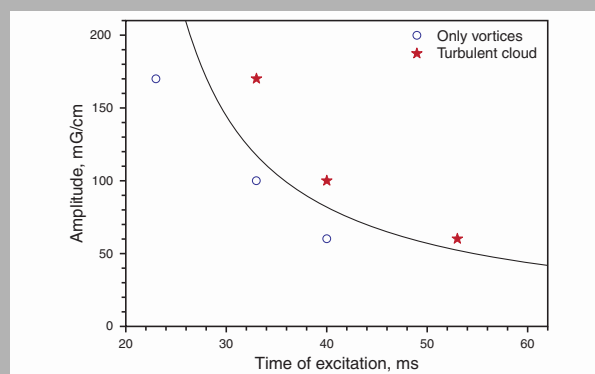


Abstract: A novel concept of quantum turbulence in finite size superfluids, such as trapped bosonic atoms, is discussed. We have used an atomic ^{87}Rb Bose-Einstein condensate (BEC) to study the emergence of this phenomenon. In our experiment, the transition to the quantum turbulent regime is characterized by a tangled vortex lines formation, controlled by the amplitude and time duration of the excitation produced by an external oscillating field. A simple model is suggested to account for the experimental observations. The transition from the non-turbulent to the turbulent regime is a rather gradual crossover. But it takes place in a sharp enough way, allowing for the definition of an effective critical line separating the regimes. Quantum turbulence emerging in a finite-size superfluid may be a new idea helpful for revealing important features associated to turbulence, a more general and broad phenomenon.



Amplitude versus elapsed time diagram of magnetically excited BEC superfluid, presenting the evolution from the non-turbulent regime, with well separated vortices, to the turbulent regimes, with tangled vortices

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Transition to quantum turbulence in finite-size superfluids

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1. Introduction

Quantum Turbulence (QT) is a phenomenon related to the vortex dynamics in superfluids and it can be realized in many different ways. In the case of liquid helium, below the λ -point, moving grids and vibrating objects can generate a fully tangled configuration of quantized vortices, which characterizes QT [1, 2]. Within the context of low temperature physics, QT has been studied since its discovery over fifty years ago [3]. One of the main motivations of studying QT is to establish its relation with turbulence in classical fluids, where there is no requirement for the vortex angular momentum quantization. For many years, QT could only be studied in liquid helium (^4He and ^3He) [4]. However, recently [5] Bose-Einstein condensate (BEC) of trapped gases has provided the main ingredients to study

QT in a more simple superfluids. The vortex nucleation in a BEC can be produced by introducing a rotation in the trapped cloud [6–8]. When quantized vortices are generated, they arrange to form the Abrikosov lattices [9]. These crystalline structures of quantized vortices result from mutual interactions of the rotating field producing a repulsion between vortex pairs, which is balanced by the trapping potential. In such structures, with a collection of vortices having the same direction and circulation sign, QT cannot spontaneously occur. Since the main characteristic of QT is a spatial tangled distribution of quantized vortices, one must have vortex lines distributed in many spatial directions to reach such a configuration.

One method to achieve this was proposed by Kobayashi and Tsubota [10]. In their proposal, combined rotations around two orthogonal axes induce the nucle-

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ation of quantized vortex lines in orthogonal directions with a clear evolution to QT. In a recent work [11] a variant of the procedure suggested in [10] was implemented. Henn et al. [11] demonstrated that a special type of oscillatory excitation imposed on a BEC generates vortices. When vortices and anti-vortices are formed and proliferate through the sample, the emergence of turbulence is observed [5] as a configuration of tangled vortices. In fact, such oscillatory excitations generate vortex-antivortex pairs [12]. The method of creating vortices by means of oscillating external fields is a particular case of the general method of creating coherent topological modes by such oscillating fields [13–15].

Quantized vortices inside a condensed atomic cloud are the necessary ingredients to produce quantum turbulence. Once the cloud is filled with vortices and anti-vortices distributed in various directions, but not arranged in a lattice, turbulence should naturally be established. The production of vortices in a BEC is therefore the first step. The majority of experimental groups have produced separate vortices and vortex arrays by introducing a single-axis rotation in a BEC. The stirring technique has been used with great success. In these experiments a similarity with experiments performed with ^4He [16] could be directly observed. For large size BECs produced by the MIT group [9] a large number of vortices with the same circulation could be produced and configurations like Abrikosov lattices (originally observed for the type-II superconductors) were clearly seen. As much as 130 vortices could be accommodated in the large MIT BEC. In such experiments, however, no tangled configurations could be observed, but only crystalline structures were produced.

