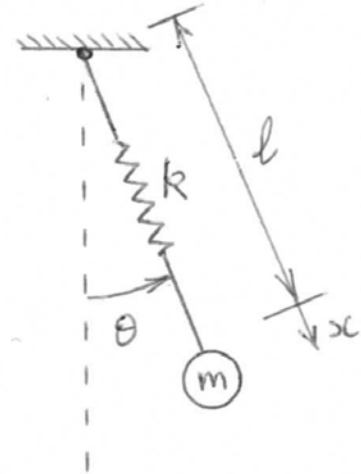


## MECH 463 -- Tutorial 10

1. A flexible pendulum consists of a light spring of unstretched length  $\ell$ , stiffness  $k$ , with a concentrated mass  $m$  attached at its lower end. Use Lagrange's equations to formulate the equations of motion of the pendulum in terms of the spring extension  $x$  and the pendulum angle  $\theta$ . Your equations will be non-linear; you are not asked to put them into matrix form nor solve them. Do not make any "small vibration" assumptions yet. Explain the meanings of the various terms in your equations. When done, simplify your equations for the case of small vibration amplitudes. Give a physical explanation of your results.



2. Three simple pendulums of equal length  $\ell$  are connected at their lower ends by springs of stiffness  $k$ . The mass of the centre pendulum is  $2m$  and of each of the other two pendulums is  $m$ . By inspection, determine the first two mode shapes of the pendulum system. Justify your choices. Determine the mass and stiffness matrices of the system. Use the orthogonality relations to determine the third mode shape.

