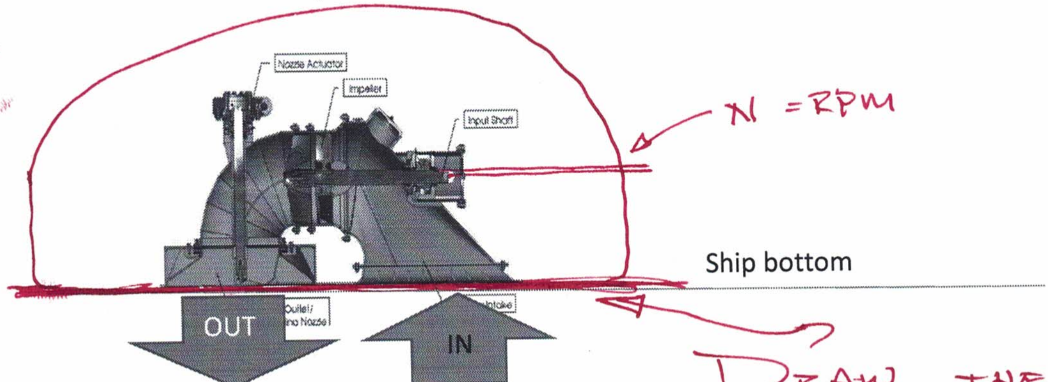
Exam 3 – Fluids

F.1 (5 marks) The NAMJet 360_Jet is a marine propulsion device that provides the ability to direct the propulsion thrust anywhere in 360-degrees. It works by drawing water into a pipe, accelerating the water with a propeller-like pump, and then discharging the accelerated water through a steerable grill.

Imagine you were asked to perform a control volume analysis of the flow through this machine. The goal of your analysis is to find the total force on a turbine blade, as this information can be used to determine total torque and give an estimate for efficiency. Draw the control volume that you would use for this analysis. Explain the reasoning behind the important choices you made in selecting that control volume boundary.

Note that no analysis is asked for – just the choice of the control volume.



DRAW THE CONTROL Volume here.

The pump becomes a black box tha "magically" imparts velocity to the flow. To find the torques & forces you will need the V_{in} & V_{OUT} and $\dot{m}$. this will give you the work done by the pump, And to get Torque you will need the RPM "N"

