Dynamical Instability of a Condensate Induced by a Rotating Thermal Gas

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(Received 10 September 2001; published 30 January 2002)

We study surface modes of the condensate in the presence of a rotating thermal cloud in an axisymmetric trap. By considering collisions that transfer atoms between the condensate and the noncondensate, we find that $m > 0$ modes, which rotate in the same sense as the thermal cloud, damp less strongly than $m < 0$ modes, where m is the polarity of the excitation. We show that above a critical angular rotation frequency, equivalent to the Landau stability criterion, $m > 0$ modes become dynamically unstable, leading to the possibility of vortex nucleation. We also generalize our stability analysis to treat the case where the stationary state of the condensate already possesses a single vortex.

DOI: 10.1103/PhysRevLett.88.070401 PACS numbers: 03.75.Fi, 05.30.Jp, 67.40.Db

The recent success at producing vortices with quantized angular momentum in trapped Bose gases [1–3] has raised interesting questions regarding the microscopic mechanisms responsible for vortex nucleation. Several papers [4–9] have suggested the idea that a possible mechanism is the excitation of low energy surface condensate modes, which have been studied in various experiments. This can be understood [4,6,7] in terms of the well-known Landau criterion [10], which predicts the excitation of condensate modes with frequency ω_m and angular momentum $m\hbar$ when the rotation rate Ω of an external perturbation satisfies the condition $\omega_m - m\Omega < 0$. This defines a critical frequency, $\Omega_{c,m} \equiv \omega_m/m$ for transfer of angular momentum to the condensate [4,5], where the particular mode excited depends upon the form of the "stirring" potential used. For example, a rotating trap anisotropy will predominantly couple to a quadrupole surface mode, which will become dynamically unstable above the Landau frequency $\Omega_{c,2}$. In related work, Sinha and Castin [11] have found dynamical instabilities about stationary condensate solutions in the rotating frame [12], and link these to vortex nucleation in a recent experiment [13]. Further experiments lend support to the role of dynamical instabilities of surface modes in the vortex formation process [3,14].

In the present Letter, we extend this discussion of vortex nucleation to finite temperatures and include for the first time the coupling of the thermal cloud to the condensate. Our calculations address the kind of experiments done at JILA [15], in which a thermal cloud is set into rotational motion above $T_{\rm BEC}$ by means of an anisotropic rotating drive. A subsequent quench through the Bose-Einstein condensation transition in the absence of the drive then leads to the formation of a condensate containing vortices (when the rotation frequency Ω of the thermal cloud exceeds a critical value). What sets the JILA study [15] apart from all other recent experiments on vortices [1,3,13,14] is that the nucleation process occurs in the presence of an *axially symmetric trap*. The JILA experiment indicates that the rotating thermal cloud is acting as a reservoir of angular momentum that is transferred to the condensate in the process of vortex formation.

It is clear that a breaking of rotational symmetry is required if such a transfer is to occur. Thermal excitation of condensate collective modes is an obvious candidate for this symmetry breaking. The density fluctuations associated with such a mode are coupled to the thermal cloud by means of the condensate mean field, as well as through the collisional exchange of atoms between the condensate and the thermal cloud. Of these two physical mechanisms, we focus on the latter in this paper. The mean-field coupling is also expected to be important, but we shall demonstrate that the collisional exchange of atoms is already sufficient to exhibit the dynamical instability of the condensate induced by the rotating thermal cloud.

Generalizing recent work by Williams and Griffin [16], we calculate the damping of condensate collective modes at a finite temperature due to the coupling via C_{12} collisions between the condensate and the noncondensate atoms. In formulating this problem, we assume that the thermal cloud is in rigid body rotation with angular frequency Ω, as in the JILA experiments [15]. Our major result is that the damping of the collective modes of frequency ω_m depends on the polarity m of the collective mode with respect to the rotation of the thermal cloud. As Ω is increased, the damping of the $m < 0$ modes increases while that of $m > 0$ modes decreases. At a critical rotation frequency $\Omega_{c,m} \equiv \omega_m/m$ of the thermal cloud, the imaginary part of the excitation frequency changes sign for the $m > 0$ modes, which thus become unstable in the sense that the excitation amplitude grows exponentially with time. This signals a continuous transfer of angular momentum from the thermal cloud to the condensate, which we identify with the onset of vortex formation.

