UNIVERSITY OF BRITISH COLUMBIA
FACULTY OF APPLIED SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

TEST #1, February 9th, 2017

MECH 222

Suggested Time: 75 minutes **Allowed Time:** 110 minutes

Materials admitted: Pencil, eraser, straightedge, Mech 2 Approved Calculator (Sharp EL-510), one 3x5 inch sheet of paper for hand-written notes.

All questions must be answered. Provide **all** work and solutions **on this test**. Orderly presentation of work is required for solutions to receive full credit. **Illegible work, or answers that do not include supporting calculations and explanations will NOT BE MARKED**.

FILL OUT THE SECTION BELOW AND WRITE YOUR NAME ON THE TOP OF ALL TEST PAGES. Do this during the examination time as additional time will not be allowed for this purpose.

NAME:____Section__________

SIGNATURE:___________________________________*

STUDENT NUMBER:___________________________

* By your signature you are affirming that you have neither given nor received aid on this examination

Question	Domain	Maximum Mark	Mark Received
1	Fluids	5	

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2	Math	10	
3	Fluids	5	
4	Math	10	
5	Fluids	10	
6	Math	10	
7	Fluids	15	
8	Thermodynamics	10	
TOTAL	-	75	

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Problem 1 – 5 marks

An object is supported in mid-air inside a submarine by a helium balloon. The submarine, in turn, is neutrally buoyant (that is, it displaces exactly its own mass of water) and is stationary at a constant depth below the surface. As the helium leaks out of the balloon into the air inside the submarine, the object settles to the deck inside the submarine. Will the submarine rise or sink in the water, and by how much? Why?

At the beginning of the story we have a submarine that is neutrally buoyant. This means that it has a volume displacement that exactly equals its mass. And the mass of the submarine consists of the mass of the steel hull, the engines, the people, the food, and everything else that is inside the submarine…including the balloon, and including the helium inside the balloon.

By the end of the story, the helium has leaked out of the balloon, but it hasn't leaked out of the submarine! The mass of stuff inside the submarine hasn't changed, it has just moved position. Maybe the crew ate some food. The mass didn't change, it just went from the food shelf to the crews' stomachs.

Bottom line: The submarine neither rises nor sinks. Nothing has changed.

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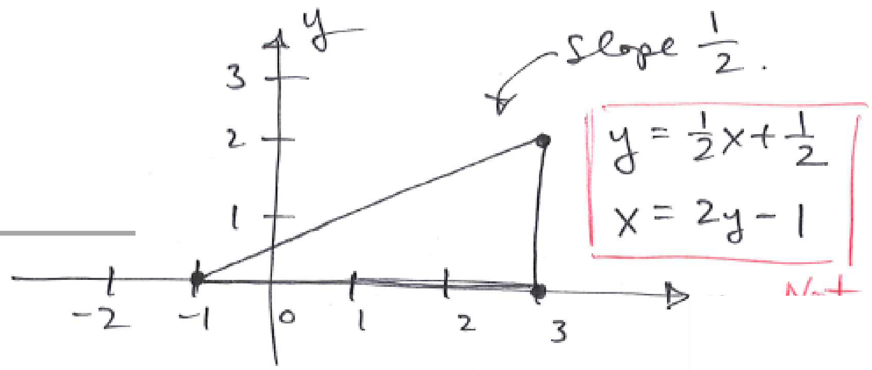
Problem 2 – 10 marks

Let D be the triangular region with vertices (-1,0), (3,2) and (3,0), and consider the function

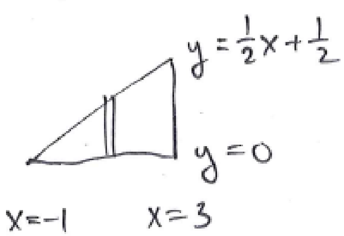
$$f(x,y)=1-xy-y^2$$

(a) [2 marks] Sketch the region D .