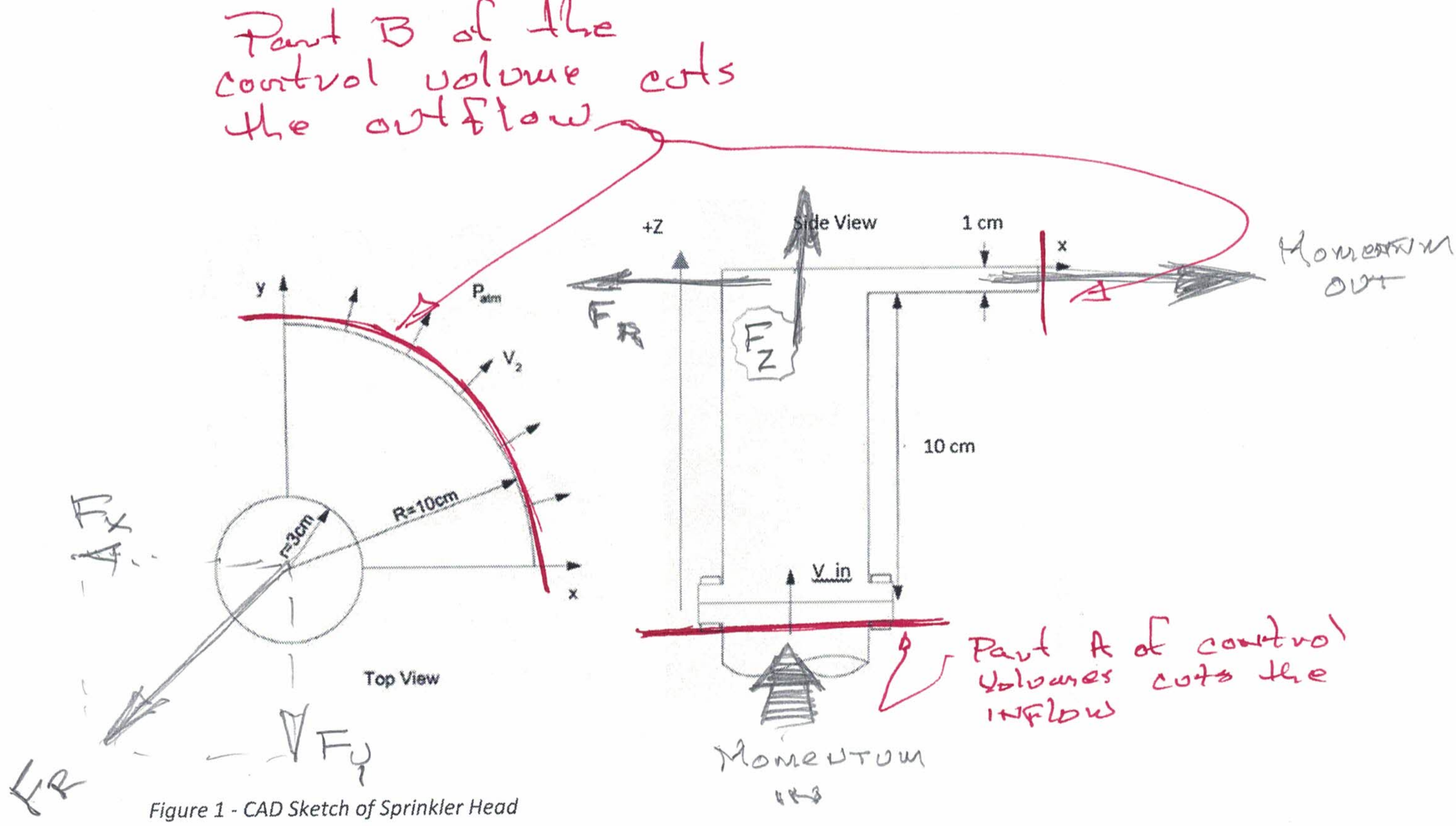


Figure 1 - CAD Sketch of Sprinkler Head

a) (2 marks) Draw a control volume (you'll need to draw on both parts of the sketch to indicate clearly what your control volume is).

b) (4 marks) Label your control volume clearly with all forces and momentum fluxes.

c) (6 marks) Use conservation of momentum for your control volume to write expressions for the force components Fx,Fy,and Fz in terms of ρ,V_in,$V2$,$P1$ and $Patm$ without (yet) finding values for all of those quantities.

d) (8 marks) Describe, with justification, what physical principles you will use to calculate theV_in,$V2$,and $P1$, and calculate them.

e) (5 marks) Finally, calculate values for the force components$Fx$and Fz , but not Fy.

c)

$$F_z = (\dot{m} V)_{in} + P_1 \cdot Area_{in}$$

$$F_x = F_y$$

$$F_R = \sqrt{F_x^2 + F_y^2} = (\dot{m} V)_{OUT}$$

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d) we will find V_{IN} & V_{OUT} from mass continuity.
we will find P_1 by Bernoulli, since all six limitations apply

d-1) find V_{in}

$$20 \frac{\text{litres}}{\text{sec}} = V_{in} \times Area_{in}$$

$$Area_{in} = \frac{\pi D_{in}^2}{4}$$

$$D_{in} = 6\,cm = 0.06\,m$$

$$Area_{in} = \frac{\pi}{4} 0.06^2 = 0.002827\,m^2$$

$$20\,\text{litres} = 0.02\,m^3$$

$$V_{in} = \frac{0.02\,\frac{m^3}{s}}{.00287\,m^2} = 7.1\,\frac{m}{s}$$

d-2) Find V_{OUT}

$$0.020 \frac{m^3}{s} = V_{OUT} \times Area_{OUT}$$

$$Area_{OUT} = \frac{1}{4} \text{ Circle } \times 1 \text{ cm}$$

$$= \frac{1}{4} \pi D \times 1\text{cm}$$

$$= \frac{20}{4} \pi = 15.71 \text{ cm}^2$$

$$= 0.001571 \text{ m}^2$$

$$V_{OUT} = \frac{0.020 \frac{m^3}{s}}{0.001571} = 12.73 \frac{m}{s}$$

d-3)

