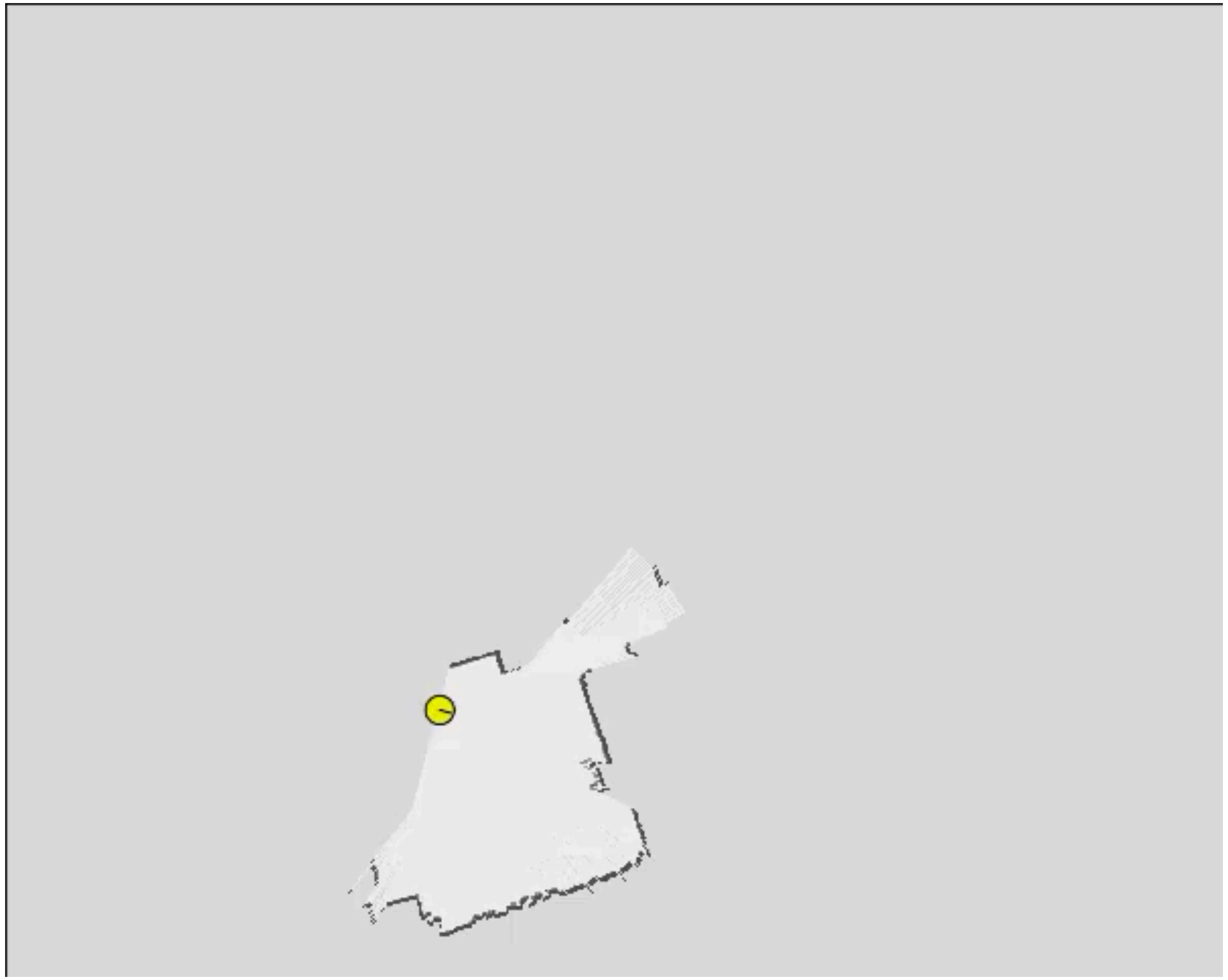
# Lecture 21
# Mobile Robotics - VI - Mapping





from Cyrill Stachniss

# Course logistics

- Project 5 is posted on 11/15 and will be due **11/29.**
- Final Project Proposal due **11/30.**

# Final (Open) Project timeline

In-class proposal presentation

Proposal Slides Due

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|
| 26 | Today 27 | 28 | 29 | 30 | 1 | 2 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 |

Poster for printing due

Poster Presentation

Final video due

# Final (Open) Project timeline

- Form your groups: Use the excel sheet in the Ed post.
  - Try to do this by today!
- Proposal Slides: (Template is provided on Ed)
  - 1-4 Slides
  - Title, Motivation, Input - Output, Deliverables, Timeline, Who is doing what?
  - Where does your project stand not the 3-axes (robots, objects, tasks)?
  - Backup plan
- In-class proposal presentation (<8mins) :
  - Teams will get feedback from the class
- Final video:
  - Describing the project idea and the outcome.
- Poster presentation: (template will be provided)
  - Presenting the project idea and the outcome to audience.

Final Project: 15%

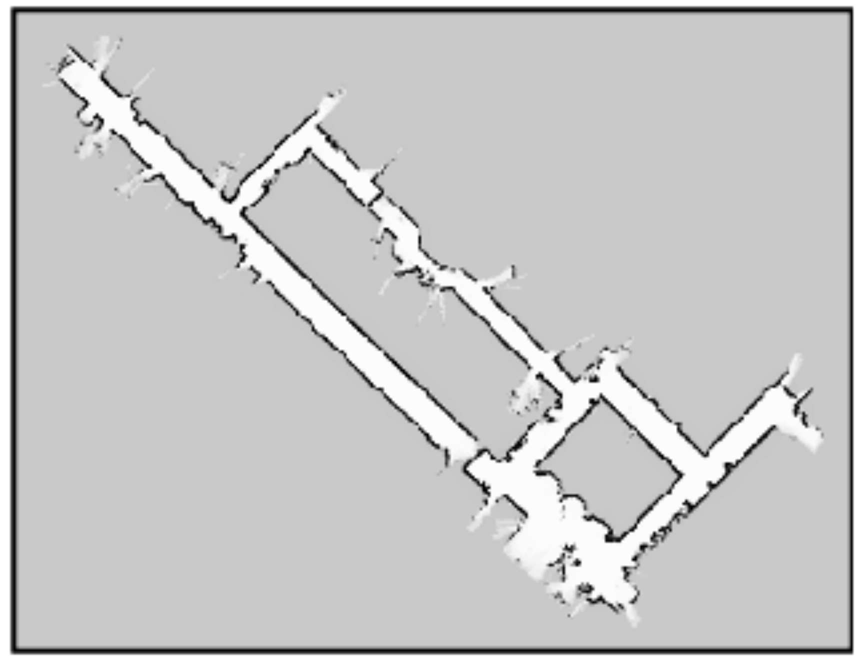
- Project proposal slides + presentation: 3%
- Final project video: 6%
- Poster presentation (evaluation by judges): 6%

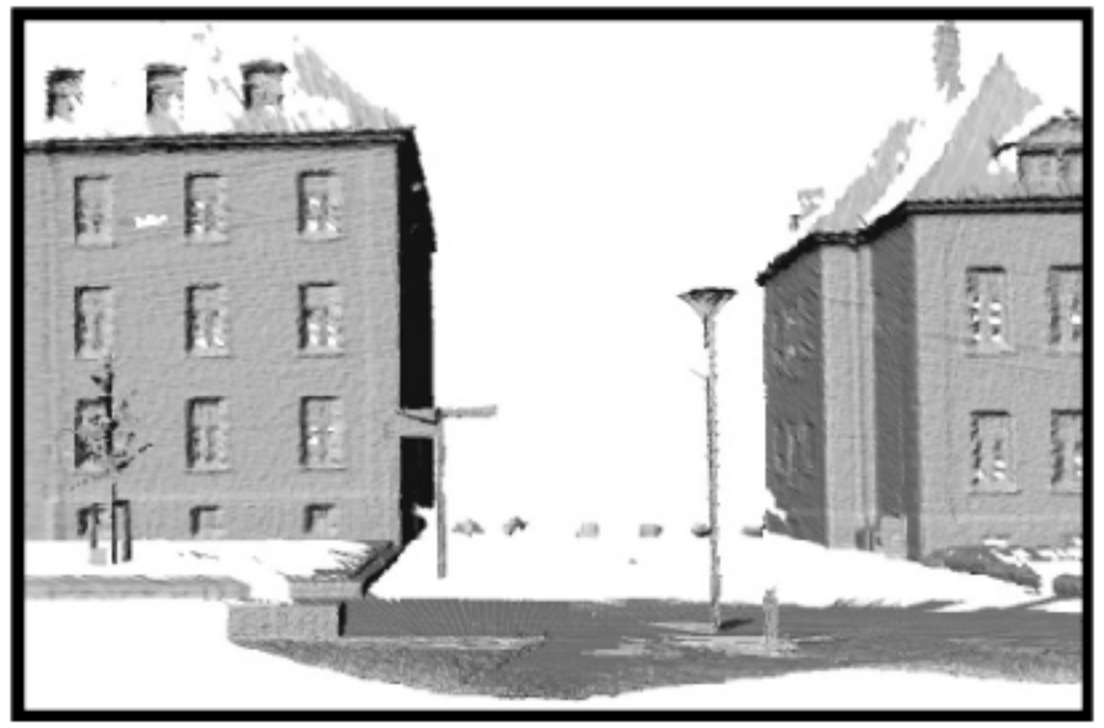
# Why Mapping?

- Learning maps is one of the fundamental problems in mobile robotics
- Maps allow robots to efficiently carry out their tasks, allow localization …
- Successful robot systems rely on maps for localization, path planning, activity planning etc.

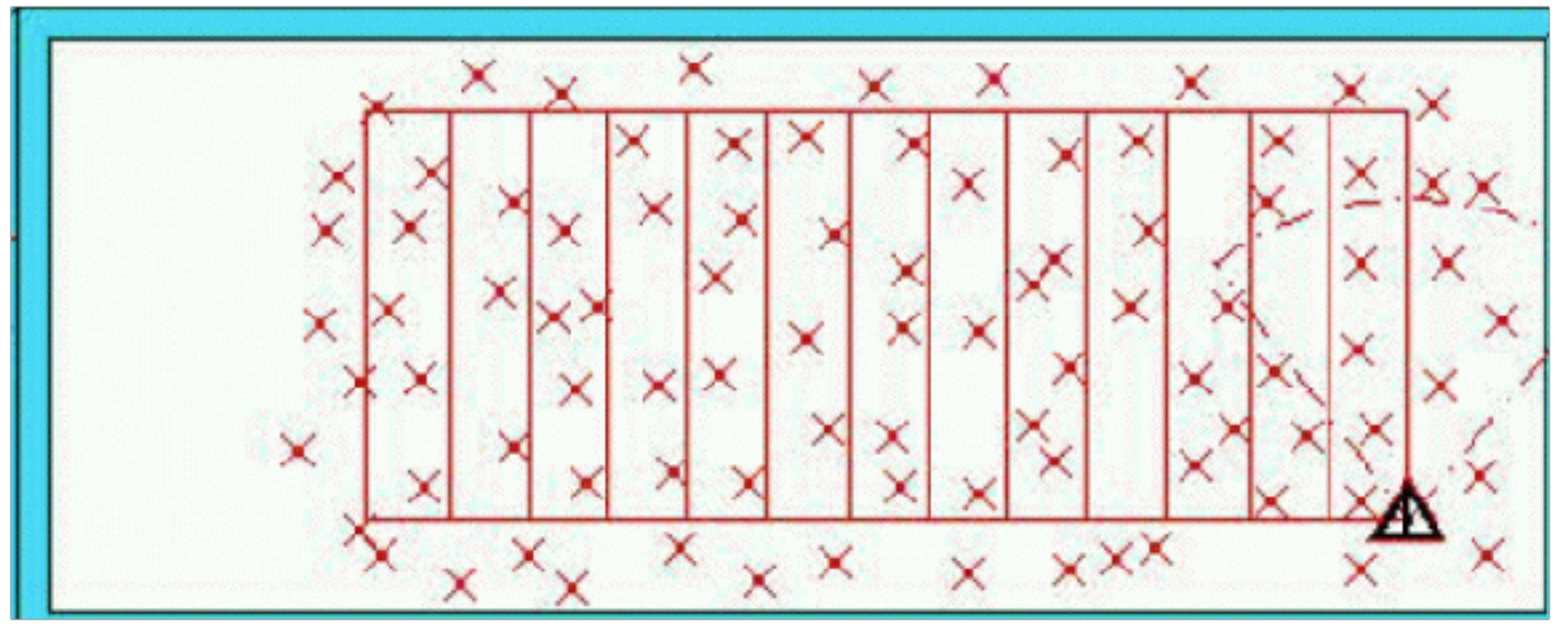
# Types of Maps

Grid maps or scans



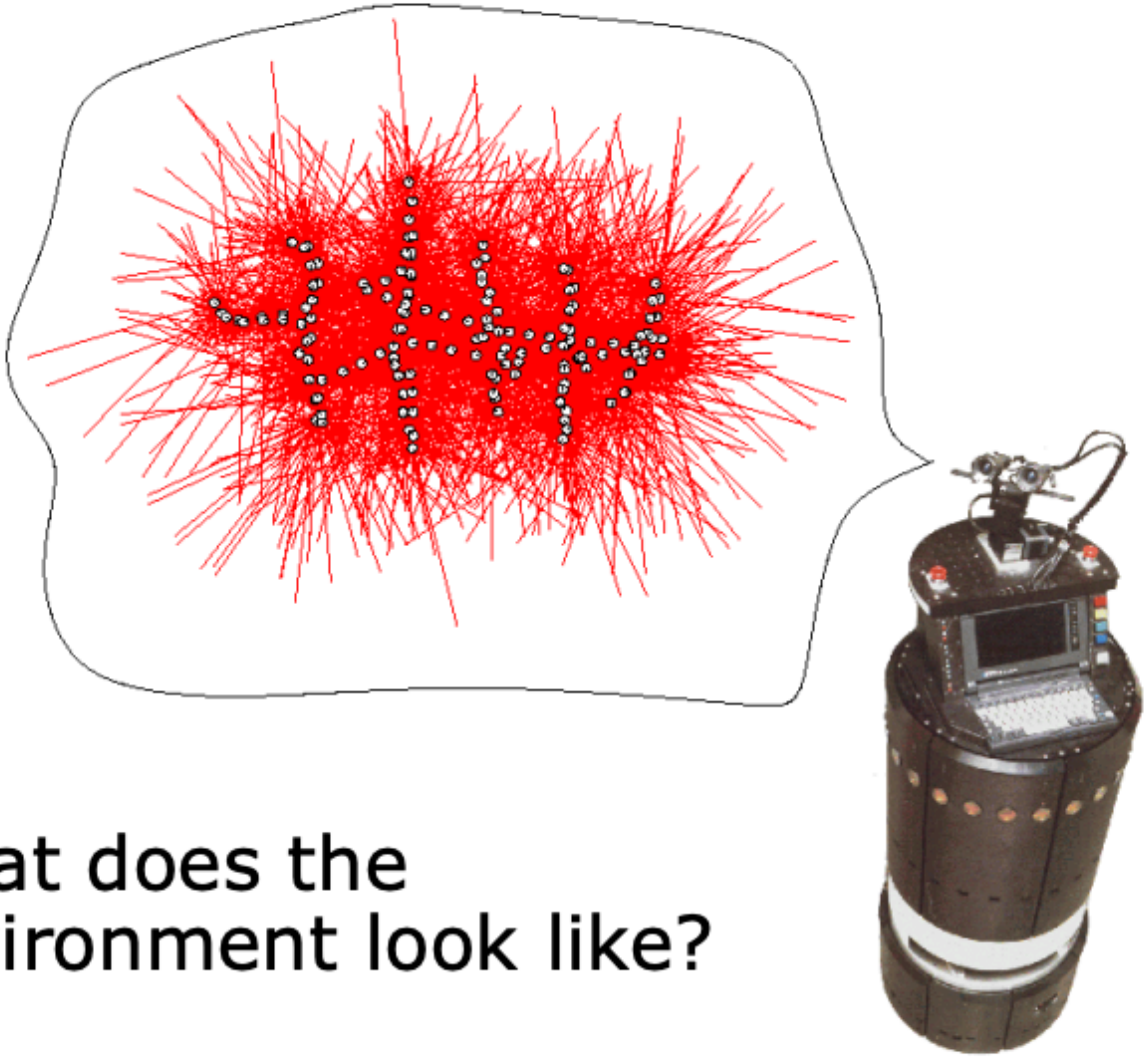


Sparse landmarks



RGB / Depth Maps

# The General Problem of Mapping



What does the environment look like?

