UNIVERSITY OF BRITISH COLUMBIA
DEPARTMENT OF MECHANICAL ENGINEERING

QUIZ 6. Wednesday November 9, 2016

MECH 221

READ THESE INSTRUCTIONS CAREFULLY
BEFORE BEGINNING THE QUIZ. GOOD LUCK!

Duration: Target = 45 minutes. Maximum = 50 minutes.

Materials admitted: Pencil, eraser, straightedge, Mech2 calculator, Mech2 formula book, one 3x5 inch index card.

There are five short-answer questions and two long-answer questions.
All questions must be answered.

Provide **all** work and solutions **on these test pages**. Scrap paper is available for your use, but **will not be collected or marked**. An additional mark of up to 5% of the exam value is available for orderly presentation of work

Illegible work, or answers that do not include supporting calculations and explanations will NOT be marked.

BE SURE TO WRITE YOUR NAME ON THE TOP OF ALL EXAM PAGES

NAME:______________________________________Section__________

SIGNATURE:________________________________

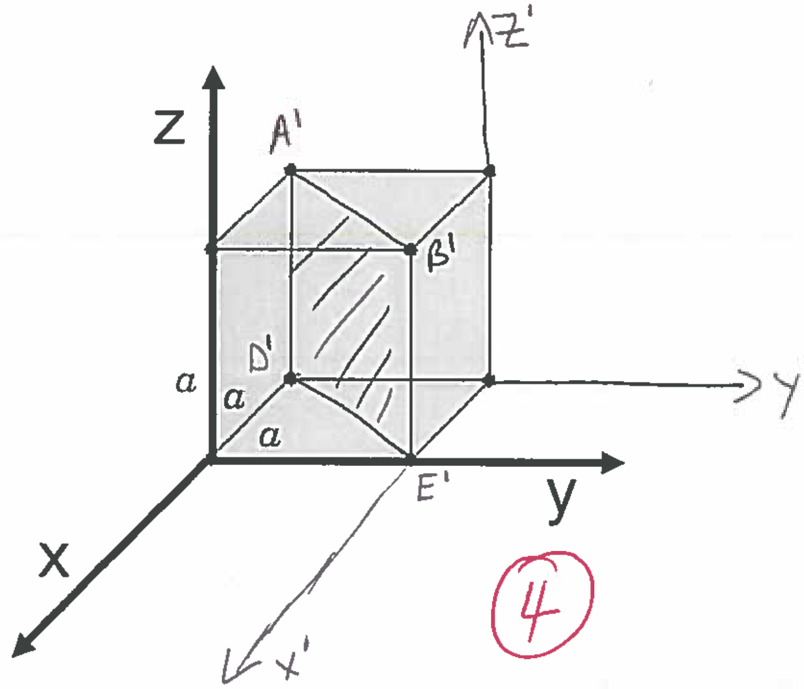
STUDENT NUMBER:_________________________

Question	Mark Received	Maximum Mark
SA 1		6
SA 2		6
SA 3		7
SA 4		6
LA 1		25
LA 1		
Presentation		
TOTAL		

Name:______________________________ Section:__________________________

Part I.

SA 1.



In the unit cell above, sketch the $(1\bar{1}0)$ plane.

If the crystal was body centre cubic, sketch the position of the atoms on the $(1\bar{1}0)$ plane assuming a hard sphere model.

First, it is most convenient to move the coordinate system to x' , y' and z' as shown above.

Next, one needs to work backwards in this problem given (hkl). Take the reciprocals of hkl to determine the intercepts, i.e.

x' intercept = 1

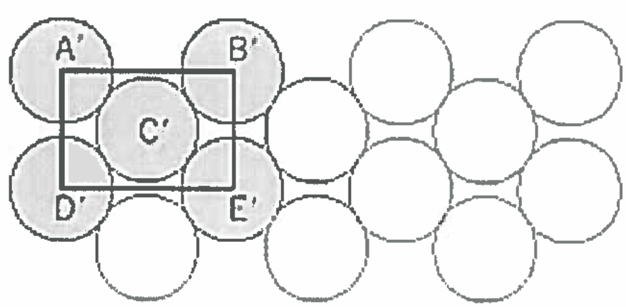
y' intercept = -1

z' intercept = ∞

This corresponds to the A'B'D'E' plane in the sketch above.

The corresponding positions of atoms on this plane for a BCC crystal is shown below:

2



Name:______________________________ Section:__________________________

SA 2. Given that iron (Fe) has a body centered cubic crystal, a density of 7.86 gcm^{-3} and a molecular weight of 55.8 $gmol^{-1}$, determine the diameter of an iron atom (Assume hard sphere model).

