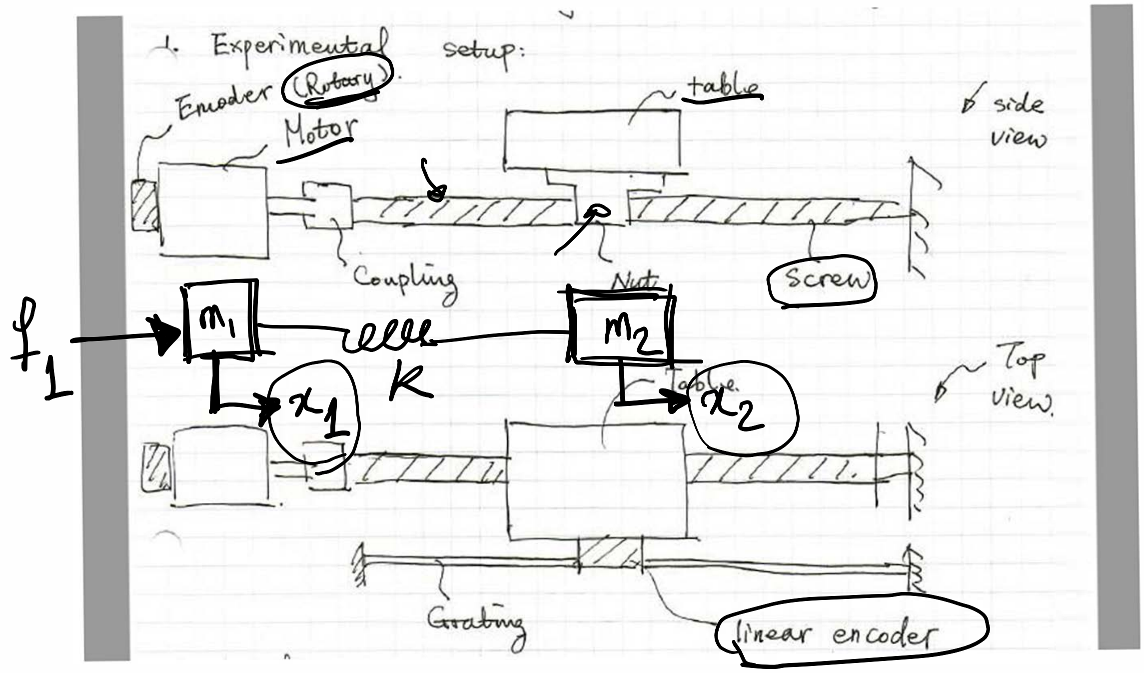
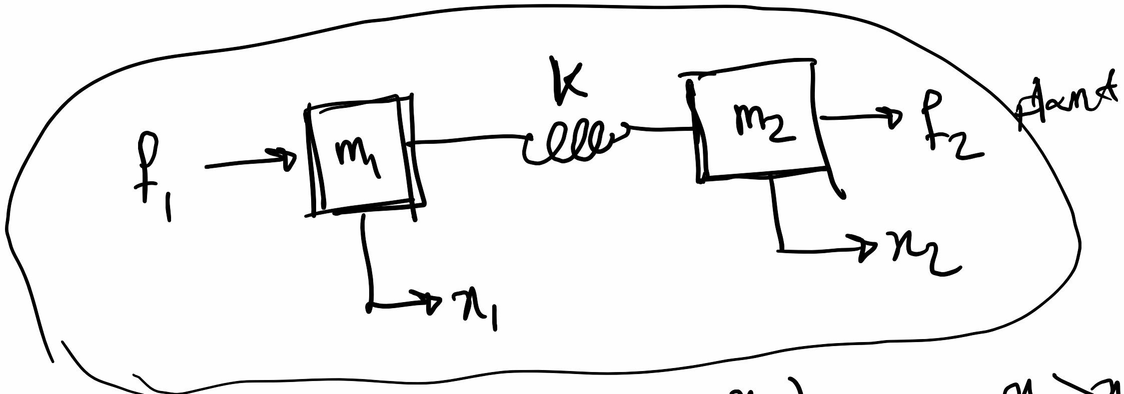
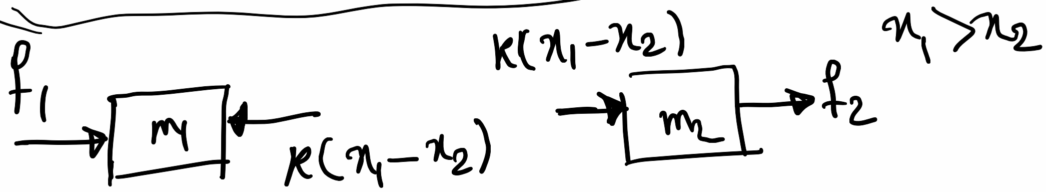






