DEPARTMENT OF MECHANICAL ENGINEERING

MECH 222 Final Examination 1 March 29, 2008

Duration: 180 minutes.

Materials admitted: Pencil, eraser, straightedge, Mech 2-Approved Calculator, Standard formula sheet (provided).

All questions must be answered. Provide **all** work and solutions **on this exam**. Illegible work or answers that do not include supporting calculations and explanations will **NOT BE MARKED**.

PLEASE WRITE YOUR NAME ON THE TOP OF ALL EXAM PAGES

NAME:____Section__________

SIGNATURE:__________________________________

STUDENT NUMBER:__________________________

Question	Mark Received	Maximum Mark
T1		15
T2		5
F1		5
F2		5
F3		5
F4		5
M1		5
M2		5
M3		5
M4		5
MF		15+15
TM		15+15
FT		15+15
Total		**150**

Name:______________________________ Section:__________________________

M1 [5 marks] The density of mosquitos in the sky [kg/m^3] is proportional to

$$f(x,y,z) = xy^2 + y^2z^3 + z^3x.$$

A swallow flies through the point (4, –2, –1) with velocity $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. Does the swallow perceive mosquito density increasing or decreasing? At what rate (density units per unit time)?

M2 [5 marks] Consider the function

$$f(x,y) = x^3 + y^2 - 6xy + 6x + 3y.$$

(a) Find all critical points of f.
(b) Choose *any one* of the critical points found in (a) and classify it as a relative minimum, relative maximum, or saddle point. (Classify only one CP, even if you find several.)

Name:______________________________ Section:__________________________

M3 [5 marks] Find the upward-pointing unit vector that is normal to this surface at the point $(1, \pi/2, 0)$:

$$\cos(xy) = e^z - 1.$$

M4 [5 marks] In deep space, a robot manipulator is moving a mass. The position of the mass is supposed to be given by

$$\mathbf{r}(t) = (4te^t, t^2, 10\pi + \sin(\pi t)), \qquad t > 0.$$

For the interval $0 < t < 1$, this equation holds. But at the instant when t=1, the mass slips out of the robot's grip and starts to drift. At what point (if any) will the mass cross the plane z=0? (Hint: "deep space" means "zero gravity".)

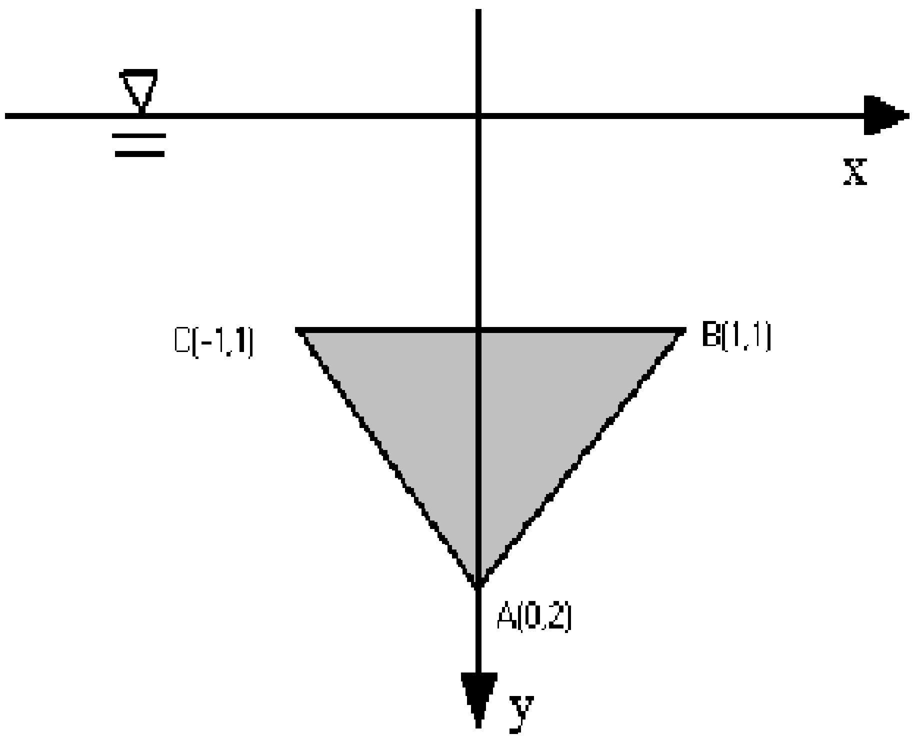
Name:______________________________ Section:__________________________

MF. [30 marks] Experimenters have installed a triangular window in the vertical side of a tank containing a toxic stratified fluid ("sludge"). Measuring y downward from the fluid's top surface, the window's corners are at the points

A: (0,2), B: (1,1), C: (-1,1). [All positions in metres.]

