Name: ____________________ **Signature:** ____________________ **UBC ID:** ______________

University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

MECH 222
Exam 2: 2 April 2015

Duration: 3 hours

This exam consists of 21 pages (including this cover sheet). Check to ensure that it is complete.

Rules Governing Formal Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
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   - (b) purposely exposing written papers to the view of other candidates or imaging devices;
   - (c) purposely viewing the written papers of other candidates;
   - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
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7. Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Question	**Marks**	**Score**
1	15	
2	15	
3	20	
4ab	10	
4c	3	
5ab	15	
5c	5	
6ab	15	
6cd	10	
6ef	12	
TOTAL	120	

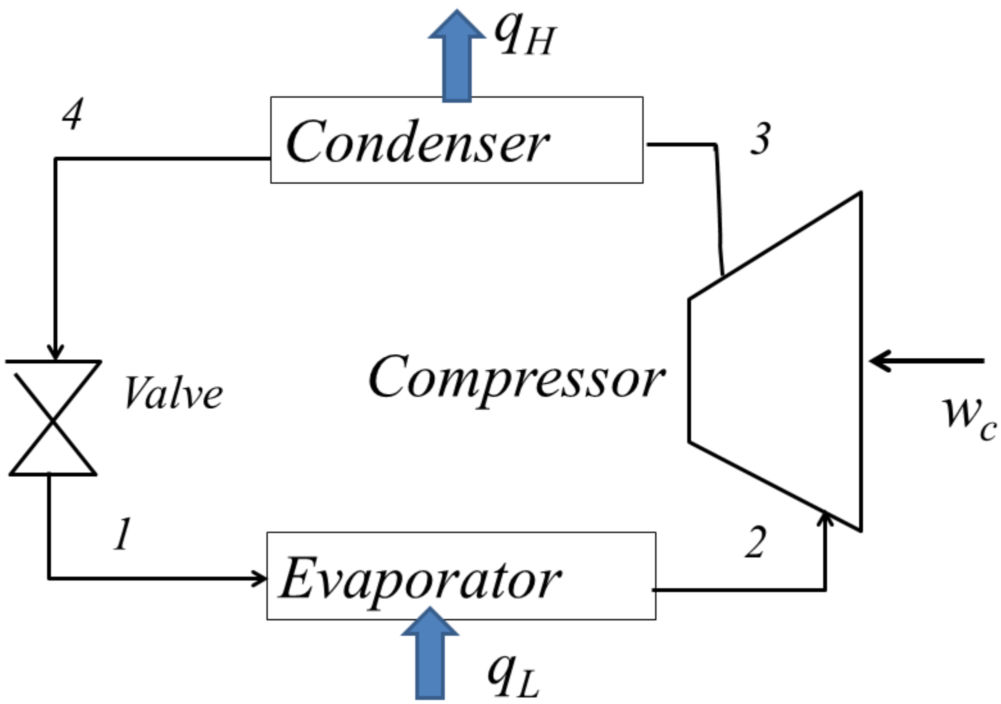
1. (15 marks) Refrigerant R-134a (tables attached) is used in a vapor-compression cycle for a building air conditioning system.

   The evaporator (States 1-2) is to be run at a pressure of 165 kPa.

   The compressor (states 2-3) is to be considered adiabatic and reversible in this problem. $T_3 = 50^\circ$ C

   The condenser (3-4) is to run at 1200 kPa. State 4 is saturated liquid.

   The pressure is let down again (4-1) through an insulated control valve.



