Name: ____________________ **Signature:** ____________________ **UBC ID:** ________________

University of British Columbia
Faculty of Applied Science
Department of Mechanical Engineering

MECH 222
Exam 2: 28 March 2013

Duration: 3 hours

This exam consists of 17 pages (including this cover sheet). Check to ensure that it is complete.

Rules Governing Formal Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
4. Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
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   - (b) purposely exposing written papers to the view of other candidates or imaging devices;
   - (c) purposely viewing the written papers of other candidates;
   - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Question	**Marks**	**Score**
1	12	
2	12	
3	15	
4	25	
5	15	
6	10	
7	25	
8	6	
TOTAL	120	

1. (Math: 12 marks) A cargo ship makes a daily trip from town P, at $(b, 0)$, to town Q, at $(0, b)$. The trip starts at time $t = 0$ and ends at time $\pi T/2$; the ship's position at time t is

$$x(t) = b\cos^3(t/T), \quad y(t) = b\sin^3(t/T), \qquad 0 \le t \le \pi T/2.$$

The ship's motion through the water is opposed by a drag force whose magnitude is given by

$$F(V) = \frac{1}{2}C_D \rho V^2 A.$$

Here V is the ship's speed, A is some characteristic area of the ship's submerged part, ρ is the density of water, and C_D is a (constant) drag coefficient.

(a) Find the total length of the ship's path as it travels from P to Q. (*Hint*: The correct answer is independent of T.)

(b) Let W denote the total work the ship must do against drag on the trip described above. Express W as a single definite integral.

(c) Evaluate the work W defined in part (b). Verify that the units of your final expression are correct.

2. (Math: 12 marks) In 1873, Johannes Diderik van der Waals (Nobel Prize, 1910) proposed the following relationship between pressure P, temperature T, and specific volume v:

$$\left(P + \frac{\alpha}{v^2}\right)\left(v - \beta\right) = RT.$$

This is "the van der Waals equation of state", which applies throughout this question. Here α, β, and R are constants that depend on the material. For air, which we will treat as a van der Waals substance,

$$R = 0.286\,\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}, \qquad \alpha = 0.162\,\frac{\mathrm{kJ}\cdot\mathrm{m}^3}{\mathrm{kg}^2}, \qquad \beta = 0.00126\,\frac{\mathrm{m}^3}{\mathrm{kg}}.$$

We are interested in the state where $T_0 = 120$ K and $v_0 = 0.004\ \mathrm{m}^3/\mathrm{kg}$.

(a) Complete the following sentence by writing correct words in the box provided.
Using the approximations $(\alpha/v^2)/P \approx 0$ and $\beta/v \approx 0$ amounts to treating a van der Waals substance as ... $\boxed{}$.

(b) Find the pressure, P_0, at the state of interest. Give an explicit decimal value, with units.

(c) Write a linear relationship between P, v, and T that provides a good approximation to the van der Waals equation of state when the parameters are close to P_0, v_0, and T_0. (A symbolic answer is fully acceptable here.)

(d) Find explicit decimal values, including units, for both $\left(\dfrac{\partial P}{\partial v}\right)_T$ and $\left(\dfrac{\partial T}{\partial v}\right)_P$ at the state (P_0, v_0, T_0).

(e) Provide a one-sentence written response to each question below.

- What does the value of $\left(\dfrac{\partial P}{\partial v}\right)_T$ say about the relationship between P and v near (P_0, v_0, T_0)?

- Is there are any significant qualitative difference between this relationship and more standard ones? If so, what is it?

- If your previous answer is "Yes," what scientific response do you recommend?

3. (Fluids: 15 marks) On Tuesday's Exam 1, you were asked to non-dimensionalize the aerodynamic drag force F on a bobsled, which depends on the bobsled's speed V, length L, and the air density ρ and viscosity μ:

$$F = F(\rho, V, L, \mu)$$

(a) On Tuesday's exam, Student A and Student B both got the Reynolds number $\mathrm{Re} = \dfrac{\rho V L}{\mu}$ as one non-dimensional parameter, but they got two different non-dimensional force coefficients, Π_A and Π_B. Specifically, Student A wrote the common form

