University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

TEST #6, November 10, 2017

MECH 221

Suggested Time: 45 min **Allowed Time:** 50 min

Materials admitted: Pencil, eraser, straightedge, Mech 2 Approved Calculator (Sharp EL-510), one 3x5 inch sheet of paper for hand-written notes.

There are 3 Short Answer Questions and 1 Long Answer Question on this test. All questions must be answered.

Provide **all** work and solutions **on this test**.

Orderly presentation of work is required for solutions to receive full credit. **Illegible work, or answers that do not include supporting calculations and explanations will NOT BE MARKED**.

FILL OUT THE SECTION BELOW. Do this during the examination time as additional time will not be allowed for this purpose.

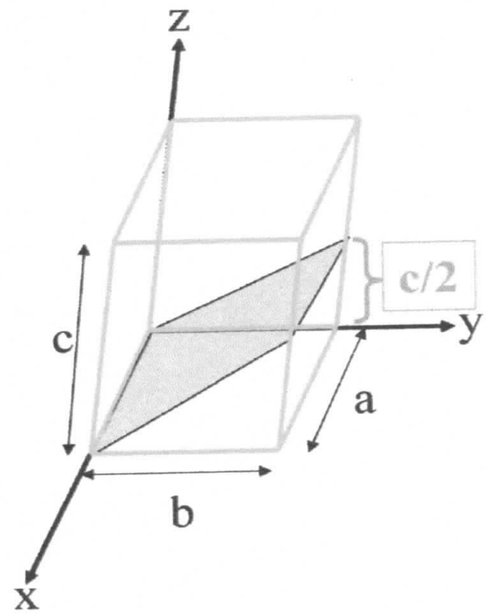
NAME:______________________________Section__________

SIGNATURE:______________________________

STUDENT NUMBER:______________________________

Question	Mark Received	Maximum Mark
SA 1		7
SA 2		6
SA 3		6
LA 1		21

SA 1. a) Determine the Miller indices for the crystal plane shown in the diagram. Show all steps in the determination.



