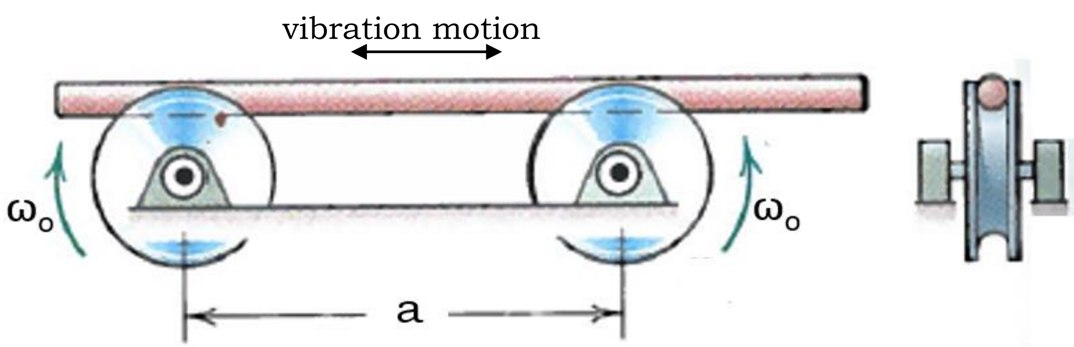
Name:______________________________ Section:___________________________

SA 1. [5 marks] In the system below the two fixed counter rotating pulleys rotate with the same speed, ω_o. If the slender bar that is resting on the pulleys is centered on the pulleys it will be stationary as the pulley's rotate beneath it. However, if the slender bar is shifted to the right (as shown below) or left, and then released, then the bar will vibrate back and forth in the horizontal plane.

a) [4 marks] Draw the free body diagram of the displaced slender rod.

b) [1 mark] Explain in one or two sentences why the slender rod vibrates.



SA2 [5 Marks]. The transient current (following a switching event) in a series *RLC* circuit is described by the following differential equation (DE):

$$\frac{d^2 i(t)}{dt^2} + 10\frac{di(t)}{dt} + 25i(t) = 0 .$$

(a) [2 Marks]. Determine the damping factor, ζ, and the undamped natural frequency, ω_o.

(b) [1 Mark]. Determine the general form of the response, $i(t)$, for $t > 0$. Do not try to solve for the coefficients.

(c) [2 Marks]. Suppose the resistance in the circuit is decreased, but the capacitance and inductance remain unchanged, yielding a slightly different DE. How would such a change affect the general form of the response $i(t)$. Briefly state your reasoning.

Name:________________________________ Section:______________________________

Prob 1 [25 marks] ***Parts (a), (b), (c) & (d) are separate questions.***

(a) [8 marks] Use the Laplace transform to solve the differential equation

$$y'' + 3y = 1 + \sin(2t)\ , \qquad y(0) = 1, \qquad y'(0) = 1$$

The following partial fraction formulas will be helpful:

$$\frac{1}{s(s^2+a)} = \frac{1}{a}\left(\frac{1}{s} - \frac{s}{s^2+a}\right) \qquad \frac{1}{(s^2+a)(s^2+b)} = \frac{1}{b-a}\left(\frac{1}{s^2+a} - \frac{1}{s^2+b}\right)$$

(b) [7 marks] Use the **definition** of the Laplace transform to **prove**

$$\mathcal{L}\{te^{2t}\} = \frac{1}{(s-2)^2}\ , \quad s > 2$$

(c) [5 marks] Find the inverse Laplace transform of

$$Y(s) = \frac{s-1}{s^2+2s+5}$$

Name:______________________________ Section:______________________________

(d) [5 marks] Suppose the Laplace transform of $y(t)$ is given by

$$Y(s) = \frac{1}{s^2 + cs + 1}$$

Find the values for the constant c such that $y(t)$ oscillates and

$$\lim_{t \to \infty} y(t) = 0$$