When the wind blows over a cylindrical object - like a smoke stack or a car antenna - the flow can cause this object to oscillate. In fact, this can even be used to make a wind-driven musical stringed-instrument, called an "Aeolian Harp."

We find in practice that the frequency of an Aeolian Harp (or of a car antenna) depends upon the diameter of the antenna and the speed of the wind. Use this fact to write a non-dimensional expression for frequency as a function of wind speed V and diameter D. (3 marks)

Based on this expression, what will happen to a given harp string if the wind speed increases - does it go up in pitch or down? (2 marks)

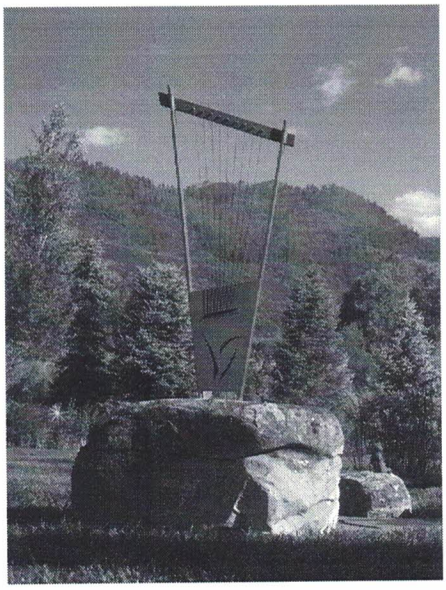
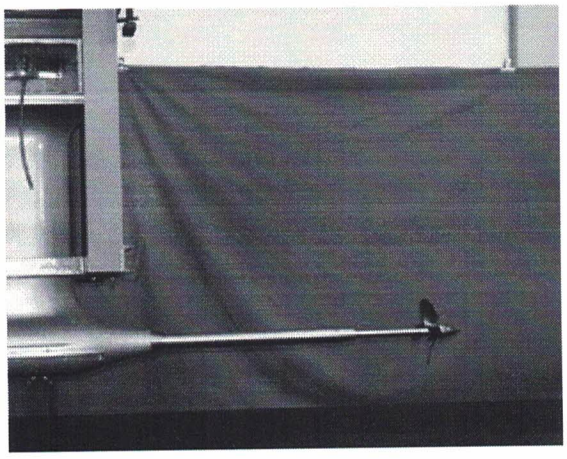


*Figure 1 An Aeolian Harp*



We wish to test a ship propeller in a water channel. The photo shows the test apparatus before placing it into the water. The propeller is characterized by its diameter "D" and its RPM "N". Our model propeller will be smaller than the full-size propeller by a factor we will call "λ," such that λ*D_model = D_prototype. We will use fresh water in the channel, although the real ship will sail in salt water (different density and viscosity). [Hint: If you get stuck, try ignoring this fact and use the same water in both cases.] We will run the water in the channel at a speed that is appropriate to simulate the ship traveling at velocity "V." In this condition the propeller generates a force "T" called "Thrust" which we use to propel the ship. We will measure this in the model test. We will also measure the torque "Q" that we have to supply to the propeller.

(2 marks) Generate a non-dimensional expression for the Thrust.

(3 marks) Generate a non-dimensional expression for the Torque.

(5 marks) At what speed do we need to run the water channel?

(7 marks) At what RPM do we need to run the model propeller?

(5 marks) For a λ–scale propeller, how much thrust should I expect (as a percentage of the full-scale thrust)?

(5 marks) For a λ–scale propeller, how much torque should I expect (as a percentage of the full-scale torque)?

(3 marks) Does this test seem practical to you? If not, why not?

Problem 3 – 5 marks

Consider the function

$$f(x,y) = x^2 + y^2 + \sin(x) + \sin(y)$$

How many critical points does the function have and what is their classification? (Note: The question is not asking to find the exact positions of the critical points but rather to determine how many critical points there are and what are they.)