The fluid density [kg/m^3] as a function of depth y is $\rho(y) = 1000 + 2ky$, where k is a constant that should appear in your answers. Write g for the acceleration of gravity: don't substitute a number yet.

i. Find the (x,y) coordinates of the centroid of the window.
ii. Find the fluid pressure as a function of depth. Answer in terms of g and k.
iii. Find the total force due to fluid pressure on the window.
iv. The answer for (iii) equals $(1333+16k/9)g$ only in the special case k=0. Explain. That is, say (i) why these answers should agree when k=0, and (ii) why they should differ when $k\neq0$.
v. Suppose the window is hinged along edge BC and a student is employed to prevent sludge from flooding the lab by applying force at point A. ("Summer research project.") What force must the student apply? Calculate with k=150 [kg/m^4] and g=9.8 [m/s^2]. Give units for your answer.



Name:____________________________ Section:__________________________

TM (a) [15 marks] A hot bar of solid steel emerges from the furnace in a steel mill at a uniform temperature of 1000 ˚C. Inequalities describing the bar's location are

$$0 \le z \le h,\ \ x^2 + y^2 \le a^2; \quad h = 1.50 \text{ m},\ \ a = 0.20 \text{ m}.$$

Cool water is sprayed over the bar. The surface cools faster than the interior, so the bar's temperature depends on position as well as time. At a certain instant, the temperature (degrees C) is

$$T(x, y, z) = 100 \ + \ 20000\,(a^2 - x^2 - y^2)\sin(\pi z/h)$$

The total energy U of the bar can be modeled as U=MCT, provided that T is the average temperature over the bar.

(i) [3 marks] Sketch the bar and compute its mass. The density of steel is 7850 kg/m^3.

(ii) [7 marks] Set up the definite integral to compute the average temperature of the bar.

(iii) [5 marks] Evaluate the integral from part (ii) to find the total heat transfer from the bar in kilojoules.

TM (b) [15 marks] How many kilograms of 10 ˚C cooling water would be needed to reduce the average temperature of the bar in part (a) from 1000 ˚C to an average temperature of 400 ˚C?

Assume that the entire process occurs at 100 kPa, and the cooling essentially stops when all of the water has changed to saturated vapor. Use a good diagram to show how this process can be modeled using control **mass** analysis.