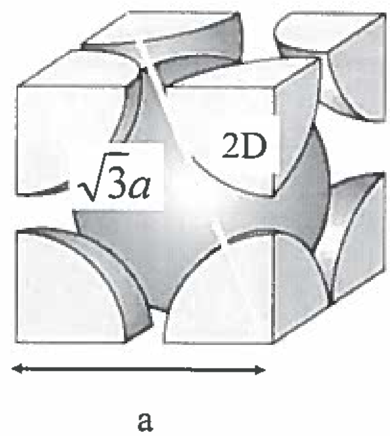
Unit Cell Length

a = lattice parameter

R = atomic radius

$$\sqrt{3}a = 2D$$

1 $$D = \frac{\sqrt{3}a}{2}$$



1

Density = ρ = (mass of atoms in unit cell)/(volume of unit cell)

1 $$\rho = \frac{nA / N_A}{V_c}$$

n = number of atoms per unit cell

A = Atomic weight g/mol

V_c = unit cell volume = a^3

N_A = Avogadro's Number (6.023 x 10^{23} atoms/mol)

$$V_c = \frac{nA/N_A}{\rho} = \frac{2 \times 55.8\ gmol^{-1} / 6.02 \times 10^{23}\ atoms\ mol^{-1}}{7.86\ gcm^{-3}}$$

$$V_c = 2.36 \times 10^{-23}\ cm^3$$

3 $$a = \sqrt[3]{V_c} = 2.86 \times 10^{-8}\ cm \times \frac{m}{100 cm} \times \frac{nm}{10^{-9} m}$$

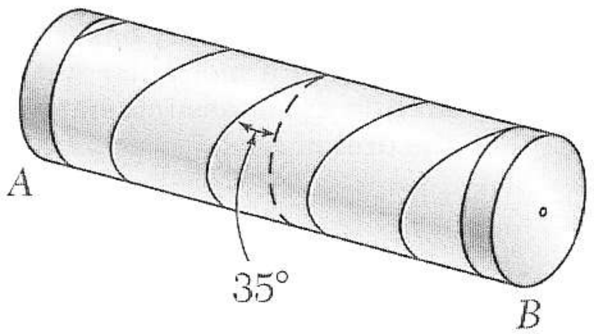
$$= 0.286\ nm$$

$$D = \frac{\sqrt{3}}{2} \times 0.286 nm = 0.248\ nm$$

$\therefore$ diameter of Fe atom is 0.248 nm.

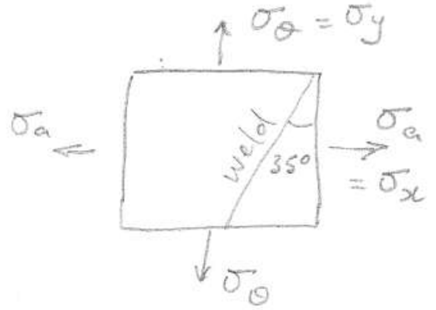
Name:______________________________ Section:___________________________

SA 3. A thin-walled cylinder of length L, diameter D, wall thickness t and interior pressure p is made by rolling and welding a plate into a 35° spiral, as shown. Derive a compact formula for the stress normal to the weld.



Cylinder stresses:

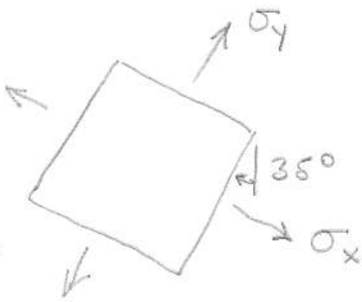
$$\sigma_\theta = \frac{pR}{t} \qquad \sigma_a = \frac{pR}{2t} \qquad \sigma_r \simeq 0$$



Identify σ_a with σ_x and σ_θ with σ_y

Stress normal to weld is reached by rotating 35° clockwise from σ_x

$$\rightarrow \theta = -35°$$



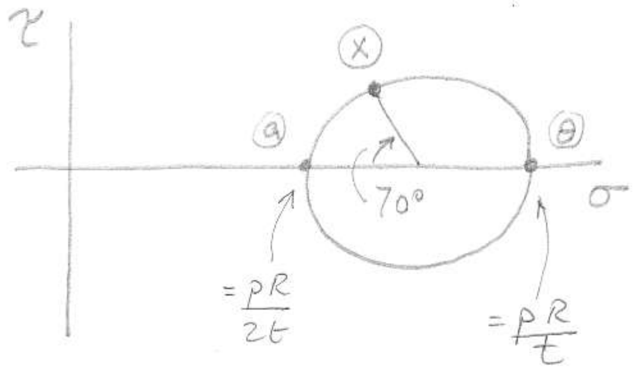
$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{\frac{pR}{2t} + \frac{pR}{t}}{2} + \frac{\frac{pR}{2t} - \frac{pR}{t}}{2}\cos(-70°) + 0$$

$$\sigma_x = \frac{3pR}{4t} - \frac{pR}{4t}\cos 70°$$

stress normal to weld

Alternatively, Mohr's Circle could be used:



Name:___________________________ Section:_________________________

SA 4. Why do we use safety factors ? What issues do we typically need to consider when choosing an appropriate safety factor ?

We use safety factors to protect engineering objects from the effects of factors that we cannot adequately consider in our design. Such factors include uncertainties in loads, operating environment and material performance. We choose to use higher safety factors when the consequences of failure are serious, e.g. human injury or large $ loss. We choose to use lower safety factors when weight or manufacturing cost are crucial.

Name:____________________________ Section:_________________________

Part II.

LA 1. A long cylinder made of 4140 steel (Length = 5 m, diameter = 1 m, wall thickness = 10 mm, E = 210 GPa, ν = 0.3, yield stress = 1000 MPa) was designed to store nitrogen gas. After the cylinder was filled with gas it was noticed that the interior pressure gradually decreased. So that enough nitrogen gas would remain in the cylinder when needed, it was decided to add some more nitrogen in the cylinder by increasing the initial pressure. However, during this filling, the cylinder exploded at a pressure of 15,000 kPa. (No people or animals were injured in the construction of this question). A subsequent investigation revealed that there had initially been two through-thickness cracks, one parallel to the length of the cylinder with a total length 20 mm and one parallel to the circumference of the cylinder with a total length of 40 mm.

i. Sketch the cylinder before it fractured.

ii. Did the material of the cylinder yield ? Explain your reasoning.

iii. Determine which of the cracks was most likely to have been the fracture initiation site. You may assume that Y=1.

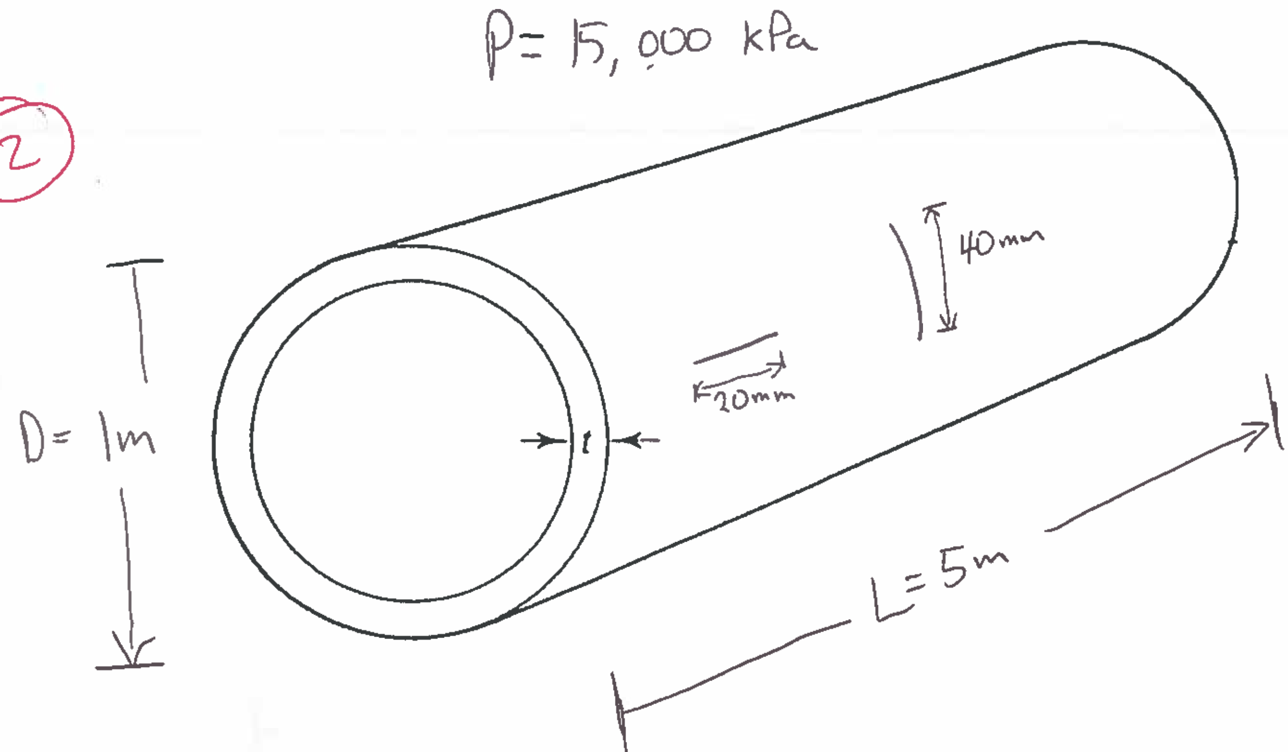
iv. Estimate the fracture toughness, K_{Ic} , of the steel (in units of $\mathrm{MPa}\sqrt{\mathrm{m}}$)