streamline:

$$\frac{1}{2}\rho V_{IN}^2 + P_1 \longrightarrow \frac{1}{2}\rho V_{OUT}^2 + P_{atm}$$

$$\frac{1}{2}\rho V_{IN}^2 + P_1 = \frac{1}{2}\rho V_{OUT}^2 + P_{atm}$$

$$P_1 = \frac{1}{2}\rho \left[V_{OUT}^2 - V_{IN}^2 \right] + P_{atm}$$

$$P_1 = 500 \left[12.73^2 - 7.1^2 \right] + P_{atm}$$

$$= 56 \text{ kPa} \quad \text{GAGE}$$

e) $F_z = \dot{m}V_{in} + P_1 \cdot Area_{in}$

$\dot{m} = 20 \ \frac{kg}{sec}$

$V_{in} = 7.1 \ m/s$

$P_1 = 56{,}000 \ N/m^2$

$Area_{in} = 0.002827 \ m^2$

$F_z = 299$ Newtons

$$F_x = F_y = \frac{\sqrt{2}}{2} F_z = \frac{\sqrt{2}}{2}\left[\dot{m} V_{out}\right]$$

$$= \frac{\sqrt{2}}{2} \cdot 20 \frac{kg}{sec} \cdot 12.73 \frac{m}{sec}$$

$$= 180 \ N$$

Q1 [5 marks] Find the equation of the tangent plane to the surface $z = \ln(2x + y)$ at the point $(-1,3)$.

$f(x,y) = \ln(2x+y)$ $\quad f_x = \frac{1}{2x+y}(2)$ $\quad f_x(-1,3) = 2$

$f(-1,3) = 0$ $\quad f_y = \frac{1}{2x+y}$ $\quad f_y(-1,3) = 1$

Equation of tangent plane:

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$z = 0 + 2(x-(-1)) + (y-3)$$

$$z = 2(x+1) + (y-3)$$

$$\boxed{2x + y - z = 1}$$

Q2 [5 marks] Find the **unit** vector $\vec{u}$ such that the directional derivative $D_{\vec{u}}f$ of the function $f(x,y,z) = x^2y + xz - zy^3$ is maximum at the point $P = (1,-1,1)$, and find the value of the directional derivative $D_{\vec{u}}f$ at P.

$\vec{u} = \frac{\nabla f(1,-1,1)}{\|\nabla f(1,-1,1)\|}$ $\qquad D_{\vec{u}}f(1,-1,1) = \|\nabla f(1,-1,1)\|$

$f_x = 2xy + z$ $\qquad \nabla f(1,-1,1) = (-1,-2,2)$

$f_x(1,-1,1) = -1$ $\qquad \|\nabla f(1,-1,1)\| = \sqrt{(-1)^2 + (-2)^2 + (2)^2} = 3$

$f_y = x^2 - 3zy^2$ $\qquad \boxed{\vec{u} = \frac{1}{3}(-1,-2,2)}$

$f_y(1,-1,1) = -2$

$f_z = x - y^3$ $\qquad \boxed{D_{\vec{u}}f(1,-1,1) = 3}$

$f_z(1,-1,1) = 2$

Solutions

MECH 222 Thermodynamics; March 02, 2017

Question (35 Marks)

A system consists of 2 kg of H_2O contained in a piston cylinder assembly that undergoes a cycle composed of followings:

Process 1-2	Constant pressure expansion
Process 2-3	Constant volume process
Process 3-4	Constant pressure compression
Process 4-1	Constant volume process

The following information are available:

At state 1, the fluid temperature is T_1 = 60 °C, and the volume of the fluid is 1 m^3.

