1. (5 marks) You are driving a fully-enclosed truck carrying live chickens (that is, there are no ventilation holes in the cargo compartment — poor chickens!). The empty truck is within the weight limit for a bridge you have to cross; with the chickens it is overweight. A bystander claims that if you could get the chickens to fly around in the truck (for example, by banging on the side of the truck to scare them), their weight would not be adding to the weight of the truck anymore and you could safely drive across the bridge. Use reasoning about fluid mechanics to support or refute this claim. A diagram may be a helpful part of your explanation. (Note: The marks for this question are tied to the explanation; a yes or no answer without explanation will be marked as zero.)

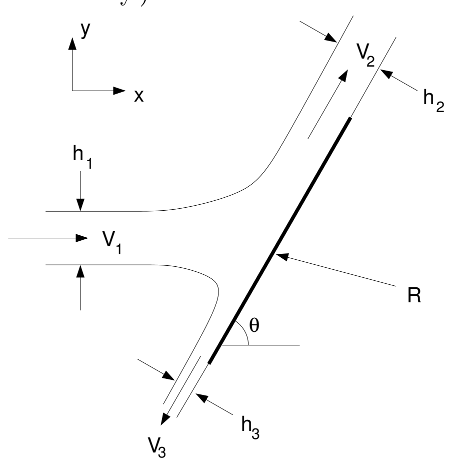
   Consider a control volume around the truck, just outside the last layer of paint on the outside. We need to know what force is required to support the truck, since that's what might break the bridge. And remember, the outside world interacts with the truck only through forces, etc, that cross the control volume boundary. In this case, it doesn't matter whether the chickens are flying or not: the total weight inside the control volume remains the same, and all of that has to be supported by the reaction force from the bridge. So you'd better go around.

   If there were ventilation holes in the sides of the truck, then it's possible that there would be (because of the chickens flapping their wings) enough movement of air in and out of the truck to support part of the weight by a momentum flux. Even this is potentially dodgy, though, because that downdraft might impinge on the bridge and exert a force on it that way.

2. (10 marks) The sketch shows a jet of water (1 m wide perpendicular to the page) hitting an inclined plate (tilted at an angle θ from the horizontal). The incoming jet has a thickness h_1 of 6 cm and a velocity V_1 of 5 m/sec. The original jet splits into two parts, as shown. A reaction force $\vec{R}$ is required to hold the plate in place.

   For all parts of this question, you may assume that flow is steady and that the velocity is uniform at 1, 2, and 3.

   (a) (6 marks) For the particular case where $V_2 = V_3 = 5\,\mathrm{m/sec}$, $h_2 = 4\,\mathrm{cm}$, $h_3 = 2\,\mathrm{cm}$ and $\theta = 60°$, find the vertical component of the reaction force R_y. (Note: drawing and labeling a control volume is mandatory.)



   Your control volume should have cut all three jets perpendicular to the flow direction, and should have included the entire plate.

   I'm including both components of $\vec{R}$ in the solutions.

   To find both components of $\vec{R}$, we'll need to write conservation of momentum in both directions. Note that, as drawn, $\vec{R}$ is up and to the left, which will guide the choice of signs when we write them down.

$$
\begin{aligned}
\overset{+}{\rightarrow} \sum F_x : \qquad -R_x &= \text{x-mom flux out} - \text{x-mom flux in} \\
+\uparrow \sum F_y : \qquad R_y &= \text{y-mom flux out} - \text{y-mom flux in}
\end{aligned}
$$

Because these are all jets of a liquid in air, the pressure is atmospheric everywhere and does not have to be included. Focusing first on the x-direction:

$$-R_x = (\rho V_2 \cos\theta)(V_2 A_2) + (-\rho V_3 \cos\theta)(V_3 A_3) - (\rho V_1)(V_1 A_1)$$

where the minus sign in the second term is there because the horizontal component of momentum out at 3 is to the left (negative). Note that, for all three terms, I grouped terms into momentum per unit volume times volume flow rate. Putting numbers into all that:

