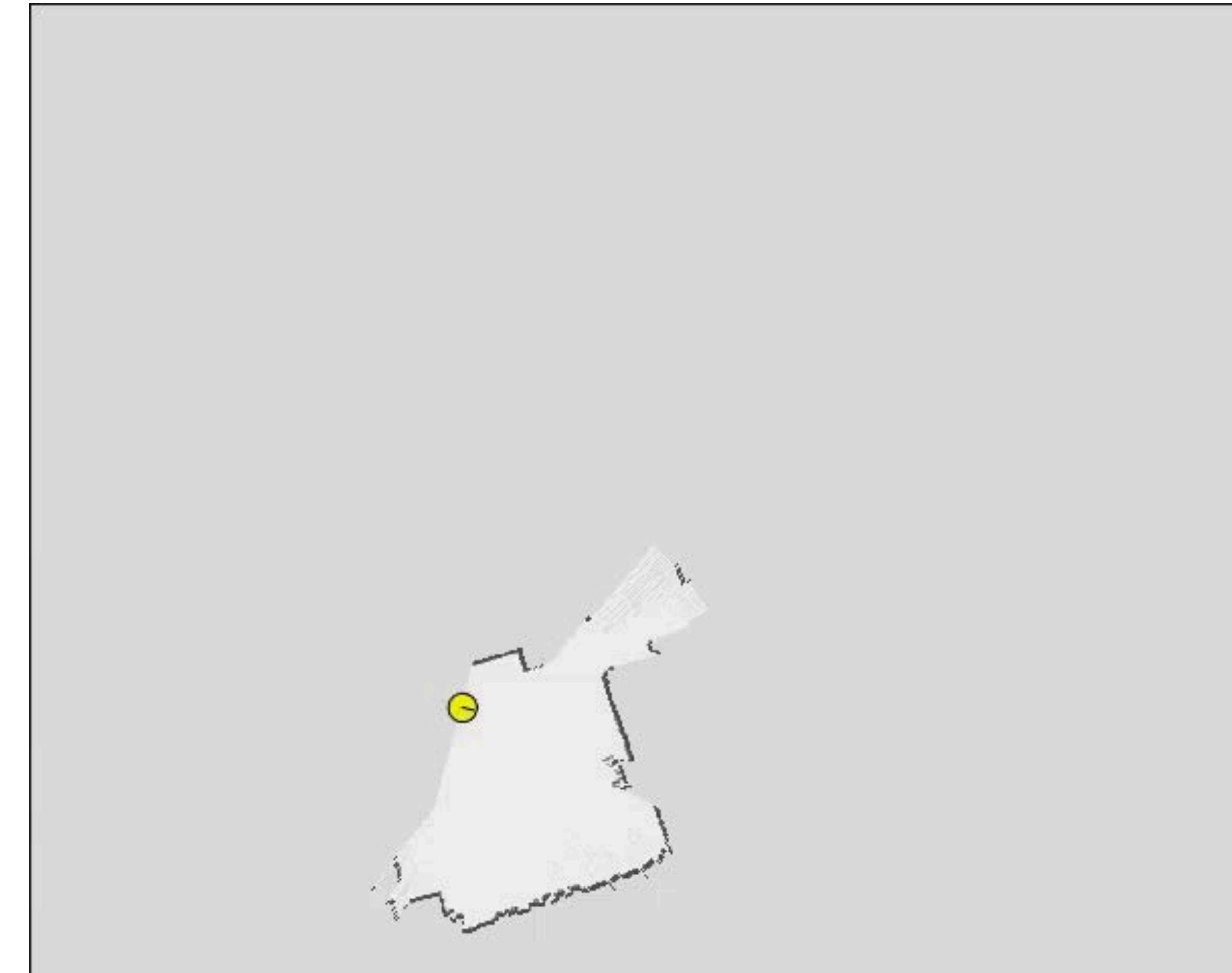


# Lecture 22

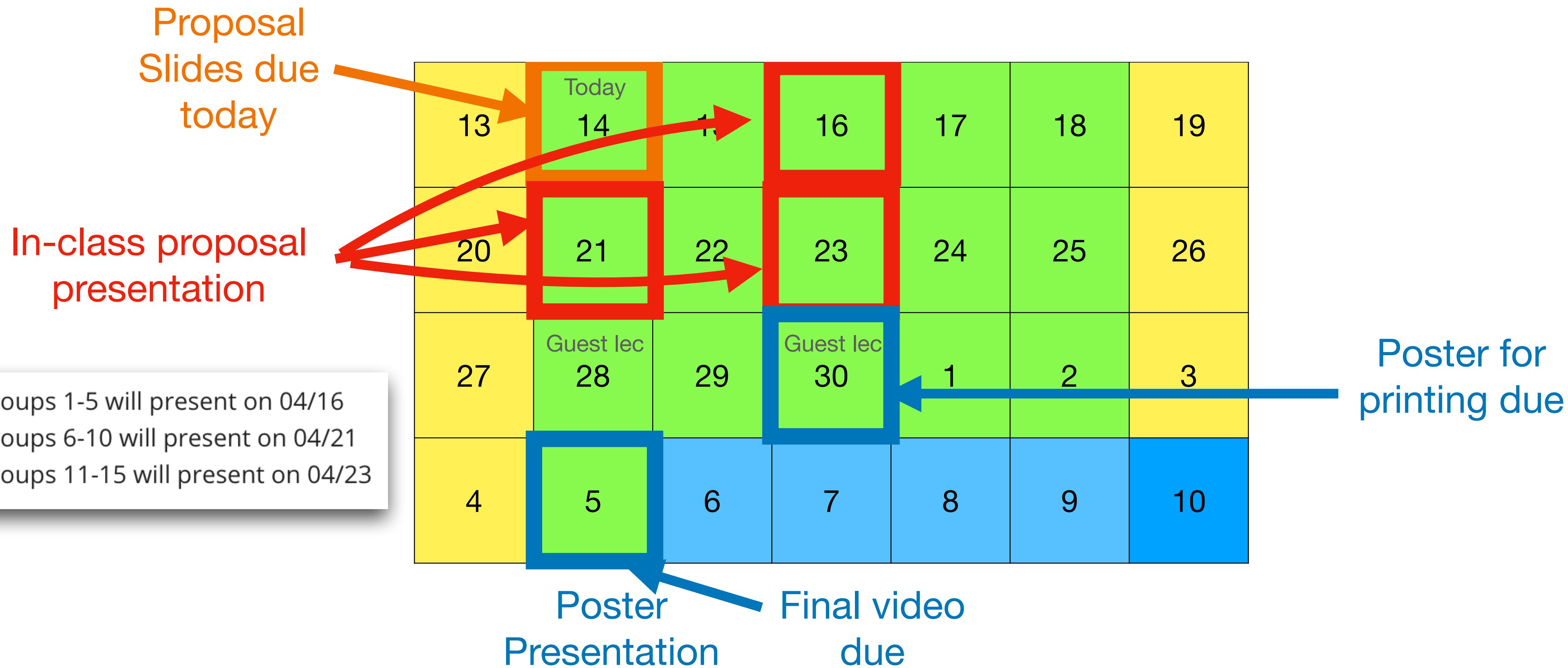
## Mobile Robotics - VII - SLAM



# Course logistics

- Quiz 11 will be posted tomorrow and will due on Wed at noon.
- **Final project proposal slides are due today 11:59 pm CT.**
- Project 7:
  - Sessions are going well.
    - *If you are late to the sessions, you will not receive full points.*
- No TA OHs between 04/07 and 04/23.
  - They will be available on demand.
  - Karthik's OH will be available to discuss final projects.
- **Final Poster Session: 05/05/2025 - Monday - 12:30pm - 2:30pm, Shepherd Labs 164 - mark your calendars**

# Final (Open) Project timeline



# Final (Open) Project timeline

- Proposal Slides: (template is provided)
  - 1-4 Slides
  - Title, Motivation, Input - Output, Evaluation, Deliverables, Timeline, Who is doing what?
  - Where does your project stand not the 3-axes (robots, objects, tasks)?
  - Backup plan
- In-class proposal presentation (<8mins) :
  - Teams will get feedback from the class
- Final video:
  - Describing the project idea and the outcome.
- Poster presentation: (template will be provided)
  - Presenting the project idea and the outcome to audience.

Final Project: 15%

- Project proposal slides + presentation: 3%
- Final project video: 6%
- Poster presentation (evaluation by judges): 6%



Have you started working on your final projects?

At this point, we expect you've settled on an idea and begun making progress.

# Frontier-based Exploration:

**Frontier-based exploration is the process of repeatedly detecting frontiers and moving towards them, until there are no more frontiers and therefore no more unknown regions.**

**What are frontiers?**

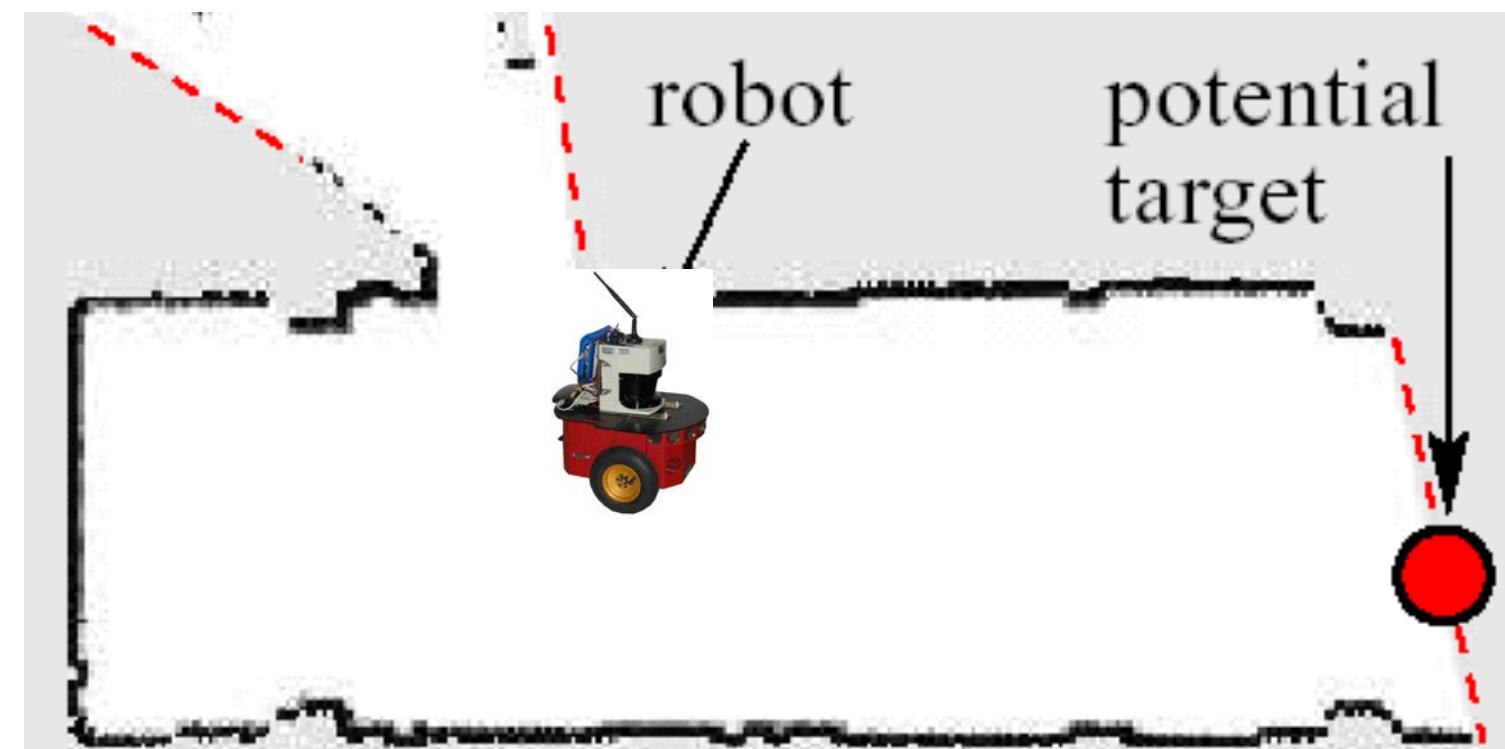
**Frontier cells define the border between known and unknown space.**





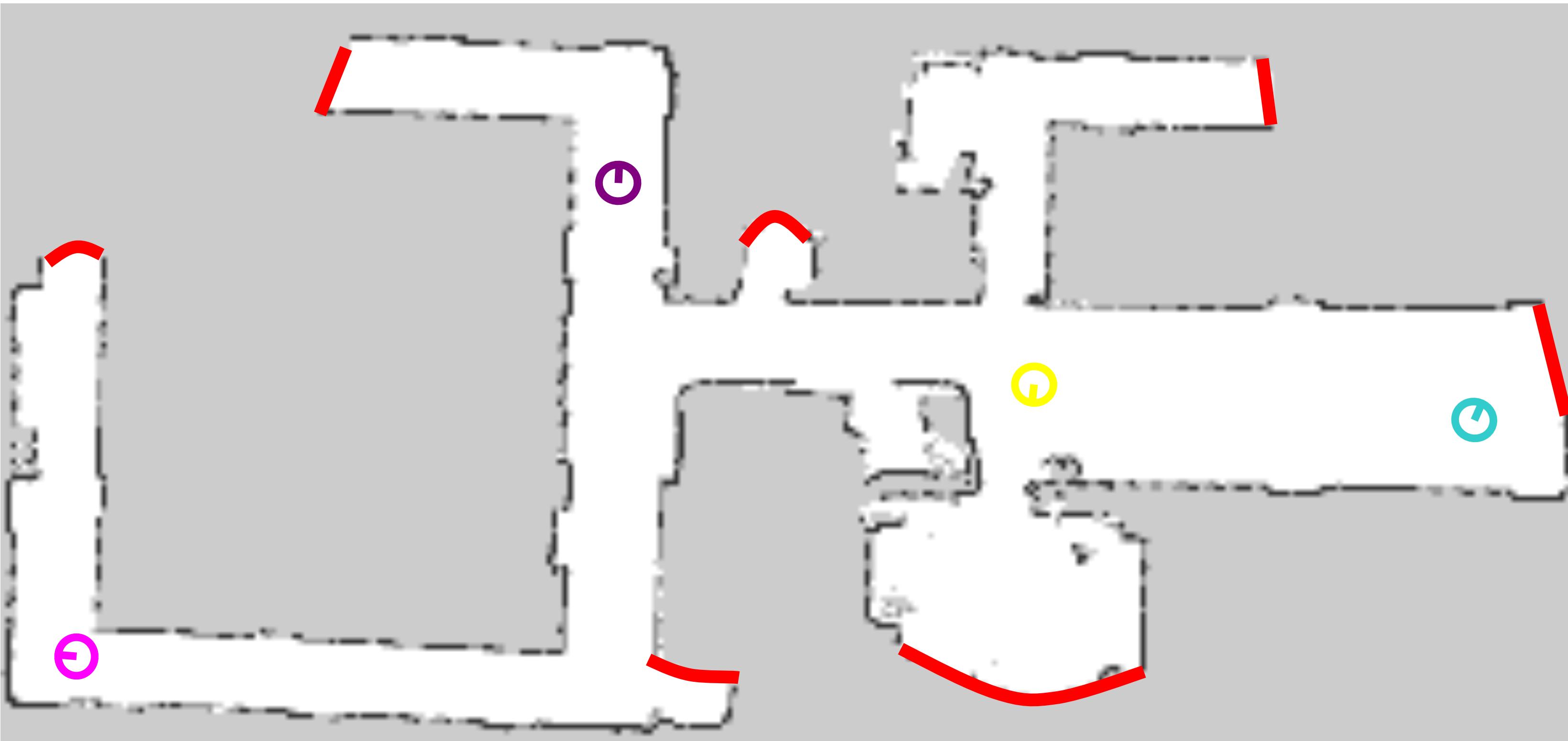
# Single Robot Exploration

- Frontiers between free space and unknown areas are potential target locations
- Going to frontiers will gain information



- Select the target that minimizes a cost function (e.g. travel time / distance /...)

# Coordinated Exploration



[Burgard et al. 00],  
[Simmons et al. 00]

# The SLAM Problem



# The SLAM Problem

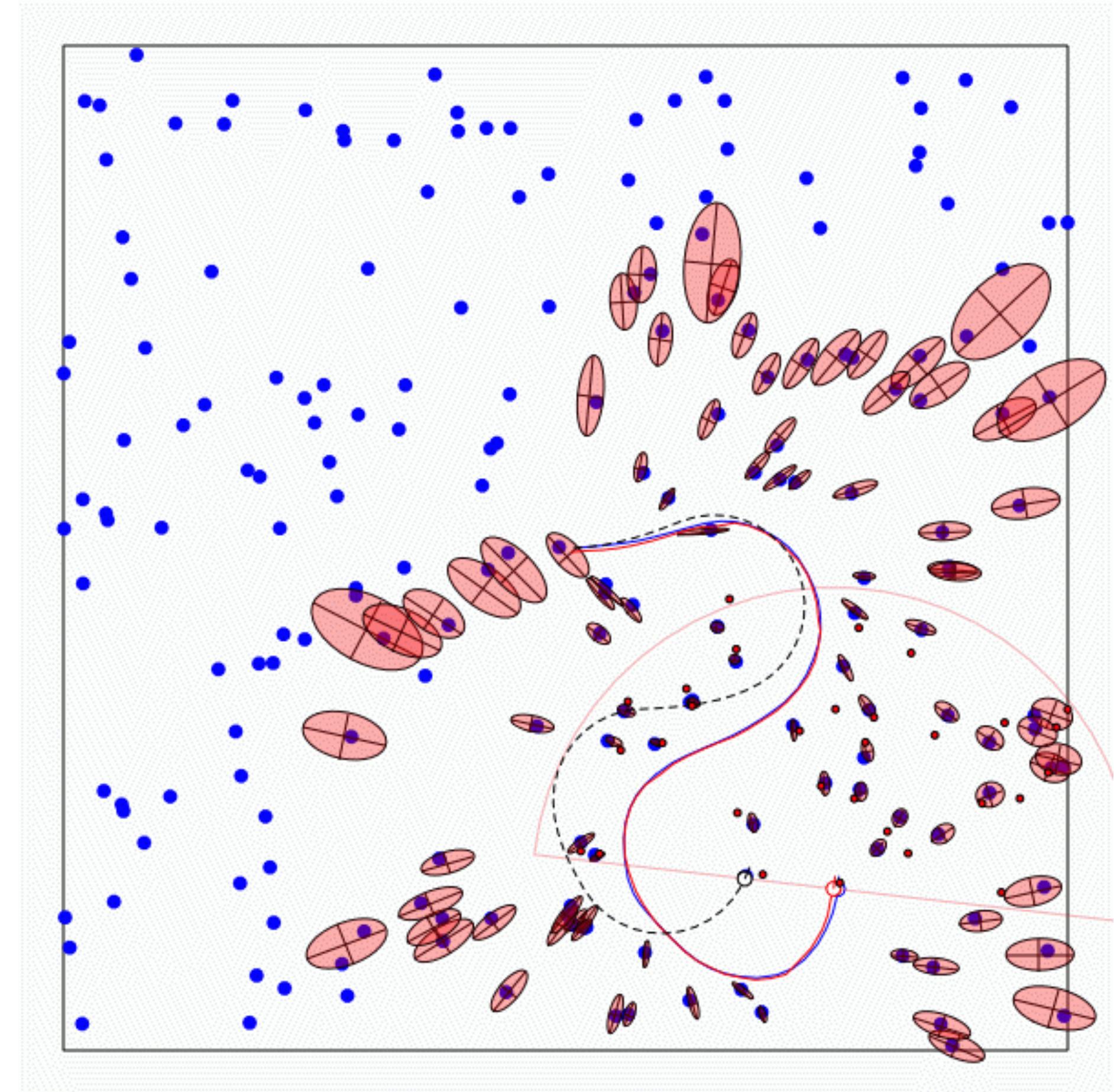
A robot is exploring an unknown, static environment.

