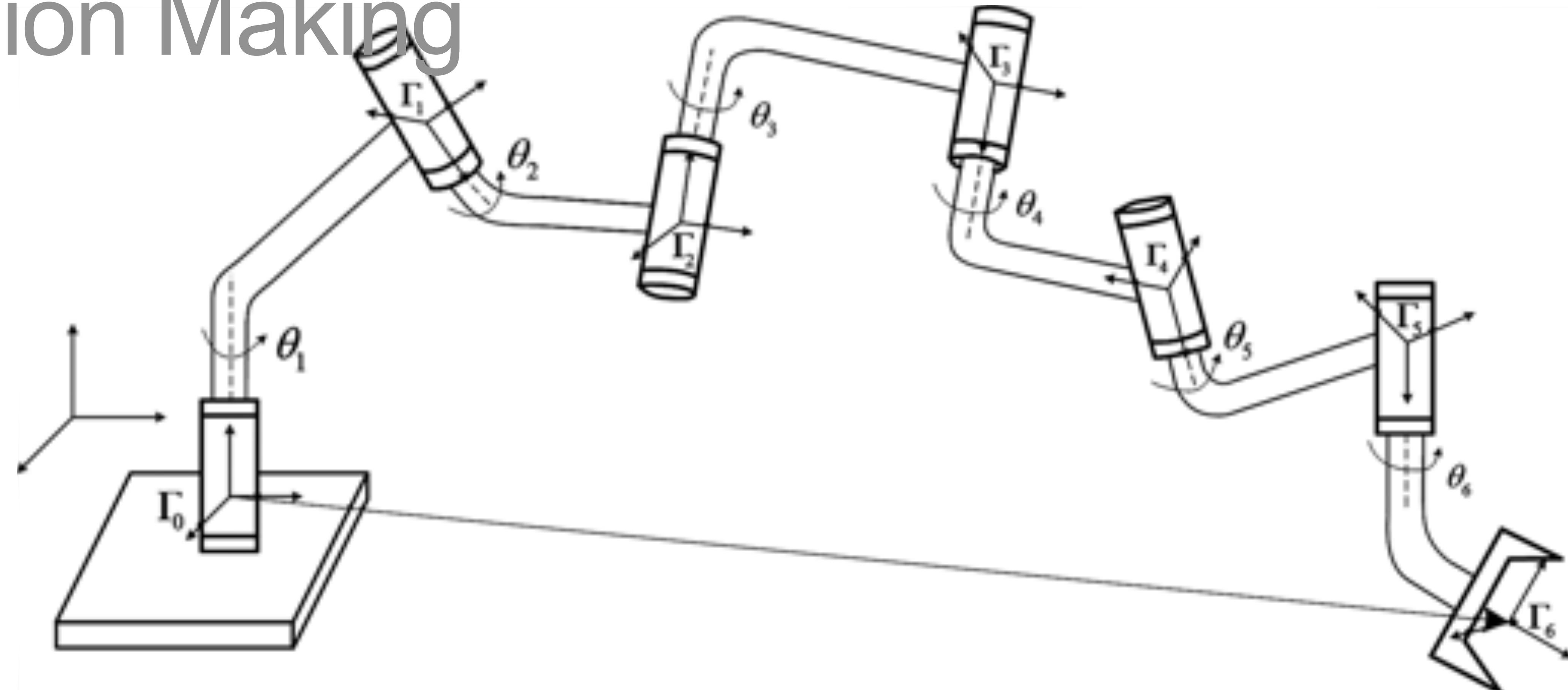


# Lecture 05

## Manipulation - I

### Forward Kinematics & Decision Making



# Course Logistics

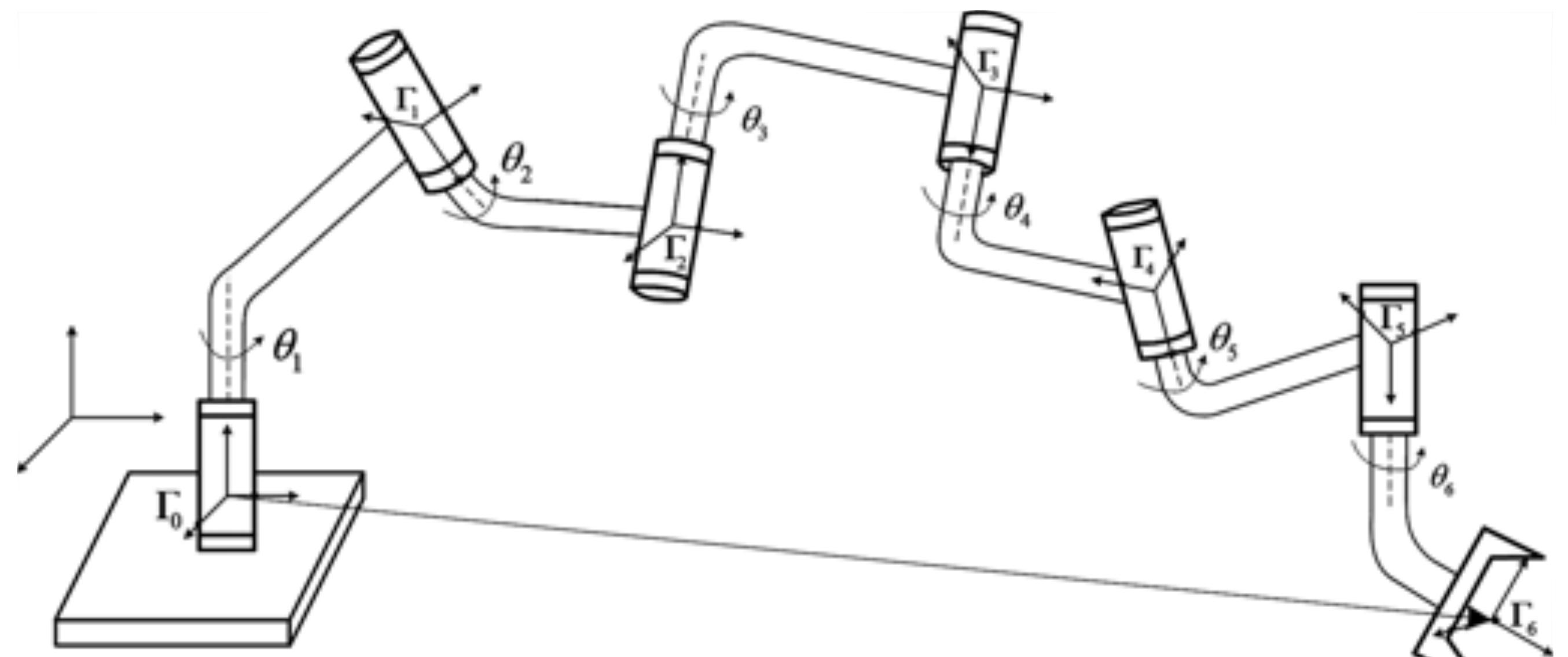
- Quiz 3 was posted today and was due before the lecture.
- Project 0 was posted on 09/13 and will be due 09/25 (extended).
- Autograder will be made available for you to test and submit your code.
  - CSE-IT is still working on it. Will be made available soon.
- Project 1 will be posted on 09/20 (today) and will be due 10/02.



# Robot Kinematics

**Goal:** Given the structure of a robot arm, compute

- **Forward kinematics:** infer the pose of the end-effector, given the state of each joint.
- **Inverse kinematics:** infer the joint states to reach a desired end-effector pose.



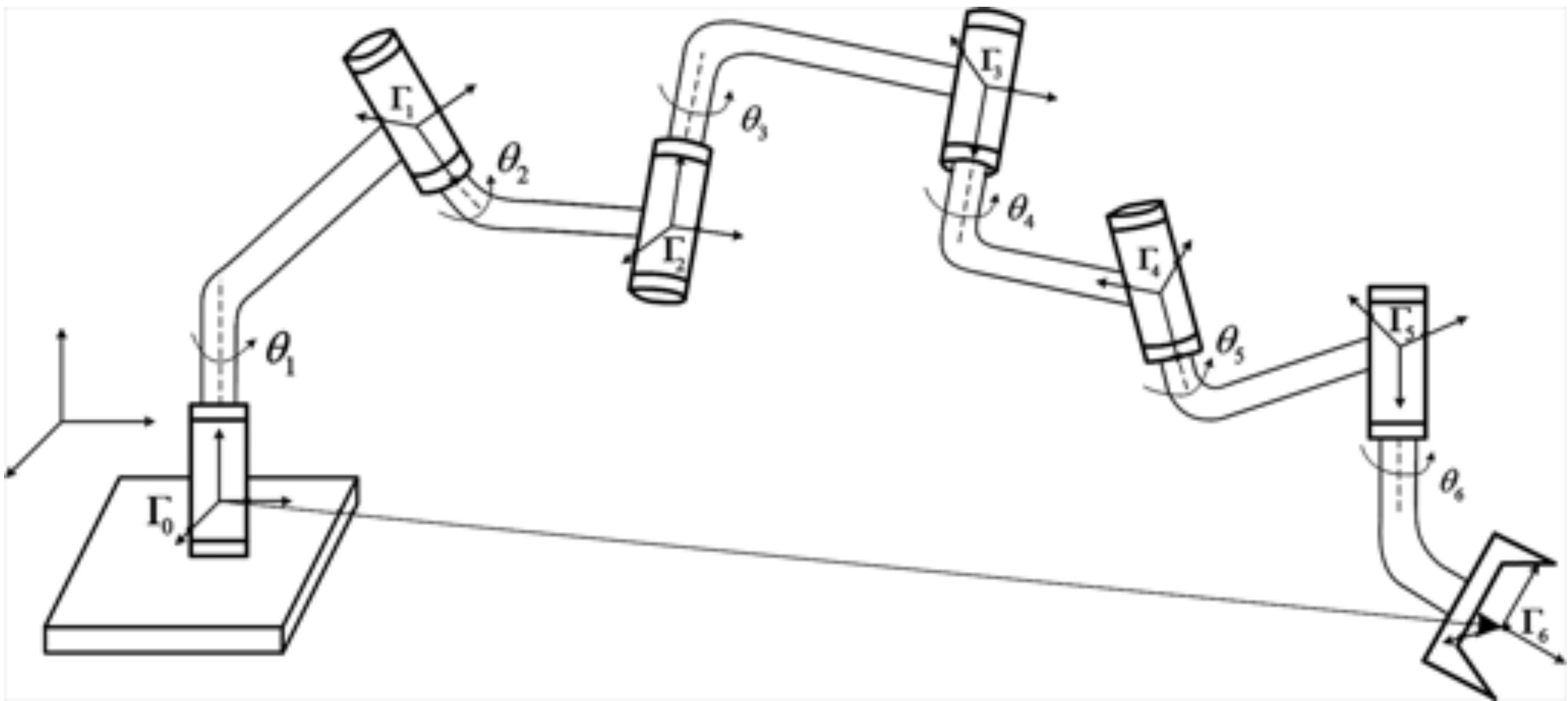
# Robot Kinematics

– **Forward kinematics**: infer the pose of the end-effector, given the state of each joint.

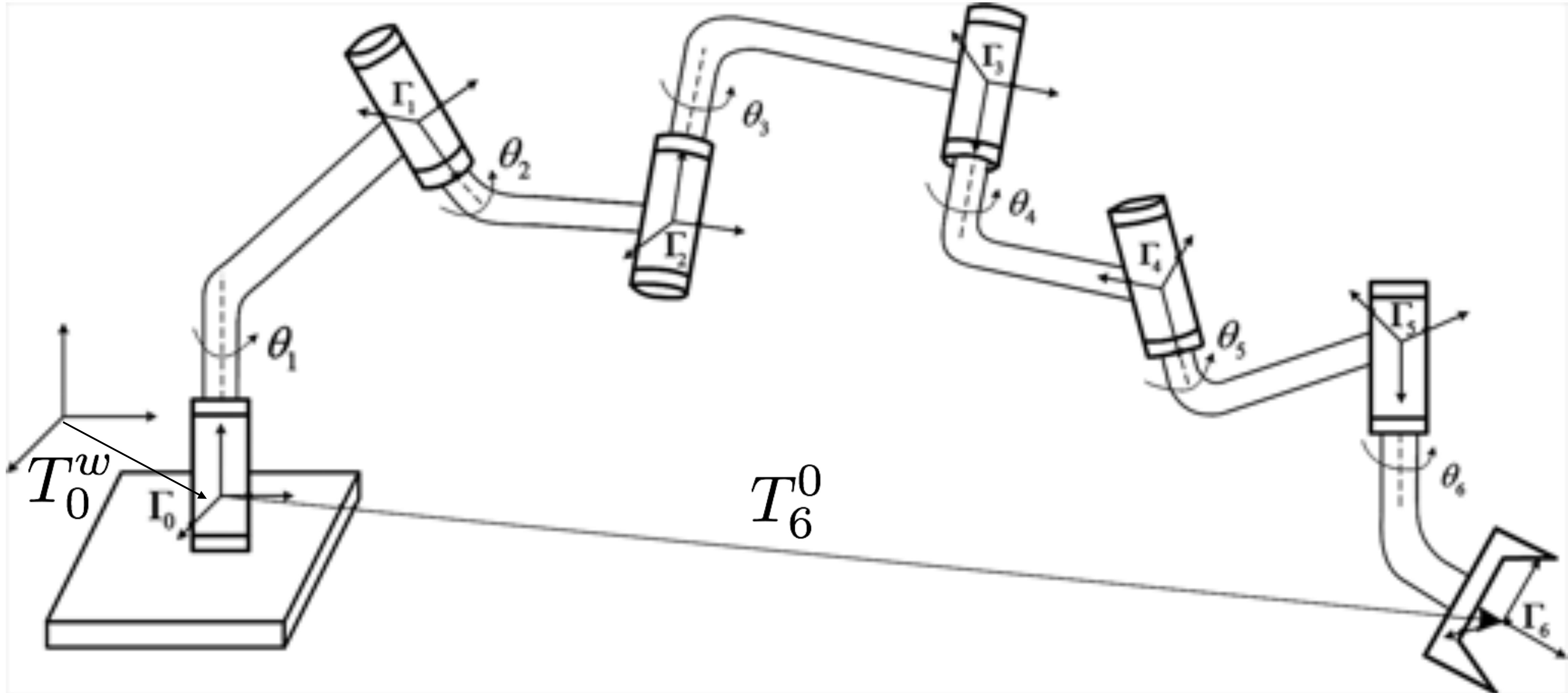
Infer: pose of each joint and link in a common world workspace

Assuming as given the:

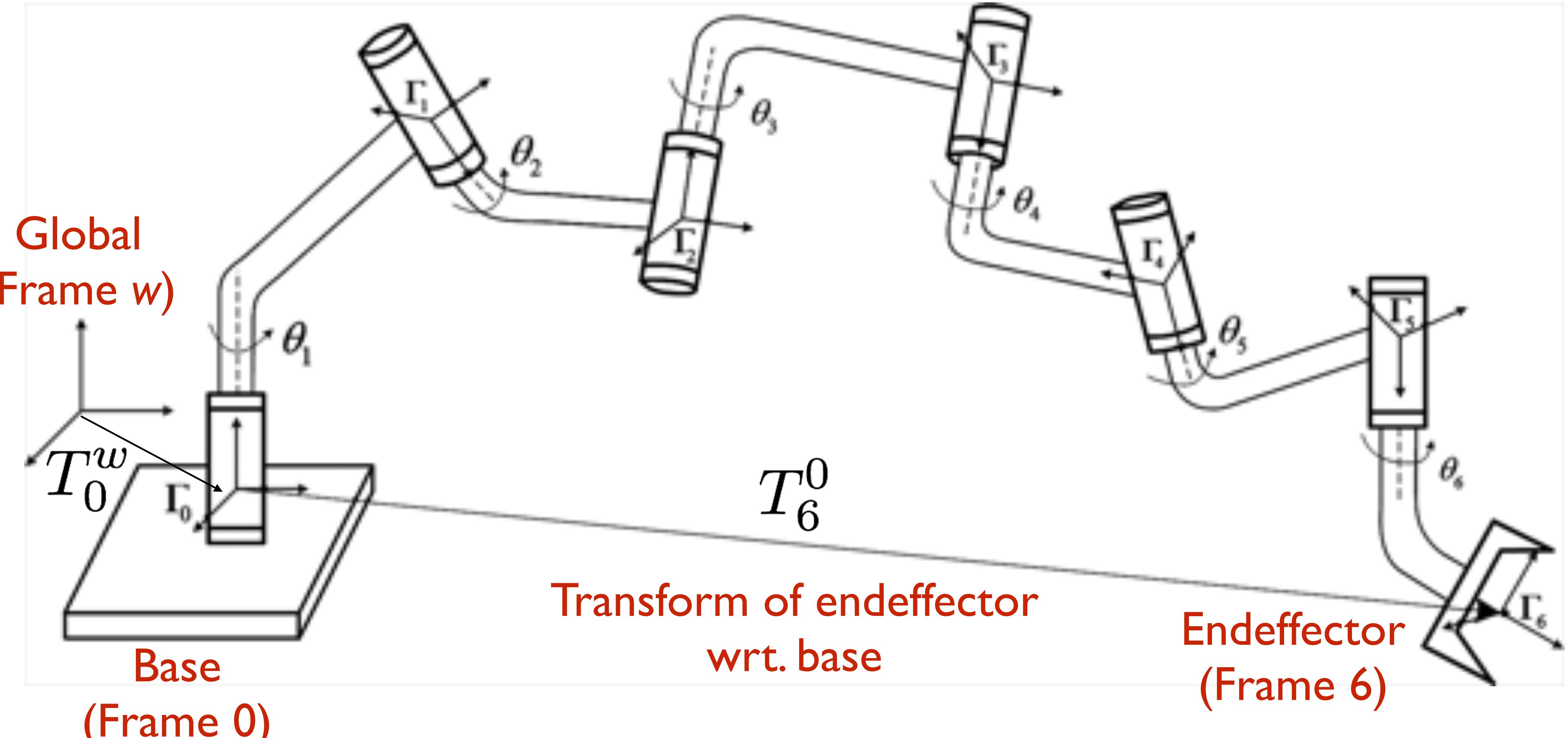
- robot's kinematic definition
- geometry of each link
- current state of all joints
  - zero configuration
  - add motor motion



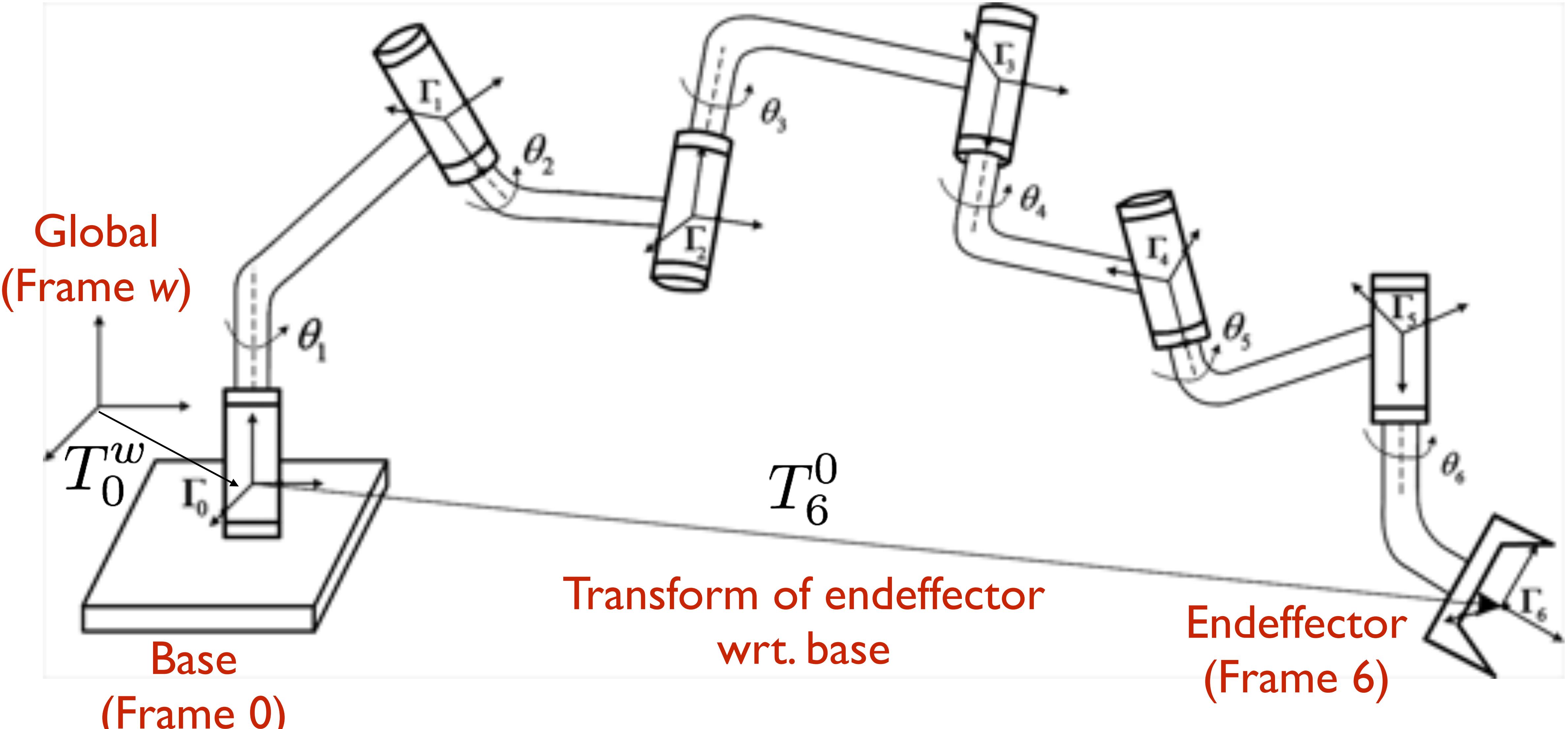
**Forward kinematics:** many-to-one mapping of robot configuration to reachable workspace endeffector poses



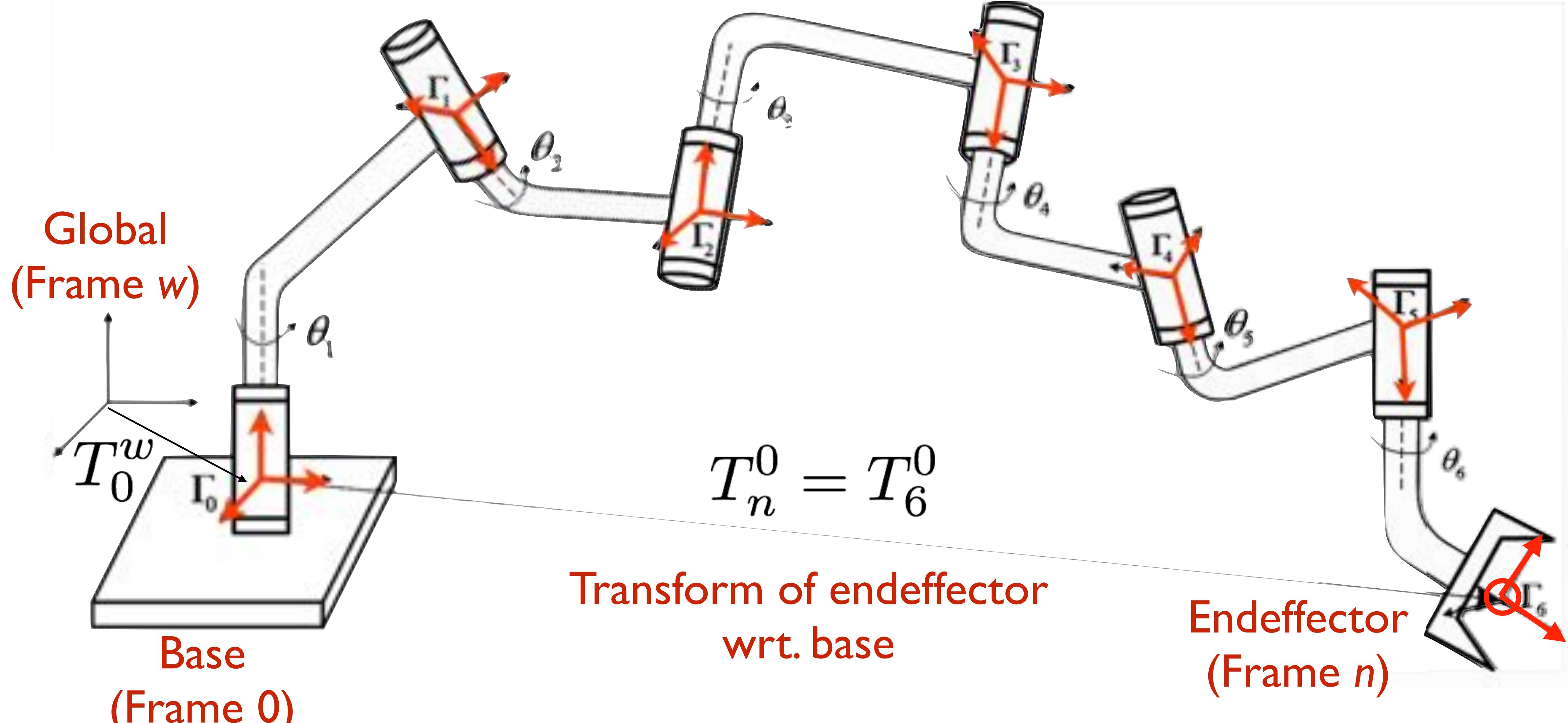
**Forward kinematics:** many-to-one mapping of robot configuration to reachable workspace endeffector poses



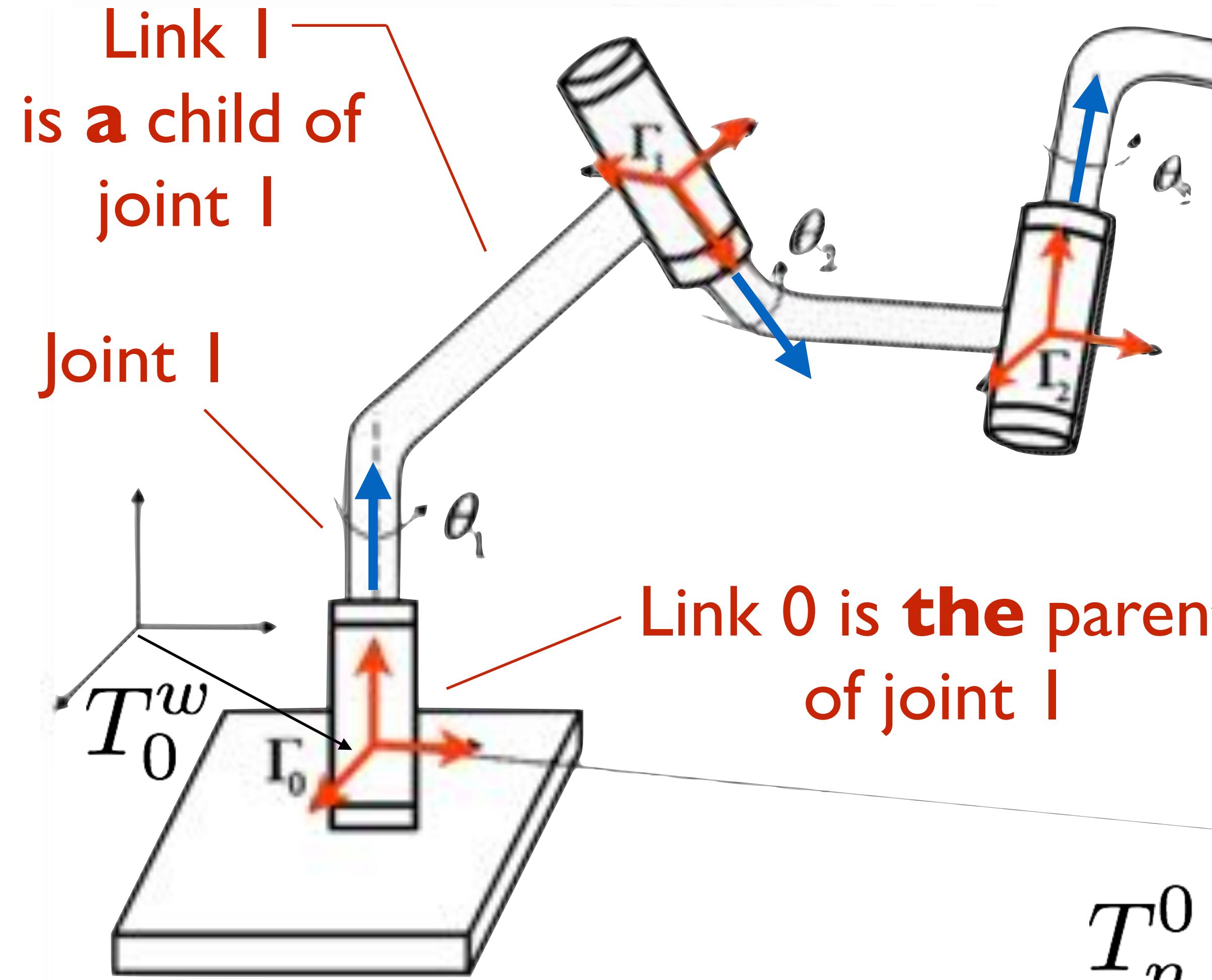
**Workspace:** 3D space defined in the global frame