Problem 4 – 8 marks

Find and classify the critical points of the function

$$f(x,y) = 2x^3 - 3x^2y - 12x^2 - 3y^2$$

Problem 5 – 8 marks

Use the method of Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y) = e^{xy}$$

subject to the constraint $x^3 + y^3 = 16$.

Problem 6 – 9 marks

Use the method of Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y, z) = x + 2y + 3z$$

subject to the constraints $x - y + z = 1$ and $x^2 + y^2 = 1$.

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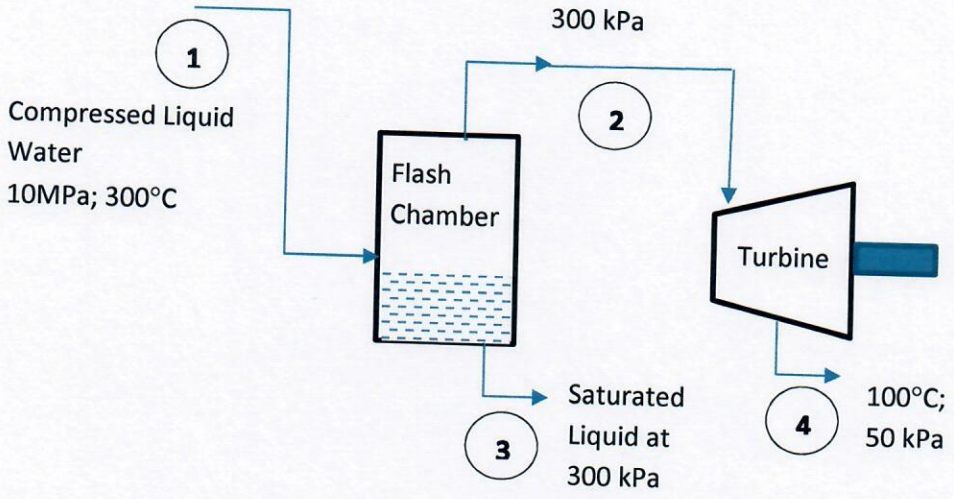
Solutions

(10 marks) In an industrial process, compressed liquid water at 10 MPa and 300°C enters an insulated flash chamber. The flash chamber pressure is adjusted to 300 kPa. Saturated vapor leaving the flash chamber expands in a turbine and exits at 50 kPa and 100°C. If $\dot{m}_1 = 50\frac{kg}{s}$, determine:

a. (5 marks) The mass flowrate at point 2, in kg/s
b. (5 marks) The rate of work generated through the turbine, in W




Please write your final answer in the Table below:

