Solutions key

Final Examination for MECH 463 Mechanical Vibrations

15th December 2012

Time: 2 hrs. 30 mts. Max. Available Mark: 60

READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Please write your **name and student number** on the answer sheets.
2. This exam consists of 4 pages including this page.
3. ANSWER ALL QUESTIONS.
4. Your mark in this exam must be **AT LEAST 30 OUT OF 60 to pass.**
5. The mark obtained on this exam will be scaled to 65% of total course mark.
6. One letter-sized formula sheet, written/typed on both sides, is allowed.
7. ONLY non-programmable calculators are allowed.
8. COLLECT the solutions KEY for a detailed breakdown of marks allocated.

This space is intentionally left blank. Continue onto the next page for the exam questions.

Question 1 **Concepts tested**: Kinematics, FBD, Forced Vibration, Equivalent Systems, Energy Methods

(a) An automobile moving on a rough road is shown in Figure.(1). The road is approximated by a sinusoid of amplitude $Y = 1$ mm and wave length $L = 5$ m. Taking $m = 1500$ kg for the mass and $k = 400$ kN/m for the suspension stiffness of the automobile, determine the horizontal speed v at which large vibrations are experienced by the passengers. Suggest possible design changes to improve ride comfort and any design limitations. **(12marks)**

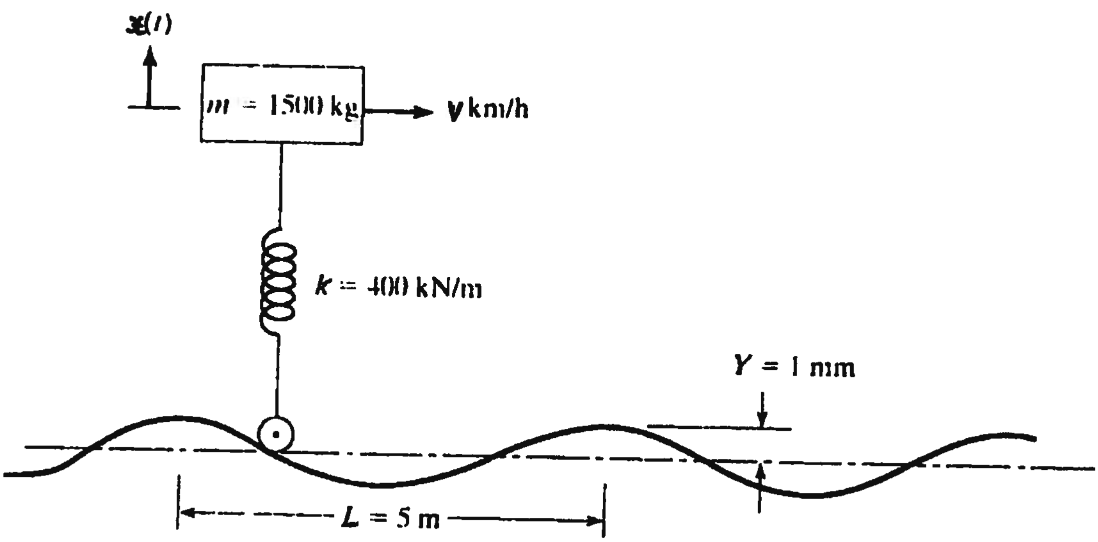


Figure 1: Figure for Question 1 part (a).

(b) Gears A and B mesh with a gear ratio $n = n_B/n_A$ and are fixed to circular shafts of equal length and diameter, each of torsional stiffness K N-m/rad. Taking the mass moments of inertia of gears as J_A and J_B kg-m^2, respectively, find the natural frequency of torsional vibrations. What kinematic constraint did you use? **(8 marks)**