## Given:

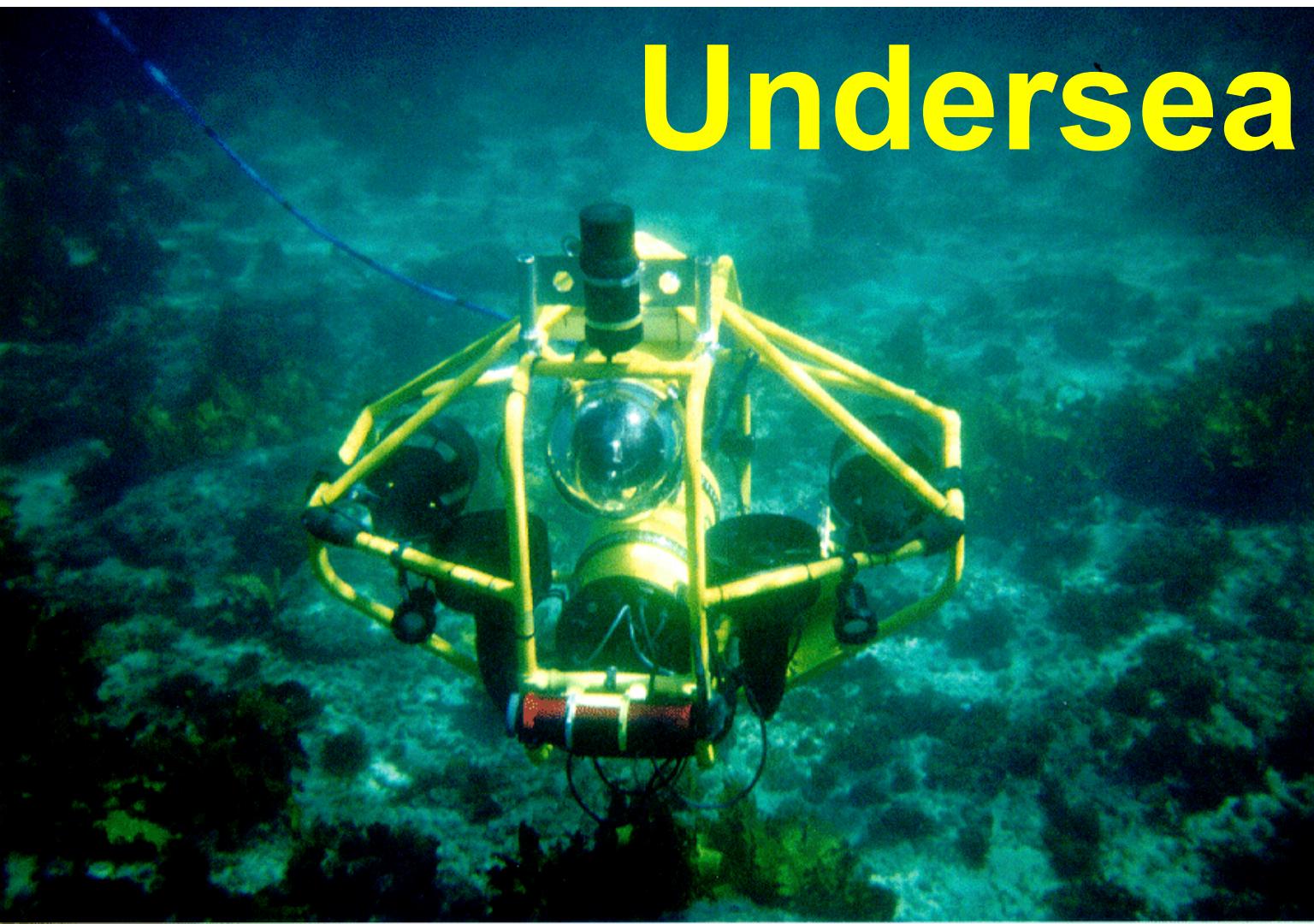
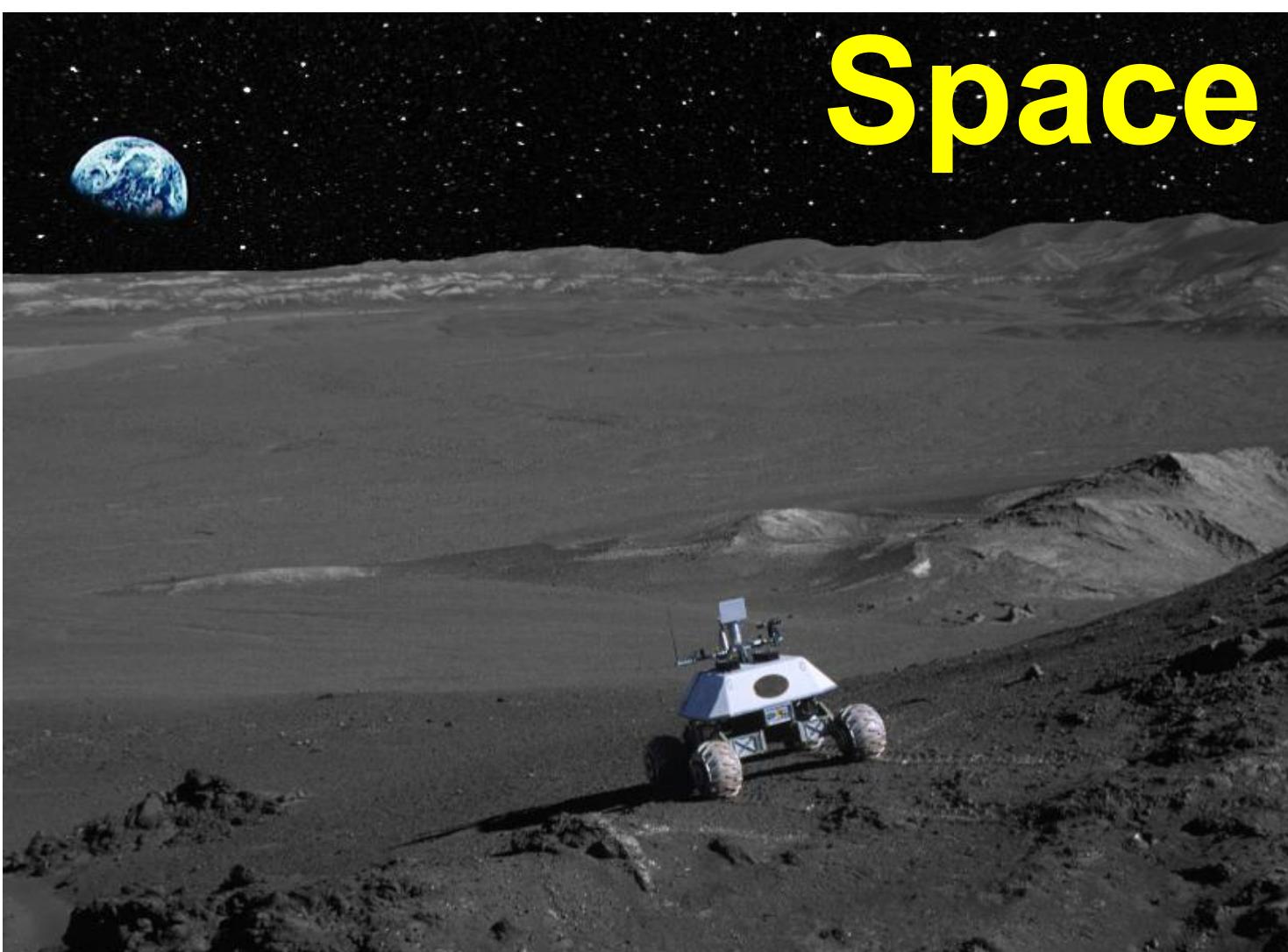
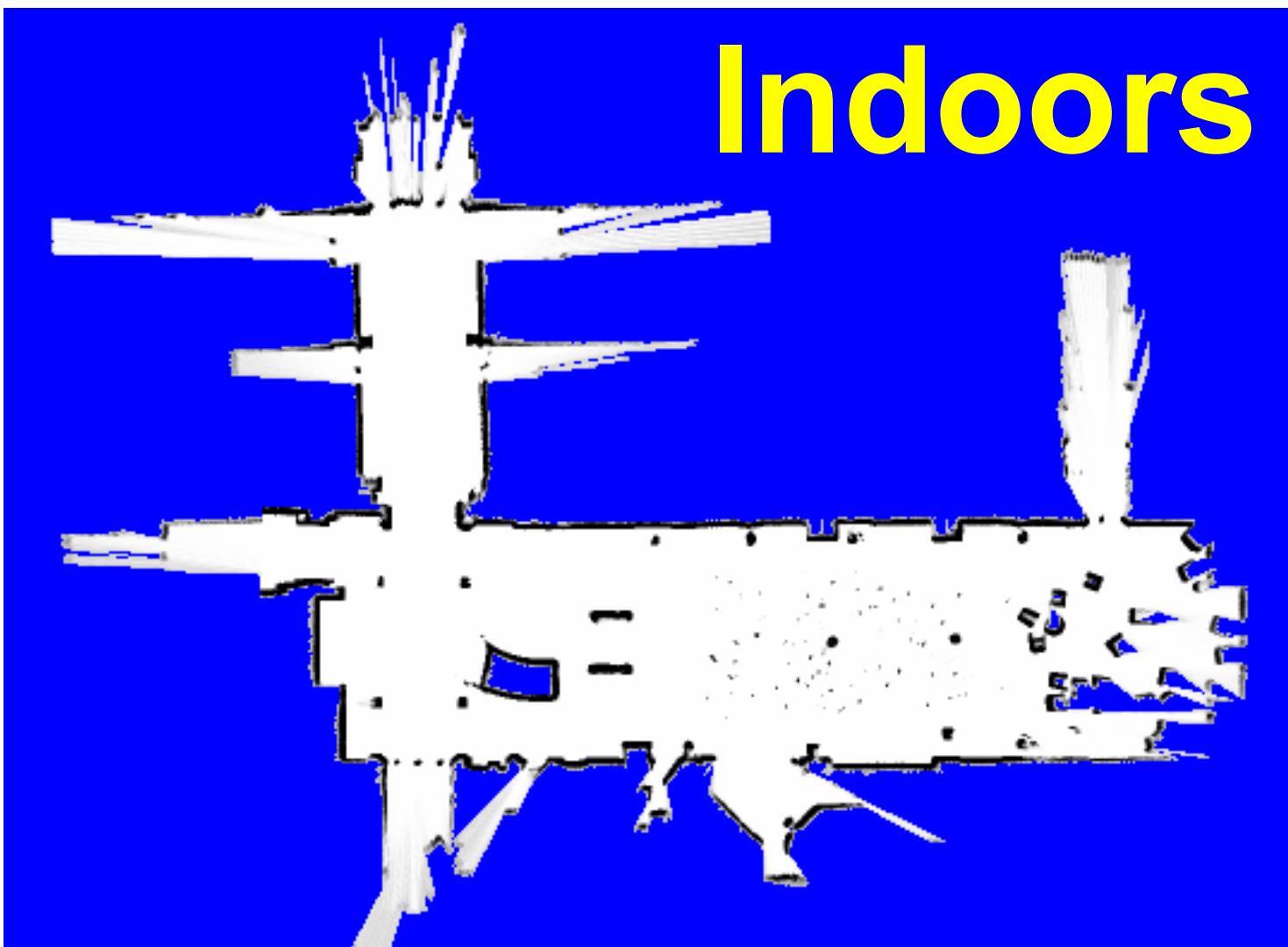
- The robot's controls
- Observations of nearby features