**Kinematic chain:** connects  $N+1$  links together by  $N$  joints;  
with a coordinate frame on each link



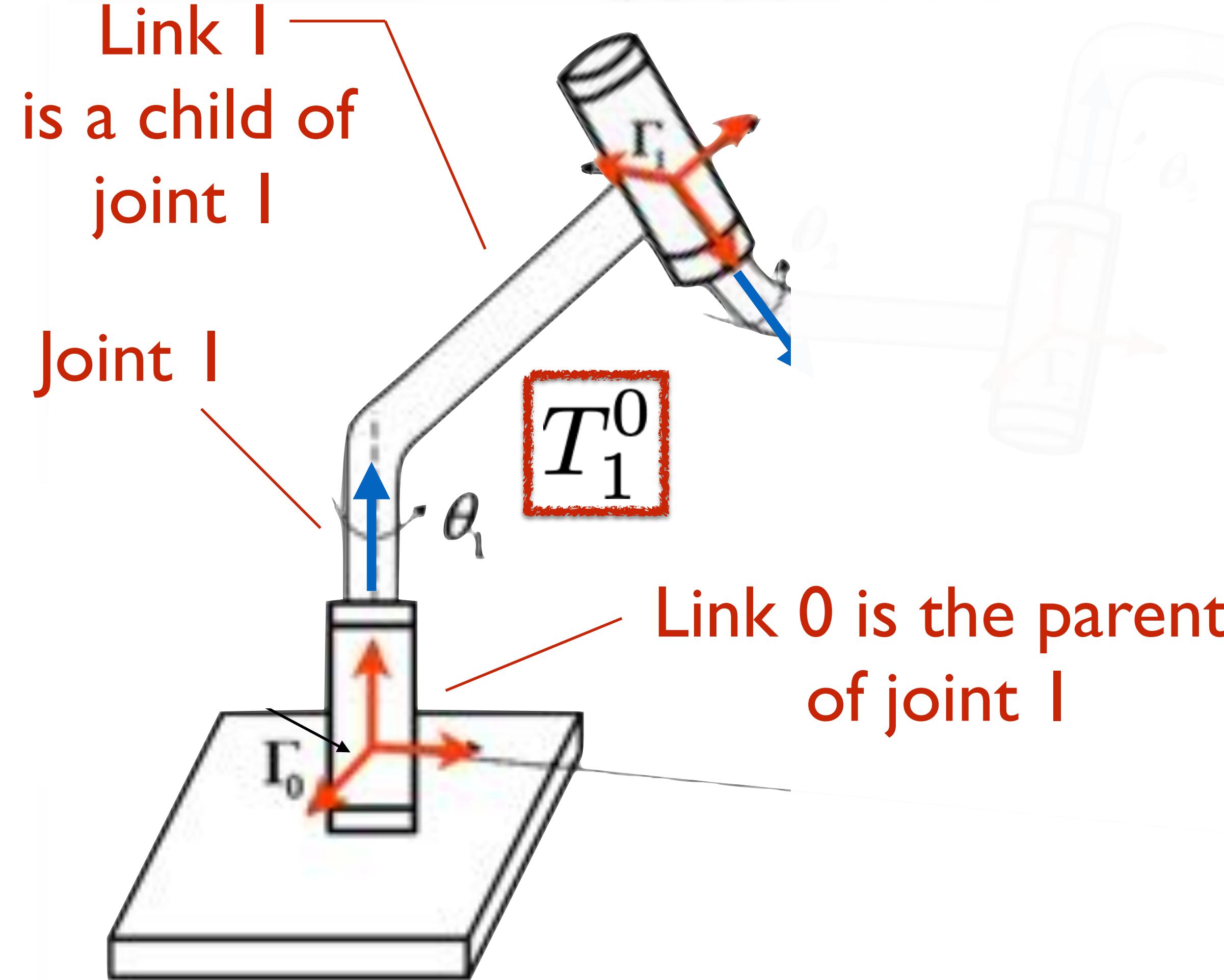
**Joint ( $q_i$ ):** relates the motion of one link (the child link) wrt. another link (the parent)  
joint motion only affects the child link



$$q_i = \begin{cases} \theta_i, & \text{if revolute} \\ d_i, & \text{if prismatic} \end{cases}$$

$$T_n^0$$

**Joint ( $q_i$ ):** relates the motion of one link (the child link) wrt. another link (the parent)  
joint motion only affects the child link, where its state



$$q_i = \begin{cases} \theta_i, & \text{if revolute} \\ d_i, & \text{if prismatic} \end{cases}$$

is used to express a 4-by-4 homogeneous transform  $\mathbf{A}_i(q_i)$ :

$$A_i = \begin{bmatrix} R_i^{i-1} & o^{i-1} \\ 0 & 1 \end{bmatrix}$$

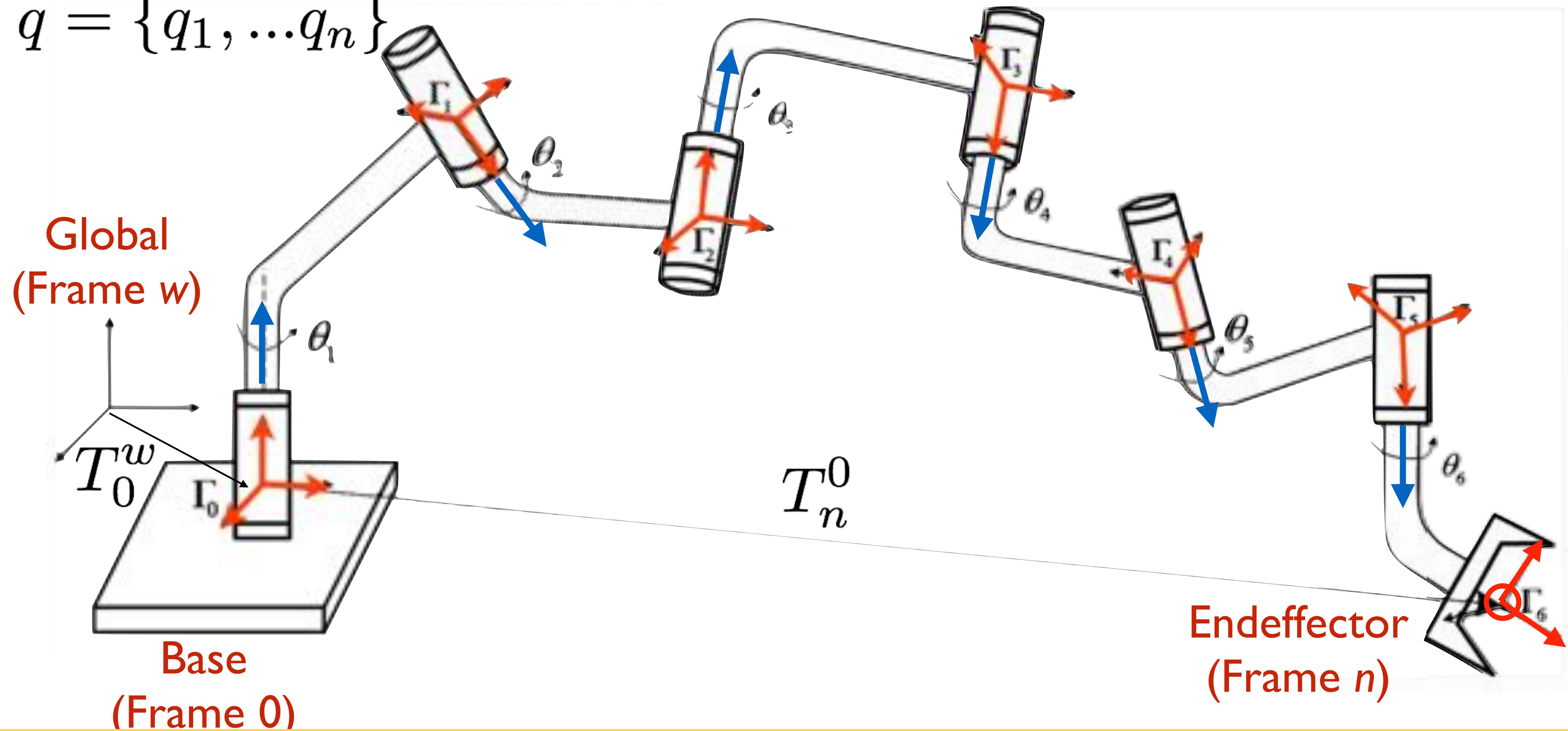
such that frames in a kinematic chain are related as by  $T_j^i$ :

$$T_j^i = \begin{cases} A_{i+1}A_{i+2}\dots A_{j-1}A_j & \text{if } i < j \\ I & \text{if } i = j \\ (T_j^i)^{-1} & \text{if } j > i \end{cases}$$

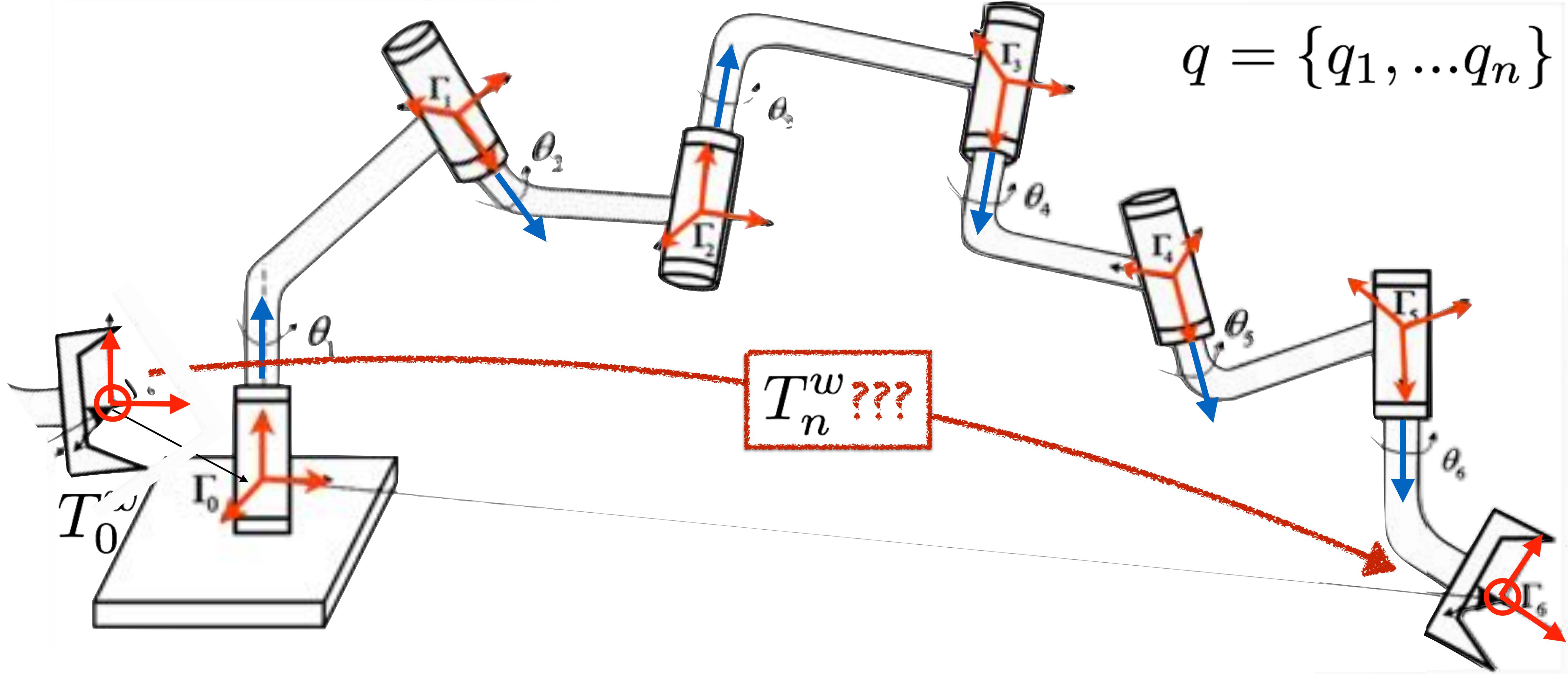
**Configuration ( $q$ ):** is the state of all joints in the kinematic chain

**Configuration space:** the space of all possible configurations

$$q = \{q_1, \dots q_n\}$$

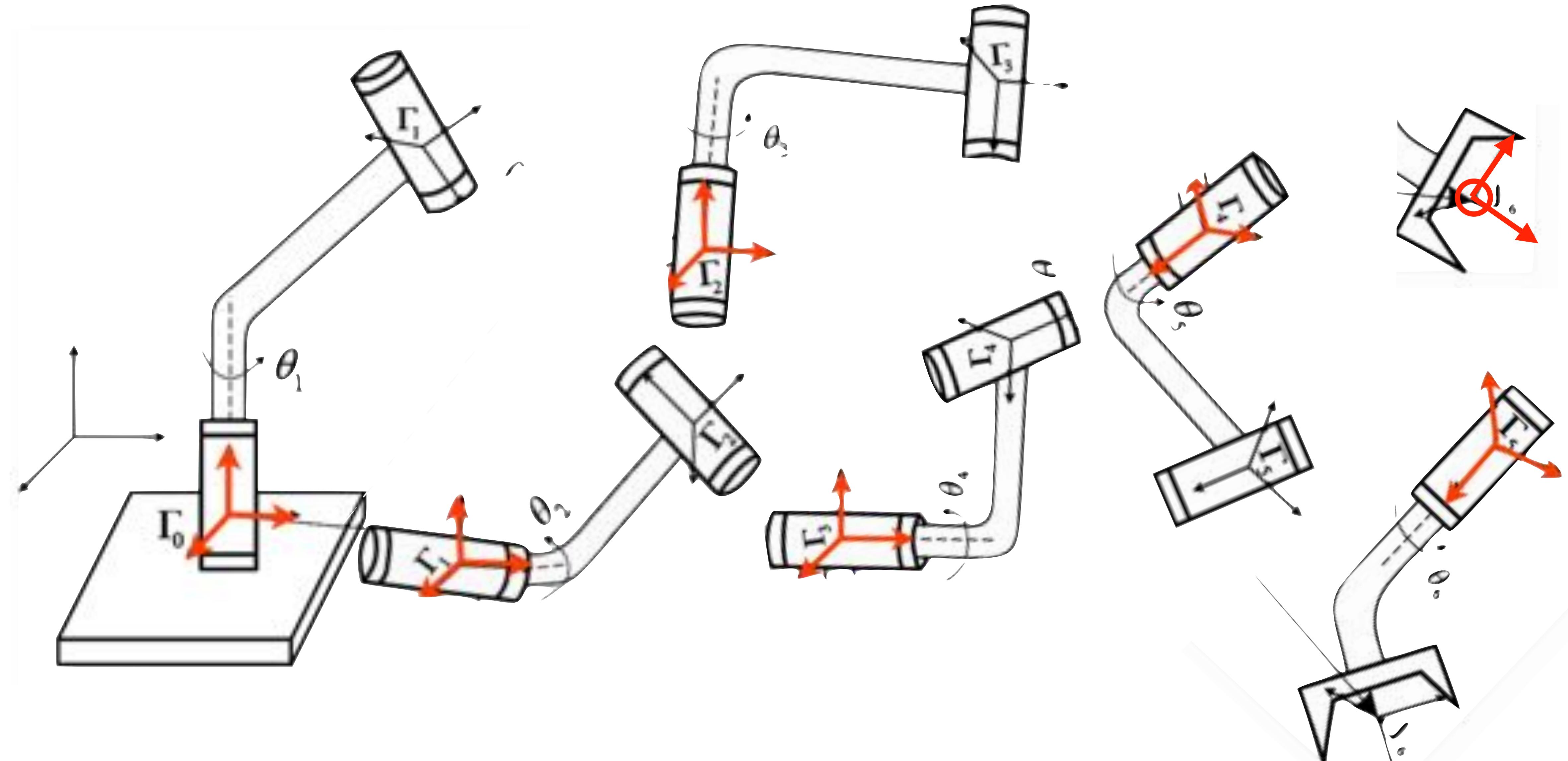


**Forward kinematics restated:** Given  $\mathbf{q}$ , find  $T^w_n$ ;  
 $T^w_n$  transforms endeffector into workspace

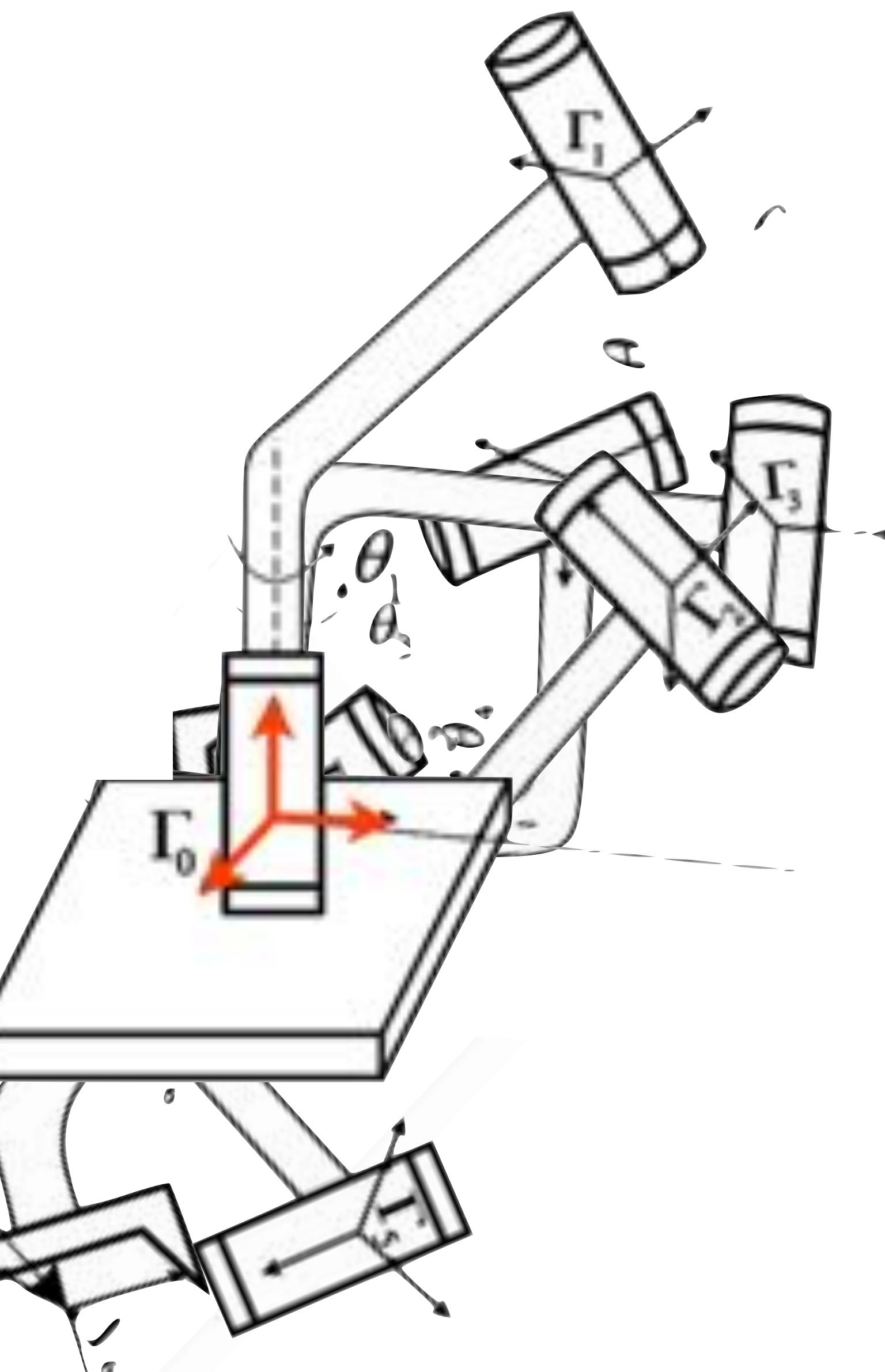


$$\mathbf{q} = \{q_1, \dots, q_n\}$$

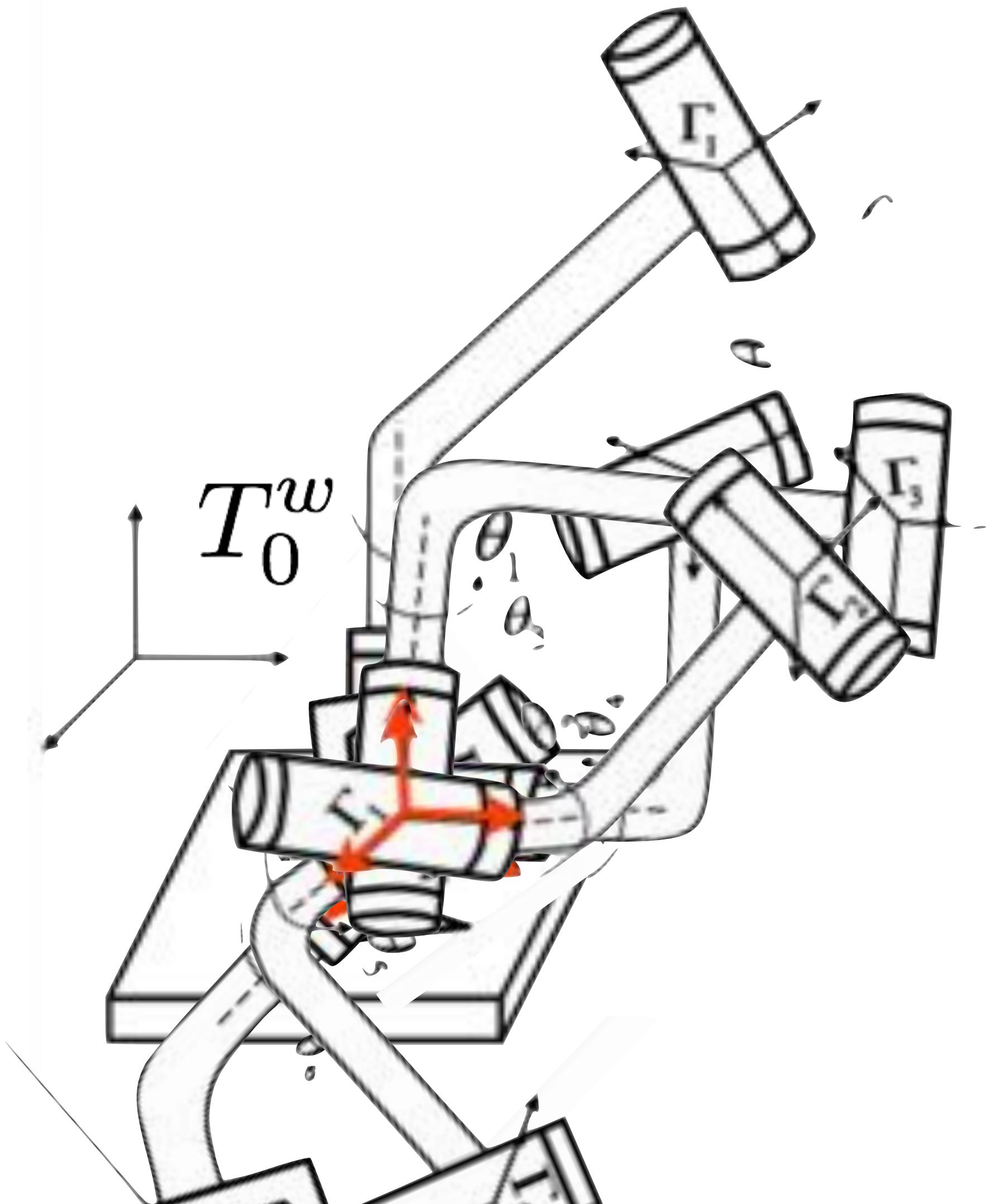
**Problem:** Every link considers itself to be the center of the universe.  
How do we properly pose link with respect to each other?

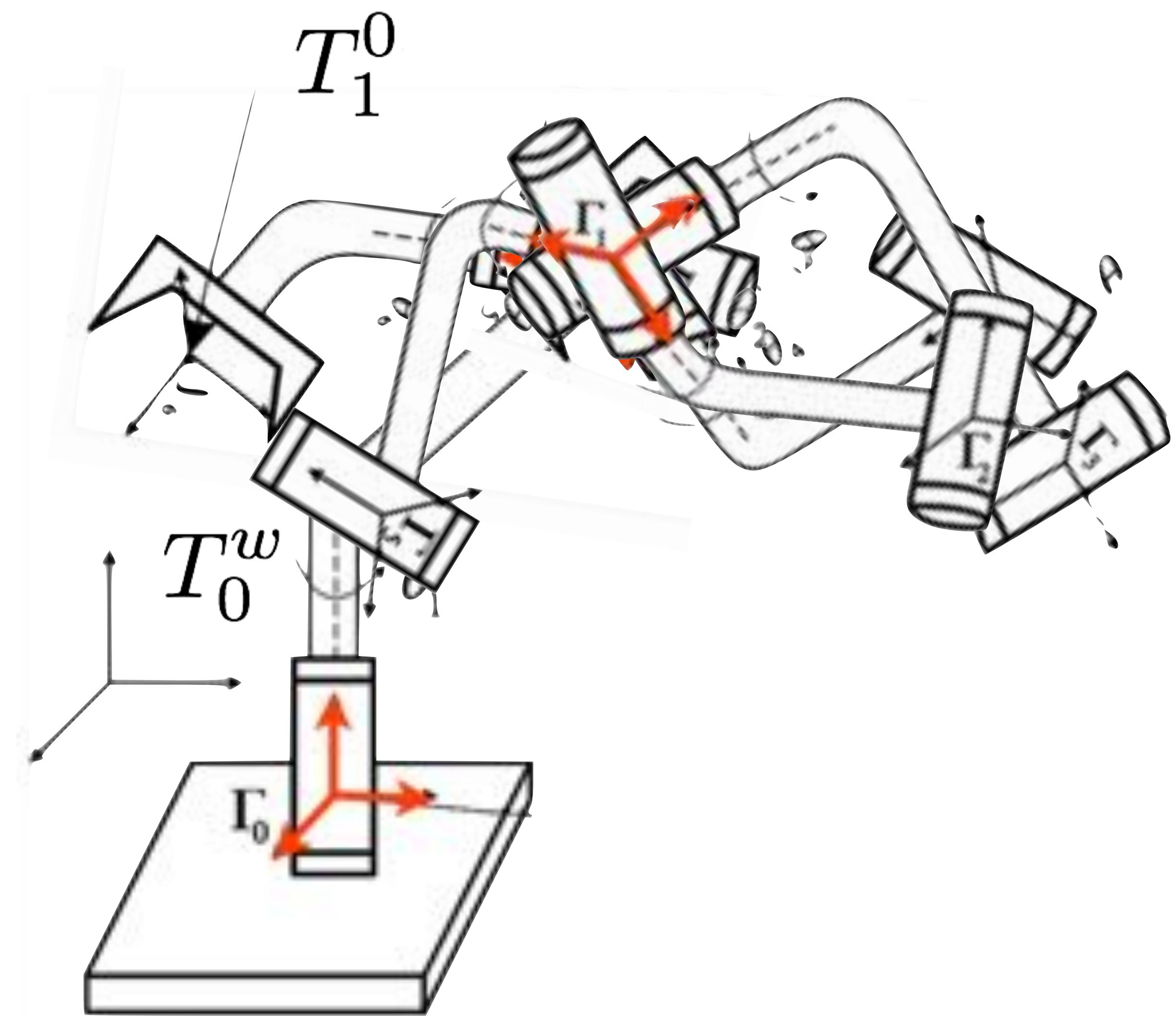


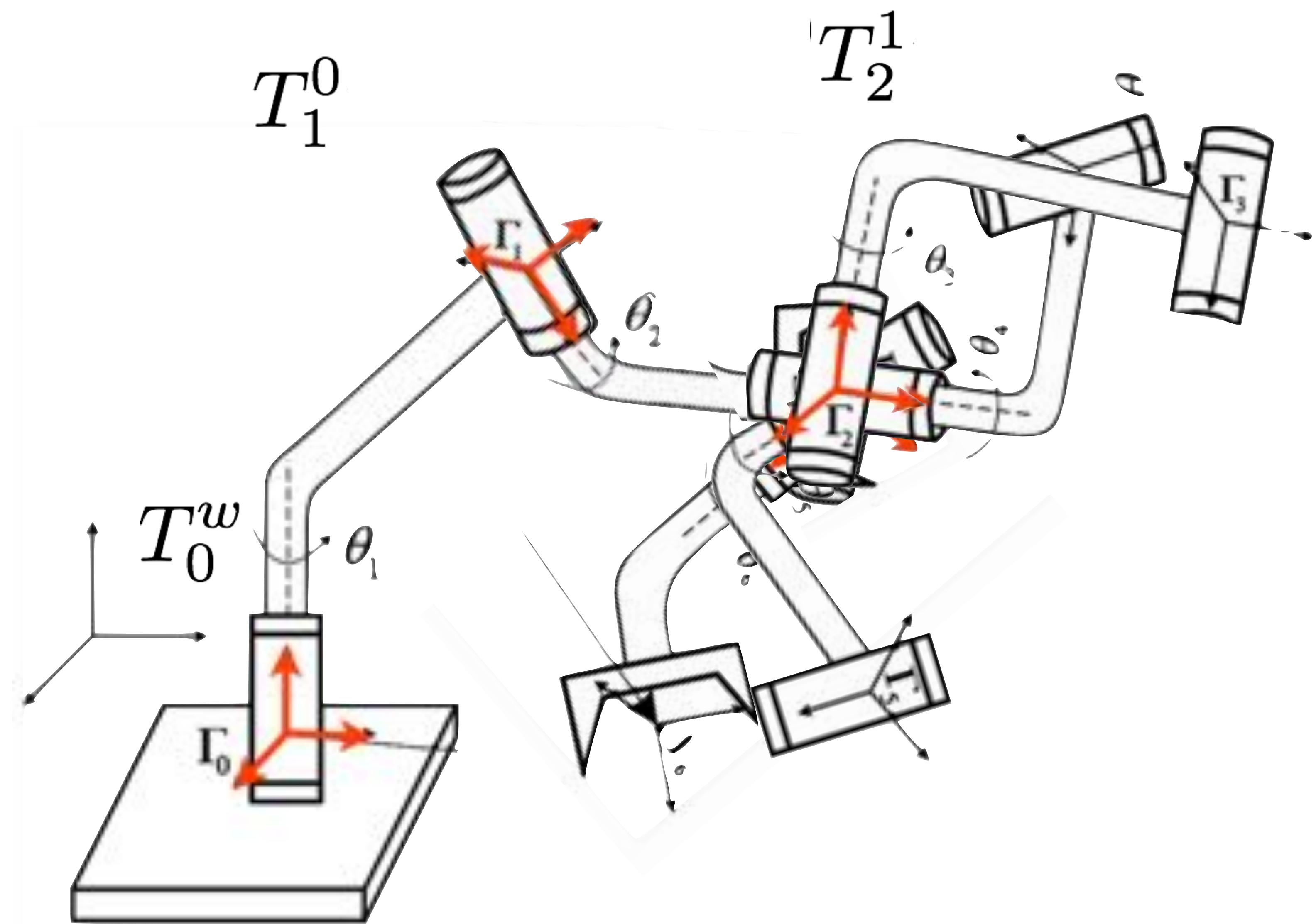
**Approach:** Consider all links to be aligned with the global origin ...

