Rose Analysis

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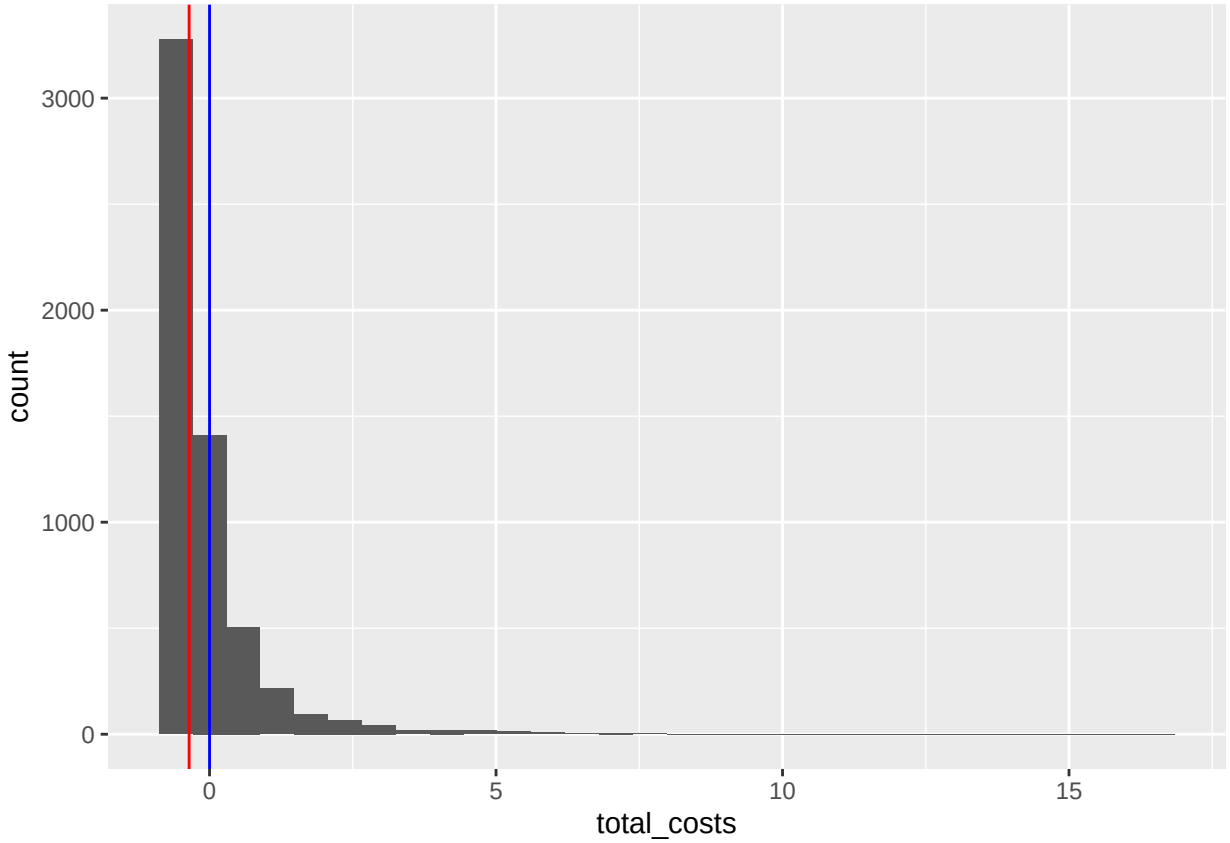
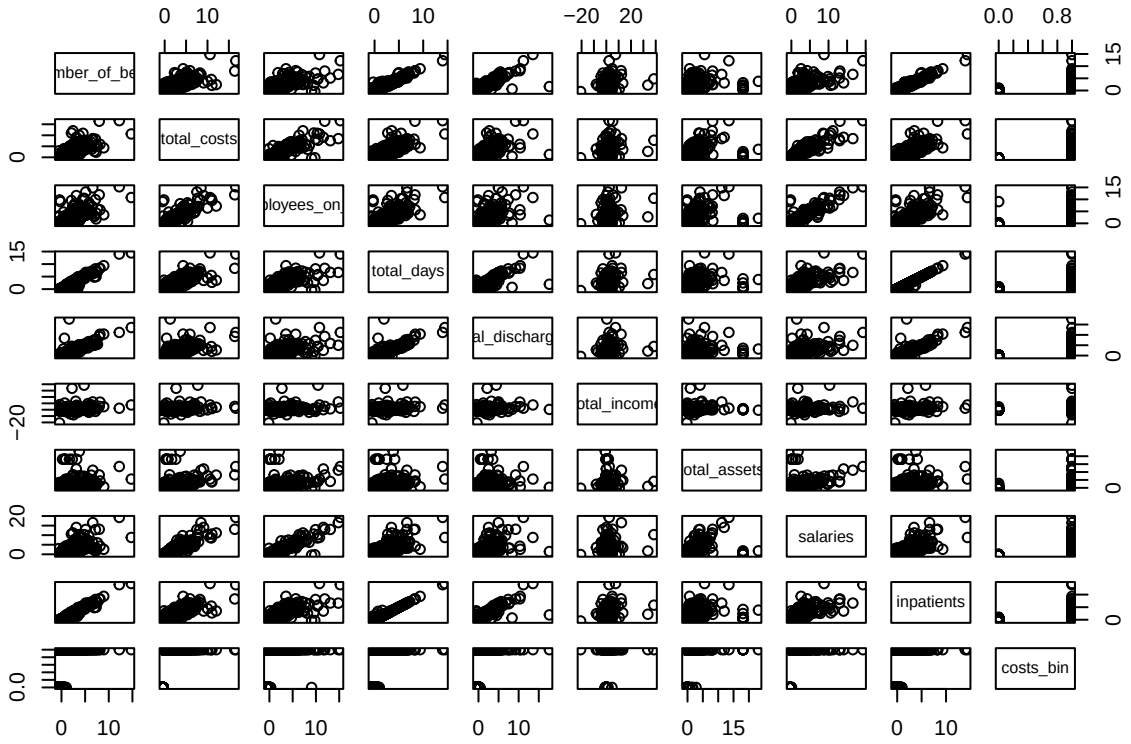
Exploratory Data Analysis