| | |
|---|---|
| $\dot{m}_2 =$ | Kg/s |
| $\dot{W} =$ | W |

TABLE B.1.4

***Compressed Liquid Water***

| Temp. (°C) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) |
|---|---|---|---|---|---|---|---|---|
| | 500 kPa (151.86 °C) | | | | 2000 kPa (212.42 °C) | | | |
| Sat. | 0.001093 | 639.66 | 640.21 | 1.8606 | 0.001177 | 906.42 | 908.77 | 2.4473 |
| 0.01 | 0.000999 | 0.01 | 0.51 | 0.0000 | 0.000999 | 0.03 | 2.03 | 0.0001 |
| 20 | 0.001002 | 83.91 | 84.41 | 0.2965 | 0.001001 | 83.82 | 85.82 | .2962 |
| 40 | 0.001008 | 167.47 | 167.98 | 0.5722 | 0.001007 | 167.29 | 169.30 | .5716 |
| 60 | 0.001017 | 251.00 | 251.51 | 0.8308 | 0.001016 | 250.73 | 252.77 | .8300 |
| 80 | 0.001029 | 334.73 | 335.24 | 1.0749 | 0.001028 | 334.38 | 336.44 | 1.0739 |
| 100 | 0.001043 | 418.80 | 419.32 | 1.3065 | 0.001043 | 418.36 | 420.45 | 1.3053 |
| 120 | 0.001060 | 503.37 | 503.90 | 1.5273 | 0.001059 | 502.84 | 504.96 | 1.5259 |
| 140 | 0.001080 | 588.66 | 589.20 | 1.7389 | 0.001079 | 588.02 | 590.18 | 1.7373 |
| 160 | — | — | — | — | 0.001101 | 674.14 | 676.34 | 1.9410 |
| 180 | — | — | — | — | 0.001127 | 761.46 | 763.71 | 2.1382 |
| 200 | — | — | — | — | 0.001156 | 850.30 | 852.61 | 2.3301 |
| | 5000 kPa (263.99 °C) | | | | 10000 kPa (311.06 °C) | | | |
| Sat | 0.001286 | 1147.78 | 1154.21 | 2.9201 | 0.001452 | 1393.00 | 1407.53 | 3.3595 |
| 0 | 0.000998 | 0.03 | 5.02 | 0.0001 | 0.000995 | 0.10 | 10.05 | 0.0003 |
| 20 | 0.001000 | 83.64 | 88.64 | 0.2955 | 0.000997 | 83.35 | 93.32 | 0.2945 |
| 40 | 0.001006 | 166.93 | 171.95 | 0.5705 | 0.001003 | 166.33 | 176.36 | 0.5685 |
| 60 | 0.001015 | 250.21 | 255.28 | 0.8284 | 0.001013 | 249.34 | 259.47 | 0.8258 |
| 80 | 0.001027 | 333.69 | 338.83 | 1.0719 | 0.001025 | 332.56 | 342.81 | 1.0687 |
| 100 | 0.001041 | 417.50 | 422.71 | 1.3030 | 0.001039 | 416.09 | 426.48 | 1.2992 |
| 120 | 0.001058 | 501.79 | 507.07 | 1.5232 | 0.001055 | 500.07 | 510.61 | 1.5188 |
| 140 | 0.001077 | 586.74 | 592.13 | 1.7342 | 0.001074 | 584.67 | 595.40 | 1.7291 |
| 160 | 0.001099 | 672.61 | 678.10 | 1.9374 | 0.001195 | 670.11 | 681.07 | 1.9316 |
| 180 | 0.001124 | 759.62 | 765.24 | 2.1341 | 0.001120 | 756.63 | 767.83 | 2.1274 |
| 200 | 0.001153 | 848.08 | 853.85 | 2.3254 | 0.001148 | 844.49 | 855.97 | 2.3178 |
| 220 | 0.001187 | 938.43 | 944.36 | 2.5128 | 0.001181 | 934.07 | 945.88 | 2.5038 |
| 240 | 0.001226 | 1031.34 | 1037.47 | 2.6978 | 0.001219 | 1025.94 | 1038.13 | 2.6872 |
| 260 | 0.001275 | 1127.92 | 1134.30 | 2.8829 | 0.001265 | 1121.03 | 1133.68 | 2.8698 |
| 280 | | | | | 0.001322 | 1220.90 | 1234.11 | 3.0547 |
| 300 | | | | | 0.001397 | 1328.34 | 1342.31 | 3.2468 |

TABLE B.1.2 (*continued*)