## Estimate:

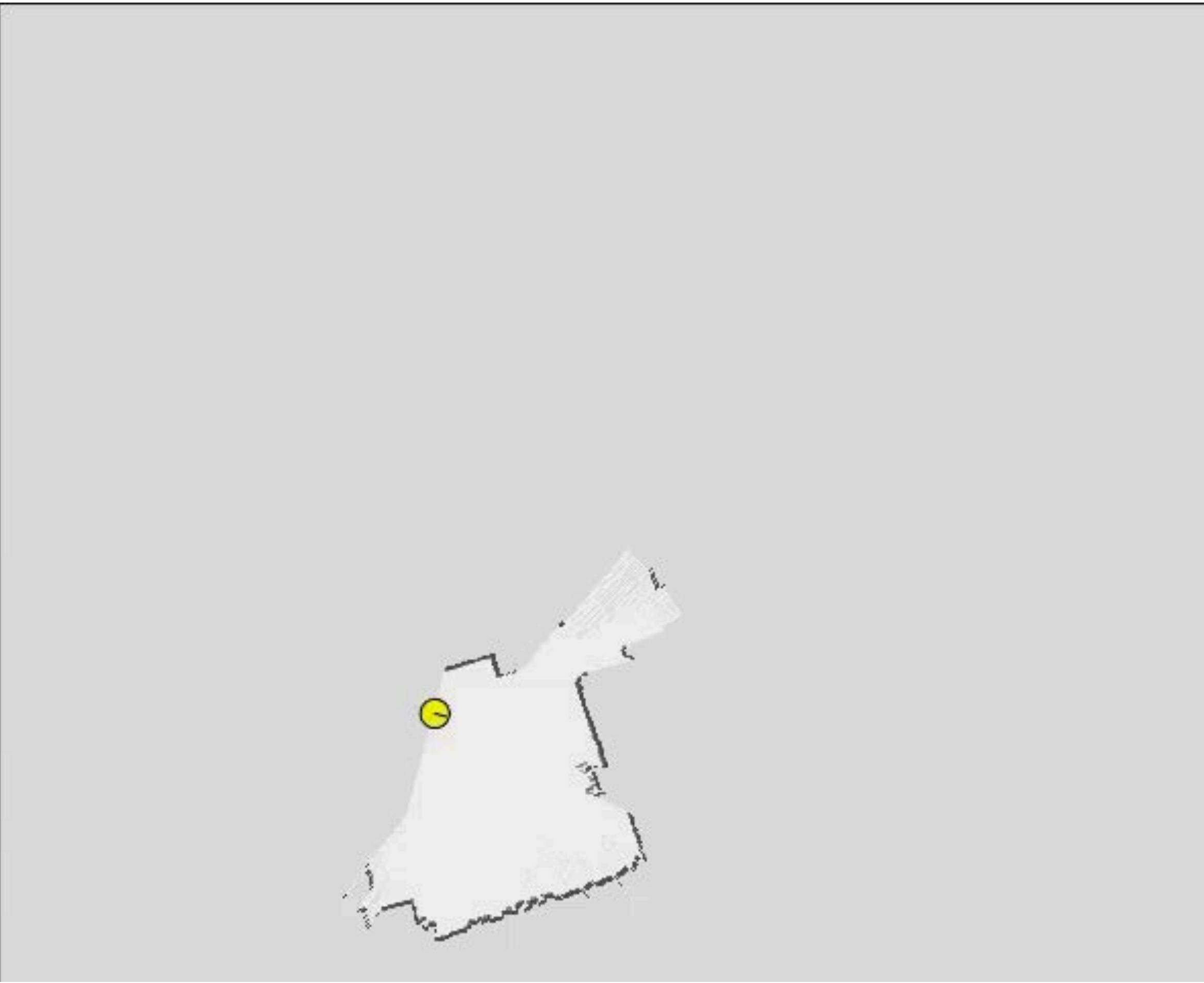
- Map of features
- Path of the robot



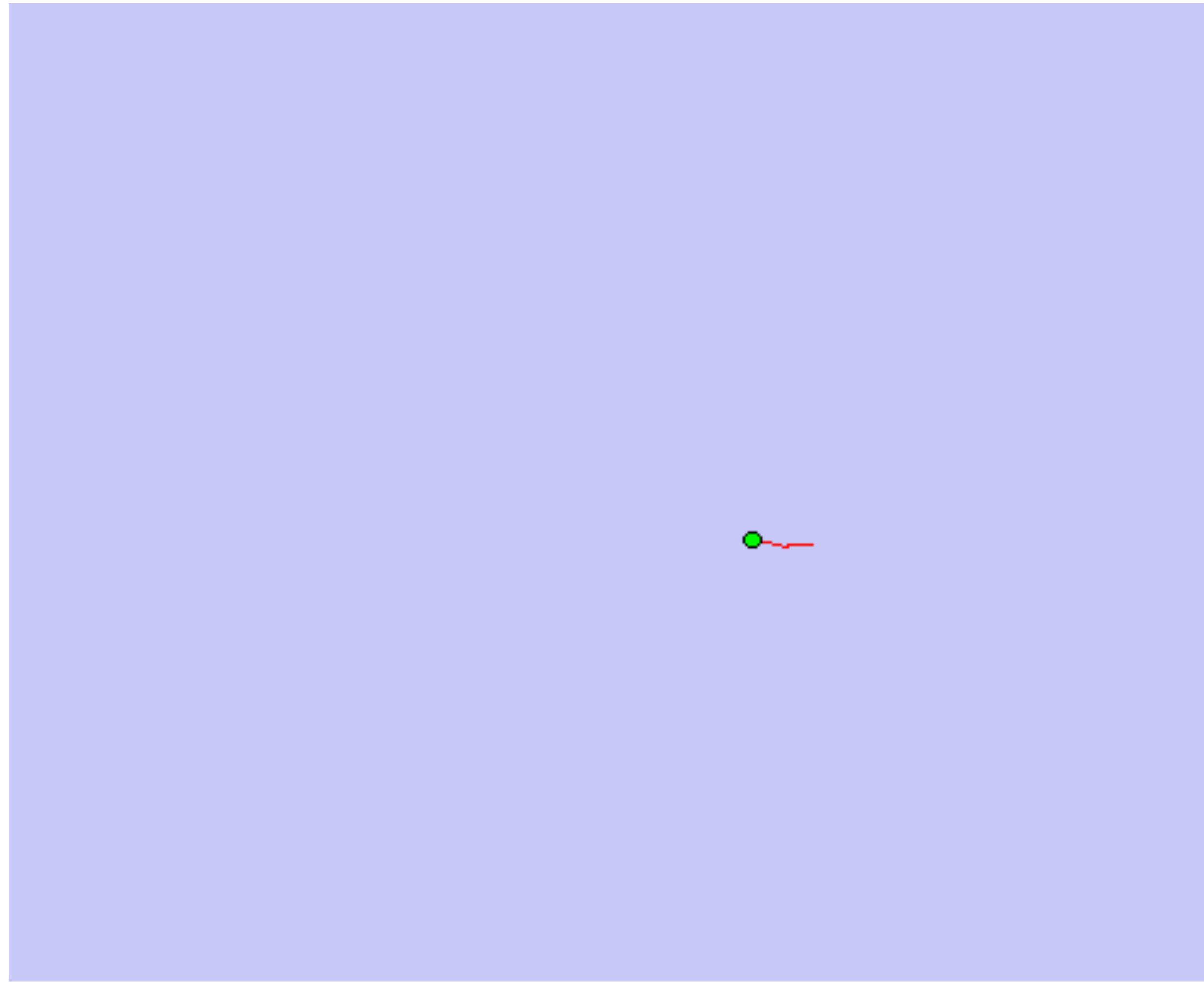
# SLAM Applications



# Mapping with Perfect Odometry

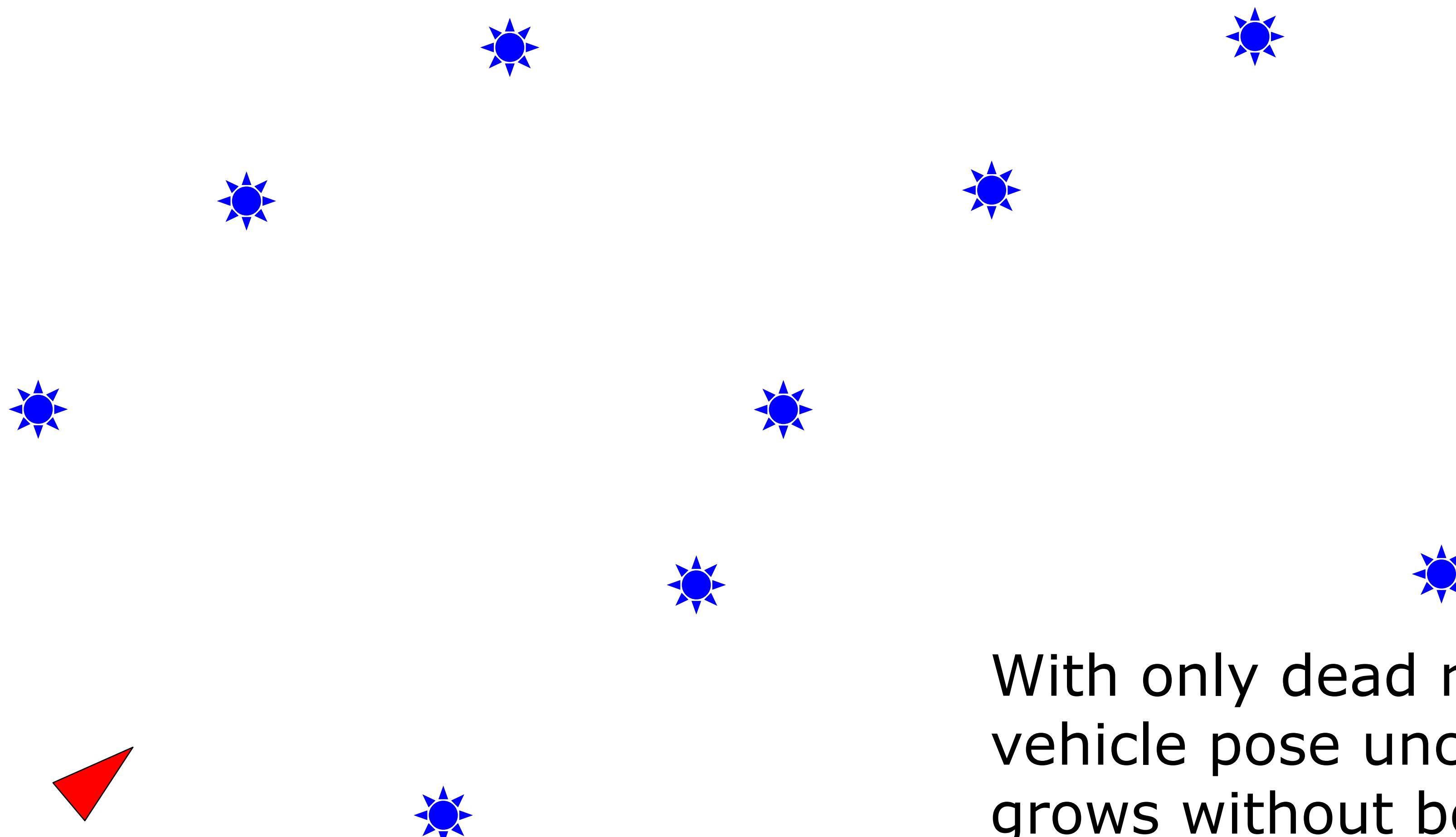


# Mapping with Raw Odometry

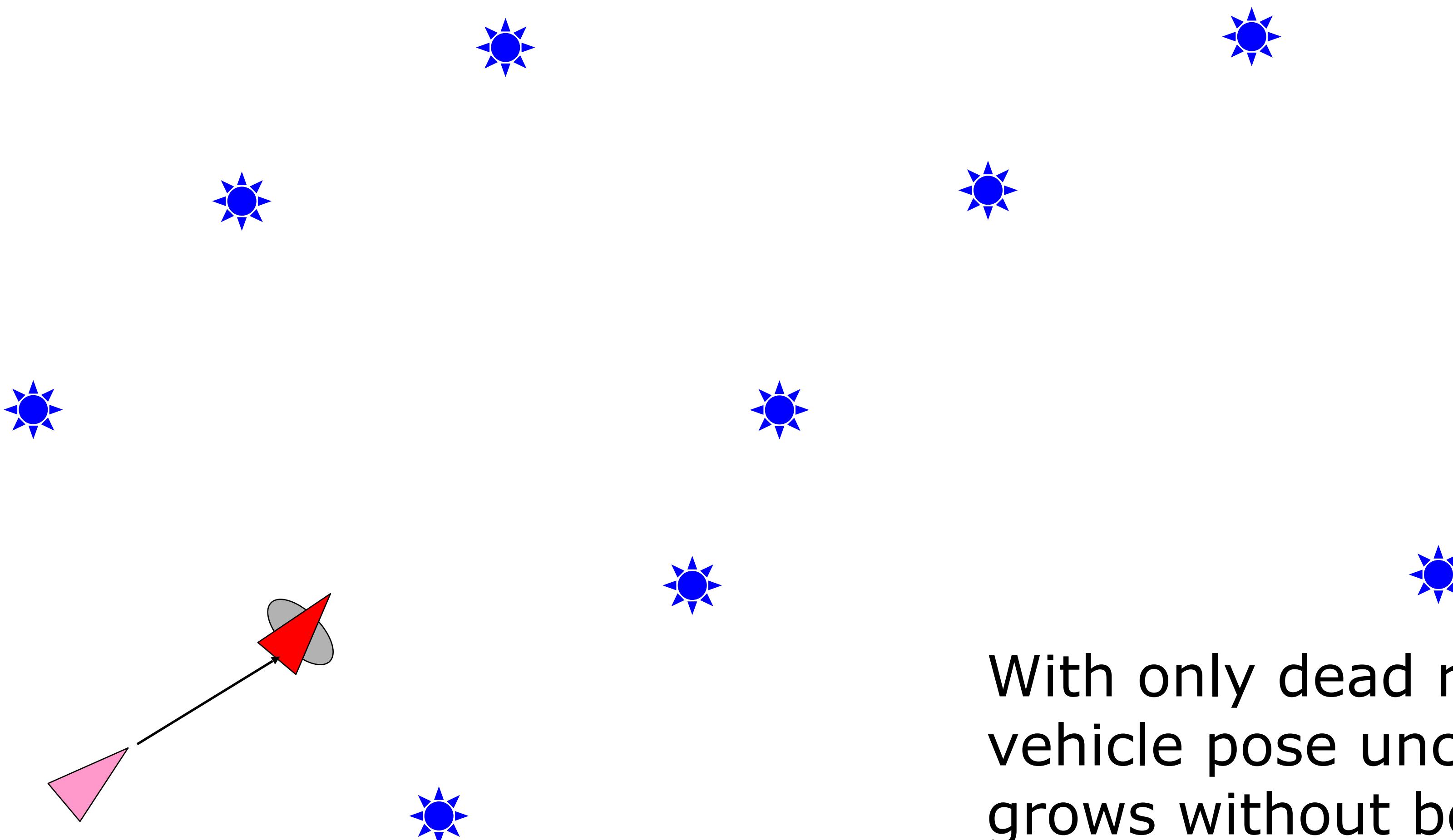


# Illustration of SLAM

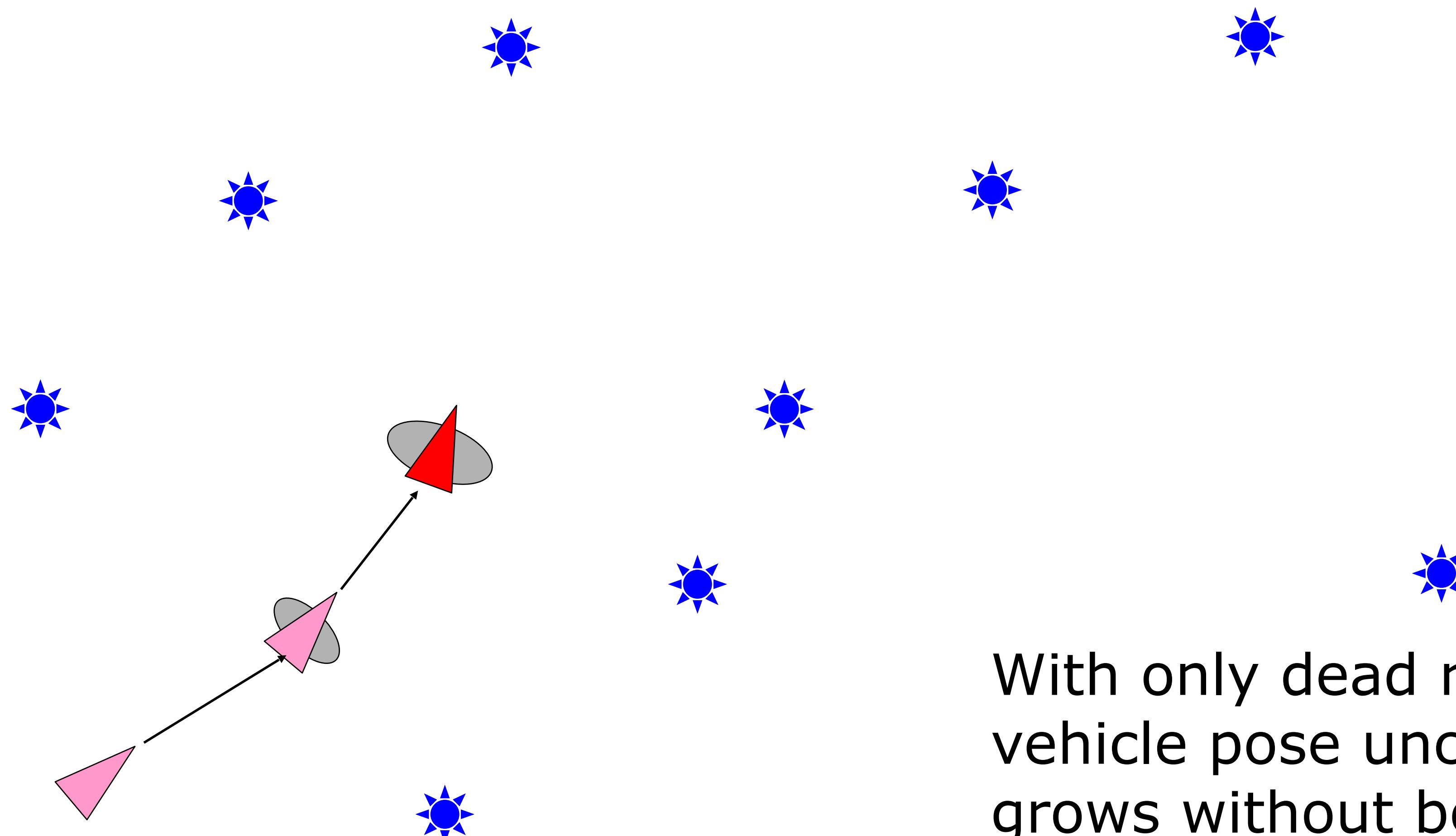
# Illustration of SLAM without Landmarks



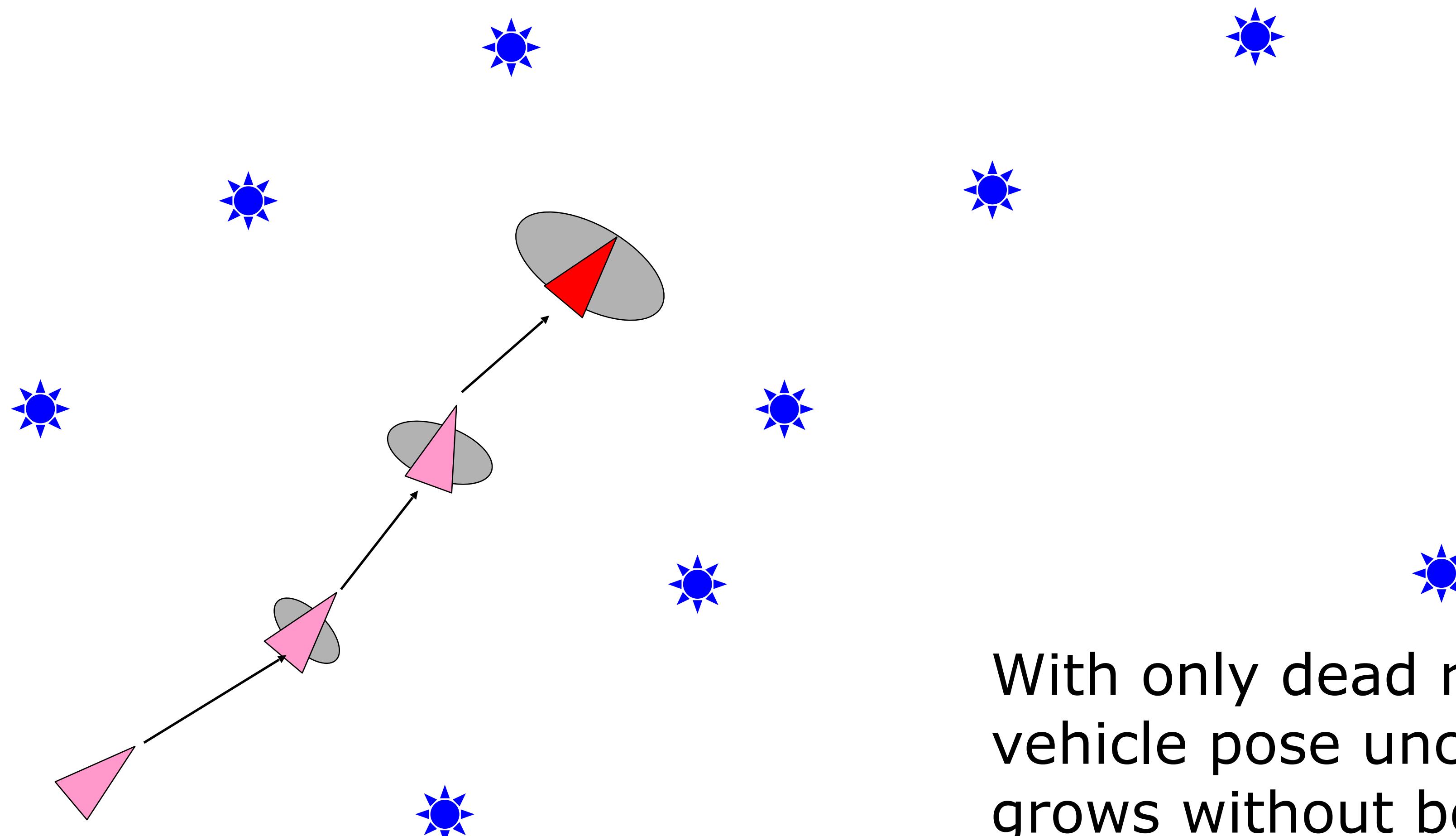
# Illustration of SLAM without Landmarks



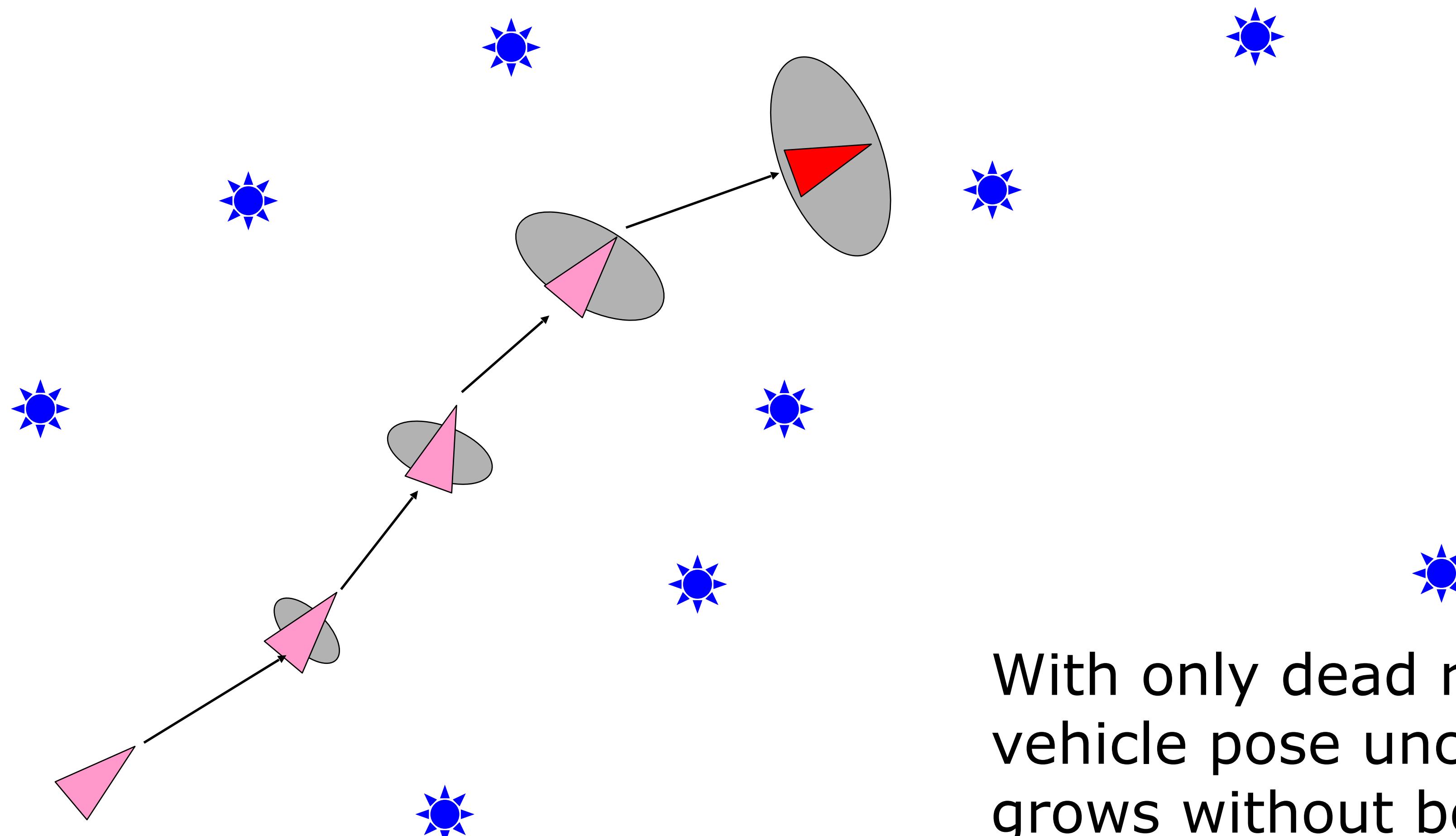
# Illustration of SLAM without Landmarks



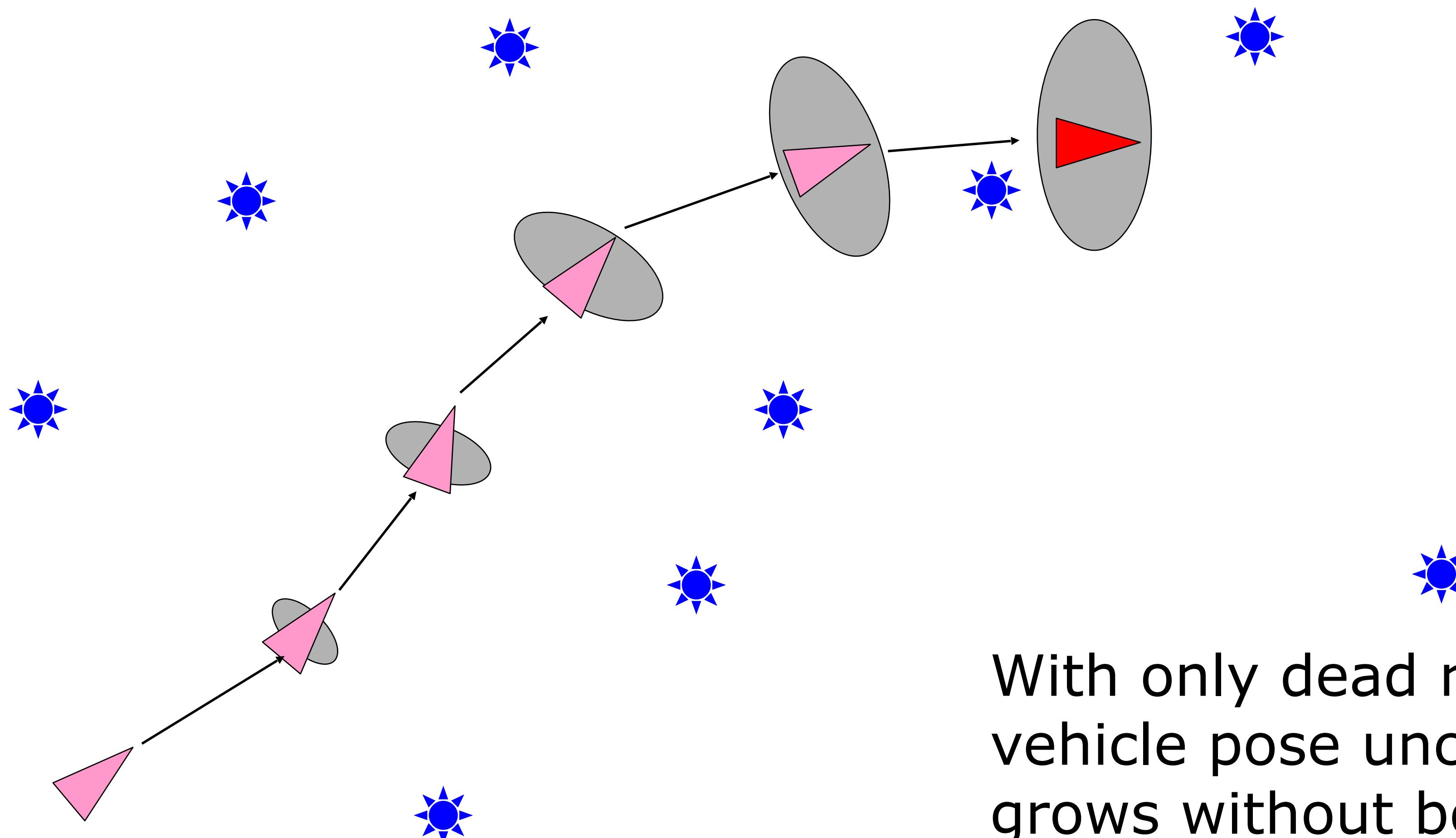
# Illustration of SLAM without Landmarks



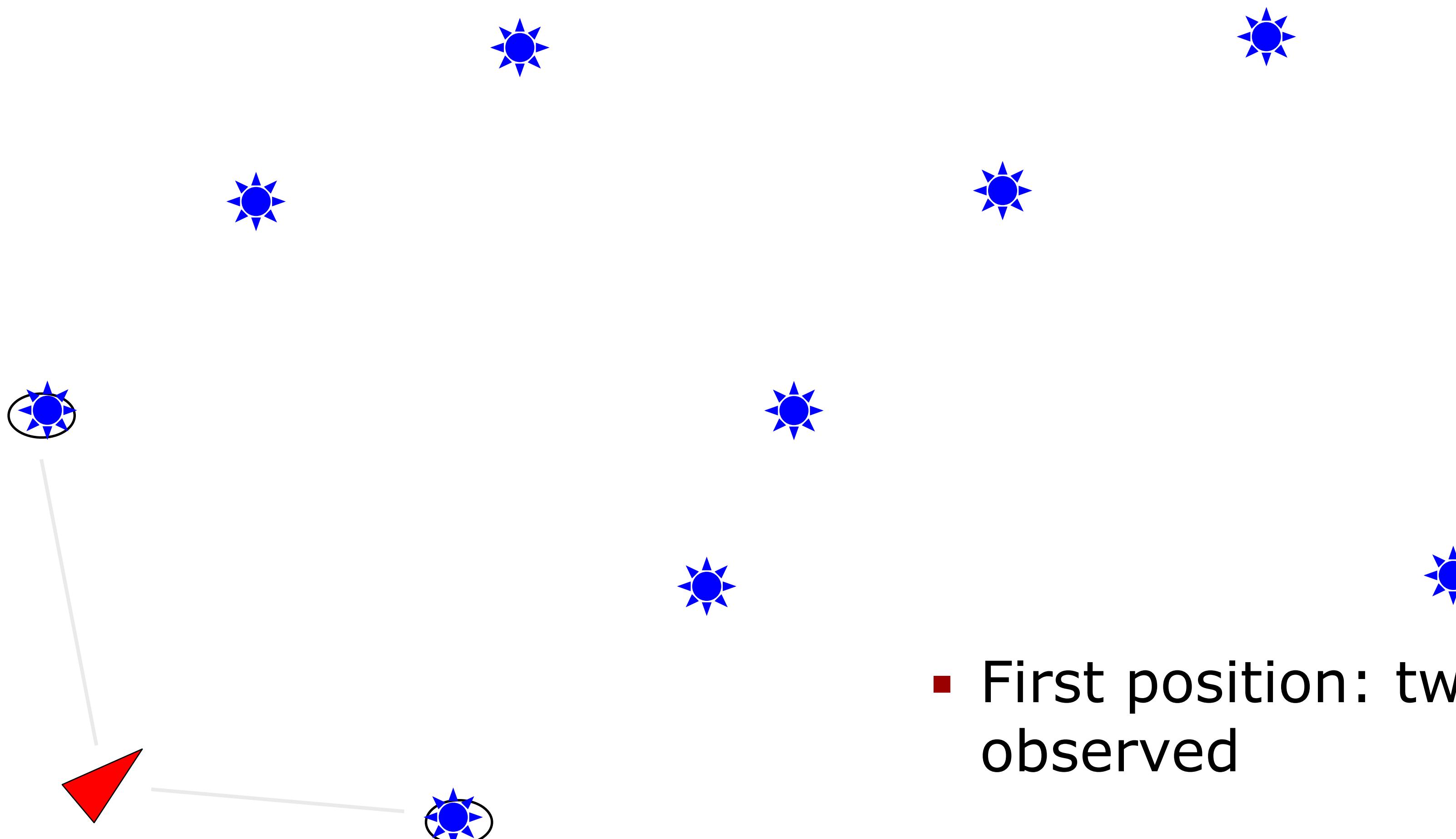
# Illustration of SLAM without Landmarks