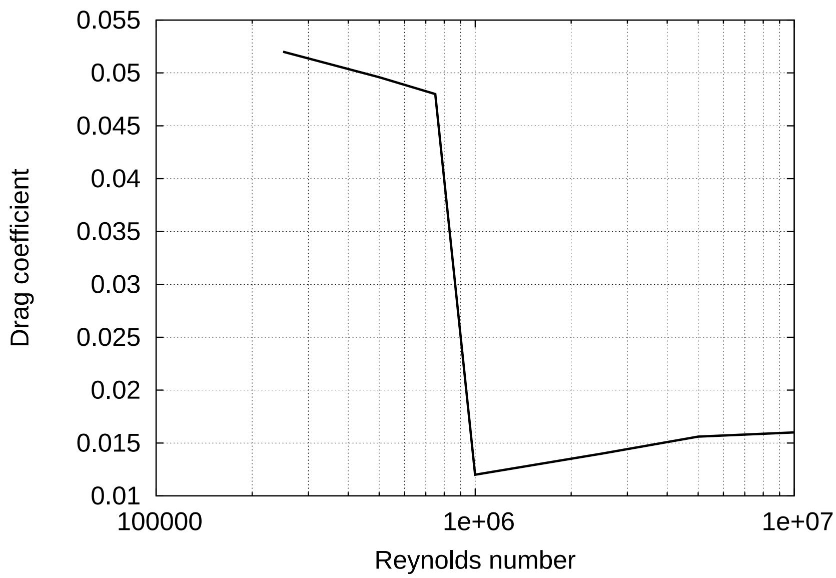
$$\Pi_A \equiv \frac{F}{\rho V^2 L^2} = f_A\left(\mathrm{Re}\right)$$

while Student B wrote

$$\Pi_B \equiv \frac{\rho F}{\mu^2} = f_B\left(\mathrm{Re}\right)$$

Are both of these answers correct? If they are, how will $f_A\left(\mathrm{Re}\right)$ and $f_B\left(\mathrm{Re}\right)$ be related? Why might one form have a significant advantage versus the other for scaling experimental results?

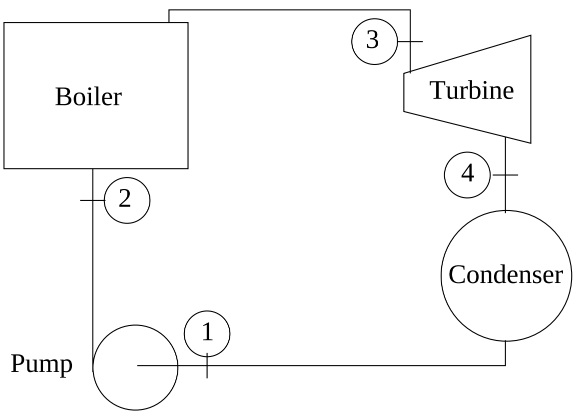
The full-sized bobsled is 6 m long. A scale model is tested in a wind tunnel, and the results post-processed to produce a plot of drag coefficient $C_D = \dfrac{F}{\frac{1}{2}\rho V^2 L^2}$ (note the inclusion of the conventional factor of $\dfrac{1}{2}$ in that definition) as a function of Reynolds number $\mathrm{Re} = \dfrac{\rho V L}{\mu}$.



(b) Early in a run, the bobsled travels at 30 km/hour. Based on the non-dimensional experimental data from the figure, estimate the drag force on the bobsled at this speed. Clearly state any assumptions you must make.

(c) Late in a run, the bobsled has accelerated to 130 km/hour. Based on the non-dimensional experimental data from the figure, estimate the drag force on the bobsled at this speed. Clearly state any assumptions you must make.

4. (Thermo: 25 marks) Consider the simple power plant shown below. The flow rate of water in the cycle is 5 kg/s. The pressure at states 2 and 3 is 5 MPa and the pressure at states 1 and 4 is 10 kPa. At state 1, we have saturated liquid water (so the quality is 0). At state 3, the temperature is 500°C. Assuming that the pump and turbine both have efficiencies of 95%, use the steam tables provided to calculate the net power output from the plant and its thermal efficiency. (Note: the units for T, v, u, h and s in the lower two tables are the same as those given in the upper table.)



