Worksheet 1

Felix Funk, MATH Tutorial - Mech 221

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Reminder: Separable ODEs.
An ordinary differential equations (ODE) is separable if one can fit the differential equation into the form

$$\frac{dy}{dx} = f(x) \cdot g(y). \tag{1}$$

Frequently, we are interested how a solution $y(x)$ to (1) varies with changes in the independent variable x in general. But sometimes, we require the differential equation to satisfy additional constraints - we want the solution to attain an initial value y_0

$$y(x_0) = y_0. \tag{2}$$

In the latter case, we speak of solving an initial value problem.

Problem: Separable ODE.
Solve

$$y'(x) = xy + x + y + 1 \tag{3}$$

such that y(0) = 1.

Exercise: Terminology.
We reduce the problem to its basic components.

1. Identify: What is the ODE? What is the desired function? What is the independent variable?

2. Cross out everything that does not apply: The equation (3) is a linear, non-linear, first-order, second-order, higher-order, constant - coefficient, homogeneous, non-homogeneous equation.

3. How can you change this equation for it to become a second-order non-linear non-homogeneous equation?

4. Is this an initial value problem? If so identify x_0, y_0:

5. Identify the functions f and g

Example: Solving separable ODE's: A recipe.
We solve the problem corresponding to (3)

1. Isolate x-terms to the right; y-terms to the left: $\frac{dy}{g(y)} = f(x)dx$.

2. Integrate both sides. $\int \frac{1}{g(y)} \cdot dy = \int f(x)dx$. Don't forget the constant c.

3. Solve for y, if possible.

4. Determine the constant c. Use the initial value (if provided).

5. Check your solution!

Exercise: Separable ODEs.
Obtain the general solution for one of the following ODEs. Solve the IVP if asked for.

1. $$y' = 2xy,$$

2. $$x' = 3xt^2 - 3t^2, x(0) = 2,$$

3. $$y' = \frac{x^2+1}{y^2+1}, y(0) = 1.$$

Hint: You might not always find an explicit solution.

Reminder: Linear ODEs.
A first order linear ODE is an equation of the form

$$y' + p(x)y = f(x). \tag{4}$$

A function r that satisfies $r'(x) = p(x)r(x)$ constitutes the relationship

$$\frac{d}{dx}[r(x)y] = r(x)y' + r(x)p(x)y = r(x)f(x). \tag{5}$$

The function r is called the integrating factor and can be calculated in the following way:

$$r(x) = e^{\int p(x)dx} \tag{6}$$

Problem: Linear First Order ODE.
Solve the equation

$$y' + 6y = e^x. \tag{7}$$

such that y(0)=1.

Example: Use integrating factors.

1. What is $p(x)$? What is $f(x)$?

$$\begin{aligned} p(x) &= \\ f(x) &= \end{aligned}$$

2. Determine the integrating factor. (You don't need to keep track of the integration constant.)

$$r(x) =$$

3. Multiply equation (7) with the integrating factor and utilize (5) $\frac{d}{dx}[r(x)y] = r(x)f(x)$

$$\frac{d}{dx}[\qquad\qquad\qquad] =$$

4. Integrate both sides with respect to x. Don't forget the integration constant.

5. Isolate y, if possible.

$$y(x) =$$

6. Determine the integration constant.

7. Check your solution!

Exercise: Linear ODEs.
Solve the following linear ODEs/ IVPs

1. $$y' + xy = x, y(0) = 0,$$

2. $$y' + cos(x)y = cos(x).$$