# Illustration of SLAM without Landmarks

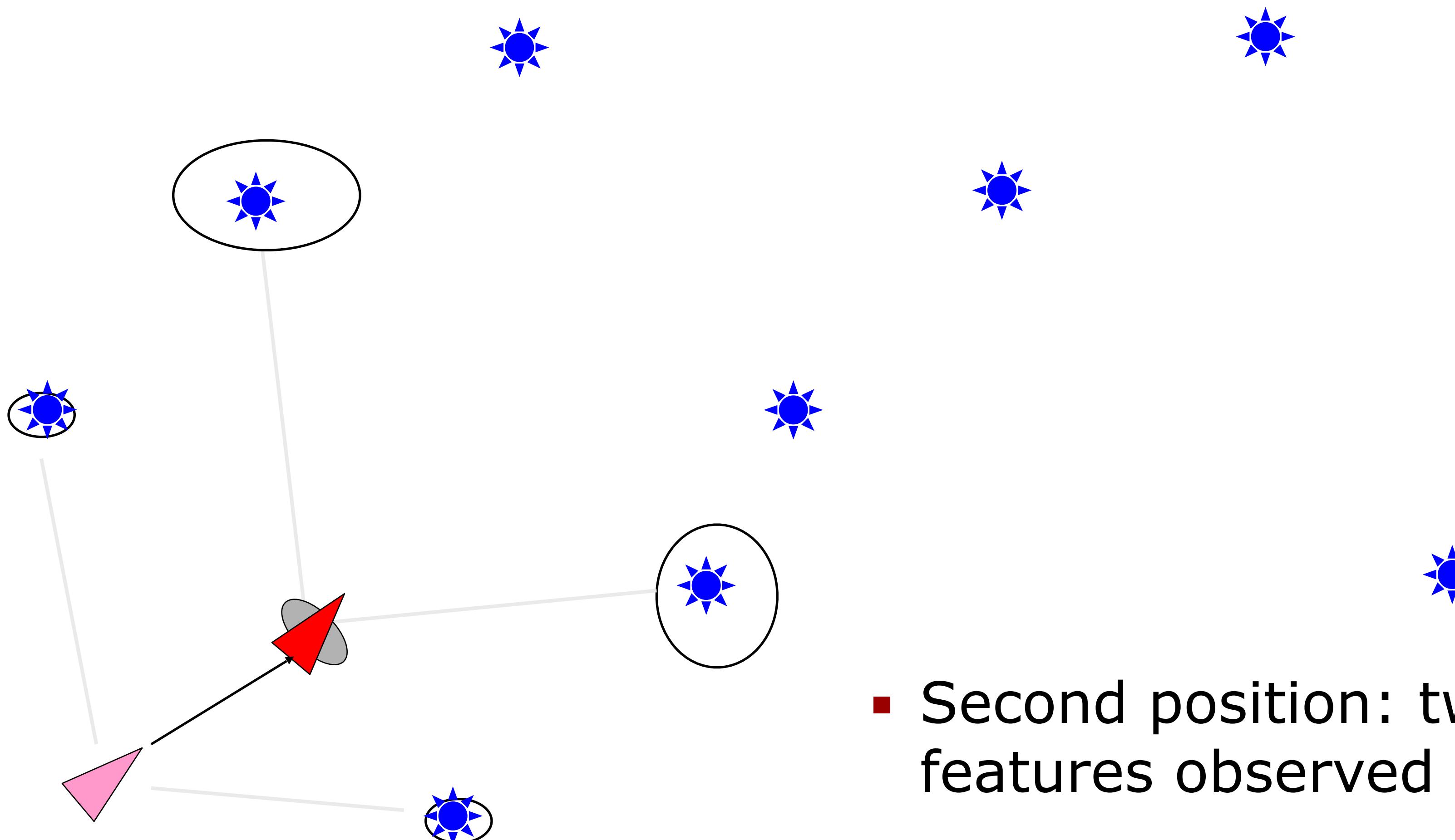


# Repeat, with Measurements of Landmarks



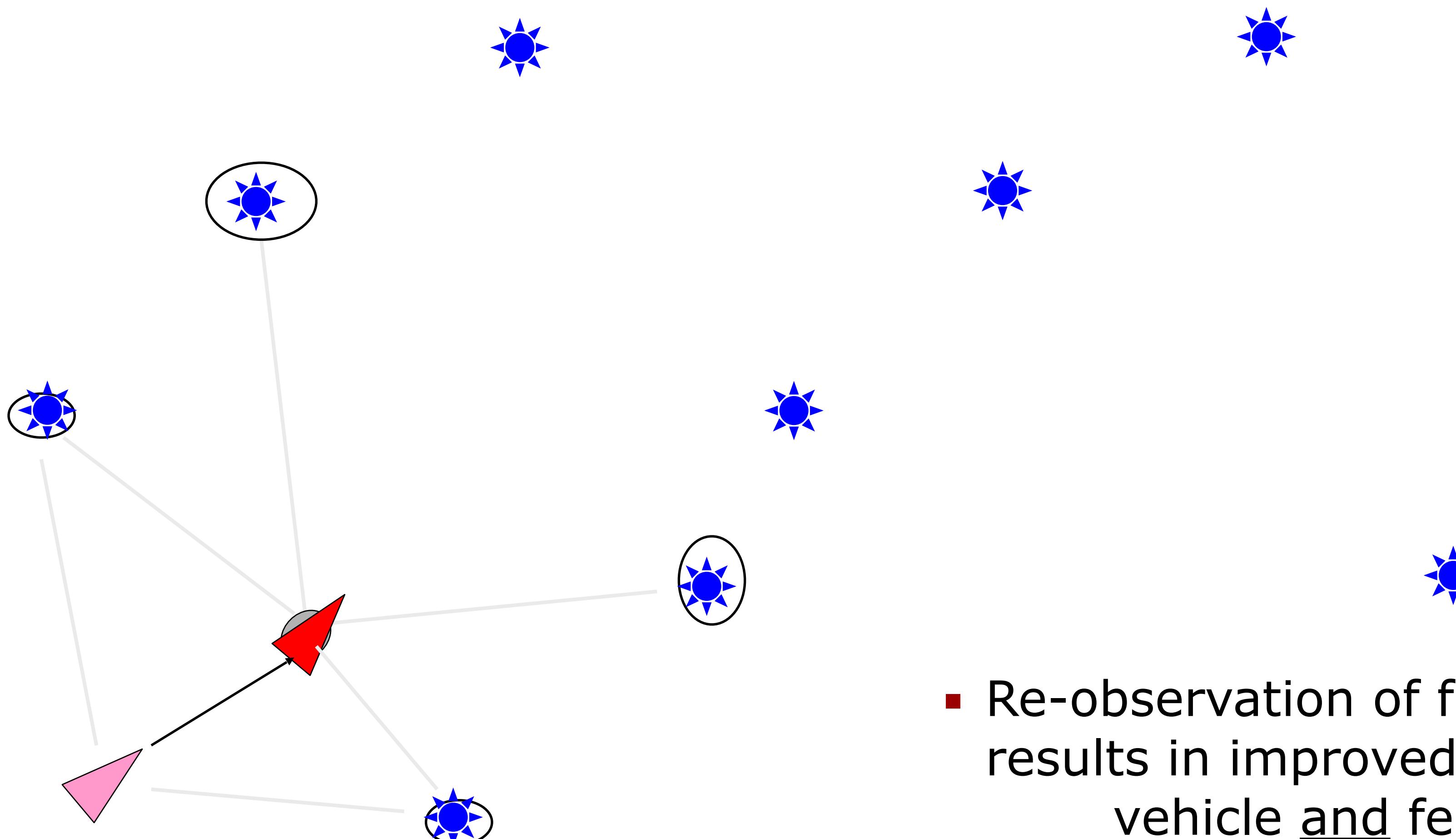
Courtesy J. Leonard

# Illustration of SLAM with Landmarks



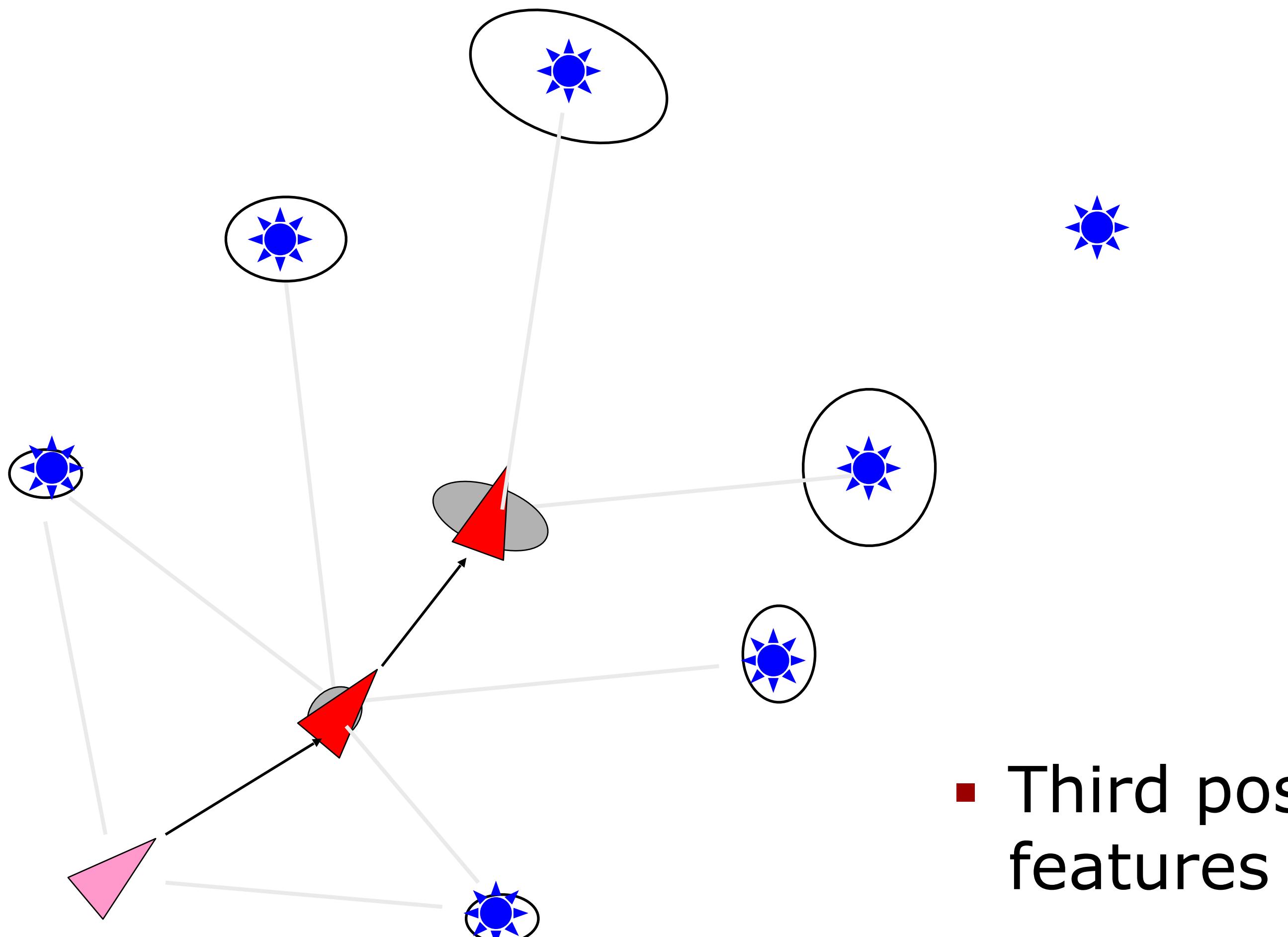
Courtesy J. Leonard

# Illustration of SLAM with Landmarks



Courtesy J. Leonard

# Illustration of SLAM with Landmarks

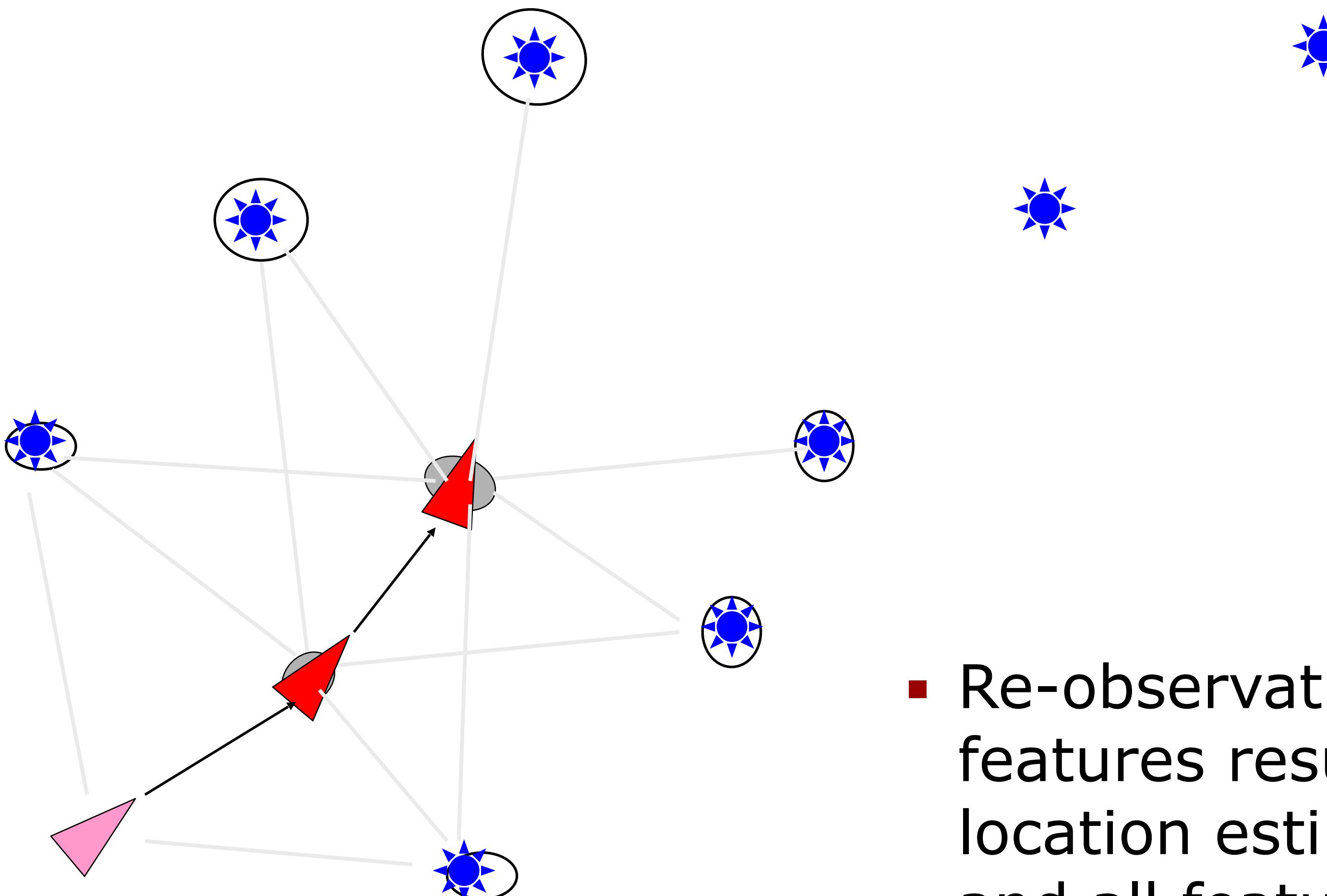


- Third position: two additional features added to map

Courtesy J. Leonard



# Illustration of SLAM with Landmarks

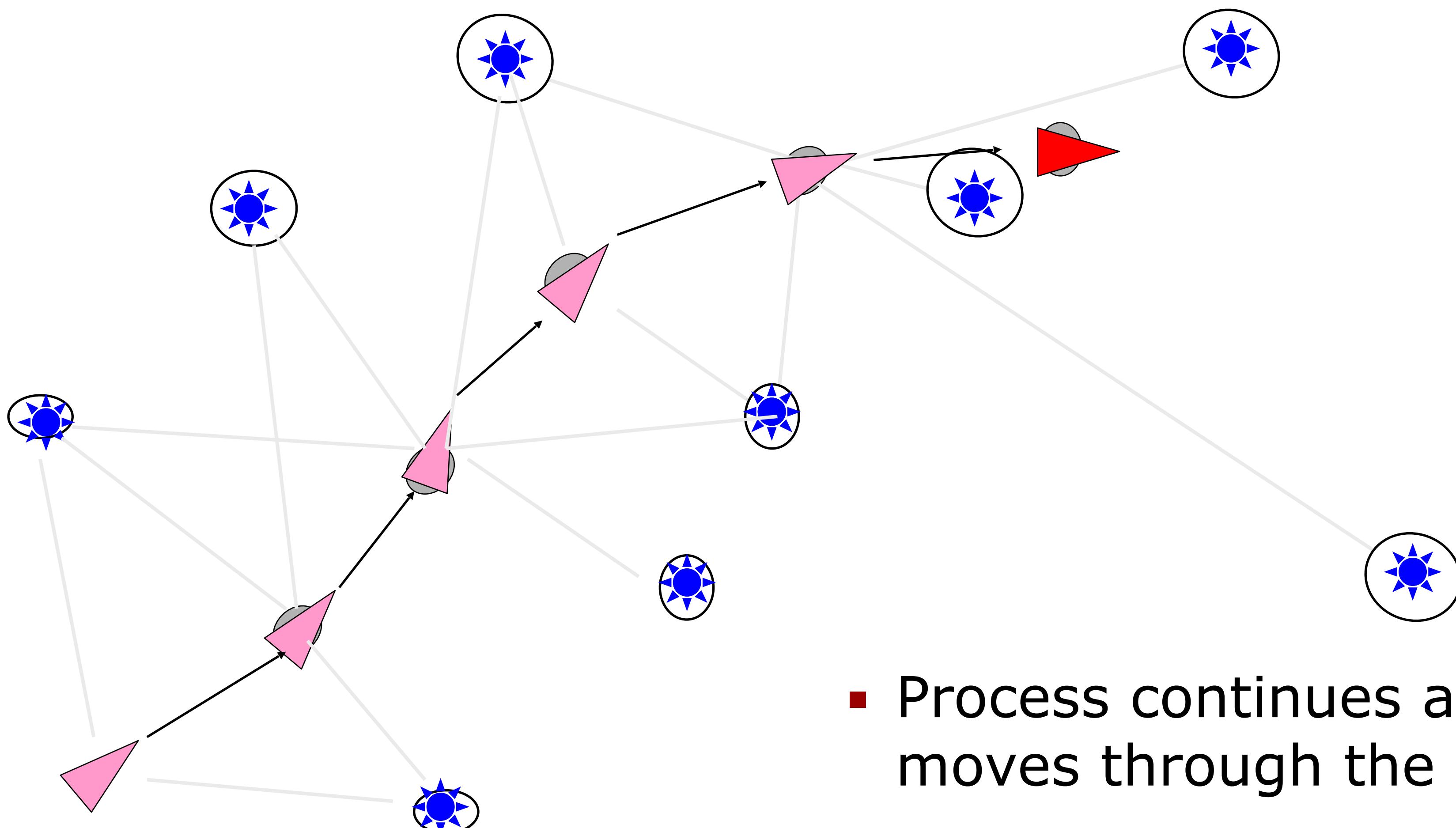


- Re-observation of first four features results in improved location estimates for vehicle and all features

