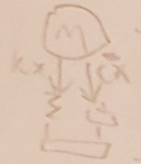
Q1: 13
Q2: 7.5
Bonus: 0.5
Total: 21



$\Sigma x \quad -m\ddot{y} - m(\ddot{x}-\ddot{y}) - k(x-y) - c(\dot{x}-\dot{y}) = 0$

Σy

$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = -m\ddot{Y}\cos\omega_f t$

$(-m\omega_f^2 + ic\omega_f + k)\,Re[De^{i\omega_f t}] = Re[m\ddot{Y}e^{i\omega_f t}]$

$D = m\ddot{Y}/(-m\omega_f^2 + ic\omega_f + k)$

$= (m\ddot{Y}/k)/((1-r^2)+i(2\xi r))$

$|D| = (m\ddot{Y}/k)/\sqrt{(1-r^2)^2+(2\xi r)^2}$

$\angle D = \tan^{-1}(-2\xi r/(1-r^2))$

$x = e^{-\xi\omega_n t}(A\cos\omega_d t - B\sin\omega_d t)$

$\omega = 2\pi f$

$\omega_n = \omega_d/\sqrt{1-\xi^2}$

$r = \omega_f/\omega_n$

$\omega_n^2 = k/m$

$\xi = c/2\sqrt{km}$

>1 over

$=1$ critical

$0<\xi<1$ under

$=0$ no

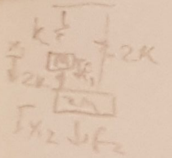
$\omega_f c/k = 2\xi r$

$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \xi 2\pi \frac{\omega_n}{\omega_d} = \frac{\xi 2\pi}{\sqrt{1-\xi^2}}$

$x = A\cos\omega t - B\sin\omega t$
$= C\cos(\omega t + \phi)$
$= Ge^{i\omega t} + He^{-i\omega t}$
$= Re[De^{i\omega t}]$

$x = x_c + x_p$

x_p = any sol that is an x



$\uparrow kx_1 \; \uparrow m\ddot{x}_1$

$\downarrow f_1 \; \downarrow 2k(x_2 - x_1)$

$\uparrow 2m\ddot{x}_2 \; \uparrow 2k(x_2 - x_1)$

$\downarrow f_2 \; \downarrow 2kx_2$

$\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}\begin{Bmatrix}\ddot{x}_1 \\ \ddot{x}_2\end{Bmatrix} + \begin{bmatrix} 3k & -2k \\ -2k & 4k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$

$f(t) = 0,\ \omega = \omega_n$

$x = X\cos(\omega t + \phi) \rightarrow (-\omega^2 m + k)X\cos(\omega t + \phi) = 0$

$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc = 0 \rightarrow \omega_n^2 = k/m,\ 4k/m$

$x = e\,u \rightarrow \begin{bmatrix} -\omega^2 m + 3k & -2k \\ -2k & -2\omega^2 m + 4k \end{bmatrix}\begin{bmatrix} 1 \\ u \end{bmatrix} = 0 \rightarrow u = 1, -\frac{1}{2}$

decouple

$x = up \rightarrow u = [u_1\ u_2] = \begin{pmatrix} 1 & 1 \\ 1 & -\frac{1}{2} \end{pmatrix}$

$Mu\ddot{p} + Kup = f$

$u^T M u\ddot{p} + u^T K u p = u^T f \qquad \left(u = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow u^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\right)$

$m^* \ddot{p} + k^* p = f^*$

$M\ddot{x} + Kx = F$

$(-\omega_f^2 M + K)X = F$

$X = (-\omega_f^2 M + K)^{-1} F$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det\begin{bmatrix} a & b \\ c & d \end{bmatrix}}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$X_1 = \frac{dF_1 - bF_2}{ad - bc}$

$X_2 = \frac{-cF_1 + aF_2}{ad - bc}$

$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad K = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

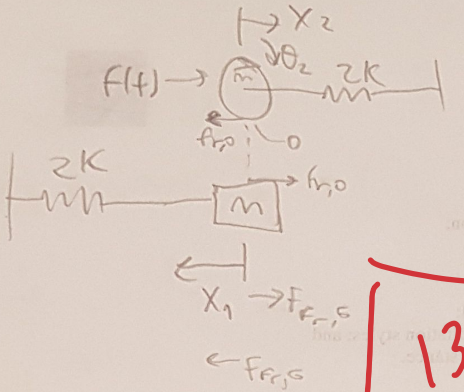
MATLAB:

```
M=[[a b]'[c d]'],
K=[[e f]'[g h]'];
[V,wz]=eig(K,M,'vector');
```

$V = U$ mode shape

$wz = \omega^2$

1a.