$$\begin{aligned}
-R_x &= (\rho V_2 \cos\theta)(V_2 A_2) + (-\rho V_3 \cos\theta)(V_3 A_3) - (\rho V_1)(V_1 A_1) \\
&= \left(998\frac{\text{kg}}{\text{m}^3}\cdot 5\frac{\text{m}}{\text{sec}}\cdot 0.5\right)\left(5\frac{\text{m}}{\text{sec}}\cdot 0.04\text{m}\cdot 1\text{m}\right) + \left(-998\frac{\text{kg}}{\text{m}^3}\cdot 5\frac{\text{m}}{\text{sec}}\cdot 0.5\right)\left(5\frac{\text{m}}{\text{sec}}\cdot 0.02\text{m}\cdot 1\text{m}\right) \\
&\quad - \left(998\frac{\text{kg}}{\text{m}^3}\cdot 5\frac{\text{m}}{\text{sec}}\right)\left(5\frac{\text{m}}{\text{sec}}\cdot 0.06\text{m}\cdot 1\text{m}\right) \\
&= 998\cdot 5^2\cdot 1\cdot(0.5\cdot 0.04 - 0.5\cdot 0.02 - 0.06) \\
&= -1248\text{N}
\end{aligned}$$

which is in fact a force to the left.

Now in the vertical direction:

$$\begin{aligned}
R_y &= (\rho V_2 \sin\theta)(V_2 A_2) + (-\rho V_3 \sin\theta)(V_3 A_3) - 0 \\
&= \left(998\frac{\text{kg}}{\text{m}^3}\cdot 5\frac{\text{m}}{\text{sec}}\cdot\frac{\sqrt{3}}{2}\right)\left(5\frac{\text{m}}{\text{sec}}\cdot 0.04\text{m}\cdot 1\text{m}\right) + \left(-998\frac{\text{kg}}{\text{m}^3}\cdot 5\frac{\text{m}}{\text{sec}}\cdot\frac{\sqrt{3}}{2}\right)\left(5\frac{\text{m}}{\text{sec}}\cdot 0.02\text{m}\cdot 1\text{m}\right) \\
&= 998\cdot 5^2\cdot 1\cdot\frac{\sqrt{3}}{2}(0.04 - 0.02) \\
&= 432\text{N}
\end{aligned}$$

So the overall reaction force is $\vec{R} = -1248\text{N}\hat{\imath} + 432\text{N}\hat{\jmath}$; this is pointing into the same quadrant as shown in the sketch.

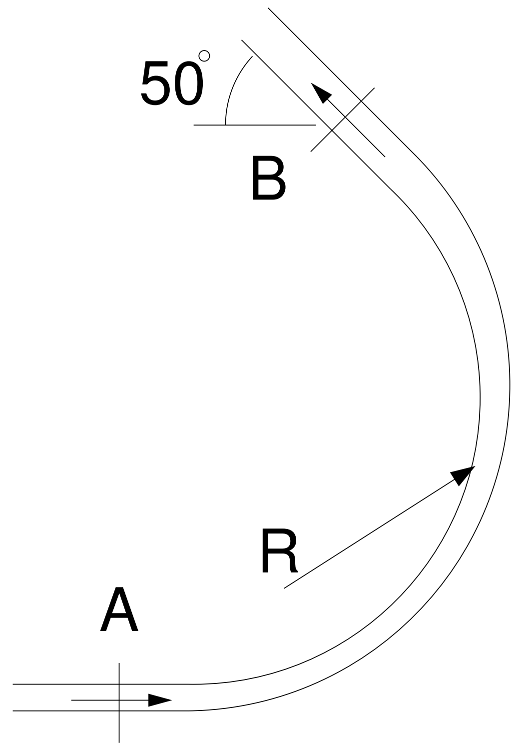
(b) Suppose that the outgoing jets still have the same flow rate as in part 2a, but that because of viscous stresses (that is, friction) on the surface of the plate, we now have $V_2 = V_3 = 4.5\,\text{m/sec}$. (*Note: You do not need to do any calculations for this part of the question. And again, the explanation is the key part of your answers.*)

i. (2 marks) Will this change the reaction force? Why or why not?
Yes, the reaction force will change. For each of the outflows, the volume flow rate remains the same (given), but the momentum per unit volume changes, so the reaction force will change.

ii. (2 marks) To calculate the reaction force, will you need to determine the viscous stress on the front of the plate? Why or why not?
No, you don't. The correct control volume for this problem passes to the right of the plate, exposing only the reaction force R. The viscous stress (and pressure) on the left face of the plate is therefore inside the control volume, and we don't have to care about details inside the control volume.
If your control volume is on the left side of the plate, then you should have written a pressure integral in your force balance in part a; since you almost certainly didn't do that, you effectively used a control volume that had the plate inside. In the extremely unlikely event that you dealt with the pressure on the surface of the plate in part a, and you think you've been wronged in the way this part of the question has been marked, come see me.

