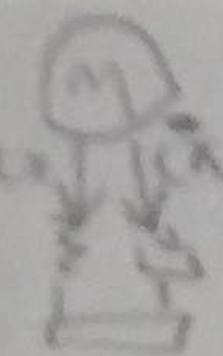


fx $-m\ddot{y} - m(\ddot{x}-\ddot{y}) - k(x-y) - c(\dot{x}-\dot{y}) = 0$

$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = -m\ddot{Y}\cos\omega_f t$

$(-m\omega_f^2 + ic\omega_f + k)\,Re[De^{i\omega_f t}] = Re[m\ddot{Y}e^{i\omega_f t}]$

fy

$D = m\ddot{Y}/(-m\omega_f^2 + ic\omega_f + k)$

$= (m\ddot{Y}/k)/((1-r^2) + i(2\xi r))$

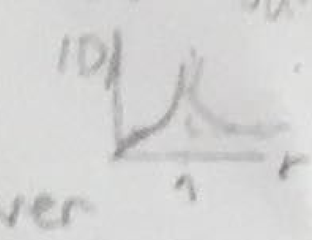
$|D| = (m\ddot{Y}/k)/\sqrt{(1-r^2)^2 + (2\xi r)^2}$

$\angle D = \tan^{-1}(-2\xi r/(1-r^2))$

$x = e^{-\xi\omega_n t}(A\cos\omega_d t - B\sin\omega_d t)$

$\omega = 2\pi f$

$\omega_n = \omega_d/\sqrt{1-\xi^2}$



$r = \omega_f/\omega_n$ — >1 over

$\omega_n^2 = k/m$ — $=1$ critical

$\xi = c/2\sqrt{km} \rightarrow 0<\xi<1$ under

$=0$ no

$\omega_f c/k = 2\xi r$

$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \xi 2\pi \frac{\omega_n}{\omega_d} = \frac{\xi 2\pi}{\sqrt{1-\xi^2}}$