In order to get a visual sense of the relationships in the data, we created a few exploratory plots.

The plot below displays the distribution of (scaled) total costs, where the red line represents the median and the blue line represents the mean. We can see that the distribution of total costs is very right-skewed. For this reason, we chose to define our binary response based on whether or not total costs was above the median, not the mean.



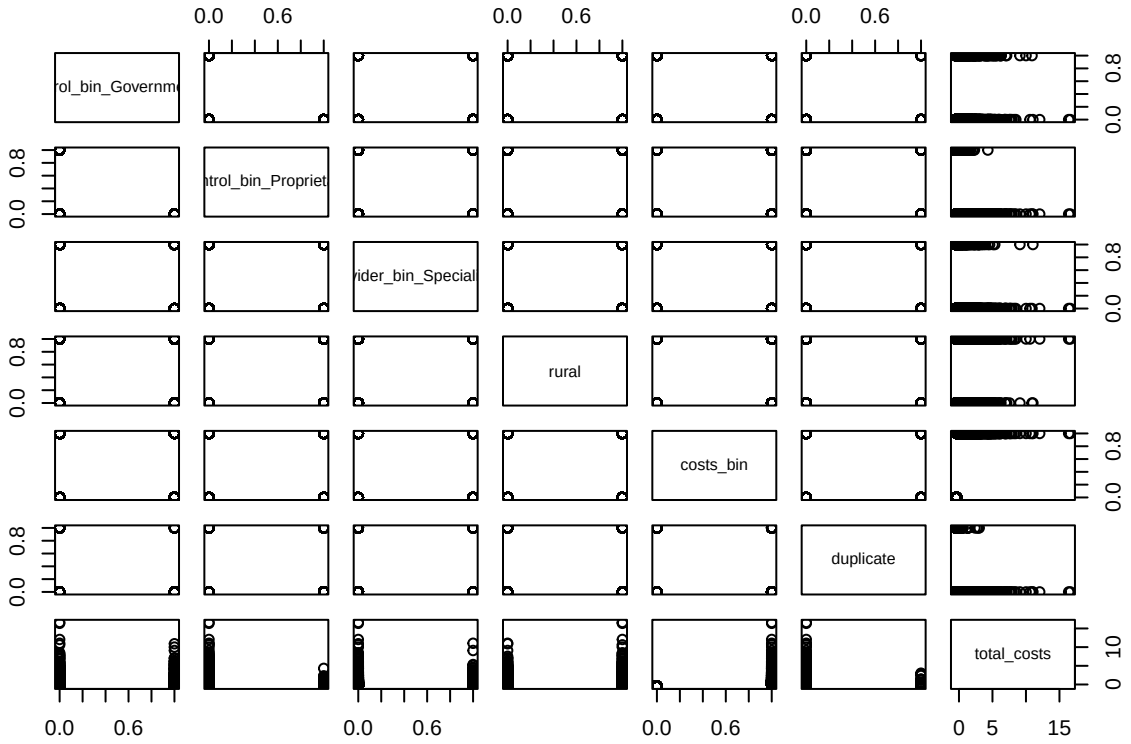
The pairs plots below visualize the relationships between all variables in the data (using the scaled data). We separated the data into two pairs plots, one including only numeric predictors and one including only categorical predictors, in order to see the relationships more clearly.



