

**SA 1. [5 marks]**

(a) [2.5 marks] Find the general solution of the equation

$$y' - y = e^t \sin(t)$$

The equation is linear:  $p(t) = -1 \Rightarrow r(t) = e^{-t}$

$$\Rightarrow y(t) = \frac{1}{e^{-t}} \left( \int e^{-t} \cdot e^t \sin(t) dt \right)$$

$$= e^t \cdot \int \sin(t) dt$$

$$= e^t (-\cos(t) + C)$$

$$\Rightarrow \boxed{y(t) = -e^t \cos(t) + C e^t, C \in \mathbb{R}}$$

(b) [2.5 marks] Find the unique solution of the equation

$$y' + y^2 = y^2 \cos(t), y(0) = 1$$

The equation is separable

$$y' = y^2 (\cos(t) - 1)$$

$$\int \frac{1}{y^2} dy = \int (\cos(t) - 1) dt$$

$$\frac{-1}{y} = \sin(t) - t + C$$

plug in  $y(0) = 1$

$$\frac{-1}{1} = \sin(0) - 0 + C$$

$$C = -1$$

$$\Rightarrow \boxed{y = \frac{-1}{\sin(t) - t - 1}}$$