Lecture 13
Planning - V - Potential Fields

Course Logistics

- **Quiz 11 was posted today and was due before the lecture.**
- Project 3 was posted on 10/11 and is due on 10/25.
  - Please look into updates posted by Chahyon Ku on Ed discussions!

RRT Algorithm

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[Kuffner, LaValle 2000]

RRT Algorithm

Extend graph towards a random configuration and repeat

BUILD_RRT(q_{init})
1 $\mathcal{T}$.init(q_{init});
2 **for** $k = 1$ **to** K **do**
3 $q_{rand} \leftarrow$ RANDOM_CONFIG();
4 EXTEND($\mathcal{T}, q_{rand}$);
5 Return $\mathcal{T}$

[Kuffner, LaValle 2000]

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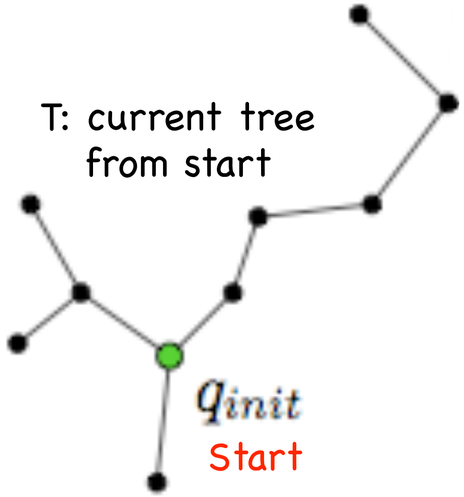
5  Return T
```



q_{rand}

Extension Goal
(randomly
selected)

Figure 3: The EXTEND operation.

[Kuffner, LaValle 2000]

RRT Algorithm

Extend graph towards a random configuration and repeat

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5  Return T
```
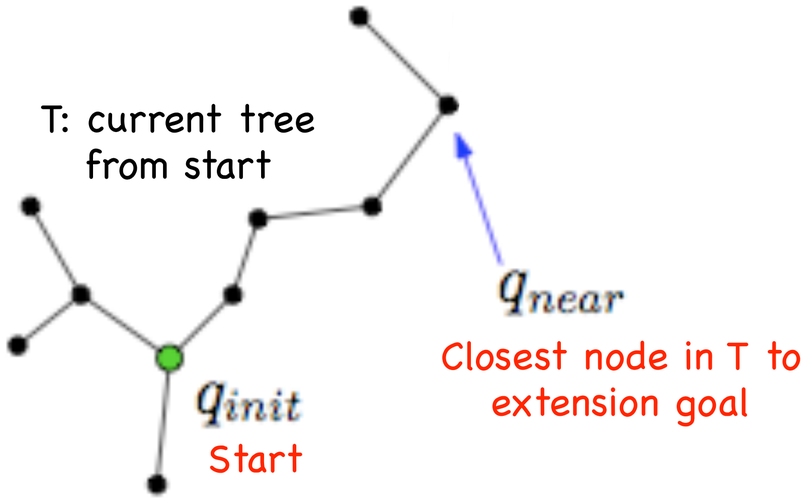

```
EXTEND(T, q)
1  q_near ← NEAREST_NEIGHBOR(q, T);
2  if NEW_CONFIG(q, q_near, q_new) then
3      T.add_vertex(q_new);
4      T.add_edge(q_near, q_new);
5      if q_new = q then
6          Return Reached;
7      else
8          Return Advanced;
9  Return Trapped;
```



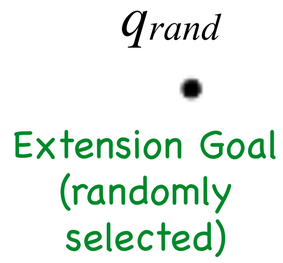


Figure 3: The EXTEND operation.

Extend graph towards a random configuration

[Kuffner, LaValle 2000]

RRT Algorithm

Extend graph towards a random configuration and repeat

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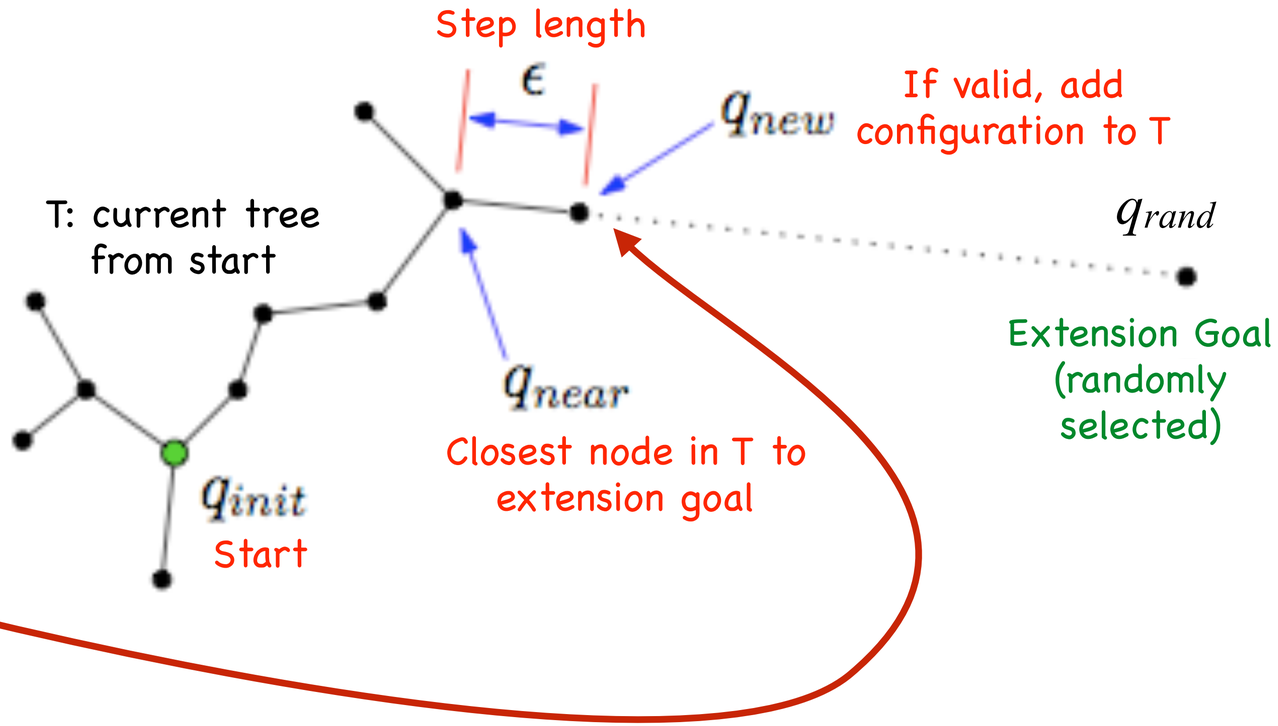
Extend graph towards a random configuration



Generate and test new configuration along vector in C-space from q_{near} to q_{rand}

RRT* Algorithm

Slide borrowed from Michigan Robotics autorob.org

RRT*

Algorithm 6: RRT*

$V \leftarrow \{x_{\text{init}}\}$; $E \leftarrow \emptyset$;
for $i = 1, \ldots, n$ **do**
  $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i$;
  $x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}})$;
  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})$;
  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**
    $X_{\text{near}} \leftarrow \texttt{Near}(G = (V, E), x_{\text{new}}, \min\{\gamma_{\text{RRT}^*}(\log(\text{card}\,(V))/\,\text{card}\,(V))^{1/d}, \eta\})$;
    $V \leftarrow V \cup \{x_{\text{new}}\}$;
    $x_{\min} \leftarrow x_{\text{nearest}}$; $c_{\min} \leftarrow \texttt{Cost}(x_{\text{nearest}}) + c(\texttt{Line}(x_{\text{nearest}}, x_{\text{new}}))$;
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Connect along a minimum-cost path
      if $\texttt{CollisionFree}(x_{\text{near}}, x_{\text{new}}) \wedge \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}})) < c_{\min}$ **then**
        $x_{\min} \leftarrow x_{\text{near}}$; $c_{\min} \leftarrow \texttt{Cost}(x_{\text{near}}) + c(\texttt{Line}(x_{\text{near}}, x_{\text{new}}))$
    $E \leftarrow E \cup \{(x_{\min}, x_{\text{new}})\}$;
    foreach $x_{\text{near}} \in X_{\text{near}}$ **do** // Rewire the tree
      if $\texttt{CollisionFree}(x_{\text{new}}, x_{\text{near}}) \wedge \texttt{Cost}(x_{\text{new}}) + c(\texttt{Line}(x_{\text{new}}, x_{\text{near}})) < \texttt{Cost}(x_{\text{near}})$
      then $x_{\text{parent}} \leftarrow \texttt{Parent}(x_{\text{near}})$;
      $E \leftarrow (E \setminus \{(x_{\text{parent}}, x_{\text{near}})\}) \cup \{(x_{\text{new}}, x_{\text{near}})\}$
return $G = (V, E)$;

Karaman & Frazzo

RRT*

Algorithm 6: RRT*

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      if $\texttt{CollisionFree}(x_{\text{new}}, x_{\text{near}}) \wedge \texttt{Cost}(x_{\text{new}}) + c(\texttt{Line}(x_{\text{new}}, x_{\text{near}})) < \texttt{Cost}(x_{\text{near}})$ **then** $x_{\text{parent}} \leftarrow \texttt{Parent}(x_{\text{near}})$;
      $E \leftarrow (E \setminus \{(x_{\text{parent}}, x_{\text{near}})\}) \cup \{(x_{\text{new}}, x_{\text{near}})\}$
return $G = (V, E)$;

FIND x_{new}

FIND neighbors to x_{new} **in** G
ADD x_{new} **to** G

FIND edge to x_{new} **from neighbors with least cost**
ADD that to G

REWIRE the edges in the neighborhood if any least cost path exists from the root to the neighbors via x_{new}

Karaman & Frazzo

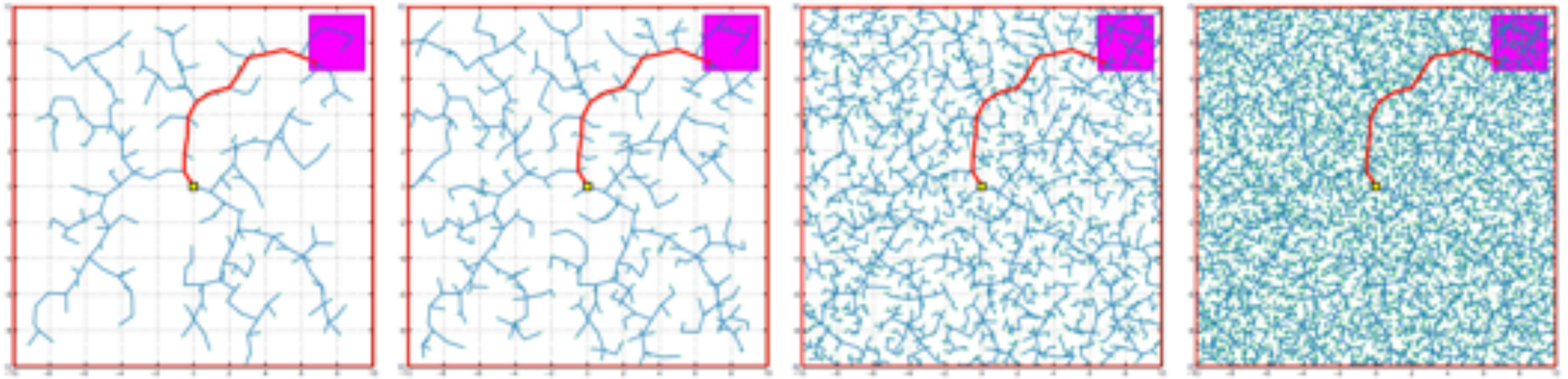
RRT*

- Asymptotically optimal
- Main idea:
  - Swap new point in as parent for nearby vertices who can be reached along shorter path through new point than through their original (current) parent

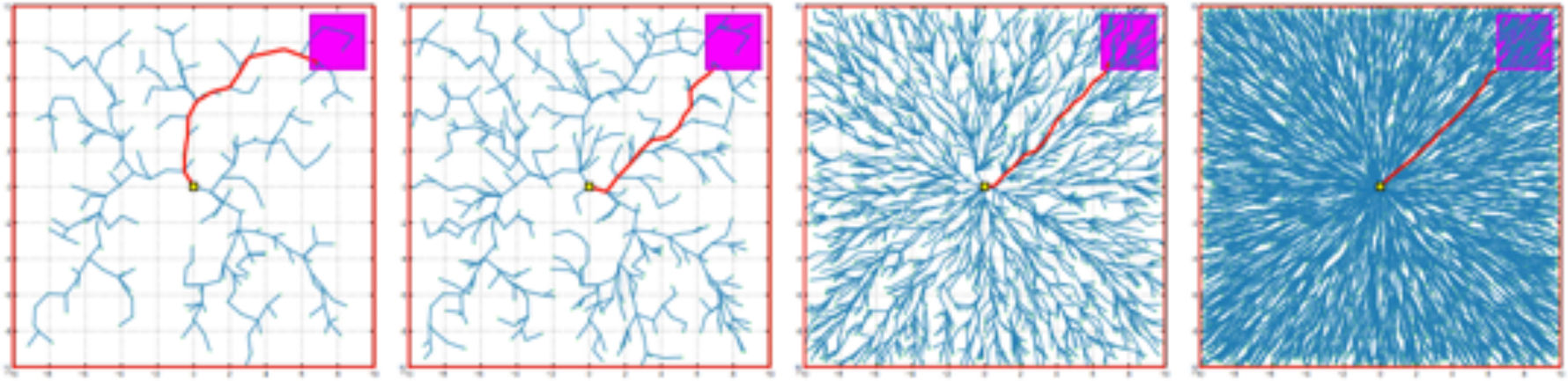
Demonstration - https://demonstrations.wolfram.com/RapidlyExploringRandomTreeRRTAndRRT/

RRT*

RRT



RRT*

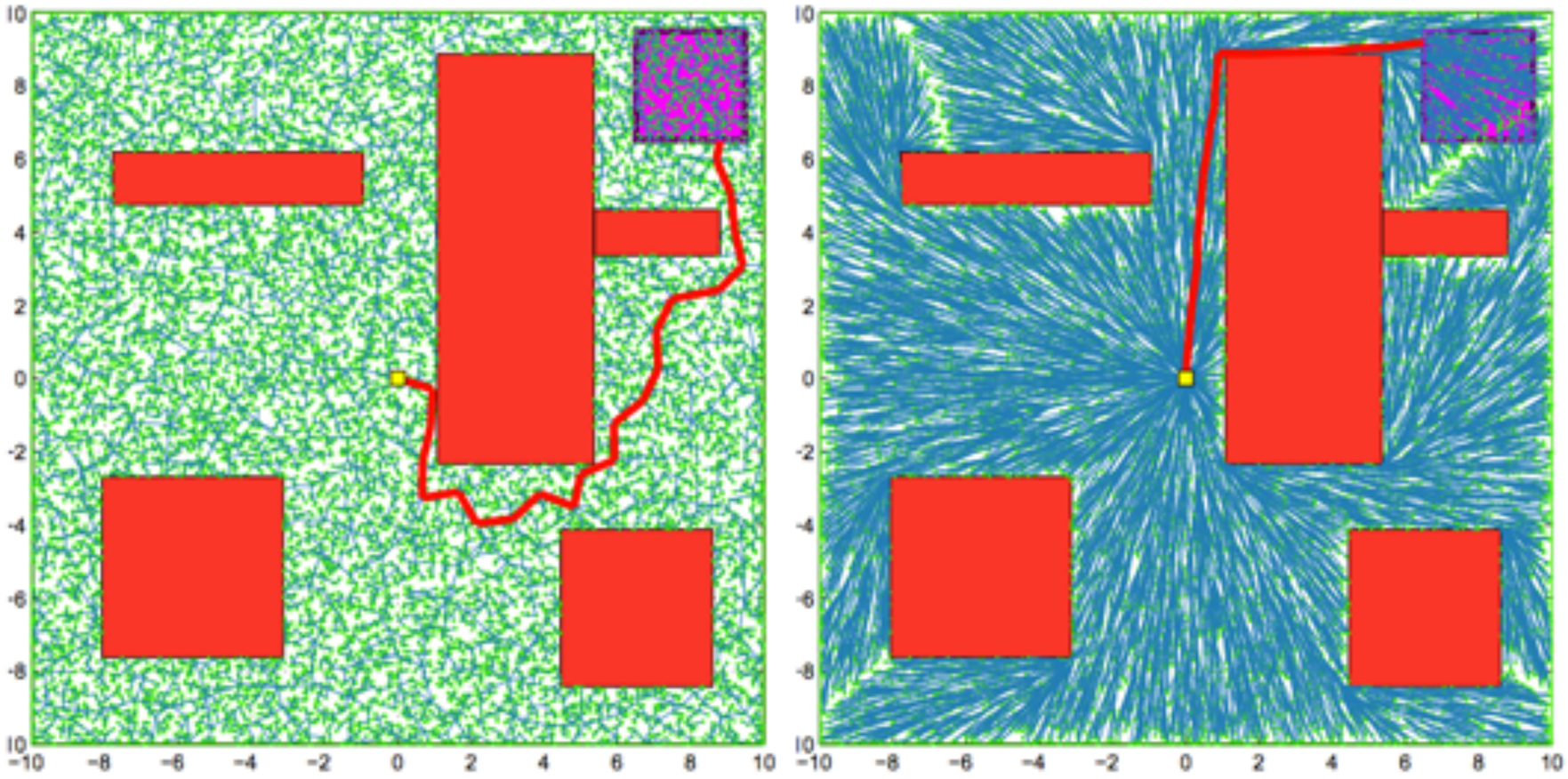


Source: Karaman and Frazzol

RRT*

RRT

RRT*



Source: Karaman and Frazzoli

Smoothing

Randomized motion planners tend to find not so great paths for execution: very jagged, often much longer than necessary.

→ In practice: do smoothing before using the path

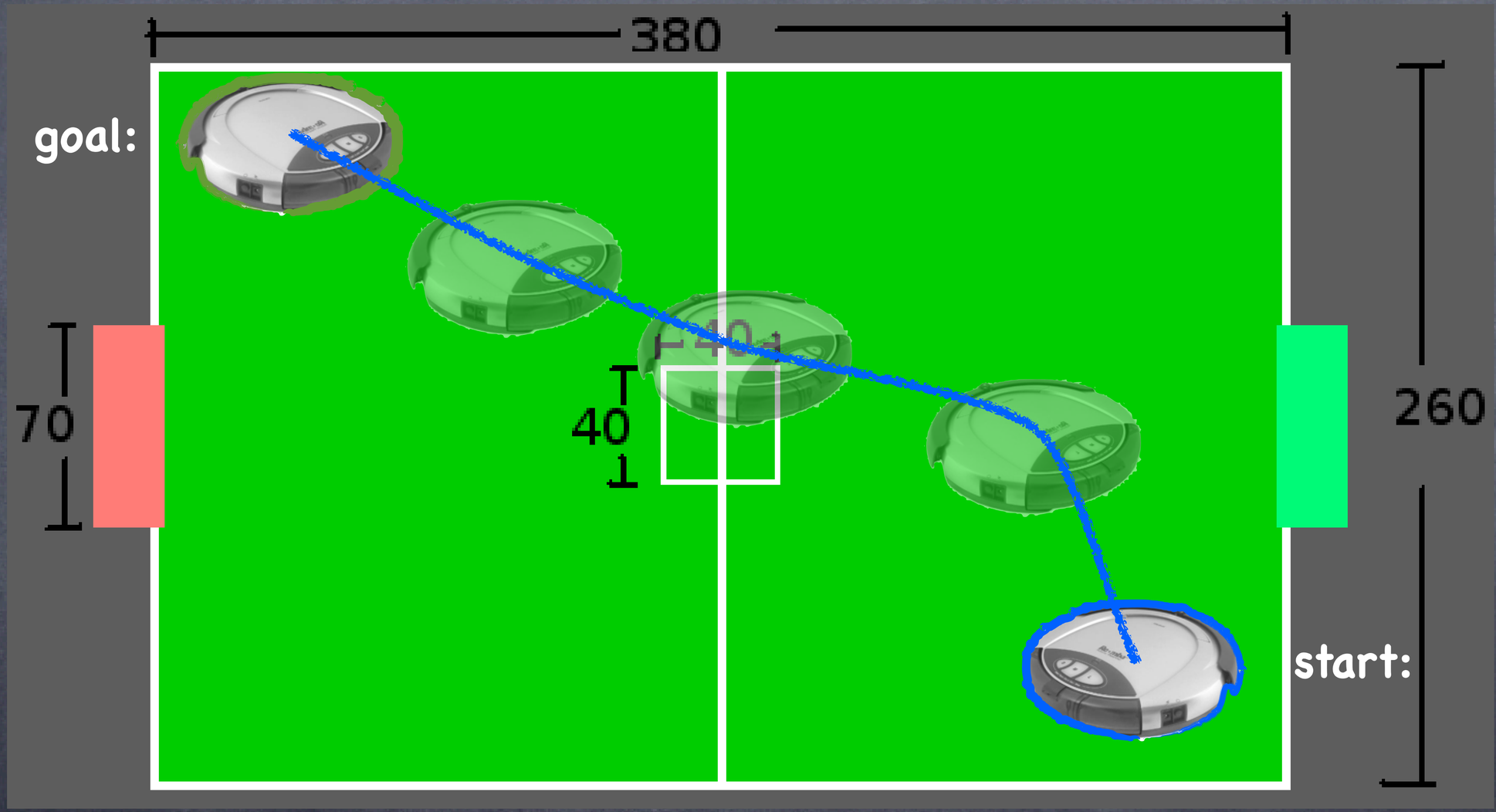
- Shortcutting:
  - along the found path, pick two vertices x_{t1}, x_{t2} and try to connect them directly (skipping over all intermediate vertices)
- Nonlinear optimization for optimal control
  - Allows to specify an objective function that includes smoothness in state, control, small control inputs, etc.

