Useful Fluids Formulae

Data

- For water, at 20°C and 1 atm: $\rho = 998$ kg/m^3, $\mu = 1.00 \cdot 10^{-3}$ N $\cdot$ sec/m^2.
- For air:
  - At sea level & standard temperature (288.2 K): $P = 101.3$ kPa, $\rho = 1.225$ kg/m^3, $\mu = 1.8{\cdot}10^{-5}$ N$\cdot$sec/m^2.
  - $R = 287 \frac{\mathrm{m}^2}{\mathrm{sec}^2 \cdot \mathrm{K}}$

Hydrostatics

- For constant density:
  - Pressure variation with depth (z measured downwards): $P - P\,(z = 0) = \rho g z$
  - Force due to pressure: $F = P_c A$ where P_c is pressure at the centroid of the surface.
  - Location of line of action: $y_p - y_c = \rho g \sin\theta \frac{I_{xx,c}}{F}$
- For variable density:
  - Pressure variation with depth (z measured downwards): $\frac{\partial P}{\partial z} = \rho g$
  - Force: $F = \iint P dA$. Moment: $M = \iint P r\, dA$, where r is the moment arm for force exerted on dA.

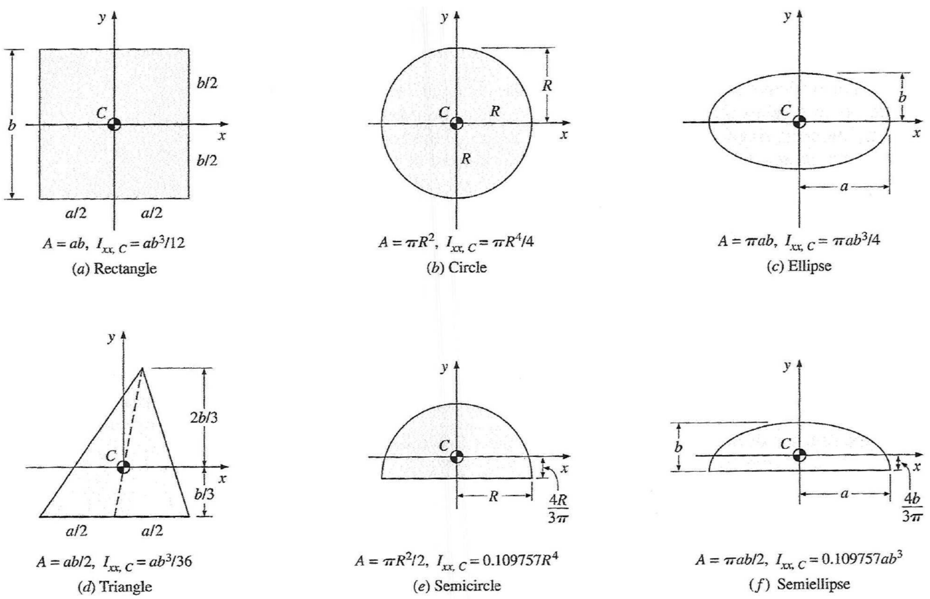

FIGURE 3–28
The centroid and the centroidal moments of inertia for some common geometries.

- Buoyant force = weight of fluid displaced
- Fluid undergoing uniform acceleration $\vec{a}$ is equivalent to new gravity vector $\vec{g} - \vec{a}$.

Control Volume Analysis

- Amount of stuff in a system B (mass, momentum, energy, etc): $B_{sys} = \iiint_{sys} \beta\rho\, dV$ where β is the amount of stuff per unit mass.
- Reynolds transport theorem relates system conservation law to control volume conservation law:

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\iiint_{CV} \beta\rho\, dV + \iint_{CS} \beta\rho\vec{U}\cdot n\, \hat{d}A$$

- Conservation of mass.
  - 1D: $\frac{\partial}{\partial t} m_{CV} = (\rho A U)_{in} - (\rho A U)_{out}$
  - 3D $\left(\vec{U} = u\hat{i} + v\hat{j} + w\hat{k}\right)$:
    * General:

$$0 = \frac{\partial}{\partial t}\iiint_{CV} \rho\, dV + \iint_{CS} \rho\vec{U}\cdot\hat{n}\, dA$$

