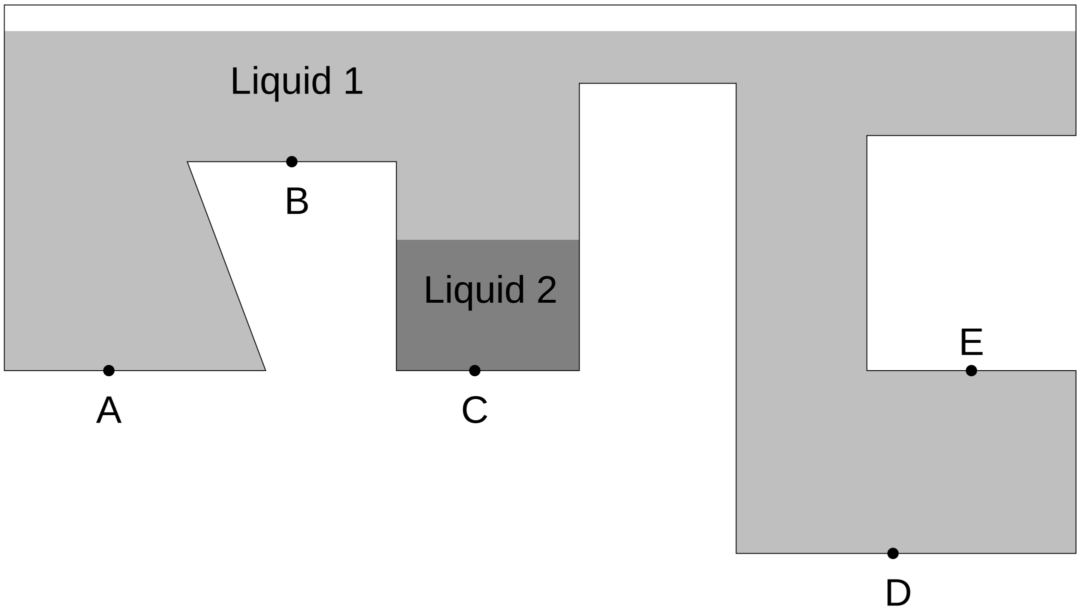
1. (5 marks) The tank in the sketch contains two liquids, as shown. Order the points from highest to lowest pressure. You can give your answer as a single ordering, as in: "$X > Y > Z$" or, in cases of ambiguity, as two or more partial orderings: "$X > Y$ and $X > Z$", indicating that you can't tell which of Y and Z has higher pressure.



Solution:

Because of depth in Liquid 1, you can say for sure that

$$P_D > P_E = P_A > P_B$$

Also, Liquid 2 is clearly more dense than Liquid 1, so

$$P_C > P_E = P_A > P_B$$

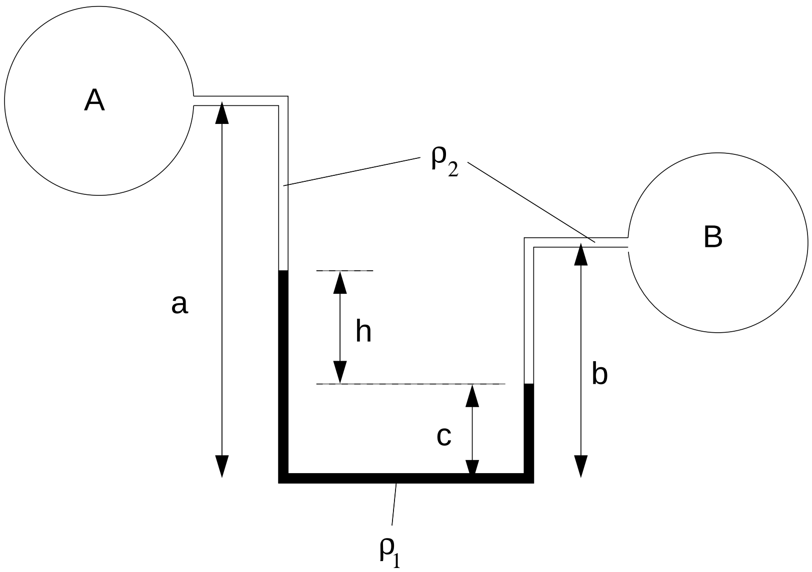
That leaves only the question of which of C and D has higher pressure, which would require more information:

$$P_C ? P_D$$

2. (5 marks) Pipes A and B in the sketch are carrying fluid in the direction perpendicular to the page. Use the manometer shown to find the pressure difference $P_B - P_A$. You may leave your answer in symbolic form if you wish. If you prefer to work with numbers, use:

$$\rho_1 = 1000 \,\mathrm{kg/m^3} \qquad \rho_2 = 800 \,\mathrm{kg/m^3}$$

$$a = 0.8\mathrm{m} \qquad b = 0.65\mathrm{m} \qquad c = 0.2\mathrm{m} \qquad h = 0.3\mathrm{m}$$



