MECH 420 SENSORS AND ACTUATORS
Assignment 4

Problems 4.2, 4.3, 4.6, and 4.14 from the textbook

Problem 1 (Problem 4.2 from Textbook)

The ideal calibration curve of a sensor is given by $y = ax^p$, where, x = measured quantity (measurand), y = measurement (sensor reading), and a and p are calibration (model) parameters

Note: In practice, x has to be determined for a measurement y, according to $(y/a)^{1/p}$.

Suppose that in a calibration process, with a set of known measurand values, the corresponding measurements are collected. Model the calibration experiment by $y = (a+v)x^p$ where v represents model error.

(a) Generate 25 points of calibration data (X_i, Y_i), $i = 1, 2, \ldots, n$ by using $a = 1.5$, $p = 2$, $v = N(0.1, 0.2^2)$ (i.e., random with Gaussian distribution of mean 0.1 and std 0.2), and n =25, with $X_1 = e$ (≈ 2.718282) and x-increments of 0.5.

(b) Estimate the parameters a and p using linear least squares error estimation (LSE) in log scale

(c) Comment on the estimation results.

Problem 2 (Problem 4.3 from Textbook)

Consider a random signal Y whose mean is μ and the variance is σ^2. The signal is measured and N data values Y_i, $i = 1, 2, \ldots, N$ are collected, independently of one another. The sample mean and sample variance are computed using this data sample according to:

Sample mean: $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$; Sample variance: $S^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$

(a) Show that these two quantities are unbiased estimates of the mean and the variance of the signal

(b) Particularly comment on this estimate for variance.

Problem 3 (Problem 4.6 from Textbook)

This problem concerns estimation of the damping parameters of a shock absorber using experimental data. In the experimental setup, one end of the shock absorber is firmly mounted on a load cell. At the other end, a velocity input is applied using a shaker (a linear actuator). The experimental setup is shown in Figure P4.6.

The velocity v that is applied by the shaker (m/s) and the resulting force f at the load cell (N) are measured and 41 pairs of data are recorded. First obtain a simulated set of data using the following MATLAB script:

```
% Problem 4.6
t=[]; v=[]; f=[];% declare storage vectors
dt=0.05; % time increment
v0=0.15; om= 3.0; b1=2.2; b2=0.2; % parameter values
t(1)=0.0; v(1)=0.0; f(1)=0.0; % initial values
for i=2:41
t(i)=t(i-1)+dt; % time increment
v(i)=v0*sin(om*t(i))+normrnd(0,0.01);  % velocity measurement
f(i)=b1*v(i)+b2*v(i)^2+normrnd(0.01,0.02);  % force measurement
end
t=t'; % convert to column vector
v=v'; %convert x data to a column vector
f=f'; %convert y data to a column vector
plot(t,v,'-')
plot(t,f,'-')
plot(v,f,'x')
```

Figure P4.6: Experimental setup of a shock absorber.

(a) List possible error sources in estimating the damping parameters
(b) Using MATLAB, curve fit the data (least-squares fit) to the linear viscous damping model $f = b_1 v + b_0$ and estimate the damping parameters b_0 and b_1. Give some statistics for estimation error and "goodness of fit."
(c) Using MATLAB, curve fit the data (least-squares fit) to the quadratic damping model $f = b_0 + b_1 v + b_2 v^2$ and estimate the damping parameters b_0 ,b_1, and b_2. Give some statistics for estimation error and "goodness of fit."
(d) Compare the results from the two fits. In particular, is a linear fit adequate or do you recommend quadratic (or still higher order) fit for this data?

Note: Provide plots of the data and the results of curve fitting.

<u>Problem 4 (Problem 4.14 from Textbook)</u>

A digital tachometer measures speed by counting the clock pulses per revolution. For 25 revolutions of a disk, the following numbers of clock pulses were recorded:

y = [803 809 789 804 802 793 798 802 818 814 793 815 804 800 804 799 799 807 807 807 803 794 804 808 802]

1 clock pulse = 0.5 ms.

Recursively estimate and plot the estimated speed (rev/s) and the associated estimation error std using Recursive LSE with the following algorithm

$$\bar{Y}_1 = Y_1$$

$$\bar{Y}_{i+1} = \frac{1}{(i+1)}(i \times \bar{Y}_i + Y_{i+1}), \quad i = 1, 2, \ldots..$$

$$S_1^2 = 0$$

$$S_{i+1}^2 = \frac{1}{i}[S_i^2 \times (i-1) + \left(Y_{i+1} - \bar{Y}_{i+1}\right)^2]$$