Name: ____________________ **Signature:** ____________________ **UBC ID:** ______________

University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

MECH 222
Exam 2: 4 April 2014

Duration: 3 hours

This exam consists of 17 pages (including this cover sheet). Check to ensure that it is complete.

Rules Governing Formal Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
4. Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   - (a) speaking or communicating with other candidates, unless otherwise authorized;
   - (b) purposely exposing written papers to the view of other candidates or imaging devices;
   - (c) purposely viewing the written papers of other candidates;
   - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Question	Marks	Score
1F	20	
1T	15	
2F	20	
2M	20	
3M	20	
3T	25	
TOTAL	120	

1. (35 marks; 20 fluids, 15 thermo) A large open tank of water is at the top of a steep slope. A thin plastic tube is run down the slope from the tank. The tube is 30 m long and has an inner diameter of 2 mm. The end of the tube ends up 10 m below the top surface of the water tank. The water in the tank is at 20° C, as is the surrounding air. The process of water draining down the tube ends up being isothermal such that we can assume constant specific heat ($4.18\,{}^{\mathrm{kJ}}/_{\mathrm{kg \cdot K}}$), density, and viscosity (given in the formula booklet) for the water. Calculate the rate of entropy generation in this process. (Note: A Moody diagram is attached to the back of the exam; feel free to detach it if you find that convenient. Also, you can neglect minor losses.)

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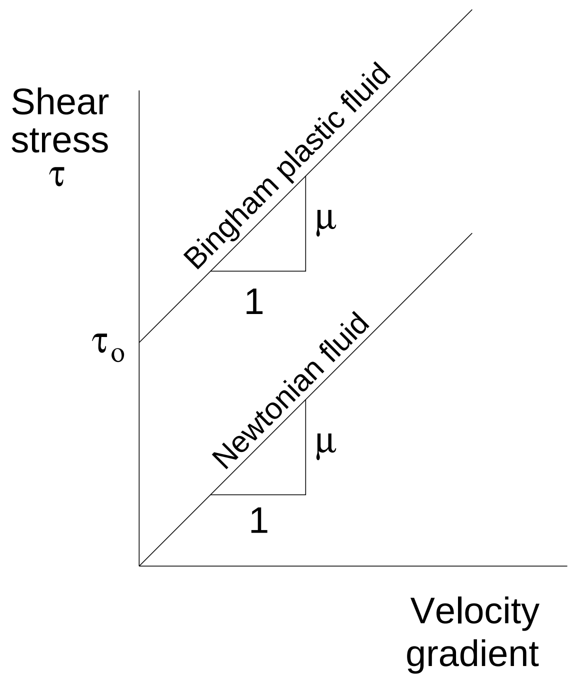
2. While things like water, air, and steam are Newtonian fluids, not all fluids are. For instance, Bingham plastic fluids (like toothpaste, mayonnaise, and mustard) can resist low amounts of shear stress before they begin to flow; at higher levels of shear stress, Bingham plastic fluids have a stress that increases linearly with increasing velocity gradient:

$$\frac{\partial u}{\partial r} = \begin{cases} 0 & \tau < \tau_0 \\ \dfrac{\tau - \tau_0}{\mu} & \tau \geq \tau_0 \end{cases}$$

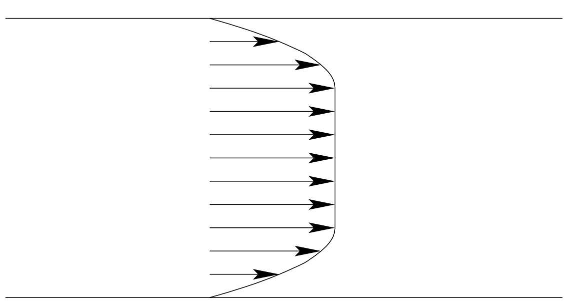
This is illustrated schematically below.



Pie filling, as it turns out, is a Bingham plastic fluid, with $\tau_0 = 33.6$ Pa, $\mu = 0.3\,{}^{\mathrm{kg}}/_{\mathrm{m \cdot sec}}$, and $\rho = 1250\,{}^{\mathrm{kg}}/_{\mathrm{m}^3}$.