The dynamics of the condensate wave function $\Phi(\mathbf{r}, t)$ in the Zaremba-Nikuni-Griffin (ZNG) kinetic theory [17] is described by a generalized Gross-Pitaevskii (GP) equation, which at finite temperatures includes the interaction with the noncondensate through a mean-field term and a non-Hermitian damping term due to C_{12} collisions that exchange atoms between the condensate and the noncondensate. The noncondensate, represented by a phase-space distribution function $f(\mathbf{r}, \mathbf{p}, t)$ obeying a semiclassical kinetic equation [17], is likewise coupled to the condensate through mean-field and collisional exchange terms. The atoms are trapped in a static harmonic trap possessing cylindrical symmetry $U_{\rm ext}(\mathbf{r}) = M\omega_\perp^2(\rho^2 + \lambda^2 z^2)/2$, where the aspect ratio of the trap is $\lambda = \omega_z/\omega_\perp$. The noncondensate atoms move in a dynamic Hartree-Fock (HF) potential $U(\mathbf{r}, t) = U_{\rm ext}(\mathbf{r}) + 2g[n_c(\mathbf{r}, t) + \tilde{n}(\mathbf{r}, t)]$, where the interaction parameter $g = 4\pi\hbar^2 a/M$, a is the s-wave scattering length, $n_c(\mathbf{r}, t) = |\Phi(\mathbf{r}, t)|^2$, and $\tilde{n}(\mathbf{r}, t)$ is the noncondensate density defined by $\tilde{n}(\mathbf{r}, t) = \int d\mathbf{p}\, f(\mathbf{r}, \mathbf{p}, t)/(2\pi\hbar)^3$.

In our idealized model of the JILA experiments, the collective excitations of the condensate are determined for an equilibrium state defined by a thermal cloud undergoing solid body rotation at the angular frequency Ω. For this situation, the equilibrium distribution $f^0(\mathbf{r}, \mathbf{p})$ of the rotating thermal cloud is given by

$$f^0 = \frac{1}{\exp\{\beta[(\mathbf{p} - M\mathbf{v}_{n0})^2/2M + U_{\rm eff} - \tilde{\mu}_0]\} - 1}, \tag{1}$$

where the noncondensate velocity field is $\mathbf{v}_{n0}(\mathbf{r}) = \mathbf{\Omega} \times \mathbf{r} = \Omega\rho\hat{\phi}$. The effective potential acting on the thermal cloud is $U_{\rm eff}(\mathbf{r}) = U_0(\mathbf{r}) - M\Omega^2\rho^2/2$, with $U_0(\mathbf{r})$ the equilibrium HF potential. The corresponding equilibrium state of the condensate is given by the solution of the time-independent generalized GP equation. Although we are primarily interested in the initial onset of vortex formation, we can readily extend our analysis to a situation in which the condensate already has a single vortex centered on the z axis with q units of quantized circulation. The equilibrium condensate wave function $\Phi^{(q)}(\rho, \phi, z) = \psi^{(q)}(\rho, z)e^{iq\phi}$ is then defined by

$$\left(-\frac{\hbar^2\nabla^2}{2M} + \frac{M}{2}v_{c0}^2 + U_{\rm ext} + gn_{c0} + 2g\tilde{n}_0\right)\psi^{(q)} = \mu_0^{(q)}\psi^{(q)}. \tag{2}$$

Here, the condensate velocity field is given by $\mathbf{v}_{c0}(\rho) = (\hbar q/M\rho)\hat{\phi}$.