3. (15 total) Water at 4° C (use $\rho = 1000\,\text{kg/m}^3$) flows through a bend in a pipe, as shown; the pipe continues beyond both A and B, well beyond the boundaries of the sketch. Measurements reveal that $A_A = 0.002\text{ m}^2$, $A_B = 0.004\text{ m}^2$, $P_A = 151$ kPa absolute, and $V_B = 8$ m/sec. Neglect gravity and any viscous effects, and assume that the flow is uniform at both A and B. Find the ***x-component*** of the total force required to hold this pipe bend in place.

- Just as with the previous question, and any other control volume question you ever do in your life, drawing and labeling a control volume is mandatory.
- There is not enough information given to deduce how much of this force is applied at A and how much at B, so don't try.



Again, I'll show both components in the solution.

We're after the total force applied to the CV, so we need to write conservation of momentum:

$$\begin{aligned} \overset{+}{\rightarrow}\sum F_x &= \text{x-mom flux out} - \text{x-mom flux in} \\ +\uparrow\sum F_y &= \text{y-mom flux out} - \text{y-mom flux in} \end{aligned}$$

For uniform flow at both entrance (A) and exit (B), we get:

$$\begin{aligned} R_x + P_A A_A + P_B A_B \cos 50^\circ &= (-\rho V_B \cos 50^\circ)(V_B A_B) - (\rho V_A)(V_A A_A) \\ R_y - P_B A_B \sin 50^\circ &= (\rho V_B \sin 50^\circ)(V_B A_B) - 0 \end{aligned}$$

A couple of things to note here:

- Pressure pushes in at both entrance (to the right) and exit (down and right); I'll be using gauge pressure for both, so the atmospheric pressure will be automatically taken care of.
- The out-going flux in the x-direction is directed to the left, so that's why the minus sign (opposite to the direction of positive forces). The in-coming flux in the x-direction is directed to the right, but subtracted because it's a flux *into* the control volume.
- The sines and cosines are taking vector components, for both pressure force and momentum flux.

We know both areas as well as P_A and V_B. We need to find P_B and V_A. We can use conservation of momentum to find the inflow velocity V_A:

$$\begin{aligned} \rho V_A A_A &= \rho V_B A_B \\ V_A &= V_B \frac{A_B}{A_A} = 16 \frac{\text{m}}{\text{sec}} \end{aligned}$$

Now we can use Bernoulli (check: steady, compressible flow, along a streamline, no shaft work, negligible heat transfer, negligible viscous effects) to find P_B:

$$\begin{aligned}
P_B + \frac{1}{2}\rho V_B^2 &= P_A + \frac{1}{2}\rho V_A^2 \\
P_B &= P_A + \frac{1}{2}\rho\left(V_A^2 - V_B^2\right) \\
&= 247 \text{ kPa absolute} = 146 \text{ kPa gauge}
\end{aligned}$$

For reference, $P_A = 50$ kPa gauge. Returning to the x-force balance:

$$\begin{aligned}
R_x + P_A A_A + P_B A_B \cos 50^\circ &= -\rho V_B^2 A_B \cos 50^\circ - \rho V_A^2 A_A \\
R_x &= -P_A A_A - P_B A_B \cos 50^\circ - \rho V_B^2 A_B \cos 50^\circ - \rho V_A^2 A_A \\
&= -50 \cdot 10^3 \cdot 0.002 - 146 \cdot 10^3 \cdot 0.004 \cdot \cos 50^\circ - 1000 \cdot 8^2 \cdot 0.004 \cdot \cos 50^\circ - 1000 \cdot 16^2 \cdot 0 \\
&= -100\text{N} - (584 \cdot 0.643)\text{N} - (256 \cdot 0.643)\text{N} - 512\text{N} \\
&= -100\text{N} - 375.4\text{N} - 164.6\text{N} - 512\text{N} \\
&= -1152 \text{ N}
\end{aligned}$$

This force is to the left (in fact, each of the terms it balances are that way). And the y-force balance is:

$$\begin{aligned}
R_y - P_B A_B \sin 50^\circ &= \rho V_B^2 A_B \sin 50^\circ - 0 \\
R_y &= P_B A_B \sin 50^\circ + \rho V_B^2 A_B \sin 50^\circ \\
&= 447 \text{ N} + 196 \text{ N} \\
&= 643 \text{ N}
\end{aligned}$$