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I promise to work honestly on this exam, to obey all instructions carefully, and not have any unfair advantage over any other students.

Start : 19:00
End : 21:30

$\downarrow x \quad -m\ddot{y} - m(\ddot{x}-\ddot{y}) - K(x-y) - C(\dot{x}-\dot{y}) = 0$

$m\ddot{z} + C\dot{z} + kz = -m\ddot{y} = -mY\cos\omega_f t$

$(-m\omega_f^2 + iC\omega_f + k)Re[De^{i\omega_f t}] = Re[m\ddot{Y}e^{i\omega_f t}]$

$D = m\ddot{Y}/(-m\omega_f^2 + iC\omega_f + k)$

$|D| = (m\ddot{Y}/K)/\sqrt{(1-r^2)^2+(2\xi r)^2}$ $= (m\ddot{Y}/k)/((1-r^2)+i(2\xi r))$

$\angle D = \tan^{-1}(-2\xi r/(1-r^2))$

$x = e^{-\xi\omega_n t}(A\cos\omega_d t - B\sin\omega_d t)$

$\omega = 2\pi f$

$\omega_n = \omega_d/\sqrt{1-\xi^2}$

$r = \omega_f/\omega_n$ >1 over

$\omega_n^2 = K/m$ =1 critical

$\xi = C/2\sqrt{km}$ → $0<\xi<1$ under, =0 no

$\omega_f C/K = 2\xi r$

$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \xi 2\pi \frac{\omega_n}{\omega_d} = \frac{\xi 2\pi}{\sqrt{1-\xi^2}}$

$x = A\cos\omega t - B\sin\omega t$
$= C\cos(\omega t + \phi)$
$= Ge^{i\omega t} + He^{-i\omega t}$
$= Re[De^{i\omega t}]$

$x = x_c + x_p$

x_p = any sol that is an x

$\uparrow kx_1 \; \uparrow m\ddot{x}_1$, $\downarrow f_1 \; \downarrow 2K(x_2-x_1)$

$\uparrow 2m\ddot{x}_2 \; \uparrow 2K(x_2-x_1)$, $\downarrow f_2 \; \downarrow 2Kx_2$

$\uparrow x_2 \downarrow f_2$

$\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}[\ddot{x}] + \begin{bmatrix} 3k & -2k \\ -2k & 4k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$ $f(t)=0, \omega=\omega_n$

$X = X\cos(\omega t + \phi) \to (-\omega^2 m + K)X\cos(\omega t + \phi) = 0$

$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc = 0 \to \omega_n^2 = k/m, 4k/m$

$x = e u \to \begin{bmatrix} -\omega^2 m + 3K & -2K \\ -2K & -2\omega^2 m + 4K \end{bmatrix}\begin{pmatrix} 1 \\ u \end{pmatrix} = 0 \to u = 1, -\frac{1}{2}$

decouple: $x = up \to u = [u_1 \; u_2] = \begin{pmatrix} 1 & 1 \\ 1 & -\frac{1}{2} \end{pmatrix}$

$Mu\ddot{p} + Kup = F$

$u^T M u \ddot{p} + u^T K u p = u^T F$ $\left(u = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \to u^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\right)$

$m^*\ddot{p} + k^* p = f^*$

$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $K = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

MATLAB:

$M = [[a\ b]'[c\ d]']$,

$K = [[e\ f]'[g\ h]']$;

$[V, w2] = eig(K, M, 'vector')$;

$V = u$ mode shape

$w2 = \omega^2$

$M\ddot{x} + Kx = F$

$(-\omega_f^2 M + K)X = F$

$X = (-\omega_f^2 M + K)^{-1}F$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det\begin{bmatrix} a & b \\ c & d \end{bmatrix}}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$X_1 = \frac{dF_1 - bF_2}{ad - bc}$

$X_2 = \frac{-cF_1 + aF_2}{ad - bc}$

$M\ddot{x} + C\dot{x} + K = 0$

$X = X_0 e^{\lambda t}$

$(M\lambda^2 + C\lambda + K)X = 0$

$\det[M\lambda^2 + C\lambda + K] = 0$

$(\lambda^2 + D)(A\lambda^2 + B\lambda + C) = 0$

$\omega_n^2 = C/A$ $(\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0)$

$\xi = B/2\sqrt{AC}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$u_3 = \begin{bmatrix} 1 \\ \alpha \\ \beta \end{bmatrix}$ $M = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix}$