University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

MECH 222

EXAM 1: 1 April 2009
MATHEMATICS SECTION

PLEASE WRITE YOUR UBC ID NUMBER ON EACH PAGE

NAME: ____________________

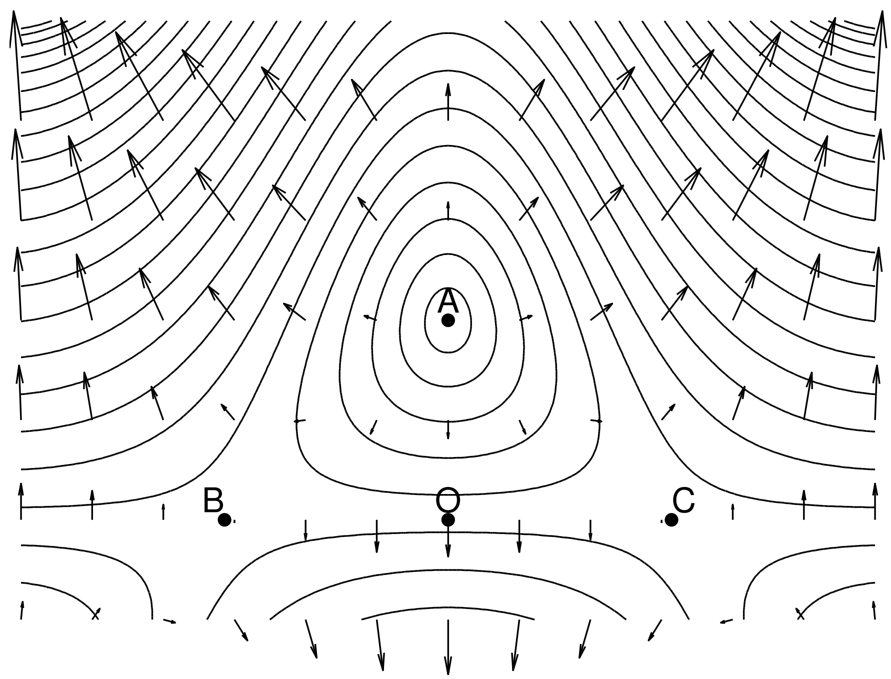
Signature: ____________________

UBC ID Number: ____________________

Group Letter Code: ____________________

Question	Marks	Score
8	10	
9	6	
10	5	
11	9	
12	10	
TOTAL	40	

8. (8 marks) The sketch below shows selected level curves for a certain function $f = f(x, y)$ near the point $O(0, 0)$. The arrows on the sketch represent the vectors $\nabla f(x, y)$ at their tail points. There are exactly three critical points for f in the region shown: A, B, and C.



(a) Using only the information in the sketch, classify each point A, B, C as a local maximizer, a local minimizer, or a saddle point. Give a terse explanation (one sentence or less).

Use $\nabla f(x, y) = \langle 2y\sin(x),\, 2y - 2\cos(x)\rangle$ and $f(B) = 1$ in parts (b)–(e).

(b) Find the coordinates of points A, B, and C.

(c) Find $f(A)$.

(d) Write a mathematical description for the level set $f(x, y) = 1$. Then draw that set onto the sketch provided above.

(e) Consider the level curve in the sketch passing through $P(\pi/2, 1)$. Find the equation for this curve's tangent line at P.

9. (6 marks) Evaluate this integral in terms of the constants a and p. You may assume $a > 0$ and $p > 0$. (One approach uses polar coordinates.)

$$J(p) = \int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} (x^2 + y^2)^p y \, dx \, dy.$$

10. (5 marks) Find the total flux of **F** outward through the surface of the sphere

$$x^2 + (y+3)^2 + (z-1)^2 = a^2,$$

where $a > 0$ is a constant (which may appear in your answer) and

$$\mathbf{F}(x, y, z) = (yx)\mathbf{i} + (zy)\mathbf{j} + (z^2)\mathbf{k}.$$

Could this **F** represent the velocity field for an incompressible fluid flow? Why or why not?

11. (9 marks) Let $\mathcal{S}$ denote the part of the surface $z = 1 + (2x + y)^2$ selected by

$$2x + y \geq 2, \qquad x \leq 1, \qquad y \leq 2.$$

Find the upward flux of $\mathbf{F}$ through $\mathcal{S}$, where

$$\mathbf{F}(x, y, z) = \langle \cos(e^{xz}), -2\cos(e^{xz}), 3x + 2y \rangle \, .$$

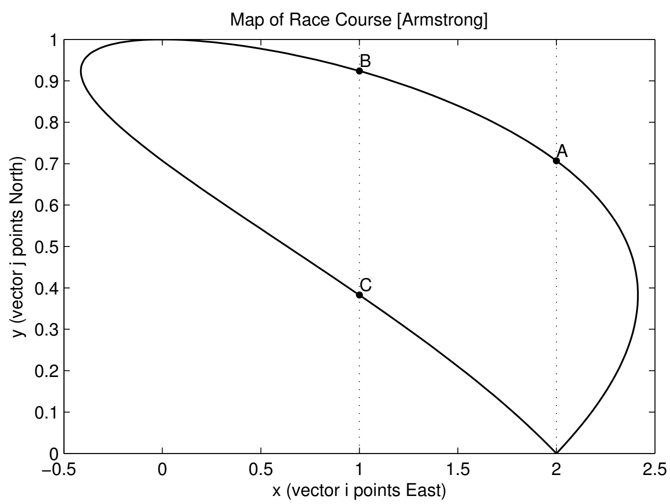
12. (10 marks) In the Virtual Tour de France, racers follow a course whose coordinates on a map are given parametrically by

$$x = 1 + \sqrt{2}\sin(2\theta + \pi/4), \quad y = \sin(\theta), \qquad 0 \le \theta \le \pi.$$

Racers travel in the direction of increasing θ. The land elevation at point (x, y) is given by

$$z = \frac{1}{20}\left(1 - x^2 - 5y^2\right).$$

All linear measurements (x, y, and z) are in the same units.



