

Inflationary Universe in Deformed Phase Space Scenario

S. M. M. Rasouli,^{1,2,*} Nasim Saba,³ Mehrdad Farhoudi,³ J. Marto,^{1,2} and P.V. Moniz^{1,2,†}

¹*Departamento de Física, Universidade da Beira Interior,
Rua Marquês d'Ávila e Bolama, 6200 Covilhã, Portugal.*

²*Centro de Matemática e Aplicações (CMA - UBI), Universidade da Beira Interior,
Rua Marquês d'Ávila e Bolama, 6200 Covilhã, Portugal.*

³*Department of Physics, Shahid Beheshti University, G.C.
Evin, Tehran, 19839, Iran.*

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We consider a noncommutative (NC) inflationary model with a homogeneous scalar field minimally coupled to gravity. The particular NC inflationary setting herein proposed, produces entirely new consequences as summarized in what follows. We first analyze the free field case and subsequently examine the situation where the scalar field is subjected to a polynomial and exponential potentials. We propose to use a canonical deformation between momenta, in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe, and while the Friedmann equation (Hamiltonian constraint) remains unaffected the Friedmann acceleration equation (and thus the Klein-Gordon equation) is modified by an extra term linear in the NC parameter. This concrete noncommutativity on the momenta allows interesting dynamics that other NC models seem not to allow. Let us be more precise. This extra term behaves as the sole explicit pressure that under the right circumstances implies a period of accelerated expansion of the universe. We find that in the absence of the scalar field potential, and in contrast with the commutative case, in which the scale factor always decelerates, we obtain an inflationary phase for small negative values of the NC parameter. Subsequently, the period of accelerated expansion is smoothly replaced by an appropriate deceleration phase providing an interesting model regarding the graceful exit problem in inflationary models. This last property is present either in the free field case or under the influence of the scalar field potentials considered here. Moreover, in the case of the free scalar field, we show that not only the horizon problem is solved but also there is some resemblance between the evolution equation of the scale factor associated to our model and that for the R^2 (Starobinsky) inflationary model. Therefore, our herein NC model not only can be taken as an appropriate scenario to get a successful kinetic inflation, but also is a convenient setting to obtain inflationary universe possessing the graceful exit when scalar field potentials are present.

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I. INTRODUCTION

As Einstein gravitational theory is not suitable to describe the universe at very high energies, alternative proposals must provide an essential new perspective. In this regard, Snyder's formulation [1, 2] for a NC setting of spacetime coordinates is of significant interest. It introduces a short length cutoff (that is called the NC parameter) which can modify the renormalizability properties of relativistic quantum field theory (see [3, 4] and references therein for a thorough review; cf., e.g., [5–24] for several specific explorations.) At the scales where quantum gravity effects would be important, NC effects could therefore be relevant¹. In particular, as inflation

proceeds from such energy scales, employing deformed phase space scenarios for investigating this dynamical stage of the universe is surely pertinent. Accordingly, we may expect that such correction from the spacetime uncertainty principle (implying a deviation from general relativity) may affect the cosmic microwave background power spectrum and hence may be identified in future cosmological observational data.

As far as the inflationary paradigm framework currently stands², it has been widely acquiesced that a scalar

*Electronic address: mrasouli@ubi.pt

†Electronic address: pmoniz@ubi.pt

¹ String/M theories have added interest into the framework discussion regarding noncommutativity, due to the natural appearance of NC spacetime [25–27] (see also [28–31] and references therein). More precisely, the spacetime uncertainty relation $\Delta t \Delta x > l_s^2$

(where t and x are physical time and space coordinates, respectively, and l_s is the string length scale), emerging in string/M theories, indicates that the spacetime could be noncommutative at particular scales [32].

² As a brief summary for the benefit of a reader, let us just add that inflationary models have been considered to overcome unsatisfactory aspects of the (standard) hot big bang scenario, such as the flatness, horizon and monopole problems [33–37]. It has been believed that the horizon problem is a quite acute problem with respect to the flatness and the monopole issues, such that every number of e-foldings that can resolve the horizon problem, automatically, can solve the others as well [38]. Moreover, such

field (usually designated as the inflaton), is the responsible for the period of accelerated expansion during that earliest epoch of the universe.

In the original proposal (of inflation) by Guth [33], it was assumed that a scalar field is trapped in a false vacuum. Subsequently, by tunneling through a quantum-mechanical barrier, it is possible for the inflaton field to exit from this local minimum value. Then, via a first order transition, it can go toward a true vacuum associated to the present universe. However, in this hypothetical process, inflation cannot terminate successfully. In order to overcome this problem, a new inflationary model had been independently proposed by Linde [34], Albrecht and Steinhard [35], which is indeed a modified version of the aforementioned scenario. In the new inflationary model, the inflaton field varies slowly in a double-well potential and undergoes a phase transition of the second order. In this slow evolutionary behavior associated to the scalar field, its corresponding potential energy dominates its own kinetic energy; via such an assumption the universe expands quasi-exponentially, associated to slow-roll approximation (SRA) conditions. In this setting, the inflationary epoch terminates when the potential energy stops dominating. The simplest example of the new inflationary model is the chaotic inflation in which the potential, with a sufficiently flat region, sustain a slow-roll regime [36].

Let us mention that noncommutativity has been employed regarding inflation in the recent literature. Concretely, in Ref. [40], it has been shown that a NC space-time affected power law inflation and could provide a large enough running of the spectra index; in Ref. [41], the effect of noncommutativity on cosmic microwave background has been investigated and it has been shown that noncommutativity may cause the spectrum of fluctuation to be non-Gaussian and anisotropic. Moreover, in Ref. [40, 42–44], the effects of NC spacetime on the power spectrum, spectral index and running spectral index of the curvature perturbations have been investigated in the inflationary universe. However, notwithstanding as well the content in Refs. [32, 40–45], where several types of NC frameworks have been proposed to study the early universe, we use instead a rather different NC relation in our herein model, as we will explain in our manuscript.

In this paper, we investigate the effects of a particular type of noncommutativity with a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model in the presence of a scalar field which is

minimally coupled to gravity. It has been proposed that the geometry of the universe at early times was not commutative, namely, we should incorporate imprints of the NC geometry in the description of the universe at those times [46]. Therefore, our purpose is to study the effects of that specific NC property (associated to the conjugate momentum sector) regarding inflationary scenarios. Employing our NC model, in which just one NC parameter is showing up linearly in the equations of motion, we show that, even in the case of free scalar field (in the absence of the scalar potential), for very small values of the NC parameter, there is a short epoch at early times in which the universe inflates. Subsequently, the universe enters in a decelerating era which can be considered as the radiation dominated epoch. We should note such a phase transition behavior can never be obtained in the commutative case, where the scale factor of the universe always decelerates. Moreover, in order to overcome to the main problem with the standard cosmology, i.e., the horizon problem, we show that in our NC kinetic inflationary universe, the relevant nominal condition is completely satisfied during the evolution of the universe. Furthermore, we discuss briefly regarding the close similarity between the herein NC inflation and the R^2 (Starobinsky) inflationary model [47, 48] at the level of the equation associated to the evolution of the scale factor and demonstrate that we can find more relevance for interpreting the NC models in very small scales. However, for the free scalar field case, the commutative model does not yield an accelerating phase, nor does it satisfy the nominal condition. In the presence of the scalar potential, we extend the standard SRA setting and employ this procedure for a few well known scalar potentials. Using small values for the NC parameter, we show that the noncommutativity affects in the values of the numbers of e-folding as well as in behavior of the slow-roll parameters, scalar field and the Hubble parameter depicted versus the logarithmic scale factor.

This work is organized as follows. In the next section, by employing the Hamiltonian formalism and proposing a particular kind of a dynamical deformation between the conjugate momentum sector, we obtain the corresponding NC field equations. In section III, we analyse our model in the absence of the scalar potential and compare the results with those obtained from the commutative case. In section IV, by employing the SRA procedure, we obtain a generalized set of the SRA parameters/relations for our herein NC model for a class of polynomial potentials. Subsequently, in section IV B, with the assistance of numerical analysis, using re-scaled variables and choosing a set of suitable initial conditions, we present the effects of our chosen NC deformation in the field equations and cosmological observables. Finally, in the last section, we summarize the main results and present a short discussion.

models also allow to predict the primordial power spectrum of the density fluctuations, which are presumed to be a seed for all the observed structure in the universe [39]. However, in any satisfactory inflationary model, the universe must exit from the rapidly accelerating phase and then proceed into a decelerating expansion stage (where normal baryonic matter, radiation, neutrinos and dark matter become dominant). Such a transition is often called the graceful exit and still constitutes a well-known problem to be faced by any inflationary model.