In equilibrium, the chemical potentials of the thermal cloud and condensate are related by $\tilde{\mu}_0 = \mu_0^{(q)} - \hbar\Omega q$. It is straightforward to prove that the form of $f^0(\mathbf{r}, \mathbf{p})$ in (1) is a stationary solution of the ZNG kinetic equation for the case of a static axially symmetric trap when the chemical potentials satisfy this relation. Using (1), the thermal cloud density is given by $\tilde{n}_0(\rho, z) = g_{3/2}[z_0(\rho, z)]/\Lambda_0^3$, where the thermal de Broglie wavelength is $\Lambda_0 \equiv (2\pi\hbar^2/Mk_BT)^{1/2}$ and the equilibrium fugacity is $z_0(\rho, z) \equiv e^{-\beta[U_{\rm eff}(\rho,z) - \tilde{\mu}_0]}$. As Ω increases, the aspect ratio of the noncondensate density increases approximately as $\omega_z/\sqrt{\omega_\perp^2 - \Omega^2}$.

Within our model, we neglect the dynamics of the noncondensate and assume the thermal cloud remains rigidly rotating, as described by (1) (see also Refs. [16,18,19] for a discussion of this type of static approximation). In effect, we are ignoring the perturbation of the rotating normal cloud induced by the mean field of the oscillating condensate. Working with the amplitude and phase of the condensate $\Phi(\mathbf{r}, t) = \sqrt{n_c(\mathbf{r}, t)}\, e^{i\theta(\mathbf{r}, t)}$, the condensate dynamics is described by the coupled quantum hydrodynamic equations [16,17],

$$\frac{\partial n_c}{\partial t} + \nabla \cdot (n_c\mathbf{v}_c) = -\Gamma_{12}^0, \tag{3}$$

$$M\frac{\partial \mathbf{v}_c}{\partial t} = -\nabla\left(\mu_c + \frac{1}{2}Mv_c^2\right), \tag{4}$$

where the local chemical potential $\mu_c(\mathbf{r}, t)$ of the condensate is

$$\mu_c = -\frac{\hbar^2}{2M}\frac{\nabla^2 n_c^{1/2}}{n_c^{1/2}} + U_{\rm ext} + gn_c + 2g\tilde{n}_0. \tag{5}$$

The condensate velocity is $\mathbf{v}_c(\mathbf{r}, t) = (\hbar/M)\nabla\theta(\mathbf{r}, t)$. The source term $\Gamma_{12}^0(\mathbf{r}, t)$ appearing in (3) describes the collisional exchange of atoms between the condensate and the thermal cloud and is given by (see also Refs. [16,17])

$$\Gamma_{12}^0 \equiv \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} C_{12}[f^0, \Phi] = \frac{n_c}{\tau_{12}}\{e_0^{\beta[\varepsilon_c - \tilde{\mu}_0 - M\mathbf{v}_c\cdot\mathbf{v}_{n0}]} - 1\}, \tag{6}$$

where the collision time $\tau_{12}(\mathbf{r}, t)$ is defined as

$$\tau_{12}^{-1} = \frac{2g^2}{(2\pi)^5\hbar^7}\int d\mathbf{p}_1 \int d\mathbf{p}_2 \int d\mathbf{p}_3 \times \delta(\mathbf{p}_c + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \times \delta(\varepsilon_c + \tilde{\varepsilon}_{p_1} - \tilde{\varepsilon}_{p_2} - \tilde{\varepsilon}_{p_3})(1 + f_1^0)f_2^0 f_3^0. \tag{7}$$

The local condensate momentum per atom is $\mathbf{p}_c(\mathbf{r}, t) \equiv M\mathbf{v}_c(\mathbf{r}, t)$ and the local energy is $\varepsilon_c(\mathbf{r}, t) = \mu_c(\mathbf{r}, t) + M\mathbf{v}_c^2/2$. The equilibrium single-particle distribution function $f_i^0 \equiv f^0(\mathbf{r}, \mathbf{p}_i)$ is as given by (1). We note that the term Γ_{12}^0 in (3) is the only source of damping in our calculations. Our static approximation for the rotating thermal cloud precludes Landau damping, which can be expected to provide additional damping of the condensate modes.

