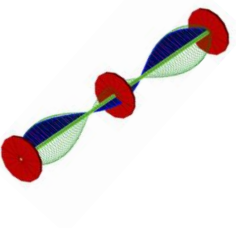

Dynamics of Rotating structures

Basic Rotordynamics

Izhak Bucher

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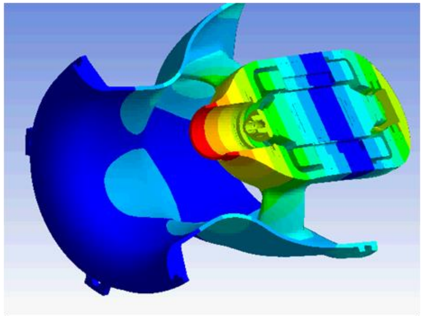
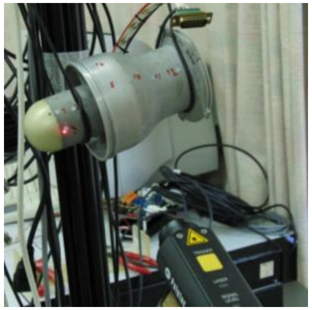
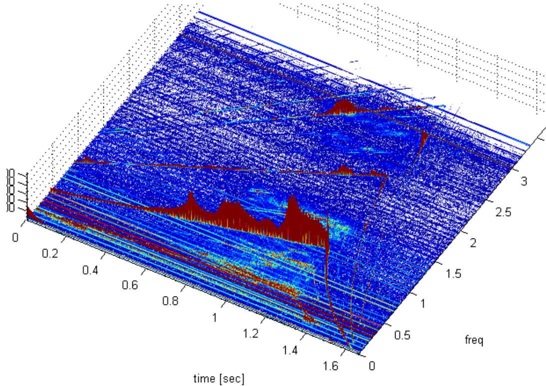
Haifa, Israel



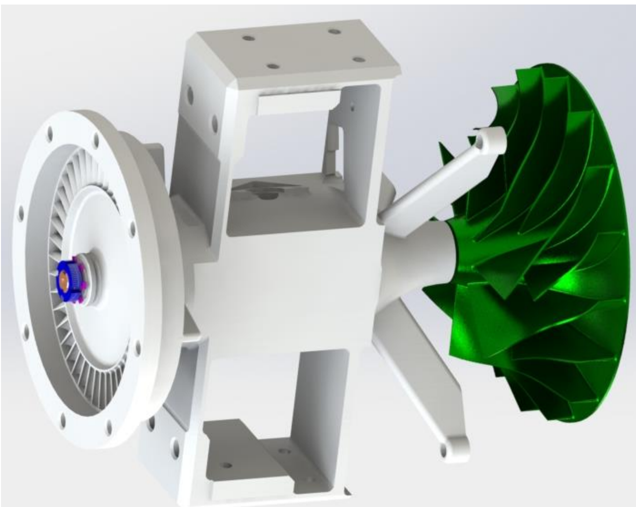


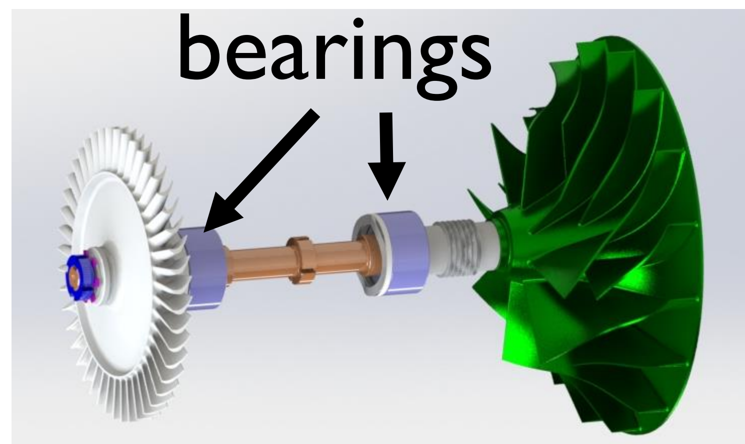


topics

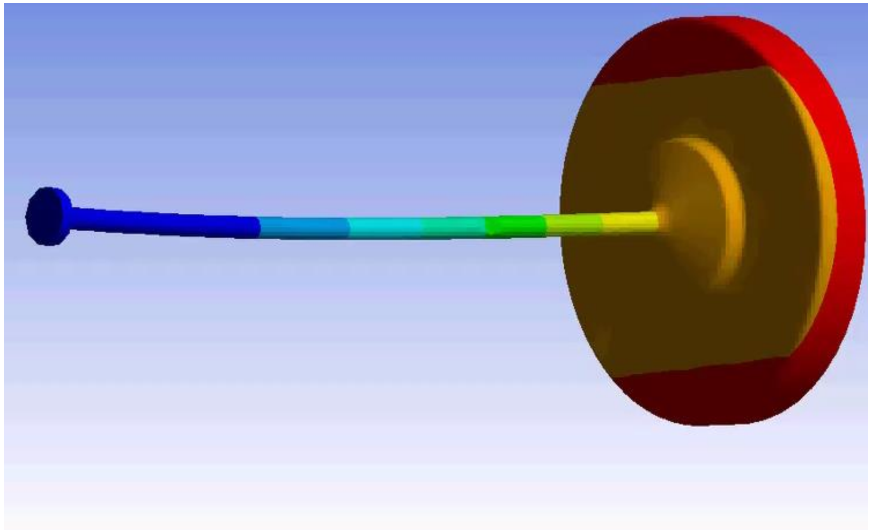
- Introduction & motivation
- Modeling rotating system dynamics
  - Jeffcott Rotor model (1 disk + shaft)
  - Whirling at constant speed
  - Self centering
  - Effect of damping & bearing forces
  - Anisotropic bearings and elliptical whirling

