MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

Mid-term Quiz

Date: Mar 1, 2021
Time: 3:00 – 4:00pm

Problem 1 (25 points)

Figure 1 shows a brushed dc motor whose axis of rotation is aligned with a unit vector $\hat{\mathbf{e}}_z$.

<u>External to the motor:</u>
V is the terminal voltage, i is the terminal current, $\omega\,\hat{\mathbf{e}}_z$ is the rotor angular velocity, and $\tau_{\text{ext}}\,\hat{\mathbf{e}}_z$ is the external torque applied to the rotor.

<u>Internal to the motor:</u>
R is the winding resistance, K is the torque constant, e is the back-emf, J is the rotor inertia, b is the mechanical damping between the stator and the rotor, and $\tau\,\hat{\mathbf{e}}_z$ is the torque transmitted from the stator to the rotor. The winding inductance is assumed to be zero ($L = 0$).

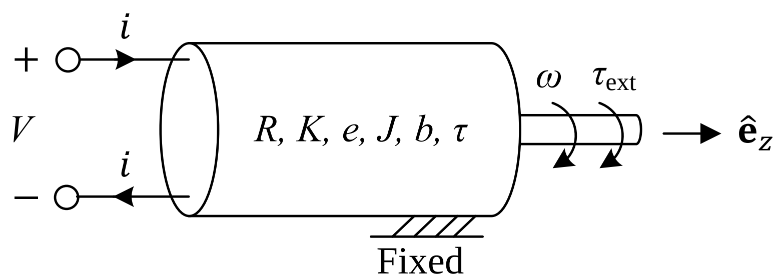


Figure 1: Brushed dc motor for the problem.

(a) (5 pt.) Draw a lumped-parameter model of the motor, where the electrical domain is modeled as a circuit diagram and the mechanical domain is modeled as a free-body diagram. The model should include all the variables and parameters shown in Figure 1.

(b) (6 pt.) Suppose a unit-step voltage V is applied across the electrical terminals. Draw the response of the rotor angular speed $\omega(t)$.

(c) (6 pt.) Suppose a unit-step external torque τ_{ext} is applied to the rotor while the electrical terminals are **open-circuited**. Draw the response of the terminal voltage $V(t)$.

(d) (8 pt.) Suppose a unit-step external torque τ_{ext} is applied to the rotor while the electrical terminals are **short-circuited**. Draw the response of the rotor angular speed $\omega(t)$.

Problem 2 (35 points)

Let us consider an op-amp circuit in Figure 2. We assume that the op-amp has infinite input impedance, zero output impedance, and open-loop transfer function $A(s)$.

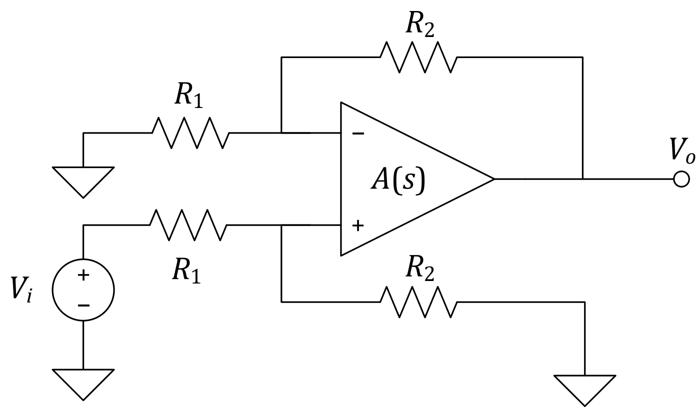


Figure 2: Op-amp circuit for the problem.

(a) (5 pt.) Draw a block diagram that shows the relation between the input voltages V_i and the the output voltage V_o. The block diagram should show a feedback loop around $A(s)$.

(b) (5 pt.) Express the loop return ratio $L(s)$ in terms of R_1, R_2, and $A(s)$.

(c) (5 pt.) For $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 1\,\mathrm{k\Omega}$, and $A(s)$ given in Figure 3, find the unity-gain crossover frequency ω_c and phase margin ϕ_m of $L(s)$.

(d) (5 pt.) For $R_1 = 1\,\mathrm{k\Omega}$ and $A(s)$ given in Figure 3, find the resistance value R_2 that makes the closed-loop transfer function $G(s) = V_o/V_i$ achieve a $-3\,\mathrm{dB}$ bandwidth of $10\,\mathrm{kHz}$.

