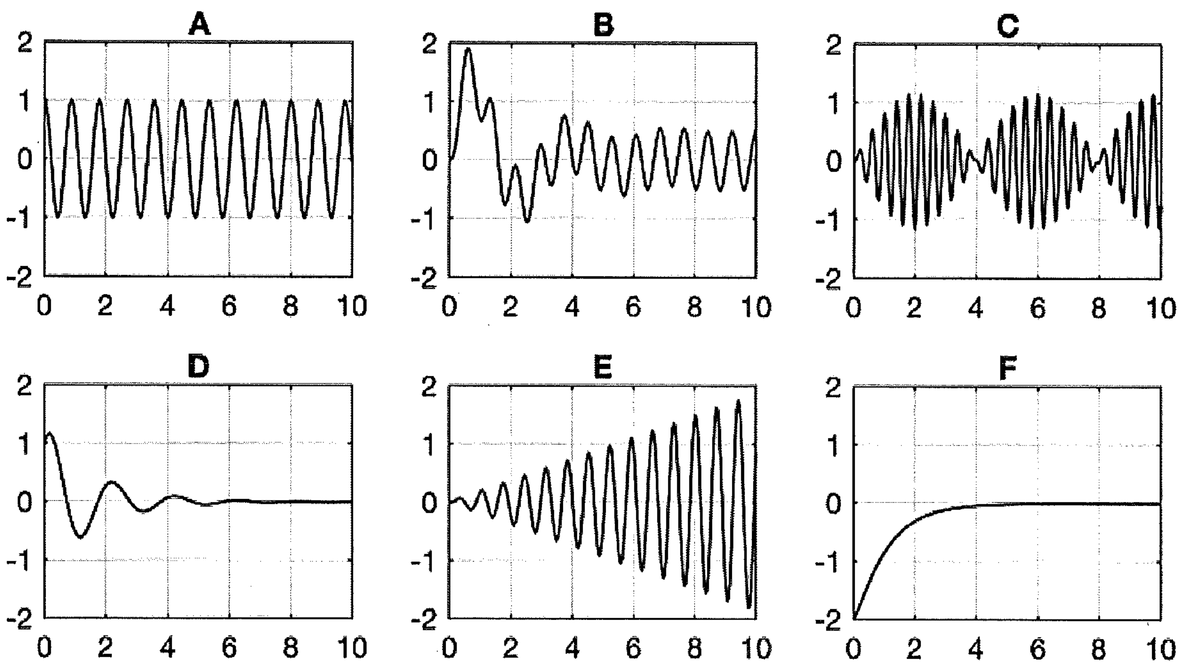
SA 2. (5 marks) Each figure shows a solution of a mass-spring-damper system. Match each figure with the corresponding differential equation in the table below.



Forced, undamped
$\omega_n \approx 16.6$

Free
Underdamped
$5^2 - 4(4)(40) < 0$

Forced
Underdamped
$1^2 - 4(1)(4) < 0$

Free
overdamped
$5^2 - 4(1)(4) > 0$

Free
Undamped
$\omega_n = \sqrt{50}$

$y'' + 275y = 30\sin(15t)$	C
$4y'' + 5y' + 40y = 0$	D
$y'' + y' + 4y = 30\sin(8t)$	B
$y'' + 5y' + 4y = 0$	F
$2y'' + 100y = 0$	A
$3y'' + 243y = 10\sin(9t)$	E

Forced
Undamped

$\omega_n = 9$
(Resonance!)

SA 3. (5 marks) Find the general solution of the differential equation

$$2y'' - 3y' + 5y = 2t - e^{3t}$$

Characteristic polynomial: $p(s) = 2s^2 - 3s + 5$

$\Rightarrow$ Roots $r = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(5)}}{2(2)} = \dfrac{3}{4} \pm \dfrac{\sqrt{31}}{4} i$

$$\Rightarrow \boxed{y_c(t) = C_1 e^{\frac{3}{4}t} \cos\left(\frac{\sqrt{31}}{4} t\right) + C_2 e^{\frac{3}{4}t} \sin\left(\frac{\sqrt{31}}{4} t\right), \quad C_1, C_2 \in \mathbb{R}}$$

Let $y_p = At + B + Ce^{3t}$

$\Rightarrow y_p' = A + 3Ce^{3t} \qquad y_p'' = 9Ce^{3t}$

$$2y_p'' - 3y_p' + 5y_p = 2(9Ce^{3t}) - 3(A + 3Ce^{3t}) + 5(At + B + Ce^{3t})$$

$$= 5At + (5B - 3A) + 14Ce^{3t} = 2t - e^{3t}$$

$\Rightarrow A = \dfrac{2}{5} \quad B = \dfrac{6}{25} \quad C = \dfrac{-1}{14}$

$$\Rightarrow \boxed{y_p(t) = \frac{2}{5}t + \frac{6}{25} - \frac{1}{14}e^{3t}}$$

$$\Rightarrow \boxed{y(t) = y_c(t) + y_p(t)}$$