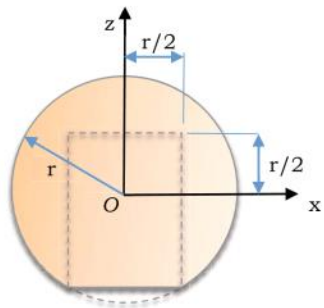
Test 4 Dynamics Solutions 2017W

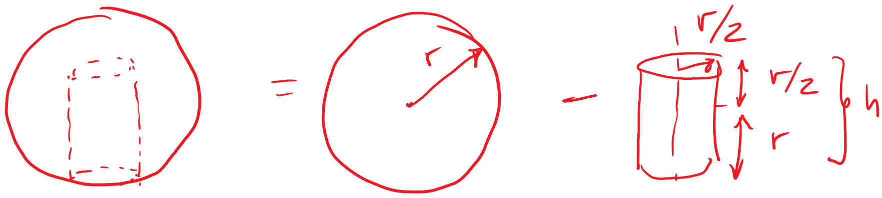
October 21, 2017 11:46 PM

SA 1. (5 marks)



Find the mass moment of inertia, I_{zz}, for the spherical ceramic femoral head ball (with a cylindrical hole) from a total hip replacement, expressed in terms of head radius, r, and uniform density, ρ.

(Recall that reasonable approximations at the intersection of round and flat objects are acceptable; select a simple approximation).



$$I_{zz\,head} = \frac{2}{5} m_s r^2 - \frac{1}{2} m_c \left(\frac{r}{2}\right)^2$$

$$m_{sphere} = \rho V = \rho \frac{4}{3}\pi r^3$$

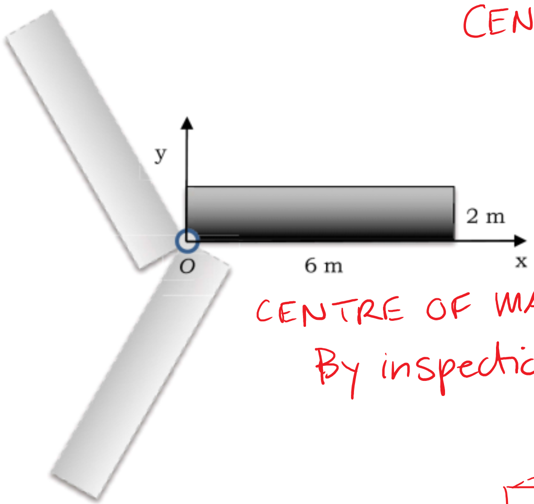
$$m_{cyl.} = \rho\left(\pi\left(\frac{r}{2}\right)^2 \underbrace{\left(r+\frac{r}{2}\right)}_{h}\right) = \rho\pi\frac{r^2}{4}\left(\frac{3r}{2}\right)$$

$$I_{zz,head} = \frac{2}{5}\left(\rho\frac{4}{3}\pi r^3\right) r^2 - \frac{1}{2}\left(\rho\pi\frac{r^2}{4}\left(\frac{3r}{2}\right)\right)\left(\frac{r^2}{4}\right)$$

$$\boxed{I_{zz\,head} = \rho\pi r^5\left(\frac{8}{15}-\frac{3}{64}\right)}$$

Prob 1. (25 marks)

(a) (~~5~~ 8 marks) Find the centroid (x_C, y_C) and centre of mass (x_G, y_G) of the **single wind turbine blade** highlighted below. Density varies such that ρ = 3(2-y) kg/m^3. Assume the blade is 0.1 m thick.



CENTROID:

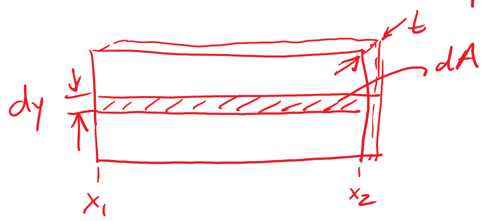
By inspection:

$$\boxed{(x_c, y_c) = (3\,m, 1\,m)}$$

CENTRE OF MASS:

By inspection: $x_G = 3\,m$ (density does not vary with x)

$$y_G = \frac{\iint_m y\,dm}{\iint_m dm}$$



$$dV = (x_2 - x_1)\,t\,dy = (6-0)(0.1)dy = 0.6\,dy$$

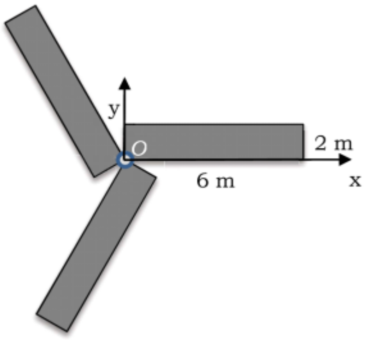
$$dm = \rho\,dV = 3(2-y)(0.6)\,dy$$

$$dm = (3.6 - 1.8y)\,dy$$

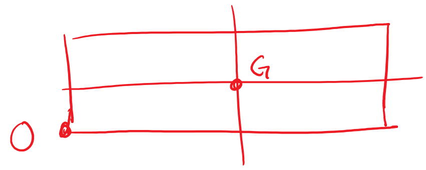
$$y_G = \frac{\int_0^2 y(3.6 - 1.8y)\,dy}{\int_0^2 (3.6 - 1.8y)\,dy} = \frac{\int_0^2 3.6y - 1.8y^2\,dy}{\int_0^2 3.6 - 1.8y\,dy} = \frac{\left[\frac{3.6y^2}{2} - \frac{1.8y^3}{3}\right]_0^2}{\left[3.6y - \frac{1.8y^2}{2}\right]_0^2}$$