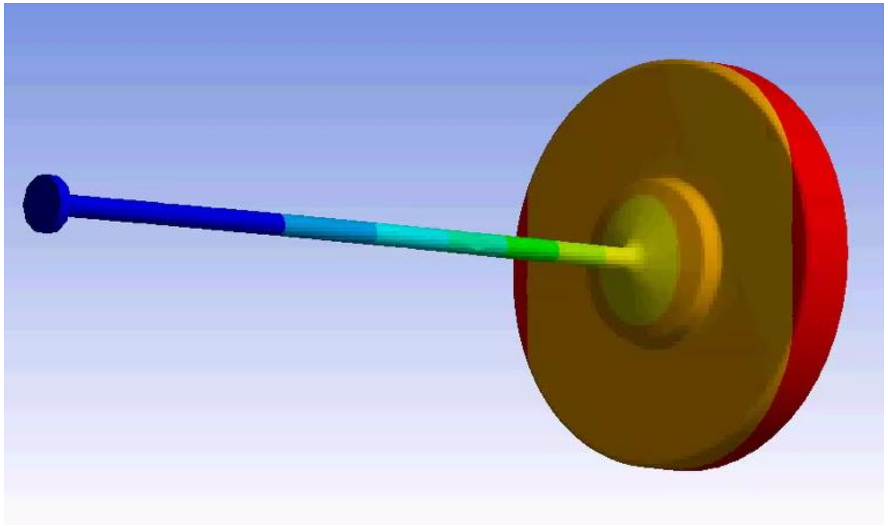
Real rotating machines





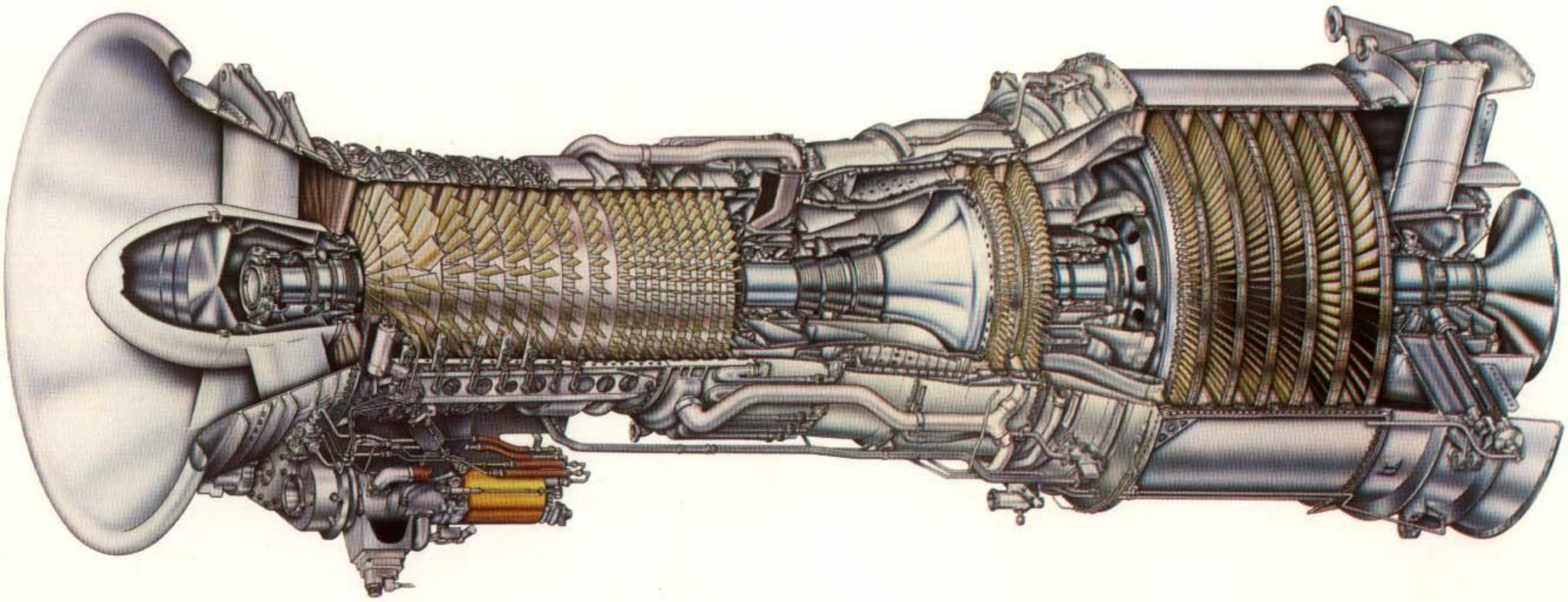
Vibrating vs. rotating





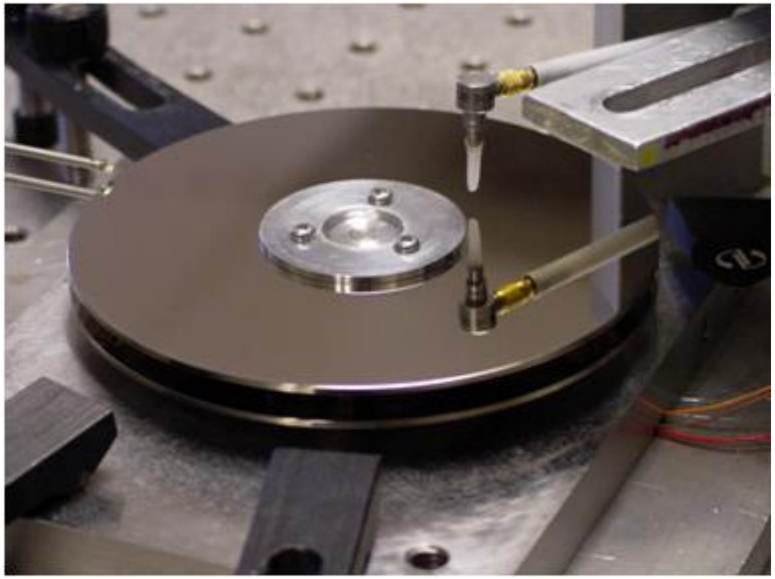
Typical machine with rotating elements

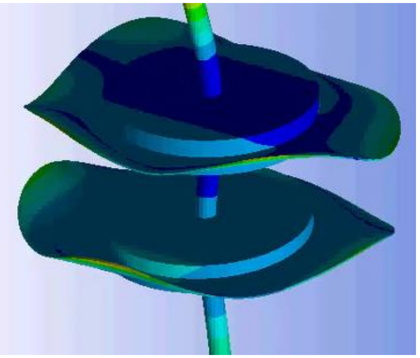
Gas turbine

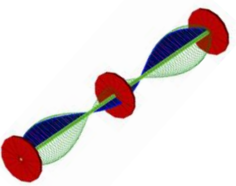


General Electric LM2500 Gas Turbine

Hard disk vibration



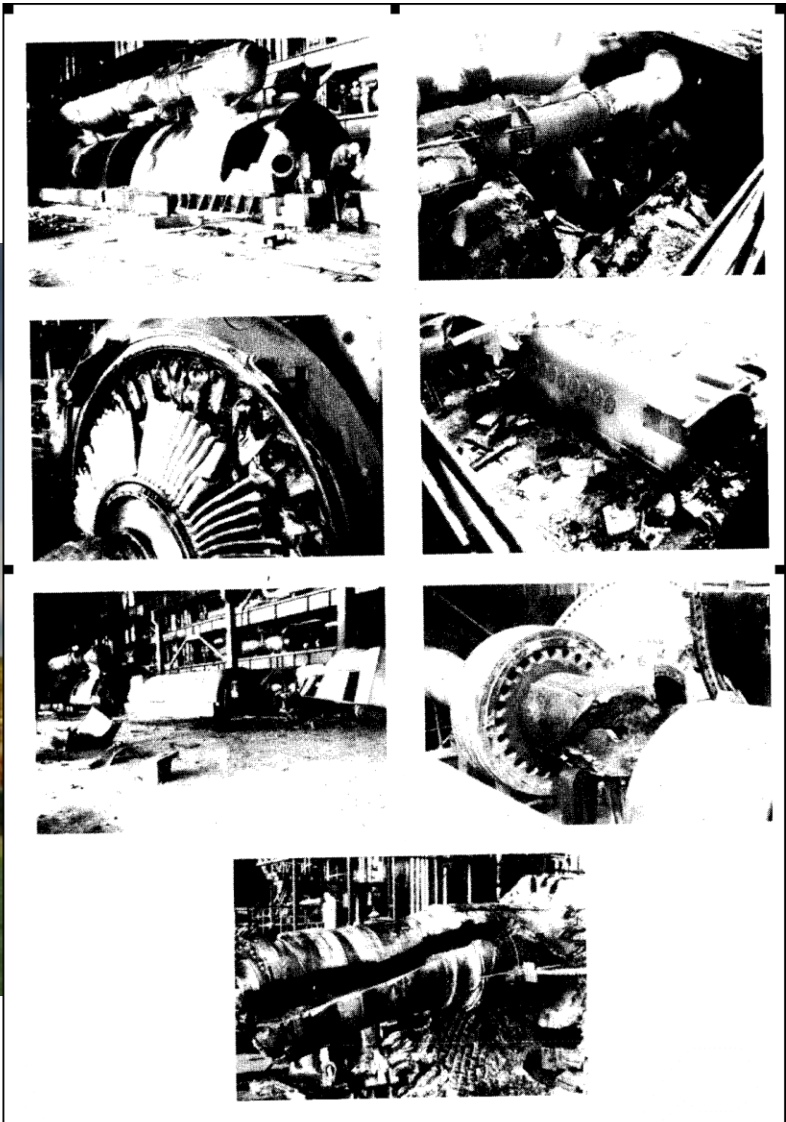


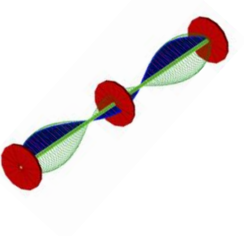


Why teach dynamics of rotating structures?

- Many machines contain rotating elements
- Rotating structures contain considerable energy
- Rotating machine operate at a range of speeds changing their behavior
- Theory of rotating machines >100 years old

Without words

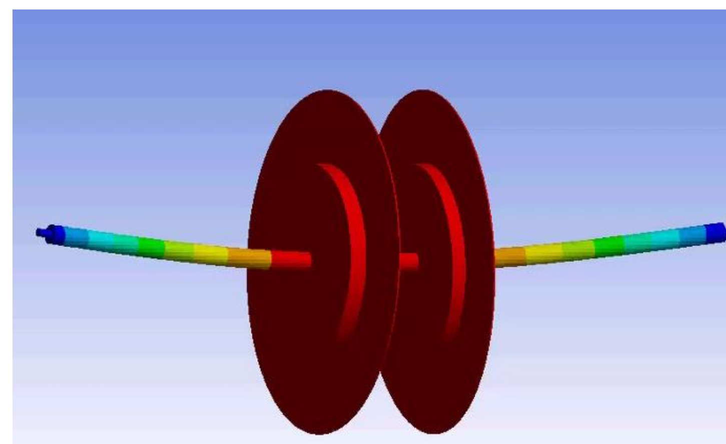




ROTATING MACHINE VIBRATION & SIMPLE MODELS

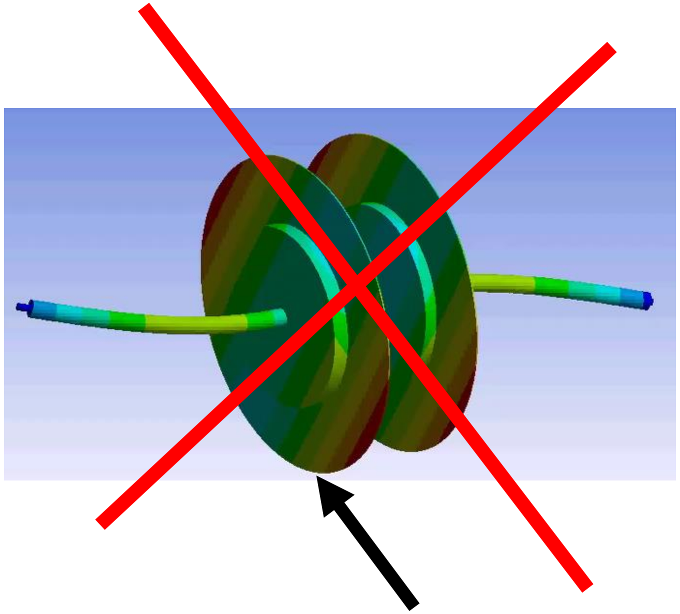
Basic assumptions

- Jeffcott rotor model
- No Gyroscopic effect
- Single mode dynamics / massless shaft
- No torsion (stiff)
- Only bending

