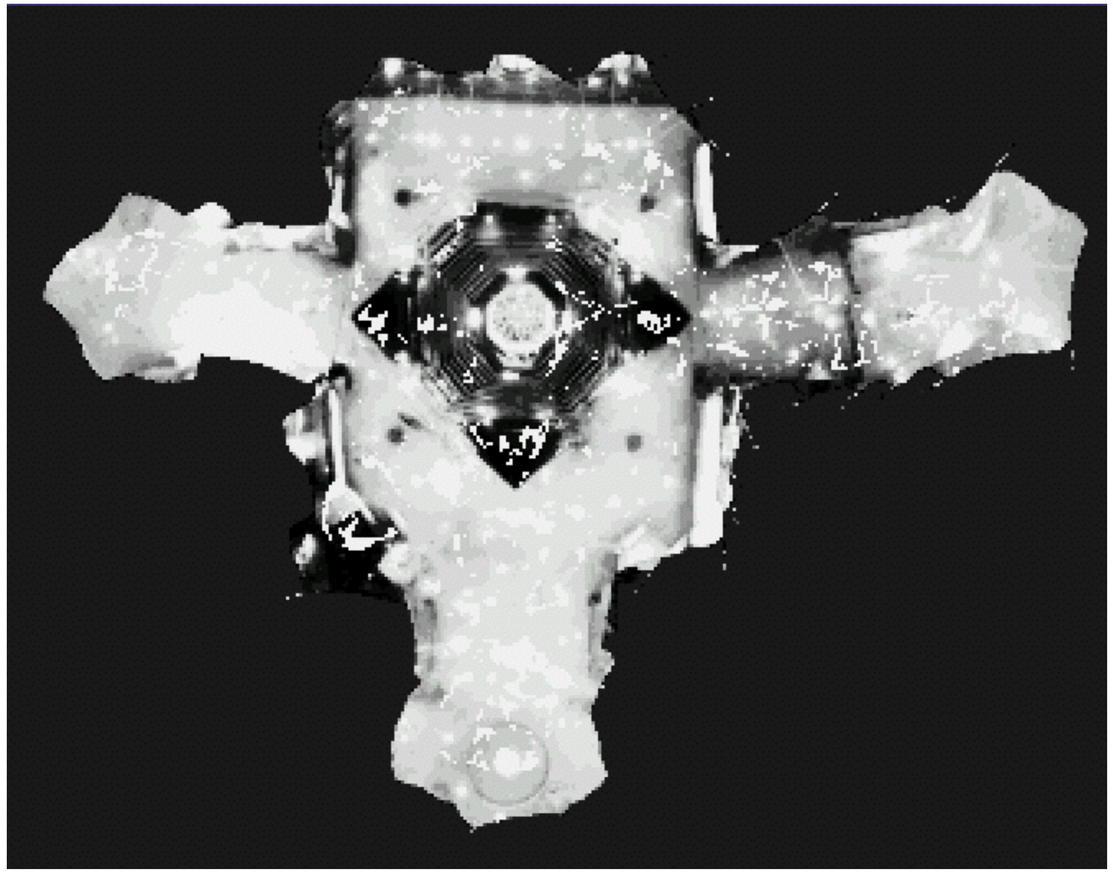
Elsewhere

Measurement z:

***P(z|x)*:**



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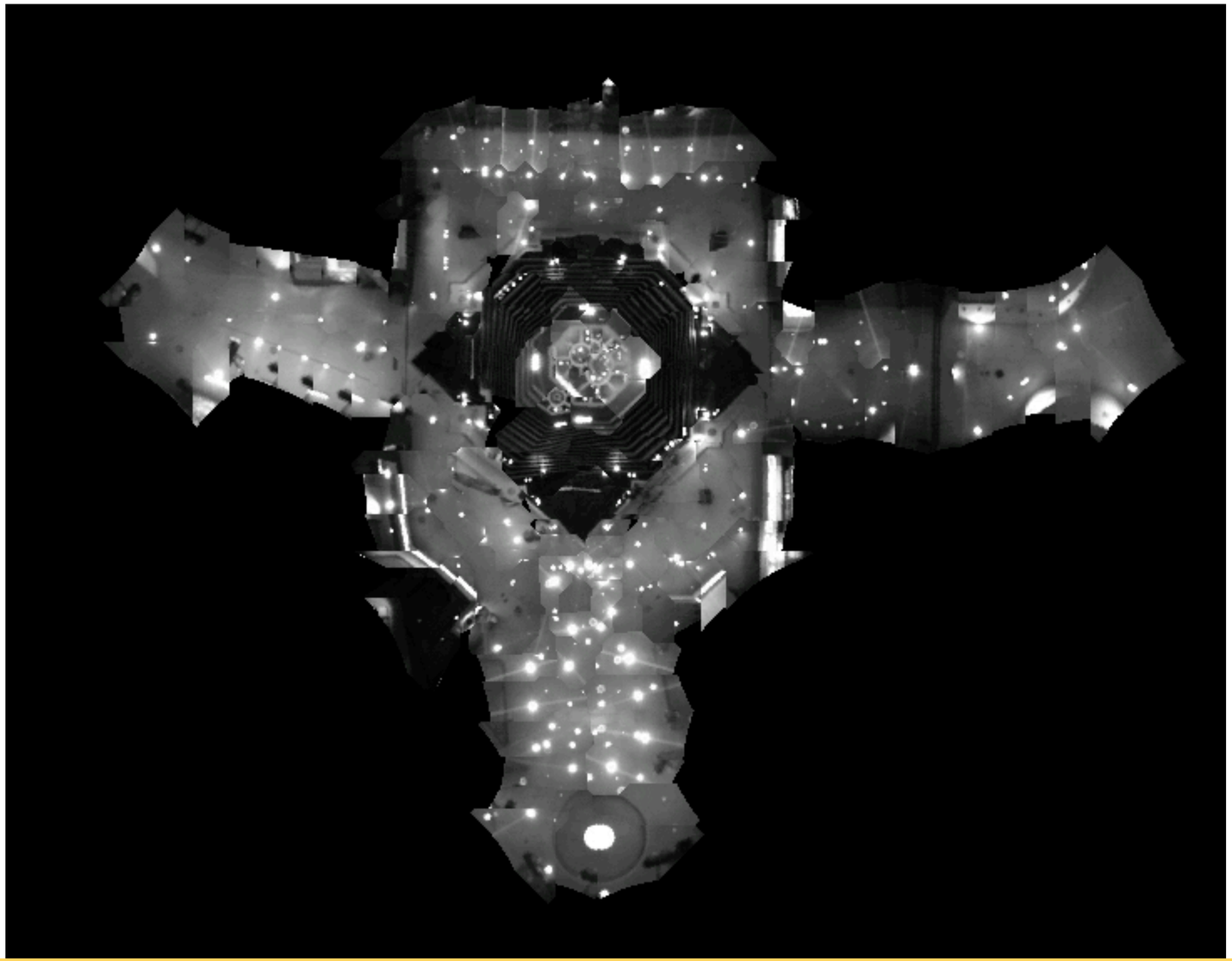
Global Localization Using Vision



