Lesson 9-1 – Equivalent Annual Cashflow Analysis

Chapter 6

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Chapter 6 Learning Objectives

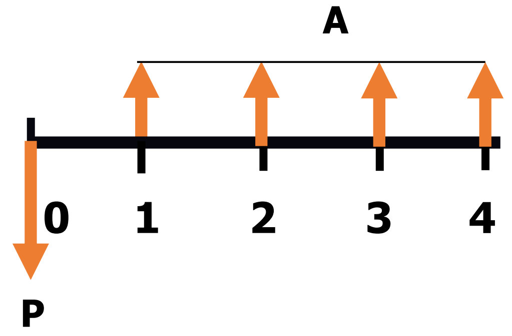
- Understand and define the equivalent annual cash flow (EACF) in terms of benefits & costs.
- Conduct an economic analysis of an investment based on EACF.
- Determine when an analysis based on EACF, rather than NPV, is appropriate.
- Use EACF to compare alternatives with equal, unequal or infinite lifetimes.
- Understand various mortgage types and terms

Annual Cash Flow Analysis

- Relationship between PV and EACF
  - Simple
- Salvage Value (touched on briefly last lesson)
- Analysis periods
  - EACFn = EACF infinite

Annual Cash Flow Calculations

- The objective is to compare alternatives based on annual cash flows.
- This requires converting present values and one-time values on the timeline to equivalent uniform values.
    - Using annual worth factors: F, P,
        - For example: A = P(A/P, i%, 4)



Annual Cash Flow Calculations

- The textbook calls these EUACs (equivalent uniform annual costs) and EUABs (equivalent uniform annual benefits). You can also use ‘EACF’ since it is clear from its sign whether a cash flow is a cost or a benefit.

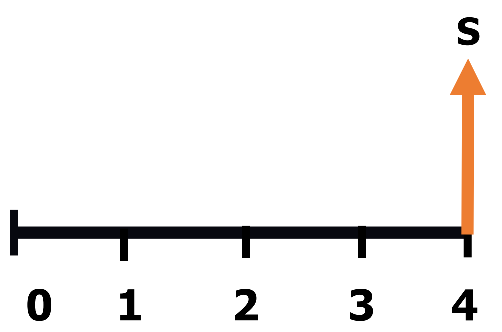
Annual Cash Flow Calculations Continued…

Some points regarding cash flow calculations:

1. The EACF is the annuity payment that gives the net present value (NPV) when discounted by the required rate of return, and vice versa.
2. The EACF increases (becomes more positive) when benefits increase, and decreases when costs increase.
3. The EACF is a simple, convenient representation for a series of irregular cash flows or for a series of cash flows with regular changes, i.e. an arithmetic gradient series or geometric series.

Addressing Salvage Value

- When there is a salvage value at the end of the life of an asset, it is represented as a one-time cash flow benefit at the end of the asset's life.
  - Example: Salvage value (S) of asset with a four-year life



- What is the EACF of a salvage cashflow?
- A = S(A/F, i, n) (sinking fund factor)

Adding them together…

- EACF of the Present value of an alternative:
- A1 = P(A/P, i, n)
- EACF of the salvage value
- A2 = F(A/F, i, n)

- EACF = A1 + A2
- This can all be boiled down to (using factors):
  - EUAC = P(A/P, i, n) – S(A/F, i, n)

    OR
  - EUAC = (P – S)(A/F, i, n) + Pi

    OR
  - EUAC = (P – S)(A/P, i, n) + Si

Annual Cash Flow Calculations Continued…

Input/output	***Situation***	***Criterion***
Fixed input	Amount of capital available is fixed	Maximize EACF
Fixed output	Amount of benefits is fixed	Maximize EACF
Neither fixed	Neither amount, capital nor benefits, is fixed	Maximize EACF

Cash Flow Calculations: Problem 1

A university student looking for new tires has located the following alternatives:

Expected Tire Life	Price/Tire
12 Months	$30.95
24 Months	$44.95
36 Months	$53.95
48 Months	$59.95

Cash Flow Calculations: Solution 1

Use 'Uniform Series Capital Recovery Factor':

$$A = P(A/P, i, n) = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

EUAC (12 month tire) = \$30.95 (A/P, 12%, 1) = \$34.66

EUAC (24 month tire) = \$44.95 (A/P, 12%, 2) = \$26.60

EUAC (36 month tire) = \$53.95 (A/P, 12%, 3) = \$22.46

EUAC (48 month tire) = \$59.95 (A/P, 12%, 4) = \$19.74

Choose the 48 Month Tire to reduce annual costs

Annual Cash Flow Analysis: Problem 2

The following data are available for three different alternatives:

	Alternative A	Alternative B	Alternative C
Initial Cost	$1000	$1500	$2000
Uniform Annual Benefits	$200	$276.20	$654.80
Useful Life in Years	--	20	5
Interest Rate	15%	15%	15%

Alternatives B and C are replaced at the end of their useful lives with identical replacements. Using annual cash flow analysis find the most attractive alternative.