(b) [2marks] Write the integral $\iint_D f(x,y)\,dA$ as an iterated integral of the form:

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$



$$\boxed{\int_{-1}^{3} \int_{0}^{\frac{1}{2}x+\frac{1}{2}} (1-xy-y^2)\,dy\,dx}$$

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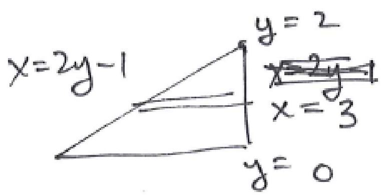
(c) [2 marks] Write the integral $\iint_D f(x,y)\,dA$ as an iterated integral of the form:

$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$



$$\boxed{\int_0^2 \int_{2y-1}^{3} (1-xy-y^2)\,dx\,dy}$$

(d) [4 marks] Compute the integral $\iint_D f(x,y)\,dA$.

$$\int_0^2 \int_{2y-1}^{3} (1-xy-y^2)\,dx\,dy$$

$$= \int_0^2 \left(x - \frac{x^2}{2}y - xy^2\right)\Big|_{2y-1}^{3} dy$$

$$= \int_0^2 \left(3 - \frac{9}{2}y - 3y^2 - \left(2y-1 - \frac{(2y-1)^2}{2}y - (2y-1)y^2\right)\right) dy$$

$$= \int_0^2 \left(4 - \frac{13}{2}y - 3y^2 + \frac{4y^3-4y^2+y}{2} + 2y^3 - y^2\right) dy$$

$$= \int_0^2 (4 - 6y - 6y^2 + 4y^3)\,dy$$

$$= 4y - 3y^2 - 2y^3 + y^4 \Big|_0^2$$

$$= 4(2) - 3(2)^2 - 2(2)^3 + 2^4 - 0 \boxed{= -4}$$

See also alternative solution on page 8

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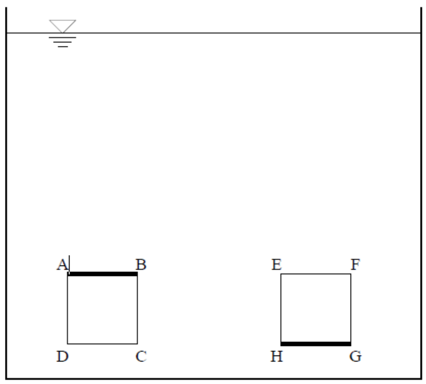
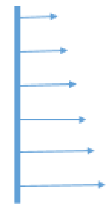
Problem 3 – 5 marks

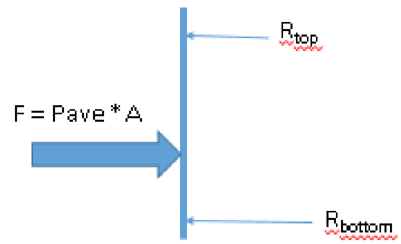
A deep tank with vertical sides is filled with liquid and has two identical square holes in its side, as shown in the figure, each covered by a hinged plate. For each hole, one horizontal edge (AB or GH) of the square is hinged (dark line in the figure) and a force is applied perpendicular to the side of the tank at the other horizontal edge (CD or EF) to hold the plate in place. For which plate is the required force higher, or are the forces equal, and why?



Pressure Distribution:



Free Body Diagram:



Without any calculations we can see that for this trapezoidal pressure distribution shown, F will be closer to the bottom than to the top. Therefor R_{top} will be smaller than R_{bottom}, and thus plate #2 (EFGH) requires less closing force.

By calculation (see p.93 of C&C):

$$dimension\ of\ hole \equiv d \times d \qquad F = d^2 * P_{ave} \qquad P_{ave} = \rho\, g\, z_{ave}$$

$$s \equiv distance\ of\ line\ AB\ below\ the\ liquid\ surface$$

$$2 + ¿\, P_0 / (\rho g)$$

$$s + d / ¿$$

$$¿$$

$$12 ¿$$

$$location\ of\ application\ of\ the\ Force \equiv z_p = s + \frac{d}{2} + \frac{d^2}{¿}$$

$$2 + ¿\, P_0 / (\rho g)$$

$$s + d / ¿$$

$$12 [¿¿]$$

$$\frac{d}{2} + \frac{d^2}{¿}$$

$$Moment\ around\ top\ line \equiv M_{top} = F * (z_p - s) = F * ¿$$

$$2+¿P_0/(\rho g)$$
$$s+d/¿$$
$$12[¿¿]$$
$$\frac{1}{2}+\frac{d}{¿}$$

$$Force\,required\,at\,bottom\,if\,hinge\,is\,at\,top=\frac{M_{top}}{d}=F*¿$$

$$2+¿P_0/(\rho g)$$
$$s+d/¿$$
$$12[¿¿]$$
$$\frac{d}{2}-\frac{d^2}{¿}$$

$$Moment\,around\,bottom\,line=F*(s+d-z_p)=F*¿$$

$$2+¿P_0/(\rho g)$$
$$s+d/¿$$
$$12[¿¿]$$
$$\frac{1}{2}-\frac{d}{¿}$$

$$Force\,required\,at\,top\,if\,hinge\,is\,at\,bottom=\frac{M_{bottom}}{d}=F*¿$$

Which is larger, ½ minus something or ½ plus something? (If the “something” is always > 0) Therefore the force required at the top if hinge is at the bottom is always less = Door #2 (EFGH)