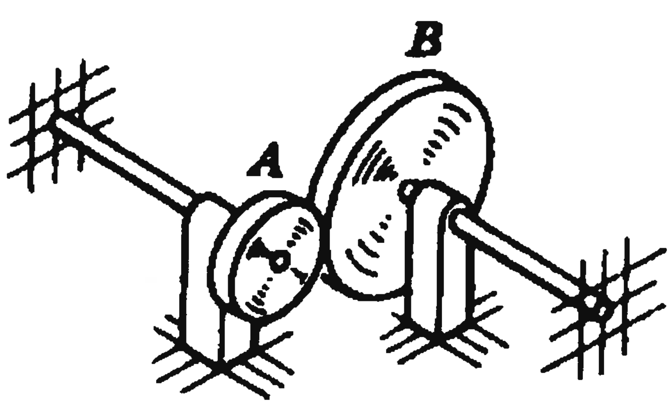
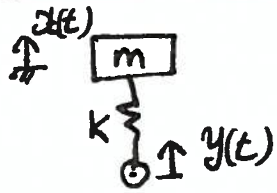
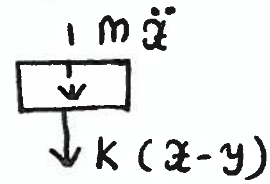


Figure 2: Figure for Question 1 part (b).

Q1) (a) MODEL



FBD



GRAVITY CANCELS STATIC SPRING FORCE

4 MARKS EQUATION OF MOTION: $\uparrow +ve \sum F_x = 0 \Rightarrow -m\ddot{x} - k(x-y) = 0$

$$\Rightarrow m\ddot{x} + kx = ky \quad \text{— (1)}$$

2 MARKS INPUT DISPLACEMENT FROM ROAD: $y(t) = ?$

USING THE DATA : $y(t) = Y \sin \frac{2\pi s}{L} = Y \sin \frac{2\pi vt}{L}$ — (2)

WHERE s = HORIZONTAL DISTANCE TRAVELLED = vt

(2) IN (1) $\Rightarrow$ $m\ddot{x} + kx = kY \sin \frac{2\pi vt}{L}$

UNDAMPED FORCED VIBRATION PROBLEM

$x = x_h + x_p$, IGNORING FREE VIBRATION PART

$$x = x_p = \frac{kY}{k - m\left(\frac{2\pi v}{L}\right)^2} \sin\left(\frac{2\pi vt}{L} - \phi\right)$$

4 MARKS x IS MAXIMUM WHEN $k - m\left(\frac{2\pi v}{L}\right)^2 = 0$ RESONANCE CONDITION

$$\Rightarrow k = \frac{4\pi^2 m v^2}{L^2} \Rightarrow v = \sqrt{\frac{kL^2}{m\pi^2 4}} = \frac{L}{2\pi}\sqrt{\frac{k}{m}}$$

USING $k = 400 \times 10^3$ N/m; $L = 5$ m; $m = 1500$ kg

$$v = \frac{5}{2\pi}\sqrt{\frac{400\times10^3}{1500}} = 12.99 \text{ m/s} = 12.99 \times \frac{60\times60}{1000} \text{ km/hr}$$

$$v = 46.78 \text{ km/hr}$$

1 MARK DESIGN MODIFICATIONS: (1) CHANGE 'k' OF SUSPENSION: $k\uparrow$ $v\uparrow$

$k\downarrow$ $v\downarrow$

(2) ADD DAMPING

(3) CHANGE 'm' OF CAR : $m\uparrow$ $v\downarrow$

$m\downarrow$ $v\uparrow$

1 MARK LIMITATIONS: (1) TOO HIGH 'k' GIVES ROUGH RIDE (SPORTY)

TOO LOW 'k' GIVES LARGE OSCILLATIONS

(2) INCREASING 'm' IS NOT FUEL EFFICIENT.

Q1 (b) ENERGY METHOD IS EFFICIENT HERE.

LET θ_A AND θ_B DENOTE ANGULAR DISPALACEMENTS OF GEARS A & B, RESPECTIVELY.

2 MARKS KINETIC ENERGY = $K.E. = \frac{1}{2} J_A \dot{\theta}_A^2 + \frac{1}{2} J_B \dot{\theta}_B^2$

2 MARKS POTENTIAL ENERGY = $P.E = \frac{1}{2} k \theta_A^2 + \frac{1}{2} k \theta_B^2$ — ①

CHOOSE θ_B AS REFERENCE CO-ORDINATE TO OBTAIN EQUIVALENT INERTIA AND STIFFNESS.

