# Tutorial 9.5. Nonlinear Systems. Solutions

$\dot{x} = x(3-x-2y)$

$\dot{y} = y(2-x-y)$

* See Strogatz's Nonlinear Dynamics and Chaos §6.4 for a discussion of this problem.

## 1. Steady-states

Set $\dot{x} = \dot{y} = 0$: This gives $x_1 = (0,0)$, $x_2 = (0,2)$, $x_3 = (3,0)$, $x_4 = (1,1)$.

## 2. Jacobian

$$J = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} = \begin{pmatrix} 3-2x-2y & -2x \\ -y & 2-x-2y \end{pmatrix}.$$

$f = 3x - x^2 - 2xy$

$g = 2y - xy - y^2$

## 3. J @ steady-states

$$J\Big|_{x_1=(0,0)} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}.$$

$$J\Big|_{x_2=(0,2)} = \begin{pmatrix} -1 & 0 \\ -2 & -2 \end{pmatrix}.$$

$$J\Big|_{x_3=(3,0)} = \begin{pmatrix} -3 & -6 \\ 0 & -1 \end{pmatrix}.$$

$$J\Big|_{x_4=(1,1)} = \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}.$$

4. $J_{X_1=(0,0)} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$ has eigenvalues $\lambda = 3, 2$ so

<u>$(0,0)$ is an unstable node</u>.

$J_{X_2=(0,2)} = \begin{pmatrix} -1 & 0 \\ -2 & -2 \end{pmatrix}$ has eigenvalues $\lambda = -1, -2$ so

<u>$(0,2)$ is a stable node</u>.

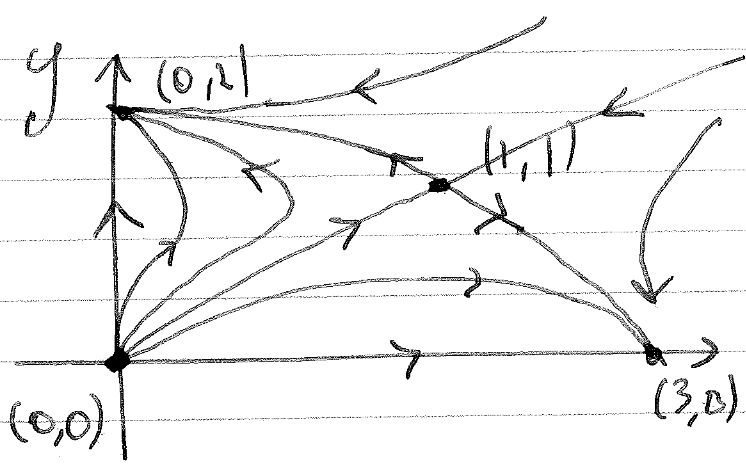
$J_{X_3=(3,0)} = \begin{pmatrix} -3 & -6 \\ 0 & -1 \end{pmatrix}$ has eigenvalues $\lambda = -3, -1$ so

<u>$(-3,-1)$ is a stable node</u>

$J_{X_4=(1,1)} = \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}$ has eigenvalues $\lambda \approx 0.4142$, $\lambda \approx -2.4142$

so <u>$(1,1)$ is a saddle point</u>.

5.




*Sketching these is always tricky!