Subject:
Date

1. a) $G_p(s) = \dfrac{100}{s(s+100)}$

$$G_p(j\omega) = \frac{100}{\underbrace{(j\omega)}_{sys_1}\underbrace{(j\omega+100)}_{sys_2}}$$

sys_1 : $\dfrac{1}{j\omega}$ $\longrightarrow$ Gain : 1 (at $\omega = 1$)

Phase: $-90°$

Pole : 0

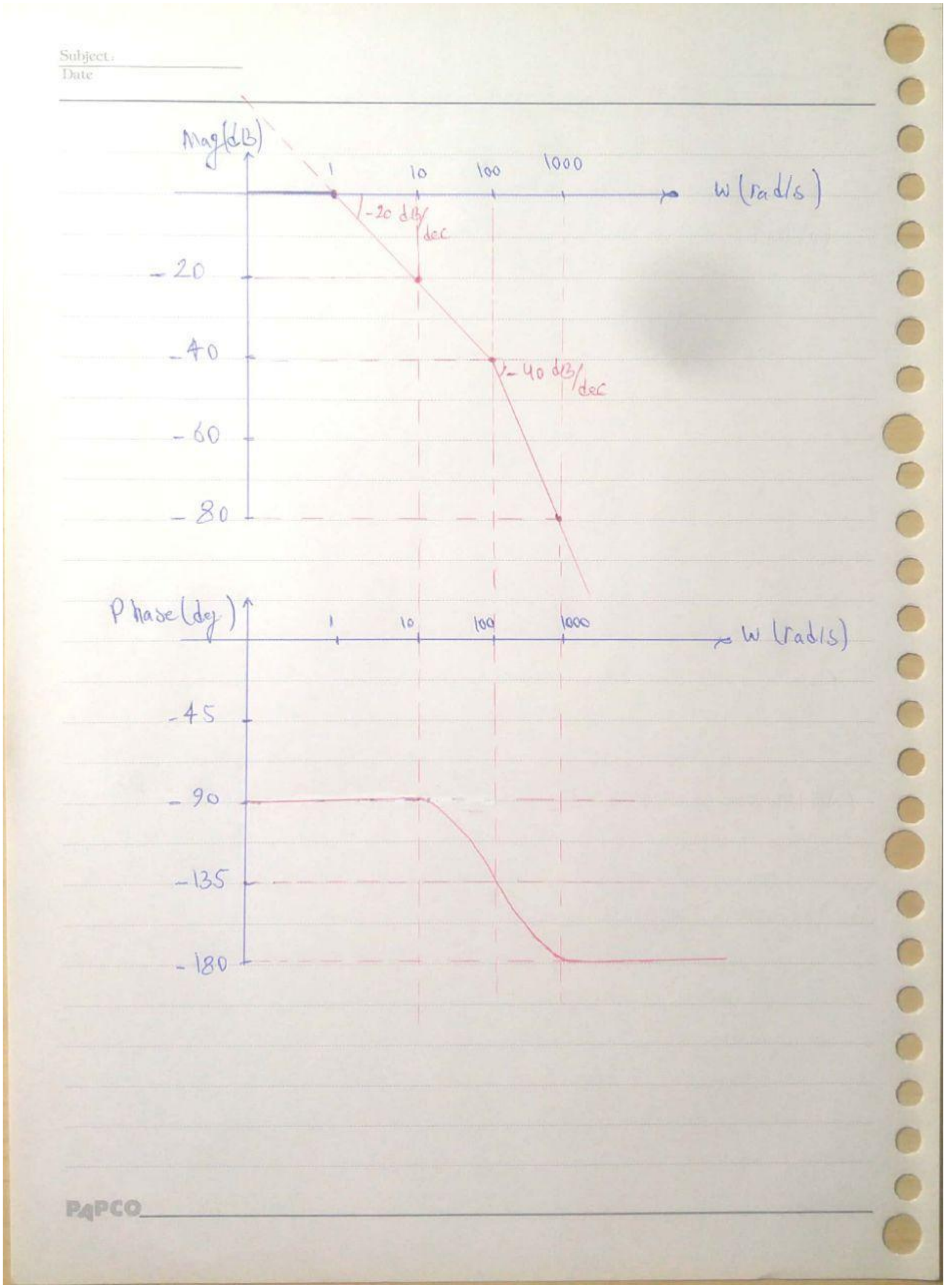
sys_2 : $\dfrac{100}{j\omega+100}$ $\longrightarrow$