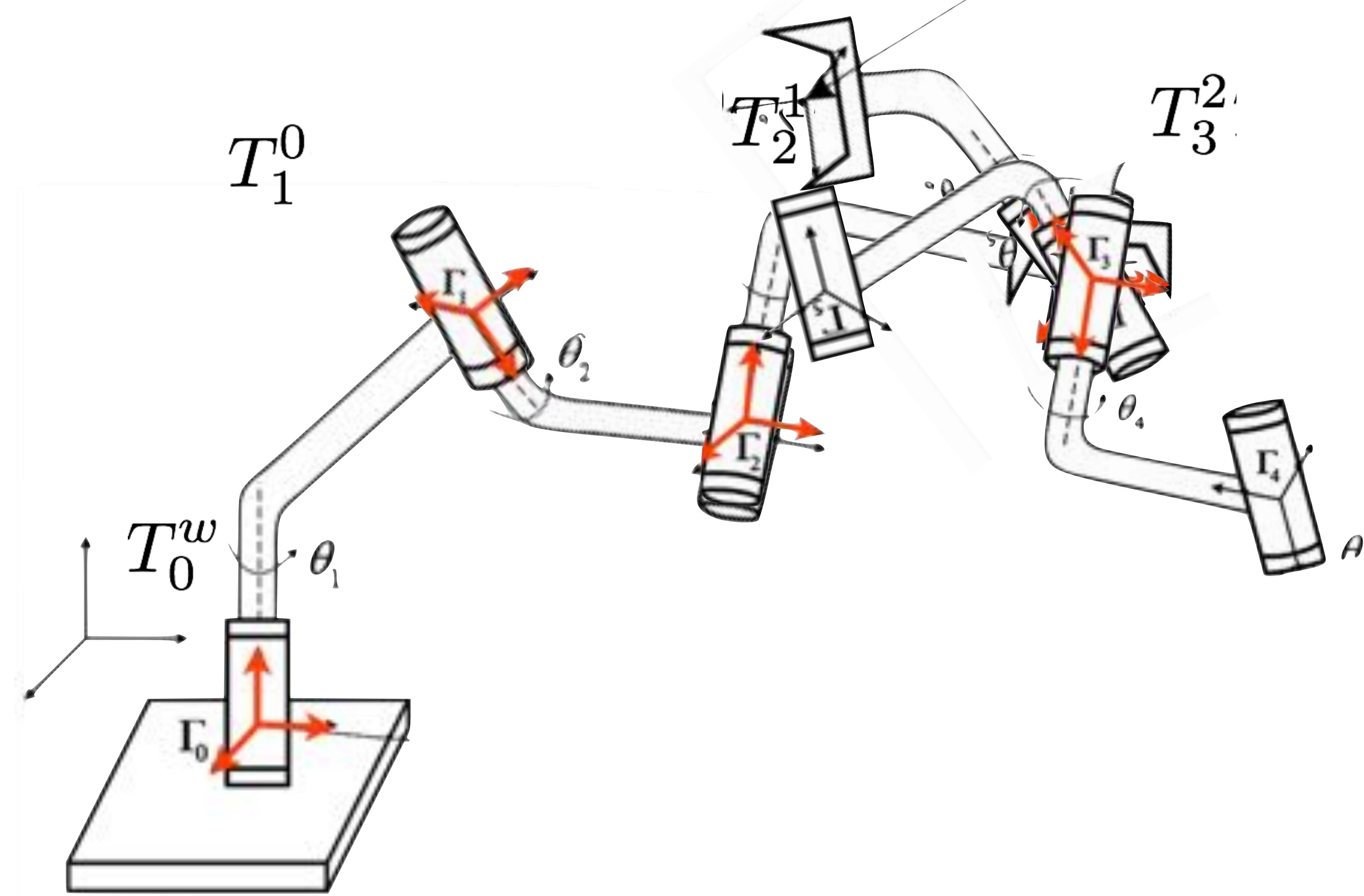


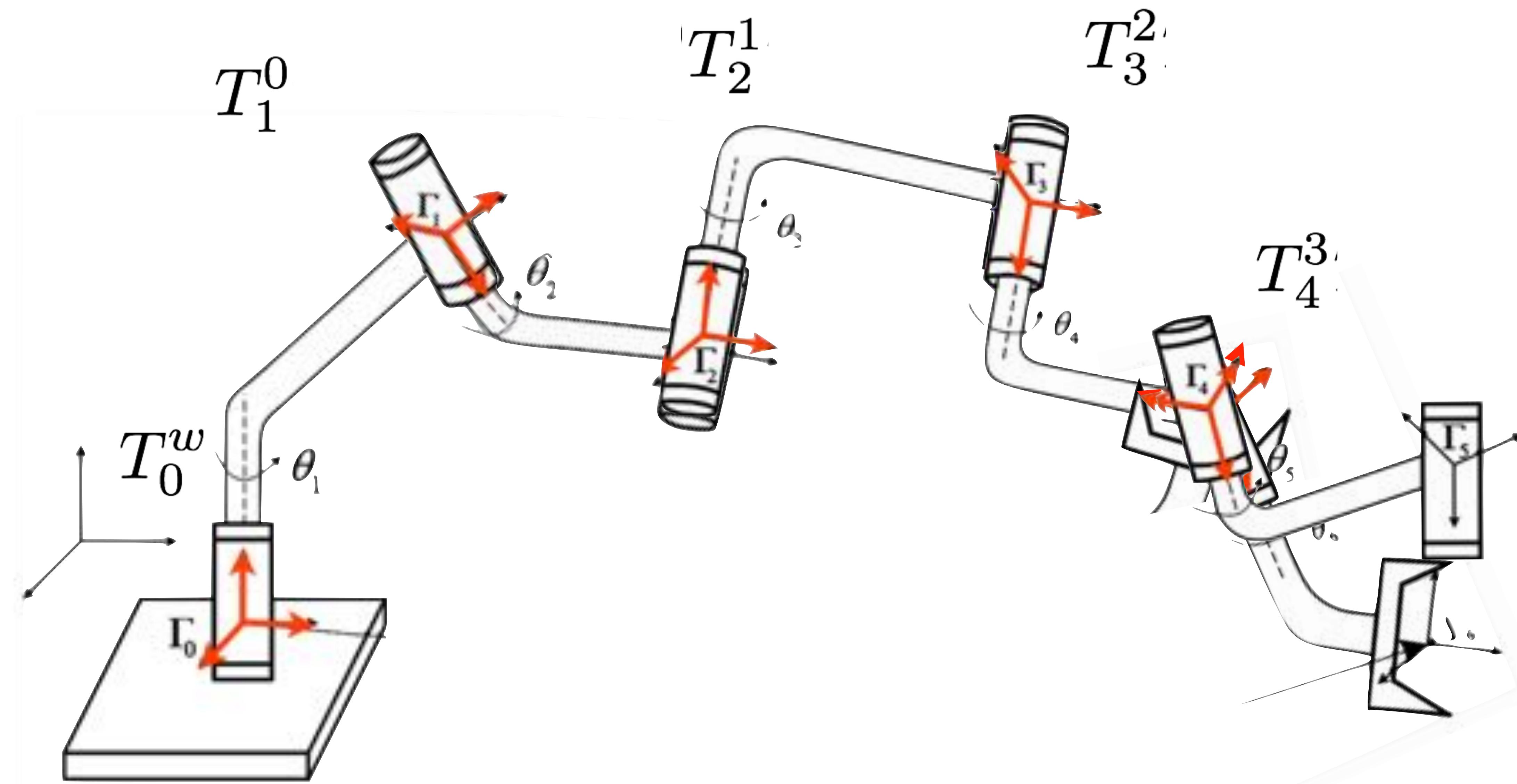
**Approach:** transform along kinematic chain bringing descendants along; each transform will consist of a rotation and a translation

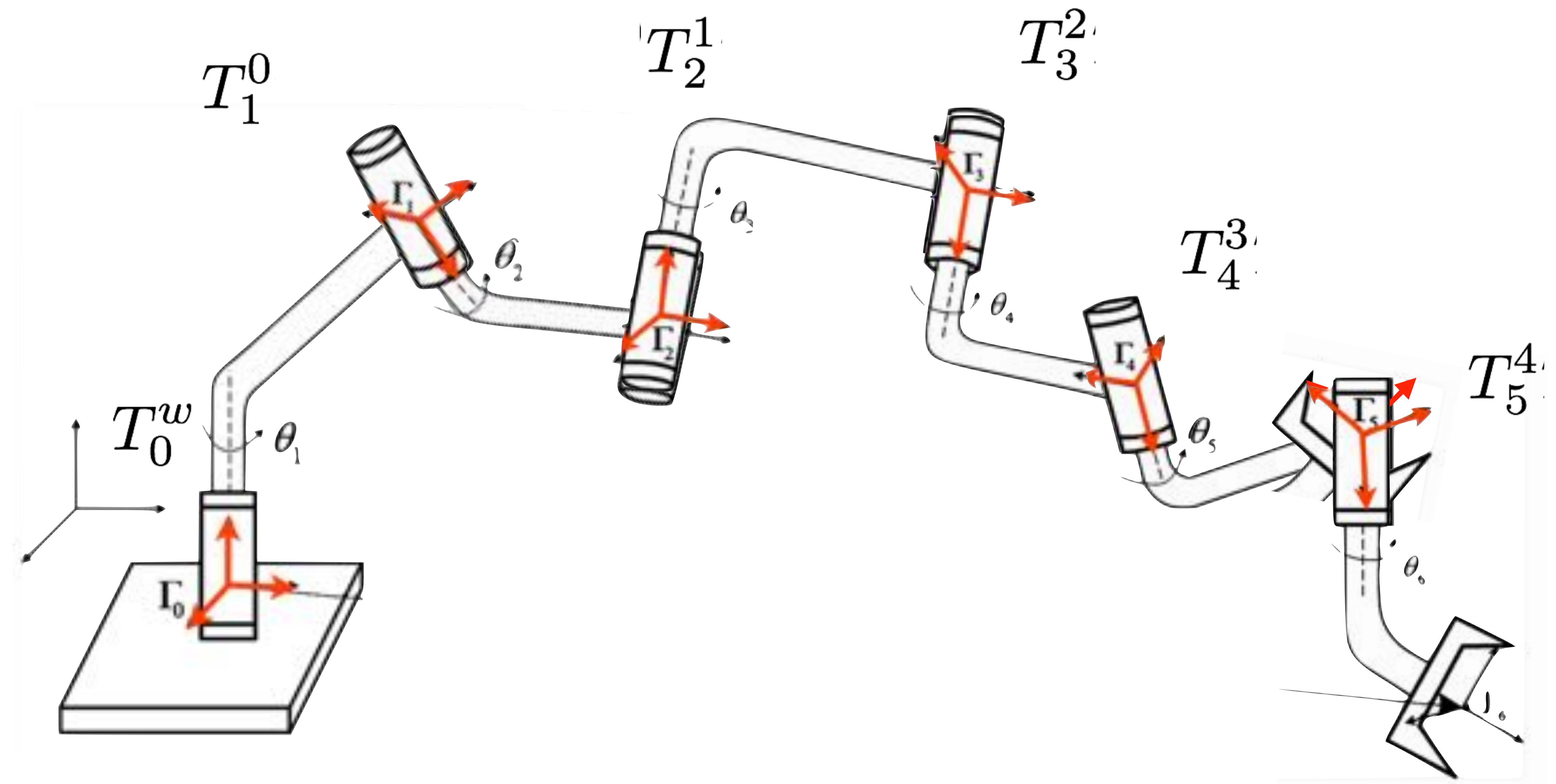


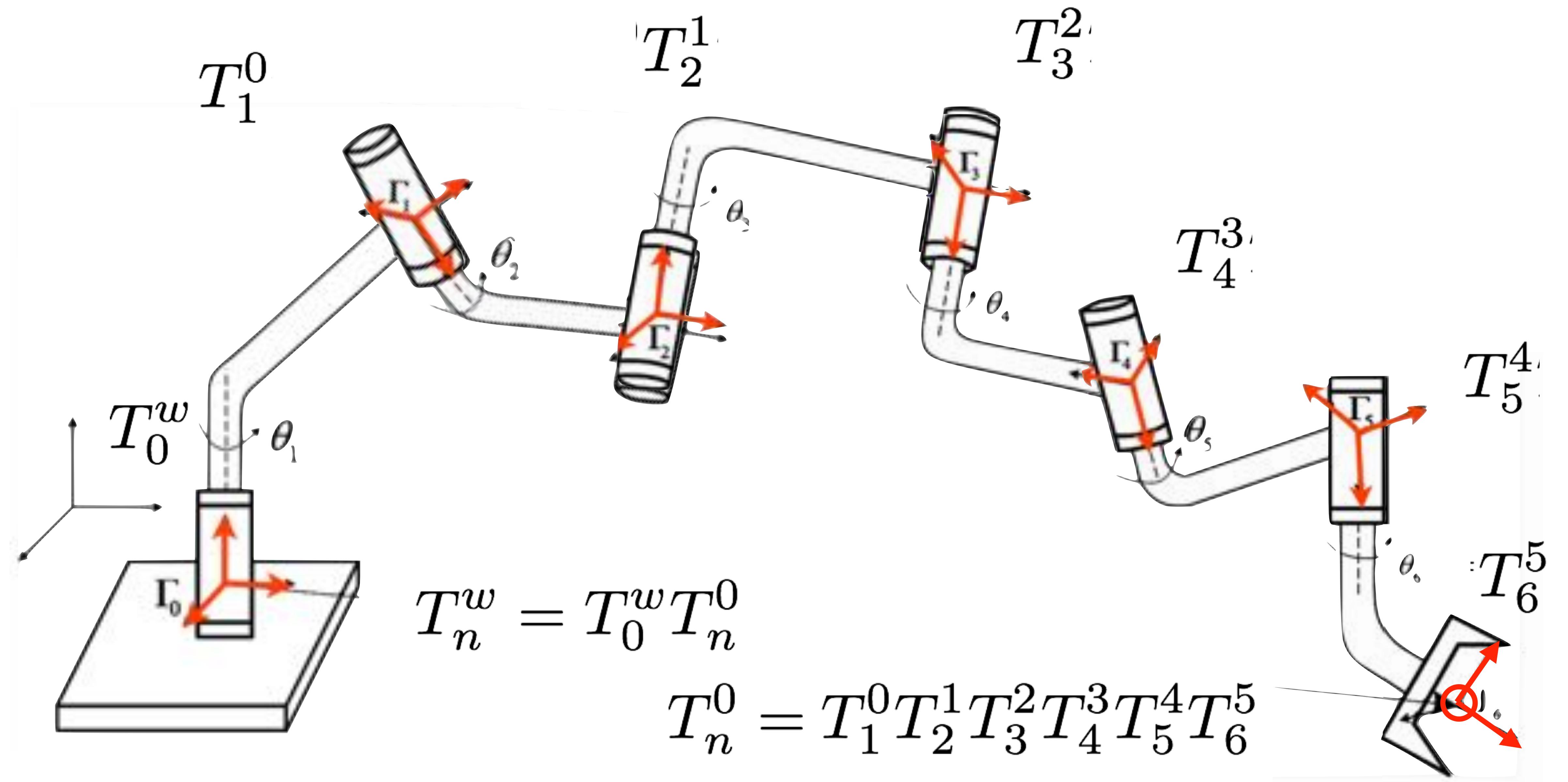




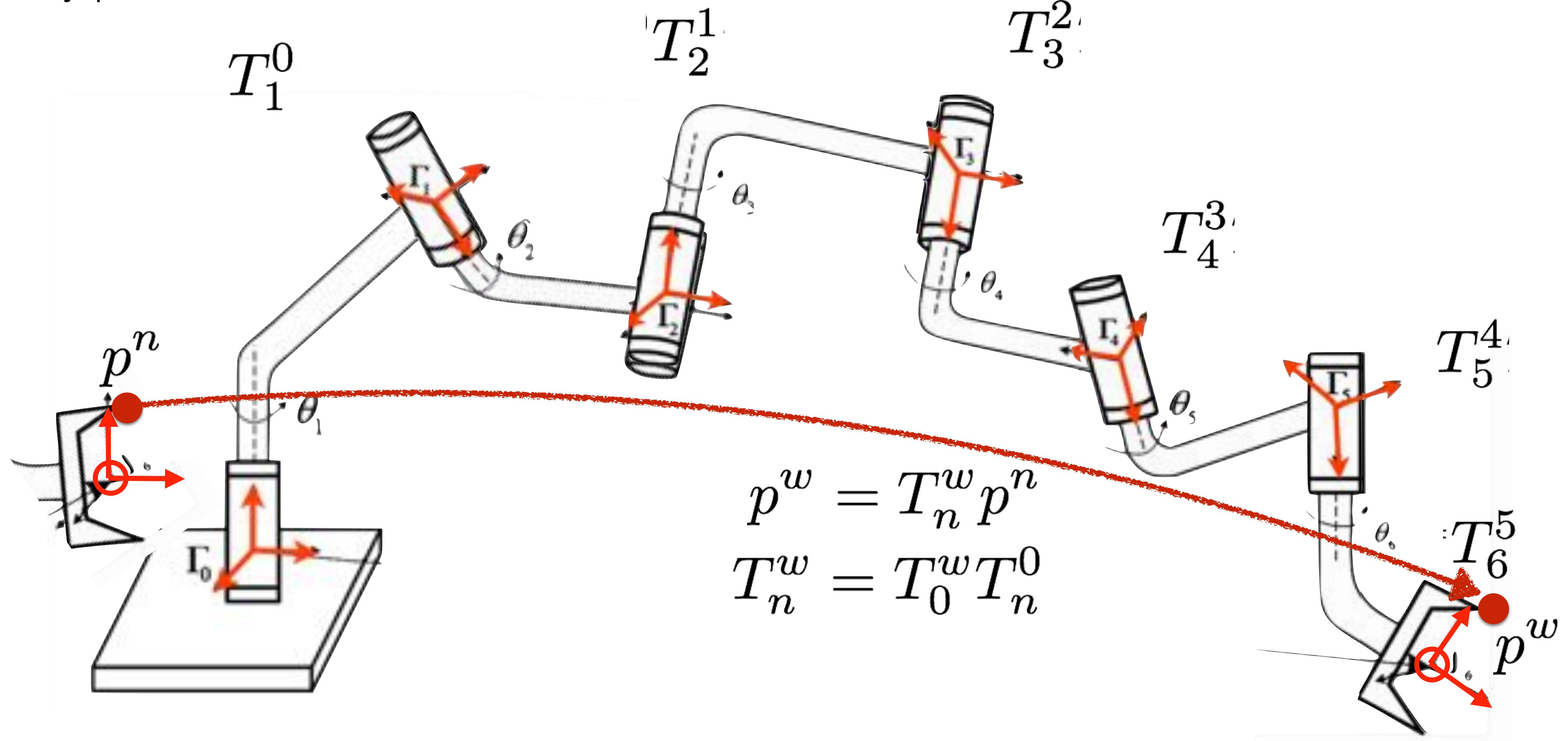




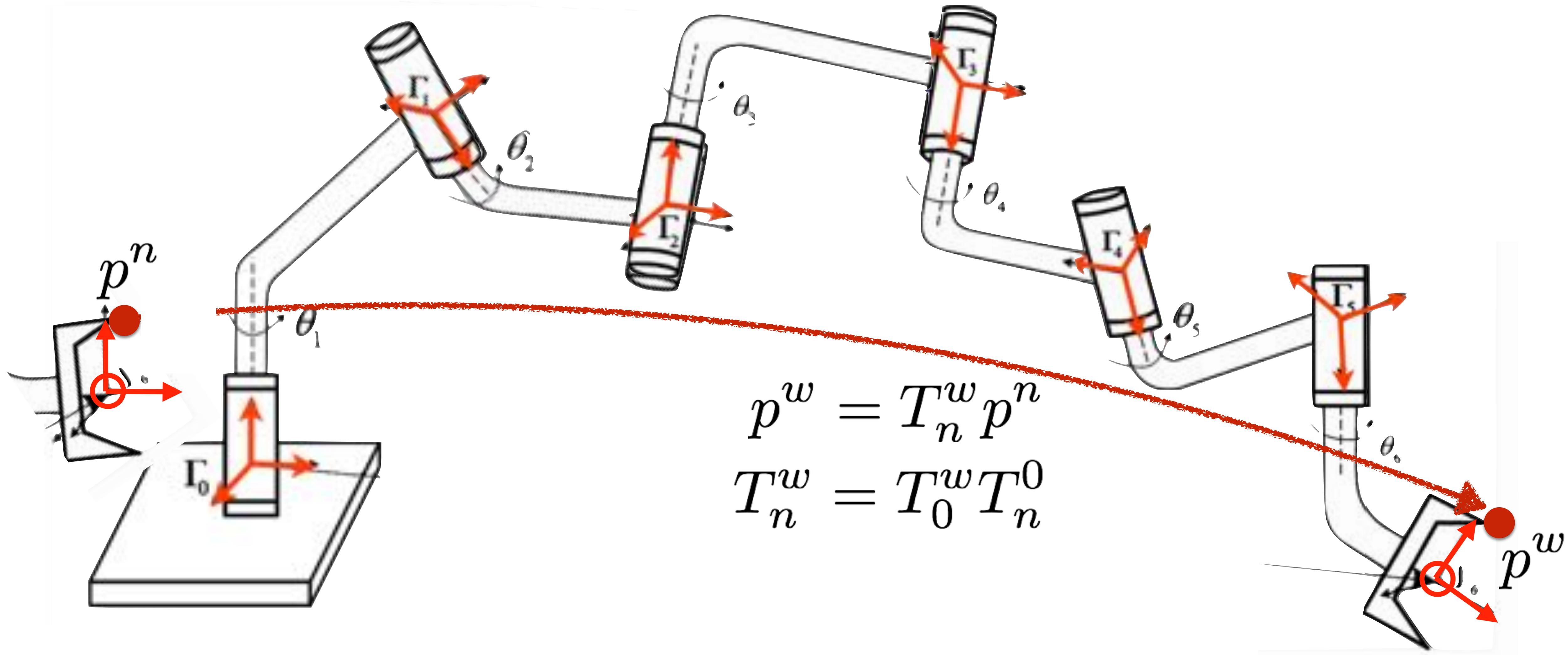




Any point on the endeffector can be transformed to its location in the world



- 1) How to represent homogeneous transforms?
- 2) How to compute transform to endeffector?

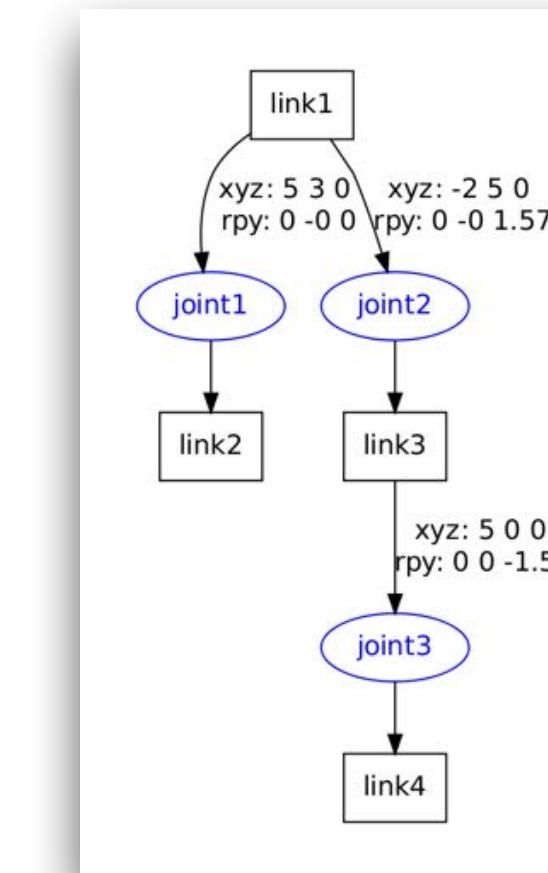


# Previously

1) How to represent homogeneous transforms?

Assuming as given the:

- geometry of each link
- robot's kinematic definition



## Homogeneous Transform

defines  $SE(2)$ : Special Euclidean Group 2

$$H = \begin{bmatrix} R_{00} & R_{01} & d_x \\ R_{10} & R_{11} & d_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{2 \times 2} & \mathbf{d}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & 1 \end{bmatrix}$$

$H \in SE(2)$     $\mathbf{R}_{2 \times 2} \in SO(2)$     $\mathbf{d}_{2 \times 1} \in \mathbb{R}^2$

## 3D Homogeneous Transform

$$H_3 = \begin{bmatrix} R_{00} & R_{01} & R_{02} & d_x \\ R_{10} & R_{11} & R_{12} & d_y \\ R_{20} & R_{21} & R_{22} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{d}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \in SE(3)$$

if  $T_1^0 \in SE(3)$  and  $T_2^1 \in SE(3)$  then composition holds:

$$\begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^1 & d_2^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1^0 R_2^1 & R_1^0 d_2^1 + d_1^0 \\ 0 & 1 \end{bmatrix}$$

such that points in Frame 2 can be expressed in Frame 0 by:

$$p^0 = T_1^0 T_2^1 p^2$$

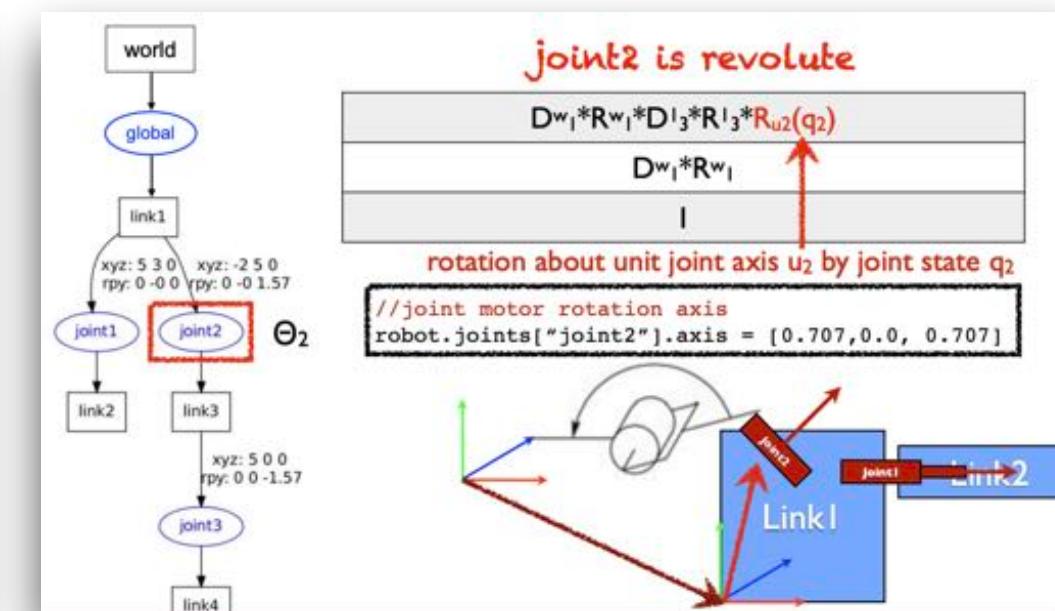
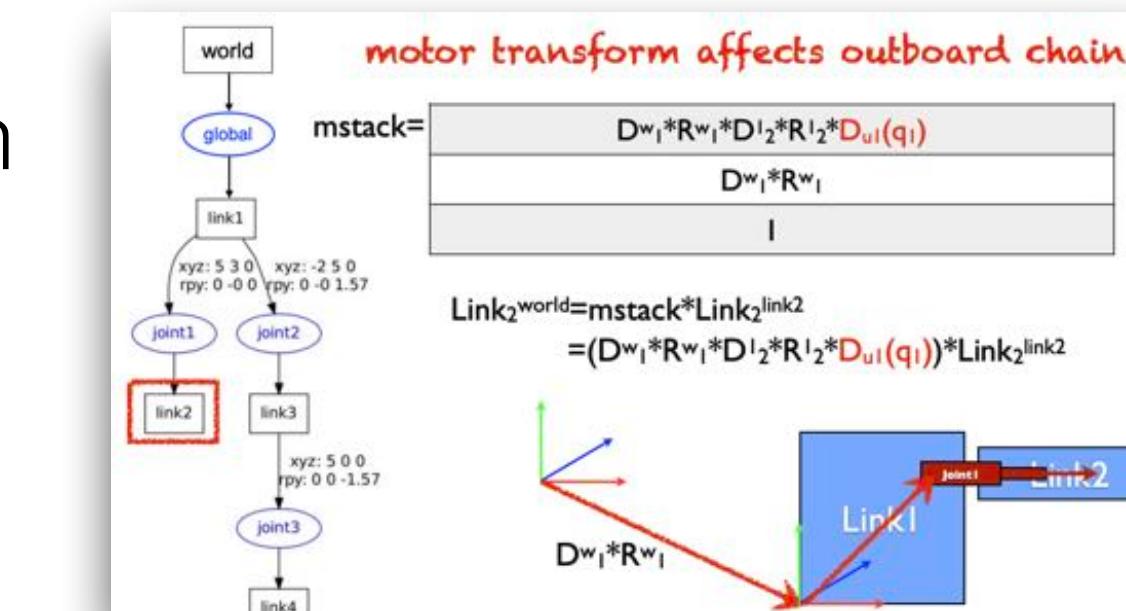
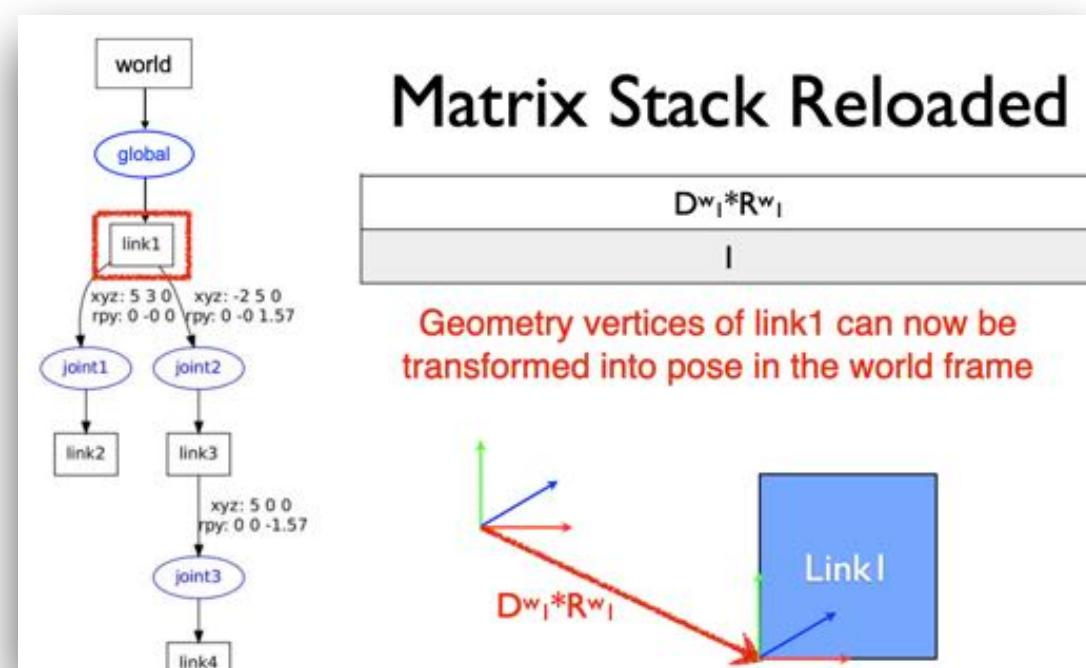
2) How to compute transform to endeffector?

## Zero configuration

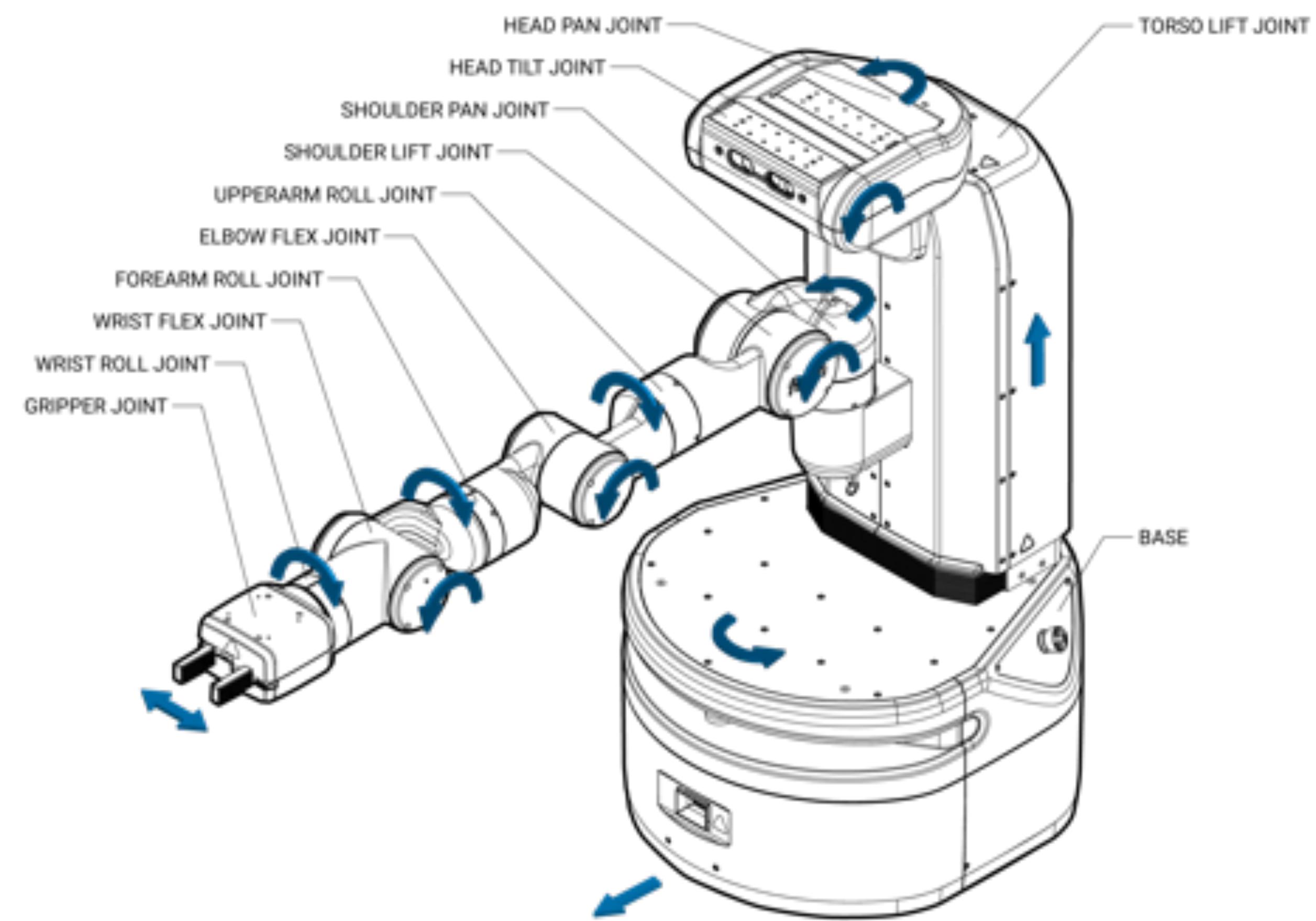
Assuming as given the:

- geometry of each link
- robot's kinematic definition
- **current state of all joints**