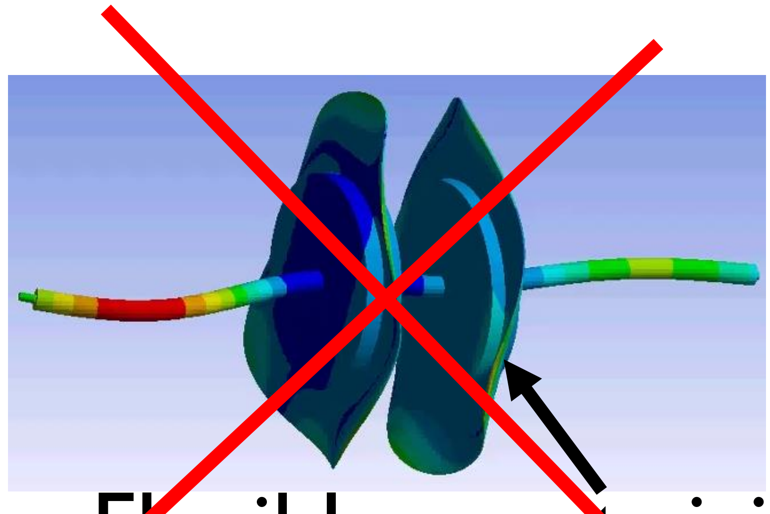


Basic model assumptions

- No consideration of higher frequencies
- No consideration of disk flexibility



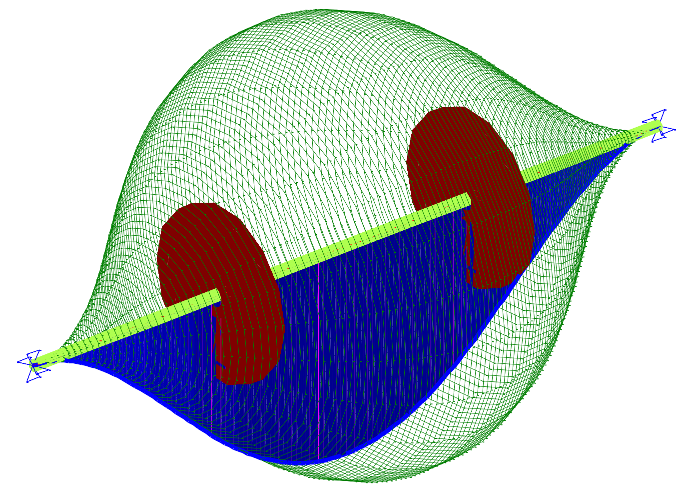
Gyroscopic effect

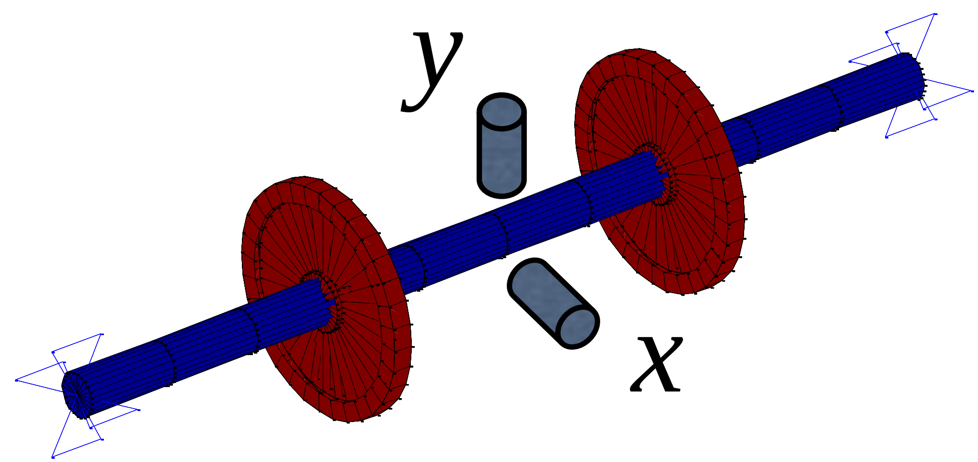


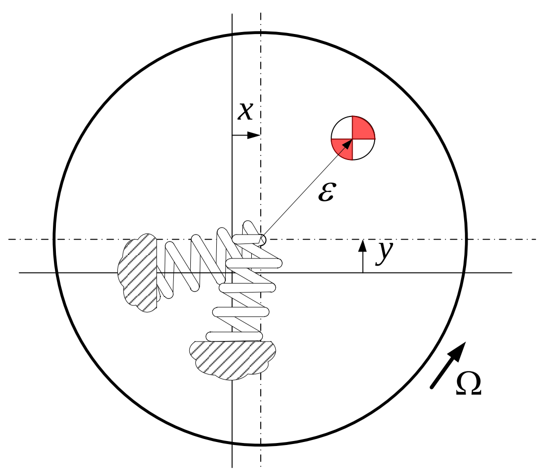
Flexible – not rigid

One degree of freedom – Jeffcott rotor

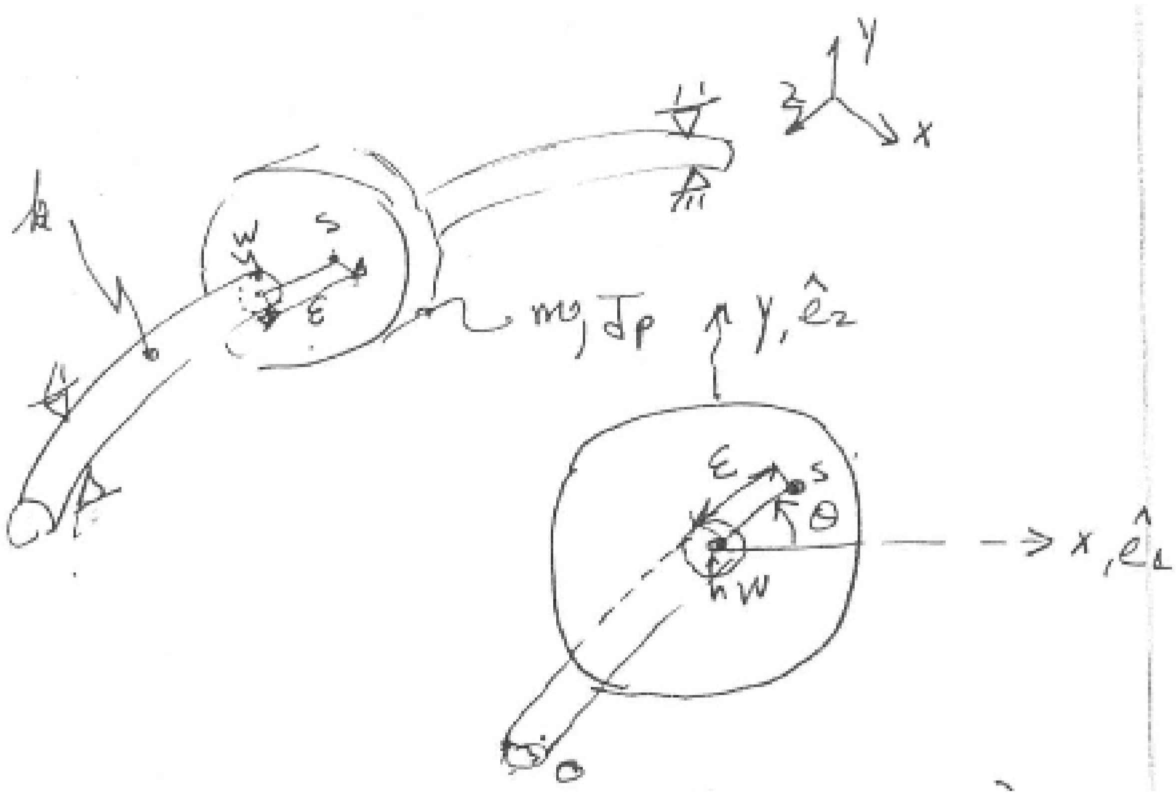
- One mode
- Response to unbalance
- Isotropic supports