Gain : 1

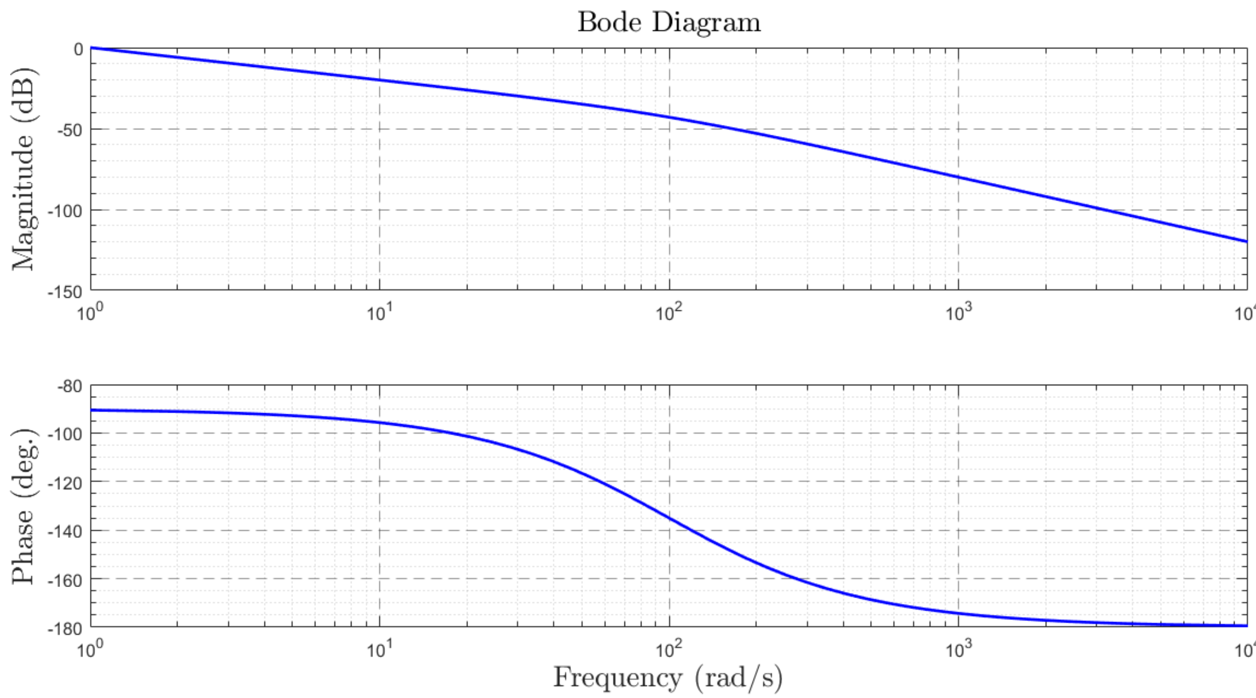
Phase: $-\tan^{-1}\left(\dfrac{\omega}{100}\right)$

Pole : -100

PAPCO


Mag (dB)
1
10
100
1000
w (rad/s)
-20 dB/dec
-20
-40
-40 dB/dec
-60
-80
Phase (deg)
1
10
100
1000
w (rad/s)
-45
-90
-135
-180

Below is the MATLAB plot for comparison.



