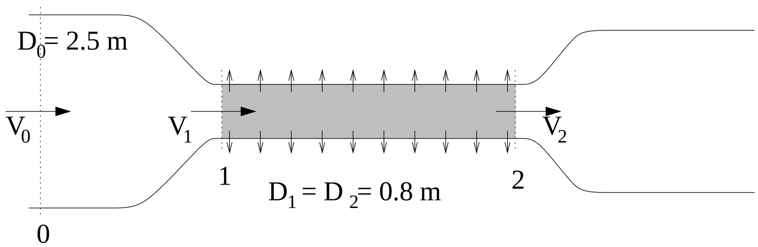
1. (10 marks) In some wind tunnels, the walls of the test section are perforated to suck out fluid and prevent the growth a viscous boundary layer along the wall that would otherwise affect the flow in the test section. The sketch shows a wind tunnel (of circular cross section) whose test section is perforated, with 1200 holes of 5 mm diameter for every square meter of surface. Air is sucked through the holes at a speed of 2 m/sec, and the velocity of air entering the test section, V_1, is 35 m/sec. Assume uniform, incompressible, steady flow of air at 20° C,



(a) (7 marks) Find V_0 and V_2 in m/sec.

I forgot to get the length of the test section labeled on the drawing; my apologies.

To find V_0, we can use conservation of mass:

$$\begin{aligned}
\rho V_0 A_0 &= \rho V_1 A_1 \\
V_0 \frac{\pi D_0^2}{4} &= V_1 \frac{\pi D_1^2}{4} \\
V_0 &= V_1 \frac{D_1^2}{D_0^2} \\
V_0 &= 3.58 \,\mathrm{m/sec}
\end{aligned}$$

To find V_2, we again use conservation of mass:

$$\begin{aligned}
\dot{m}_{in} &= \dot{m}_{out} \\
\rho V_1 A_1 &= \rho V_2 A_2 + \rho V_s A_s
\end{aligned}$$

where $V_s = 2\,\mathrm{m/sec}$ and A_s are the velocity and area for the leakage flow through the sides. The total lateral area of the test section is

$$A_l = \pi D_2 L$$

and there are 1200 holes of 5 mm diameter per square meter. The total area for all 1200 of those holes is $0.02356\,\mathrm{m}^2$, so the open fraction for the wall is 0.02356. That means that the area for all the holes in the sides of the tunnel is

$$\begin{aligned}
A_s &= 0.02356 \cdot A_l \\
&= 0.02356 \cdot \pi D_2 L
\end{aligned}$$

Putting this all together, we get:

$$\begin{aligned}
V_2 &= V_1 - \frac{V_s A_s}{A_2} \\
&= V_1 - \frac{0.02356 \cdot V_s \pi D_2 L}{\pi D_2^2/4} \\
&= V_1 - \frac{0.09425 \cdot V_s L}{D_2} \\
&= 35 - \frac{0.09524 \cdot 2 \cdot 1}{0.8} \\
&= 34.76 \,\mathrm{m/sec}
\end{aligned}$$

(b) (3 marks) What is the vector momentum flux at the inlet to the test section?

The momentum per unit volume is $\rho V_1 \hat{\imath}$, and the volume flow rate is $V_1 A_1$, so the momentum flux is

$$\begin{aligned}\rho V_1^2 A_1 \rightarrow &= 1.225 \cdot 35^2 \cdot \frac{\pi \cdot 0.8^2}{4} \rightarrow \\ &= 754\text{N} \rightarrow\end{aligned}$$