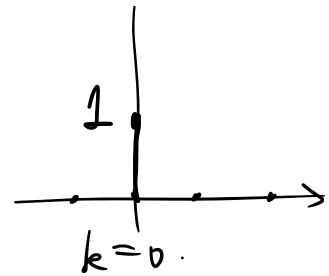
L18 – Digital Control System

- DT Systems (LTI).

$$x[k] \longrightarrow \boxed{H(z)} \longrightarrow y[k]$$

- Impulse Resp.

$x[k] = \underline{\delta[k]}$. "Kronecher delta"



$y[k] = \underline{h[k]}$.

- Transfer ftn. $\longleftrightarrow$ $H(s) = \mathcal{L}\{h(t)\}$.

$$H(z) = \mathcal{Z}\{h[k]\}.$$

$$= \sum_{k=-\infty}^{\infty} h[k] \cdot z^{-k}$$

$\underline{h[k] = 0 \text{ for } k<0}$. "Causal"

$$= \sum_{k=0}^{\infty} h[k] z^{-k}.$$

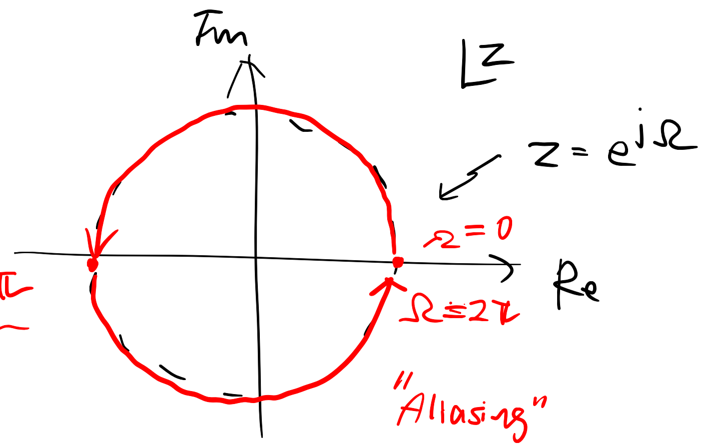
- DT Freq Resp.

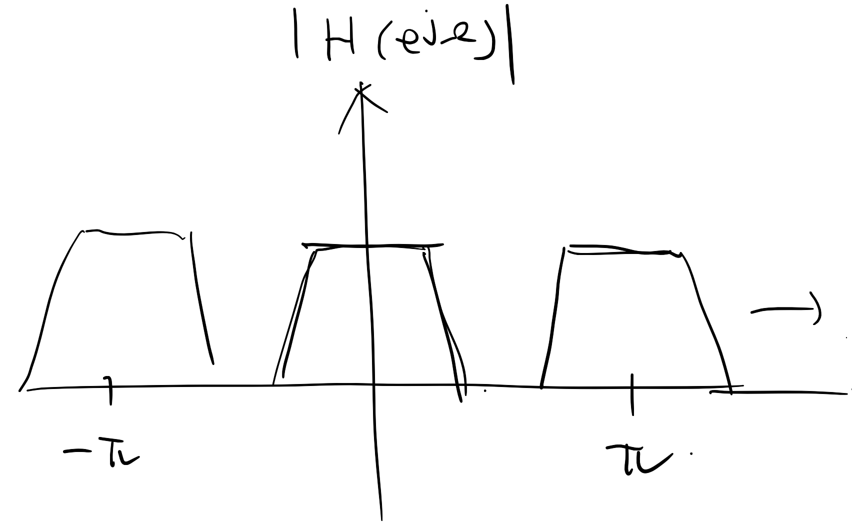
$$H(e^{j\Omega}) = \mathcal{F}\{h[k]\} = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\Omega k} = H(z)\Big|_{z=e^{j\Omega}}$$

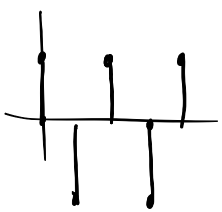
DT Fourier trans.

$H(z)$ "

$H(e^{j\Omega})$





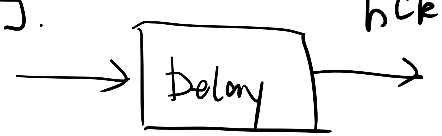


DTFT is periodic.

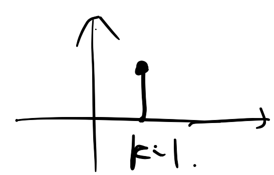
$$H(e^{j\Omega}) = H(e^{j(\Omega+2\pi)}).$$

- Unit delay

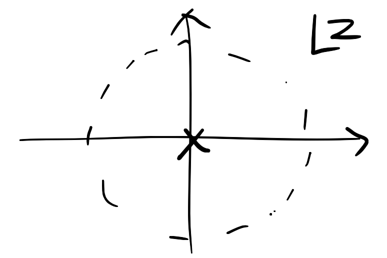
$\delta[k]$.