II. NONCOMMUTATIVE COSMOLOGICAL SCENARIO

Our background spacetime is described by the spatially flat FLRW universe

$$ds^2 = -\mathcal{N}^2(t)dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (2.1)$$

where t is the cosmic time and x, y, z are the Cartesian coordinates; $\mathcal{N}(t)$ is a lapse function and $a(t)$ is the scale factor. We employ the well known Lagrangian density

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right], \quad (2.2)$$

in which the scalar field ϕ is minimally coupled to gravity. In (2.2), g is the determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar, $V(\phi)$ is a scalar potential, the Greek indices run from zero to three and we have assumed the units in which $c = 1$ and $\hbar = 1$.

By inserting the expression for the Ricci scalar [associated to the metric (2.1)] into Lagrangian (2.2), we get

$$\mathcal{L} = -\frac{3}{8\pi G} \mathcal{N}^{-1} a \dot{a}^2 + \frac{1}{2} \mathcal{N}^{-1} a^3 \dot{\phi}^2 - \mathcal{N} a^3 V, \quad (2.3)$$

where an overdot represents a derivative with respect to t and we have omitted the total time derivative term. It can be shown that the Hamiltonian of the model is given by [20]

$$\mathcal{H} = -\frac{2}{3} \pi G \mathcal{N} a^{-1} P_a^2 + \frac{1}{2} \mathcal{N} a^{-3} P_\phi^2 + \mathcal{N} a^3 V(\phi), \quad (2.4)$$

where P_a and P_ϕ are the momenta conjugates associated to the scale factor and the scalar field, respectively. We henceforth take the comoving gauge where $\mathcal{N} = 1$. The equations of motion associated to the commutative case (correspond to the phase space coordinates $\{a, \phi; P_a, P_\phi\}$) are obtained by considering the ordinary phase space structure in which

$$\{a, P_a\} = 1 \quad \text{and} \quad \{\phi, P_\phi\} = 1 \quad (2.5)$$

and the other brackets vanish. Consequently, employing the corresponding Hamiltonian (2.4), we get

$$\dot{a} = \{a, \mathcal{H}\} = -\frac{4\pi G}{3} a^{-1} P_a, \quad (2.6)$$

$$\begin{aligned} \dot{P}_a = \{P_a, \mathcal{H}\} &= -\frac{2\pi G}{3} a^{-2} P_a^2 + \frac{3}{2} a^{-4} P_\phi^2 \\ &\quad - 3a^2 V(\phi), \end{aligned} \quad (2.7)$$

$$\dot{\phi} = \{\phi, \mathcal{H}\} = a^{-3} P_\phi, \quad (2.8)$$

$$\dot{P}_\phi = \{P_\phi, \mathcal{H}\} = -a^3 V'(\phi), \quad (2.9)$$

where the prime represents the derivative with respect to the argument.

As far as our NC setting is regarded, let us elaborate on it from hereafter. The literature includes, concerning canonical deformation by means of Poisson brackets, either the Moyal product (i.e., the star-product, see,

e.g., [49] and references therein) or the generalized uncertainty principle [50]. Applying these and other NC frameworks (at classical or quantum levels) into cosmological settings, have enabled to explore important challenges. For instance, it allowed to reasonably address UV/IR mixing, as means to describe and relate in a non-trivial manner physical phenomena at large and short distances (or equivalently, high and low energy regimes). This has been achieved as an outcome of employing NC quantum field theories; see, e.g., Refs. [51, 52]. Consequently, importing NC features into (classical or quantum) cosmology can be soundly motivated as opening quite promising avenues to explore. In the present work, we shall restrict ourselves to a classical geometrical framework, where the corresponding NC effects will be obtained by using classical canonical noncommutativity features into Poisson brackets.

Let us therefore employ a specific type of a canonical noncommutativity, which is obtained by means of an appropriate deformation on the classical phase space variables. Our choice has been seldom [20] used in the literature, namely in inflationary settings but has computational advantages. We will explain that the corresponding equations of motion (associated to the deformed scenario) can still be obtained by employing the Hamiltonian (2.4), being evaluated on variables which satisfy the deformed Poisson bracket. Therefore, let us apply the deformed Poisson bracket between the canonical conjugate momenta as³

$$\{P_a, P_\phi\} = \theta \phi^3, \quad (2.10)$$

where the NC parameter θ has been assumed as a constant. We should note that it is also possible to assume other choices for the right hand side of (2.10), still satisfying the dimensionality of $\{P_a, P_\phi\}$, but the present suggestion (2.10) reveals to be particularly interesting because it is linear (most simple dependence) in terms of the deformation parameter and the NC parameter does not appear in the Friedmann equation (Hamiltonian constraint). In addition, more motivations concerning noncommutativity between the momenta can be found in [18]. Moreover, employing the NC ingredient (2.10) for studying the gravitational collapse of a homogeneous scalar field produced interesting results [20]. It is worthwhile to note that if instead of the NC Poisson bracket (2.10) for momenta, we used a NC upon only the scale factor and the scalar field, then any NC effects will be

³ In this work (see also footnote 7), we have used the units where $\hbar = 1 = c$, therefore, from the Plank length, $L_P = \sqrt{\hbar G/c^3}$, the dimension of G is $[G] = L_P^2$. We assumed that the scale factor and the lapse function to be dimensionless parameters, and the dimensions of coordinates and the Lagrangian are $[x^\mu] = L_P$ and $[\mathcal{L}] = L_P^{-4}$. Consequently, we can show that $[\phi] = L_P^{-1}$, $[P_a] = L_P^{-3}$, $[P_\phi] = L_P^{-2}$, and consequently $[\{P_a, P_\phi\}] = L_P^{-2}$. Therefore, assuming (2.10) yields the dimension of the deformation parameter as $[\theta] = L_P$.

absent for a vanishing potential [53]. However, an important outcome in our model, is that with (2.10) we still get modified field equations for the case where the scalar potential is absent as we will elaborate about. In summary, we believe that this dynamical noncommutativity between the momenta provides more interesting dynamics to describe the evolution of the universe, at least in the early times, than other choice of modified Poisson brackets.

Before proceeding, let us just clarify that the phase space structure (2.5) is still employed, with the modified configuration being brought from the relation (2.10), which is the sole responsible as a canonical NC feature; it induces a set of modified equations as the novel framework to explore, as we will elaborate in the following.

It is then straightforward to show that the modified equations of motion with respect to the Hamiltonian (2.4) are given by

$$\dot{P}_a = -\frac{2\pi G}{3}a^{-2}P_a^2 + \frac{3}{2}a^{-4}P_\phi^2 - 3a^2V(\phi) + \theta(a^{-3}\phi^3P_\phi), \quad (2.11)$$

$$\dot{P}_\phi = -a^3V'(\phi) + \theta\left(\frac{4\pi G}{3}a^{-1}\phi^3P_a\right). \quad (2.12)$$

We note that as equations (2.6) and (2.8), under the chosen noncommutativity, are not modified, we have forbore from rewriting them. Moreover, in order to obtain equations (2.11) and (2.12), we have employed the following formulas⁴

$$\{P_a, f(P_a, P_\phi)\} = \theta\phi^3 \frac{\partial f}{\partial P_\phi}, \quad (2.13)$$

$$\{P_\phi, f(P_a, P_\phi)\} = -\theta\phi^3 \frac{\partial f}{\partial P_a}. \quad (2.14)$$

Obviously, the standard commutative equations are recovered in the limit $\theta \rightarrow 0$.

The equations of motion associated to our herein NC framework can be written as the standard form as

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \equiv \frac{8\pi G}{3}\rho_{\text{tot}}, \quad (2.15)$$

$$\begin{aligned} 2\frac{\ddot{a}}{a} + H^2 &= -8\pi G\left[\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right) + \frac{\theta\phi^3\dot{\phi}}{3a^2}\right] \\ &\equiv -8\pi Gp_{\text{tot}}, \end{aligned} \quad (2.16)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \theta H\left(\frac{\phi^3}{a^2}\right) = 0, \quad (2.17)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and we have employed the Hamiltonian constraint $\mathcal{H} = 0$.

Moreover, in this NC model, the energy density and pressure associated to the scalar field have been denoted by ρ_{tot} and p_{tot} , respectively. Let us also introduce $p_{\text{nc}} \equiv \frac{\theta\phi^3\dot{\phi}}{3a^2}$, which denotes the sole *explicit* term representing the direct NC effects in the total pressure. We should note that not only the p_{nc} *explicitly* depends on the NC parameter, but also the two first terms of p_{tot} as well as the ρ_{tot} *implicitly* depend on the NC parameter. We emphasize that there is no appropriate manner to separate the commutative portion unless setting $\theta = 0$.