$\Rightarrow$ At wheel

$m\ddot{x}_2$ $\quad F\cos(w_f t)$

$F_{fr,0}$ $\quad J\ddot{\theta}_2 \rightarrow \theta = \frac{x}{r} \rightarrow \ddot{\theta} = \frac{\ddot{x}}{r}$

kx_2

$\Rightarrow$ At plate

$m\ddot{x}_1$

kx_1

$F_{fr,0}$

$F_{fr,G}$

$\boxed{13/13}$

$-\frac{1}{2}$

At rollers

$F_{fr,G}$

$$\Sigma M_0 = 0 = 2kx_2 r + \left(\frac{mr^2}{2}\right)\frac{\ddot{x}_2}{r} + m\ddot{x}_2 r - F\cos(w_f t) r$$

$$F\cos(w_f t) = \frac{3}{2}m\ddot{x}_2 + 2kx_2$$

$$\Sigma F_x = 0 = -F\cos(w_f t) + m\ddot{x}_2 + \cancel{F_{fr,0}} + 2kx_2 - m\ddot{x}_1 - 2kx_1 - \cancel{F_{fr,0}}$$
$$- \cancel{F_{fr,G}} + \cancel{F_{fr,G}}$$

$$F\cos(w_f t) = m\ddot{x}_2 - m\ddot{x}_1 + 2kx_2 - 2kx_1$$

$$\begin{bmatrix} 0 & \frac{3}{2}m \\ -m & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 2k \\ -2k & 2k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F \\ F \end{bmatrix}\cos(w_f t)$$

C- Pick $x = X\cos(w_f t + \phi)$ as solution, substitute into equation

$$(-w_f^2 M + K)X = F$$

$$X = (-w_f^2 M + K)^{-1} F = \begin{pmatrix} 0 & -w_f^2 \frac{3}{2} m + 2K \\ w_f^2 m - 2K & -w_f^2 m + 2K \end{pmatrix}^{-1} F$$

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{1}{\det \begin{pmatrix} 0 & -w_f^2 \frac{3}{2} m + 2K \\ w_f^2 m - 2K & -w_f^2 m + 2K \end{pmatrix}} \begin{pmatrix} -w_f^2 m + 2K & w_f^2 \frac{3}{2} m - 2K \\ -w_f^2 m + 2K & 0 \end{pmatrix} \begin{pmatrix} F \\ F \end{pmatrix}$$

$$X_1 = \frac{((-w_f^2 m + 2K) - (w_f^2 \frac{3}{2} m - 2K))F}{a}$$

$$a = (-w_f^2 \tfrac{3}{2} m + 2K)(w_f^2 m - 2K)$$

$$X_2 = \frac{((-w_f^2 m + 2K) - 0)F}{a}$$

$$\frac{X_1}{X_2} = \text{mag fac} = \frac{(-w_f^2 m + \cancel{2K} + w_f^2 \frac{3}{2} m - \cancel{2K})}{-w_f^2 m + 2K}$$

$$= \frac{1}{-4 + \frac{2K}{w_f^2 m}} = \left(\frac{1}{2}\right) \frac{w_f^2 m}{-w_f^2 m + 2K}$$

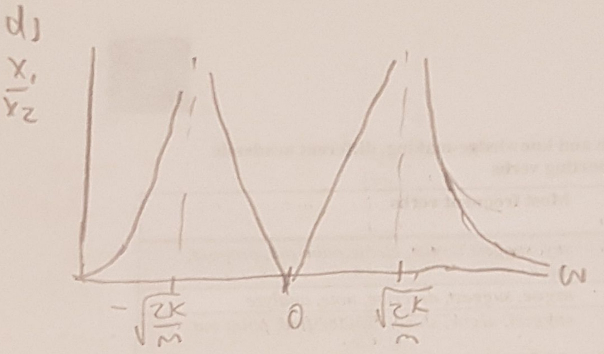
For zero response, $X_1 = X_2 \rightarrow \frac{X_1}{X_2} = -1$ (defined originally as opposite

$$-2(-w_f^2 m + 2K) = w_f^2 m$$

$$\frac{2k}{3m} = w_f^2$$

-1

d)



for $\frac{X_1}{X_2} \to +\infty$

$(-\omega_f^2 m + 2k) = 0$

$\omega_f^2 = \frac{2k}{m}$

$\omega_f = +\sqrt{\frac{2k}{m}}, -\sqrt{\frac{2k}{m}}$

(???)

e. to get natural frequency, say $f(t) = 0$ and $X = X\cos(\omega_n t)$

$(-\omega_n^2 M + K) X = 0$

$\det(-\omega_n^2 M + K) = 0$

$$\det \begin{vmatrix} 0 & -\omega_n^2 \frac{3}{2} m + 2k \\ -\omega_n^2 m + 2k & -\omega_n^2 m + 2k \end{vmatrix} = 0$$