Slide borrowed from Dieter Fox

Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- Graph Search (fixed graph)
  - Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first
- Sampling-based Search (build graph):
  - Probabilistic Road Maps, Rapidly-exploring Random Trees
- **Optimization and local search:**
  - **Gradient descent, Potential fields, Simulated annealing, Wavefront**

Navigation (again)



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Potential field

(like a game of "warmer-colder")

goal:
volcanic

start:
cold

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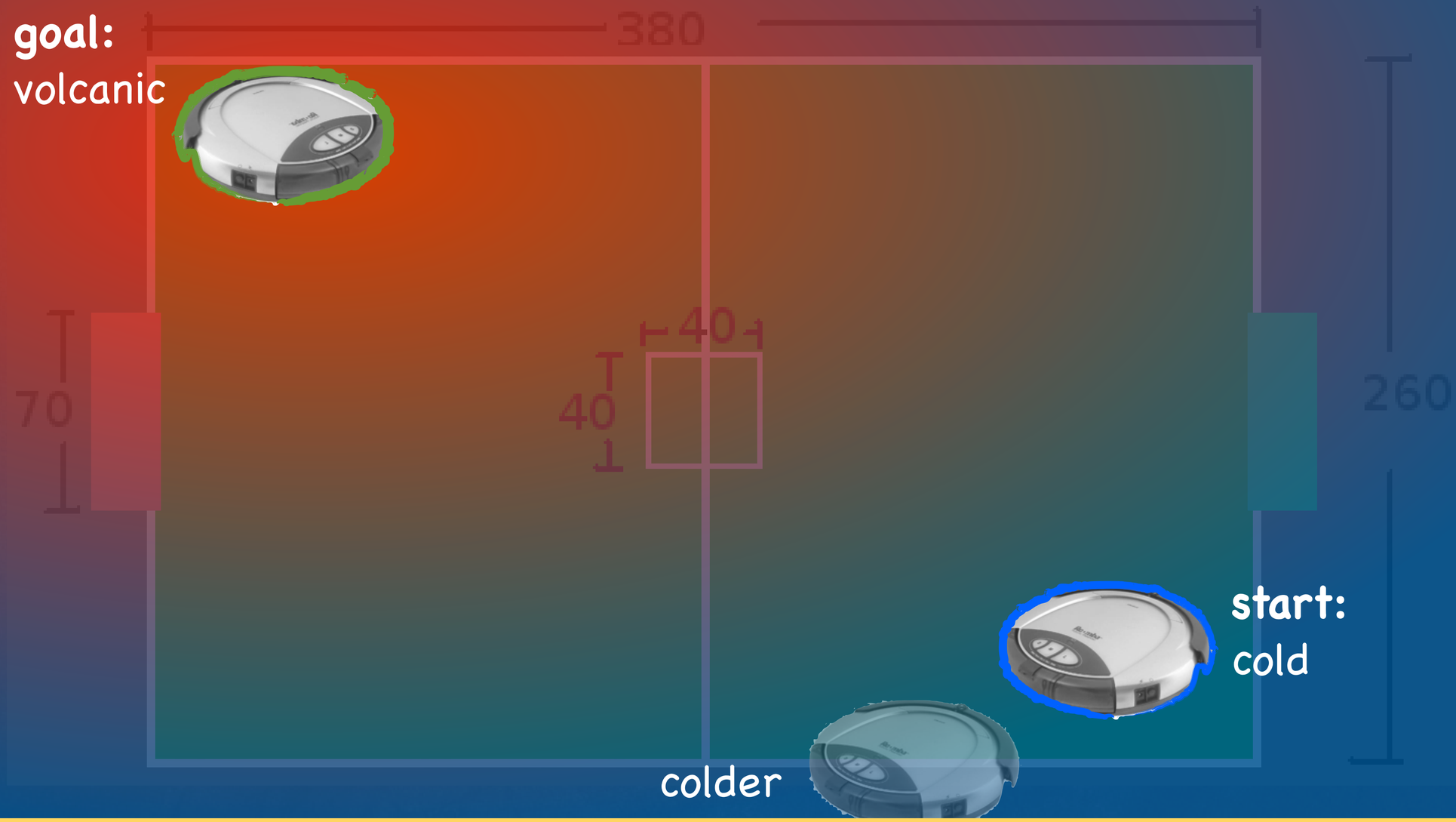
Potential field

(like a game of "warmer-colder")



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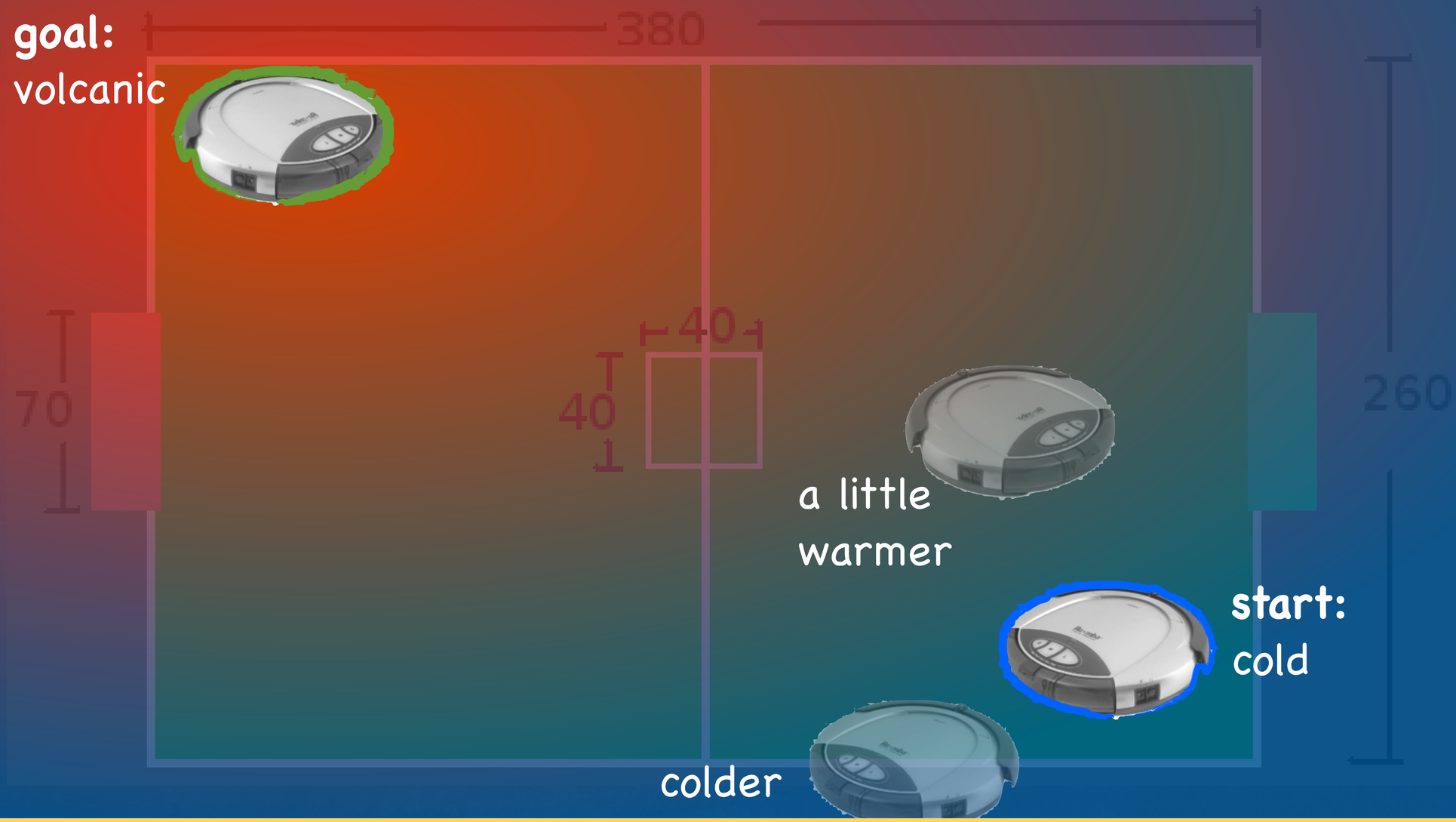

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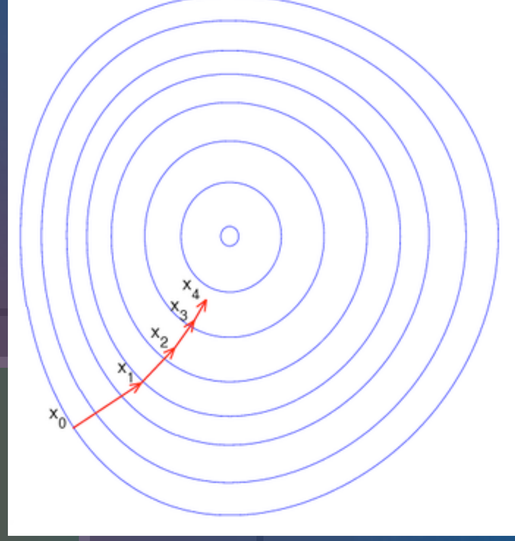

Gradient descent:
Energy potential converges at goal

goal:
volcanic

a little warmer

start:
cold

colder

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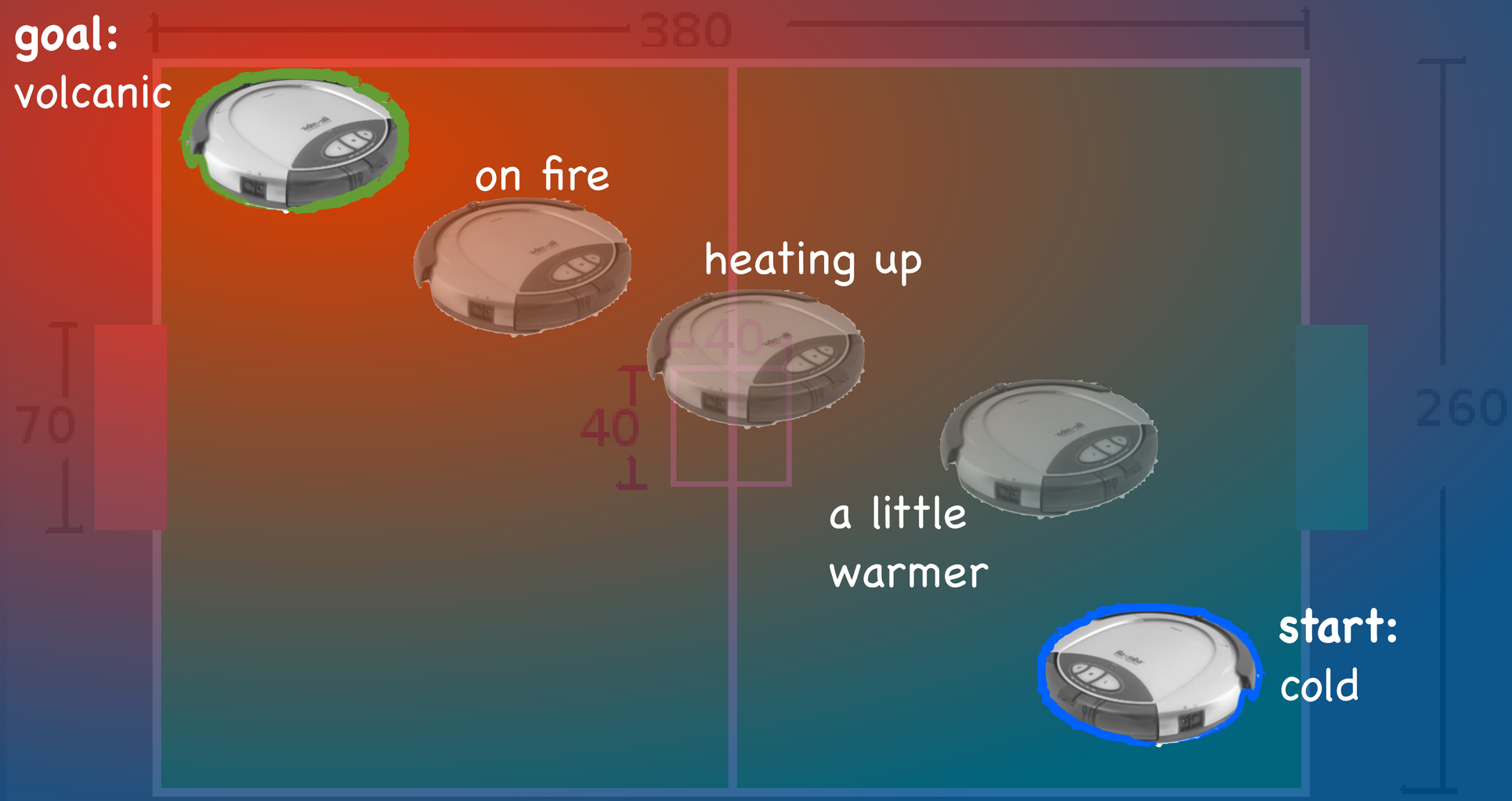
Potential field

(like a game of "warmer-colder")



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How do we define a potential field?

Potential Field

- A potential field is a differentiable function $U(q)$ that maps configurations to scalar "energy" value
- At any q, gradient $\nabla U(q)$ is the vector that maximally increases U
- At goal q_{goal}, energy is minimized such that $\nabla U(q_{goal}) = 0$
- Navigation by descending field - $\nabla U(q)$ *to goal*

Gradient Descent Algorithm:

$q_{path}[0] \leftarrow q_{start}$

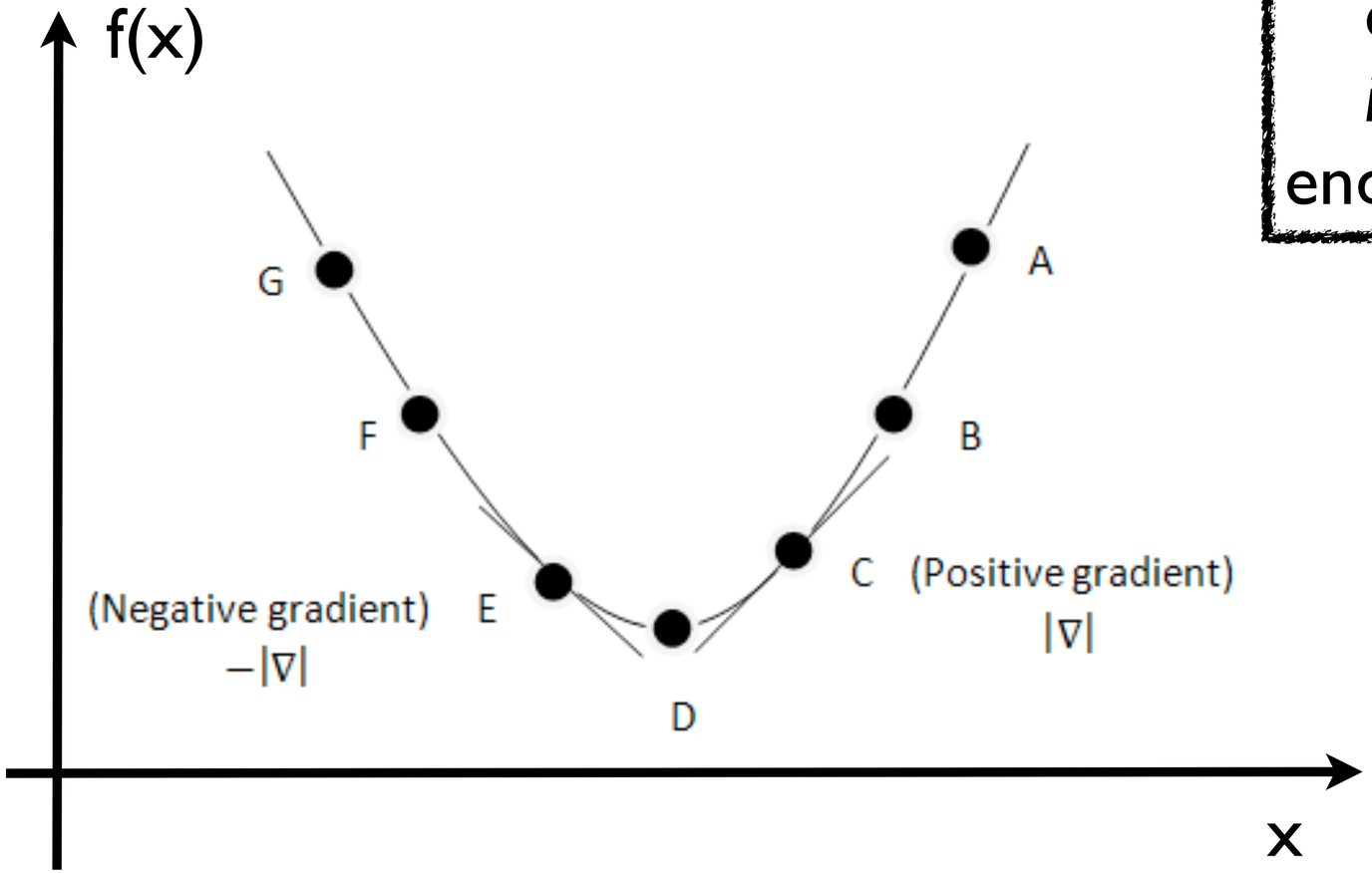
$i \leftarrow 0$

while $(||\nabla U(q[i])|| > \varepsilon)$

  $q_{path}[i+1] \leftarrow q_{path}[i] - \alpha \nabla U(q_{path}[i])$

  $i \leftarrow i+1$

end



Derivative assumed to be direction of steepest ascent away from goal

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \gamma_n \nabla F(\mathbf{x}_n)$$

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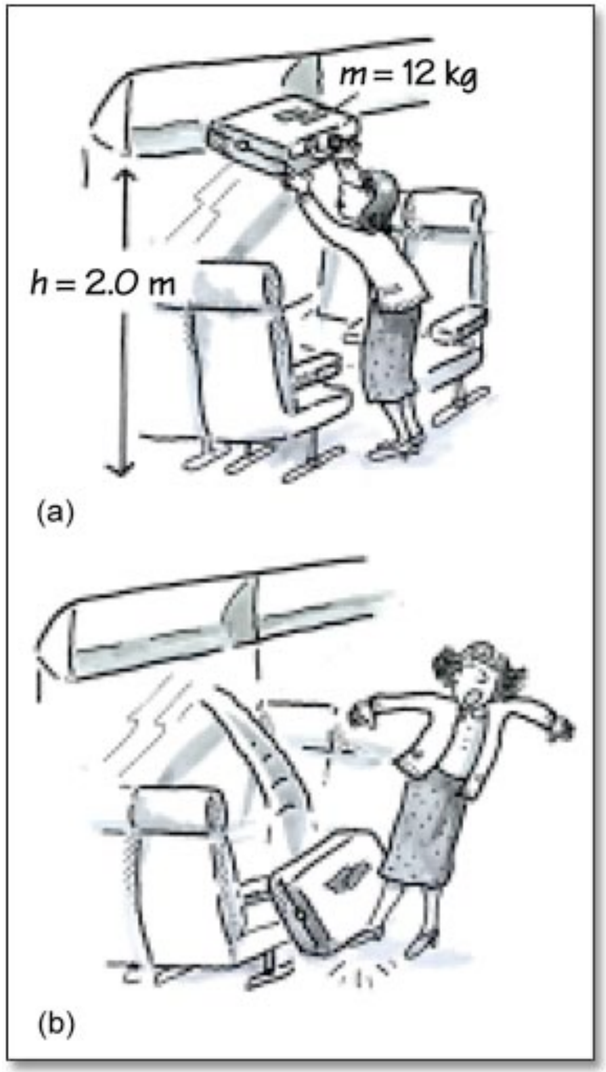
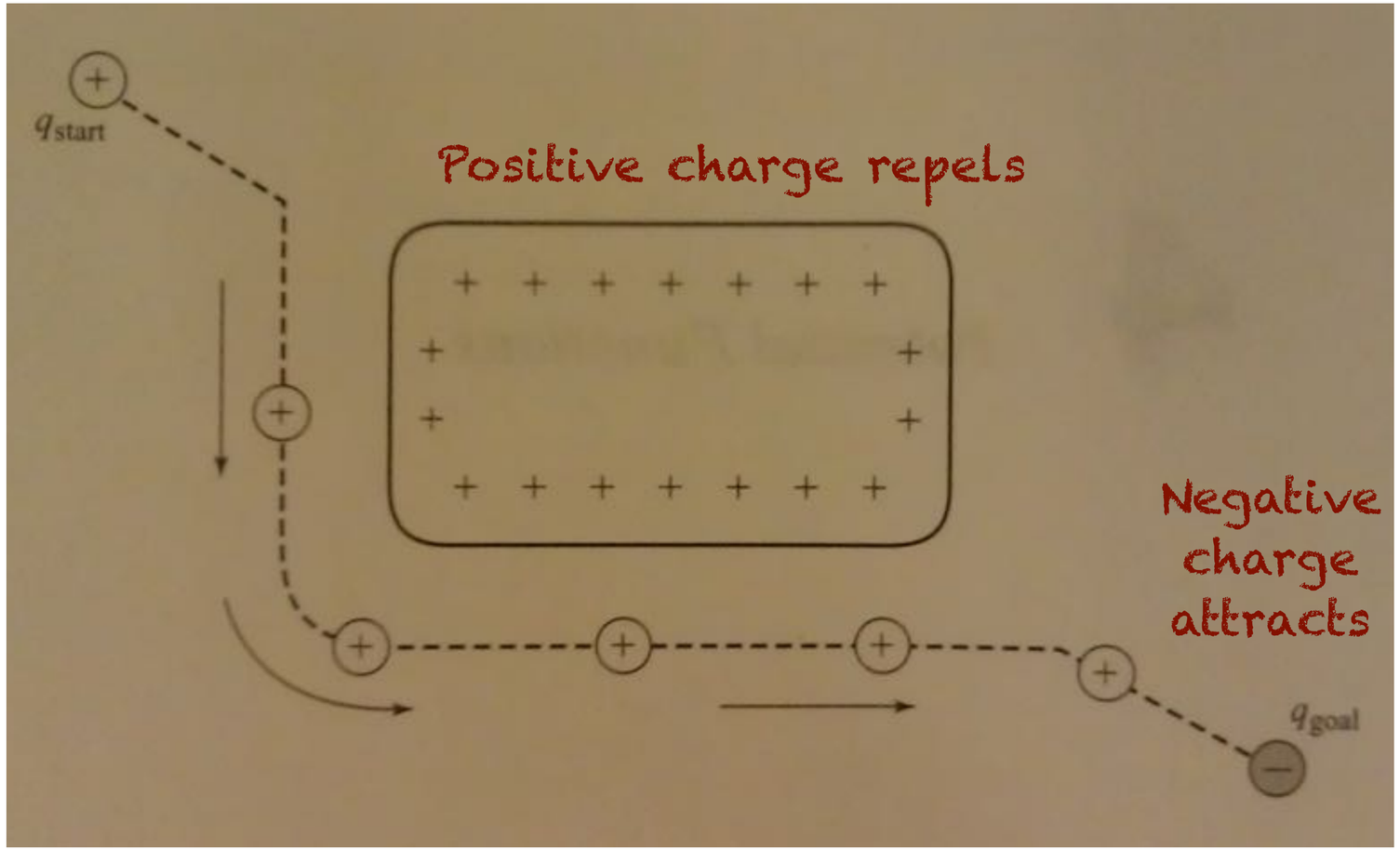
Potential Energy



- Energy stored in a physical system
- Kinetic motion caused by system moving to lower energy state
- For objects acting only w.r.t. gravity
- potential_energy = mass*height*gravity

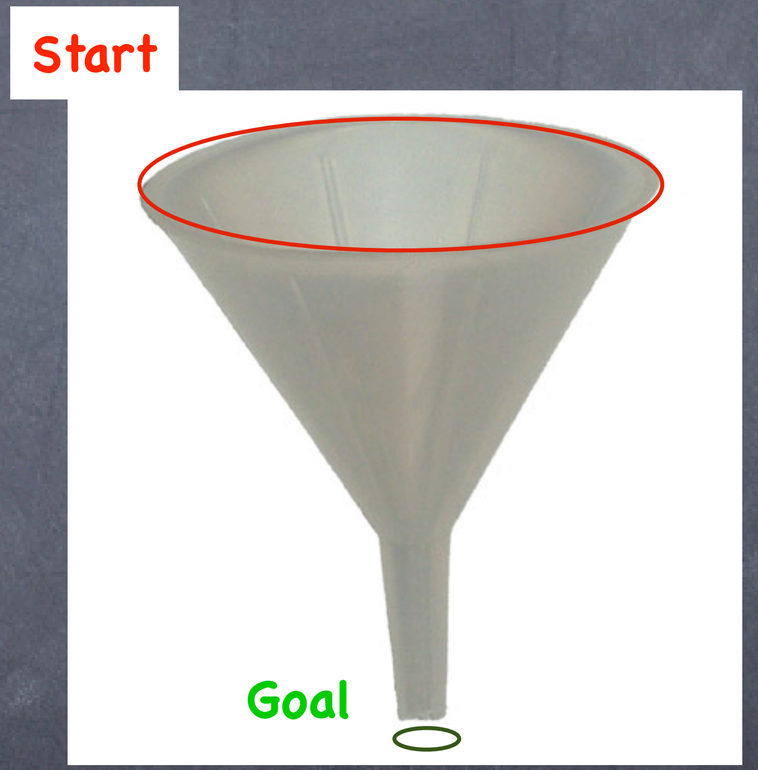
Charged Particle Example

Positively charged particle follows potential energy to goal



Convergent Potentials

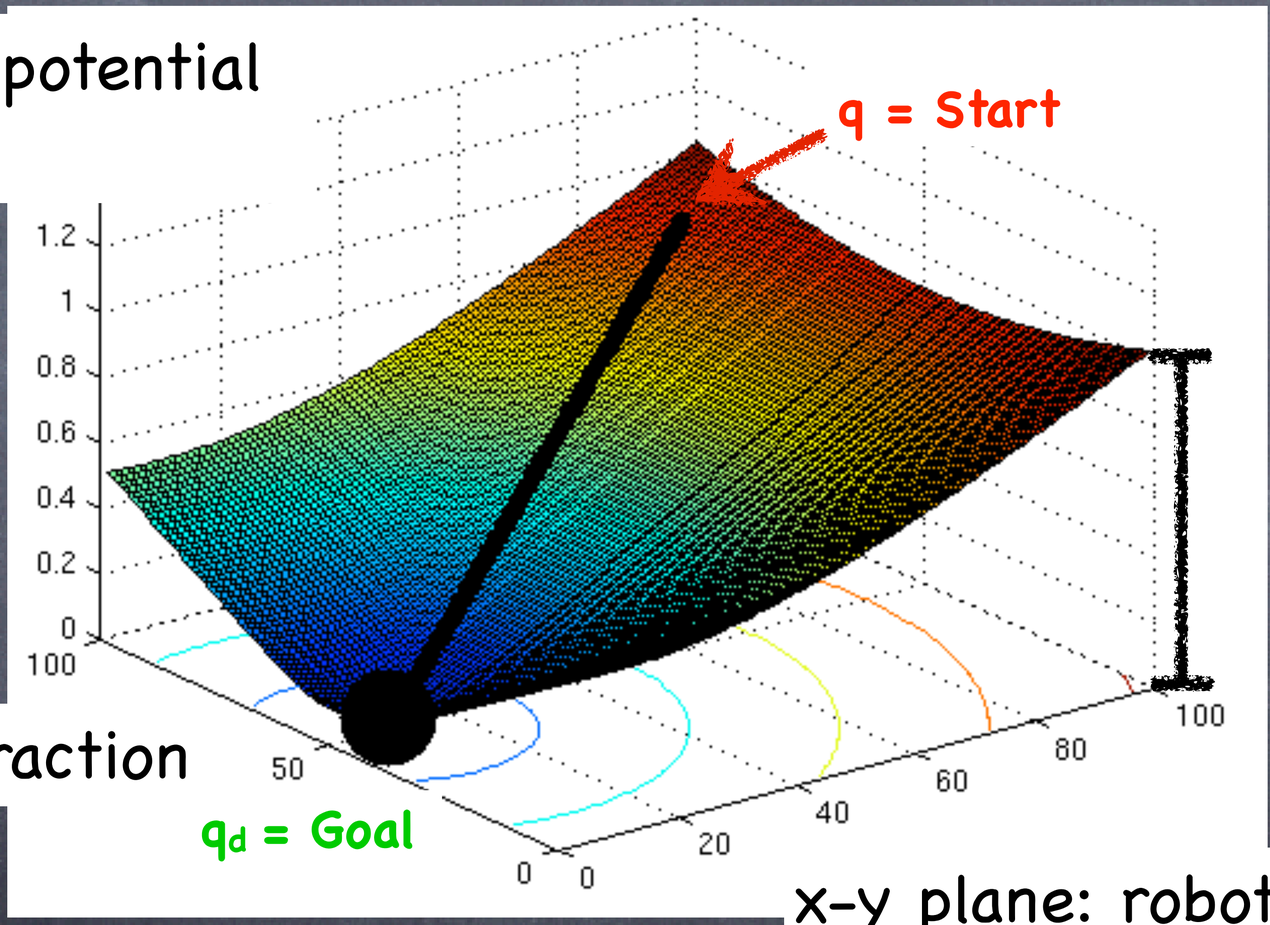
let's call these "attractor landscapes"



basin of attraction

2D potential navigation

z: height indicates potential at location



basin of attraction

x-y plane: robot position

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"Cone" Attractor

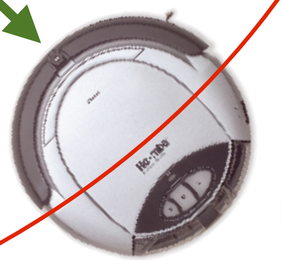
top view

q_d = Goal

"Attractor"

$\nabla U(q) = w(q - q_d)/\|q - q_d\|$

q = Start



w: weight

$(q - q_d)$: direction

$\|q - q_d\|$: distance

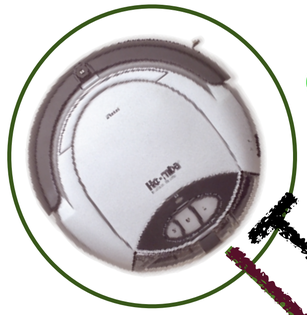
Start

Goal

side view

"Cone" Attractor

top view



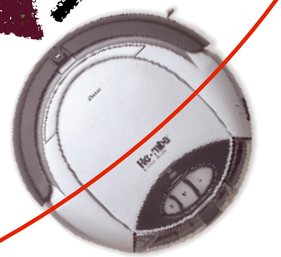
q_d = Goal

"Attractor"

$\|q - q_d\|$

$(q - q_d)$

$\nabla U(q) = w(q - q_d)/\|q - q_d\|$



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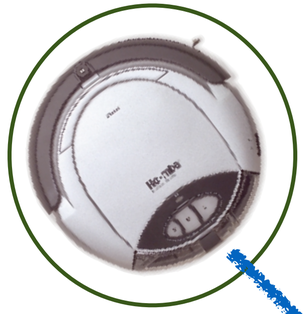
Start

Goal

side view

"Cone" Attractor

top view



q_d = Goal

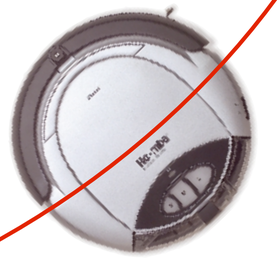
"Attractor"

unit vector

$(q - q_d)/\|q - q_d\|$

$\nabla U(q) = w(q - q_d)/\|q - q_d\|$

q = Start



w: weight

$(q_d - q)$: direction

$\|q_d - q\|$: distance

Start

Goal

side view

"Cone" Attractor

top view

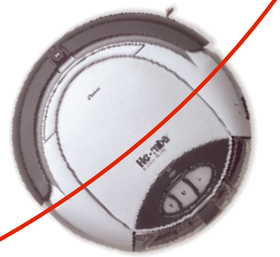
q_d = Goal

"Attractor"

w

$\nabla U(q) = w(q - q_d)/\|q - q_d\|$

x = Start



w: weight (< 1)

$(q - q_d)$: direction

$\|q - q_d\|$: distance

Start

Goal

side view

Can we modulate the range of a potential field?