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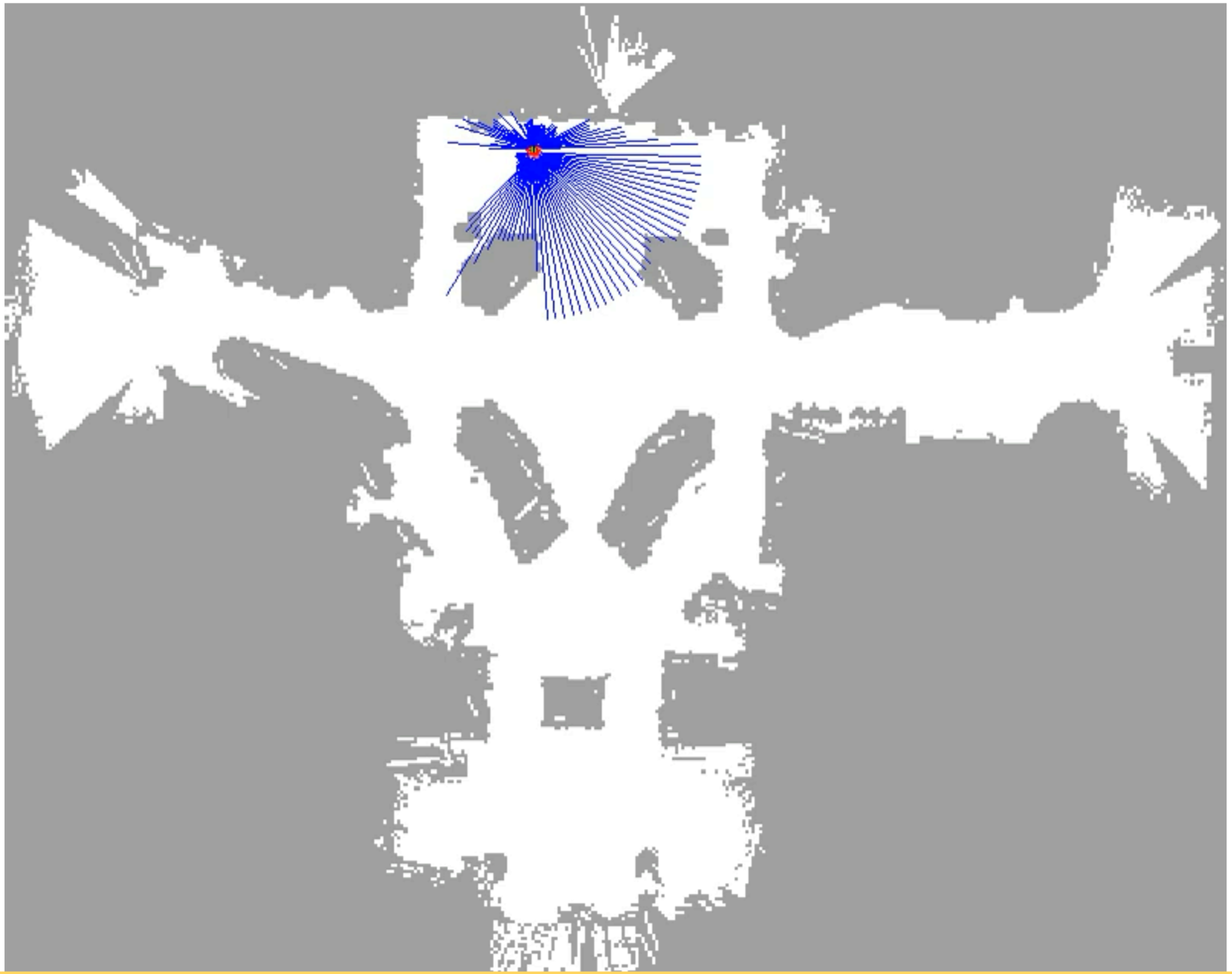
Recovery from Failure



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Next Lecture:
More PF and Mapping