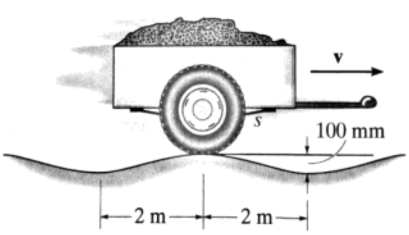
Test 5 Dynamics Solutions 2017W

November 1, 2017 9:12 PM

MECH 221 Test 5 2017W
Dynamics solutions

SA 1. [5 marks]



A 250-kg trailer is pulled with constant speed over the surface of a bumpy road, which may be approximated as a cosine curve with an amplitude of 50 mm and wavelength of 4 m. Two springs, s, with stiffness 800 N/m each, support the trailer. Neglect the mass of the wheels. Assume v = 10 km/h.

a) Find ω_n

Parallel springs:

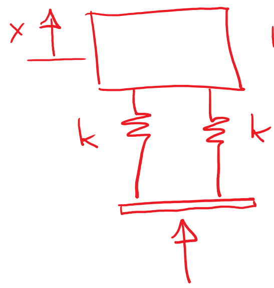
$k_{eq} = k + k$

$= 1600\ N/m$

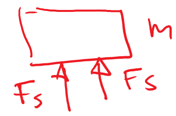
$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{1600\ N/m}{250\ kg}}$$

$= 2.53\ rad/s$

analogous to:



DERIVATION



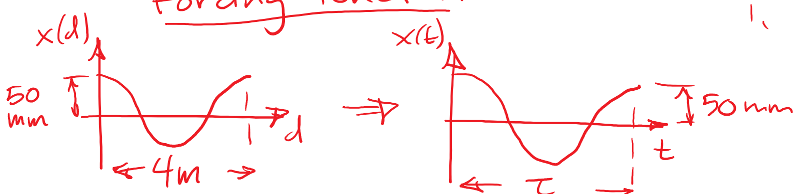
$\Sigma F_x:\ 2F_s = ma_x$

$-2kx = m\ddot{x}$

$m\ddot{x} + \underbrace{2k}_{k_{eq}} x = 0$

b) Find ω_0

Forcing function:



in terms of distance along road

in terms of time

* The relationship btwn distance version + time version is velocity. $v = 10\ km/hr$ $= 2.78\ m/s$

$$\tau = \frac{\lambda}{v} = \frac{4m}{2.78\ m/s} = 1.44s$$

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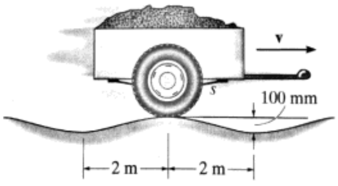
$$\boxed{\omega_0 = \frac{2\pi}{\tau} = \frac{2\pi}{1.44} = 4.36\ rad/s}$$

(λ = wavelength)

c) Find the magnification factor (MF)



$$MF = \frac{1}{1-\left(\frac{\omega_0}{\omega_n}\right)^2}$$

$$= \frac{1}{1-\left(\frac{4.36\ rad/s}{2.53\ rad/s}\right)^2}$$

$$\boxed{MF = -0.51}$$ (unitless)

This means x_c and x_p (free and forced components) move in opposition, and therefore you see an amplitude of about $\frac{1}{2} F_0/k$. (~ half static displacement)

d) At what velocity will the trailer experience resonance?

resonance occurs at $\omega_0 = \omega_n$

$\omega_0 = \omega_n = \frac{2\pi}{\tau}$ and $\tau = \frac{\lambda}{v}$

$\Rightarrow \omega_n = \frac{2\pi}{\lambda} \cdot v$

$\Rightarrow v = \frac{\omega_n \lambda}{2\pi} = \frac{2.53\ rad/s\ (4m)}{2\pi}$

$$\boxed{v_{resonance} = 1.61\ m/s = 5.80\ km/hr}$$

(we need to pass through this velocity to get to 10 km/hr - why doesn't the trailer shake apart?)