Lesson 5 – Video 2
Simple and Compound, Interest

Interest Rate

- For discrete cash flows, there are two types of interest:
    1. Simple Interest: interest is calculated once only and paid at the end of the term
    2. Compound Interest: interest is calculated periodically and accumulated into the balance based on the compounding periods.
        - “The Miracle of Compound Interest”
        - Interest on top of interest

Definitions

- P or PV = Present Value
- F or FV = Future Value
- i – interest rate per period
- I – interest (dollar value): I_n – interest for single period n
- n – number of periods

1. Simple Interest

- Simple Interest is interest applied only to the original sum, and only at the end of the term (on the maturity date)
- Interest is never calculated on previous interest calculations.
- Total interest (I) = $P \times i \times n$
    - Where: P = present sum/value or principle, i = interest rate per period, and n = # of time periods
    - For example: P = $100; i = 9% per year; 6 months

$$I = \$100 \times 9\%/\text{period} \times \tfrac{1}{2}\ \text{period} = \$4.50$$

Simple Interest Continued…

- Simple annual interest rate = 9% (change to decimal)

End-of-year	Amount Borrowed	Interest for Period	Amount Owed at EOY
0	\$1000		
1		\$90	\$1090
2		\$90	\$1180
3		\$90	\$1270

$$\$1180 = \$1000 + \$1000(0.09)2$$

Simple Interest Continued…

- Amount of money due or made at the end of a loan is the total money after n periods or 'Future value' = F

$$F = P + Pin$$

$$\boxed{\$1180 = \$1000 + \$1000(0.09)2}$$

2. Compound Interest

- Compound interest is computed on the unpaid principal and on the unpaid interest every compounding period and it is accumulated with the outstanding amount.
- "Interest on top of interest"
- Total interest: $I_n = P(1 + i)^n - P$
  - where P = present sum of money/principle amount; i = interest rate per period; n = number of compounding periods
- For example: P = \$100; i = 9% compounded annually; five-year term

$$I_5 = \$100 \times (1+0.09)^5 - \$100 = \$53.86$$

Compound Interest Formula

- Remember the definitions: P, F, i, and n

- Value at time zero, start of Period 1: P
  - Interest at end of first period: P*i
  - Future value at end of first period: $F = P+P*i = P(1+i)$
- Value at start of Period 2: $P(1+i)$
  - Interest at end of second period: $iP(1+i)$
  - Future value at end of second period: $P(1+i)+iP(1+i) = P(1+i)^2$
- Continue the series for an arbitrary length n, $F = P(1+i)^n$
- Single Payment Compound Amount: $F = P(1+i)^n$