$u_1^T M u_3 = 0$, $u_2^T M u_3 = 0$ $\Big\}$ $u_3 = \begin{bmatrix} 1 \\ \cdot \\ 1 \end{bmatrix}$ matrix symmetry

$M\ddot{x} + C\dot{x} + K = F\cos\omega_f t = Re(Fe^{i\omega_f t})$

$(-\omega_f^2 M + i\omega_f C + K)Xe^{i\omega_f t} = Fe^{i\omega_f t}$

$x = Re(Xe^{i\omega_f t})$ $[-\omega_f^2 M + i\omega_f C + K][X] = [F]$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}[X] = [F]$

$X_1 = \frac{dF_1 - bF_2}{ad - bc} = A + iB$

$X_2 = \frac{-cF_1 + aF_2}{ad - bc} = C + iD$

$x = Re(Xe^{i\omega_f t}) = Re((A+iB)(\cos\omega_f t + i\sin\omega_f t))$

$X_1 = A\cos\omega_f t - B\sin\omega_f t$

$X_2 = C\cos\omega_f t - D\sin\omega_f t$

$|X| = \sqrt{A^2+B^2} = \sqrt{C^2+D^2}$

phase $= \arctan\left(\frac{B}{A}\right) = \arctan\left(\frac{D}{C}\right)$

Lagranges (ignore $\frac{\partial R}{\partial q}$)

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = \Sigma F_i \cdot \frac{\partial v_i}{\partial \dot{q}} + \Sigma M_j \cdot \frac{\partial \omega_j}{\partial \dot{q}}$

T - kin energy

$\frac{mv^2}{2}, \frac{I\omega^2}{2}, \frac{m\dot{x}^2}{2}, \frac{J\dot{\theta}^2}{2}$

V - pot E

$mgy, \frac{Kx^2}{2}, P_o L$ string tension q = each DOF/unknown var

v_i = vel at F_i ω_j = ang vel at M_j

$V = \{k_{11}q_1^2 + k_{22}q_2^2 \cdots k_{nn}q_n^2\}\frac{1}{2}$

$+(k_{12}q_1q_2 + k_{13}q_1q_3 \cdots + k_{1n}q_1q_n)$

$+(k_{23}q_2q_3 + k_{2n}q_2q_n)$

$\vdots$

$+(\quad + k_{n-1,n}q_{n-1}q_n)$

$+(k_{nn}q_n^2)]$

$\frac{\partial V}{\partial q_i} = k_{i1}q_1 + k_{i2}q_2 \cdots + k_{in}q_n$

$K = \begin{bmatrix} k_{11} & k_{12} & \cdots \\ k_{21} & k_{22} & \cdots \\ \vdots & \vdots & \ddots & k_{nn} \end{bmatrix}$ $k_{12} = k_{21}$

$V = \frac{1}{2}q^T K q$

$m\ddot{x} + Kx = F\cos(\omega_f t)$

$x = x_c + x_p$

$x_p = C\cos\omega_f t$

$C = \frac{F}{-\omega_f^2 m + K}$

get init cond plug to get A & B

$x = \underbrace{A\cos\omega_n t + B\sin\omega_n t}_{x_c} + x_p$

$\frac{\partial u}{\partial x}dx$

$P \to P + \frac{\partial P}{\partial x}dx$

$\rho A dx \ddot{u}$

strain $= \frac{(dx + \frac{\partial u}{\partial x}dx) - dx}{dx} = \frac{\partial u}{\partial x}$

stress $= E(\text{strain}) = E\left(\frac{\partial u}{\partial x}\right) = \frac{P}{A} \to P = AE\frac{\partial u}{\partial x} \to \frac{\partial P}{\partial x} = AE\frac{\partial^2 u}{\partial x^2}$

FBD $\to 0 = P + \frac{\partial P}{\partial x}dx - P - \rho A dx \ddot{u} = 0$

$0 = AE\frac{\partial^2 u}{\partial x^2} - \rho A\frac{\partial^2 u}{\partial t^2} = 0$