# The General Problem of Mapping

- Formally, mapping involves, given the sensor data,

$$d = \{u_1, z_1, u_2, z_2, \ldots, u_n, z_n\}$$

to calculate the most likely map

$$m^* = \arg\max_m P(m \mid d)$$

# Mapping as a Chicken and Egg Problem

- So far we learned how to estimate the pose of the vehicle given the data and the map.
- Mapping, however, involves to simultaneously estimate the pose of the vehicle and the map.
- The general problem is therefore denoted as the simultaneous localization and mapping problem (SLAM).
- Throughout this section we will describe how to calculate a map given we know the pose of the vehicle.

# Problems in Mapping

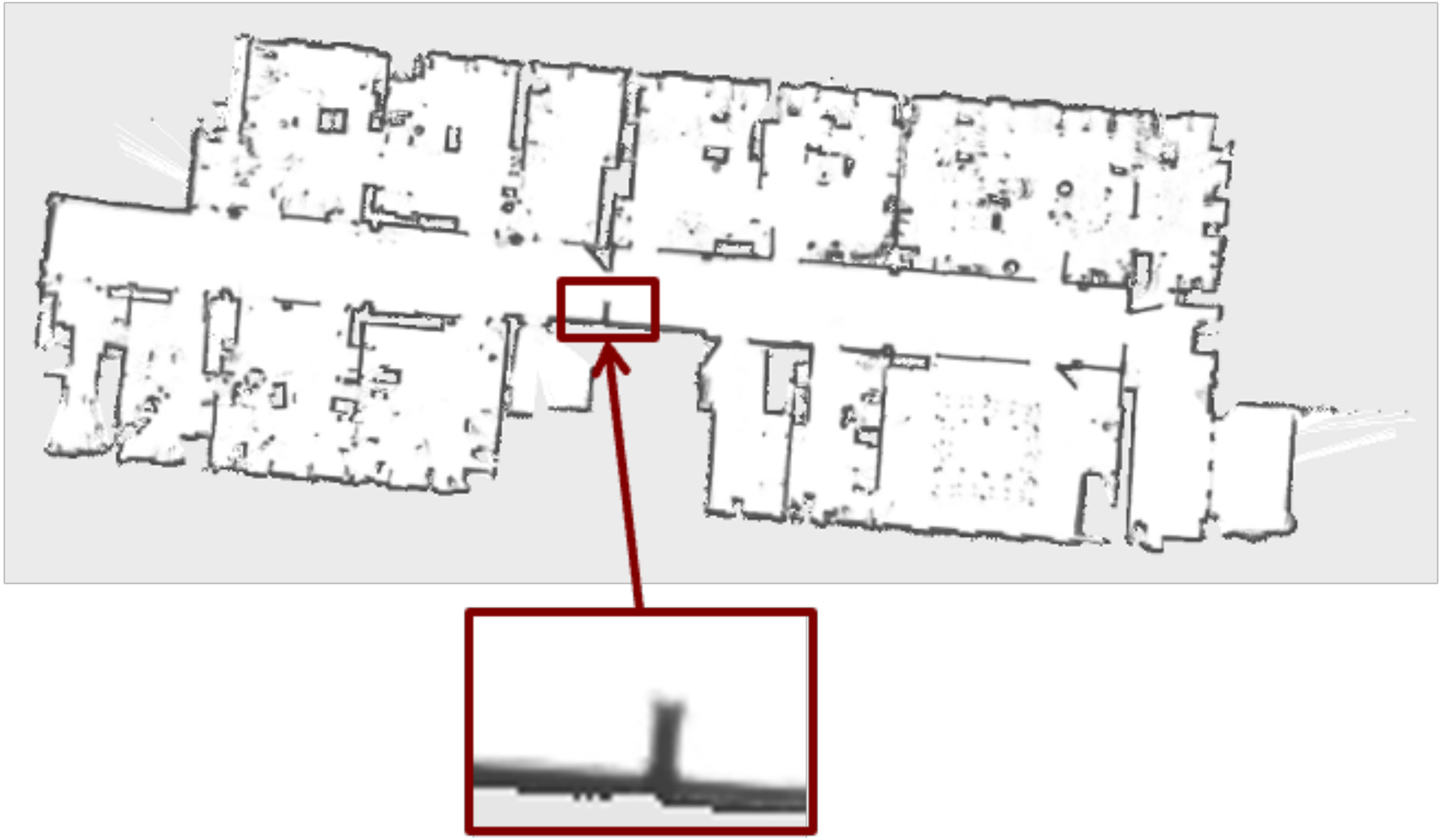
- Sensor interpretation
  - How do we extract relevant information from raw sensor data?
  - How do we represent and integrate this information over time?
- Robot locations have to be known
  - How can we estimate them during mapping?

# Occupancy Grid Mapping

# Grid Maps

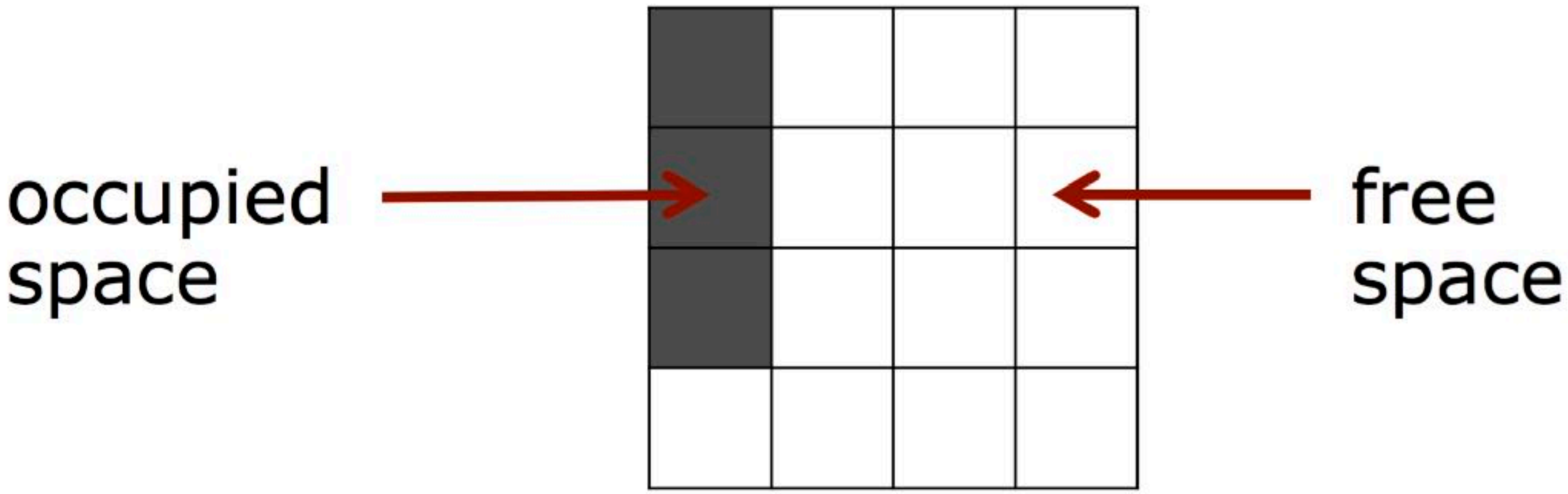
- Discretize the world into cells
- Grid structure is rigid
- Each cell is assumed to be occupied or free space
- Non-parametric model
- Large maps require substantial memory resources
- Do not rely on a feature detector

# Example



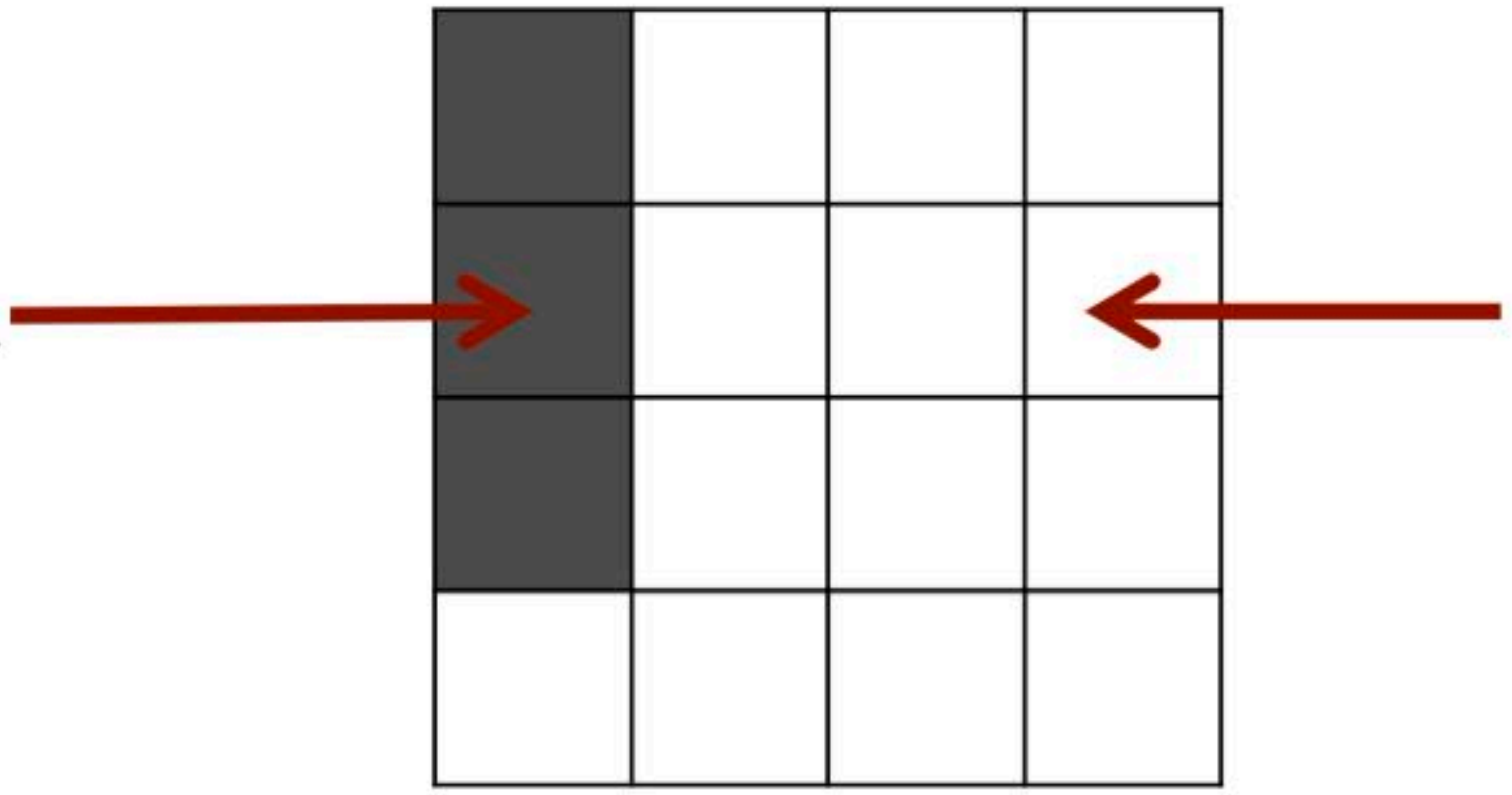
# Assumption 1

- The area that corresponds to a cell is either completely free or occupied



# Representation

- Each cell is a **binary random variable** that models the occupancy

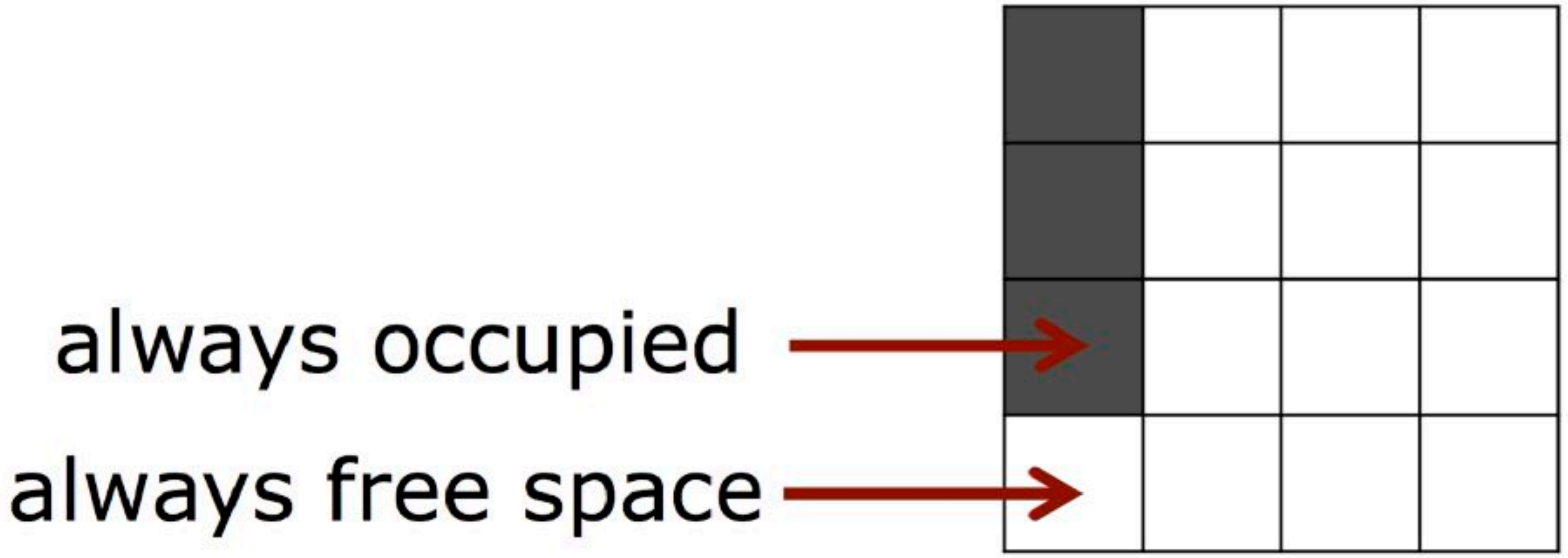
$p(m_i) \to 1$  $p(m_j) \to 0$

# Occupancy Probability

- Each cell is a **binary random variable** that models the occupancy
- Cell is occupied: $p(m_i) = 1$
- Cell is not occupied: $p(m_i) = 0$
- No knowledge: $p(m_i) = 0.5$