$x = A\cos\omega t - B\sin\omega t$

$= C\cos(\omega t + \phi)$

$= Ge^{i\omega t} + He^{-i\omega t}$

$= Re[De^{i\omega t}]$

$x = x_c + x_p$

x_p = any sol that is an x

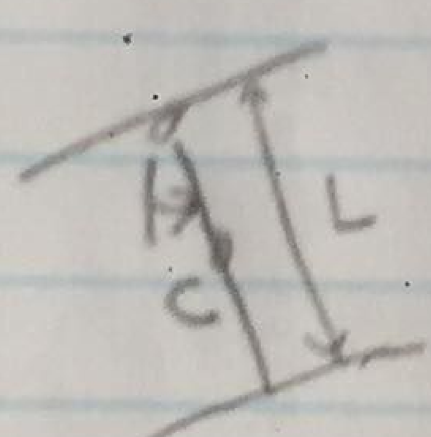
#63205165
Ratthanaoon Prakitpong

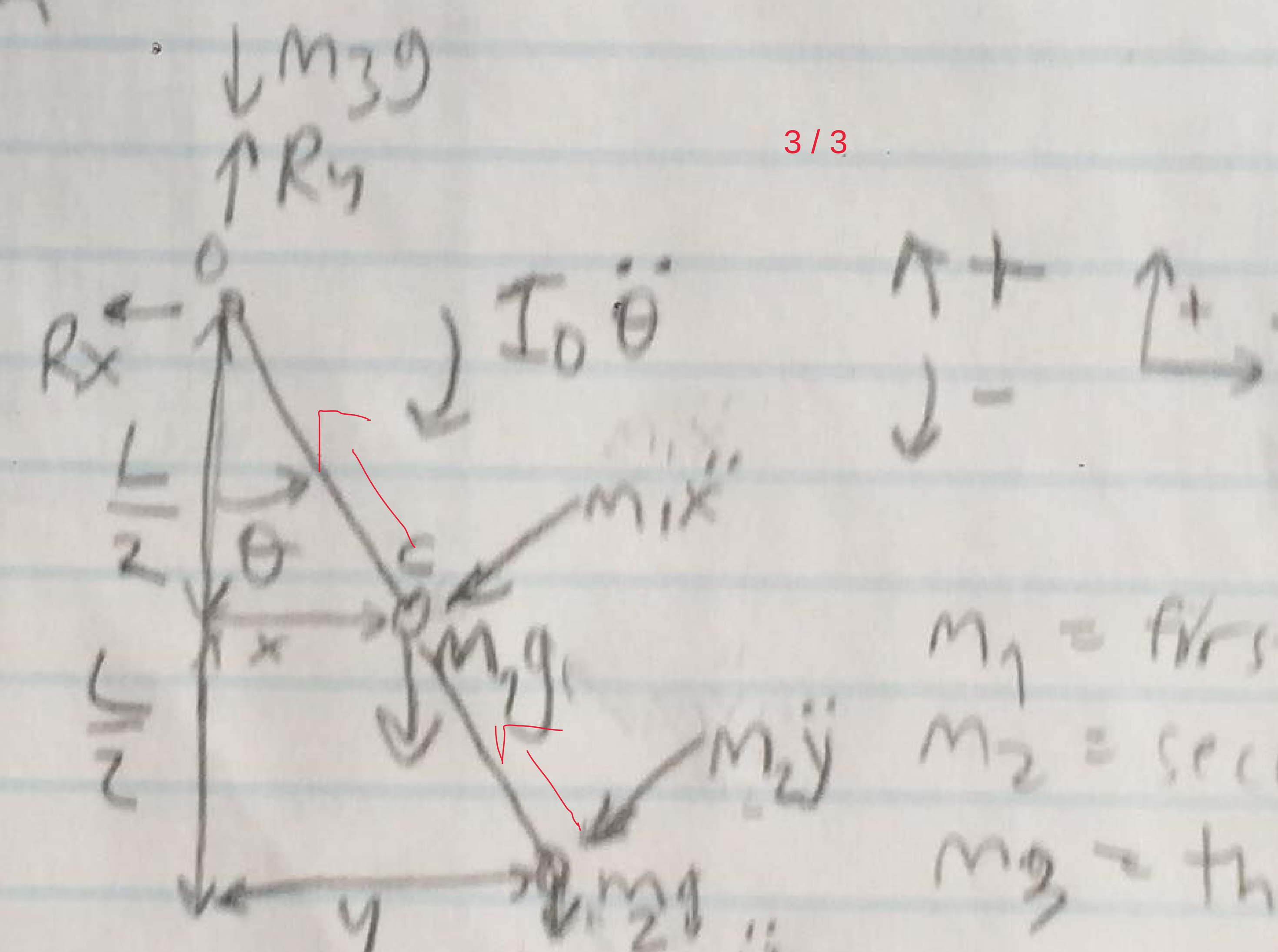
Honour:
I promise to work honestly on this exam to obey all instructions, and not have any unfair advantage over any other students

Total: 5.5 / 10

Start 2:00 PM
Finish 2:50 PM

3 / 3

1a) 

$\Rightarrow$ 

I_0 = of center rod

m_1 = first rod
m_2 = second rod
m_3 = third rod

b) $\Sigma M_{o1} = 0 = -I_0\ddot{\theta} - mg\left(\theta\frac{L}{2}\right) - m\left(\ddot{\theta}\frac{L}{2}\right)\frac{L}{2}$
first rod

$0 = \frac{mL^2}{12}\ddot{\theta} + \frac{mgL}{2}\theta + m\frac{L^2}{4}\ddot{\theta} = \frac{2L}{3}\ddot{\theta} + g\theta$ $\left[\begin{matrix} x \approx \theta\frac{L}{2} \\ \ddot{x} = \ddot{\theta}\frac{L}{2}\end{matrix}\right]$

$\Sigma M_{o2} = 0 = -(mL^2)\ddot{\theta} - mg(\theta L) - m(\ddot{\theta}L)L$ $\left[\begin{matrix} y \approx \theta L \\ \ddot{y} = \ddot{\theta}L\end{matrix}\right]$
second rod

$0 = +mL^2\ddot{\theta} + mgL\theta + mL^2\ddot{\theta} = 2L\ddot{\theta} + g\theta$