## Add motor motion



# Can a joint move infinitely far?



# Joint Limits

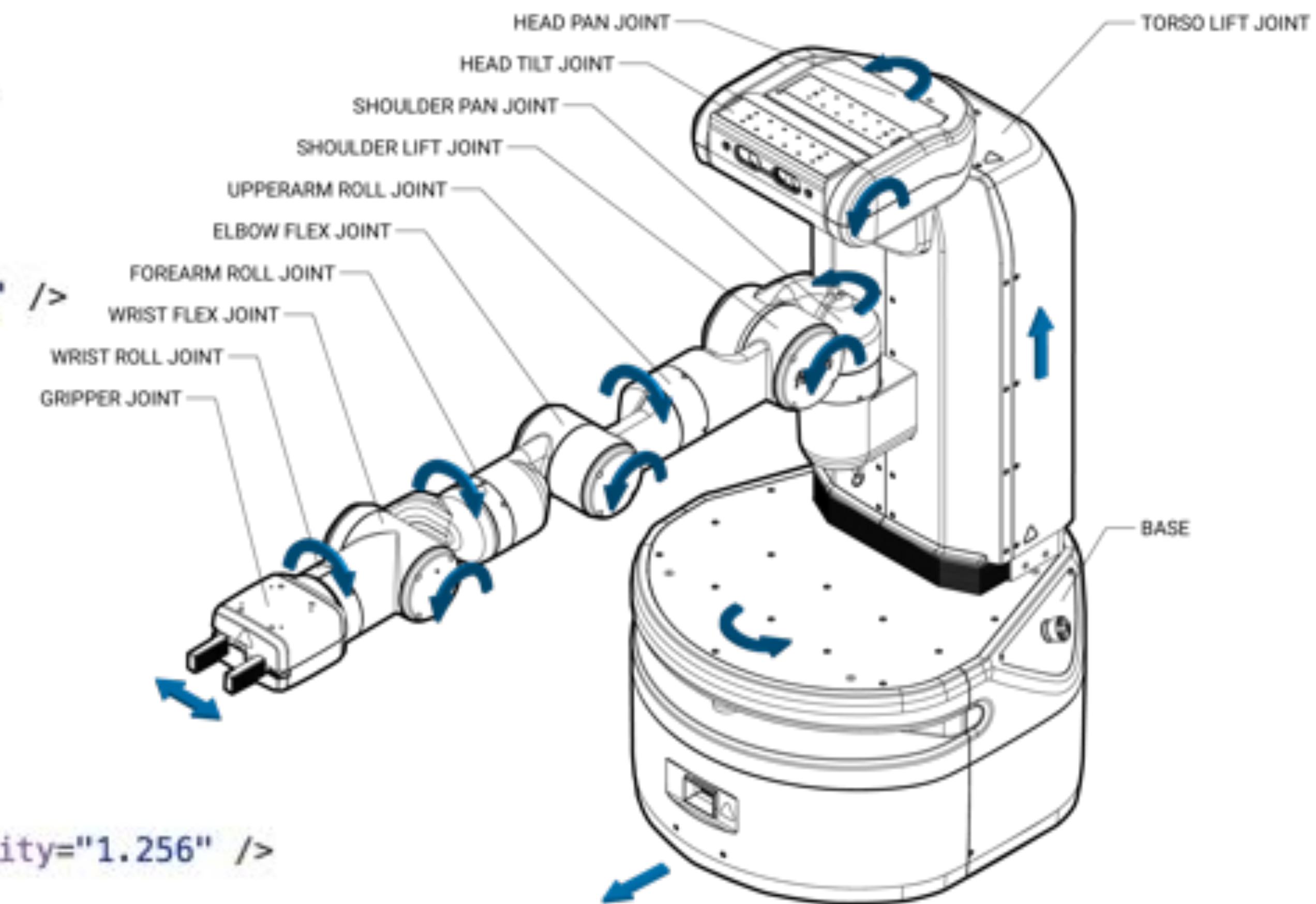
Prismatic joint description

```
<joint name="torso_lift_joint" type="prismatic">
  <origin rpy="-6.123E-17 0 0" xyz="-0.086875 0 0.37743" />
  <parent link="base_link" />
  <child link="torso_lift_link" />
  <axis xyz="0 0 1" />
  <limit effort="450.0" lower="0" upper="0.4" velocity="0.1" />
</joint>
```

Revolute joint description

```
<joint name="shoulder_pan_joint" type="revolute">
  <origin rpy="0 0 0" xyz="0.119525 0 0.34858" />
  <parent link="torso_lift_link" />
  <child link="shoulder_pan_link" />
  <axis xyz="0 0 1" />
  <dynamics damping="1.0" />
  <limit effort="33.82" lower="-1.6056" upper="1.6056" velocity="1.256" />
</joint>
```

Continuous joints have no limits



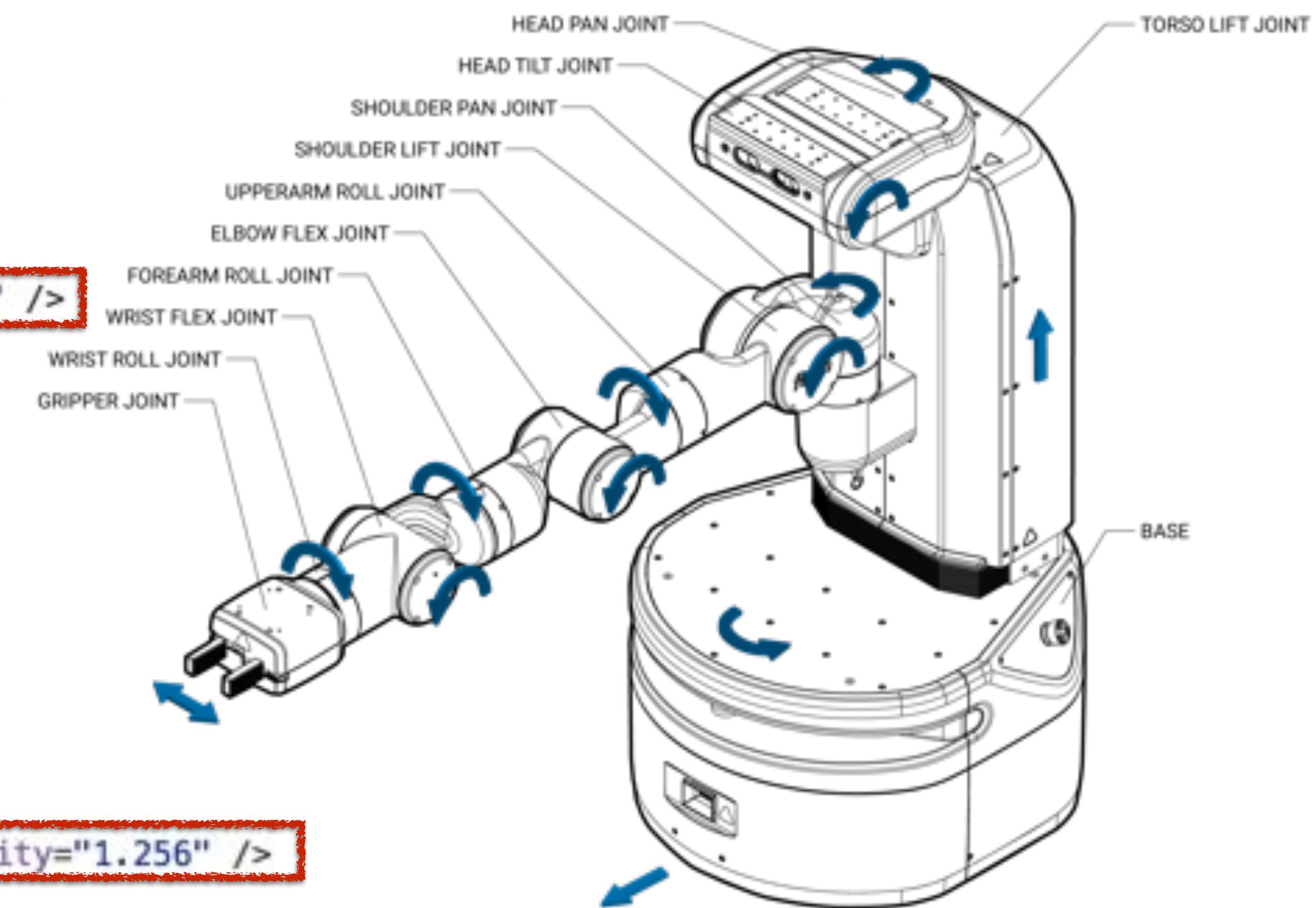
# Joint Limits

## Prismatic joint description

```
<joint name="torso_lift_joint" type="prismatic">
  <origin rpy="-6.123E-17 0 0" xyz="-0.086875 0 0.37743" />
  <parent link="base_link" />
  <child link="torso_lift_link" />
  <axis xyz="0 0 1" />
  <limit effort="450.0" lower="0" upper="0.4" velocity="0.1" />
<dynamics damping="100.0" /></joint>
```

## Revolute joint description

```
<joint name="shoulder_pan_joint" type="revolute">
  <origin rpy="0 0 0" xyz="0.119525 0 0.34858" />
  <parent link="torso_lift_link" />
  <child link="shoulder_pan_link" />
  <axis xyz="0 0 1" />
  <dynamics damping="1.0" />
  <limit effort="33.82" lower="-1.6056" upper="1.6056" velocity="1.256" />
</joint>
```



```

robot.joints.torso_lift_joint = {parent:"base_link", child:"torso_lift_link"};
robot.joints.torso_lift_joint.axis = [0,0,1];
robot.joints.torso_lift_joint.type = "prismatic";
robot.joints.torso_lift_joint.origin = {xyz: [-0.086875,0,0.37743], rpy:[-6.123E-17,0,0]};
robot.joints.torso_lift_joint.limit = {lower:0, upper:0.4};

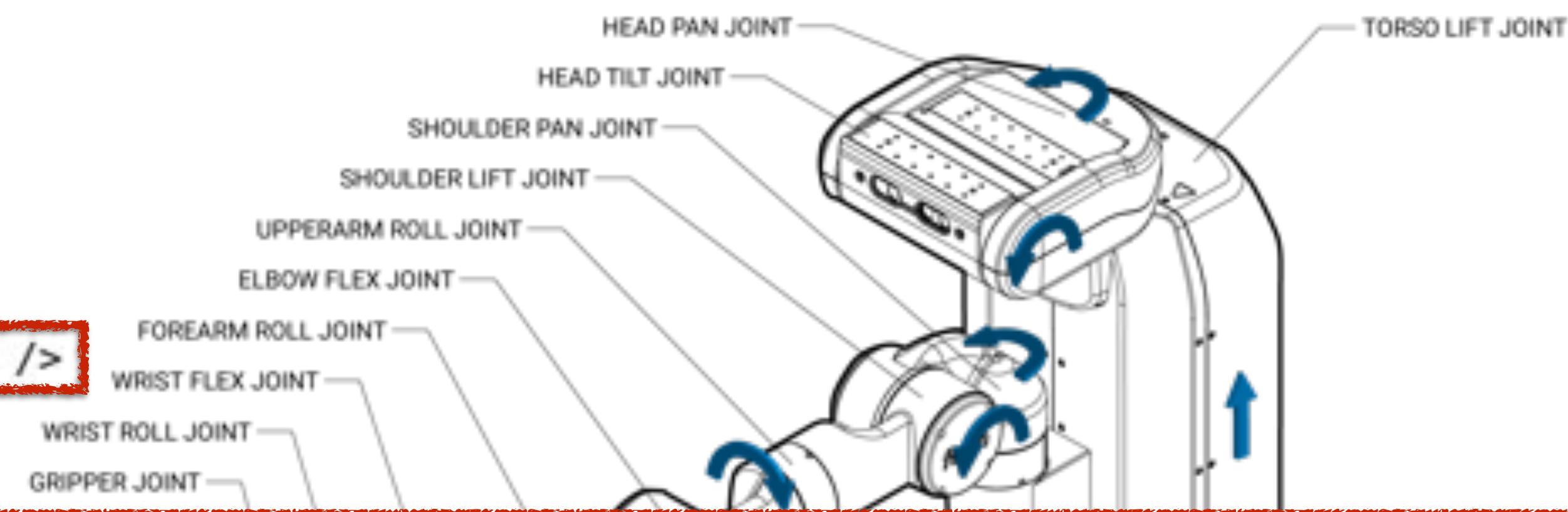
```

## Prismatic joint description

```

<joint name="torso_lift_joint" type="prismatic">
  <origin rpy="-6.123E-17 0 0" xyz="-0.086875 0 0.37743" />
  <parent link="base_link" />
  <child link="torso_lift_link" />
  <axis xyz="0 0 1" />
  <limit effort="450.0" lower="0" upper="0.4" velocity="0.1" />
  <dynamics damping="100.0" /></joint>

```



## Revolute joint description

```

<joint name="shoulder_pan_joint" type="revolute">
  <origin rpy="0 0 0" xyz="0.119525 0 0.34858" />
  <parent link="torso_lift_link" />
  <child link="shoulder_pan_link" />
  <axis xyz="0 0 1" />
  <dynamics damping="1.0" />
  <limit effort="33.82" lower="-1.6056" upper="1.6056" velocity="1.256" />
</joint>

```

```

robot.joints.shoulder_pan_joint = {parent:"torso_lift_link", child:"shoulder_pan_link"};
robot.joints.shoulder_pan_joint.axis = [0,0,1];
robot.joints.shoulder_pan_joint.type = "revolute";
robot.joints.shoulder_pan_joint.origin = {xyz: [0.119525,0,0.34858], rpy:[0,0,0]};
robot.joints.shoulder_pan_joint.limit = {lower:-1.6056, upper:1.6056};

```



# Important notes



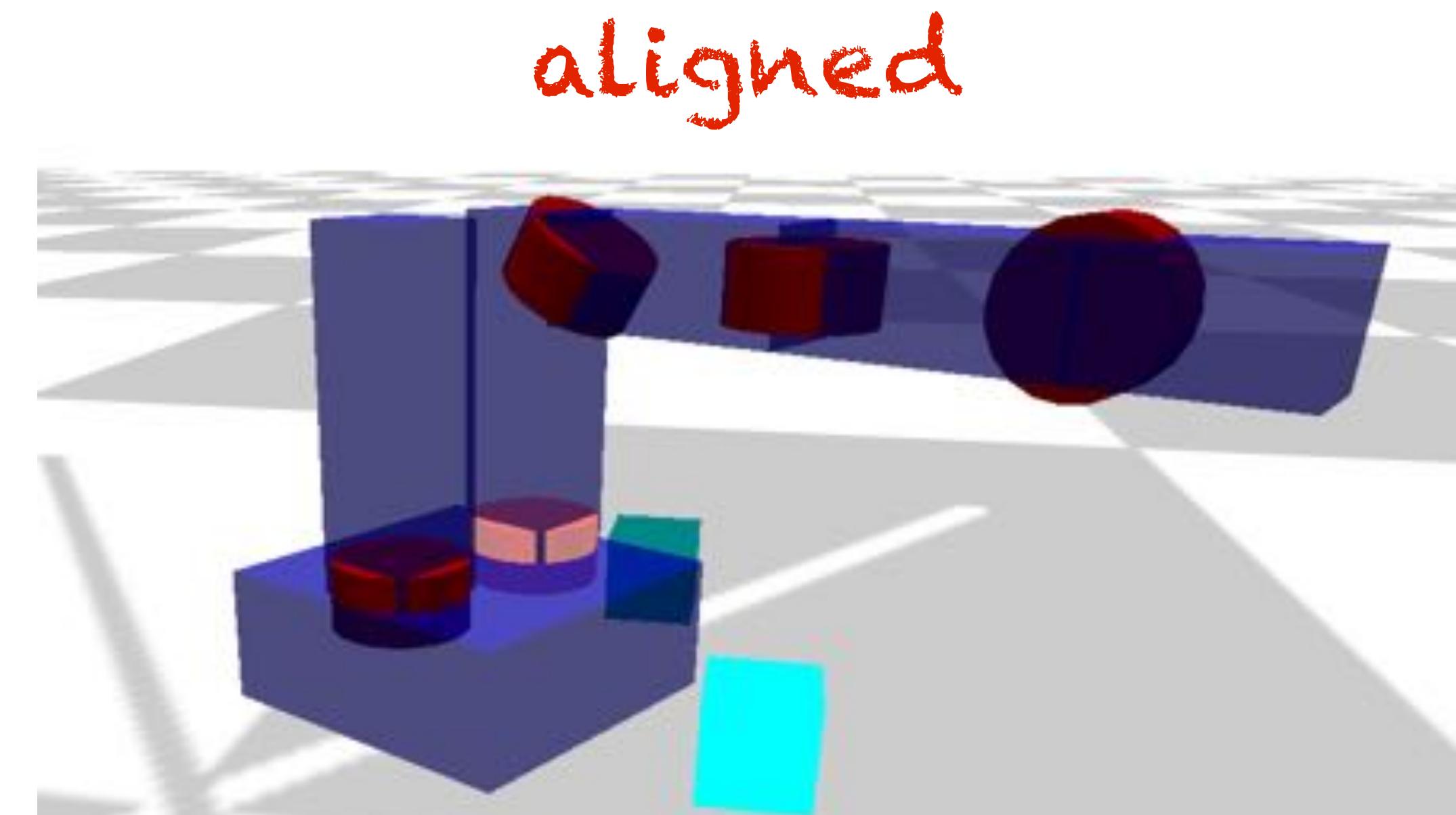
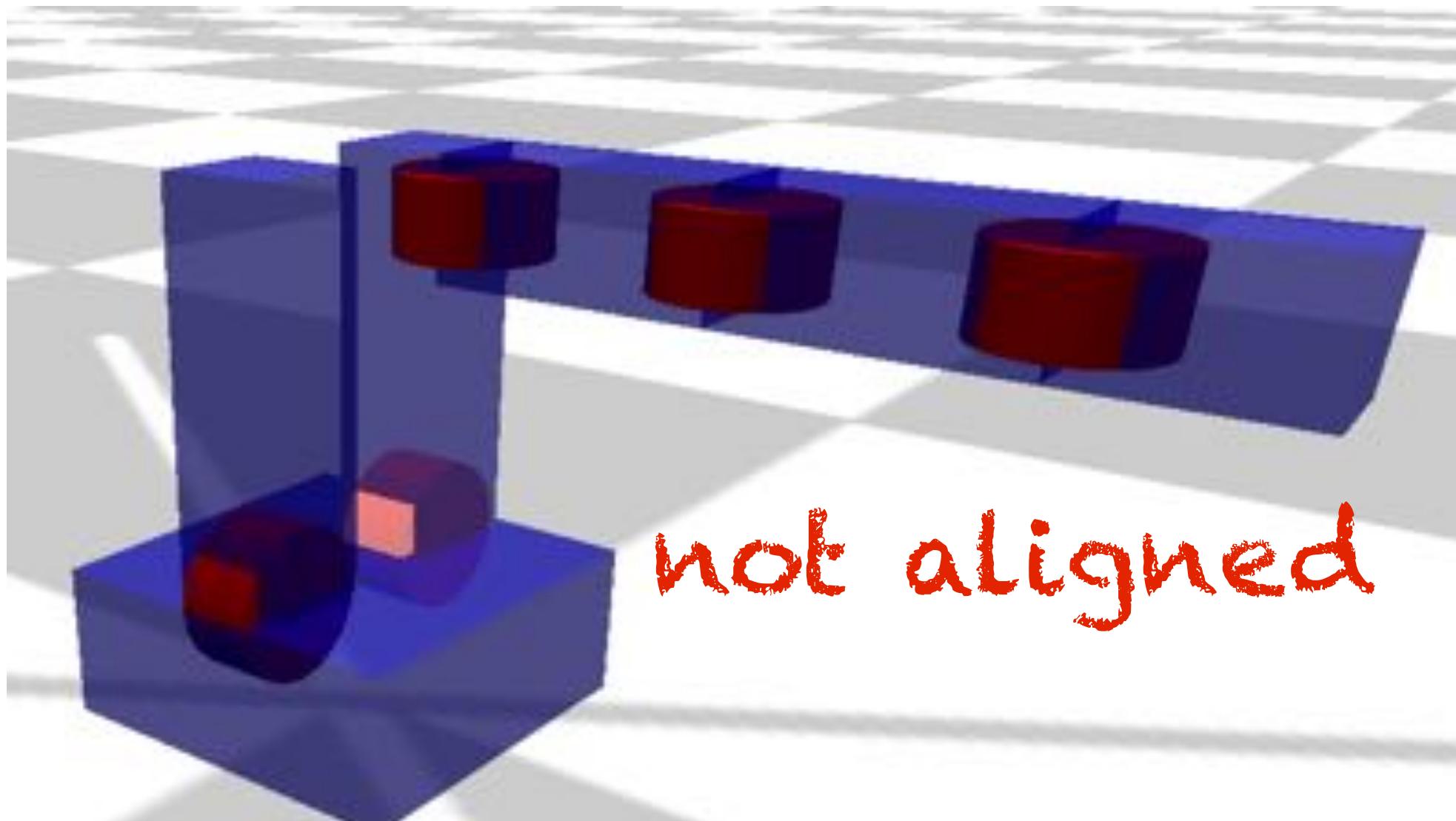
# Important notes

- Rotation order I use: **XYZ** ( $R_zR_yR_x$ )
- `vector_cross()`: code stencil tests for and uses this function
- Base controls: must be implemented for interactive control
- The “`.origin`” field of links and joints are used to store transforms without consideration of joint motion (provided only for debugging)
- A joint and its child link will share the same coordinate frame



# KinEval joint cylinder rendering

- threejs creates cylinders with axes aligned along y-axis
- you need to implement `vector_cross()` for KinEval to render joint cylinders properly along joint axis



# Global controls for base

- Assume we have a base that is holonomic wrt. ground plane
  - holonomic: can move in any direction
  - kineval\_userinput.js assumes:

How to perform this  
base movement?

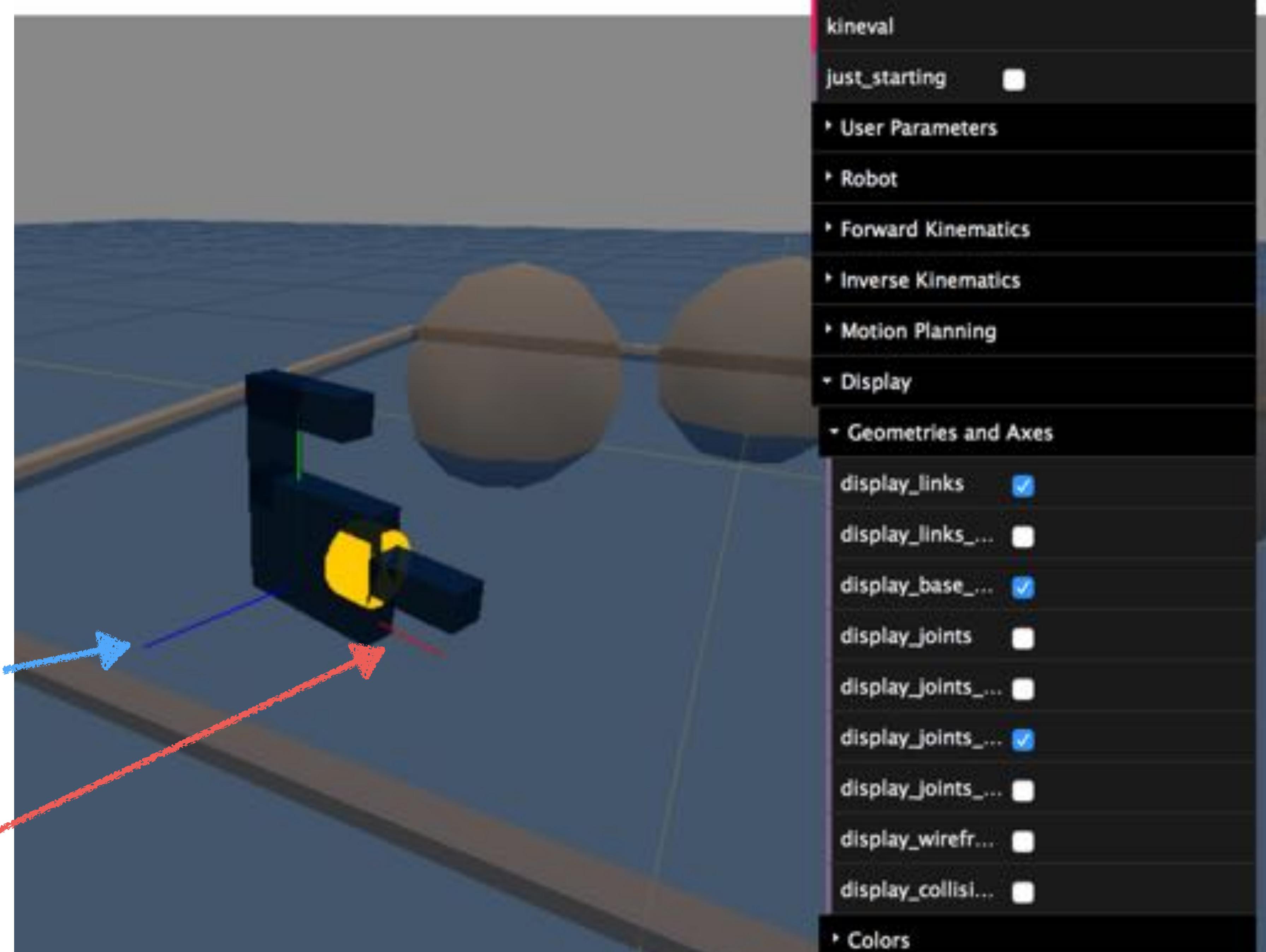


Transform vectors for heading (local z-axis) and lateral (local x-axis) of robot base into world coordinates

Store transformed vectors in variables “robot\_heading” and “robot\_lateral”

Forward heading  
of the robot

Lateral heading  
of the robot



# Alternatives for FK

- Denavit-Hartenberg Convention
- Product of Exponentials with Matrix Stack    **AutoRob & 5551**
- Screw vectors as Dual Quaternions