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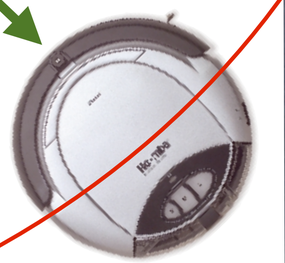
"Bowl" Attractor

top view

$\nabla U(q) = \exp(-\|q - q_d\|/w)\,(q - q_d)$

q_d = Goal



q = Start

Start

Goal

side view

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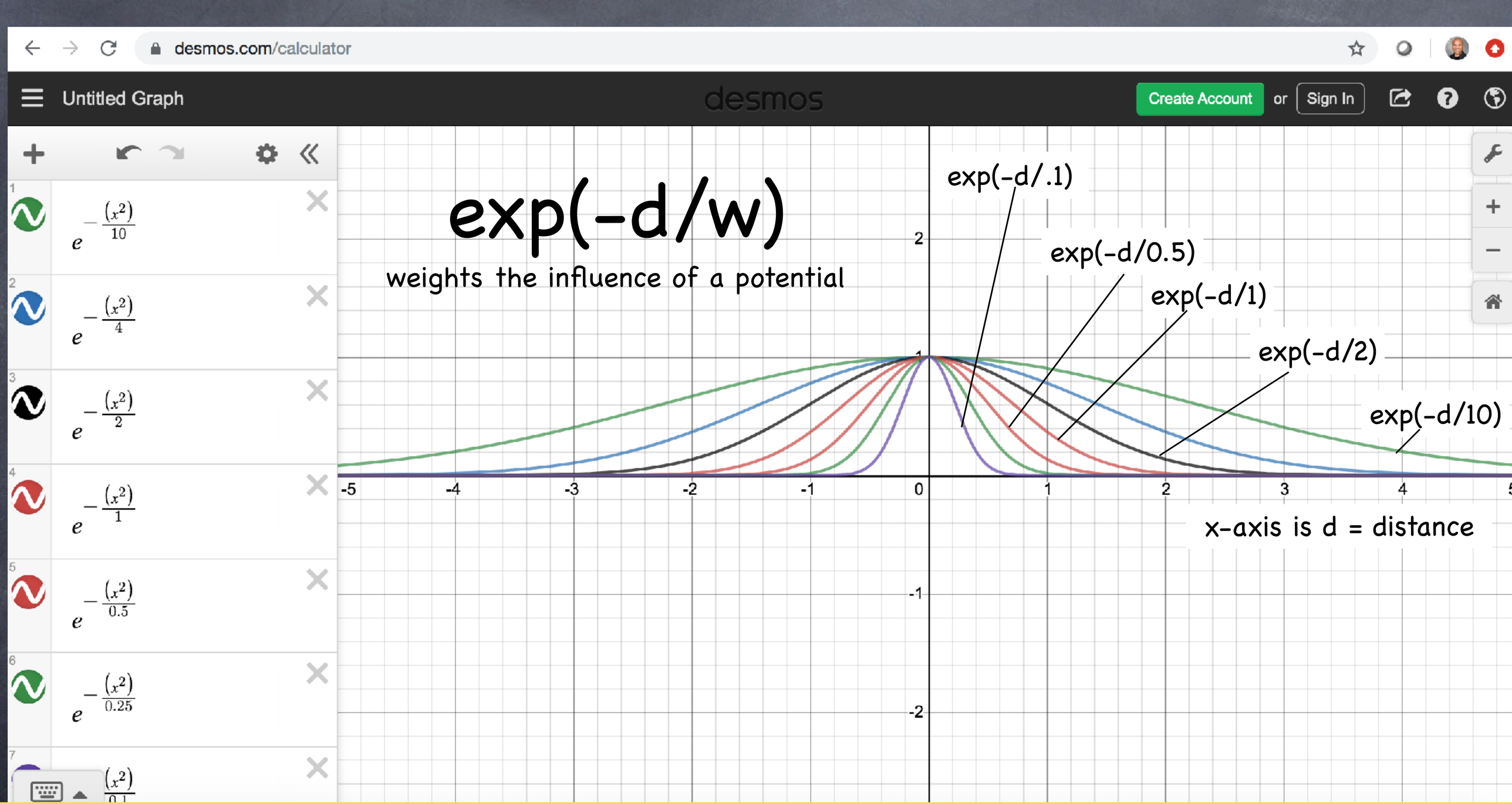

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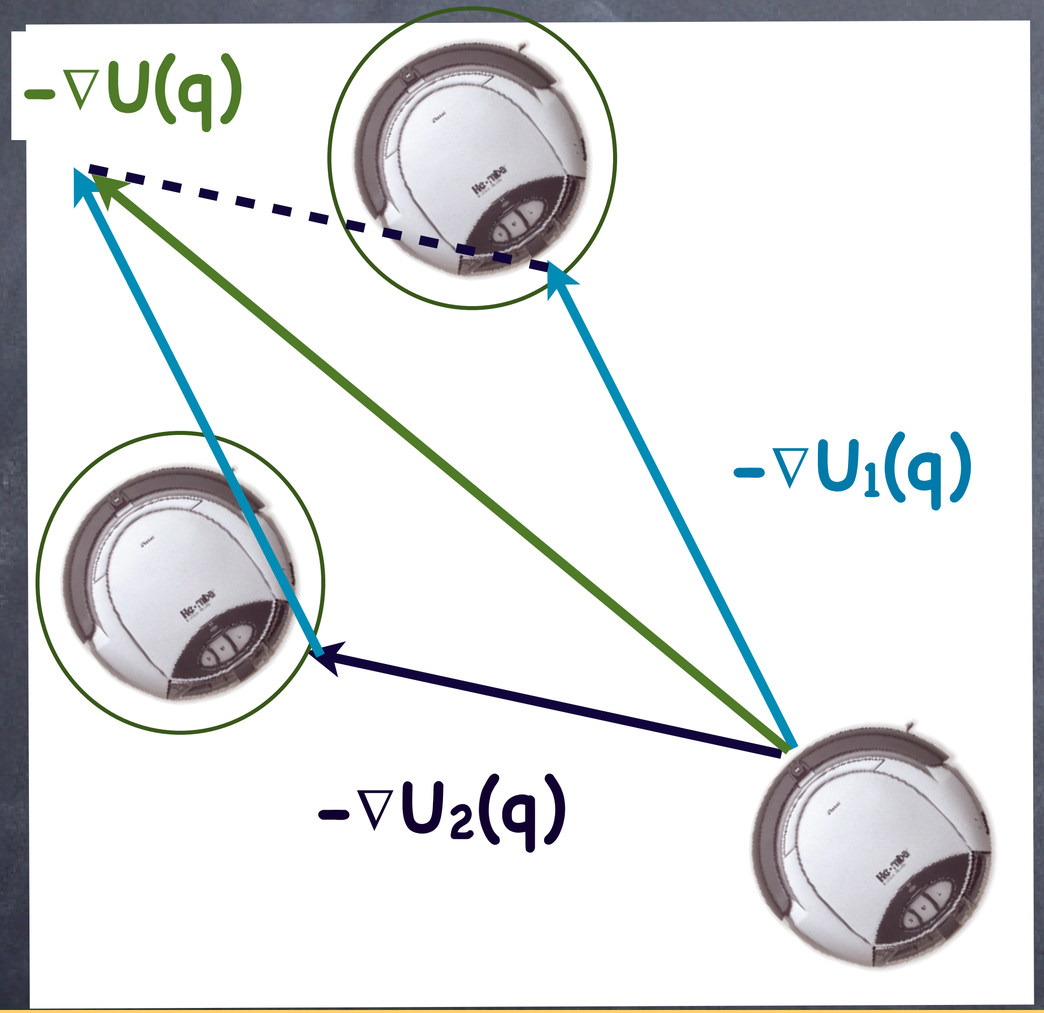
Can we combine multiple potentials?

Multiple potentials



- Output of potential field is a vector
- Combine multiple potentials through vector summation

$$U(q) = \Sigma_i U_i(q)$$

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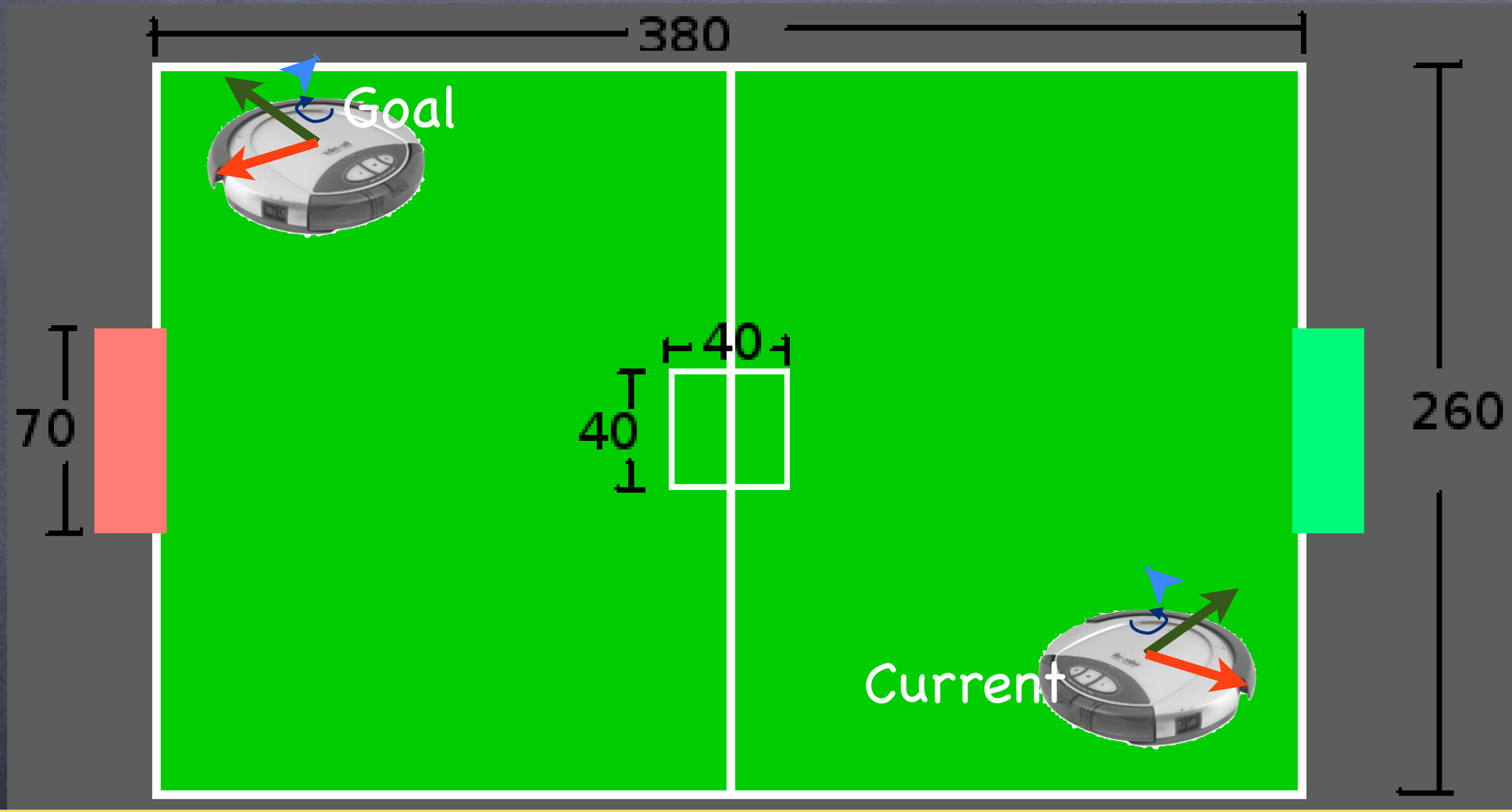
describe performance for this case



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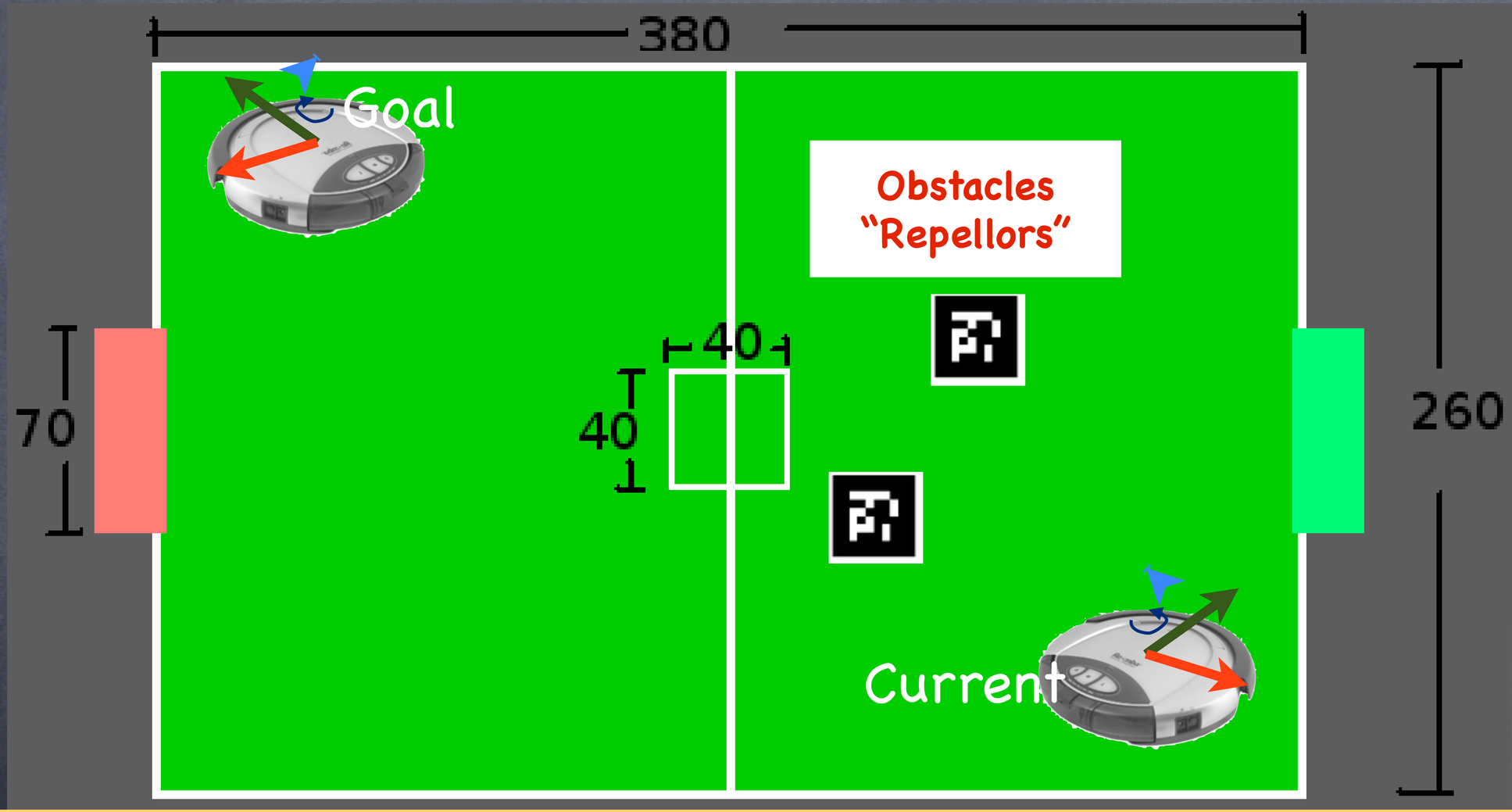
describe performance for this case



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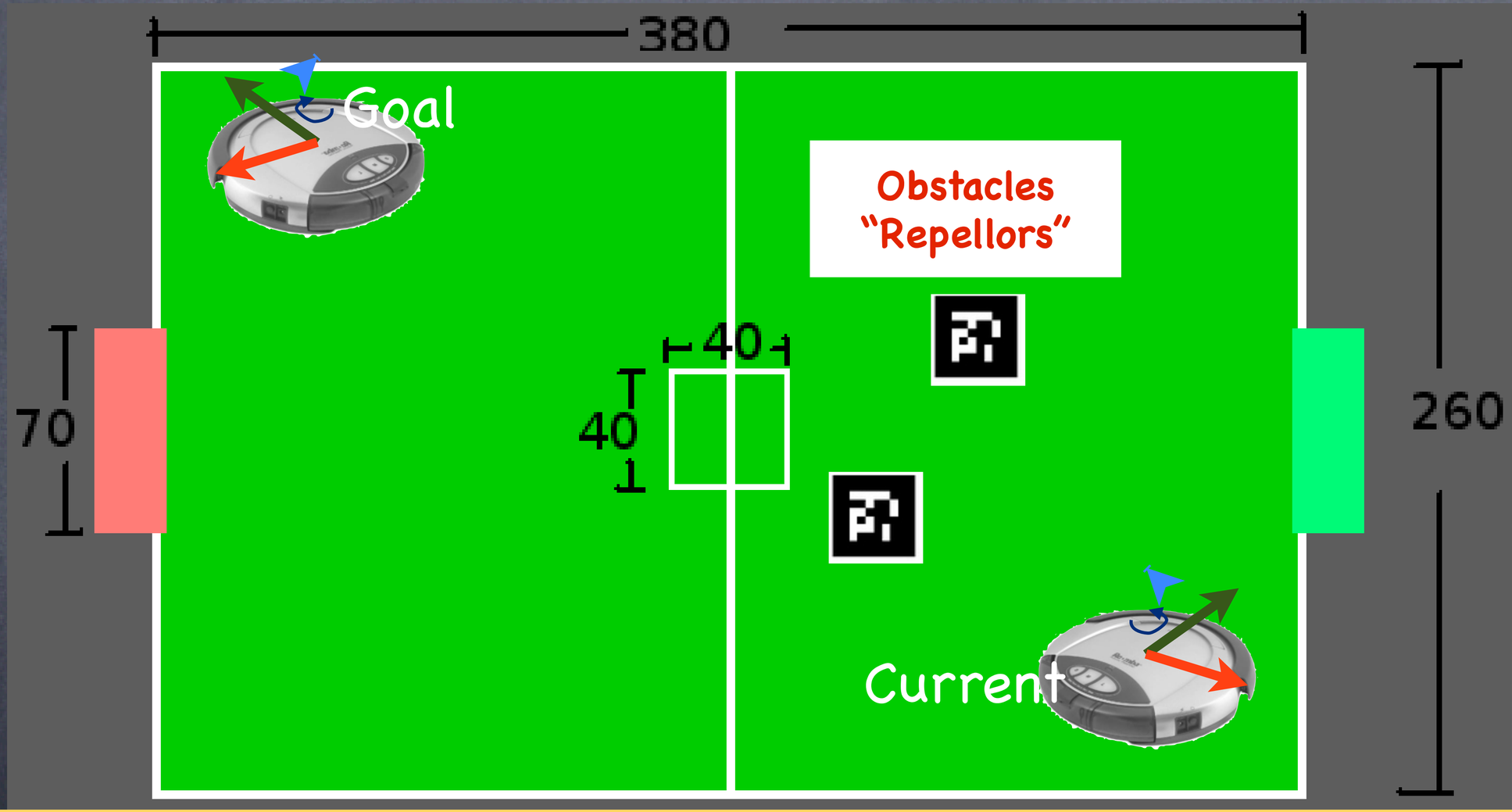
describe performance for this case

how do we deal with repellors?