# Assumption 2

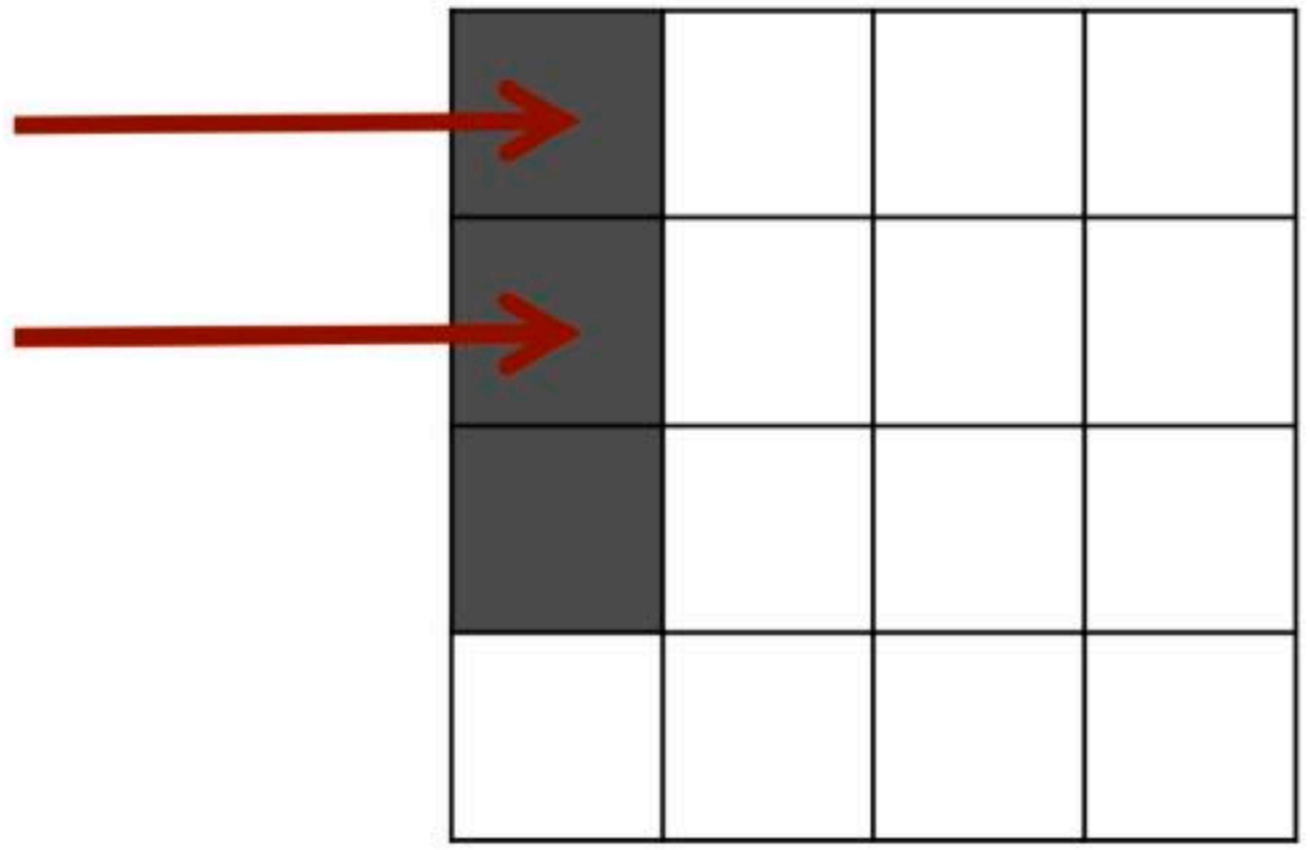
- The world is **static** (most mapping systems make this assumption)



# Assumption 3

- The cells (the random variables) are **independent** of each other

no dependency
between the cells



# Representation

- The probability distribution of the map is given by the product over the cells

$$p(m) = \prod_i p(m_i)$$

map

cell

# Representation

- The probability distribution of the map is given by the product over the cells

$$p(m) = \prod_i p(m_i)$$

example map
(4-dim state)

4 individual cells

# Estimating a Map From Data

- Given sensor data $z_{1:t}$ and the poses $x_{1:t}$ of the sensor, estimate the map

$$p(m \mid z_{1:t}, x_{1:t}) \quad = \quad \prod_i p(\underset{\substack{\uparrow \\ \text{binary random variable}}}{m_i} \mid z_{1:t}, x_{1:t})$$

binary random variable

➡ Binary Bayes filter (for a static state)

# Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

# Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

# Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

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$$p(z_t \mid m_i, x_t) \overset{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)}{p(m_i \mid x_t)}$$

# Static State Binary Bayes Filter

$$
\begin{aligned}
p(m_i \mid z_{1:t}, x_{1:t}) &\overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\; p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})} \\
&\overset{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\; p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})} \\
&\overset{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t)\; p(z_t \mid x_t)\; p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid x_t)\; p(z_t \mid z_{1:t-1}, x_{1:t})}
\end{aligned}
$$

# Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\; p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\; p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t)\; p(z_t \mid x_t)\; p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid x_t)\; p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(m_i \mid z_t, x_t)\; p(z_t \mid x_t)\; p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i)\; p(z_t \mid z_{1:t-1}, x_{1:t})}$$

# Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid x_t)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Markov}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Do exactly the same for the opposite event:

$$p(\neg m_i \mid z_{1:t}, x_{1:t}) \overset{\text{the same}}{=} \frac{p(\neg m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(\neg m_i \mid z_{1:t-1}, x_{1:t-1})}{p(\neg m_i)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

# Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{p(\neg m_i \mid z_{1:t}, x_{1:t})} = \frac{\frac{p(m_i|z_t,x_t)\,\cancel{p(z_t|x_t)}\,p(m_i|z_{1:t-1},x_{1:t-1})}{p(m_i)\,\cancel{p(z_t|z_{1:t-1},x_{1:t})}}}{\frac{p(\neg m_i|z_t,x_t)\,\cancel{p(z_t|x_t)}\,p(\neg m_i|z_{1:t-1},x_{1:t-1})}{p(\neg m_i)\,\cancel{p(z_t|z_{1:t-1},x_{1:t})}}}$$

# Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{p(\neg m_i \mid z_{1:t}, x_{1:t})}$$

$$= \frac{p(m_i \mid z_t, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})\, p(\neg m_i)}{p(\neg m_i \mid z_t, x_t)\, p(\neg m_i \mid z_{1:t-1}, x_{1:t-1})\, p(m_i)}$$

$$= \frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)} \frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})} \frac{1 - p(m_i)}{p(m_i)}$$

# Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$
\begin{aligned}
&\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})} \\
&\quad = \frac{p(m_i \mid z_t, x_t)\ p(m_i \mid z_{1:t-1}, x_{1:t-1})\ p(\neg m_i)}{p(\neg m_i \mid z_t, x_t)\ p(\neg m_i \mid z_{1:t-1}, x_{1:t-1})\ p(m_i)} \\
&\quad = \underbrace{\frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)}}_{\text{uses } z_t} \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}}_{\text{recursive term}} \underbrace{\frac{1 - p(m_i)}{p(m_i)}}_{\text{prior}}
\end{aligned}
$$

# From Ratio to Probability

We can turn the ratio into a probability:

$$
\begin{aligned}
\frac{p(x)}{1-p(x)} &= Y \\
p(x) &= Y - Y\,p(x) \\
p(x)\,(1+Y) &= Y \\
p(x) &= \frac{Y}{1+Y} \\
p(x) &= \frac{1}{1+\frac{1}{Y}}
\end{aligned}
$$

# From Ratio to Probability

- Using $p(x) = \left[1 + Y^{-1}\right]^{-1}$ directly leads to

$$
\begin{aligned}
& p(m_i \mid z_{1:t}, x_{1:t}) \\
& \quad = \left[1 + \frac{1 - p(m_i \mid z_t, x_t)}{p(m_i \mid z_t, x_t)} \frac{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid z_{1:t-1}, x_{1:t-1})} \frac{p(m_i)}{1 - p(m_i)}\right]^{-1}
\end{aligned}
$$

**For reasons of efficiency, one performs the calculations in the log odds notation**

# Log Odds Notation

- The log odds notation computes the logarithm of the ratio of probabilities

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})}$$

$$= \underbrace{\frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)}}_{\text{uses } z_t} \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}}_{\text{recursive term}} \underbrace{\frac{1 - p(m_i)}{p(m_i)}}_{\text{prior}}$$

$$\Rightarrow \quad l(m_i \mid z_{1:t}, x_{1:t}) = \log\left(\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})}\right)$$

# Log Odds Notation

- Log odds ratio is defined as

$$l(x) \quad = \quad \log \frac{p(x)}{1 - p(x)}$$

- and with the ability to retrieve $p(x)$

$$p(x) \quad = \quad 1 - \frac{1}{1 + \exp l(x)}$$

# Occupancy Mapping in Log Odds Form

- The product turns into a sum

$$
\begin{aligned}
&l(m_i \mid z_{1:t}, x_{1:t}) \\
&\quad = \underbrace{l(m_i \mid z_t, x_t)}_{\text{inverse sensor model}} + \underbrace{l(m_i \mid z_{1:t-1}, x_{1:t-1})}_{\text{recursive term}} - \underbrace{l(m_i)}_{\text{prior}}
\end{aligned}
$$

- or in short

$$
l_{t,i} \quad = \quad \text{inv\_sensor\_model}(m_i, x_t, z_t) + l_{t-1,i} - l_0
$$

# Occupancy Mapping Algorithm

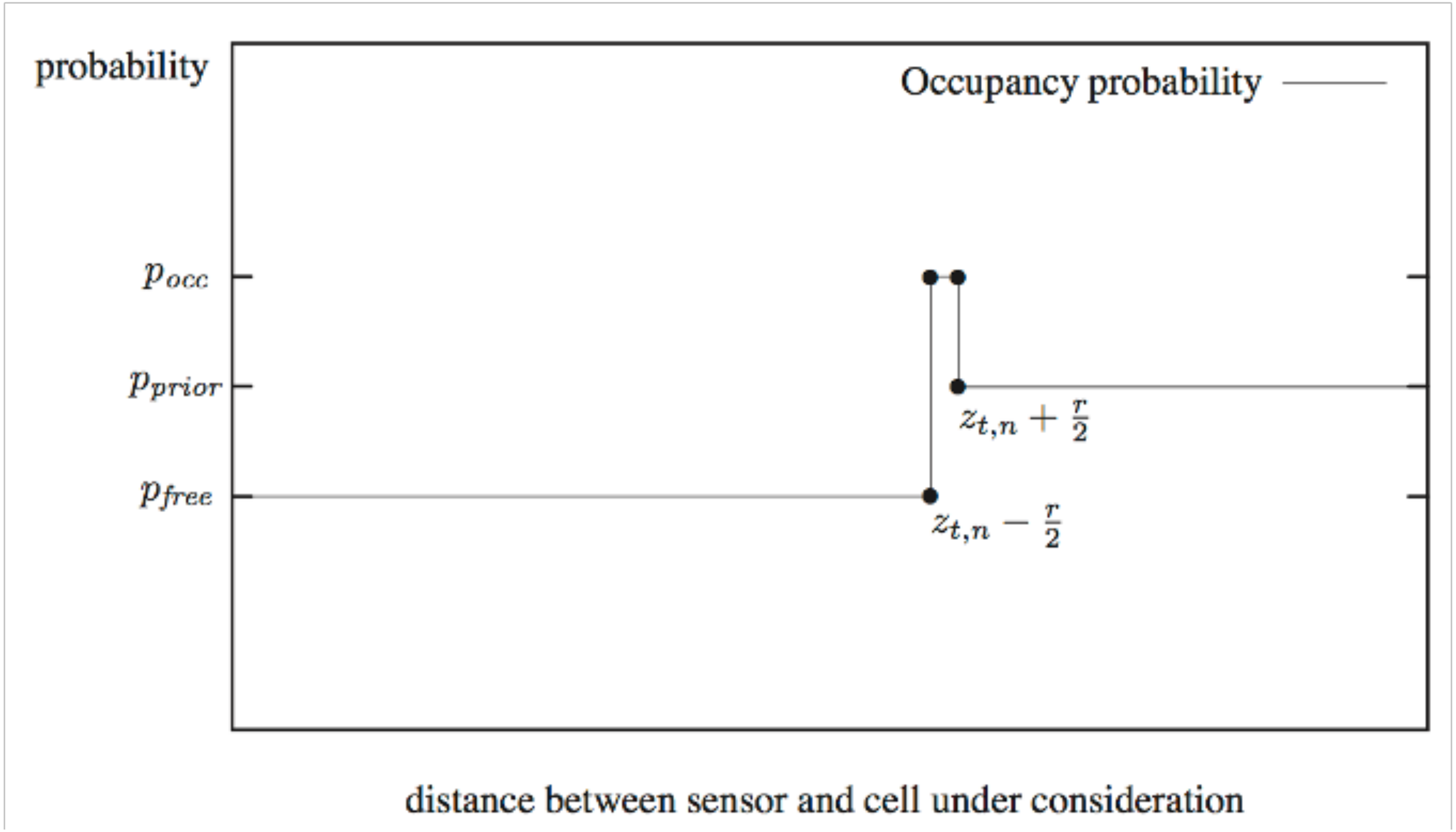
**occupancy_grid_mapping**($\{l_{t-1,i}\}, x_t, z_t$)**:**

```
for all cells m_i do
    if m_i in perceptual field of z_t then
        l_{t,i} = l_{t-1,i} + inv_sensor_model(m_i, x_t, z_t) - l_0
    else
        l_{t,i} = l_{t-1,i}
    endif
endfor
return {l_{t,i}}
```

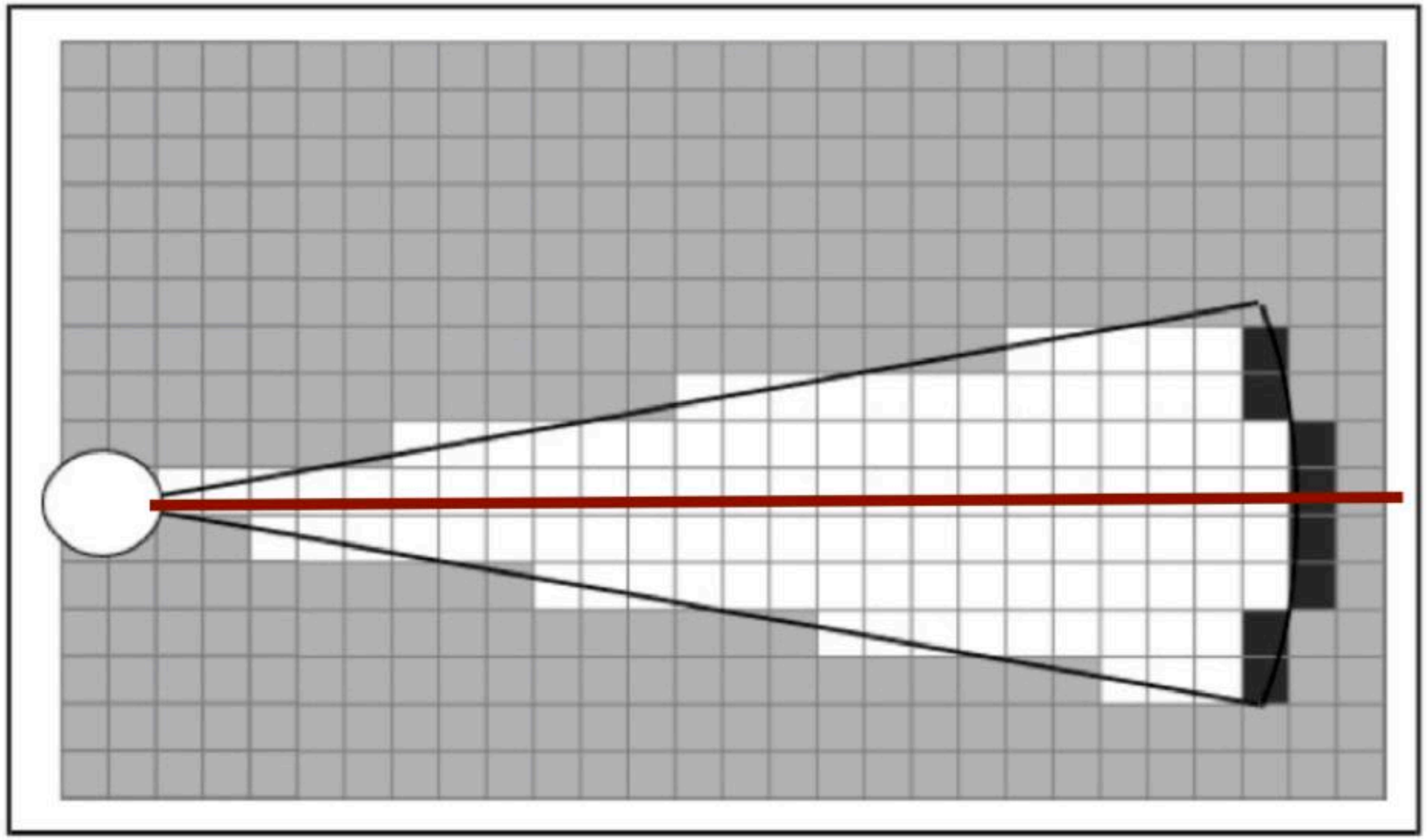
$$
\begin{array}{l}
\textit{for all cells } m_i \textit{ do} \\
\quad \textit{if } m_i \textit{ in perceptual field of } z_t \textit{ then} \\
\quad\quad l_{t,i} = l_{t-1,i} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - l_0 \\
\quad \textit{else} \\
\quad\quad l_{t,i} = l_{t-1,i} \\
\quad \textit{endif} \\
\textit{endfor} \\
\textit{return } \{l_{t,i}\}
\end{array}
$$