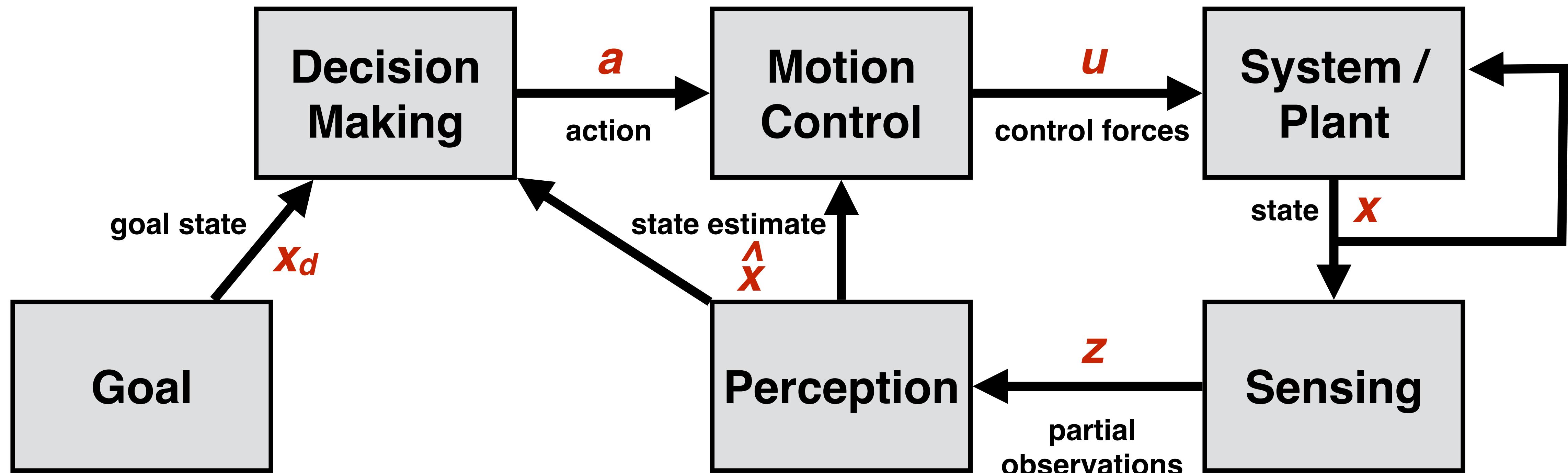
**[Kenwright 2012, Daniilidis 1999]**



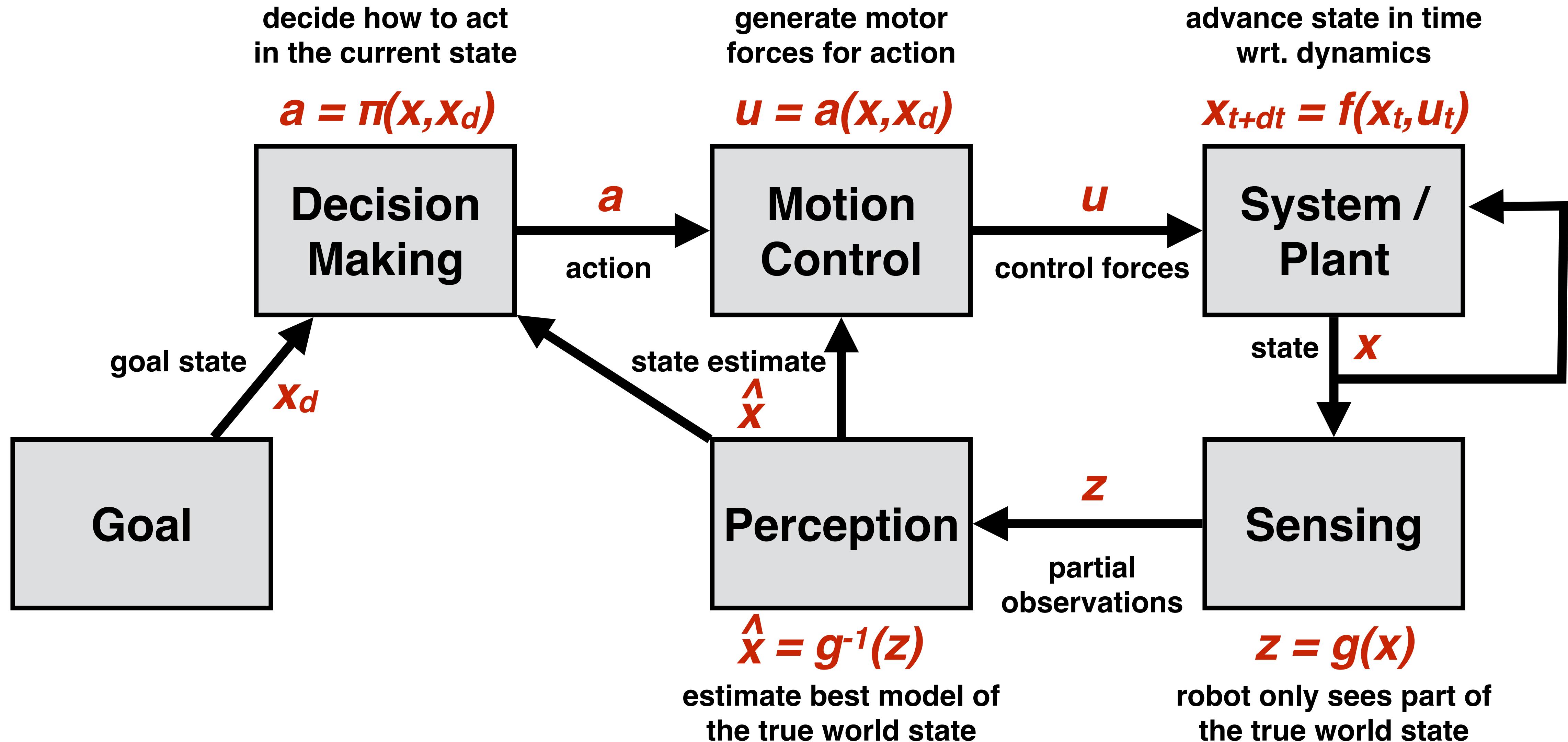
# Decision Making



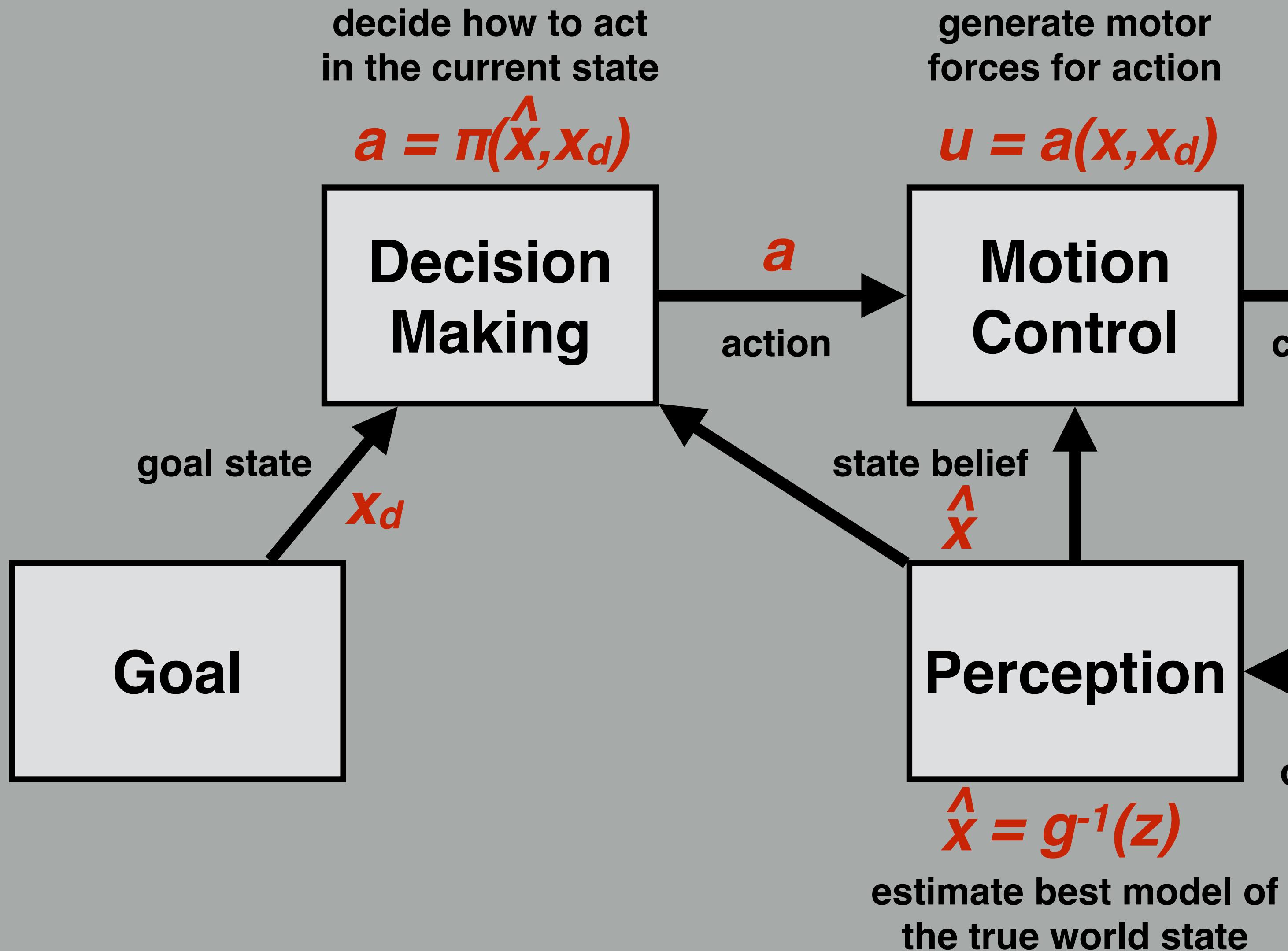
# Robot Control Loop



# Robot Control Loop



# Autonomy



# Embodiment

advance state in time  
wrt. dynamics

$$x_{t+dt} = f(x_t, u_t)$$

**System /  
Plant**

$$\text{state } x$$

**Sensing**

$$z = g(x)$$

robot only sees part of  
the true world state



# App

## Task

decide how to act  
in the current state

$$a = \pi(\hat{x}, x_d)$$

**Decision Making**

*a*

action

goal state

$x_d$

**Goal**

## State-Action

generate motor  
forces for action

$$u = a(x, x_d)$$

**Motion Control**

*u*

control forces

state belief

$\hat{x}$

**Perception**

$$\hat{x} = g^{-1}(z)$$

estimate best model of  
the true world state

## Embodiment

advance state in time  
wrt. dynamics

$$x_{t+dt} = f(x_t, u_t)$$

**System /  
Plant**

*x*

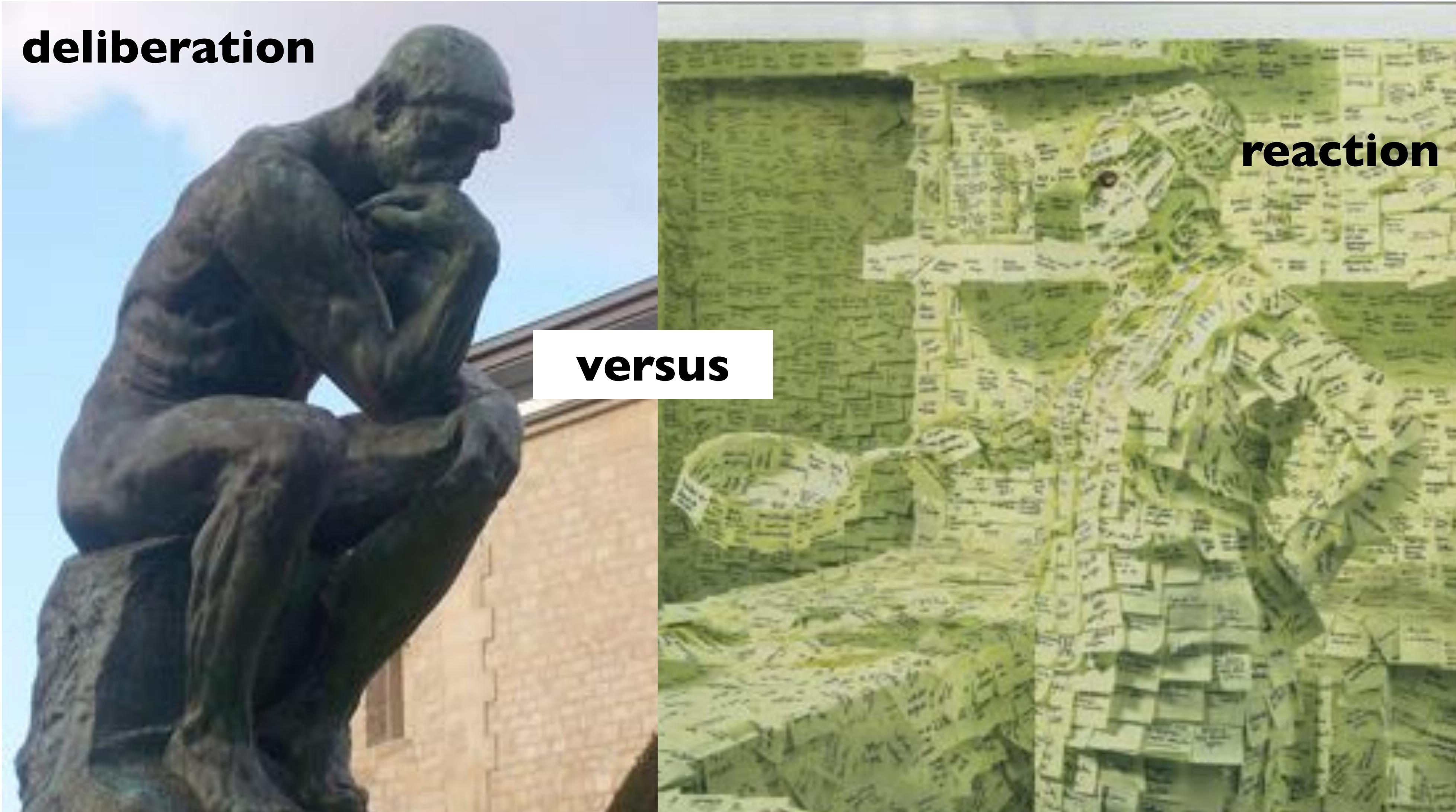
**Sensing**

$$z = g(x)$$

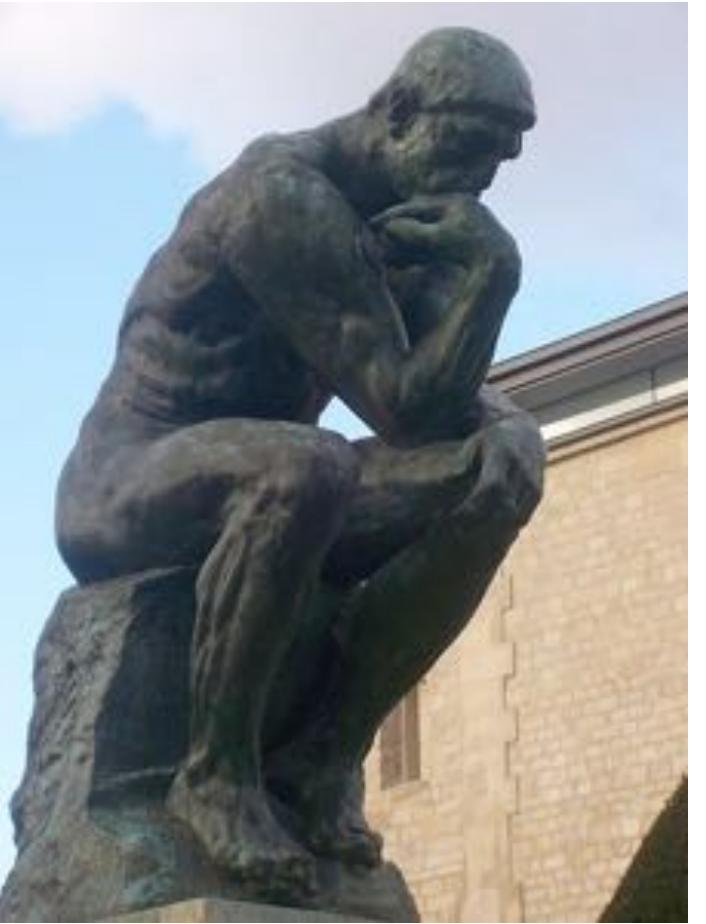
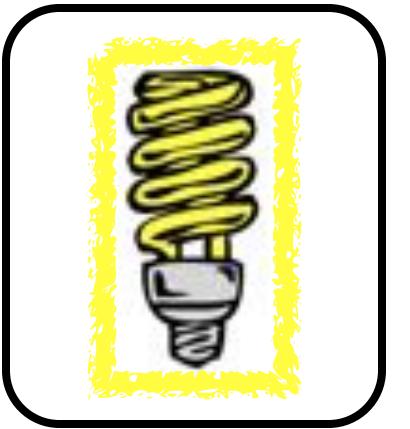
robot only sees part of  
the true world state



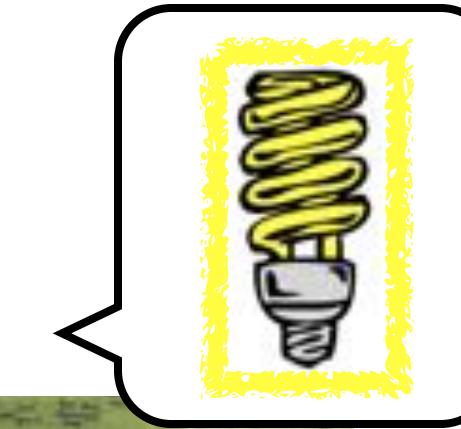
# Robot Decision Making



# Should your robot's decision making



OR

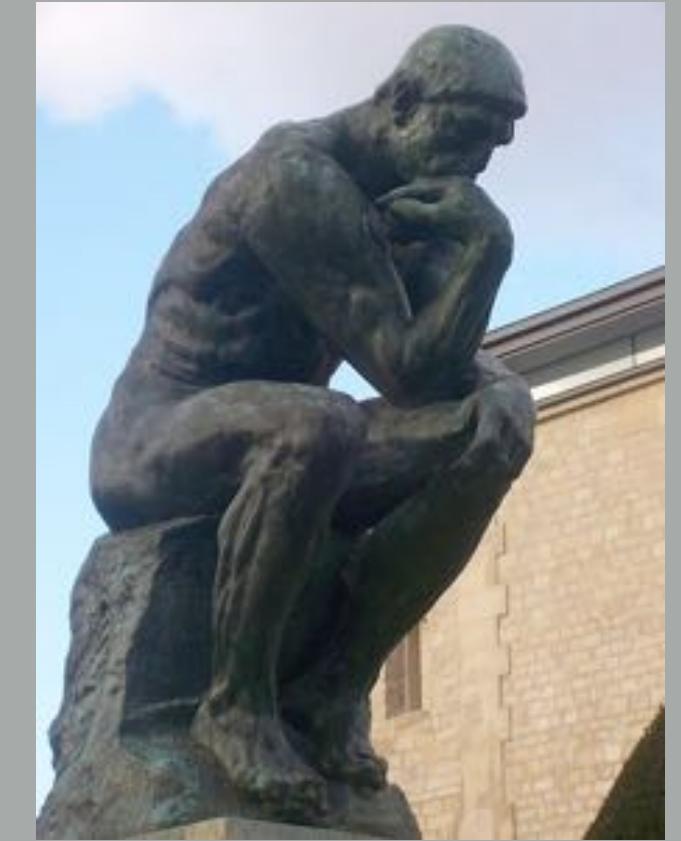
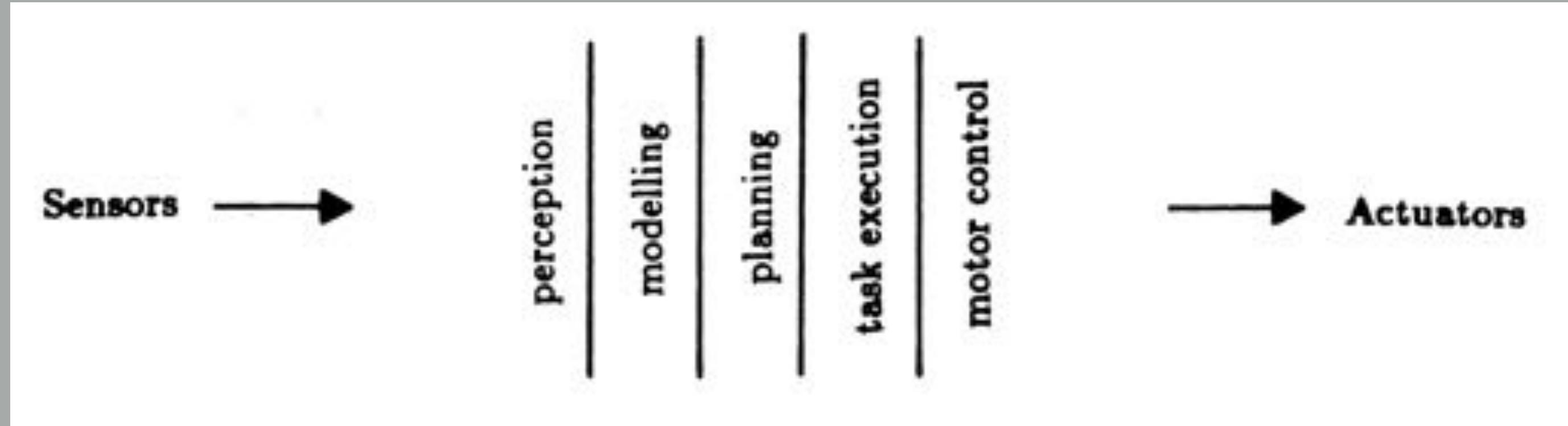


fully think through  
solving a problem?

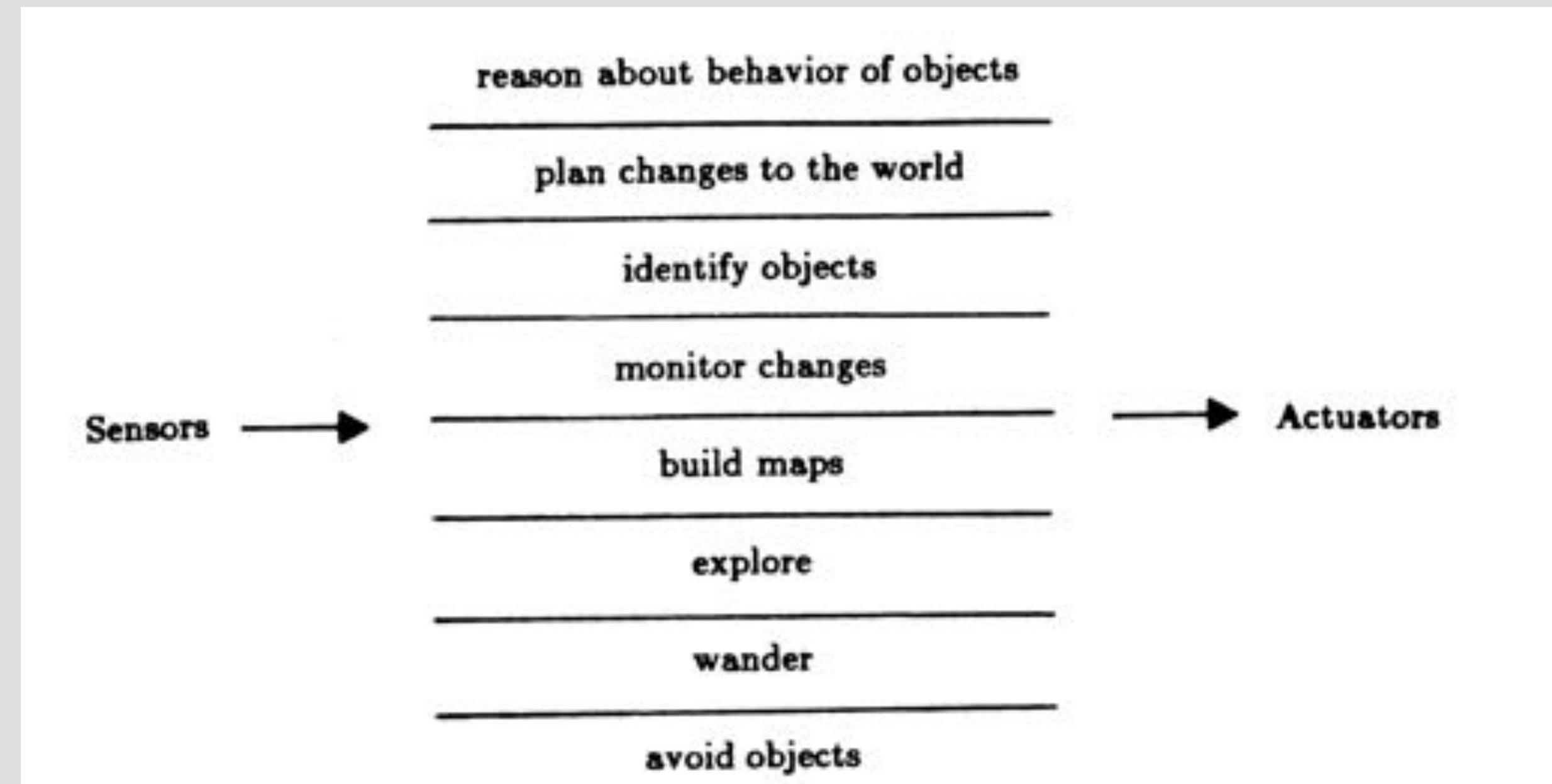
react quickly to  
changes in its world?

# Deliberation v. Reaction

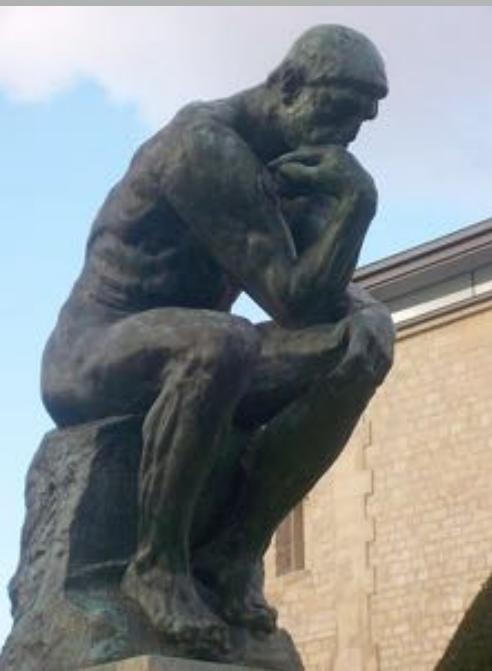
**deliberative:**  
sense-plan-act,  
path planning  
motion planning



**reaction:**  
controllers acting in parallel  
subsumption,  
Finite State Machine



# Deliberation-Reaction spectrum



**DELIBERATIVE**

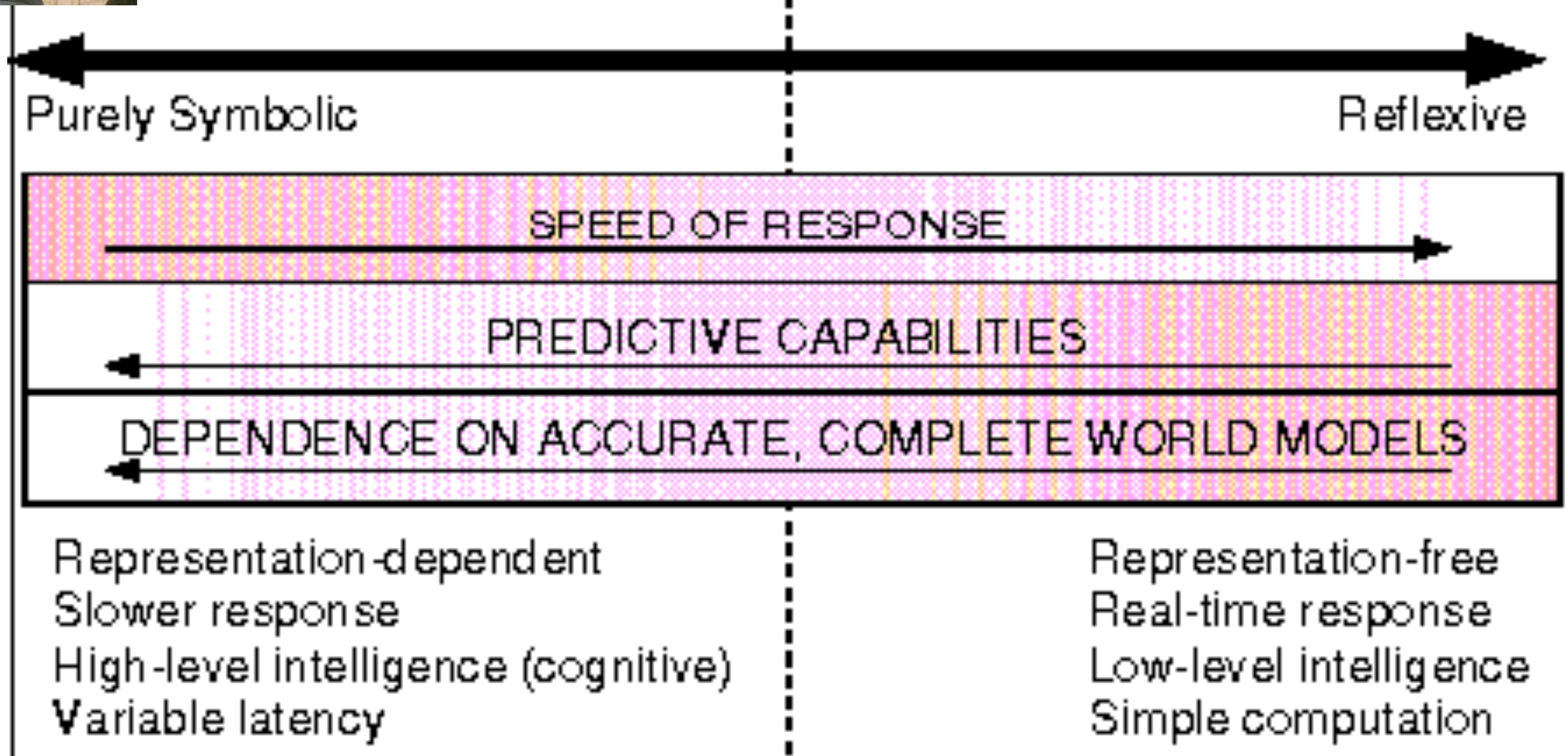
**REACTIVE**

Complete  
Adaptive  
Optimal  
Slower

Faster  
Cheaper  
More robust  
Forgetful

Requires  
complete model  
of the world

Requires a  
complete design  
of the problem



# Examples?



**DELIBERATIVE**



**REACTIVE**

example???

Purely Symbolic

Reflexive

# Examples?



**DELIBERATIVE**



**REACTIVE**

Purely Symbolic

Reflexive



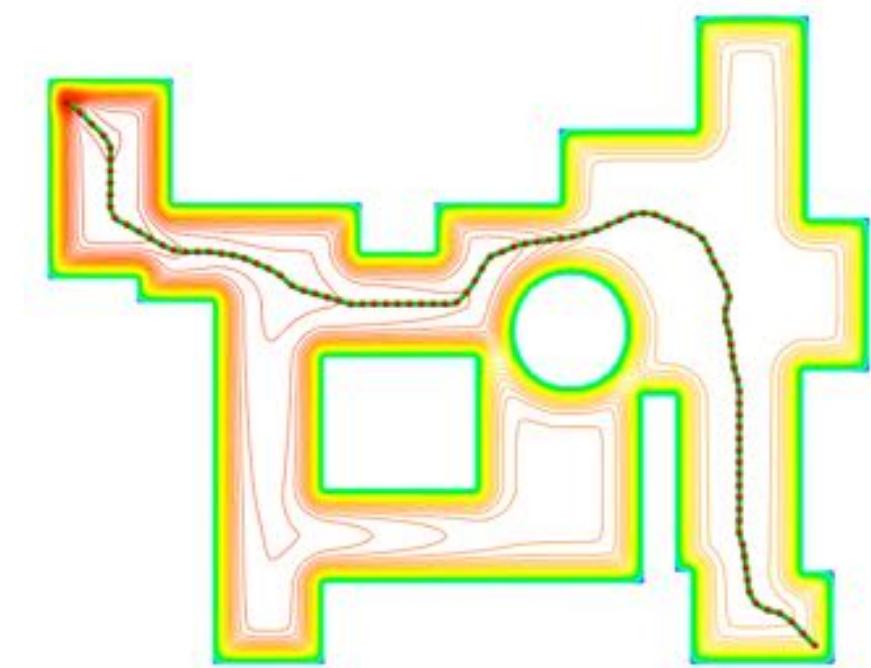
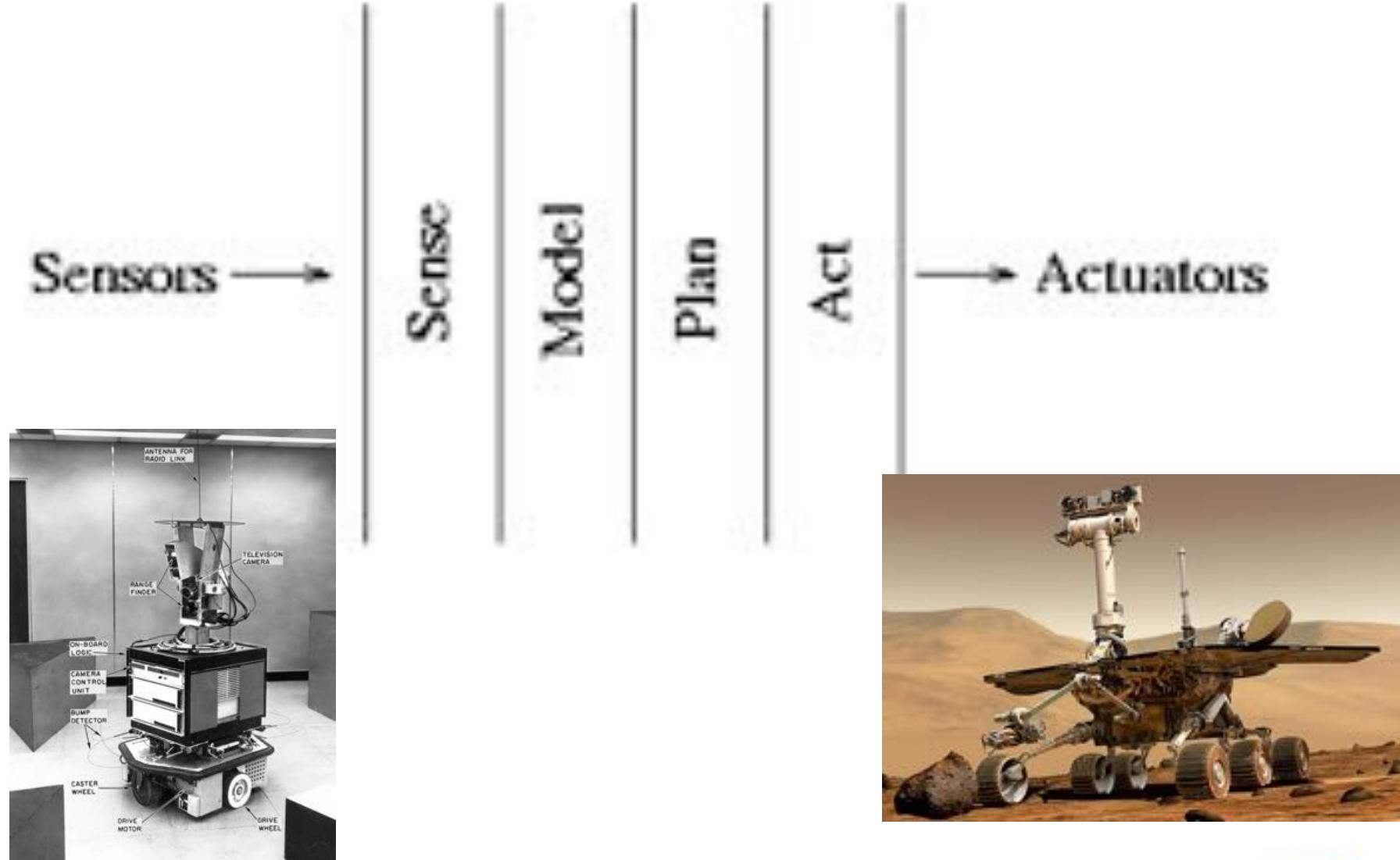
<https://www.youtube.com/watch?v=jCB3pd-wBw0>



# Deliberation

## “Sense-Plan-Act” paradigm

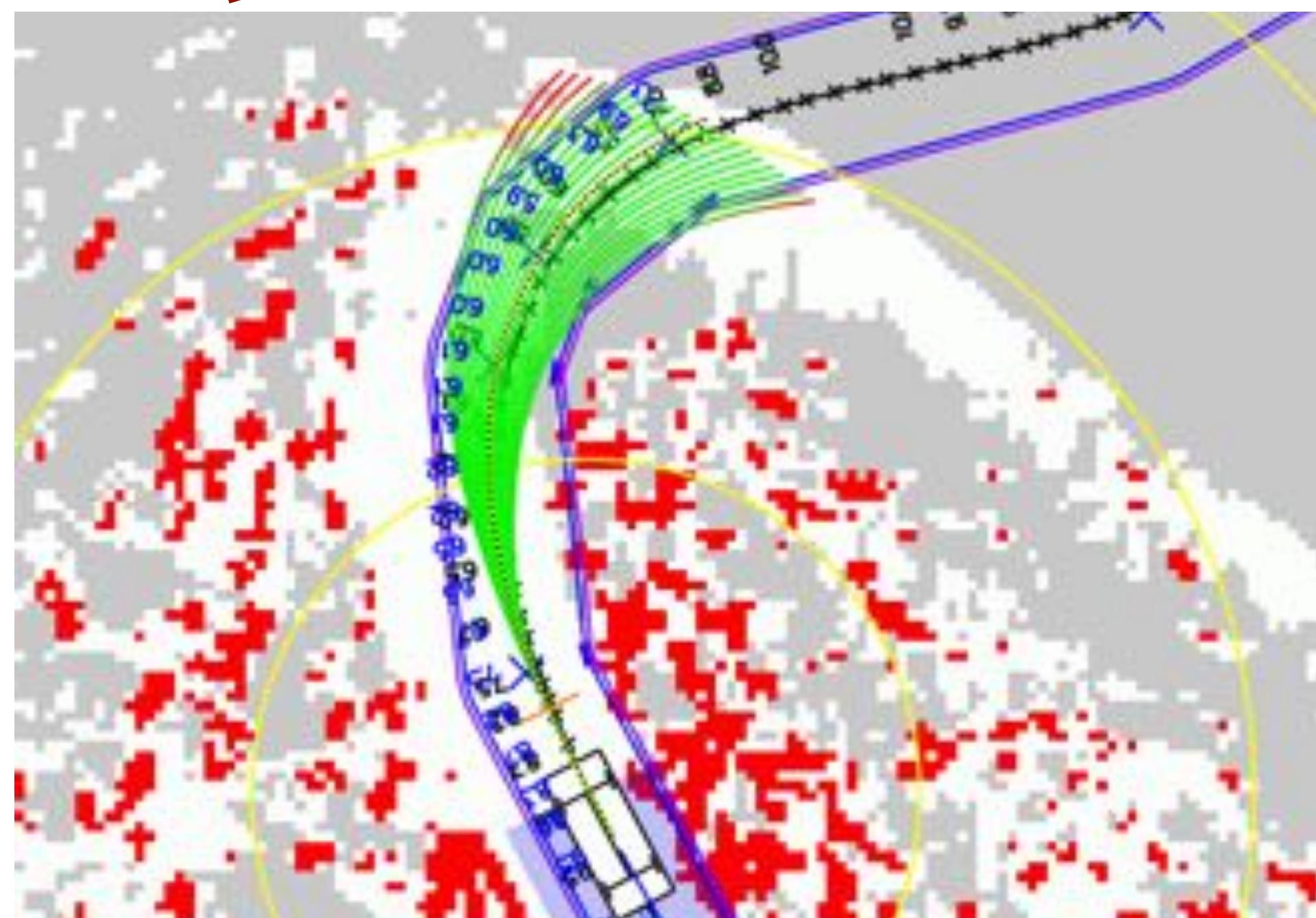
- sense: build most complete model of world
  - GPS, SLAM, 3D reconstruction, affordances
- plan: search over all possible outcomes
  - Graph search, Roadmap planning
- act: execute plan through motor forces
  - PID control, Model predictive control



# Stanley (Grand Challenge)



Navigation



Road detection

2005

# MIT Talos (Urban Challenge)



2007





2013



Deliberation  
requires a model of the world



Color+Depth Camera



# Simultaneous Localization and Mapping



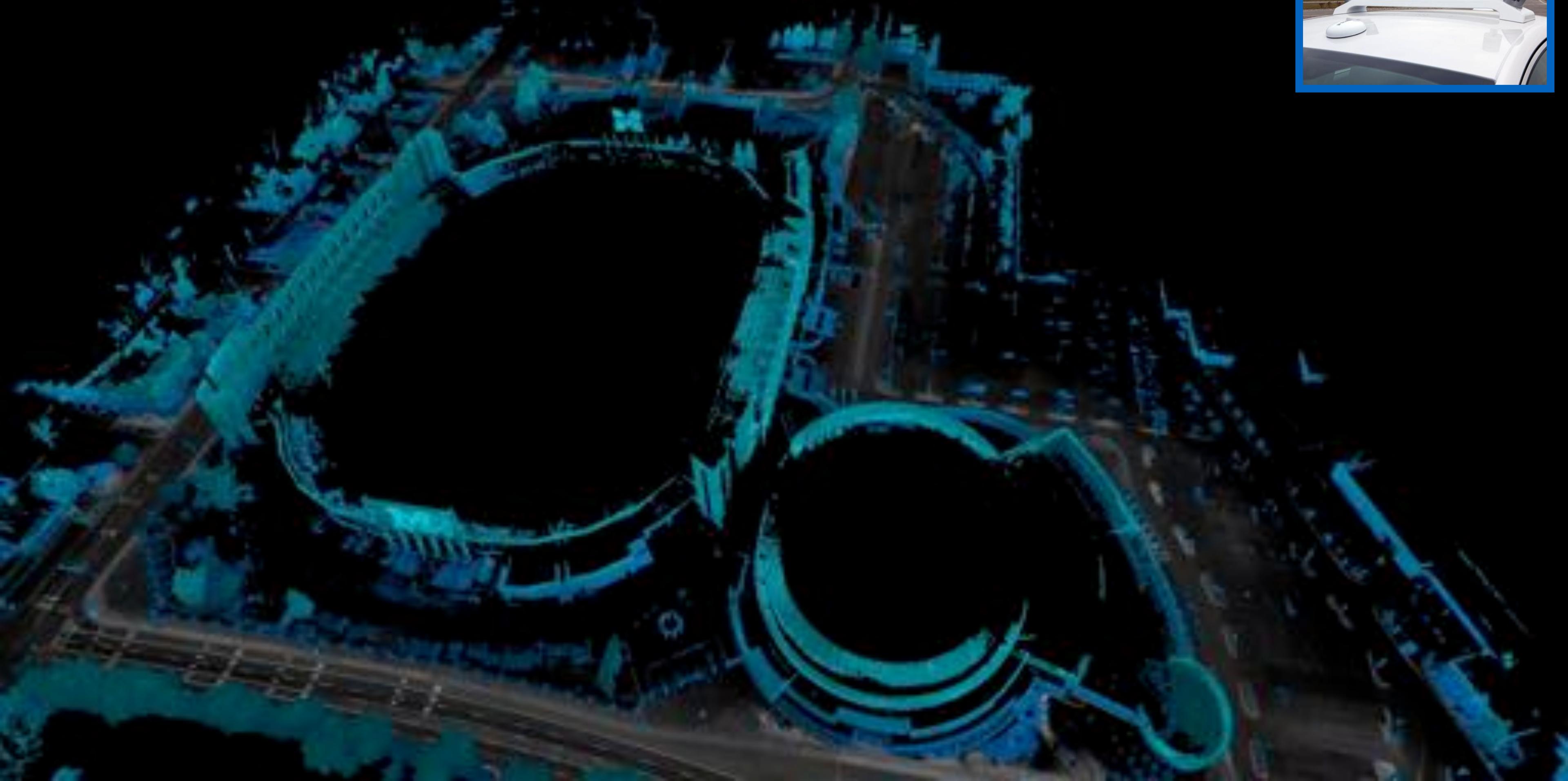


Autonomous robot navigation  
from previously built map





Michigan Next Generation Vehicle (Eustice, Olson et al.)