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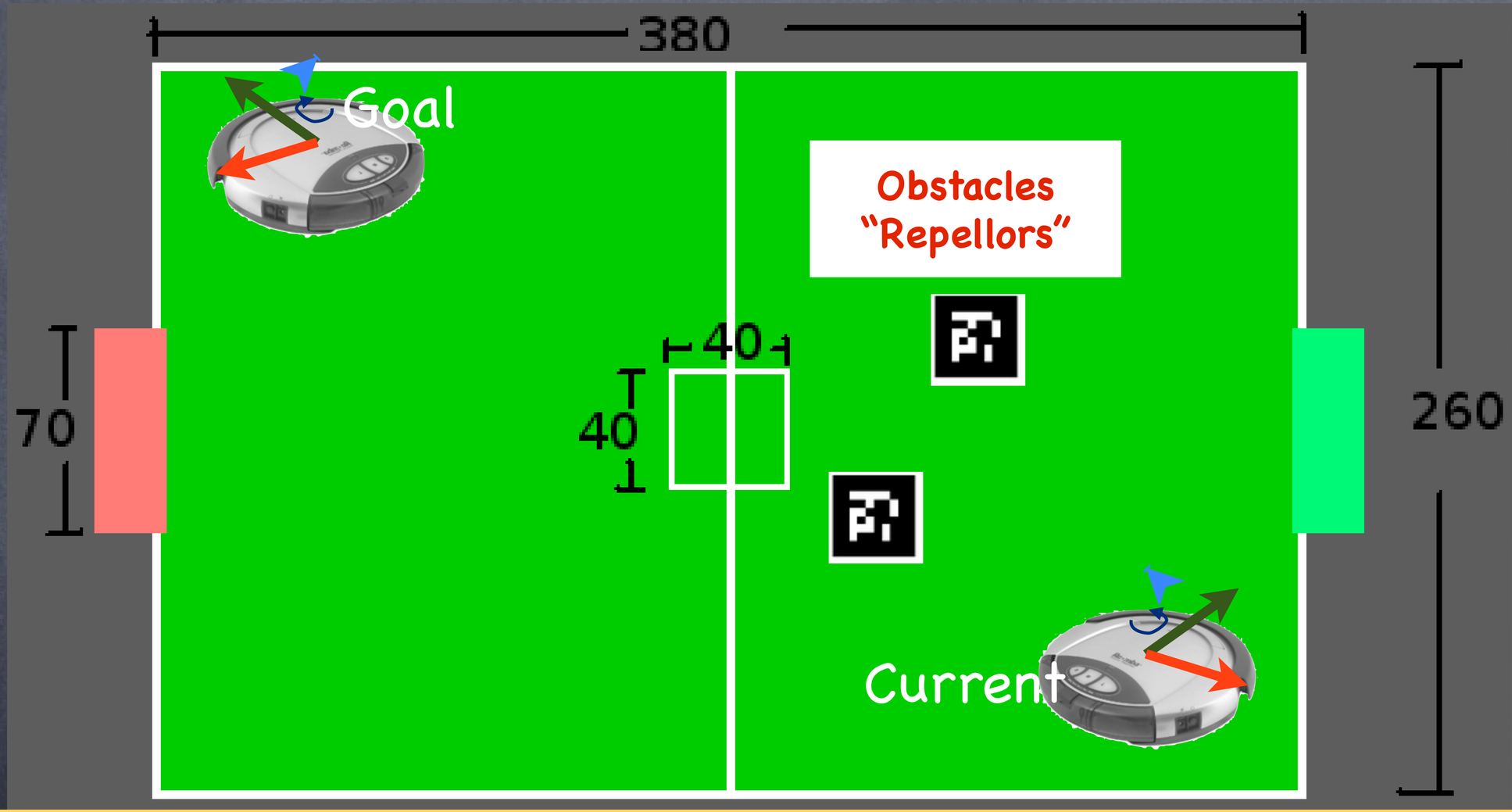
add sum of repulsive potentials

$$U(q) = U_{attracts}(q) + U_{repellors}(q)$$



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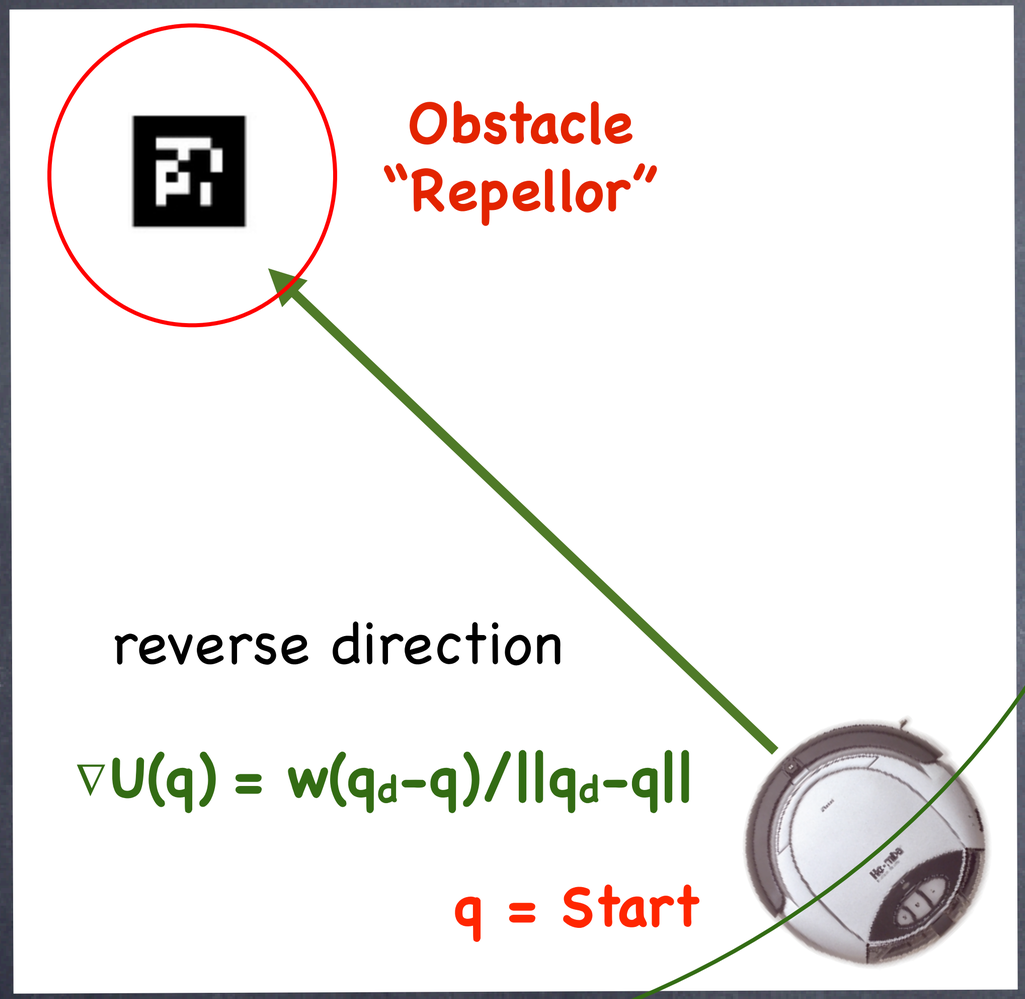
"Cone" Repellor

potential problems?



side view

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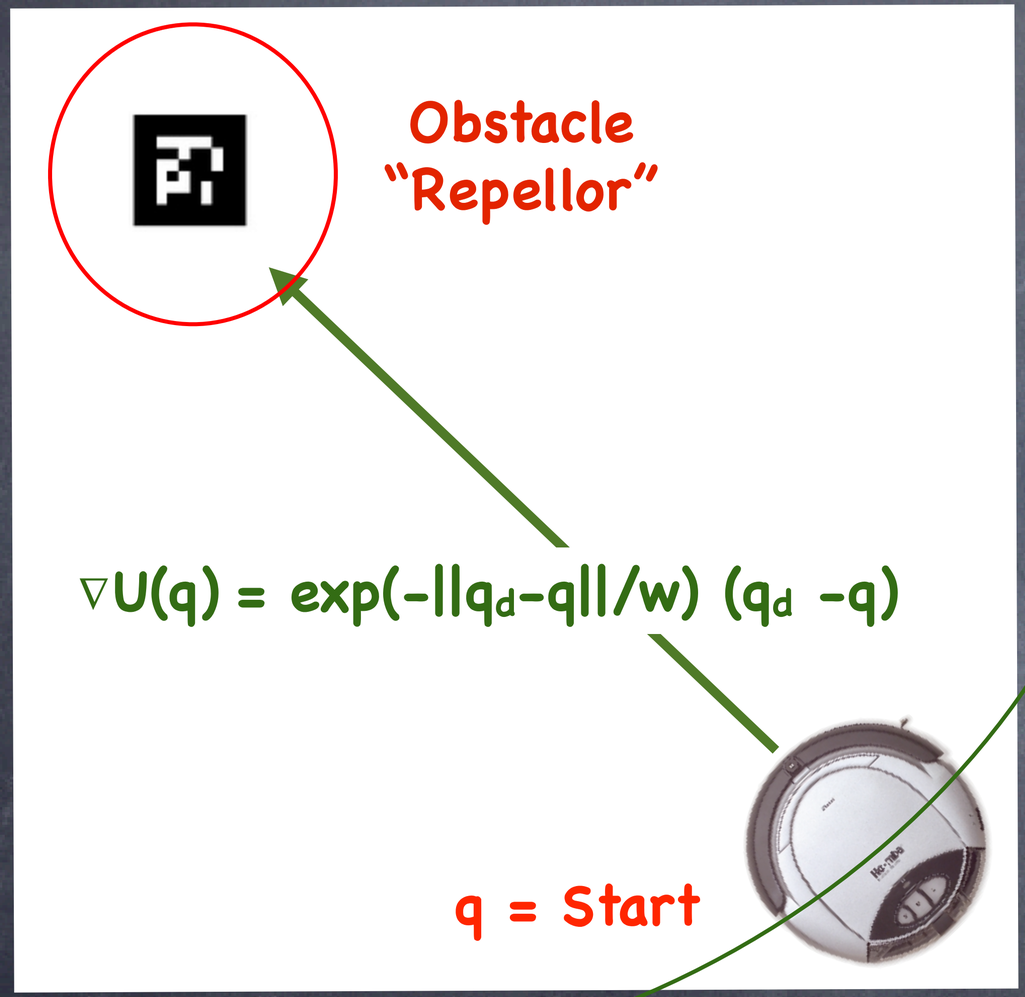
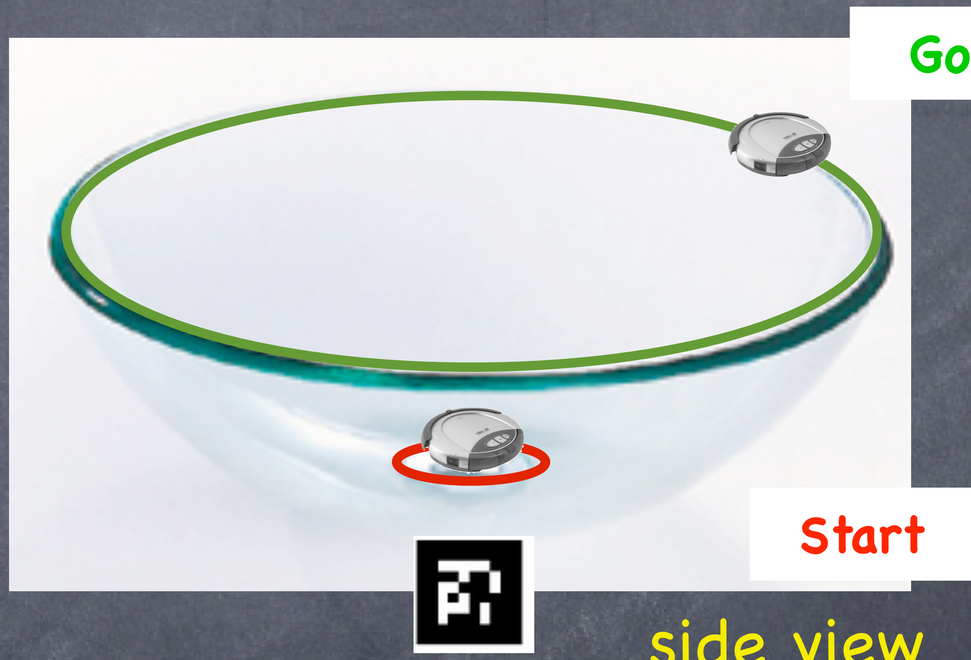
"Bowl" Repellor



top view



side view

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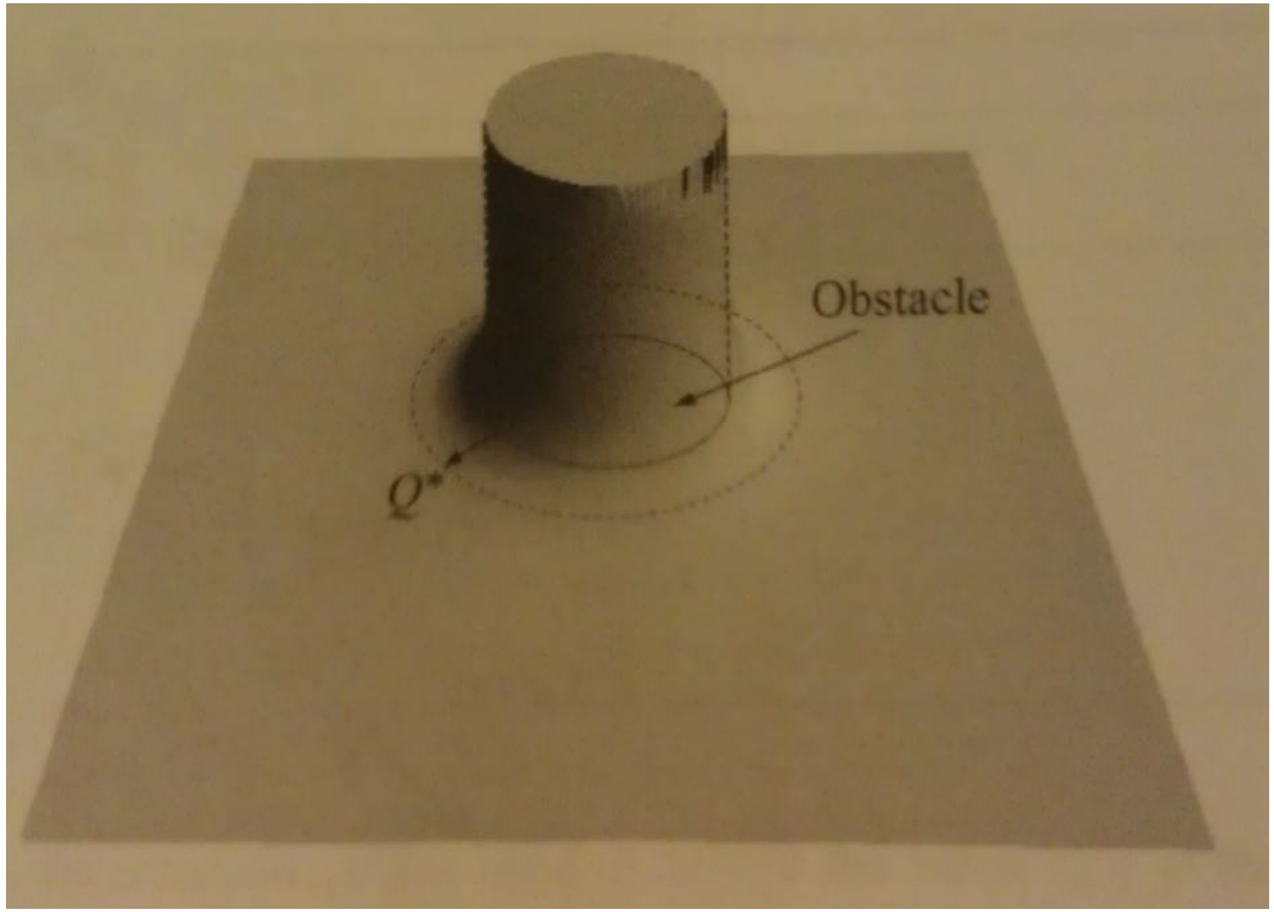
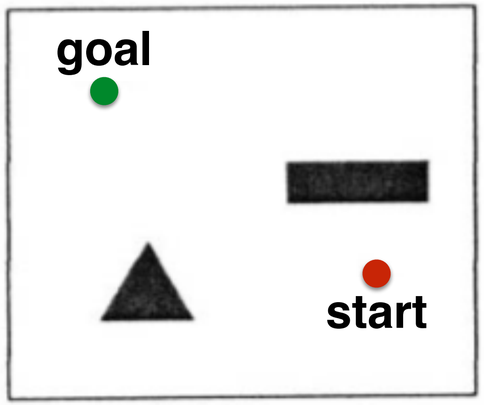
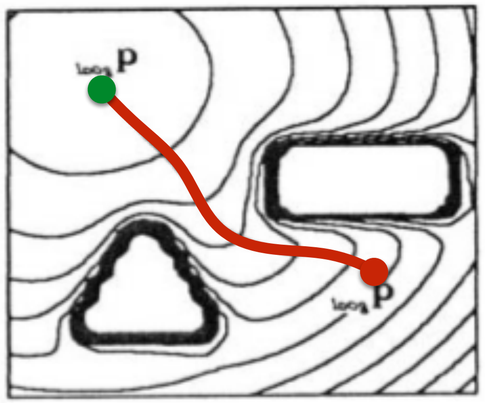
repellor should only have local influence,
repelling only around boundary improves path



2 Obstacle example

configuration space





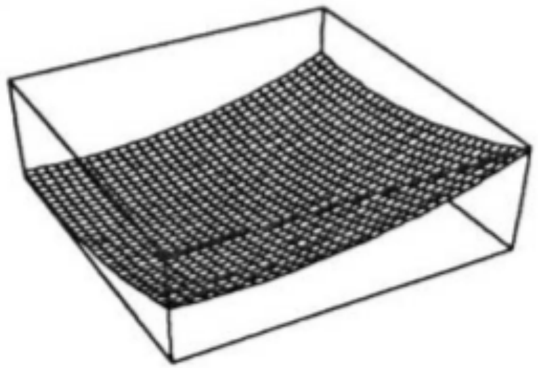
path from descent on gradient field

attractor field

repellor fields

combined potential

resulting gradient field



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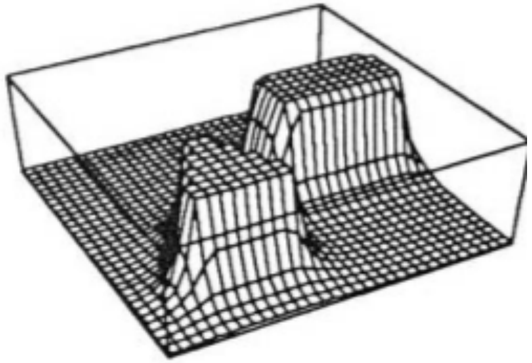

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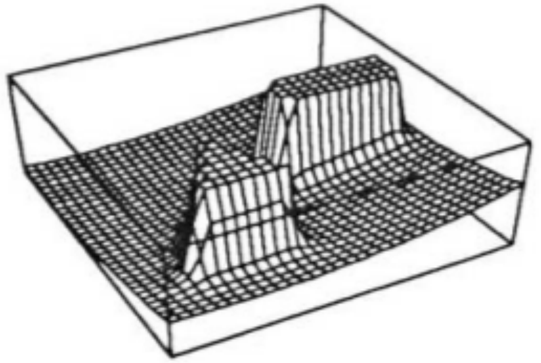
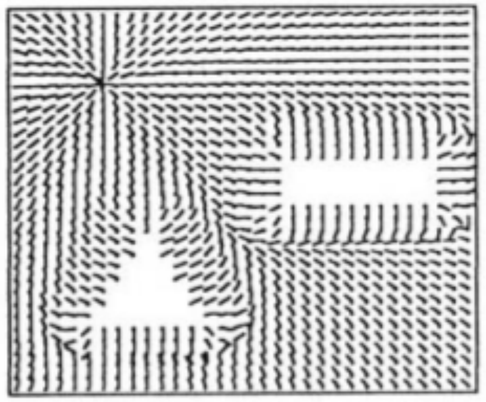
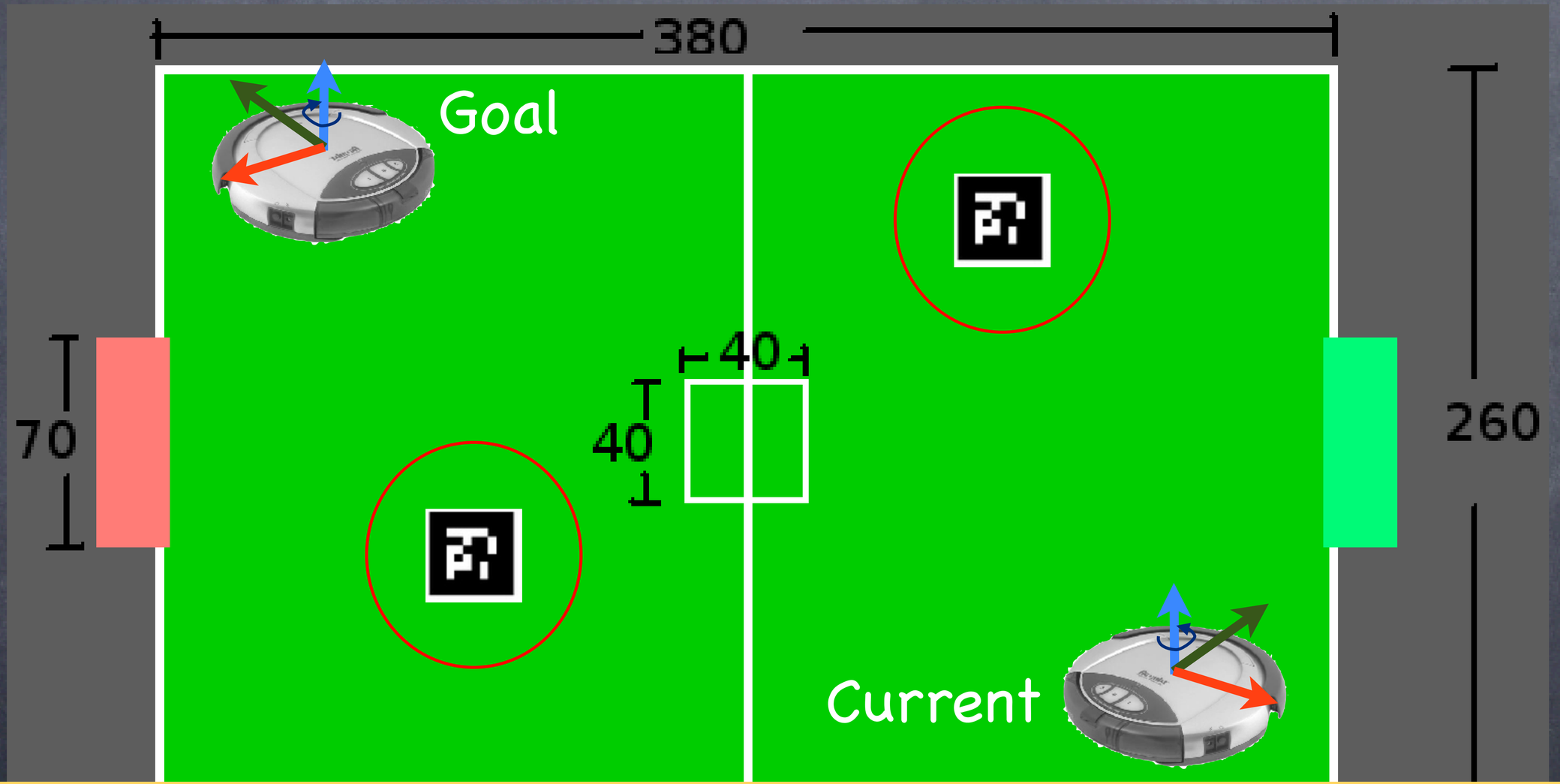




describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



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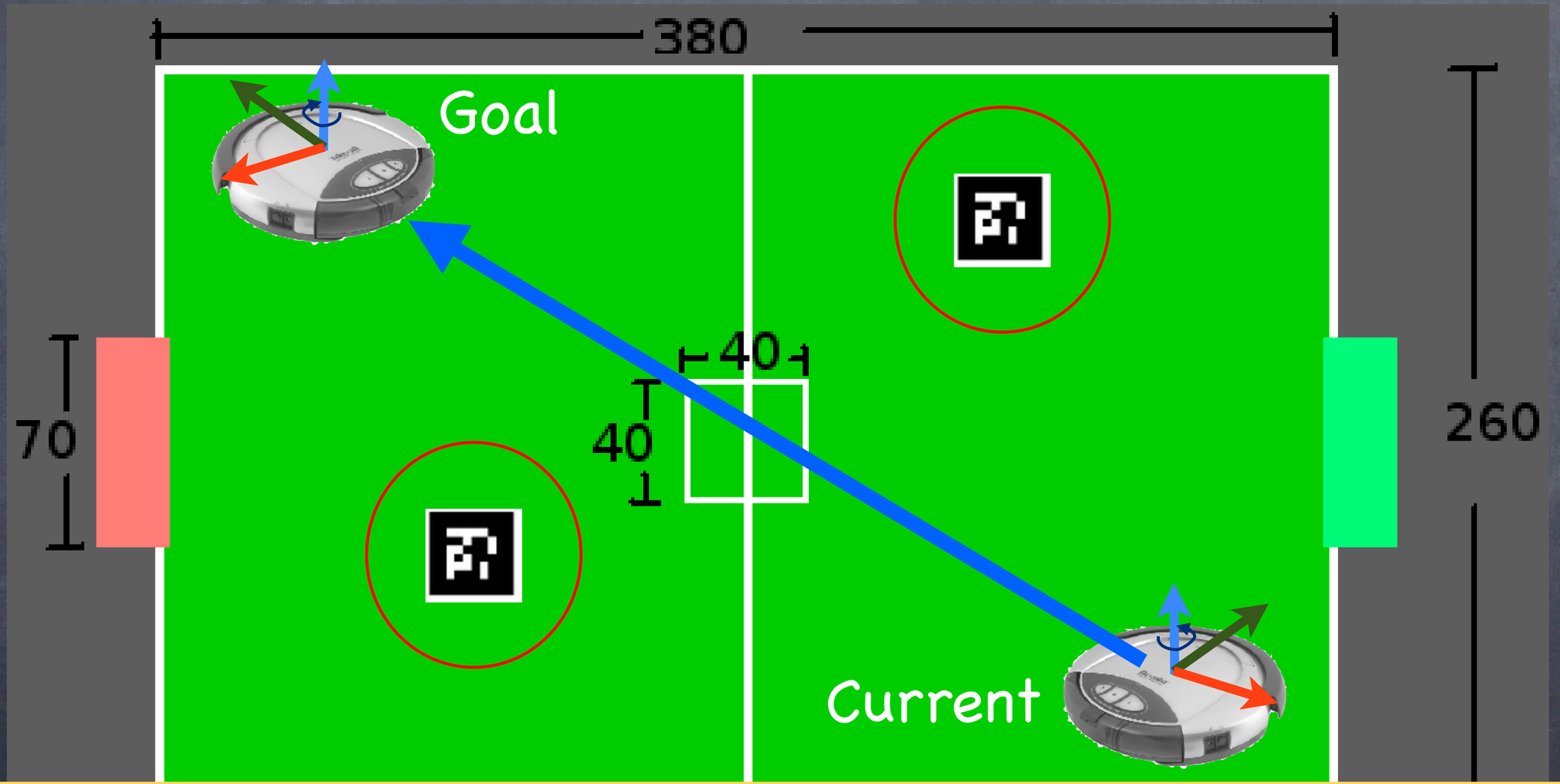
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