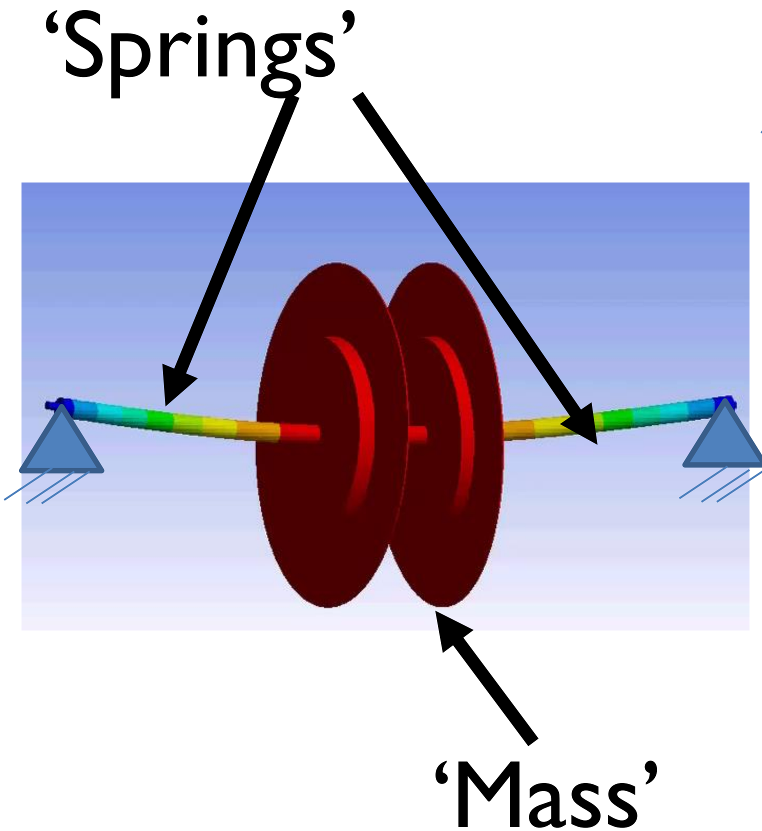


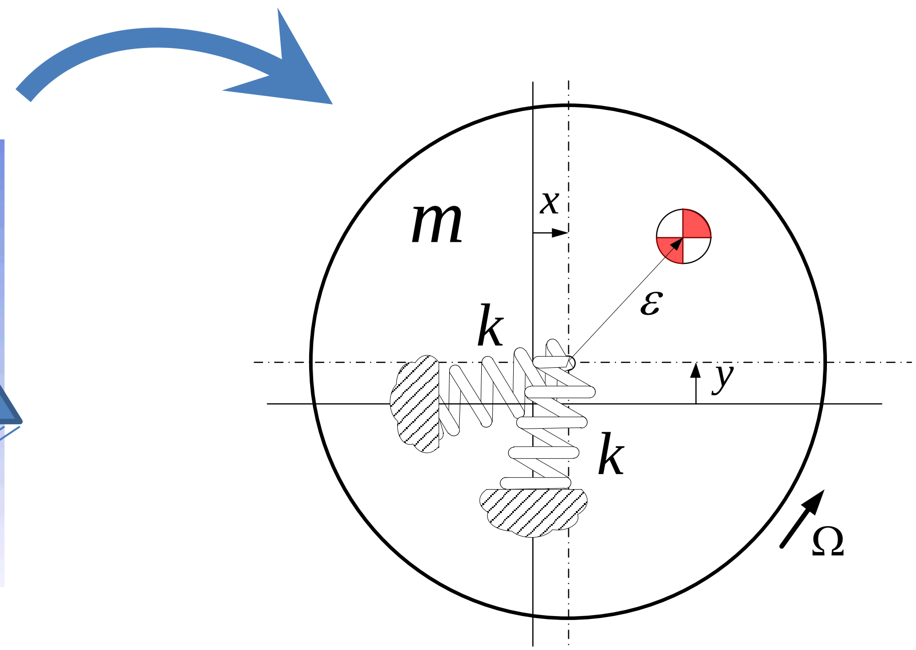


model



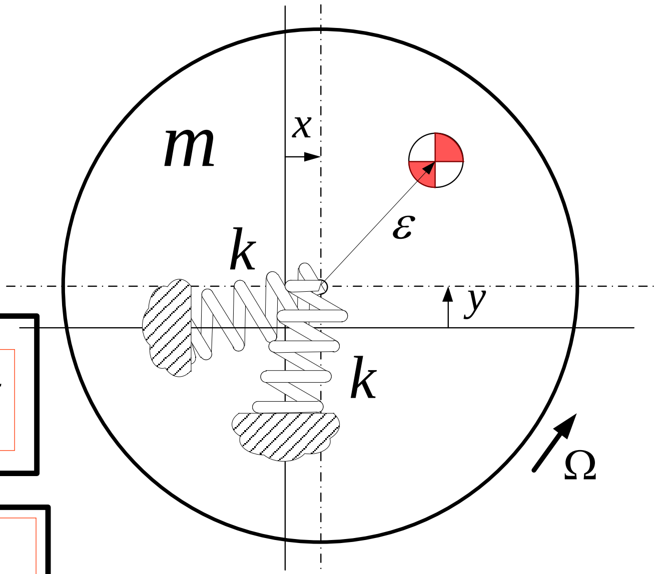
Jeffcott Rotor model





Jeffcott rotor

- constant speed $\dot{\Omega} = 0$
- Unbalance



$$x_{cg} = x + \varepsilon \cos \Omega t \qquad y_{cg} = y + \varepsilon \sin \Omega t$$

$$\frac{d}{dt}\left(m \frac{d}{dt} x_{cg} \right) = -kx \qquad \frac{d}{dt}\left(m \frac{d}{dt} y_{cg} \right) = -ky$$

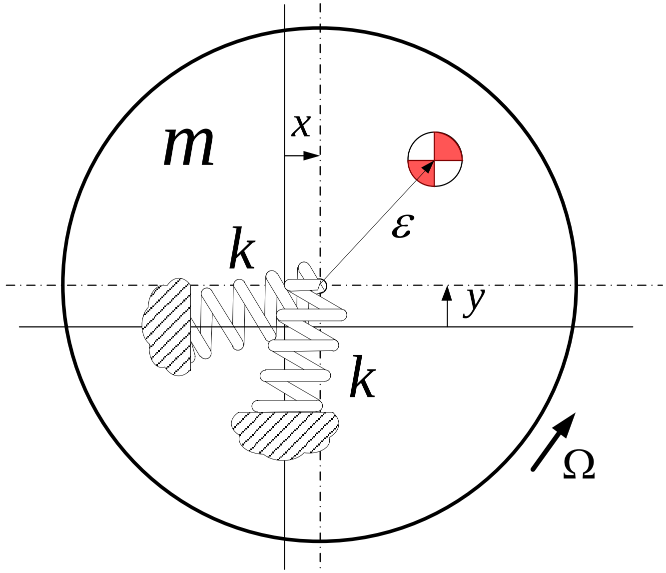
$$m\ddot{x} + kx = m\varepsilon\Omega^2 \cos \Omega t$$

$$m\ddot{y} + ky = m\varepsilon\Omega^2 \sin \Omega t$$

$$r \triangleq x + iy$$

$$m\ddot{r} + kr = m\varepsilon\Omega^2 e^{i\Omega t}$$

Jeffcott rotor – steady state response



$$m\ddot{r} + kr = m\varepsilon\Omega^2 e^{i\Omega t}$$

Put in Eq.

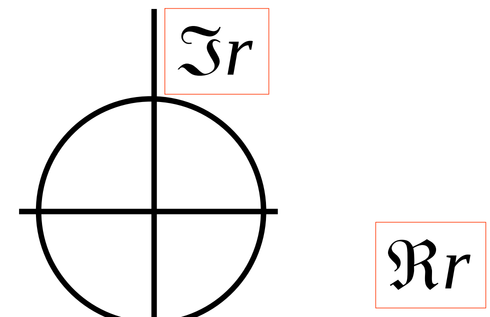
$$r = Ae^{i\Omega t} + Be^{-i\Omega t}$$

$$r = \frac{\varepsilon\Omega^2}{\omega_n^2 - \Omega^2} e^{i\Omega t}$$

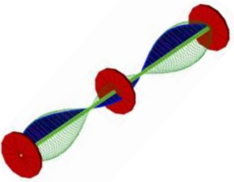
$$\omega_n^2 = \frac{k}{m}$$

Isotropic! supports

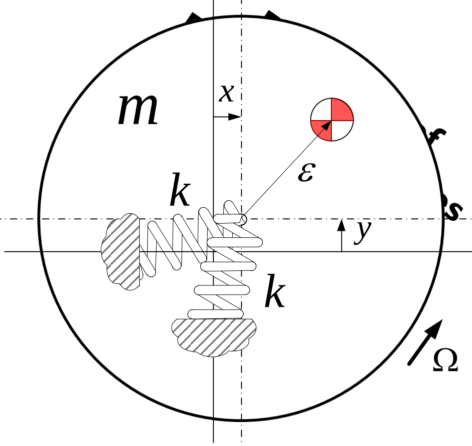
orbit



Prof Izhak Bucher

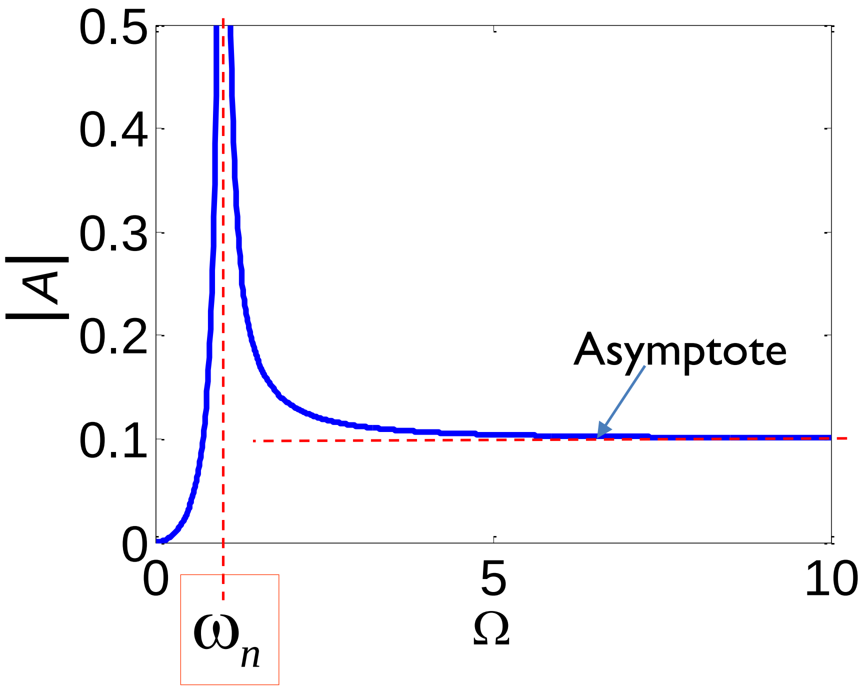


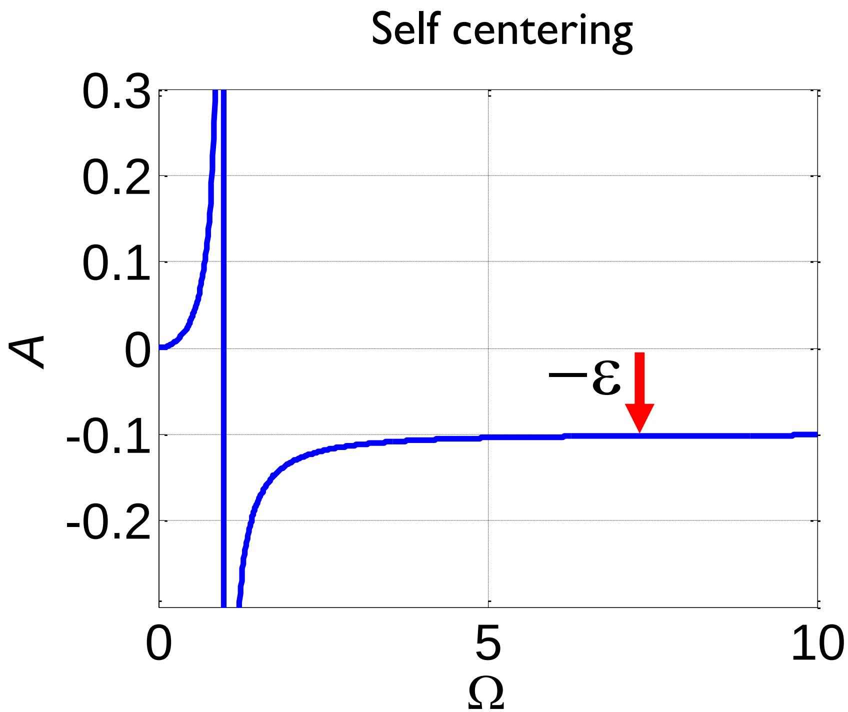
Response vs speed



$$r = \frac{\varepsilon \Omega^2}{\omega_n^2 - \Omega^2} e^{i\Omega t} = A e^{i\Omega t}$$

$$\omega_n^2 = \frac{k}{m}$$





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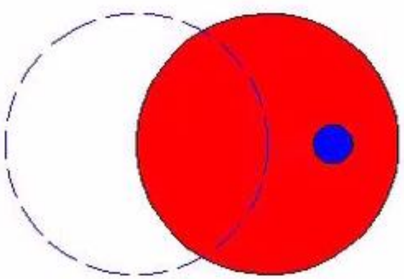
Animate whirl