    * Incompressible: $\iint_{CS} \vec{U}\cdot\hat{n}\, dA = 0$ or $\nabla\cdot\vec{U} = 0$.
    * Steady: $\iint_{CS} \rho\vec{U}\cdot\hat{n}\, dA = 0$
- Conservation of momentum.
  - 1D uniform flow at inlets and outlets:

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\vec{U}\, dV + \sum_{\text{outlets}} \dot{m}_o\vec{U}_o - \sum_{\text{inlets}} \dot{m}_i\vec{U}_i = \sum\vec{F}$$

  - General case:

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\vec{U}\, dV + \iint_{CS} \rho\vec{U}\left(\vec{U}\cdot\hat{n}\right)\, dA = \sum\vec{F}$$

- Bernoulli's equation.
  - General version:

$$\int_1^2 \frac{\partial U}{\partial t} ds + \int_1^2 \frac{dP}{\rho} + \frac{1}{2}\left(U_2^2 - U_1^2\right) + g\left(z_2 - z_1\right) = 0$$

  - Steady, incompressible version:

$$\left[p + \frac{1}{2}\rho U^2 + \rho g z\right]_1 = \left[p + \frac{1}{2}\rho U^2 + \rho g z\right]_2 = \text{constant}$$

Common Dimensionless Numbers

$\mathrm{Re} = \frac{\rho U L}{\mu}$ $\quad \mathrm{Fr} = \frac{U}{\sqrt{gL}}$ $\quad C_D = \frac{D}{\frac{1}{2}\rho U^2 L^2}$ $\quad \mathrm{Pr} = \frac{c_p\mu}{k}$ $\quad \mathrm{Nu} = \frac{hL}{k}$

Pipe Flow

Entrance length: $\frac{L}{D} \approx 0.06\mathrm{Re}_D$ (laminar) $\frac{L}{D} \approx 4.4\mathrm{Re}_D^{1/6}$ (turbulent)

$\mathrm{Re}_D \approx 2300$ @ transition. Friction factor: $h_f = f\frac{L}{D}\frac{U^2}{2g}$ Minor losses: $h_m = K_m\frac{U^2}{2g}$

Laminar pipe flow: $f = 64/\mathrm{Re}_D$ Turbulent pipe flow: $\frac{1}{\sqrt{f}} = -1.8\log_{10}\left[\frac{6.9}{\mathrm{Re}_D} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$

Pressure loss along a pipe:

$$\left[p + \frac{1}{2}\rho U^2 + \rho g z\right]_1 = \left[p + \frac{1}{2}\rho U^2 + \rho g z\right]_2 + \rho g\left(h_f + h_m\right)$$

UBC MECH 222 MATH FORMULAS

VECTOR IDENTITIES

For $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k},\ \mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k},\ \mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k},$

$$\mathbf{u} \bullet \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = |\mathbf{u}|\,|\mathbf{v}|\cos(\theta), \quad 0 \le \theta \le \pi$$

$$|\mathbf{u}| = \sqrt{\mathbf{u} \bullet \mathbf{u}} = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \langle u_2v_3 - u_3v_2,\, u_3v_1 - u_1v_3,\, u_1v_2 - u_2v_1 \rangle$$

$$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\sin\theta$$

$$\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) = \mathbf{v} \bullet (\mathbf{w} \times \mathbf{u}) = \mathbf{w} \bullet (\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \bullet \mathbf{w})\mathbf{v} - (\mathbf{u} \bullet \mathbf{v})\mathbf{w}$$

DISTANCES AND PROJECTIONS

From point (x_0, y_0, z_0) to plane $Ax + By + Cz = D$: $\quad s = \dfrac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$

$$\mathbf{F} = \mathbf{proj_u}(\mathbf{F}) + \mathbf{orth_u}(\mathbf{F}) \qquad \mathbf{proj_u}(\mathbf{F}) = \left(\frac{\mathbf{F} \bullet \mathbf{u}}{\mathbf{u} \bullet \mathbf{u}}\right)\mathbf{u}$$

