Question 1

(a) Giving reasons, discuss how the sensitivity analysis and the error analysis of an engineering system can be done using the same analytical basis. What are some limitations to this approach of error analysis? **(10%)**

Note: Error analysis in the present context involves determination of how the error in one component of the system affects the performance of another component in the system, and determination of the overall error in the system performance in terms of the errors in the individual components.

(b) A pressure sensor that uses microelectromechanical systems (MEMS) technology and that is connected to a resistance bridge is shown in Figure Q2. The sensor has a circular diaphragm of radius R and thickness t, which is made of n-silicon (single crystal silicon doped with a donor impurity) of Young's modulus E and Poisson's ratio υ. Four MEMS strain gauges of resistance $R1$, $R2$, $R3$, and $R4$ are implanted in the diaphragm as shown so that when a pressure P is applied at the top side of the diaphragm, the resistances $R2$ and $R3$ decrease (due to compressive strain) while the resistances $R1$ and $R4$ increase (due to tensile strain). The strain gauges are made of p-silicon (single crystal silicon doped with an acceptor impurity).

The four strain gauges are connected to a resistance bridge of reference dc voltage v_{ref}, as shown in Figure Q2. When there is no pressure (i.e., P = 0) the diaphragm is unstrained, and the bridge is balanced (i.e., the output voltage v_o of the bridge is zero). It can be shown that, when a non-zero pressure is present, the bridge output may be expressed as

$$\frac{v_o}{v_{ref}} = \alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E} P$$

where,

$\alpha =$ diaphragm constant

$k =$ bridge constant = [bridge output]/[bridge output when only one strain gauge is active]

$G =$ gauge factor (sensitivity) of a strain gauge

Determine expressions (in terms of the parameters in the above sensor equation) for:

(i) Overall sensitivity of the sensor (i.e., $\frac{\partial v_o}{\partial P}$)

(ii) Sensitivity of the sensor for a unit reference voltage.

Compute numerical values for these two sensitivities (in the units V/MPa and V/V/MPa) using the following parameter values:

α = 3/8, k = 4, G = 150.0, diaphragm radius = 1 mm, diaphragm thickness = 50 μm, υ = 0.25, Young's modulus = 150.0 GPa, and $v_{ref} = 20.0$ V

(25%)

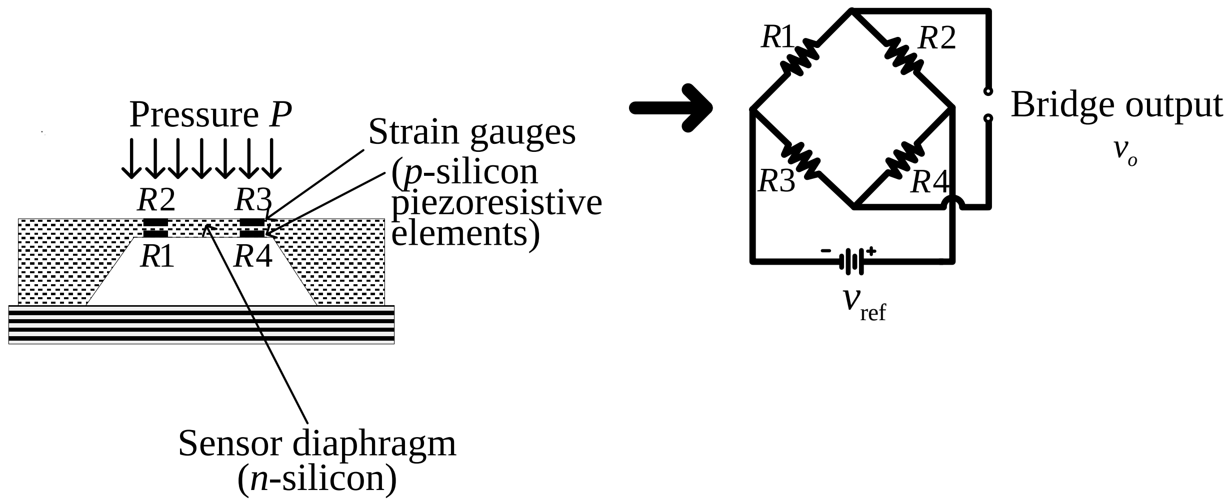

Figure Q2: MEMS-based pressure sensor that uses a resistance bridge.