Subject:
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1. b)

$$G_p(s) = \frac{1000\,(s+100)}{s^2 + 10001s + 10000}$$

$$\Delta = b^2 - 4ac = 10001^2 - 10000 \times 4 > 0$$

root 1 : -1 , root 2 : -10^4

$$\Rightarrow \quad G_p(s) = \frac{1000\,(s+100)}{(s+1)\,(s+10^4)}$$

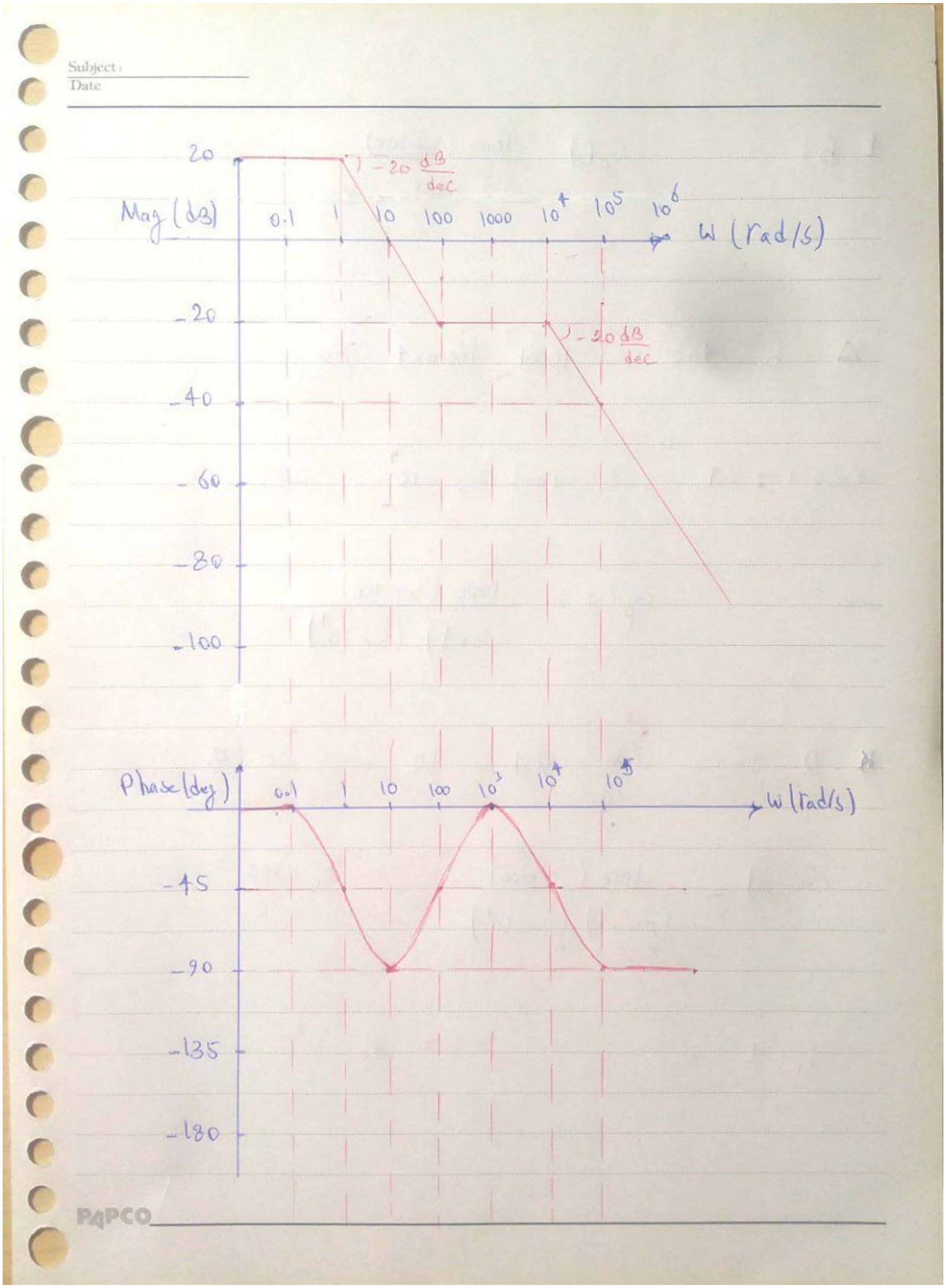
$$\text{DC gain} = \lim_{s \to 0} G_p(s) = 10 = \boxed{20\text{ dB}}$$

$$G(j\omega) = \frac{1000\,\overbrace{(j\omega+100)}^{\text{sys 2}}}{\underbrace{(j\omega+1)}_{\text{sys 1}}\,\underbrace{(j\omega+10^4)}_{\text{sys 3}}}$$

