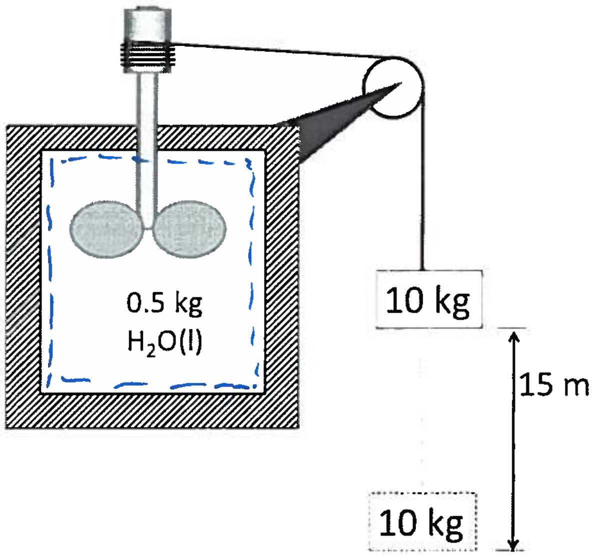
1. (8 marks) Consider Joule's apparatus, sketched below (not to scale). An insulated container holds 0.5 kg of liquid water at 25 C and 1 bar, and a paddle. Rope from a drum runs over a pulley to a 10 kg mass, which falls 15 meters in the process of going from state 1 to state 2.



(a) (2 marks) Find the temperature increase of the water (our control mass) in the process 1-2 as the weight falls 15 m.

$E_2 - E_1 = Q - W$ NO $\Delta KE, \Delta PE$ for water

$U_2 - U_1 = 0 - W$ $W = (10\ kg)(9.81)\,15\ m$

$W = 1471.5$ Joules

Property model for water: v = constant and $du = C\,dT$ $C \sim 4.186\ kJ/kg\text{-}K$

$\therefore\ mC(T_2 - T_1) = 1.4715\ kJ$

$T_2 - T_1 = \dfrac{1.472}{(0.5)(4.186)} = 0.7\,^\circ C$

(b) (2 marks) What is the heat transfer to the water in the process?

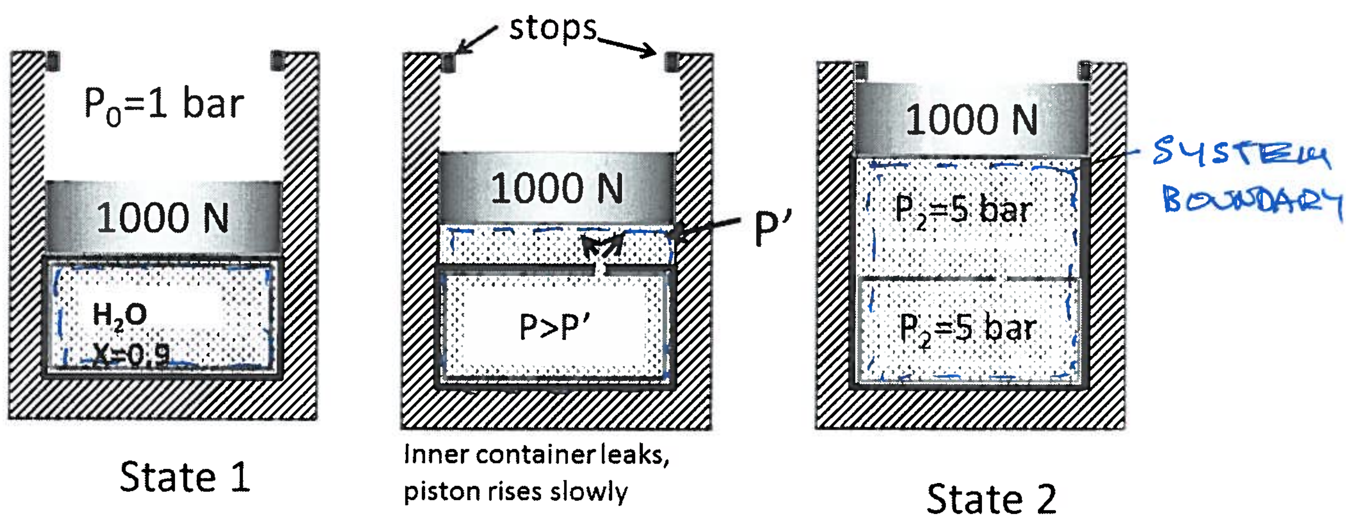
0

(c) (4 marks) What is the entropy change of the water?