**highly efficient, we only have to compute sums**

# Inverse Sensor Model for Laser Range Finders

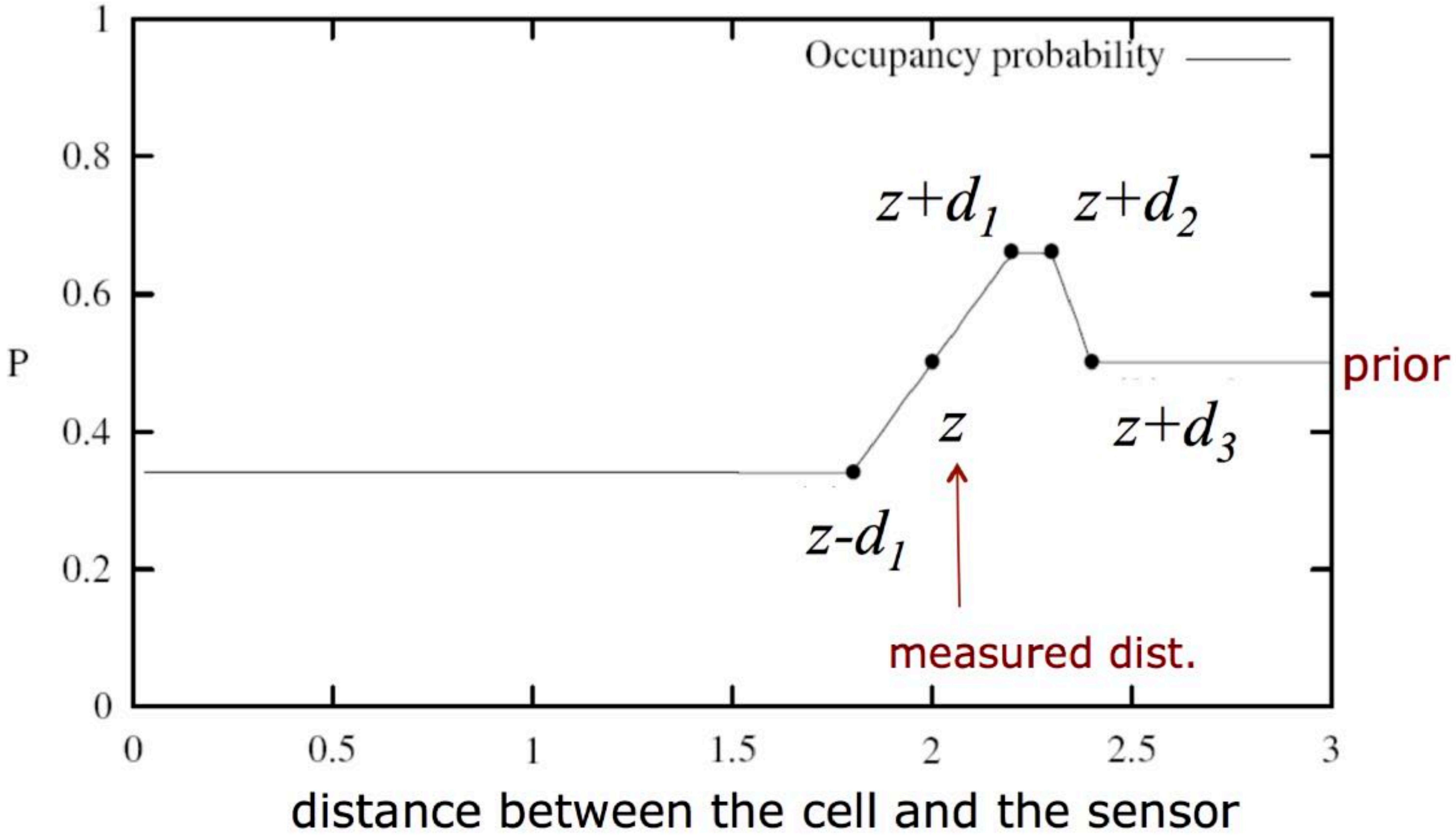


# Inverse Sensor Model for Sonar Range Sensors



In the following, consider the cells along the optical axis (red line)

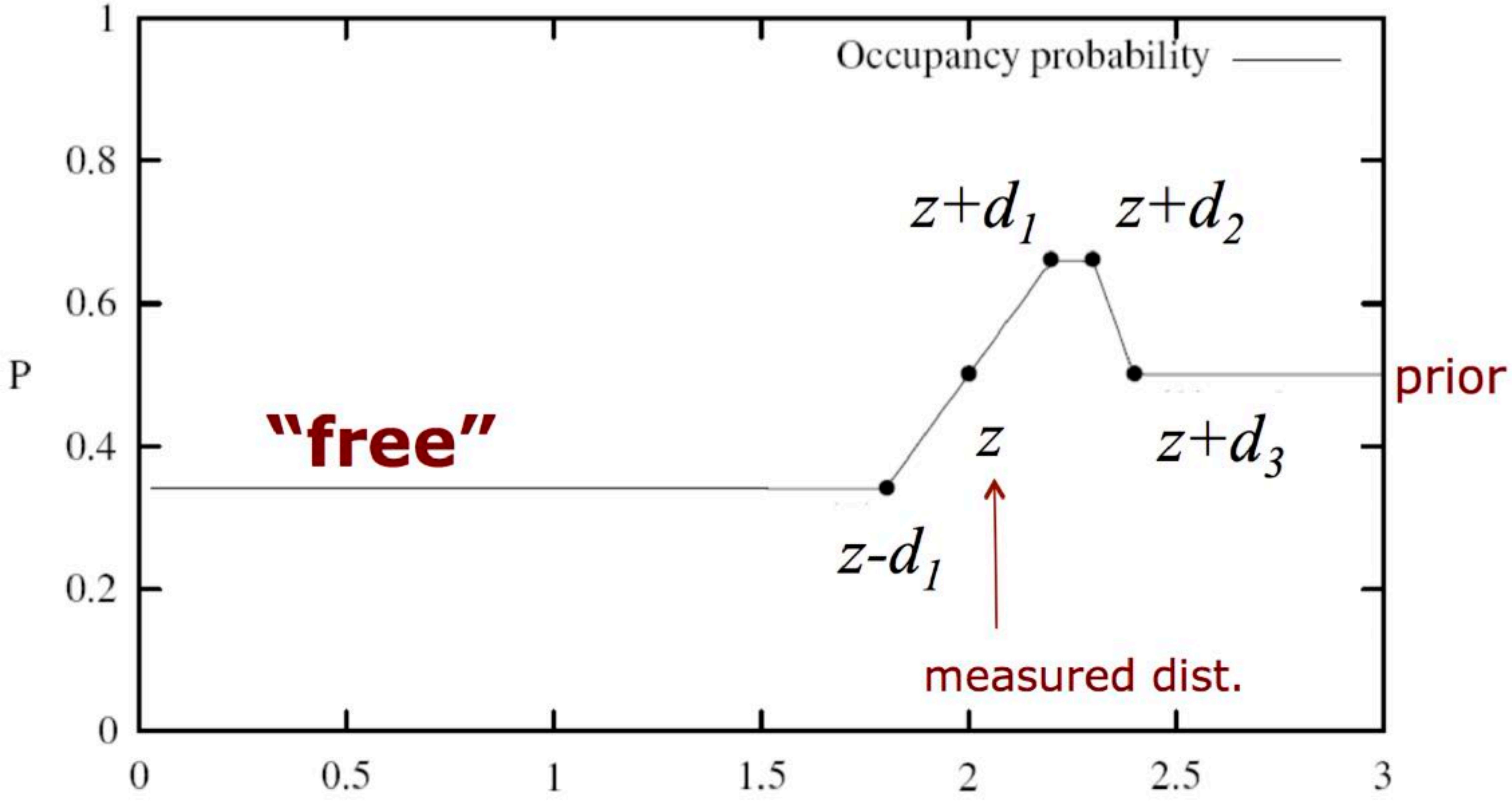
# Occupancy Value Depending on the Measured Distance





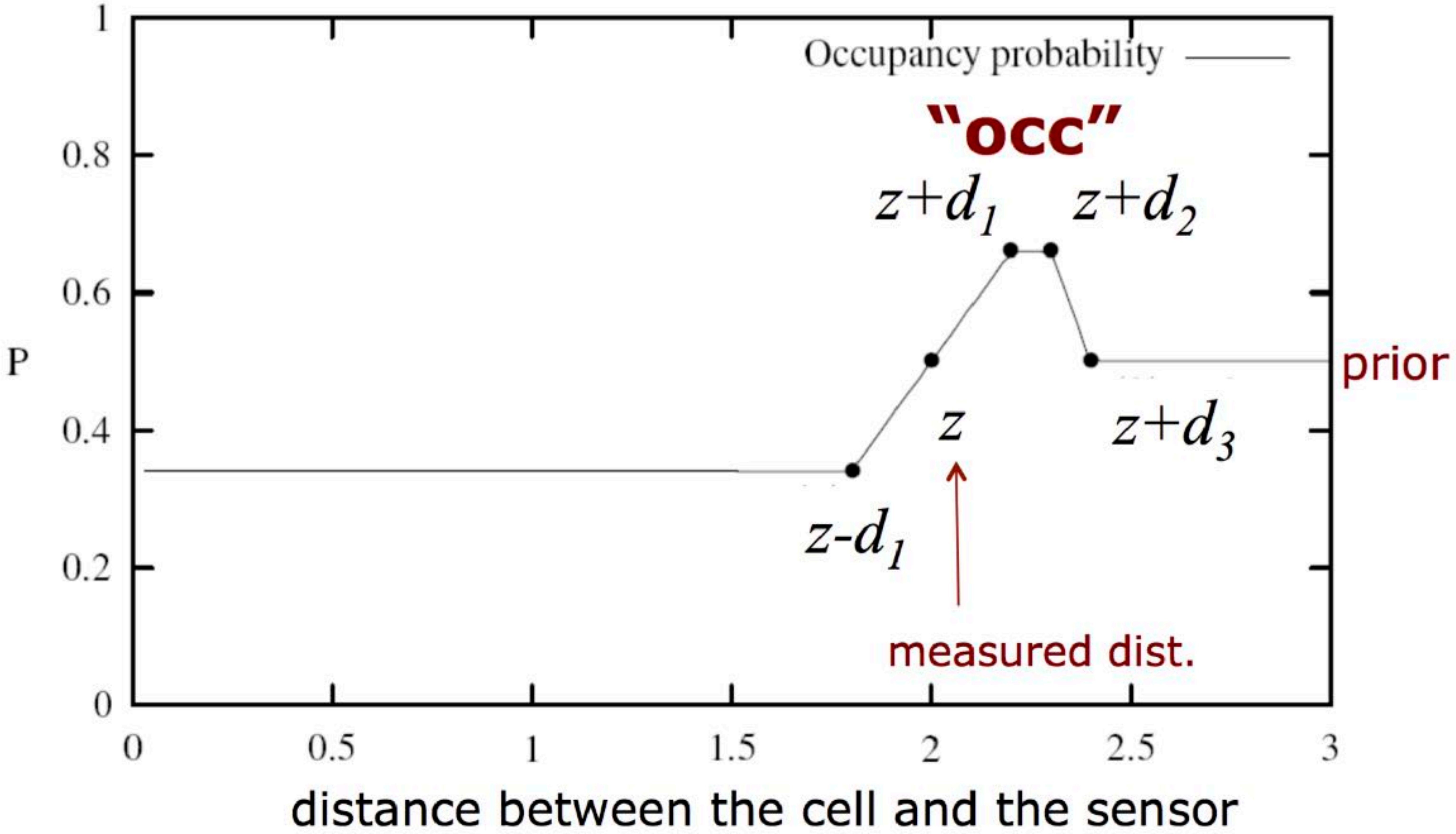

*Slide borrowed from Cyrill Stachniss*

# Occupancy Value Depending on the Measured Distance




distance between the cell and the sensor

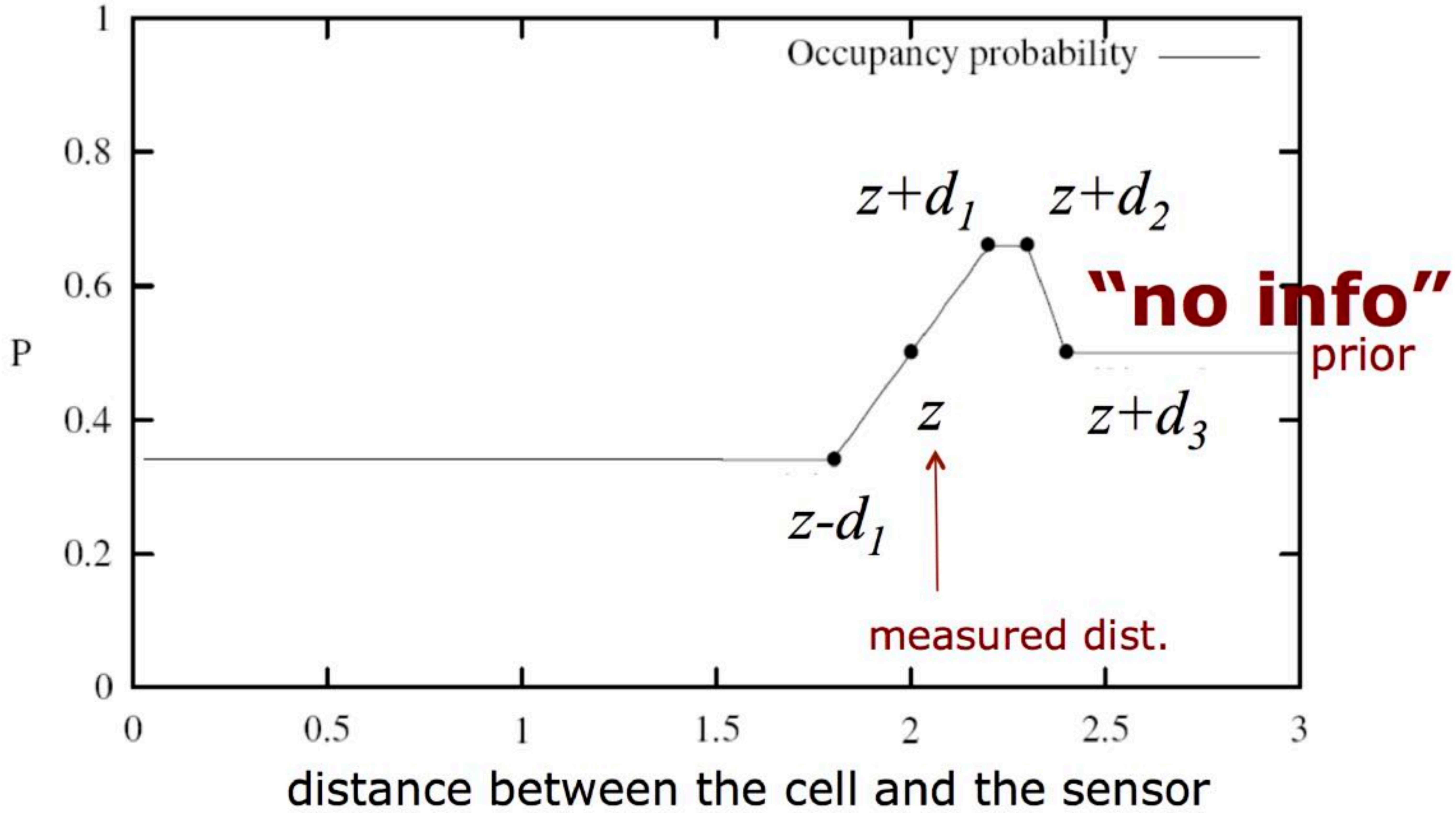
# Occupancy Value Depending on the Measured Distance