In the plot above visualizing the relationships between all numeric predictors and the response, we find that total costs appears to have fairly strong positive linear relationships with the following predictors: employees on payroll, total days, bed days, total discharges, and salaries. Total costs appears to have weak or no relationship with total income and total assets. For the binary response, the predictors which have positive relationships with total cost have a pattern such that binary costs = 0 corresponds to a higher concentration of points with low values of the predictor, and binary costs = 1 corresponds to a wider range of values for the predictor. This same pattern shows up also for total assets and total income, which did not look associated with continuous total cost.

Further, we find very high pairwise positive collinearity between (1) total days, bed days, and total discharges, and (2) salaries and employees on payroll.

Interestingly, despite total costs being so skewed, the relationships between each predictor and the response look very linear, and none of them appear non-linear.



Looking at the categorical predictors above, it is less obvious to see relationships between the predictors and response, but there does appear to be some relationship between duplicate and total costs as well as between proprietary control and total costs. For each of these relationships, it seems that when the predictor value equals zero, the total costs are low, while when the predictor value equals one, there is more. of a range of total costs values. However, it is hard to tell if this is a true relationship or if it simply results from unequal numbers of observations in each predictor category. Most total costs values are small, so if there are a smaller number of data points in one category, it would be likely that most of those have small total costs values, while if there are more points in a category, it would look like a wider range.

Focusing on the plot of total costs against binary cost (the two response variables), we notice that all total costs values below the median are very small, while those above the median have a much wider range of values. This reflects the skewness of total costs visualized above.

Quantitative Outcome Analyses

Marginal simple linear regressions

Assumptions

Linear Regression analysis has four key assumptions:

1. Linear Relationship between predictor and response
2. Independence of Errors
3. Constant Variance

4. Errors are normally distributed

In order to assess these assumptions, we created pairwise scatterplots between the response (total costs) and each predictor. We see from the scatterplots that the relationship between the response and each numeric predictor appears approximately linear, and there are no obvious violations of non-constant variance. A histogram of total costs shows that the response variable is notably right-skewed, indicating possible concern that the errors may be non-normal. However, the linear relationships between each predictor and the response indicate that linear regression is a suitable model. We tried log-transforming total costs to better meet the normality of errors assumption, but we found that this transformation made the relationships between each predictor and the response notably non-linear, indicating that this transformation is not useful to improve the fit of the data to the model assumptions.

For independence of errors, it is relatively reasonable to assume that the total costs of one hospital would not impact those of another hospital, so this assumption is reasonably met. There could be small violations if, for example, two hospitals are in close proximity and one has more capacity than the other. In this scenario the one with lower capacity may redirect patients to the higer capacity one, making the costs decrease for the smaller one and increase for the larger one.

Results

The results of the simple linear regression models are summarized in the table below. The first column of the table represents the estimated Mean Squared Error of Prediction (MSEP) by cross validation. The second column represents the MSEP for the held-out test set. The third column represents the coefficient estimate for the predictor. The fourth column represents the p-value associated with the t-test for whether or not each coefficient estimate is equal to zero.

method	cv_error	test_error	coef_est	p_value
Marginal LR number_of_beds	0.253	0.226	0.861	0.000
Marginal LR fte_employees_on_payroll	0.132	0.129	0.956	0.000
Marginal LR total_days	0.205	0.191	0.891	0.000
Marginal LR total_discharges	0.298	0.264	0.835	0.000
Marginal LR total_income	0.939	0.993	0.403	0.000
Marginal LR total_assets	0.731	0.409	0.553	0.000
Marginal LR salaries	0.103	0.229	0.979	0.000
Marginal LR inpatients	0.200	0.189	0.893	0.000
Marginal LR control_bin_Governmental	0.996	1.037	-0.025	0.480
Marginal LR control_bin_Proprietary	0.955	1.003	-0.441	0.000
Marginal LR provider_bin_Specialized	0.980	1.023	-0.337	0.000
Marginal LR rural	0.992	1.039	-0.129	0.000
Marginal LR duplicate	0.996	1.038	-0.121	0.188

We notice that almost all of the p-values equal zero, indicating strong evidence that each predictor is truly associated with the response, total costs. The indicator variable for Government control (`control_bin_Governmental`) has a large p-value, but the indicator variable for Proprietary control (`control_bin_Proprietary`) has a small p-value, indicating evidence that in general, the type of control is truly associated with total costs. The only other predictor with a large p-value is duplicate. This indicates that we do not have evidence that change of ownership of a hospital during the fiscal year is related to the total costs of the hospital.