Therefore, in analogy with standard cosmology, the equation of state can be written as

$$w_{\text{tot}} = \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = \frac{\dot{\phi}^2 - 2V(\phi) + \frac{2\theta\phi^3\dot{\phi}}{3a^2}}{\dot{\phi}^2 + 2V(\phi)}. \quad (2.18)$$

If we set $\theta = 0$ (here and in the field equations), we get the same equation of state associated to the standard models, namely, $w_{\text{tot}} = w_\phi$, where $w_\phi < -1/3$, which corresponds to $\dot{\phi}^2 < V(\phi)$, being associated with the quintessence cosmological model for the late times. Whilst, a dominant potential energy with respect to the kinetic term, can lead to an inflationary epoch at very early times.

By using the conservation equation

$$\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0, \quad (2.19)$$

time derivative of the Hubble parameter (that will be needed later on) is

$$\dot{H} = -4\pi G\left(\dot{\phi}^2 + \frac{\theta\phi^3\dot{\phi}}{3a^2}\right). \quad (2.20)$$

As expected, let us repeat again, in all of the above equations if we set $\theta = 0$, then, each equation reduces to its corresponding commutative counterpart.

In addition, one evident but very pertinent impact of the NC deformation studied in this work is that the NC dependent terms in Eqs. (2.16)-(2.20) are, at least, proportional to the inverse square of the scale factor $a(t)$. Therefore, it is expected that the NC effect should be noticeable at the initial stage of inflation and very residual at its end.

III. KINETIC INFLATION AND THE HORIZON PROBLEM

In this section, we want to present and analyze the NC effects when the scalar field potential is absent and compare them with those found from the standard framework.

By assuming $V = 0$, from (2.15), it is easy to show that the scale factor is related to the scalar field as

$$a(t) = a_0 e^{\kappa\phi(t)}, \quad (3.1)$$

⁴ In [20], two different approaches have been used to retrieve the equations of motion.

where $a_0 > 0$ is an integration constant and $\kappa = \pm \sqrt{\frac{4\pi G}{3}} = \pm \frac{\sqrt{2}}{2}l$, where $l = 4.7 \times 10^{-33} \text{cm}$ is the Planck length and we should take the positive (upper) sign. Moreover, in this section, we work with the units where $8\pi G = 1$. Furthermore, relation (3.1) yields

$$H = \kappa \dot{\phi}(t). \quad (3.2)$$

By substituting (3.1) and (3.2) into the modified Klein-Gordon equation (2.17), we write

$$\ddot{\phi} + 3\kappa\dot{\phi}^2 + \kappa\theta \left(\frac{\dot{\phi}\phi^3}{a_0^2 e^{2\kappa\phi(t)}} \right) = 0. \quad (3.3)$$

In order to discuss the NC consequences within our model, contrasting with those obtained from a standard cosmological scenario (in the absence of potential), let us obtain the solution associated to the commutative case. It is straightforward to show that, for $\theta = 0$, we get the following relations for the scalar field and scale factor [54]

$$\phi(t) = \frac{1}{3\kappa} \ln [3\kappa(c_1 t + t_0)], \quad (3.4)$$

$$a(t) = a_0 [3\kappa(c_1 t + t_0)]^{\frac{1}{3}},$$

where $c_1 > 0$ and $t_0 > 0$ are integration constants and $t > -\frac{t_0}{c_1}$. Let us in what follows describe the behaviour of the quantities associated to this case. At $t \rightarrow -t_0/c_1$, we get $\phi \rightarrow -\infty$; the scalar field increases with the cosmic time and goes to $+\infty$ for very large times. Moreover, for all times we see that $\frac{\dot{\phi}}{\phi} < 0$, namely, the scalar field always decelerates. Therefore, the energy decreases with the cosmic time and tends to zero after an infinite expansion. In addition, the scale factor of the universe starts its decelerating expansion from a nonzero value. Concerning the time behaviour of quantities associated to the commutative case, we present a few examples, by using particular initial conditions, in figures 1, 2 and 3 (the upper panels).

For $\theta \neq 0$, solving the complicated differential equation (3.3) analytically is impossible. However, it is feasible to derive the general conditions under which the universe can accelerate. Moreover, we can obtain a condition concerning the horizon problem as

$$d_\gamma \equiv a(t) \int \frac{dt}{a(t)} > H^{-1}, \quad (3.5)$$

(where d_γ is the particle horizon distance) associated to an inflationary universe, which will be obtained in our herein NC model. Therefore, we first deal about these general conditions and then we will investigate and analyse the consequences produced by our numerical endeavors.

In the absence of the scalar potential, equation (2.17) can also be written as

$$\frac{d(a^3 \dot{\phi})}{dt} = -a_0 \kappa \theta \dot{\phi}^3 e^{\kappa\phi(t)} \dot{\phi}, \quad (3.6)$$

where we have used (3.1) and (3.2). Integrating (3.6) over dt , we obtain

$$\dot{\phi} = -\frac{\theta}{a_0^2 \kappa^3 e^{2\kappa\phi(t)}} [(\kappa\phi)^3 - 3(\kappa\phi)^2 + 6\kappa\phi - 6] + \frac{c}{a_0^3 e^{3\kappa\phi(t)}}, \quad (3.7)$$

where, again, we have used (3.1) and (3.2); c is an integration constant and it equals to the initial value of $a^3 \dot{\phi}$. It is clear that $\dot{\phi}$ depends (explicitly) also linearly on the NC parameter. By substituting $\dot{\phi}$ form (3.7) into (2.16), θ^2 will also be present in the relation associated to the second (time) derivative of the scale factor, which is consistent. As mentioned, what is important is that the NC parameter has the correct linear dependence in the (standard form of) the equations of motion.

Employing (3.1) and (3.2) and (3.7) in (2.16), it is straightforward to show that

$$\frac{\ddot{a}}{a} = -\frac{2c^2 \kappa^2}{a_0^6 e^{6\kappa\phi(t)}} + \theta \Lambda(\phi) + \theta^2 \Psi(\phi), \quad (3.8)$$

where

$$\Lambda(\phi) \equiv \frac{3c}{a_0^5 \kappa e^{5\kappa\phi(t)}} (\kappa\phi - 2) [\kappa\phi(\kappa\phi - 2) + 4], \quad (3.9)$$

$$\Psi(\phi) \equiv -\frac{1}{a_0^4 \kappa^4 e^{4\kappa\phi(t)}} [(\kappa\phi)^2(\kappa\phi - 6) + 12(\kappa\phi - 1)] \times [(\kappa\phi)^2(\kappa\phi - 3) + 6(\kappa\phi - 1)]. \quad (3.10)$$

From (3.8), we observe that the acceleration/deceleration condition of the scale factor completely depends on the evolution of the scalar field, which, in turn, is obtained from a nonlinear differential equation (3.3). Note that to obtain $\frac{\ddot{a}}{a}$ for the commutative case, θ must be set equal to zero in both (3.3) and (3.8). In what follows, when we will use the numerical analysis to get the evolution of the scale factor, we will see how the dynamical relation (3.8) works.

It is worthwhile to discuss concerting a required condition pertinent to inflation. Employing (3.2) and (3.7), we can easily show that

$$d_\gamma = \frac{a(t)}{c\kappa} \int a^2(t) H(t) dt + \frac{\theta a(t)}{c\kappa^3} \int [(\kappa\phi)^3 - 3(\kappa\phi)^2 + 6\kappa\phi - 6] dt. \quad (3.11)$$

Employing integration by parts for the first integral in the right hand side and employing (3.1), we obtain

$$d_\gamma = \frac{a_0^3}{2c\kappa} e^{3\kappa\phi(t)} + \frac{a_0 \theta e^{\kappa\phi(t)}}{c\kappa^3} \int [(\kappa\phi)^3 - 3(\kappa\phi)^2 + 6\kappa\phi - 6] dt. \quad (3.12)$$

Using relations (3.2), (3.7) and (3.12), we get

$$d_\gamma^{\text{nc}} = \frac{a_0^3 e^{3\kappa\phi(t)}}{2c\kappa} + \frac{a_0\theta e^{\kappa\phi(t)}}{c\kappa^3} \int [(\kappa\phi)^3 - 3(\kappa\phi)^2 + 6\kappa\phi - 6] dt + \frac{a_0^3\kappa^2 e^{3\kappa\phi(t)}}{a_0\theta e^{\kappa\phi(t)} [(\kappa\phi)^3 - 3(\kappa\phi)^2 + 6\kappa\phi - 6] - c\kappa^3} \quad (3.13)$$

where

$$d_\gamma^{\text{nc}} \equiv d_\gamma - H^{-1}, \quad (3.14)$$

To satisfy the nominal condition for an inflationary universe in our herein model, the condition $d_\gamma^{\text{nc}} > 0$ must be satisfied. In what follows, we will investigate this condition for our numerical solutions. We should note that as d_γ^{nc} completely depends on the scalar field, therefore, to plot the time behavior of d_γ^{nc} , we must solve the differential equation (3.3). Obviously, to get d_γ^{nc} corresponds to the commutative case, we must set $\theta = 0$ in both (3.3) and (3.13). We expect that the NC modifications in (3.13) may assist properly to get an appropriate nominal condition for our inflationary model.