At State 2, the water is at saturated vapor condition

At state 3, the pressure is 50,000 Pa

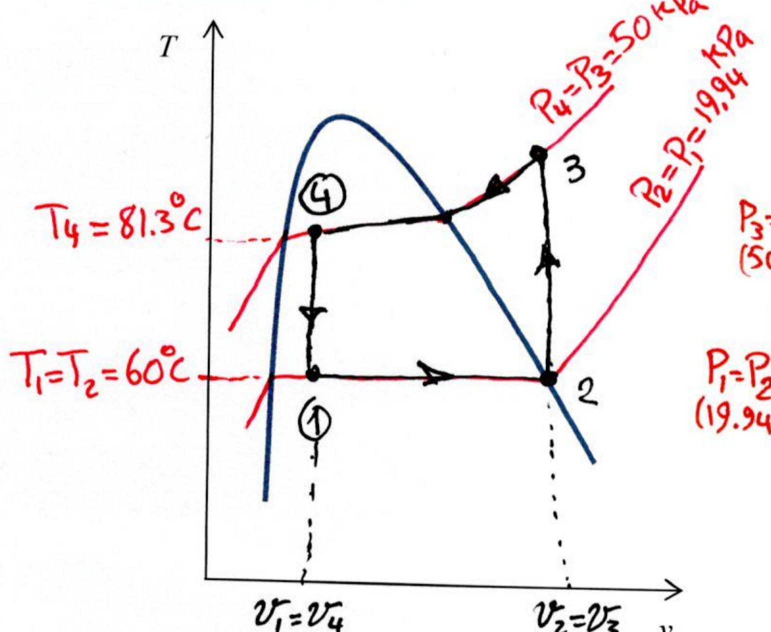
Neglect kinetic energy and potential energy effects.

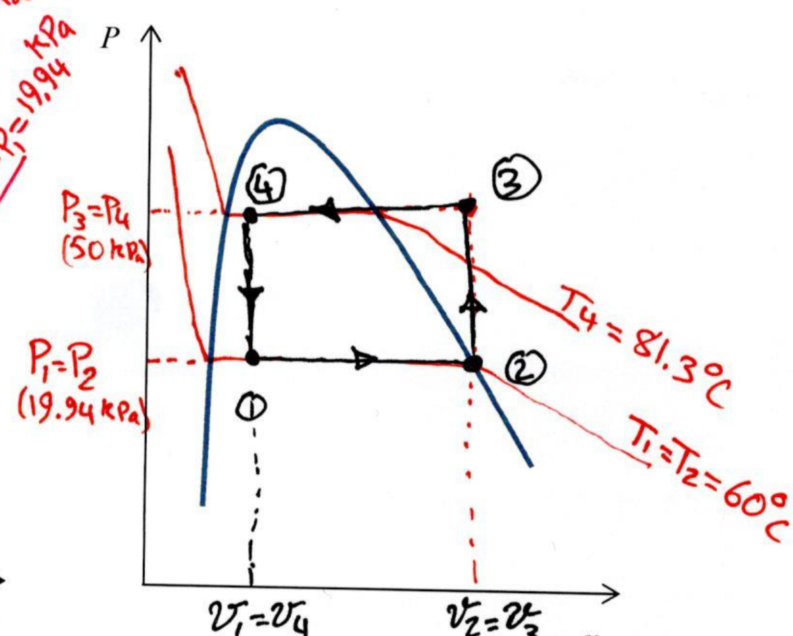
a) Sketch the cycle on T-v diagram (4.5 marks)

b) Sketch the cycle on P-v diagram (4.5 marks)

c) Determine the net amount of work for the cycle in kJ (10 marks)

d) Determine the heat transfer for each process in kJ: $Q_{1\text{-}2}$, $Q_{2\text{-}3}$, $Q_{3\text{-}4}$ and $Q_{4\text{-}1}$ (16 marks)





Please write your final answers in the Table below:

$W_{net\text{-}cycle}$ =		kJ
$Q_{1\text{-}2}$ =		kJ
$Q_{2\text{-}3}$ =		kJ
$Q_{3\text{-}4}$ =		kJ
$Q_{4\text{-}1}$ =		kJ

(1) $T_1 = 60°C$
$T_{sat} = 60°C \xrightarrow{\text{Table B.1.1.}} \begin{cases} v_f = 0.001017 \ m^3/kg \\ v_g = 7.67071 \ m^3/kg \\ u_f = 251.09 \ kJ/kg \\ u_g = 2456.63 \ kJ/kg \end{cases}$ $v_f < v < v_g$ two phase

