MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

Final Exam

Date: Apr 21, 2021
Time: 3:30 – 5:30pm

Problem 1 (8 points)

Figure 1 shows the frequency responses of four LTI systems. For each frequency response, find the corresponding step response from Figure 2 and briefly explain the reasoning.

Frequency response	(a)	(b)	(c)	(d)
Step response				

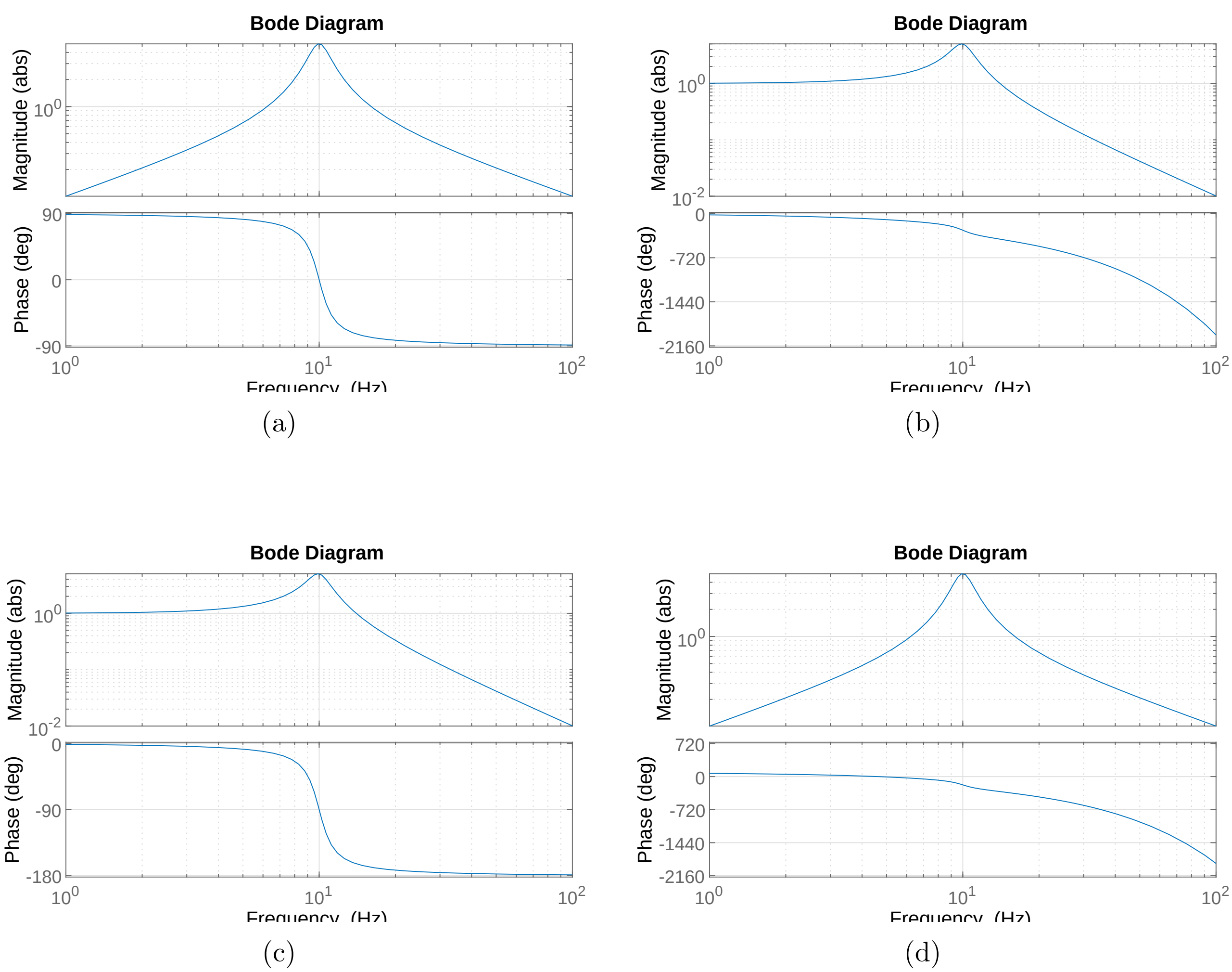

Figure 1: Frequency responses.