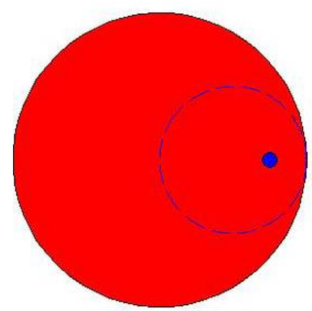
$\omega_n > \Omega$

$A > 0$



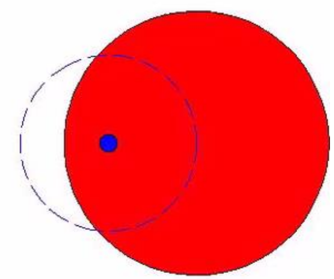
$\omega_n < \Omega$

$A < 0$



$\omega_n \ll \Omega$

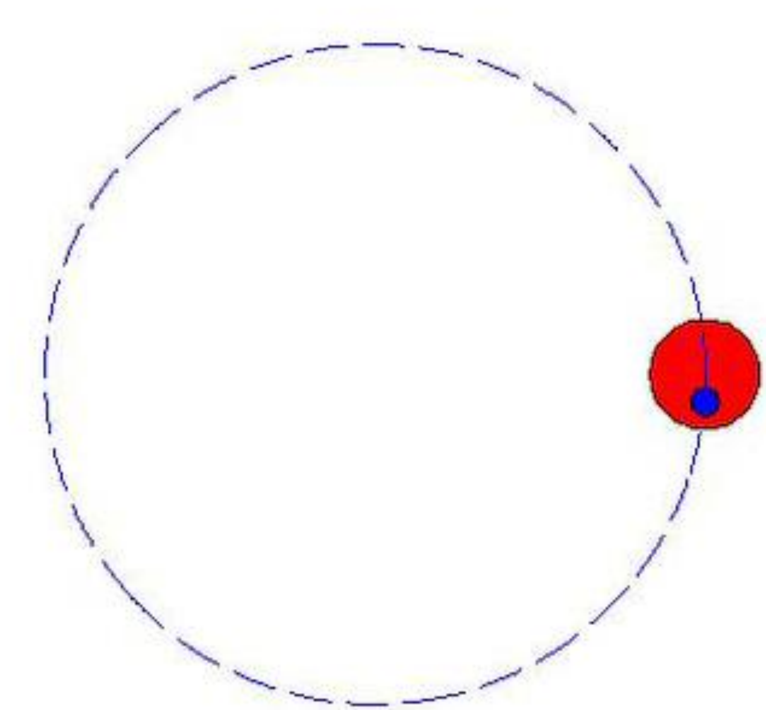
$A = -\varepsilon < 0$



Animate whirl

$\omega_n \approx \Omega$

$A \rightarrow \infty$

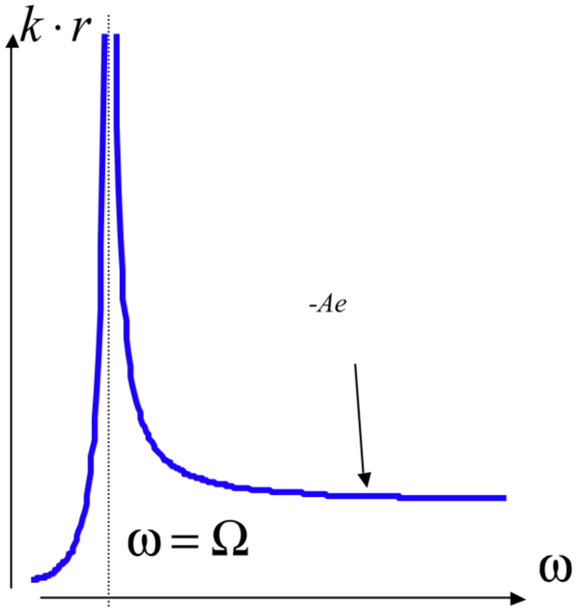


$phase\, lag \quad \dfrac{\pi}{2}$

forces @ bearings

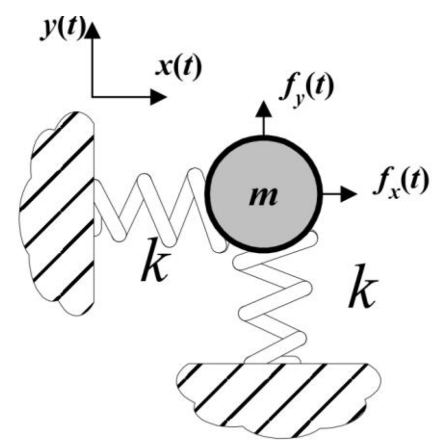
- Reaction force

$$f\_bearing = kr(t) = kA\,exp(i\Omega t) = k\frac{\frac{\Omega^2}{\omega_n^2}e}{\left(1-\frac{\Omega^2}{\omega_n^2}\right)}exp(i\Omega t)$$



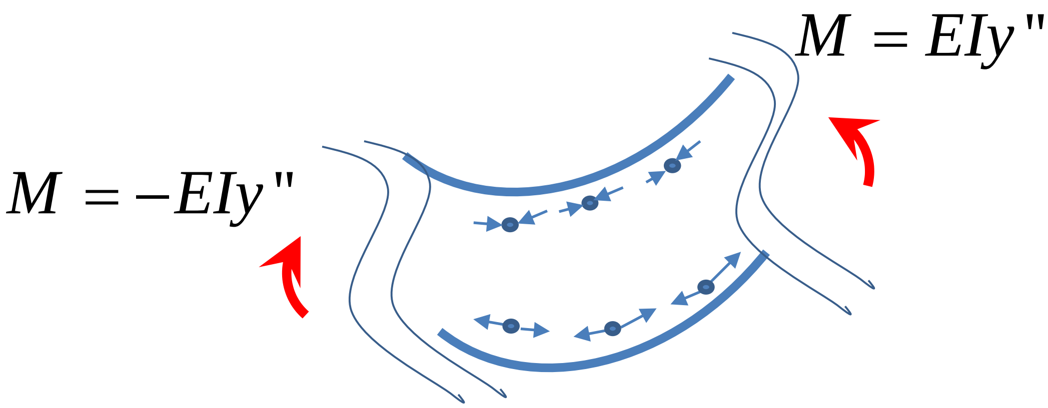
$$f_x(t) = k\frac{\frac{\Omega^2}{\omega_n^2}e}{\left(1-\frac{\Omega^2}{\omega_n^2}\right)}cos\,\Omega t$$

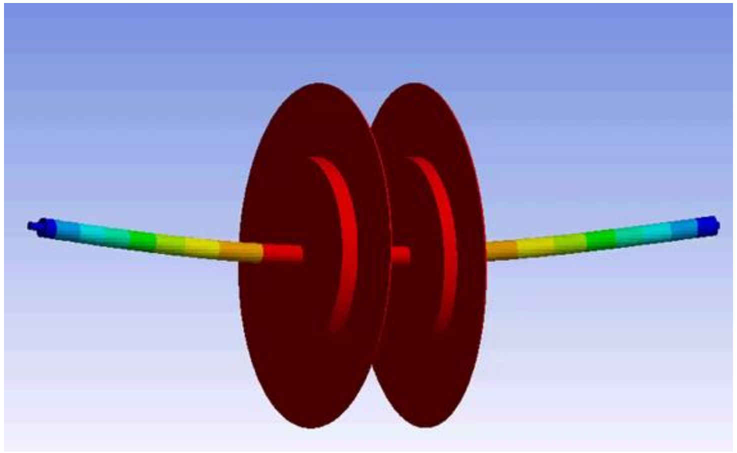
$$f_y(t) = k\frac{\frac{\Omega^2}{\omega_n^2}e}{\left(1-\frac{\Omega^2}{\omega_n^2}\right)}sin\,\Omega t$$

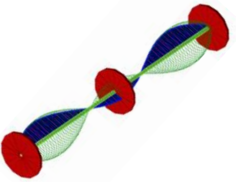


Shaft stress

- Is shaft in tension or compression?
- Is stress alternating (fatigue), at what rate?

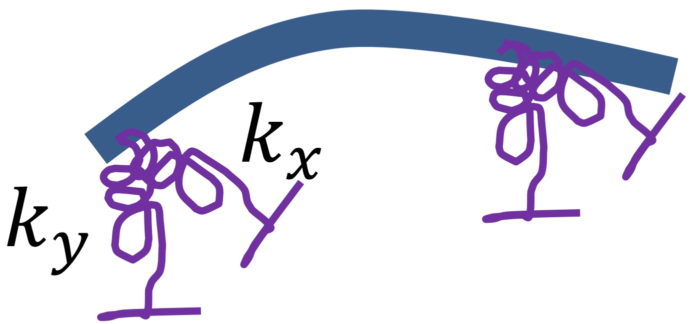


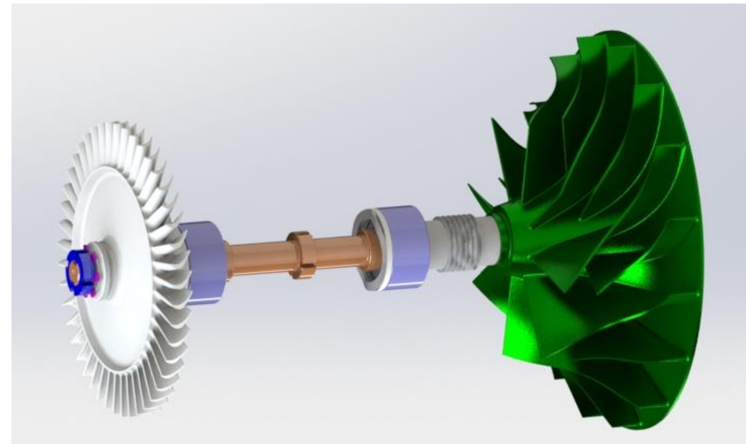




Rotors in anisotropic bearings

unequal stiffness





Total stiffness (springs in series)