Annual Cash Flow Analysis: Solution 2

<u>Alternative A</u>

EUAB – EUAC = 200 – 1000(0.15) = \$50

<u>Alternative B</u> – use capital recovery factor

EUAB – EUAC = 276.2 – 1500(A/P, 15%, 20)

= 276.2 – 1500(0.1598) = \$36.5

<u>Alternative C</u> – use capital recovery factor

EUAB – EUAC = 654.8 – 2000(A/P, 15%, 5)

= 654.8 – 2000(0.2983) = \$58.2

Choose Alternative C

Annual Cash Flow Calculations Continued…

- Analysis period cases:
  1. The analysis period is equal to the useful lifetimes of all the alternatives, which are also equal.
  2. The analysis period is a common integer multiple of the useful lifetimes of the alternatives.
  3. The analysis period is for a continuing requirement, i.e. the analysis period is indefinite or infinite.
  4. The analysis period is specified otherwise, e.g. the planned lifetime of the project, or the period over which the investor expects to own/operate the investment.

Analysis Periods

1. Alternatives have equal lives
   - If the lives are equal, the analysis period is based on the same lifetime
   - This case rarely occurs in real life
2. Alternatives have unequal lives
   - If the lives are unequal, the analysis period is based on alternate lifetimes
     - No LCM is required as in present worth analysis
     - Multiples of service life are equivalent to one service life with annual worth analysis—therefore: It doesn't matter
     - See Example 6-7 in text.

Annual cash flow analysis …

- Case 1: Analysis period = useful lifetime of all the alternatives (which are all equal)
  - Base the comparison on the common useful lifetime or analysis period.
  - We can compare the NPVs directly or compare the EACFs of the alternatives.
  - This case rarely occurs in real life.

Case 1 Example

- A mine is considering a regular dump body or a lightweight body for its haul trucks. They both would scrapped after five years. The mine's rate of interest is 9%.

	Regular	**Lightweight**
Cost	$ 400,000	$ 300,000
Annual Maintenance	$ 30,000	$ 50,000
Salvage	$ 100,000	$ 80,000

Case 1 Example

- For each option, EAUC = Ac+Am-As
- Option A: 400000(A/P, 9%, 5) + 30000 – 100000(A/F, 9%, 5)
- = $102837 + $30000 -$16709 = $116,128

- Option B: 300000(A/P, 9%, 5) + 50000 – 80000(A/F, 9%, 5)
- = 77128 + 50000 -13367 = $113,760

- Choose the lightweight body.

9%				Compound
	Single Payment			Uniform P
	Compound Amount Factor Find *F* Given *P*	Present Worth Factor Find *P* Given *F*	Sinking Fund Factor Find *A* Given *F*	Capital Recovery Factor Find *A* Given *P*
n	*F/P*	*P/F*	*A/F*	*A/P*
1	1.090	.9174	1.0000	1.0900
2	1.188	.8417	.4785	.5685
3	1.295	.7722	.3051	.3951
4	1.412	.7084	.2187	.3087
5	1.539	.6499	.1671	.2571

Annual cash flow analysis …

- Case 2: Analysis period = a common integer multiple of the useful lifetimes of the alternatives (NPW) OR single lifetimes (EACF)
  - The useful lifetimes of the alternatives vary and the analysis period is a common integer multiple of those lifetimes.
  - We can compare the NPVs directly if we repeat lifetimes of the alternatives to match the common integer multiple, or analyze to an alternate timeframe.
  - We can compare the EACFs of the alternatives for single lifetimes since the EACF for one lifetime = the EACF for any integer multiple of lifetimes.
  - ASSUMPTION: repeatability: the benefits and costs remain valid for multiple lifetimes; this may be unrealistic in real life.

Case 2 Example

- The Navy is upgrading the Fire Control Computer on one of its warships. It can purchase a used copy of the same computer installed now for $2,500.000, and it is expected to last 8 years and have no salvage value.

- Or it can purchase a new Fire Control Computer system. The new system would cost $4,000,000 to purchase but is expected to last 16 years and would have a salvage value of $2,000,000.
- Assuming a 12% interest rate, which of these would be the cheaper option?