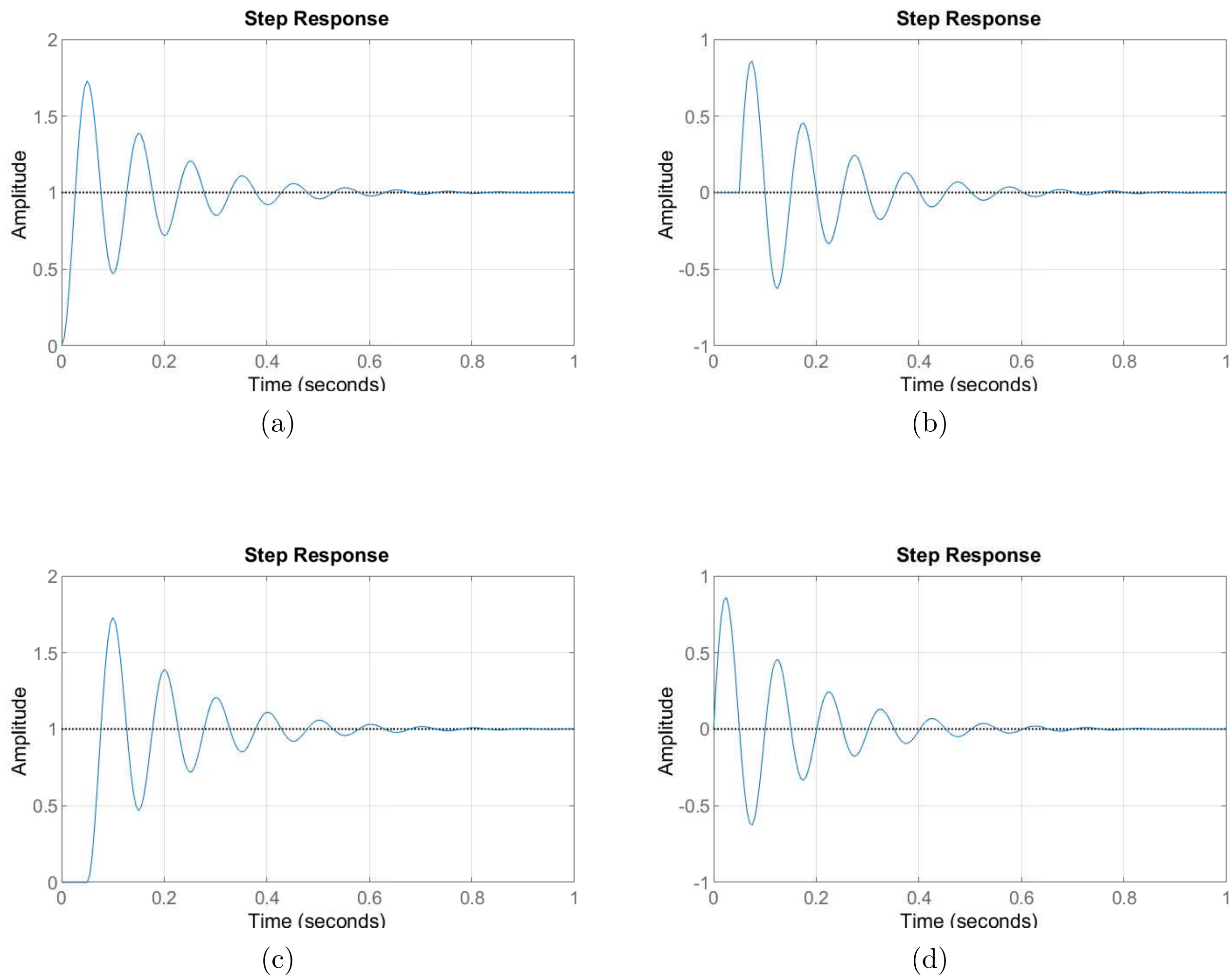

Figure 2: Step responses.