Shaft + bearing + foundation

Usually $k_x \neq k_y$

$$\omega_x^2 = \frac{k_x}{m}$$

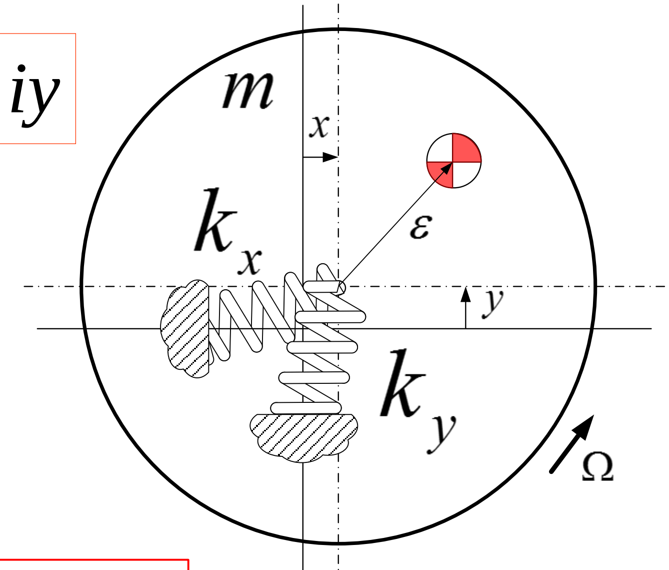
$$\omega_y^2 = \frac{k_y}{m}$$

Anisotropic bearings

$$\ddot{x} + \omega_x^2 x = \varepsilon\Omega^2 \cos\Omega t$$

$$\ddot{y} + \omega_y^2 y = \varepsilon\Omega^2 \sin\Omega t$$

$$r \triangleq x + iy$$



$$\omega_x^2 \triangleq \omega_n^2 + \omega_\Delta^2, \;\; \omega_y^2 \triangleq \omega_n^2 - \omega_\Delta^2,$$

$$\ddot{r} + \omega_n^2 r + \omega_\Delta^2 \bar{r} = \Omega^2 \varepsilon e^{i\Omega t}$$

$$\Rightarrow \quad r = r_+ e^{i\Omega t} + r_- e^{-i\Omega t}$$

$$r_+ = \frac{\Omega^2 \varepsilon}{2} \frac{\left(\omega_x^2 + \omega_y^2 - 2\Omega^2\right)}{\left(\omega_x^2 - \Omega^2\right)\left(\omega_y^2 - \Omega^2\right)}$$

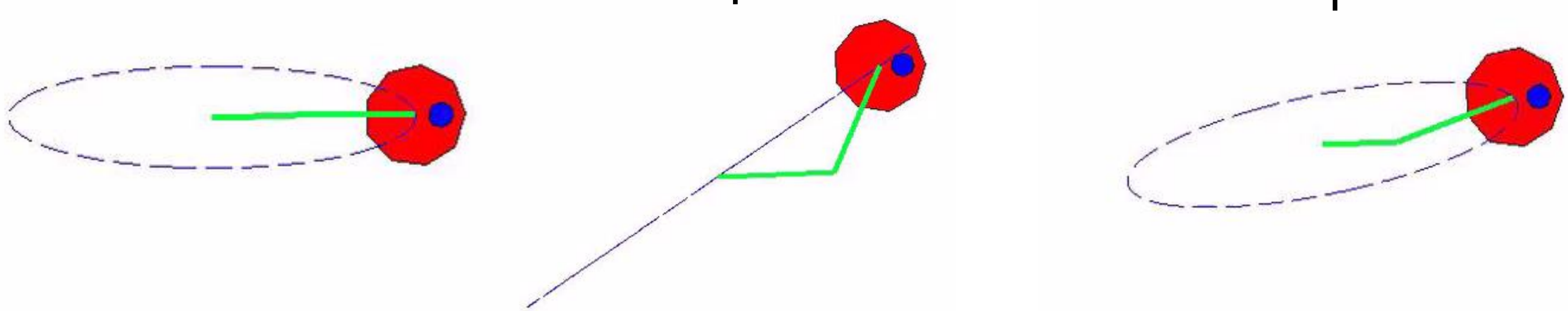
$$r_- = \frac{\Omega^2 \varepsilon}{2} \frac{\left(\omega_y^2 - \omega_x^2\right)}{\left(\omega_x^2 - \Omega^2\right)\left(\omega_y^2 - \Omega^2\right)}$$

Forward & backward whirl

- Forward whirl takes place when $r_+ > r_-$
- Backward whirl $r_+ < r_-$

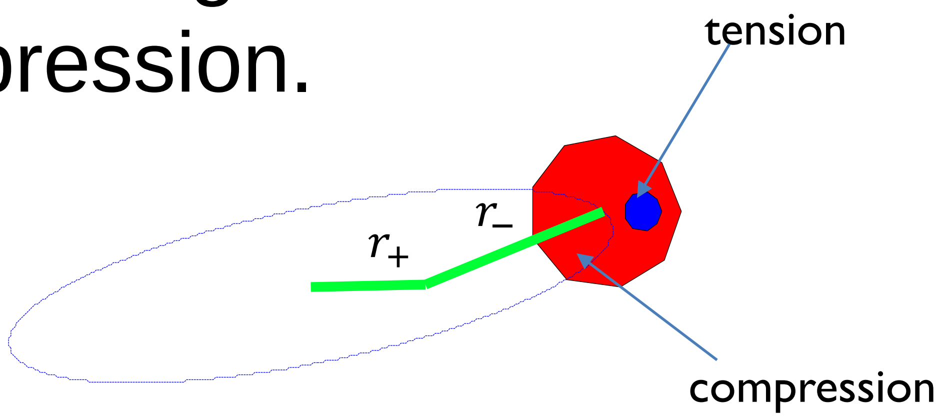
$$\boxed{r = r_+ e^{i\Omega t} + r_- e^{-i\Omega t}}$$

$r_+ > r_-$ $\quad$ $r_+ = r_-$ $\quad$ $r_+ < r_-$

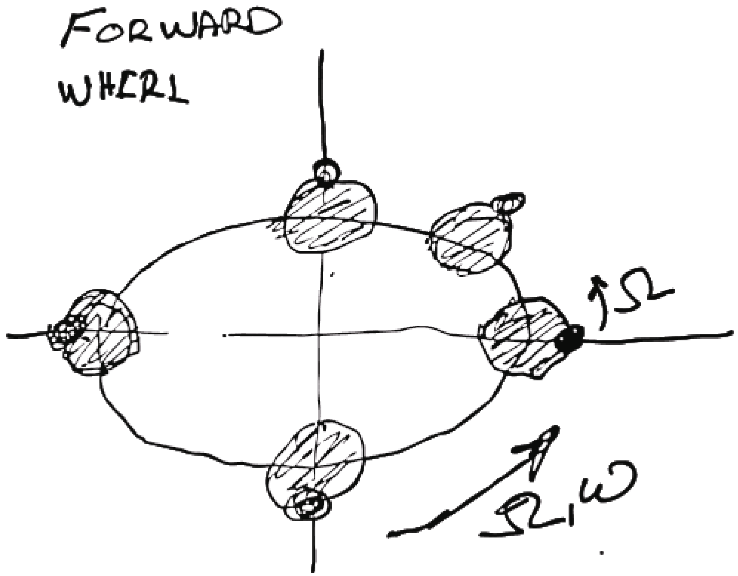


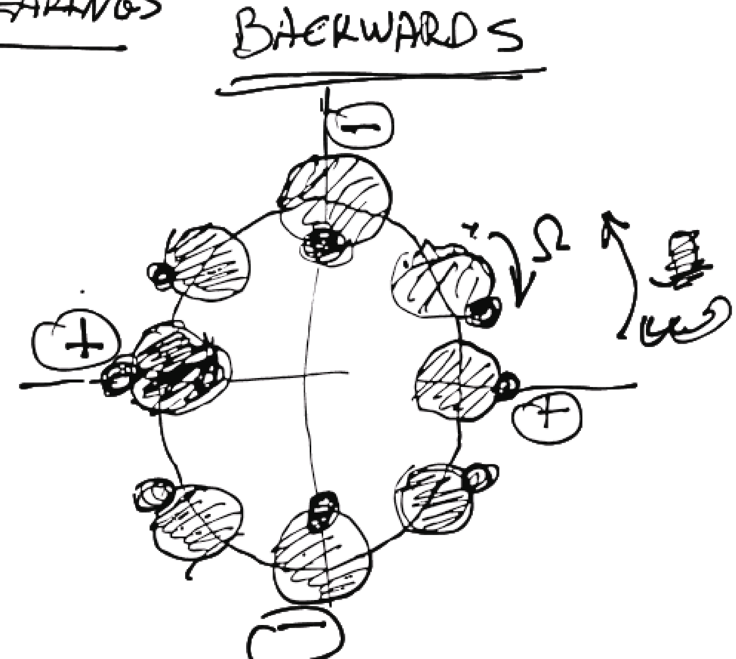
Stress & anisotropic bearings

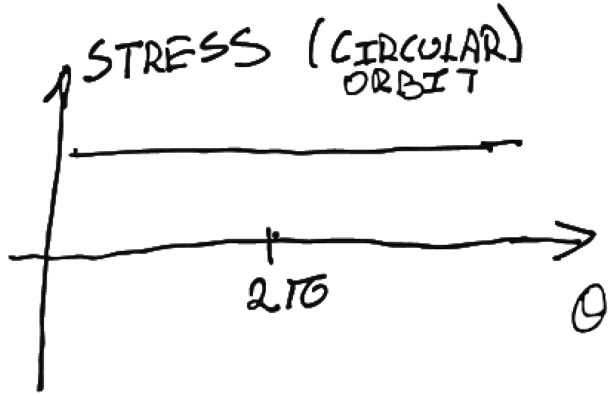
- Consider the blue dot (cg) as a material fiber.
- As long as it is further than the dashed line, it is in tension
- If it is closer to the origin than the dashed line, it is in compression.



