THE UNIVERSITY OF BRITISH COLUMBIA
FACULTY OF APPLIED SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 221

TEST #4, October 27th, 2016

Suggested Time: 75 minutes **Allowed Time:** 110 minutes

Materials admitted: Pencil, eraser, straightedge, MECH 2 Approved Calculator (Sharp EL-510), one 3x5 inch index card or sheet of paper for hand-written notes.

There are 5 Short Answer Questions and 2 Long Answer Problems on this test. All questions must be answered.

Provide **all** work and solutions **on this test**. Additional marks of up to 5% of the test value is available for orderly presentation of work throughout the test. **Illegible work, or answers that do not include supporting calculations and explanations will NOT BE MARKED**.

FILL OUT THE SECTION BELOW AND WRITE YOUR NAME ON THE TOP OF ALL TEST PAGES. Do this during the examination time as additional time will not be allowed for this purpose.

NAME:____Section__________

SIGNATURE:__________________________________

STUDENT NUMBER:__________________________

Question	Mark Received	Maximum Mark
SA 1		5
SA 2		5
SA 3		5
SA 4		5
SA 5		5
Prob 1		25
Prob 2		25
Presentation		

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SA1 [5 marks]. Classify (but do not solve) each equation as undamped, overdamped, underdamped, or critically damped, and find the corresponding plot of the solution.

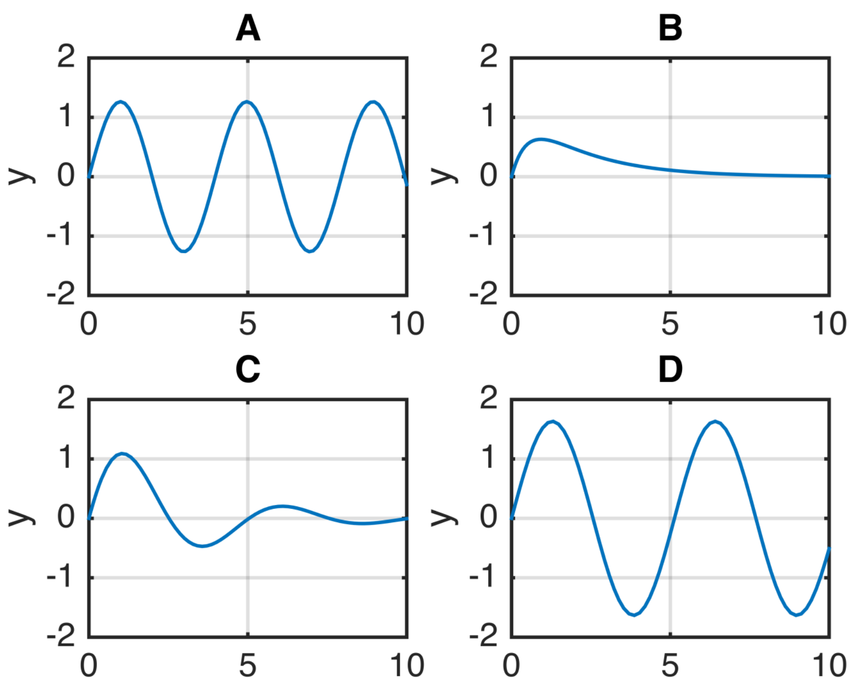
A system $my'' + cy' + ky = 0$ is undamped if $c = 0$, otherwise the classification only depends on the discriminant of the characteristic polynomial: $c^2 - 4mk$.

i. $3y'' + 2y' + 5y = 0,\ y(0) = 0,\ y'(0) = 2$
Classification: Underdamped: $2^2 - 4(3)(5) = -56 < 0$
Plot: C

ii. $2y'' + 5y = 0,\ y(0) = 0,\ y'(0) = 2$
Classification: Undamped: $c = 0$
Plot: A (higher natural frequency compared to iii)

iii. $2y'' + 3y = 0,\ y(0) = 0,\ y'(0) = 2$
Classification: Undamped: $c = 0$
Plot: D (lower natural frequency compared to ii)

iv. $2y'' + 5y' + 2y = 0,\ y(0) = 0,\ y'(0) = 2$
Classification: Overdamped: $5^2 - 4(2)(2) = 9 > 0$
Plot: B



