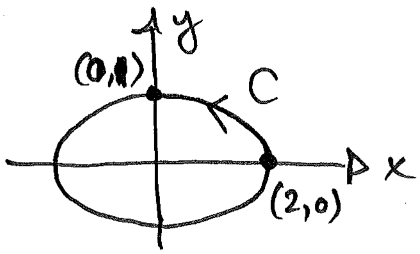
Name:______________________________ Section:__________________________

Question 1 [5 marks]

Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ of the vector field

$$\mathbf{F}(x, y) = (-2y^2, x^2)$$

over the curve C from (2,0) to (0,1) along the ellipse $x^2 + 4y^2 = 4$.



$C: \vec{r}(t) = (2\cos(t), \sin(t)) \quad 0 \leq t \leq \pi/2.$

$\vec{r}\,'(t) = (-2\sin(t), \cos(t))$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi/2} \left(-2(\sin(t))^2, (2\cos(t))^2\right) \cdot (-2\sin(t), \cos(t))\, dt$$

$$= \int_0^{\pi/2} \left(4\sin^3(t) + 4\cos^3(t)\right) dt$$

$$= 4\int_0^{\pi/2} \sin^3(t)\, dt + 4\int_0^{\pi/2} \cos^3(t)\, dt$$

$$= 4\left(\tfrac{2}{3}\right) + 4\left(\tfrac{2}{3}\right) = \boxed{\frac{16}{3}}$$

Note: $\vec{F}(x,y) = (-2y^2, x^2)$ is not conservative

since $\frac{\partial}{\partial y}(-2y^2) \neq \frac{\partial}{\partial x}(x^2)$.

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Question 2 [5 marks]

Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ of the vector field

$$\mathbf{F}(x, y) = (y^3 + 2x, 3xy^2)$$

over the curve C defined by $xy^2 + x^3 + y^2 = 1$ from (1,0) to (0,1).

Let $P(x,y) = y^3 + 2x$ and $Q(x,y) = 3xy^2$

so that $\vec{F}(x,y) = P(x,y)\underline{i} + Q(x,y)\underline{j}$

Then $\frac{\partial P}{\partial y} = 3y^2$ and $\frac{\partial Q}{\partial x} = 3y^2$

(and are continuous everywhere) therefore

$\vec{F}$ is <u>conservative</u>.

$\int P\,dx = xy^3 + x^2 + g(y)$

and $\frac{\partial}{\partial y}(xy^3 + x^2 + g(y)) = 3xy^2 + g'(y) = \underbrace{3xy^2}_{Q}$

$\Rightarrow$ Let $g(y) = 0$ and so $f(x,y) = xy^3 + x^2$

is a potential function for $\vec{F}$.

$\Rightarrow$ By the Fundamental Theorem for Line Integrals

$$\int_C \vec{F} \cdot d\vec{r} = f(0,1) - f(1,0)$$

$$= 0 - 1 \boxed{= -1}$$