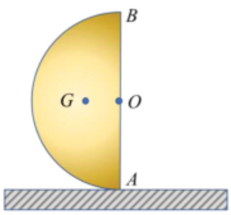
Test 7 Dynamics Solutions (2017W)

November 15, 2017 5:54 PM

221-test7-DYN (2017W)

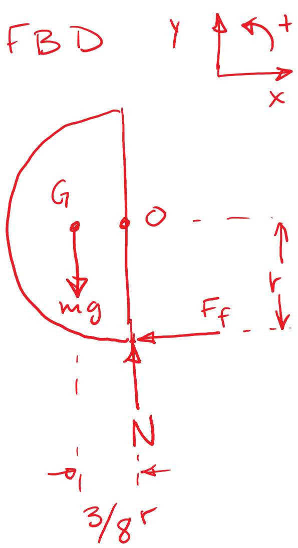
SA 1. (5 marks)



A hemisphere with mass m and radius r is released from rest in the position shown. The hemisphere is just on the edge of slipping at point A in the instant it is released. **Draw the free body diagram, and write each of the equations of motion at the instant of release in terms of the angular acceleration, α.** (You do not have to solve these equations of motion for α or μ_s.)

Note that the distance between G and O is $\frac{3}{8}r$, and the moment of inertia about point A is $I_A = 1.4mr^2$.

① FBD



② EOM

$\Sigma F_x: \ -F_f = m a_{Gx}$ Ⓐ

$\Sigma F_y: \ N - mg = m a_{Gy}$ Ⓑ

$\Sigma M_A: \ mg\left(\frac{3}{8}r\right)\hat{k} = \underbrace{I_A}_{1.4mr^2}\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$ Ⓒ

3 eqns, 5 unknowns

$a_{Gx}, a_{Gy}, F_f, N, \alpha$

③ kinematic constraints

Since A is just on the point of slipping (but not yet slipping), we can say:

$F_f = \mu_s N$ and $\vec{a}_A = 0$

Since we are starting from rest: $\vec{\omega} = 0$

$\therefore \ \vec{a}_G = \vec{\alpha} \times \vec{r}_{G/A} = \alpha\hat{k} \times \left(-\frac{3}{8}r\hat{\imath} + r\hat{\jmath}\right)$

$= \underbrace{-\frac{3}{8}\alpha r}_{a_{Gy}}\hat{\jmath} \ \underbrace{-\alpha r}_{a_{Gx}}\hat{\imath}$

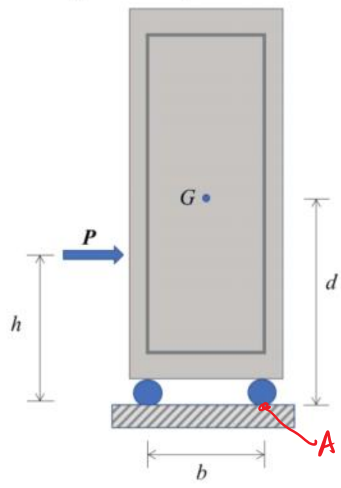
$\therefore$ EOMs become:

Ⓐ: $-\mu_s N = -m\alpha r$

Ⓑ: $N - mg = -\frac{3}{8}m\alpha r$

Ⓒ: $mg\left(\frac{3}{8}r\right) = 1.4m\alpha r^2$

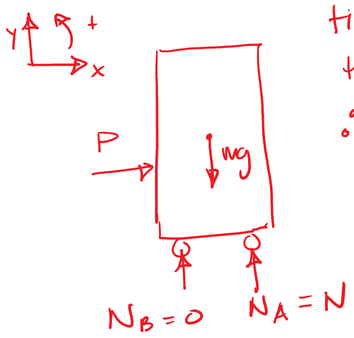
SA 2. (5 marks)



A 25 kg cabinet is mounted on wheels allowing it to move freely on the floor. If a 200N force (***P***) is applied, determine the acceleration of the centre of gravity, and the maximum value of h in mm for which the cabinet will not tip to the right (away from ***P***).

d = 700 mm
b = 300 mm

① FBD



asked about tipping to the right, ∴ $N_B = 0$

② EoM: $\Sigma F_x : P = m a_{Gx}$ Ⓐ

$\Sigma F_y : N - mg = m \cancel{a_{Gy}} \to 0$ (moving horizontally) Ⓑ

$\Sigma M_A : P \cdot h \hat{k} - mg \frac{b}{2} \hat{k} = m \vec{r}_{G/A} \times \vec{a}_G + I_A \cancel{\alpha}$ Ⓒ

