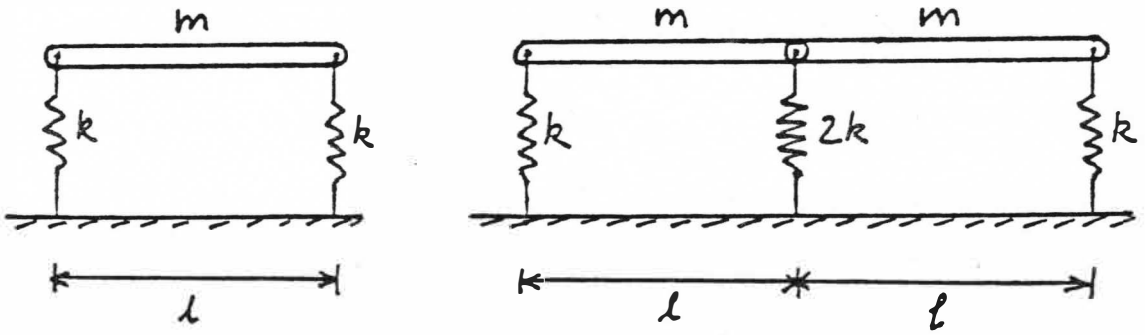
# MECH463 -- Homework 4

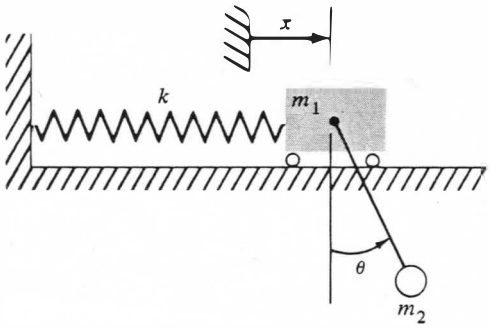
1. (a) A uniform rod of total mass m and length $\ell$ is supported at each end by a spring of stiffness k. By inspection, draw the two mode shapes and determine the natural frequencies for small oscillations.

   (b) Two uniform rods, each of mass m and length $\ell$ are pinned together and are supported at their free ends by springs of stiffness k, and at the pin joint by a spring of stiffness 2k. By inspection, draw the three mode shapes and determine the natural frequencies for small oscillations.

   Ans. $\omega^2$ = 2k/m, 3k/m, 6k/m




2. A part of a machine can be idealized as a mass $m_1$ which is free to slide along a horizontal surface. It is attached horizontally through a spring of stiffness k. A pendulum component of mass $m_2$ and length L is attached to the first mass. Using the coordinate system shown, formulate the matrix equation of motion, and make it symmetrical if necessary. For the case $m_1 = m_2 = m$, $k = mg/L$, solve each of the matrix equations for the natural frequencies and mode shapes. Confirm that the results are equivalent.




   Ans. $\omega^2$ = ½(3 ± √5) k/m

3. Formulate the equations of motion and the characteristic equation for the double pendulum shown in the diagram, with $m_1 = m_2 = m$ and $L_1 = L_2 = L$. Assume small displacements. Determine the natural frequencies and mode shapes.

   Ans. $\omega^2 = (2 \mp \sqrt{2})\, g/L$  $u = \pm \sqrt{2}$