$\Sigma M_{o3} = 0 \Rightarrow 0$ (centroid at o)
third rod

1.5 / 2

$\Sigma M_o = 0 = \left(\frac{2}{3}L + 2L\right)\ddot{\theta} + (g+g)\theta = \frac{8L}{3}\ddot{\theta} + 2g\theta = \frac{4L}{3}\ddot{\theta} + g\theta$
superposition $= \Sigma M_{o1} + \Sigma M_{o2} + \Sigma M_{o3}$

you should have used solution No.2 and shown the needed steps in details.

c) $W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\frac{4L}{3}}} = \sqrt{\frac{3g}{4L}}$

0 / 1

d) $\sum M_0 = \sum M_{01} + \sum M_{03} = \frac{2L}{3}\ddot{\theta} + g\theta$

$$W_n = \sqrt{\frac{K}{m}} = \cancel{\sqrt{\frac{3g}{2L}}}$$

6 2

0.5 / 2

e) $W_{n123} = \sqrt{3g/4L}$

$W_{n13} = \sqrt{3g/2L}$

$W_{n23} = \sqrt{g/2L} = \sqrt{3g/6L}$

This makes sense since with super position, W_{n123} will be a combination of W_{n1} & W_{n2} & W_{n3}.

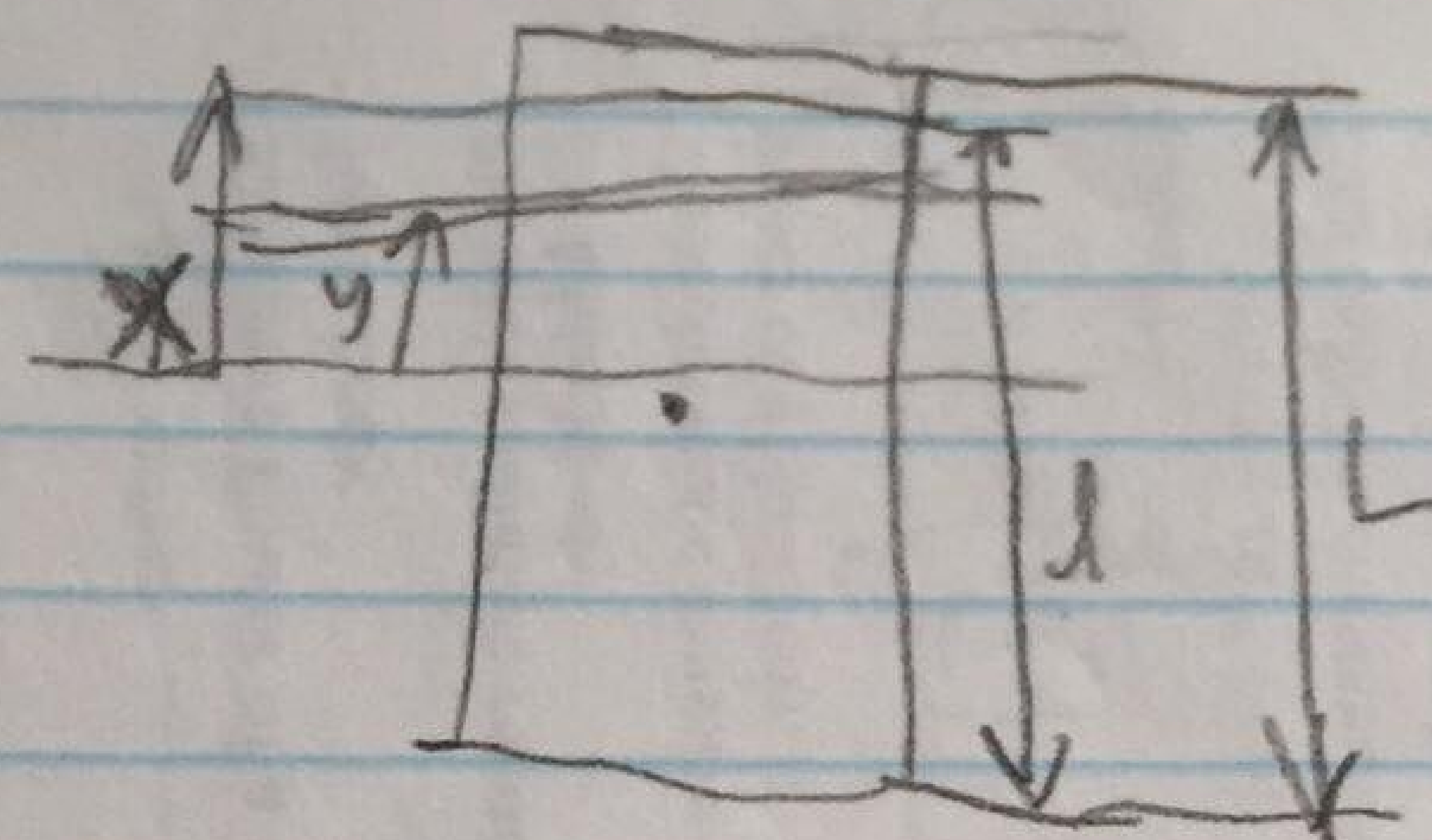
Mass appears in the denominator of frequency formula, so decreases in mass causes an increase in natural frequency.