$$\sum \vec{F} = m_1\ddot{x}_1$$

$$f_2 + k(x_1 - x_2) = m_2\ddot{x}_2$$

$$f_1 - k(x_1 - x_2) = m_1\ddot{x}_1$$

$$\begin{cases} m_1\ddot{x}_1 + k(x_1 - x_2) = f_1 \\ m_2\ddot{x}_2 - k(x_1 - x_2) = f_2 \end{cases} \Rightarrow \underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{M} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{K} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Lagrangian method

nDOF system

Lagrangian's equations (non-conservative system)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} - \frac{\partial \mathcal{F}}{\partial \dot{x}_i} = Q_{i,nv} \quad \boxed{i = 1, 2, \dots, n}$$

power dissipation

generalized force due to nonviscous forces

$$L = T - V$$

kinetic, potential

Rayleigh's dissipation function

$$\mathcal{F} = -\frac{1}{2}P = -\frac{1}{2}c\dot{x}^2$$

damper

power dissipated

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$$

$$V = \frac{1}{2}k(x_2 - x_1)^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = Q_1 = f_1$$

$$\frac{\partial L}{\partial \dot{x}_1} = \frac{\partial}{\partial \dot{x}_1}\left(\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 - \frac{1}{2}k(x_2 - x_1)^2\right)$$

$$= m_1\dot{x}_1$$

$$\frac{\partial L}{\partial x_1} = k(x_2 - x_1)$$

$$\Rightarrow \frac{d}{dt}\left(m_1\dot{x}_1\right) - k(x_2 - x_1) = f_1$$

$$\boxed{m_1\ddot{x}_1 - k(x_2 - x_1) = f_1}$$

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{M} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{K} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\mathcal{L}\left\{ \frac{dx(t)}{dt} \right\} = sX(s) - \cancel{x(0)}^{\nearrow 0} \qquad t=0$$

$$\mathcal{L}\left\{ \frac{d^2x(t)}{dt} \right\} = s^2X(s) - s\cancel{x(0)}^{\nearrow 0} - \cancel{\dot{x}(0)}^{\nearrow 0}$$

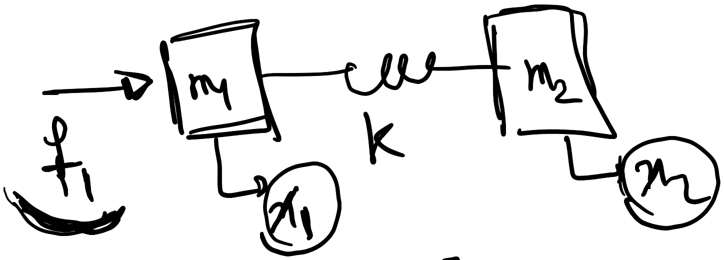
$$M s^2 \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + K \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix}$$

$$\underbrace{(M s^2 + K)} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix}$$

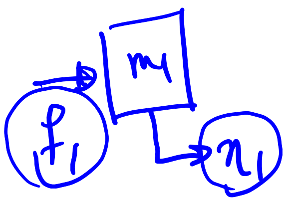
$X_1 \to F_1$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = (M s^2 + K)^{-1} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