To study the collective modes, we consider the density $\delta n_c(\mathbf{r}, t)$ and velocity $\delta\mathbf{v}_c(\mathbf{r}, t)$ fluctuations of the condensate about equilibrium $n_c(\mathbf{r}, t) = n_{c0}(\mathbf{r}) + \delta n_c(\mathbf{r}, t)$ and $\mathbf{v}_c(\mathbf{r}, t) = \mathbf{v}_{c0}(\mathbf{r}) + \delta\mathbf{v}_c(\mathbf{r}, t)$. In the Thomas-Fermi (TF) limit, the linearized equations of motion for the condensate fluctuations are given by

$$\frac{\partial \delta n_c}{\partial t} + \nabla \cdot (n_{c0}\delta\mathbf{v}_c + \mathbf{v}_{c0}\delta n_c) = -\delta\Gamma_{12}, \tag{8}$$

$$\frac{\partial \delta\mathbf{v}_c}{\partial t} = -\nabla\left(\frac{g}{M}\delta n_c + \mathbf{v}_{c0}\cdot\delta\mathbf{v}_c\right). \tag{9}$$

It is straightforward to show that the linearized form of $\Gamma_{12}^0(\mathbf{r}, t)$ given in (6) is

$$\delta\Gamma_{12} = \frac{1}{\tau'}\left[\delta n_c + \frac{M}{g}\,\delta\mathbf{v}_c \cdot (\mathbf{v}_{c0} - \mathbf{v}_{n0})\right], \qquad (10)$$

where $1/\tau'(\mathbf{r}) \equiv g n_{c0}(\mathbf{r})/[k_B T \tau_{12}^0(\mathbf{r})]$. Here $\tau_{12}^0(\mathbf{r})$ is given by the expression in (7), but with the dynamic or time-dependent condensate quantities replaced by the corresponding equilibrium ones. We remark that the second term in (10) has the form of a mutual friction term depending on the relative velocity $\mathbf{v}_{c0}(\mathbf{r}) - \mathbf{v}_{n0}(\mathbf{r})$ between the condensate and normal gas, which couples to the velocity fluctuation of the condensate $\delta\mathbf{v}_c$. For $\Omega = 0$ and $\mathbf{v}_{c0} = 0$, the results in (3)–(7) reduce to those in Ref. [16].

In the TF approximation, the condensate density is given by $n_{c0}(\mathbf{r}) = (\mu_0^{(q)} - U_{\rm ext} - 2g\tilde{n}_0 - \frac{1}{2}Mv_{c0}^2)/g$. To simplify the calculation, we neglect the effect of the noncondensate mean field on the condensate profile [16]. In addition, we neglect the effect of the vortex core on the overall shape of the condensate. This approximation is valid when $\xi^2/R_\perp^2 \ll 1$, where ξ is the healing length and $R_\perp$ is the Thomas-Fermi condensate radius [20]. We therefore approximate the condensate density as $n_{c0} = (\mu_{\rm TF} - U_{\rm ext})/g$, where $\mu_{\rm TF}(T) = \hbar\omega_\perp[15\lambda N_c(T)a/d_\perp]^{2/5}/2$ and $d_\perp = \sqrt{\hbar/M\omega_\perp}$.

In an axially symmetric trap, the density and velocity fluctuations of the collective modes take the form $\delta n_c(\mathbf{r}, t) = \delta n_m(\rho, z)e^{i(m\phi - \omega_m t)}$ and $\delta\mathbf{v}_c(\mathbf{r}, t) = (\hbar/M)\nabla\delta\theta_m(\rho, z)e^{i(m\phi - \omega_m t)}$. Substituting these expressions into (8) and (9) gives

$$\hbar\left[\hat{S} + \frac{im}{\tau'}\left(\frac{V}{\rho} - \Omega\right)\right]\delta\theta_m - ig\left(\omega_m - \frac{mV}{\rho} + \frac{i}{\tau'}\right)\delta n_m = 0, \qquad (11)$$

