MECH 420 SENSORS AND ACTUATORS
Solutions to Assignment 5

Sol-Problem 1 (Problem 5.5 from Textbook)

(a) It is advantageous to have a single measuring device to determine more than one kinematic variable under the following circumstances:

(i) Measured signal is well behaved (e.g., narrow-band or periodic) and signal-to-noise ratio (SNR) is high

(ii) Cost of additional measuring devices and associated instrumentation is high in comparison to the cost of data processing (hardware and/or software)

(iii) The weight, size, and loading of additional instrumentation on the measurand crucially affect the accuracy of the measurement (mechanical and electrical loading)

(iv) Physical difficulties such as poor accessibility, room for cables, and mounting gear can restrict the number of measuring devices that can be employed

(v) No sensor is available for direct measurement of the variable.

(b) It is disadvantages to use a single measuring device to determine more than one kinematic variable under the following circumstances:

(i) Signal is noisy and processing of the signal (e.g., differentiation) accentuates the noise further, thus deteriorating the signal-to-noise ratio

(ii) Additional data processing cost is higher than the cost of extra instrumentation

(iii) Required processing speed, sampling rate, etc., cannot be achieved with the existing hardware (This becomes a problem particularly when the frequency range of interest is fairly high)

(iv) The desired signal has an error-compounding characteristic (i.e., drift—error in the measurand at an instant of time grows with time).

As an example, consider the motion sensing problem in an industrial robotic manipulator. If an optical encoder is used to measure a joint angle, the corresponding angular velocity may be determined by pulse rate counting for high speeds and by pulse width detection for low speeds.

The same procedure may be employed if a magnetic probe and a toothed wheel are used. In both cases there is a finite limit on the displacement resolution, which depends on window pitch of the track or tooth pitch. Particularly at low speeds, this arrangement may not yield sufficiently accurate measurements. Therefore, stepping up the speed using a gear mechanism prior to measurement may be necessary. This will introduce backlash, friction and additional mechanical loading, however. Alternatively, a potentiometer or a resolver (an analog device for measuring angular displacement) in conjunction with an appropriate ADC and digital processing may be used. This approach is limited by the digital sampling and processing capabilities of the utilized hardware/software.

If one uses a resolver or a potentiometer as well as a tachometer, accuracy checks can be incorporated for the consistency of measured and computed motion variables (Note: Sensor fusion can be done as well, with further benefits of accuracy and reliability). This will introduce hardware (and software) redundancy, and can assist in fault diagnosis of the sensors and of the system itself. But mechanical loading errors and instrumentation costs will be higher.

As a second example, consider a mobile robot. If navigation acceleration is measured using an accelerometer and double integrated to obtain position, the result might not be acceptable for many reasons. Since low frequency signals are considered in this application, the accelerometer will have low frequency distortion errors. Furthermore, if acceleration is measured and sampled, an error in each sample will introduce a displacement error that will become compounded over a period of time (due to the integrating action).

Another example is ride-quality control of an automated ground transit vehicle using an active suspension system. If the ride quality is specified in terms of jerk, it has to be measured. Since direct measurement is not possible, acceleration measurement and data processing may be used for this purpose.

<u>Sol-Problem 2 (Problem 5.11 from Textbook)</u>

$$\frac{v_o}{v_{ref}} = \left[\frac{(\theta/\theta_{\max})(R_L/R_c)}{(R_L/R_c) + (\theta/\theta_{\max}) - (\theta/\theta_{\max})^2}\right]$$

Let $x = (\theta/\theta_{\max})$ and $y = v_o/v_{\text{ref}}$

Normalized sensitivity $S = \dfrac{\partial y}{\partial x} \cdot \dfrac{x}{y} = \dfrac{(\gamma + x^2)}{(\gamma + x - x^2)}$ where $\gamma = \dfrac{R_L}{R_c}$

Some sensitivity values are given in Table S5.11.

The sensitivity curve is sketched in Figure S5.11, for $\gamma = 0.5$. It is noted from the curve that the maximum sensitivity occurs at x = 1. The maximum value is $S_{\max} = 1 + \frac{1}{\gamma}$.