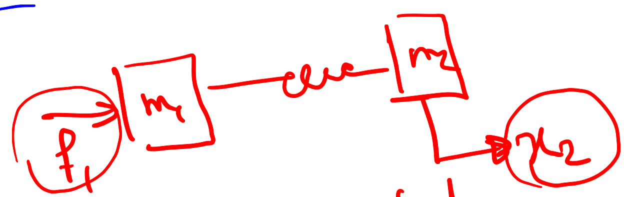
$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \underbrace{\frac{1}{m_1 m_2 s^4 + (m_1 + m_2) k s^2} \begin{bmatrix} m_2 s^2 + k & k \\ k & m s^2 + k \end{bmatrix}} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$



$$H_1(s) = \frac{X_1(s)}{F_1(s)} = \frac{m_2 s^2 + k}{m_1 m_2 s^4 + (m_1+m_2) k s^2}, \quad H_2 = \frac{X_2(s)}{F_1(s)} = \frac{k}{m_1 m_2 s^4 + (m_1+m_2) k s^2}$$



collocated



non-collocated

poles $\quad s^2\left(m_1 m_2 s^2 + (m_1+m_2)k\right) = 0$

poles $\quad$ same poles

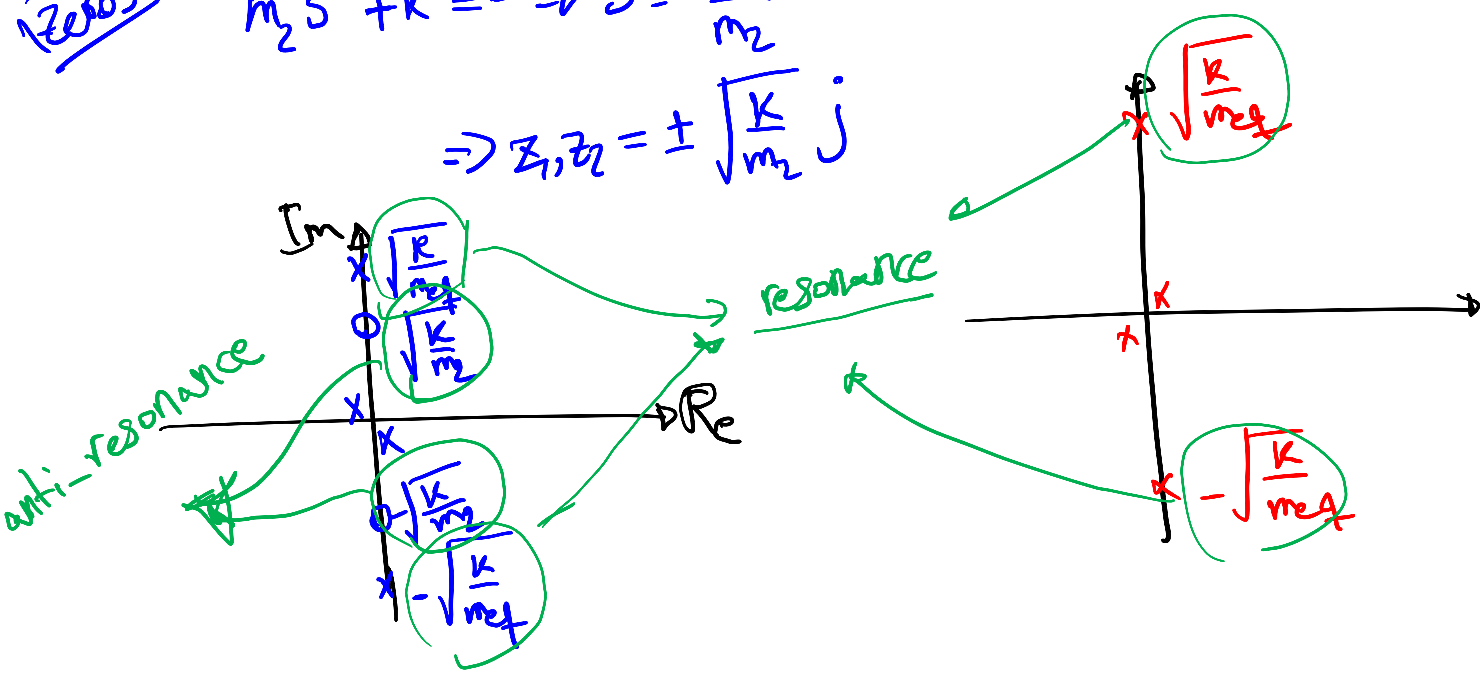
$P_{1,2} = 0$

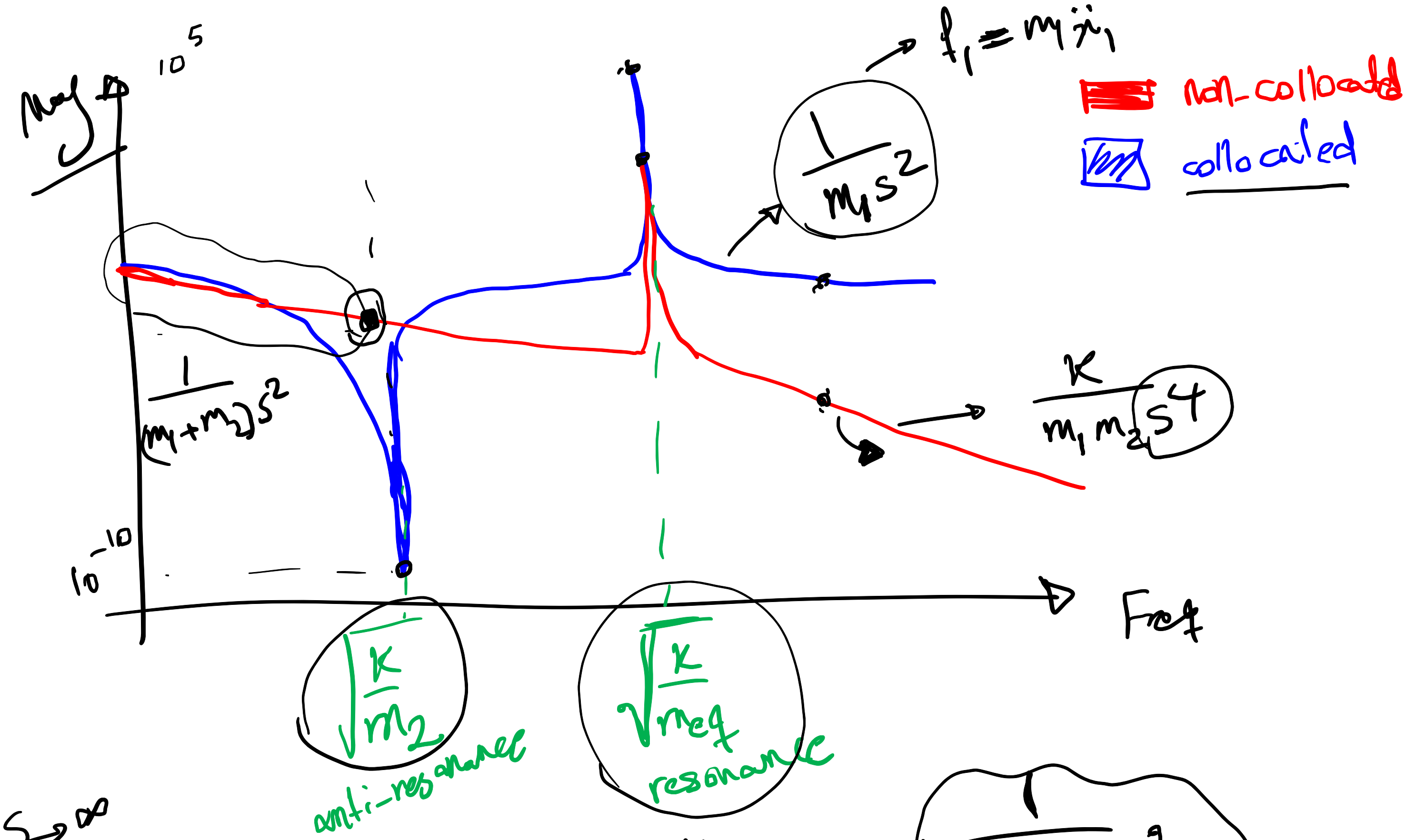
$$s^2 = -\frac{k}{\frac{m_1 m_2}{m_1+m_2}} \Rightarrow P_{3,4} = \pm\sqrt{\frac{k}{m_{eq}}}\, j$$

