MECH 222 THERMODYNAMICS FORMULAE

KEY CONSTANTS AND CONVERSION FACTORS

$$R_{\text{air}} = 287\,\text{J/kg/K} \qquad 1\,\text{atm} = 101.325\,\text{kPa}$$
$$[\text{K}] = [^\circ\text{C}] + 273.15 \qquad 1\,\text{bar} = 100\,\text{kPa}$$

GIBBS' PHASE RULE

$$\overbrace{\underbrace{S}_{\substack{\text{DoF}\\\text{in}\\\text{state}}} + \underbrace{\underbrace{X}_{\substack{\text{DoF}\\\text{in}\\\text{extent}}}}_{=\underbrace{P}_{\text{Phases}}}}^{\overbrace{N}^{\substack{\text{Total}\\\text{DoF}}}} = \underbrace{C}_{\substack{\text{Indep.}\\\text{chem}\\\text{comp.}}} + 2$$

TRANSPORT EQUATIONS

Energy Balance

$$\overbrace{\mathrm{d}(U + KE + PE)}^{\text{Local Change}} = \sum_i \overbrace{\delta Q_i}^{\text{Heat}} - \overbrace{\sum_{j \neq \cancel{P},\cancel{\vec{g}}} \delta W_j}^{\substack{\text{Work against}\\\text{forces other}\\\text{than pressure}\\\text{and gravity}}} \overbrace{- \; P \;\; \mathrm{d}V}^{\substack{\text{Boundary}\\\text{Work}}} + \overbrace{\sum_k \quad \delta m_k \theta_k}^{\substack{\text{Advection and}\\\text{Flow Work}}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(U + KE + PE) = \sum_i \dot{Q}_i - \sum_{j \neq \cancel{P},\cancel{\vec{g}}} \dot{W}_j - P\frac{\mathrm{d}}{\mathrm{d}t}V + \sum_k \dot{m}_k \theta_k$$

$$\Delta(U + KE + PE) = \sum_i Q_i - \sum_{j \neq \cancel{P},\cancel{\vec{g}}} W_j - \int P \;\; \mathrm{d}V + \sum_k \int \delta m_k \theta_k$$

$$\text{where} \quad \theta_k \equiv \underbrace{u_k + P_k v_k}_{h_k} + ke_k + pe_k$$

Mass and Entropy Balances

$$\overbrace{\mathrm{d}m}^{\substack{\text{Local}\\\text{Change}}} = \overbrace{\sum_k \quad \delta m_k}^{\text{Advection}} \qquad \overbrace{\mathrm{d}\mathbb{S}}^{\substack{\text{Local}\\\text{Change}}} = \overbrace{\sum_i \frac{\delta Q_i}{T_i}}^{\substack{\text{Transport}\\\text{by Heat}}} + \overbrace{\sum_k \quad \delta m_k s_k}^{\text{Advection}} + \overbrace{\delta \mathbb{S}_{\text{gen}}}^{\substack{\text{Local}\\\text{Source}}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}m = \sum_k \dot{m}_k \qquad \frac{\mathrm{d}}{\mathrm{d}t}\mathbb{S} = \sum_i \frac{\dot{Q}_i}{T_i} + \sum_k \dot{m}_k s_k + \dot{\mathbb{S}}_{\text{gen}}$$

$$\Delta m = \sum_k \int \delta m_k \qquad \Delta \mathbb{S} = \sum_i \frac{Q_i}{T_i} + \sum_k \int \delta m_k s_k + \mathbb{S}_{\text{gen}}$$

FORMULA-BASED PROPERTY EVALUATION

Phase	Identities	Constant-(c_P, c_v) approximation of… Δu	Δh	Δs
Incompressible Solid / Liquid	$c_P = c_v \equiv c$	$c\Delta T$	$c\Delta T + v\Delta P$ (often $\approx c\Delta T$)	$c \ln\left(\frac{T_2}{T_1}\right)$
Ideal Gas	$c_P = c_v + R$	$c_v \Delta T$	$c_P \Delta T$	$c_P \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$ or $c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$ or $c_v \ln\left(\frac{P_2}{P_1}\right) + c_P\ln\left(\frac{v_2}{v_1}\right)$

TABLE-BASED PROPERTY EVALUATION

Phase	v	$(u - u_{\text{ref}})$	$(h - h_{\text{ref}})$	$(s - s_{\text{ref}})$
Liquid	$v(P,T)$ or $v_f(T)$	$u(P,T)$ or $u_f(T)$	$h(P,T)$ or $v_f(T) \cdot (P - P_{\text{sat}}(T)) + h_f(T)$	$s(P,T)$ or $s_f(T)$
Saturated Mixture	$\phi_f(T \text{ or } P) + x \cdot \phi_{fg}(T \text{ or } P)$ (where ϕ represents v, u, h, or s)			
Vapour (Non-Ideal Gas)	$\phi(P,T)$ (where ϕ represents v, u, h, or s)			
[Ideal] Gas	$\frac{RT}{P}$	$u(T)$	$h(T)$	$s^\circ(T) - R\ln\left(\frac{P}{P_{\text{ref}}}\right)$