In this letter, we analyze the process of generating QT in BEC, focusing on the consequences of having a finite-size sample which constitutes an important property of superfluids originating from cold trapped atoms. In this case, the formed vortices spread inside the trap until turbulence is developed. We analyze this effect in terms of the amplitude and time of excitation by the oscillatory perturbation. We suggest a simple model demonstrating the existence of an effective transition line between non-turbulent and turbulent regimes. A comparison with experimental data is presented, showing a good qualitative agreement. We start with a brief description of the emergence of QT in a BEC, followed by a general analysis and a comparison with experimental observations. Finally, arguments are presented concerning the importance of the phenomenon of quantum turbulence in finite superfluids.

2. Experimental observation

The vortex nucleation can be done by imposing an external oscillating field on the condensed trapped atomic cloud, producing an effect equivalent to rotation. Indeed, those oscillations generate vortex-antivortex pairs [13–15]. Vortex nucleation is expected to take place when the perturbation provides enough energy.

Our system is composed of ^{87}Rb atoms forming a BEC described earlier [5, 11]. We used a combination of oscillations introduced by the superposition of coils to the conventional trapping field. The anisotropic harmonic potential of a quadrupole-Ioffe-configuration (QUIC) trap is approximately given by

$$V = \frac{1}{2}m(\omega_\rho^2\rho^2 + \omega_z^2z^2).$$

In our case, $\omega_\rho/\omega_z = 9$, which corresponds to a cigar shaped trap. Superimposing off-axis coils on this trap modifies the potential so that the long axis is rotated while the minimum is displaced. This type of excitation can produce different numbers of vortices, depending on the oscillation amplitude and excitation time [11]. This type of excitation should create both vortices and anti-vortices. Observations of three vortices inside an atomic cloud provide good evidence for the existence of vortex and anti-vortex pairs, as discussed elsewhere [12]. Since the vortex and anti-vortex pairs are not exactly parallel to each other (oscillations are not confined to a plane) the vortex and anti-vortex are expected to live longer than if they would be parallel. We have fixed the excitation frequency of 200 Hz and considered a range of amplitude and time intervals of the applied external field. The results are observed after a time-of-flight (TOF) of 20 ms. The overall cloud characteristics are determined by the trapping harmonic potential, whose frequencies are $f_z = 23$ Hz and $f_\rho = 207$ Hz. The cigar shaped BEC contains about 2×10^5 atoms with a typical condensate fraction ranging from 40 to 70%.

As the oscillation amplitude/time increases, the vortices start to be nucleated in agreement with [11]. The number of vortices in the cloud is counted as the clearly distinguishable dark regions in the absorption image after TOF and it changes with the amplitude and time interval of the excitation [17]. We have varied the amplitude from 0 to 200 mG/cm and the excitation time up to 60 ms. We are interested in studying the crossover region from the observation of vortices to the turbulent cloud. The region in the diagram, separating both regimes, as well as the typical images characterizing the so-called non-turbulent and turbulent regimes are shown in Fig. 1 and Fig. 2.

In order to distinguish between the non-turbulent and turbulent regimes, we have adopted the following criterion. In Fig. 1a, we present a density profile, where one can clearly see the vortices as the dark regions (valleys) spread inside the cloud. In this case, the typical Thomas-Fermi aspect ratio inversion is observed during the TOF (free fall), as expected in a *non-turbulent* gas. On the other hand, the turbulent regime is characterized by tangled vortex lines spread all over the cloud. As a result, the dark region contrast essentially fades away, as is seen in the absorption image of Fig. 1b. Besides that, the behavior of a QT cloud during the TOF is different than that of a regular BEC. The axes aspect ratio is kept essentially unchanged [5]. The tangled vortices make the whole system isotropic [18]. Combining these characteristics, we define