$V_1 = 1 m^3$; $m = 2 kg$

$v_1 = \frac{V_1}{m} = \frac{1}{2} = 0.5 \ m^3/kg$

$$x_1 = \frac{v - v_f}{v_g - v_f} = \frac{0.5 - 0.001017}{7.67071 - 0.001017} = 0.065$$

$$u_1 = (1 - x_1) u_f + x_1 u_g = (1 - 0.065) 251.09 + 0.065 \times 2456.63$$

$$u_1 = 394.58 \ kJ/kg$$

(2) $P_2 = P_1 = P_{sat}\Big|_{T_1 = 60°C} = 19.941 \ kPa$; $v_2 = v_g = 7.6707 \ m^3/kg$

$u_2 = u_g = 2456.63$

(3) $\left.\begin{array}{l} v_3 = v_2 = 7.6707 \ m^3/kg \\ P_3 = 50 \ kPa \end{array}\right\} \xrightarrow{\text{Table B.1.1}} v_3 > v_g\Big|_{P_3}$ Superheated

Superheated Table $\xrightarrow{\text{B.1.3}}$

$P = 50$ kPa		
v	T	u
7.13364	500°C	3,131.94
8.05748	600°C	3,302.22

$$u_3 = 3131.94 + \frac{3131.94 - 3302.22}{7.13364 - 8.05748}(7.6707 - 7.13364)$$

$$u_3 = 3230.93 \ kJ/kg$$

$$T_3 = 500 + \frac{500 - 600}{7.13364 - 8.05748}(7.6707 - 7.13364) \Rightarrow T_3 = 558.13 \ °C$$

④ $P_4 = P_3 = 50\ kPa$

$v_4 = v_1 = 0.5\ m^3/kg$

at $P_{sat} = 50\ kPa \longrightarrow$
$$\begin{cases} v_g = 3.24034\ m^3/kg & u_f = 340.42\ kJ/kg \\ v_f = 0.001030\ m^3/kg & u_g = 2483.85\ kJ/kg \\ T_{sat} = 81.33^\circ C & \end{cases}$$

$v_f < v_4 < v_g$ Two phase

$$x_4 = \frac{v_4 - v_f}{v_g - v_f} = \frac{0.5 - 0.001030}{3.24034 - 0.001030} = 0.154$$

$$u_4 = (1-x_4)\,u_f + x_4\,u_g = (1-0.154)\,340.42 + 0.154 \times 2483.85$$

$u_4 = 670.51\ kJ/kg$

c) $W_{\substack{Net\\Cycle}} = ?$ the net work is the area of the Rectangle Presented on the P-v Diagram time mass of the Water.

$$|W_{Net}| = m\,(P_3 - P_1)(v_3 - v_1) = 2(50 - 19.94)(7.6707 - 0.5)$$

$$|W_{Net}| = 431.1\ kJ.$$

Note: $W_{1-2} = m \int_{v_1}^{v_2} P\,dv$

Alternatively

$$W_{Net} = W_{1-2} + \underbrace{W_{2-3}}_{0\ \ v_2 = v_3} + W_{3-4} + \underbrace{W_{4-1}}_{0\ \ v_4 = v_1}$$

$$= P_1(v_2 - v_1)\,m + P_3(v_4 - v_3)\,m = P_1(v_3 - v_1)\,m + P_3(v_1 - v_3)\,m$$

$$= m\,(P_3 - P_1)(v_1 - v_3) = \underline{-431.1\ kJ}$$ Work is done on the system

d) $Q_{1-2} = ?$

$$m(u_2 - u_1) = Q_{1-2} - W_{1-2}$$

$$Q_{1-2} = m(u_2 - u_1) + m P_1 (v_2 - v_1)$$
$$= 2(2456.63 - 394.58) + 2 \times 19.941 (7.6707 - 0.5)$$

$$Q_{1-2} = 4,124.1 + 285.98 = 4,410.1 \text{ kJ.}$$

$Q_{2-3} = ?$ $\quad m(u_3 - u_2) = Q_{2-3} - \cancel{W_{2-3}} \xrightarrow{} 0$

$$Q_{2-3} = 2(3230.93 - 2456.63) = 1,548.6 \text{ kJ.}$$

$Q_{3-4} = ?$

$$Q_{3-4} = m(u_4 - u_3) + m P_3 (v_4 - v_3)$$
$$= 2(670.51 - 3230.93) + 2 \times 50 (0.5 - 7.6707)$$
$$= -5120.84 - 717.07 = -5837.91 \text{ kJ.}$$