***Saturated Water Pressure Entry***

| | | Enthalpy, kJ/kg | | | Entropy, kJ/kg-K | | |
|---|---|---|---|---|---|---|---|
| **Press. (kPa)** | **Temp. (°C)** | **Sat. Liquid $h_f$** | **Evap. $h_{fg}$** | **Sat. Vapor $h_g$** | **Sat. Liquid $s_f$** | **Evap. $s_{fg}$** | **Sat. Vapor $s_g$** |
| 0.6113 | 0.01 | 0.00 | 2501.3 | 2501.3 | 0 | 9.1562 | 9.1562 |
| 1.0 | 6.98 | 29.29 | 2484.89 | 2514.18 | 0.1059 | 8.8697 | 8.9756 |
| 1.5 | 13.03 | 54.70 | 2470.59 | 2525.30 | 0.1956 | 8.6322 | 8.8278 |
| 2.0 | 17.50 | 73.47 | 2460.02 | 2533.49 | 0.2607 | 8.4629 | 8.7236 |
| 2.5 | 21.08 | 88.47 | 2451.56 | 2540.03 | 0.3120 | 8.3311 | 8.6431 |
| 3.0 | 24.08 | 101.03 | 2444.47 | 2545.50 | 0.3545 | 8.2231 | 8.5775 |
| 4.0 | 28.96 | 121.44 | 2432.93 | 2554.37 | 0.4226 | 8.0520 | 8.4746 |
| 5.0 | 32.88 | 137.79 | 2423.66 | 2561.45 | 0.4763 | 7.9187 | 8.3950 |
| 7.5 | 40.29 | 168.77 | 2406.02 | 2574.79 | 0.5763 | 7.6751 | 8.2514 |
| 10 | 45.81 | 191.81 | 2392.82 | 2584.63 | 0.6492 | 7.5010 | 8.1501 |
| 15 | 53.97 | 225.91 | 2373.14 | 2599.06 | 0.7548 | 7.2536 | 8.0084 |
| 20 | 60.06 | 251.38 | 2358.33 | 2609.70 | 0.8319 | 7.0766 | 7.9085 |
| 25 | 64.97 | 271.90 | 2346.29 | 2618.19 | 0.8930 | 6.9383 | 7.8313 |
| 30 | 69.10 | 289.21 | 2336.07 | 2625.28 | 0.9439 | 6.8247 | 7.7686 |
| 40 | 75.87 | 317.55 | 2319.19 | 2636.74 | 1.0258 | 6.6441 | 7.6700 |
| 50 | 81.33 | 340.47 | 2305.40 | 2645.87 | 1.0910 | 6.5029 | 7.5939 |
| 75 | 91.77 | 384.36 | 2278.59 | 2662.96 | 1.2129 | 6.2434 | 7.4563 |
| 100 | 99.62 | 417.44 | 2258.02 | 2675.46 | 1.3025 | 6.0568 | 7.3593 |
| 125 | 105.99 | 444.30 | 2241.05 | 2685.35 | 1.3739 | 5.9104 | 7.2843 |
| 150 | 111.37 | 467.08 | 2226.46 | 2693.54 | 1.4335 | 5.7897 | 7.2232 |
| 175 | 116.06 | 486.97 | 2213.57 | 2700.53 | 1.4848 | 5.6868 | 7.1717 |
| 200 | 120.23 | 504.68 | 2201.96 | 2706.63 | 1.5300 | 5.5970 | 7.1271 |
| 225 | 124.00 | 520.69 | 2191.35 | 2712.04 | 1.5705 | 5.5173 | 7.0878 |
| 250 | 127.43 | 535.34 | 2181.55 | 2716.89 | 1.6072 | 5.4455 | 7.0526 |
| 275 | 130.60 | 548.87 | 2172.42 | 2721.29 | 1.6407 | 5.3801 | 7.0208 |
| 300 | 133.55 | 561.45 | 2163.85 | 2725.30 | 1.6717 | 5.3201 | 6.9918 |
| 325 | 136.30 | 573.23 | 2155.76 | 2728.99 | 1.7005 | 5.2646 | 6.9651 |
| 350 | 138.88 | 584.31 | 2148.10 | 2732.40 | 1.7274 | 5.2130 | 6.9404 |
| 375 | 141.32 | 594.79 | 2140.79 | 2735.58 | 1.7527 | 5.1647 | 6.9174 |
| 400 | 143.63 | 604.73 | 2133.81 | 2738.53 | 1.7766 | 5.1193 | 6.8958 |
| 450 | 147.93 | 623.24 | 2120.67 | 2743.91 | 1.8206 | 5.0359 | 6.8565 |
| 500 | 151.86 | 640.21 | 2108.47 | 2748.67 | 1.8606 | 4.9606 | 6.8212 |
| 550 | 155.48 | 655.91 | 2097.04 | 2752.94 | 1.8972 | 4.8920 | 6.7892 |
| 600 | 158.85 | 670.54 | 2086.26 | 2756.80 | 1.9311 | 4.8289 | 6.7600 |
| 650 | 162.01 | 684.26 | 2076.04 | 2760.30 | 1.9627 | 4.7704 | 6.7330 |
| 700 | 164.97 | 697.20 | 2066.30 | 2763.50 | 1.9922 | 4.7158 | 6.7080 |
| 750 | 167.77 | 709.45 | 2056.98 | 2766.43 | 2.0199 | 4.6647 | 6.6846 |
| 800 | 170.43 | 721.10 | 2048.04 | 2769.13 | 2.0461 | 4.6166 | 6.6627 |