Let us focus on a numerical analysis to depict the behaviour of the above mentioned quantities. Our numerical endeavours are summarized in what follows.

For very small negative⁵ values of the NC parameter, we have observed that:

- For a small interval of the cosmic time at early times, in contrast with the commutative case, we found an accelerating expansion for both the scale factor and scalar field, namely, we obtained $\frac{\dot{a}}{a}, \frac{\dot{\phi}}{\phi} > 0$ and $\frac{\ddot{a}}{a}, \frac{\ddot{\phi}}{\phi} > 0$.

Subsequently, after this short time acceleration, both the scale factor as well as scalar field begin to decelerate with the cosmic time, see figures 1 and 2. However, for the commutative case, both of them always decelerate. We can interpret such interesting behaviors as follows: a very short time interval in the early universe, in which the scale factor accelerates, can be associated to a substantial epoch of inflation. Immediately after this inflationary epoch, the scale factor decelerates, which can be assigned to the radiation epoch. Such an appropriate transition from an inflationary epoch to a radiation dominated era is called graceful exit.

Concretely, our NC model, contrary to the corresponding standard scenario, even in the case of a vanishing scalar potential, can describe, at least

qualitatively, a realistic inflationary phase for the universe. These interesting consequences are associated to NC effects, which involve solely a NC parameter, which appears linearly in the (modified) set of field equations.

- In figures 3, we have shown the time behaviour of what we specified as the total kinetic energy ρ_{tot} , total pressure p_{tot} as well as explicit NC pressure p_{nc} .

We observe that ρ_{tot} increases during the inflationary epoch to reach its maximum value, and subsequently, it turns to decrease during the radiation dominated era and afterwards. Whereas, p_{tot} and p_{nc} always get negative values, such that they decrease during the inflationary epoch to reach their corresponding minimum values; immediately afterwards, they increase during the radiation dominated era and afterwards. Moreover, all these quantities tends to zero for large values of the cosmic time.

Let us interpret such unusual⁶ time evolution.

First, we should note that all of these quantities depend on the NC parameter. Unfortunately, complicated and *implicit* dependence (of these quantities) on the NC parameter does not allow us to separate analytically a strictly NC behaviour. For instance, in equation (2.16), it seems that p_{tot} is composed of two parts, in which the NC component is separated completely as $p_{\text{nc}} = \frac{\theta\phi^3\dot{\phi}}{3a^2}$ from what seems, naively, the commutative part. However, our numerical simulation shows that, for $\theta \neq 0$, even for vanishing scalar potential, when $p_{\text{tot}} - p_{\text{nc}}$ is plotted against the cosmic time, it does not correspond to the associated quantity in the commutative model. More precisely, when $V = 0$, we get $p_{\text{tot}} - p_{\text{nc}} = \rho_{\text{tot}}$, which is plotted as a red curve in the middle panel of figure 3, whose time behavior is completely different from that is shown in the upper panel (of the same figure) for $p_\phi = \rho_\phi$ (the pressure associated to the commutative case). Concretely, the pressure p_{nc} merely plays an *explicit* NC role and it cannot be interpreted as the whole NC component in p_{tot} . Such result confirms the implicit dependence on the NC parameter for those observables. However, as expected, when we set $\theta = 0$ in the numerical computation, we recover exactly the consistent behaviours associated to their corresponding commutative counterparts.

Secondly, by comparing the time behaviors (associated to the commutative and NC cases) for known

⁵ The small negative values of the NC parameter, in the case where the scalar field is positive, yield an negative p_{nc} which can be conjectured to drive an accelerated expansion.

⁶ As in the standard cosmological models, we expect that the energy density of the universe should decrease while the cosmic time increases.

quantities, we observe that, for small negative values of θ , during the so induced inflationary epoch the NC effect plays its role more drastically with respect to the radiation-dominated epoch.

Disclaimer: In order to check the degree of accuracy for every set of numerical results, we have depicted the numerical error in our solutions when they have to satisfy the conservation equation (2.19), see, e.g., figure 3, lower panel.

- Up to now, we have claimed that our NC model not only yields an accelerating epoch at early times but also such an accelerating universe, after a very short time, enters to radiation dominated decelerating phase. However, to get a successful inflationary scenario, we should resolve the main problem with the standard cosmology, namely the horizon problem. In other word, we should examine our numerical results at least for the mentioned nominal condition, which we wrote it as $d_\gamma^{\text{nc}} > 0$ (where d_γ^{nc} is given by (3.13)). Our numerical results show that: (i) it is never satisfied for the commutative case, as expected; (ii) while, for the NC case, for different small (negative) values of θ , by employing the same initial conditions used to get the above described inflationary universe, we have shown that it is satisfied during all times of the evolution of the universe, see figures 5.

In addition, our numerical results for the free field case, have shown that (i) choosing different values of the NC parameter, (ii) by taking the same consistent values for the other initial conditions, then the time behaviour of the NC quantities and their time derivatives are effectively changed. For instance, let us consider the behaviour of scale factor against the cosmic time: we have found that the smaller the value of $|\theta|$, the larger the time interval and the smaller the number of e-folding associated to the accelerating epoch (of the very early times), respectively; see, e.g., figures 4.

Before closing this section, let us add a further feature concerning this important case.

It is straightforward to show that in the absence of the scalar field potential, the evolution of the scale factor is given by

$$\frac{\dot{a}\ddot{a}}{a^2} + \left[2 - \frac{3}{\ln(\frac{a}{a_0})} \right] \frac{\dot{a}^2\ddot{a}}{a^3} - \left(\frac{\ddot{a}}{a} \right)^2 + 2 \left[2 - \frac{3}{\ln(\frac{a}{a_0})} \right] \left(\frac{\dot{a}}{a} \right)^4 = 0, \quad (3.15)$$

where a_0 is an integration constant. Note that the NC parameter does not appear explicitly in the field equations. However, when $\theta = 0$, instead of the above equation, we proceed to compute the equation corresponding to the standard case for the free scalar field, namely,

$$\frac{\ddot{a}}{a} + 3\frac{\dot{a}\ddot{a}}{a^2} - 4\left(\frac{\dot{a}}{a}\right)^3 = 0. \quad (3.16)$$

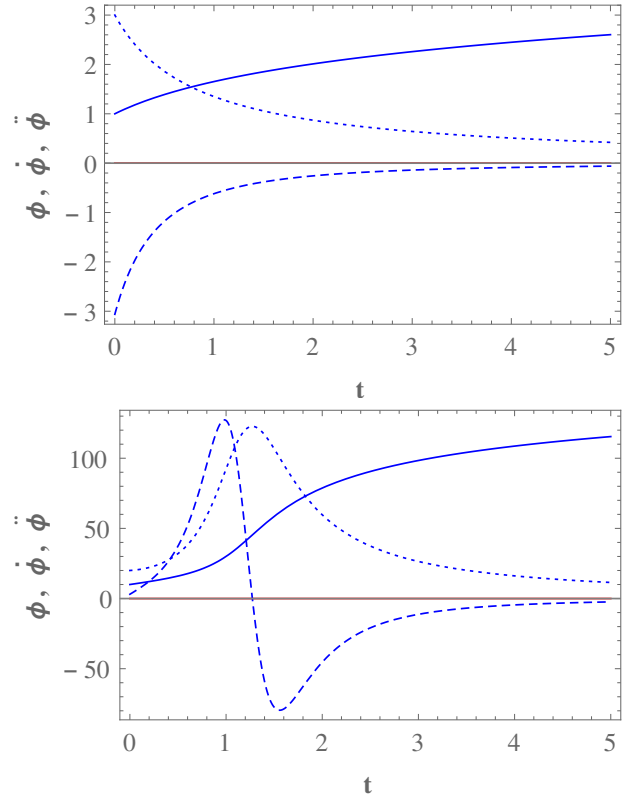


Figure 1: The behaviour of $\phi(t)$ (solid curves), $\dot{\phi}(t)$ (dotted curves) and $\ddot{\phi}(t)$ (dashed curves) against cosmic time for commutative case (upper panel) and for NC case (lower panel). The red line is associated to $\phi(t) = 0$ to clearly show when those quantities are positive or negative. We have set $8\pi G = 1$, $\kappa = \frac{\sqrt{6}}{6}$, $a_0 = 0.01$, $\phi(0) = 1 = \dot{\phi}(0)$ and $\theta = -0.0008$ for the NC case. For more clarity, we have rescaled the curves.