0 for no tip

$\vec{a}_G = a_{Gx} \hat{i}$

$\vec{r}_{G/B} = -\frac{b}{2} \hat{i} + d \hat{j}$

∴ Ⓒ $\Rightarrow \Sigma M_A = m \left(-\frac{b}{2} \hat{i} + d \hat{j}\right) \times a_{Gx} \hat{i} = m a_{Gx} d \hat{k}$

Ⓐ $\Rightarrow a_{Gx} = \frac{P}{m} = \frac{200\,N}{25\,kg} = 8\,m/s^2$

Ⓒ $\Rightarrow P \cdot h - mg\frac{b}{2} = m a_{Gx} d$

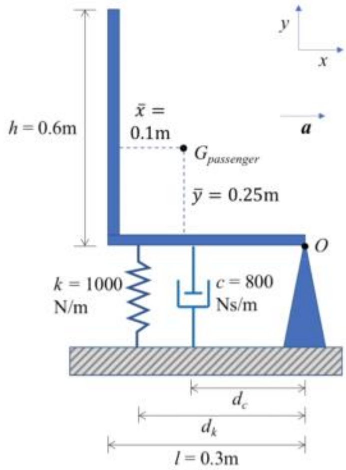
$$h = \frac{m\left(g\frac{b}{2} + a_{Gx} d\right)}{P}$$

$$= \frac{25\,kg\left[(9.81\,m/s^2)(0.15\,m) + 8\,m/s^2 (0.7\,m)\right]}{200\,N}$$

$$\boxed{h = 0.884\,m = 884\,mm}$$

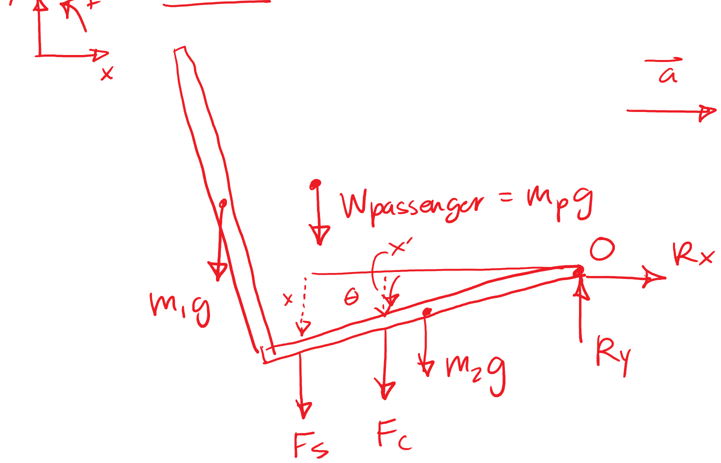
LA 1. [25 marks] Engineers are designing a passenger seat in a high-speed train. When the train is in motion, let $\boldsymbol{a}$ denote the train's acceleration along a straight section of track. The seat rotates about the point O. Let θ denote the angular position of the seat (measured counter-clockwise). Pictured is position $\theta = 0$.

A spring (k = 1000 N/m) and damper (c = 800 Ns/m) are placed under the seat at distances d_k and d_c (measured in meters) from O. The seat and seatback have a constant width (into the page), dimensions $l = 0.3m$ and $h = 0.6m$, and a mass per unit length of 15 kg/m.



(a) [4 marks] Assume that an 80 kg passenger sitting in the seat acts like a point mass located at $G_{passenger}$ ($\bar{x} = 0.10m$ and $\bar{y} = 0.25m$) and is rigidly attached to and moving with the seat due to a seat belt. Draw a free body diagram for a small perturbation of the system, θ.

