Image Deblurring and Denoising for Various Fidelity Terms and Regularizers

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1 Introduction

make your images pretty

2 Background of Methodology and Implementation

The general approach of to an image deblurring or denoising problem is the minimization of a loss function of the form

$$\min_x f(\mathcal{A}(x) - b) + g(x), \tag{1}$$

where $\mathcal{A}(x)$ is a convolution of the desired image x with a gaussian kernel creating a blur, $f : \mathbb{R}^{m\times n} \to [0, \infty)$ represents some continuous measure of distance between the corrupt image b and the desired image x, and $g : \mathbb{R}^{m\times n} \to [0, \infty)$ is some regularization on the allowed amount of noise in x. The term $f(\mathcal{A}(x) - b)$ is referred to as the *fidelity term*, and the term $g(x)$ is referred to as the *regularization*. We can represent the blurring convolution $\mathcal{A}(\cdot) : \mathbb{R}^{m\times n} \to \mathbb{R}^{m\times n}$ by left matrix multplication with some matrix A, so we do so for convenience from here forward. In addition, since we require some pixel value $x_{ij} \in [0, 1]$, we can add an additional term to the loss function as

$$L_b(x) = f(Ax - b) + g(x) + \delta(x|[0, 1]), \tag{2}$$

where $\delta(x|[0, 1])$ is an indiciator function for the unit interval.

here we can explain the general prox-gradient approach, and go into details for specific cases in the following subsections

2.1 Total Variation Regularization

The usual Total-Variation deblurring model, as seen in (BECK/TOUBELLE REF) can be formulated as

$$\min_x \|Ax - b\|^2 + 2\lambda \mathrm{TV}(x), \tag{3}$$

where $\|\cdot\|$ is taken as either a Frobenius norm or a 2-norm, depending on applications, $\lambda > 0$ is a regularization parameter, and $\mathrm{TV}(x)$ is the Total-Variation semi-norm. Two choices similar choices exist for the TV-norm: the so-called isotropic type, and the l_1 type. In this work, we work exclusively with the l_1-based TV-norm, defined as

$$TV_{l_1}(x) = \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} \left(|x_{i,j} - x_{i+1,j}| + |x_{i,j} - x_{i,j+1}|\right) + \sum_{i=1}^{m-1} |x_{i,n} - x_{i+1,n}| + \sum_{j=1}^{n-1} |x_{m,j} - x_{m,j+1}|,$$

for $x \in \mathbb{R}^{m\times n}$, and where the reflexive boundary conditions

$$\begin{aligned} x_{m+1,j} - x_{m,j} &= 0, \text{ for all } j \\ x_{i,n+1} - x_{i,n} &= 0, \text{ for all } i \end{aligned}$$

are assumed. Our approach considers a more general problem

$$\min_x f(Ax - b) + 2\lambda \mathrm{TV}_{l_1}(x), \tag{4}$$

where $f : \mathbb{R}^{m\times n} \to [0, \infty)$ is any continuous functional which gives a measurement of the size of the fidelity term.

2.2 1-Norm Wavelet Regularization

3 Testing

3.1 Something

3.2 Something Else

4 Results

5 Discussion

References