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Saturated R-134a

Temp. (°C)	Press. (kPa)	Enthalpy, kJ/kg: Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Entropy, kJ/k-K: Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
−70	8.3	119.47	235.15	354.62	0.6645	1.1575	1.8220
−65	11.7	123.18	234.55	357.73	0.6825	1.1268	1.8094
−60	16.3	127.53	233.33	360.86	0.7031	1.0947	1.7978
−55	22.2	132.37	231.63	364.00	0.7256	1.0618	1.7874
−50	29.9	137.62	229.54	367.16	0.7493	1.0286	1.7780
−45	39.6	143.18	227.14	370.32	0.7740	0.9956	1.7695
−40	51.8	148.98	224.50	373.48	0.7991	0.9629	1.7620
−35	66.8	154.98	221.67	376.64	0.8245	0.9308	1.7553
−30	85.1	161.12	218.68	379.80	0.8499	0.8994	1.7493
−26.3	101.3	165.80	216.36	382.16	0.8690	0.8763	1.7453
−25	107.2	167.38	215.57	382.95	0.8754	0.8687	1.7441
−20	133.7	173.74	212.34	386.08	0.9007	0.8388	1.7395
−15	165.0	180.19	209.00	389.20	0.9258	0.8096	1.7354
−10	201.7	186.72	205.56	392.28	0.9507	0.7812	1.7319
−5	244.5	193.32	202.02	395.34	0.9755	0.7534	1.7288
0	294.0	200.00	198.36	398.36	1.0000	0.7262	1.7262
5	350.9	206.75	194.57	401.32	1.0243	0.6995	1.7239
10	415.8	213.58	190.65	404.23	1.0485	0.6733	1.7218
15	489.5	220.49	186.58	407.07	1.0725	0.6475	1.7200
20	572.8	227.49	182.35	409.84	1.0963	0.6220	1.7183
25	666.3	234.59	177.92	412.51	1.1201	0.5967	1.7168
30	771.0	241.79	173.29	415.08	1.1437	0.5716	1.7153
35	887.6	249.10	168.42	417.52	1.1673	0.5465	1.7139
40	1017.0	256.54	163.28	419.82	1.1909	0.5214	1.7123
45	1160.2	264.11	157.85	421.96	1.2145	0.4962	1.7106
50	1318.1	271.83	152.08	423.91	1.2381	0.4706	1.7088
55	1491.6	279.72	145.93	425.65	1.2619	0.4447	1.7066
60	1681.8	287.79	139.33	427.13	1.2857	0.4182	1.7040
65	1889.9	296.09	132.21	428.30	1.3099	0.3910	1.7008
70	2117.0	304.64	124.47	429.11	1.3343	0.3627	1.6970
75	2364.4	313.51	115.94	429.45	1.3592	0.3330	1.6923
80	2633.6	322.79	106.40	429.19	1.3849	0.3013	1.6862
85	2926.2	332.65	95.45	428.10	1.4117	0.2665	1.6782
90	3244.5	343.38	82.31	425.70	1.4404	0.2267	1.6671
95	3591.5	355.83	64.98	420.81	1.4733	0.1765	1.6498
100	3973.2	374.74	32.47	407.21	1.5228	0.0870	1.6098
101.2	4064.0	390.98	0	390.98	1.5658	0	1.5658

Superheated R-134a

Temp. (°C)	v (m^3/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)	v (m^3/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	800 kPa (31.30°C)				1000 kPa (39.37°C)			
Sat.	0.02571	395.15	415.72	1.7150	0.02038	399.16	419.54	1.7125
40	0.02711	403.17	424.86	1.7446	0.02047	399.78	420.25	1.7148
50	0.02861	412.23	435.11	1.7768	0.02185	409.39	431.24	1.7494
60	0.03002	421.20	445.22	1.8076	0.02311	418.78	441.89	1.7818
70	0.03137	430.17	455.27	1.8373	0.02429	428.05	452.34	1.8127
80	0.03268	439.17	465.31	1.8662	0.02542	437.29	462.70	1.8425
90	0.03394	448.22	475.38	1.8943	0.02650	446.53	473.03	1.8713
100	0.03518	457.35	485.50	1.9218	0.02754	455.82	483.36	1.8994
110	0.03639	466.58	495.70	1.9487	0.02856	465.18	493.74	1.9268
120	0.03758	475.92	505.99	1.9753	0.02956	474.62	504.17	1.9537
130	0.03876	485.37	516.38	2.0014	0.03053	484.16	514.69	1.9801
140	0.03992	494.94	526.88	2.0271	0.03150	493.81	525.30	2.0061
150	0.04107	504.64	537.50	2.0525	0.03244	503.57	536.02	2.0318
160	0.04221	514.46	548.23	2.0775	0.03338	513.46	546.84	2.0570
170	0.04334	524.42	559.09	2.1023	0.03431	523.46	557.77	2.0820
180	0.04446	534.51	570.08	2.1268	0.03523	533.60	568.83	2.1067
	1200 kPa (46.31°C)				1400 kPa (52.42°C)			
Sat.	0.01676	402.37	422.49	1.7102	0.01414	404.98	424.78	1.7077
50	0.01724	406.15	426.84	1.7237	—	—	—	—
60	0.01844	416.08	438.21	1.7584	0.01503	413.03	434.08	1.7360
70	0.01953	425.74	449.18	1.7908	0.01608	423.20	445.72	1.7704
80	0.02055	435.27	459.92	1.8217	0.01704	433.09	456.94	1.8026
90	0.02151	444.74	470.55	1.8514	0.01793	442.83	467.93	1.8333
100	0.02244	454.20	481.13	1.8801	0.01878	452.50	478.79	1.8628
110	0.02333	463.71	491.70	1.9081	0.01958	462.17	489.59	1.8914
120	0.02420	473.27	502.31	1.9354	0.02036	471.87	500.38	1.9192
130	0.02504	482.91	512.97	1.9621	0.02112	481.63	511.19	1.9463
140	0.02587	492.65	523.70	1.9884	0.02186	491.46	522.05	1.9730
150	0.02669	502.48	534.51	2.0143	0.02258	501.37	532.98	1.9991
160	0.02750	512.43	545.43	2.0398	0.02329	511.39	543.99	2.0248
170	0.02829	522.50	556.44	2.0649	0.02399	521.51	555.10	2.0502
180	0.02907	532.68	567.57	2.0898	0.02468	531.75	566.30	2.0752