Solution:

One easy way to do this is to work your way systematically from one end of the system to the other:

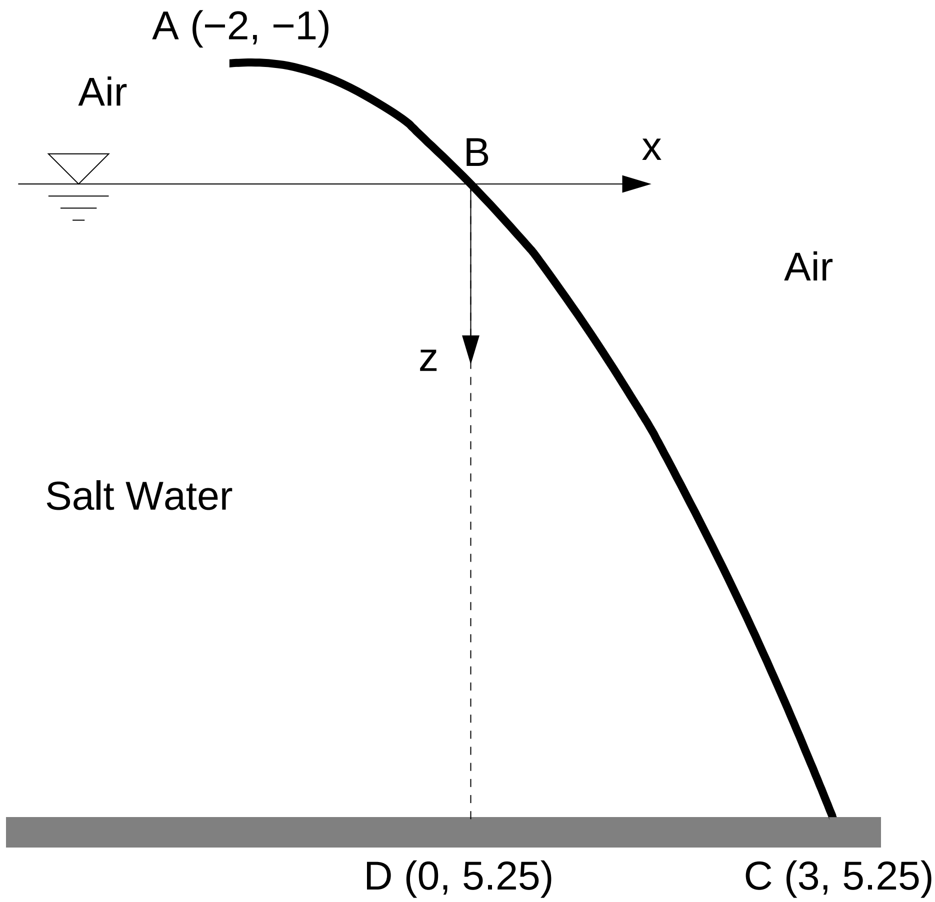
$$\begin{aligned}
P_A + \rho_2 g\,(a-c-h) + \rho_1 g h - \rho_2 g\,(b-c) &= P_B \\
\rho_1 g h + \rho_2 g\,(a-b-h) &= P_B - P_A \\
P_B - P_A &= 1000\,\mathrm{kg/m^3}\cdot 9.81\,\mathrm{m/s^2}\cdot 0.3\,\mathrm{m} \\
&\quad +800\,\mathrm{kg/m^3}\cdot 9.81\,\mathrm{m/s^2}\cdot(0.8\,\mathrm{m}-0.3\,\mathrm{m}-0.65\,\mathrm{m}) \\
&= 1766\,\mathrm{Pa}
\end{aligned}$$

That $\rho_2 g\,(a-b-h)$ can be thought of as going down through fluid 2 by a distance $a-b$ (the height difference between pipe A and pipe B), and up through fluid 2 by a distance h in the right arm of the manometer. The bit between the fluid 1 / fluid 2 interface on the left and the height of pipe B cancels out.

3. (30 marks) An aquarium has a tropical reef display whose front is curved, as shown in the sketch. The tank is 5 m wide in the direction perpendicular to the page, and the shape of the curved wall can be written as:

$$z = \frac{(x+2)^2}{4} - 1$$

using the coordinate system in the sketch. All distances, including the coordinates labeled on the sketch, are in meters. The density of salt water is $1030\,\mathrm{kg/m^3}$.



(a) (5 marks) Sketch the pressure distribution along the side of the tank ABC.

(b) (5 marks) Write the weight of the water under the curved tank wall (that is, the water in the region BCD in the sketch) as a double integral with the appropriate domain, and evaluate this integral.