Slide borrowed from Cyrill Stachniss

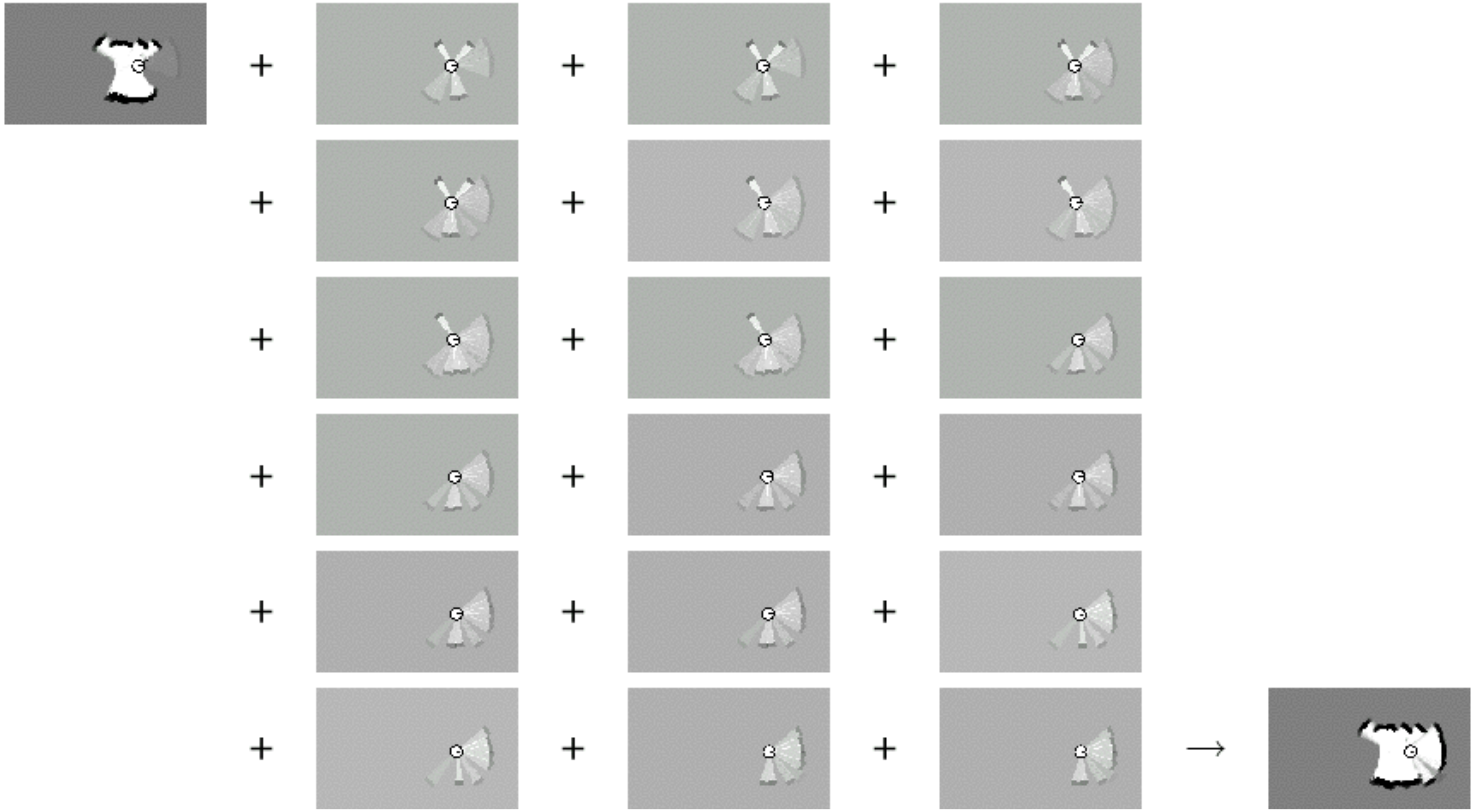
# Occupancy Value Depending on the Measured Distance






*Slide borrowed from Cyrill Stachniss*

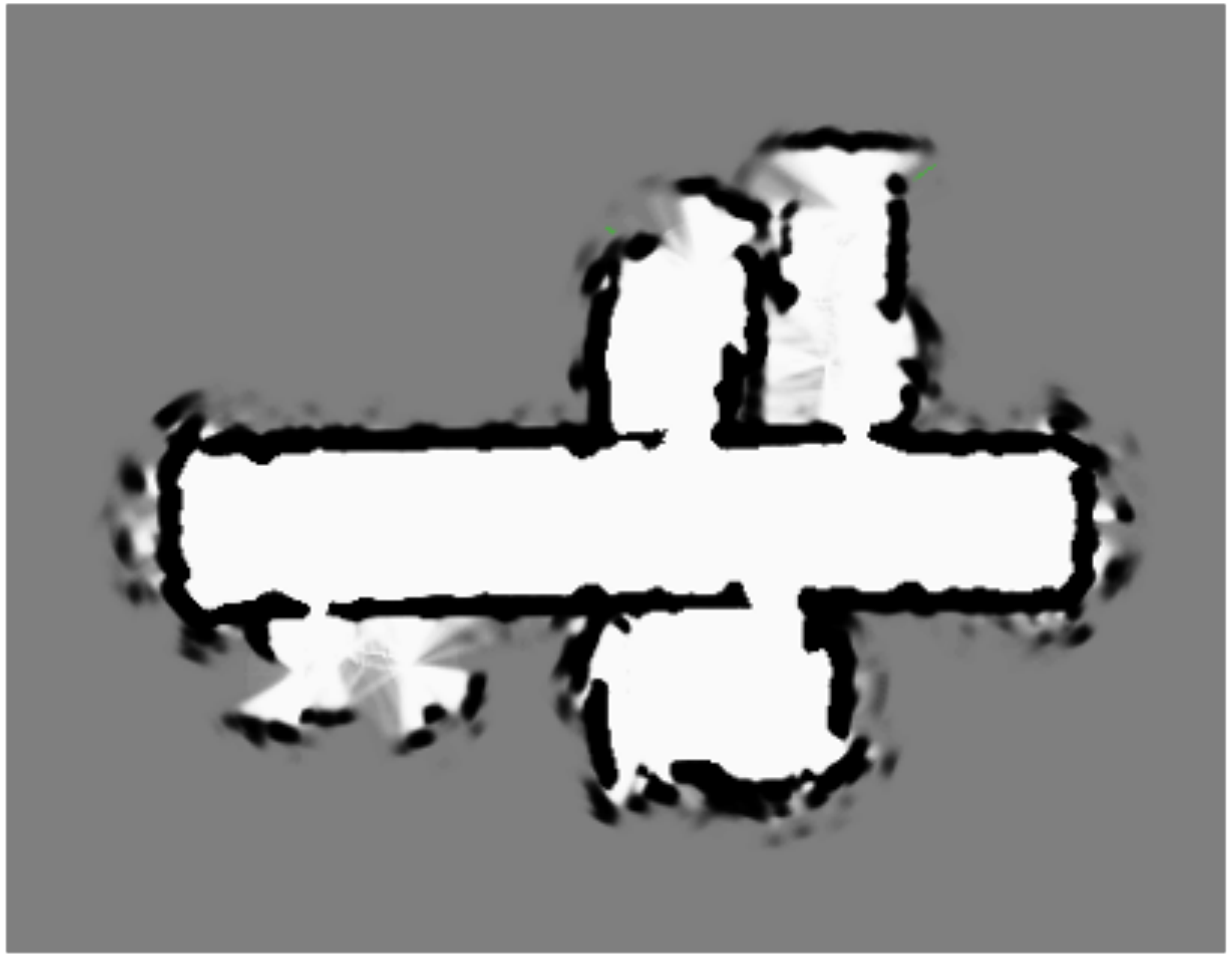
# Incremental Updating of Occupancy Grids (Example)



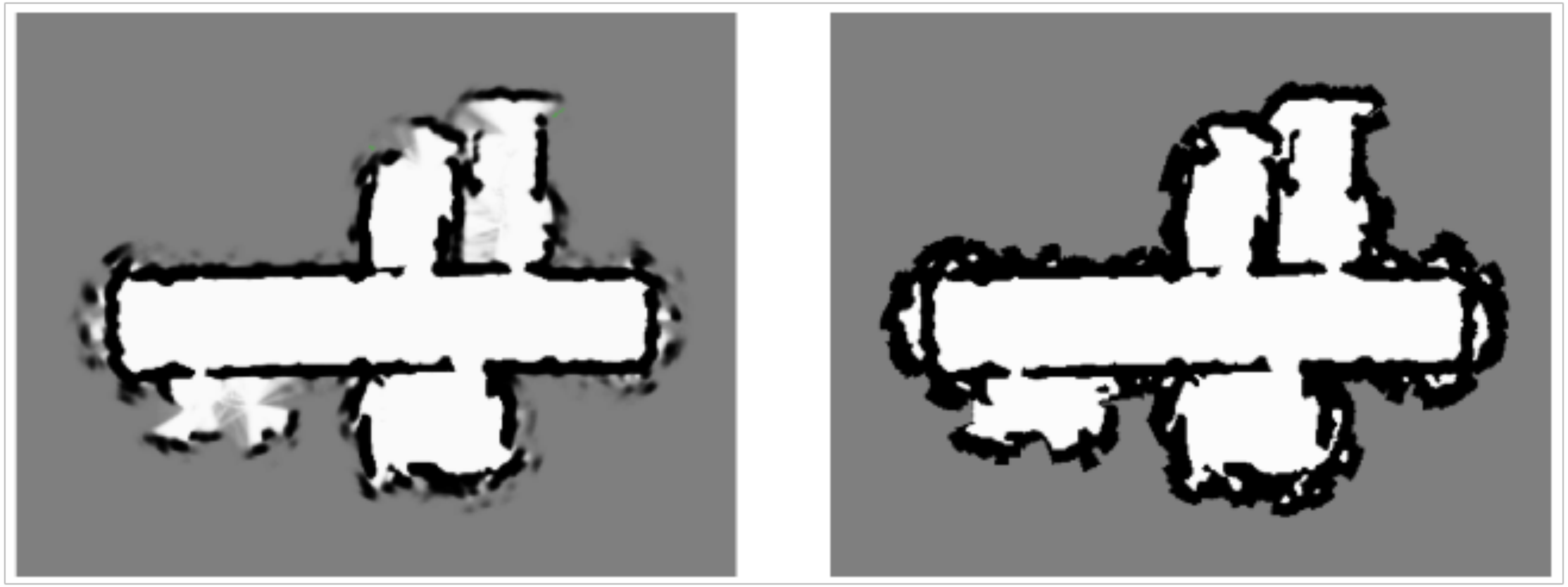



*Slide borrowed from Dieter Fox*

# Resulting Map Obtained with 24 Sonar Range Sensors



# Resulting Occupancy and Maximum Likelihood Map

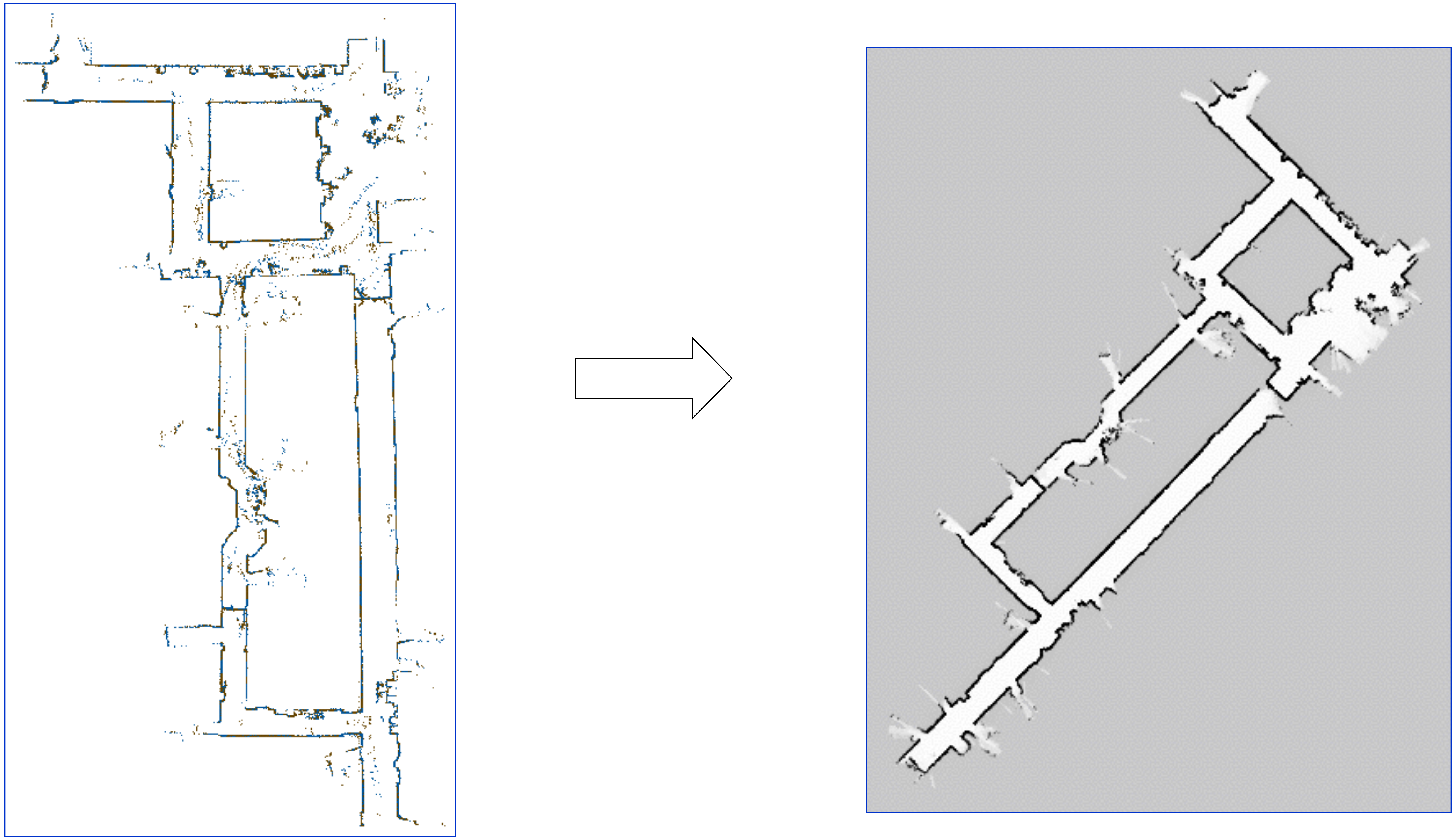


The maximum likelihood map is obtained by rounding the probability for each cell to 0 or 1.