# Autonomous Transportation

Michigan Next Generation Vehicle (Eustice, Olson et al.)



# Examples?



**DELIBERATIVE**



**REACTIVE**

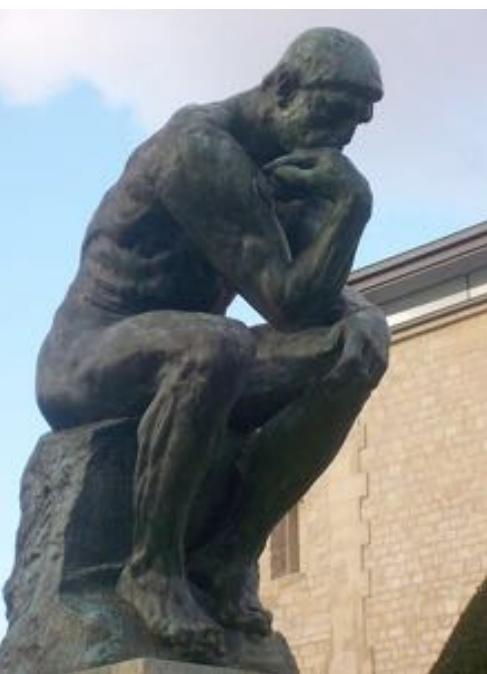
Purely Symbolic

Reflexive



more common  
example???

# Examples?



**DELIBERATIVE**



**REACTIVE**

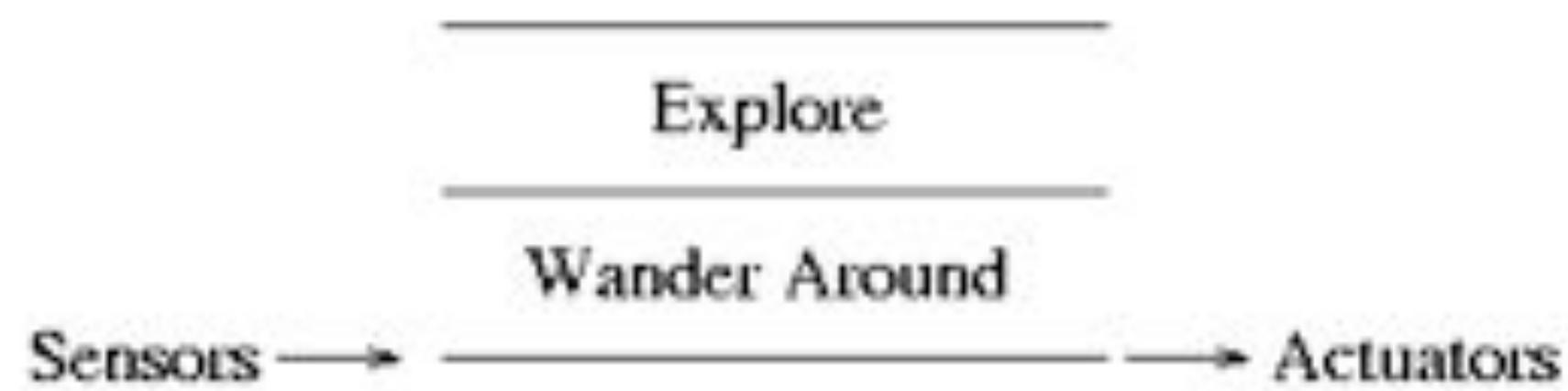
Purely Symbolic

Reflexive

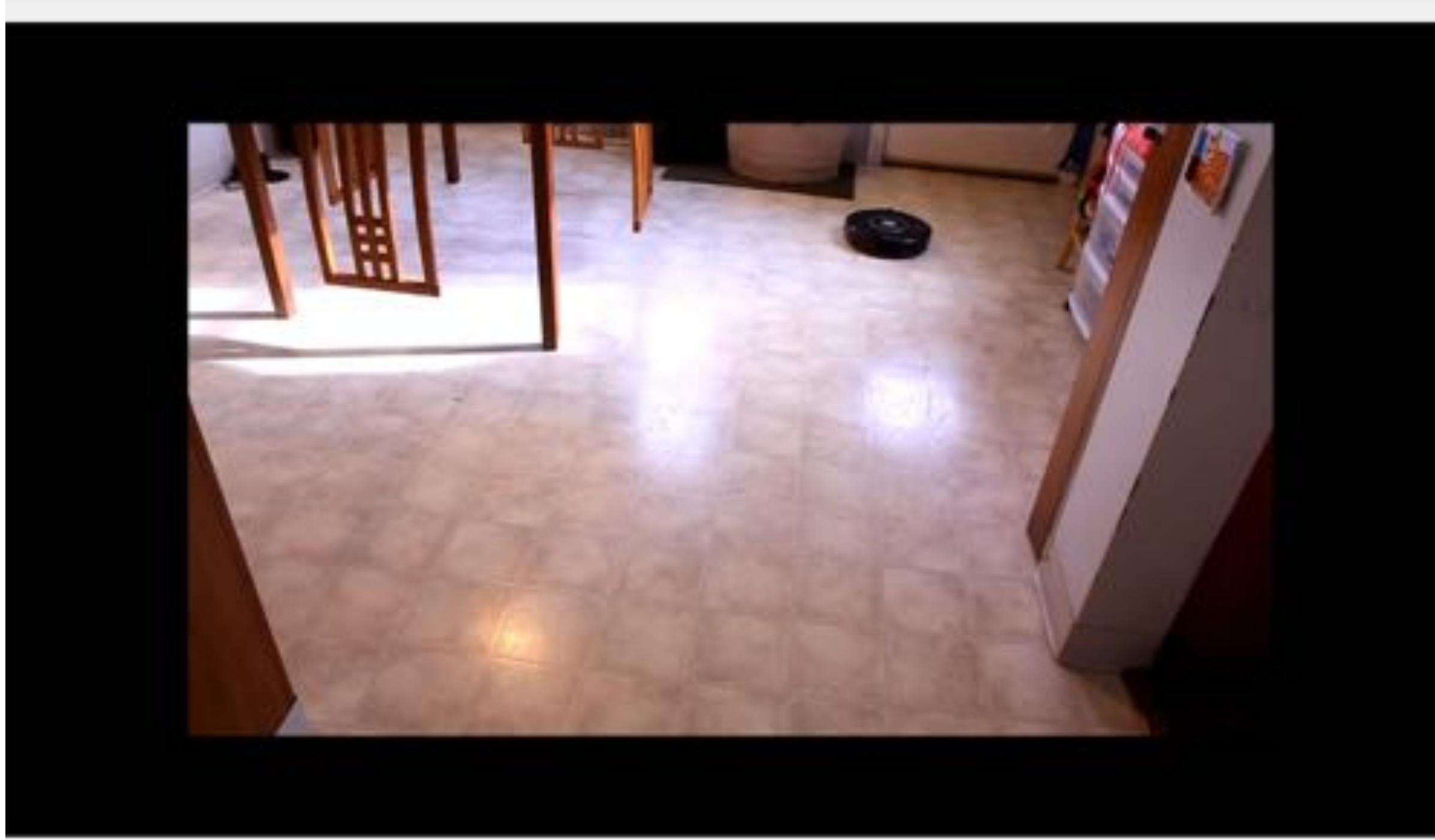


# Reaction

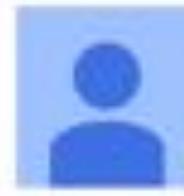
- No representation of state
  - Typically, fast hardcoded rules
- Embodied intelligence
  - behavior  $\leftarrow$  control + embodiment
  - Stigmergy (e.g, ant scouts using pheromones)
- Finite State Machines
  - most common
- Subsumption architecture
  - prioritized reactive policies



# Roomba cleaning pattern



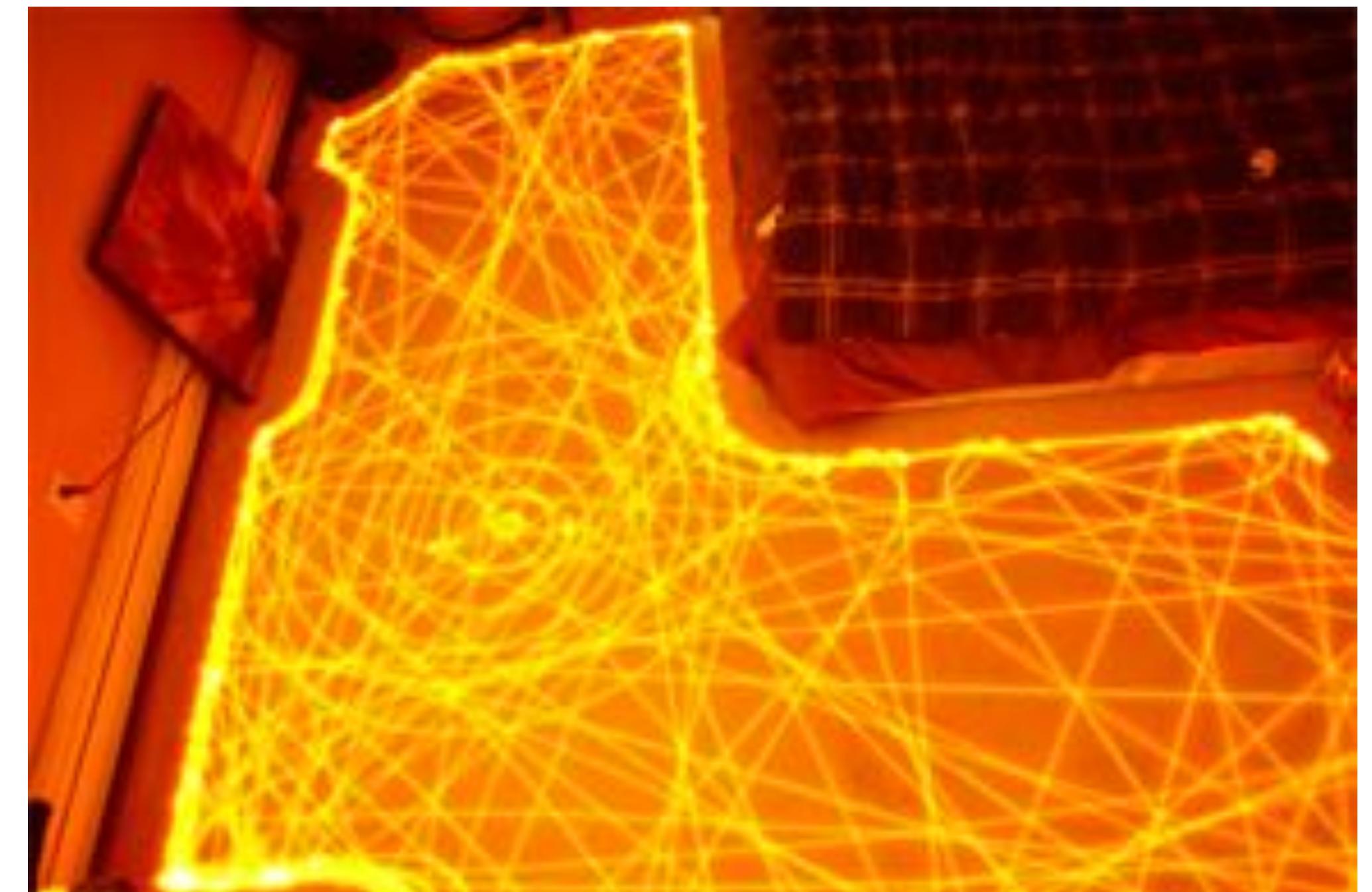
roomba cleaning "pattern"



miro ledajaks



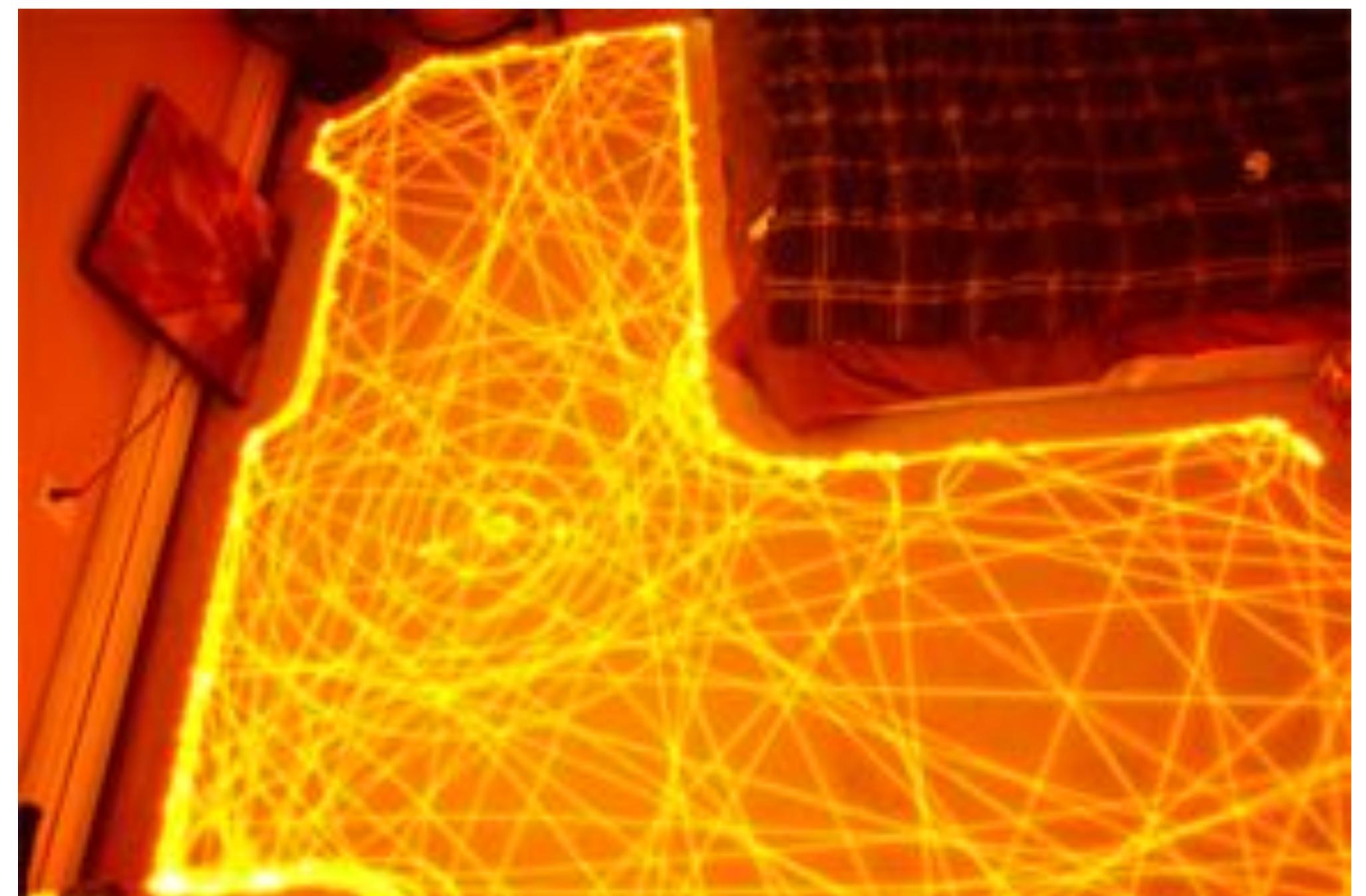
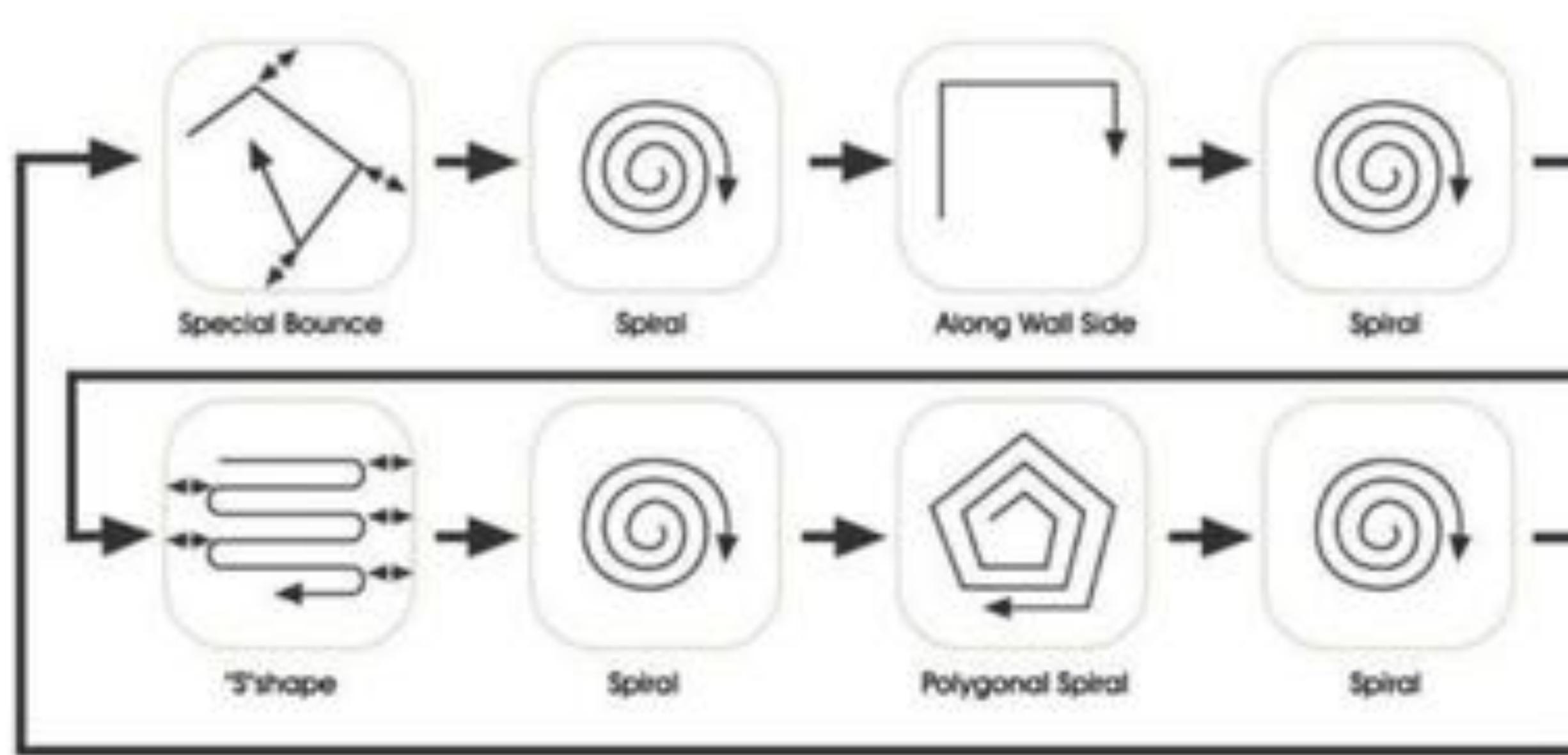
196 views



<https://www.youtube.com/watch?v=G4ocrevf4ng>



# Vacuuming Finite State Machine



# Manipulation Gaits



Collections of robust manipulation controllers

# How do we computationally represent reactive control?

# Finite State Machines

- Components
  - alphabet (or inputs)
    - “observations” in robotics
  - states (some robot action)
  - transitions (between states)
  - stopping condition
- Commonly, implemented as switch-case or if-else within a while loop

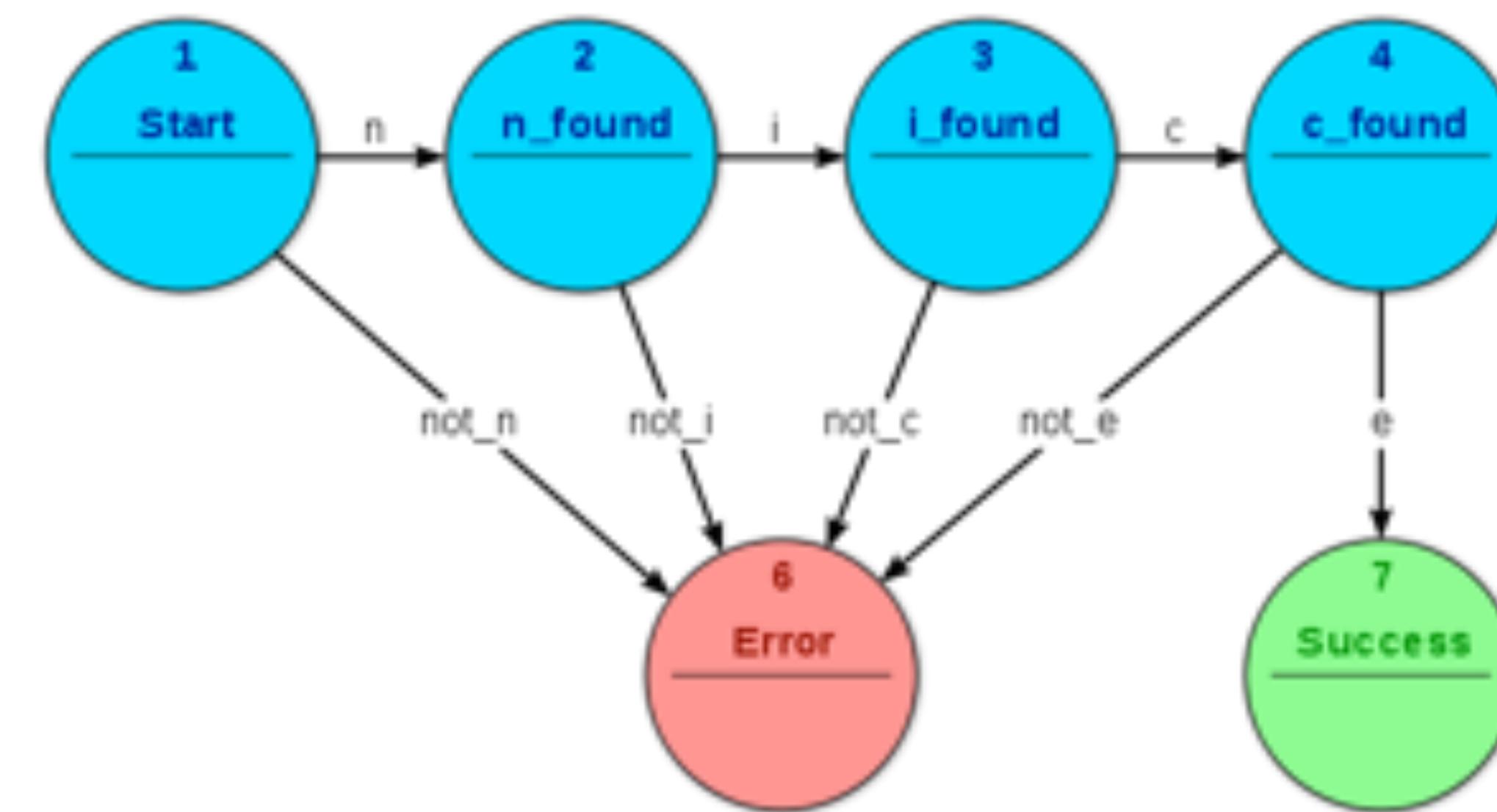


[http://en.wikipedia.org/wiki/Switch\\_statement](http://en.wikipedia.org/wiki/Switch_statement)

# “nice” recognizer

- recognize the string “nice” from input

- if input is “nice”
    - output **success**
  - if input not “nice”
    - output **error**



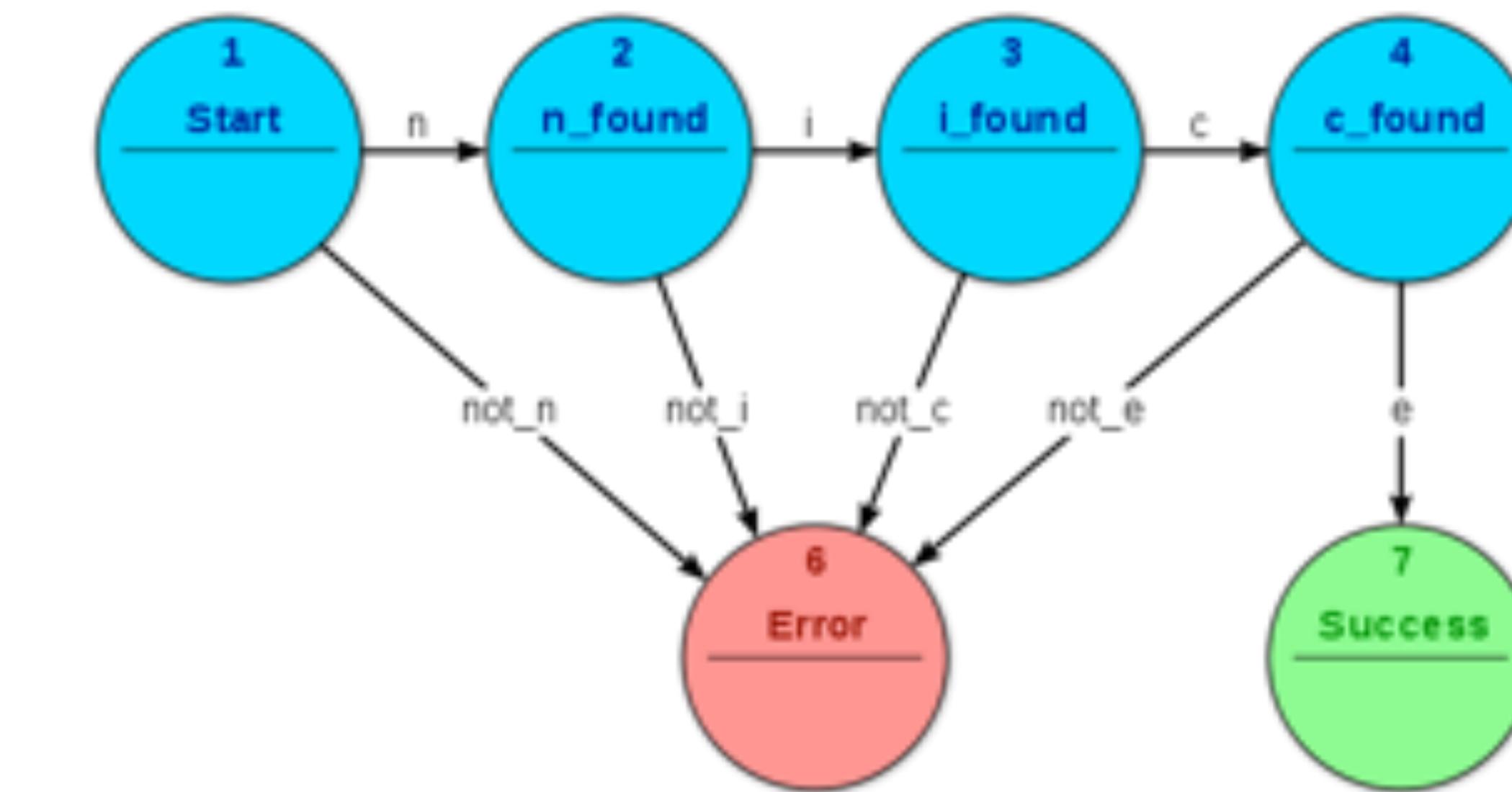
- robotics uses
  - preconditions (enter state)
  - postconditions (exit state)

```

state ← start
while state != success and state != error
    token ← <next string character from input>
    switch (state):
        case start:
            if token = "n" then state ← n_found
            else state ← error
            break
        case n_found:
            if token = "i" then state ← i_found
            else state ← error
            break
        case i_found:
            if token = "c" then state ← c_found
            else state ← error
            break
        case c_found:
            if token = "e" then state ← success
            else state ← error
            break
    end while loop
    output ← state

