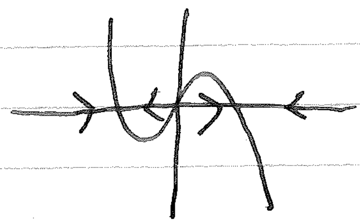
Tutorial 9 : Autonomous Equations Solutions.

$$x' = \underbrace{-x(x-2)(x+2)}_{f(x)=}$$



1. The steady-states are $x_1^* = 0$, $x_2^* = 2$, $x_3^* = -2$.

2. $f(x) = -x(x^2-4) = -x^3 + 4x$.

$\Rightarrow f'(x) = -3x^2 + 4$

$\dfrac{d\eta}{dt} = (-3x^2+4)\eta$. is the linearized equation.

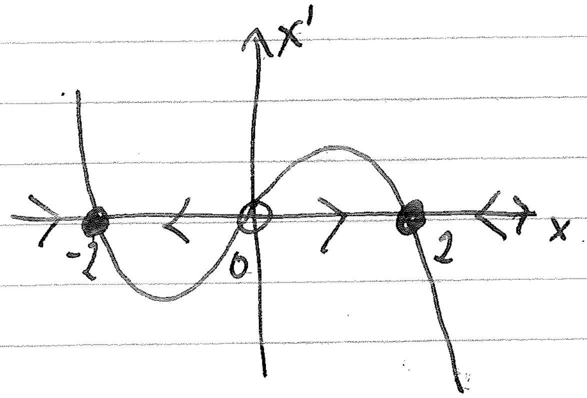
$\underline{x_1^* = 0}$: $\dfrac{d\eta}{dt} = +4\eta$. Since $f'(x_1^*) = f'(0) = +4$, $x_1^* = 0$ is $\underline{\text{unstable}}$.

$\underline{x_2^* = 2}$: $\dfrac{d\eta}{dt} = (-3\cdot 4 + 4)\eta = -8\eta$.

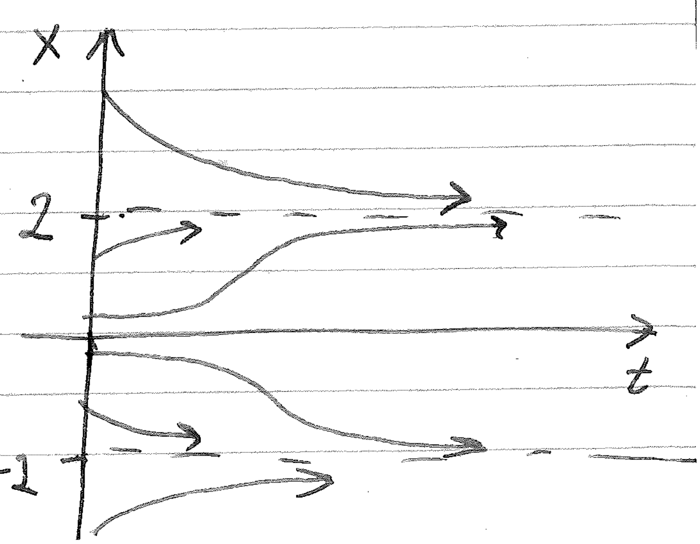
$\underline{x_3^* = -2}$: $\dfrac{d\eta}{dt} = -8\eta$.

Since $-8 < 0$, x_2^* & x_3^* are ~~are~~ $\underline{\text{stable}}$

3.



