Solutions, 222 Weekly Exam 5 Fluids, March 20, 2014

1. (10 marks) On last week's test, you were asked to non-dimensionalize the droplet size D from a hand-pumped kitchen oil sprayer, which depends on the gauge pressure P in the spray bottle, the size of the nozzle (let's call its diameter δ to avoid confusion), and the density ρ, viscosity μ, and surface tension σ of the oil. One possible result of doing this non-dimensionalization is

$$\frac{D}{\delta} = fn_2\left(\frac{\rho\delta^2 P}{\mu^2}, \frac{\sigma}{P\delta}\right)$$

You want to test the sprayer with water instead of olive oil, because water is cheaper and easier to clean off the equipment you will use to measure droplet size.

(a) (5 marks) What conditions must be true for you to be able to scale your experimental results to the actual sprayer? Check all that apply.

- ✓ The experimental sprayer must be a precise scale model of the actual sprayer.
- ✗ If your experimental sprayer is smaller than the actual sprayer, the pressure in the experimental sprayer must be proportionately lower.
- ✗ If your experimental sprayer is smaller than the actual sprayer, the pressure in the experimental sprayer must be proportionately higher.
- ✗ The fluid properties of the test liquid must be the same as the fluid properties of olive oil.
- ✓ The dimensionless surface tension $\frac{\sigma}{P\delta}$ must be the same for the experimental and actual sprayers.

You do some preliminary experiments with an off-the-shelf oil sprayer, and determine that the droplet size is independent of viscosity. You are overjoyed, because now you can simplify your non-dimensional relationship to

$$\frac{D}{\delta} = fn_3\left(\frac{\sigma}{P\delta}\right)$$

(b) (2 marks) You would like larger droplets in the experiment than in the actual oil sprayer, because this will make the droplet size easier to measure. Should your experimental model be larger or smaller than the actual sprayer, *and why?*

Your model should be larger. Once you've matched the non-dimensional surface tension term (input to the function) — which you must do for your results to scale — your droplet size will be proportional to the model size. So a larger model implies larger drops.

(c) (3 marks) Without thinking about the scaling issue from part b, you build a larger experimental model to test because it's easier to build the precise nozzle shape that you want. Your model is 5 times larger than the actual oil sprayer will be. You expect the operating pressure in the actual oil sprayer to be 30 kPa gauge. What pressure should you use in your experimental model?

We're looking at matching an independent variable, in this case the surface tension term:

$$\begin{aligned}
\frac{\sigma_f}{P_f \delta_f} &= \frac{\sigma_m}{P_m \delta_m} \\
P_m &= P_f \frac{\sigma_m}{\sigma_f} \frac{\delta_f}{\delta_m} \\
&= 30\,\mathrm{kPa} \frac{0.072\,{}^{\mathrm{N}}/_{\mathrm{m}}}{0.032\,{}^{\mathrm{N}}/_{\mathrm{m}}} \frac{1}{5} \\
&= 13.5\,\mathrm{kPa}
\end{aligned}$$