Point A lies on the line $x = 2$; points B and C on $x = 1$.

(a) Find the y-coordinates of points A, B, and C. (Either exactly, or with 5-digit accuracy.)

(b) Is a racer passing through point A going uphill or downhill? At what slope? Explain.

(c) Is a racer passing through point C going uphill or downhill? At what slope? Explain.

University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

MECH 222

Exam 1 (Math): 7 April 2010

PLEASE WRITE YOUR UBC ID NUMBER ON EACH PAGE

NAME: ______________________________

Signature: ______________________________

UBC ID Number: ______________________________

Group Letter Code: ______________________________

Question	Marks	Score
1	5	
2	5	
3	10	
4	10	
5	10	
TOTAL	40	

1. (5 marks) Write an equation for the plane that is tangent to the surface

$$z\cos(xz) + \sin(xy) = 1$$

at the point $P(\pi/2, 1, 1)$.

2. (5 marks) Match each English-only phrase below with the one (and only) mathematical expression that best captures the concept or calculation it describes.
Example: Description (0) applies to expression (e), as indicated below.

English-only explanations:

(0) Acceleration of a point whose position is a known function of time.
(1) Area of a triangle.
(2) Rate of change of temperature in a given direction.
(3) Work done by a force on a point as it traces a given path.
(4) Test critical points of profit function for max/min/saddle.
(5) Solid volume between two given surfaces.
(6) Total outward flux through the boundary of a given control volume.
(7) Total mass of gold in a core sample, based on gold's linear density along the bore hole.

Math-only expressions:

(a) $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{r}$ ____________________

(b) $\iint_{\mathcal{S}} f(x,y,z)\, dS$ ____________________

(c) $\iiint_{E} \nabla \bullet \mathbf{F}\, dV$ ____________________

(d) $\lim_{h\to 0} \dfrac{f(\mathbf{a}+h\mathbf{u}) - f(\mathbf{a})}{h}$ ____________________

(e) $\dfrac{d^2\mathbf{r}}{dt^2}$ Given description (0)

(f) $\int_{\mathcal{C}} f\, ds$ ____________________

(g) $\nabla g(a,b,c) \bullet \langle x-a, y-b, z-c\rangle = 0$ ____________________

(h) $\left(\dfrac{\mathbf{u} \bullet \mathbf{w}}{\mathbf{w} \bullet \mathbf{w}}\right) \mathbf{w}$ ____________________

(i) $\dfrac{1}{2}\,|\mathbf{u} \times \mathbf{w}|$ ____________________

(j) $\iint_{D} [h(x,y) - g(x,y)]\; dA(x,y)$ ____________________

(k) $\begin{bmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial y^2} \end{bmatrix}$ ____________________

3. (10 marks) Let $$I = \int_0^{\sqrt{3}} \int_{y/\sqrt{3}}^{1} \frac{y e^{x^2+y^2}}{\sqrt{x^2+y^2}}\,dx\,dy + \int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y e^{x^2+y^2}}{\sqrt{x^2+y^2}}\,dy\,dx.$$

   (a) Convert I to an iterated double integral in polar coordinates.

   (b) Find the exact value of I.

4. (10 marks) Fresh water fills a long pipe that lies along the x-axis. The pipe has a rectangular cross-section, defined by

$$-a \le y \le a, \qquad -b \le z \le b,$$

where a and b are positive constants. The water's flow velocity (all units are SI) is

$$\mathbf{V}(x,y,z) = u(x,y,z)\,\widehat{\mathbf{i}} + 0\,\widehat{\mathbf{j}} + 0\,\widehat{\mathbf{k}}, \quad \text{where} \quad u(x,y,z) = K(y^2-a^2)(z^2-b^2).$$

(a) (6 marks) Find the x-component of the momentum flux across the plane $x = 0$.