Problem 2 (12 points)

Briefly answer the following questions.

(1) Explain the pros and cons of linear power amplifiers.

(2) Explain the pros and cons of switching power amplifiers.

(3) Draw an equivalent circuit model of a power MOSFET being used as a switch.

(4) A half-bridge circuit consists of two power MOSFETs connected in series across a DC link. Explain what happens when the two MOSFETs turn on simultaneously.

Problem 3 (40 points)

Figure 3 show an analog circuit, which consists of an op-amp whose input impedance is infinite ($Z_i \to \infty$), output impedance is zero ($Z_o = 0$), and open-loop gain is A.

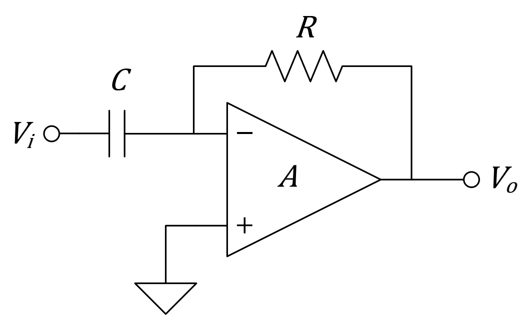

Figure 3: Op-amp circuit.

Part I

Answer the following questions assuming that the op-amp open-loop gain is $A \to \infty$.

(1) (3 pt.) Derive the transfer function from V_i to V_o

$$G(s) = \frac{V_o(s)}{V_i(s)}.$$

(2) (3 pt.) Draw the Bode plot of $G(s)$.

(3) (3 pt.) Draw the step response of $G(s)$.

(4) (3 pt.) Find the output voltage $V_o(t)$ when the input voltage is a persistent sinusoid

$$V_i(t) = \cos(\omega t).$$

Part II

Answer the following questions assuming that the op-amp open-loop gain is $A = A(s)$.

(5) (6 pt.) Draw a block diagram that shows the feedback relation between V_i and V_o.

(6) (3 pt.) Determine the loop return ratio $L(s)$ and complementary sensitivity function $T(s)$.

(7) (3 pt.) Derive the transfer function from V_i to V_o

$$G(s) = \frac{V_o(s)}{V_i(s)}$$

and express it in terms of $T(s)$.

(8) (6 pt.) Suppose the circuit parameters are given as

$$A(s) = \frac{10^7}{s} \qquad\qquad C = 0.01\,\mu\text{F} \qquad\qquad R = 1\,\text{k}\Omega.$$

Manually draw the Bode plot of $L(s)$, and mark the approximate values of the unity-gain crossover frequency ω_c and phase margin ϕ_m.

(9) (6 pt.) Manually draw the Bode plot of $T(s)$, and mark the approximate values of the resonant frequency ω_r and resonant peak M_r.

(10) (4 pt.) Sketch the step response of $G(s)$ and explain key features, such as initial value, initial slope, final value, oscillation, etc.

Problem 4 (40 points)

Figure 4 shows a current controller for a brushed dc motor, which consists of a real-time computer and op-amp circuits. The op-amps are assumed to be ideal ($Z_i \to \infty, Z_o = 0, A \to \infty$).

A discrete-time control $C(z)$ is implemented at a fixed sampling rate $f_s = 1/T$. The ADC converts $y(t)$ to $y[k]$ via instantaneous sampling

$$y[k] = y(t)|_{t=kT}$$

and the DAC converts $u[k]$ to $u(t)$ via zero-order hold

$$u(t) = u[k] \quad \text{for} \quad kT \le t < (k+1)T.$$

Answer the following questions.