v. The steel used in the cylinder was strengthened by a combination of deformation at room temperature (strain hardening, reducing the grain size of the steel and solid solution hardening. Explain the basic principles that cause these mechanisms to strengthen the steel.

vi. Calculate the changes in length and diameter of the cylinder just before it had exploded.

Name:______________________ Section:______________________

i)



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i) Calculate hoop and axial stresses.

$$\sigma_H = \frac{PD}{2t} \quad , \quad \sigma_A = \frac{PD}{4t}$$

$$\sigma_H = \frac{15{,}000 \text{ kPa} \times 1000\text{mm}}{2 \times 10\text{mm}}$$

$$= 750{,}000 \text{ kPa or } 750 \text{ MPa}$$

$$\sigma_A = \frac{15{,}000 \text{ kPa} \times 1000 \text{ mm}}{4 \times 10 \text{ mm}} = 375 \text{ MPa}$$

The cylinder does not yield since the maximum stress of 750 MPa is less than the yield stress (1000 MPa). Note: this ignores the effect of the orthogonal stress.

2

Name:________________________ Section:________________________

iii) Need to calculate the stress intensity factor for each of the two cracks.

Crack parallel to the length, stress = hoop stress

$$K = \sigma_H \sqrt{\pi a}$$

3

$$= 750\,\text{MPa} \sqrt{\pi \times 0.01\,\text{m}}$$

$$= 132.9\ \text{MPa}\sqrt{\text{m}}$$

Crack perpendicular to the length, stress = axial stress

$$K = \sigma_A \sqrt{\pi a}$$

2

$$= 375\,\text{MPa} \sqrt{\pi \times 0.02\,\text{m}}$$

$$= 94.0\ \text{MPa}\sqrt{\text{m}}$$

1

As the crack parallel to the length of the cylinder has the highest stress intensity factor, this should be the fracture initiation site.

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iv) K_{IC} is defined as the stress intensity at fracture. In this case, fracture occurred at a K of $132.9\ \text{MPa}\sqrt{\text{m}}$. Therefore, this should be an estimate of K_{IC}

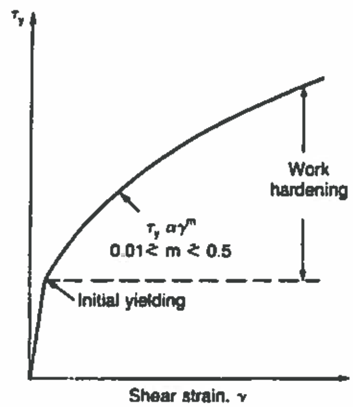
2

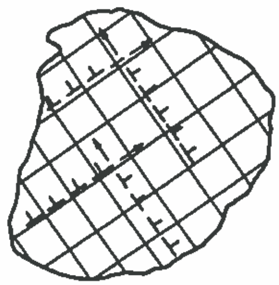
$$K_{IC} = 132.9\ \text{MPa}\sqrt{\text{m}}$$

Name:________________________ Section:________________________

v. The principle which underlies the strengthening of crystalline materials such as 4140 steel is to make dislocation motion more difficult.

Strain hardening





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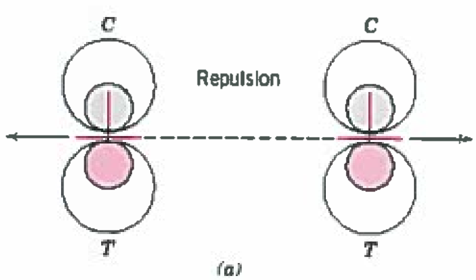
In this case, dislocation motion is made more difficult due to the interaction between dislocations on parallel or perpendicular planes. Either way, the stress to move the dislocation must increase due to this interaction. For example, there is a repulsive interaction between the two dislocations below. If the dislocation on the left is to move forward the stress will have to be increased due to the interaction with the dislocation on the left.