(b) (4 marks) Suppose a mesh screen across the pipe has the general shape given by

$$\mathcal{S}: \qquad x = h(y,z), \quad -a \le y \le a,\ -b \le z \le b.$$

Use the integral expressing the x-component of momentum flux through $\mathcal{S}$, namely,

$$\iint_{\mathcal{S}} \rho u \mathbf{V} \bullet \widehat{\mathbf{n}}\, dS,$$

to give a math-based explanation of why the momentum flux through every such surface has the same value, which is precisely the value obtained in part (a).
[A correct pure-math explanation is required for full marks, but partial credit will be given for a really clear Engineering-based explanation, if it mentions key fluids properties.]

(Extra work space.)

5. (10 marks) A scalar constant $a > 0$ is given. The path $\mathcal{C}$ starts at $O(0,0,0)$, runs straight up to $A(0,0,a)$, and then arcs down to $B(1,1,0)$ along the parabola

$$z = a(1 - x^2), \quad y = x.$$

Evaluate the following quantities in terms of a:

(a) $I = \displaystyle\int_{\mathcal{C}} 2y\, ds$

(b) $J = \displaystyle\int_{\mathcal{C}} z^2\, dx + \sin(\pi y)\, dy + 2xz\, dz$

This examination has 7 pages including this cover

The University of British Columbia

Final Examination – 30 March 2011

Mathematics 253(201)

Multivariable Calculus for Mech 2

Closed book examination | Time: about 1/3 of 180 minutes

Name______________________________ **Signature** ______________________________

UBC Student Number________________

Special Instructions:

To receive full credit, all answers must be supported with clear and correct derivations. A standard Mech 2 calculator is allowed. A formula sheet is provided with the test.

Rules governing examinations

1. Each candidate must be prepared to produce, upon request, a UBCcard for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
  (a) having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
  (b) speaking or communicating with other candidates; and
  (c) purposely exposing written papers to the view of other candidates or imaging devices.
The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		10
3		10
4		10
Total		40

[10] **1.** Make a good sketch of the domain of integration and then evaluate

$$I = \int_{-a}^{a} \int_{0}^{\sqrt{a^2-x^2}} \frac{2h}{\left(b^2+x^2+y^2\right)^2}\, dy\, dx.$$

Here a, b, and h are positive constants which may appear in your answer.

Continued on page 3

[10] **2.** Consider the surface $\mathcal{S}$ defined by

$$(*) \qquad xz - xy^2 + yz - y^3 = -2.$$

Notice that the point $P(1,1,0)$ lies on $\mathcal{S}$.

(a) Find the constants a and c for which the vector $\langle a, 1, c\rangle$ is perpendicular to $\mathcal{S}$ at P.

(b) Find the constant k for which the vector $\langle 1, 1, k\rangle$ is tangent to $\mathcal{S}$ at P.

(c) If x, y, and z are physical quantities always related by $(*)$, find the values at P for

$$\left(\frac{\partial y}{\partial z}\right)_x \quad \text{and} \quad \left(\frac{\partial x}{\partial y}\right)_z.$$

Continued on page 4

[10] **3.** Let $\mathcal{S}$ denote the part of the surface

$$z = a^2 - x^2 - y^2$$

selected by the simultaneous inequalities

$$y \geq 0, \qquad z \geq 0.$$

Evaluate the upward flux of $\mathbf{F}$ through $\mathcal{S}$, given

$$\mathbf{F}(x, y, z) = x^8 e^{z^2} \tan^{-1}(y)\,\mathbf{i} + \mathbf{j} + (\sqrt{z} + y)\,\mathbf{k}.$$

Continued on page 5

(Extra work space)

Continued on page 6

[10] **4.** Use the information given in the bullet-list below to evaluate

$$J = \int_{\mathcal{C}} yz^2\,dx + (xz^2 - z)\,dy + w(x,y,z)\,dz.$$

- The vector field $\mathbf{F}(x,y,z) = yz^2\,\mathbf{i} + (xz^2 - z)\mathbf{j} + w(x,y,z)\mathbf{k}$ is conservative.
- One has $w(x,0,z) = z^2$ for every real x and z.
- The curve $\mathcal{C}$ is given by $x = t^2, \quad y = e^{t^2}, \quad z = (1+t)^{2t}, \qquad 0 \le t \le 1.$