$Q_{4-1} = ?$

$$Q_{4-1} = m(u_1 - u_4) + \cancel{W_{4-1}} \xrightarrow{} 0$$
$$= 2(394.58 - 670.51) = -551.86 \text{ kJ.}$$

Note: for a cycle

$$\Delta E = \oint \delta Q - \oint \delta W = 0 \Rightarrow \oint \delta Q = \oint \delta W$$

$$\oint \delta Q = Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1} = 4,410.1 + 1,548.6 + (-5837.91) + (-551.86)$$
$$= -431.1 \text{ kJ.} \checkmark$$

$$\oint \delta W = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1} = -431.1 \text{ kJ} \checkmark$$

TABLE B.1

Thermodynamic Properties of Water

TABLE B.1.1

Saturated Water

Temp. (°C)	Press. (kPa)	Specific Volume, m³/kg Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g	Internal Energy, kJ/kg Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapor u_g
0.01	0.6113	0.001000	206.131	206.132	0	2375.33	2375.33
5	0.8721	0.001000	147.117	147.118	20.97	2361.27	2382.24
10	1.2276	0.001000	106.376	106.377	41.99	2347.16	2389.15
15	1.705	0.001001	77.924	77.925	62.98	2333.06	2396.04
20	2.339	0.001002	57.7887	57.7897	83.94	2318.98	2402.91
25	3.169	0.001003	43.3583	43.3593	104.86	2304.90	2409.76
30	4.246	0.001004	32.8922	32.8932	125.77	2290.81	2416.58
35	5.628	0.001006	25.2148	25.2158	146.65	2276.71	2423.36
40	7.384	0.001008	19.5219	19.5229	167.53	2262.57	2430.11
45	9.593	0.001010	15.2571	15.2581	188.41	2248.40	2436.81
50	12.350	0.001012	12.0308	12.0318	209.30	2234.17	2443.47
55	15.758	0.001015	9.56734	9.56835	230.19	2219.89	2450.08
60	19.941	0.001017	7.66969	7.67071	251.09	2205.54	2456.63
65	25.03	0.001020	6.19554	6.19656	272.00	2191.12	2463.12
70	31.19	0.001023	5.04114	5.04217	292.93	2176.62	2469.55
75	38.58	0.001026	4.13021	4.13123	313.87	2162.03	2475.91
80	47.39	0.001029	3.40612	3.40715	334.84	2147.36	2482.19
85	57.83	0.001032	2.82654	2.82757	355.82	2132.58	2488.40
90	70.14	0.001036	2.35953	2.36056	376.82	2117.70	2494.52
95	84.55	0.001040	1.98082	1.98186	397.86	2102.70	2500.56
100	101.3	0.001044	1.67185	1.67290	418.91	2087.58	2506.50
105	120.8	0.001047	1.41831	1.41936	440.00	2072.34	2512.34
110	143.3	0.001052	1.20909	1.21014	461.12	2056.96	2518.09
115	169.1	0.001056	1.03552	1.03658	482.28	2041.44	2523.72
120	198.5	0.001060	0.89080	0.89186	503.48	2025.76	2529.24
125	232.1	0.001065	0.76953	0.77059	524.72	2009.91	2534.63
130	270.1	0.001070	0.66744	0.66850	546.00	1993.90	2539.90
135	313.0	0.001075	0.58110	0.58217	567.34	1977.69	2545.03
140	361.3	0.001080	0.50777	0.50885	588.72	1961.30	2550.02
145	415.4	0.001085	0.44524	0.44632	610.16	1944.69	2554.86
150	475.9	0.001090	0.39169	0.39278	631.66	1927.87	2559.54
155	543.1	0.001096	0.34566	0.34676	653.23	1910.82	2564.04
160	617.8	0.001102	0.30596	0.30706	674.85	1893.52	2568.37
165	700.5	0.001108	0.27158	0.27269	696.55	1875.97	2572.51
170	791.7	0.001114	0.24171	0.24283	718.31	1858.14	2576.46
175	892.0	0.001121	0.21568	0.21680	740.16	1840.03	2580.19
180	1002.2	0.001127	0.19292	0.19405	762.08	1821.62	2583.70
185	1122.7	0.001134	0.17295	0.17409	784.08	1802.90	2586.98
190	1254.4	0.001141	0.15539	0.15654	806.17	1783.84	2590.01