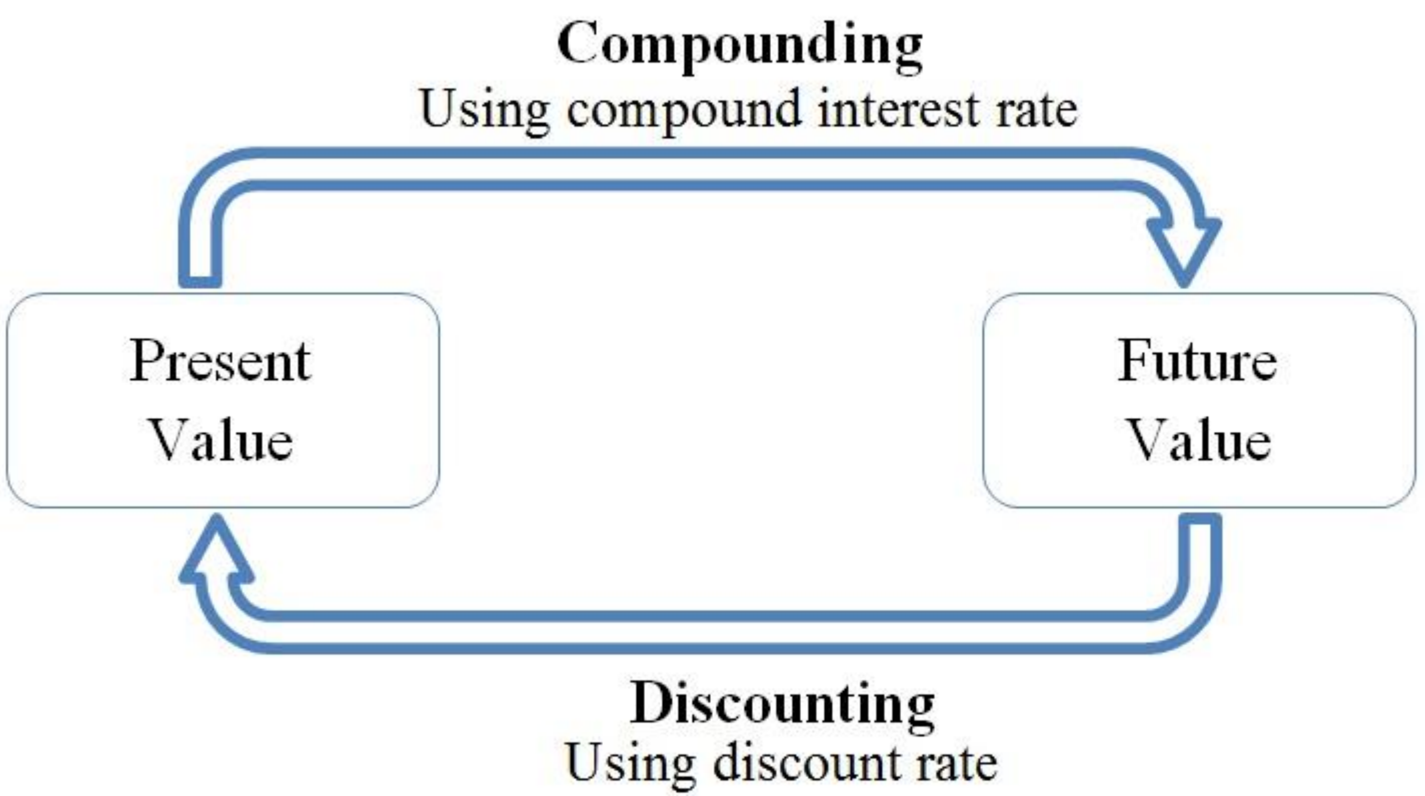
Compound Interest Formula

- Single Payment Compound Amount: $F = P(1+i)^n$

- Can also write in a functional notation:
- $F = P(F/P, i, n)$, the SINGLE PAYMENT COMPOUND AMOUNT FACTOR
  - Note this factor is constant for any given combination of I and n
  - Can simply plug in a give P and solve for F
  - Values can be read easily from compound interest tables

- We can also flip this around to solve for P: $P = F(1+i)^{-n}$
- Or, in functional notation:
- $P = F(P/F, i, n)$, the SINGLE PAYMENT PRESENT WORTH FACTOR

Compounding and Discounting



- *Credit: Farid Tayari*

Definition of 'Discount Rate'

- In finance, discounted cash flow (DCF) analysis is a method of valuing a project, company, or asset using the concepts of the time value of money. All future cash flows are estimated and discounted to give their present values (PVs)—the sum of all future cash flows, both incoming and outgoing, is the net present value (NPV), which is taken as the value or price of the cash flows in question
- **The discount rate refers to the interest rate used in DCF analysis to determine the present value of future cash flows.**
- The greater the uncertainty of future cash flows, the higher the discount rate.

Compound Interest Continued…

- How much to invest?
- $F = P(1+i)^n$

  $P = F(1+i)^{-n}$

- For example: FV = \$1,000,000; i = 4%; 25-year term

$$P = \$1000000(1 + 0.04)^{-25} = \$375,116.80$$

Single-Payment Compound Interest

Year	Beginning balance	Interest for period	Ending balance
1	P	iP	$P(1+i)$
2	$P(1+i)$	$iP(1+i)$	$P(1+i)^2$
3	$P(1+i)^2$	$iP(1+i)^2$	$P(1+i)^3$
n	$P(1+i)^{n-1}$	$iP(1+i)^{n-1}$	$P(1+i)^n$

- Notation:
  - i = interest rate per period
  - n = number of interest periods
  - P = a present sum of money = single payment present value factor $P = F(1+i)^{-n}$
  - F = a future sum of money = single payment future value factor $F = P(1+i)^n$
- Assume:
  - Calculations are done at the end of period