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SA2 [5 marks]. Find the general solution of the equation

$$2y'' + 4y' + 5y = 1 - e^{-2t}.$$

The characteristic polynomial is

$$p(s) = 2s^2 + 4s + 5$$

and the roots are

$$r = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} = \frac{-4 \pm \sqrt{4^2 - 4(2)(5)}}{2(2)} = -1 \pm \frac{\sqrt{6}}{2} i$$

Therefore the complimentary solution is

$$y_c = C_1 e^{-t} \cos\left(\frac{\sqrt{6}}{2} t\right) + C_2 e^{-t} \sin\left(\frac{\sqrt{6}}{2} t\right), C_1, C_2 \in \mathbb{R}$$

A particular solution must be of the form $y_p = A + Be^{-2t}$. Compute $y'_p = -2Be^{-2t}$ and $y''_p = 4Be^{-2t}$ and plug into the equation

$$2(4Be^{-2t}) + 4(-2Be^{-2t}) + 5(A + Be^{-2t}) = 5A + 5Be^{-2t}$$

Therefore

$$y_p = \frac{1}{5} - \frac{1}{5} e^{-2t}$$

and the general solution is $y = y_c + y_p$.

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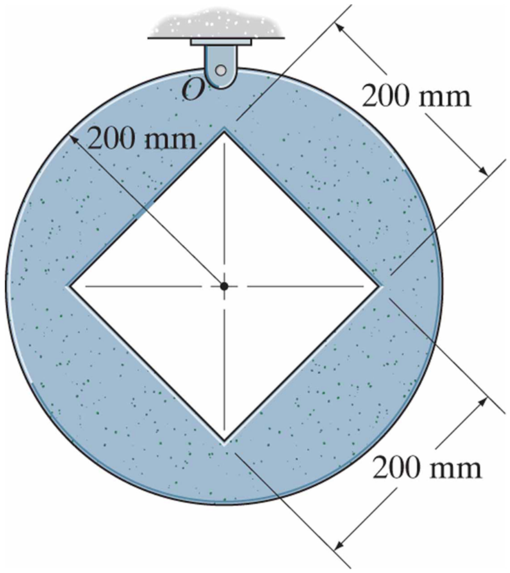
SA 3 [5 Marks]. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point *O*. The material has a mass per unit area of 20 kg/m².



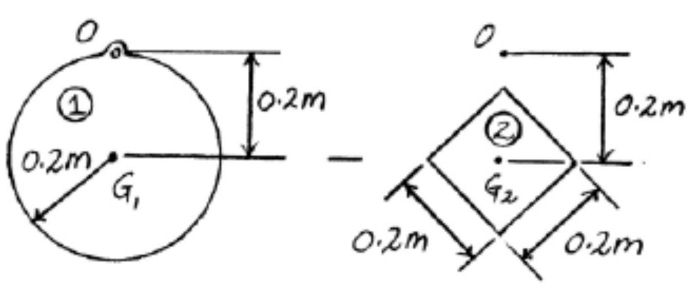
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point *O* are also indicated.

Mass Moment of Inertia: The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the plate about an axis perpendicular to the page and passing through point *O* for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113 \text{ kg} \cdot \text{m}^2$$ **Ans.**



(a)

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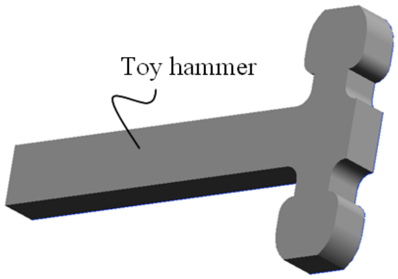
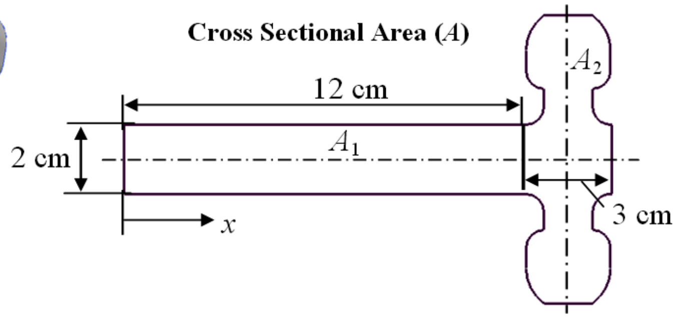
SA 4 [5 Marks]. A curious young woman wants to find the cross sectional area (A) of her toy hammer shown below. All she has are a pencil, a plain sheet of paper and a ruler. You are called to help her solve the problem by applying your MECH 221 Dynamics knowledge.





a) [2 marks]. Explain in **ONE** or **TWO** sentence (s) how you would find the location of the centre of mass ($\bar{x}$) of the hammer using **ONLY** the girl's pencil (and the hammer).