Courtesy J. Leonard



# Illustration of SLAM with Landmarks

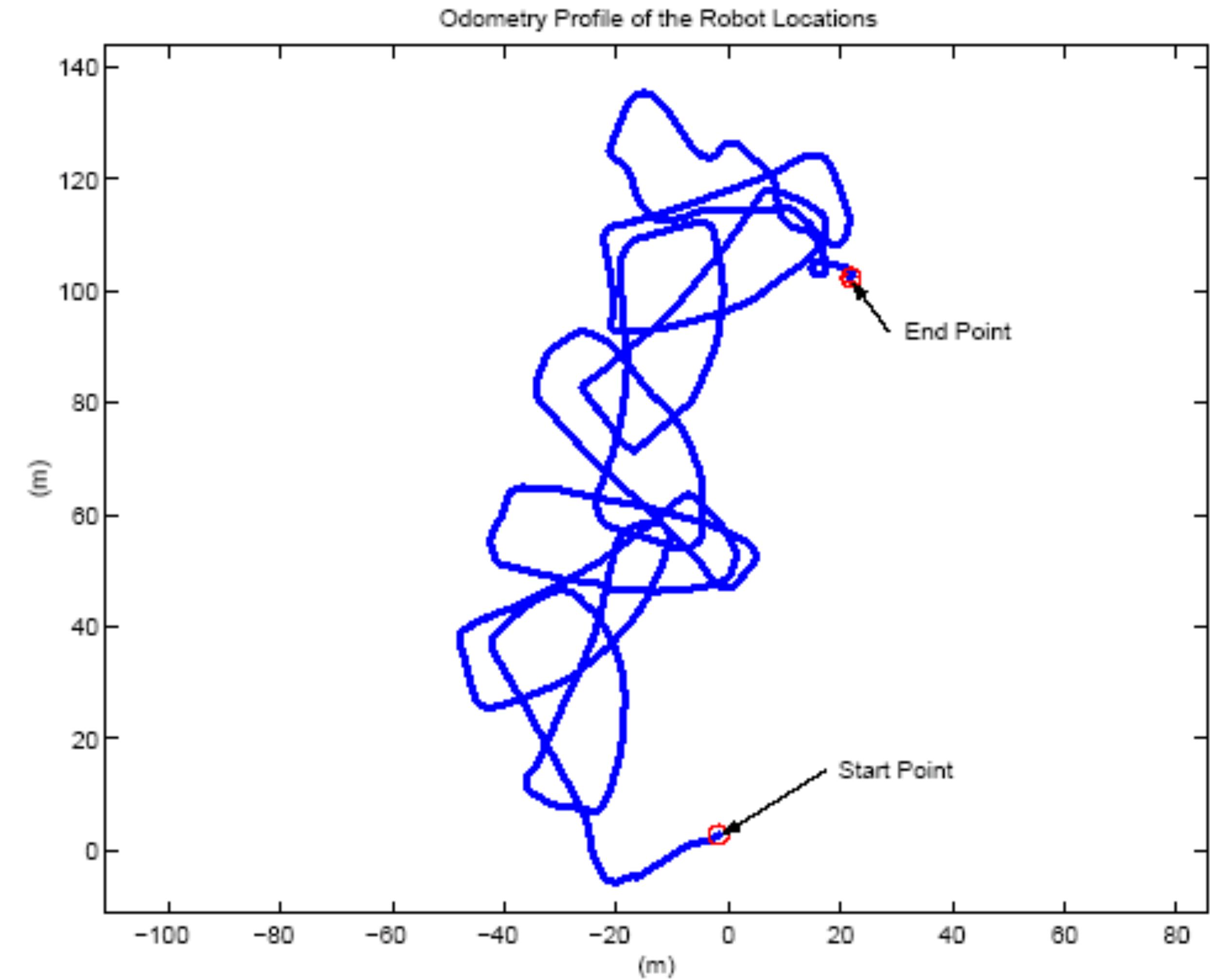


- Process continues as the vehicle moves through the environment

Courtesy J. Leonard



# SLAM Using Landmarks



Courtesy J. Leonard

# Test Environment (Point Landmarks)



Courtesy J. Leonard



# View from Vehicle



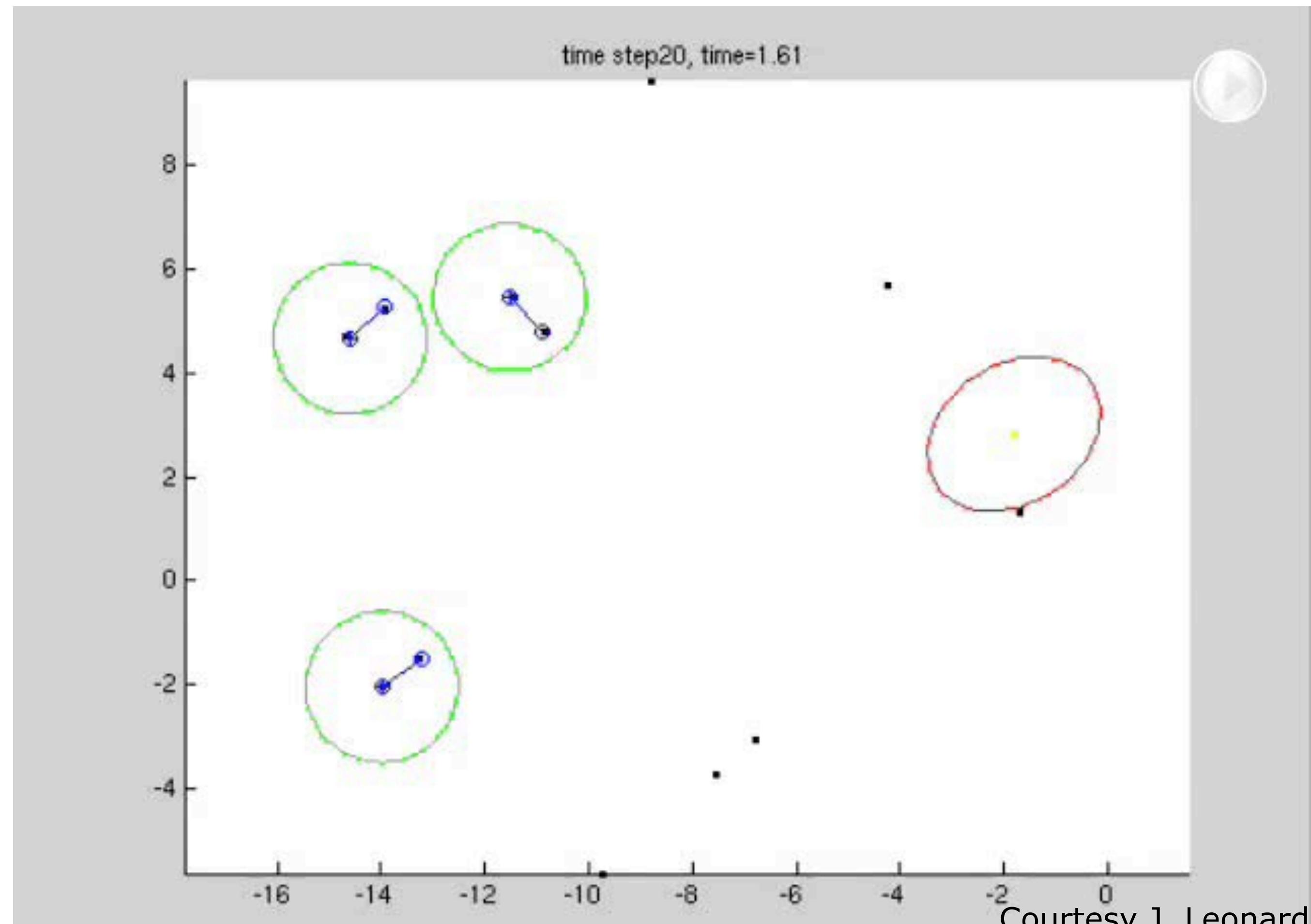
Courtesy J. Leonard

# SLAM Using Landmarks

1. Move
2. Sense
3. Associate measurements with known features
4. Update state estimates for robot and previously mapped features
5. Find new features from unassociated measurements
6. Initialize new features
7. Repeat



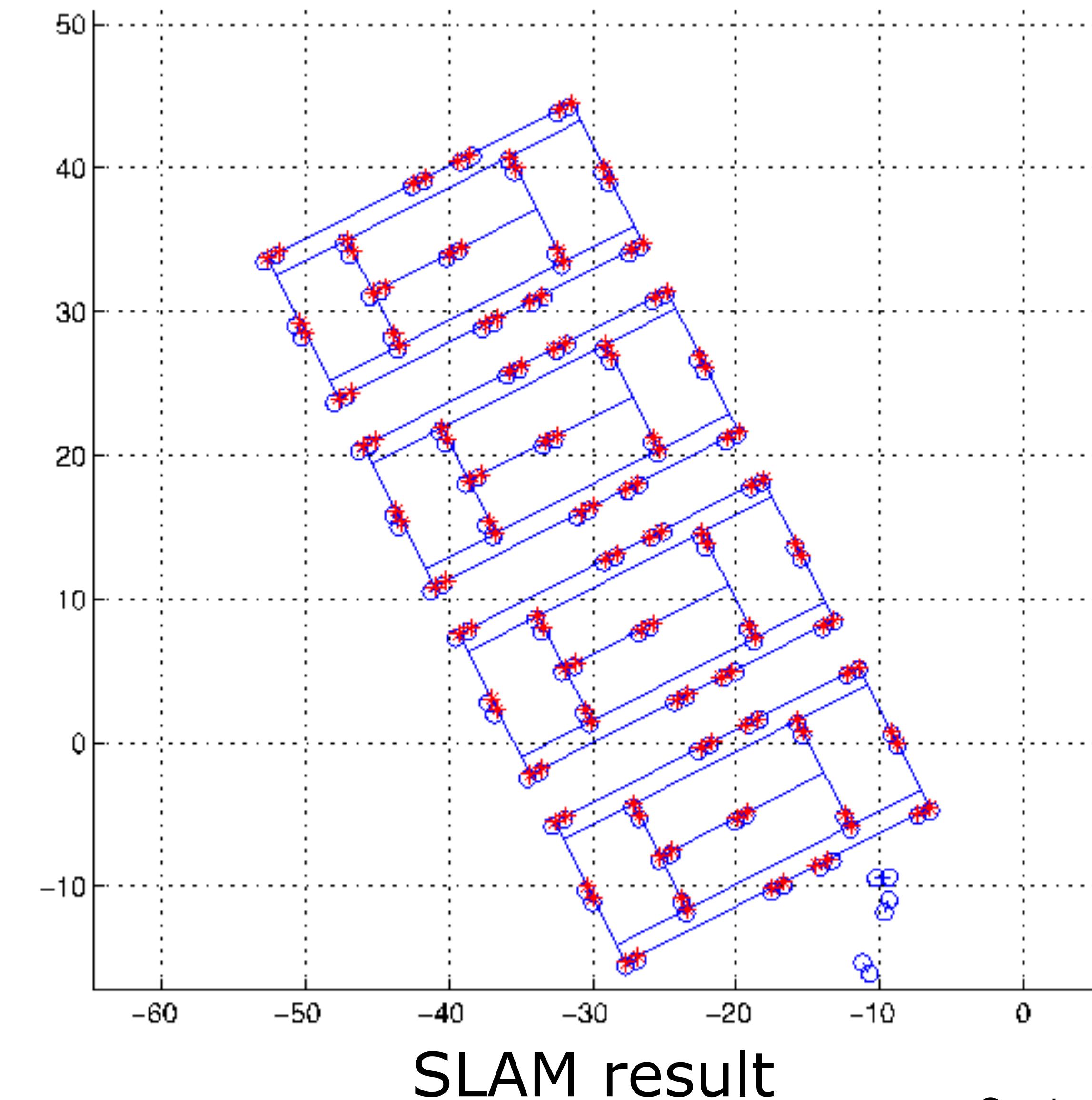
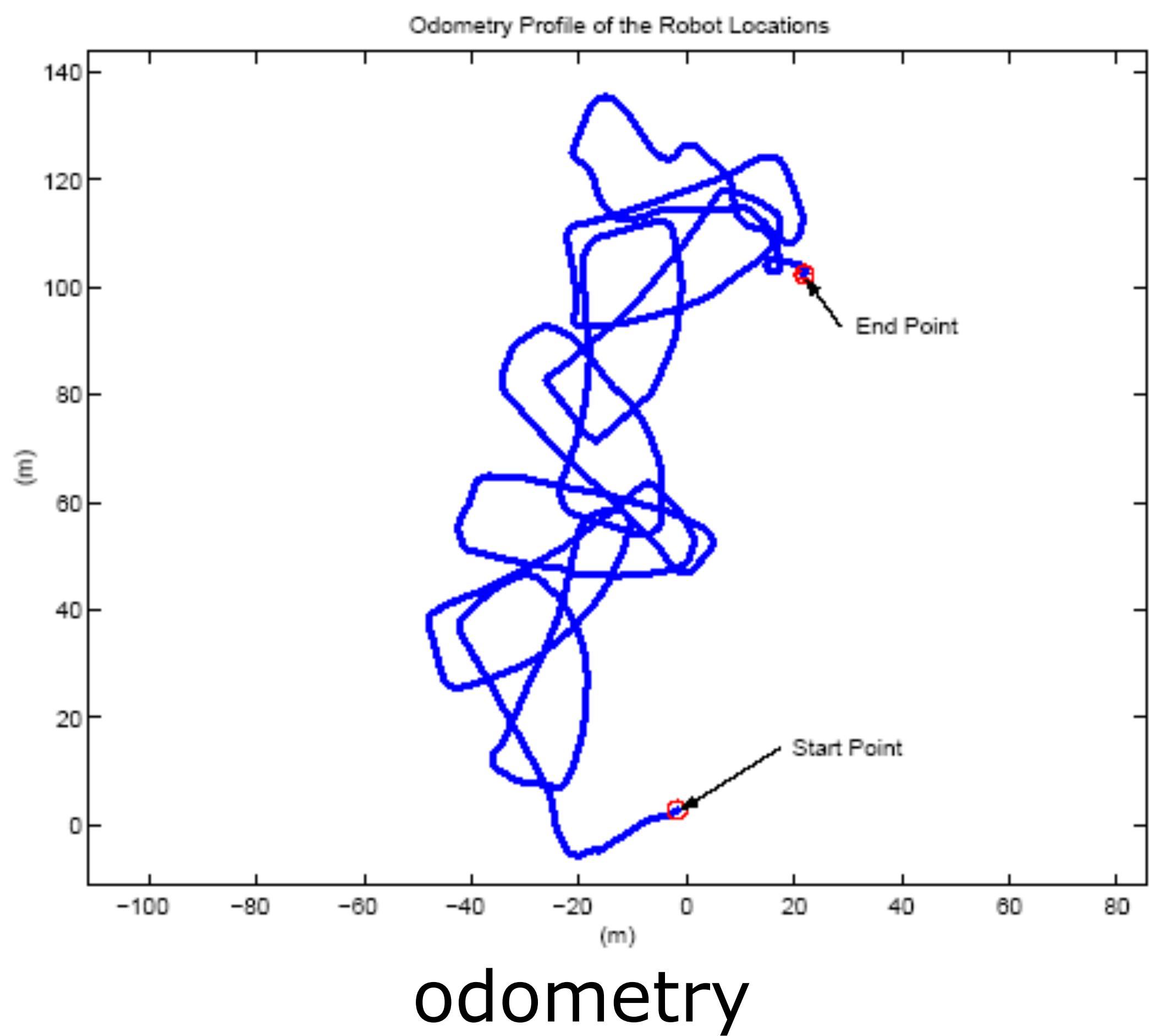
MIT Indoor Track



Courtesy J. Leonard



# Comparison with Ground Truth



Courtesy J. Leonard



# Simultaneous Localization and Mapping (SLAM)

- Building a map and locating the robot in the map at the same time
- Chicken-and-egg problem



# Definition of the SLAM Problem

## Given

- The robot's controls

$$u_{1:T} = \{u_1, u_2, u_3, \dots, u_T\}$$

- Observations

$$z_{1:T} = \{z_1, z_2, z_3, \dots, z_T\}$$

## Wanted

- Map of the environment

$m$

- Path of the robot

$$x_{0:T} = \{x_0, x_1, x_2, \dots, x_T\}$$

# Three Main Paradigms

Kalman  
filter

Graph-  
based

Particle  
filter

# EKF SLAM

- Application of the EKF to SLAM
- Estimate robot's pose and locations of landmarks in the environment
- Assumption: known correspondences **and ??**
- State space (for the 2D plane) is

$$x_t = \left( \underbrace{\begin{array}{c} x, y, \theta \\ \text{robot's pose} \end{array}}_{\text{robot's pose}} , \underbrace{\begin{array}{c} m_{1,x}, m_{1,y} \\ \text{landmark 1} \end{array}}_{\text{landmark 1}} , \dots , \underbrace{\begin{array}{c} m_{n,x}, m_{n,y} \\ \text{landmark n} \end{array}}_{\text{landmark n}} \right)^T$$

# EKF SLAM: State Representation

- Map with  $n$  landmarks:  $(3+2n)$ -dimensional Gaussian
- Belief is represented by

$$\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix} \underbrace{\left( \begin{array}{ccc} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} \\ \hline \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} \\ \vdots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} \end{array} \right)}_{\mu} \underbrace{\left( \begin{array}{ccccc} \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{m_{n,x}} & \sigma_{m_{n,y}} \\ \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \hline \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \cdots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \cdots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \cdots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \cdots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{array} \right)}_{\Sigma}$$

# EKF SLAM: State Representation

- More compactly

$$\underbrace{\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \dots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \dots & \Sigma_{m_1 m_n} \\ \vdots & \ddots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \dots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

# EKF SLAM: State Representation

- Even more compactly (note:  $x_R \rightarrow x$ )

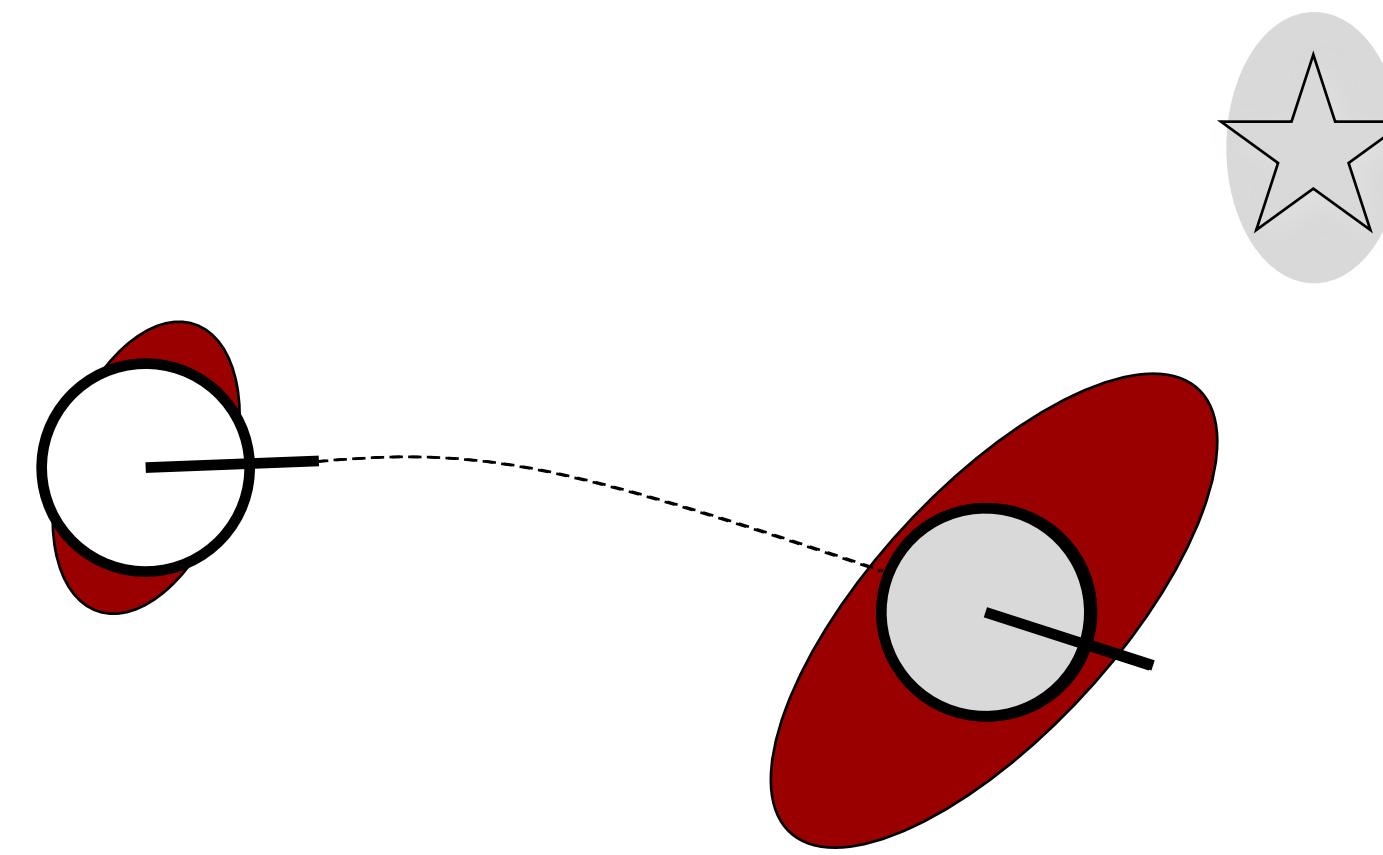
$$\begin{pmatrix} x \\ m \end{pmatrix} \quad \begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix}$$