Table S5.11: Some key sensitivity values.

Fractional Displacement (*x*)	**Normalized Sensitivity (*S*)**
0	1
1/4	$1 - \frac{2}{(16\gamma + 3)}$
1/2	1
3/4	$1 + \frac{6}{(16\gamma + 3)}$
1	$1 + \frac{1}{\gamma}$

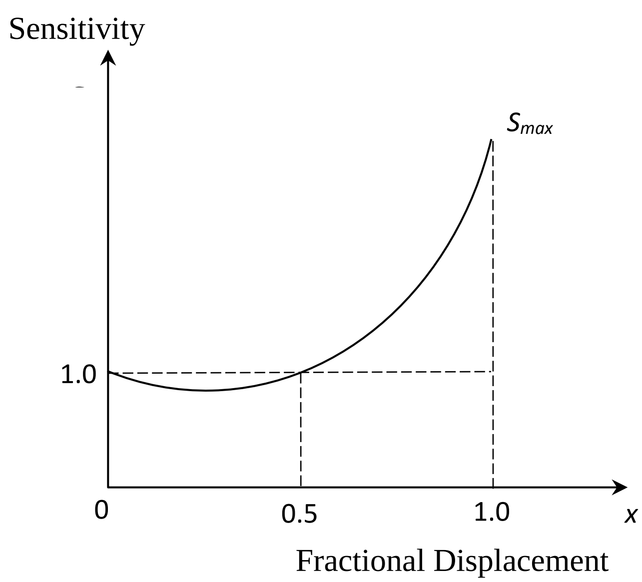

Figure S5.11: Sensitivity curve of a rotatory potentiometer for $R_L / R_c = 0.5$.

Sol-Problem 3 (Problem 5.23 from Textbook)

See Figure S5.23. Note that the cases **(a)** and **(b)** both produce the same signal envelope. Hence direct demodulation will not recover the direction characteristics of the core displacement. Phase-sensitive demodulation would be necessary.