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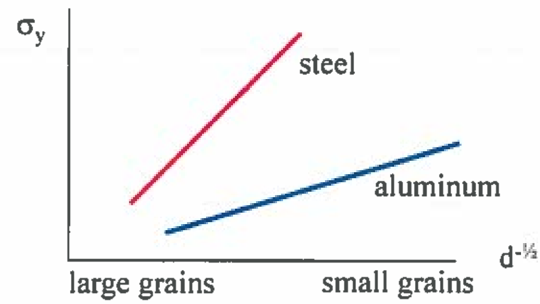
Grain size strengthening

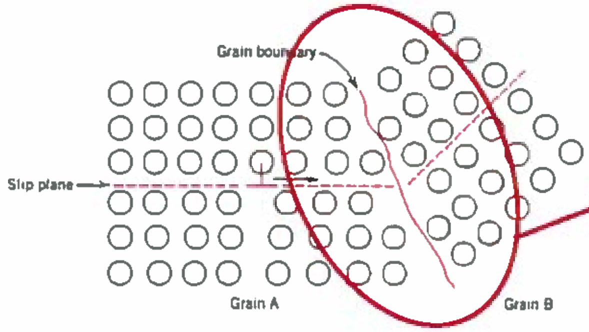
For this case, the grain boundary acts as a barrier to the dislocation since the edge dislocation shown on the next page cannot be transmitted from Grain A to Grain B. This is increases the stress necessary to move the dislocation. Grain size strengthening is given by the Hall-Petch equation:

Name:______________________________ Section:__________________________

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$



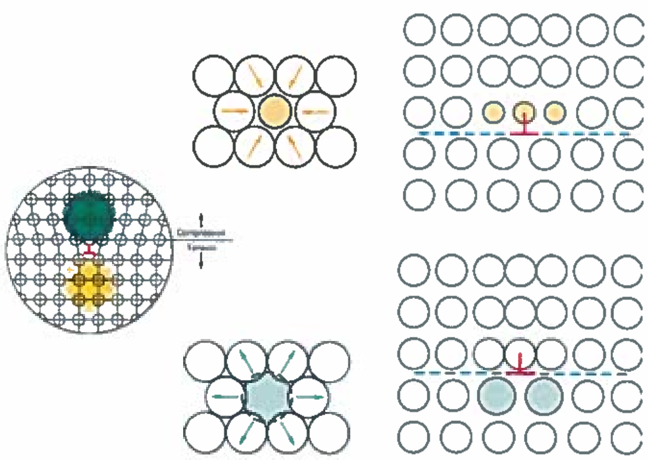
- Grain boundaries act as barrier to dislocations
- As grain size decreases, number of barriers to dislocations increases
- Therefore strength increases



By definition, slip planes are discontinuous across grain boundaries, and change directions

Solid solution hardening

For this case, the elastic stress field of the dislocation interacts with either smaller or larger solute atoms dissolved in the lattice of the host.



Smaller substitutional atom
- Tensile stresses
- Alloy atoms are attracted to upper side of dislocation

2

Larger substitutional atom
- Compressive stresses
- Alloy atoms are attracted to lower side of dislocation

- Two means by which there is an interaction
  - Alloy atoms diffuse to dislocation and pin it down initially
  - Moving dislocation approaches solute atom

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(vi) $\sigma_\theta = \frac{PR}{t}$ $\sigma_a = \frac{PR}{2t}$ $\sigma_r \simeq 0$ Cylinder stresses

Hooke's Law:

$$\varepsilon_\theta = \frac{\sigma_\theta}{E} - \nu\frac{\sigma_a}{E} - \nu\frac{\sigma_r}{E} = \frac{PR}{2Et}(2-\nu)$$

$$\varepsilon_a = \frac{\sigma_a}{E} - \nu\frac{\sigma_\theta}{E} - \nu\frac{\sigma_r}{E} = \frac{PR}{2Et}(1-2\nu)$$

Change in length $\Delta L = L\,\varepsilon_a = \frac{PRL}{2Et}(1-2\nu)$

$$\Delta L = \frac{15\times10^6\times0.5\times5}{2\times210\times10^9\times0.01}(1-2\times0.3) = \underline{3.6\text{ mm}}$$

Change in diameter $\Delta D = D\,\varepsilon_\theta = \frac{PR^2}{Et}(2-\nu)$

$$\Delta D = \frac{15\times10^6\times0.5^2}{210\times10^9\times0.01}(2-0.3) = \underline{3.0\text{ mm}}$$