$\mu$                                $\Sigma$

# EKF SLAM: Filter Cycle

1. State prediction
2. Measurement prediction
3. Measurement
4. Data association
5. Update

# EKF SLAM: State Prediction

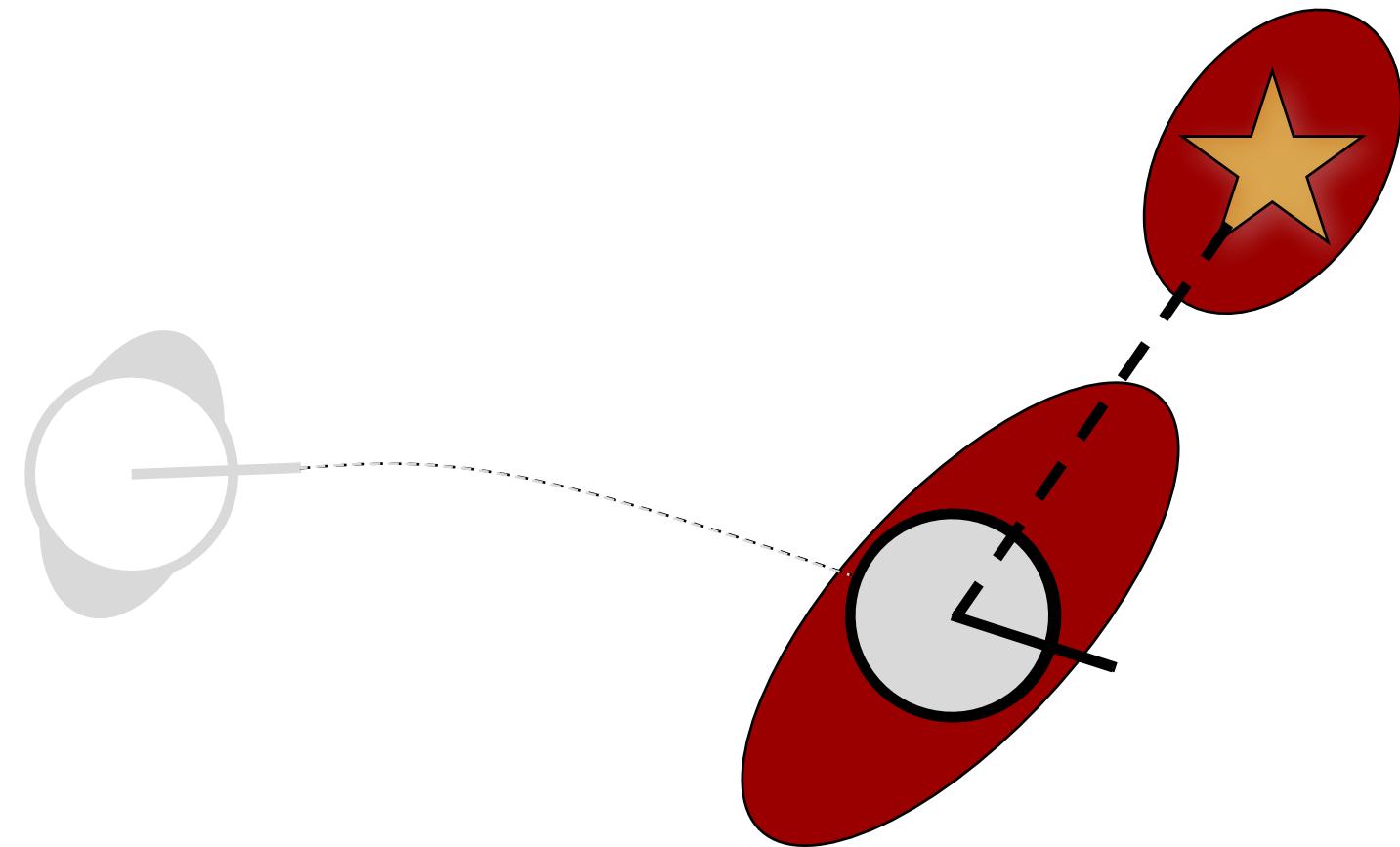


$$\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix} \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss



# EKF SLAM: Measurement Prediction

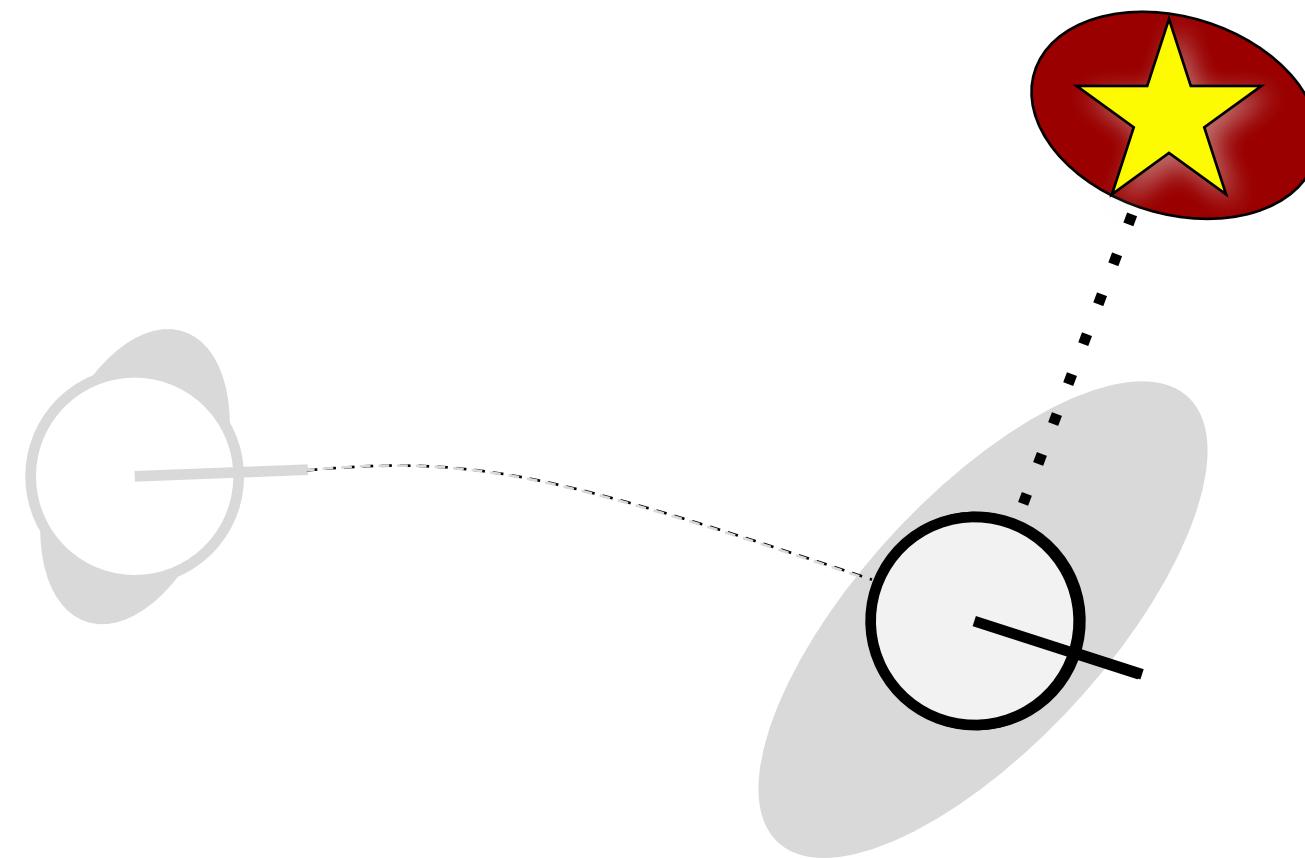


$$\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix} \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss



# EKF SLAM: Obtained Measurement

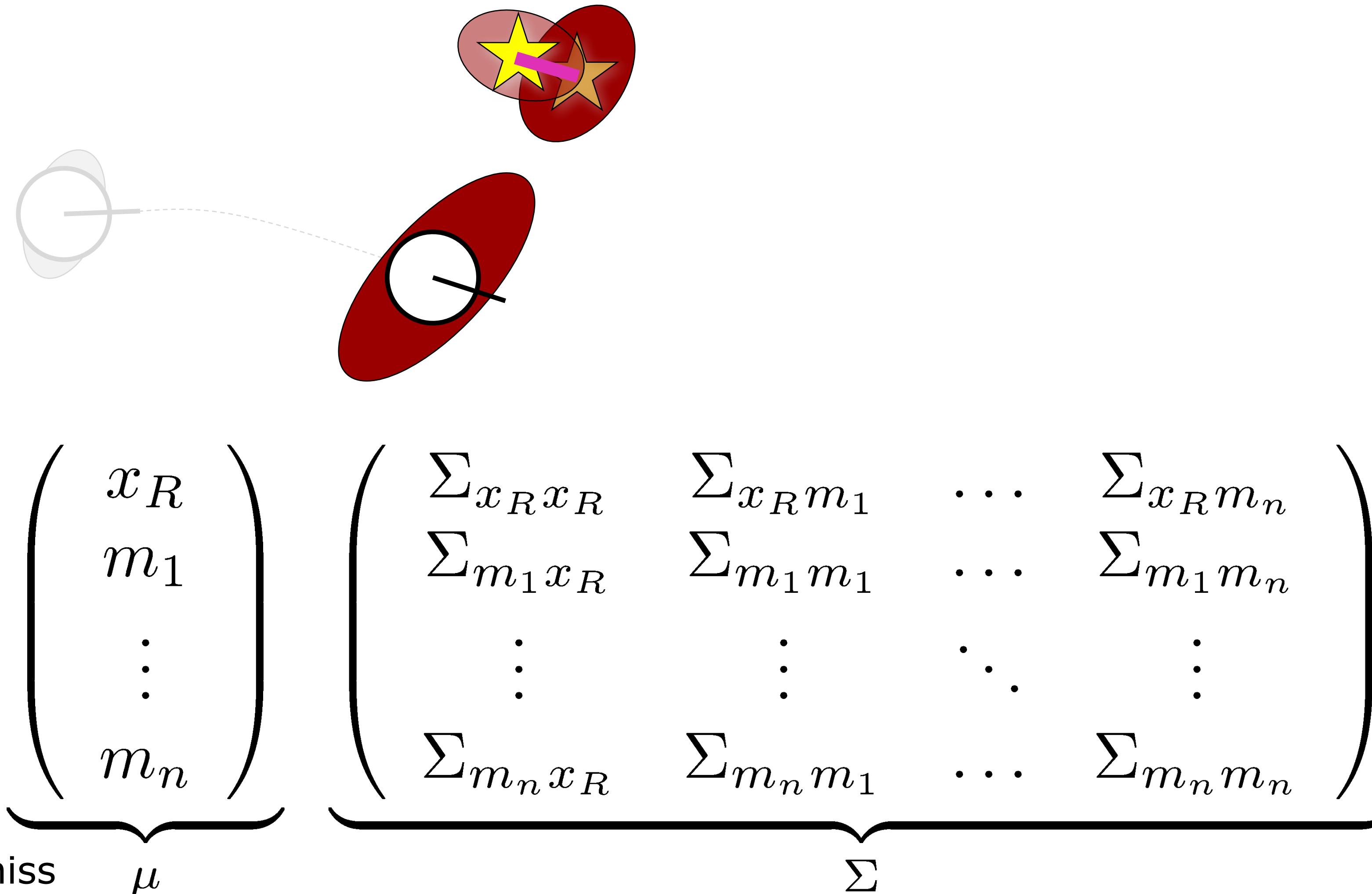


$$\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix} \underbrace{\begin{pmatrix} \Sigma_{x_R x_R} & \Sigma_{x_R m_1} & \cdots & \Sigma_{x_R m_n} \\ \Sigma_{m_1 x_R} & \Sigma_{m_1 m_1} & \cdots & \Sigma_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m_n x_R} & \Sigma_{m_n m_1} & \cdots & \Sigma_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

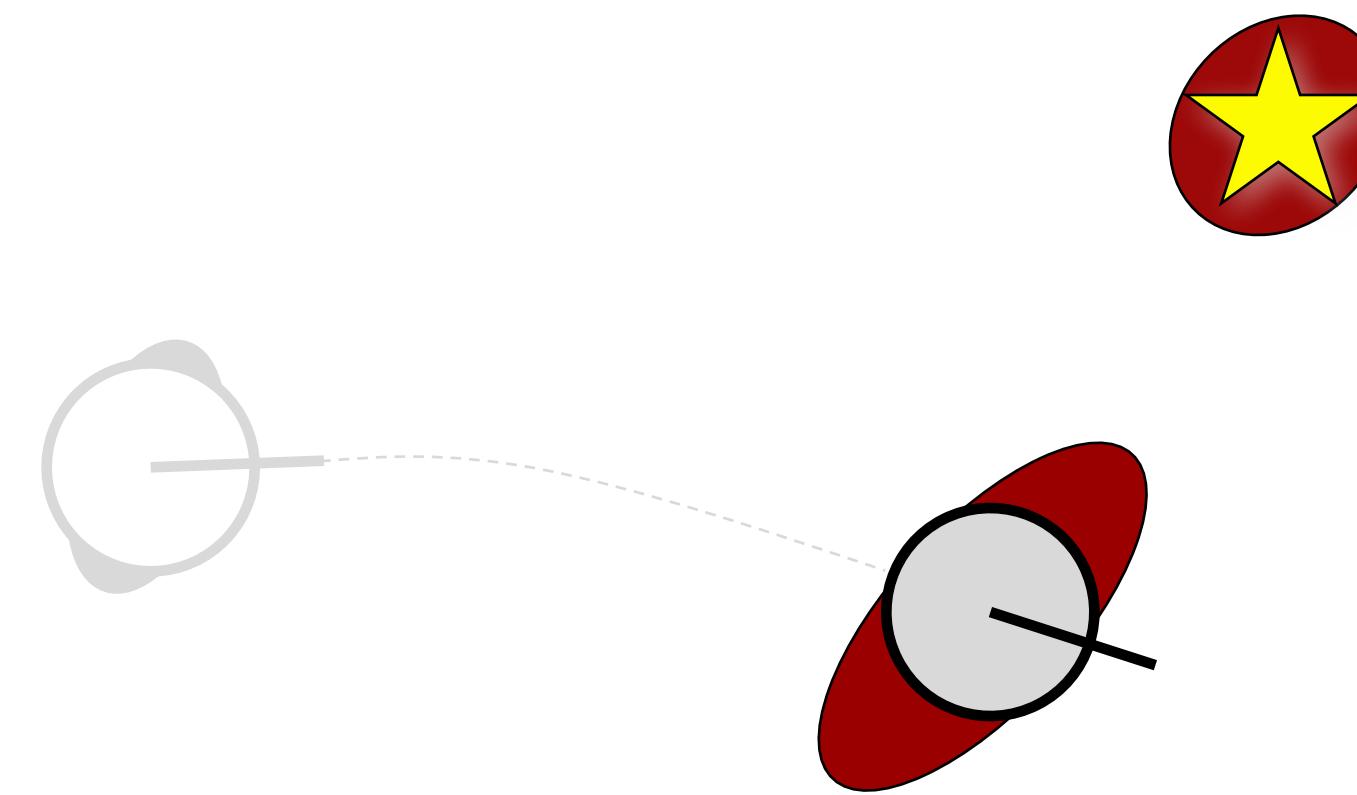
Courtesy: Cyrill Stachniss



# EKF SLAM: Data Association and Difference Between $h(x)$ and $z$



# EKF SLAM: Update Step

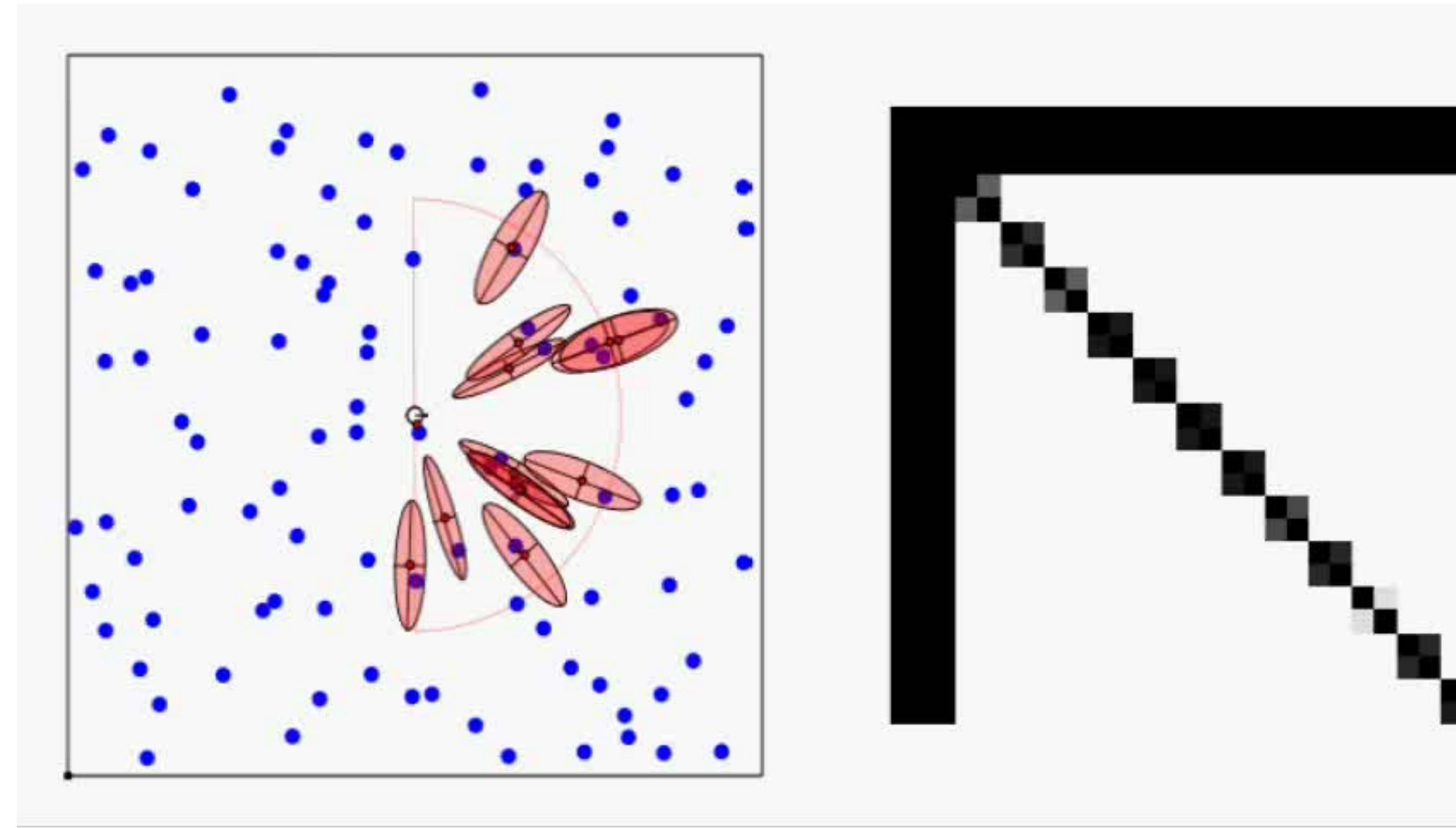


$$\begin{pmatrix} x_R \\ m_1 \\ \vdots \\ m_n \end{pmatrix} \quad \underbrace{\begin{pmatrix} \sum_{x_R x_R} & \sum_{x_R m_1} & \cdots & \sum_{x_R m_n} \\ \sum_{m_1 x_R} & \sum_{m_1 m_1} & \cdots & \sum_{m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{m_n x_R} & \sum_{m_n m_1} & \cdots & \sum_{m_n m_n} \end{pmatrix}}_{\Sigma}$$

Courtesy: Cyrill Stachniss

$\mu$

# EKF SLAM Correlations



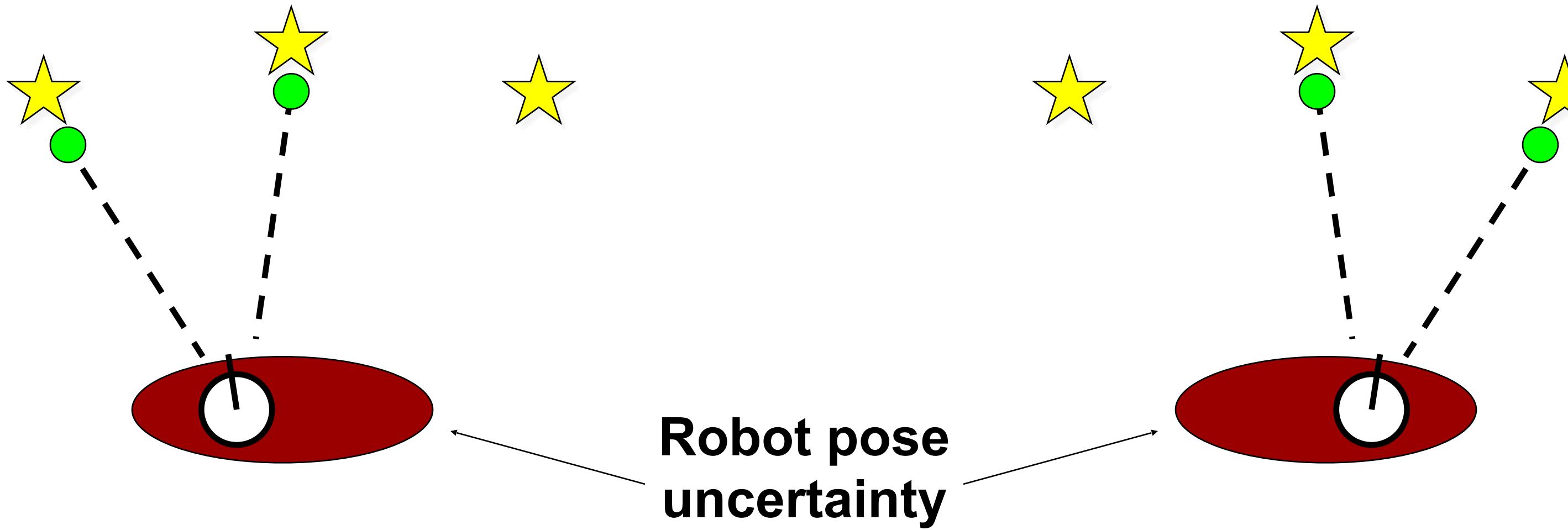
Blue path = true path   **Red path** = estimated path   **Black path** = odometry

- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] ...
- **Single hypothesis data association**

Courtesy: M. Montemerlo



# Data Association in SLAM

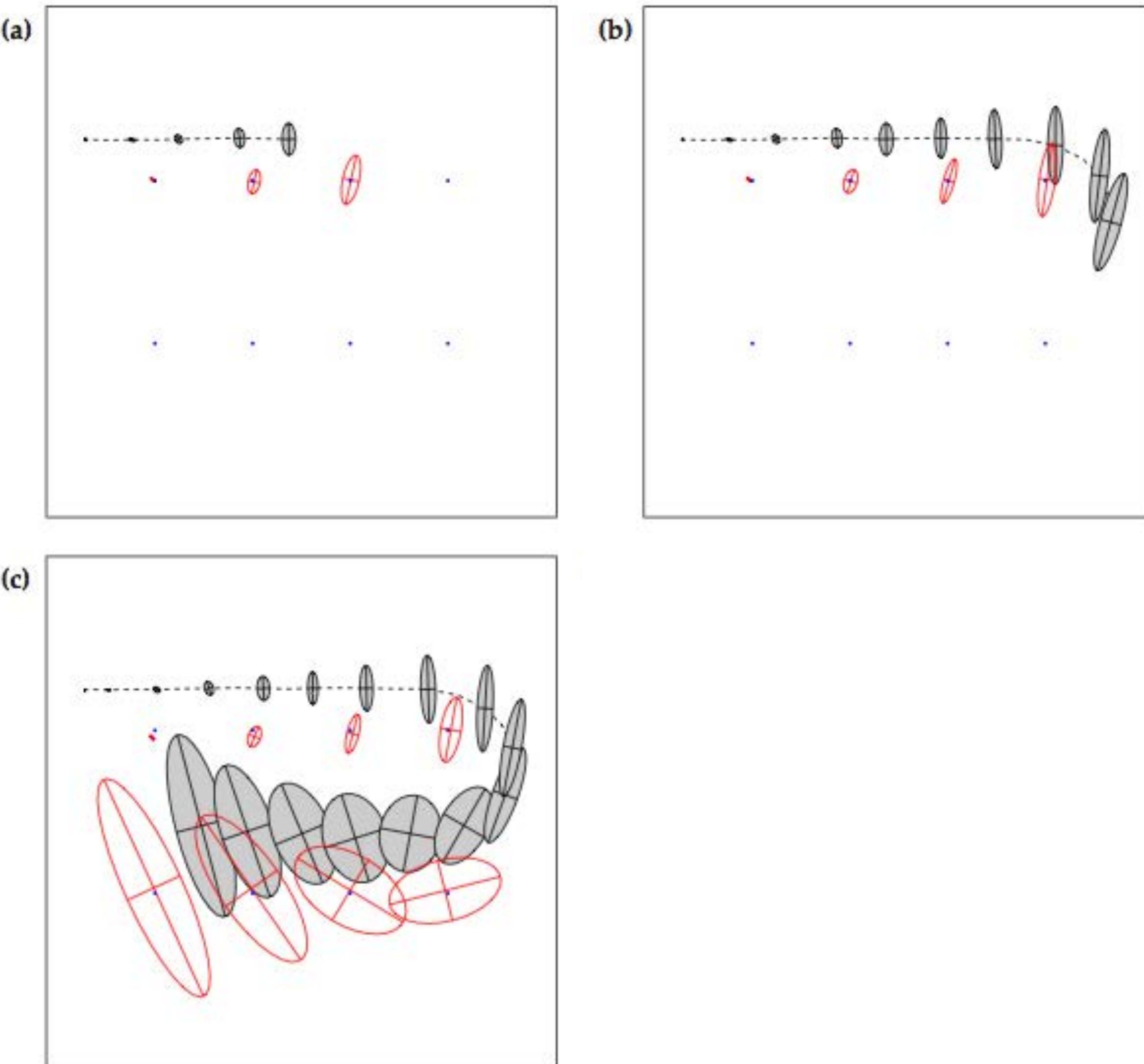


- In the real world, the mapping between observations and landmarks is **unknown**
- Picking wrong data associations can have **catastrophic** consequences
  - EKF SLAM is brittle in this regard
  - Pose error correlates data associations

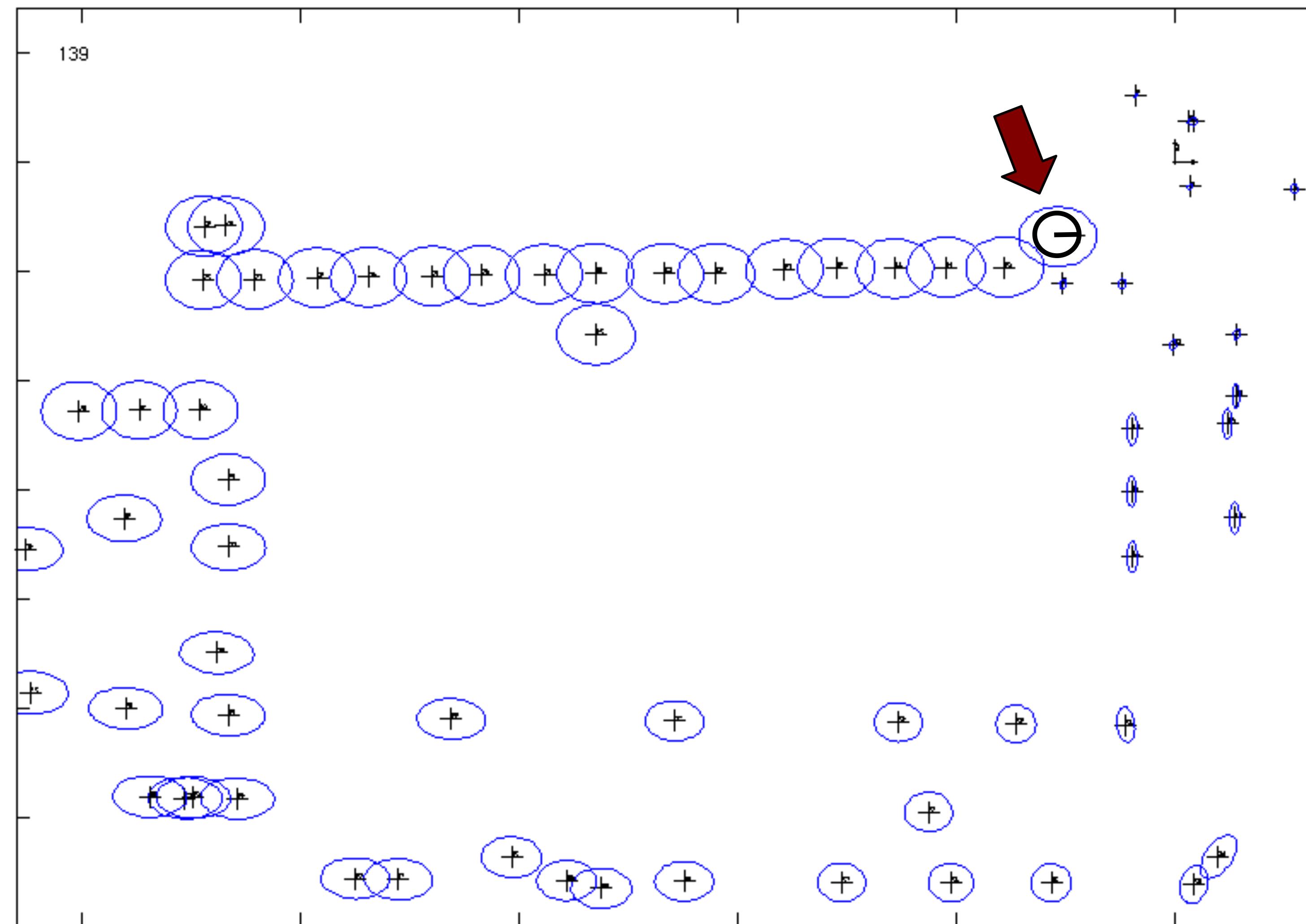
# Loop-Closing

- Loop-closing means recognizing an already mapped area
- Data association under
  - high ambiguity
  - possible environment symmetries
- Uncertainties **collapse** after a loop-closure  
(whether the closure was correct or not)

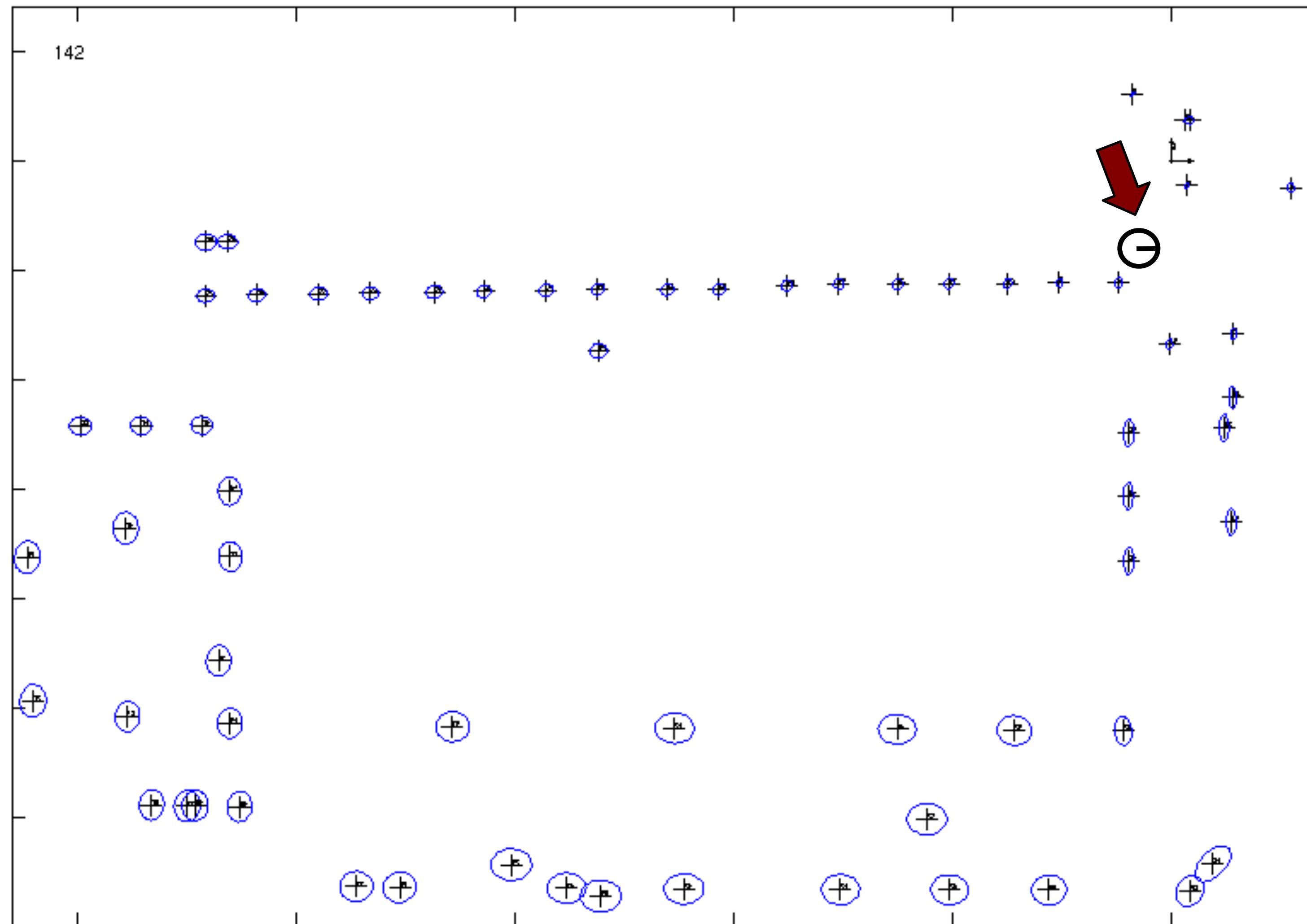
# Online SLAM Example



# Before the Loop-Closure



# After the Loop-Closure



# Example: Victoria Park Dataset



Courtesy: E. Nebot

# Example: Victoria Park Dataset



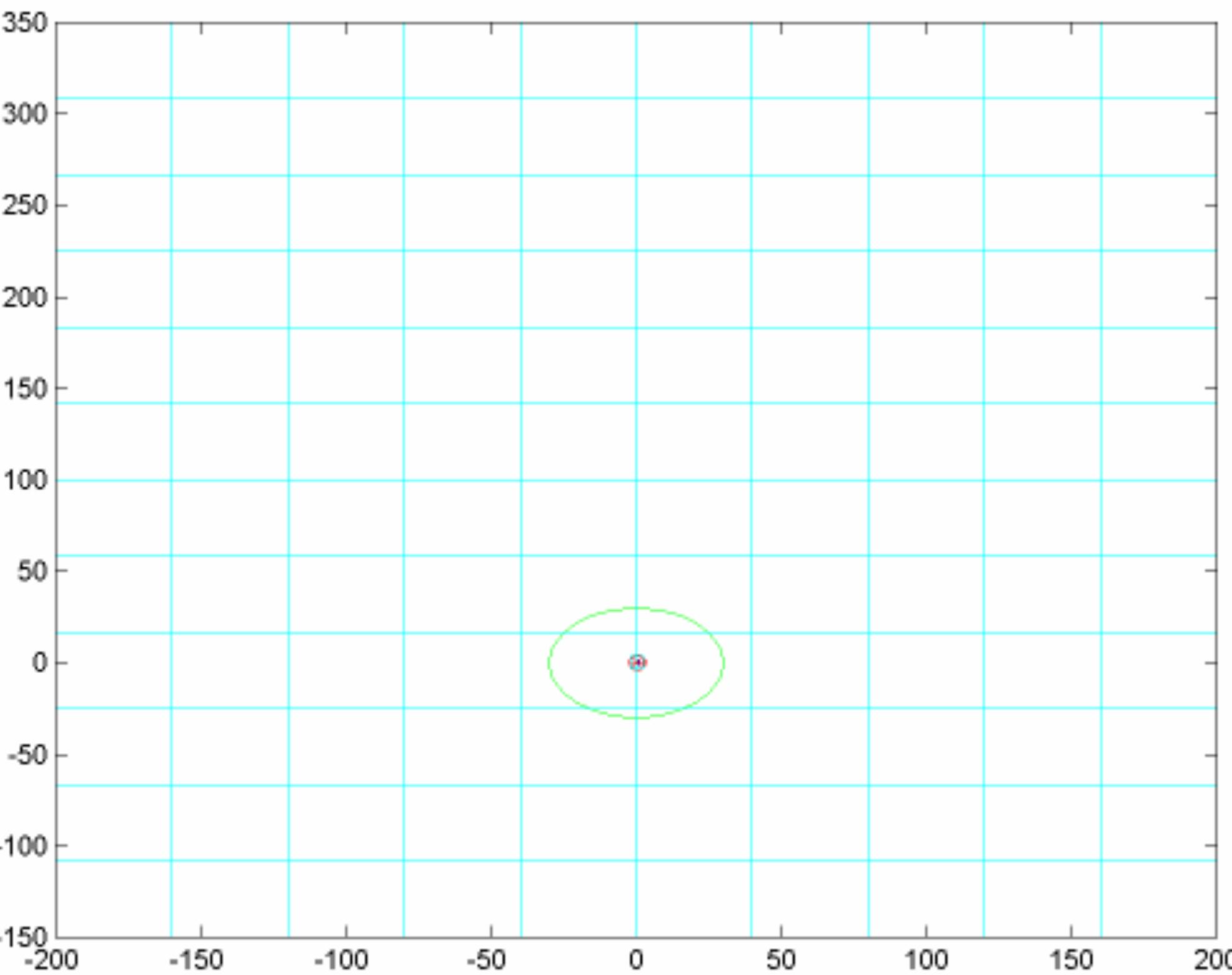
Courtesy: E. Nebot

# Victoria Park: Data Acquisition



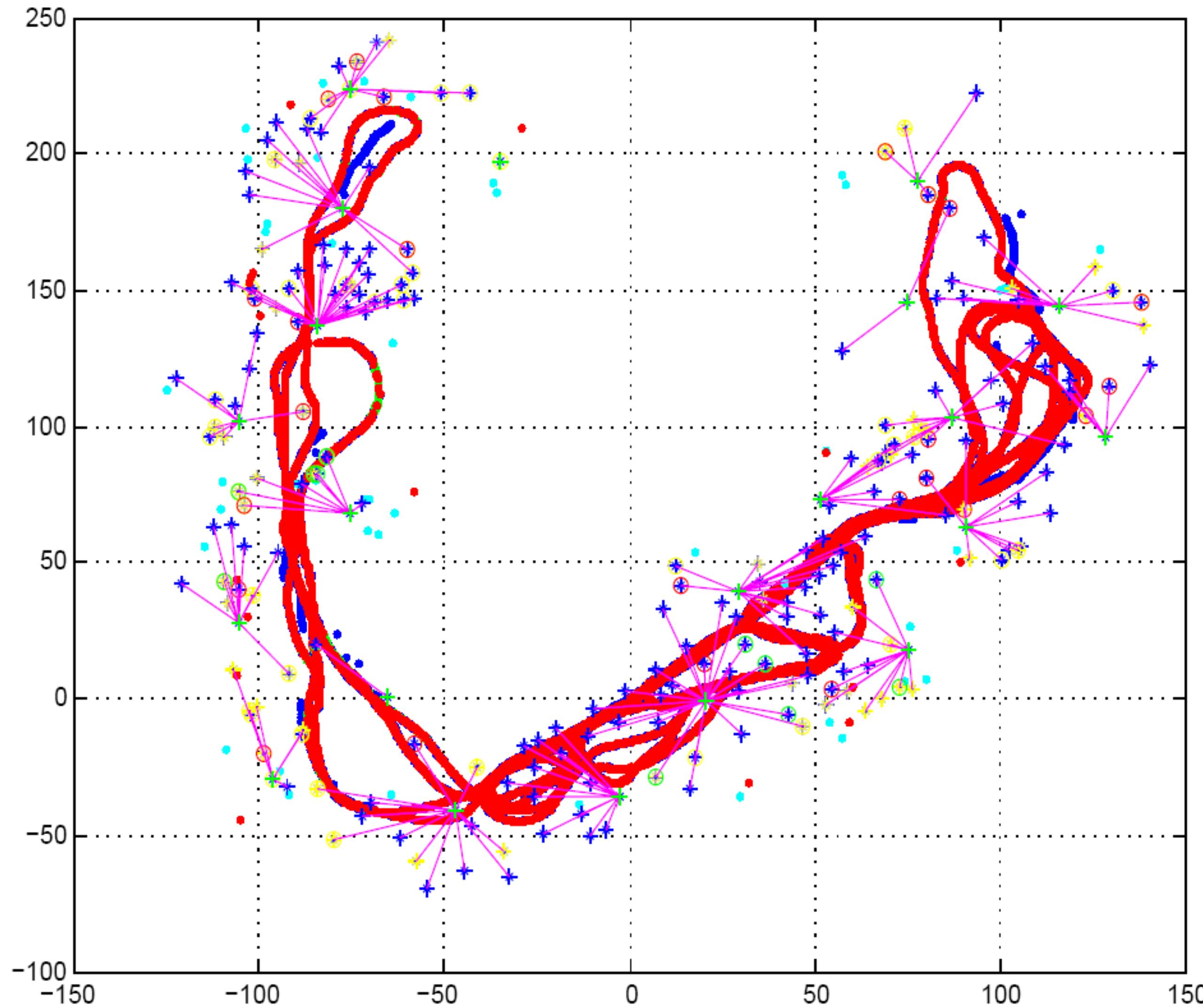
Courtesy: E. Nebot

# Victoria Park: EKF Estimate



Courtesy: E. Nebot

# Victoria Park: EKF Estimate



# Victoria Park: Landmarks



Courtesy: E. Nebot

# Andrew Davison: MonoSLAM



# EKF SLAM Summary

- Quadratic in the number of landmarks:  
 $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.