In a factory that makes and cans pie filling, pie filling flows through a smooth round horizontal pipe with cross-sectional area $A = 0.004\,\mathrm{m}^2$ at a rate of 170 liters / minute. The velocity profile in the pipe (illustrated below) is given by

$$u(r) = \begin{cases} U_m & r < R/2 \\ 4U_m \dfrac{r(R-r)}{R^2} & r \geq R/2 \end{cases}$$

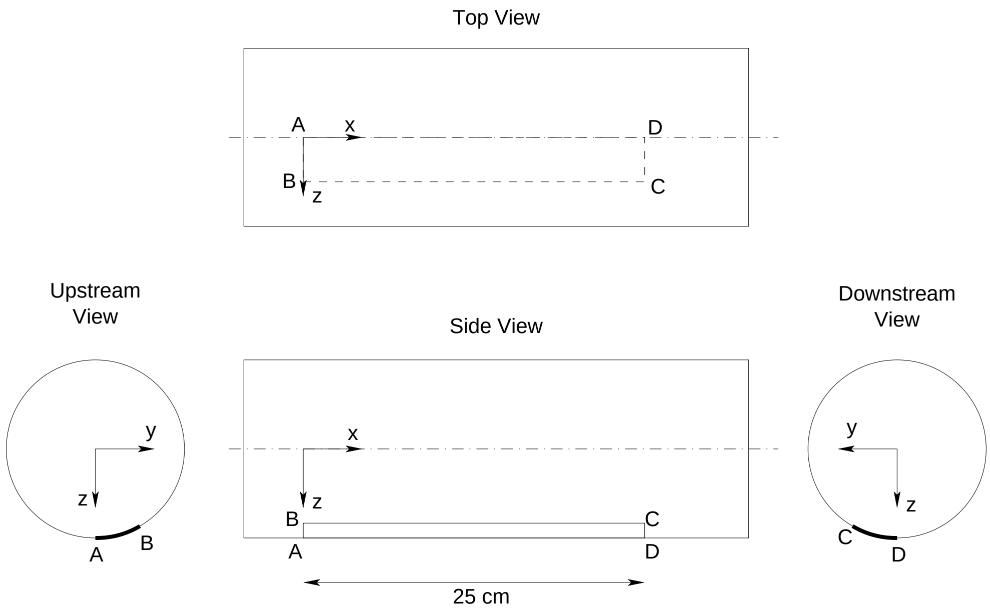


If you wish, you can detach this page from the exam for easy reference on subsequent pages. Be sure to put your student number on it if there's anything on the page you'd like marked.

(a) (10 marks; half math, half fluids) Find the maximum velocity of pie filling in the pipe U_m under these conditions. You are encouraged, though not required, to leave your answer in terms of flow rate and pipe area (or radius); if you compute a number, be sure to include the correct units.

(b) (10 marks; half math, half fluids) Find the momentum flux of pie filling through the cross section of the pipe at A. You are encouraged, though not required, to leave your answer in terms of density ρ, maximum velocity $U_{\max}$, and pipe area (or radius); if you compute a number, be sure to include the correct units.

(c) (10 marks Fluids) The pipe has a viewport ABCD, extending from the bottom center to 30 degrees above the bottom, as shown in the sketch. The length of the viewport along the pipe is 25 cm.



The gauge pressure along the centerline of the pipe in the coordinate system of the sketch is

$$P\left(x,y,0\right) = 4\,\text{kPa} - 3500x$$

Find the pressure force on the viewport. (Note: you do not need to find the line of action of the force.)

