Test 9 Dynamics Solutions (2017W)

November 30, 2017 3:21 PM

221-test9-DYN-(2017W)

University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

TEST #9, November 30, 2017

MECH 221

Suggested Time: 1hr 40 min **Allowed Time:** 1hr 50 min

Materials admitted: Pencil, eraser, straightedge, Mech 2 Approved Calculator (Sharp EL-510), one 3x5 inch sheet of paper for hand-written notes.

There are 4 Short Answer Questions and 2 Long Answer Questions on this test. All questions must be answered.

Provide **all** work and solutions **on this test**. Do not mark in the QR code area of the page – this may cause sorting issues, leading to your work for that page not being graded.

Orderly presentation of work is required for solutions to receive full credit. **Illegible work, or answers that do not include supporting calculations and explanations will NOT BE MARKED**.

FILL OUT THE SECTION BELOW. Do this during the examination time as additional time will not be allowed for this purpose.

NAME:____________________________________Section__________

SIGNATURE:______________________________

STUDENT NUMBER:________________________

Question	Mark Received	Maximum Mark
SA 1		5
SA 2		5
SA 3		5
SA 4		5
LA 1		25
LA 2		25

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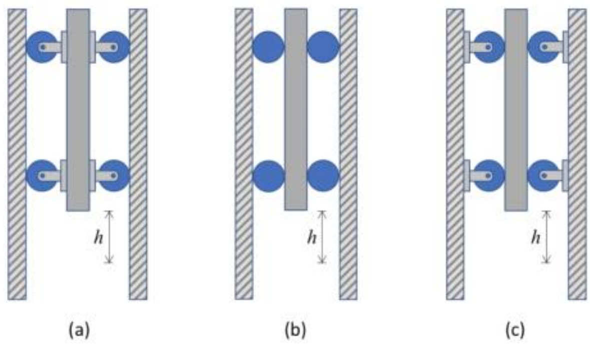
LA 1. [25 marks]



Part 1 [24 marks, 8 marks each case]: A bar of mass m_b = 5 kg and length l = 1 m, is guided down a chute between four disks, each with mass of m_d = 2 kg and radius r = 75 mm. Assume that the forces on the disks are sufficient to prevent slipping, there is no friction at the pin joints, and the bar is released from rest. **Find the velocity of the bar in each case** after it has dropped a distance h = 0.25 m. Be sure to include free-body diagrams.

State 1

in all cases, $T_1 = 0$

$V_1 = 0$ (datum through CoG of all bodies)

case (a)

state 2

$T_{2\,bar} = \frac{1}{2} m_b v^2$

$I_G = \frac{1}{2} m_d r^2$

FBD system:

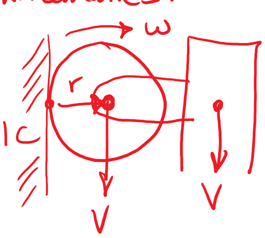
recall: can omit forces internal to the system (e.g. reactions @ pins)

$T_{2\,disk} = \frac{1}{2} m_d v^2$ (same vel. @ CoG, since centre pin rigidly attached to bar)

$+ \frac{1}{2} I_G \omega^2$

$= \frac{1}{2} m_d v^2 + \frac{1}{2}\left(\frac{1}{2} m_d \underbrace{r^2)\omega^2}_{v^2}$

kinematics:



$v_G = v$ & $v = \omega r$

$T_2 = \frac{1}{2} m_b v^2 + 4\left(\frac{1}{2} m_d v^2 + \frac{1}{4} m_d v^2\right)$

$= \frac{1}{2} m_b v^2 + 3 m_d v^2 = \left(\frac{1}{2} m_b + 3 m_d\right) v^2$

All move down together $\therefore V_2 = -(m_b + 4m_d) g h$

Cons of energy: $T_1 + \cancel{V_1}^{0} = T_2 + V_2$

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$\left(\frac{1}{2} m_b + 3 m_d\right) v^2 = (m_b + 4 m_d) g h$

no external forces doing work

(a) con'd. $V = \sqrt{\dfrac{(m_b+4m_d)gh}{(\frac{1}{2}m_b+3m_d)}} = \sqrt{\dfrac{(5kg+4(2kg))(9.81 m/s^2)(0.25m)}{\frac{1}{2}(5kg)+3(2kg)}}$

$= \sqrt{\dfrac{31.88}{8.5}} = 1.94 m/s$ $\boxed{\vec{v} = -1.94 m/s \uparrow}$ Case (a)

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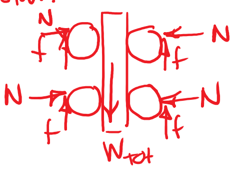
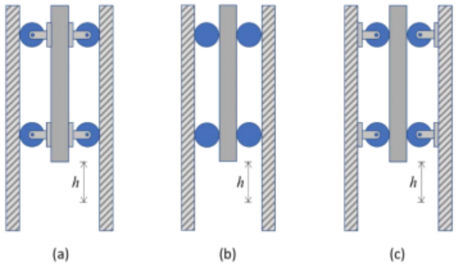
Case (b)

FBD System:



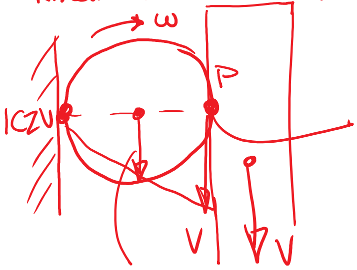
omit internal forces



$4\left(\frac{1}{8}+\frac{1}{16}\right)$

$4\left(\frac{3}{16}\right) = \frac{3}{4}$

Kinematics:



no slipping

∴ same velocity @ pt P ($v_p = v$)