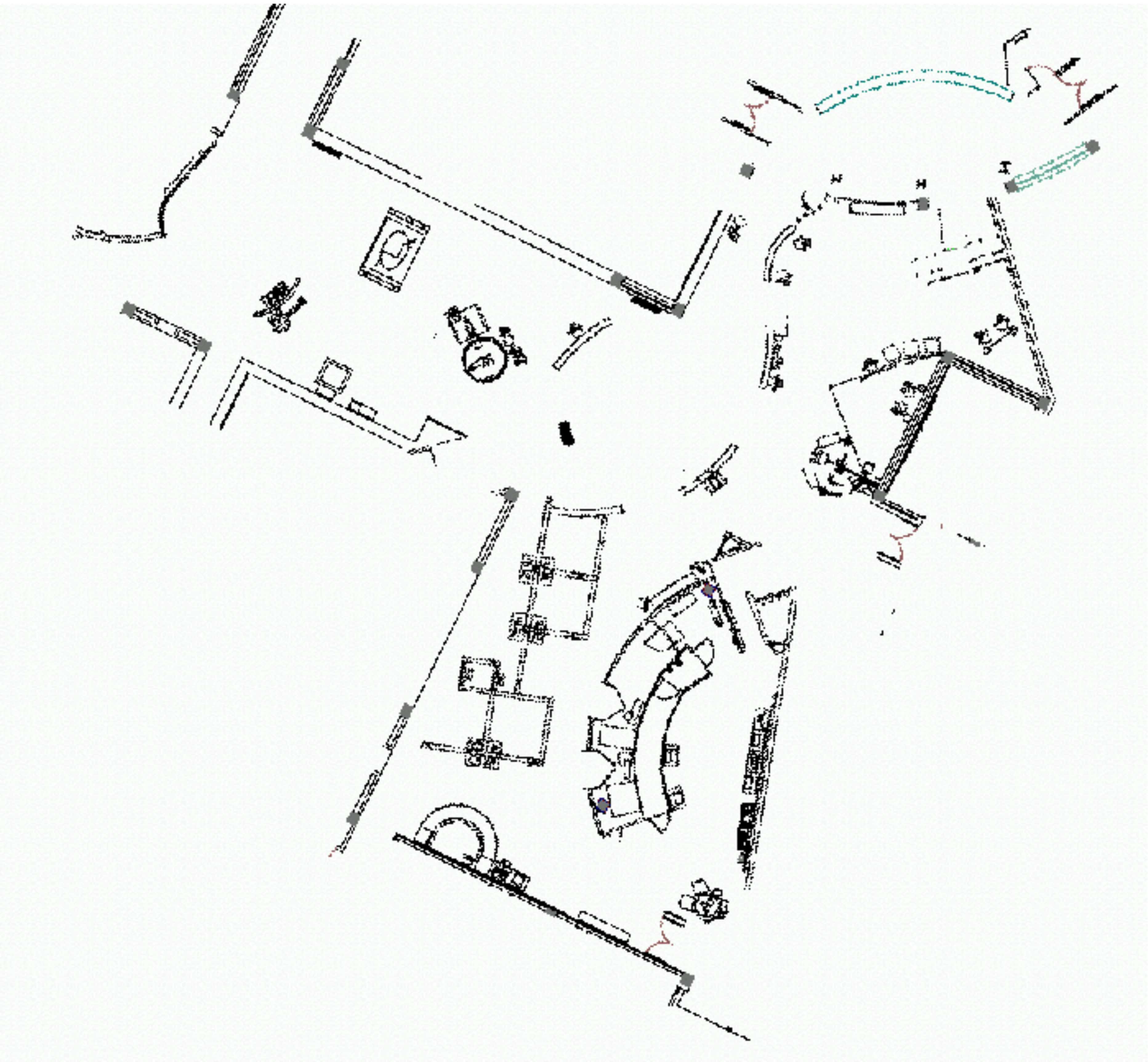


*Slide borrowed from Cyrill Stachniss*

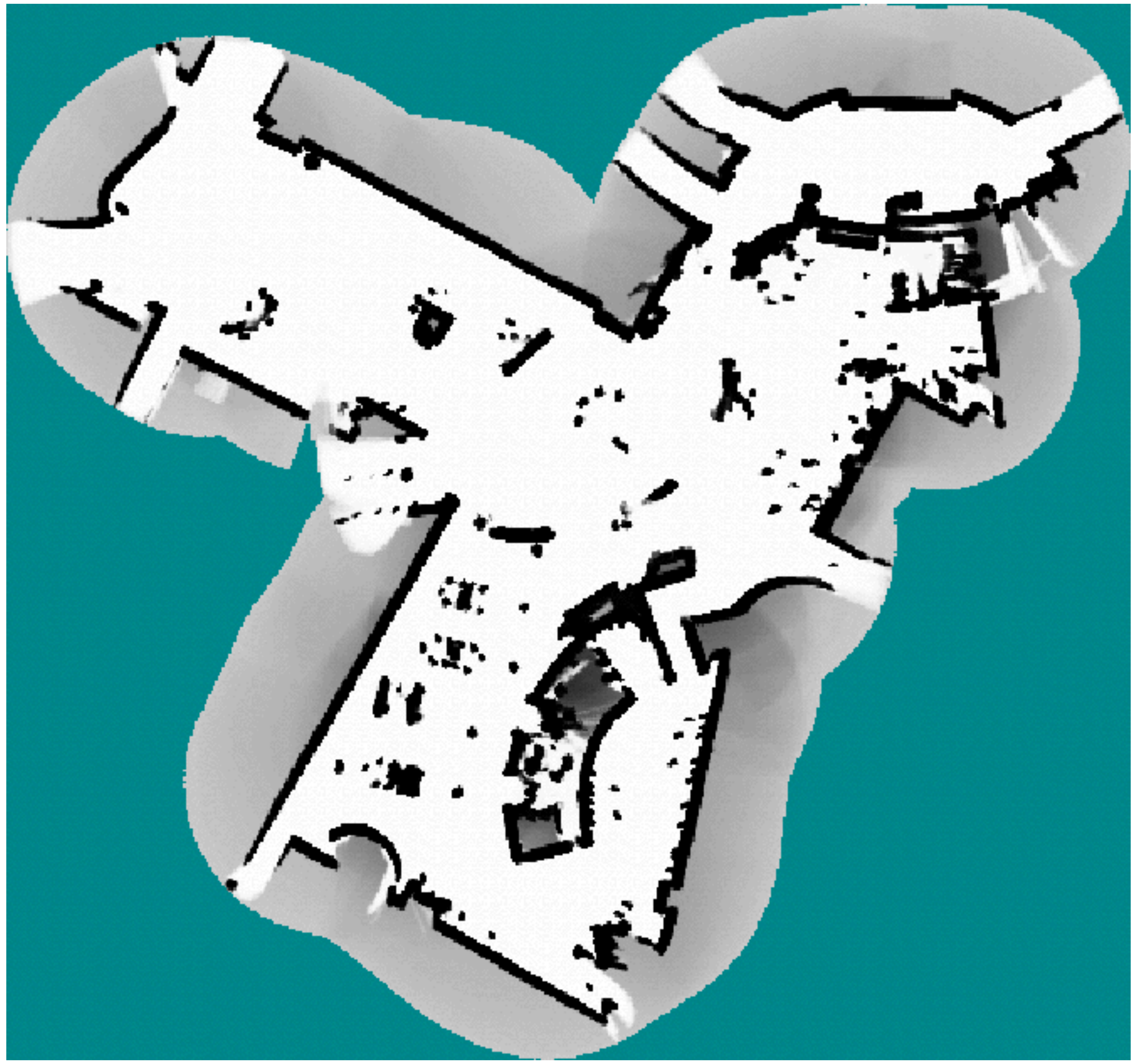
# Occupancy Grids: From scans to maps



# Tech Museum, San Jose

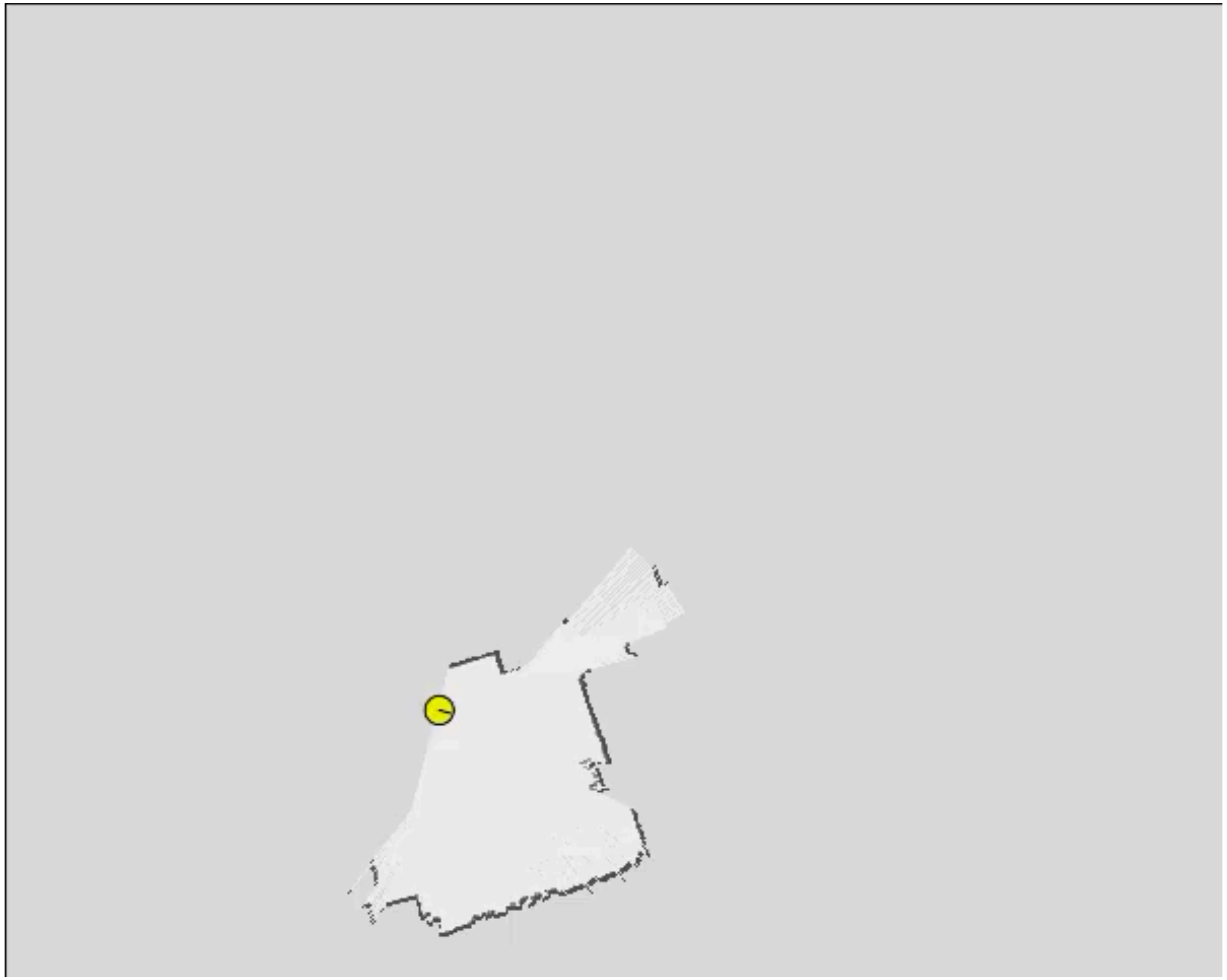


CAD map



occupancy grid map

# Uni Freiburg Building 106



# Occupancy Grid Map Summary

- Occupancy grid maps discretize the space into independent cells
- Each cell is a binary random variable estimating if the cell is occupied
- Static state binary Bayes filter per cell
- Mapping with known poses is easy
- Log odds model is fast to compute
- No need for predefined features