$h[k] = \delta[k-1]$.

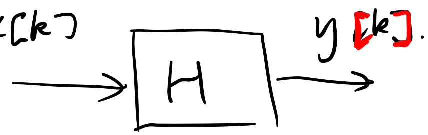


$$H(z) = \sum_{k=-\infty}^{\infty} \delta[k-1]\, z^{-k} = z^{-1} = \frac{1}{z}.$$



$$H(e^{j\Omega}) = e^{-j\Omega} \quad \longleftrightarrow \quad e^{-j\omega T}$$

"CT delay"

Example. 

$$y[k] = x[k-1] - \frac{1}{2}\, y[k-1].$$

"Difference equation"

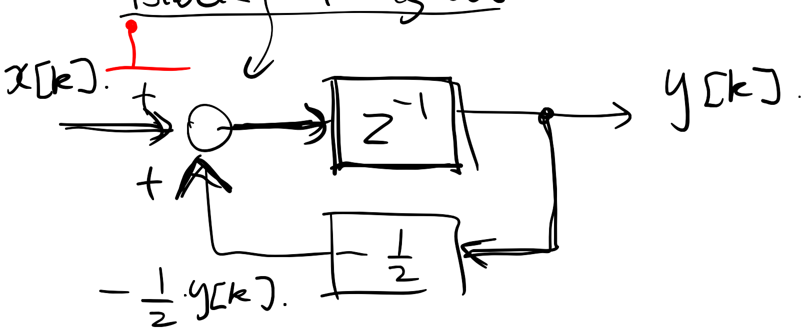
$$\mathcal{Z}\{\cdot\} \Rightarrow \quad Y(z) = z^{-1} X(z) - \frac{1}{2} z^{-1} Y(z)$$

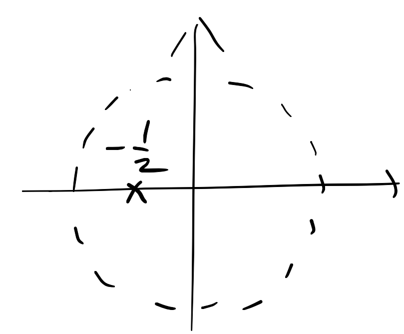
$$\left(1 + \frac{1}{2} z^{-1}\right) Y(z) = z^{-1} X(z).$$

$$\therefore \frac{Y(z)}{X(z)} = \boxed{\frac{z^{-1}}{1 + \frac{1}{2} z^{-1}}} = \boxed{\frac{1}{z + \frac{1}{2}}}.$$

$x[k] - \frac{1}{2} y[k].$

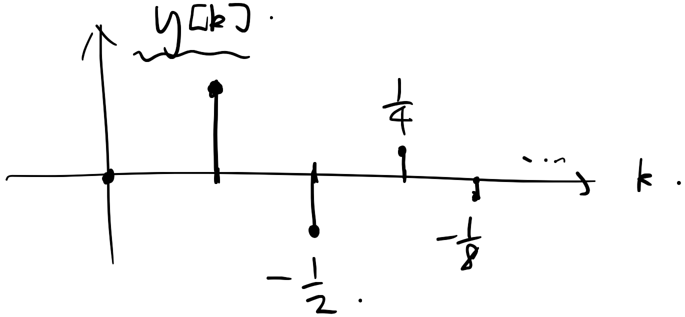
Block Diagram





Impulse Response

$x[k] = \delta[k]$.