zeros $\quad$ no zeros

zeros $\quad m_2 s^2 + k = 0 \Rightarrow s^2 = -\frac{k}{m_2}$

$$\Rightarrow z_1, z_2 = \pm\sqrt{\frac{k}{m_2}}\, j$$





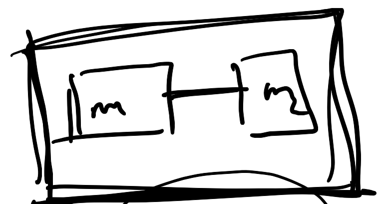
$s \to \infty$

$s \to 0$

$$H_{11}(s) = \frac{m_2 s^2 + k}{s^2(m_1 m_2 s^2 + (m_1+m_2)k)} \simeq \frac{K}{(m_1+m_2)K s^2} \simeq \frac{1}{(m_1+m_2)s^2}$$

$s \to 0$

$$H_{21}(s) \simeq \frac{1}{(m_1+m_2)s^2}$$



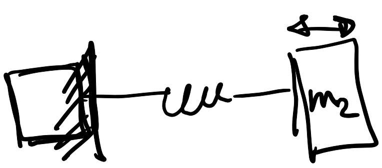
$M = m_1 + m_2$

$$\frac{1}{M s^2} = \frac{X_1}{F_1}$$

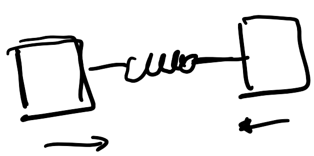
$$M s^2 X_1 = F_1$$

$$f_1 = M \ddot{x}_1$$

at anti-resonance



at resonance

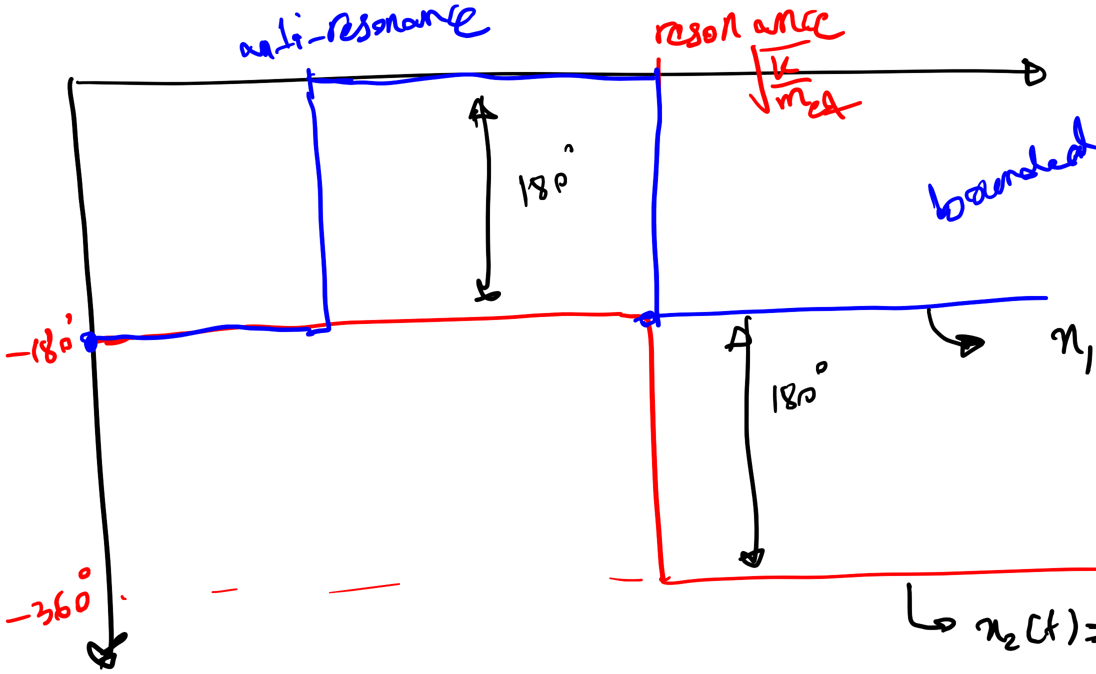


180°

after resonance

$\boxed{m_1}$ move freely disconnected from m_2

$\boxed{m_2}$



$$x_1(t) = A\sin(\omega t)$$

$$x_2(t) = A_2\sin(\omega t - 180^\circ)$$

$$= -A_2\sin(\omega t)$$