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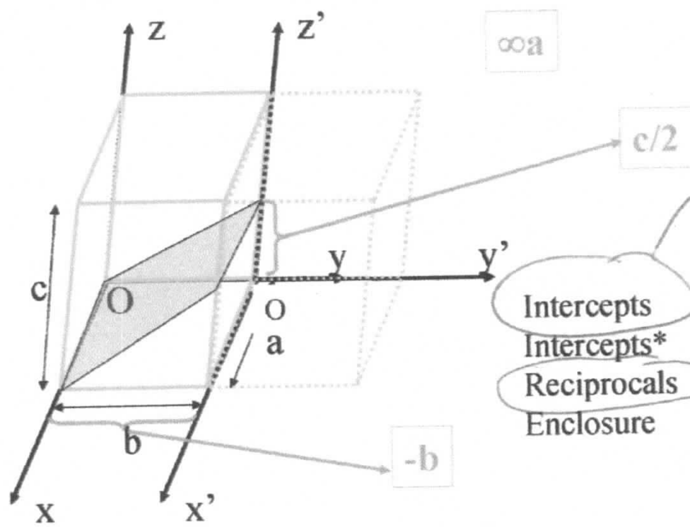
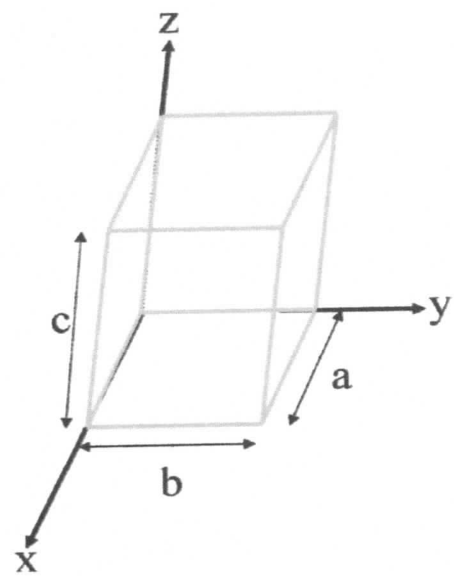
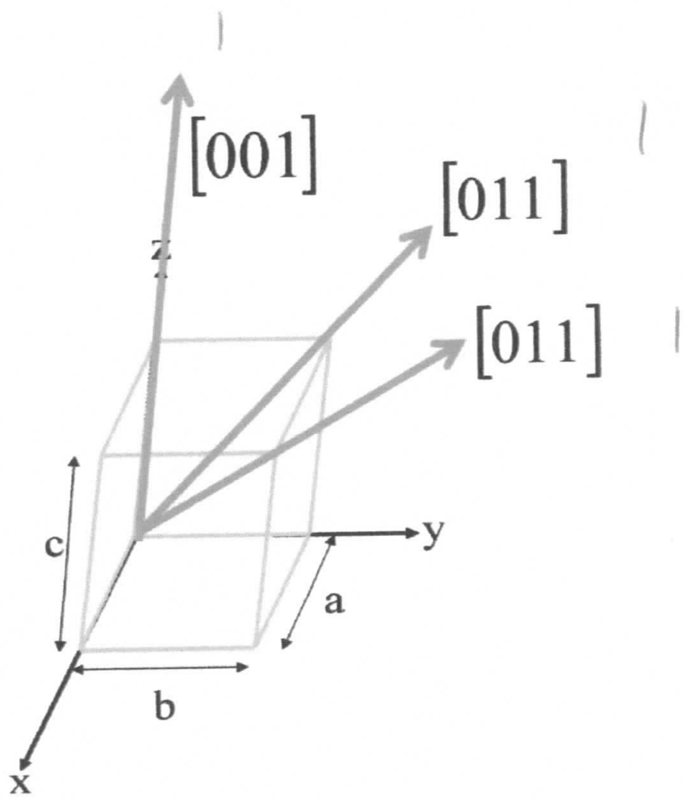


	x	y	z
Intercepts	∞a	-b	c/2
Intercepts*	∞	-1	½
Reciprocals	0	-1	2
Enclosure		$(0\bar{1}2)$	

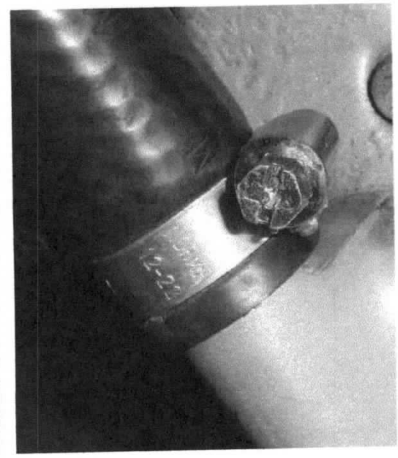
* In terms of lattice parameters a,b,c

b) – Draw the [001], [011] and [111] crystal directions in the crystal reference frame shown in the diagram.





SA2. A hose clamp is used to secure a rubber hose on to a car engine. The clamp is 10mm wide, 0.7mm thick. When tightened it can provide a maximum circumferential tensile force of 1kN. The hose outer diameter is 40mm. Calculate the maximum pressure that the clamp can exert on the surface of the hose.

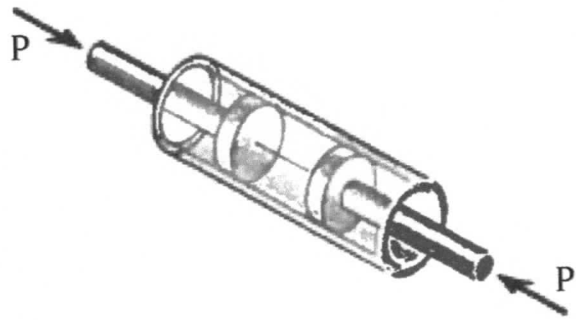


The hose clamp behaves like a thin-walled cylinder.