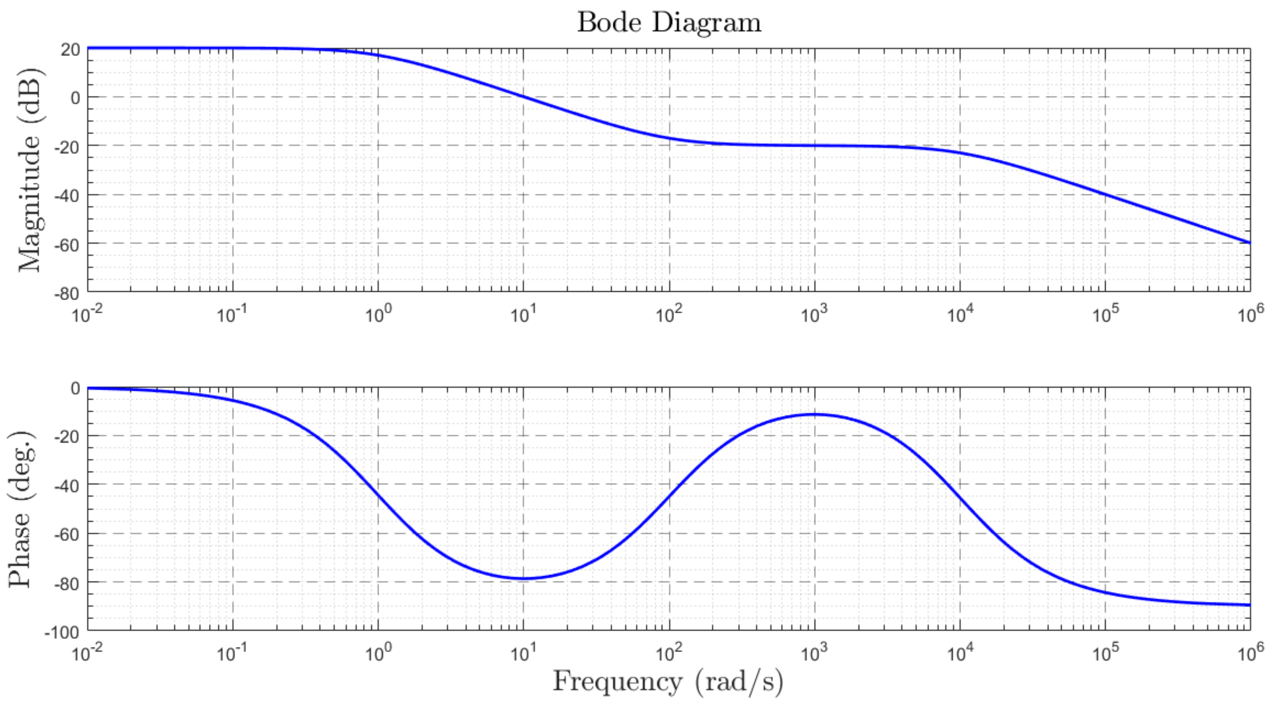
$Z_1 = 100$

$P_1 = 1$ $\qquad$ $P_2 = 10^4$

PAPCO


Subject:
Date
Mag (dB)
20
-20
-40
-60
-80
-100
0.1
1
10
100
1000
10^4
10^5
10^6
ω (rad/s)
-20 dB/dec
-20 dB/dec
Phase (deg)
-45
-90
-135
-180
0.1
1
10
100
10^3
10^4
10^5
ω (rad/s)
PAPCO

Below is the MATLAB plot for comparison.



