

# Image Denoising and Deblurring

## Applied Math 515 Final Project

Samuel Rudy, Kelsey Maass, Riley Molloy, and Kevin Mueller

March 12 2016

# Contents

# Image Denoising and Deblurring



# Mathematical Formulation

$$Ax + w = b$$

True image

Observed image

Blur operator

Noise

# Naive Solution

$$x = A^{-1}b$$



True image



Blurred image



Recovered image

# Naive Solution

$$x = A^{-1}(b-w)$$



True image



Blurred and noisy image



Recovered image

Since the naive solution is contaminated by round-off error and noise, a better approach is to solve the problem:

$$\min_x \underbrace{f(Ax - b)}_{\text{Fidelity term}} + \underbrace{\lambda R(x)}_{\text{Regularization}}$$

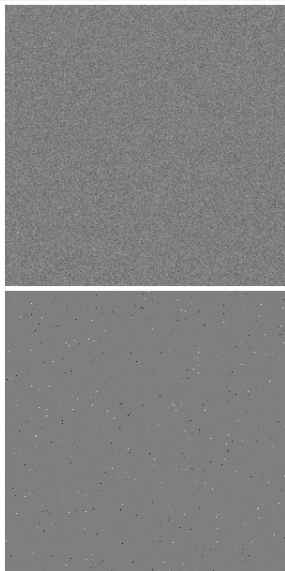
### Fidelity Term

$$f = \begin{cases} \|\cdot\|_F^2 \\ h_\gamma(\cdot) \\ \gamma^{-1} \log(\cosh(\gamma \cdot)) \end{cases}$$

### Regularization Term

$$R = \begin{cases} TV(x) \\ \|Wx\|_1 \end{cases}$$

# Fidelity Term Penalty Functions



What is  $Ax - b$  Why use different functions than frobenius norm? Use pictures as motivation. Show quadratic and huber penalty functions, show different distributions?



# Image Assumptions and Regularizers

Either assume image is sparse in wavelet domain, or assume image is smooth

Talk about choice of  $g$  Haar, FFT What is TV?

Show two different definitions of TV from paper.

# Kelsey's stuff here

- Loss function
- choice of  $\lambda$
- choice of wavelet
- prox gradient method

# Total Variation Regularization

## Loss Function

$$L_b(x) = f(Ax - b) + \lambda \|x\|_{TV} + \delta(x|_{[0,1]})$$

## Proximal Gradient Step

$$\begin{aligned} x^{k+1} &= \text{prox}_{\mathcal{L}^{-1}(\lambda \|\cdot\|_{TV} + \delta_{[0,1]})} \underbrace{(x^k - \mathcal{L}^{-1}A^T \nabla f(Ax^k - b))}_{u^k} \\ &= \arg \min_z (\|u^k - z\|_F^2 + \lambda \|z\|_{TV} + \delta(z|_{[0,1]})) \\ &= P_{[0,1]} \left( \arg \min_z (\|u^k - z\|_F^2 + \lambda \|z\|_{TV}) \right) \end{aligned}$$

# Dual Form of Total Variation

A Few Definitions

weee

Total Variation

blarg

# Dual Form of TV Denoising with $\|\cdot\|_F^2$

# Optimization of Dual Form

# Questions?



Codes used to generate figures

<https://github.com/snagcliffs/Amath575project>



Guckenheimer, J., Holmes, P. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer-Verlag, 1983. Print.



Oliveira, D., Leonel, E. (2008) *Braz. J. Phys.* 38(1):62-64



Grassberger, P., Procaccia, I. (1983) *Phys. Rev. Letters*. 50(5):346-349