Hold the pencil parallel to the ground and Balance the hammer on the pencil with the pencil and hammer perpendicular to one another. The balance point is under the centre of mass of the hammer.

b) [3 marks]. You find that $\bar{x}$ is located at $x = 9$ cm. You also obtain the dimensions shown on the figure using the ruler. Assuming that the density and thickness of the hammer are **uniform** throughout the cross section, calculate the area $A = A_1 + A_2$.

Remember: $\bar{x} = \dfrac{\sum_i \bar{x}_i m_i}{m}$

Since density is uniform we can assume Area=mass. i.e assume density = 1 Kg/m²

$$\bar{x} = \frac{\sum_i \bar{x}_i m_i}{m} \rightarrow 9=[(6)(12 \times 2)+13.5(A_2)] \,/\, (12\times2 + A_2)=$$

$$9=[144+13.5 A_2]/(24+ A_2)$$

$$216+9 A_2=144+13.5 A_2$$

$$A_2=16 \text{ cm}^2$$

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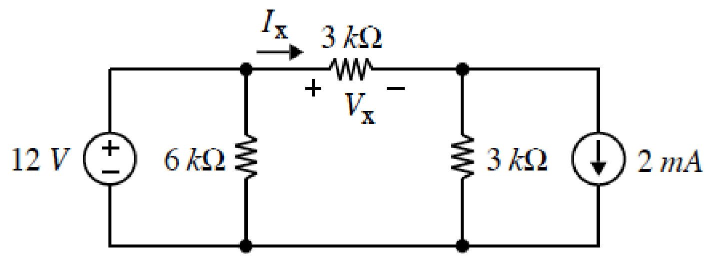
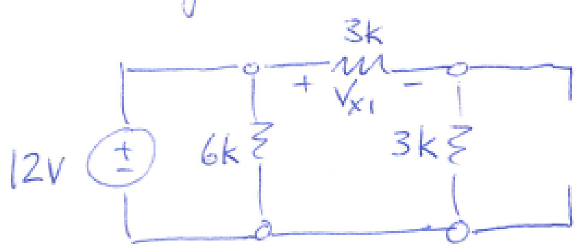
SA 5 [5 Marks]. Consider the following circuit.

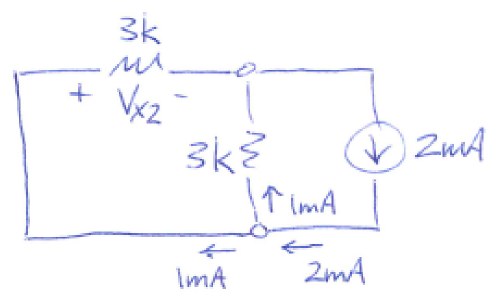


a) [2 marks]. Sketch the two independent equivalent circuits that could be used to solve for V_x by *superposition*.

Voltage source alone:



Current source alone:



b) [3 marks]. Use superposition to determine V_x and use the result to compute I_x.

$$V_{x1} = \frac{(12V)(3k)}{(3k)+(6k)} = 6V$$

$$V_{x2} = (1mA)(3k) = 3V$$

i.e. 2mA divides equally between 2 3k resistors

$$V_x = V_{x1} + V_{x2} = 9V$$

$$I_x = \frac{V_x}{3k} = \frac{9V}{3k} = 3mA$$

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Prob 1 [25 Marks]. Consider the two circuits shown below.