TABLE B.1.3

***Superheated Vapor Water***

| Temp. (°C) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) |
|---|---|---|---|---|---|---|---|---|
| | $P = 10$ kPa (45.81 °C) | | | | $P = 50$ kPa (81.33 °C) | | | |
| Sat. | 14.67355 | 2437.89 | 2584.63 | 8.1501 | 3.24034 | 2483.85 | 2645.87 | 7.5939 |
| 50 | 14.86920 | 2443.87 | 2592.56 | 8.1749 | — | — | — | — |
| 100 | 17.19561 | 2515.50 | 2687.46 | 8.4479 | 3.41833 | 2511.61 | 2682.52 | 7.6947 |
| 150 | 19.51251 | 2587.86 | 2782.99 | 8.6881 | 3.88937 | 2585.61 | 2780.08 | 7.9400 |
| 200 | 21.82507 | 2661.27 | 2879.52 | 8.9037 | 4.35595 | 2659.85 | 2877.64 | 8.1579 |
| 250 | 24.13559 | 2735.95 | 2977.31 | 9.1002 | 4.82045 | 2734.97 | 2975.99 | 8.3555 |
| 300 | 26.44508 | 2812.06 | 3076.51 | 9.2812 | 5.28391 | 2811.33 | 3075.52 | 8.5372 |
| 400 | 31.06252 | 2968.89 | 3279.51 | 9.6076 | 6.20929 | 2968.43 | 3278.89 | 8.8641 |
| 500 | 35.67896 | 3132.26 | 3489.05 | 9.8977 | 7.13364 | 3131.94 | 3488.62 | 9.1545 |
| 600 | 40.29488 | 3302.45 | 3705.40 | 10.1608 | 8.05748 | 3302.22 | 3705.10 | 9.4177 |
| 700 | 44.91052 | 3479.63 | 3928.73 | 10.4028 | 8.98104 | 3479.45 | 3928.51 | 9.6599 |
| 800 | 49.52599 | 3663.84 | 4159.10 | 10.6281 | 9.90444 | 3663.70 | 4158.92 | 9.8852 |
| 900 | 54.14137 | 3855.03 | 4396.44 | 10.8395 | 10.82773 | 3854.91 | 4396.30 | 10.0967 |
| 1000 | 58.75669 | 4053.01 | 4640.58 | 11.0392 | 11.75097 | 4052.91 | 4640.46 | 10.2964 |
| 1100 | 63.37198 | 4257.47 | 4891.19 | 11.2287 | 12.67418 | 4257.37 | 4891.08 | 10.4858 |
| 1200 | 67.98724 | 4467.91 | 5147.78 | 11.4090 | 13.59737 | 4467.82 | 5147.69 | 10.6662 |
| 1300 | 72.60250 | 4683.68 | 5409.70 | 14.5810 | 14.52054 | 4683.58 | 5409.61 | 10.8382 |