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2(d) (Alternative Solution)

$$\int_{-1}^{3}\int_{0}^{\frac{1}{2}x+\frac{1}{2}} (1-xy-y^2)\,dy\,dx$$

$$= \int_{-1}^{3} \left(y - \frac{xy^2}{2} - \frac{y^3}{3}\right)\Big|_{0}^{\frac{1}{2}x+\frac{1}{2}} dx$$

$$= \int_{-1}^{3} \left(\frac{1}{2}x+\frac{1}{2} - \frac{x}{2}\left(\frac{x+1}{2}\right)^2 - \frac{1}{3}\left(\frac{x+1}{2}\right)^3\right) dx$$

$$= \frac{1}{24}\int_{-1}^{3} \left(12(x+1) - 3(x^3+2x^2+x) - (x^3+3x^2+3x+1)\right) dx$$

$$= \frac{1}{24}\int_{-1}^{3} (11+6x-9x^2-4x^3)\,dx$$

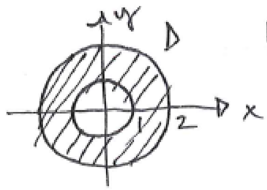
$$= \frac{1}{24}\left(11x+3x^2-3x^3-x^4\right)\Big|_{-1}^{3}$$

$$= \frac{1}{24}\left(-102-(-6)\right) = \boxed{-4}$$

Alternative solution to 2(d)

Problem 4 – 10 marks

Find the center of mass of the lamina covering the region D located inside the circle $x^2+y^2=4$ and outside the circle $x^2+y^2=1$, with density $\rho(x,y)=3-x$.



$$M = \iint_D \rho(x,y)\,dA$$

$$= \int_0^{2\pi}\int_1^2 (3 - r\cos\theta)\,r\,dr\,d\theta$$

$\rho(x,y) = 3 - x$

$= 3 - r\cos\theta$

$$= \int_0^{2\pi}\int_1^2 (3r - r^2\cos\theta)\,dr\,d\theta$$

$$= \int_0^{2\pi}\left(\frac{3}{2}r^2 - \frac{1}{3}r^3\cos\theta\right)\Big|_1^2\,d\theta$$

$$= \int_0^{2\pi}\left(\frac{3}{2}(2^2-1^2) - \frac{1}{3}(2^3-1^3)\cos\theta\right)d\theta$$

$$= \frac{9}{2}\theta - \frac{7}{3}\sin\theta\,\Big|_0^{2\pi} = 9\pi$$

Since $\rho(x,y)$ is independent of y and D is symmetric about origin, $\boxed{\bar{y} = 0.}$

$$\bar{x} = \frac{1}{M}\iint_D x\,\rho(x,y)\,dA = \frac{1}{9\pi}\int_0^{2\pi}\int_1^2 r\cos\theta(3 - r\cos\theta)\,r\,dr\,d\theta$$

$$= \frac{1}{9\pi}\int_0^{2\pi}\int_1^2 (3r^2\cos\theta - r^3\cos^2\theta)\,dr\,d\theta$$

$$= \frac{1}{9\pi}\int_0^{2\pi}\left(r^3\cos\theta - \frac{r^4}{4}\cos^2\theta\right)\Big|_1^2\,d\theta.$$

$$= \frac{1}{9\pi}\left(\cancel{(2^3-1^3)\int_0^{2\pi}\cos\theta\,d\theta}^{\,0} - \frac{(2^4-1^4)}{4}\int_0^{2\pi}\cos^2\theta\,d\theta\right)$$

$$= -\frac{1}{9\pi}\left(\frac{15}{4}\right)\left(\frac{1}{2}\theta + \frac{1}{4}\sin(2\theta)\right)\Big|_0^{2\pi} = \frac{-15}{9\pi(4)}(\pi) = \frac{-5}{12}$$

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$\Rightarrow \boxed{\bar{x} = -5/12}$

