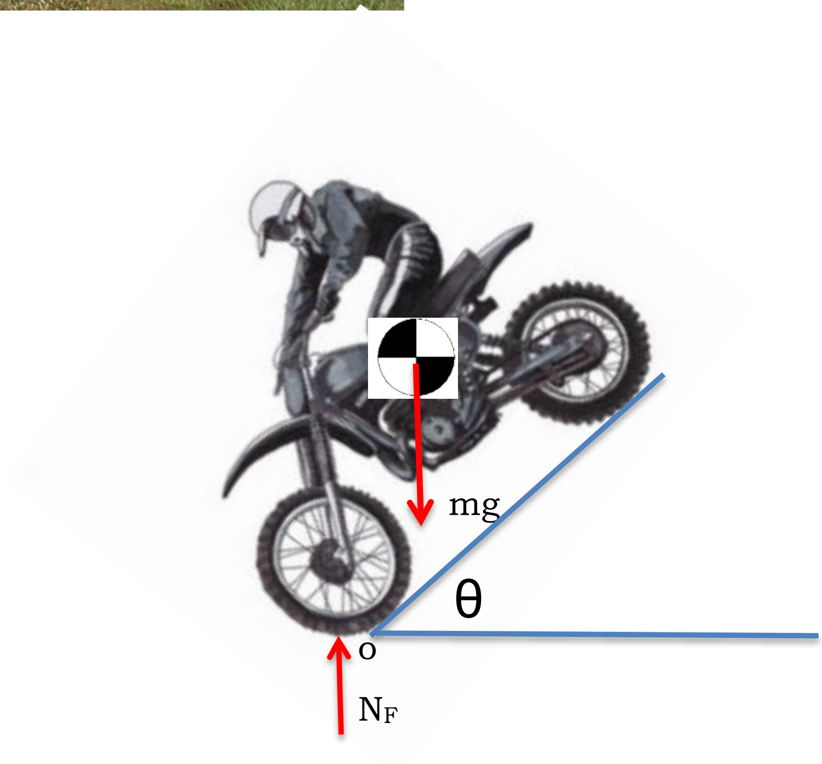
SA 1. [5 marks] At the instant shown in the photograph below the motorcycle is stationary.

a) [2 marks] Draw the free body diagram of the motorcycle and rider. What physical measurements would you have to take to define the quantities in the free body diagram?

b) [3 mark] Write the equations of motion of the motorcycle.



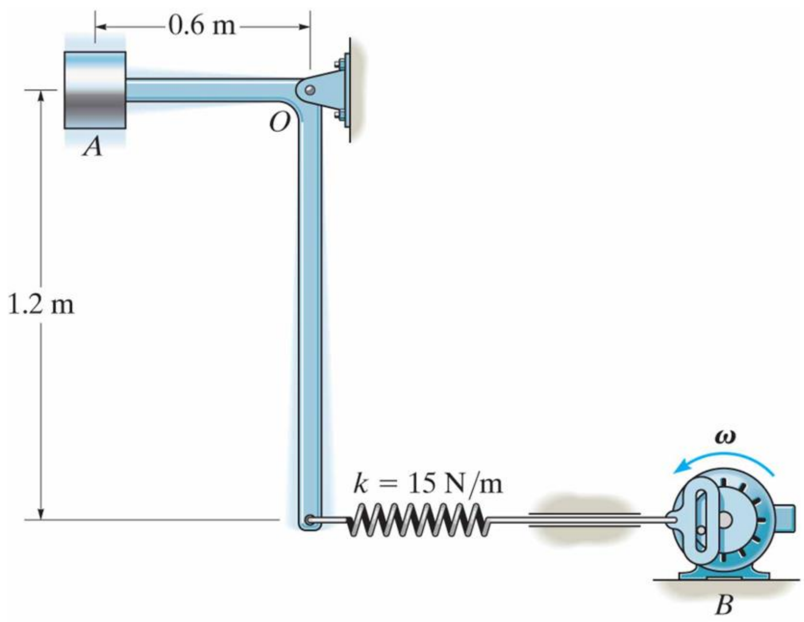
a) We would need to know the location of the centre of gravity of the combined motorcycle and rider and the mass of the combined motorcycle and rider and the vector from the centre of gravity to the front wheel contact point $\mathbf{r}_{o/CG}$.

b)