| | 100 kPa (99.62 °C) | | | | 200 kPa (120.23 °C) | | | |
| Sat. | 1.69400 | 2506.06 | 2675.46 | 7.3593 | 0.88573 | 2529.49 | 2706.63 | 7.1271 |
| 150 | 1.93636 | 2582.75 | 2776.38 | 7.6133 | 0.95964 | 2576.87 | 2768.80 | 7.2795 |
| 200 | 2.17226 | 2658.05 | 2875.27 | 7.8342 | 1.08034 | 2654.39 | 2870.46 | 7.5066 |
| 250 | 2.40604 | 2733.73 | 2974.33 | 8.0332 | 1.19880 | 2731.22 | 2970.98 | 7.7085 |
| 300 | 2.63876 | 2810.41 | 3074.28 | 8.2157 | 1.31616 | 2808.55 | 3071.79 | 7.8926 |
| 400 | 3.10263 | 2967.85 | 3278.11 | 8.5434 | 1.54930 | 2966.69 | 3276.55 | 8.2217 |
| 500 | 3.56547 | 3131.54 | 3488.09 | 8.8341 | 1.78139 | 3130.75 | 3487.03 | 8.5132 |
| 600 | 4.02781 | 3301.94 | 3704.72 | 9.0975 | 2.01297 | 3301.36 | 3703.96 | 8.7769 |
| 700 | 4.48986 | 3479.24 | 3928.23 | 9.3398 | 2.24426 | 3478.81 | 3927.66 | 9.0194 |
| 800 | 4.95174 | 3663.53 | 4158.71 | 9.5652 | 2.47539 | 3663.19 | 4158.27 | 9.2450 |
| 900 | 5.41353 | 3854.77 | 4396.12 | 9.7767 | 2.70643 | 3854.49 | 4395.77 | 9.4565 |
| 1000 | 5.87526 | 4052.78 | 4640.31 | 9.9764 | 2.93740 | 4052.53 | 4640.01 | 9.6563 |
| 1100 | 6.33696 | 4257.25 | 4890.95 | 10.1658 | 3.16834 | 4257.01 | 4890.68 | 9.8458 |
| 1200 | 6.79863 | 4467.70 | 5147.56 | 10.3462 | 3.39927 | 4467.46 | 5147.32 | 10.0262 |
| 1300 | 7.26030 | 4683.47 | 5409.49 | 10.5182 | 3.63018 | 4683.23 | 5409.26 | 10.1982 |
| | 300 kPa (133.55 °C) | | | | 400 kPa (143.63 °C) | | | |
| Sat. | 0.60582 | 2543.55 | 2725.30 | 6.9918 | 0.46246 | 2553.55 | 2738.53 | 6.8958 |
| 150 | 0.63388 | 2570.79 | 2760.95 | 7.0778 | 0.47084 | 2564.48 | 2752.82 | 6.9299 |
| 200 | 0.71629 | 2650.65 | 2865.54 | 7.3115 | 0.53422 | 2646.83 | 2860.51 | 7.1706 |