EFFICIENCY

Device	Conversion Efficiency: General	Conversion Efficiency: Reversible
Heat Engine	$\eta_{\text{HE}} \equiv \dfrac{W_{\text{net}}^{\text{out}}}{Q_{\text{hi}}^{\text{in}}}$	$\eta_{\text{HE}}^{\text{rev}} = 1 - \dfrac{T_{\text{lo}}}{T_{\text{hi}}}$
Heat Pump	$\beta_{\text{HP}} \equiv \dfrac{Q_{\text{hi}}^{\text{out}}}{W_{\text{net}}^{\text{in}}}$	$\beta_{\text{HP}}^{\text{rev}} = \dfrac{T_{\text{hi}}}{T_{\text{hi}} - T_{\text{lo}}}$
Refrigerator	$\beta_{\text{R}} \equiv \dfrac{Q_{\text{lo}}^{\text{in}}}{W_{\text{net}}^{\text{in}}}$	$\beta_{\text{R}}^{\text{rev}} = \dfrac{T_{\text{lo}}}{T_{\text{hi}} - T_{\text{lo}}}$

Adiabatic Device	Isentropic Efficiency
Nozzle	$\eta_s^{\text{nozz}} \equiv \dfrac{ke_{\text{out}}^{\text{a}}}{ke_{\text{out}}^{\text{s}}}$
Pump	$\eta_s^{\text{pump}} \equiv \dfrac{\dot{W}_{\text{in}}^{\text{s}}}{\dot{W}_{\text{in}}^{\text{a}}}$
Compressor	$\eta_s^{\text{comp}} \equiv \dfrac{\dot{W}_{\text{in}}^{\text{s}}}{\dot{W}_{\text{in}}^{\text{a}}}$
Turbine	$\eta_s^{\text{nozz}} \equiv \dfrac{\dot{W}_{\text{out}}^{\text{a}}}{\dot{W}_{\text{out}}^{\text{s}}}$

REVERSIBLE STEADY-FLOW WORK

$$\dot{W}^{\text{rev}} = \dot{m}\left[\int_{\text{out}}^{\text{in}} v\,\mathrm{d}P + (ke_{\text{in}} - ke_{\text{out}}) + (pe_{\text{in}} - pe_{\text{out}})\right]$$

REVERSIBLE HEAT TRANSFER

$$q = \frac{Q^{\text{rev}}}{m} = \int_1^2 T\,\mathrm{d}s \qquad \text{(Closed system)}$$

$$q = \frac{\dot{Q}^{\text{rev}}}{\dot{m}} = \int_1^2 T\,\mathrm{d}s \qquad \text{(Open system, steady flow)}$$

AIR-STANDARD POWER CYCLES

Cycle	System	Property held constant during… Compression	Heating	Expansion	Cooling	Conversion Efficiency
Carnot	Closed	s	T	s	T	$1 - \dfrac{T_{\text{lo}}}{T_{\text{hi}}}$
Otto	Closed	s	v	s	v	$1 - r^{1-\gamma}$
Diesel	Closed	s	P	s	v	$1 - r^{1-\gamma}\left[\dfrac{r_c^{\gamma} - 1}{\gamma\,(r_c - 1)}\right]$
Brayton	Steady-Flow	s	P	s	P	$1 - r_p^{\frac{1-\gamma}{\gamma}}$

THE POLYTROPIC MODEL PROCESS

Applicability

Process		
Substance	Conditions	n
Any	Isobaric (const. P)	0
Ideal Gas	Isothermal (const. T)	1
Ideal Gas	Adiabatic, only boundary work *or* Isentropic (const. s)	$\gamma \equiv \frac{c_p}{c_v}$
Any	Isochoric (const. V)	∞

Property Ratios

$$\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = \left(\frac{v_2}{v_1}\right)^{\frac{1}{-1}} \underbrace{= \left(\frac{T_2}{T_1}\right)^{\frac{1}{n-1}}}_{\text{Ideal gas only}}$$

Work-Related Integrals

$$\int_1^2 P\mathrm{d}v = \begin{cases} \dfrac{1}{1-n}\left(P_2v_2 - P_1v_1\right) \underbrace{= \dfrac{R}{1-n}\left(T_2 - T_1\right)}_{\text{Ideal gas only}} & (n \neq 1) \\ -Pv\ln\left(\dfrac{P_2}{P_1}\right) = Pv\ln\left(\dfrac{v_2}{v_1}\right) & (n = 1) \end{cases}$$

$$\int_{\text{out}}^{\text{in}} v\mathrm{d}P = \begin{cases} \dfrac{n}{1-n}\left(P_{\text{out}}v_{\text{out}} - P_{\text{in}}v_{\text{in}}\right) \underbrace{= \dfrac{nR}{1-n}\left(T_{\text{out}} - T_{\text{in}}\right)}_{\text{Ideal gas only}} & (n \neq 1) \\ -Pv\ln\left(\dfrac{P_{\text{out}}}{P_{\text{in}}}\right) = Pv\ln\left(\dfrac{v_{\text{out}}}{v_{\text{in}}}\right) & (n = 1) \end{cases}$$

Where, when $n = 1$, $\quad Pv = P_1v_1 = P_2v_2 \underbrace{= RT_1 = RT_2}_{\text{Ideal gas only}}$