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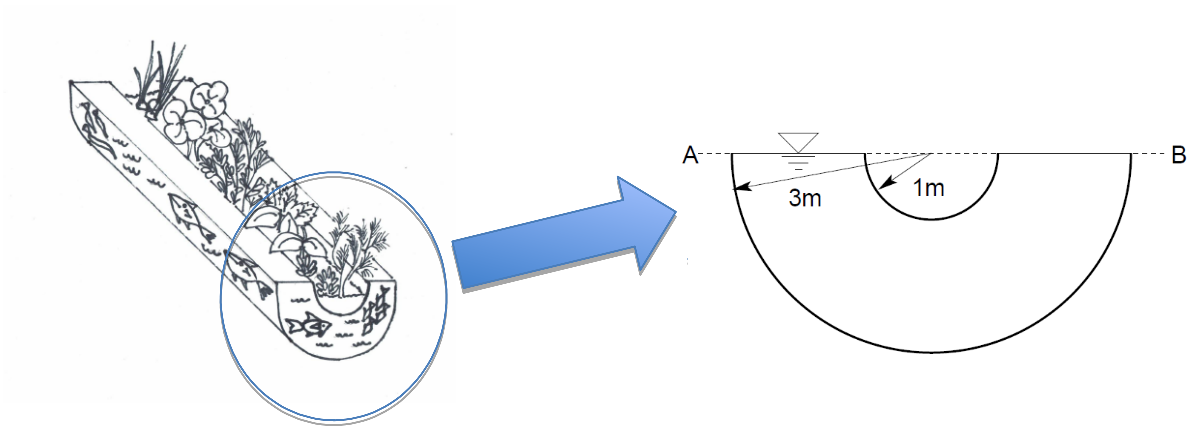
Problem 5 – 10 marks

The sketch shows the end view of an unusually shaped aquarium, which is filled with salt water (density ρ = 1025kg/m3). Determine the magnitude of the hydrostatic pressure **force** and the moment that hydrostatic pressure produces about the line AB.



The "A Ha!" here is to realize that you can do this as two simple semi-circles: "One that is and one that isn't." Once you get that the arithmetic becomes easy:

Circle that is: This is the force of the water on the big 3m circle:

Circle that isn't: This is the force on the 1m circle, which has been erroneously included in the big circle. We need to find out how much this force and moment is, and remove it from the big circle, so that we get only the "doughnut."

	IS		ISN'T
Area = R^2 pi / 2	pi*9/2		pi*1/2
Pave = P_centroid = P@4R/3pi			
Rho*g*4R/(3*pi)	9.81*1025*4*(3/3)/pi		9.81*1025*4*(1/3)/pi
Force = Pave * Area =	9*9.81*1025*4*3/6		9.81*1025*4*1/6
	180.994 kN	-	6.703 kN
Net force on aquarium	174.291 kNm		
Ixx = 0.1098 R4	0.1098*3^4		0.1098
Yp = Yc + Ixx / (A*Yc)	4*(R/3)/pi + 0.1098*R^4/(4*(R/3)/pi * pi*R^2/2)		
	4*R/(3*/pi) + 0.1098*R*6/4		
	4/pi+0.1098*18/4		4/(3pi)+0.1098*6/4
	1.767 m		0.589 m
Moment = Force * Yp	319.879 kNm	-	3.949 kNm
Total Moment =	315.93 kNm		
Net lever arm of total force =	1.813 m		

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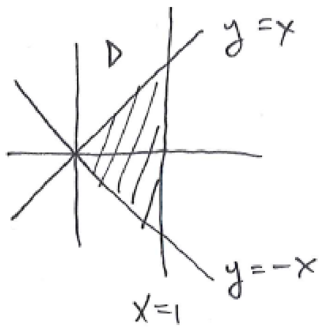
Problem 6 – 10 marks

Compute the integral

$$\iint y^2\sqrt{1+x^4}\,dA$$

over the region D bound by the lines $y=x$, $y=-x$ and $x=1$.



