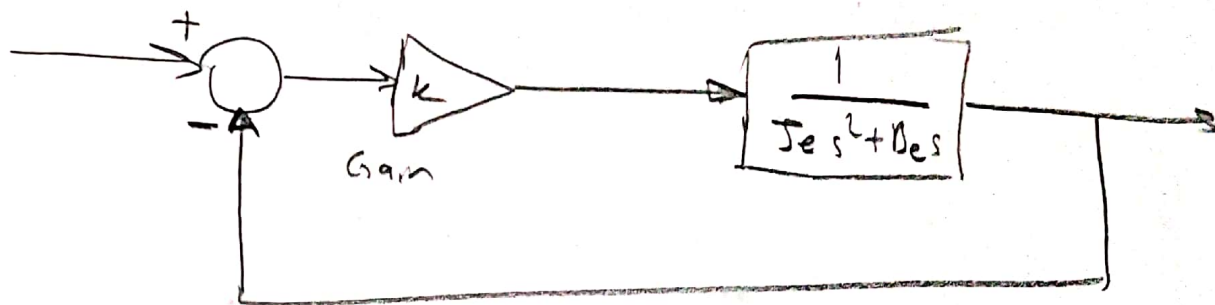


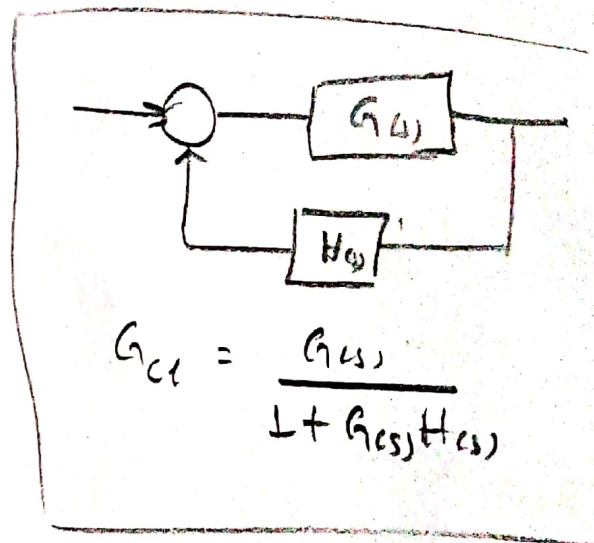
MECH 467/541 - Tutorial 7



1)

$$G_{cl(s)} = \frac{k \cdot \frac{1}{J_e s^2 + B_e s}}{1 + k \cdot \frac{1}{J_e s^2 + B_e s}}$$

$$= \frac{k}{J_e s^2 + B_e s + k}$$



2)

$$G_{cl(s)} = \frac{k}{J_e s^2 + B_e s + k} = \frac{k/J_e}{s^2 + \frac{B_e}{J_e} s + \frac{k}{J_e}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{k}{J_e} \Rightarrow \omega_n = \sqrt{\frac{k}{J_e}}$$

3)

$$2\zeta\omega_n = 2\zeta \sqrt{\frac{k}{J_e}} = \frac{B_e}{J_e} \Rightarrow \zeta = \frac{B_e}{2J_e} \sqrt{\frac{J_e}{k}}$$

4) Char eqn.

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$P_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$= -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$$

5) Q: How does ζ relate to the angle that these lines make with real axis?

$$\psi = \tan^{-1} \left(\frac{\omega_n\sqrt{1-\zeta^2}}{\zeta\omega_n} \right) = \tan^{-1} \left(\frac{\sqrt{1-\zeta^2}}{\zeta} \right)$$

6)

$$(-\zeta\omega_n)^2 + (\omega_n\sqrt{1-\zeta^2})^2 = \omega_n^2$$

7)

$$G(s) = \frac{K}{s^2 + \beta s + k}$$

$$P_{1,2} = \frac{-\frac{\beta}{s} \pm \sqrt{\frac{\beta^2}{s^2} - \frac{4K}{s}}}{2}$$