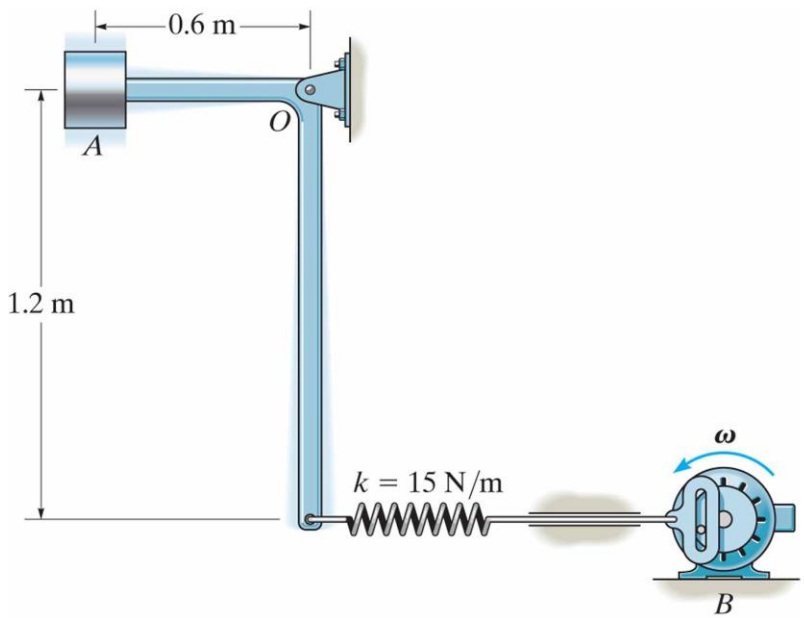
SA 1. [5 marks] At the instant shown in the photograph below the motorcycle is stationary.

a) [2 marks] Draw the free body diagram of the motorcycle and rider. What physical measurements would you have to take to define the quantities in the free body diagram?

b) [3 mark] Write the equations of motion of the motorcycle.

Prob 1 [25 Marks] The small block at A has width 0.2m and height 0.3m, and unknown mass m. The block is mounted on the slender bent rod with mass 5 kg/m. Before switching on the motor at B, the system is at equilibrium as shown in the figure below. The length of the shorter end of the rod is 0.5m.



A) [2 marks] Draw the free body diagram of the system before the motor at B is switched on.

B) [8 marks] Use the small angle approximation to **prove** that the equation of motion of the system (before the motor at B is switched on) is approximately

$$(0.37083m + 3.0883)\ddot{\theta} + 56.916\theta = 0$$

where m is the unknown mass of the block, and θ is the angular position (positive in the counter-clockwise direction) relative to position shown in the figure. **Do not solve this equation.**

C) [2 marks] Find the value of the mass m that makes the natural frequency of the system equal to 2 radians per second.

D) [5 marks] The mass of the block is now known to be 5kg, the motor at B is switched on and the motion at B is given by $\delta_B = 0.2\cos(16t)$. The position δ_B is measured in meters (the positive direction is to the right) and t is measured in seconds. Draw the free body diagram and determine the equation of motion.

E) [8 marks] Find the general solution of the equation in part D.

THE UNIVERSITY OF BRITISH COLUMBIA
FACULTY OF APPLIED SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 221

TEST #7, November 17th, 2016

Suggested Time: 100 minutes **Allowed Time:** 110 minutes

Materials admitted: Pencil, eraser, straightedge, MECH 2 Approved Calculator (Sharp EL-510), one 3x5 inch index card or sheet of paper for hand-written notes.

There are 4 Short-Answer Questions and 2 Long-Answer Problems on this test. All questions must be answered.

Provide **all** work and solutions **on this test**. Orderly presentation of work is required for solutions to receive full credit. **Illegible work, or answers that do not include supporting calculations and explanations will NOT BE MARKED**.

FILL OUT THE SECTION BELOW AND WRITE YOUR NAME ON THE TOP OF ALL TEST PAGES. Do this during the examination time as additional time will not be allowed for this purpose.

