Mathematics and Statistics for AI

Assignment

1. Basic

Assignment 1: Vectors and Distance Metrics

In the One Hour AI Solutions platform, we represent client AI problems and engineer skills as vectors to find the best match.

Problem Statement: Two AI engineers have the following skill vectors (representing expertise in: machine learning, data preprocessing, statistical analysis, visualization, cloud deployment) on a scale of 0-10:

- Engineer A: [9, 5, 8, 4, 7]
- Engineer B: [6, 9, 5, 8, 5]

A new client project has requirements represented as: [8, 7, 7, 6, 4]

Tasks:

1. Calculate the Euclidean distance between the client's requirements and each engineer's skills.
2. Based on this metric, which engineer is a better match for the project?
3. How would the result change if we used Manhattan distance instead?

Assignment 2: Statistical Analysis of Service Times

The One Hour AI Solutions platform tracks the time it takes for engineers to solve different types of AI problems.

Problem Statement: The platform has collected data on resolution times (in minutes) for algorithm optimization problems:

35, 48, 42, 63, 52, 38, 45, 57, 41, 50

Tasks:

1. Calculate the mean, median, and standard deviation of these resolution times.
2. If the platform wants to provide a time estimate to clients, what range would cover approximately 68% of cases (assuming normal distribution)?
3. If the platform promises clients that problems will be solved "within the average time plus 20 minutes," what percentage of problems would be completed within this guarantee?

2. Intermediate

Optimization with Gradient Descent

The One Hour AI Solutions platform is optimizing a pricing model for its services based on various factors. The error in their current model is represented by the function:

Problem Statement: The platform's cost function for price prediction is:

$J(w_1, w_2) = (w_1 - 4)^2 + 2(w_2 - 3)^2 + w_1w_2$

where w_1 relates to problem complexity and w_2 relates to required expertise.

Tasks:

1. Calculate the gradient $\nabla J(w_1, w_2)$ by finding the partial derivatives with respect to each weight.
2. Starting with initial weights $(w_1, w_2) = (1, 1)$ and using a learning rate of 0.1, perform 3 iterations of gradient descent.
3. Explain how this optimization mathematically improves the platform's pricing model.

3. Advanced

Bayesian Probability for Client Classification

The One Hour AI Solutions platform wants to better predict which clients might need extended consultations versus quick solutions.

Problem Statement: The platform has analyzed its historical data and found:

- 70% of all clients are businesses (B)
- 30% of all clients are individual developers (I)
- 80% of business clients require extended consultations (E)
- 40% of individual developers require extended consultations (E)
- 60% of business clients require additional data preprocessing (D)
- 30% of individual developers require additional data preprocessing (D)

Tasks:

1. Calculate P(B|E) - the probability that a client is a business, given that they required an extended consultation.
2. Calculate P(I|D) - the probability that a client is an individual developer, given that they required additional data preprocessing.
3. If a new client requires both extended consultation and additional data preprocessing, what is the probability they are a business client? (Assume E and D are independent given the client type)
4. Explain how these probabilities can help optimize resource allocation on the platform.