Case 2 Example

- EAUCa = 2,500,000(A/P, 12%, 8)
- =$503,257

12%				Compound
	Single Payment			Uniform P
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*
1	1.120	.8929	1.0000	1.1200
2	1.254	.7972	.4717	.5917
3	1.405	.7118	.2963	.4163
4	1.574	.6355	.2092	.3292
5	1.762	.5674	.1574	.2774
6	1.974	.5066	.1232	.2432
7	2.211	.4523	.0991	.2191
8	2.476	.4039	.0813	.2013
9	2.773	.3606	.0677	.1877
10	3.106	.3220	.0570	.1770

- EAUCb = 4,000,000(A/P, 12%, 16) – 2,000,000(A/F, 12%, 16)
- =(4000000-2000000)(A/P, 12%, 16) + 2000000(0.12)
- = $526,780

12%				Compound
	Single Payment			Uniform P
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*
1	1.120	.8929	1.0000	1.1200
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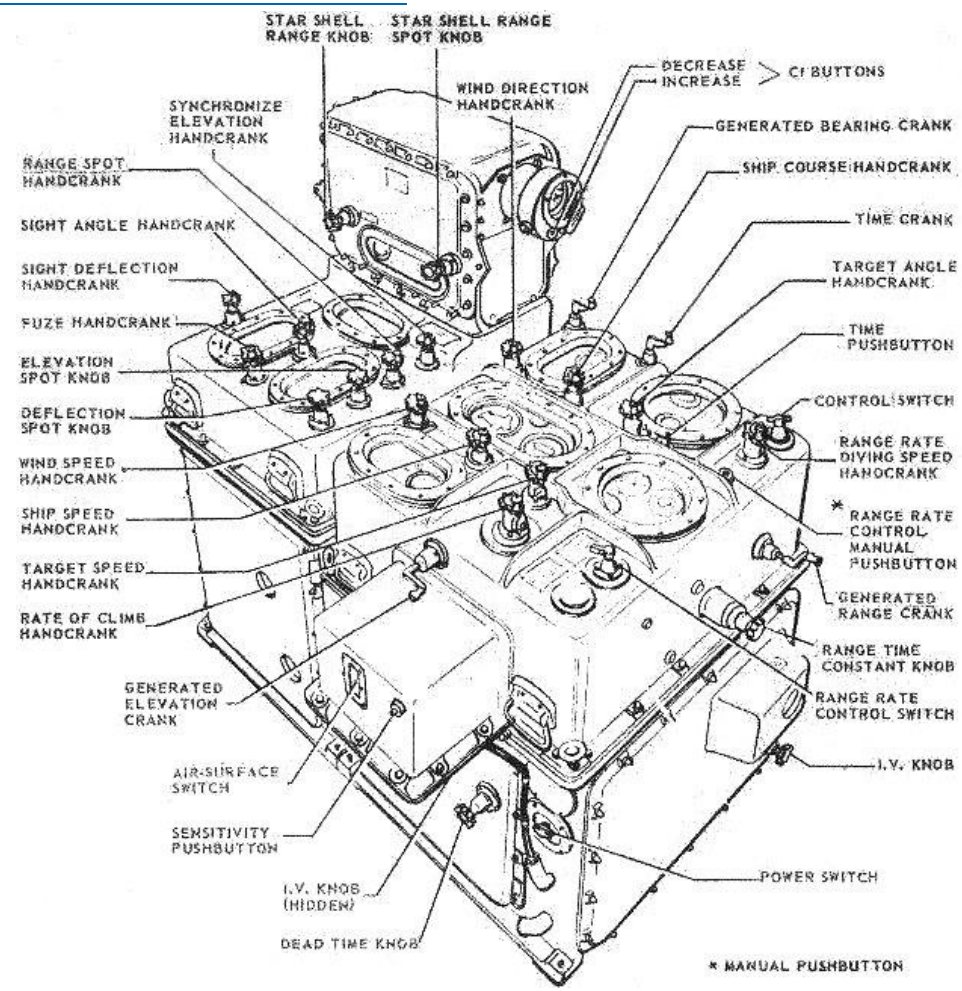
5	1.762	.5674	.1574	.2774
6	1.974	.5066	.1232	.2432
7	2.211	.4523	.0991	.2191
8	2.476	.4039	.0813	.2013
9	2.773	.3606	.0677	.1877
10	3.106	.3220	.0570	.1770
11	3.479	.2875	.0484	.1684
12	3.896	.2567	.0414	.1614
13	4.363	.2292	.0357	.1557
14	4.887	.2046	.0309	.1509
15	5.474	.1827	.0268	.1468
16	6.130	.1631	.0234	.1434



Annual cash flow analysis …

- Case 3: Analysis period for a continuing requirement (indefinite or infinite analysis period)
  - It can be assumed that the project will last for a long, indefinite or infinite time period.
  - Since the length of the analysis period is not important, we can analyze the alternatives using
    - a common integer multiple of the lifetimes of the alternatives, as in Case 2, or
    - an infinite analysis period.
  - Again, the EACF for one lifetime = the EACF for any integer multiple of lifetimes.