2 MARKS USING NO SLIP CONDITION $n_A \dot{\theta}_A = n_B \dot{\theta}_B$

$$\Rightarrow \dot{\theta}_A = \frac{n_B}{n_A} \dot{\theta}_B = n \dot{\theta}_B$$

IIIly $\theta_A = n\theta_B$ — ②

② IN ① GIVES

$$K.E. = \frac{1}{2} J_{eq} \dot{\theta}_B^2 = \frac{1}{2} J_A (n \dot{\theta}_B)^2 + \frac{1}{2} J_B \dot{\theta}_B^2$$

$$\Rightarrow J_{eq} = J_A n^2 + J_B$$

2 MARKS

$$P.E. = \frac{1}{2} K_{eq} \theta_B^2 = \frac{1}{2} k \theta_A^2 + \frac{1}{2} k \theta_B^2 = \frac{1}{2} k n^2 \theta_B^2 + \frac{1}{2} k \theta_B^2$$

$$\Rightarrow K_{eq} = k(n^2+1)$$

$\therefore$ NATURAL FREQUENCY = $\omega_n = \sqrt{\frac{K_{eq}}{J_{eq}}} = \sqrt{\frac{k(1+n^2)}{J_A n^2 + J_B}}$ rad/s

KINEMATIC CONSTRAINT USED TO RELATE MOTIONS OF GEARS A AND B IS NO-SLIP AT THE MESHING INTERFACE.

Question 2 **Concepts tested**: FBDs, Shaky Table Lab

An electric motor, front view shown in Figure.(3), has a mass of $m = 20$ kg and is set on four identical springs, situated at four corners, each with a spring of modulus 1.6 N/mm. The radius of gyration of the motor assembly is $r = 100$ mm about the shaft axis. Note that mass moment of inertia is given by $J = mr^2$. The running speed of motor is 3000 rpm. The spacing between springs is 250 mm in the front view and the plan.

(a) Using appropriate Free Body Diagrams (FBDs), determine the natural frequencies for the vertical (up-down) vibrations and torsional vibrations (tilting) about the shaft axis: passing through the centre point of Figure.(3). **State your assumptions and clearly label the FBDs indicating appropriate co-ordinate(s).** **(14 marks)**

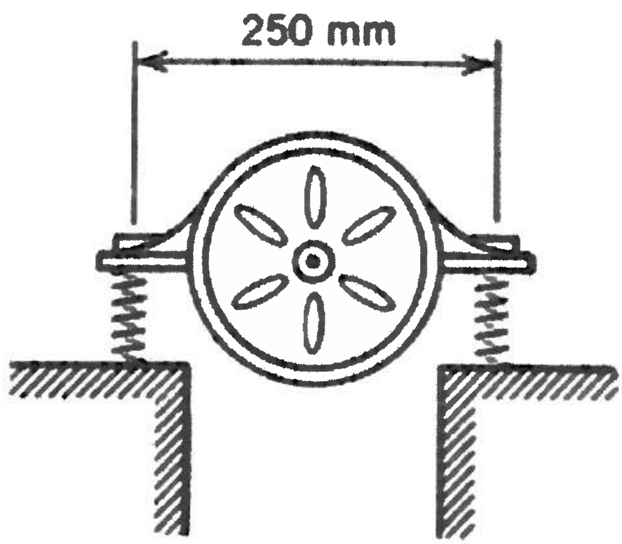


Figure 3: Figure for part Question 2, front view of the motor resting on four identical springs.

(b) Where will you place an additional mass in order to decrease the natural frequencies of the vertical and torsional vibrations? Where will you place an additional stiffness if the design requirement is to increase the natural frequencies for both vertical and torsional vibrations? **(3 marks)**

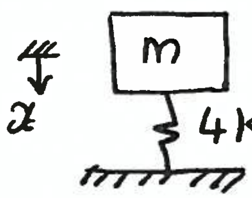
(c) Where will you place an additional mass in order to decrease the natural frequency of the vertical vibration **without changing the torsional natural frequency**? **(3 marks)**

Q2 (a) VERTICAL (UP-DOWN) MOTION:

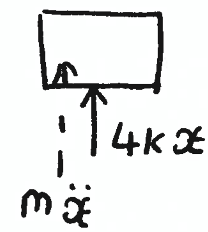
ALL SPRINGS EXPERIENCE SAME VERTICAL DISPLACEMENT AND HENCE ARE IN PARALLEL.