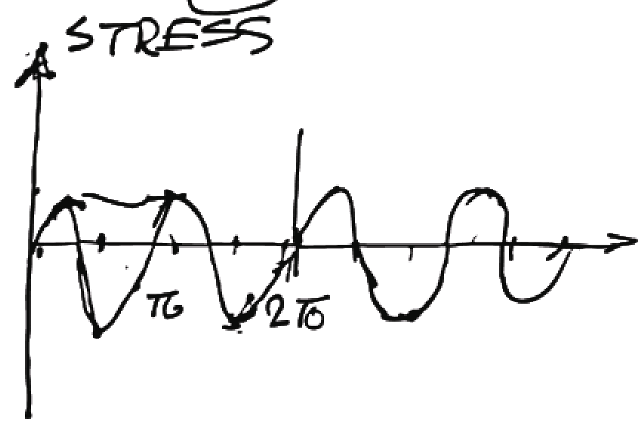
ANISOTROPIC BEARINGS







$$\theta = \int \Omega \, dt$$

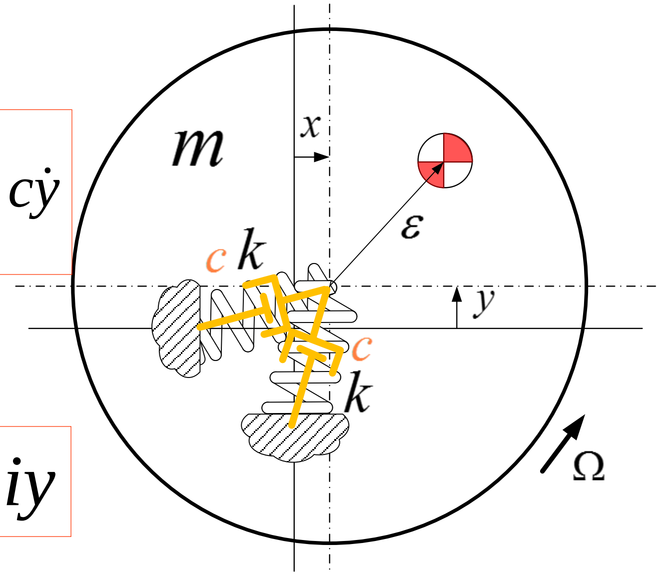


Damped *Rotors*

Damping + unbalance

- Unbalance

$$\frac{d}{dt}\left(m\frac{d}{dt}x_{cg}\right)=-kx-c\dot{x} \qquad \frac{d}{dt}\left(m\frac{d}{dt}y_{cg}\right)=-ky-c\dot{y}$$



$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2x=\varepsilon\Omega^2\cos\Omega t$$

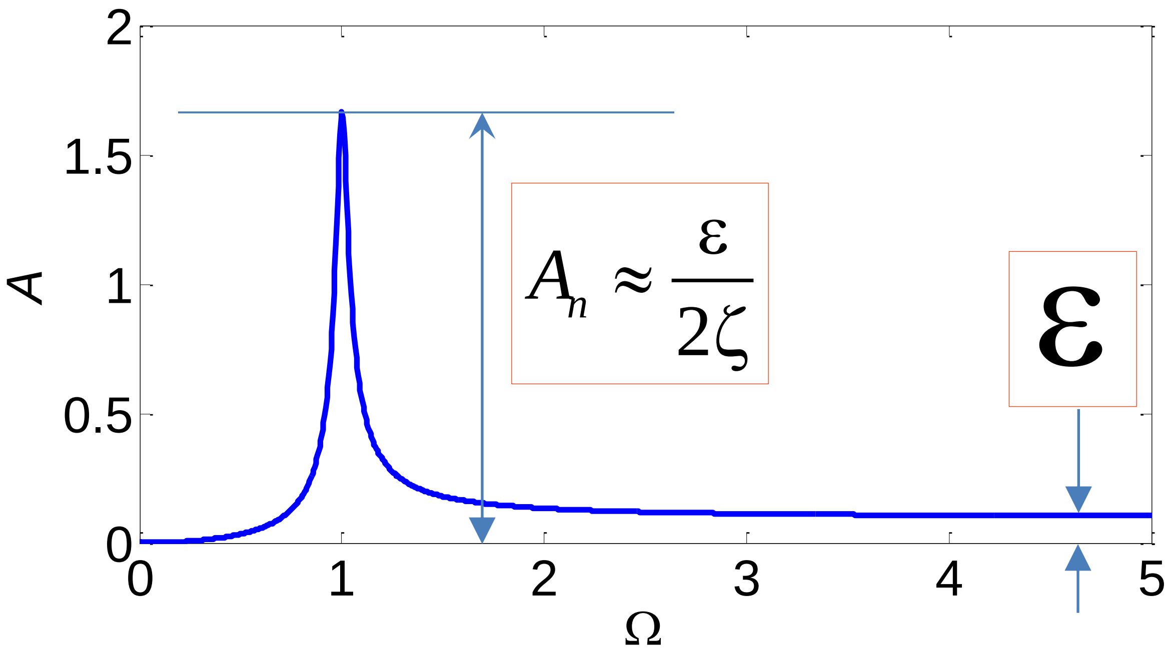
$$\ddot{y}+2\zeta\omega_n\dot{y}+\omega_n^2y=\varepsilon\Omega^2\sin\Omega t$$

$$r\triangleq x+iy$$

$$\ddot{r}+2\zeta\omega_n\dot{r}+\omega_n^2r=\varepsilon\Omega^2e^{i\Omega t}$$

$$\Rightarrow \quad r=\frac{\varepsilon\Omega^2}{\omega_n^2-\Omega^2+i2\zeta\omega_n\Omega}e^{i\Omega t}$$

Amplitude vs speed



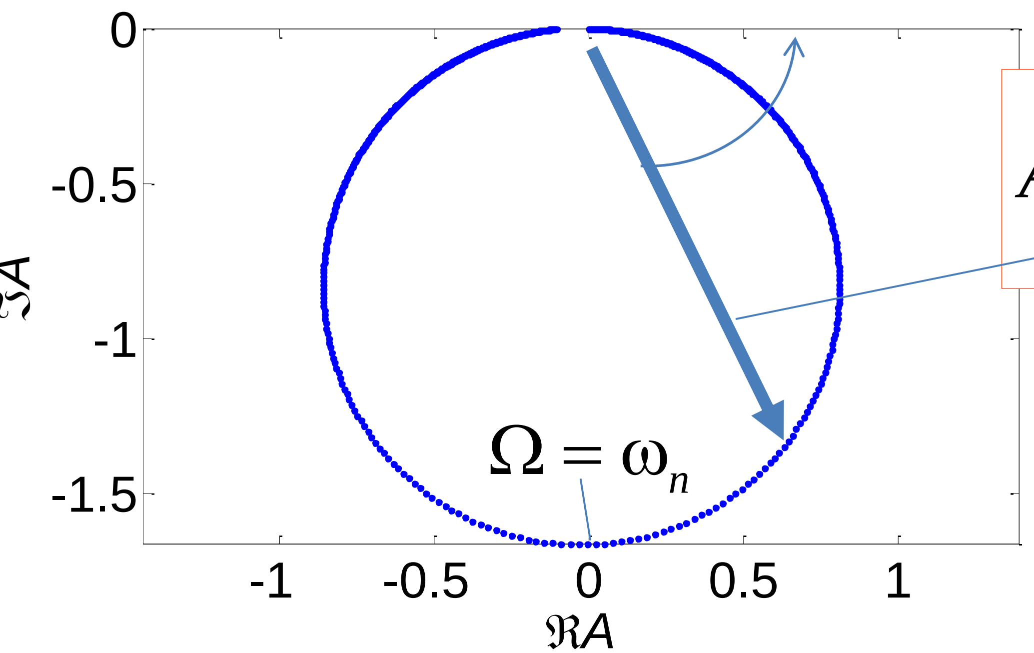
- Does damping affect self centering?

$$r = \frac{\varepsilon\Omega^2}{\omega_n^2 - \Omega^2 + i2\zeta\omega_n\Omega} e^{i\Omega t}$$

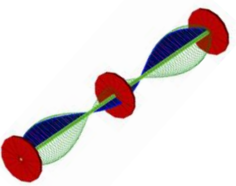
$$\Omega \gg \omega_n$$

$$r = \frac{\varepsilon\Omega^2}{-\Omega^2} e^{i\Omega t} = -\varepsilon e^{i\Omega t}$$

Polar plot



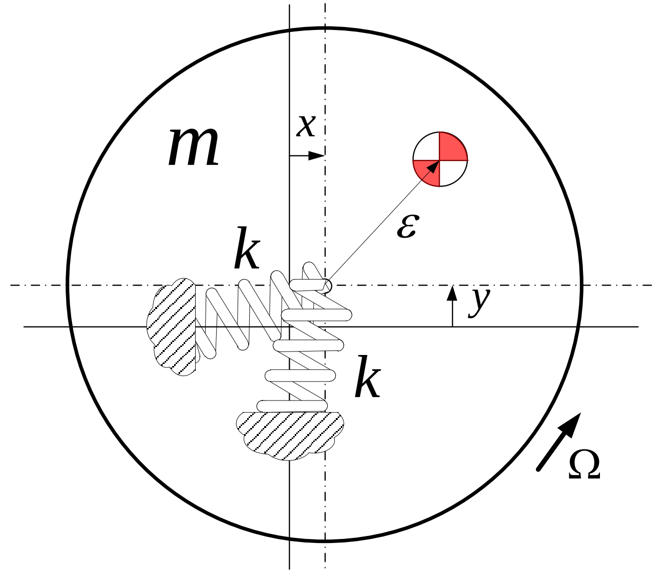
$$A = \frac{\varepsilon \Omega^2}{\omega_n^2 - \Omega^2 + i 2\zeta\omega_n\Omega}$$



Rotors in body-fixed coordinates

Jeffcott rotor – steady state response

$$m\ddot{r} + kr = m\varepsilon\Omega^2 e^{i\Omega t}$$

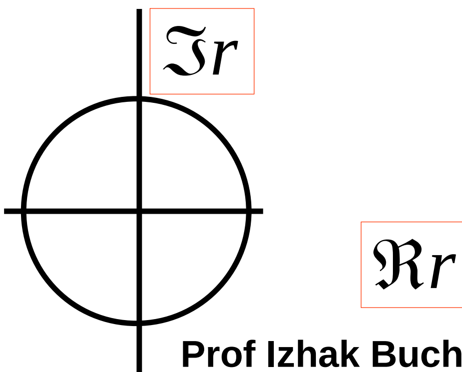


Put in Eq.