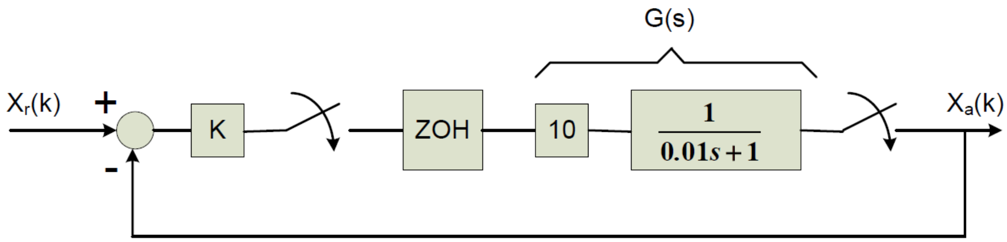


Fig. 1. Close loop control system of a discrete system

2) Obtain the zero-order hold equivalent of $G(s)$ with $T_s = 1\ ms$.

$$G_{pZOH}(z) = ZOH\{G_p(s)\} = (1-z^{-1})Z\left\{\frac{10}{s(0.01s+1)}\right\} = (1-z^{-1})Z\left\{\frac{1000}{s(s+100)}\right\}$$
$$= (1-z^{-1})Z\left\{\frac{10}{s}+\frac{-10}{s+100}\right\} = (1-z^{-1})\left\{\frac{10}{1-z^{-1}}+\frac{-10}{1-e^{-0.1}z^{-1}}\right\}$$
$$= (1-z^{-1})\left\{\frac{10(1-e^{-0.1})z^{-1}}{(1-z^{-1})(1-e^{-0.1}z^{-1})}\right\} = \frac{0.9516z^{-1}}{1-0.9048z^{-1}}$$

3) Obtain the closed loop transfer function of the whole system in z-domain.

$$\mathrm{G_{cl}}(\mathrm{z}^{-1}) = \frac{\mathrm{KG_{pZOH}(z)}}{1+\mathrm{KG_{pZOH}(z)}} = \frac{0.6\times\frac{0.9516\mathrm{z}^{-1}}{1-0.9048\mathrm{z}^{-1}}}{1+0.6\times\frac{0.9516\mathrm{z}^{-1}}{1-0.9048\mathrm{z}^{-1}}} = \frac{0.5710\mathrm{z}^{-1}}{1-0.3339\mathrm{z}^{-1}}$$

4) Calculate the response X_a at the first three sample times.

$$\frac{X_a(k)}{X_r(k)} = \mathrm{G_{cl}}(\mathrm{z}^{-1}) = \frac{0.5710\mathrm{z}^{-1}}{1-0.3339\mathrm{z}^{-1}} \rightarrow X_a(k)[1-0.3339\mathrm{z}^{-1}] = 0.5710\mathrm{z}^{-1}X_r(k)$$

$$X_a(k) = 0.5710\mathrm{z}^{-1}X_r(k) + 0.3339\mathrm{z}^{-1}X_a(k)$$

$$\boxed{=>\ X_a(k) = 0.571X_r(k-1) + 0.3339X_a(k-1)}$$

X_r is a unit step input. Assume zero initial conditions.

k=0: $X_r(-1) = 0$, $X_a(-1) = 0$ => $X_a(0) = 0$

k=1: $X_r(0) = 1$, $X_a(0) = 0$ => $X_a(1) = 0.5710$

k=2: $X_r(1) = 1$, $X_a(1) = 0.5710$ => $X_a(2) = 0.7617$

k=3: $X_r(2) = 1$, $X_a(2) = 0.7617$ => $X_a(3) = 0.8253$

5) Find the final value and steady state error of the system subject to a unit step input.

Using final value theorem in discrete domain:

$$Xa_{ss} = \lim_{z \to 1}(1 - z^{-1})X_r(z)G_{cl}(z) = \lim_{z \to 1}(1 - z^{-1})\frac{1}{(1 - z^{-1})}\frac{0.5710\mathrm{z}^{-1}}{1 - 0.3339\mathrm{z}^{-1}} = \frac{0.5710}{1 - 0.3339} = 0.8572$$

$$e_{ss} = \lim_{z \to 1}(1 - z^{-1})X_r(z)[1 - G_{cl}(z)] = \lim_{z \to 1}(1 - z^{-1})\frac{1}{(1 - z^{-1})}\left[1 - \frac{0.5710\mathrm{z}^{-1}}{1 - 0.3339\mathrm{z}^{-1}}\right]$$

$$= 1 - \frac{0.5710}{1 - 0.3339} = 0.1428$$