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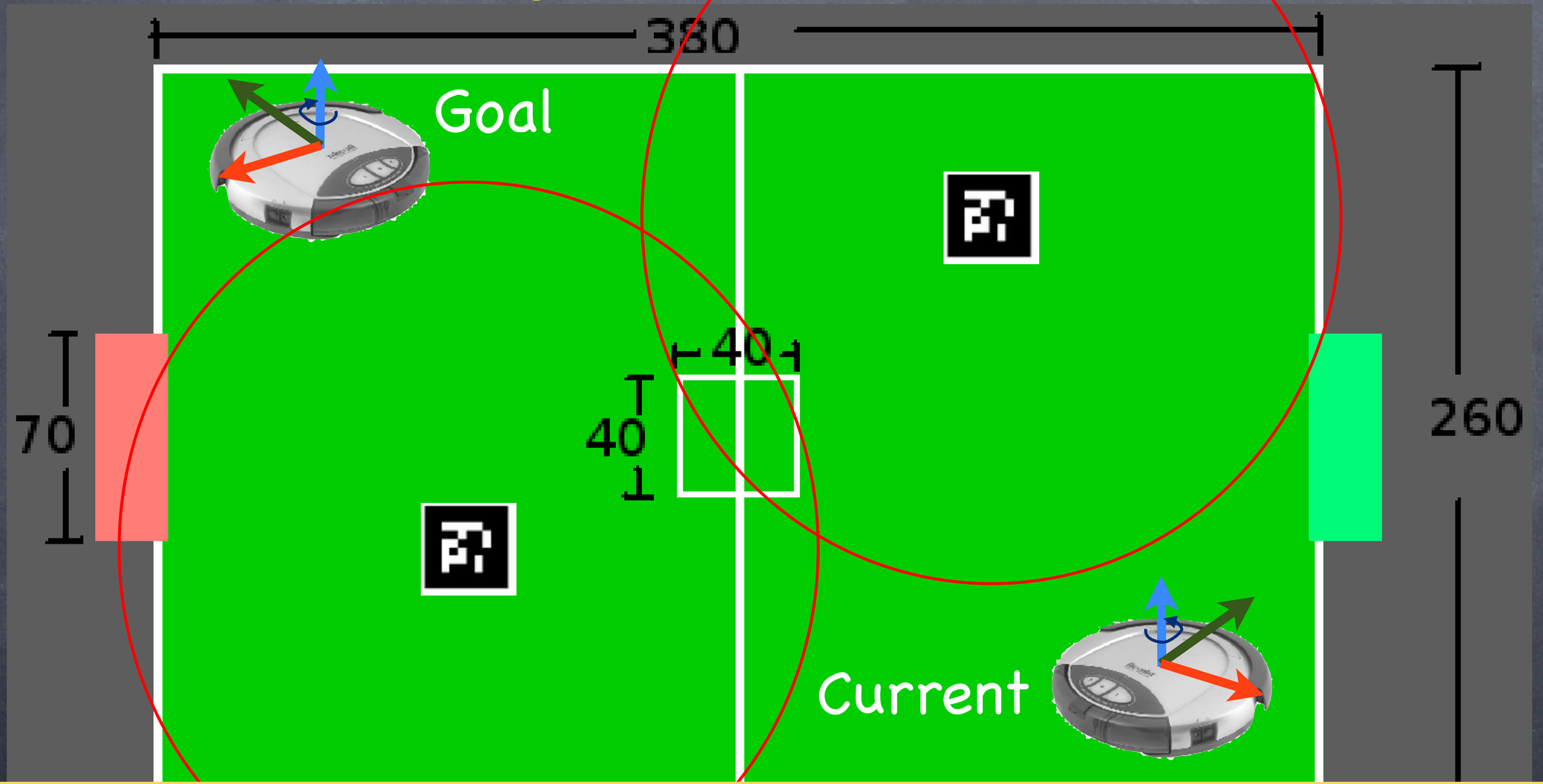

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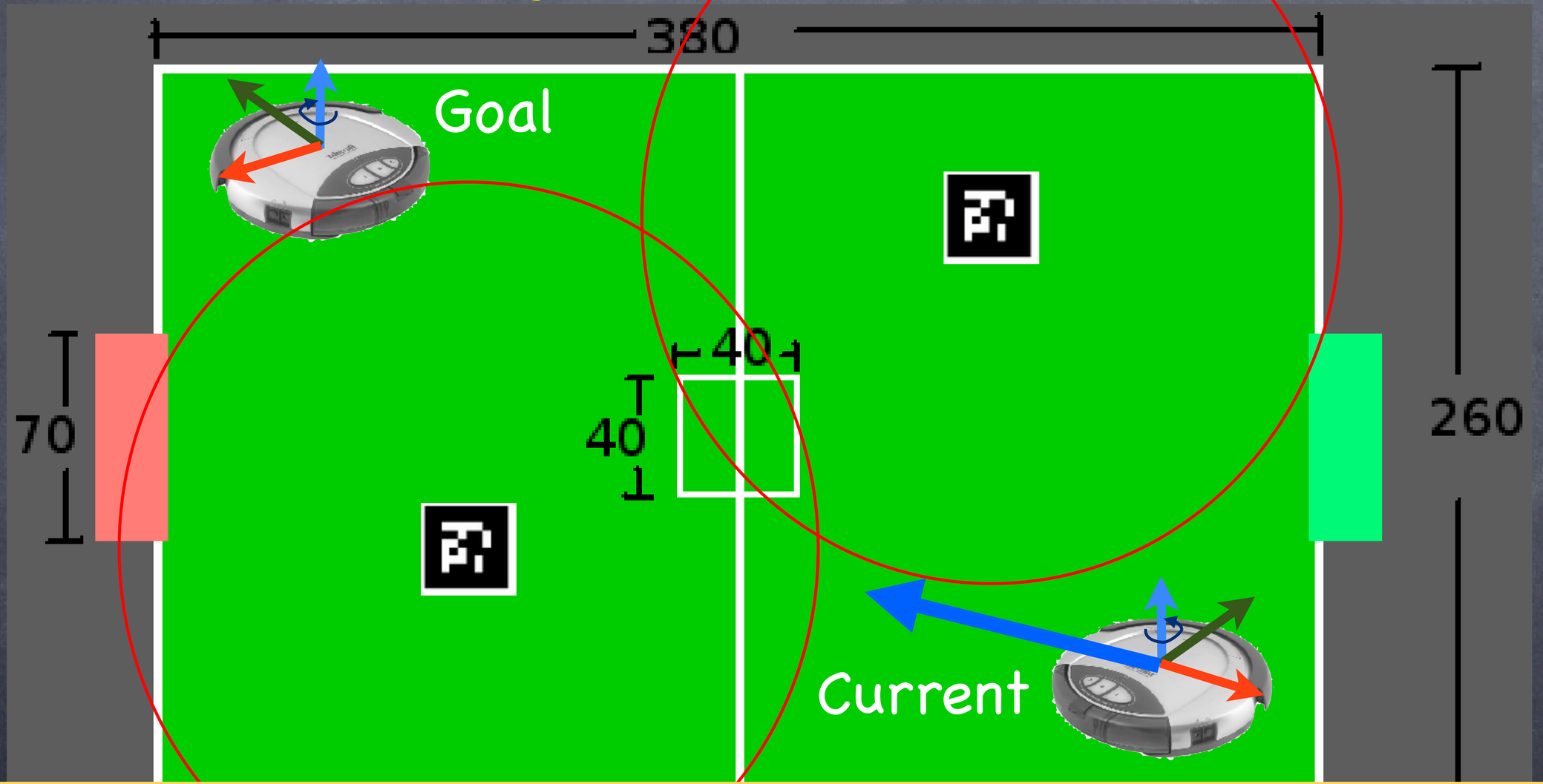

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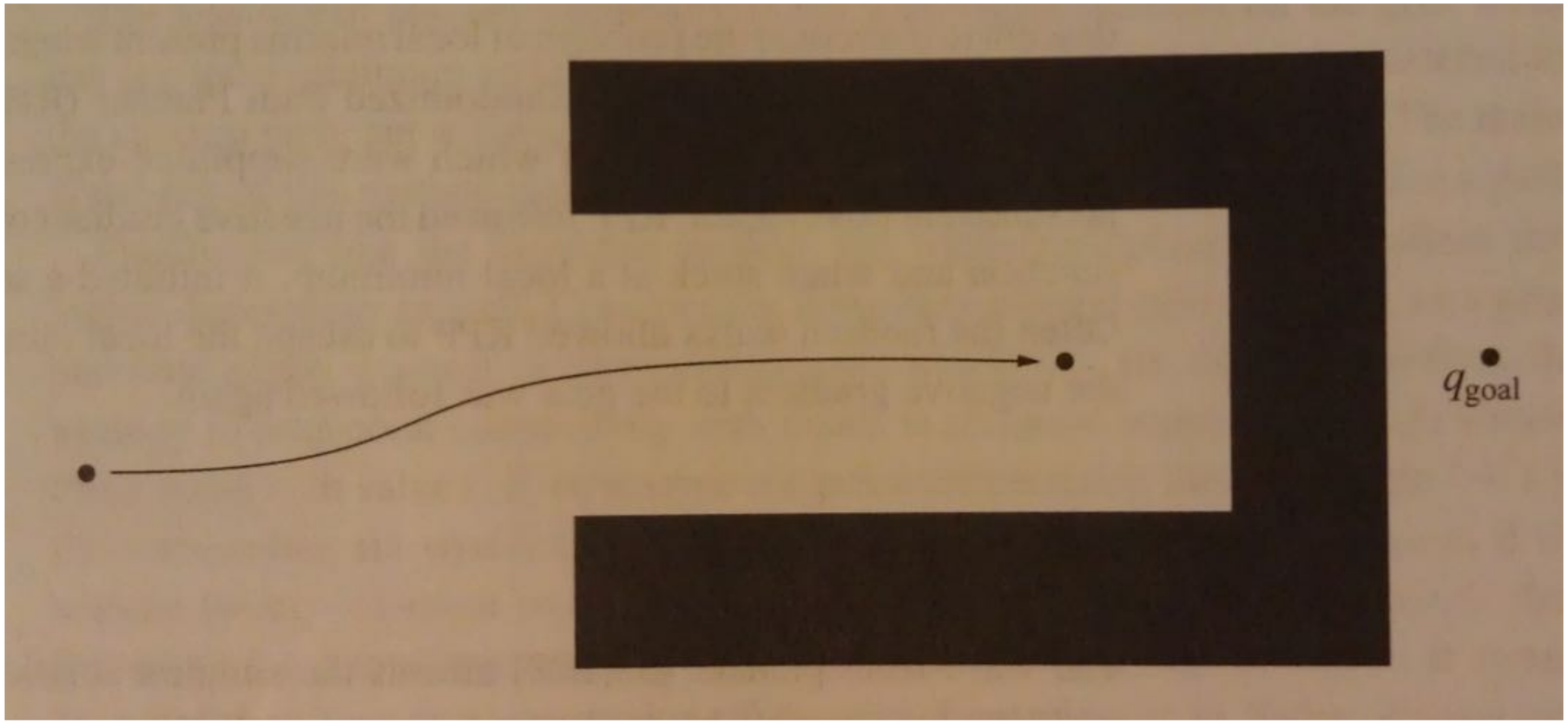
Local Minima



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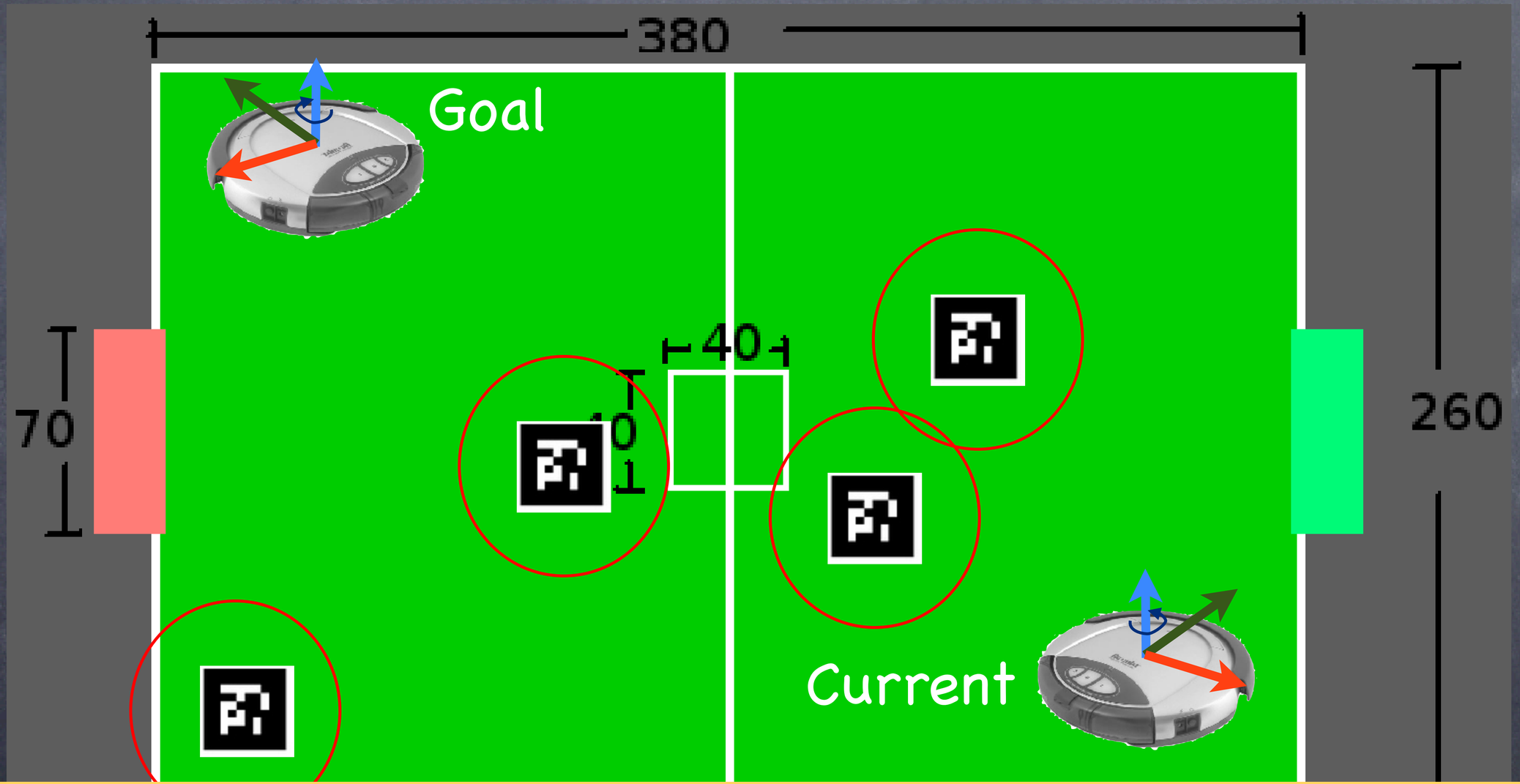
describe performance for this case



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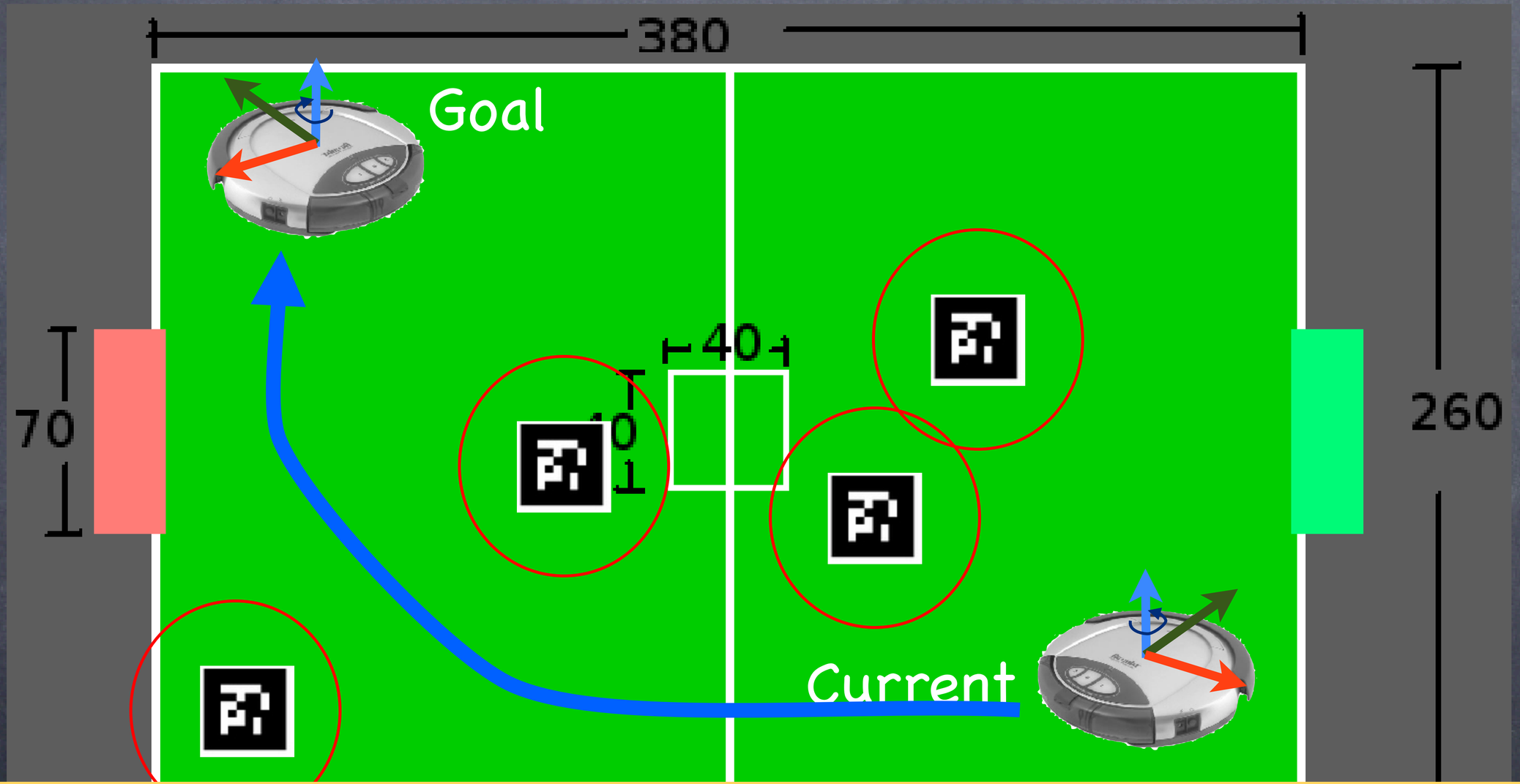
describe performance for this case



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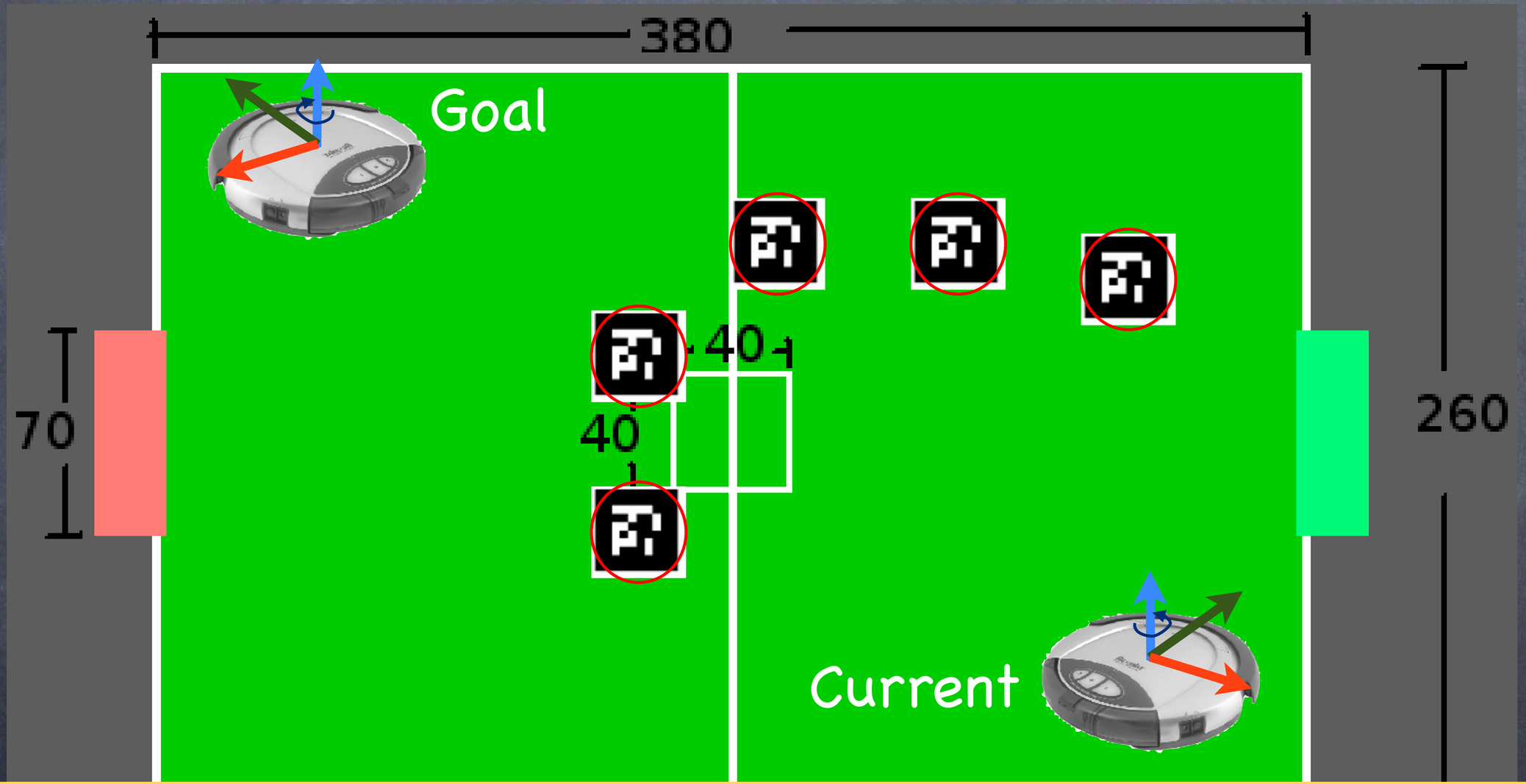
describe performance for this case