DERIVATIVES

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

$$\mathbf{F}(x,y,z) = P(x,y,z)\,\mathbf{i} + Q(x,y,z)\,\mathbf{j} + R(x,y,z)\,\mathbf{k}$$

$$\nabla\phi(x,y,z) = \mathbf{grad}\,\phi(x,y,z) = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial y}\mathbf{j} + \frac{\partial\phi}{\partial z}\mathbf{k}$$

$$\nabla \bullet \mathbf{F}(x,y,z) = \mathbf{div}\ \mathbf{F}(x,y,z) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\nabla \times \mathbf{F}(x,y,z) = \mathbf{curl}\,\mathbf{F}(x,y,z) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k}$$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \bullet (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \bullet \mathbf{G} - \mathbf{F} \bullet (\nabla \times \mathbf{G})$$

$$\nabla \bullet (\phi\mathbf{F}) = (\nabla\phi) \bullet \mathbf{F} + \phi(\nabla \bullet \mathbf{F})$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \bullet \mathbf{G}) - \mathbf{G}(\nabla \bullet \mathbf{F}) - (\mathbf{F} \bullet \nabla)\mathbf{G} + (\mathbf{G} \bullet \nabla)\mathbf{F}$$

$$\nabla \times (\phi\mathbf{F}) = (\nabla\phi) \times \mathbf{F} + \phi(\nabla \times \mathbf{F})$$

$$\nabla(\mathbf{F} \bullet \mathbf{G}) = \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F}) + (\mathbf{F} \bullet \nabla)\mathbf{G} + (\mathbf{G} \bullet \nabla)\mathbf{F}$$

$$\nabla \times (\nabla\phi) = \mathbf{0} \qquad (\mathbf{curl\,grad} = \mathbf{0})$$

$$\nabla \bullet (\nabla \times \mathbf{F}) = 0 \qquad (\mathbf{div\,curl} = 0)$$

$$\nabla^2\phi(x,y,z) = \nabla \bullet \nabla\phi(x,y,z) = \mathbf{div\,grad}\,\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

VECTOR-VALUED FUNCTIONS OF ONE VARIABLE

$$\frac{d}{dt}(\lambda(t)\mathbf{u}(t)) = \lambda'(t)\mathbf{u}(t) + \lambda(t)\mathbf{u}'(t)$$

$$\frac{d}{dt}(\mathbf{u}(t) \bullet \mathbf{v}(t)) = \mathbf{u}'(t) \bullet \mathbf{v}(t) + \mathbf{u}(t) \bullet \mathbf{v}'(t)$$

$$\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t)) = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$$

$$\frac{d}{dt}|\mathbf{u}(t)| = \frac{\mathbf{u}(t) \bullet \mathbf{u}'(t)}{|\mathbf{u}(t)|}, \quad \mathbf{u}(t) \ne \mathbf{0}$$

$$\frac{d}{dt}(\mathbf{u}(\lambda(t))) = \lambda'(t)\mathbf{u}'(\lambda(t))$$

Position $\mathbf{r} = \mathbf{r}(t)$ gives velocity $\mathbf{v}(t) = \mathbf{r}'(t)$, speed $v(t) = |\mathbf{v}(t)|$, acceleration $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$. In this case,

$$d\mathbf{r} = \langle dx, dy, dz \rangle = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle dt = \frac{d\mathbf{r}}{dt}\,dt = \mathbf{v}(t)\,dt, \qquad ds = |d\mathbf{r}| = \left|\frac{d\mathbf{r}}{dt}\right| dt = |\mathbf{v}(t)|\,dt = v(t)\,dt.$$

LINEAR APPROXIMATIONS

Differentiability test—function f, point $\mathbf{a}$: $\quad 0 = \lim_{\mathbf{x}\to\mathbf{a}} \frac{E(\mathbf{x})}{|\mathbf{x}-\mathbf{a}|}$, where $E(\mathbf{x}) = f(\mathbf{x}) - f(\mathbf{a}) - \nabla f(\mathbf{a}) \bullet (\mathbf{x}-\mathbf{a})$