Regarding the significance of the modified evolutionary equation of the scale factor (3.15), let us take a quick glance over the R^2 (Starobinsky) inflationary model [47, 48], in which the Einstein equations have been solved in the presence of effective quantum corrections. This model does have a graceful exit from an acceleration phase (associated to the inflationary era) and it is consistent with observational data associated to the spectrum of the primordial perturbations [55, 56]. Briefly, this model has been considered as one of the successful inflationary models regarding the observational constraints imposed by the recent Planck data [55, 56]. In this model, the evolutionary equation for the scale factor, for the case of spatially flat FLRW line-element, is given by [47, 48]

$$2\frac{\dot{a}\ddot{a}}{a^2} + 2\frac{\dot{a}^2\ddot{a}}{a^3} - \left(\frac{\ddot{a}}{a} \right)^2 - \left(\frac{M_0^2}{H_0^2} + 3 \right) \left(\frac{\dot{a}}{a} \right)^4 + M^2 \left(\frac{\dot{a}}{a} \right)^2 = 0, \quad (3.17)$$

where $H_0^2 = 360\pi/Gk_2$ and $M_0^2 = -360\pi/Gk_3$, where $k_2 > 0$ and $k_3 < 0$ are numerical coefficients.

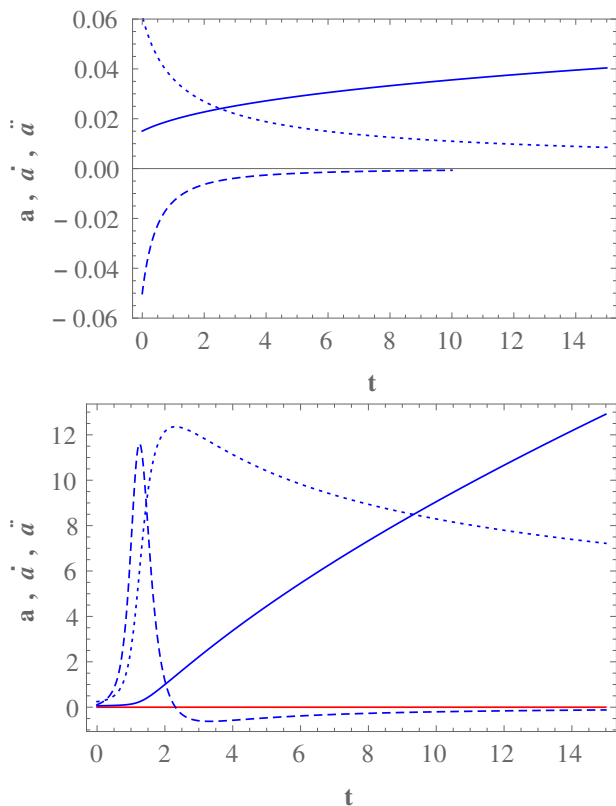


Figure 2: The behaviour of $a(t)$ (solid curves), $\dot{a}(t)$ (dotted curves) and $\ddot{a}(t)$ (dashed curves) against cosmic time for commutative case (upper panel) and for NC case (lower panel). The red line in the lower panel is associated to $a(t) = 0$ to clearly show when those quantities are positive or negative. We have set $8\pi G = 1$, $\kappa = \frac{\sqrt{6}}{6}$, $a_0 = 0.01$, $\phi(0) = 1 = \dot{\phi}(0)$ and $\theta = -0.0008$ for the NC case. For more clarity, we have re-scaled the curves.

It is not feasible to establish a full matching correspondence between our herein NC framework and $R + R^2$ model, specifically at the Lagrangian level. Nevertheless, at the level of the field equations (3.15) and (3.17) [or their corresponding phase space plane, by assuming appropriate approximations with respect to the corresponding epoches], we can speculate in extracting the following. As the integration constant a_0 relates the scale factor and the scalar field (which, in turn, is obtained from Klein-Gordon equation and it depends on the NC parameter) via relation (3.1) for a constant time, it may be possible to consider associating the NC parameter to the coefficients k_2 and k_3 .

IV. NONCOMMUTATIVE SETTING AND SLOW-ROLL APPROXIMATIONS

Let us now in what follows, discuss the case where a potential is present.

The SRA [57], which leads to reliable consequences

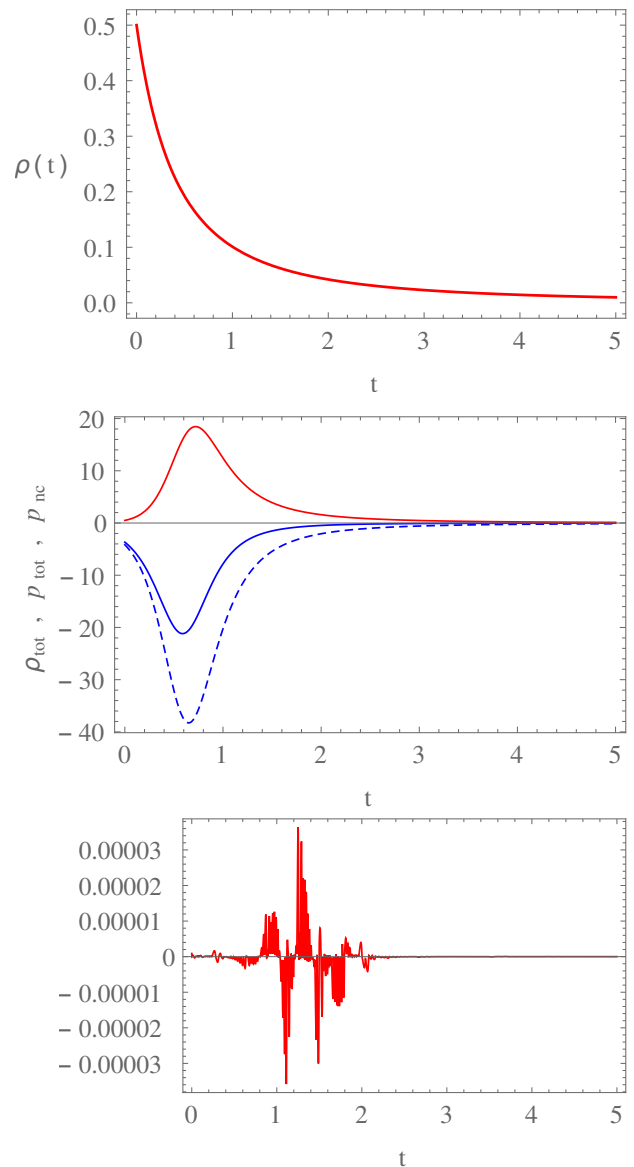


Figure 3: The behaviour of the kinetic energy (red curves), total pressure (solid blue curve) and NC pressure (dashed blue curve) against cosmic time for commutative case (upper panel) and for NC case (middle panel). The lower panel shows the numerical error for the time evolution associated to satisfying the conservation equation (2.19). We have set $8\pi G = 1$, $\kappa = \frac{\sqrt{6}}{6}$, $a_0 = 0.01$, $\phi(0) = 1 = \dot{\phi}(0)$ and $\theta = -0.0008$ for the NC case.

when employing smooth potentials, have been traditionally applied as an approximation method in inflationary cosmology. However, there are also other alternative approximations, such as the WKB [58], the Green function [59] and the improved WKB [60] methods, that have been used by some researchers in the study of inflationary scenarios. Nevertheless, in this work, we employ the SRA for our herein NC model and then analyse the results according to this context.