# OctoMap

## A Probabilistic, Flexible, and Compact 3D Map Representation for Robotic Systems

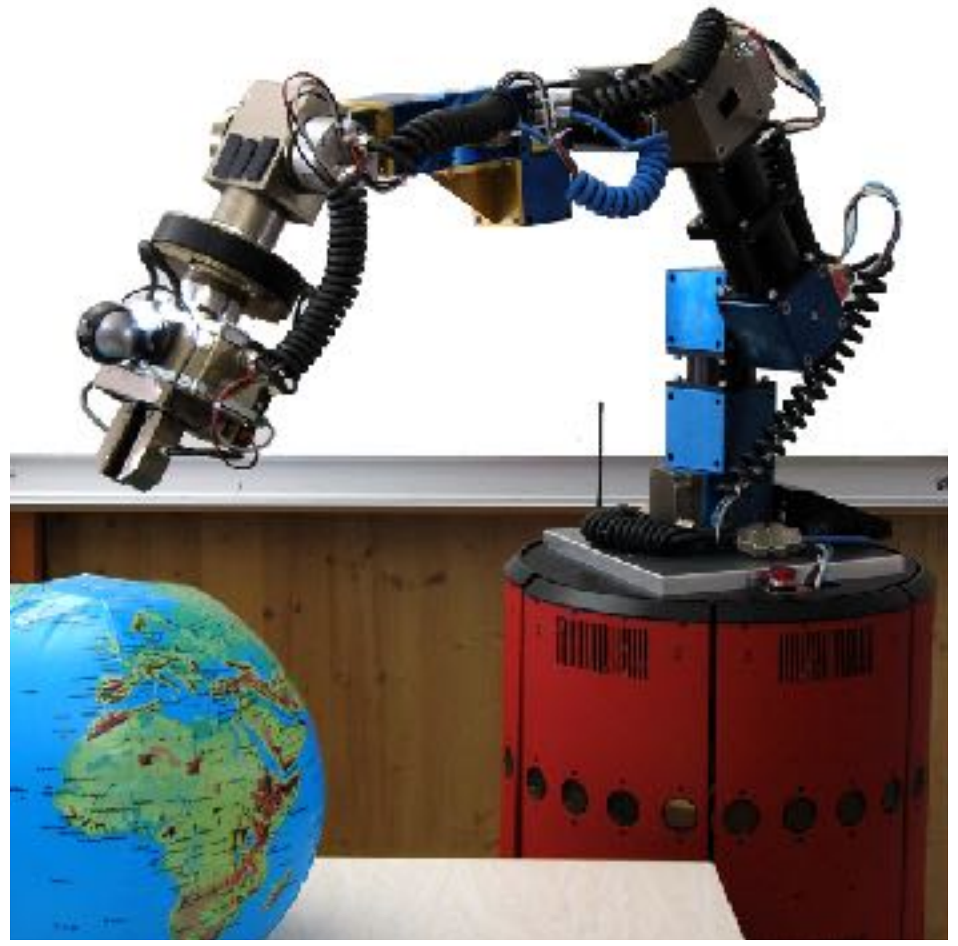

K.M. Wurm, *A. Hornung*,

M. Bennewitz, C. Stachniss, W. Burgard

University of Freiburg, Germany

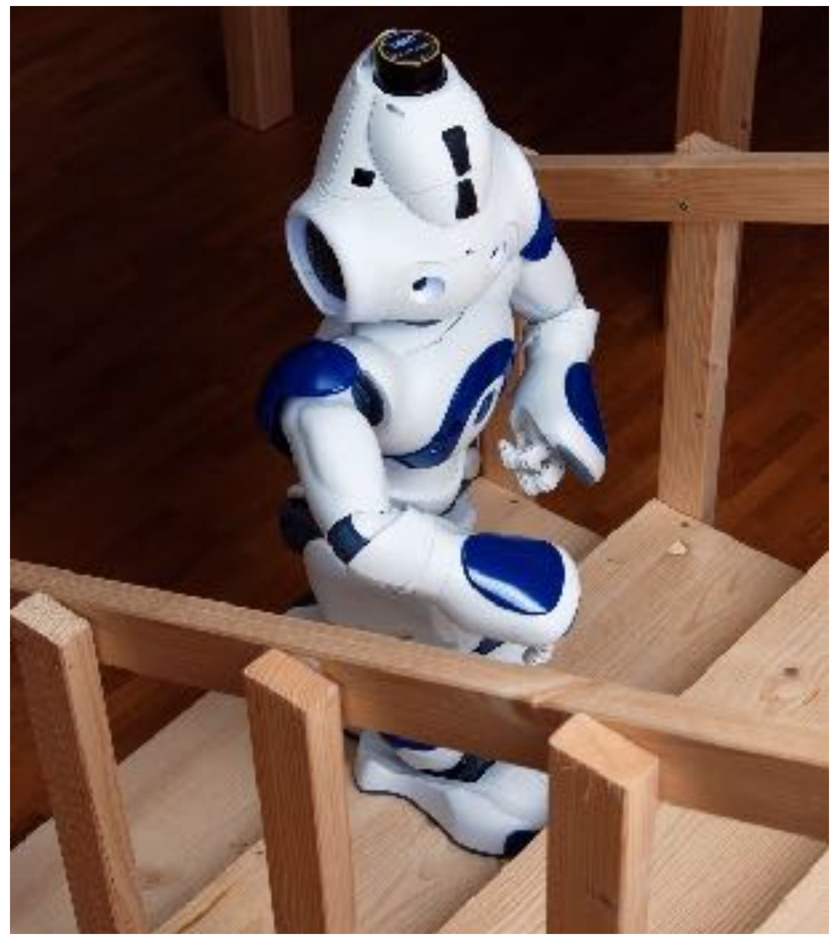

**http://octomap.sf.net**

# Robots in 3D Environments

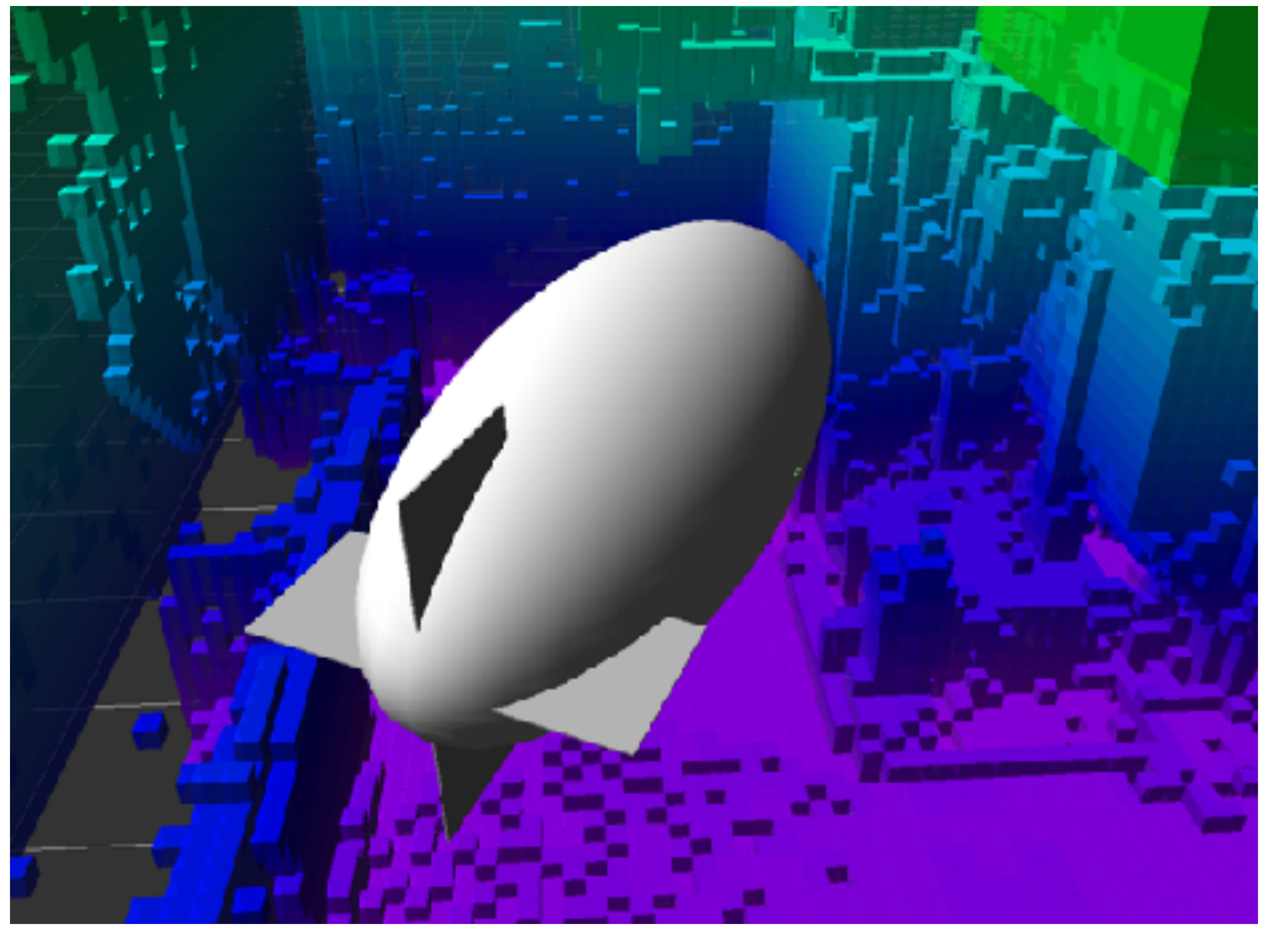


Mobile manipulation

Outdoor navigation



Humanoid robots



Flying robots

# 3D Map Requirements

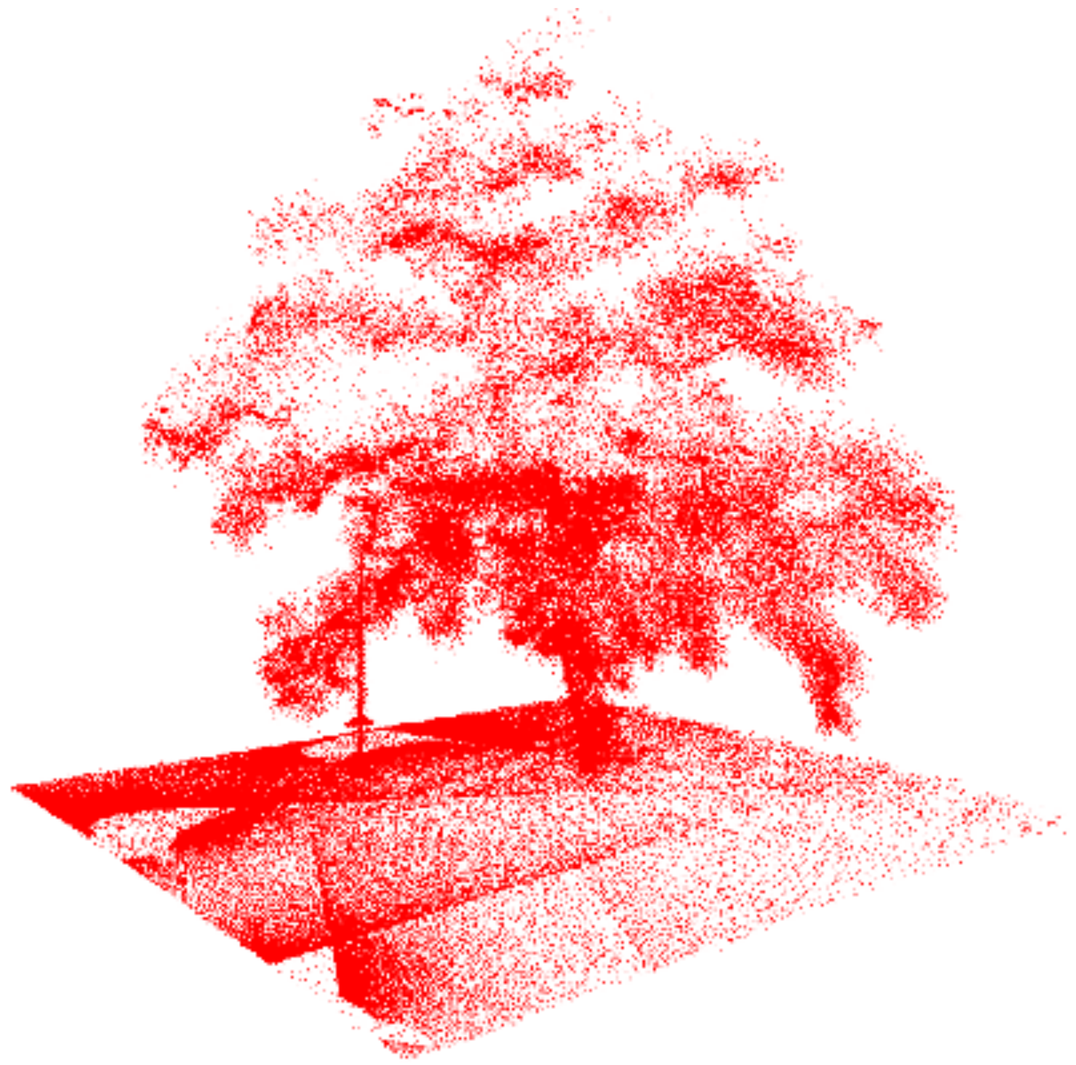
- Full 3D Model
  - Volumetric representation
  - Free-space
  - Unknown areas (e.g. for exploration)
- Can be updated
  - Probabilistic model
    (sensor noise, changes in the environment)
  - Update of previously recorded maps
- Flexible
  - Map is dynamically expanded
  - Multi-resolution map queries
- Compact
  - Memory efficient
  - Map files for storage and exchange



*Slide borrowed from Dieter Fox*

# Map Representations

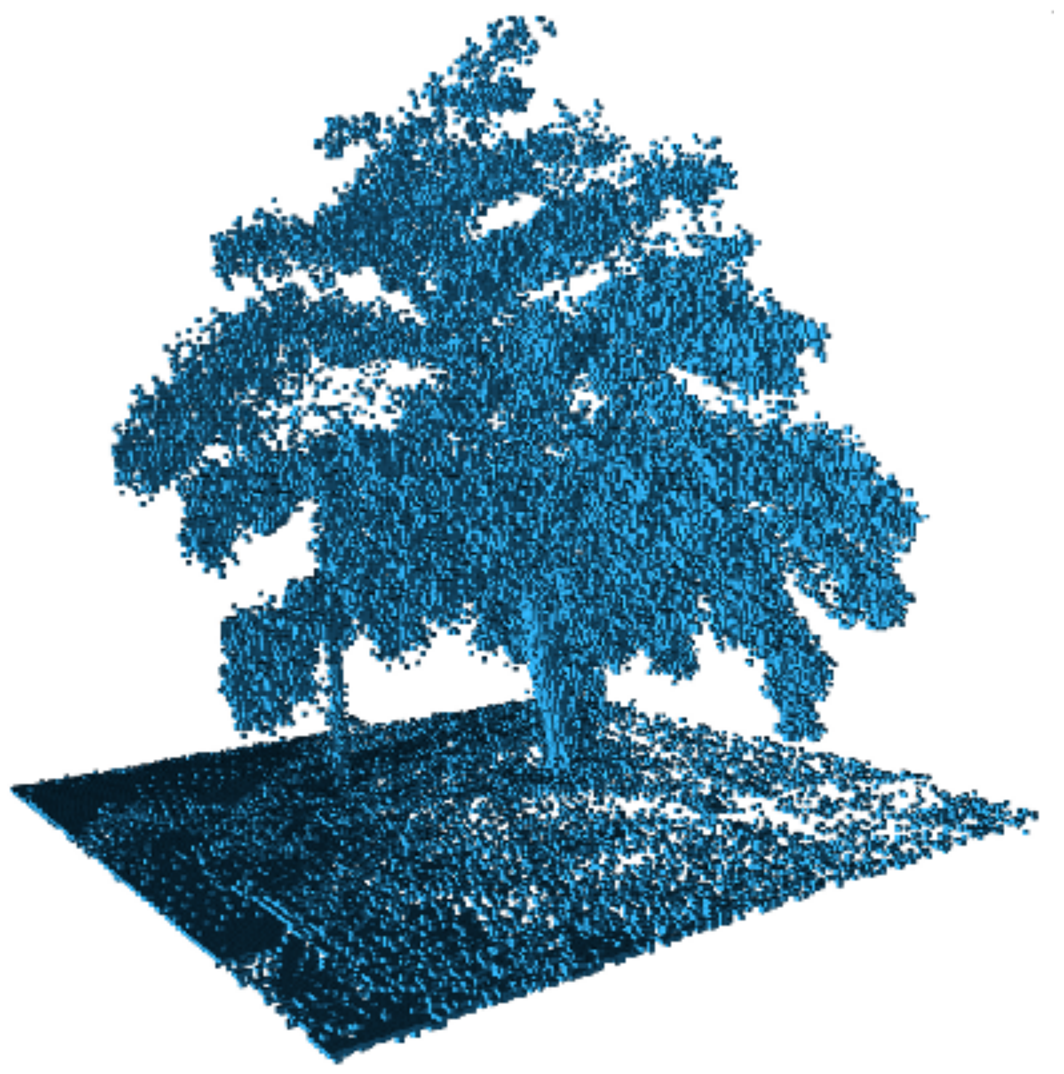
Pointclouds



- Pro:
  - No discretization of data
  - Mapped area not limited
- Contra:
  - Unbounded memory usage
  - No direct representation of free or unknown space

# Map Representations

3D voxel grids



- Pro:
  - Probabilistic update
  - Constant access time
- Contra:
  - Memory requirement
    - Extent of map has to be known
    - Complete map is allocated in memory