Tangent Hyperplane for $G(\mathbf{x}) = 0$ at $\mathbf{a}$: $\quad 0 = \nabla G(\mathbf{a}) \bullet (\mathbf{x}-\mathbf{a})$

Linearization of f around $\mathbf{a}$: $\quad L(\mathbf{x}) = f(\mathbf{a}) + (\mathbf{x}-\mathbf{a}) \bullet \nabla f(\mathbf{a})$

Differentials for $w = f(x,y,z)$: $\quad dw = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy + \frac{\partial f}{\partial z}\,dz = \nabla f \bullet d\mathbf{r}$

SURFACE NORMALS AND AREA ELEMENTS

For any oriented surface normal $\mathbf{n} \neq \mathbf{0}$, $\quad d\mathbf{S} = \widehat{\mathbf{n}}\,dS = \frac{\mathbf{n}}{|\mathbf{n}\bullet\mathbf{k}|}\,dx\,dy = \frac{\mathbf{n}}{|\mathbf{n}\bullet\mathbf{j}|}\,dx\,dz = \frac{\mathbf{n}}{|\mathbf{n}\bullet\mathbf{i}|}\,dy\,dz, \qquad dS = |d\mathbf{S}|$

Level Surface $G(x,y,z) = 0$: $\quad$ normal $\mathbf{n} = \pm\nabla G(x,y,z)$ $\quad$ (choose sign to orient)

Parametric Surface $\mathbf{r} = \mathbf{r}(u,v)$: $\quad$ normal $\mathbf{n} = \pm\left(\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}\right)$ $\quad$ (choose sign to orient)

POLAR AND CYLINDRICAL COORDINATES

Transformation: $x = r\cos\theta,\quad y = r\sin\theta,\quad z = z$ $\qquad$ Volume element: $dV = r\,dr\,d\theta\,dz$

Surface area element (on $r = a$): $dS = a\,d\theta\,dz$ $\qquad$ Surface area element (on $z = 0$): $dS = r\,dr\,d\theta$

SPHERICAL COORDINATES

Transformation: $x = \rho\sin\phi\cos\theta,\quad y = \rho\sin\phi\sin\theta,\quad z = \rho\cos\phi$

Volume element: $dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$ $\qquad$ Surface area element (on $\rho = a$): $dS = a^2\sin\phi\,d\theta\,d\phi$

INTEGRATING DERIVATIVES: THE FUNDAMENTAL THEOREM OF CALCULUS

1-variable FTC: $\quad \int_a^b f'(t)\,dt = f(b) - f(a)$

Let $\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k}$, $\qquad C:\quad \mathbf{r} = \mathbf{r}(t),\ a \le t \le b$

Work from Potential:
$$\int_C \nabla\phi \bullet d\mathbf{r} = \int_C \frac{\partial \phi}{\partial x}\,dx + \frac{\partial \phi}{\partial y}\,dy + \frac{\partial \phi}{\partial z}\,dz = \phi(\mathbf{r}(b)) - \phi(\mathbf{r}(a))$$

Green's Theorem:
$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \oint_{\partial D} \mathbf{F}\bullet d\mathbf{r} = \oint_{\partial D} P(x,y)\,dx + Q(x,y)\,dy$$

Stokes's Theorem:
$$\iint_S \nabla\times\mathbf{F}\bullet\widehat{\mathbf{n}}\,dS = \oint_{\partial S} \mathbf{F}\bullet d\mathbf{r} = \oint_{\partial S} P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz$$

Divergence Theorem:
$$\iiint_E \nabla\bullet\mathbf{F}\,dV = \oiint_{\partial E} \mathbf{F}\bullet\widehat{\mathbf{n}}\,dS$$

AVERAGE VALUES: f ON CURVE $\mathcal{C}$, g ON SURFACE $\mathcal{S}$, h ON SOLID E

$$\overline{f} = \frac{\int_C f\,ds}{\int_C 1\,ds} \qquad \overline{g} = \frac{\iint_S g\,dS}{\iint_S 1\,dS} \qquad \overline{h} = \frac{\iiint_E h(x,y,z)\,dV}{\iiint_E 1\,dV}$$