TABLE B.1.2
Saturated Water Pressure Entry

		Specific Volume, m³/kg			Internal Energy, kJ/kg		
Press. (kPa)	Temp. (°C)	Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapor u_g
0.6113	0.01	0.001000	206.131	206.132	0	2375.3	2375.3
1	6.98	0.001000	129.20702	129.20802	29.29	2355.69	2384.98
1.5	13.03	0.001001	87.97913	87.98013	54.70	2338.63	2393.32
2	17.50	0.001001	67.00285	67.00385	73.47	2326.02	2399.48
2.5	21.08	0.001002	54.25285	54.25385	88.47	2315.93	2404.40
3	24.08	0.001003	45.66402	45.66502	101.03	2307.48	2408.51
4	28.96	0.001004	34.79915	34.80015	121.44	2293.73	2415.17
5	32.88	0.001005	28.19150	28.19251	137.79	2282.70	2420.49
7.5	40.29	0.001008	19.23674	19.23775	168.76	2261.74	2430.50
10	45.81	0.001010	14.67254	14.67355	191.79	2246.10	2437.89
15	53.97	0.001014	10.02117	10.02218	225.90	2222.83	2448.73
20	60.06	0.001017	7.64835	7.64937	251.35	2205.36	2456.71
25	64.97	0.001020	6.20322	6.20424	271.88	2191.21	2463.08
30	69.10	0.001022	5.22816	5.22918	289.18	2179.22	2468.40
40	75.87	0.001026	3.99243	3.99345	317.51	2159.49	2477.00
50	81.33	0.001030	3.23931	3.24034	340.42	2143.43	2483.85
75	91.77	0.001037	2.21607	2.21711	394.29	2112.39	2496.67
100	99.62	0.001043	1.69296	1.69400	417.33	2088.72	2506.06
125	105.99	0.001048	1.37385	1.37490	444.16	2069.32	2513.48
150	111.37	0.001053	1.15828	1.15933	466.92	2052.72	2519.64
175	116.06	0.001057	1.00257	1.00363	486.78	2038.12	2524.90
200	120.23	0.001061	0.88467	0.88573	504.47	2025.02	2529.49
225	124.00	0.001064	0.79219	0.79325	520.45	2013.10	2533.56
250	127.43	0.001067	0.71765	0.71871	535.08	2002.14	2537.21
275	130.60	0.001070	0.65624	0.65731	548.57	1991.95	2540.53
300	133.55	0.001073	0.60475	0.60582	561.13	1982.43	2543.55
325	136.30	0.001076	0.56093	0.56201	572.88	1973.46	2546.34
350	138.88	0.001079	0.52317	0.52425	583.93	1964.98	2548.92
375	141.32	0.001081	0.49029	0.49137	594.38	1956.93	2551.31
400	143.63	0.001084	0.46138	0.46246	604.29	1949.26	2553.55
450	147.93	0.001088	0.41289	0.41398	622.75	1934.87	2557.62
500	151.86	0.001093	0.37380	0.37489	639.66	1921.57	2561.23
550	155.48	0.001097	0.34159	0.34268	655.30	1909.17	2564.47
600	158.85	0.001101	0.31457	0.31567	669.88	1897.52	2567.40
650	162.01	0.001104	0.29158	0.29268	683.55	1886.51	2570.06
700	164.97	0.001108	0.27176	0.27286	696.43	1876.07	2572.49
750	167.77	0.001111	0.25449	0.25560	708.62	1866.11	2574.73
800	170.43	0.001115	0.23931	0.24043	720.20	1856.58	2576.79