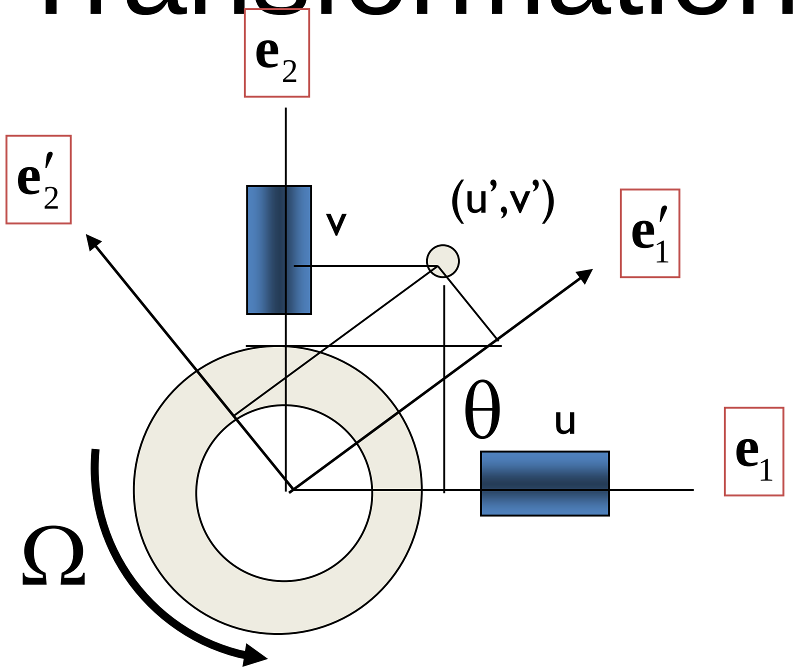
$$r = Ae^{i\Omega t} + BAe^{-i\Omega t}$$

$$r = \frac{\varepsilon\Omega^2}{\omega_n^2 - \Omega^2} e^{i\Omega t} \qquad \omega_n^2 = \frac{k}{m}$$

orbit



Transformation of axes



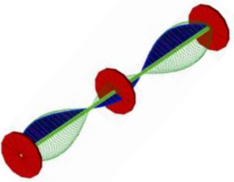
$$\theta = \int_0^t \Omega(\tau)\, d\tau$$

vector

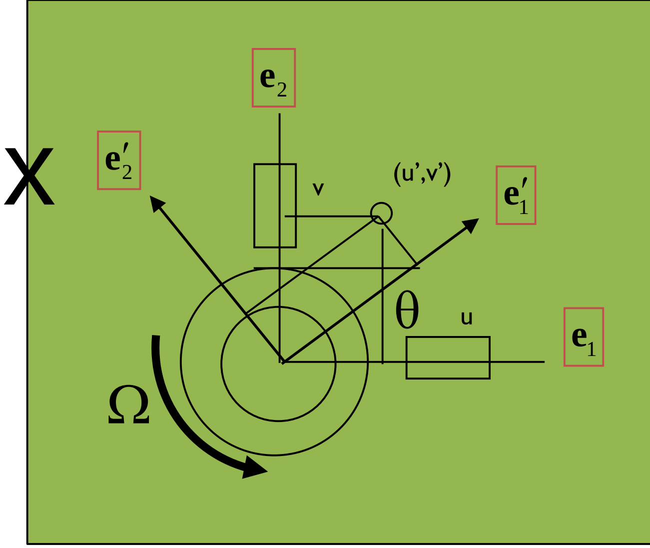
$$\mathbf{r} = u\mathbf{e}_1 + v\mathbf{e}_2 = u'\mathbf{e}'_1 + v'\mathbf{e}'_2$$

Complex representation

$$r = u + \mathrm{i}\, v \qquad \rho = u' + \mathrm{i}\, v'$$



Transformation matrix (proof)



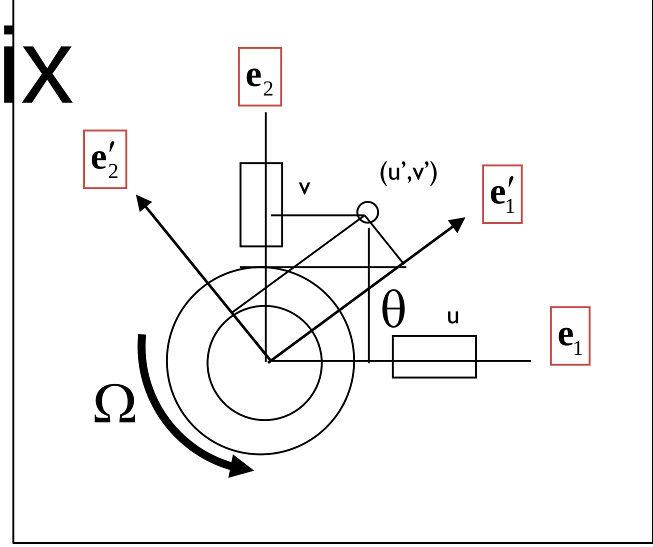
$$\boxed{\mathbf{r}\cdot\mathbf{e}_1' = u\mathbf{e}_1\cdot\mathbf{e}_1' + v\mathbf{e}_2\cdot\mathbf{e}_1' = u'\mathbf{e}_1'\cdot\mathbf{e}_1' + v'\mathbf{e}_2'\cdot\mathbf{e}_1'}$$

$$\boxed{\mathbf{r}\cdot\mathbf{e}_1' = u\cos\theta + v\sin\theta = u'}$$

$$\boxed{\mathbf{r}\cdot\mathbf{e}_2' = u\mathbf{e}_1\cdot\mathbf{e}_2' + v\mathbf{e}_2\cdot\mathbf{e}_2' = u'\mathbf{e}_1'\cdot\mathbf{e}_2' + v'\mathbf{e}_2'\cdot\mathbf{e}_2'}$$

$$\boxed{\mathbf{r}\cdot\mathbf{e}_2' = -u\sin\theta + v\cos\theta = v'}$$

Transformation matrix (proof)



$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u' \\ v' \end{pmatrix}$$

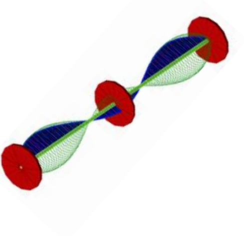
$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} u' \\ v' \end{pmatrix}$$

← Matrix transformation of cords

$$u'\cos\theta - v'\sin\theta + \mathrm{i}\left(u'\sin\theta + v'\cos\theta\right) = u + \mathrm{i}\, v$$

$$\rho e^{\mathrm{i}\,\theta} = r$$

← Complex transformation of cords



EQ of motion in body cords

- Stationary cords & transformation

$$\boxed{\ddot{r}+\omega_n^2 r=\varepsilon\Omega^2 e^{i\Omega t}} \qquad \boxed{\rho e^{i\theta}=\rho e^{i\Omega t}=r}$$

$$\boxed{\left(\ddot{\rho}+2i\,\Omega\dot{\rho}-\rho\Omega^2\right)e^{i\Omega t}+\omega_n^2\rho e^{i\Omega t}=\varepsilon\Omega^2 e^{i\Omega t}}$$

$$\boxed{\left(\ddot{\rho}+2i\,\Omega\dot{\rho}-\rho\Omega^2\right)+\omega_n^2\rho=\varepsilon\Omega^2}$$

A strain gauge would measure that

$$\boxed{\rho=\overbrace{\frac{\varepsilon\Omega^2}{\omega_n^2-\Omega^2}}^{\text{steady}}+\overbrace{\rho_1 e^{i\lambda_1 t}+\rho_2 e^{i\lambda_2 t}}^{\text{transient}}}$$

transient in body cords

$$\left(\ddot{\rho} + 2i\,\Omega\dot{\rho} - \rho\Omega^2\right) + \omega_n^2\rho = \cancel{\varepsilon\Omega^2} \boxed{= 0}$$

Propose a solution $\boxed{\rho = \rho_0 e^{\lambda t}}$

$$\left(\lambda^2 + 2i\lambda\Omega + \omega_n^2 - \Omega^2\right)\rho_0 e^{\lambda t} = 0 \quad \Rightarrow \quad \lambda_{1,2} = -i\left(\omega_n \pm \Omega\right)$$

A strain gauge would measure that

$$\rho = \underbrace{\frac{\varepsilon\Omega^2}{\omega_n^2 - \Omega^2}}_{\text{steady}} + \underbrace{\rho_1 e^{-i(\omega_n+\Omega)t} + \rho_2 e^{i(\omega_n-\Omega)t}}_{\text{transient}}$$

When a disturbance occurs, **the apparent frequency is shifted**

List of reference books

Most relevant books

- **Dynamics of rotating systems,** 2005
  - Giancarlo Genta
- **Rotordynamik (German Edition),** 2007
  - Robert Gasch, Rainer Nordmann, Herbert Pfützner

Specific books (rotating machines)

- Ehrich, Fredric F. editor, Handbook of rotordynamics
- Childs, Dara, Turbomachinery rotordynamics : 1993.
- Lalanne, Michel, Rotordynamics prediction in engineering
- Vance, John M. Machinery Vibration and Rotordynamics
- Adams, Maurice L. Rotating machinery vibration : 2001.
- Wowk, Victor, Machinery vibration : balancing 1995.
- Kramer, Erwin, Dynamics of rotors and foundations, 1993.