# EKF Algorithm

1. **Extended\_Kalman\_filter**( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):

2. Prediction:

$$3. \bar{\mu}_t = g(u_t, \mu_{t-1}) \quad \longleftrightarrow \quad \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$$

$$4. \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t \quad \longleftrightarrow \quad \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

5. Correction:

$$6. K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} \quad \longleftrightarrow \quad K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$7. \mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t)) \quad \longleftrightarrow \quad \mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$

$$8. \Sigma_t = (I - K_t H_t) \bar{\Sigma}_t \quad \longleftrightarrow \quad \Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

9. Return  $\mu_t, \Sigma_t$

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t} \quad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

# Literature

## EKF SLAM

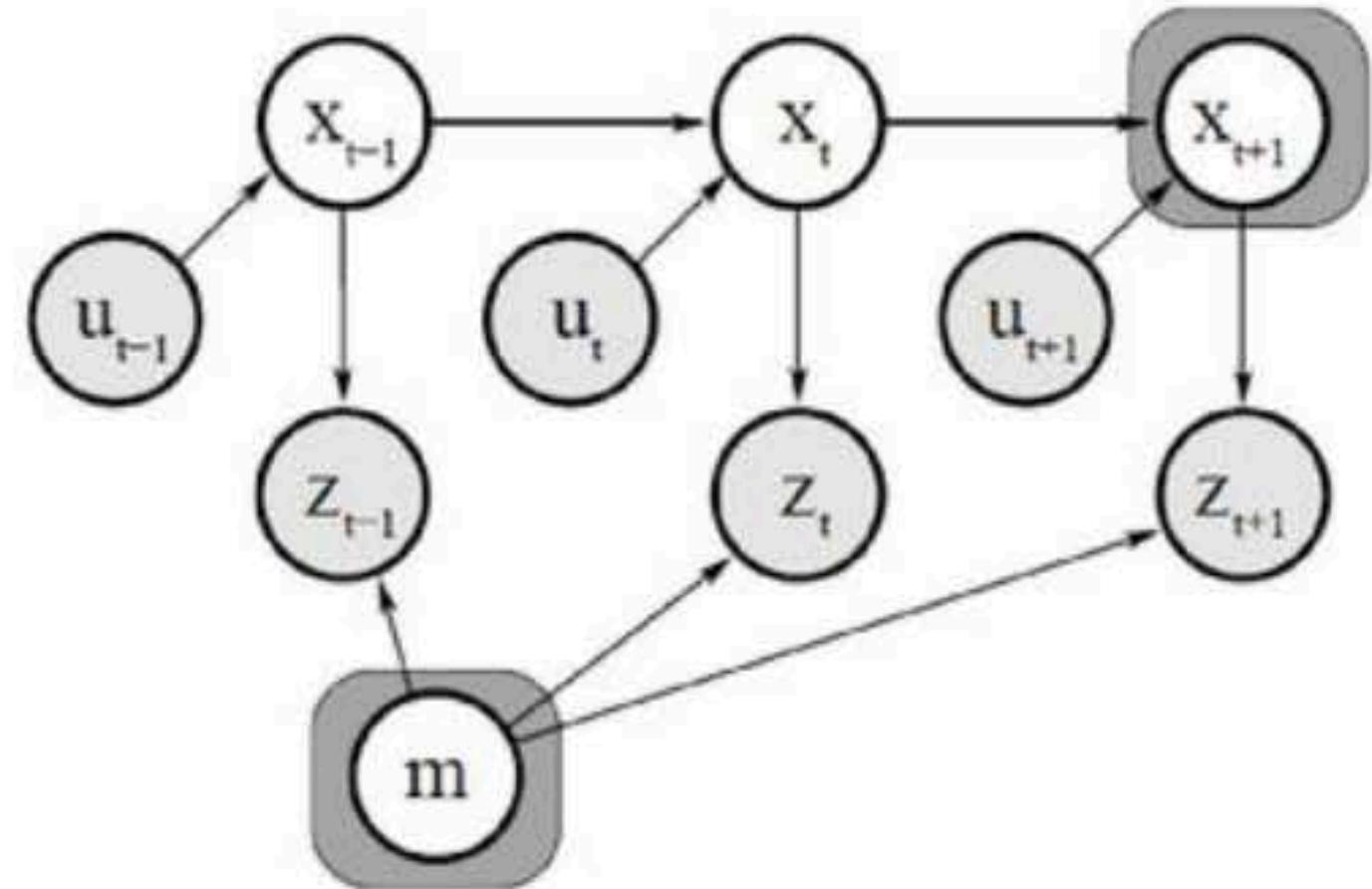
- “Probabilistic Robotics”, Chapter 10
- Smith, Self, & Cheeseman: “Estimating Uncertain Spatial Relationships in Robotics”
- Dissanayake et al.: “A Solution to the Simultaneous Localization and Map Building (SLAM) Problem”
- Durrant-Whyte & Bailey: “SLAM Part 1” and “SLAM Part 2” tutorials



# Online vs Full SLAM

Online SLAM problem

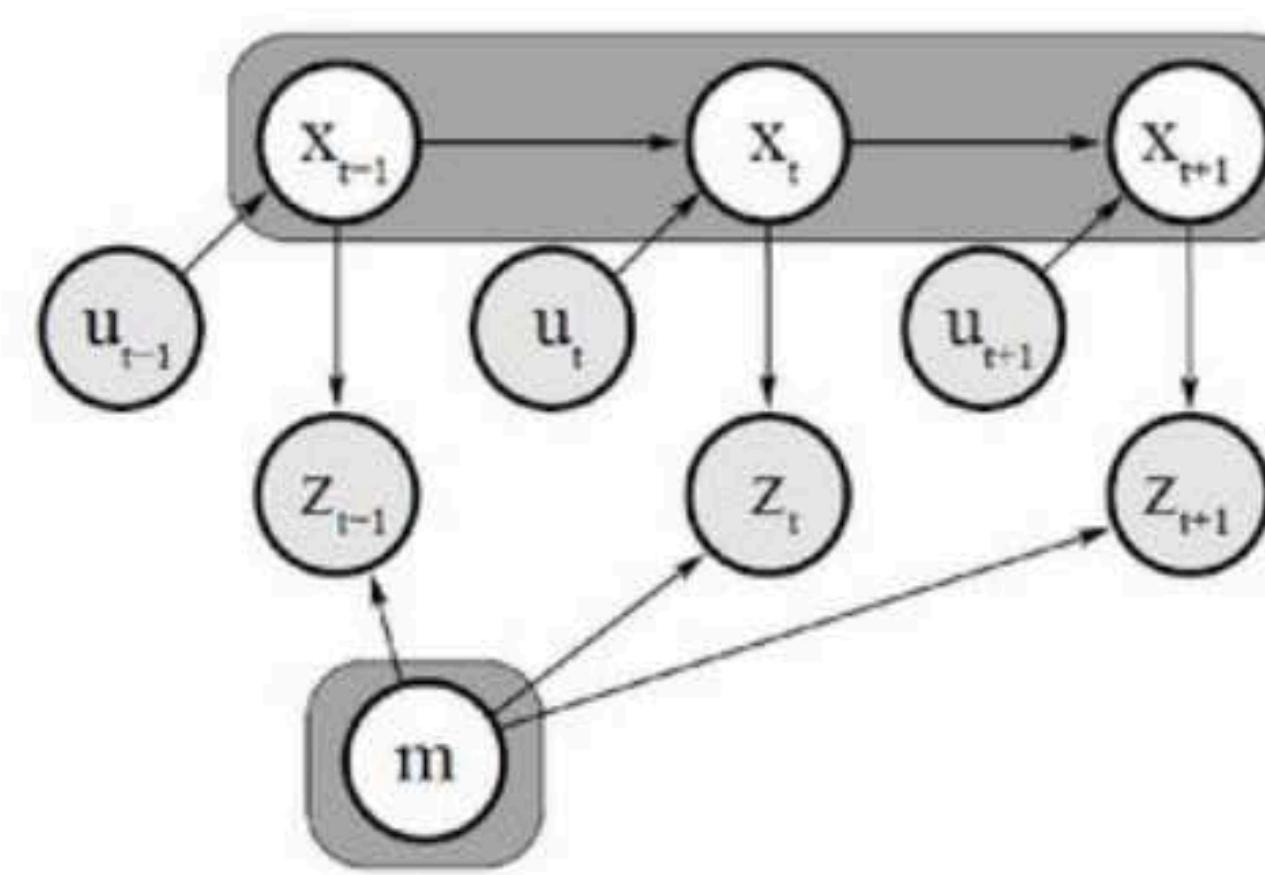
$$p(x_t, m | z_{1:t}, u_{1:t})$$



Estimate map  $m$  and current position  $x_t$

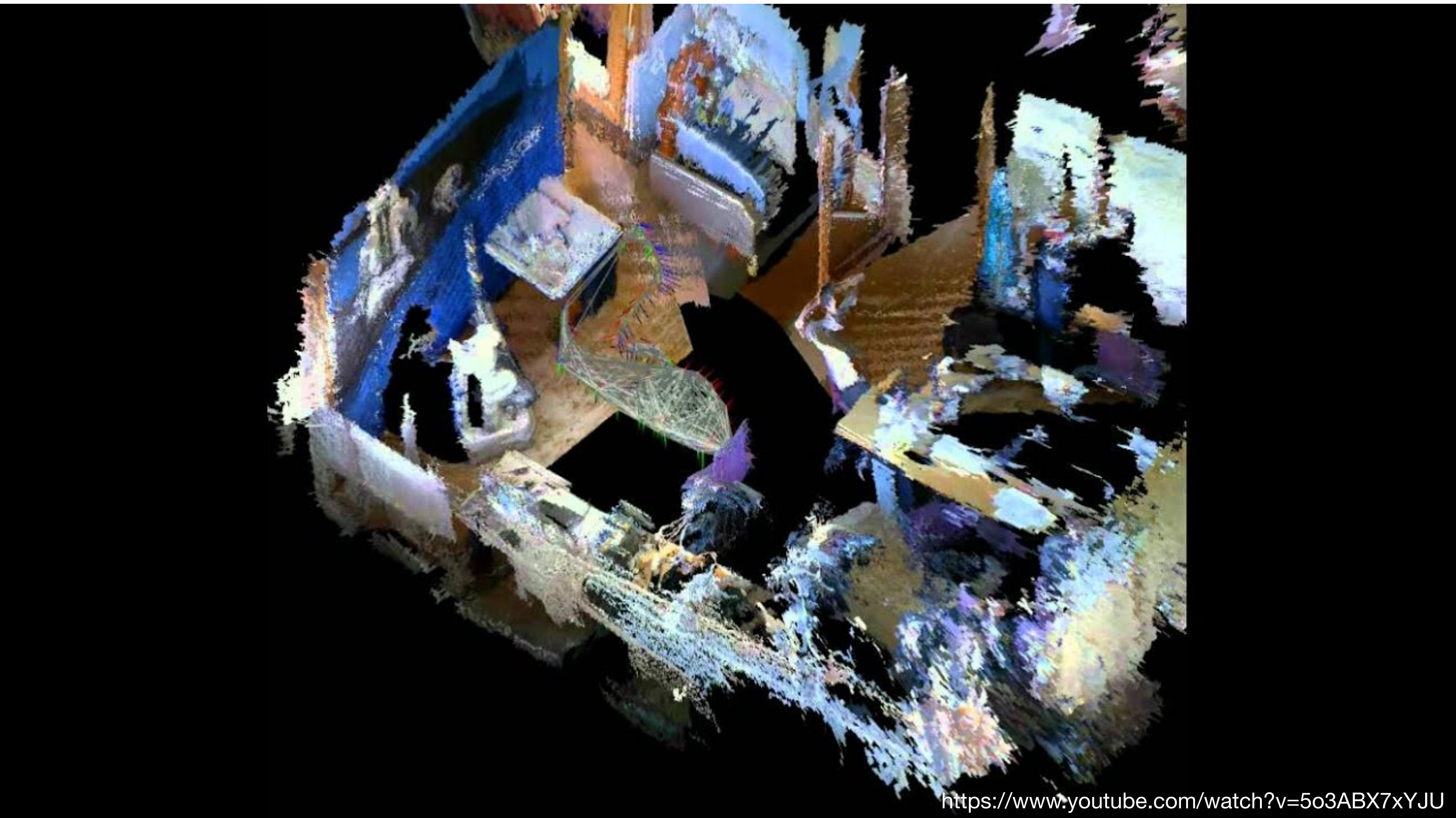
Full SLAM problem

$$p(x_{1:t}, m | z_{1:t}, u_{1:t})$$



Estimate map  $m$  and driven path  $x_{1:t}$

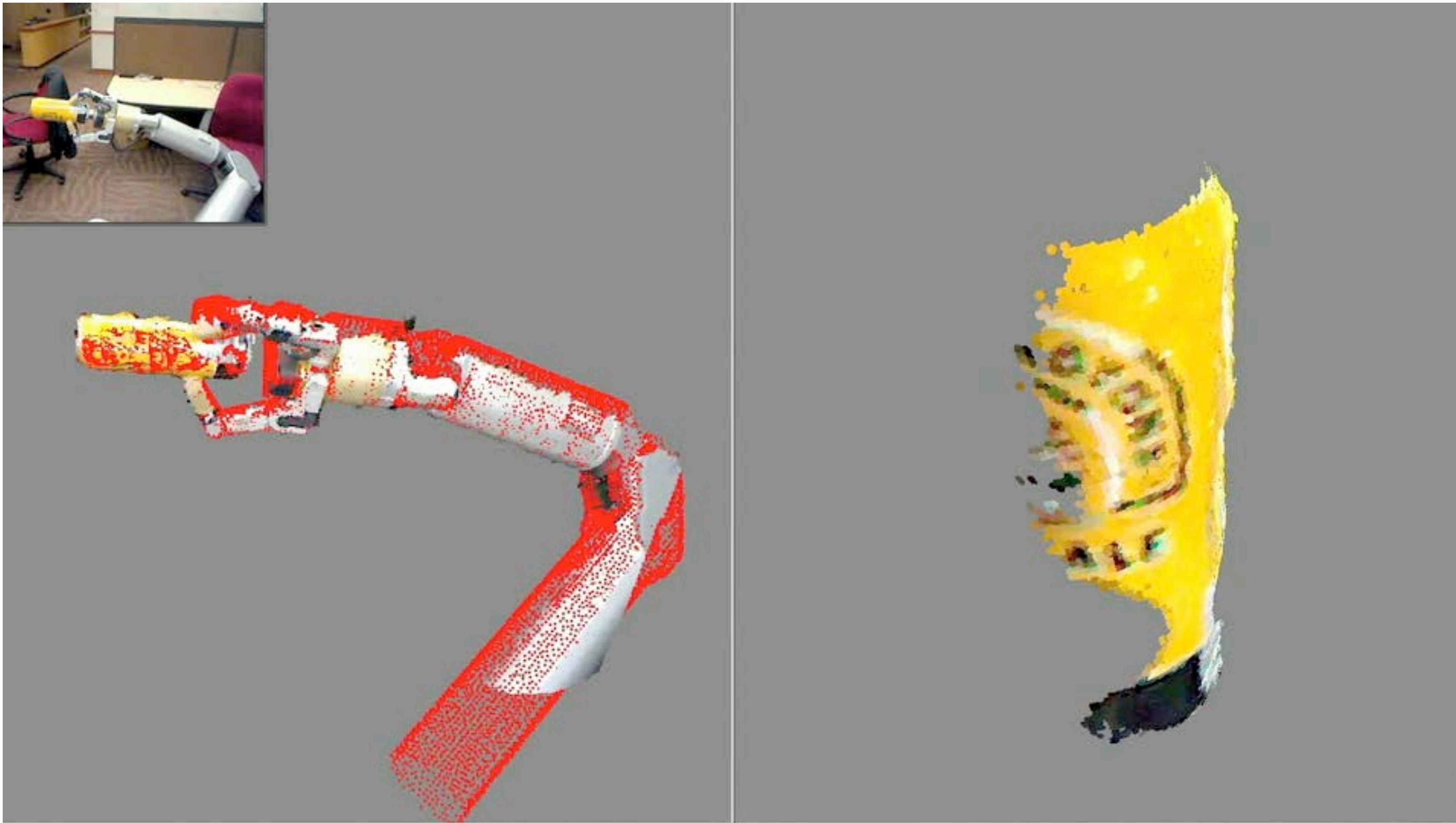
# RGBD Mapping



<https://www.youtube.com/watch?v=5o3ABX7xYJU>



# Active Object Modeling: Joint Tracking and Modeling



Robotic In-Hand 3D Object Modeling, [UW Robotics and State Estimation Lab](#) Michael Krainin, Peter Henry, Xiaofeng Ren, Dieter Fox, and Brian Curless



# Thats the end of the course lectures!



## Representations

- 1. Transformations
- 2. Rotations & Quaternions



## Planning

- 1. Path Planning
- 2. Bugs
- 3. Configuration space
- 4. Sampling based planners
- 5. Potential Fields
- 6. Collision Detection



## Motion Control



## Mobile Robotics

- 1. Probabilistic Robotics
- 2. Sensor and Motion models
- 3. Kalman Filter, Particle Filters
- 4. Localization
- 5. Mapping
- 6. SLAM



# Final Project Proposals and Guest Lectures

04/16	Open Ended Final Project Pitches
04/21	Open Ended Final Project Pitches
04/23	Open Ended Final Project Pitches
04/28	Guest Lecture - Adam Imdieke (PhD student)
04/30	Guest Lecture - Xun Tu (PhD student)
05/05	<b>Poster Day</b>

- Groups 1-5 will present on 04/16
- Groups 6-10 will present on 04/21
- Groups 11-15 will present on 04/23

## CRAY Colloquium: Is Data All You Need?: Large Robot Action Models and Good Old Fashioned Engineering

The computer science colloquium takes place on Mondays from 11:15 a.m. - 12:15 p.m. This week's speaker, **Ken Goldberg** (University of California, Berkeley), will be giving a talk titled "**Is Data All You Need?: Large Robot Action Models and Good Old Fashioned Engineering**".

### Abstract

Enthusiasm has been skyrocketing for humanoids based on recent advances in "end-to-end" large robot action models. Initial results are promising, and several collaborative efforts are underway to collect the needed demonstration data. But is data really all you need?

Although end-to-end Large Vision, Language, Action (VLA) Models have potential to generalize and reliably solve all problems in robotics, initial results have been mixed. It seems likely that the size of the VLA state space and dearth of available demonstration data, combined with challenges in getting models to generalize beyond the training distribution and the inherent challenges in interpreting and debugging large models, will make it difficult for pure end-to-end systems to provide the kind of robot performance that investors expect in the near future.

**Start date**

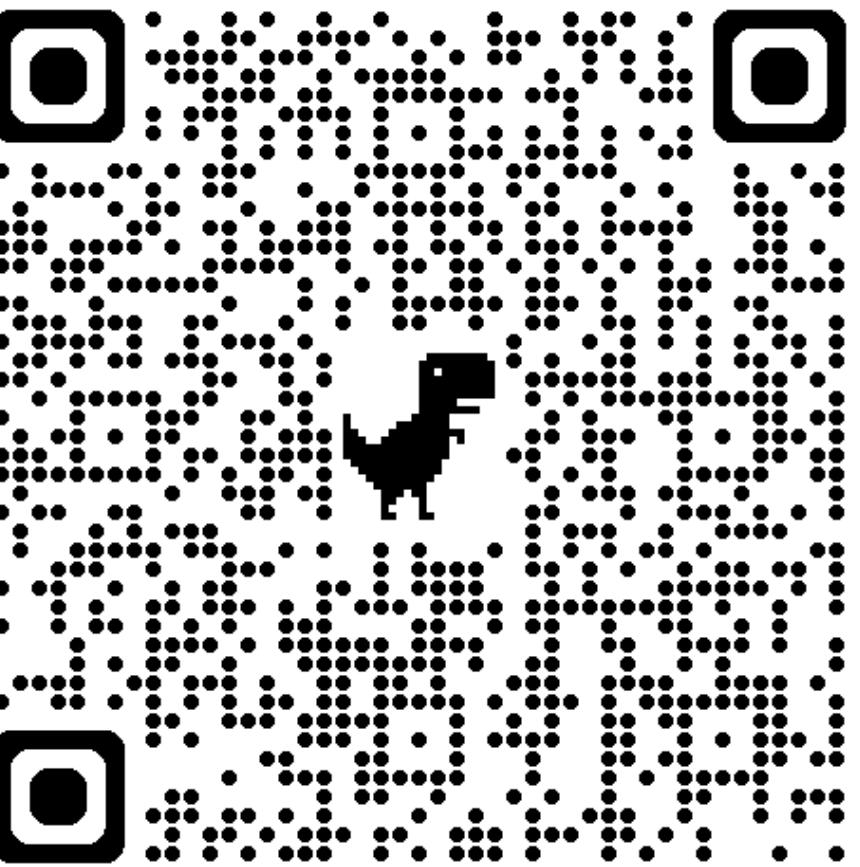
Monday, April 21, 2025, 11:15 a.m.

**End date**

Monday, April 21, 2025, 12:15 p.m.

**Location**

[Keller Hall 3-180](#)

**Share**

# SRTs

- Please take a moment to fill the Student Rating of Teaching
  - If the QR code does not work.
  - Please search for “SRT” in your email and fill the one for CSCI5551.
- Please encourage your classmates and teammates to finish the rating.
  - If we get more than 95% response rate, I will get everyone in the class 1 quiz point.



CSCI 5551 001 Robotics

Students

<https://go.blueja.io/xRdARz2wb0WHoD80bJbqAg>