$$= \frac{\frac{3.6(2)^2}{2} - \frac{1.8(2)^3}{3}}{3.6(2) - \frac{1.8(2)^{\cancel{2}}}{\cancel{2}}} = \frac{2.4}{3.6} = \frac{2}{3}\,m \qquad \boxed{(x_G, y_G) = (3\,m, \tfrac{2}{3}\,m)}$$

(b) ~~(10~~ 9 **marks)** Find I_{zz} at the centre hub, O, for the **complete 3-blade wind turbine** below. Assume equally-spaced blades. Assume density of the blade is constant at 5 kg/m³ for this part of the question. Assume the blades are 0.1 m thick.



First, I_{zz} @ G for one blade:



By inspection:

$(x_G, y_G) = (x_c, y_c)$

$= (3m, 1m)$

From table:

$I_{zz,G} = \frac{1}{12} m (a^2 + b^2) = \frac{1}{12} m (2^2 + 6^2)$

thin plate

$m_{\text{one blade}} = \rho V = (5\ kg/m^3)(2m)(6m)(0.1m)$

$= 6\ kg$

$I_{zz,G} = \frac{1}{12}(6)(4+36) = 20\ kg\text{-}m^2$

Second, $I_{zz,o}$ (one blade) (parallel axes):

$I_{zz,o} = I_{zz,G} + md^2$ $\qquad d^2 = (x_G^2 + y_G^2) = (3^2 + 1^2) = 10$

$= 20\ kg\text{-}m^2 + 6\ kg(10m^2)$

$= 80\ kg\text{-}m^2$

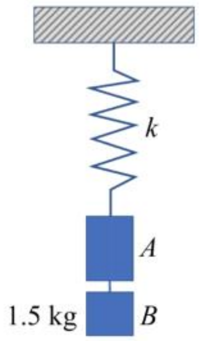
Third, add $I_{zz,o}$ for 3 blades:

$I_{zz,o\ \text{total}} = 3\, I_{zz,o} = 3(80\ kg\text{-}m^2)$

$\boxed{I_{zz,o\ \text{total}} = 240\ kg\text{-}m^2}$

8

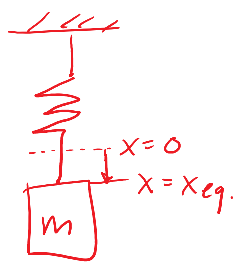
(c) **(~~10~~ marks)** A spring has two masses, *A* and *B*, hanging down from it. The period of vibration with both masses attached is 0.6 s, and with only *A* attached (*B* removed) is 0.5 s. The mass of *B* is 1.5 kg. **Find the mass of *A* and the spring constant, *k*.**

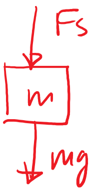


Note: if you start with writing $\ddot{x} + \frac{k}{m}x = 0$ for each case, that is acceptable

FOR EITHER CASE:

Equilibrium





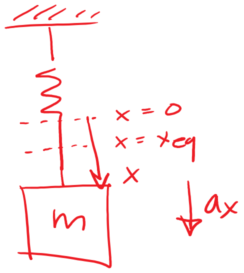
$F_s = -kx_{eq}$

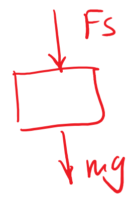
ΣF_x: $F_s + mg = 0$

$-kx_{eq} + mg = 0$

$\Rightarrow kx_{eq} = mg$

Perturbation:





$F_s = -k(x + x_{eq})$

ΣF_x: $F_s + mg = ma_x$

$-kx \underbrace{- k\cancel{x_{eq}} + \cancel{mg}}_{\text{equal/opp}} = ma_x$

$\Rightarrow ma_x + kx = 0$

$\ddot{x} + \frac{k}{m}x = 0$

$\tau = \frac{2\pi}{\omega_n}$ (period)

$\omega_n = \frac{2\pi}{\tau}$, $\omega_n^2 = \frac{4\pi^2}{\tau^2}$

A+B CASE:

$\ddot{x} + \frac{k}{m_A + m_B}x = 0$, $\omega_n^2 = \frac{k}{m_A + m_B} = \frac{4\pi^2}{(0.6)^2}$

$\frac{k}{m_A + m_B} = 109.6$ (1)

A ONLY CASE:

$\ddot{x} + \frac{k}{m_A}x = 0$, $\omega_n^2 = \frac{k}{m_A} = \frac{4\pi^2}{(0.5)^2}$

$k = 157.8\, m_A$ (2)

From (1) + (2):

$109.6\, m_A + 109.6(1.5) = 157.8\, m_A$

$\Rightarrow m_A = \frac{109.6(1.5)}{(157.8 - 109.6)} = \boxed{3.4\text{kg}}$

$k = 157.8(3.4)$

$\boxed{k = 536.5 \text{ kg-m}^2}$