Circumferential tension $T = \sigma_\theta . A = \frac{pR}{t} . \underset{\substack{\uparrow \\ \text{width}}}{w} t = pRw$

$\rightarrow p = \frac{T}{Rw} = \frac{1000}{0.020 \times 0.010} = \underline{5\ MPa}$

SA 3. A double piston system for pressurizing oil is fitted within a brass tube of inside diameter 50mm and wall thickness 1mm. The distance between the pistons is 80mm. A force P = 20 kN is applied to the pistons. Compute the resulting change in length of the part of the tube that is between the two pistons.



Internal pressure $p = \frac{P}{A} = \frac{P}{\pi D^2/4} = \frac{4P}{\pi D^2}$

Stresses : $\sigma_\theta = \frac{pR}{t} = \frac{4P}{\pi D^2} \cdot \frac{D/2}{t} = \frac{2P}{\pi D t}$ $\quad \sigma_a = 0 \quad \sigma_r = 0$

↑because ends free

Hooke's Law :

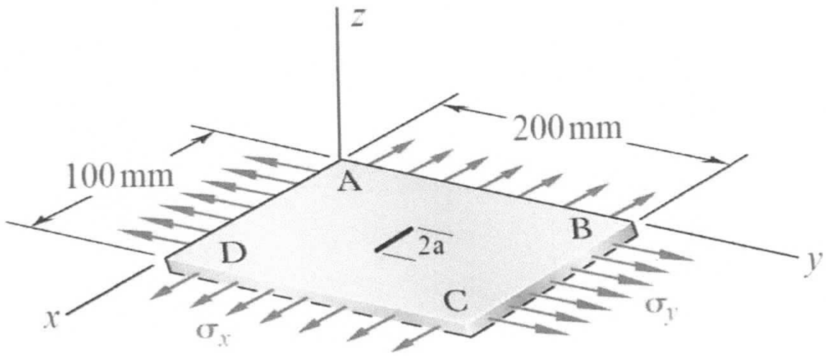
$$\varepsilon_a = \frac{\overset{=0}{\cancel{\sigma_a}}}{E} - \nu \frac{\sigma_\theta}{E} - \nu \frac{\overset{=0}{\cancel{\sigma_r}}}{E} = -\nu \frac{2P}{\pi D E t}$$

Length change $\Delta L = L \varepsilon_a = \frac{-2\nu P L}{\pi D E t}$

For brass: $E = 105\ GPa$ $\quad \nu = 0.35$

$$\Delta L = \frac{-2 \times 0.35 \times 20000 \times 0.080}{\pi \times 0.050 \times 105 \times 10^9 \times 0.001} = \underline{0.068\,mm}$$

LA1. A rectangular steel plate (made from grade 4340), 200mm x 100mm x 1mm is rigidly fixed along all four edges. The resulting structure is redundant, so stresses can exist within it without any external load. The in-plane stresses are measured when the ambient temperature is 30 °C and are found to be σ_x = 50MPa and σ_y = 200MPa. The plate is made from grade 4340 steel, for which E = 210 GPa, ν = 0.3, α = 11 x 10^{-6}/°C, K_{IC} = 25 MPa $\sqrt{m}$, $\sigma_{yield\ stress}$ = 1500 MPa



There is crack of length 2a = 8mm in the centre of the plate. You are asked to determine the temperature at which the crack will start to grow.

(a) Explain in non-mathematical terms why the plate structure is redundant.

(b) Describe in words what happens when the temperature of the plate changes.

(c) Describe the steps that you plan to take to solve this question.

(d) Determine the stress σ_y needed to initiate crack growth.

(e) Determine the corresponding stress σ_x.

(f) Determine the temperature required to reach stress state (d).