In the commutative case, in order to attain an infla-

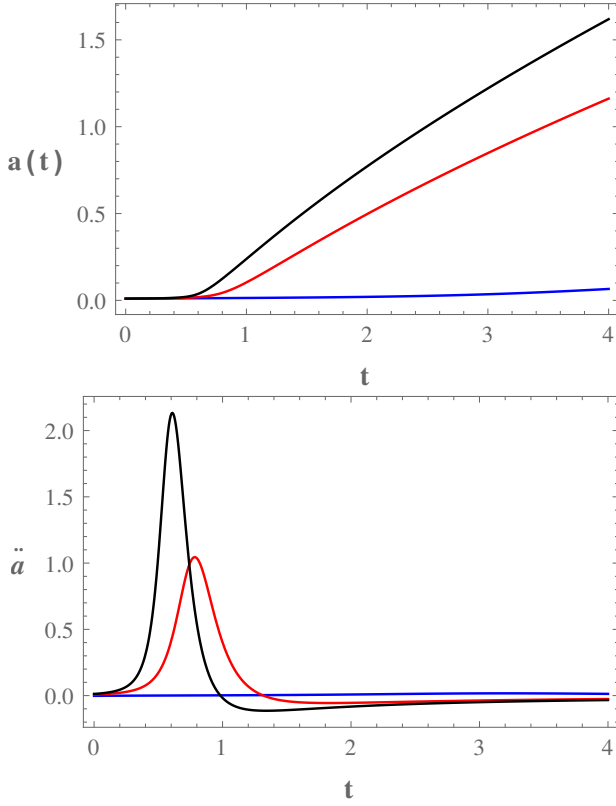


Figure 4: The behaviour of the scale factors (upper panel) and their corresponding second time derivatives (lower panel) against cosmic time for the NC case where $\theta = -0.00009$ (blue curves), $\theta = -0.0007$ (red curves) and $\theta = -0.001$ (black curves). The black dashed line is associated to $\ddot{a} = 0$ to clearly show when those quantities are positive or negative. We have set $8\pi G = 1$, $\kappa = \frac{\sqrt{6}}{6}$, $a_0 = 0.01$, $\phi(0) = 1 = \dot{\phi}(0)$.

tionary accelerating universe, the potential energy of inflation must dominate the kinetic energy of the system. Consequently, to get a sufficient amount of inflation, a flat potential associated to an inflationary scenario is needed. Concretely, it is required to impose the slow-roll conditions

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi) \quad \text{and} \quad \ddot{\phi} \ll 3H\dot{\phi}, \quad (4.1)$$

with the slow-roll parameters being defined as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \quad \text{and} \quad \eta \equiv -\frac{|\ddot{\phi}|}{|\dot{\phi}|H}, \quad (4.2)$$

where, in the case of $\theta = 0$, using (2.20), we get obviously $\eta = -|\ddot{H}|/(2|\dot{H}|H)$. In this case, from conditions (4.1), it is straightforward to show that, during inflation, these parameters should satisfy $\epsilon \ll 1$ and $|\eta| \ll 1$ [38]; at the end of inflationary phase, these parameters increase with the order of unit and thus, the SRA breaks down.

However, for the NC configuration, in order to satisfy the condition $\epsilon \ll 1$, from equations (2.15) and (2.20),

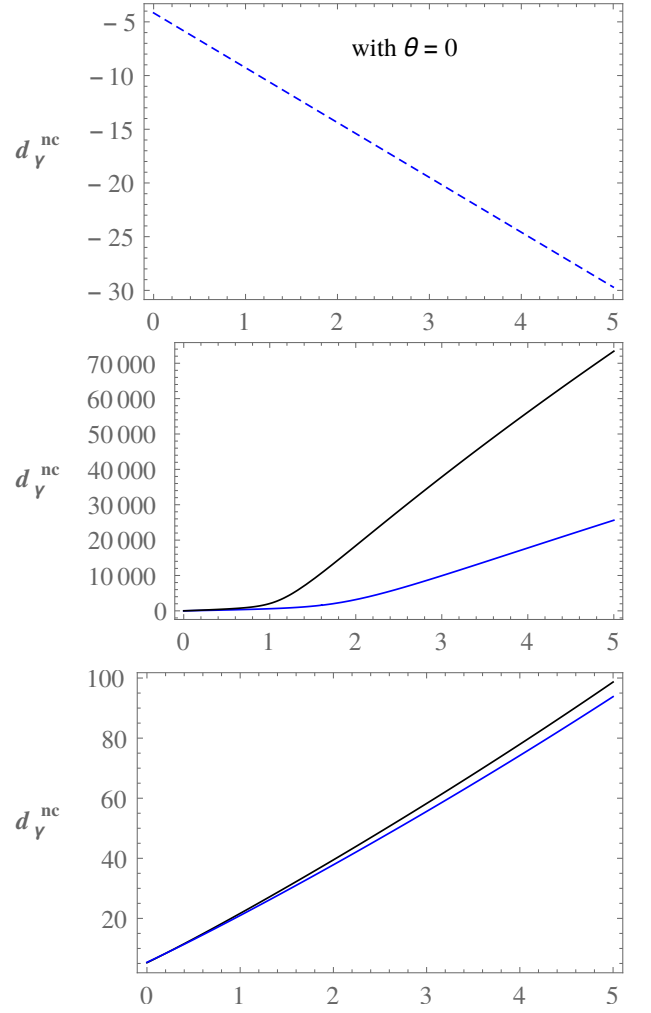


Figure 5: The time behaviour of d_γ^{nc} for the commutative (upper panel) and NC cases (middle and lower panels). In the middle panel, the black and blue curves are associated to $\theta = -0.001$ and $\theta = -0.0005$, respectively; While in the lower panel, they are associated to $\theta = -0.00001$ and $\theta = -0.00009$, respectively. We have employed the other initial conditions equal to those used to plot the previous figures. Namely, have set $8\pi G = 1$, $\kappa = \frac{\sqrt{6}}{6}$, $a_0 = 0.01$, $\phi(0) = 1 = \dot{\phi}(0)$ and $c = a_0^3 \dot{\phi}(0)$.

we get the condition

$$\dot{\phi}^2 \ll |V(\phi) - \frac{1}{2}\theta a^{-2}\phi^3 \dot{\phi}|. \quad (4.3)$$

Now, by imposing these conditions, the most slowly varying terms in equations of motion (2.15), (2.16) and (2.17) become negligible and hence, it remains

$$H^2 \approx \frac{8\pi G}{3}V(\phi), \quad (4.4)$$

$$2\frac{\ddot{a}}{a} + H^2 \approx 8\pi G \left[V(\phi) - \frac{\theta\phi^3\dot{\phi}}{3a^2} \right], \quad (4.5)$$

$$3H\dot{\phi} + V'(\phi) + \theta\dot{a}\left(\frac{\phi}{a}\right)^3 \approx 0. \quad (4.6)$$

Likewise, by employing Eqs. (4.4) and (4.6), the time derivative of the Hubble parameter, according to Eq. (2.20), reduces to

$$\dot{H} \approx -\frac{4\pi G}{3} \left[\frac{V'^2(\phi)}{V(\phi)} + \theta \frac{\sqrt{3V(\phi)}V'(\phi)}{3V(\phi)} \frac{\phi^3}{a^2} \right]. \quad (4.7)$$

Hence, the slow-roll parameters in (4.2), will read now as

$$\epsilon \approx \epsilon_1 + \theta \frac{\sqrt{3V(\phi)}V'(\phi)}{6V^2(\phi)} \frac{\phi^3}{a^2} \quad (4.8)$$

and

$$\eta \approx \eta_1 - \epsilon + \theta \sqrt{3V(\phi)} \frac{\phi^3}{a^2} \left(\frac{1}{\phi V(\phi)} + \frac{2 + \epsilon}{3V'(\phi) + \theta \sqrt{3V(\phi)} \frac{\phi^3}{a^2}} \right), \quad (4.9)$$

where, analogous to the standard model [61], we have defined ϵ_1 and η_1 as

$$\epsilon_1 \equiv \frac{V'^2(\phi)}{2V^2(\phi)} \quad (4.10)$$

and

$$\eta_1 \equiv \frac{V''(\phi)}{V(\phi)}. \quad (4.11)$$

Moreover, note that relation (4.8) is obtained by employing equations (4.4) and (4.7) in the first definition of (4.2), and relation (4.9) has been derived by taking the time derivative of equation (4.6), then employing equations (4.4) and (4.6) into the second definition of (4.2). It is clear that by setting $\theta = 0$ in (4.8) and (4.9), the NC slow-roll parameters reduce to their corresponding commutative ones.

A. Behaviors of physical quantities in the presence of the polynomial scalar field potential

Let us therefore investigate in this section the behaviors of cosmological quantities, with the aid of a numerical analysis, while considering two particular cases of the following general scalar potential

$$V(\phi) = \lambda M^4 (M/\phi)^n, \quad (4.12)$$

where $\lambda > 0$ is a parameter, M is some mass scale and n is a positive or negative integer constant. Potential (4.12) has been studied in the context of chameleon field theory [62–64]; in particular, when $\lambda = 1$ and $n > 0$, it is called the Ratra-Peebles potential, that is used in the intermediate inflation [65, 66] and in the quintessence models. Furthermore, when $n \neq -4$, M can be scaled such that, without loss of generality, we can set λ equals

to unity [62]. Whereas for $n = -4$, M drops out and the ϕ^4 theory is resulted [63]. Also note that, action (2.2) with a scalar potential $V(\phi)$ can be considered as the Einstein representation of the well-known Brans-Dicke theory whose corresponding Jordan frame exists with a trapped field and a coupling function $\omega(\phi)$ [67].