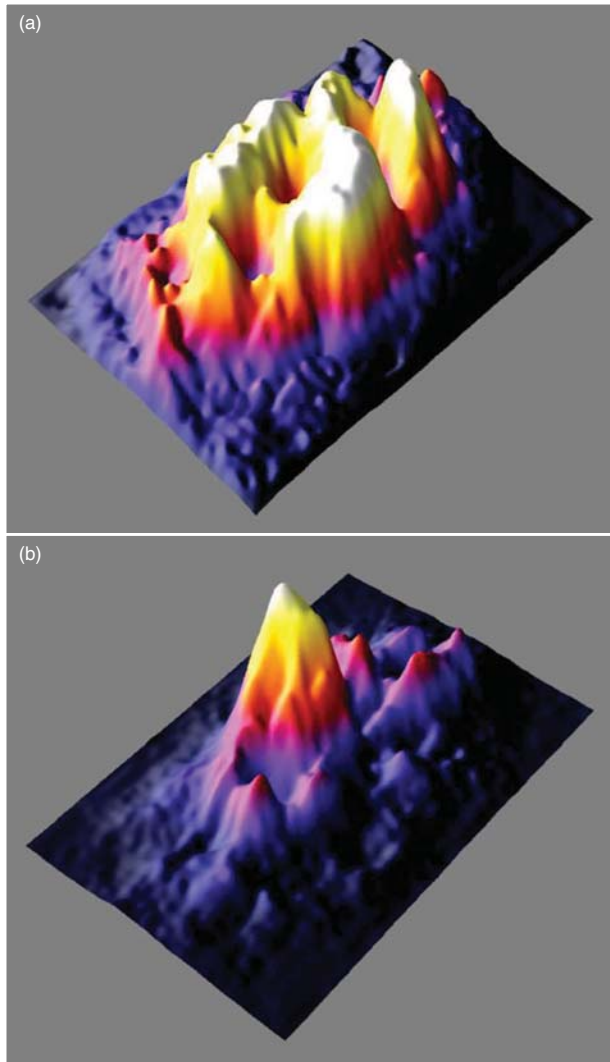


Figure 1 (online color at www.lphys.org) Atomic optical density images of: (a) non-turbulent cloud with well-defined separate vortices and (b) turbulent cloud, where the partial absorption changes along the image due to the existence of tangled vortices. The images were taken after 15 ms of free expansion

a *turbulent* cloud. This is the criterion for distinguishing between the turbulent and non-turbulent regimes.

3. Theoretical model

According to Fig. 2, there exists a well-defined parameter region, where the non-turbulent regime evolves to the turbulent regime. To understand the observed behavior, we propose a simple model based on energy-balance arguments, which are quite general and do not depend on the actual mechanism type needed for the vortex creation or its dynamics inside the cloud. The nucleation of vortices

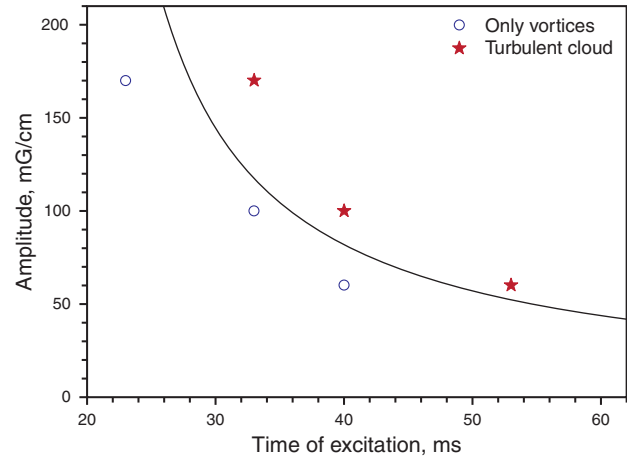


Figure 2 (online color at www.lphys.org) Diagram, on the excitation amplitude-time plane, demonstrating the crossover between the non-turbulent atomic cloud, with well-defined separate vortices, and the turbulent cloud, with tangled vortices. The plotted line is based on Eq. (7) considering halfway points with the same amplitude as experimental critical points

is due to the instability of collective excitations arising from the energy pumped by external oscillating perturbations [18, 19].

We start by noting that there should exist a certain energy amount that is necessary to pump into the superfluid atomic cloud for the vortex formation. Following [20], we write down the energy needed for the vortex nucleation as

$$E_{vort} = \frac{\hbar^2}{ml_0^2} \ln \frac{l_0}{\xi}, \quad (1)$$

where

$$\xi = \frac{1}{\sqrt{8\pi n a_s}}$$

is the healing length, n is the BEC peak density, a_s is the s-wave scattering length, and l_0 is the vortex line length. We assume that the latter is approximately equal to the effective cloud's harmonic oscillator length,

$$l_0 = a_{ho} = \sqrt{\frac{\hbar}{m(\omega_\rho^2 \omega_z)^{1/3}}}. \quad (2)$$

Then, let R_{pump} be the rate of the total energy pumped into the cloud by the external oscillating field, and η be the energy fraction converted to rotation. Therefore the total energy needed for the vortex formation can be written as

$$E_{pump} = \eta R_{pump}(t - t_0), \quad (3)$$

where t is the elapsed excitation time and t_0 is the minimal time period needed for the first vortex creation. The first

vortex is created when $E_{\text{pump}} \approx E_{\text{vort}}$. If after the time t of pumping, the number of vortices N_{vort} is formed, then the energy balance implies that

$$\eta R_{\text{pump}}(t - t_0) = N_{\text{vort}} E_{\text{vort}}, \quad (4)$$

and the expected number of vortices is

$$N_{\text{vort}} = \frac{\eta R_{\text{pump}}}{E_{\text{vort}}}(t - t_0). \quad (5)$$

