University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

TEST #8, November 23, 2017

MECH 221

Suggested Time: 1hr 40 min **Allowed Time:** 1hr 50 min

Materials admitted: Pencil, eraser, straightedge, Mech 2 Approved Calculator (Sharp EL-510), one 3x5 inch sheet of paper for hand-written notes.

There are 5 Short Answer Questions and 2 Long Answer Questions on this test. All questions must be answered.

Provide **all** work and solutions **on this test**.

Orderly presentation of work is required for solutions to receive full credit. **Illegible work, or answers that do not include supporting calculations and explanations will NOT BE MARKED**.

FILL OUT THE SECTION BELOW. Do this during the examination time as additional time will not be allowed for this purpose.

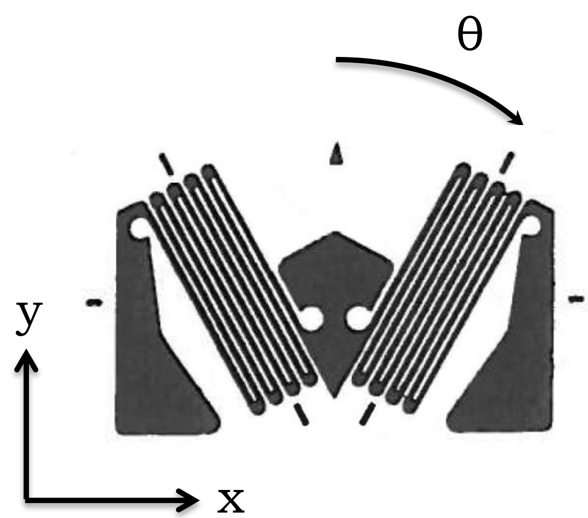
NAME:____________________________________Section_________

SIGNATURE:_______________________________

STUDENT NUMBER:________________________

Question	Mark Received	Maximum Mark
SA 1		5
SA 2		5
SA 3		6
SA 4		8
SA 5		9
LA 1		21
LA 2		18

LA 2. The diagram shows a "stress gauge", which is a type of double strain gauge specially designed to measure the stress σ_y, independent of the perpendicular σ_x stress. Each of the two strain gauges measures the strain in the direction along the length of the inclined lines shown in the diagram. The combined gauge is attached to a material with elastic constants E and ν. You are told that the angle θ = 33° and you are asked to find the Poisson's ratio for which the stress gauge is designed.



(a) Write formulas for the strain that would be measured by the right-side strain gauge if stress σ_x acts alone, for cases when θ = 0° and θ = 90°. Then draw a graph of the expected strain vs. angle relationship for θ values between 0° and 90°.

(b) Derive a formula for the strain vs. θ relationship illustrated in part (a). Use Mohr's Circle of Stress for your derivation.

(c) Derive a formula for the angle θ that makes the strain gauge insensitive to σ_x.

(d) In the actual stress gauge, θ = 33°. For what Poisson's ratio is it designed ?

(e) Derive a formula for the measured strain when a stress σ_y is applied.

(f) (Bonus) Any ideas why two strain gauges are used ?

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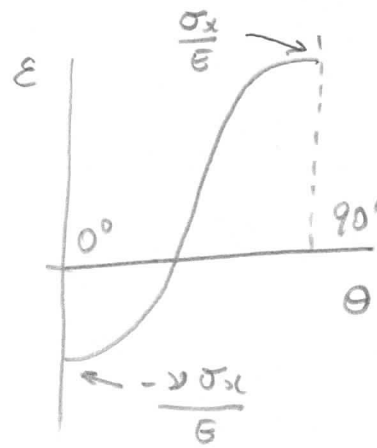


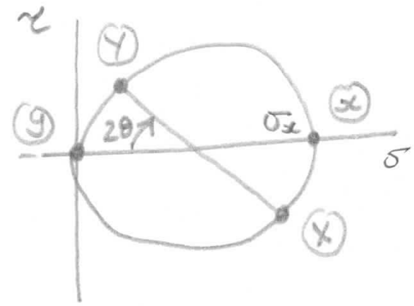
(a) For σ_x acting alone $\rightarrow \sigma_y = 0$

$$\left.\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \rightarrow \frac{\sigma_x}{E} \\ \varepsilon_y &= -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} \rightarrow -\frac{\nu\sigma_x}{E} \end{aligned}\right] \text{ for } \sigma_y = 0$$

When $\theta = 0°$, strain gauge measures $\varepsilon_y = -\frac{\nu\sigma_x}{E}$

When $\theta = 90°$, strain gauge measures $\varepsilon_x = \frac{\sigma_x}{E}$

Positive and negative θ give same results, so slopes at $\theta = 0°$ and $90°$ are zero.



(b) Let Y be the measurement direction of the strain gauge and X the perpendicular direction

From Mohr's Circle:

$$\sigma_X = \frac{\sigma_x}{2}(1 + \cos 2\theta) \qquad \sigma_Y = \frac{\sigma_x}{2}(1 - \cos 2\theta)$$

Hooke's Law: $\varepsilon_Y = \frac{\sigma_Y}{E} - \nu\frac{\sigma_X}{E} = \underline{\frac{\sigma_x}{2E}\left((1-\nu) - (1+\nu)\cos 2\theta\right)}$

(c) Strain gauge insensitive to σ_x when $\varepsilon_Y = 0$

$$\rightarrow (1-\nu) - (1+\nu)\cos 2\theta = 0$$

$$\rightarrow \cos 2\theta = \frac{1-\nu}{1+\nu} \quad \rightarrow \quad \underline{\theta = \frac{1}{2}\operatorname{acos}\left(\frac{1-\nu}{1+\nu}\right)}$$

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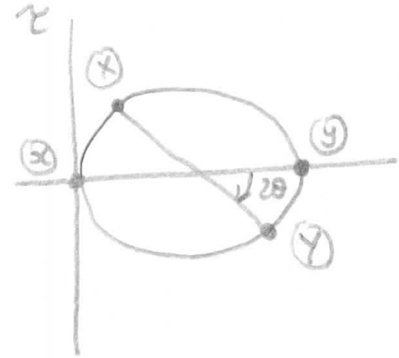
(d) If $\theta = 33°$ $\quad \cos 2\theta = \cos 66° = 0.407 = \frac{1-\nu}{1+\nu}$

$\rightarrow 0.407 + 0.407\nu = 1 - \nu \rightarrow \nu = \frac{1-0.407}{1+0.407} = \underline{0.42}$

(e) When σ_y is applied:



$\sigma_x = \frac{\sigma_y}{2}(1-\cos 2\theta)$

$\sigma_y = \frac{\sigma_y}{2}(1+\cos 2\theta)$

Hooke's Law:

$\varepsilon_Y = \frac{\sigma_Y}{E} - \nu\frac{\sigma_x}{E} = \frac{\sigma_y}{2E}\left((1-\nu) + (1+\nu)\cos 2\theta\right)$

sub. $\cos 2\theta = \frac{1-\nu}{1+\nu}$

$\varepsilon_Y = \frac{\sigma_y}{2E}\left((1-\nu) + (1+\nu)\frac{1-\nu}{1+\nu}\right)$

$\underline{\varepsilon_Y = \frac{\sigma_y}{E}(1-\nu)}$ for the given $\theta = 33°$

(f) If the two strain gauges are connected in series, the measurement will be the average of the strain gauge readings. For an applied shear stress τ_{xy} the normal stresses seen by the two strain gauges are opposite, so they will sum to zero. Thus, the double strain gauge is insensitive to σ_x and τ_{xy}.