Refrigeration cycle problem continued.

(a) (4 marks) Write the First Law, appropriately simplified, for the 4 control volumes of this cycle.

(b) (3 marks) Sketch the T-s diagram for this cycle. Indicate what property (if any) is constant along each segment of the cycle.

(c) (8 marks) Determine the COP (q_L/w_c). Along the way, you will need to complete most (but not all!) of the table below.

State	In to:	Temp. °C	Pressure kPa	Enthalpy kJ/kg	Entropy kJ/kg/K
1	evaporator		165		
2	compressor		165		
3	condenser	50	1200		
4	valve		1200		

2. (15 marks) On Exam 1, you were asked to non-dimensionalize the behavior of a cup anemometer, for which the rotation speed Ω (in radians / second) for a cup anemometer depends on the wind speed U, the air density ρ and viscosity μ (in kg/m-sec), the diameter d of the cups and the length l of the rods on which the cups are mounted. One reasonable result for this problem is:

$$\frac{\Omega l}{U} = f\left(\frac{\rho U d}{\mu}, \frac{l}{d}\right) \tag{1}$$

where the non-dimensional rotation rate that you would ordinarily get has been multiplied by l/d to get the ratio of the speed at which the cups are moving to the wind speed.

You were asked on Exam 1 to use scaling to find the experimental conditions for a small scale model (4 cm cups as opposed to the 30 cm cups for the full-scale anemometer) and a wind speed of 6 m/sec; under those conditions, the test wind speed is 45 m/sec.

(a) (7 marks) A full-scale wind speed of 6 m/sec on Exam 1 isn't very fast: only about 20 km/hr. To be a useful wind gauge at an airport, we need to be able to measure speeds as high as 120 km/hr accurately. What test speed would this correspond to for your scale model anemometer? What problems do you anticipate if you try to test under these conditions? What could you do to address these problems?

(b) (4 marks) You test your scale model in air at a range of wind speeds and with arms of several different lengths. After analyzing the data, you determine that the Reynolds number has little effect on the (dimensionless) rotation speed and that Ω is proportional to $l^{-3/2}$. Using this physical information and the non-dimensional relationship in Eq. 1, write Ω in terms of the wind speed, anemometer dimensions, and a constant α.

(c) (4 marks) Using the data in this table, estimate the constant α to two significant digits.

d (m)	L (m)	U (m/sec)	Ω (rad/sec)
0.04	0.10	10.0	44.5
0.04	0.14	11.0	29.3
0.04	0.175	11.0	21.0

3. (20 marks) The vector field $\mathbf{F} = (e^{-y^2z^2} - 2x)\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z+1)\hat{\mathbf{k}}$ is divergence free (incompressible): $\nabla \cdot \mathbf{F} = 0$ (as you can easily check). A tube which is initially a cylinder $x^2 + y^2 = 1$, $0 \le z \le 2$, becomes warped (in its lower half), resulting in a surface S_1 described by:

$$S_1 : \qquad x^2 + y^2 = \left\{ \begin{array}{ll} 1 + \frac{1}{100}\sin(4\pi z) & 0 \le z \le 1 \\ 1 & 1 \le z \le 2 \end{array} \right\}, \quad 0 \le z \le 2.$$