$\int \sqrt{1+x^4}\,dx$ is difficult...
integrate with respect to y first.

$$\iint_D y^2\sqrt{1+x^4}\,dA$$

$$= \int_0^1 \int_{-x}^{x} y^2\sqrt{1+x^4}\,dy\,dx$$

$$= \int_0^1 \left(\tfrac{1}{3}y^3\sqrt{1+x^4}\right)\Big|_{-x}^{x} dx$$

$$= \tfrac{2}{3}\int_0^1 x^3\sqrt{1+y^4}\,dx$$

$$= \frac{2}{3}\,\frac{2}{3}\,\frac{(1+x^4)^{3/2}}{4}\Bigg|_0^1$$

$$= \frac{1}{9}\left(2^{3/2}-1\right)$$

$$\boxed{= \frac{2\sqrt{2}-1}{9}}$$

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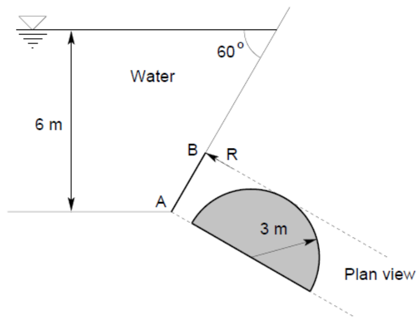
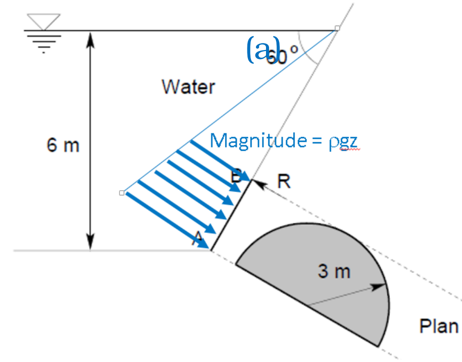
Problem 7 – 15 marks

The sketch shows a side view of a tank filled with fresh water. The gate AB is semi-circular, with a radius of three meters. This gate is hinged at A and held closed by a force R applied at B, perpendicular to the tank wall.





(a) (5 marks) On the sketch, draw in the pressure distribution on the gate. Indicate the magnitude by labeling at least one of the arrows that you draw. See above

(b) (10 marks) Find the magnitude of the net pressure force on the gate.

F = Pave * π r^2 /2

Pave = "P at the centroid" = rho g z_centroid

Z_centroid = Formula sheet = 4R/3π

Remember to include the 60 degree angle: z_centroid = 6m – cos(60)*(4 * 3m / (3π)) = 5.36 m

Rho g z_centroid = 1000 kg/m3 * 9.81 m/s2 * 5.36 m = 52.615 kN/m2

Force = Pressure * Area = 52.615 kN/m2 * π r^2 /2 = 52.615 kN/m2 * π 3^2 /2 = 744 kN

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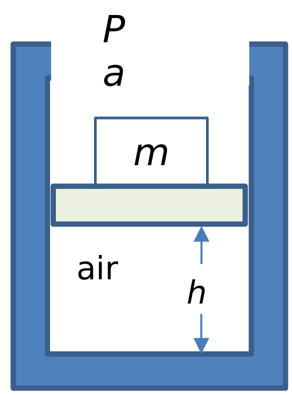
Problem 8 – 10 marks

Consider a cylinder/piston assembly containing 4 g of air. The piston is frictionless and weightless. Assuming for the given conditions provided on the accompanying figure the system is in equilibrium with its surroundings; find

a) The absolute pressure of the air inside the cylinder (5 Marks)
b) The temperature of the air inside the cylinder (5 Marks)

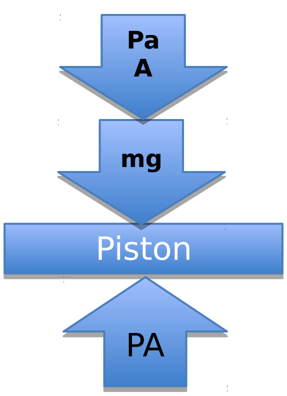
The local atmospheric pressure is P_a = 100 kPa
Piston diameter, d = 30 cm
Mass on top of the piston, m = 5 kg
Piston height, h = 5 cm
Air gas constant is 0.287 kJ/kg-K



Please write your final answers in the Table below:

Part a	$P=$	100.694 kPa
Part b	$T=$	310.1 K

Solution:



(a) At equilibrium the forces acting on the piston are balanced, so:

PaA + mg = PA

$$A=\frac{\pi d^2}{4}=\frac{\pi 0.3^2}{4}=0.0707\,m^2$$

$$100\times10^3*0.0707+5*9.81=P*0.0707 \qquad 7119.05=P*0.0707$$

$$P=100{,}693.78\,Pa \qquad P=100.694\,kPa$$

(b) Assume air behaves as an ideal gas: $PV=m_{air}RT$

$$V=A*h=0.0707*0.05=0.003535\,m^3$$

$$T = \frac{PV}{m_{air}T} = \frac{100.694 * .003535}{.004 * 0.287} = 310.1\,K$$

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