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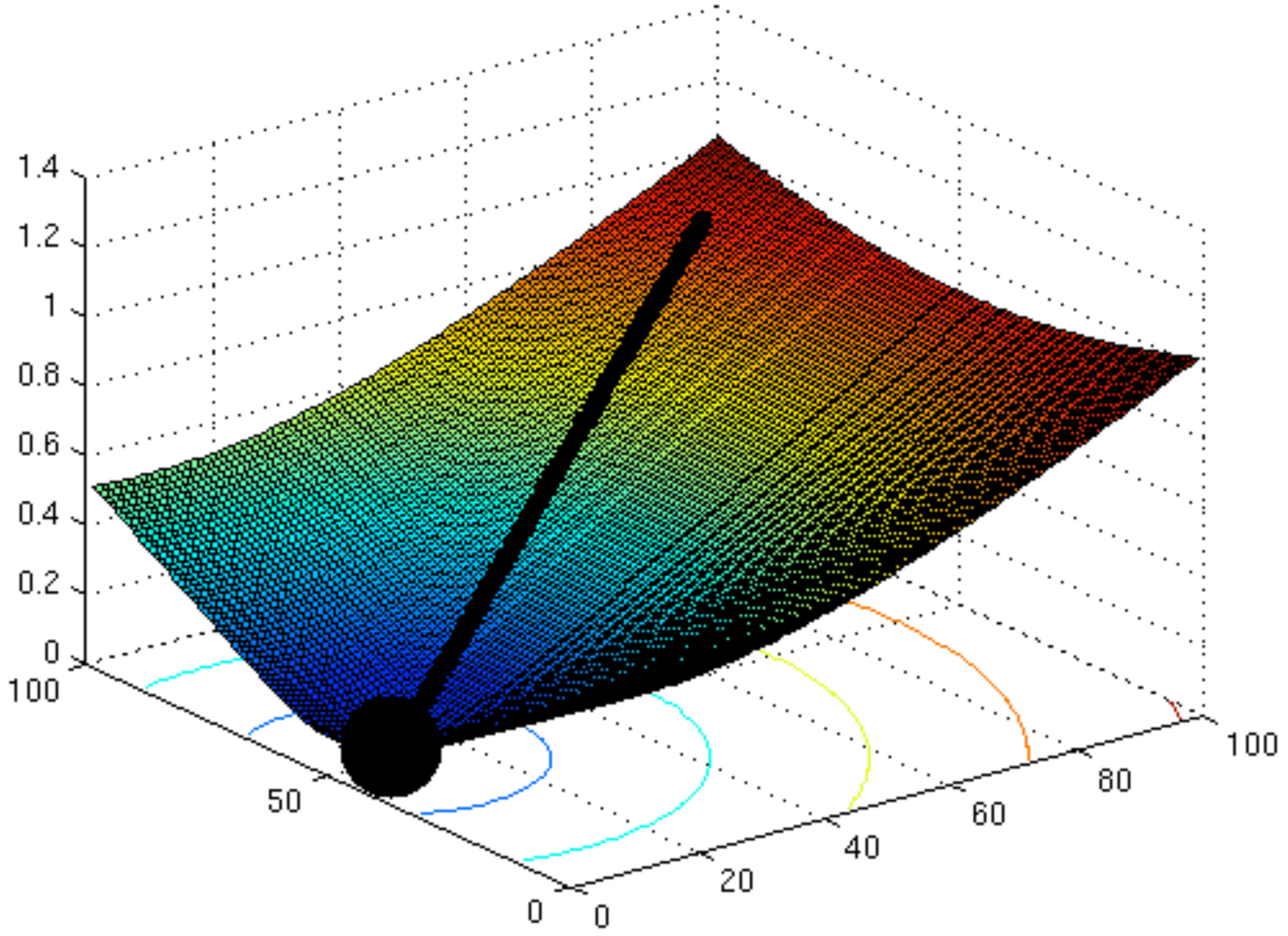
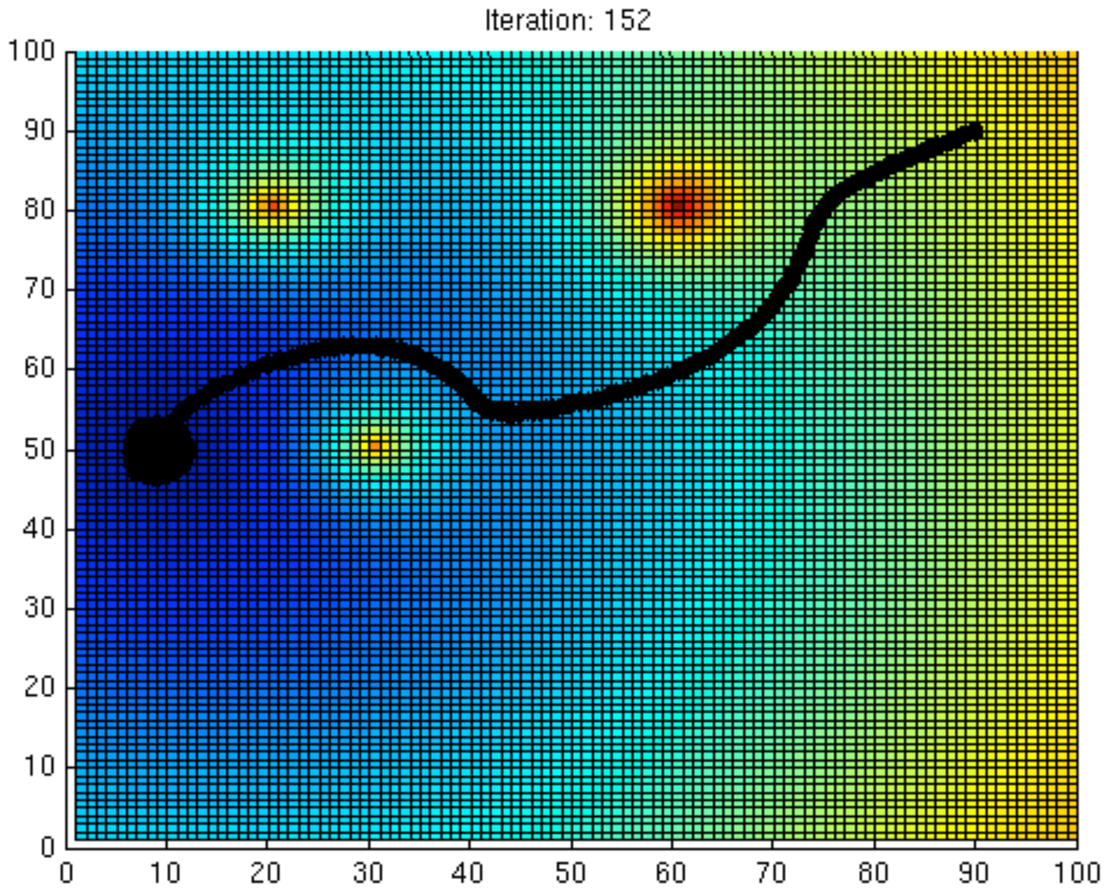
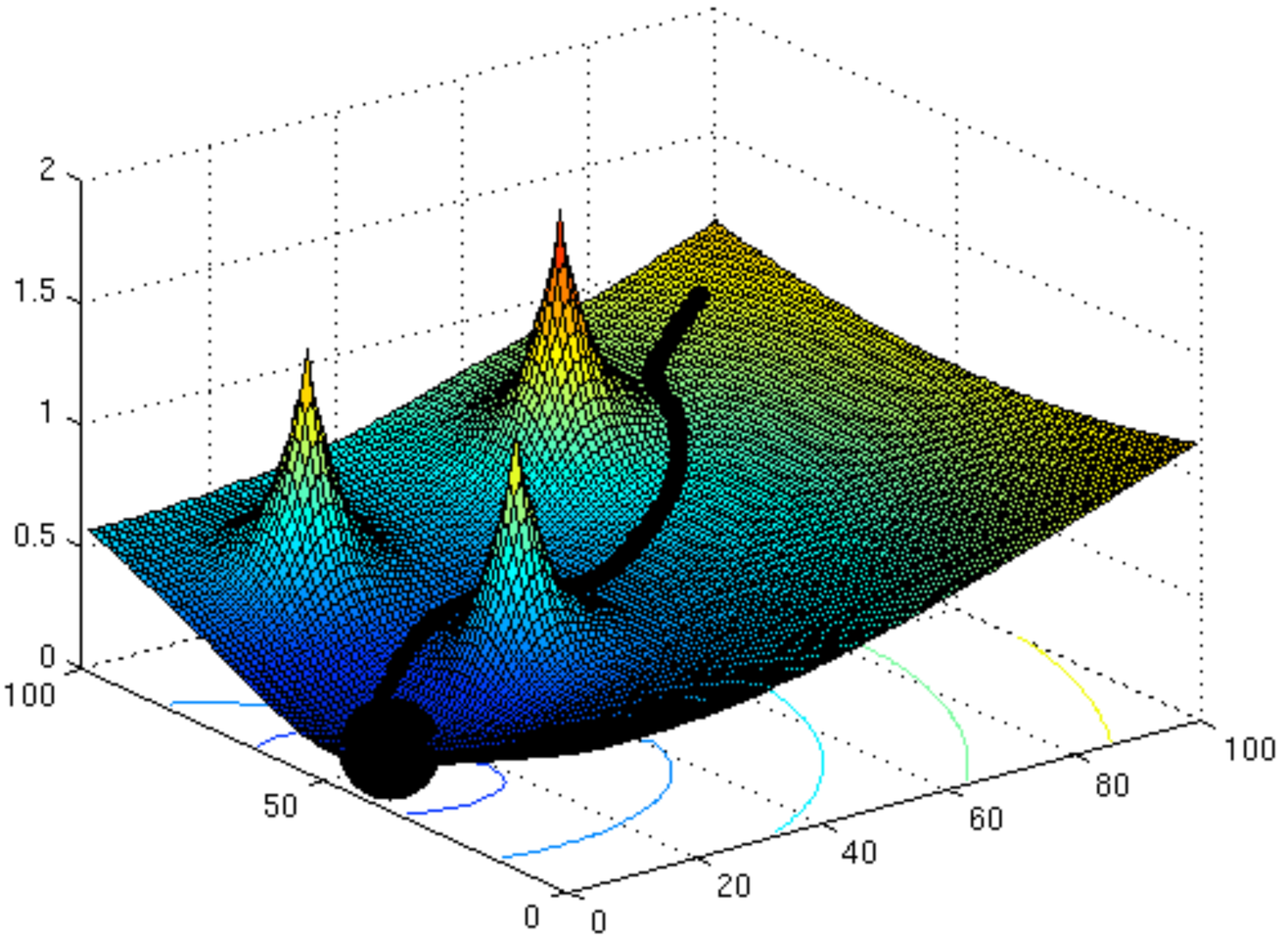
matlab example



pfield.m [1 5 8 12]

matlab example

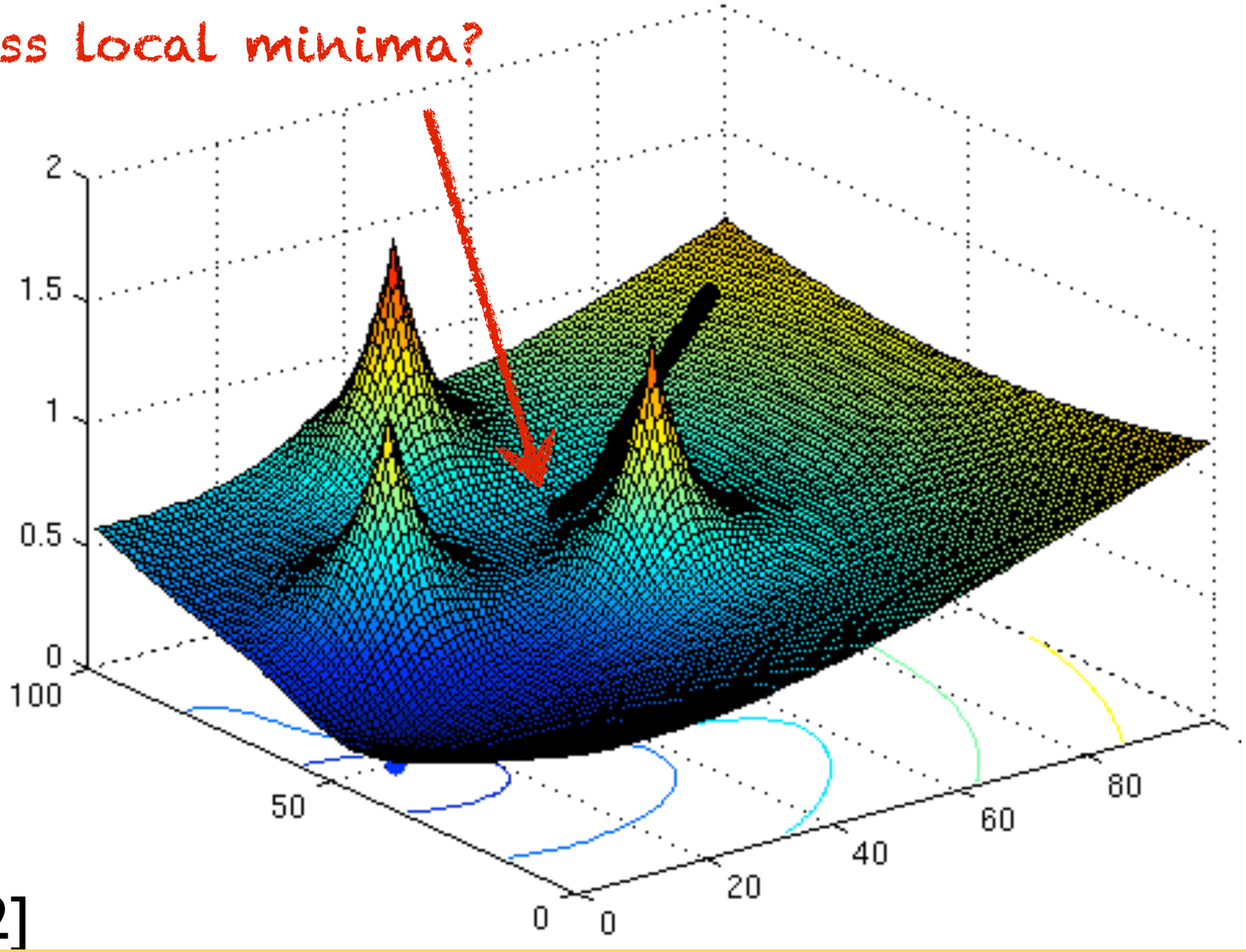




pfield.m [1 5 8 12]

matlab example

How to address local minima?



pfield.m [1 5 8 12]

How can we get out of local minima?

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How can we get out of local minima?

Go back to planning.

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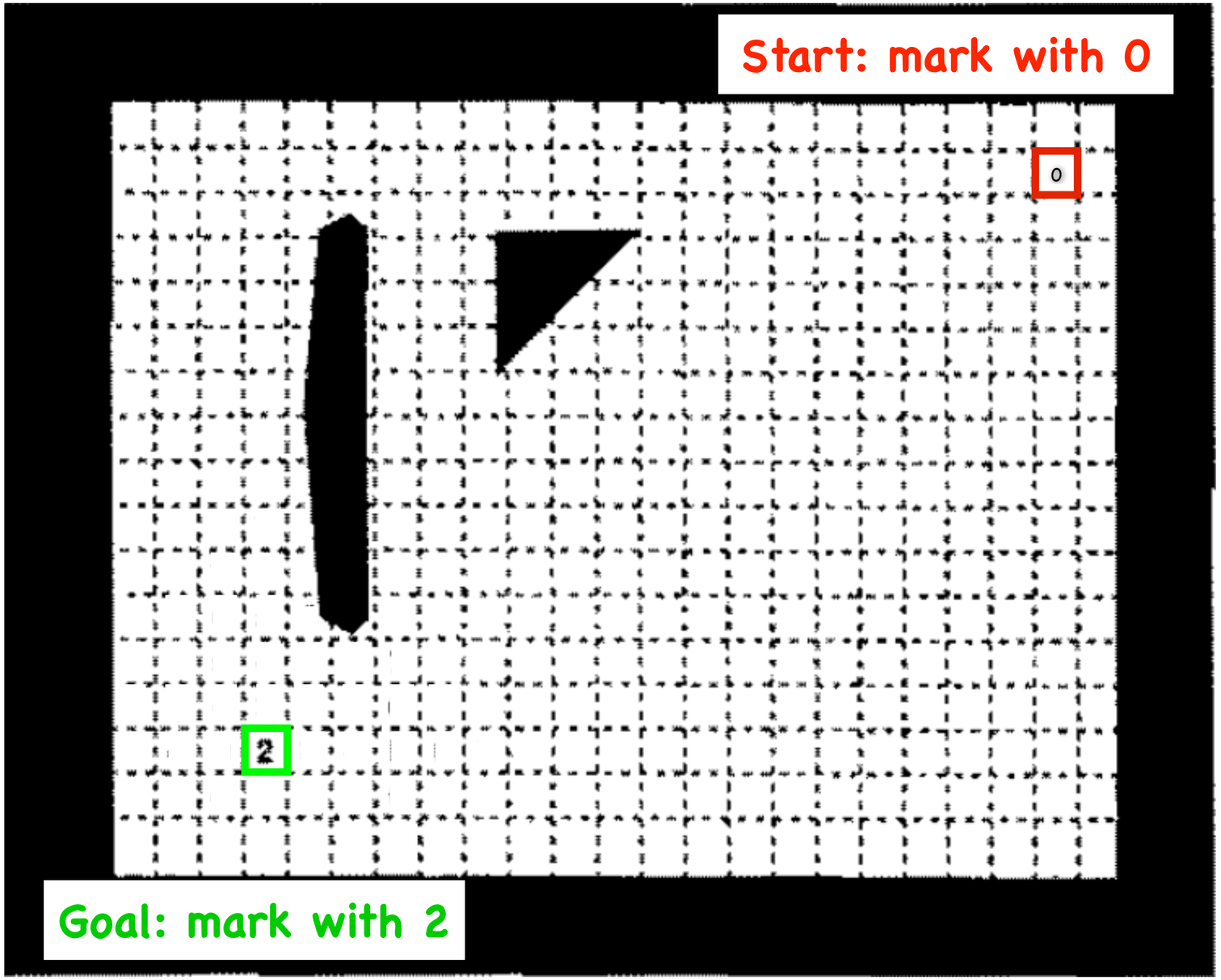
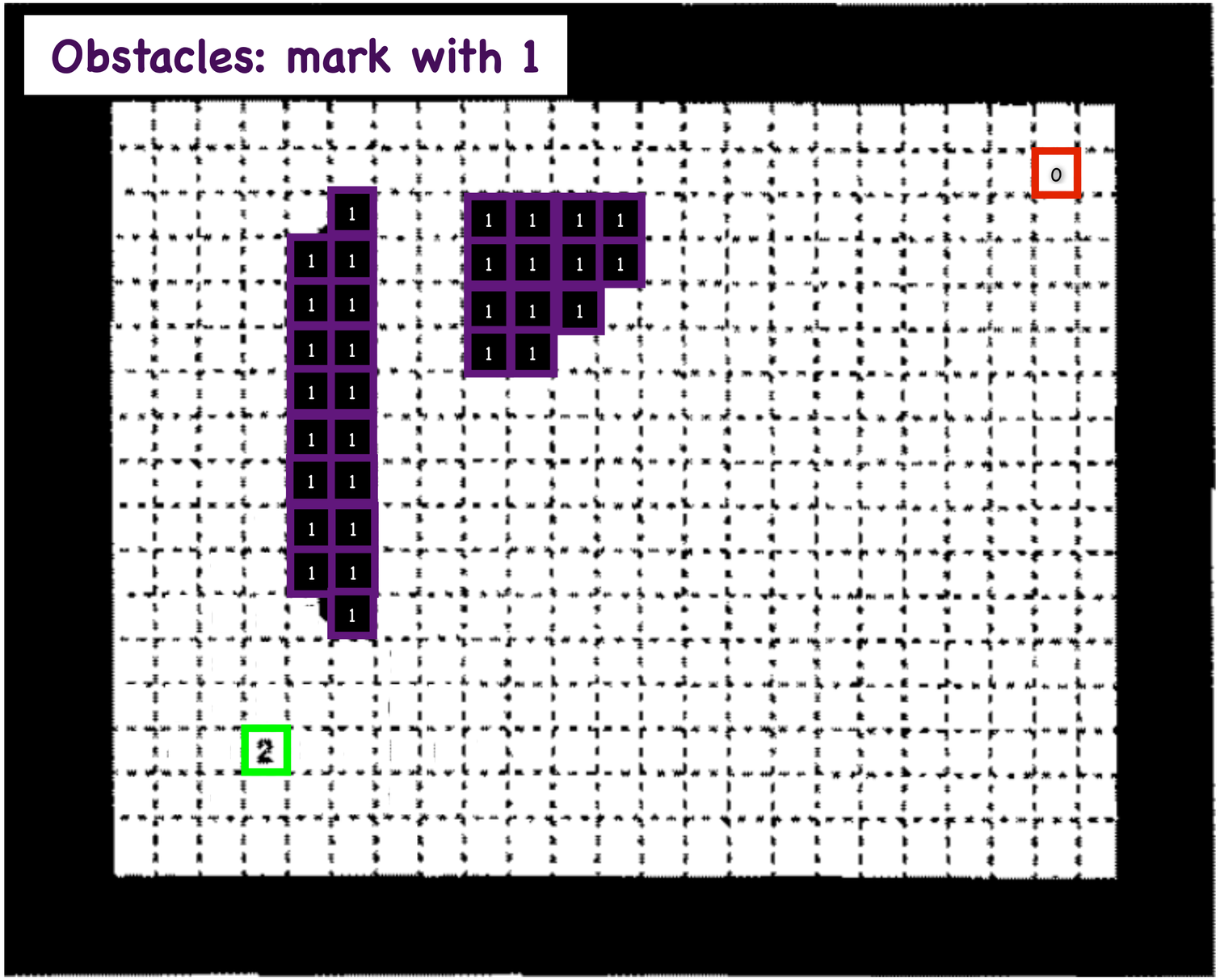
Wavefront Planning

- Discretize potential field into grid
  - Cells store cost to goal with respect to potential field
  - Computed by Brushfire algorithm (essentially BFS)
- Grid search to find navigation path to goal


Start: mark with 0
0
2
Goal: mark with 2

Obstacles: mark with 1



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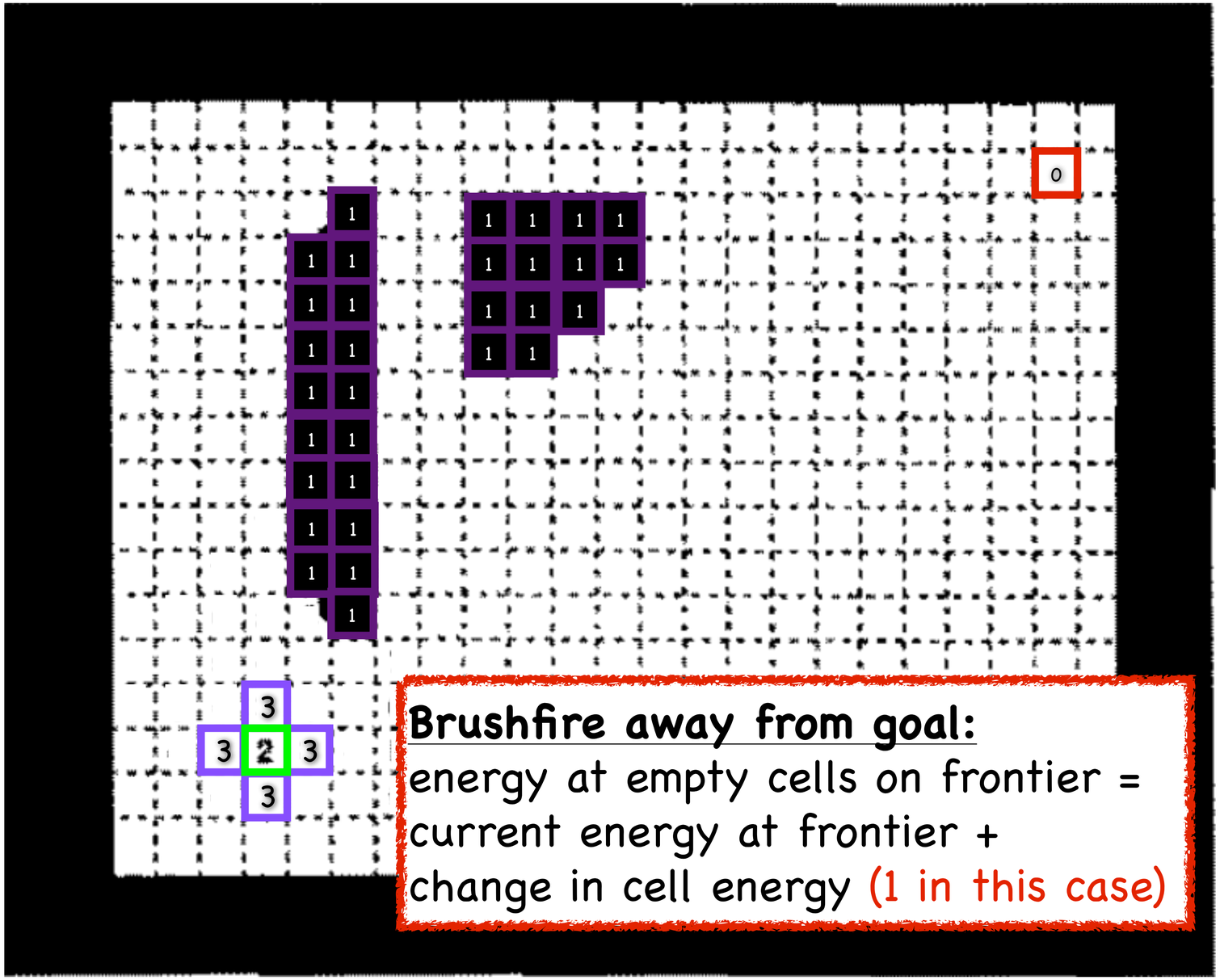
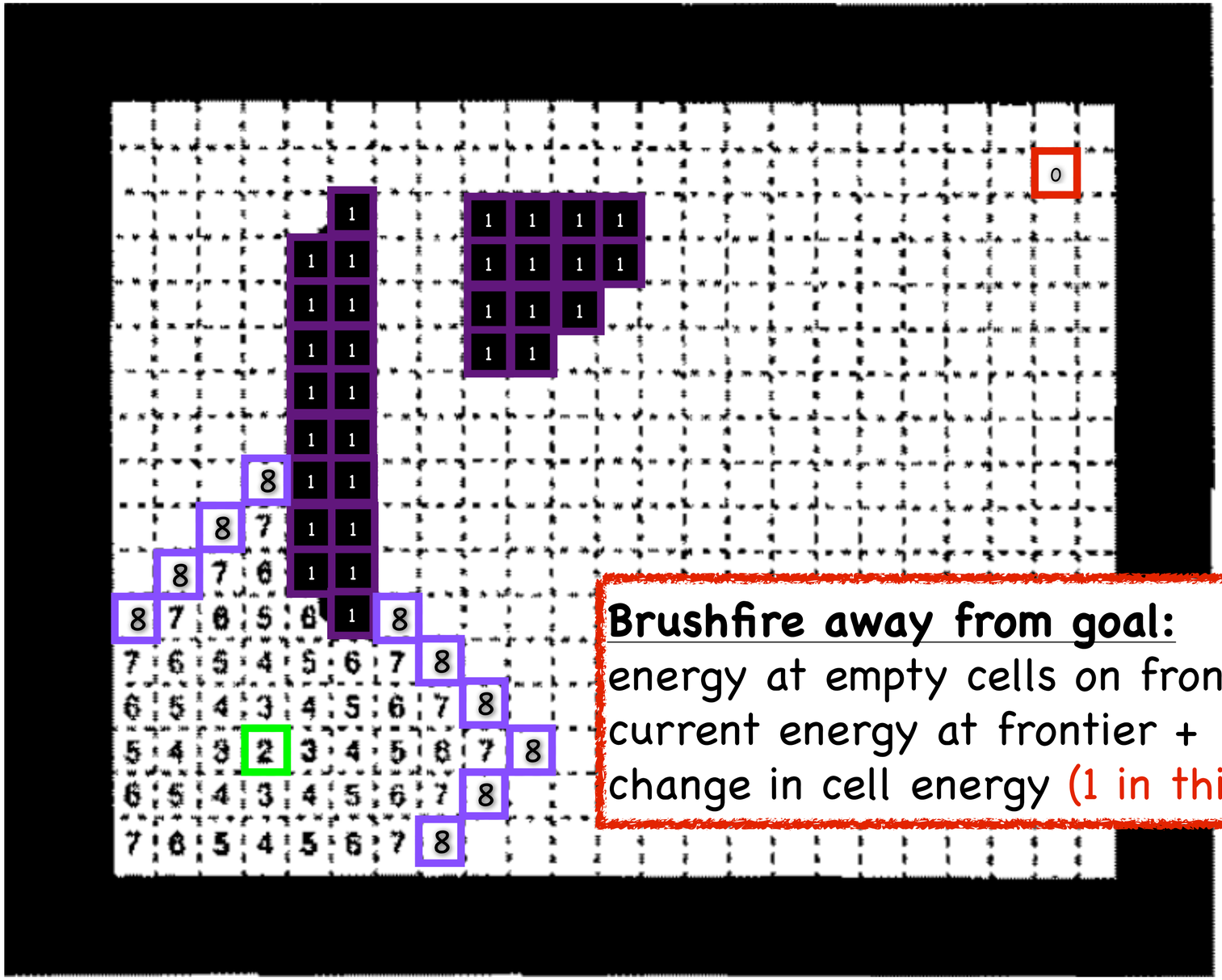
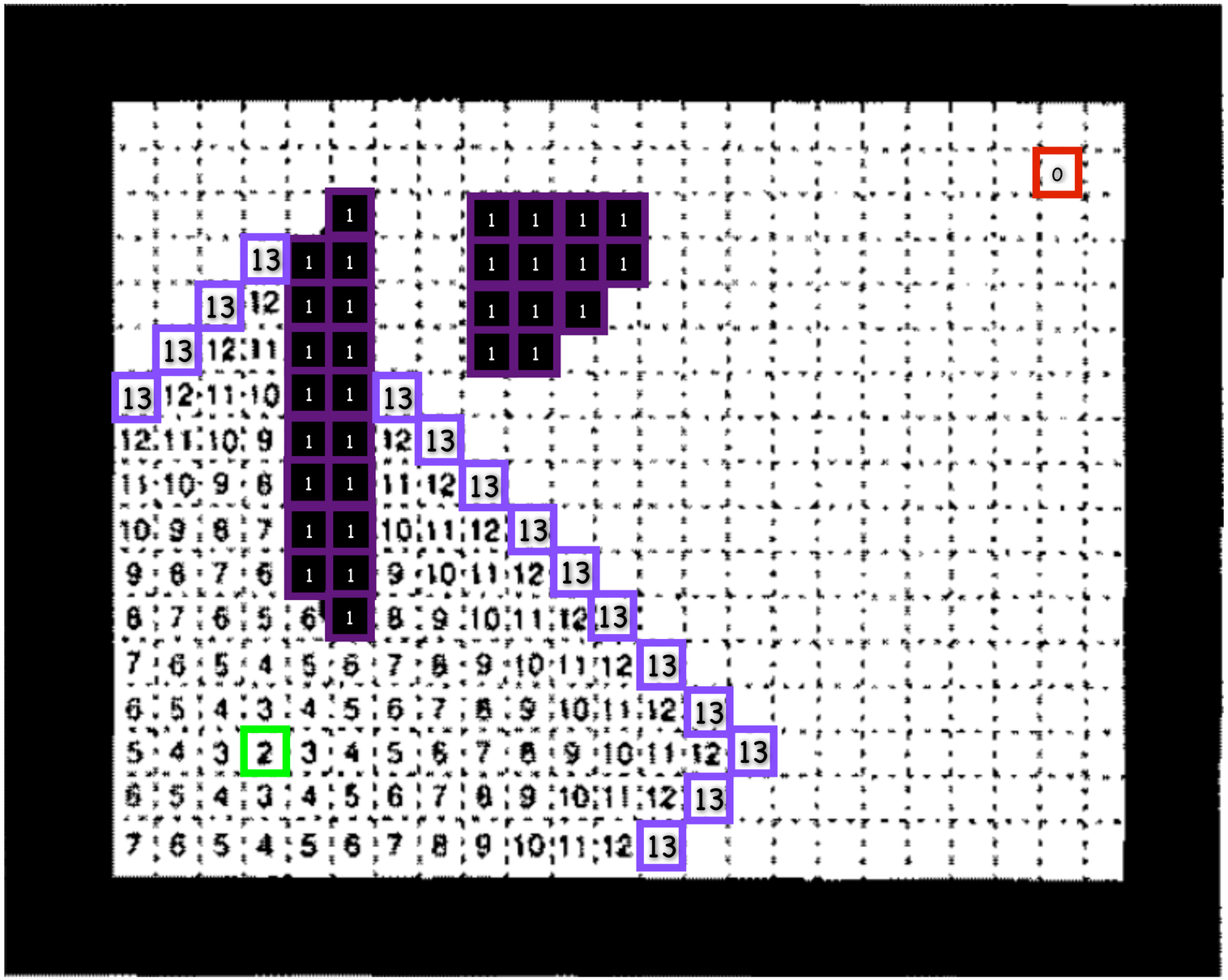