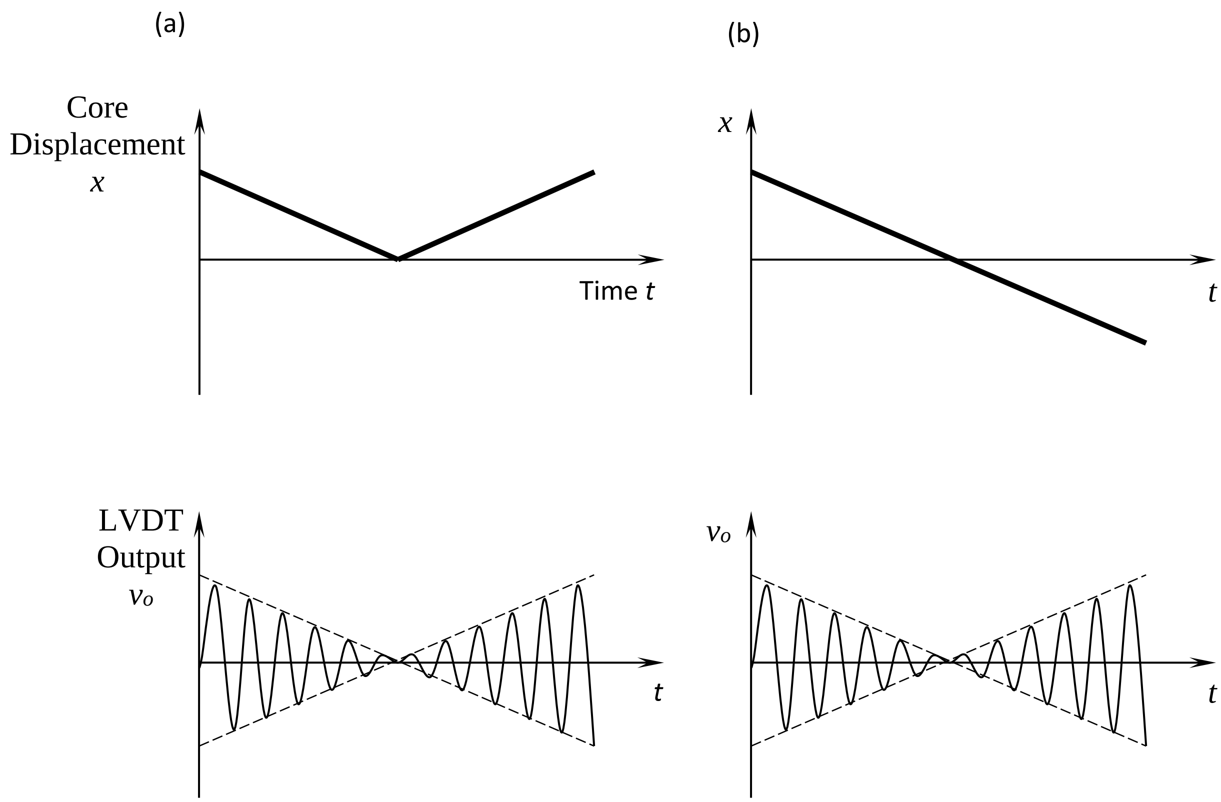

Figure S5.23: Behavior of the output of an LVDT under transient motion of the core.

Sol-Problem 4 (Problem 5.36 from Textbook)

It is known that the moisture content increases the permittivity (dielectric constant) of a material. For high-density materials, this relationship can be rather linear. Nevertheless, an empirical relationship can be obtained for the variation of the dielectric constant. Suppose that a parallel-plate capacitor has dielectric packing such as paper between its plates. We can measure the capacitance using a bridge circuit (i.e., a capacitance bridge) starting from the balanced state) or an oscillator circuit (i.e., resonant frequency of an *LC* circuit, which depends on *C*). This gives the dielectric constant of the capacitor (for a given gap and face area). This in turn determines the moisture content in the dielectric medium, which directly depends on the relative humidity.

Advantages: Simple and relatively low cost
Disadvantages: Slow (large time constant) in view of the time taken for the dielectric medium to absorb moisture. Measurements have to be taken at steady state. Aging of the dielectric medium will affect the accuracy. Sensitivity is low and the device is nonlinear in general (proper signal conditioning and calibration are needed).

Sol-Problem 5 (Problem 5.39 from Textbook)-rev

Since C_c is used to model the internal capacitance of the piezoelectric element, we may ignore C in Figure P 5.39. Even if we keep C, the following analysis does not change because it is in series with q.

We have:

R = charge leakage resistance; C_c = cable capacitance; C_p = piezoelectric crystal capacitance.

Current balance equation: $\dot{q} = \frac{v_o}{R} + (C_p + C_c)\,\dot{v}_o$

Laplace transformation gives: $sq = \left(\frac{1}{R} + Cs\right) v_o$; where, $C = C_p + C_c$.

The transfer function is: $\frac{v_o}{q} = \frac{Rs}{[1+RCs]} = \frac{Rj\omega}{1+\tau j\omega}$ in the frequency domain.

where, $\tau = RC$ = time constant.

Note: The transfer function $\rightarrow$ constant for large ω and/or large τ.

Given: $C = 300 + 700 = 1000$ pF; $R = 1\ x\ 10^{11}$ Ω.

Substitute: $\tau = 1000 \times 10^{-12} \times 1 \times 10^{11} = 100$ s.

Note: The effects of all the extraneous parameters (R; C_c; C_p) can be fully compensated for by using a charge amplifier (because, q would be at zero potential then).

Sol-Problem 6 (Problem 5.47 from Textbook)

Let T_1, T_2, and T_3 be the torques measured at locations 1, 2, and 3, respectively. Then, with reference to the free-body diagram in Figure S5.47, we can write torque balance equations for the four members of the manipulator; namely, the bearing at 'A', the rotor, the bearing at 'B', and link 2.

Torque balance at 'A': $T_3 = T_A$ (i)

where, T_A = frictional torque at bearing A.