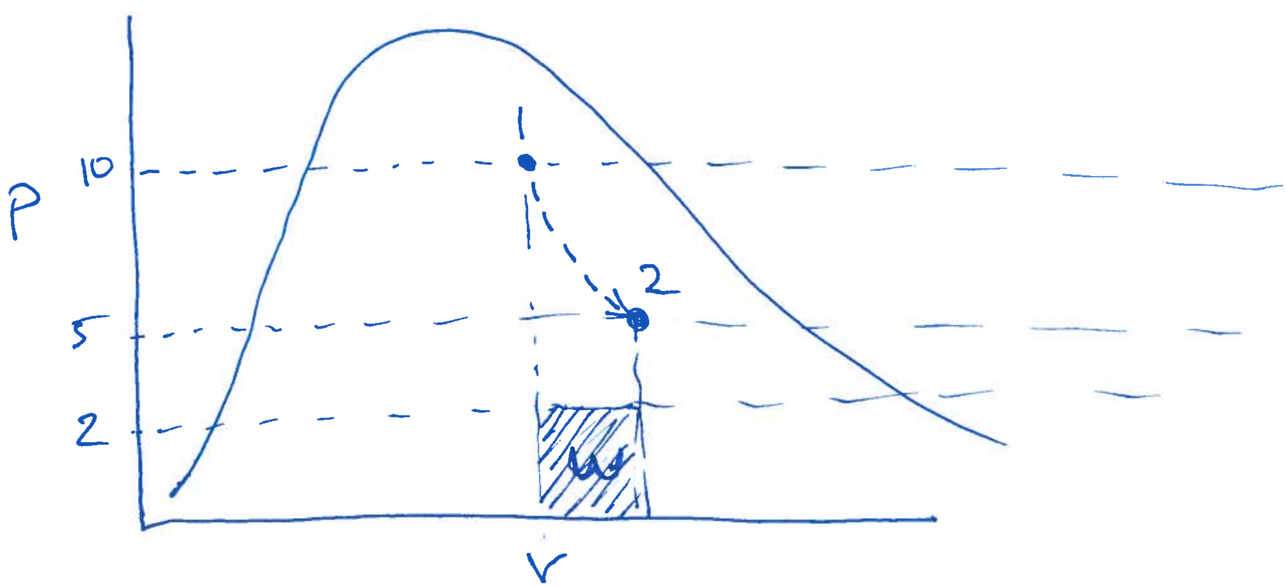
$S_2 - S_1 = mC \ln \dfrac{T_2}{T_1}$ must use T in K!

$= (0.5)(4.186) \ln\left(\dfrac{273.15+25+.7}{273.15+25}\right) = 4.9 \times 10^{-3}$ kJ/K

4. (20 marks) An insulated cylinder and piston (weight 1000 N; area $A = 0.01\ m^2$) contains an inner chamber under the piston. Intially this inner container has steam at 10 bar and quality of $x_1 = 0.9$. Atmospheric pressure is 1 bar=100 kPa. This container develops a small leak, so the steam slowly lifts the piston. During this process, the pressure under the piston is P', which is less than the pressure in the lower chamber. Eventually, the "stops" are reached, at which point the volume no longer changes. In the final State 2, the pressure has risen to 5 bar, uniform throughout the cylinder.



(a) (5 marks) Mark your system boundary on the sketches above and sketch the process on a P-v diagram (*Hint: assume the final state is in the 2-phase region*).



(b) (3 marks) How would you compute the specific work w done by the steam if you knew v_2?

Process is not reversible but while the piston is moving, the pressure below it is

$$P' = P_o + \frac{F}{A} = 2 \text{ bar}$$

$$\therefore \quad w = 200 \text{ kPa} (v_2 - v_1) \quad \frac{kJ}{kg}$$

(c) (9 marks) Find a relation between the quality x_2 and the relevant properties (expressed as symbols), assuming that heat transfer through the piston and cylinder is negligible.

First Law $\Delta KE, \Delta PE = 0$, work with specific properties

$$u_2 - u_1 = -w$$

$$u_f + x_2(u_{fg}) - u_1 = P'(v_1 - v_2)$$

State 1 given in problem, quality x_2 unknown.

$$u_f + x_2 u_{fg} - u_1 = P'(v_1 - v_f - x_2 v_{fg})$$

u_f, $u_{fg} = u_g - u_f$, v_f, $v_{fg} = v_g - v_f$ are for saturated steam at 5 bar.

$$x_2(u_{fg} + P'v_{fg}) = u_1 - u_f + P'(v_1 - v_f)$$

$$x_2 = \frac{u_1 - u_f + P'(v_1 - v_f)}{u_{fg} + P'v_{fg}}$$

(d) (3 marks) Evaluate x_2 , v_2 and u_2 .

State 1 $P = 10$ bar $T = 179.89$ °C

$v_f = .001127$ m³/kg
$v_g = 0.19$
$u_f = 761.6$ kJ/kg
$u_g = 2582.8$ kJ/kg

Given $x_1 = 0.9$
$u_1 = 2400.68$ kJ/kg
$v_1 = 0.17111$ m³/kg

State 2 $P = 5$ bar
$T = 151.84$ °C

$v_f = .001093$
$v_g = .37$

$u_f = 639.6$ kJ/kg
$u_g = 2560.7$ kJ/kg

$$x_2 = \frac{2400.68 - 639.6 + 200\,\text{kPa}\,(.17111 - .001093)}{2560.7 - 639.6 + 200\,(0.37 - .001093)}$$

$$x_2 = \frac{1761.08 + 34.003}{1921.1 + 73.78} = \frac{1795.08}{\cancel{1958.88}\ 1994.88} = 0.899$$

$v_2 = .001093 + .899(.37 - .001093) = 0.333$ m³/kg

$u_2 = 639.6 + .899(2560.7 - 639.6) = 2366$ kJ/kg

7. (7 marks) Derive the formula for the entropy change of an ideal gas when it changes state from T_1, v_1 to T_2, v_2, starting with:

- the ideal gas law $Pv = RT$
- the relationship between energy and temperature for a gas $du = C_v dT$
- the assumption of reversibility $\delta w = P dv$ $T ds = \delta q$
- any other laws, balances, facts or math that you think are needed.

(*Hint: Your final result will look something like the expression on the formula sheet involving T and P, but you can't use that formula as your starting point*).

First law, $\Delta KE, \Delta PE = 0$

$$du = \delta q - \delta w$$

$$du = T ds - P dv$$

$$C_v dT = T ds - P dv \Rightarrow ds = C_v \frac{dT}{T} + \frac{P dv}{T}$$

$$\frac{P}{T} = \frac{R}{v} \Rightarrow ds = C_v \frac{dT}{T} + R \frac{dv}{v}$$

For process $1 \to 2$ integrate the differentials:

$$\int_1^2 ds = C_v \int_1^2 \frac{dT}{T} + R \int_1^2 \frac{dv}{v}$$

$$\boxed{s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}}$$