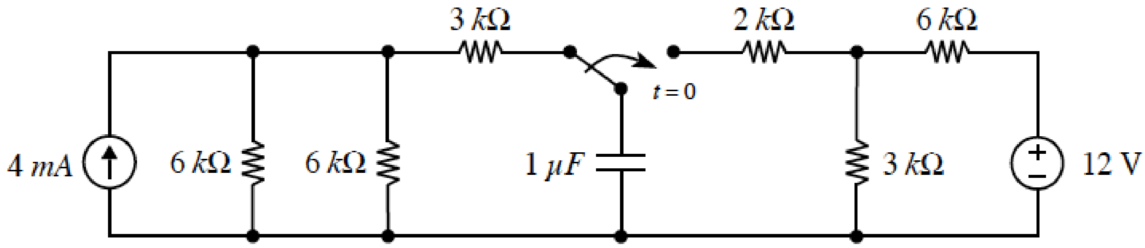


Figure 1

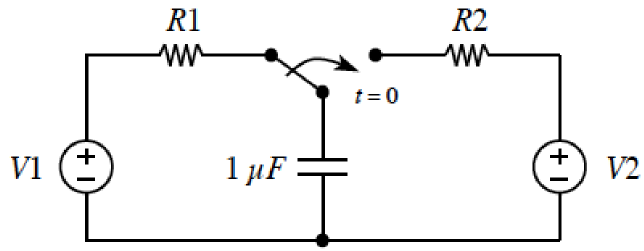
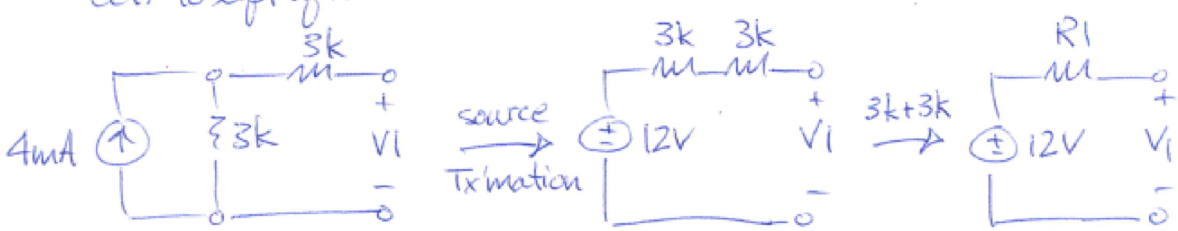


Figure 2

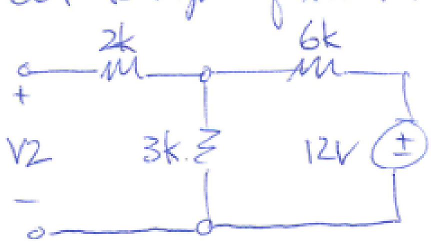
a) [6 marks]. Apply *Thévenin's* and/or *Norton's* theorems to determine values for *V*1, *R*1, *R*2 and *V*2 in the circuit in Figure 2 such that it is equivalent to the circuit in Figure 1, as seen by the capacitor.

Cct. to left of switch & load C:



$V1 = 12V$ $\qquad$ $R1 = 6k$

Cct. to right of switch & load C:



$$V2 = \frac{(12V)(3k)}{(3k)+(6k)} = 4V = \text{open-cct. voltage}$$

$$R2 = (2k) + (3k \| 6k) = 4k$$

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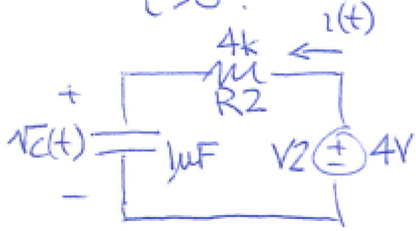
Prob 1, *Cont'd.*

b) [3 marks]. Assume the circuit has reached a steady state prior to $t = 0$. Derive a differential equation based on the circuit of Fig. 2 that describes the capacitor voltage, for $t > 0$. (*i.e.*, after the switch is changed).

$t>0$:



KVL: $(R2)i(t) + v_C(t) - V2 = 0$

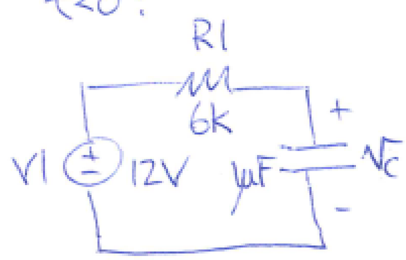
$$i(t) = C\frac{dv_C(t)}{dt}$$

$$\Rightarrow (R2)(C)\frac{dv_C(t)}{dt} + v_C(t) = V2$$

D.E. $$\frac{dv_C(t)}{dt} + \underbrace{\frac{1}{(R2)(C)}}_{\frac{1}{(4k)(1\mu F)}} v_C(t) = \underbrace{\frac{V2}{(R2)(C)}}_{\frac{4V}{(4k)(1\mu F)}}$$

c) [5 marks]. Assume a solution of the form $v_C(t) = v_C(\infty) + \left[v_C(0+) - v_C(\infty)\right]e^{-t/\tau}$. Determine the parameters and write the final solution.