$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} \to c$ = wave speed $= \sqrt{\frac{E}{\rho}}$

wave eq

$V = mg\left(\frac{l}{2} - \sqrt{\left(\frac{l}{2}\right)^2 - \left(\frac{x}{2}\right)^2}\right)$ $l - \sqrt{l^2 - x^2} = x^2/2l$

$\simeq \frac{mg x^2}{4l}$ $-l + \sqrt{l^2 + x^2} = x^2/2l$

Transient

$m\ddot{x} + Kx = F$

F = step $\to x = \frac{F_0}{k}(1 - \cos\omega t)$

$x(0) = \dot{x}(0) = 0$

f = ramp $= at \to x = \frac{a}{K\omega}(\omega t - \sin\omega t)$

$x(0) = \dot{x}(0) = 0$

Proportional damping

$M\ddot{q} + C\dot{q} + Kq = F$

$C = \alpha M + \beta K$

$q = Up$

U mode shape matrix

$u^T M u \ddot{p} + u^T(\alpha M + \beta K)u\dot{p} + u^T K u p = u^T F$

$m^*\ddot{p} + (\alpha m^* + \beta K^*)\dot{p} + K^* p = u^T F$

diagonal

Rayleigh Quotient

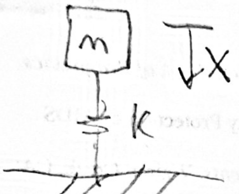
$M\ddot{x} + Kx = 0$

ω_R^2 = rayleigh Q $= \frac{V^T K V}{V^T M V}$

$V = \begin{pmatrix} 1 \\ v \end{pmatrix}$ = guess mode shape

$\frac{d\omega_R^2}{dv} = 0$ & min ω_R^2 where V is smaller

1a.



→ no rebound = end of spring remains grounded
→ pick datum such that compression from gravity is ignored, but consider compression from initial speed v
→ initial compression is positive

b.

from Fbd:

$m\ddot{x} + kx = 0$ ← F = 0 since no leg control = no force acting

given initial speed, kinetic energy converted to spring compression when kangaroo landed (t = 0):

$$\frac{mv^2}{2} = \frac{kx^2}{2} \rightarrow x(t=0) = \sqrt{\frac{mv^2}{k}}$$

c. No leg control → F = 0
pick a solution of x & substitute!

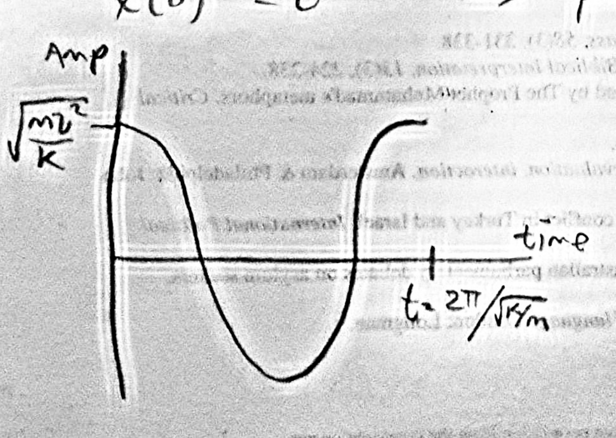
$x = X\cos(\omega_n t)$

$(-\omega_n^2 m + k)X\cos(\omega_n t) = 0$

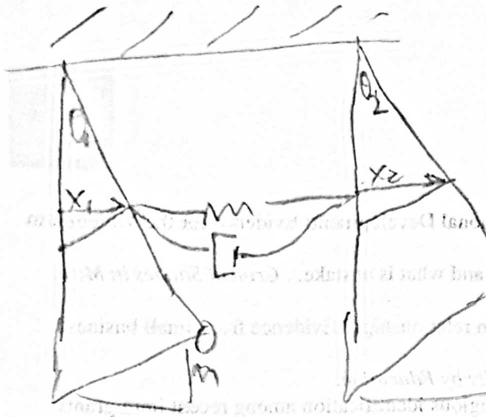
$\omega_n = \sqrt{k/m} \rightarrow x = X\cos(\sqrt{\frac{k}{m}}t)$

d. $x(0) = \sqrt{\frac{mv^2}{k}} \rightarrow \sqrt{\frac{mv^2}{k}} = X\cos(0) \rightarrow \sqrt{\frac{mv^2}{k}} = X$

