Image Denoising and Deblurring

Applied Math 515 Final Project

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Contents

Image Denoising and Deblurring

Mathematical Formulation

Artificial Blur/Noise

- Blur added via convolution with Gaussian kernel.
- Gaussian or Student-t noise added to blurred image.

Image Recovery

Find an image x balancing two properties

1. Convolution of x with blur kernel is similar to b
2. Some measure of noise on x is regularized

This slide kind of sucks as is :) Anyone wanna fix it?

A General Loss Function

$$L_b(x) = \underbrace{f(Ax-b)}_{\text{Fidelity Term}} + \underbrace{\lambda g(x)}_{\text{Noise Regularization}} + \underbrace{\delta(x|[0,1])}_{\text{Range of Pixel Values}}$$

Fidelity Term

$$f = \begin{cases} \|\cdot\|_F^2 \\ h_\gamma(\cdot) \\ \gamma^{-1}\log(\cosh(\gamma\cdot)) \end{cases}$$

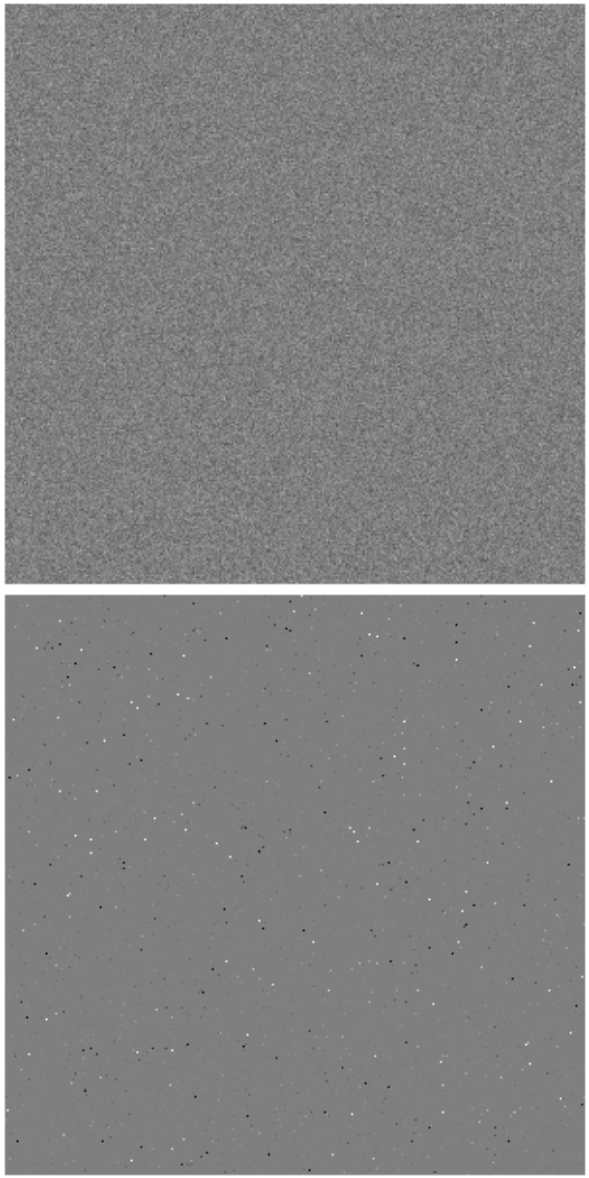
Regularization Term

$$g = \begin{cases} TV(x) \\ \|Wx\|_1 \end{cases}$$

Regularizer

Talk about choice of g Haar, FFT What is TV?
Show two different definitions of TV from paper.

Fidelity Function



What is Ax - b Why use different functions than frobenius norm? Use pictures as motivation

Kelsey's stuff here

Total Variation Regularization

Loss Function

$$L_b(x) = f(Ax - b) + \lambda \|x\|_{TV} + \delta(x|[0,1])$$

Proximal Gradient Step

$$\begin{aligned} x^{k+1} &= \text{prox}_{\mathcal{L}^{-1}(\lambda\|\cdot\|_{TV} + \delta_{[0,1]})}(\underbrace{x^k - \mathcal{L}^{-1}A^T\nabla f(Ax^k - b)}_{u^k}) \\ &= \arg\min_z \left(\|u^k - z\|_F^2 + \lambda\|z\|_{TV} + \delta(z|[0,1]) \right) \\ &= P_{[0,1]}\left(\arg\min_z \left(\|u^k - z\|_F^2 + \lambda\|z\|_{TV} \right) \right) \end{aligned}$$

Dual Form of Total Variation

A Few Definitions

weee

Total Variation

blarg

Dual Form of TV Denoising with $\|\cdot\|_F^2$

Optimization of Dual Form

Questions?

- Codes used to generate figures
  `https://github.com/snagcliffs/Amath575project`
- Guckenheimer, J., Holmes, P. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer-Verlag, 1983. Print.
- Oliveira, D., Leonel, E. (2008) *Braz. J. Phys.* 38(1):62-64
- Grassberger, P., Procaccia, I. (1983) *Phys. Rev. Letters.* 50(5):346-349