DEFINITE INTEGRALS AND SPECIAL FUNCTIONS

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt \qquad \int_0^\infty e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$$

$$\int_0^{\pi/2} \sin x\,dx = \int_0^{\pi/2} \cos x\,dx = 1 \qquad \int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$$

$$\int_0^{\pi/2} \sin^3 x\,dx = \int_0^{\pi/2} \cos^3 x\,dx = \frac{2}{3} \qquad \int_0^{\pi/2} \sin^4 x\,dx = \int_0^{\pi/2} \cos^4 x\,dx = \frac{3\pi}{16}$$

$$\int_0^{\pi/2} \sin^5 x\,dx = \int_0^{\pi/2} \cos^5 x\,dx = \frac{8}{15} \qquad \int_0^{\pi/2} \sin^6 x\,dx = \int_0^{\pi/2} \cos^6 x\,dx = \frac{5\pi}{32}$$

TRIGONOMETRIC IDENTITIES

$$\sin^2 x + \cos^2 x = 1 \qquad \sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$

$$\sec^2 x = 1 + \tan^2 x \qquad \csc^2 x = 1 + \cot^2 x \qquad \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \qquad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \qquad \sin\left(\frac{\pi}{2}\right) = 1 = \cos(0)$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right)$$

$$\sin(0) = 0 = \cos\left(\frac{\pi}{2}\right) \qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$$

INDEFINITE INTEGRALS

$$\int x \sin(bx)\,dx = \frac{\sin(bx)}{b^2} - \frac{x\cos(bx)}{b} \qquad \int x\cos(bx)\,dx = \frac{\cos(bx)}{b^2} + \frac{x\sin(bx)}{b}$$

$$\int x e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1) \qquad \int \tan x\,dx = \ln|\sec x|$$

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}\,(a\sin bx - b\cos bx)}{a^2 + b^2} \qquad \int e^{ax}\cos(bx)\,dx = \frac{e^{ax}\,(a\cos bx + b\sin bx)}{a^2 + b^2}$$

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x \qquad \int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$

$$\int \sec^2 x\,dx = \tan x \qquad \int \tan^2 x\,dx = \tan x - x$$

$$\int \sin^3 x\,dx = \frac{1}{3}\cos^3 x - \cos x \qquad \int \cos^3 x\,dx = \sin x - \frac{1}{3}\sin^3 x$$

$$\int (a^2 - x^2)^{3/2}\,dx = \frac{x}{8}(5a^2 - 2x^2)\sqrt{a^2 - x^2} + \frac{3a^4}{8}\sin^{-1}\frac{x}{a} \qquad \int (x^2 \pm a^2)^{3/2}\,dx = \frac{x}{8}(2x^2 \pm 5a^2)\sqrt{x^2 \pm a^2} + \frac{3a^4}{8}\ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) \qquad \int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{x + a}{x - a}\right| \quad (a > 0) \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} \quad (a > 0)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \quad (a > 0) \qquad \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

$$\int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 - x^2}} \qquad \int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

Adapted from R. A. Adams, *Calculus, A Complete Course*, Addison-Wesley, 2003.

MECH 222
Thermodynamics
Formula Sheets
"2016"

Pressure definition	$P = \frac{F}{A}$	(mathematical limit for small A)
Specific volume	$v = \frac{V}{m}$	
Density	$\rho = \frac{m}{V}$	
Static pressure variation	$\Delta P = \rho g H = -\int \rho g\, dh$	
Absolute temperature	$T[\mathrm{K}] = T[^\circ\mathrm{C}] + 273.15$	
	$T[\mathrm{R}] = T[\mathrm{F}] + 459.67$	
Specific total energy	$e = u + \frac{1}{2}V^2 + gz$	

Concepts from Physics

Newton's law of motion	$F = ma$
Acceleration	$a = \frac{d^2x}{dt^2} = \frac{d\mathbf{V}}{dt}$
Velocity	$\mathbf{V} = \frac{dx}{dt}$