$$-i\hbar\left(\omega_m - \frac{mV}{\rho}\right)\delta\theta_m + g\delta n_m = 0, \qquad (12)$$

where $\hat{S} \equiv g[\nabla \cdot (n_{c0}\nabla) - m^2 n_{c0}/\rho^2]/M$ and $V \equiv \hbar q/M\rho$. These equations can be combined into a single equation for δn_m, following the procedure used in Ref. [20]. We relate $\delta\theta_m$ to δn_m, making use of the following approximation to (12):

$$\delta\theta_m \simeq -i\,\frac{g}{\hbar\omega_m}\left(1 + \frac{mV}{\omega_m\rho}\right)\delta n_m. \qquad (13)$$

This is valid for $\rho \gg \xi$, consistent with neglecting the effect of the vortex core on the static condensate density profile. We then obtain a single wave equation for the condensate density fluctuation for mode m:

$$(\omega_m^2 + \hat{S} + \hat{S}_q + i\hat{S}_\tau)\delta n_m = 0, \qquad (14)$$

where

$$\hat{S}_q \equiv \frac{2m\hbar q}{\omega_m M\rho^2}\left[\omega_\perp^2 - \omega_m^2 + \frac{2gn_{c0}}{M\rho}\left(\frac{1}{\rho} - \frac{\partial}{\partial\rho}\right)\right], \qquad (15)$$

and

$$\hat{S}_\tau \equiv (\omega_m - m\Omega)\,\frac{1}{\tau'}. \qquad (16)$$

In order to obtain an approximate solution of (14), we treat the effect of the vortex $\hat{S}_q$ and the damping $\hat{S}_\tau$ perturbatively [16] by expanding in the zeroth-order TF solutions given by $\hat{S}\delta n_S = -\omega_S^2\delta n_S$ [21]. Equation (14) can then be simplified to [16]

$$\omega_m^2 - [\omega_m^{(q)}]^2 + i(\omega_m - m\Omega)\left\langle\frac{1}{\tau'}\right\rangle = 0, \qquad (17)$$

where the spatial average of an operator χ is defined by $\langle\chi\rangle \equiv \int d\mathbf{r}\,\delta n_S(\mathbf{r})\chi(\mathbf{r})\delta n_S(\mathbf{r})/\int d\mathbf{r}\,\delta n_S^2(\mathbf{r})$. The frequency $\omega_m^{(q)}$ is the collective mode frequency of a vortex state [20,22],

$$[\omega_m^{(q)}]^2 \equiv \omega_S^2(1 + qm\Delta_m), \qquad (18)$$

where $\omega_S \equiv \omega_m^{(0)}$ is the TF frequency for $q = 0$, with

$$\Delta_m \equiv \frac{2\hbar}{\omega_S^3 M}\left\langle(\omega_S^2 - \omega_\perp^2)\frac{1}{\rho^2} + \frac{2g}{M}\frac{n_{c0}}{\rho^3}\left(\frac{\partial}{\partial\rho} - \frac{1}{\rho}\right)\right\rangle. \qquad (19)$$

The solution of (17) to lowest order in the damping is $\omega_m = \omega_m^{(q)} - i\Gamma_m^{(q)}$, where the damping rate is given by

$$\Gamma_m^{(q)} = \frac{1}{2}\left\langle\frac{1}{\tau'}\right\rangle\left(1 - \frac{m\Omega}{\omega_m^{(q)}}\right). \qquad (20)$$

We note that the finite T result in (18) is formally identical to the $T = 0$ result in Ref. [20]. The only difference is the use of $N_c(T)$ for the number of condensate atoms. As discussed in Refs. [20,22], collective modes with opposite polarity m are split in frequency since the vortex circulation breaks the azimuthal symmetry. Modes rotating in the same sense as the vortex ($m > 0$) are shifted higher in frequency and modes rotating against the flow of the vortex ($m < 0$) are shifted down, in agreement with experiments [2,15].