(c) Using the *absolute error* method determine an expression for the fractional error in the bridge output (v_o) in terms of the fractional errors in G, R, t, υ, E, v_{ref}, and P.

Note: Assume that α and k are error free. The errors in G, R, t, υ, E, and v_{ref} are *sensor error*. The error in P comes from errors such as noise and disturbances that enter the process before the pressure signal is measured by the sensor (i.e., *process error*).

(20%)

Solution

Solution 1

(a) In analysing the sensitivity of one parameter/variable on another parameter/variable in a system, we may apply a small increment (a differential) to the first parameter/variable and determine the corresponding increment in the second parameter/variable. The ratio of the two increments is in fact the derivative of the second quantity with respect to the first quantity.

For error analysis, these increments may be interpreted as the “errors” in the corresponding parameters/variables, and the same analytical procedure as for the sensitivity analysis may be used.

Limitations of this approach include the following:

1. An analytical model (or at least an experimental model) must be available, to represent the relationship between various parameters/variables of interest in the system (for determining the corresponding derivatives).

2. The required first derivatives are assumed to exist, which may not be the case in some practical situations (e.g., Coulomb friction, dead zone, saturation, where the derivative is either zero or infinity)
3. The procedure assumes small increments in the sense that the second order terms in a Taylor series expansion are neglected. In other words, only the first derivatives are used, which in fact means the use of a "linear" model. Hence, if the errors or changes in the considered quantities are large, the method can become less accurate or meaningless.

(b) The system model is $\frac{v_o}{v_{ref}} = \alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E} P$. This may be expressed as

$$v_o = \alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E} v_{ref} P.$$

(i) The overall sensitivity is $\frac{\partial v_o}{\partial P} = \alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E} v_{ref}$

Substitute numerical values:

$$\alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E} v_{ref} = \frac{3}{8} \times 4 \times 150.0 \times \frac{(1\times 10^{-3})^2 \times (1-0.25^2)}{(50\times 10^{-6})^2 \times 150 \times 10^9} \times 20.0 \text{ V/Pa}$$

Overall sensitivity = 11.25×10^{-6} V/Pa = 11.25 V/MPa

(ii) The sensitivity with respect to a unit reference voltage is

$$\frac{\partial^2 v_o}{\partial v_{ref} \partial P} = \frac{1}{v_{ref}} \frac{\partial v_o}{\partial P} = \alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E}$$

Substitute numerical values:

$$\alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E} = \frac{11.25 \times 10^{-6}}{20} \text{ V/V/Pa}$$

$= 0.5625 \times 10^{-6}$ V/V/Pa = 0.5625 V/V/MPa

(c) The system model is $v_o = \alpha k G \frac{R^2(1-\upsilon^2)}{t^2 E} v_{ref} P$

Take the natural logarithm:

$$\ln v_o = \ln \alpha + \ln k + \ln G + 2\ln R - 2\ln t + \ln(1-\upsilon^2) - \ln E + \ln v_{ref} + \ln P$$

Take the differentials of the individual terms (*Note*: Neglect the differentials of α and k because they do not have errors). We get

$$\frac{\delta v_o}{v_o} = \frac{\delta G}{G} + 2\frac{\delta R}{R} - 2\frac{\delta t}{t} - \frac{2\upsilon \delta \upsilon}{(1-\upsilon^2)} - \frac{\delta E}{E} + \frac{\delta v_{ref}}{v_{ref}} + \frac{\delta P}{P}$$

Or,

$$\frac{\delta v_o}{v_o} = \frac{\delta G}{G} + 2\frac{\delta R}{R} - 2\frac{\delta t}{t} - \frac{2\upsilon^2}{(1-\upsilon^2)} \frac{\delta \upsilon}{\upsilon} - \frac{\delta E}{E} + \frac{\delta v_{ref}}{v_{ref}} + \frac{\delta P}{P}$$

Now, in terms of fractional errors, we have

$$e_{v_o} = e_G + 2e_R - 2e_t - \frac{2\upsilon^2}{(1-\upsilon^2)} e_\upsilon - e_E + e_{v_{ref}} + e_P$$

When expressed as absolute values (the errors will be additive regardless of the actual sign), we have

$$\left|e_{v_o}\right| = \left|e_G\right| + 2\left|e_R\right| + 2\left|e_t\right| + \frac{2\upsilon^2}{(1-\upsilon^2)} \left|e_\upsilon\right| + \left|e_E\right| + \left|e_{v_{ref}}\right| + \left|e_P\right|$$