$\dot{x}(0) = 0 \rightarrow$ possible sin term in x function is zero



2a.



$$x_2 = \frac{l}{2}\theta_2 \approx \frac{l}{2}\sin\theta_1$$

$$x_1 = \frac{l}{2}\theta_1 \approx \frac{l}{2}\sin\theta_2$$

$$\Delta x = x_2 - x_1 = \frac{l}{2}(\theta_2 - \theta_1)$$

$$\approx \frac{l}{2}(\sin\theta_2 - \sin\theta_1)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot q}\right) - \frac{\partial T}{\partial q} + \frac{\partial R}{\partial \dot q} + \frac{\partial V}{\partial q} = \Sigma F_i \cdot \frac{\partial v_i}{\partial \dot q} + \Sigma M_j \cdot \frac{\partial \omega_j}{\partial \dot q}$$

$$\left.\begin{array}{l}\Sigma F_i = 0 \\ \Sigma M_j = 0\end{array}\right\} \text{ for all eqn's}$$

$$T = \frac{M}{2}(l\dot\theta_1)^2 + \frac{M}{2}(l\dot\theta_2)^2$$

$$V = \frac{K}{2}\left(\frac{l}{2}(\theta_2 - \theta_1)\right)^2 + Mgl(1-\cos\theta_1) + mgl(1-\cos\theta_2)$$

$$= \frac{Kl^2}{8}(\theta_2^2 - 2\theta_2\theta_1 + \theta_1^2) + Mgl(1-\cos\theta_1) + mgl(1-\cos\theta_2)$$

$$R = \frac{c}{2}\left(\frac{l}{2}(\dot\theta_2 - \dot\theta_1)\right)^2$$

$$= \frac{cl^2}{8}(\dot\theta_2^2 - 2\dot\theta_2\dot\theta_1 + \dot\theta_1^2)$$

$$q = \theta_1$$

$$\frac{\partial T}{\partial \dot\theta_1} = Ml\dot\theta_1$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_1}\right) = Ml\ddot\theta_1$$

$$\frac{\partial T}{\partial \theta_1} = 0$$

$$\frac{\partial R}{\partial \dot\theta_1} = \frac{cl^2}{8}(-2\dot\theta_2 + 2\dot\theta_1)$$

$$\frac{\partial V}{\partial \theta_1} = \frac{kl^2}{8}(-2\theta_2 + 2\theta_1) + mgl\sin\theta_1$$

$$= -\frac{kl^2}{4}\theta_2 + \left(\frac{kl^2}{4} + mgl\right)\theta_1$$

small angle approx

Equation:

$$ml\ddot{\theta}_1 + -\frac{cl^2}{4}\dot{\theta}_2 + \frac{cl^2}{4}\dot{\theta}_1 + -\frac{kl^2}{4}\theta_2 + \left(\frac{kl^2}{4} + mgl\right)\theta_1 = 0$$

$q = \theta_2$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = ml\ddot{\theta}_2$$

$$\frac{\partial T}{\partial \theta_2} = 0$$

$$\frac{\partial R}{\partial \dot{\theta}_2} = \frac{cl^2}{8}(-2\dot{\theta}_1 + 2\dot{\theta}_2)$$

$$\frac{\partial V}{\partial \theta_2} = -\frac{kl^2}{4}\theta_1 + \left(\frac{kl^2}{4} + mgl\right)\theta_2$$

Eqn:

$$ml\ddot{\theta}_2 + \frac{cl^2}{4}\dot{\theta}_2 - \frac{cl^2}{4}\dot{\theta}_1 + \left(\frac{kl^2}{4} + mgl\right)\theta_2 - \frac{kl^2}{4}\theta_1 = 0$$