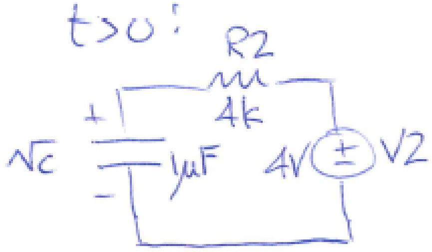
$t<0$:



in steady state, $v_C = V1 = 12V = v_C(0-)$

$\Rightarrow v_C(0+) = 12V$

$t>0$:



in steady state, $v_C = V2 = 4V$

$\Rightarrow v_C(\infty) = 4V$

$\tau = (4k)(1\mu F) = 4ms$

$$v_C(t) = 4V + [\underbrace{12V - 4V}_{8V}]\, e^{-t/4ms}$$

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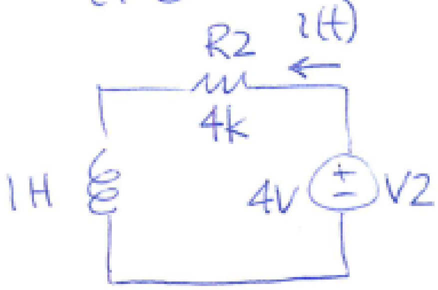
Prob 1, *Cont'd.*

d) [3 marks]. Suppose the capacitor is replaced by a single inductor of 1 *H*. Derive a differential equation based on the circuit of Fig. 2 that describes the inductor current, for $t > 0$. (*i.e.*, after the switch is changed).

$t>0$:



KVL: $(R2)\,i(t) + V_L(t) - V2 = 0$

$$V_L(t) = L\frac{di}{dt}$$

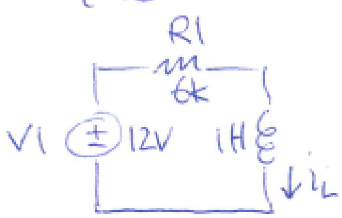
$$\Rightarrow (R2)\,i(t) + L\frac{di(t)}{dt} = V2$$

D.E. $$\frac{di(t)}{dt} + \frac{R2}{L}\,i(t) = \frac{V2}{L}$$

$$\frac{4k}{1H} \qquad \frac{4V}{1H}$$

e) [5 marks]. Assume a solution of the form $i_L(t) = i_L(\infty) + \left[i_L(0+) - i_L(\infty)\right]e^{-t/\tau}$. Determine the parameters and write the final solution.

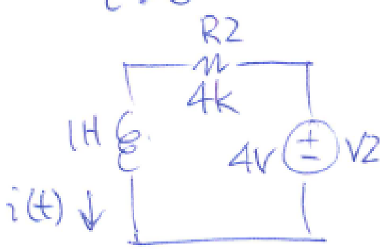
$t<0$:



in steady state, $i_L = \frac{V1}{R1} = \frac{12V}{6k} = 2mA$

$$\Rightarrow i(0-) = i(0+) = 2mA$$

$t>0$:



in steadystate, $i_L = \frac{V2}{R2} = \frac{4V}{4k} = 1mA$

$$\Rightarrow i(\infty) = 1mA$$

$$\tau = \frac{L}{R} = \frac{1H}{4k} = 0.25\,ms$$

$$i(t) = 1mA + \left[\underbrace{2mA - 1mA}_{1mA}\right]e^{-t/0.25ms}$$

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Prob 1, *Cont'd.*

f) [3 marks]. What is the initial value of the voltage across the inductor at time $t = 0+$ (*i.e.*, the instant after the switch is changed)?