Solution:

Because the width of the wall (perpendicular to the page in the sketch) is constant, the weight can be written as a double integral:

$$W = \iint_{BCD} 5\rho g \, dz \, dx = \int_0^3 \int_{z_w(x)}^{5.25} 5\rho g \, dz \, dx$$

where $z_w(x) = \frac{(x+2)^2}{4} - 1$. Evaluating, we get:

$$\begin{aligned} W &= 5\rho g \int_0^3 \left(5.25 - \frac{(x+2)^2}{4} + 1 \right) dx \\ &= 5\rho g \left(\frac{25}{4}x - \frac{(x+2)^3}{12} \right)_0^3 \\ &= 5\rho g \left(\frac{75}{4} - \frac{125}{12} \right) = \frac{500}{12}\rho g \\ &= 421\text{kN} \end{aligned}$$

(c) (5 marks) Repeat this process to find the x-location of the center of mass of the water in region BCD.

Solution:

From the definition of the centroid, we have:

$$\bar{x} = \frac{\iint x\,dA}{\iint 1\,dA}$$

Both top and bottom can obviously be multiplied by the width of the wall without affecting the answer. The integral on the bottom is related to the weight:

$$W = \rho g w A$$

where w is the width of the wall. Therefore, $A = 8.33\bar{3}\,\mathrm{m}^2$ (that's 25/3).

Now, the integral on the top has the same limits as we used for the weight, so:

$$\begin{aligned}
\bar{x} &= \frac{3}{25}\int_0^3 \int_{z_w(x)}^{5.25} x\,dz\,dx \\
&= \frac{3}{25}\int_0^3 \left(\frac{25}{4}x - \frac{(x+2)^2}{4}x\right) dx \\
&= \frac{3}{25}\int_0^3 \left(\frac{25}{4}x - \frac{x^3+4x^2+4x}{4}\right) dx \\
&= \frac{3}{25}\frac{1}{4}\int_0^3 -x^3 - 4x^2 + 21x\,dx \\
&= \frac{3}{100}\left(-\frac{x^4}{4} - \frac{4x^3}{3} + 21\frac{x^2}{2}\right)_0^3 \\
&= \frac{3}{100}\left(-\frac{81}{4} - \frac{4\cdot 27}{3} + 21\cdot\frac{9}{2}\right) \\
&= 1.1475\text{m}
\end{aligned}$$

(d) (7 marks) Find the net pressure force on the side of the tank ABC; write this in terms of its horizontal and vertical components, with the direction clearly indicated for each.

Solution:

For the horizontal force, we calculate the pressure at the centroid of the vertical projection of the wetted surface and multiply by the area of that projection:

$$\begin{aligned}
F_H &= P_c A \\
&= \left(\rho g \frac{5.25}{2}\right)(5.25\cdot 5) \\
&= 696\text{kN} \rightarrow
\end{aligned}$$

The vertical force has two parts: the pressure force (upwards) along CD and the weight of the water (downwards); you calculated this in part b. The resultant of these two is:

$$\begin{aligned}
F_V &= P\,(x=5.25)\,A_{CD} - W \\
&= (\rho g\cdot 5.25)\,(3\cdot 5) - 421\text{kN} \\
&= 375\text{kN} \uparrow
\end{aligned}$$

(e) (8 marks) Find the moment that the pressure force exerts about the base of the curved side C. You may neglect the weight of the tank wall.

Solution:

Now it's time to sum up all the moments for all the forces, which means we'll also need lines of action. I'll sum moments with positive clockwise about C (because horizontal pressure force and vertical pressure force at the bottom of the tank produce moments that way), and with M_C defined as positive *counter*clockwise (clearly the direction it has to be; correct pressure sketches in part a illustrate that quite clearly).

$$\circlearrowright \sum M : \qquad F_H r_H + F_{CD} r_{CD} - W r_W - M_C = 0$$

where the r's are all moment arms. For the horizontal force, $r_H = 5.25 - y_p$, and

$$\begin{aligned} y_p &= y_c + \frac{\rho g I_{xx}}{F_H} \\ &= \frac{5.25}{2} + \frac{\rho g \frac{5.25^3 \cdot 5}{12}}{\rho g \frac{5.25}{2} (5.25 \cdot 5)} \\ &= \frac{5.25}{2} + \frac{\frac{5.25}{12}}{\frac{1}{2}} = \frac{2}{3} \cdot 5.25 = 3.5\text{m} \end{aligned}$$

For the vertical pressure force at the bottom of the tank, $r_{CD} = 1.5$m, because the pressure is uniform on the bottom. Finally, r_W is the distance to centroid of the water under the parabola, which you found in part c; $r_W = 3 - 1.1475 = 1.8525$m.

Putting this all together, we get:

$$\begin{aligned} M_C &= F_H r_H + F_{CD} r_{CD} - W r_W \\ &= 696\text{kN} \cdot 1.75\text{m} + 796\text{kN} \cdot 1.5\text{m} - 421\text{kN} \cdot 1.8525\text{m} \\ &= 1.632 \text{ MN} \cdot \text{m} \circlearrowleft \end{aligned}$$