$0 - (\omega_n^2 m - 2k)(-\omega_n^2 \frac{3}{2} m + 2k) = 0$

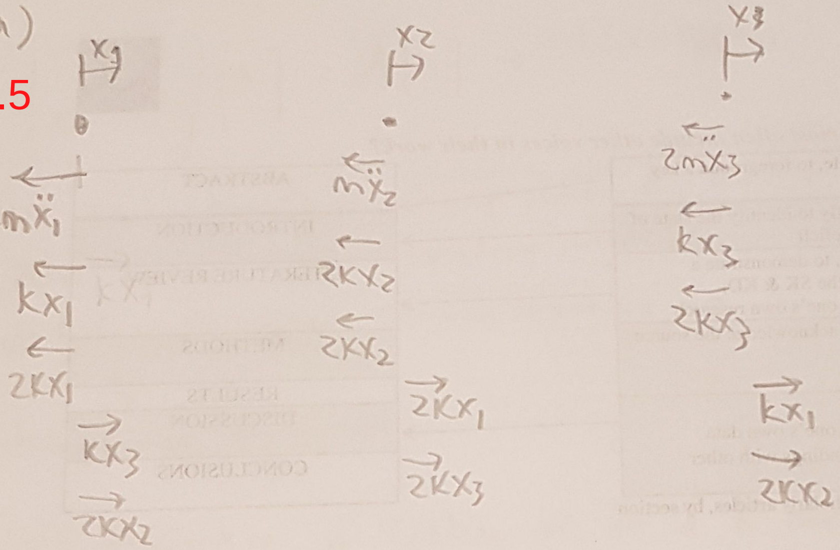
$\omega_{n,1} = \sqrt{\frac{2k}{m}}$

$\omega_{n2} = \sqrt{\frac{4K}{3m}}$

+2

2a)

1.5



$F_1 = 0$

$F_2 = 0$

$F_3 = 0$

$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 2m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 3k & -2k & -k \\ -2k & 4k & -2k \\ -k & -2k & 3k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [0]$$

2

Editor - C:\Users\Frienddo\Desktop\MECH4\MECH463\exams\yes.m

yes.m

1

```
%q2b
clear all;
close all;
m = 1;
k = 1;
M = [[2*m 0 0]' [0 m 0]' [0 0 2*m]'];
K = k*[[3 -2 -1]' [-2 4 -2]' [-1 -2 3]'];
[V,w2] = eig(K,M,'vector');
V(:,:) = V(:,:) ./ V(1,:);
disp(V);
disp(w2);
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> yes
    1.0000    1.0000    1.0000
    1.0000    0.0000   -4.0000
    1.0000   -1.0000    1.0000

    0.0000
    2.0000
    5.0000
```

2c) To solve for eigen values:

2.5

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \end{bmatrix} = \lambda\begin{bmatrix} x \end{bmatrix} \rightarrow -\lambda I x + \cancel{AX} = 0$$

we can see that the process is analogous to how we solve for natural frequency, where:

$$\det(-w_n^2 M + K) = 0 \leftarrow -w_n^2 M x + Kx = 0$$

$$\downarrow$$

$$\det(K - w_n^2 M) = 0$$

Only difference is scaling of M & K matrices, which are accounted for by specifying 'vector' in MATLAB.

Also analogously, ~~find~~ a vector that pairs to eigen value is eigenvector, just like how mode shape equation pairs to natural frequency, where:

$$X = C\begin{bmatrix} 1 \\ u \end{bmatrix} \text{ only for } (-w_n^2 M + K)X = 0 \text{ at } w_n = w_{n1}, w_{n2}$$

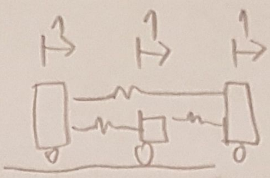
$$\approx$$

$$V = C\begin{bmatrix} 1 \\ u \end{bmatrix} \text{ only for } (-\lambda I + A)V = 0 \text{ at } \lambda = \lambda_1, \lambda_2$$

2d)

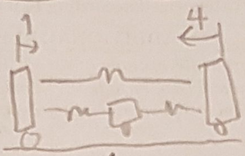
From MATLAB

$\omega_{n_1}^2 = 0 \rightarrow u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow$



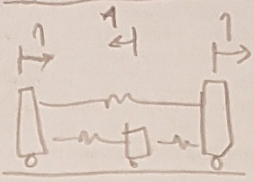
everything moves together as a single unit, no oscillate

$\omega_{n_2}^2 = 2 \rightarrow u_2 = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} \rightarrow$



x_1 & x_3 oscillate as x_2 remain nodal

$\omega_{n_3}^2 = 5 \rightarrow u_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \rightarrow$



as x_1 & x_3 moves uniformly, x_2 oscillates in other direction to maintain force balance

0.5

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I did not cheat.