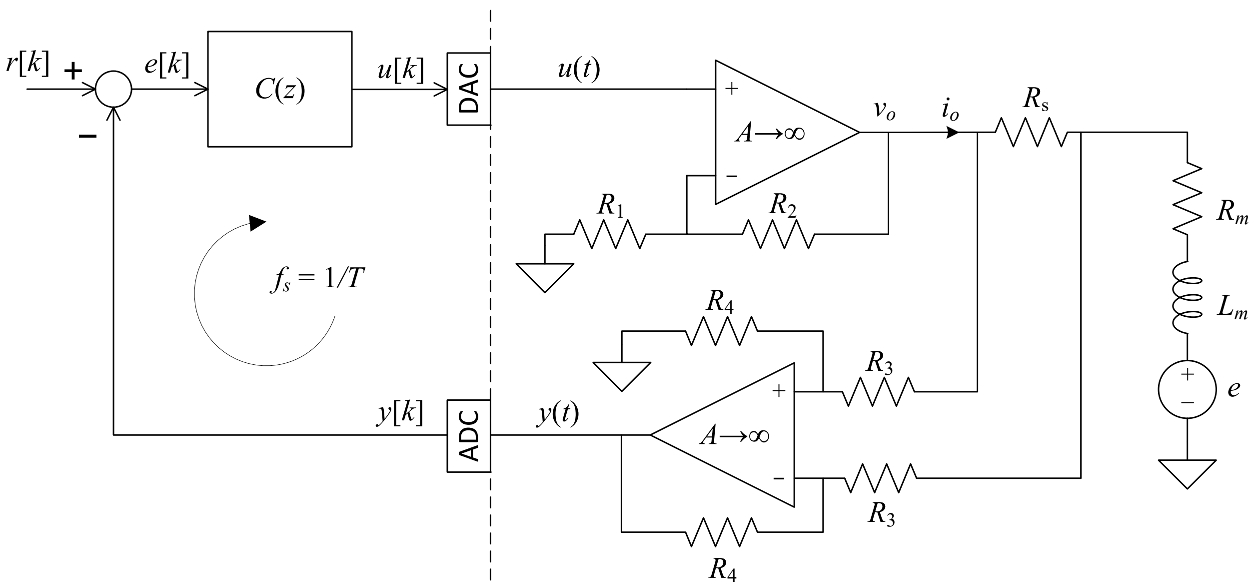

Figure 4: Current controller for a brushed dc motor.

(1) (5 pt.) Draw a block diagram that shows the relation between the control effort $u(t)$, current measurement $y(t)$, and back-emf e. Assume that R_s is much smaller than other resistors.

(2) (2 pt.) Derive the transfer function from $u(t)$ to $y(t)$

$$H_1(s) = \frac{Y(s)}{U(s)}.$$

(3) (3 pt.) We can approximate the DAC as a continuous-time transfer function $H_2(s)$. Express $H_2(s)$ in terms of the sampling period T.

(4) (10 pt.) Suppose the system parameters are given as

$R_1 = 1\,\text{k}\Omega$	$R_2 = 4\,\text{k}\Omega$	$R_3 = 1\,\text{k}\Omega$	$R_4 = 20\,\text{k}\Omega$
$R_m = 1\,\Omega$	$L_m = 1\,\text{mH}$	$R_s = 10\,\text{m}\Omega$	$1/T = 6\,\text{kHz}$.

Determine the approximate continuous-time plant transfer function

$$P(s) = H_1(s)H_2(s)$$

and draw the Bode plot of $P(s)$. Clearly mark the break frequency and $-180°$ phase crossover frequency.

(5) (10 pt.) Design a continuous-time PI control

$$C(s) = K_p\left(1 + \frac{\omega_i}{s}\right)$$

that makes the loop return ratio $L(s) = C(s)P(s)$ achieve the following specifications.

- The slope of the magnitude curve is -1 ($-20\,\text{dB/decade}$) for all frequencies.
- Phase margin is $\phi_m = 60°$.

Clearly show the values for K_p, ω_i, and the unity-gain crossover frequency ω_c.

(6) (5 pt.) Design a discrete-time PI control $C(z)$ that approximates $C(s)$ via the backward rectangular method (numerical integration).

(7) (5 pt.) Draw the block diagram of $C(z)$ that includes only one unit-delay block. Implement an integrator anti-windup that bounds the state of the integrator.