2b.

q2_vars.m

```
1 -    mr = 0.095;
2 -    r = 0.085;
3 -    br = 0.001;
4 -    mp = 0.024;
5 -    Lp = .129;
6 -    bp = .00005;
7 -    g = 9.81;
8 -    Jr = mr*r*r/3;
9 -    Jp = mp*Lp*Lp/3;
10 -   l = Lp/2;
11 -   Jt = Jr*Jp - (mp*r*l)^2;
```

Fig 1: Matlab code

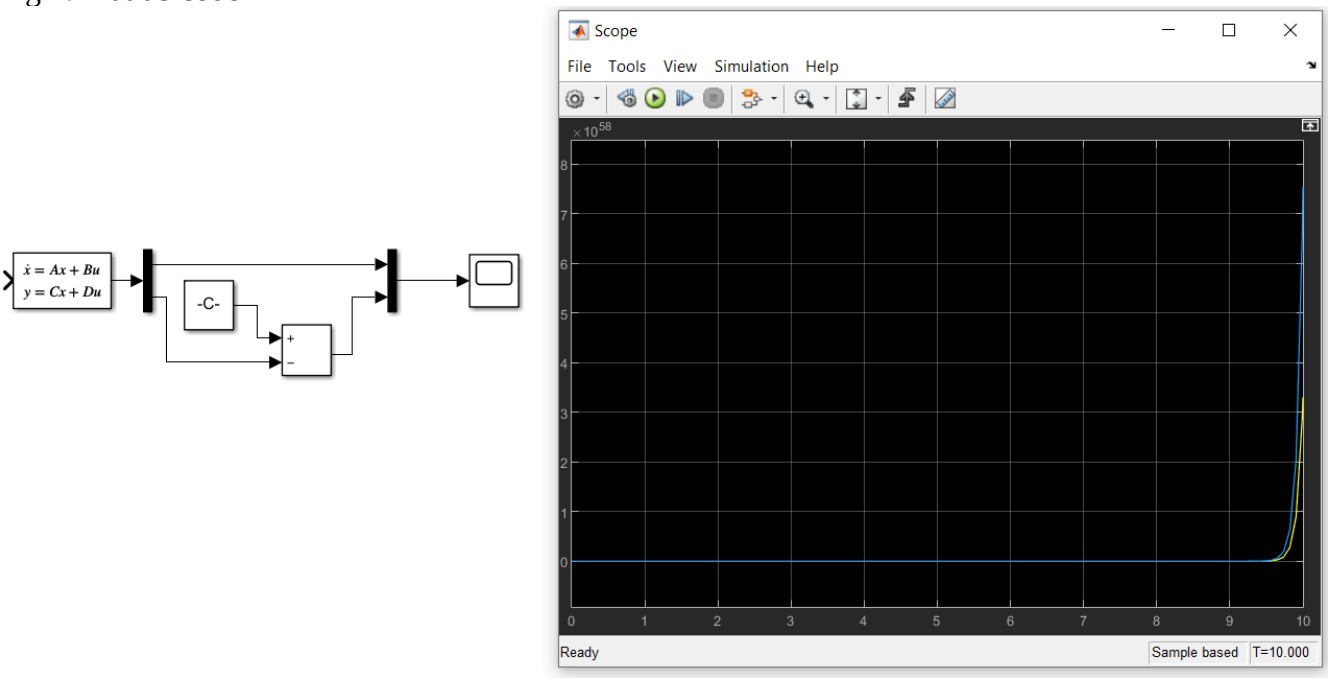

Fig 2: Simulink model, and plot (theta over time (yellow) and alpha over time (blue))

2c.

q2_vars.m

```
mr = 0.095;
r = 0.085;
br = 0.001;
mp = 0.024;
Lp = .129;
bp = .00005;
g = 9.81;
Jr = mr*r*r/3;
Jp = mp*Lp*Lp/3;
l = Lp/2;
Jt = Jr*Jp - (mp*r*l)^2;
A = [0 1 0 0; 0 -Jp*br/Jt ((mp*l)^2)*r*g/Jt mp*r*l*bp/Jt; 0 0 0 1; 0 mp*r*l*br/Jt -Jr*mp*g*l/Jt -Jr*bp/Jt];
B = [0; Jp/Jt; 0; -mp*r*l/Jt];
A_inv = [0 1 0 0; 0 -Jp*br/Jt -((mp*l)^2)*r*g/Jt -mp*r*l*bp/Jt; 0 0 0 -1; 0 -mp*r*l*br/Jt -Jr*mp*g*l/Jt -Jr*bp/Jt];
B_inv = [0; Jp/Jt; 0; mp*r*l/Jt];
C = [1 0 0 0; 0 0 1 0];
D = [0; 0];
[~,eigval] = eig(A);
[~,eigval_inv] = eig(A_inv);
disp("normal pendulum's eigen values:");
disp(eigval);
disp("an eigenvalue is zero, so we need to check rank(eigval*I-A) =? n-m");
disp("we know n=4, and m=1 (no repeated eig vals), so n-m=3");
disp("normal pendulum's rank(0*I - A):");
disp(rank(-A));
disp("since rank is 3 which is equal to n-m=3, normal pendulum is marginally stable");
disp("inverted pendulum's eigen values:");
disp(eigval_inv);
disp("one of the eigenvalues is >0, so inverted pendulum is unstable");
```

```
>> q2_vars
normal pendulum's eigen values:
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -4.8479 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -3.0749 +15.1276i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -3.0749 -15.1276i

an eigenvalue is zero, so we need to check rank(eigval*I-A) =? n-m
we know n=4, and m=1 (no repeated eig vals), so n-m=3
normal pendulum's rank(0*I - A):
     3

since rank is 3 which is equal to n-m=3, normal pendulum is marginally stable
inverted pendulum's eigen values:
         0         0         0         0
         0  -20.8385         0         0
         0         0   -4.0042         0
         0         0         0   13.8450

one of the eigenvalues is >0, so inverted pendulum is unstable
```

Fig 3: Matlab code and computed values