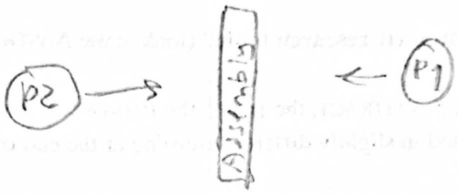
MATRIX form:

$$\theta = \begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix}$$

$$\underbrace{\begin{bmatrix} ml & 0 \\ 0 & ml \end{bmatrix}}_{\downarrow\ \underset{\sim}{M}}[\ddot{\theta}] + \underbrace{\begin{bmatrix} \frac{cl^2}{4} & -\frac{cl^2}{4} \\ -\frac{cl^2}{4} & \frac{cl^2}{4} \end{bmatrix}}_{\downarrow\ \underset{\sim}{C}}[\dot{\theta}] + \underbrace{\begin{bmatrix} \frac{kl^2}{4} + mgl & -\frac{kl^2}{4} \\ -\frac{kl^2}{4} & \frac{kl^2}{4} + mgl \end{bmatrix}}_{\downarrow\ \underset{\sim}{K}}[\theta] = [0]$$

b. Equations are symmetrical. This is because $m_1 = m_2$ and $l_1 = l_2$. Think about it this way.

Top view



person 1 & 2 wouldn't notice a difference when looking at this system from opposite views.

c. say that $\theta = \Theta e^{\lambda t}$, and substitute into equation:

$$\big(\underbrace{\tilde{m}\lambda^2 + c\lambda + K}_{\text{has to be } 0}\big)\Theta e^{\lambda t} = 0 \Rightarrow \frac{4}{l}\left(\tilde{m}\lambda^2 + \tilde{c}\lambda + \tilde{K}\right) = 0$$

$$\det\left[\frac{4}{l}\left(\tilde{m}\lambda^2 + \tilde{c}\lambda + \tilde{K}\right)\right] = \det\begin{bmatrix} 4m\lambda^2 + cl\lambda + kl + 4mg & -cl\lambda - kl \\ -cl\lambda - kl & 4m\lambda^2 + cl\lambda + kl + 4mg \end{bmatrix} =$$

$$0 = \left(4m\lambda^2 + cl\lambda + (kl + 4mg)\right)^2 - (cl\lambda + kl)^2$$

$$0 = 16m^2\lambda^4 + 4mcl\lambda^3 + 4m\lambda^2(kl + 4mg) - \cancel{c^2l^2\lambda^2} + 2ckl^2k + \cancel{k^2l^2}$$

$$+ 4mcl\lambda^3 + \cancel{c^2l^2\lambda^2} + cl\lambda(\cancel{k}l + 4mg)$$

$$+ 4m\lambda^2(kl + 4mg) + cl\lambda(\cancel{kl} + 4mg) + \cancel{k^2l^2} + 8mgkl + 16m^2g^2$$

d

$$0 = 16m^2\lambda^4 + 8mcl\lambda^3 + 8m(kl + 4mg)\lambda^2 + 8mgcl\lambda + 8(mgkl + 2m^2g^2)$$

$$0 = 2m\lambda^4 + cl\lambda^3 + (kl + 4mg)\lambda^2 + gcl\lambda + (gkl + 2mg^2)$$

e.

... Part C

since we know that this can act like a regular pendulum, a solution has to be $\lambda = \sqrt{\frac{g}{l}} \rightarrow (\lambda^2 - \frac{g}{l})$

$$0 = (\lambda^2 - \frac{g}{l})(i\lambda^2 + j\lambda + k)$$

$$2m\lambda^2 - cl\lambda$$

$$\lambda^2 - \frac{g}{l} \overline{) 2m\lambda^4 - cl\lambda^3 + (kl + 4mg)\lambda^2 + gcl\lambda + (gkl + 2mg^2)}$$

$$2g\frac{m}{l} = \lambda^2$$

$$-cl\lambda^3 + gcl\lambda$$

I can't solve this algebraically. Likely wrong approach or wrong calculation.