Quality	$x = m_{\text{vap}}/m$	(vapor mass fraction)
	$1 - x = m_{\text{liq}}/m$	(liquid mass fraction)
Average specific volume	$v = (1-x)v_f + xv_g$	(only two-phase mixture)
Equilibrium surface	P–v–T	Tables or equation of state
Ideal gas law	$Pv = RT$	$PV = mRT = n\overline{R}T$

Universal gas constant	$\overline{R} = 8.3145$ kJ/kmol K
Gas constant	$R = \overline{R}/M$ kJ/kg K
Compressibility factor Z	$Pv = ZRT$
Reduced properties	$P_r = \frac{P}{P_c}$ $\quad T_r = \frac{T}{T_c}$

Total energy	$E = U + \mathrm{KE} + \mathrm{PE} = mu + \frac{1}{2}m\mathbf{V}^2 + mgZ$
Kinetic energy	$\mathrm{KE} = \frac{1}{2}m\mathbf{V}^2$
Potential energy	$\mathrm{PE} = mgZ$
Specific energy	$e = u + \frac{1}{2}\mathbf{V}^2 + gZ$

Enthalpy $\quad h \equiv u + Pv$

Two-phase mass average
$$u = u_f + xu_{fg} = (1-x)u_f + xu_g$$
$$h = h_f + xh_{fg} = (1-x)h_f + xh_g$$

Specific heat, heat capacity $\quad C_v = \left(\frac{\partial u}{\partial T}\right)_v;\ C_p = \left(\frac{\partial h}{\partial T}\right)_p$

Solids and liquids: Incompressible, so v = constant $\cong v_f$ (or v_i) and v small
$$C = C_v = C_p$$
$$u_2 - u_1 = C(T_2 - T_1)$$
$$h_2 - h_1 = u_2 - u_1 + v(P_2 - P_1)$$ (Often the second term is small.)
$$h = h_f + v_f(P - P_{\text{sat}});\ u \cong u_f$$ (saturated at same T)

Ideal gas
$$h = u + Pv = u + RT$$ (only functions of T)
$$C_v = \frac{du}{dT};\ C_p = \frac{dh}{dT} = C_v + R$$
$$u_2 - u_1 = \int C_v\, dT \cong C_v(T_2 - T_1)$$
$$h_2 - h_1 = \int C_p\, dT \cong C_p(T_2 - T_1)$$

Energy equation rate form	$\dot{E} = \dot{Q} - \dot{W}$ (rate = +in − out)
Energy equation integrated	$E_2 - E_1 = {}_1Q_2 - {}_1W_2$ (change = +in − out)
	$m(e_2 - e_1) = m(u_2 - u_1) + \frac{1}{2}m(\mathbf{V}_2^2 - \mathbf{V}_1^2) + mg(Z_2 - Z_1)$
Multiple masses, states	$E = m_A e_A + m_B e_B + m_C e_C + \cdots$
Work	Energy in transfer: mechanical, electrical, and chemical
Heat	Energy in transfer caused by ΔT
Displacement work	$W = \int_1^2 F\,dx = \int_1^2 P\,dV = \int_1^2 \mathcal{S}\,dA = \int_1^2 T\,d\theta$
Specific work	$w = W/m$ (work per unit mass)
Power, rate of work	$\dot{W} = F\mathbf{V} = P\dot{V} = T\omega$ ($\dot{V}$ displacement rate)
	Velocity $\mathbf{V} = r\omega$, torque $T = Fr$, angular velocity $= \omega$
Polytropic process	$PV^n =$ constant or $Pv^n =$ constant
Polytropic process work	${}_1W_2 = \frac{1}{1-n}(P_2V_2 - P_1V_1)$ (if $n \neq 1$)
	${}_1W_2 = P_1V_1 \ln \frac{V_2}{V_1}$ (if $n = 1$)
Conduction heat transfer	$\dot{Q} = -kA\frac{dT}{dx} \simeq kA\frac{\Delta T}{L}$
Conductivity	k (W/m K)
Convection heat transfer	$\dot{Q} = hA\,\Delta T$
Convection coefficient	h (W/m² K)
Radiation heat transfer (net to ambient)	$\dot{Q} = \varepsilon\sigma A(T_s^4 - T_{\text{amb}}^4)$ ($\sigma = 5.67 \times 10^{-8}$ W/m² K⁴)
Rate integration	${}_1Q_2 = \int \dot{Q}\,dt \approx \dot{Q}_{\text{avg}}\,\Delta t$
Volume flow rate	$\dot{V} = \int \mathbf{V}\,dA = A\mathbf{V}$ (using average velocity)
Mass flow rate	$\dot{m} = \int \rho\mathbf{V}\,dA = \rho A\mathbf{V} = A\mathbf{V}/v$ (using average values)
Flow work rate	$\dot{W}_{\text{flow}} = P\dot{V} = \dot{m}Pv$
Flow direction	From higher P to lower P unless significant KE or PE exists