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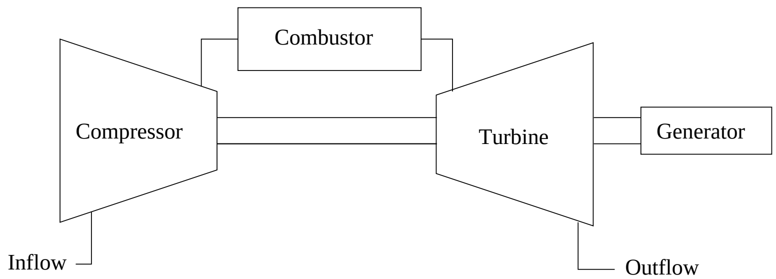
(d) (10 marks Math) Express the moment exerted by pressure on the viewport $ABCD$ about the line AB as an iterated surface integral. The positive sense of moment should be a moment that pushes the viewport out from the bottom of the pipe. **You do not need to evaluate this integral.**

(e) Bonus (10 marks Fluids): The pressure gradient along the pipe given in part (c) is not actually correct. Find the correct pressure gradient (equivalently, the change in pressure per unit length) in the pipe for these flow conditions.

3. The device depicted below is being used to generate power. It is found that the efficiency of the turbine has a significant dependence on the pressure ratio and the temperature flowing into it. To create a simplified model of the system, assume that the working fluid is an ideal gas with a constant ratio of specific heats, k; assume that the compressor stage is isentropic (as its performance seems to not depend on the variables we can control and any other model here would be too complicated!). The combustor is modelled as a simple heat exchanger and we can control the change in temperature, ΔT_c, across this. We can also control the pressure ratio, r_p, across the compressor (which is the same as the pressure ratio across the turbine).



   (a) (10 marks Thermo) Show that the thermal efficiency of this device, given the assumptions made above, can be expressed as a function of only the Inflow temperature T_1, the pressure ratio, the efficiency of the turbine η_t and the temperature difference across the combustor ΔT_c.

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The function in question 3a can be made to look like this:

$$\eta_{th} = C\left[\eta_t\left(r_p^a + 1/C - 1 - Cr_p^{-a}\right) + 1 - r_p^a\right]$$

with $C = \dfrac{T_1}{\Delta T_c}$ and $a = \dfrac{k-1}{k}$. The device presently operates with r_p=14 and ΔT_c=400K. The inflow temperature is 300K, and to get the model equation above to fit measurements taken of the system, $k = 1.5$ seems to work best. Measurements also suggest that the efficiency of the turbine can be approximated with the function:

$$\eta_t = \left(1 - \left(\frac{r_p}{30}\right)^3\right) \cdot \left(1 - \left(\frac{\Delta T_c}{3000}\right)^3\right).$$

(b) (15 marks Math) A technician working with the turbine claims that the overall device efficiency would be higher if we reduced the pressure ratio and increased the change in temperature across the combustion chamber. What does the model above suggest about this: is he right? Do *not* evaluate the above function in answering this question.

