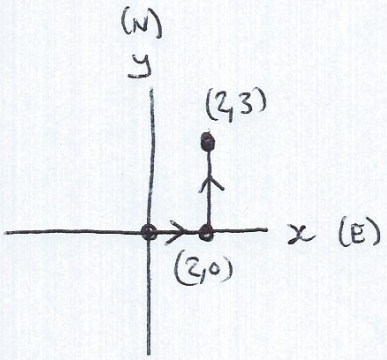
1. (10 marks) You are standing at the origin $(0,0)$ of a coordinate system (x,y) in which (as usual) the x direction is east and the y direction is north (all distances in m.). The elevation of the surrounding land is given by a function $f(x,y)$.

   (a) (5 marks) Walking 2 $m.$ east, you gain 0.5 $m.$ of elevation. Then walking a further 3 $m.$ north, you gain a further 2.8 $m.$ elevation. Using linear approximation, estimate $\nabla f(0,0)$.



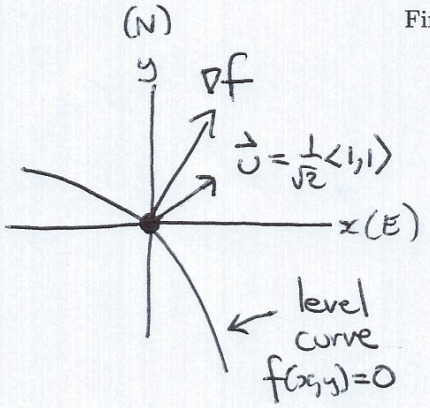
- $0.5 = f(2,0) - f(0,0) \approx f_x(0,0)\cdot 2 \Rightarrow f_x(0,0) \approx 0.25$
- $2.8 + 0.5 = f(2,3) - f(0,0) \approx f_x(0,0)\cdot 2 + f_y(0,0)\cdot 3$
  $\approx 0.5 + f_y(0,0)\cdot 3 \Rightarrow f_y(0,0) \approx \frac{2.8}{3} \approx 0.93$

$\Rightarrow \nabla f(0,0) \approx \langle 0.25, 0.93 \rangle$

   (b) (5 marks) Suppose you happen to know the following two things:

   - the curve $\frac{x^2}{40} + x + \frac{y^2}{10} + 2\sqrt{3}y = 0$ is a level curve (contour line) of constant elevation $f(x,y)$;
   - if you walk directly north-east from $(0,0)$, your rate of ascent is 0.7 (vertical $m.$ per horizontal $m.$).

   Find $\nabla f(0,0)$



- $\nabla f(0,0)$ is normal to the level curve at $(0,0)$
- $\nabla\left[\frac{x^2}{40} + x + \frac{y^2}{10} + 2\sqrt{3}y\right] = \left\langle \frac{x}{20} + 1, \frac{y}{5} + 2\sqrt{3} \right\rangle$
  $= \langle 1, 2\sqrt{3} \rangle$ (at $(0,0)$) is also normal to the level curve
  $\Rightarrow \nabla f(0,0) = \alpha\langle 1, 2\sqrt{3} \rangle$ for some α
- $0.7 = D_{\vec{u}} f(0,0) = \nabla f(0,0) \cdot \frac{1}{\sqrt{2}}\langle 1,1\rangle = \frac{\alpha}{\sqrt{2}}(1 + 2\sqrt{3})$

$\Rightarrow \alpha = \frac{\sqrt{2}(0.7)}{1+2\sqrt{3}}$

$\Rightarrow \nabla f(0,0) = \left\langle \frac{\sqrt{2}(0.7)}{1+2\sqrt{3}}, \frac{2\sqrt{2}\sqrt{3}(0.7)}{1+2\sqrt{3}} \right\rangle \approx \langle 0.22, 0.77 \rangle$