Instantaneous Process

Continuity equation	$\dot{m}_{\text{C.V.}} = \sum \dot{m}_i - \sum \dot{m}_e$
Energy equation	$\dot{E}_{\text{C.V.}} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \sum \dot{m}_i h_{\text{tot}\,i} - \sum \dot{m}_e h_{\text{tot}\,e}$
Total enthalpy	$h_{\text{tot}} = h + \frac{1}{2}\mathbf{V}^2 + gZ = h_{\text{stagnation}} + gZ$

Steady State

No storage: $\dot{m}_{\text{C.V.}} = 0$; $\dot{E}_{\text{C.V.}} = 0$

Continuity equation	$\sum \dot{m}_i = \sum \dot{m}_e$ (in = out)
Energy equation	$\dot{Q}_{\text{C.V.}} + \sum \dot{m}_i h_{\text{tot}\,i} = \dot{W}_{\text{C.V.}} + \sum \dot{m}_e h_{\text{tot}\,e}$ (in = out)
Specific heat transfer	$q = \dot{Q}_{\text{C.V.}}/\dot{m}$ (steady state only)
Specific work	$w = \dot{W}_{\text{C.V.}}/\dot{m}$ (steady state only)
Steady-state, single-flow energy equation	$q + h_{\text{tot}\,i} = w + h_{\text{tot}\,e}$ (in = out)

Transient Process

Continuity equation	$m_2 - m_1 = \sum m_i - \sum m_e$
Energy equation	$E_2 - E_1 = {}_1Q_2 - {}_1W_2 + \sum m_i h_{\text{tot}\,i} - \sum m_e h_{\text{tot}\,e}$
	$E_2 - E_1 = m_2\left(u_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2\right) - m_1\left(u_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1\right)$

Heat engine	$W_{HE} = Q_H - Q_L$;	$\eta_{HE} = \frac{W_{HE}}{Q_H} = 1 - \frac{Q_L}{Q_H}$	
Heat pump	$W_{HP} = Q_H - Q_L$;	$\beta_{HP} = \frac{Q_H}{W_{HP}} = \frac{Q_H}{Q_H - Q_L}$	
Refrigerator	$W_{REF} = Q_H - Q_L$;	$\beta_{REF} = \frac{Q_L}{W_{REF}} = \frac{Q_L}{Q_H - Q_L}$	

Real heat engine	$\eta_{HE} = \frac{W_{HE}}{Q_H} \le \eta_{\text{Carnot HE}} = 1 - \frac{T_L}{T_H}$
Real heat pump	$\beta_{HP} = \frac{Q_H}{W_{HP}} \le \beta_{\text{Carnot HP}} = \frac{T_H}{T_H - T_L}$
Real refrigerator	$\beta_{REF} = \frac{Q_L}{W_{REF}} \le \beta_{\text{Carnot REF}} = \frac{T_L}{T_H - T_L}$

Clausius inequality $\quad \int \frac{dQ}{T} \le 0$

Entropy $\quad ds = \frac{dq}{T} + ds_{\text{gen}}; \quad ds_{\text{gen}} \ge 0$

Rate equation for entropy $\quad \dot{S}_{\text{c.m.}} = \sum \frac{\dot{Q}_{\text{c.m.}}}{T} + \dot{S}_{\text{gen}}$