Brushfire away from goal:
energy at empty cells on frontier =
current energy at frontier +
change in cell energy (1 in this case)


0
Brushfire away from goal:
energy at empty cells on frontier =
current energy at frontier +
change in cell energy (1 in this case)



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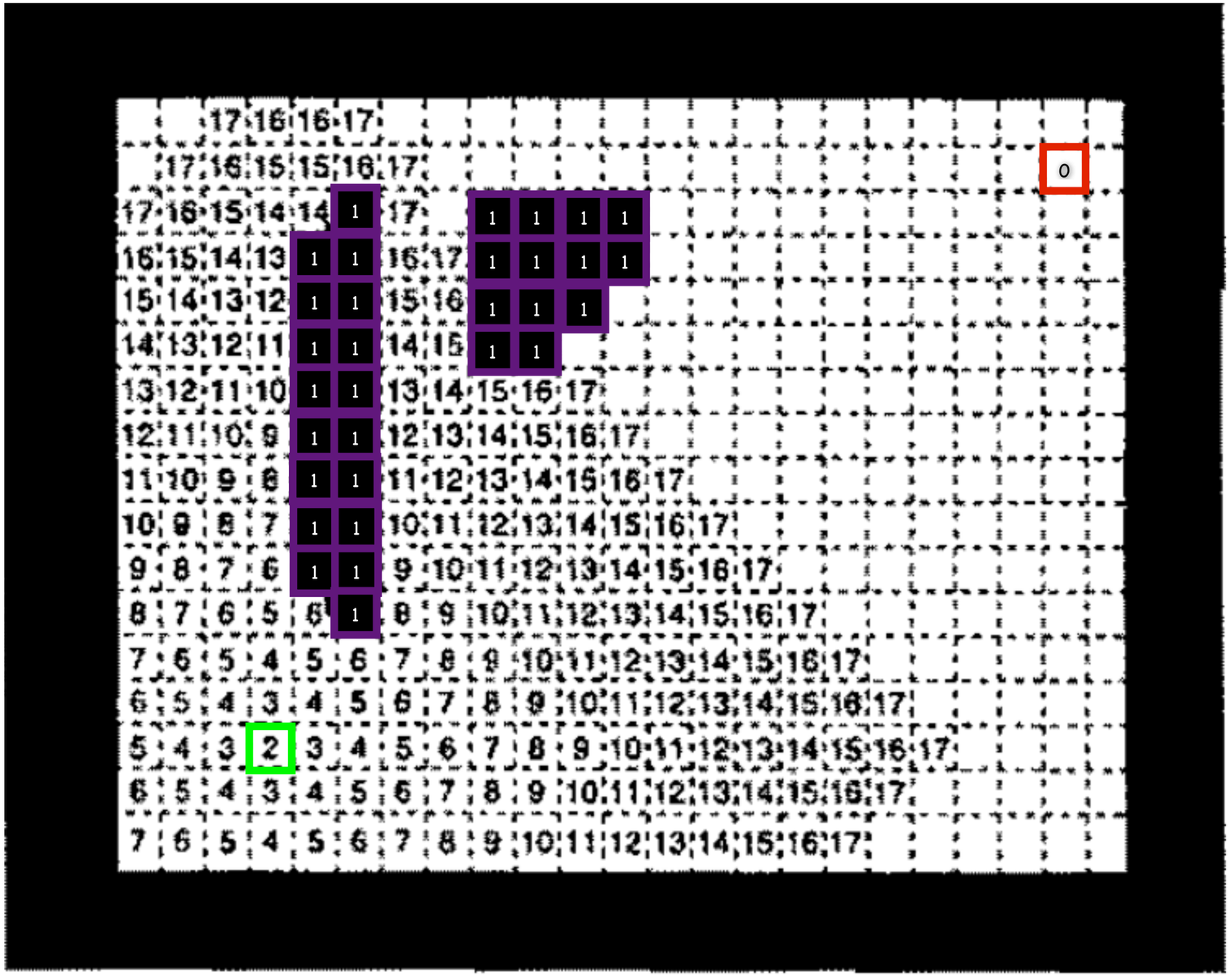

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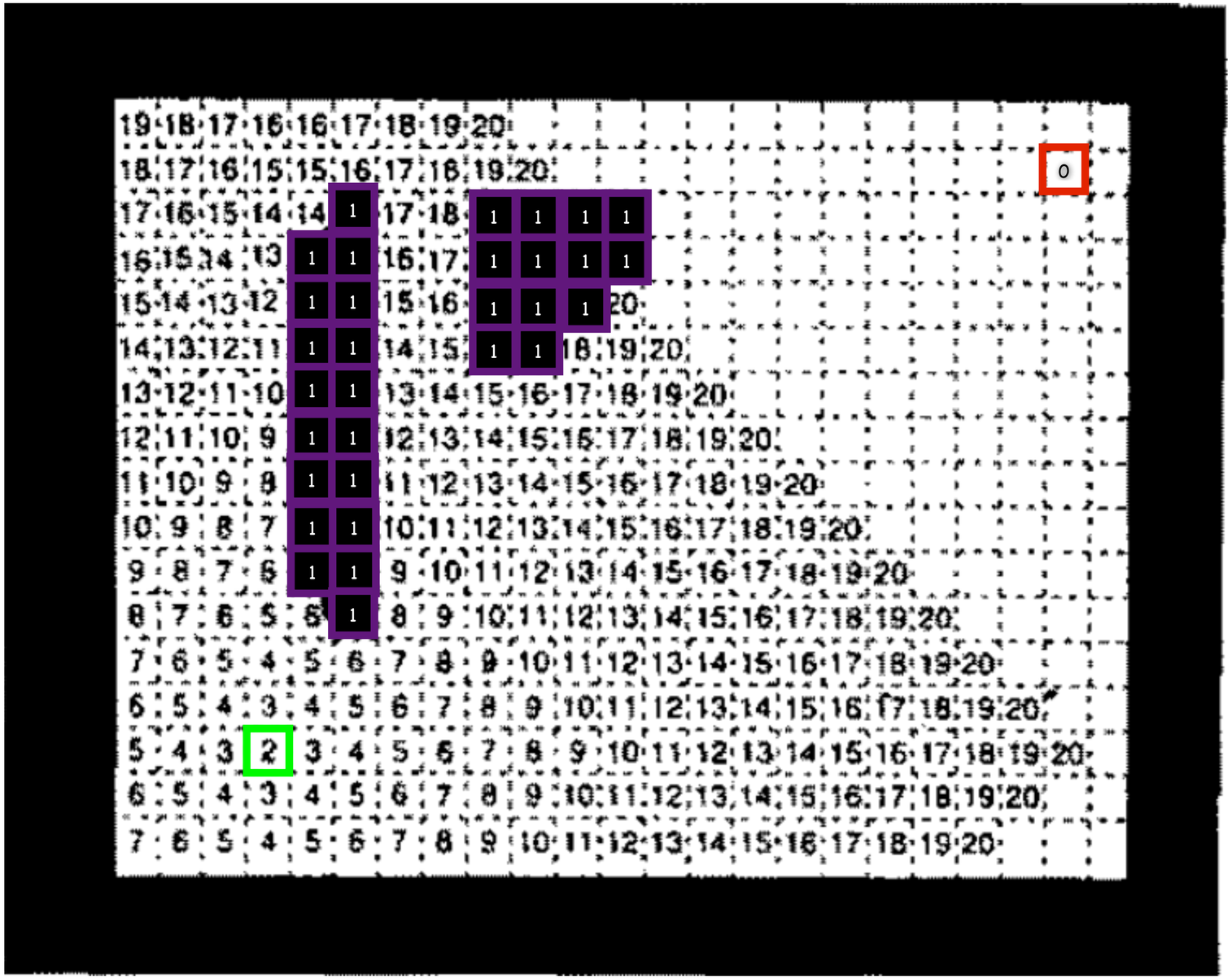
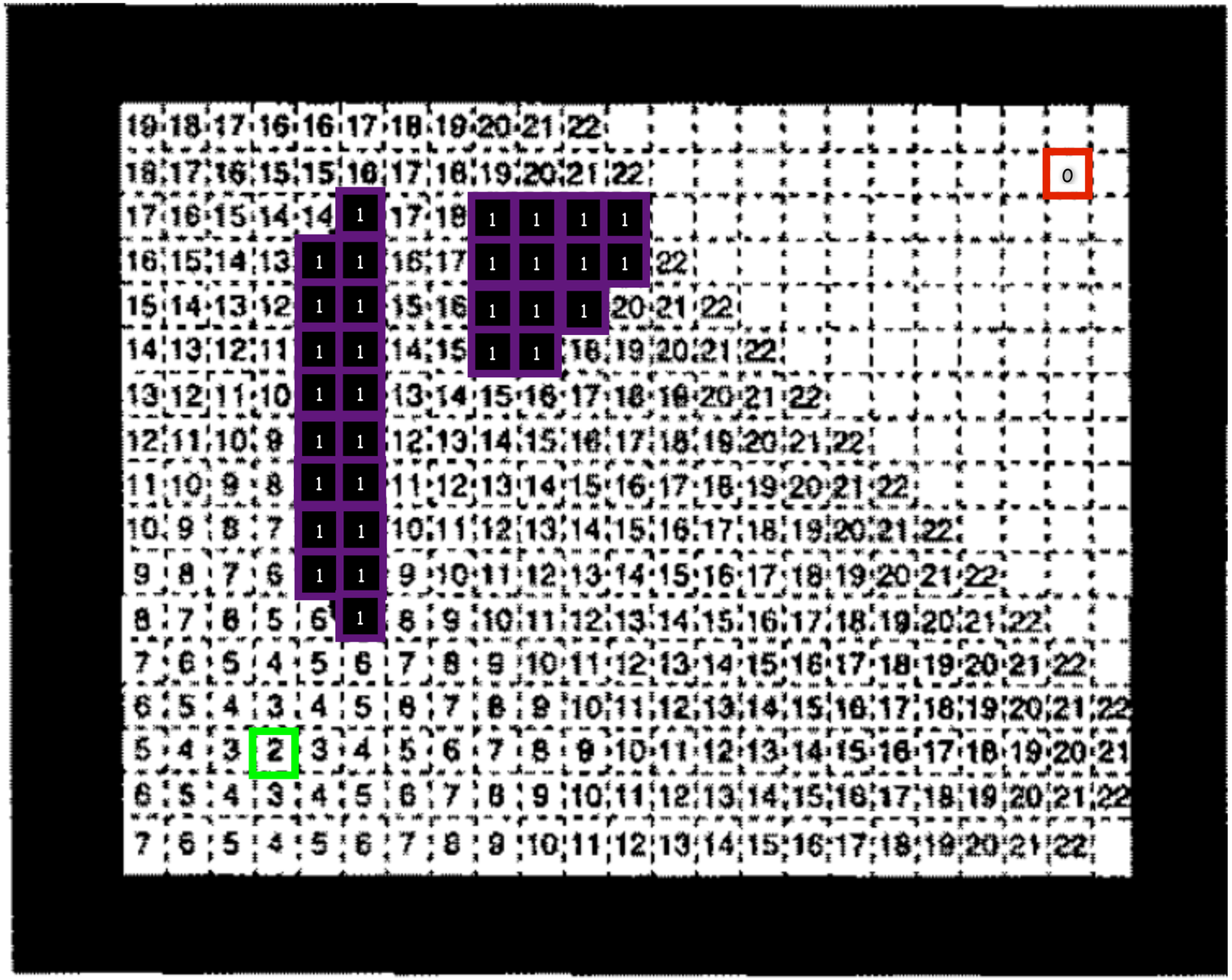

0
2



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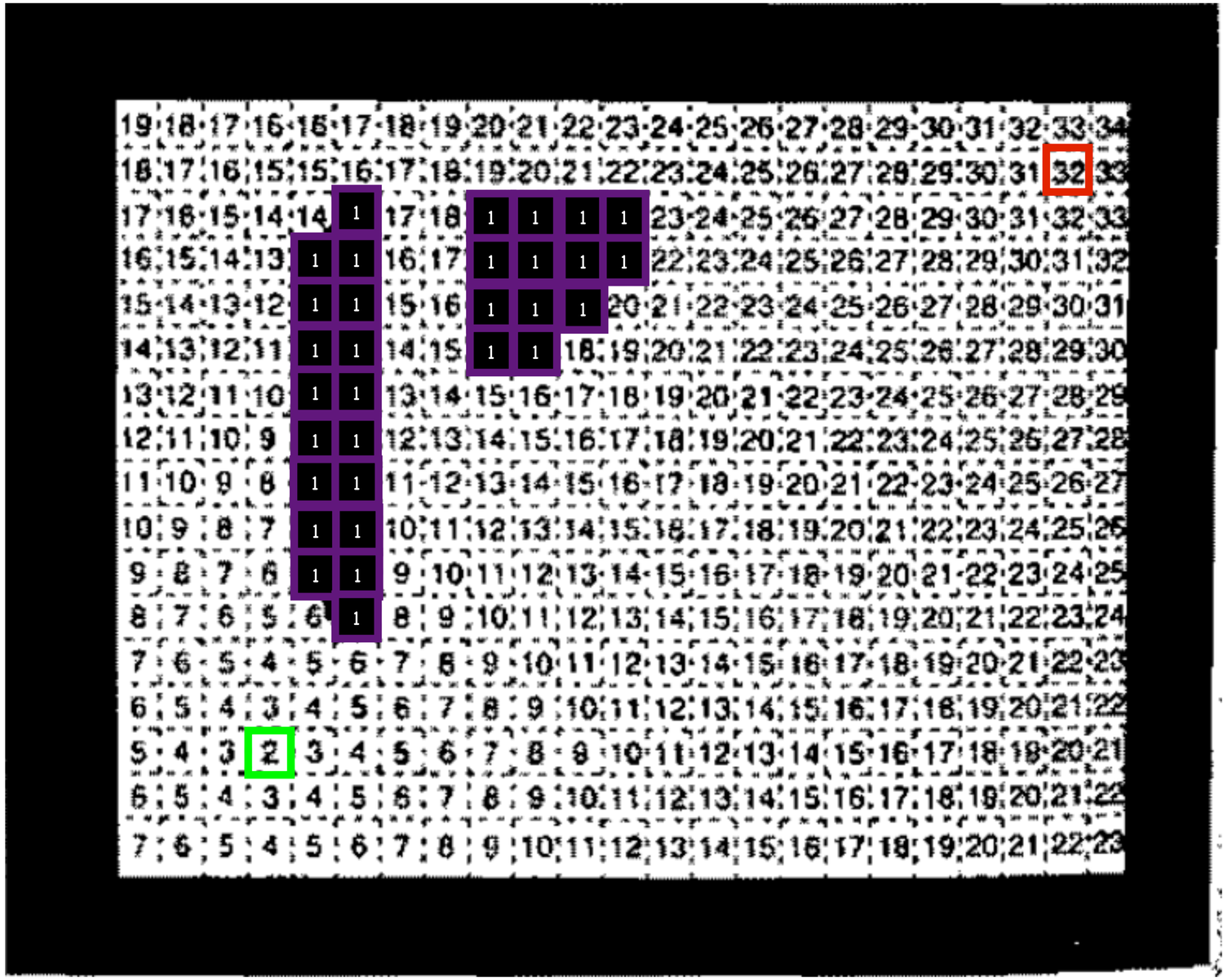
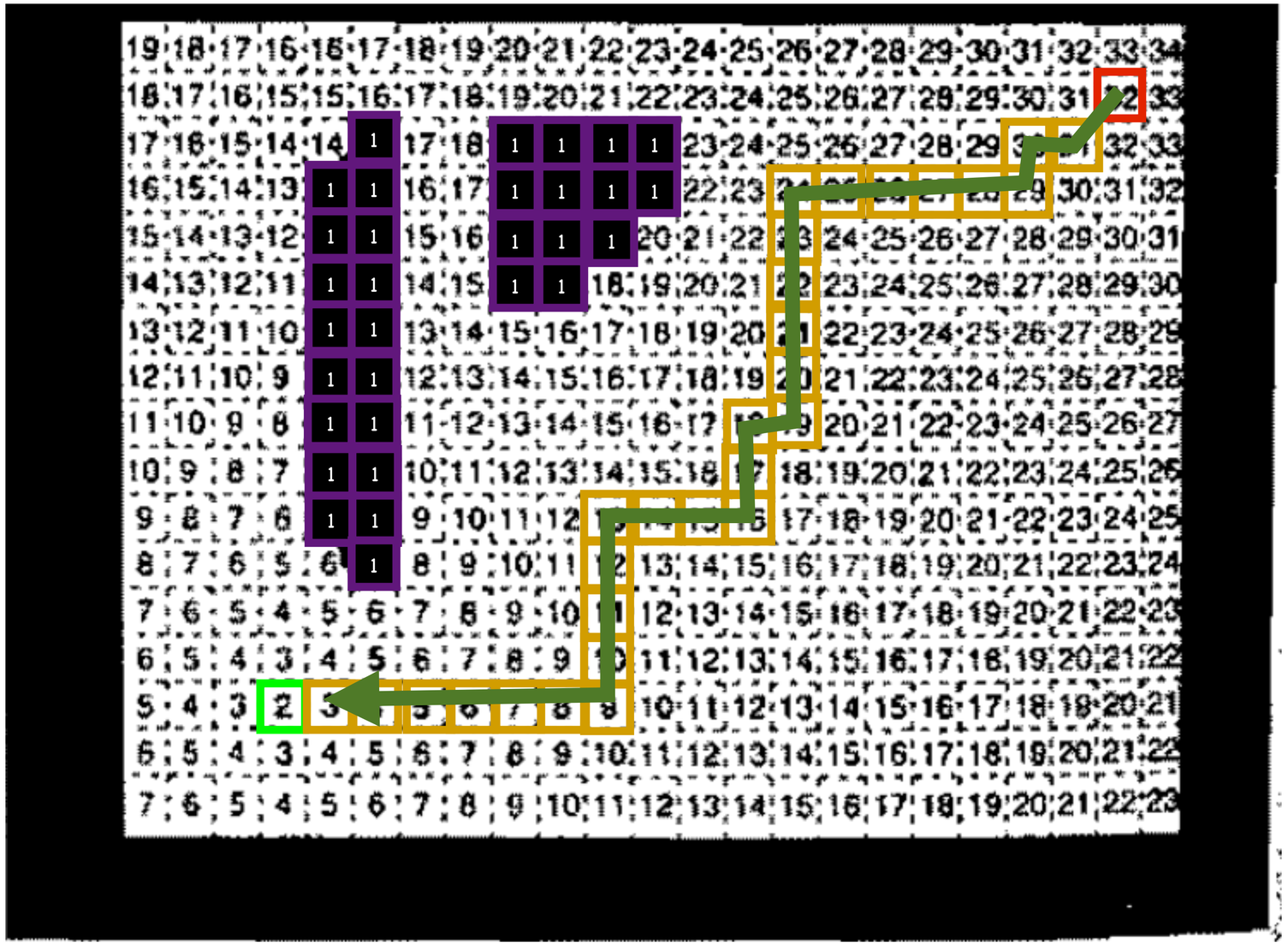