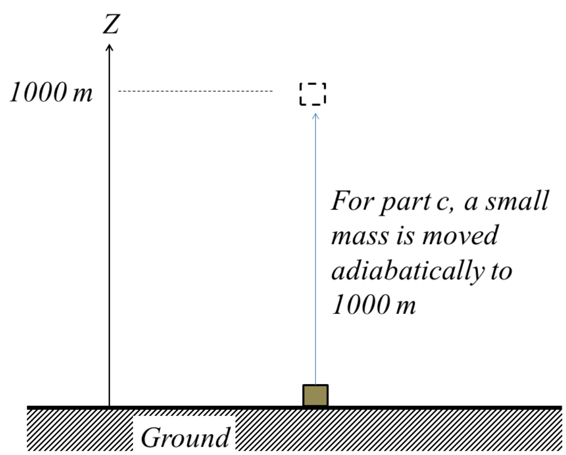
Orient S_1 with its normal directed outside the cylinder (away from $x = y = 0$).

(a) (10 marks) Give a rough sketch of S_1, and use the Divergence Theorem to compute the flux integral $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$.

(b) (10 marks) A curved blade removes the part of the warped tube S_1 lying above $z = 2 - y^2$. Denote the remaining part (the part below $z = 2 - y^2$) by S_2. Give a rough sketch of S_2, and use the Divergence Theorem to compute the flux integral $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$.

4. (13 marks) Every day, all over the world, meteorologists send balloons into the sky to measure the vertical temperature profile of the atmosphere $T(z)$. For a particular location and day, it was found that temperature actually increases with height[1] according to $T(z) = T_0 + az$ with $a = .002$ K$/m$ and ground-level temperature $T_0 = 280$K.

   (a) (8 marks) Derive an expression for the pressure profile $P(z)$ for a linear temperature profile, using $P(z = 0) = P_0 = 101$ kPa. Obtain a result using symbols. Model the thermodynamic properties of air in the usual way.



[1] Typically, air temperature decreases with altitude.

(b) (2 marks) Compute the air temperature T, pressure P, and density ρ at an elevation of $z = 1000\,\mathrm{m}$.

(c) (3 marks) Suppose a parcel of air is carried from the ground to 1000 m, slowly enough that it has nearly the pressure of the surrounding atmosphere (i.e., $P(z)$), but fast enough that the parcel does not transfer any heat with the surroundings. When the parcel gets to the new elevation, would it tend to float further up, or sink back towards the ground? Explain.

5. (20 marks) Some aircraft parts are machined from cast ingots of aluminum. An ingot is a block of metal formed by pouring molten metal into a mold. The shape of the ingot is the box: $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le \pi/2$ (dimensions in meters). We are planning to produce millions of these, so the prototype casting process will be examined in detail. Prior to casting, a number of thermocouples are placed in the mold, so that after the casting occurs, we can track the temperature at different locations.

   After the metal has solidified, it seems that the measurements fit the function

   $T(x, y, z, t)[^oC] = F(t)G(x, y, z)$

   with $F(t) = 500e^{-\alpha t}$, $G(x, y, z) = x\cos(z)$ and $\alpha = k/(\rho C)$.

   We are skeptical of measurements, so we will use some math and thermodynamics to decide whether the measurements are physically reasonable. Three key ideas you will need are:

   - The specific internal energy of this aluminum is simply $u = CT$, where C=600 J/kg/^oC.
   - The density of the metal is uniform and independent of time, $\rho = 3000\ kg/m^3$.
   - Recall that the heat flux in 1-dimensional conduction, in W/m^2 is $q = -k\frac{dT}{dx}$ where x is a spatial coordinate [m] and $k = 150$ is the conductivity [W/m/K]. This relationship generalizes to 3-dimensions as $\mathbf{q} = -k\nabla T = -kF(t)\nabla G(x, y, z)$. This means that the heat flux through some surface element is $\mathbf{q} \cdot \mathbf{dS}$.

   (a) (7 marks) For time t=0, find the total internal energy U in the ingot and dU/dt.

(b) (8 marks) At time $t = 0$ compute the total heat transfer through the surface of ingot, from the conduction equation above. Are any surfaces of the ingot insulated?