This gives us a good estimate for the number of vortices observed in the atomic cloud, as a function of the excitation elapsed time, which can be compared to the time dependence reported in [17]. Here, we have not explicitly taken into account the vortex-antivortex annihilation, but this effect could be incorporated into the value of the coefficient η . This estimate depends neither on the cloud motion during the excitation, nor on the presence of collective modes that certainly arise [21]. Assuming that the turbulence onset takes place when the atomic cloud is heavily populated by vortices, and $N_{\text{vort}}\xi$ is of the order of the characteristic trap size, we conclude that turbulence should arise when the number of vortices is

$$N_{\text{vort}} \approx \frac{l_0}{\xi}.$$

Near the frontier between the two regimes, the above equations result in the critical behavior of R_{pump} , generating turbulence:

$$R_{\text{pump}} = \frac{l_0 E_{\text{vort}}}{\xi \eta (t - t_0)}. \quad (6)$$

The energy pump rate, R_{pump} , is proportional to the ratio between the external field amplitude, A , and the oscillating (pump) frequency. Therefore, the critical pumping amplitude can be expressed as

$$A_c(t) = \frac{C}{t - t_0}. \quad (7)$$

The borderline separating the non-turbulent and the turbulent regimes is shown in Fig. 2. For our experimental system, we have found $C \approx 1.6$ G/cm, $t_0 \approx 17$ ms, $\xi \approx 0.06$ μm , $l_0 \approx 1.08$ μm , and $E_{\text{vort}} \approx 20$ nK $\times k_B$, as the characteristic values. With these quantities, we expect $N_{\text{vort}} \approx 20$ for the QT onset, which is in good agreement with the recent work [17], where the turbulence is observed when the number of vortices is close to the value 20 found above.

4. Role of the finite size of atomic systems

The existence of the critical borderline, separating the non-turbulent and turbulent regions in the diagram of Fig. 2, is closely related to the finite-size of the trapped superfluid system, whose effective size is given by the characteristic

anisotropic harmonic trap length l_0 . Quantum turbulence develops when the trapped atomic cloud becomes densely filled with vortices. Then the energy, pumped into the system, transforms not only into the newly formed vortices, but also to their rapid motion, with the formation of their tangled distribution, accompanied by the appearance of reconnections and the formation of Kelvin waves. At this point, the absorption images become hazy, which is a manifestation of the arising turbulence [22].

The standard experiments with ^4He and ^3He do not fulfill the conditions, where finite-size effects would become crucially important. Hence, the superfluid clouds, formed by trapped atomic BECs, represent a whole new class of systems, whose properties essentially depend on finite-size effects, as well as on interactions.

Effects, related to finite temperature, should also be important, though, at this time, we do not have enough data for performing the corresponding analysis. The pumped energy is, certainly, partially transformed into the thermal cloud, which is necessary to take into account for a more detailed consideration [23]. It can also be that the losses, caused by the thermal cloud, could be responsible for the value of the delay time t_0 . The existence of the critical line in Fig. 2 requires that t be larger than t_0 . In order to generate vortices at $t \approx t_0$, one needs a very large amplitude. But a strong or long pumping should produce a large admixture of the thermal fraction [18].

5. Conclusions

Summarizing, we have presented experimental data and offered an explanation for the transition between a non-turbulent and turbulent regimes observed in a superfluid formed by a Bose-Einstein condensate of trapped atoms. The transition region occurs on the plane of the amplitude-time parameters of the related excitation. The character of the observed transition region is closely connected with the finiteness of the superfluid, since quantum turbulence arises when the sample becomes densely saturated with vortex lines. A simple model allows us to qualitatively understand the observed main features of the phenomenon.

Further studies, both experimental and theoretical, are necessary for the better understanding of this phenomenon of quantum turbulence in finite-size superfluids. This concept of quantum turbulence in finite systems can also be applied to small ^4He droplets, though their experimental realization can be much more complicated than the creation of atomic clouds in traps. The possible existence of finite-size effects in turbulent superfluids opens a novel direction in the investigation of finite systems, such as trapped atoms and liquid droplets.

At low temperatures, fermions, depending on the sign of their interactions, can form either superfluid molecular BEC or paired superconductor-type fluid, both of which can exhibit superfluid properties [24, 25]. It is, therefore, feasible to produce quantum turbulence in such systems

and to observe finite-size effects in fermionic turbulent fluids, similar to those observed in trapped bosonic superfluids.

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