Torque balance at the rotor: $T_m = T_1 + T_3 + J_m\ddot{\theta}_m$ (ii)

Torque balance at 'B': $T_1 - T_2 = T_B$ (iii)

where, T_B = frictional torque at bearing B.

Torque balance for link 2: $T_2 = J_l\ddot{\theta}_l$ (iv)

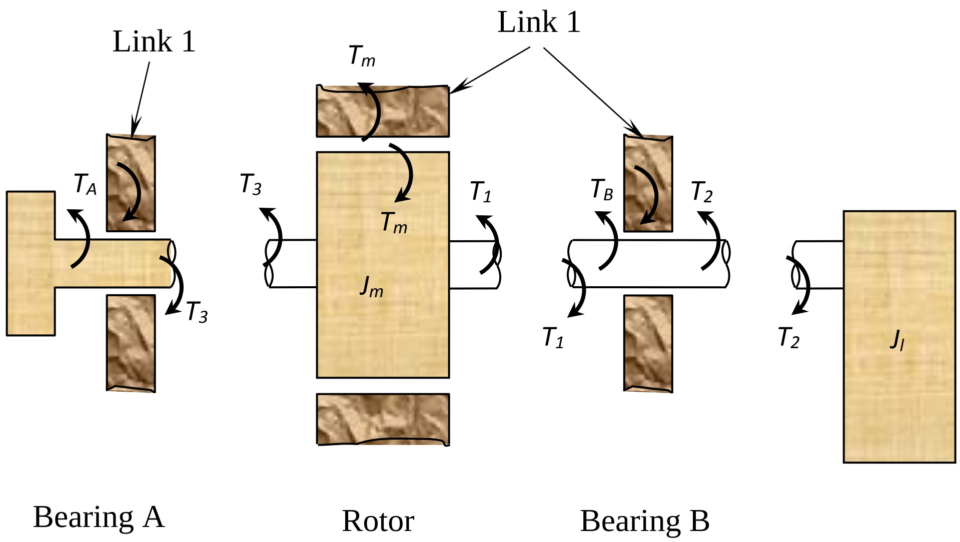

Figure S5.47: Free-body diagram of robot joint.

Then, we have the following:

Torque transmitted to link 2 $= T_2$

Frictional torque at bearing 'A' $= T_3$

Frictional torque at bearing 'B' $= T_1 - T_2$

Reaction torque at link 1 $= T_m - T_A - T_B = T_m - T_3 - (T_1 - T_2) = T_m - (T_3 + T_1) + T_2$

$$= T_m - (T_m - J_m \ddot{\theta}_m) + J_l \ddot{\theta}_l = J_m \ddot{\theta}_m + J_l \ddot{\theta}_l = J_m \ddot{\theta}_m + T_2$$

Sol-Problem 7 (Problem 5.56 from Textbook)

(a) Various components of the measuring device are identified in Figure S5.56.

(b) Force is applied to the tip of the cantilever; it converts force into a displacement. The LVDT converts this displacement into an output (ac) voltage v_o. This is simply an LVDT load cell.

Measurand: Force f
Output: Voltage v_o

(c) Advantages and disadvantages of LVDT would apply for this sensor. In addition,

(i) Bandwidth is limited by the fundamental natural frequency of the cantilever (due to the cantilever mass).

(ii) Measured force is limited by the strength of the cantilever.

(iii) Linearity depends on elastic properties and geometry of the cantilever.

(iv) In the given configuration, only reaction forces or static loads can be measured. The method can be extended, however, for transmittal forces, and torques as well.

(v) Displacements have to be small in order to reduce support motion (motion transmissibility) effects.

Note: If a bimetallic cantilever is used, this becomes a temperature sensor.