4 MARKS

MODEL:



FBD:



EQUATION OF MOTION: $\downarrow \sum_{+ve} F_x = 0 \Rightarrow -m\ddot{x} - 4kx = 0$

$$\Rightarrow m\ddot{x} + 4kx = 0$$

3 MARKS NATURAL FREQUENCY $= \omega_1 = \sqrt{\frac{4K}{m}} = \sqrt{\frac{4 \times 1.6 \times 10^3}{20}} = \underline{\underline{17.89 \text{ rad/s}}}$

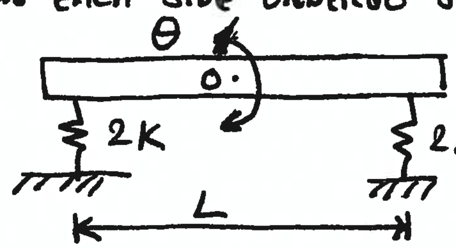
Note: $K = 1.6$ N/mm $= 1.6 \times 10^3$ N/m ; $m = 20$ kg

TORSIONAL (TILTING) MOTION:

SPRINGS ON EACH SIDE UNDERGO SAME VERTICAL DISPLACEMENT.

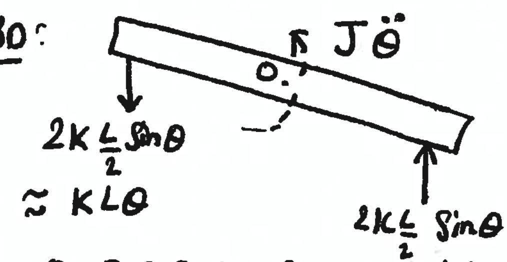
MODEL:

3 MARKS



Θ IS +VE CLOCKWISE

FBD:



$\sin\theta \approx \theta$ FOR SMALL θ

EQUATION OF MOTION: $\curvearrowright \sum_{+ve} M_O = 0 \Rightarrow -J\ddot{\theta} - KL\theta \times \frac{L}{2} - KL\theta \times \frac{L}{2} = 0$

2 MARKS $\Rightarrow J\ddot{\theta} + 2\frac{KL^2}{2}\theta = 0 \Rightarrow J\ddot{\theta} + KL^2\theta = 0$

NATURAL FREQUENCY $= \omega_2 = \sqrt{\frac{KL^2}{J}}$

$K = 1.6 \times 10^3$ N/m ; $L = 250$ mm $= 0.25$ m ; $J = mr^2 = 20 \times (100 \times 10^{-3})^2$

2 MARKS

$$\therefore \omega_2 = \sqrt{\frac{1.6 \times 10^3 \times (0.25)^2}{20 \times (100 \times 10^{-3})^2}} = \underline{\underline{22.36 \text{ rad/s}}}$$

WE ASSUMED THAT GRAVITY IS NOT IMPORTANT AND C.G. OF THE MOTOR IS IN THE MIDPLANE. OTHERWISE, 4 EQUAL SPRINGS CAN'T MAINTAIN THE MOTOR IN A LEVELLED CONFIGURATION. BOTH VERTICAL & TORSIONAL MOTIONS WILL BE COUPLED.

Q2 (b) FOR MAXIMUM EFFECTIVENESS TO REDUCE THE NATURAL FREQUENCIES OF VERTICAL AND TORSIONAL OSCILLATIONS PLACE FOUR IDENTICAL MASSES AT THE FOUR CORNERS. THIS SYMMETRIC ARRANGEMENT ENSURES THAT THE KINETIC ENERGY IS REDUCED WITH SMALLER MASSES.

3 MARKS

C.G. STILL REMAINS TO BE IN THE MID-PLANE. SAME HOLDS FOR STIFFNESS. ADDING STIFFNESS INCREASES NATURAL FREQUENCIES.