(a) The plate is redundant because it has greater than the minimum number of supports. Consequently, it has the possibility to have internal forces without external loads. This feature can be seen by imagining the case where the plate is slightly too short to fit between the fixed supports. Internal tension is then needed to make the plate fit.

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(b) If the plate temperature increases it will seek to expand thermally. However, the constraint on the plate from the fixed support will prevent this. A compressive stress will be created so that the sum of the thermal expansion and mechanical contraction equals zero. For a temperature decrease, a tensile stress will be created.

(c)
- use fracture mechanics theory to determine critical stress σ_y
- use principle of superposition to consider the redundant and thermal stresses separately
- identify how much more thermal stress is needed
- Use Hooke's Law to identify relationship between ΔT and σ_x, σ_y
- find ΔT to produce the needed extra σ_y
- find corresponding σ_x.

(d) For a through-thickness crack in the centre of a plate, no edge effects because $a/W < 0.1$ $\rightarrow Y = 1$

$$\rightarrow K = \sigma\sqrt{\pi a}$$

Consider only stresses perpendicular to the crack:

Fracture occurs when $K = K_{IC}$

$$\sigma_y = \frac{K_{IC}}{\sqrt{\pi a}} = \frac{25\times10^6}{\sqrt{\pi\, 0.004}} = \underline{223 \text{ MPa}}$$

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(e) Principle of Superposition:

total stress = redundant stress + thermal stress

$\rightarrow$ thermal stress = total stress − redundant stress

$$\sigma_y(\text{thermal}) = 223 - 200 = 23 \text{ MPa}$$

Hooke's Law: $$\varepsilon_x = \frac{\sigma_x(\text{thermal})}{E} - \nu\frac{\sigma_y(\text{thermal})}{E} - \nu\frac{\overset{=0}{\cancel{\sigma_z}}}{E} + \alpha\Delta T = 0$$

$$\varepsilon_y = -\nu\frac{\sigma_x(\text{thermal})}{E} + \frac{\sigma_y(\text{thermal})}{E} - \nu\frac{\overset{=0}{\cancel{\sigma_z}}}{E} + \alpha\Delta T = 0$$

Subtracting: $(1+\nu)\,\sigma_x(\text{thermal}) - (1-\nu)\,\sigma_y(\text{thermal}) = 0$

$$\rightarrow \sigma_x(\text{thermal}) = \sigma_y(\text{thermal}) = 33 \text{ MPa}$$

$$\rightarrow \sigma_x(\text{thermal}) = \sigma_y(\text{thermal}) = \frac{-\alpha E\Delta T}{1-\nu}$$

$$\Delta T = \frac{-(1-\nu)\,\sigma_y(\text{thermal})}{\alpha E} = \frac{-(1-0.3)\times 23\times10^6}{11\times10^{-6}\times 210\times10^9} = -6{\cdot}7\,^\circ C$$

Crack growth temperature = initial temperature + ΔT

$$= 30 - 6{\cdot}7 = \underline{23{\cdot}3\,^\circ C}$$

(f) Principle of superposition:

total stress = redundant stress + thermal stress

$$\sigma_x = 50 + 23 = \underline{73 \text{ MPa}}$$

(g) In attempt to prolong the life of the plate, the crack was drilled out, i.e. replaced by a circular hole. If the cyclic stress was ±190 MPa, estimate the number of thermal cycles to failure given the fatigue data for 4340 steel below. Note: the solution to this problem does not rely on the calculations for part a) to f).

$$\sigma_m = \sigma_o\left[1 + 2\left(\frac{a}{\rho_t}\right)^{1/2}\right]$$

3

For a round hole a = ρ_t

Therefore, $\sigma_m = 3\ \sigma_o$ or in terms of this problem the stress amplitude would be

= 3 x 190 MPa = 570 MPa

Reading the data off the graph blow, the number of cycles to failure would be approximately 40,000.