Continued on page 7

(Extra work space)

The End

Name______________________________ **Signature** ______________________________

UBC Student Number________________

This examination has 7 pages including this cover

The University of British Columbia

Final Examination – 28 March 2012

Mathematics 253(201)

Multivariable Calculus for Mech 2

Closed book examination Time: about 1/3 of 180 minutes

Special Instructions:

To receive full credit, all answers must be supported with clear and correct derivations. A standard Mech 2 calculator is allowed. A formula sheet is provided with the test. No other aids are allowed.

Rules governing examinations

1. Each candidate must be prepared to produce, upon request, a UBCcard for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
  (a) having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
  (b) speaking or communicating with other candidates; and
  (c) purposely exposing written papers to the view of other candidates or imaging devices.
The plea of accident or forgetfulness shall not be received.
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6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		8
2		8
3		8
4		8
5		8
Total		40

[8] **1.** You want to photograph the evening sky with your iPhone camera. To make sure the camera doesn't shake, you gently set it on the ground at the point $P(0, 1, 2)$ and start the countdown timer. Near the point P, the z-axis points straight up and the ground surface is given by

$$xz^2 + 4yz + 2\sin(xy) = 8.$$

The camera's optics collect all the incoming light rays that make an angle of $45°$ or less with the vector perpendicular to the camera's flat front side; of course the camera's front is parallel to its back, and the back is tangent to the ground at P.

At the instant when the picture is recorded, a rare bird is at the point $Q(1, 2, 5.2)$. Does the bird appear in your picture? Why or why not?

Continued on page 3

[8] **2.** We are prospecting for Neodymium (Nd). Drilling core samples is slow and expensive, so we have no way of knowing that the true concentration of Nd at (x, y, z) is

$$f(x, y, z) = 5x^2y + 7y^2z + 3z^2x \qquad \text{mg/kg.}$$

(Each of x, y, and z is measured in km.) We only have enough samples near the point $P(1, -1, 1)$ to estimate both f and ∇f at this point.

(a) Find the true values of f and ∇f at P.

(b) Using only the information from (a), predict which of the points within 0.1 km of P has the highest concentration of Nd. Also predict the concentration of Nd at that point. (Give decimal answers, with 4 significant figures.)

(c) Repeat part (b) for points within 0.5 km of P.

Continued on page 4

[8] **3.** Find the centroid coordinates for the uniform solid E defined by

$$a(x^2 + y^2) - a \le z \le b - b(x^2 + y^2).$$

Assume a and b are positive constants; include a good sketch.

Continued on page 5

[8] **4.** (a) [4 marks] Suppose $\mathbf{F} = \langle c_1, c_2, c_3 \rangle$ is a constant vector. Explain why

$$\iint_{\mathcal{S}} \mathbf{F} \bullet d\mathbf{S} = 0,$$

where $\mathcal{S}$ is defined by $x^2 + y^2 + z^2 = a^2$ for some constant $a > 0$, and oriented using outward normals. For full credit, use only geometry and symmetry; there is a penalty for appealing to any of the divergence theorem, flow Physics, spherical coordinates, or a projection argument.

(b) [4 marks] Find the upward flux of $\mathbf{F} = \left\langle 2x^2y, -11xy^2, 0 \right\rangle$ through the part of the surface $z = xy$ where $0 \leq x \leq 1$, $0 \leq y \leq 1$.

Continued on page 6

[8] **5.** Find the constants a, b, and c for which the total work done by the force

$$\mathbf{F}(x,y,z) = (axy + 3yz)\,\mathbf{i} + \left(x^2 + 3xz + by^2z\right)\mathbf{j} + \left(bxy + cy^3 + 3z^2\right)\mathbf{k}$$

is 0 along any path whose endpoint equals its starting point. Using these constants, evaluate the work done by $\mathbf{F}$ on a particle that moves from $A(0,1,-1)$ to $B(2,1,1)$.

Continued on page 7

(Extra work space)

The End