1. Math

2. Thermo

3. (10 marks) The modern alternative to kitchen aerosol oil sprayers that use compressed gas is refillable oil sprayers that are pressurized by pumping by hand. These sprayers do not always produce a spray with small enough droplets. You have a new design in mind for a spray nozzle for such a sprayer, and you want to test it out. To avoid the mess of cleaning up sprayed oil, you want to test with water, which means you need to do a dimensional analysis to help you design your experiment.

   You know that the size of the oil droplets (given by their diameter D) depends on the gauge pressure P in the spray bottle, the size of the nozzle (let's call its diameter δ to avoid confusion), and the density ρ, viscosity μ, and surface tension σ of the oil. That is, you know that

$$D = fn\left(P, \delta, \rho, \mu, \sigma\right)$$

   Write a functional relationship for the non-dimensional drop size in terms of other non-dimensional parameters. Note: the dimensions of the surface tension σ are $\frac{\mathrm{m}}{\mathrm{t}^2}$ (that is, $\mathrm{kg}/\mathrm{sec}^2$ in SI).

   We begin by writing down the dimensions of all the parameters:

D	P	δ	ρ	μ	σ
L	$\frac{m}{t^2 L}$	L	$\frac{m}{L^3}$	$\frac{m}{Lt}$	$\frac{m}{t^2}$

   Now I'll use δ to eliminate length from all the other variables:

D/δ	$P\delta$	$\rho\delta^3$	$\mu\delta$	σ
1	$\frac{m}{t^2}$	m	$\frac{m}{t}$	$\frac{m}{t^2}$

   And now $\rho\delta^3$ to eliminate mass:

D/δ	$\frac{P}{\rho\delta^2}$	$\frac{\mu}{\rho\delta^2}$	$\frac{\sigma}{\rho\delta^3}$
1	$\frac{1}{t^2}$	$\frac{1}{t}$	$\frac{1}{t^2}$

   None of these look terribly promising to eliminate time. I'll use the pressure term, which is easy with surface tension but requires a square root for the viscosity term:

$$\frac{\mu}{\rho\delta^2}\sqrt{\frac{\rho\delta^2}{P}} = \frac{\mu}{\sqrt{\rho\delta^2 P}}$$