Saturated Steam: Pressure Table

		Specific Volume m³/kg		Internal Energy kJ/kg			Enthalpy kJ/kg			Entropy kJ/kg K		
Press. kPa P	Temp. °C T	Sat. Liquid v_f	Sat. Vapor v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapor u_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
0.6113	0.01	0.001 000	206.14	.00	2375.3	2375.3	.01	2501.3	2501.4	.0000	9.1562	9.1562
1.0	6.98	0.001 000	129.21	29.30	2355.7	2385.0	29.30	2484.9	2514.2	.1059	8.8697	8.9756
1.5	13.03	0.001 001	87.98	54.71	2338.6	2393.3	54.71	2470.6	2525.3	.1957	8.6322	8.8279
2.0	17.50	0.001 001	67.00	73.48	2326.0	2399.5	73.48	2460.0	2533.5	.2607	8.4629	8.7237
2.5	21.08	0.001 002	54.25	88.48	2315.9	2404.4	88.49	2451.6	2540.0	.3120	8.3311	8.6432
3.0	24.08	0.001 003	45.67	101.04	2307.5	2408.5	101.05	2444.5	2545.5	.3545	8.2231	8.5776
4.0	28.96	0.001 004	34.80	121.45	2293.7	2415.2	121.46	2432.9	2554.4	.4226	8.0520	8.4746
5.0	32.88	0.001 005	28.19	137.81	2282.7	2420.5	137.82	2423.7	2561.5	.4764	7.9187	8.3951
7.5	40.29	0.001 008	19.24	168.78	2261.7	2430.5	168.79	2406.0	2574.8	.5764	7.6750	8.2515
10	45.81	0.001 010	14.67	191.82	2246.1	2437.9	191.83	2392.8	2584.7	.6493	7.5009	8.1502
15	53.97	0.001 014	10.02	225.92	2222.8	2448.7	225.94	2373.1	2599.1	.7549	7.2536	8.0085
20	60.06	0.001 017	7.649	251.38	2205.4	2456.7	251.40	2358.3	2609.7	.8320	7.0766	7.9085
25	64.97	0.001 020	6.204	271.90	2191.2	2463.1	271.93	2346.3	2618.2	.8931	6.9383	7.8314
30	69.10	0.001 022	5.229	289.20	2179.2	2468.4	289.23	2336.1	2625.3	.9439	6.8247	7.7686
40	75.87	0.001 027	3.993	317.53	2159.5	2477.0	317.58	2319.2	2636.8	1.0259	6.6441	7.6700

Superheated Vapor

P = 5.0 MPa (263.99)

T	v	u	h	s
Sat.	.039 44	2597.1	2794.3	5.9734
275	.041 41	2631.3	2838.3	6.0544
300	.045 32	2698.0	2924.5	6.2084
350	.051 94	2808.7	3068.4	6.4493
400	.057 81	2906.6	3195.7	6.6459
450	.063 30	2999.7	3316.2	6.8186
500	.068 57	3091.0	3433.8	6.9759
600	.078 69	3273.0	3666.5	7.2589
700	.088 49	3457.6	3900.1	7.5122
800	.098 11	3646.6	4137.1	7.7440
900	.107 62	3840.7	4378.8	7.9593
1000	.117 07	4040.4	4625.7	8.1612
1100	.126 48	4245.6	4878.0	8.3520
1200	.135 87	4456.3	5135.7	8.5331
1300	.145 26	4672.0	5398.2	8.7055

Compressed Liquid

P = 5 MPa (263.99)

T	v	u	h	s
Sat.	.001 285 9	1147.8	1154.2	2.9202
0	.000 997 7	.04	5.04	.0001
20	.000 999 5	83.65	88.65	.2956
40	.001 005 6	166.95	171.97	.5705
60	.001 014 9	250.23	255.30	.8285
80	.001 026 8	333.72	338.85	1.0720
100	.001 041 0	417.52	422.72	1.3030
120	.001 057 6	501.80	507.09	1.5233
140	.001 076 8	586.76	592.15	1.7343
160	.001 098 8	672.62	678.12	1.9375
180	.001 124 0	759.63	765.25	2.1341
200	.001 153 0	848.1	853.9	2.3255
220	.001 186 6	938.4	944.4	2.5128
240	.001 226 4	1031.4	1037.5	2.6979
260	.001 274 9	1127.9	1134.3	2.8830