0.5 / 1

$$W_{123} = \frac{W_{n13} + W_{n12}}{2}$$

$$\sqrt{\frac{1}{4}} = \frac{\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{6}}}{2}$$

2a)



$\downarrow c\dot{x}$ $\downarrow m\ddot{x}$ $\downarrow f_{buoyant}$ $\downarrow f_g$

At equilibrium (1):

$$f_b = mg = \rho A l \rightarrow A = \frac{mg}{\rho l}$$

At scenario (3):

$$f_b + f_g = \rho A(l+x-y) - mg$$

$$= \cancel{\rho}\left(\frac{mg}{\cancel{\rho} l}\right)(l+x-y) - mg$$

$$= \frac{mg}{l}(l+x-y) - mg = \frac{mg}{l}(x-y)$$

$$\Sigma F = 0 = m\ddot{x} + c\dot{x} + f_b + f_g$$

$$\frac{mg}{l} y = m\ddot{x} + c\dot{x} + \frac{mg}{l} x$$

$$= \frac{mg}{l}(x-y)$$

$$\Sigma F = 0 = m\ddot{x} + c\dot{x} + f_b + f_g$$

$$\frac{mg}{l} y = m\ddot{x} + c\dot{x} + \frac{mg}{l} x$$

$$\frac{mg}{l} Y \cos w_f t = m\ddot{x} + c\dot{x} + \frac{mg}{l} x$$

4/4

b) $w_n = \sqrt{k/m} = \sqrt{(mg/l)/m} = \sqrt{g/l}$ like a pendulum

3/3

c) $k = mg/l$

$$Re\left[kY e^{iw_f t}\right] = (-m w_f^2 + i c w_f + k) Re\left[D e^{iw_f t}\right]$$

2.5/3

$$D = \frac{kY}{(-m w_f^2 + i c w_f + k)} \left(\frac{1/k}{1/k}\right)$$

$$= \frac{Y}{\left(-\frac{m}{k} w_f^2 + \frac{i c w_f}{k} + 1\right)}$$

$$= \frac{Y}{\left(-\frac{w_f^2}{w_n} + i c r w_n/k + 1\right)}$$

$$= Y / ((1-r^2) + i(2\xi r))$$

$$\left[\begin{aligned} &= c w_f / k \\ &= (c/k)(r w_n) \\ &= (c/k)(r\sqrt{k/m}) \\ &= \left(\frac{c\sqrt{k}}{k\sqrt{m}}\left(\frac{2}{2}\right)\right) r \\ &= 2\left(\frac{c}{2\sqrt{km}}\right) r \\ &= 2\xi r \end{aligned}\right]$$

