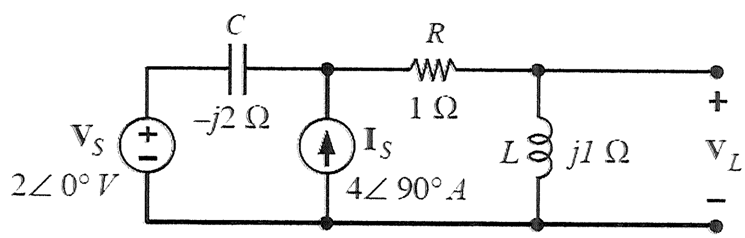
Prob 2. (25 marks)

Consider the circuit shown below which is operating in the (sinusoidal) steady-state at an angular frequency, ω = 1000 rad/s.



(a) [4 marks] Determine the actual component values of the capacitor, C (in farads), and the inductor, L (in henries). Also give the time-domain expressions $v_S(t)$ and $i_S(t)$ for the voltage source and current source, respectively.

$\omega = 1000$ rad/s

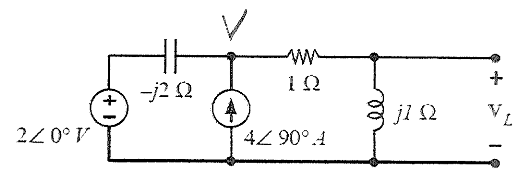
$Z_C = -j\frac{1}{\omega C} = -j2 \Rightarrow C = 5\times10^{-4}$ F.

$Z_L = j\omega L = j1 \Rightarrow L = 1\times10^{-3}$ H

$V_s = 2\angle 0° \Rightarrow v_s(t) = 2\cos(1000t)$ V.

$I_s = 4\angle 90° \Rightarrow i_s(t) = 4\cos(1000t + 90°)$ A.

(b) [10 marks] Determine the phasor voltage $\mathbf{V}_L$, across the inductor using *either* Nodal Analysis *or* Mesh Analysis, and give the corresponding time-domain expression, $v_L(t)$.



Nodal Anal:

$$\text{KCL: } \frac{V - 2\angle 0^\circ}{2\angle -90^\circ} - 4\angle 90^\circ + \frac{V - V_L}{1} = 0$$

$$V_L = V\,\frac{1\angle 90^\circ}{\sqrt{2}\angle 45^\circ} \quad \rightarrow \quad V = V_L\sqrt{2}\angle -45^\circ$$

$$\frac{V_L\sqrt{2}\angle -45^\circ}{2\angle -90^\circ} - \frac{2\angle 0^\circ}{2\angle -90^\circ} - 4\angle 90^\circ + V_L\sqrt{2}\angle -45^\circ - V_L = 0$$

$$V_L\left(\underbrace{0.707\angle 45^\circ}_{0.5 + j0.5} + \underbrace{\sqrt{2}\angle -45^\circ}_{1 - j1} - 1\right) = 4\angle 90^\circ + 1\angle 90^\circ$$

$$V_L\underbrace{(0.5 - j0.5)}_{0.707\angle -45^\circ} = 5\angle 90^\circ$$

$$V_L = \frac{5\angle 90^\circ}{0.707\angle -45^\circ} = 7.07\angle 135^\circ \text{ V.}$$

$$\therefore\ V_L(t) = 7.07\cos(1000t + 135^\circ) \text{ V.}$$

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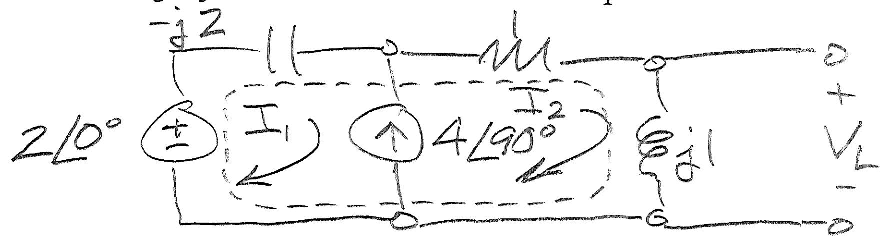
Mesh Anal:



KVL: $-2\angle 0° + I_1(2\angle -90) + I_2(1) + I_2(1\angle 90°) = 0$

(supermesh) $I_2 - I_1 = 4\angle 90° \longrightarrow I_1 = I_2 - 4\angle 90°$

$$I_2(2\angle -90°) - (4\angle 90°)(2\angle -90°) + I_2(1\angle 0°) + I_2(1\angle 90) = 2\angle 0°$$

$$I_2(\underbrace{2\angle -90°}_{0-j2} + \underbrace{1\angle 0°}_{1+j0} + \underbrace{1\angle 90°}_{0+j1}) = \underbrace{2\angle 0°}_{2+j0} + \underbrace{8\angle 0°}_{8+j0}$$

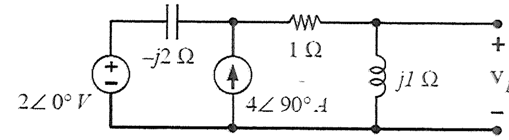
$$I_2(1-j1) = 10+j0$$

$$I_2 = \frac{10\angle 0°}{\sqrt{2}\angle -45°} = 7.07\angle 45° \text{ A.}$$

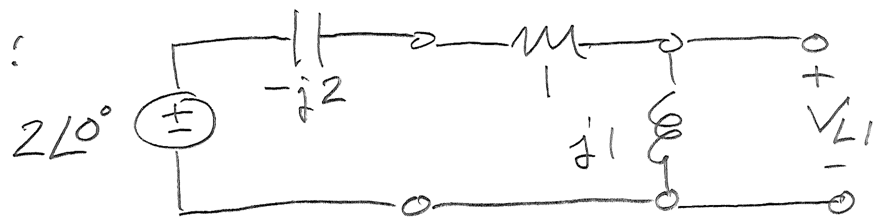
$$\therefore V_L = (7.07\angle 45°)(1\angle 90°) = 7.07\angle 135° \text{ V.}$$

$$\therefore v_L(t) = 7.07\cos(1000t + 135°) \text{ V.}$$

(c) [9 marks] Determine the phasor voltage $\mathbf{V}_L$, across the inductor using Superposition. You do not need to give the corresponding time-domain expression, $v_L(t)$.

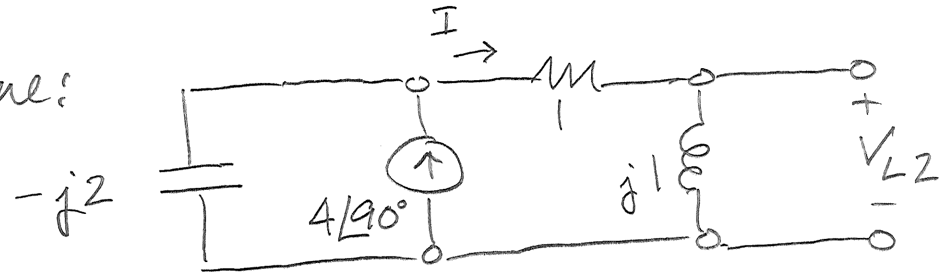


V_S alone:



$$V_{L1} = \frac{(2\angle 0°)(1\angle 90°)}{(1+j1-j2)} = \frac{(2\angle 0°)(1\angle 90°)}{\sqrt{2}\angle -45°} = 1.414\angle 135°\ V.$$
$$= -1+j1$$

I_s alone:



$$I = \frac{(4\angle 90°)(2\angle -90°)}{(1+j1-j2)} = \frac{8\angle 0°}{\sqrt{2}\angle -45°} = 5.66\angle 45°\ A.$$

$$V_{L2} = (5.66\angle 45°)(1\angle 90) = 5.66\angle 135° = -4+j4\ V$$

$$V_L = V_{L1} + V_{L2} = (-1+j1) + (-4+j4) = -5+j5\ V.$$
$$= 7.07\angle 135°\ V.$$

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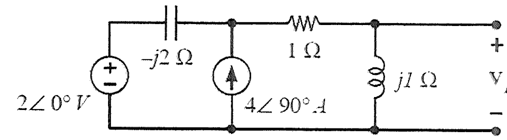
(d) [2 marks] If the two sources were <u>not</u> operating at the same frequency, briefly (in 2 or 3 sentences) describe how you would approach the determination of $v_L(t)$.



$v_L(t)$ must be determined by superposition. Here, the phasor component of v_L do to v_S needs to be calc'd using Z values at the freq. of v_S, and the phasor component due to i_S needs to be calc'd using Z values at the freq. of i_S. The 2 phasor components need to be converted to the t-domain separately to yield $v_{L1}(t)$ and $v_{L2}(t)$, since the superposition (sum) must be computed in the t-domain

i.e. $v_L(t) = v_{L1}(t) + v_{L2}(t)$.