(c) (5 marks) The energy and heat transfer should be related by some Law. Use this to check that the temperature function is physically possible. If you had trouble with parts (a) and (b), at least make it clear how you would use those results to make the check.

6. (37 marks) A rectangular duct, shown in the sketch, carries a flow of air (which we will treat as a perfect gas with $R = 287\,{}^{\mathrm{J}}/_{\mathrm{kg\cdot K}}$, $C_p = 1000\,{}^{\mathrm{J}}/_{\mathrm{kg\cdot K}}$, and $C_v = 713\,{}^{\mathrm{J}}/_{\mathrm{kg\cdot K}}$).

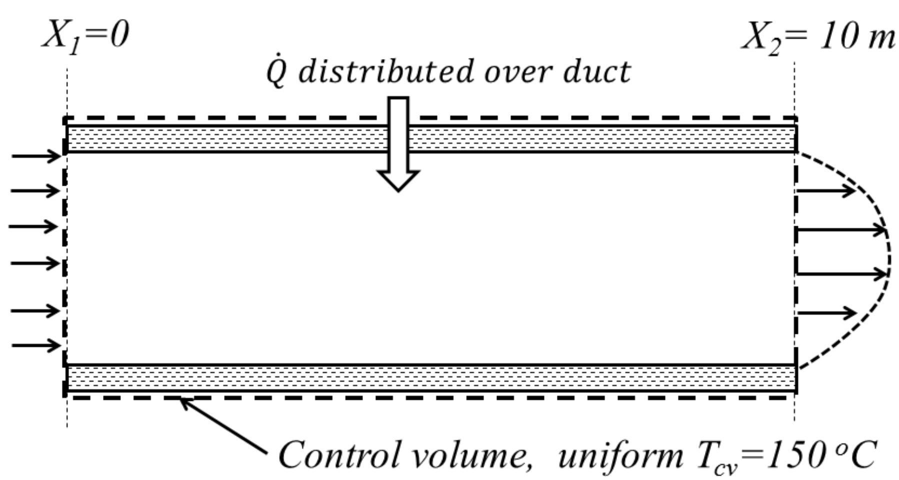
- The duct is ten meters long, and its cross section is a 0.1×0.1 meter square.
- At the duct inlet (station 1 in the sketch), the air temperature is 30° C, the pressure is 10 kPa gauge, and the air velocity is a uniform $U_1 = 25\,{}^{\mathrm{m}}/_{\mathrm{sec}}$.
- At the duct outlet (station 2), the velocity is given by

$$u\left(x_2, y, z\right) = 1600 U_m \left(y - 10y^2\right)\left(z - 10z^2\right)$$

  where U_m is the maximum velocity at the exit plane.
- Because of heat transfer, the temperature of the air inside the duct is $T\left(x, y, z\right) = 30 + 4x$, in degrees Celsius.
- Because the flow is viscous and turbulent, the pressure varies along the duct according to:

$$\frac{dP}{dx} = -\alpha \overline{V}^2$$

  where $\overline{V}$ is the average velocity in the duct at that position and $\alpha = 1.2\,{}^{\mathrm{kg}}/_{\mathrm{m}^4}$.
- At all locations along the duct, density, temperature and pressure are uniform *across* the duct, although they may all vary *along the length* of the duct. That is, ρ, P, and T are functions of x only.



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(a) (5 marks)Find U_m in terms of U_1, ρ_1, and ρ_2. (We'll get back to ρ_2 in part 6d)

(b) (10 marks) Find the x-component of the force (F_x in the sketch) required to hold the duct in place. Write your answer in terms of U_1, U_m, ρ_1, ρ_2, P_1, and P_2. (We'll get back to P_2 in part 6d also.)

(c) (3 marks) Is the average velocity $\overline{V}$ constant along the duct, or does it vary with position? Explain.

(d) (7 marks) Using the known air temperature, the perfect gas law, conservation of mass, and whatever other principles you need, write a differential equation for $P(x)$ as a function of x and given quantities only. *Do not solve this equation.*

(e) (5 marks)Neglecting changes in kinetic energy along the duct, find the heat addition (in watts) required to produce this temperature distribution.

(f) (7 marks) Write an expression for the entropy generation in the indicated control volume. Not all quantities needed to evaluate $\dot{S}_{gen}$ have been determined yet. Explain how you would find the missing required property(ies).