D/δ	$\frac{\mu}{\sqrt{\rho\delta^2 P}}$	$\frac{\sigma}{P\delta}$
1	1	1

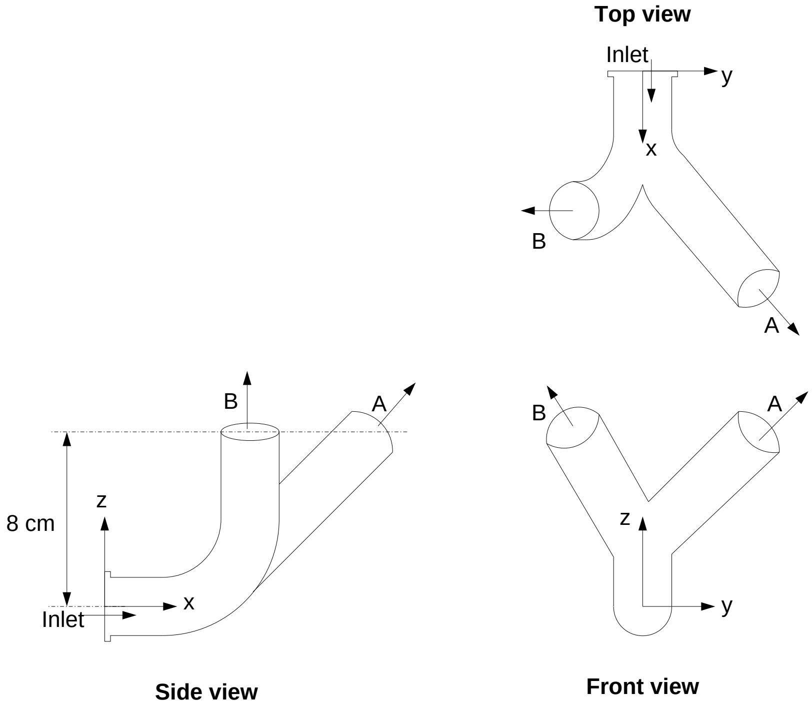
   In the end, I can write

$$\frac{D}{\delta} = fn_2\left(\frac{\mu}{\sqrt{\rho\delta^2 P}}, \frac{\sigma}{P\delta}\right)$$

4. Math

5. (25 marks) A decorative fountain has a nozzle that produces two jets of water, as shown in the sketch. Use the information given below to find the total force (all components) required at the inlet flange to hold the nozzle in place.

   - Both jets leave the nozzle into atmospheric pressure air.
   - Both jets have a cross-sectional area of 1 cm^2 and an exit speed of 5 m/sec.
   - Water leaves Jet A in the direction $\frac{\sqrt{2}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath} + \frac{1}{2}\hat{k}$.
   - Water leaves Jet B in the direction $-\frac{1}{2}\hat{\jmath} + \frac{\sqrt{3}}{2}\hat{k}$.
   - The inflow to the nozzle also has an area of 1 cm^2 and is straight in the x-direction.
   - The total weight of the nozzle and the water in it is 8 Newtons.
   - Viscous losses in the nozzle are negligibly small.



Solution: To find total force, we need to do a control volume analysis. By far the easiest control volume is one that cuts through the inlet flange and surrounds the entire rest of the nozzle. The only external forces on the control volume are the force restraining the nozzle, the pressure force at the inlet (using gauge pressure), and the weight (acting in the negative z-direction, as implied by the Side View/Front View labels and axes on those views). There are also momentum fluxes at the inlet and both exits.

We are given flow conditions at the exits but not at the inlet, where we will need to find the velocity (for momentum flux) and the pressure (for pressure force). To find the inflow velocity, we apply mass conservation:

$$\begin{aligned} \dot{m}_{\text{in}} &= \dot{m}_{\text{out}} \\ \rho V_{\text{in}} A_{\text{in}} &= \rho\left(V_{\text{out,A}} A_{\text{out,A}} + V_{\text{out,B}} A_{\text{out,B}}\right) \end{aligned}$$

The inlet and both outlets have the same area, so both density and area will cancel here, and mass conservation requires that

$$V_{\text{in}} = V_{\text{out,A}} + V_{\text{out,B}} = 5 + 5 = 10\frac{\text{m}}{\text{sec}}$$

As for the inlet pressure, we know that there are streamlines from the inlet to each outlet, which suggests that we may be able to apply Bernoulli's equation. Checking other requirements for Bernoulli: steady (yes), incompressible (yes), no viscous losses (given as an assumption), no work done (yes), no heat transfer (nothing said explicitly about this, but there's no obvious reason for any heat transfer at all, so we can reasonably assume that any heat transfer will be negligibly small compared with the flow kinetic energy). So, after all of that, yes, it looks like we can correctly use Bernoulli's equation:

$$\begin{aligned}
P_{\text{in}} + \frac{1}{2}\rho V_{\text{in}}^2 + \rho g z_{\text{in}} &= P_{\text{A}} + \frac{1}{2}\rho V_{\text{A}}^2 + \rho g z_{\text{A}} \\
P_{\text{in}} - P_{\text{atm}} &= \frac{1}{2}\rho\left(V_{\text{A}}^2 - V_{\text{in}}^2\right) + \rho g\left(z_{\text{A}} - z_{\text{in}}\right) \\
&= \frac{1}{2}\cdot 998 \cdot \left(5^2 - 10^2\right) + 998 \cdot 9.81 \cdot (0.08) \\
&= -37.4 \text{ kPa} + 783 \text{ Pa} \\
&= -36.6 \text{ kPa}
\end{aligned}$$

The gravity terms is small (about 2% of the pressure difference).