$\Rightarrow$

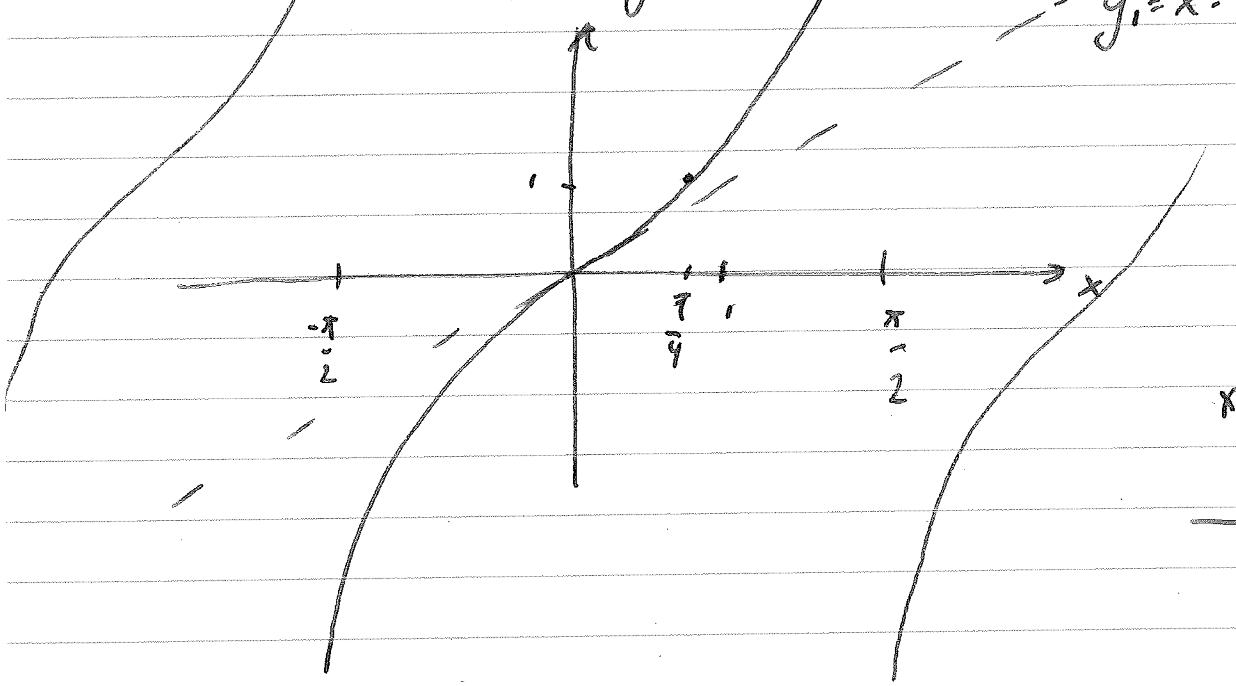


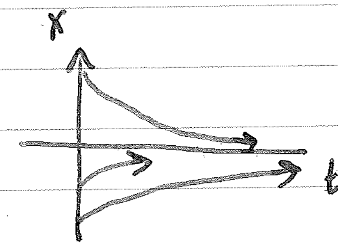
4. $x' = \underbrace{x - \tan x}_{f(x)}$.

Too lazy for analysis : let's do it graphically!

Q: What does $f(x) = x - \tan x$ look like?

A: Plot $y_1 = x$ and $y_2 = \tan x$ on the same axis.





So let's focus on $-\pi/2 < x < \pi/2$ ... to avoid infinite x'...

Between $-\pi/2 < x < 0$, ~~$x > 0$~~ $\tan x < x < 0$

and $0 > x > \pi/2$, $\tan x > x > 0$.

At $x = 0$, $\tan x = x$.

Thus, $f(x) = 0$ when $x = 0$.

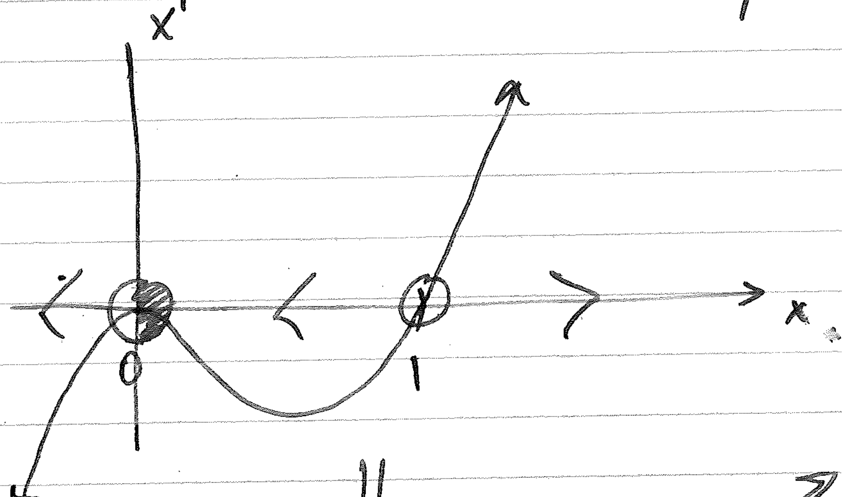
> 0 when $x \in (-\pi/2, 0)$

< 0 when $x \in (0, \pi/2)$.

$\Rightarrow$ $x = 0$ is the only steady-state. It's stable since $f(x) > 0$ for $x < 0$ and $f(x) < 0$ for $x > 0$.

5. $x' = -x^2(1-x) = -x^2 + x^3$.
$\underline{f(x) =}$

Steady states $x = 0, 1$. Think: $f(x)$ is a cubic polynomial with roots at 0 and 1. The root at 0 has multiplicity 2. The coefficient of x^3 is positive!



* You can ~~solve~~ do this with a linear stability analysis!

$\Downarrow$