In this section, polynomial chaotic inflation, in which the scalar potentials are given by $V(\phi) = M^2 \phi^2$ (massive scalar field) and $V(\phi) = \lambda \phi^4$ (self-interacting scalar field) constitute the focus of our interest. It has been established that, for standard models, with the mentioned scalar potentials, inflation occurs while the scalar field rolls down towards the potential minimum.

By substituting potential (4.12) into equations (2.15), (2.16) and (2.17), we obtain

$$H^2 = \frac{8\pi G}{3} \left(\frac{1}{2} \dot{\phi}^2 + \frac{\lambda M^{4+n}}{\phi^n} \right), \quad (4.13)$$

$$2\frac{\ddot{a}}{a} + H^2 = -8\pi G \left[\left(\frac{1}{2} \dot{\phi}^2 - \frac{\lambda M^{4+n}}{\phi^n} \right) + \frac{\theta \phi^3 \dot{\phi}}{3a^2} \right], \quad (4.14)$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{n\lambda M^{4+n}}{\phi^{n+1}} + \theta\dot{a}\left(\frac{\phi}{a}\right)^3 = 0. \quad (4.15)$$

It is easy to show that these equations can be rewritten as

$$\left(3 - 4\pi G \dot{\phi}^{*2} \right) H^2 = \frac{8\pi G \lambda M^{4+n}}{\phi^n}, \quad (4.16)$$

$$2H\dot{H} + \left(3 + 4\pi G \dot{\phi}^{*2} \right) H^2 = 8\pi G \left(\frac{\lambda M^{4+n}}{\phi^n} - \frac{\theta \phi^3 \dot{\phi}^*}{3e^{2N}} \right), \quad (4.17)$$

$$\ddot{\phi}^{**} + \left(3 + \frac{\dot{H}^*}{H} \right) \dot{\phi}^* - \frac{n\lambda M^{4+n}}{H^2 \phi^{n+1}} + \frac{\theta \phi^3}{H e^{2N}} = 0, \quad (4.18)$$

where the asterisk $*$ denotes the derivative with respect to the logarithmic scale factor $N = \ln a$. In this setting, the slow-roll parameters can be rewritten as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{\dot{H}^*}{H} \quad \text{and} \quad \eta \equiv -\frac{|\ddot{H}|}{2|\dot{H}|H} = \frac{\ddot{H}^*}{2\epsilon} - \epsilon. \quad (4.19)$$

In what follows, by means of numerical methods, we investigate the behaviors of the cosmological quantities such as the slow-roll parameter ϵ and the Hubble parameter within the framework of deformed phase space. Then, we compare them with their corresponding counterparts in the commutative case. The SRA setting is taken as a method to obtain the analytical interpretation of these cosmological quantities.

It is important to note that, as the results of our numerical endeavors show, the last term in (4.18), which includes the NC parameter, behaves like an extra friction (or antifriction) term in classical mechanics. Moreover, we should note that among those three equations of motion, only two of them are independent. Hence, in order to solve them numerically, we will employ the Friedmann equation for the consistency of initial conditions as well as the consistency check of the integration routine; we consider the other two as dynamical field equations.

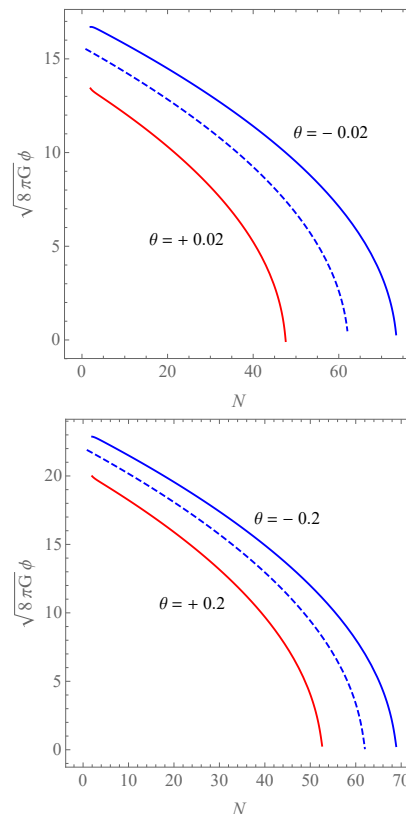


Figure 6: We present a numerical evaluation of the inflaton scalar field ϕ versus N for the commutative (dashed line) and NC (solid line) cases, where $\phi_i \simeq 15.6$, $\dot{\phi}_i \simeq -0.11$ when $n = -2$ (upper panel) whereas $\phi_i \simeq 22$, $\dot{\phi}_i \simeq -0.11$ when $n = -4$ (lower panel).

close inspection of Eq. (4.8). Taking a negative (positive) value for the NC parameter θ increases (decreases) the number of e-folding with respect to the commutative case.

B. Noncommutative case with the exponential scalar field potential

It has been shown that the case of the canonical scalar field model with an exponential potential,

$$V(\phi) = V_0 e^{-\tilde{\kappa}\phi} \quad (\tilde{\kappa}^2 = 8\pi G), \quad (4.20)$$

yields power law inflation (PLI) [68–70]. It has been also extensively known that there are two important problems with canonical PLI scenario [71]: (i) The range of the tensor-to-scalar ratio r predicted in these models is well above the limit reported by the Planck data. (ii) These models suffer from the graceful exit problem. In the scope of the present work we only address to the second problem, namely studying numerically the evolution of the scale factor when a small negative NC parameter θ is switched on. In Fig. 9 we have depicted the time evolution of the first derivative of the scale factor

Figs. 6 to 8 show the behaviors of the inflation scalar field, slow-roll parameter ϵ and the Hubble parameter associated to the commutative and NC cases as a function of the e-folding number N , for $n = -4$ and $n = -2$ as typical examples for the potential (4.12). Based on our numerical graphs, we observe that, for a set of suitable initial conditions,⁷ which (except θ) are the same for both the commutative and NC cases, the inflationary scenario associated to the commutative case is retrieved with the correct number of e-folding $N \approx 60$. Whilst, for the NC counterpart, the number of e-folding either increases or decreases. In fact, this behavior can be expected upon a

⁷ Following the footnote 3, the figures have been plotted by taking a new dimensionless scalar field as $\varphi = \sqrt{8\pi G}\phi$. In such units, we have taken the NC parameter from the interval $-1 < \theta < 1$.

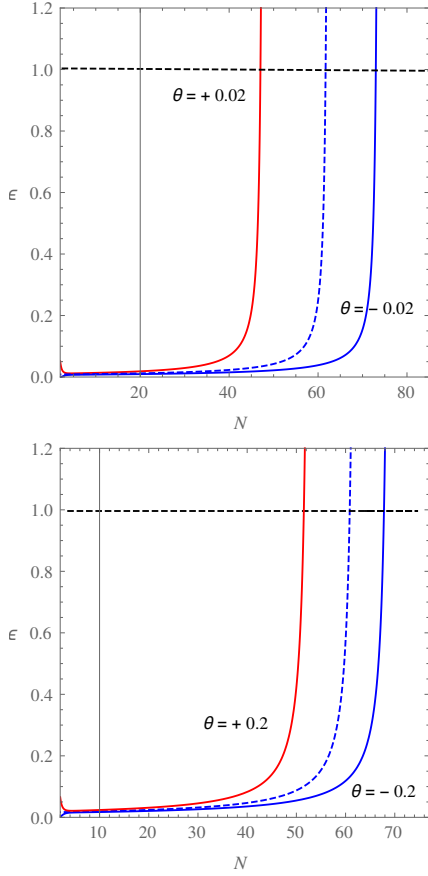


Figure 7: We present a numerical evaluation of ϵ versus N for the commutative (dashed line) and NC (solid line) cases, where $\phi_i \simeq 15.6$, $\dot{\phi}_i \simeq -0.11$ when $n = -2$ (upper panel) whereas $\phi_i \simeq 22$, $\dot{\phi}_i \simeq -0.11$ when $n = -4$ (lower panel). The slow roll parameter ϵ goes to 1 when inflation ends.

for the NC and commutative cases. As expected, in the commutative case the numerical simulation shows an accelerated expansion of the universe in a regime according to the well known power law evolution of the scale factor. In contrast, when a small negative θ is taken in Eq. (2.16), and for the same initial conditions taken for the commutative case, the scale factor derivative evolves in a transition between an initial accelerated phase and a subsequent deceleration. Therefore, the same scenario is emerging, as before, when we assumed the free field case or with the scalar field under the influence of a polynomial potential.

In face of the overall qualitative behavior of the evolution of the scale factor in the NC context studied here, we can assume that the smooth transition between a period of accelerated and a decelerated phase of expansion of the universe was induced by means of the NC effects. Moreover, this approach has the potential benefit of providing an alternative way of dealing with the graceful exit problem that weakens well motivated inflationary scenarios.