NAME:______________________________________Section__________

SIGNATURE:________________________________

STUDENT NUMBER:__________________________

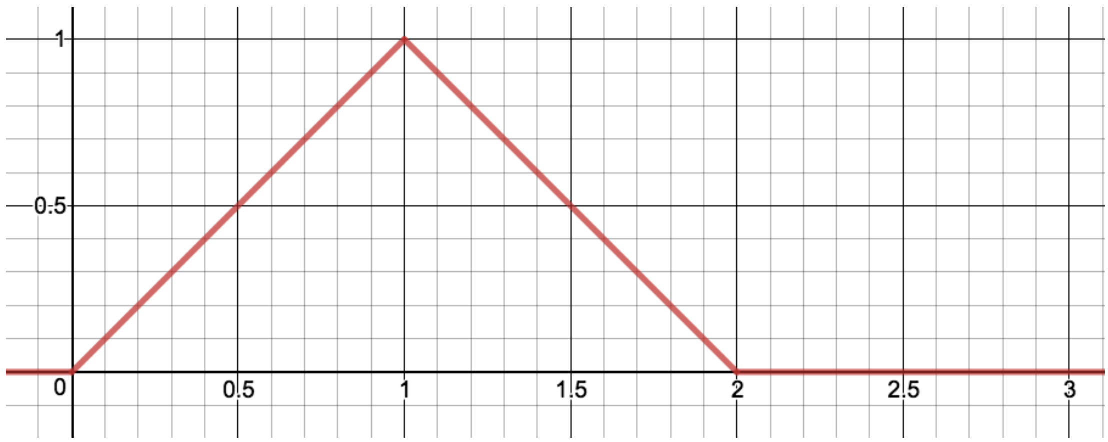
Question	Mark Received	Maximum Mark
SA 1		5
SA 2		5
SA 3		5
SA 4		5
Prob 1		25
Prob 2		25

Name:______________________________ Section:__________________________

SA1 [5 marks]. Find the Laplace transform $Y(s)$ of the solution $y(t)$ of the differential equation

$$2y'' + 3y' - y = f(t), \;\; y(0) = 0, \; y'(0) = 0$$

where the forcing function $f(t)$ is given by



(Note: Do not solve for $y(t)$.)

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SA2 [5 marks]. Suppose an output signal $y(t)$ is related to an input signal $x(t)$ by the differential equation

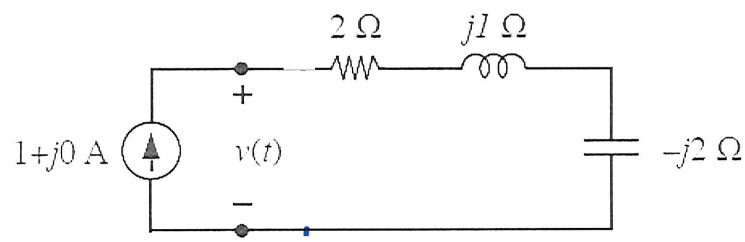
$$y'' + 3y' + y = 3x' - x\,.$$

(a) [1 mark]. Find the transfer function $H(s) = \dfrac{Y(s)}{X(s)}$.

(b) [4 marks]. Suppose $x(t) = \sin(\omega t)$. Find values $\omega > 0$ such that the amplitude of the steady state response is less than 1. In other words, find values $\omega > 0$ such that

$$\lim_{t\to\infty} \left| y(t) \right| = \left| y_{ss}(t) \right| < 1$$

SA 4 [5 Marks]. Consider the following circuit that is operating in the sinusoidal steady-state at angular frequency $\omega = 500$ rad/s.



(a) [3 marks]. Determine the time-domain voltage $v(t)$.

(b) [2 marks]. Determine the values of the *inductance* (in Henries) and *capacitance* (in Farads).

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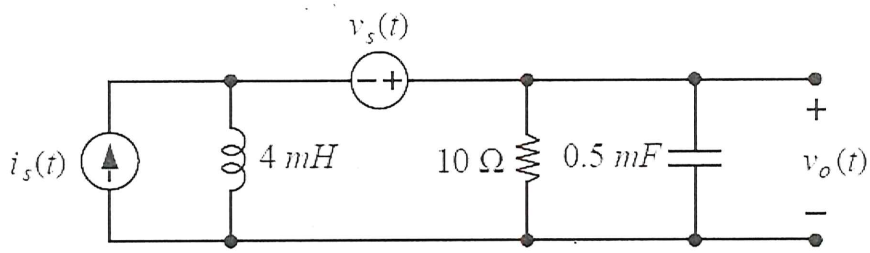
Prob 1 [25 Marks]. Consider the circuit shown below that is operating in the steady-state with $i_s(t) = 4\cos(1000t)\ A$ and $v_s(t) = 12\cos(1000t + 90°)\ V$.



(a) [3 marks]. Determine the phasor impedances for the inductor and capacitor, and write the time-domain source functions $i_s(t)$ and $v_s(t)$ as phasors $\mathbf{I}_S$ and $\mathbf{V}_S$.

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Prob 1, *Cont'd.*

(b) [7 marks]. Use *nodal analysis* to determine $\mathbf{V}_O$, the phasor representation of $v_O(t)$. (*Suggestion: it may prove useful to redraw the circuit here with all of the components labeled in phasor notation.*)

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Name:______________________________ Section:__________________________

Prob 1, *Cont'd.*

(c) [12 marks]. Use *superposition analysis* to determine the phasor voltage $\mathbf{V}_O$, and write the corresponding time-domain function $v_o(t)$.

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Prob 1, *Cont'd.*

(d) [3 marks]. Suppose the source functions $i_s(t)$ and $v_s(t)$ were instead operating at <u>different</u> frequencies, say ω_1 and ω_2. Briefly, in just a few sentences, outline how you would approach solving for $v_0(t)$.