```

“nice” recognizer



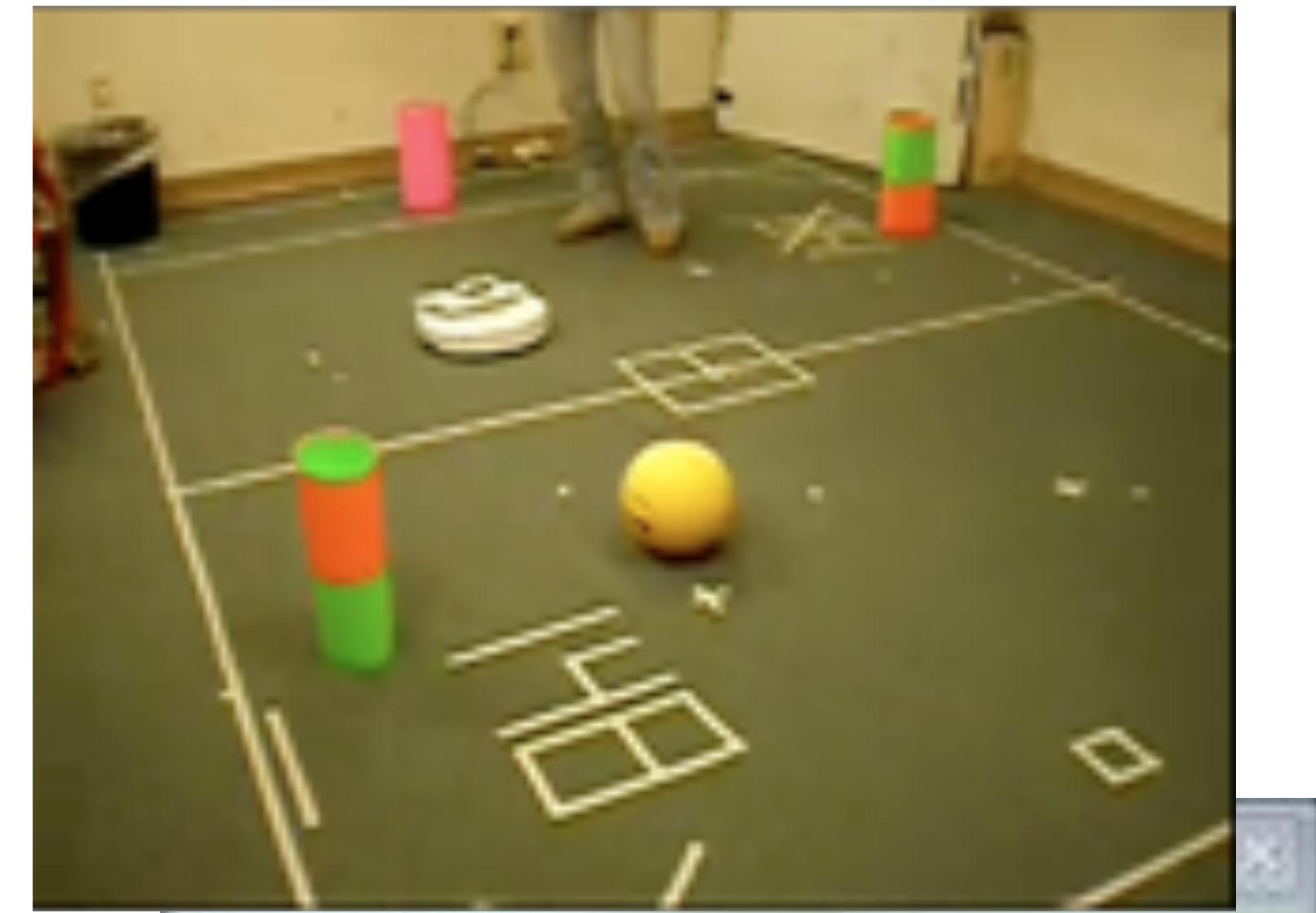
Consider input: “nice”

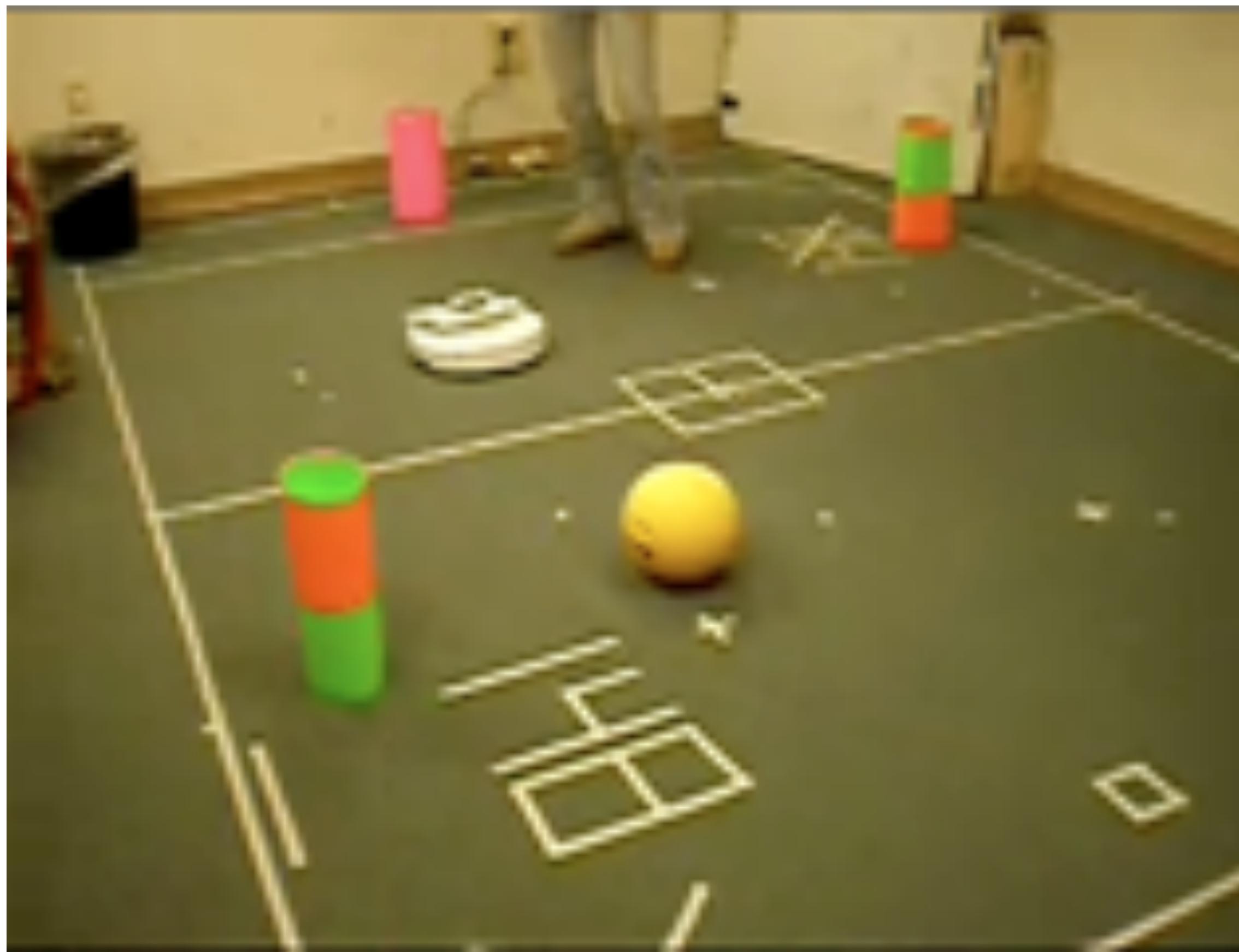
Consider input: “robotics”

Consider input: “niece”

# Move to objects in sequence?

- How to move a mobile robot to a given sequence of objects?
  - yellow ball
  - green/orange landmark
  - pink landmark
  - orange/green landmark



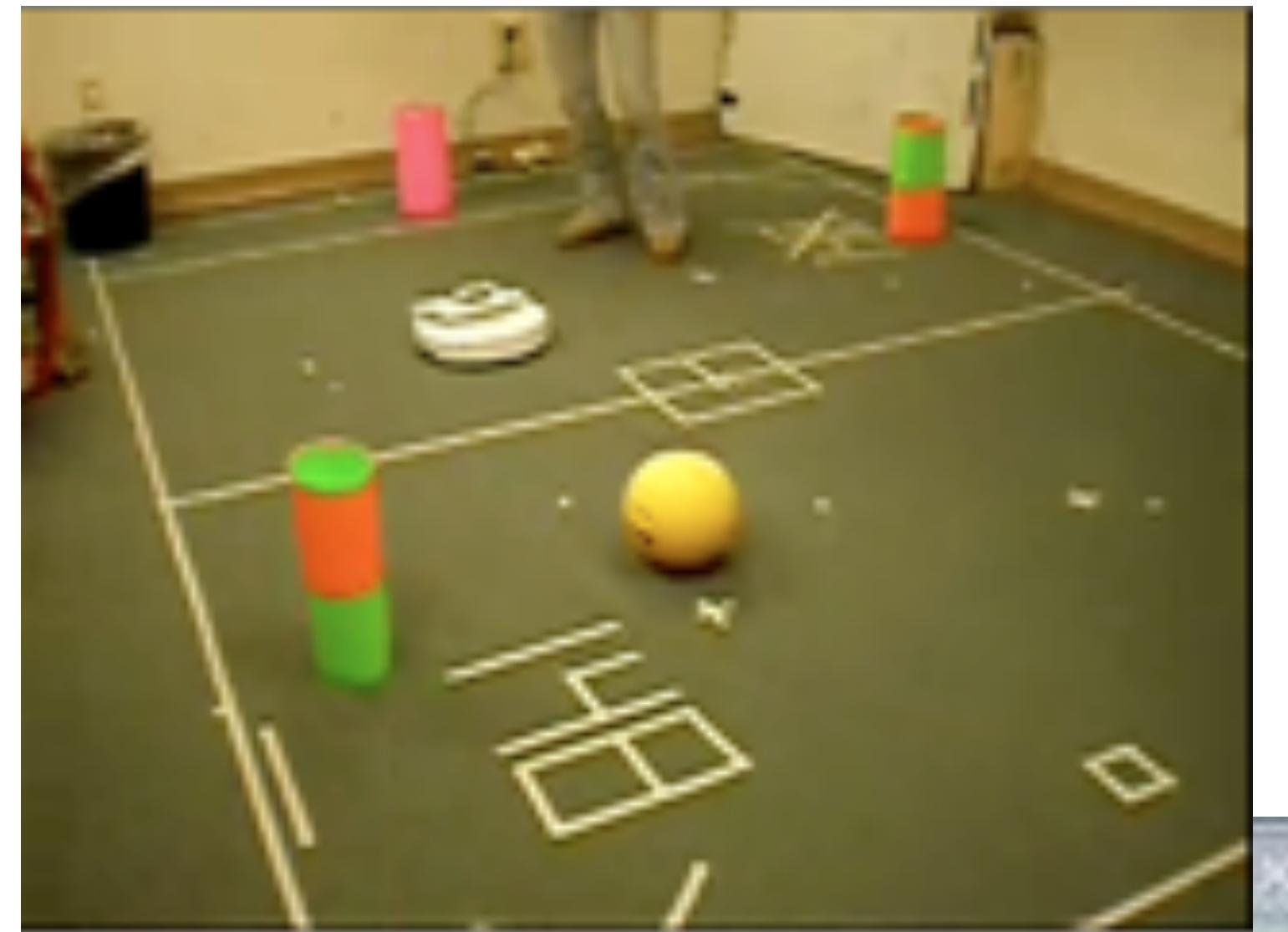


# Object Seeking

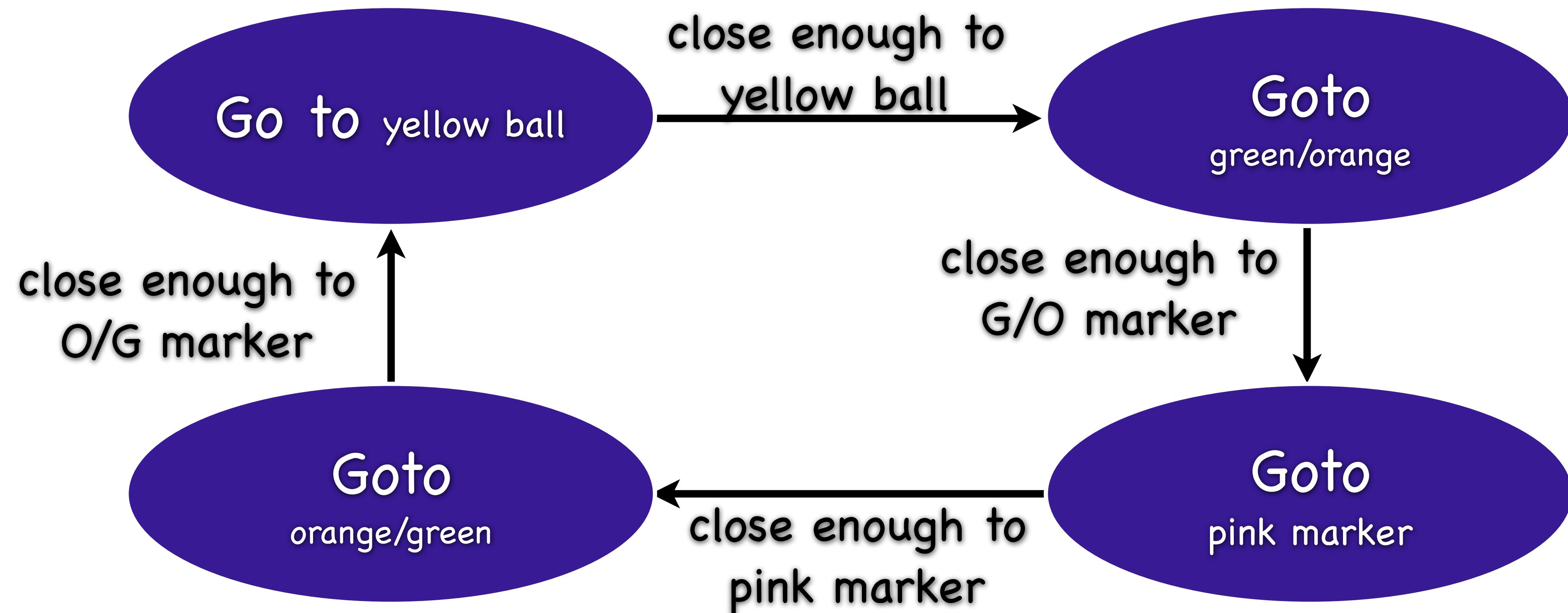
<http://www.youtube.com/watch?v=-hOA0jMUggg>

# Move to objects in sequence?

- What are the states?
- What are the transitions?
- Preconditions for states?
- Postconditions for states?

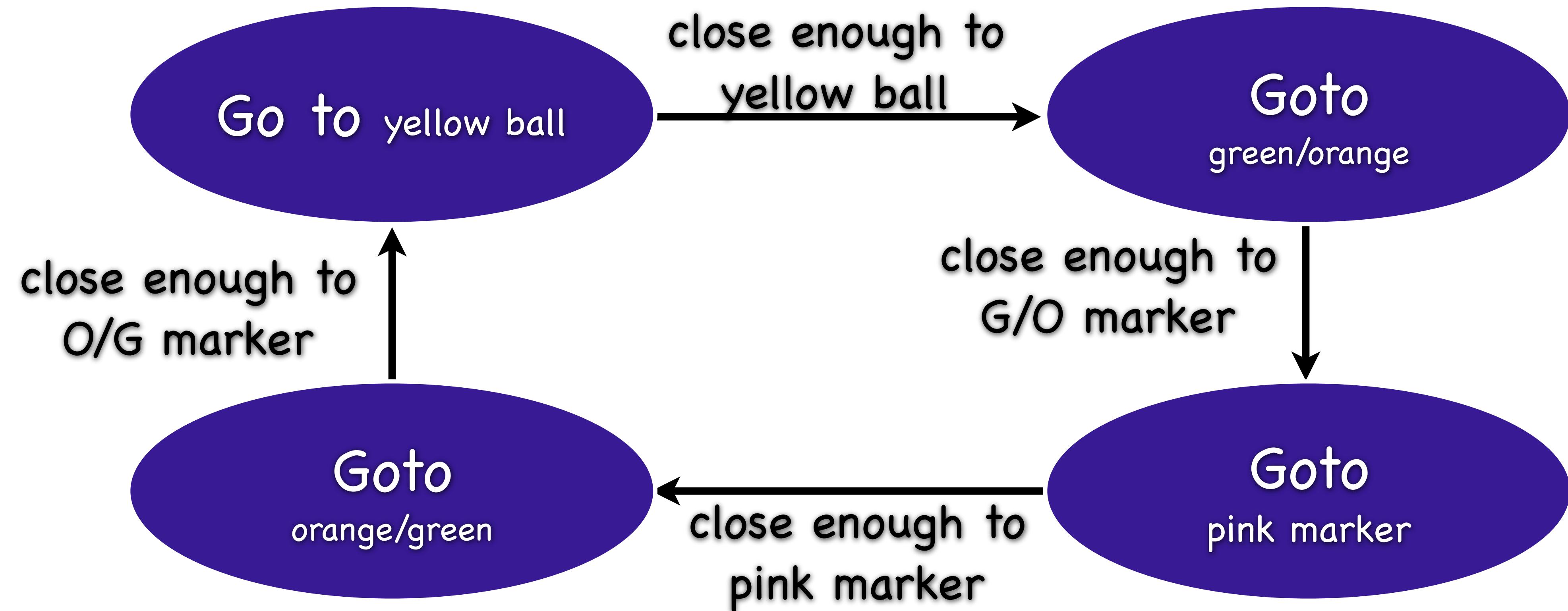


# Object seeking FSM



# Object seeking FSM

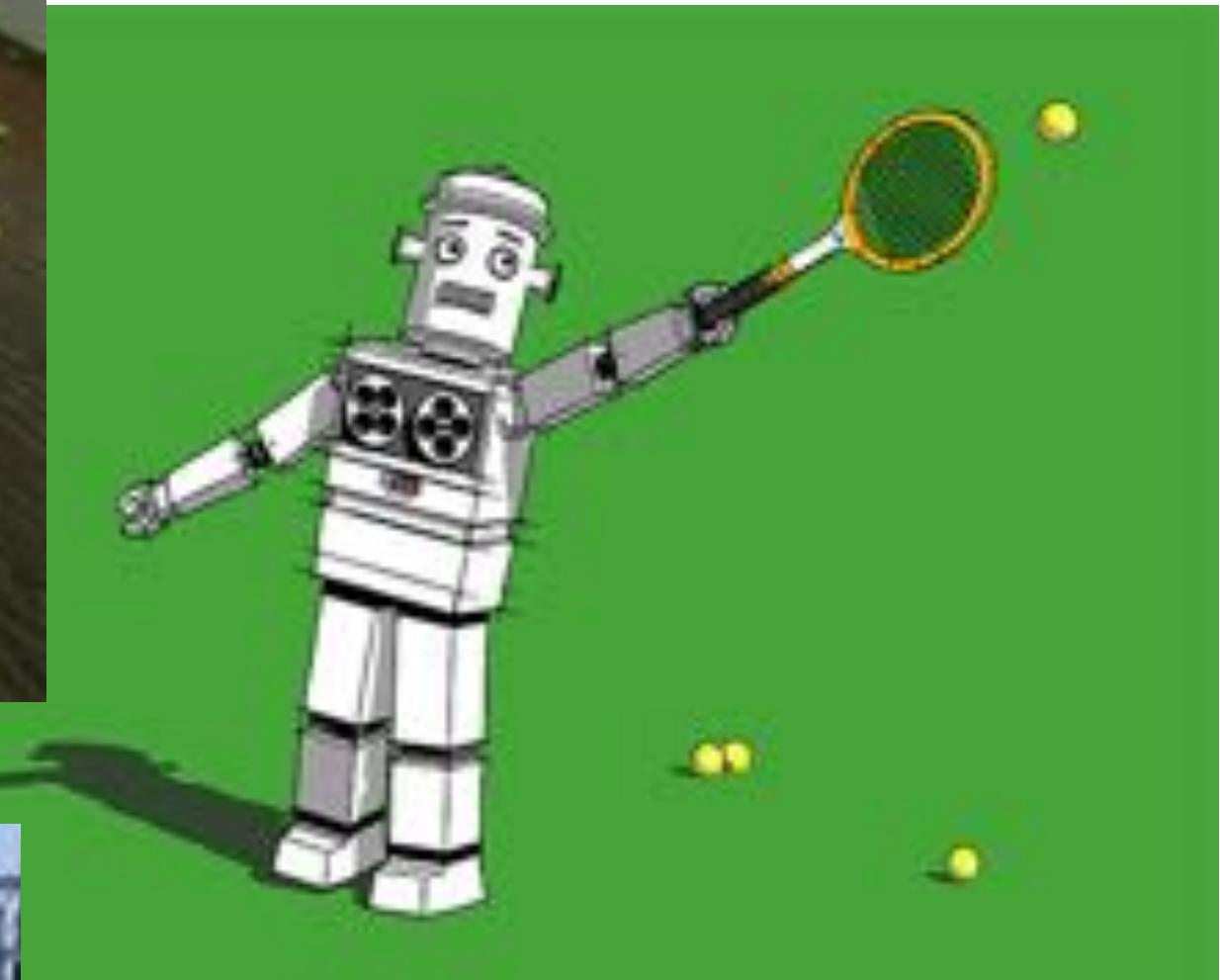
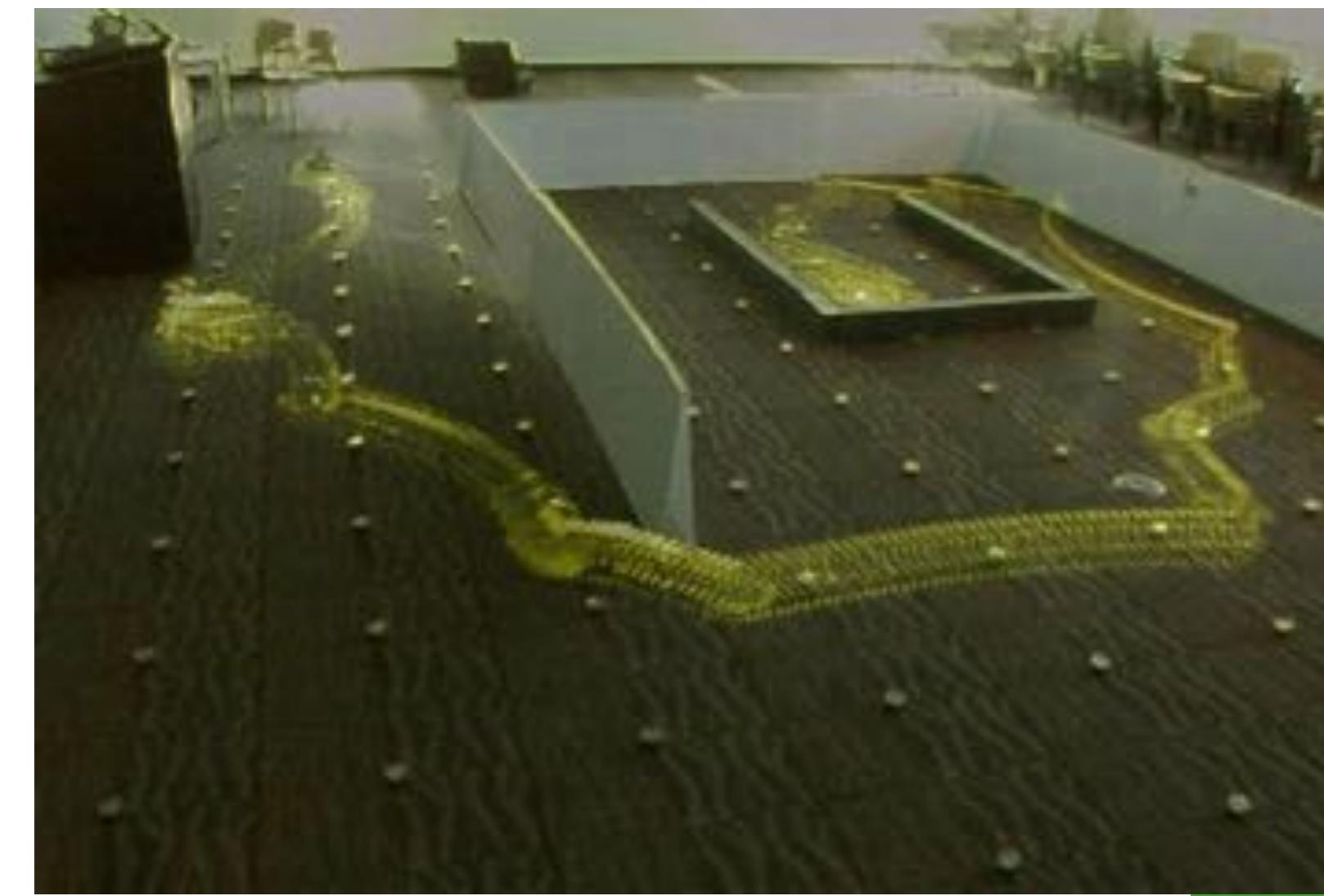
How to implement state?

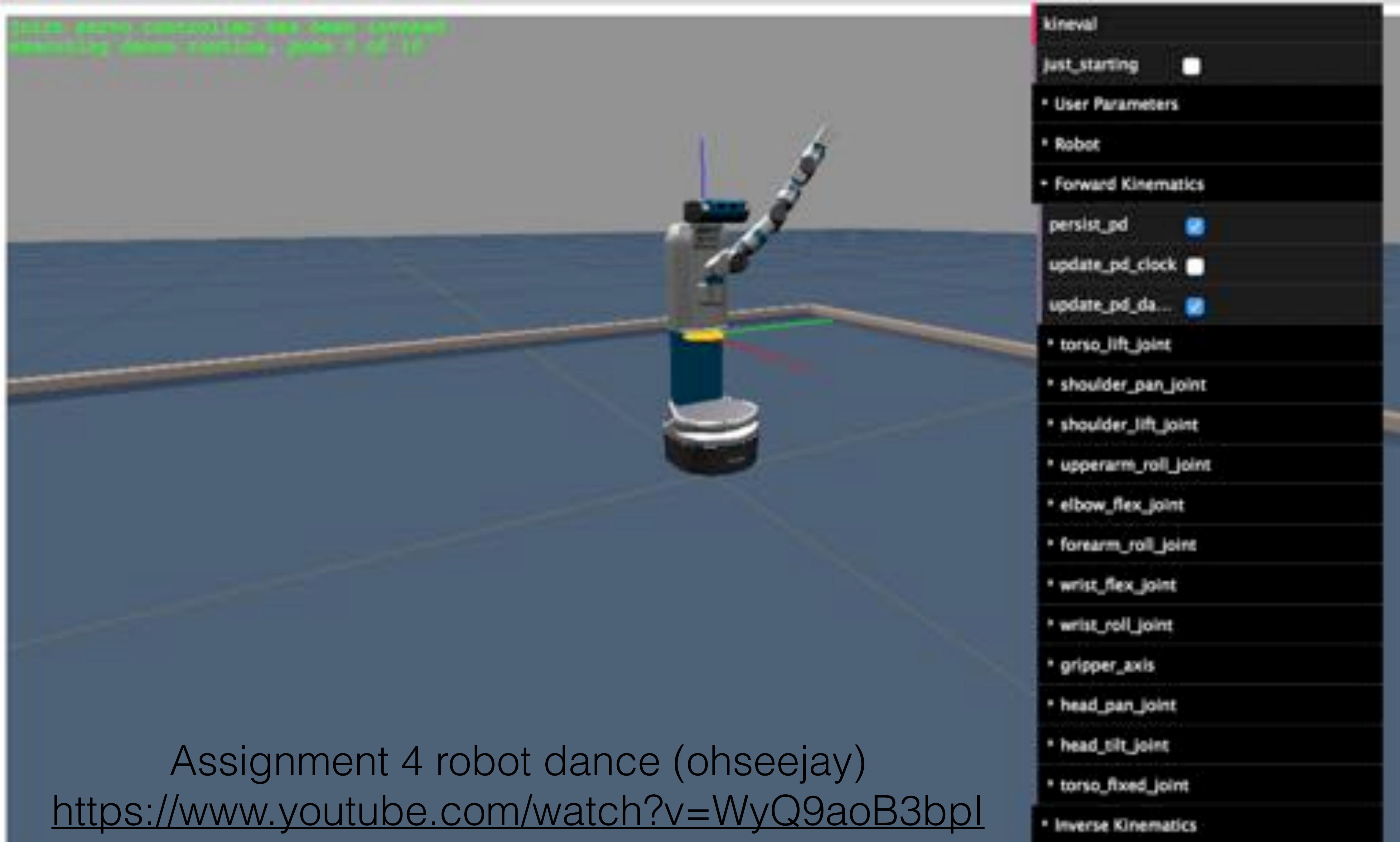


How to detect “close enough”?

# FSMs for Other Tasks

- Robot foraging?
- Robot tennis/pong?
- Pushing a ball into a goal?
- Vacuuming a room
- Driving a car?
- Robot dancing!