$$\left(\frac{i}{i}\lambda^2 + \frac{j}{i}\lambda + \frac{k}{i}\right) = (\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2) = 0$$

$$\omega_n^2 = k/i \rightarrow \omega_n = \sqrt{\frac{k}{i}}$$

$$2\xi\omega_n = j/i \rightarrow \xi = \frac{j}{2\sqrt{ik}}$$

get mode shape u_1 & u_2 from equating $\theta = X\begin{bmatrix} 1 \\ u_1 \end{bmatrix} e^{\lambda t}$

$$= X\begin{bmatrix} 1 \\ u_2 \end{bmatrix} e^{\lambda t}$$

and solving for u_1 & u_2

d. say $\theta = UP$ where $u = [u_1 \quad u_2]$ & P = principal coords

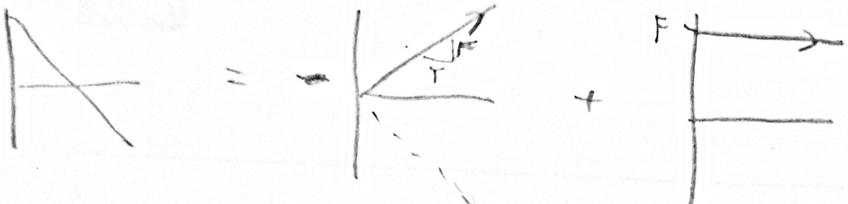
$$\underset{\sim}{M}\ddot{\theta} + \underset{\sim}{C}\dot{\theta} + k = 0$$

$$U^T\underset{\sim}{M}UP + U^T\underset{\sim}{C}UP + U^TkUP = 0$$

$M^*P + C^*P + K^*P = 0$ ← coords are principal coords & M^* C^* K^* are diagonal

e. damping ratio found in part C

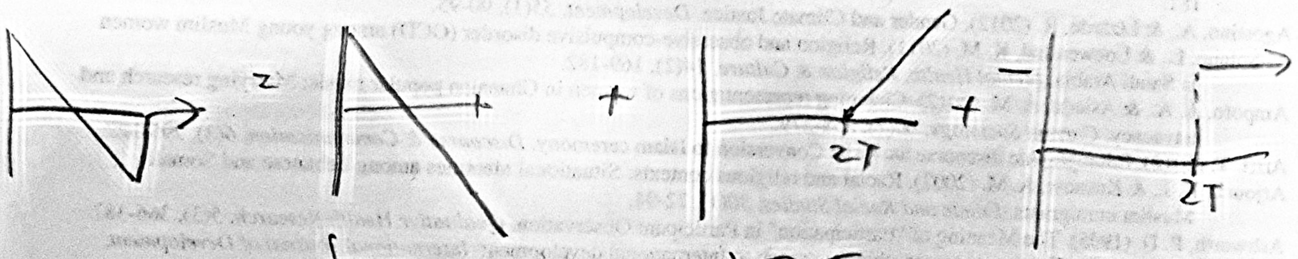
3a. for $0 < t < 2T$



$a = \frac{F}{T}$ $F_0 = F$

$$X = \frac{-F}{TKW}(wt - \sin wt) + \frac{F}{K}(1 - \cos wt)$$

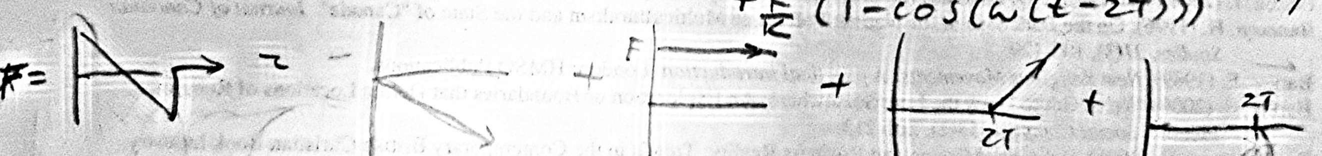
b.