FBD:

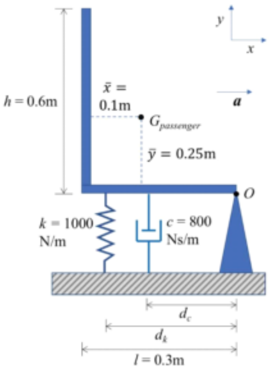


where: $F_s = -kx$ $\quad m_1 = 15\ kg/m \cdot h$

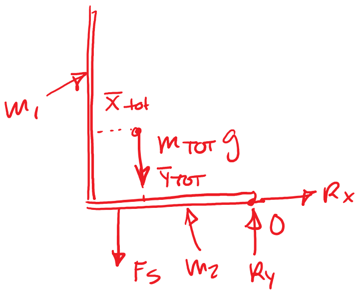
$F_c = -c\dot{x}'$ $\quad m_2 = 15\ kg/m \cdot l$

(b) [8 marks] The train accelerates at the rate a (m/s^2) as shown. Prove that the equation of motion of the seat/passenger system is

$$10.225\ddot{\theta} + 800d_c^2\dot{\theta} + 1000d_k^2\theta = 22.7a$$



equilibrium:



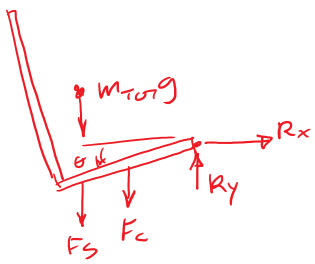
$$\Sigma M_O = F_s d_k + m_{TOT} g \bar{x}_{TOT} = 0$$

$$F_s = -k x_{eq}$$

$$\therefore \quad m_{TOT} g \bar{x}_{TOT} = k x_{eq} d_k$$

$$(m_{TOT} = m_1 + m_2 + m_p)$$

perturbation:



$$\Sigma M_O: \; F_s d_k + F_c d_c + m_{TOT} g \bar{x}_{tot} = I_{O_{TOT}} \alpha + |m_{TOT} \vec{r}_{G_{TOT}/O} \times \vec{a}_O|$$

$$F_s = -k(x + x_{eq}) \qquad \vec{a}_O = a\hat{i}$$

$$F_c = -c\dot{x}' \qquad \bar{y}_{TOT} = \frac{\frac{h}{2} m_1 + \bar{y} m_p + \cancel{0 \cdot m_2}}{m_{TOT}}$$

$$\vec{r}_{G_{TOT}/O} = \bar{x}_{TOT}\hat{i} + \bar{y}_{TOT}\hat{j} \qquad = \frac{0.3m(9kg) + 0.25m(80kg)}{m_{TOT}}$$

$$= \frac{22.7}{m_{TOT}}$$

$$\vec{r}_{G_{TOT}/O} \times \vec{a}_O = (\bar{x}_{TOT}\hat{i} + \bar{y}_{tot}\hat{j}) \times a\hat{i} = -a\bar{y}_{TOT}\hat{k}$$

$$I = I_{1,O} + I_{2,O} + I_{P,O}$$

$$I_{1,O} = \frac{1}{12} m_1 h^2 + m_1\left(\frac{h^2}{2} + l^2\right) = \frac{1}{3} m_1 h^2 + m_1 l^2$$

$$I_{2P} = \frac{1}{3} m_2 l^2 = \frac{1}{3}$$

$$I_{P,O} = m_p\left((l - \bar{x})^2 + \bar{y}^2\right)$$

$m_1 = (15\ kg/m)h = 9kg$

$m_2 = (15kg/m)l = 4.5kg$

$x = d_k \sin\theta \approx d_k\theta$

$x' = d_c \sin\theta \approx d_c\theta$

$\dot{x}' = d_c\dot{\theta}$

$$I_O = \frac{1}{3} m_1 h^2 + m_1 l^2 + \frac{1}{3} m_2 l^2 + m_p\left((l - \bar{x})^2 + \bar{y}^2\right)$$

$$= \frac{1}{3}(9kg)(0.6m)^2 + (9kg)(0.3m)^2 + \frac{1}{3}(4.5kg)(0.3m)^2 + 80kg\left((0.3m - 0.1m)^2 + (0.25m)^2\right)$$

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$$= 10.225\ kg \cdot m^2$$

$$\Sigma M_O \Rightarrow -k d_k^2\theta - k\cancel{d_k x_{eq}} - c d_c^2\dot{\theta} + \cancel{m_{TOT} g \bar{x}_{TOT}} = I_O\ddot{\theta} - m_{TOT} a \bar{y}_{TOT}$$

$$I_O\ddot{\theta} + c d_c^2\dot{\theta} + k d_k^2\theta = m_{TOT}\bar{y}_{TOT} a = \cancel{m_{TOT}}\left(\frac{22.7}{\cancel{m_{TOT}}}\right) a$$

$$\Rightarrow \boxed{10.225\ddot{\theta} + 800 d_c^2\dot{\theta} + 1000 d_k^2\theta = 22.7a}$$