Compound Interest Continued…

- Compound annual interest rate = 9%

End-of-year	Amount Borrowed	Interest for Period	Amount Owed at EOY
0	\$1000		
1		\$90.00	\$1090.00
2		\$98.10	\$1188.10
3		\$106.93	\$1295.03

$$\boxed{\$1188.10 = \$1000(1+0.09)^2}$$

Compound Interest – PV Function

- Excel provides a handy function to do these calculations without building up a table

- =PV(rate, nper, pmt, [fv], [type])
  - Rate: interest rate per period
  - Nper: number of periods to calculate
  - Pmt: any payment made per period. Set to zero for single payment calculations, will look at this more soon
  - Fv: future value, optional.
  - Type: 0 or omitted – payments due at end of period
    1 – payments due at beginning of period. Leave omitted for now

- Tricky bits:
  - Ensure your number of periods and interest rate per period are aligned (same as with other methods)
  - Signs signs signs – positive and negatives signs are very particular

Single Payment PV Function Example

Simple & Compound Interest: Problem

How much more would you owe at the end of 4 years using simple interest at 9% per year, versus compound interest at 9% per year, if you borrowed $150,000 now?

Simple & Compound Interest: Solution

Simple Interest

$F = P + Pin = 150{,}000 + 150{,}000(0.09)(4)$

$F = \$204{,}000$

Compound Interest

$F = P(1 + i)^n$

$F = 150{,}000(1.09)^4$

$F = \$211{,}737.24$

211,737.24 – 204,000 = $7737.24

You would owe $7737.24 more with a compound interest arrangement.

Simple & Compound Comparison

- The difference is negligible over a short period of time; but over a long period of time it may a considerable amount.
  - After 25 years
    - Simple $325
    - Compound $862
  - After 50 years
    - Simple $550
    - Compound $7435
  - After 100 years
    - Simple $1000
    - Compound $552904

	Principal:	$ 100.00
	Interest rate:	9.00%
Period (years)	Simple Interest	Compound Interest
0	$100.00	$100.00
1	$109.00	$109.00
2	$118.00	$118.81
3	$127.00	$129.50
4	$136.00	$141.16
5	$145.00	$153.86
6	$154.00	$167.71
7	$163.00	$182.80
8	$172.00	$199.26
9	$181.00	$217.19
10	$190.00	$236.74
11	$199.00	$258.04
12	$208.00	$281.27
13	$217.00	$306.58
14	$226.00	$334.17

Single-Payment Compound Interest: Future Example

Problem:

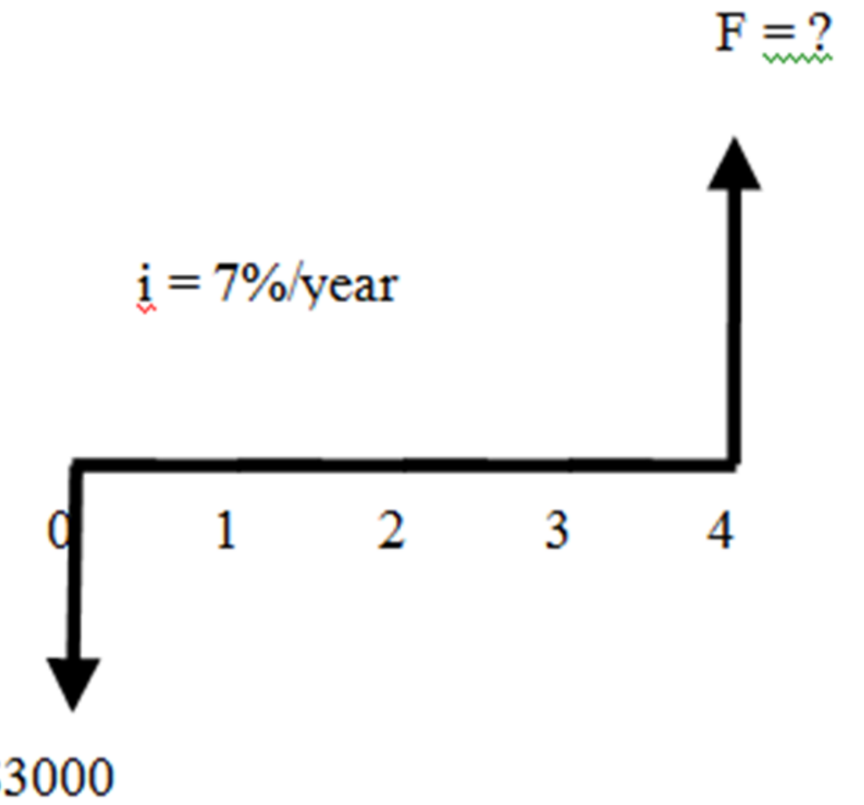
\$3000.00 deposited in a bank account at 7% per year interest would be how much after four years?