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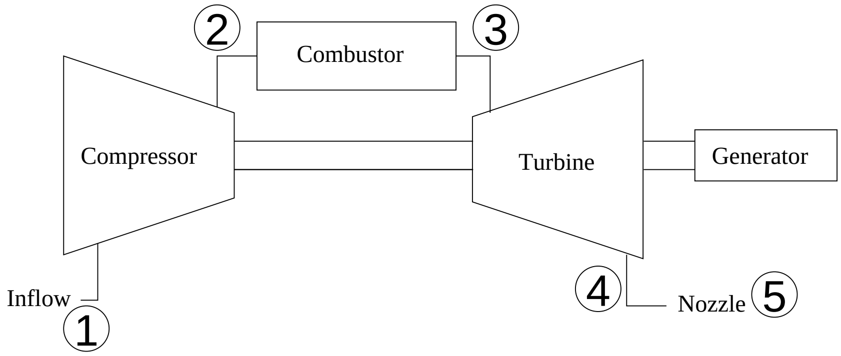
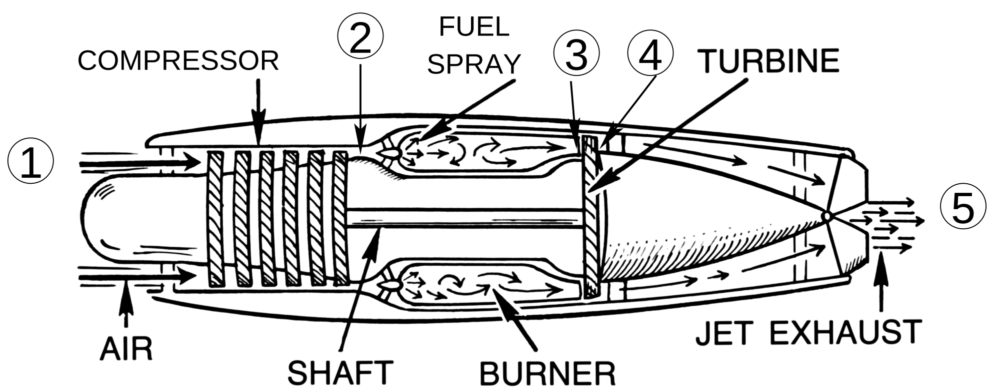
Questions 5, 6, and 7 are all related to the same physical device and operating conditions. However, they are designed to be solved independently.

A jet engine is in essence an air-standard Brayton cycle in which the turbine expands the combustion products only part way to atmospheric pressure. The turbine produces enough power to run the compressor; auxiliary components neglected here; and a generator to provide electrical power for the airplane. The exhaust gases from the turbine are then accelerated by expansion in a nozzle to produce thrust.





These questions deal with a simple approximation to the behavior of a small jet engine, just after take-off, when the airplane is traveling at 100 m/sec; the ambient air around the airplane is at 100 kPa, 15° C. The mass flow rate of air (which we will treat as an ideal gas with $R = 0.287$ kJ/kg·K and constant $C_{p0} = 1.004$ kJ/kg·K) into the engine is 21 kg/sec.

Feel free to remove this page from your exam for easy reference in problems 5–7.

5. (Thermo: 15 marks) The pressure ratio across the compressor is 10. After leaving the compressor, the air is “heated” in a combustion chamber to 1250 K (we’ll continue to treat it as air with the same properties). It then flows into a turbine. The turbine produces enough power to drive the compressor, plus another 2 MW for auxiliary equipment and electricity. The compressor and turbine each have efficiencies of 90%. What is the pressure and temperature of the air leaving the turbine?

   Note: throughout the compressor and turbine, kinetic energy changes are very small compared with enthalpy changes, and so can be neglected.

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6. (10 marks; Thermo: 5, Fluids: 5) After leaving the turbine, the air flows into a specially shaped nozzle that accelerates the air to provide thrust. In the nozzle, flow is isentropic. The nozzle exit area is 0.05 m^2 and the pressure at the exit from the nozzle is atmospheric. Find the temperature, density, and average velocity of the exhaust gases at the nozzle exit.

   - For this question, do not use your findings from Question 5. Instead, assume that the pressure and temperature in state 4 are $P_4 = 216$ kPa and $T_4 = 790$ K.
   - You may assume a uniform thermodynamic state at the nozzle exit.

7. (25 marks; Fluids: 15, Math: 10) At the exit of the nozzle, the velocity profile for the air is well approximated by

$$u(r) = U_{\max}\left(1 - \frac{r^4}{R^4}\right)$$

where R is the inside radius of the circular nozzle and $U_{\max}$ is a constant; other velocity components are zero. Use this velocity profile in a control volume analysis to find the thrust of the engine. Be sure to draw and label your control volume, and state clearly any assumptions you make.

- For this question, do not use your findings from Question 6. Instead, assume that the density of air at the nozzle exit is 0.6 kg/m^3. Note that this makes it unsafe to rely on whatever average velocity you may have found in Question 6.
- For other given quantities, continue to use the data given.

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8. (Math: 6 marks) To celebrate a successful conclusion to Mech 222, a professor decides to build a monument in the shape of the solid $\mathcal{R}$ defined below:

$$x \geq 0, \quad y \geq 0, \quad z \geq 0, \quad x + y + z \leq a.$$

The design requires the total flux of $\mathbf{F}$ through all faces of $\mathcal{R}$ to be exactly 222, where

$$\mathbf{F} = 2x^2\,\mathbf{i} + 4yz\,\mathbf{j} + \frac{\tan^{-1}(xy)}{\cosh^2(x + \sin y)}\,\mathbf{k}.$$

Find a.