Once start reached, follow brushfire potential to goal



Example with Local Minima

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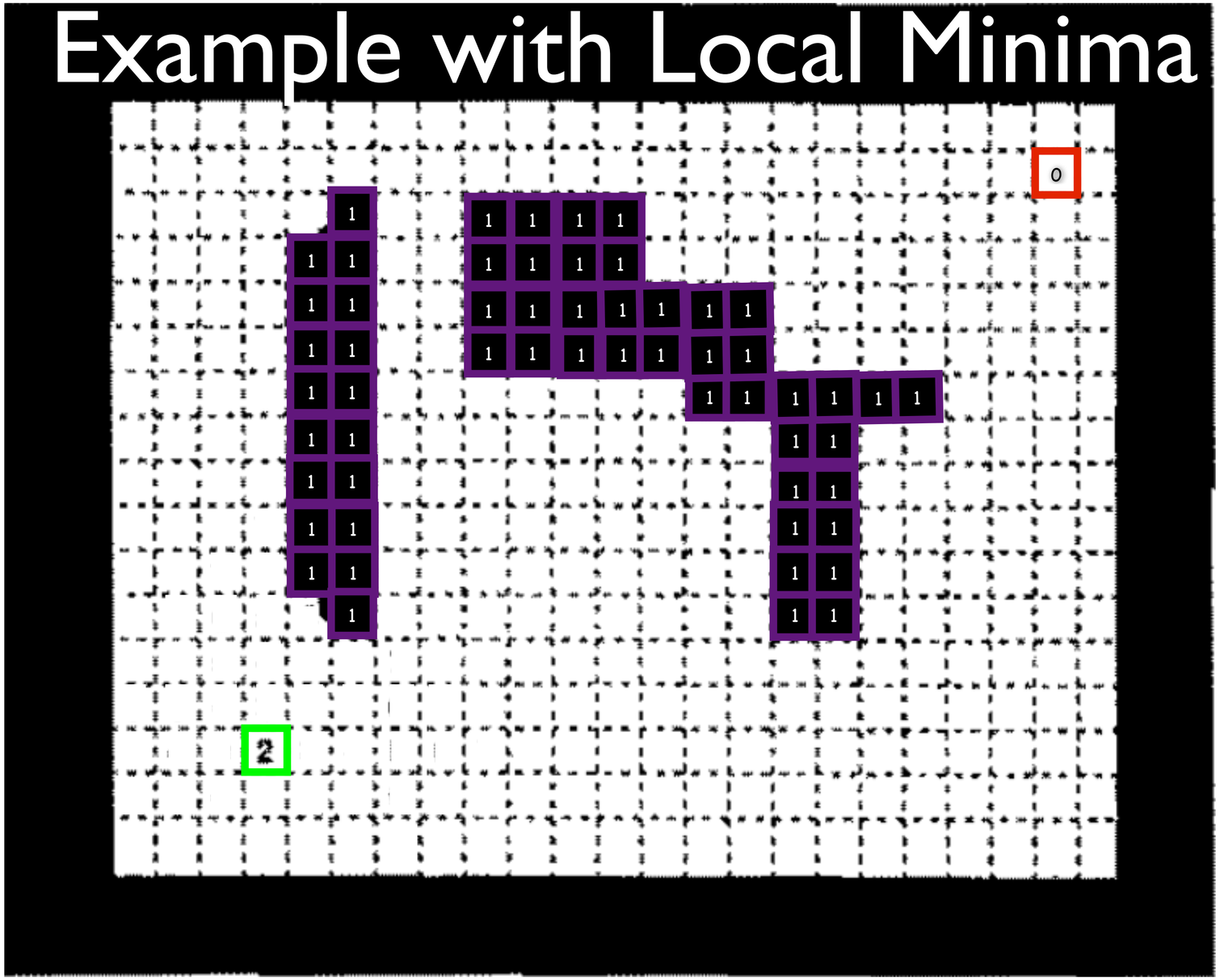
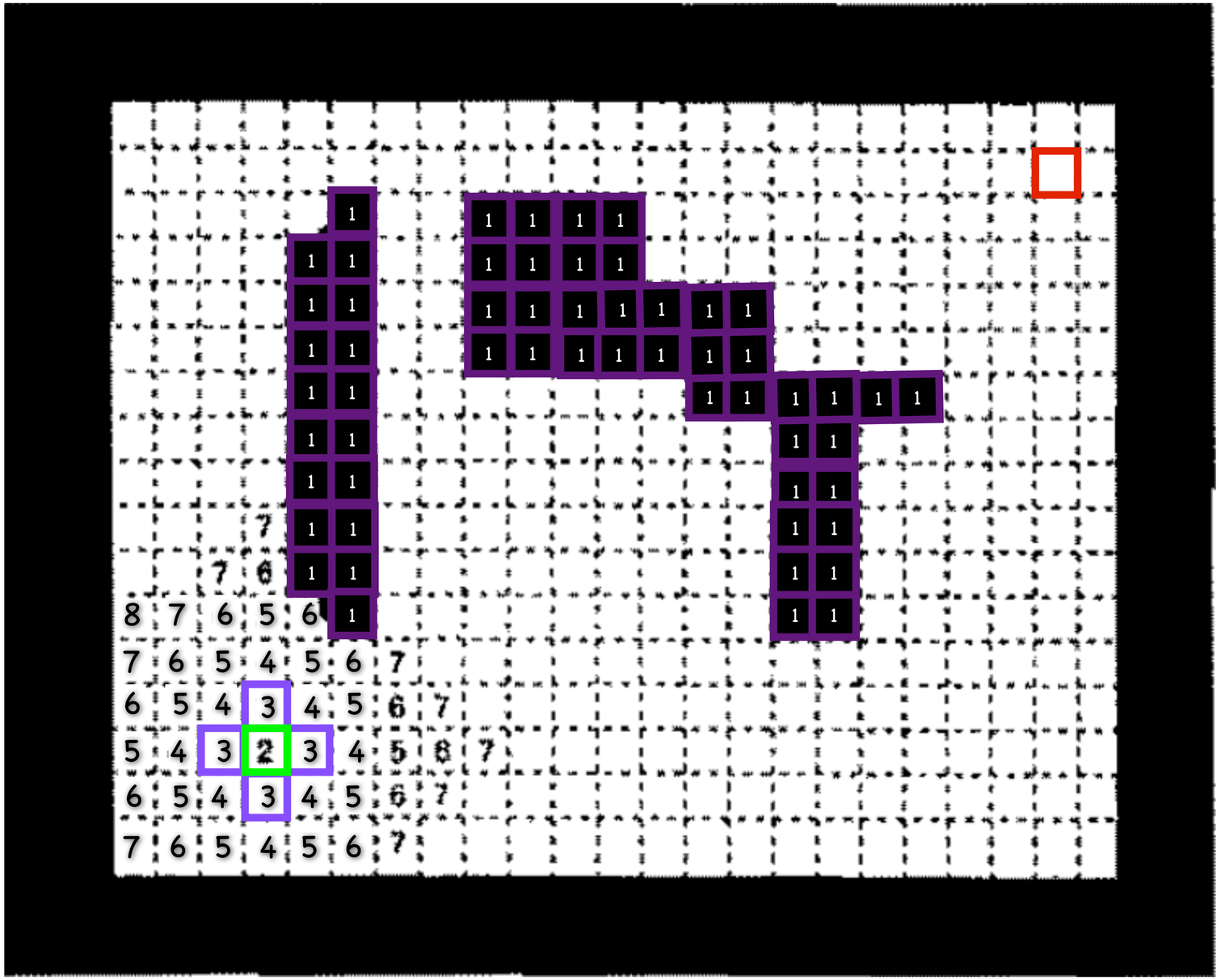
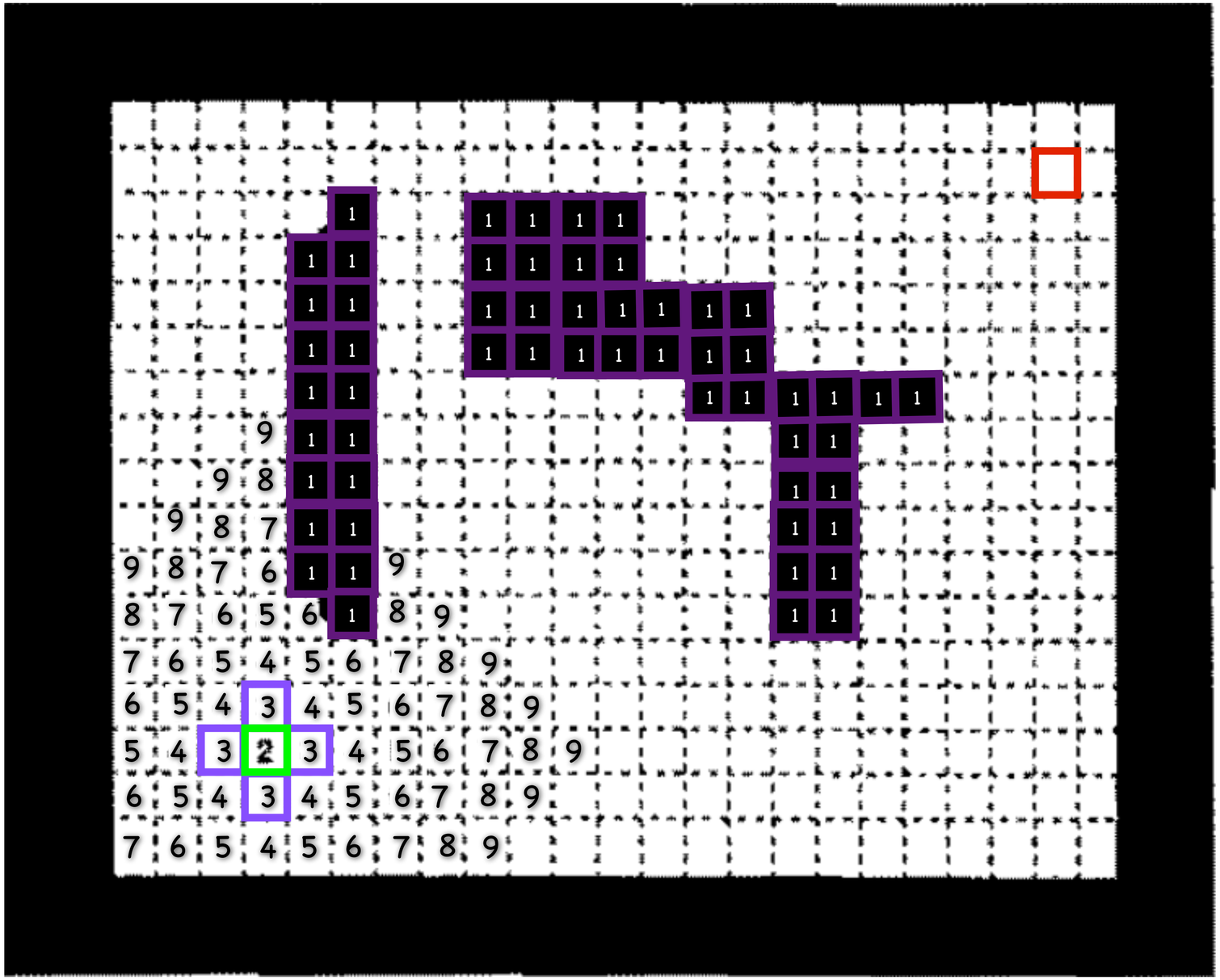
Example with Local Minima







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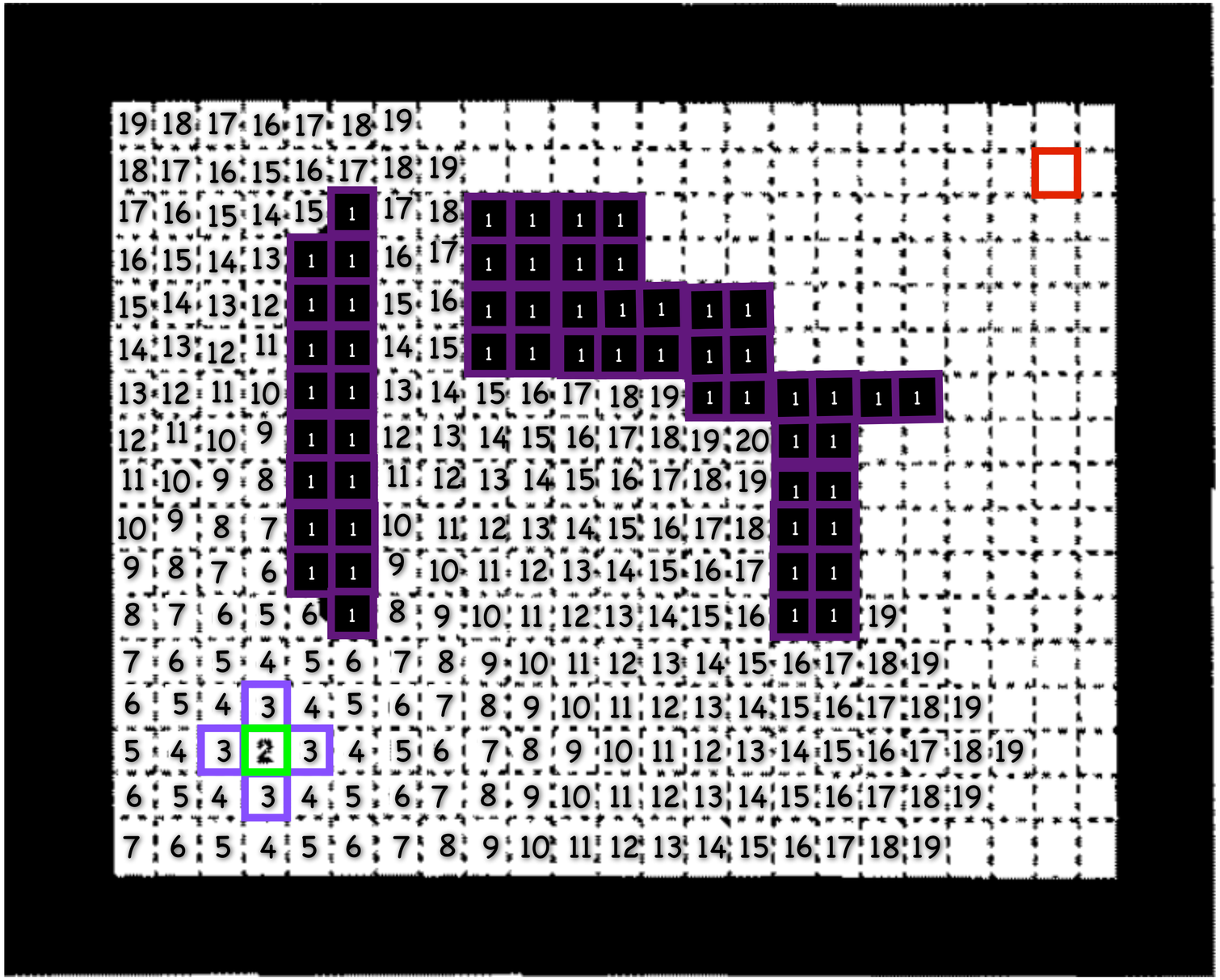

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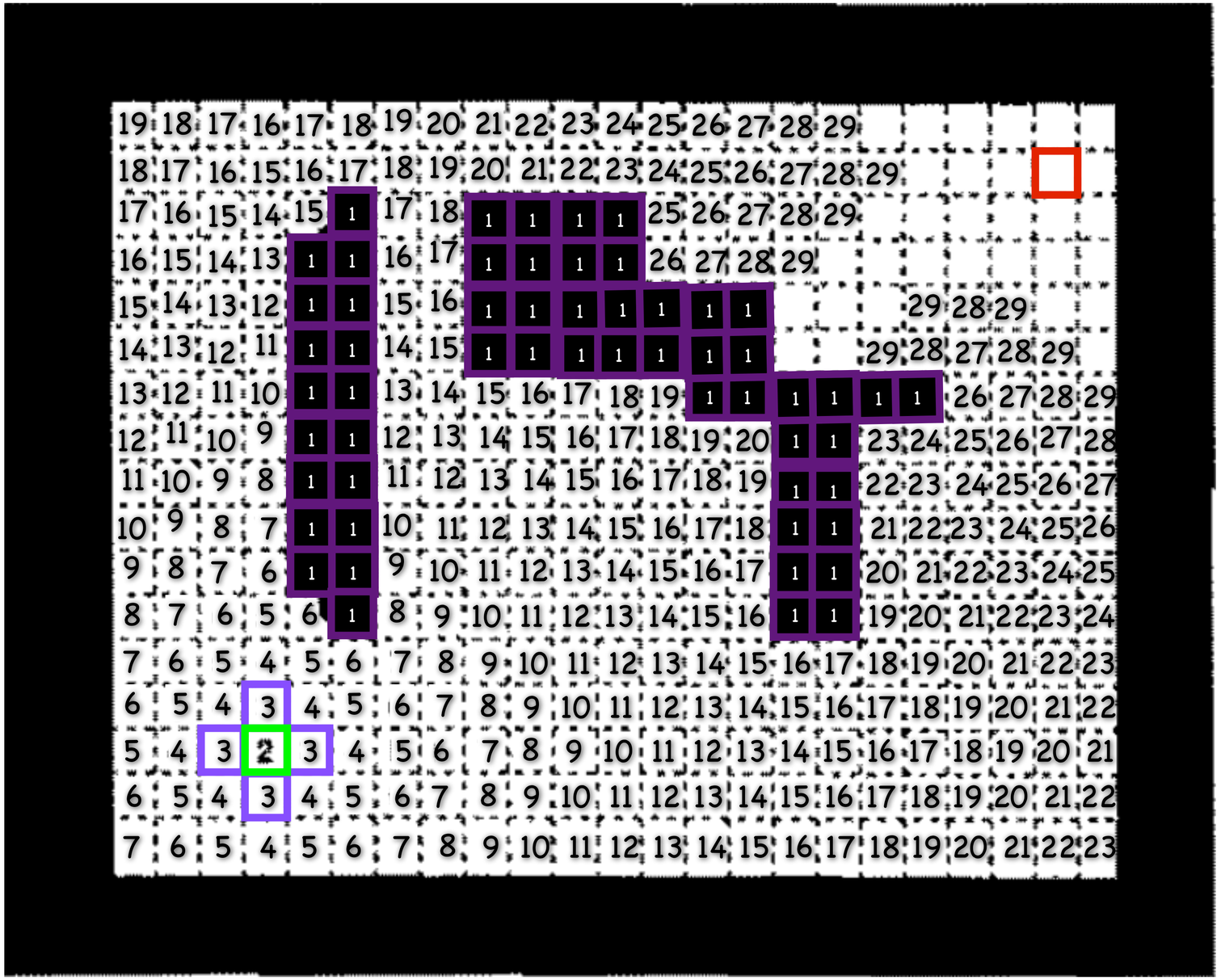

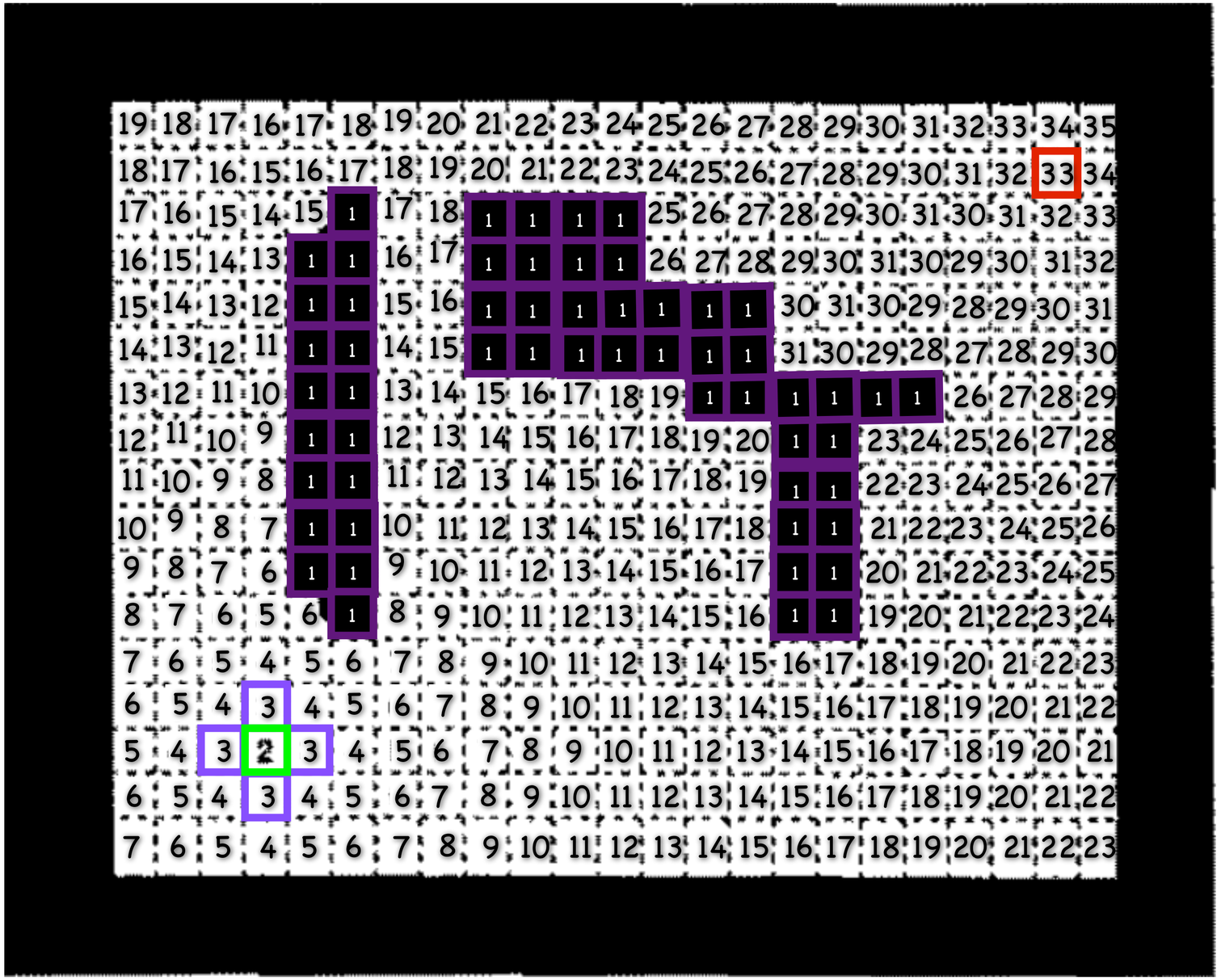

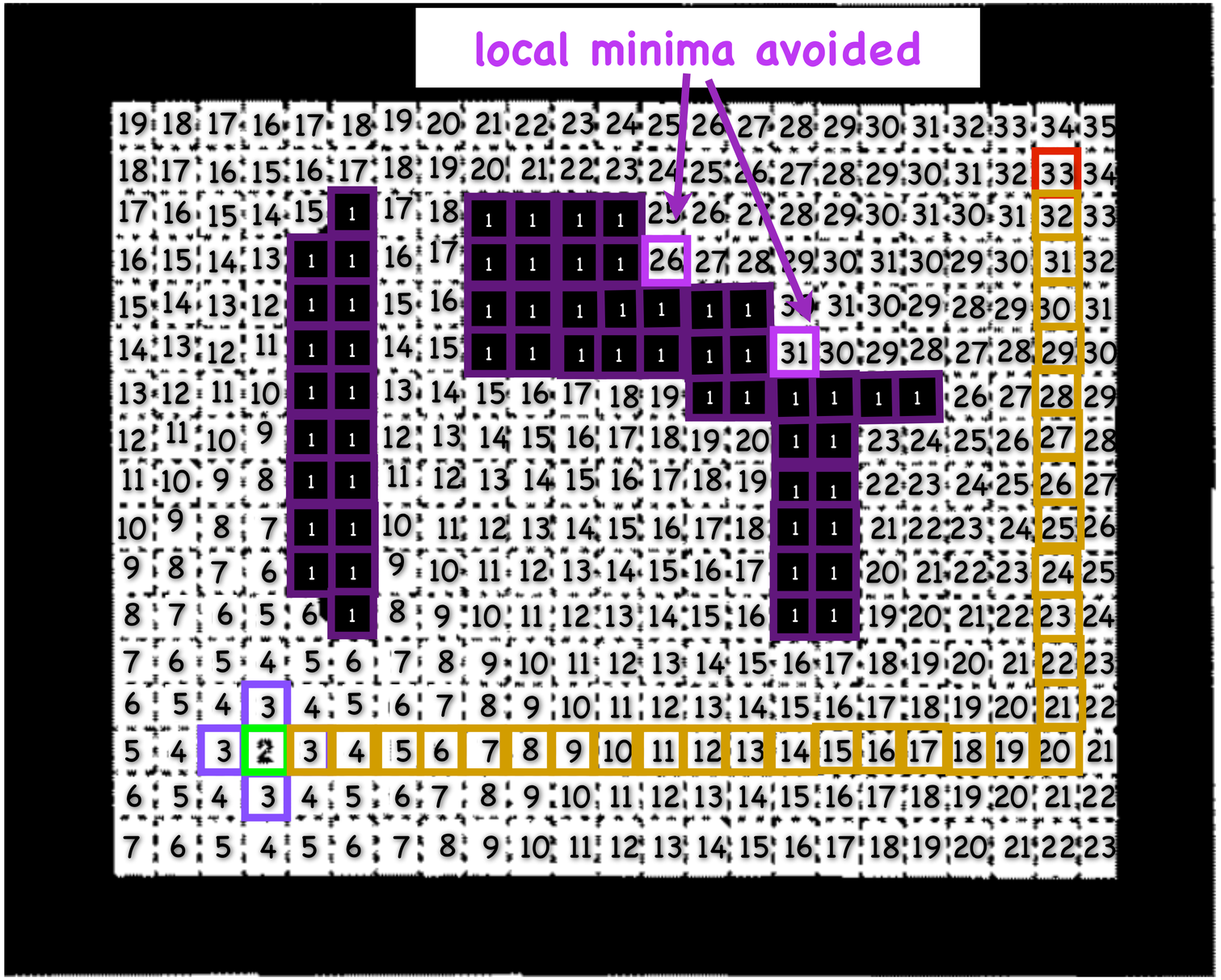

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Navigation Recap

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Navigation Recap

Bug X
- Complete
- Non-optimal
- Planar

Subsumption and FSMs
- Fast but not adaptive
- Emphasis on good design

Potential Fields
- Complete in special cases
- Non-optimal
- General C-spaces
- Scales w/dimensionality

Grid Search/Wavefront
- Complete
- General C-spaces
- Limited dimensionality

Random walk
- Will find path eventually

Sampling roadmaps/RRT
- Probabilistically complete
- General
- Tractable (with good sampling)
- Scales w/dimensionality
- Not necessarily optimal

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Next Lecture
Planning - VI - Collision Detection