Solution:

$F = P(1+i)^n$

$F = 3000(1+0.07)^4$

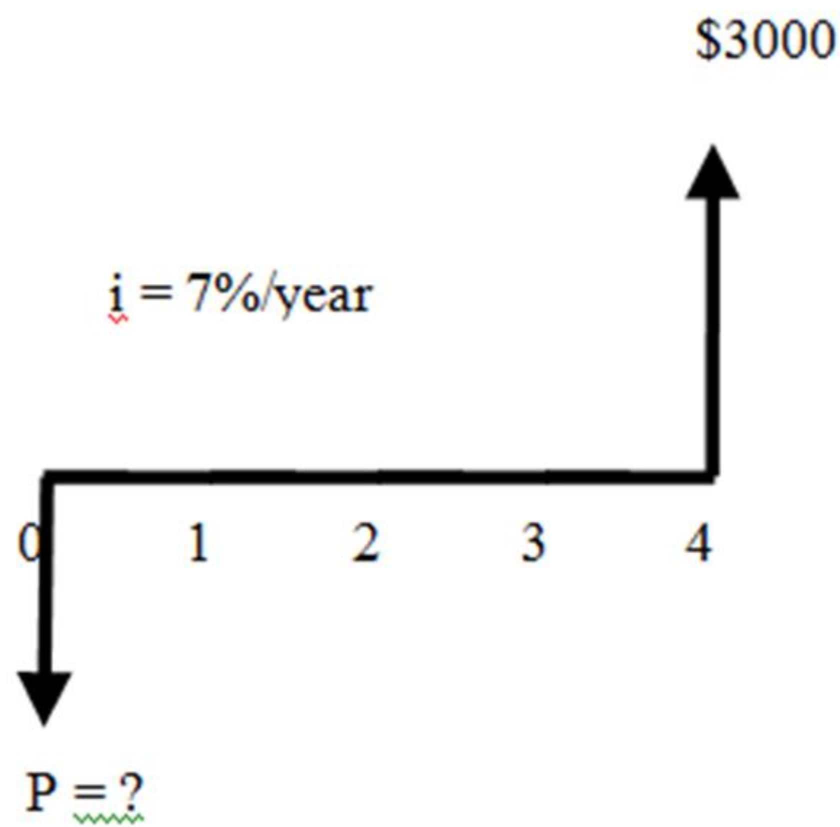
$F = \$3932.39$



Single-Payment Compound Interest: Present Example

Problem:

If you want to have \$3000.00 in the bank after four years at 7% per year interest, what would you have to deposit now?



Solution:

$F = P(1+i)^n$

$P = F(1+i)^{-n}$

$P = 3000(1+0.07)^{-4}$

$P = \$2288.69$

Calculating Compound Interest

- Formula
  - $F = P(1+i)^n$
  - Moderate precision, fairly easy to remember formula
- Spreadsheet functions
  - =FV(rate, nper, pmt, [pv], [type])
  - High precision, lots of additional factors to remember
- Compound factors and interest tables (online, or back of text)
  - F = P(F/P, i, n)
  - Lower precision, quick and easy to use

$2\frac{1}{2}\%$

	Single Payment	
n	Compound Amount Factor Find F Given P F/P	Present Worth Factor Find P Given F P/F
1	1.025	.9756
2	1.051	.9518
3	1.077	.9286
4	1.104	.9060
5	1.131	.8839