$$v_L(t) = L\frac{di(t)}{dt}$$

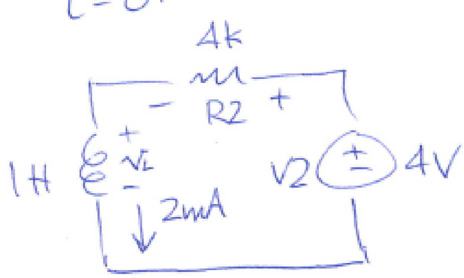
where $i(t) = 1mA + 1mA\, e^{-t/0.25ms}$

$$v_L(0+) = L\frac{di(t)}{dt}\bigg|_{t=0+} = (1H)\left(\frac{-1mA}{0.25ms}\right)e^{-t/0.25ms}\bigg|_{t=0^+}$$

$$= -4V$$

Alternatively:

$t = 0^+$



KVL: $v_L(0+) = V2 - (2mA)(4k)$

$= 4V - 8V$

$= -4V$

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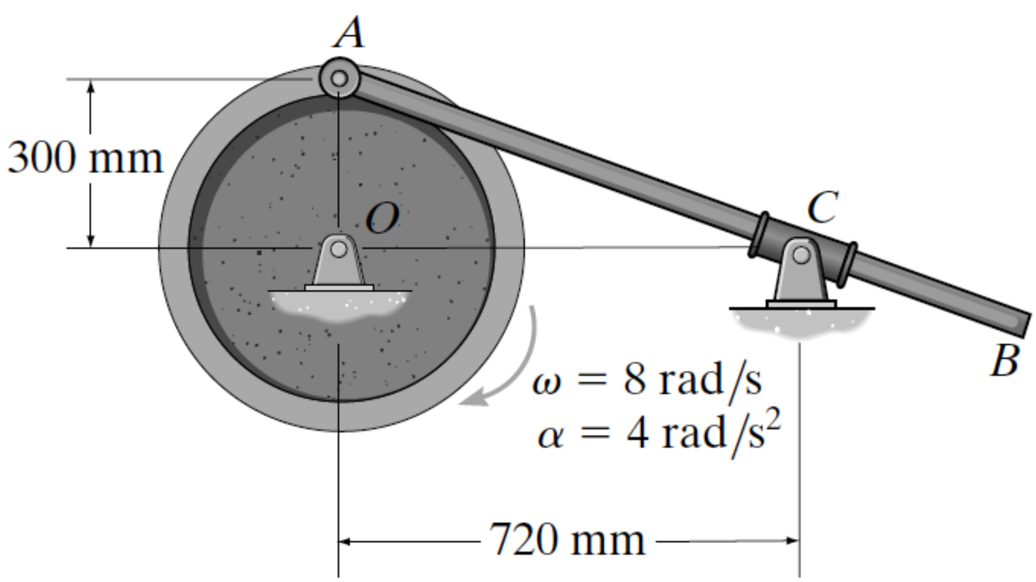
Prob 2 [25 marks]. The wheel below is rotating about point *O* with the angular velocity and angular acceleration shown. The rod slides freely through the smooth collar.

a) [5 marks]. First, determine the absolute velocity ($\mathbf{V}_A$) and acceleration ($\mathbf{a}_A$) of point A.

b) [5 marks]. Determine the angular velocity of the rod ($\boldsymbol{\omega}_{AB}$).

c) [15 marks]. Determine the angular acceleration of the rod ($\boldsymbol{\alpha}_{AB}$)



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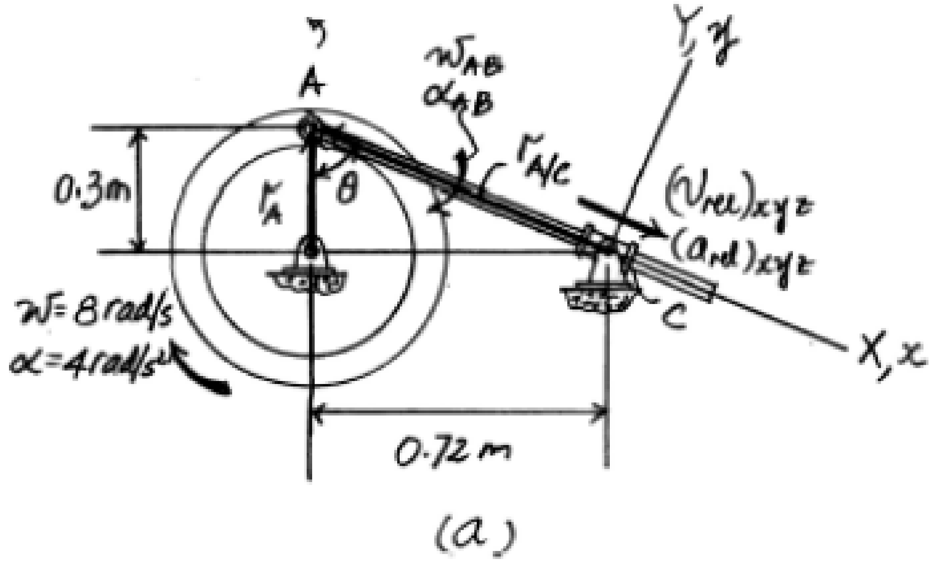