(c) HERE WE WANT THE MASS TO BE AT THE CENTRE, SO THAT IT LIES ON THE SHAFT AXIS. J REMAINS UNCHANGED SINCE MASS IS ADDED AT A POINT OF ZERO DISPLACEMENT IN TORSIONAL MODE. FOR THE VERTICAL MOTION, HOWEVER, KINETIC ENERGY IS REDUCED. HENCE, TORSIONAL FREQUENCY REMAINS UNCHANGED WHILE VERTICAL RESONANCE FREQUENCY IS REDUCED.

3 MARKS

NOTE THAT ANY UNSYMMETRICAL MODIFICATION OF STIFFNESS OR MASS WILL LEAD TO COUPLING BETWEEN VERTICAL AND TORSIONAL MODES.

Question 3 **Concepts tested**: General Excitation, Vibration Concepts

(a) A compressed air cylinder is connected to the spring-mass system in Figure.(4). Due to a small leak in the valve, the pressure on the piston $p(t)$, builds up as indicated. Find the **forced vibration response** of the piston for the following data: $m = 10$ kg, $k = 1000$ N/m,and $d = 0.1$ m. **For a force $f(t) = Fe^{at}$ you can guess a particular solution as $x_p(t) = Xe^{at}$.** **(12 marks)**

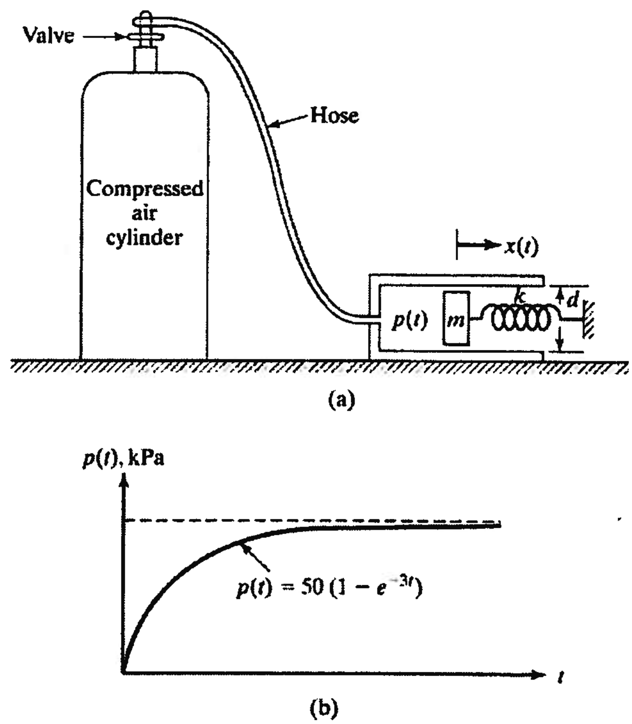


Figure 4: Figure for Question 3 part (a).

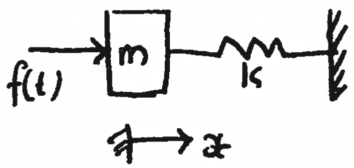
(b) What are the requirements for a dissipative force and hence explain why a spring force $f = kx$ is not dissipative? **(2marks)**

(c) What is the relation among the displacement, velocity and acceleration transmissibilities in the steady state vibrations? Will they change for transient vibrations? **(3marks)**

(d) What is the principle of superposition? How is it used in finding the forced vibration response of linear systems using Fourier series and Convolution integral? **(3marks)**

ALL THE VERY BEST IN YOUR FUTURE ENDEAVOURS!