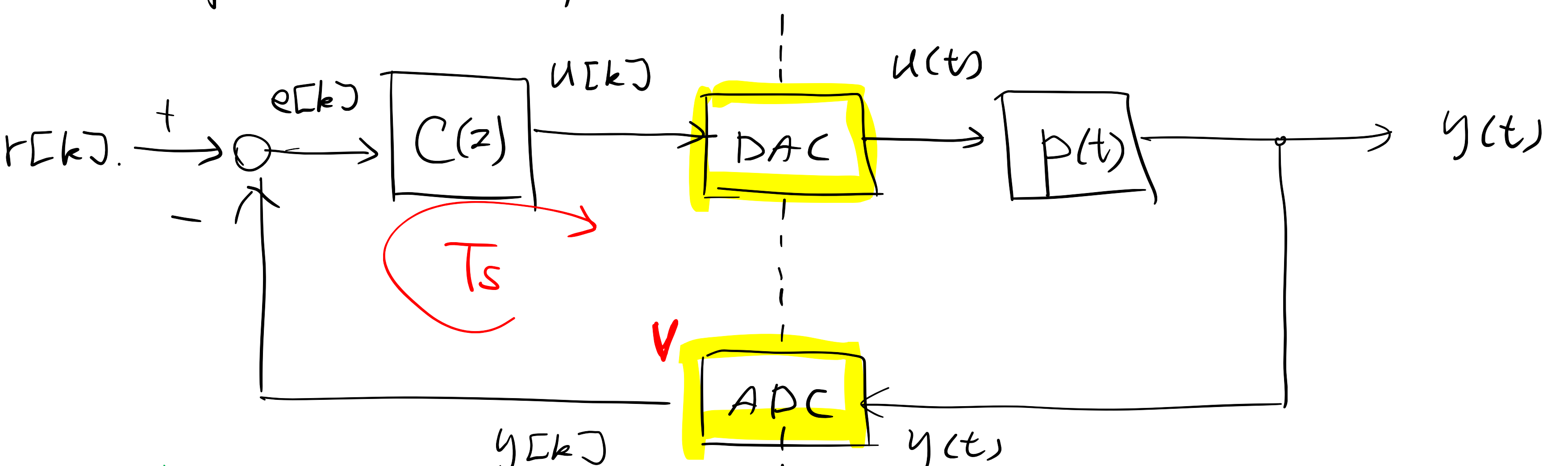
Freq Resp

$$H(z) = \frac{1}{z+\frac{1}{2}}.$$

$$H(z)\Big|_{z=e^{j\Omega}} = \frac{1}{e^{j\Omega}+\frac{1}{2}}.$$

$$|H(e^{j\Omega})| \Rightarrow \frac{1}{\sqrt{\frac{5}{4}+\cos\Omega}}.$$

∘ Sampled-data System. (DT control of CT systems)



· DT signals
· DT syst (difference eqn)

· CT signals
· CT syst (differential eqn)

- Analog to digital conv (ADC).

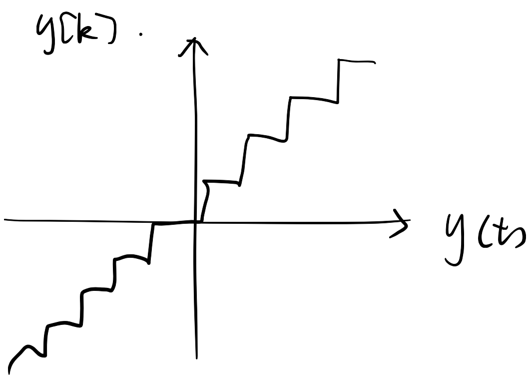
"Ideal" $y[k] = y(kT_s)$ "Instantaneous sample"

- Sample rate [Hz = S/s] : $\frac{1}{T_s}$.

{ Simultaneous Sampli "∨"
{ Multiplexed ∨

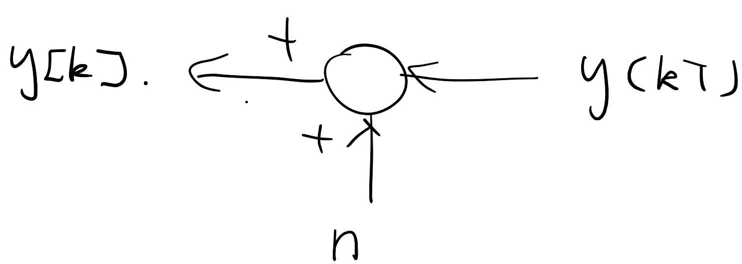
- Resolution.

Actual ADC. finite resolution.



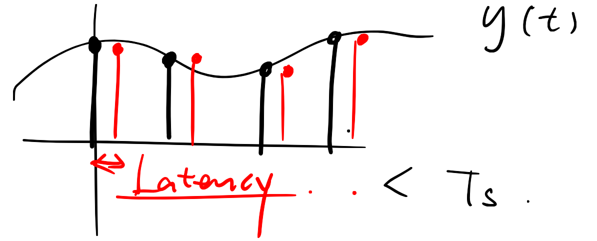
"quantization".

$\left(\text{DT signal} + \text{Quantization} \right)$ = Digital signal.



- Latency.

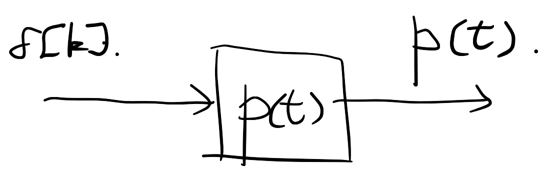
  - $y[k] = y(kT)$.