Looking at the cross validation (CV) and test errors, we see that they are smallest for the models including number of beds, number of employees, total days, salaries, and inpatients. This indicates that these predictors are more strongly associated with the response than the others.

Multiple linear regression

Assumptions

The assumptions for multiple linear regression are the same as those for simple linear regression outlined in the previous section.

Results

The results for the main effects multiple linear regression model are summarized below.

Observations	5141
Dependent variable	total_costs
Type	OLS linear regression

F(13,5127)	5871.91
R^2	0.94
Adj. R^2	0.94

	Est.	S.E.	t val.	p
(Intercept)	0.02	0.01	3.00	0.00
number_of_beds	-0.01	0.02	-0.61	0.54
fte_employees_on_payroll	0.11	0.01	8.64	0.00
total_days	0.06	0.05	1.16	0.25
total_discharges	0.02	0.01	1.46	0.15
total_income	0.03	0.00	7.20	0.00
total_assets	0.04	0.00	9.99	0.00
salaries	0.58	0.01	51.17	0.00
inpatients	0.23	0.06	4.25	0.00
control_bin_Governmental	-0.01	0.01	-1.54	0.12
control_bin_Proprietary	-0.03	0.01	-3.84	0.00
provider_bin_Specialized	-0.07	0.01	-6.65	0.00
rural	0.01	0.01	1.72	0.08
duplicate	0.02	0.02	0.77	0.44

Standard errors: OLS

The adjusted R^2 value of 0.94 indicates that 94 percent of the variability in total costs can be explained by the predictors. This R^2 value is quite high. The estimate of the test MSEP using CV is 0.071, and the test MSEP based on the held-out set is 0.106. These error rates are lower than any of the individual marginal linear regression error rates, indicating that when we include all predictors in one model, we can predict total costs more accurately than if we only include one predictor.

Looking at the coefficient estimates and p-values in the table above, we see that most predictors still have very small p-values, but some which were significant in the marginal linear regressions become insignificant in the multiple linear regression. Specifically, total days, number of beds, and total discharges have large p-values in the multiple regression when all three had very small p-values in the marginal models. This indicates that there is likely correlations between the predictors such that once we have already accounted for some of them, others do not add much further information. This is consistent with the pairs plot which shows strong linear relationships between several pairs of predictors.

Multiple linear regression with interactions and transformations:

Based on our exploratory data analysis, there were no obvious transformations or interactions which were needed in order to satisfy the linear regression assumptions. However, we added a few transformations and interactions for experimentation purposes. For transformations, we added a quadratic term for salaries and total days. For interactions, we added one interaction between number of employees and total income as well as between number of employees and provider type.

Assumptions

The assumptions for multiple linear regression are the same as those for simple linear regression outlined in the previous section.

Results

The results for the multiple linear regression model with interactions and transformations are summarized below.

Observations	5141
Dependent variable	total_costs
Type	OLS linear regression

F(17,5123)	4556.17
R^2	0.94
Adj. R^2	0.94

	Est.	S.E.	t val.	p
(Intercept)	0.02	0.01	2.88	0.00
number_of_beds	-0.02	0.02	-1.35	0.18
fte_employees_on_payroll	0.10	0.01	7.57	0.00
total_discharges	0.01	0.01	0.52	0.60
total_income	0.05	0.01	7.82	0.00
total_assets	0.04	0.00	10.08	0.00
inpatients	0.21	0.06	3.89	0.00
control_bin_Governmental	-0.02	0.01	-1.79	0.07
control_bin_Proprietary	-0.02	0.01	-2.61	0.01
provider_bin_Specialized	-0.07	0.01	-6.38	0.00
rural	0.01	0.01	1.49	0.14
duplicate	0.02	0.02	0.64	0.53
poly(total_days, 2, raw = TRUE)1	0.05	0.05	0.94	0.35
poly(total_days, 2, raw = TRUE)2	0.01	0.00	5.09	0.00
poly(salaries, 2, raw = TRUE)1	0.66	0.02	42.19	0.00
poly(salaries, 2, raw = TRUE)2	-0.01	0.00	-6.87	0.00
fte_employees_on_payroll:total_income	-0.00	0.00	-4.44	0.00
fte_employees_on_payroll:provider_bin_Specialized	-0.01	0.01	-0.62	0.54