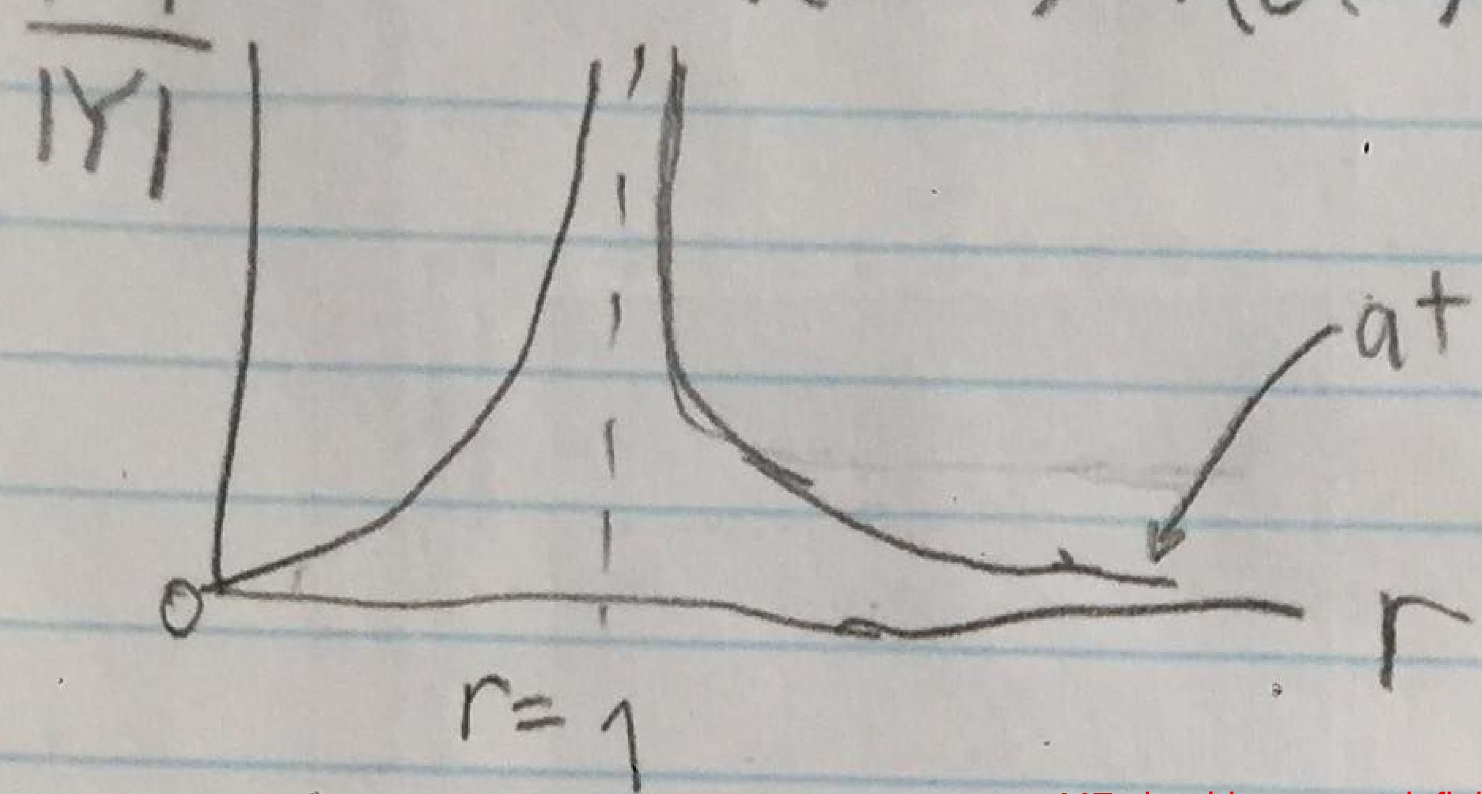
$$|D| = \frac{Y}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

where $r = \omega_f/\omega_n$

$\zeta = c/2\sqrt{km}$

$\omega_n = \sqrt{k/m}$

$$MF = \frac{|D|}{|Y|} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$



at big r, $MF \rightarrow \frac{1}{\sqrt{r^4+r^2}} = \frac{1}{r^2} = 0$

MF should not go to infinity because it is a damped system (-0.5)

TOTAL = 9.5/10