The expression for the damping (20) of the collective modes of a condensate interacting with a rigidly rotating thermal gas is our main new result. Interestingly, it predicts that modes of opposite polarity m are affected by the rotating thermal cloud in quite distinct ways. The damping of modes that rotate against the flow of the thermal cloud ($m < 0$) increases linearly with angular frequency Ω, while the damping of modes that rotate in the same sense as the thermal cloud ($m > 0$) decreases with increasing Ω. This prediction should be easily observable. For $m > 0$ excitations, the damping in (20) goes to zero when Ω reaches a critical value defined by

$$\Omega_{c,m}^{(q)} \equiv \frac{\omega_m^{(q)}}{m}. \qquad (21)$$

For $\Omega > \Omega_{c,m}^{(q)}$, the damping changes sign, indicating the onset of a dynamical instability (i.e., $\delta n_m \sim e^{+|\Gamma_m|t}$).

The result in (21) is equivalent to the usual Landau criterion for the excitation of surface modes by an external anisotropic perturbation as discussed in Ref. [4]. In contrast, our finite-temperature mechanism is effective even in an axisymmetric trap. Physically, one would expect all of the surface modes to be thermally occupied, so that, for a noncondensate rotation rate of Ω, instabilities can be induced in any of these modes as long as $\Omega > \Omega_{c,m}^{(q)}$. In other words, at least one mode will be unstable when Ω exceeds a critical frequency defined by the minimum of a plot of $\omega_m^{(q)}/m$ versus m [4]. This critical rotation frequency is generally larger than that required to ensure thermodynamic stability of a vortex state [5], so that it is likely that the primary role of surface mode instabilities is to facilitate tunneling of one or more vortex lines into the condensate bulk. This has recently [23] been demonstrated using energy arguments for the particular case of a $m = 2$ quadrupole mode. When a single $q = 1$ vortex is already present, our result (18) shows that the critical rotation frequencies increase due to corresponding upward shifts in the $m > 0$ frequencies [20,22]. Again, taking the $m = 2$ mode as a concrete example, our TF analysis predicts $\Omega_{c,2}^{(0)} = \omega_\perp/\sqrt{2}$ without a vortex, while $\Omega_{c,2}^{(1)} = \omega_\perp\sqrt{(1 + 2\Delta_2)/2}$ for a $q = 1$ vortex state. The expression in (19) can be shown to reduce to [20,22] $\Delta_2 = (7/2\sqrt{2})d_\perp^2/R_\perp^2$. After nucleation, the vortices will eventually equilibrate into a thermodynamically stable lattice configuration, as discussed in [8,9,24].

The present paper evaluates only the damping from C_{12} collisions. We expect that Landau damping, which would arise if we included thermal cloud fluctuations [16,25], would also exhibit the same dynamical instability for $\Omega > \Omega_{c,m}^{(q)}$. The damping term in (10) can be interpreted as a kind of mutual friction, although it is quite different from the usual mutual friction arising in the two-fluid hydrodynamic domain, such as discussed in Ref. [26].

In summary, stimulated by recent work at $T = 0$ on the nucleation of vortices by a rotating anisotropic harmonic potential, we have generalized the discussion to finite temperatures and considered the case where the driving field is a rigidly rotating thermal cloud [15]. Our calculation is explicitly based on the collisional exchange coupling between this rotating thermal cloud and the condensate which gives rise to mode damping. This damping can change sign when the thermal cloud angular frequency Ω reaches a critical value given by the Landau criterion for excitation of collective modes of angular momentum m along the z axis. Importantly, our result also applies to the stability of a singly quantized vortex against excitations of surface collective modes, where it is associated with dynamic nucleation of additional vortices.

We thank the authors of Ref. [15] for stimulating discussions about the JILA experiment and David Feder for a critical reading of an early draft. This research was supported by NSERC of Canada and JSPS of Japan.

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