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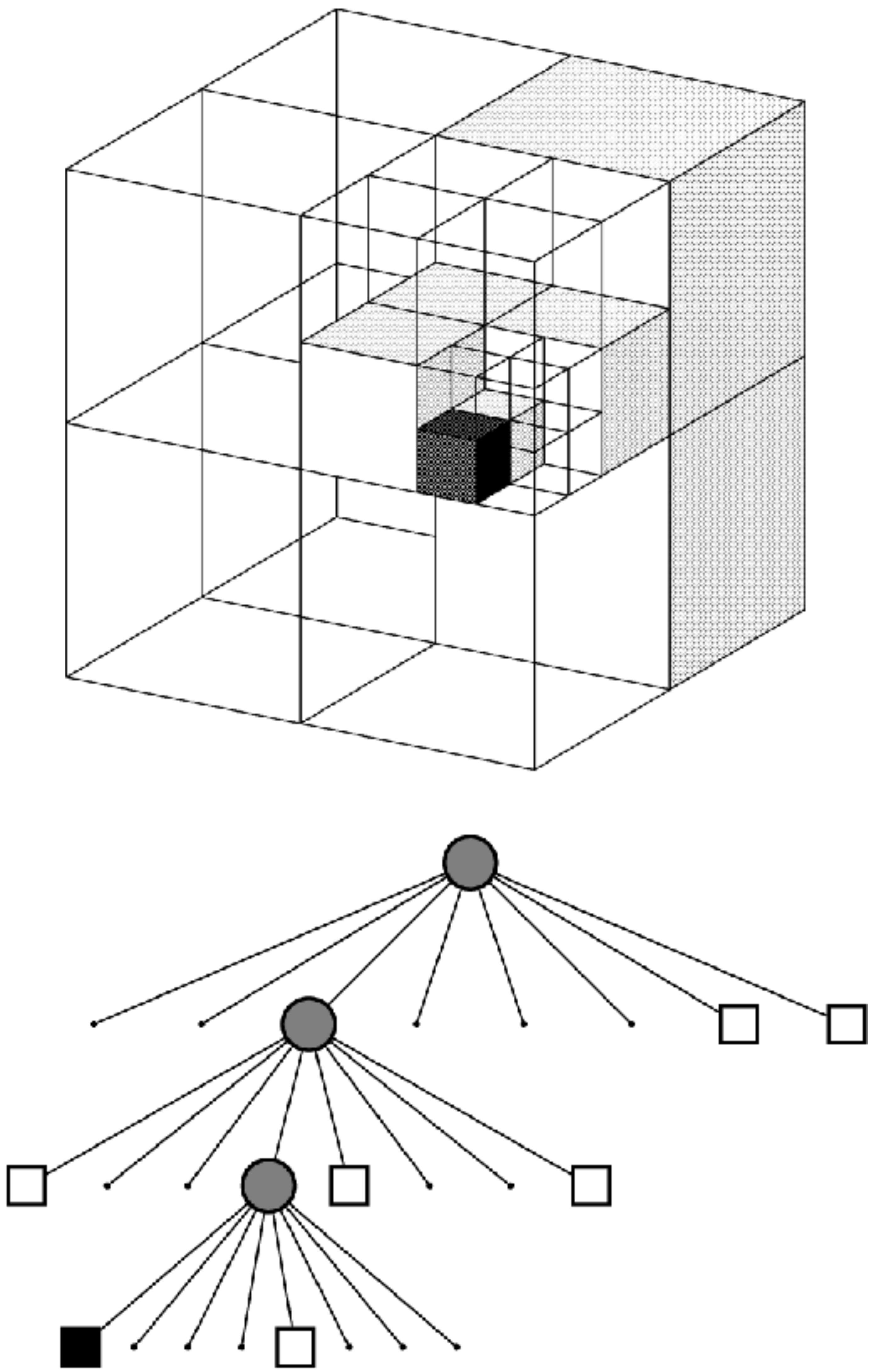
# Map Representations

## Octrees

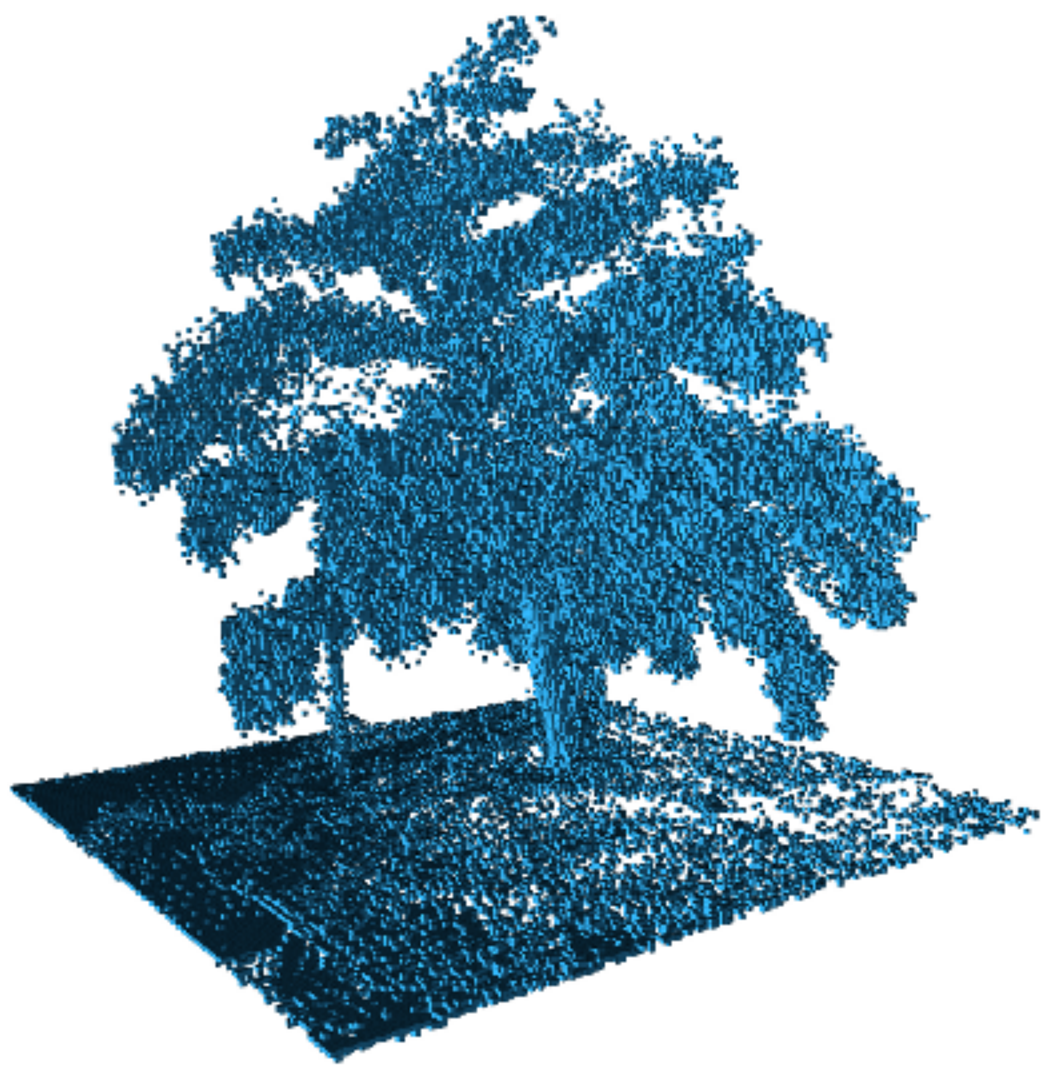
- Tree-based data structure
- Recursive subdivision of space into octants
- Volumes allocated as needed
- Multi-resolution



# Map Representations

## Octrees

- Pro:
  - Full 3D model
  - Probabilistic
  - Flexible, multi-resolution
  - Memory efficient

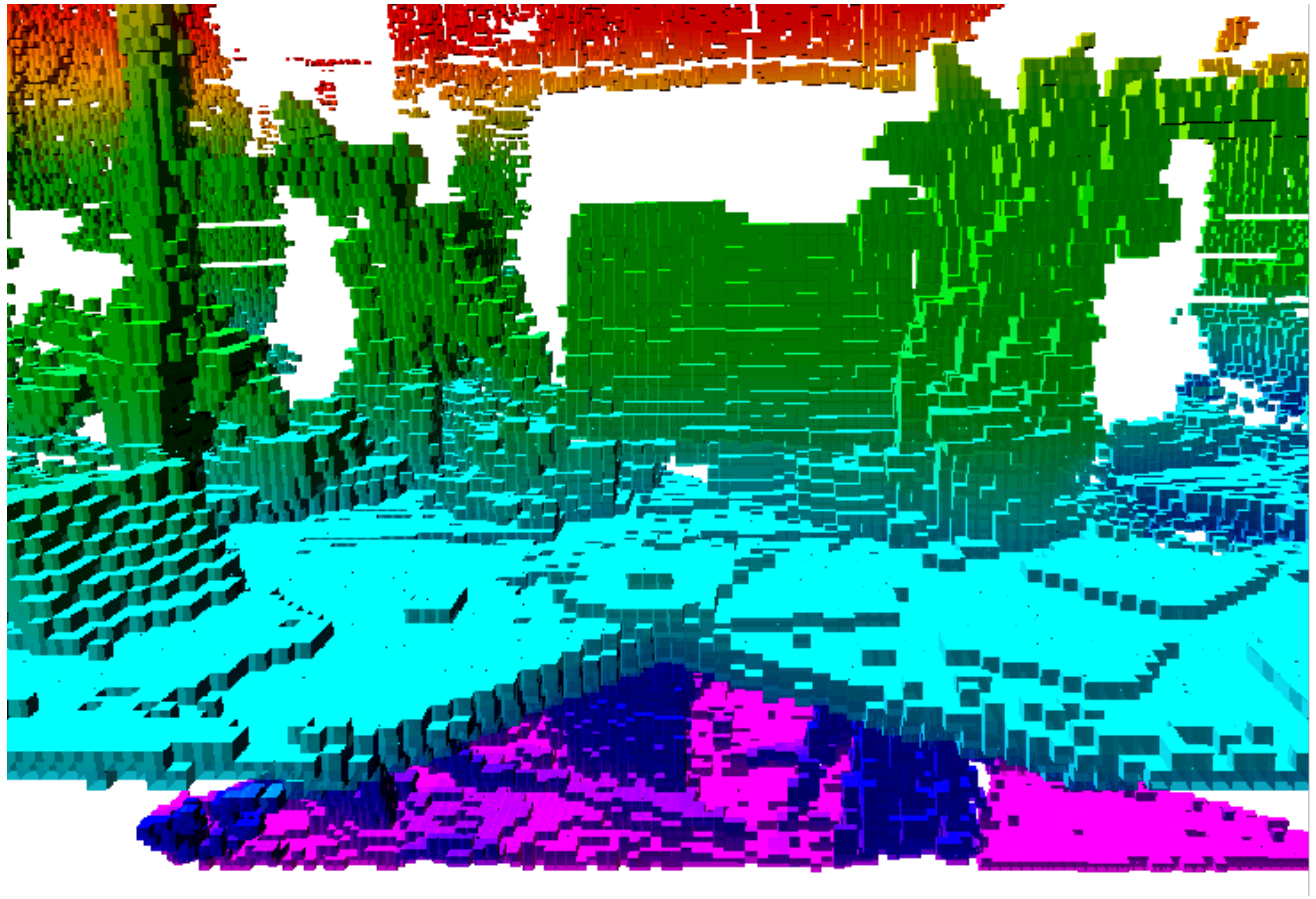


- Contra:
  - Implementation can be tricky (memory, update, map files, …)

- Open source implementation as C++ library available at http://octomap.sf.net

# Examples

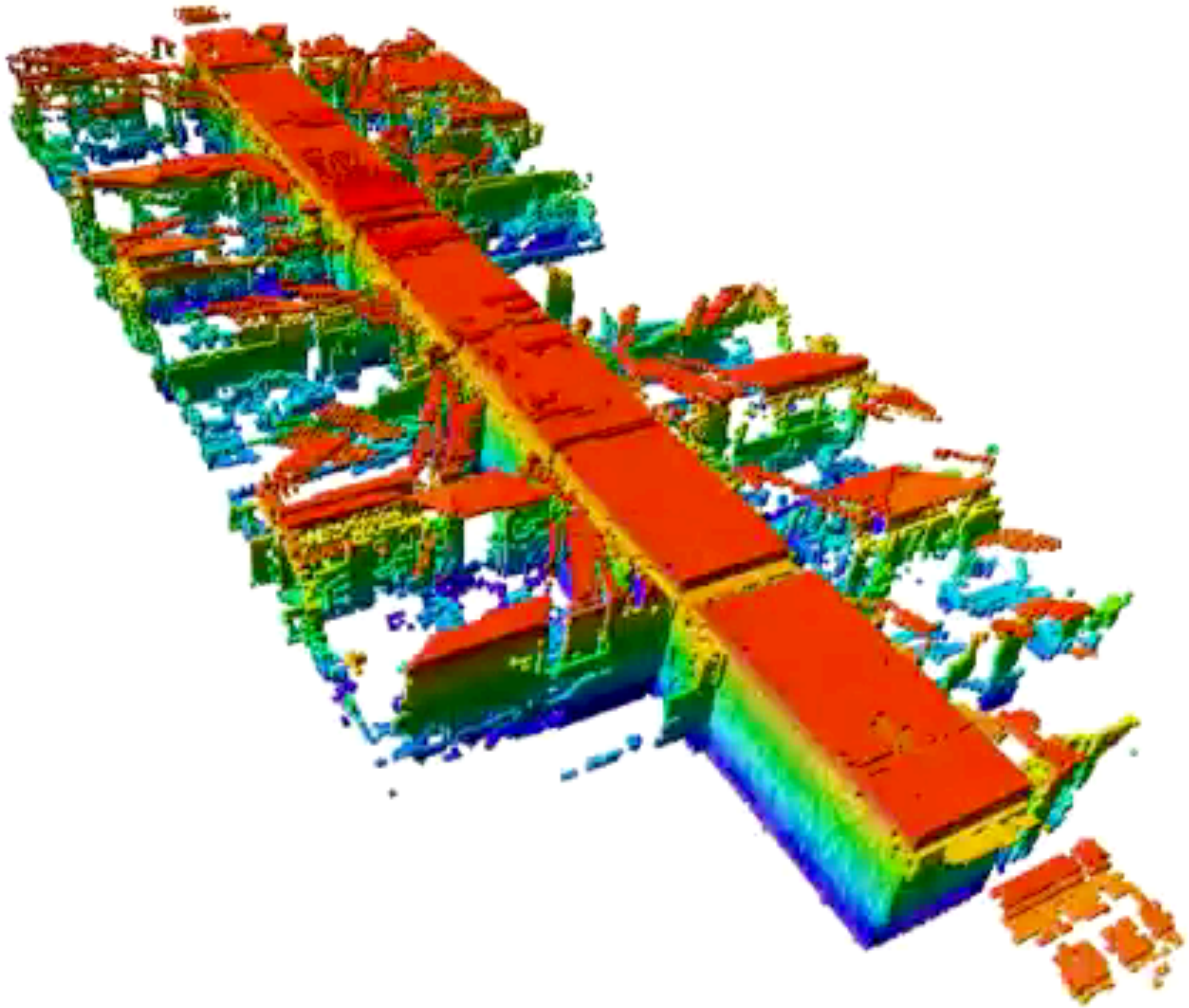
- Cluttered office environment



Map resolution: 2 cm

# Examples: Office Building

- Freiburg, building 079



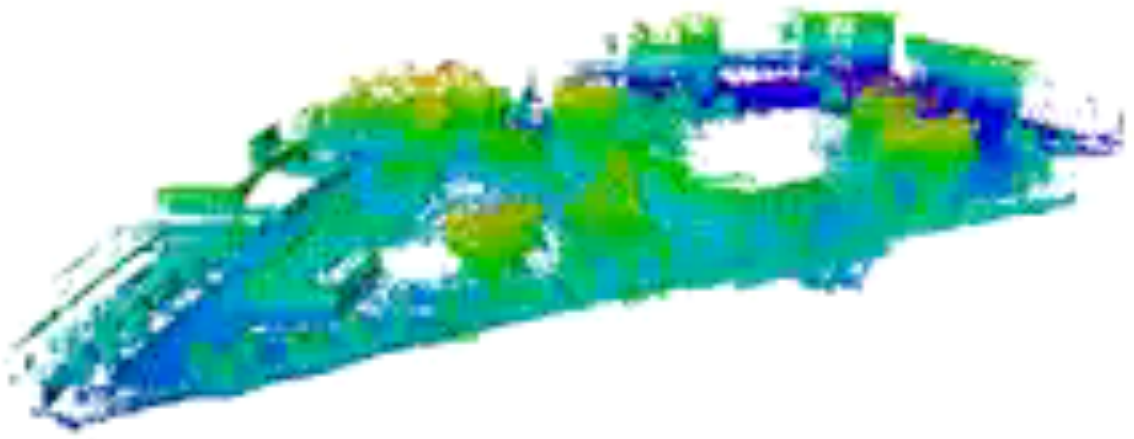


*Slide borrowed from Dieter Fox*

# Examples: Large Outdoor Areas

- Freiburg computer science campus

  (292 x 167 x 28 $m^3$, 20 cm resolution)



# Examples: Tabletop



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# Frontier-based Exploration:

Frontier-based exploration is **the process of repeatedly detecting frontiers and moving towards them, until there are no more frontiers and therefore no more unknown regions.**

## What are frontiers?

**Frontier cells define the border between known and unknown space.**

# Next Lecture:
# SLAM