TABLE B.1.2

*Saturated Water Pressure Entry*

| Press. (kPa) | Temp. ( C) | Specific Volume, $m^3/kg$ | | | Internal Energy, kJ/kg | | |
|---|---|---|---|---|---|---|---|
| | | Sat. Liquid $v_f$ | Evap. $v_{fg}$ | Sat. Vapor $v_g$ | Sat. Liquid $u_f$ | Evap. $u_{fg}$ | Sat. Vapor $u_g$ |
| 0.6113 | 0.01 | 0.001000 | 206.131 | 206.132 | 0 | 2375.3 | 2375.3 |
| 1 | 6.98 | 0.001000 | 129.20702 | 129.20802 | 29.29 | 2355.69 | 2384.98 |
| 1.5 | 13.03 | 0.001001 | 87.97913 | 87.98013 | 54.70 | 2338.63 | 2393.32 |
| 2 | 17.50 | 0.001001 | 67.00285 | 67.00385 | 73.47 | 2326.02 | 2399.48 |
| 2.5 | 21.08 | 0.001002 | 54.25285 | 54.25385 | 88.47 | 2315.93 | 2404.40 |
| 3 | 24.08 | 0.001003 | 45.66402 | 45.66502 | 101.03 | 2307.48 | 2408.51 |
| 4 | 28.96 | 0.001004 | 34.79915 | 34.80015 | 121.44 | 2293.73 | 2415.17 |
| 5 | 32.88 | 0.001005 | 28.19150 | 28.19251 | 137.79 | 2282.70 | 2420.49 |
| 7.5 | 40.29 | 0.001008 | 19.23674 | 19.23775 | 168.76 | 2261.74 | 2430.50 |
| 10 | 45.81 | 0.001010 | 14.67254 | 14.67355 | 191.79 | 2246.10 | 2437.89 |
| 15 | 53.97 | 0.001014 | 10.02117 | 10.02218 | 225.90 | 2222.83 | 2448.73 |
| 20 | 60.06 | 0.001017 | 7.64835 | 7.64937 | 251.35 | 2205.36 | 2456.71 |
| 25 | 64.97 | 0.001020 | 6.20322 | 6.20424 | 271.88 | 2191.21 | 2463.08 |
| 30 | 69.10 | 0.001022 | 5.22816 | 5.22918 | 289.18 | 2179.22 | 2468.40 |
| 40 | 75.87 | 0.001026 | 3.99243 | 3.99345 | 317.51 | 2159.49 | 2477.00 |
| 50 | 81.33 | 0.001030 | 3.23931 | 3.24034 | 340.42 | 2143.43 | 2483.85 |
| 75 | 91.77 | 0.001037 | 2.21607 | 2.21711 | 394.29 | 2112.39 | 2496.67 |
| 100 | 99.62 | 0.001043 | 1.69296 | 1.69400 | 417.33 | 2088.72 | 2506.06 |
| 125 | 105.99 | 0.001048 | 1.37385 | 1.37490 | 444.16 | 2069.32 | 2513.48 |
| 150 | 111.37 | 0.001053 | 1.15828 | 1.15933 | 466.92 | 2052.72 | 2519.64 |
| 175 | 116.06 | 0.001057 | 1.00257 | 1.00363 | 486.78 | 2038.12 | 2524.90 |
| 200 | 120.23 | 0.001061 | 0.88467 | 0.88573 | 504.47 | 2025.02 | 2529.49 |
| 225 | 124.00 | 0.001064 | 0.79219 | 0.79325 | 520.45 | 2013.10 | 2533.56 |
| 250 | 127.43 | 0.001067 | 0.71765 | 0.71871 | 535.08 | 2002.14 | 2537.21 |
| 275 | 130.60 | 0.001070 | 0.65624 | 0.65731 | 548.57 | 1991.95 | 2540.53 |
| 300 | 133.55 | 0.001073 | 0.60475 | 0.60582 | 561.13 | 1982.43 | 2543.55 |
| 325 | 136.30 | 0.001076 | 0.56093 | 0.56201 | 572.88 | 1973.46 | 2546.34 |
| 350 | 138.88 | 0.001079 | 0.52317 | 0.52425 | 583.93 | 1964.98 | 2548.92 |
| 375 | 141.32 | 0.001081 | 0.49029 | 0.49137 | 594.38 | 1956.93 | 2551.31 |
| 400 | 143.63 | 0.001084 | 0.46138 | 0.46246 | 604.29 | 1949.26 | 2553.55 |
| 450 | 147.93 | 0.001088 | 0.41289 | 0.41398 | 622.75 | 1934.87 | 2557.62 |
| 500 | 151.86 | 0.001093 | 0.37380 | 0.37489 | 639.66 | 1921.57 | 2561.23 |
| 550 | 155.48 | 0.001097 | 0.34159 | 0.34268 | 655.30 | 1909.17 | 2564.47 |
| 600 | 158.85 | 0.001101 | 0.31457 | 0.31567 | 669.88 | 1897.52 | 2567.40 |
| 650 | 162.01 | 0.001104 | 0.29158 | 0.29268 | 683.55 | 1886.51 | 2570.06 |
| 700 | 164.97 | 0.001108 | 0.27176 | 0.27286 | 696.43 | 1876.07 | 2572.49 |
| 750 | 167.77 | 0.001111 | 0.25449 | 0.25560 | 708.62 | 1866.11 | 2574.73 |
| 800 | 170.43 | 0.001115 | 0.23931 | 0.24043 | 720.20 | 1856.58 | 2576.79 |