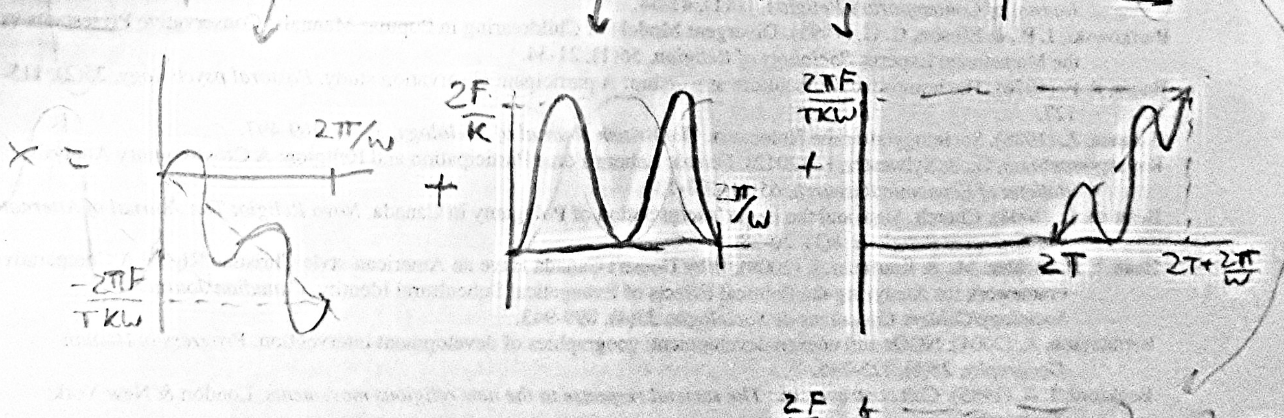


part A $t \to t - 2T$ $t = t - 2T$

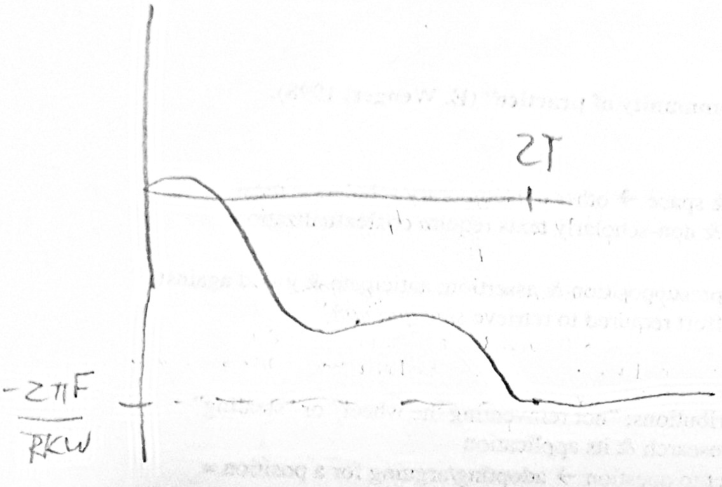
$$X = \frac{-F}{TKW}(wt - \sin wt) + \frac{F}{K}(1 - \cos wt) + \frac{F}{TKW}\left(w(t-2T) - \sin(w(t-2T))\right) + \frac{F}{K}\left(1 - \cos(w(t-2T))\right)$$

c. $F =$





combining plots:



d.

at $t<2T$:

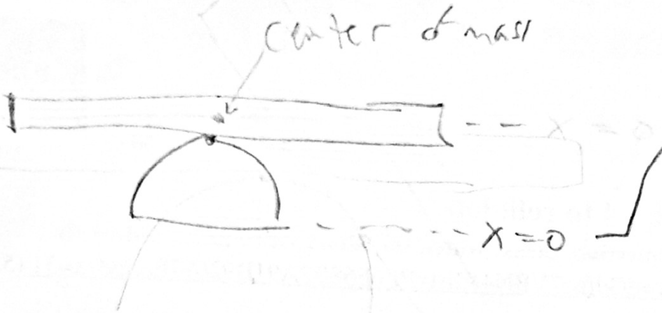
Amp of F response $\rightarrow \frac{F}{\pi K\omega} = \frac{F}{\pi K \pi/4} = \frac{F}{\pi K}$

Amp of F_{wt} response $\rightarrow \frac{F}{K}$

Amp of x $\rightarrow \frac{F}{\pi K} + \frac{F}{K} = \left(\frac{1}{\pi}+1\right)\frac{F}{K}$ at the moment both responses sync up

at $t>2T$, all responses cancel out, Amplitude is zero

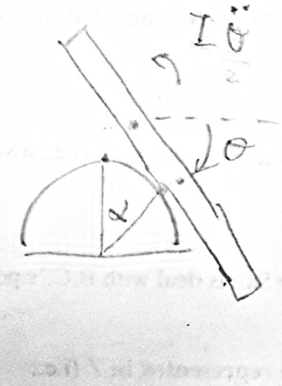
4a.