Standard errors: OLS

The adjusted R^2 value of 0.94 indicates that this model with transformations and interactions explains about the same proportion of variablity in total costs as the main effects model. One interesting finding, however, is

that the p-value for the quadratic term for total days has a significant p-value despite the main-effect for total days not being significant.

The estimate of the test MSEP using CV is 0.114, and the test MSEP based on the held-out set is 0.081. These error values are very similar to those for the main effects model.

Regression Tree (with pruning)

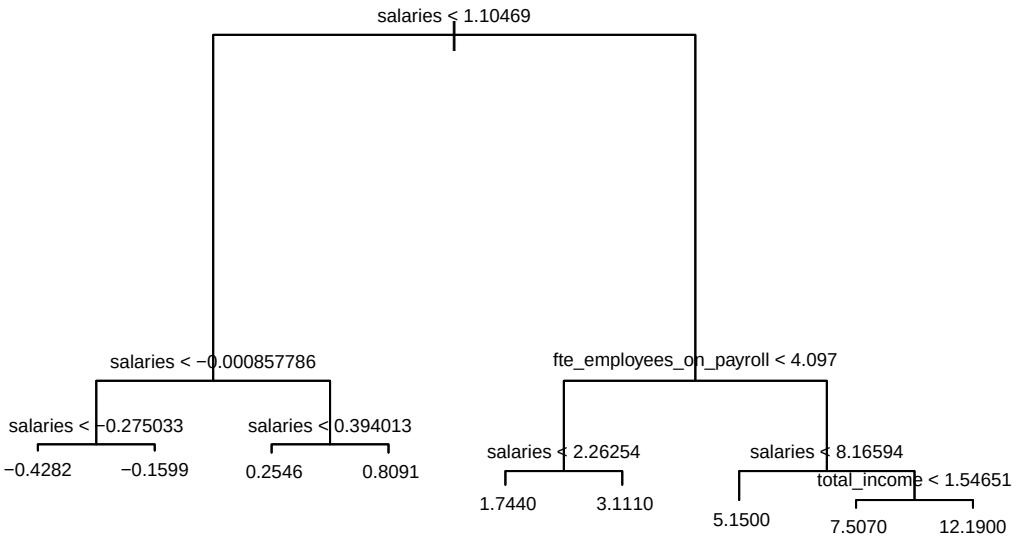
Assumptions

For the regression tree, the only assumption we are making is that the response is continuous. There are no parametric assumptions.

Results

A plot of the pruned regression tree is displayed below. The optimal pruning size was found to be 9 via cross validation.

We see that almost all splits in the tree are made based on the salaries predictor, indicating that salaries is the most important predictor. There are only two other predictors included in the tree, which are number of employees and total income.



The estimate of the test MSEP using CV is 0.144, and the test MSEP based on the held-out set is 0.100. These error values are very similar to those for the multiple linear regression models.

Bagging

Assumptions

Bagging involves taking averages across multiple regression trees, so there are no additional assumptions.

Results

The variable importance metrics for bagging are summarized in the table below. The Mean MSE Increase refers to the average percent increase in MSE when the predictor is excluded. This is computed by permuting the out-of-bag portion of the data. The Node Purity refers to the increase in node purity accounted for by the predictor. For both of these importance measures, a larger value indicates higher importance.

	Mean MSE Increase	Node Purity Increase
number_of_beds	11.8680452	33.3839272
fte_employees_on_payroll	18.3643027	760.4461010
total_days	13.9319735	45.7010248
total_discharges	16.3807968	56.1347241
total_income	1.4405305	37.7461785
total_assets	9.7844706	102.9728149
salaries	63.2554951	3996.3916487
inpatients	16.5813826	48.8786582
control_bin_Governmental	-2.2733782	3.6162787
control_bin_Proprietary	3.7671529	0.2821212
provider_bin_Specialized	6.1487162	4.1912112
rural	0.2925356	2.2098496
duplicate	0.9475105	0.4331920

We see that salaries by far has the highest node purity increase and also has the highest mean increase in MSE when the variable is removed. The second most important variable seems to be number of employees. These results are consistent with the plot of the single regression tree, and they indicate that much of the variability in hospital costs can be accounted for by the money spent on paying employees.