Entropy equation $\quad m(s_2 - s_1) = \int_1^2 \frac{\delta Q}{T} + {}_1S_{2\text{ gen}}; \quad {}_1S_{2\text{ gen}} \ge 0$

Total entropy change $\quad \Delta S_{\text{net}} = \Delta S_{\text{cm}} + \Delta S_{\text{surr}} = S_{\text{gen}} \ge 0$

Lost work $\quad W_{\text{lost}} = \int T\, dS_{\text{gen}}$

Actual boundary work $\quad {}_1W_2 = \int P\, dV - W_{\text{lost}}$

Gibbs relations $\quad T\, ds = du + P\, dv$

$T\, ds = dh - v\, dP$

Solids, Liquids

$v = \text{constant}, \quad dv = 0$

Change in s $\quad s_2 - s_1 = \int \frac{du}{T} = \int C \frac{dT}{T} \approx C \ln \frac{T_2}{T_1}$

Ideal Gas

Standard entropy $\quad s_T^0 = \int_{T_0}^{T} \frac{C_{p0}}{T}\, dT$

Change in s $\quad s_2 - s_1 = s_{T2}^0 - s_{T1}^0 - R \ln \frac{P_2}{P_1}$

$s_2 - s_1 = C_{p0} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$ (For constant $C_p,\ C_v$)

$s_2 - s_1 = C_{v0} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$ (For constant $C_p,\ C_v$)

Ratio of specific heats $\quad k = C_{p0}/C_{v0}$

Polytropic processes $\quad Pv^n = \text{constant}; \quad PV^n = \text{constant}$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n = \left(\frac{v_1}{v_2}\right)^n = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$n = 0$;	$P = \text{constant}$;	Isobaric
$n = 1$;	$T = \text{constant}$;	Isothermal
$n = k$;	$s = \text{constant}$;	Isentropic
$n = \pm\infty$;	$v = \text{constant}$;	Isochoric or isometric

Specific work $\quad {}_1w_2 = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1) \qquad n \ne 1$

${}_1w_2 = P_1v_1 \ln \frac{v_2}{v_1} = RT_1 \ln \frac{v_2}{v_1} = RT_1 \ln \frac{P_1}{P_2} \qquad n = 1$

The work is moving boundary work $w = \int P\, dv$

Rate equation for entropy	rate of change = +in − out + generation $\dot{S}_{\text{c.v.}} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{\text{c.v.}}}{T} + \dot{S}_{\text{gen}}$
Steady-state single flow	$s_e = s_i + \int_i^e \frac{\delta q}{T} + s_{\text{gen}}$
Reversible shaft work	$w = -\int_i^e v\,dP + \frac{1}{2}\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2 + gZ_i - gZ_e$
Reversible heat transfer	$q = \int_i^e T\,ds = h_e - h_i - \int_i^e v\,dP$ (from the Gibbs relation)
Bernoulli equation	$v(P_i - P_e) + \frac{1}{2}\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2 + gZ_i - gZ_e = 0$ (v = constant)
Polytropic process work	$w = -\frac{n}{n-1}(P_e v_e - P_i v_i) = -\frac{nR}{n-1}(T_e - T_i) \qquad n \neq 1$ $w = -P_i v_i \ln \frac{P_e}{P_i} = -RT_i \ln \frac{P_e}{P_i} = RT_i \ln \frac{v_e}{v_i} \qquad n = 1$ The work is shaft work $w = -\int_i^e v\,dP$ and for ideal gas
Isentropic efficiencies	$\eta_{\text{turbine}} = w_{Tac}/w_{Ts}$ (Turbine work is out) $\eta_{\text{compressor}} = w_{Cs}/w_{Cac}$ (Compressor work is in) $\eta_{\text{pump}} = w_{Ps}/w_{Pac}$ (Pump work is in) $\eta_{\text{nozzle}} = \frac{1}{2}\mathbf{V}_{ac}^2 \Big/ \frac{1}{2}\mathbf{V}_s^2$ (Kinetic energy is out)