TABLE B.1.3
Superheated Vapor Water

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	$P = 10$ kPa (45.81°C)				$P = 50$ kPa (81.33°C)			
Sat.	14.67355	2437.89	2584.63	8.1501	3.24034	2483.85	2645.87	7.5939
50	14.86920	2443.87	2592.56	8.1749	—	—	—	—
100	17.19561	2515.50	2687.46	8.4479	3.41833	2511.61	2682.52	7.6947
150	19.51251	2587.86	2782.99	8.6881	3.88937	2585.61	2780.08	7.9400
200	21.82507	2661.27	2879.52	8.9037	4.35595	2659.85	2877.64	8.1579
250	24.13559	2735.95	2977.31	9.1002	4.82045	2734.97	2975.99	8.3555
300	26.44508	2812.06	3076.51	9.2812	5.28391	2811.33	3075.52	8.5372
400	31.06252	2968.89	3279.51	9.6076	6.20929	2968.43	3278.89	8.8641
500	35.67896	3132.26	3489.05	9.8977	7.13364	3131.94	3488.62	9.1545
600	40.29488	3302.45	3705.40	10.1608	8.05748	3302.22	3705.10	9.4177
700	44.91052	3479.63	3928.73	10.4028	8.98104	3479.45	3928.51	9.6599
800	49.52599	3663.84	4159.10	10.6281	9.90444	3663.70	4158.92	9.8852
900	54.14137	3855.03	4396.44	10.8395	10.82773	3854.91	4396.30	10.0967
1000	58.75669	4053.01	4640.58	11.0392	11.75097	4052.91	4640.46	10.2964
1100	63.37198	4257.47	4891.19	11.2287	12.67418	4257.37	4891.08	10.4858
1200	67.98724	4467.91	5147.78	11.4090	13.59737	4467.82	5147.69	10.6662
1300	72.60250	4683.68	5409.70	14.5810	14.52054	4683.58	5409.61	10.8382
	100 kPa (99.62°C)				200 kPa (120.23°C)			
Sat.	1.69400	2506.06	2675.46	7.3593	0.88573	2529.49	2706.63	7.1271
150	1.93636	2582.75	2776.38	7.6133	0.95964	2576.87	2768.80	7.2795
200	2.17226	2658.05	2875.27	7.8342	1.08034	2654.39	2870.46	7.5066
250	2.40604	2733.73	2974.33	8.0332	1.19880	2731.22	2970.98	7.7085
300	2.63876	2810.41	3074.28	8.2157	1.31616	2808.55	3071.79	7.8926
400	3.10263	2967.85	3278.11	8.5434	1.54930	2966.69	3276.55	8.2217
500	3.56547	3131.54	3488.09	8.8341	1.78139	3130.75	3487.03	8.5132
600	4.02781	3301.94	3704.72	9.0975	2.01297	3301.36	3703.96	8.7769
700	4.48986	3479.24	3928.23	9.3398	2.24426	3478.81	3927.66	9.0194
800	4.95174	3663.53	4158.71	9.5652	2.47539	3663.19	4158.27	9.2450
900	5.41353	3854.77	4396.12	9.7767	2.70643	3854.49	4395.77	9.4565
1000	5.87526	4052.78	4640.31	9.9764	2.93740	4052.53	4640.01	9.6563
1100	6.33696	4257.25	4890.95	10.1658	3.16834	4257.01	4890.68	9.8458
1200	6.79863	4467.70	5147.56	10.3462	3.39927	4467.46	5147.32	10.0262
1300	7.26030	4683.47	5409.49	10.5182	3.63018	4683.23	5409.26	10.1982
	300 kPa (133.55°C)				400 kPa (143.63°C)			
Sat.	0.60582	2543.55	2725.30	6.9918	0.46246	2553.55	2738.53	6.8958
150	0.63388	2570.79	2760.95	7.0778	0.47084	2564.48	2752.82	6.9299
200	0.71629	2650.65	2865.54	7.3115	0.53422	2646.83	2860.51	7.1706