The estimate of the test MSEP using CV is 0.071, and the test MSEP based on the held-out set is 0.125. These error values are very similar to those for the multiple linear regression models and the single regression tree.

Random Forest

Assumptions

Random Forest involves taking averages across multiple regression trees, so there are no additional assumptions.

Results

The variable importance metrics for random forest are summarized in the table below. Their interpretation is the same as for bagging.

	Mean MSE Increase	Node Purity Increase
number_of_beds	8.0630251	357.9234663

	Mean MSE Increase	Node Purity Increase
fte_employees_on_payroll	17.3195365	1183.0500536
total_days	10.4914422	511.1054764
total_discharges	12.1221984	394.9144282
total_income	2.0034503	68.3684855
total_assets	11.3589813	333.5909214
salaries	19.1913953	1502.9611312
inpatients	7.0567410	698.8954939
control_bin_Governmental	0.3970834	5.1174297
control_bin_Proprietary	3.6940791	3.6041057
provider_bin_Specialized	5.3137978	8.6376733
rural	2.3303358	5.2574499
duplicate	2.1940374	0.9407285

We see that the ordering of variable importance is similar to that for bagging, with salaries and number of employees being the top two most important.

The estimate of the test MSEP using CV is 0.012, and the test MSEP based on the held-out set is 0.053. These error values are notably lower than those for the previous methods.

Boosting

Assumptions

Boosting involves manipulating and combining multiple regression trees, so there are no additional assumptions.

Results

The tuning parameter was chosen to be 0.1 by cross validation.

The relative influence of each predictor is summarized in the table below. The relative influence for boosting is similar to variable importance for bagging and random forest, but the method for computing it is slightly different. For boosting, the relative influence is computed as follows: for each split in the tree, compute the decrease in MSE. Then, average the improvement for each variable across all trees where that variable is included. A higher relative influence corresponds to a larger average decrease in MSE. A main difference in the computation compared to the Mean MSE Decrease from bagging and random forest is that for boosting, we are computing the mean decrease based on the entire training set, not only the out of bag portions. There is another method for computing importance which uses out of bag samples only, but the method described above is more widely used, so we chose that one.

	Relative Influence
salaries	53.613258
fte_employees_on_payroll	21.110887
total_days	7.176102
inpatients	5.713005
total_discharges	5.202203
total_assets	3.664882
total_income	1.828420
number_of_beds	1.691243
control_bin_Governmental	0.000000
control_bin_Proprietary	0.000000

	Relative Influence
provider_bin_Specialized	0.000000
rural	0.000000
duplicate	0.000000

We see that salaries and number of employees have the highest influence by far compared to the others, consistent with the importance metrics from the previous methods.

The estimate of the test MSEP using CV is 0.092, and the test MSEP based on the held-out set is 0.079. These error values are on the lower side compared to the previous methods, but not as low as for random forest.

Neural Network

Assumptions

The only assumption for the neural network is that the response is continuous. There are no parametric assumptions.

Results

The estimate of the test MSEP using CV is 0.072, and the test MSEP based on the held-out set is 0.088. These error values are on the lower side compared to the previous methods, but not as low as for random forest.

Summary Table Quantitative Outcome

The below table summarises the error rates for all methods applied to the quantitative response, including all predictors.

method	cv_error	test_error
Marginal LR number_of_beds	0.253	0.226
Marginal LR fte_employees_on_payroll	0.132	0.129
Marginal LR total_days	0.205	0.191
Marginal LR total_discharges	0.298	0.264
Marginal LR total_income	0.939	0.993
Marginal LR total_assets	0.731	0.409
Marginal LR salaries	0.103	0.229
Marginal LR inpatients	0.200	0.189
Marginal LR control_bin_Governmental	0.996	1.037
Marginal LR control_bin_Proprietary	0.955	1.003
Marginal LR provider_bin_Specialized	0.980	1.023
Marginal LR rural	0.992	1.039
Marginal LR duplicate	0.996	1.038
Linear Regression (Main Effects)	0.071	0.106
Linear Regression (Transformations)	0.114	0.081
Regression Tree	0.144	0.100
Bagging	0.071	0.125
Random Forest	0.012	0.053

method	cv_error	test_error
Boosting	0.092	0.079
Neural Network	0.072	0.088

We note that the random forest method has the lowest MSEP.