Q3 (a) MODEL:



FBD:

2 MARKS

$f = p(t) \times$ AREA OF CROSS SECTION

EQUATION OF MOTION: $\underset{+ve}{\rightarrow} \sum F_x = 0 \Rightarrow -m\ddot{x} - kx + f = 0$

$$\Rightarrow m\ddot{x} + kx = f$$

$$m = 10 \text{ kg} \; ; \; k = 1000 \frac{N}{m} \; ; \; f = p \times \frac{\pi d^2}{4}$$

3 MARKS

$$f = 50\left(1 - e^{-3t}\right) \times 10^3 \frac{N}{m^2} \frac{\pi}{4} (0.1)^2 m^2$$

$$= \frac{50\pi}{4} (0.1)^2 \times 10^3 \times \left(1 - e^{-3t}\right) = 392.67 \left(1 - e^{-3t}\right) N$$

DENOTE $\quad f = F\left(1 - e^{at}\right)$ WITH $F = 392.67$ & $a = -3$

FORCED VIBRATION RESPONSE: $m\ddot{x} + kx = F - Fe^{at}$

3 MARKS

RESPONSE DUE TO Fe^{at} : PUT $x_1 = X_1 e^{at}$ IN (GUESS)

$$m\ddot{x}_1 + kx_1 = Fe^{at} \qquad a = -3$$

$$\Rightarrow \left[ma^2 + k\right] X_1 e^{at} = Fe^{at} \Rightarrow X_1 = \frac{F}{k + ma^2}$$

$$\therefore x_1 = \frac{F}{k + ma^2} e^{at}$$

2 MARKS

RESPONSE DUE TO F: $\quad F = Fe^{0t} \qquad a = 0$

$$\therefore x_2 = \frac{F}{k + m(0)^2} e^{0t} = \frac{F}{k}$$

1 MARK

RESPONSE DUE TO $F - Fe^{at}$: $\quad x_p = \frac{F}{k} - \frac{F}{k + ma^2} e^{at}$

USING PRINCIPLE OF SUPER POSITION.

USING THE DATA PROVIDED

$$x_p(t) = \frac{392.67}{1000} - \frac{392.67}{1000 + 10 \times (-3)^2} e^{-3t}$$

Note: $F = 392.67$
$a = -3$

1 MARK $x_p(t) = 0.3927 - 0.3602 e^{-3t}$ m

FREE VIBRATION RESPONSE $x_h = C_1 \cos \omega_n t + C_2 \sin \omega_n t$ $\omega_n = \sqrt{\frac{k}{m}} = 10$ rad/s IS IGNORED.

2 MARKS (b) A DISSIPATIVE FORCE SHOULD EXTRACT NET ENERGY OUT OF THE SYSTEM IN EACH COMPLETE CYCLE THROUGH WORK DONE BY THE SYSTEM IN MOVING AGAINST THE FORCE. SPRING FORCE EXTRACTS ZERO NET WORK / ENERGY AND HENCE IS NOT DISSIPATIVE.

(c) IN THE STEADY STATE: VELOCITY $= \omega \times$ DISPLACEMENT

ACCELERATION $= \omega^2 \times$ DISPLACEMENT

HENCE $[TR]_{ACCELERATION} = \frac{\omega^2 X}{\omega^2 Y} = \frac{X}{Y} = [TR]_{DISPLACEMENT}$

2 MARKS AND $[TR]_{VELOCITY} = \frac{\omega X}{\omega Y} = \frac{X}{Y} = [TR]_{DISPLACEMENT}$

$\therefore [TR]_{DISPLACEMENT} = [TR]_{VELOCITY} = [TR]_{ACCELERATION}$

1 MARK IN THE TRANSIENT VIBRATION THESE RELATIONS DO NOT HOLD. MOREOVER TR IS TIME DEPENDENT.

1 MARK (d) PRINCIPLE OF SUPERPOSITION IS USED IN FINDING THE RESPONSE OF A LINEAR SYSTEM SUBJECTED TO A SET OF FORCES BY ADDING THE RESPONSE OF THE SYSTEM SUBJECTED TO EACH FORCE IN THE SET ON ITS OWN.

1 MARK IN FOURIER SERIES THE FORCE IS BROKEN UP INTO A SUM OF HARMONIC FORCES. THE RESPONSE DUE TO EACH HARMONIC FORCE IS CALCULATED AND ADDED.

1 MARK IN CONVOLUTION INTEGRAL THE FORCE IS BROKEN INTO A SUM OF SHIFTED IMPULSES. THE RESPONSE DUE TO EACH IMPULSE IS FOUND AND ADDED.

— THE END —

HAPPY HOLIDAYS & NEW YEAR 2013! _SP-