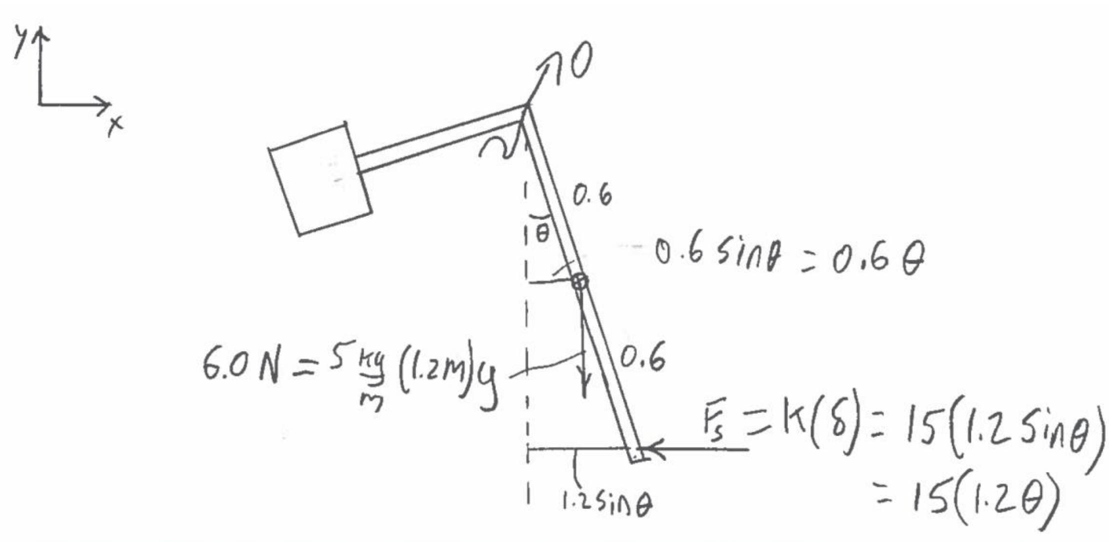
$$\sum M_o = I_o\ddot{\theta}$$

$$r_{CG/o} \times mg = I_o\ddot{\theta}$$

Prob 1 [25 Marks] The small block at A has width 0.2m and height 0.3m, and unknown mass m. The block is mounted on the slender bent rod with mass 5 kg/m. Before switching on the motor at B, the system is at equilibrium as shown in the figure below. The length of the shorter end of the rod is 0.5m.



A) [2 marks] Draw the free body diagram of the system before the motor at B is switched on.



B) [8 marks] Use the small angle approximation to **prove** that the equation of motion of the system (before the motor at B is switched on) is approximately

$$(0.37083m + 3.0883)\ddot{\theta} + 56.916\theta = 0$$

where m is the unknown mass of the block, and θ is the angular position (positive in the counter-clockwise direction) relative to position shown in the figure. **Do not solve this equation.**

The rotation of the rigid body about the point O is:

$$\sum M_O = I_O\ddot{\theta}$$

The sum of the moments about O are:

$$\sum M_O = -\,5(1.2)(9.81)(0.6)\theta - 1.2^2(15)\theta = -56.916\theta$$

Here we used the small angle approximation $\sin\theta \approx \theta$. The moment of inertia of the body about O is

$$I_O = \frac{1}{12}m(0.2^2 + 0.3^2) + m(0.6)^2 + \frac{1}{12}5(0.5)(0.5)^2 + 5(0.5)(0.25)^2$$
$$+ \frac{1}{12}5(1.2)(1.2)^2 + 5(1.2)(0.6)^2 = 0.37083m + 3.0883$$

The equation of motion is

$$(0.37083m + 3.0883)\ddot{\theta} + 56.916\theta = 0$$

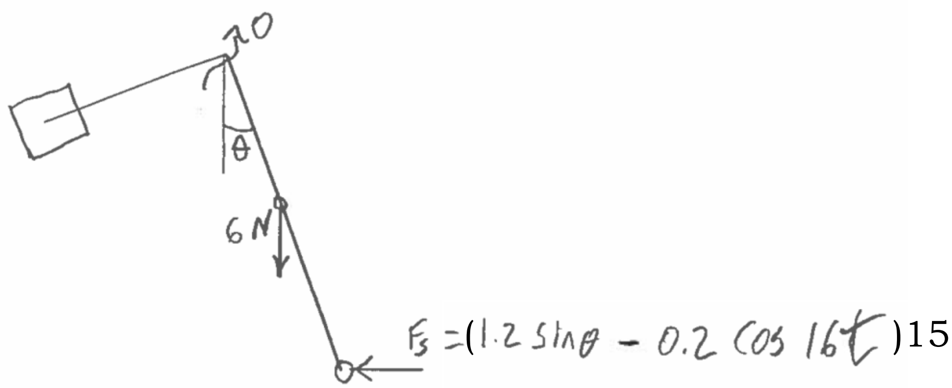
C) [2 marks] Find the value of the mass m that makes the natural frequency of the system equal to 2 radians per second.

From the equation in part B, we see that the natural frequency is

$$\omega_n = \sqrt{\frac{56.916}{0.37083m + 3.0883}}$$

Set $\omega_n = 2$ and solve for $m = 30.043\ kg$.

D) [5 marks] The mass of the block is now known to be 5kg, the motor at B is switched on and the motion at B is given by $\delta_B = 0.2\cos(16t)$. The position δ_B is measured in meters (the positive direction is to the right) and t is measured in seconds. Draw the free body diagram and determine the equation of motion.



as before but with the $0.2\cos 16t\ m$ *forcing function on the right side of the spring*

$$(0.37083m + 3.0883)\ddot{\theta} + 56.916\theta = 3.6\cos 16t$$

E) [8 marks] Find the general solution of the equation in part D.

The equation is given by

$$4.9425\ddot{\theta} + 53.316\theta = 3.6\cos(16t)$$
$$\ddot{\theta} + 10.787\theta = 0.72838\cos(16t)$$

The natural frequency in this case is $\omega_n = 3.2844$ and the complimentary solution is

$$\theta_c(t) = C_1\sin(3.2844t) + C_2\cos(3.2844t)\ , \quad C_1, C_2 \in \mathbb{R}$$

A particular solution is of the form $\theta_p(t) = A\cos(16t)$ and we compute

$$\ddot{\theta}_p + 10.787\theta_p = A(-(16^2) + 10.787)\cos(16t) = 0.72838\cos(16t)$$

therefore $A(-(16^2) + 10.787) = 0.72838$ and so $A = -0.0029794$. The general solution is

$$\theta(t) = -0.0029794\cos(16t) + C_1\sin(3.2844t) + C_2\cos(3.2844t)$$

for $C_1, C_2 \in \mathbb{R}$.