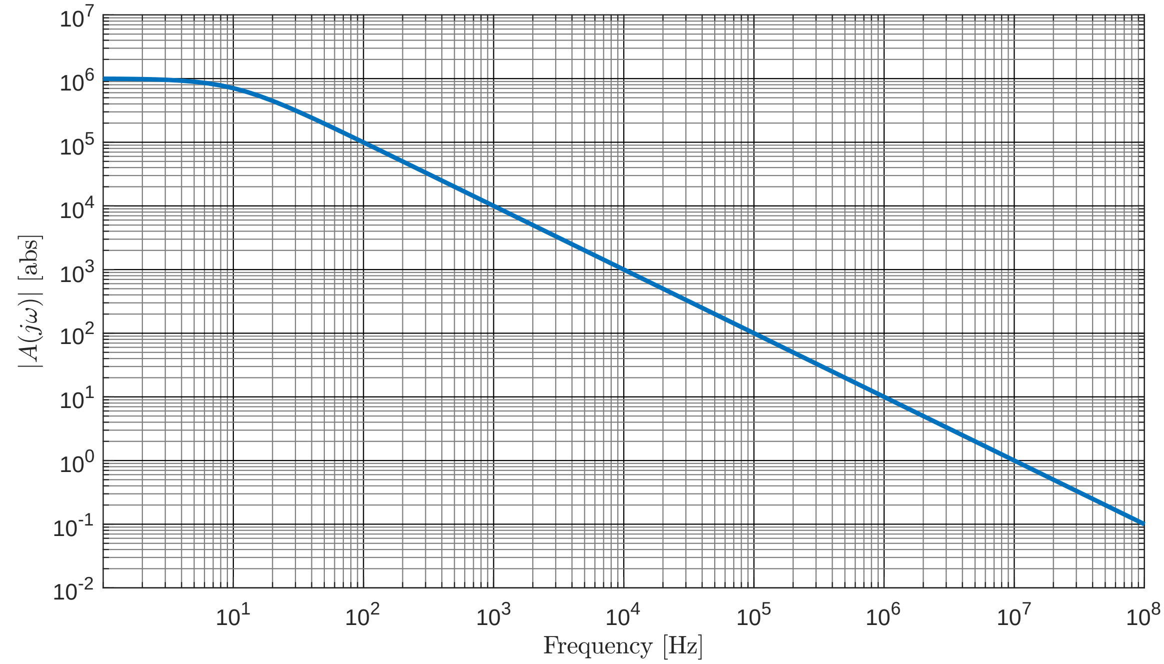
(e) (5 pt.) For the circuit designed in part (d), determine the dc gain of $G(s) = V_o/V_i$.

(f) (10 pt.) Suppose the circuit designed in part (d) is excited with an input voltage

$$V_i(t) = \cos\bigl(2\pi \times 10^7\, t\bigr),$$

which is a $10\,\mathrm{MHz}$ persistent sinusoid defined for $-\infty < t < \infty$. Find the magnitude M_o and phase ϕ_o of the output voltage

$$V_o(t) = M_o \cos\bigl(2\pi \times 10^7\, t + \phi_o\bigr).$$



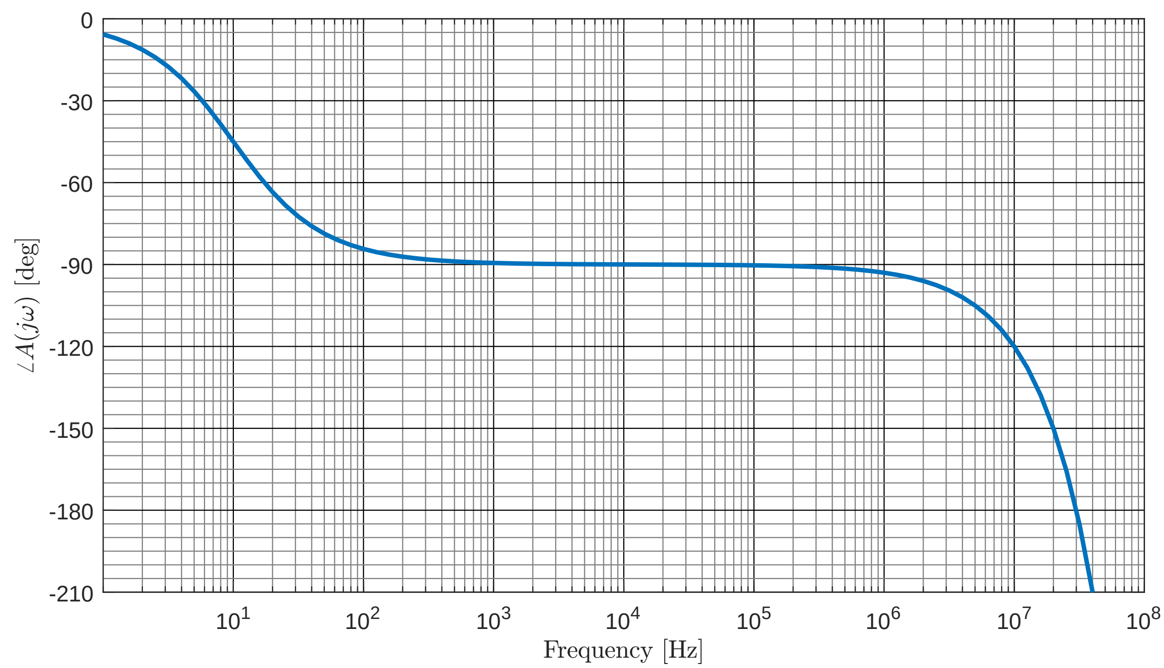


Figure 3: Bode plot of $A(s)$.