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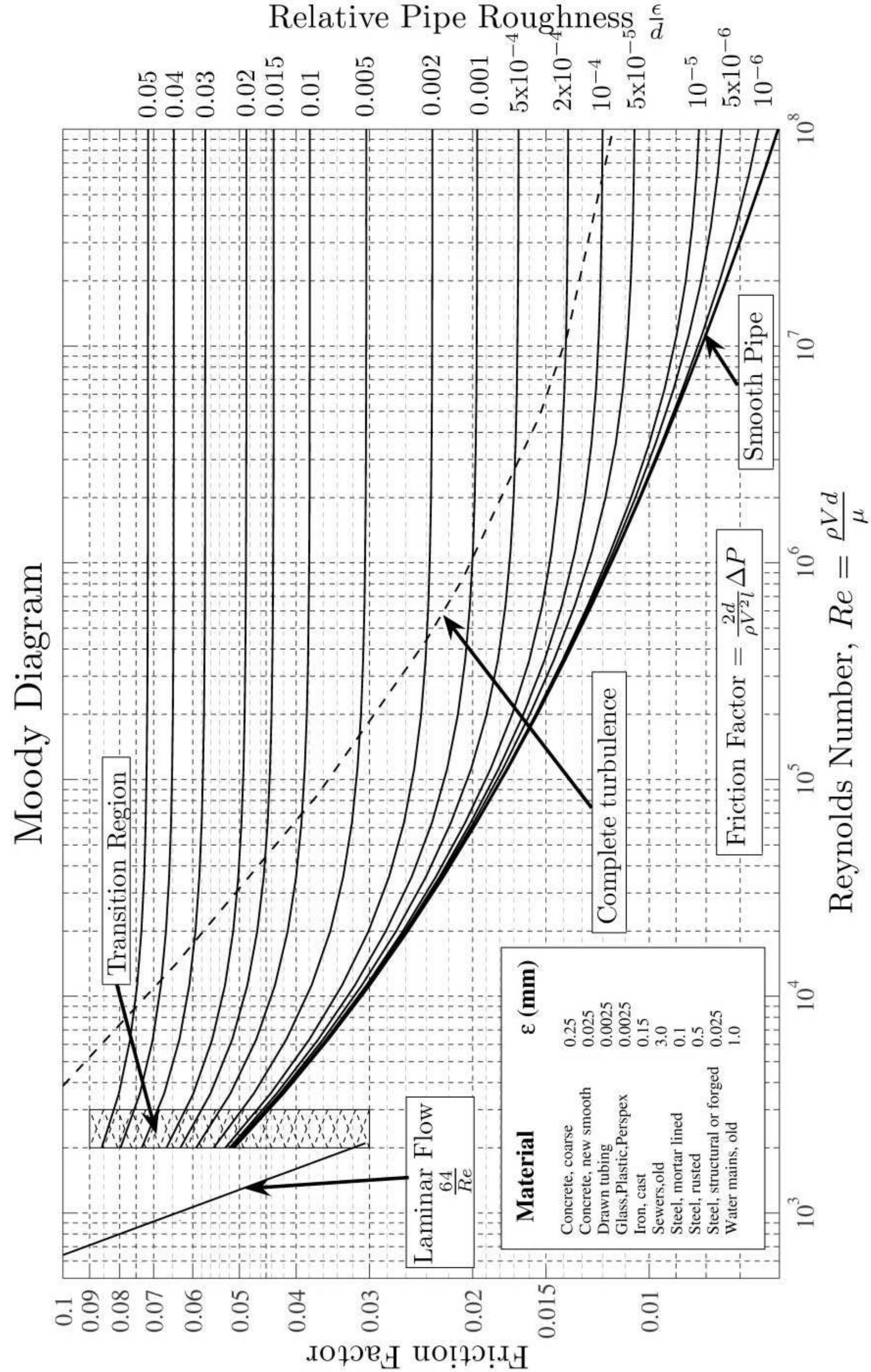
(c) (5 marks Math) From your work in part (a) you know that the current operating parameters are not optimal. Describe briefly how you would go about finding the optimal operating conditions, assuming you had free rein to change both the pressure ratio and temperature difference across the combustor.

(d) (15 marks Thermo) Prepare a short (∼200 words) essay to describe what issues you think might arise from using the model developed above to optimize performance of the device. You might consider design issues such as the effect of higher temperatures in the combustor on pollutants and their effect on the early stages of the turbine and also possible inaccuracies in using the model given the assumptions that were made in developing it.


Moody Diagram
Relative Pipe Roughness $\frac{\epsilon}{d}$
0.05
0.04
0.03
0.02
0.015
0.01
0.005
0.002
0.001
5x10^{-4}
2x10^{-4}
10^{-4}
5x10^{-5}
10^{-5}
5x10^{-6}
10^{-6}
Transition Region
Complete turbulence
Laminar Flow $\frac{64}{Re}$
Smooth Pipe
Friction Factor = $\frac{2d}{\rho V^2 l}\Delta P$
Reynolds Number, $Re = \frac{\rho V d}{\mu}$
Friction Factor
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.015
0.01
10^3
10^4
10^5
10^6
10^7
10^8
Material
ε (mm)
Concrete, coarse 0.25
Concrete, new smooth 0.025
Drawn tubing 0.0025
Glass,Plastic,Perspex 0.0025
Iron, cast 0.15
Sewers,old 3.0
Steel, mortar lined 0.1
Steel, rusted 0.5
Steel, structural or forged 0.025
Water mains, old 1.0