Infinite Analysis Period Continued…

- The difference in annual cost between a long life and an infinite life is normally small, unless an unusually low interest rate is used.
  - Example: $5.5 Million cost at 6%/year
    - Infinite Life: EUAC = 5.5M(0.06) = $330,000
    - Long Life (85 years): EUAC = 5.5M(A/P, 6%, 85) = $332,000
  - Difference in time is large (85 compared to infinity) but the EUAC is small.

Analysis Periods Continued…

3. Infinite Analysis

- Since multiples of finite service lives are equivalent to one service life, an infinite analysis of finite service lives yield:
  - $\text{EUAC}_{\text{infinite analysis period}} = \text{EUAC}_{\text{for limited life } n}$
- However, when an alternative with an infinite life is evaluated over an infinite analysis period:
  - $\text{EUAC}_{\text{infinite analysis period}} = P(A/P, i, \infty)$ + Any other annual costs
- When $n = \infty$, $A = Pi$, therefore:
  - $\text{EUAC}_{\text{infinite analysis period}} = Pi$ + Any other annual costs

Case 3 Example

- Suppose you want to build a dam . Your consulting engineers propose two alternatives. A concrete dam will cost $10,000,000 and require $500,000 in maintenance annually. An earthfill dam will cost $15,000,000 and require $200,000 in maintenance annually. Assuming the dams will last forever, which option should you choose at 5.5% interest? What if interest rates were forecast instead to be 13%?

- Image source: http://community.dur.ac.uk/~des0www4/cal/dams/emba/eintr.htm

Case 3 Example

At 5.5% Interest

Concrete: EAUC = P*i + Am = 10000000(0.055) + 500000

= $1,050,000

Earthfill: EAUC = P*i +Am = 1500000(0.055) + 200000

= $1,025,000

At 13% Interest

Concrete: EAUC = P*i + Am = 10000000(0.13) + 500000

= $1,800,000

Earthfill: EAUC = P*I +Am = 1500000(0.13) + 200000

= $2,150,000

Annual cash flow analysis …

- Case 4: Analysis period specified otherwise
  - An appropriate analysis period is specified, e.g. the planned project lifetime.
  - For each alternative, repeat lifetimes as necessary.
    - The final lifetime may be partial, to match the analysis period.
  - Generally, we need information on how the value of the assets in which we invested changes with time.
    - Adjust the cashflows to reflect analysis period =/= alternative lifetime
  - We can compare the NPVs of the alternatives with repeated lifetimes.
  - This is the most common case in real life.
  - Simplest to use NPV analysis and then convert NPV to EACF

Example

The following data are available for two different alternatives:

Analysis period = 7 years

	Alternative A	Alternative B
Initial Cost	$6000	$8000
Uniform Annual Benefits	$4000	$4000
Useful Life in Years	5	7
Salvage Value	0	$0
Interest Rate	18%	18%

Example

<u>Alternative A</u>

NPV = -$6000 + $4000(P/A, 18%, 7) - $6000(P/F, 18%, 5) + Salvage Value at year 7 (if any)

NPV = $6623 EACF = NPV(A/P, 18%, 7) = $1738

<u>Alternative B</u>

NPV = -$8000 + $4000(P/A, 7, 18%)

NPV = $7246 EACF = NPV(A/P, 18%, 7) = $1901

- Second lifetime of alternative A is only partial – 2 of 5 years
- NPV and EACF reflect paying additional $6000 for only two years of benefits.

Example

If we assume projects are repeatable indefinitely, can use EACF of 1 lifetime = EACF of repeated multiples

<u>Alternative A</u>

EUAB – EUAC = \$4000– \$6000(A/P, 18%, 5) = \$2081

<u>Alternative B</u>

EUAB – EUAC = \$4000 – \$8000(A/P, 18%, 7)

= \$4000 – 8000(0.2624) = \$1901

Why does the EACF for alternative A increase but B remain the same?

Relationship to NPW

- Direct relationship between the present worth and the equivalent uniform annual worth
  - Convert NPV to EACF(/EAUW) by multiplying by the 'Capital Recovery Factor': $\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$
  - Convert EUAW to NPW by multiplying by the 'Series Present Worth Factor': $\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$

CR Formula: $A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$ CR Factor: $\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$

SPW Formula: $P = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ SPW Factor: $\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$