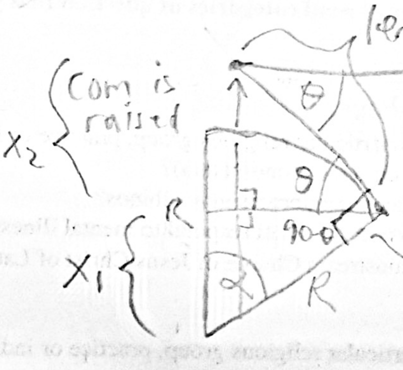


contact point at com

com at $x = R$

Assume very thin rod





$len = 2\pi R\left(\frac{\theta}{2\pi}\right) = \theta R$

$\pi = \alpha + \frac{\pi}{2} + \left(\frac{\pi}{2} - \theta\right)$

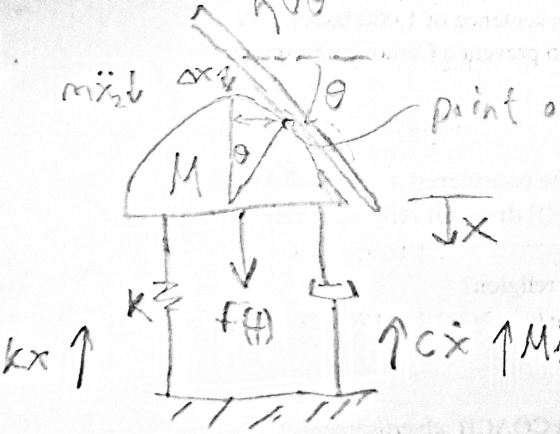
$\theta = \alpha$

$$X = x_1 + x_2$$
$$= R\cos\alpha + \theta R\sin\theta$$
$$\approx R\cos\theta + \theta R\sin\theta$$
$$\approx R + R\theta^2 \leftarrow \text{small angle approx}$$

$x_2 = \Delta x$ from rotation $= R - (R + R\theta^2)$

$= -R\theta^2$ ← neg since coord system says going down is positive

b.



$$x_2 = -R\theta^2 + x$$
$$\dot{x}_2 = -R2\theta\dot{\theta} + \dot{x}$$
$$\ddot{x}_2 = -R2\dot{\theta}^2 + -R2\theta\ddot{\theta} + \ddot{x}$$

4c. don't consider 2nd order term $\rightarrow \ddot{x}_2 = -R2\dot{\theta}\dot{\theta} + \ddot{x}$

$\Sigma F = 0 ?: f(t) - kx - c\dot{x} - M\ddot{x} - m(-R2\theta\ddot{\theta} + \ddot{x})$

$f(t) = (m+M)\ddot{x} + c\dot{x} + kx - mR2\theta\ddot{\theta}$

$\Sigma M_0 = 0 =$

No time to finish

$x = \begin{bmatrix} x \\ \theta \end{bmatrix}$

d. $\underset{\sim}{M}[\ddot{x}] + \underset{\sim}{C}[\dot{x}] + \underset{\sim}{K} = [F]$

$x = Re(\lambda e^{\lambda t})$

$[-\lambda^2 \underset{\sim}{M} + i\lambda \underset{\sim}{C} + k][x] = 0 \Rightarrow$ no forced get nat. freq.

$\det(\lambda^2 \underset{\sim}{M} + \lambda \underset{\sim}{C} + \underset{\sim}{K}) = 0$

$(\lambda^2 + D)(A\lambda^2 + B\lambda + C) = 0$

where $w_n = \sqrt{C/A}$ & $\zeta = B/2\sqrt{AC}$

sub w_n back into equation to find u_1 & u_2

$\begin{bmatrix} 1 \\ u_1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ u_2 \end{bmatrix}$

e. $x = Re(xe^{iw_ft}) = Re((A+iB)(\cos w_ft + i\sin w_ft))$

$[-w_f^2 \underset{\sim}{M} + iw_f \underset{\sim}{C} + K][x] = [F]$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}[x] = (F)$

kramer's rule to get $\underset{=A+iB}{X_1}$ & $X_2 = C+iD$

$x_1(t) = A\cos w_ft + B\sin w_ft$

$x_2(t) = C\cos w_ft + D\sin w_ft$