$V_{G\,disk} = \dfrac{V}{2}$ (similar triangles)

∴ $\omega = \dfrac{V}{2r}$ and $h_{disk} = \dfrac{h}{2}$

$T_2 = \frac{1}{2}m_b V^2 + 4\left(\frac{1}{2}m_d\left(\frac{V}{2}\right)^2 + \frac{1}{2}I_d\omega^2\right)$

$= \frac{1}{2}m_b V^2 + 4\left(\frac{1}{8}m_d V^2 + \frac{1}{2}\left(\frac{1}{2}m_d r^2\right)\left(\frac{V}{2r}\right)^2\right)$

$= \frac{1}{2}m_b V^2 + \frac{3}{4}m_d V^2 = \left(\frac{1}{2}m_b + \frac{3}{4}m_d\right)V^2$

$V_2 = -m_b gh - \cancel{4}^{2}\left(m_d g\frac{h}{\cancel{2}}\right) = -(m_b+2m_d)gh$

cons of energy

$\cancel{T_1+V_1}^{0} = T_2 + V_2 \Rightarrow \left(\frac{1}{2}m_b+\frac{3}{4}m_d\right)v^2 = (m_b+2m_d)gh$

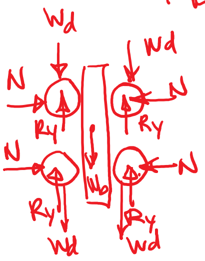
$\Rightarrow v = \sqrt{\dfrac{(m_b+2m_d)gh}{\frac{1}{2}m_b+\frac{3}{4}m_d}} = \sqrt{\dfrac{(5kg+2(2kg))(9.81 m/s^2)(0.25m)}{\frac{1}{2}(5kg)+\frac{3}{4}(2kg)}}$

$v = \sqrt{\dfrac{22.07}{4}} = 2.35 m/s$ $\boxed{\vec{v} = -2.35 m/s \uparrow}$ Case (b)

con'd

case(c)

FBD system:



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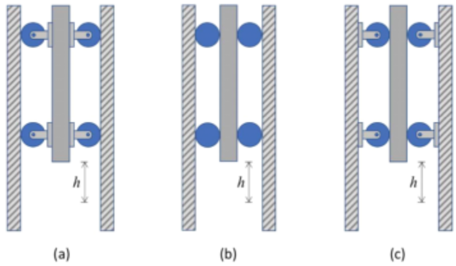
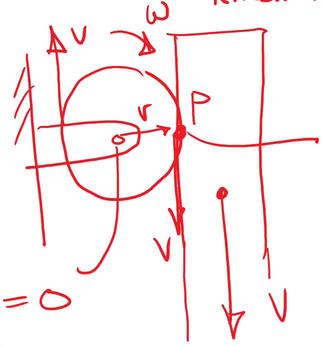


kinematics



no slipping

∴ same velocity at pt P on disk

$v_P = v$

$V_G = 0$

$\therefore v = \omega r$

$T_2 = \frac{1}{2} m_b v^2 + 4\left(\frac{1}{2} I_G \omega^2\right)$ (recall $V_{G\,disk} = 0$)

$= \frac{1}{2} m_b v^2 + 4\left(\frac{1}{2}\left(\frac{1}{2} m_d r^2\right)\omega^2\right)$ — $\underbrace{r^2\omega^2}_{v^2}$

$= \left(\frac{1}{2} m_b + m_d\right) v^2$

$V_2 = -m_b g h$ (disks do not move down)

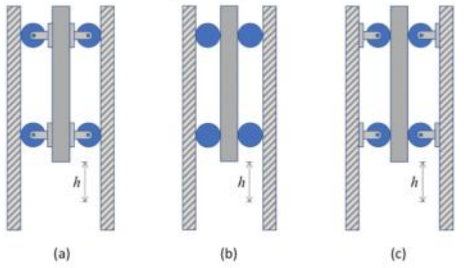
Cons. of energy

$T_1 + \cancel{V_1}^{0} = T_2 + V_2 \Rightarrow \left(\frac{1}{2} m_b + m_d\right) v^2 = m_b g h$

$$v = \sqrt{\frac{m_b g h}{\frac{1}{2} m_b + m_d}} = \sqrt{\frac{(5\,kg)(9.81\,m/s^2)(0.25\,m)}{\frac{1}{2}(5\,kg) + 2\,kg}}$$

$$= \sqrt{\frac{12.26}{4.5}} = 1.65\,m/s \quad \boxed{\vec{v} = -1.65\,m/s \uparrow}$$

Part 2 [1 mark]: Over time, the pin joints wear and develop friction that results in a moment about each pin – **how would the value of velocity (after moving distance *h*) qualitatively compare to that of the frictionless pin in each case?** (No calculation required).



Kinetic friction does negative work (opposes the motion of the disks).

$$T_1 + V_1 - U_{friction} = T_2 + V_2$$

It removes energy from the system, so that not all the stored gravitational potential energy gets converted to kinetic energy. Since there is now less energy in the system, $|\bar{v}|$ for cases (a) and (c) will be less than the values given above. Case (b) will not change (no pins).