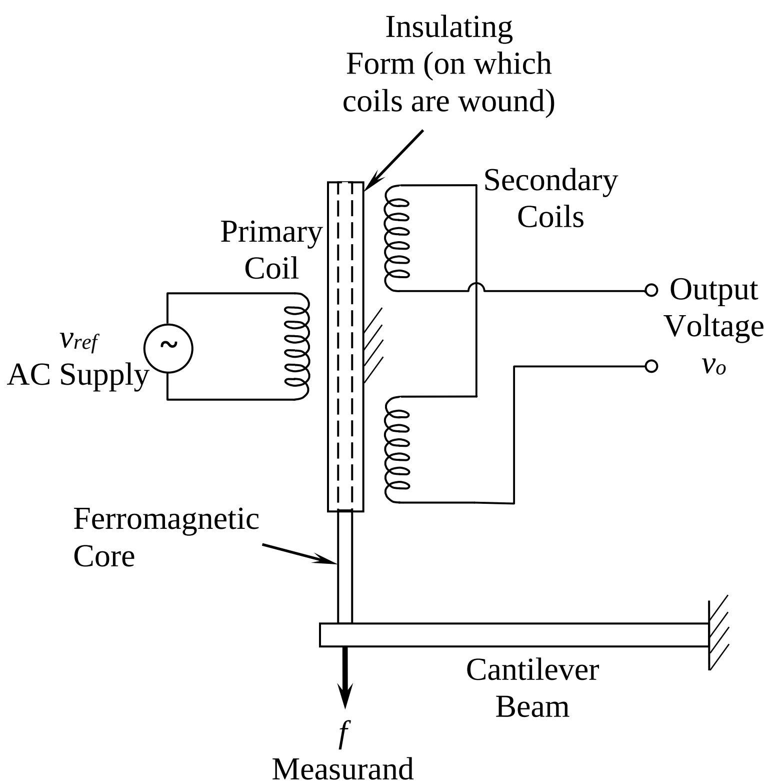

Figure S5.56: (a) Components of the measuring device.

(d) This load cell can be used to determine power-transmission characteristics of rotating machinery (e.g., motors, generators). A schematic diagram of an appropriate arrangement is shown in Figure S5.56(b).

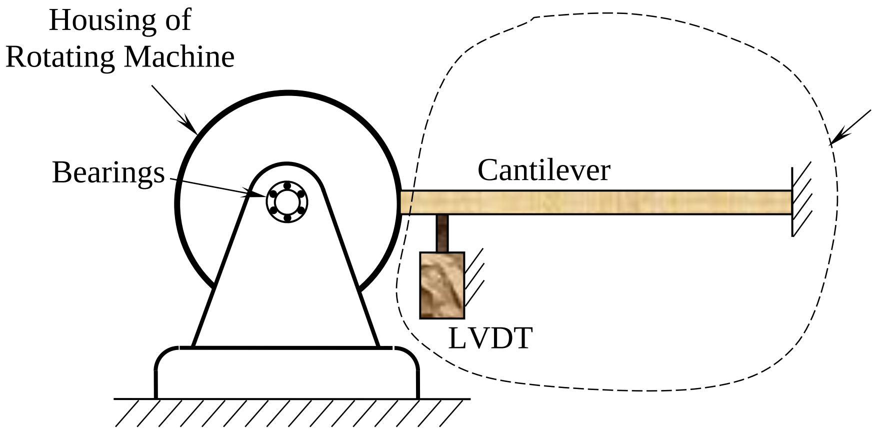

Figure S5.56: (b) Application of an LVDT load cell.

Sol-Problem 8 (Problem 5.66 from Textbook)

The angular momentum $J\omega$ about the spin axis changes at rate Ω about the axis that is orthogonal to both spin axis and gimbal (θ) axis. This corresponds to a gyroscopic torque $J\omega\Omega$, which has to be supported by a torque about the gimbal axis as provided by the torsional spring and the torsional damper. Hence, $J\omega\Omega = k\theta + b\dot{\theta}$ → $\Omega = \dfrac{k\theta + b\dot{\theta}}{J\omega}$

Assuming that b is very small, we have: $\theta = \dfrac{J\omega}{k}\Omega$ (i)

It is seen from (i) that one has to increase J and ω and decrease k to increase sensitivity. This can result in large oscillations of the gimbal with respect to the support. To reduce this problem, increase bearing damping on the gimbal axis.