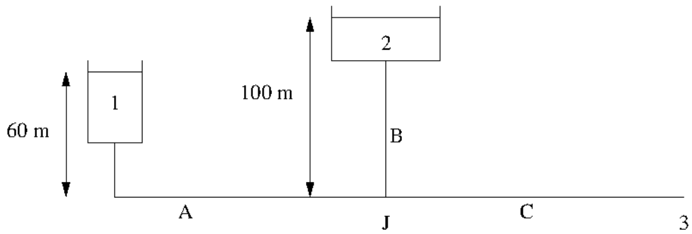
1	/30

1. (30 marks) Two reservoirs, containing water at 20° C, are connected by pipes to a common outlet (point 3 in the sketch, where the pressure is atmospheric). This problem will look at this system in several different ways, so read the information associated with each part of the question carefully, as some information that is given in one part of the question will specifically ***not*** be used in later parts.



	1	2	3
Surface height relative to J	60 m	100 m	0 m
Total pipe length to J	800 m	1000 m	600 m
Pipe diameter	20 cm	20 cm	20 cm
$f\frac{L}{d}$ (part b)	72	90	54
$\rho g \Delta z$	587 kPa	979 kPa	

(a) (9 marks) *Finding pressure change along the pipe from 1 to J.* For this part of the question, assume that the flow velocity in pipe A is 2 m/sec. Include the effects of minor losses at the entrance to the pipe (K=0.3), one 90° elbow (K=0.5), and one open gate valve (K=3). The pipe is cast iron (surface roughness 0.15 mm). Find the pressure as the flow in pipe A reaches the pipe junction J.

This is a straightforward pressure-loss-in-a-pipe question. We have to account for both major and minor losses, and for the change in elevation. Writing the energy / modified Bernoulli equation between a point on the surface of reservoir 1 and point J, we get:

$$P_1 + \rho g z_1 = P_J + \rho g z_J + \frac{1}{2}\rho V_A^2 + \frac{1}{2}\rho V_A^2 \left(f\frac{L}{d} + \sum K \right)$$

which of course we can simplify by taking advantage of the facts that $P_1 = 0$ gauge and $z_J = 0$. The next thing we need is a friction factor, which in turn means we need the Reynolds number and surface roughness ratio (0.15 mm/20 cm = 0.00075). Reynolds number, as always for pipe flow, uses diameter:

$$\begin{aligned} Re &= \frac{\rho V d}{\mu} \\ &= \frac{998\,{}^{\mathrm{kg}}\!/\!_{\mathrm{m}^3} \cdot 2\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \cdot 0.20\,\mathrm{m}}{1.003 \cdot 10^{-3}\,{}^{\mathrm{kg}}\!/\!_{\mathrm{m\cdot sec}}} = 398,000 \end{aligned}$$

And now the friction factor is 0.0192. So we get:

$$\begin{aligned} \rho g \cdot 60\,\mathrm{m} &= P_J + \frac{1}{2}\rho V_A^2 \left(1 + 0.0192\frac{800}{0.20} + (0.3 + 0.5 + 3) \right) \\ P_J &= 425\,\mathrm{kPa\,gauge} \end{aligned}$$

This is about 70% of the hydrostatic pressure due to elevation. This number is at least plausible: it's between the hydrostatic pressure available and atmospheric.

Finding the real flow speeds in all three pipes. For the rest of the problem, we are going to neglect all minor losses, and assume that the friction factor in all three pipes is 0.018 (that's not the right number, but it isn't far off).

(b) (6 marks) *Finding the flow speeds if P_J is known.* Suppose that we know that the pressure at J is $P_J = 450\,\mathrm{kPa}$ gauge (it isn't, but assume that for now). Based only on the pressure change in each pipe, and ignoring conservation of mass for the moment, find V_A, V_B, and V_C. (Hint: you can use the same approach you used for part a, solving for a different variable, to get V_A, and then do similar things for the other two pipes.)