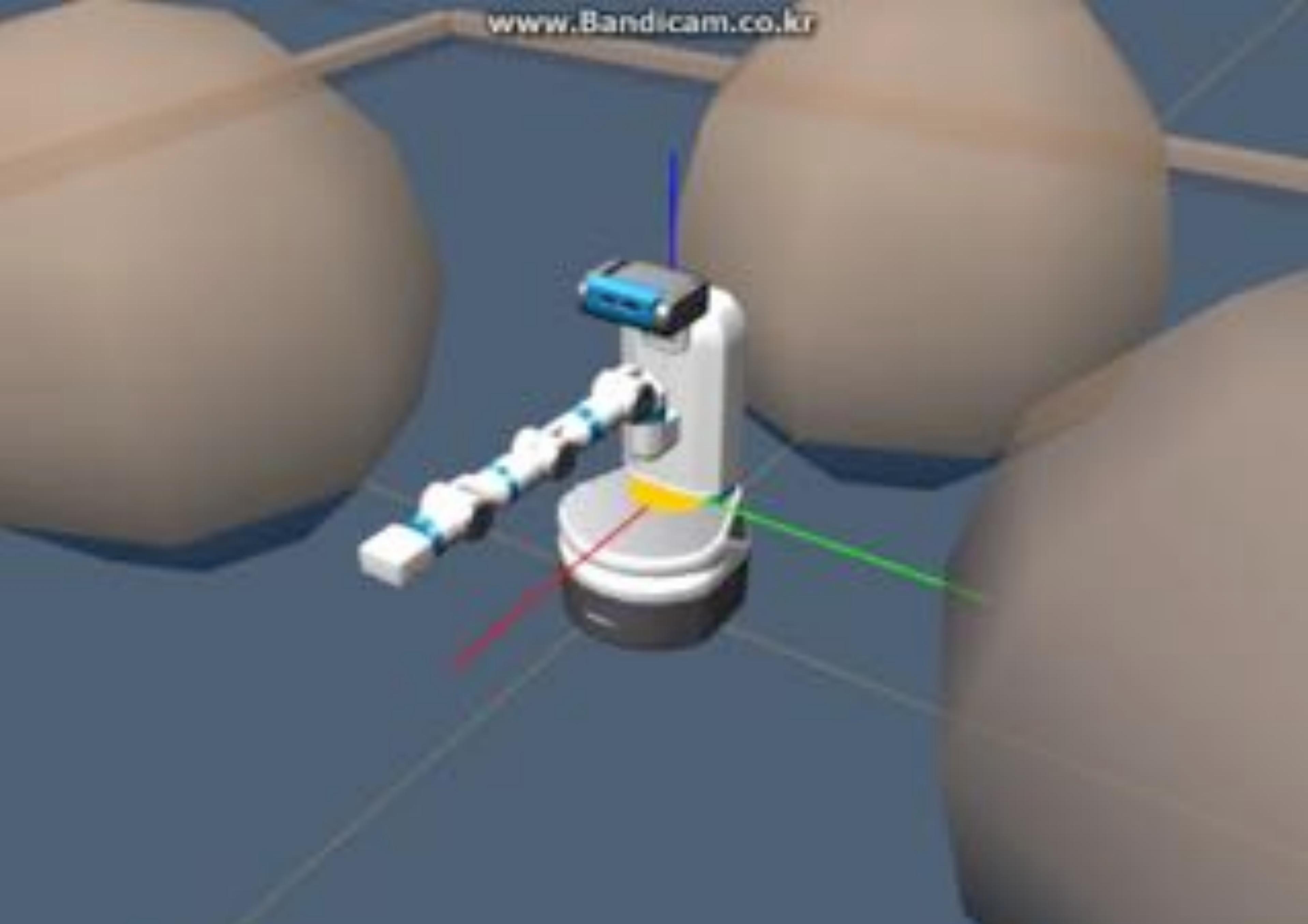
Assignment 4 robot dance (ohseejay)

<https://www.youtube.com/watch?v=WyQ9aoB3bpl>



sreesha

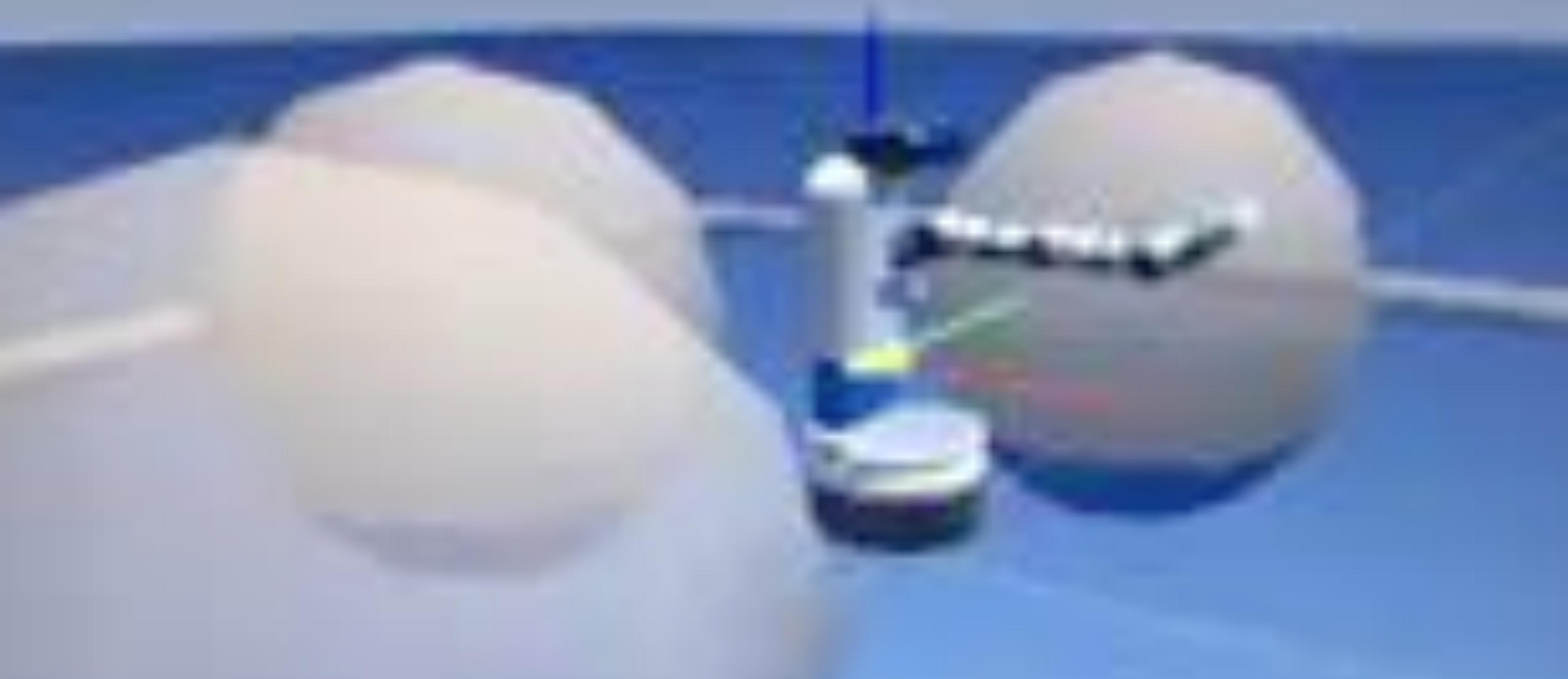


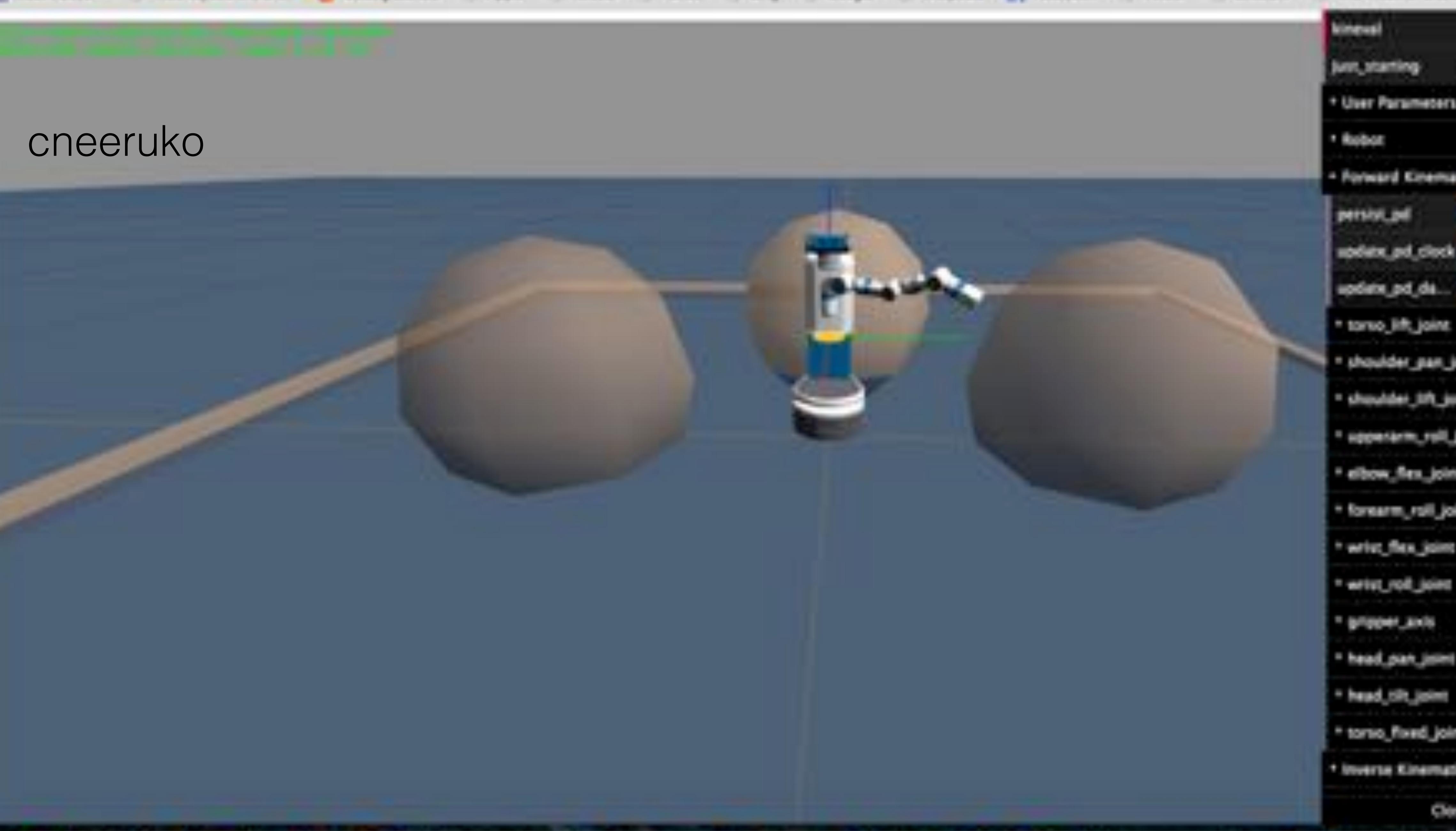


heostar



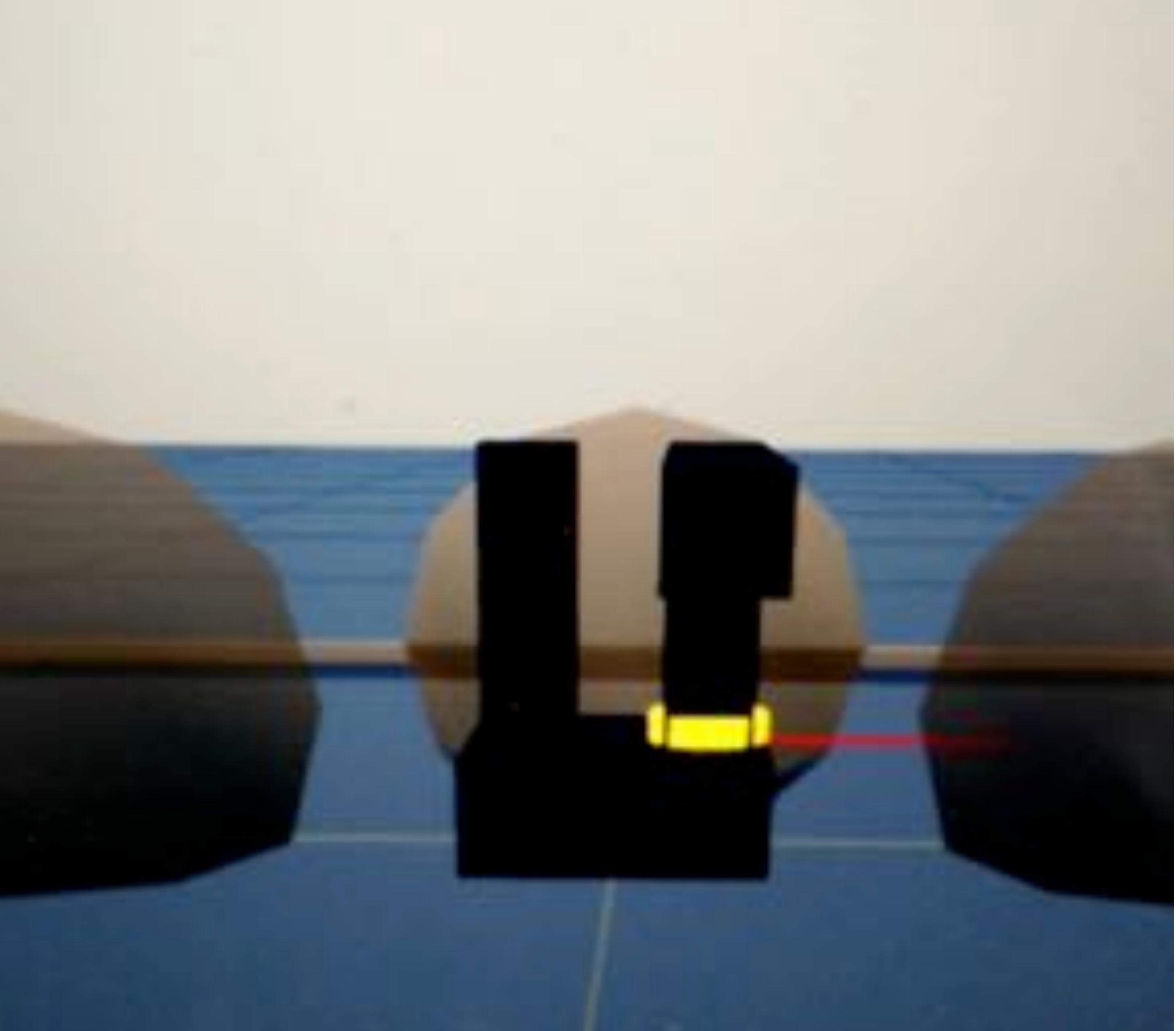
noah





tgroeche





ankit



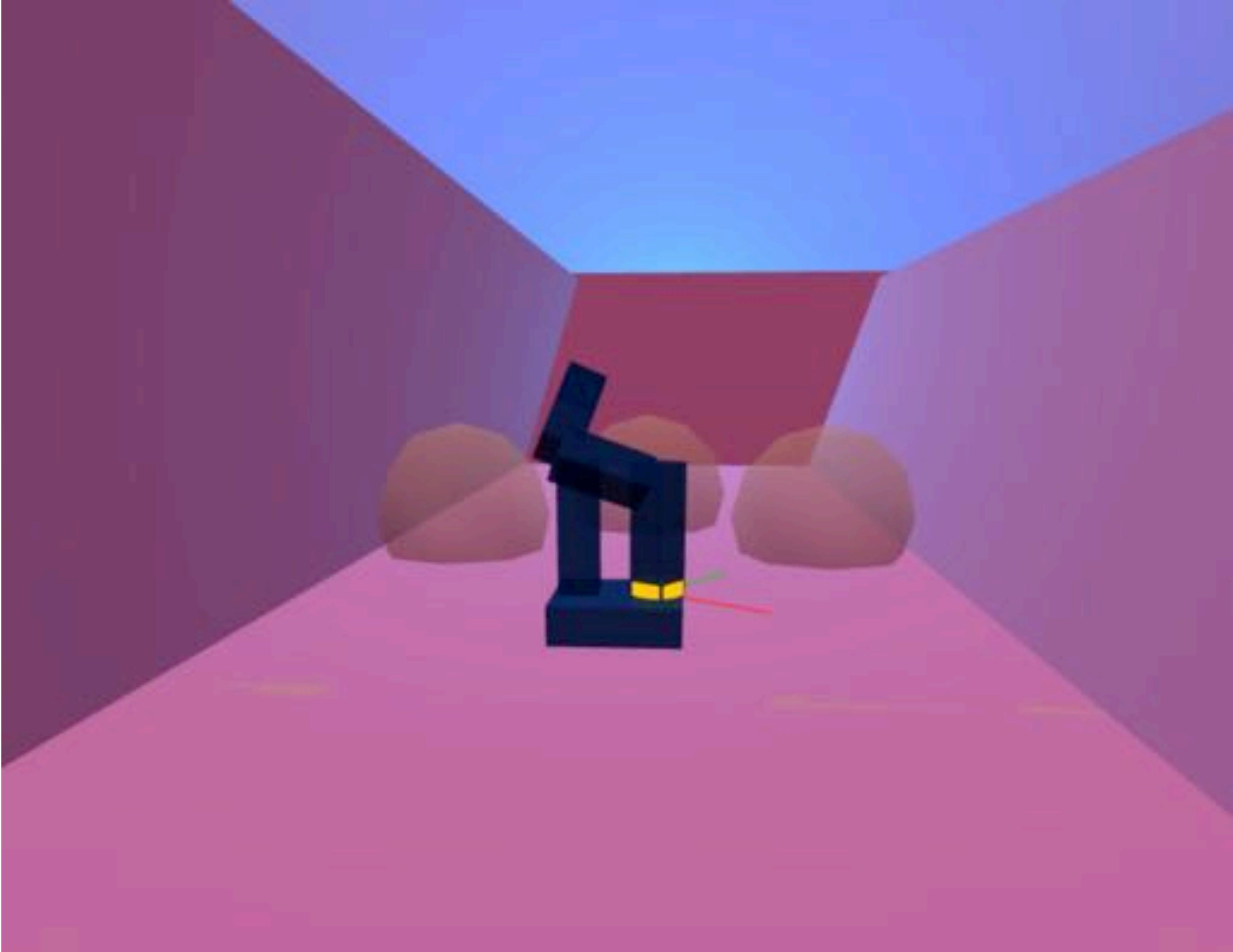


222



cszechy





cszechy  
ohseejay



# Let's generalize FSMs for robot control

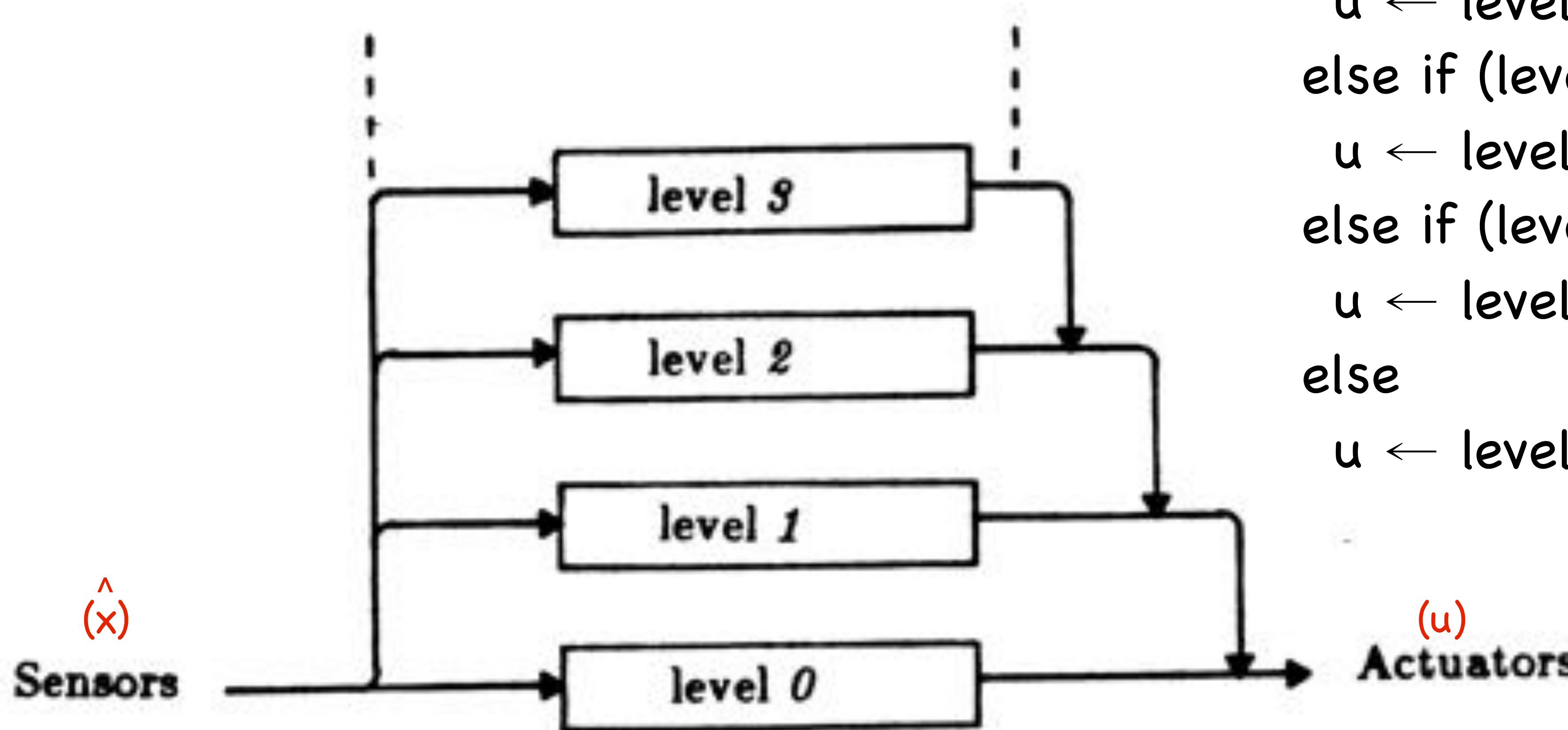
# Subsumption Architecture

[Brooks 1986]

- Generalization of FSM-based control
- Collection of modular reactive controllers in a priority hierarchy
- Controllers can be FSMs
- Large nested if-else statement
- Most robots are controlled by some form of subsumption



# Subsumption Architecture



```
if (level3_condition)  
    u ← level3_control  
else if (level2_condition)  
    u ← level2_control  
else if (level1_condition)  
    u ← level1_control  
else  
    u ← level0_control
```

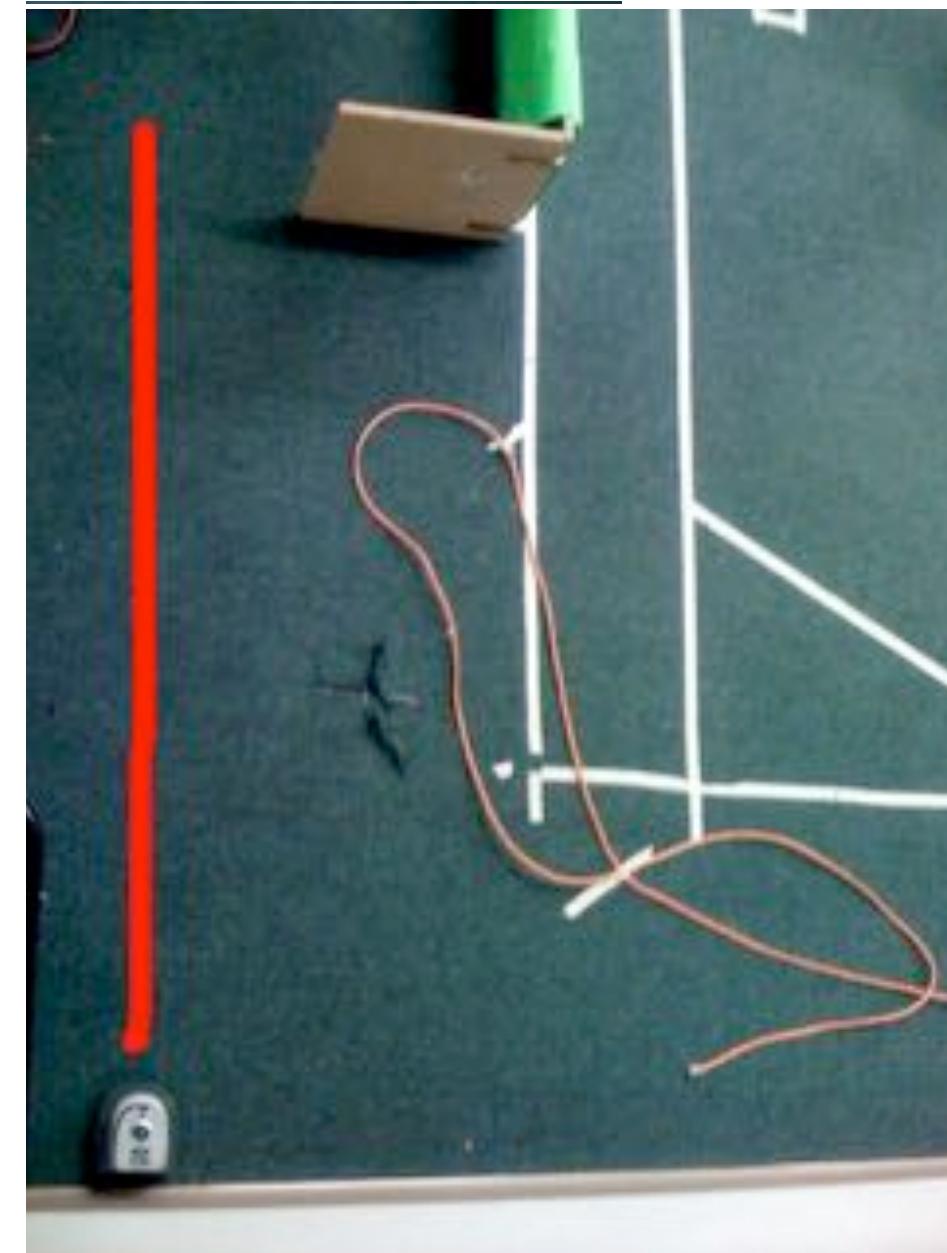
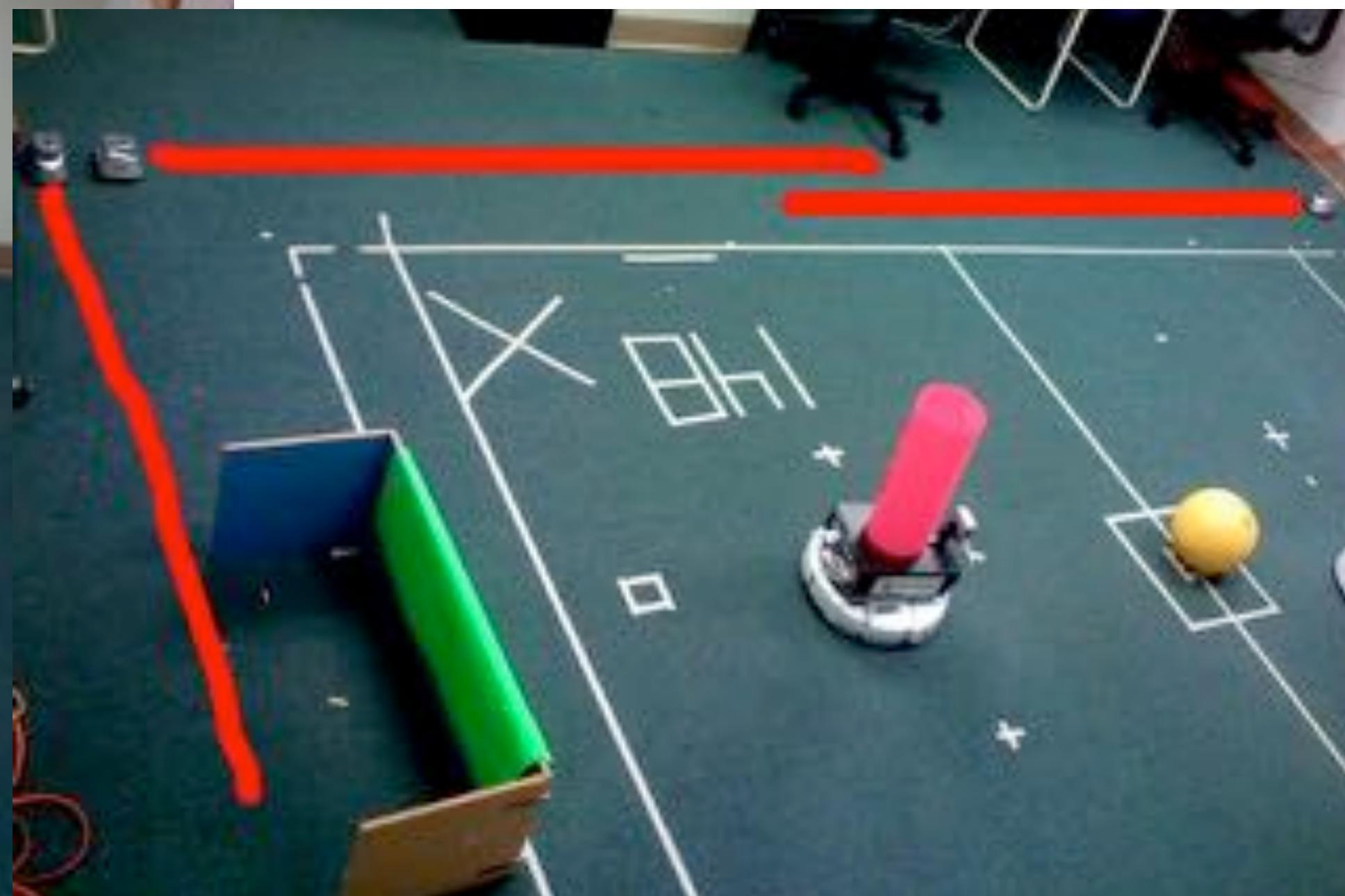
# Subsumption Design Process

- 1. Divide your problem into basic competencies** ordered simple to more complex.  
Designate a level for each basic competency.
- 2. Subdivide each level into multiple simple components** that interact through shared variables. Limit the sharing of variables among levels to avoid incomprehensible code.
- 3. Implement each module as a separate light-weight thread.** You might think of setting the priorities for these threads s.t. modules in a given level have the same priority.
- 4. Implement "arbitration" processes** for suppression and inhibition as one or more separate that serve to control access to shared variables. You might want to control access using semaphores.



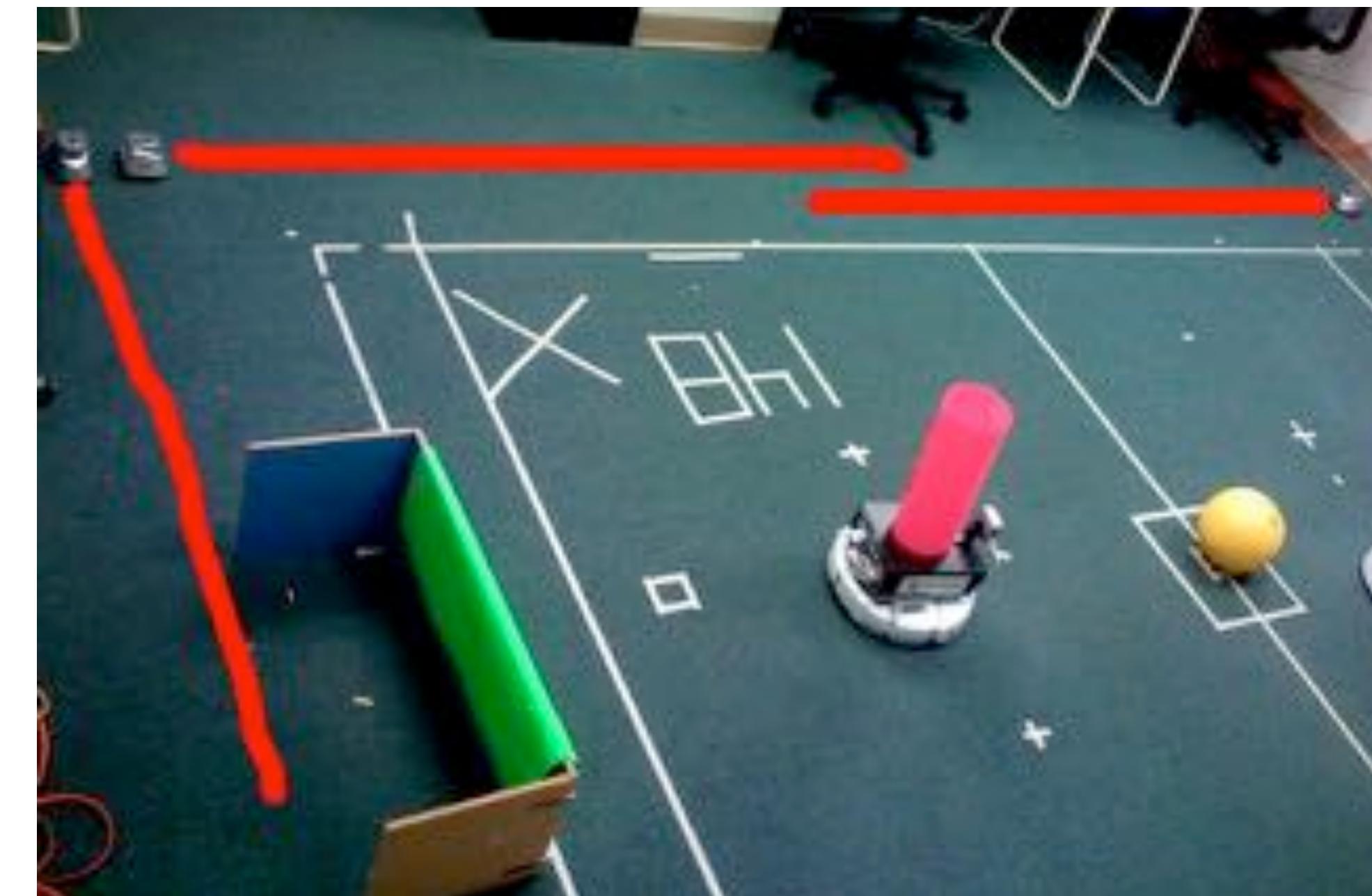
# Subsumption for robot soccer

- Propose modules and priority?



# What behavior will result?

1. Avoid IR Wall
2. Avoid Robot
3. Avoid Fiducial
4. Bumper Hit
5. Go To Opposite Goal
6. Go To Any Goal
7. Line Up On Ball
8. Go To Ball
9. Score Goal
10. At Ball
11. Look For Ball



# Snappy's Subsumption: Goal Scoring

1. Avoid IR Wall
2. Avoid Robot
3. Avoid Fiducial
4. Bumper Hit
5. Go To Opposite Goal
6. Go To Any Goal
7. Line Up On Ball
8. Go To Ball
9. Score Goal
10. At Ball
11. Look For Ball

Goal Scoring Challenge - Put ball on orange



# Snappy's Subsumption: Navigate to Ball

1. Avoid IR Wall
2. Avoid Robot
3. Avoid Fiducial
4. Bumper Hit
5. Go To Opposite Goal
6. Go To Any Goal
7. Line Up On Ball
8. Go To Ball
9. Score Goal
10. At Ball
11. Look For Ball



# Snappy in competition



Are there other methods of  
decision making?

# Types of Decision Making

- Deliberative (Planner-based) Control
  - “Think hard, act later.”
- Reactive Control
  - “Don’t think, (re)act.”
- Hybrid Control
  - “Think and act separately & concurrently.”
- Behavior-Based Control
  - “Think the way you act.”



# Next lecture: Inverse Kinematics