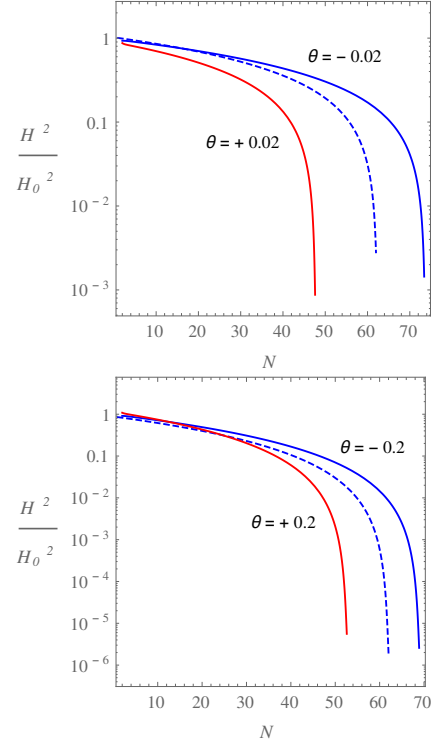


Figure 8: We present a numerical evaluation of the Hubble parameter H versus N for the commutative (dashed line) and NC (solid line) cases, where $\phi_i \simeq 15.6$, $\dot{\phi}_i \simeq -0.11$ when $n = -2$ (upper panel) whereas $\phi_i \simeq 22$, $\dot{\phi}_i \simeq -0.11$ when $n = -4$ (lower panel). The Hubble parameter has been rescaled to its initial value, i.e. H_0 , when inflation starts.

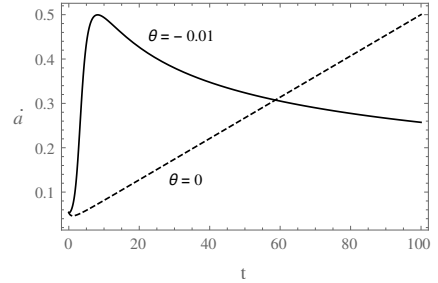


Figure 9: The behavior of the scale factor first derivative, against cosmic time, for a scalar field under the influence of an exponential potential (4.20). In the NC case (full line), the accelerated and decelerated phases are present and smoothly connected. In the commutative case (dashed line) we have the traditional accelerated phase of a PLI regime. We have set $8\pi G = 1$, $\kappa = \frac{\sqrt{6}}{6}$, $a_0 = 0.01$, $\phi(0) = 1 = \dot{\phi}(0)$.

V. CONCLUSIONS

It has been proposed that a deformation in the phase space structure can be considered as an appropriate approach (i) with which to discuss quantum gravity effects and (ii) from which to predict cosmological phenomena at very small scales.

In this work, we assumed the spatially flat FLRW line element as the background geometry and the well known Lagrangian in which the gravity and the scalar field are minimally coupled. Then, we proposed a particular type of dynamical deformation for the canonical momenta of the scale factor and of the scalar field. The main motivations for this choice are: (i) the simplicity of an extra term linear in the NC parameter θ affecting the Friedmann acceleration equation as well as the Klein-Gordon equation; (ii) the Friedmann (Hamiltonian) constraint remains unaffected. These last two aspects enable us, in contrast to what happens in [53], to access NC effects for the free field case and, therefore, to build a kinetic inflation scenario. We should note that the interesting dynamics produced by our NC choice can describe leastwise the early universe more appropriate than that provided by other possible noncommutativity, which can be proposed between the present variables. Moreover, we should note that all the consequences produced by this NC idea are entirely new and have not been presented elsewhere.

Using the Hamiltonian formalism, we have obtained the NC equations of motion, which are reduced to the corresponding ones in the commutative case, when the NC parameter goes to zero.

As explained in section III, our model, which bears just a one (linear) NC parameter, can generate a suitable interesting inflationary scenario with a graceful exit, in the absence of a scalar field potential. Moreover, in this inflationary epoch, we have shown that the relevant nominal condition is perfectly satisfied during evolution of the universe at all times. We should note that our (numerical) results have been obtained by taking very small values of the NC parameter. Moreover, when the NC parameter vanishes, we recover all the corresponding results associated to the standard commutative case. For the latter regime, in the absence of the scalar potential, we have shown, analytically and numerically, that there is no accelerating epoch for the universe, nor is there any satisfaction for the nominal condition. Notwithstanding the previously stated, the NC case provides an interesting element for analysis. In fact, from the modified evolutionary equation of the scale factor (3.15), we can, for the sake of discussion, (formally) consider the R^2 (Starobinsky) inflationary model [47], contrast equations (3.15) and (3.17): as the integration constant a_0 relates the scale factor and the scalar field (which, in turn, is obtained from Klein-Gordon equation and it depends on the NC parameter) via relation (3.1) for a constant time, we may speculate to relate the NC parameter to the coefficients k_2 and k_3 .

By employing the SRA procedure, we have retrieved the approximation conditions for our herein NC setting when a potential is present for the scalar field. These relations can be considered as the generalized versions of those in the standard commutative setting, such that when the (constant) NC parameter tends to zero, all the slow-roll relations/parameters are reduced to those intro-

duced in the commutative standard case.

Subsequently, by assuming a typical potential, we have rewritten the NC equations in terms of the scale factor as well as the logarithmic scale factor. More concretely, we have considered the polynomial chaotic inflation, in which the scalar potentials are given by $V(\phi) = M^2\phi^2$ (massive scalar field) and $V(\phi) = \lambda\phi^4$ (self-interacting scalar field).

By choosing a few sets of suitable initial conditions and working in a re-scaled units in which the NC parameter can be of order unity⁸, we explored, numerically, the behavior of the scalar field, slow-roll parameter ϵ and the Hubble parameter during the inflation (associated to the commutative and NC cases) against the e-folding number N . With the same initial conditions (except for θ) for both the commutative and NC cases, our numerical results have shown that, in the commutative case, we can obtain an inflationary universe in which the number of e-folding takes value 60. However, for the NC counterpart, the number of e-folding either increases or decreases.

In what follows, it is worthwhile to mention a few points regarding the strengths as well as shortcomings of our herein model, which should be compared with the other (classical) NC scenarios in the literature:

- In our model, all the NC field equations were modified through a sole linear function of the NC parameter, such that they reduce to those in the standard case when θ goes to zero.
- We have seen that the explicit NC corrections are weighted as products $\theta(\frac{\phi^3\dot{\phi}}{3a^2})$ and $\theta(\frac{\phi^3\ddot{a}}{a^3})$ in equations (2.16) and (2.17), respectively, while the Friedmann equation is not modified. Specifically, not only these corrections appear as different modifications in the field equations, but they can also influence the cosmological evolution in the early inflationary stage (when $a(t)$ is small) as well as late times (where \ddot{a} could be very large, although in this case a strong suppression of the NC term is due to the $a^3(t)$ dependence). We should mention that the Poisson bracket deformation described in Eq. (2.10) can be taken as a limit for small $a(t)$, suitable to study early time cosmology and that for large values of $a(t)$ another appropriate limit could provide NC effects for the late time cosmology.

Consequently, we claim that our model can be an appropriate model to investigate the inflationary scenario as well as late time accelerating universe.

- Moreover, we should notice a possible (formal) resemblance between the consequences obtained numerically (in the case of the free scalar potential) for evolution of the scale factor in our herein NC

⁸ For instance, we have taken two values of the allowed NC parameter to plot the figures.

model and the corresponding ones associated to the kinetic inflation in Brans-Dicke theory in the presence of a different choice of noncommutativity [21].

- One of the most important shortcomings with the standard PLI models is the graceful exit problem. We have numerically shown that, by employing the same initial conditions as used in the commutative case, this problem is solved, appropriately, in our NC model. Concretely, we have shown that the short time accelerating scale factor in the presence of the exponential scalar field does connect smoothly to a decelerating epoch.

Finally, concerning the shortcoming of the present model, let us point following. One of the most important achievements of the standard inflationary scenarios is predicting the (quantum) fluctuations behavior. In fact, within just a few remaining concrete inflationary scenarios (studied within standard commutative settings), still viable after the Planck 2015 data survey [55, 56], there were crucial observable quantities, such as the scalar and tensor power spectrum as well as the

scale invariant spectral index, which demonstrated a very good agreement with the observational data.

For our herein model, investigating the fluctuations and their dynamics is a very meaningful and significant issue, for not only to compare with the observational data but also, to study the stability and viability of the model. Moreover, investigating the effects of inhomogeneous arbitrary initial conditions for late time behaviors would also be a substantial outlook to proceed from the herein NC model. However, undertaking such significant questions requires an evident amount of complicated calculations, to compute perturbations for our herein NC model; this has been left out of the scope of the present work and it would be studied in subsequent works.

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