Re-writing the pressure drop along pipe A, without minor losses, and tossing out terms that are zero:

$$\rho g z_1 = P_J + \frac{1}{2}\rho V_A^2 + \frac{1}{2}\rho V_A^2 f\frac{L_A}{d}$$

Solving this for V_A, we get:

$$V_A = \sqrt{\frac{\rho g z_1 - P_J}{\frac{1}{2}\rho\left(1 + f\frac{L_A}{d}\right)}} \tag{1}$$

Substituting numbers into this, we get:

$$V_A = 1.94\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}}$$

For pipe B, we can repeat exactly the same process, with different data, to get:

$$V_B = 3.41\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}}$$

For pipe C, the modified Bernoulli's equation gives us:

$$\begin{aligned} P_J + \rho g z_J + \frac{1}{2}\rho V_C^2 &= P_3 + \rho g z_3 + \frac{1}{2}\rho V_C^2 + \frac{1}{2}\rho V_C^2 f \frac{L_C}{d} \\ P_J &= \frac{1}{2}\rho V_C^2 f \frac{L_C}{d} \end{aligned}$$

where the last line represents cancellation of the common kinetic energy term and eliminating terms that are zero. Then

$$V_C = 4.09\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}}$$

These results obviously violate conservation of mass, since $V_A + V_B - V_C = 1.26\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}}$, when conservation of mass requires that to be zero.

(c) (3 marks) *The effect of changing P_J.* If you ***increase*** P_J, what effect will that have on the flow velocities in the three pipes? Check one box in each row.

	Increase	Decrease	Stay the same	Need more info
Velocity in A		✓		
Velocity in B		✓		
Velocity in C	✓			

(d) (12 marks) *Finding actual flow rates.* Make another guess for P_J, using your thinking in part iii to decide whether it should be higher or lower than 450 kPa. Based on this information and your result from part 1b, provide a better estimate of P_J (which could be used for the next iteration if we had lots of time). Explain your strategy, including how you will know when you're done.

Increasing P_J will decrease both of the flows towards the junction while increasing flow out of the junction, which is what we need. I'll try changing P_J to 500 kPa gauge (a 50 kPA increase) and see what I get. In this case

$$\begin{aligned} V_A &= 1.55\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \\ V_B &= 3.25\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \\ V_C &= 4.31\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \\ V_A + V_B - V_C &= 0.490\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \end{aligned}$$

So I've got two data points: $P_J = 450$ kPa $\longrightarrow \sum V = 1.26\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}}$ and $P_J = 500$ kPa $\longrightarrow \sum V = 0.49\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}}$. So this change in pressure changed $\sum V$ by 0.77. We know, based on conservation of mass and the given pipe dimensions, that $V_A + V_B - V_C = 0$ must be true for the actual system. Assuming linear behavior, this means I need to increase P_J by an addition $\frac{0.49}{0.77} \cdot 50$ kPa, making $P_J = 532$ kPa gauge.

Checking this (which you didn't need to do), I get

$$\begin{aligned} V_A &= 1.23\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \\ V_B &= 3.14\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \\ V_C &= 4.44\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \\ V_A + V_B - V_C &= -0.072\,{}^{\mathrm{m}}\!/\!_{\mathrm{sec}} \end{aligned}$$

So this extrapolation overshoots by a little bit. $P_J = 528$ kPa gauge pretty much nails it (to within 0.002 m/sec total velocity into the junction). Answers to this part will vary a bit depending on how much you varied pressure in your original guess.

We could also start with Eqn. 1 for V_A, then take its derivative with respect to pressure at the junction P_J. We can do the same for V_B and V_C. Then we can combine those to get

$$\frac{d}{dP_J}\left(V_A + V_B - V_C\right).$$

Now we have a linearization of the flow rate imbalance as a function of pressure at the junction, which we can use to get a good starting estimate.