  - Sometimes Latency > Ts. (delta–sigma ADC).
  - Latency ≠ Ts.

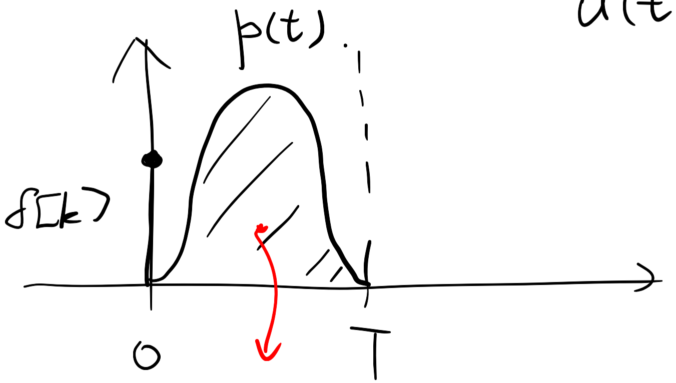
ADC. { Successive – approx resister (SAR) low latency.
Delta –sigma. High Res.

- Digital to Analog. (DAC). ZOH ADC

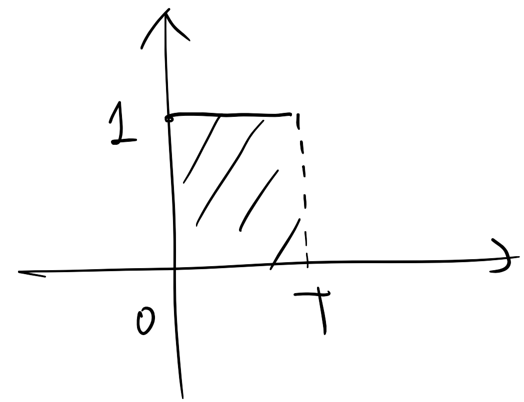


two types { ① pulse Amplitude (PAM).
② pulse width (PWM)

① PAM.

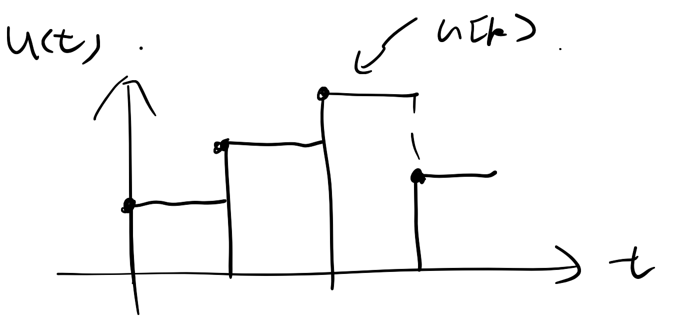


$$u(t) = \sum_{k=-\infty}^{\infty} u[k]\, p(t-kT).$$



$$\int_{-\infty}^{\infty} p(t) = T.$$

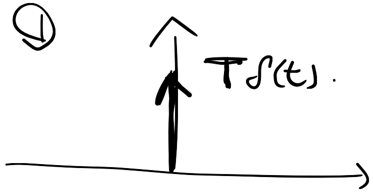
→ Zero-order hold. ADC



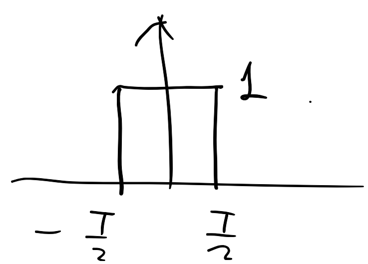
Ideal LPF

· Other types of PAM.

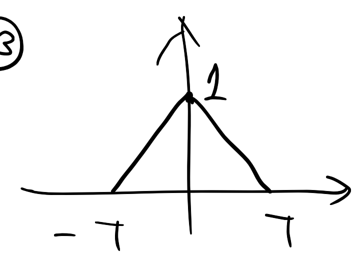
①



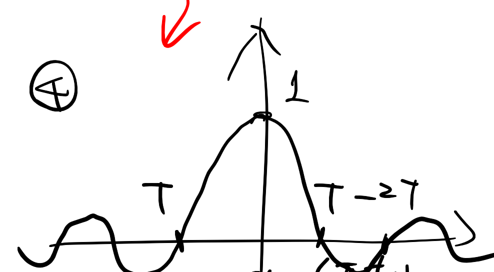
②



③



④



$$p(t) = \frac{\sin(\pi t/T)}{\pi t/T}.$$