Summing forces and momentum in the x-direction, and taking advantage of having a uniform velocity to simplify momentum flux calculations. The $\frac{\sqrt{2}}{2}$ is the component of momentum flux out at A that is in the x-direction. Note that I'm assuming the F_x is positive (to the right):

$$\begin{aligned}
+\rightarrow \sum F_x : \quad R_x + P_A A_A &= \left(\rho V_{\text{A}}\frac{\sqrt{2}}{2}\right)\left(V_A A_{\text{A}}\right) - \left(\rho V_{\text{in}}\right)\left(V_{\text{in}} A_{\text{in}}\right) \\
F_x &= -P_A A_A + \rho V_{\text{A}}^2 A_{\text{A}}\frac{\sqrt{2}}{2} - \rho V_{\text{in}}^2 A_{\text{in}} \\
&= -(-36600\text{Pa}) \cdot 10^{-4}\text{m}^2 + 998\frac{\text{kg}}{\text{m}^3}\cdot\left(5\frac{\text{m}}{\text{sec}}\right)^2 \cdot 10^{-4}\text{m}^2 \cdot \frac{\sqrt{2}}{2} \\
&\quad -998\frac{\text{kg}}{\text{m}^3}\cdot\left(10\frac{\text{m}}{\text{sec}}\right)^2 \cdot 10^{-4}\text{m}^2 \\
&= -4.56 \text{ N}
\end{aligned}$$

This is the net force on the fluid in the x-direction. Because of the change in velocity between inflow and outflow, this force is actually in the upstream direction: a force has to be applied in this direction to slow the water down.

Summing forces and momentum in the y-direction is easy: there's only the reaction force and momentum fluxes (again, the fractions are the y-component of the momentum flux):

$$\begin{aligned}
+\uparrow \sum F_y : \qquad R_y &= \left(\frac{1}{2}\rho V_{\text{A}}\right)\left(V_{\text{A}} A_{\text{A}}\right)\frac{1}{2} + \left(-\frac{1}{2}\rho V_{\text{B}}\right)\left(V_{\text{B}} A_{\text{B}}\right) \\
R_y &= 0
\end{aligned}$$

In retrospect, we could also have gotten this result from symmetry.

Finally, summing forces and momentum in the z-direction (here also the fractions are the z-component of the momentum flux):

$$
\begin{aligned}
+\nearrow \sum F_z: \quad F_z - W &= \left(\frac{1}{2}\rho V_{\mathrm{A}}\right)\left(V_{\mathrm{A}} A_{\mathrm{A}}\right) + \left(\frac{\sqrt{3}}{2}\rho V_{\mathrm{B}}\right)\left(V_{\mathrm{B}} A_{\mathrm{B}}\right) \\
F_z &= \frac{1}{2}\rho V_{\mathrm{A}}^2 A_{\mathrm{A}} \frac{1}{2} + \frac{\sqrt{3}}{2}\rho V_{\mathrm{B}}^2 A_{\mathrm{B}} + W \\
&= 998 \cdot 5^2 \cdot 10^{-4} \cdot \frac{1}{2} + 998 \cdot 5^2 \cdot 10^{-4} \cdot \frac{\sqrt{3}}{2} + 8 \\
&= 11.4 \text{ N}
\end{aligned}
$$

The signs here all make sense: the water going out each of the outlets has to be pushed upwards to get it going in that direction, and the weight certainly must be balanced by an upward force.

The total force exerted on the nozzle at the flange is:

$$
\begin{aligned}
\vec{F} &= F_x \hat{\imath} + F_y \hat{\jmath} + F_z \hat{k} \\
&= \left(-4.56\hat{\imath} + 0\hat{\jmath} + 11.4\hat{k}\right)
\end{aligned}
$$