$\mathbf{v}_C = \mathbf{a}_C = 0$ $\qquad\qquad$ $\boldsymbol{\omega}_{AB} = -\omega_{AB}\mathbf{k}$ $\qquad\qquad$ $\dot{\boldsymbol{\omega}}_{AB} = -\alpha_{AB}\,\mathbf{k}$

From the geometry shown in Fig.*a*,

$$r_{A/C} = \sqrt{0.3^2 + 0.72^2} = 0.78 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{0.72}{0.3}\right) = 67.38°$$

For the motion of point *A* with respect to the *xyz* frame,

$\mathbf{r}_{A/C} = [-0.78\mathbf{i}]$ m $\qquad\qquad$ $(\mathbf{v}_{rel})_{xyz} = (v_{rel})_{xyz}\,\mathbf{i}$ $\qquad\qquad$ $(\mathbf{a}_{rel})_{xyz} = (a_{rel})_{xyz}\,\mathbf{i}$

Since the wheel *A* rotates about a fixed axis, $\mathbf{v}_A$ and $\mathbf{a}_A$ with respect to the *XYZ* reference frame can be determined from

$$\begin{aligned}\mathbf{v}_A &= \boldsymbol{\omega} \times \mathbf{r}_A \\ &= (-8\mathbf{k}) \times (-0.3\cos 67.38°\mathbf{i} + 0.3\sin 67.38°\mathbf{j}) \\ &= [2.215\mathbf{i} + 0.9231\mathbf{j}] \text{ m/s}\end{aligned}$$

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A - \omega^2\mathbf{r}_A$$

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$$= (-4\mathbf{k}) \times (-0.3\cos 67.38°\mathbf{i} + 0.3\sin 67.38°\,\mathbf{j}) - 8^2(-0.3\cos 67.38°\mathbf{i} + 0.3\sin 67.38°\mathbf{j})$$

$$= [8.492\mathbf{i} - 17.262\mathbf{j}]\ \text{m/s}^2$$

Velocity: Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$2.215\mathbf{i} + 0.9231\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (-0.78\mathbf{i}) + (v_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$2.215\mathbf{i} + 0.9231\mathbf{j} = (v_{\text{rel}})_{xyz}\,\mathbf{i} + 0.78\omega_{AB}\,\mathbf{j}$$

Equating the i and j components yields

$$(v_{\text{rel}})_{xyz} = 2.215\ \text{m/s}$$

$$0.78\omega_{AB} = 0.9231 \qquad \omega_{AB} = 1.183\ \text{rad/s} = 1.18\ \text{rad/s}$$ **Ans.**

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\boldsymbol{\omega}}_{AB} \times \mathbf{r}_{A/C} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C}) + 2\boldsymbol{\omega}_{AB} \times (\mathbf{v}_{\text{rel}})_{xyx} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$8.492\mathbf{i} - 17.262\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (-0.78\mathbf{i}) + (-1.183\mathbf{k}) \times [(-1.183\mathbf{k}) \times (-0.78\mathbf{i})] + 2(-1.183\mathbf{k}) \times (2.215\mathbf{i}) + (a_{\text{rel}})_{xyz}\mathbf{i}$$

$$8.492\mathbf{i} - 17.262\mathbf{j} = \left[(a_{\text{rel}})_{xyz} + 1.092\right]\mathbf{i} + \left(0.78\alpha_{AB} - 5.244\right)\mathbf{j}$$

Equating the j components yields

$$-17.262 = 0.78\alpha_{AB} - 5.244$$

$$\alpha_{AB} = -15.41\ \text{rad/s}^2 = 15.4\ \text{rad/s}^2 \circlearrowright$$ **Ans.**