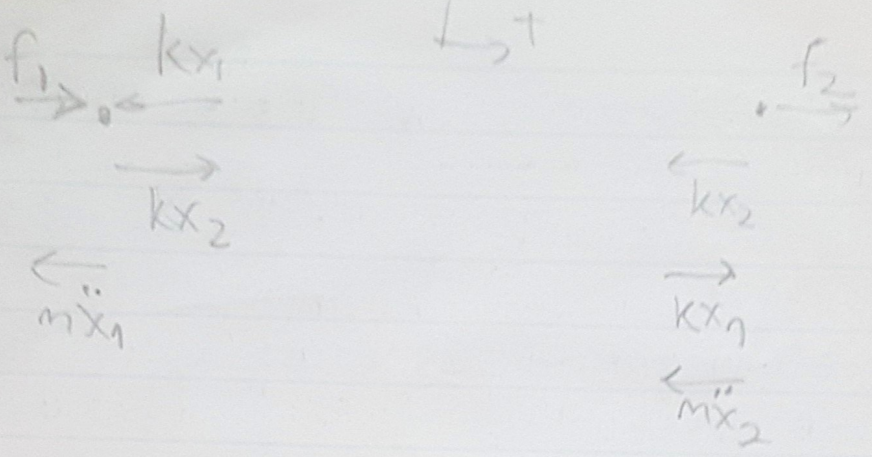
1a.



$\Sigma f_1 = 0$

$0 = -m\ddot{x}_1 - kx_1 + kx_2 + f_1$

$\Sigma f_2 = 0$

$0 = -m_2\ddot{x}_2 - kx_2 + kx_1 + f_2$

$$m_1\ddot{x}_1 - kx_2 + kx_1 = f_1$$
$$m_2\ddot{x}_2 + kx_2 - kx_1 = f_2$$

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \qquad K = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

1b.

$$(Ms^2 + K)X = F$$

$$X = (Ms^2 + K)^{-1}F$$

$$= \underbrace{H(s)}$$

$$H(s) = \begin{bmatrix} m_1s^2 + k & -k \\ -k & m_2s^2 + k \end{bmatrix}^{-1}$$

$$= \frac{1}{(m_1s^2 + k)(m_2s^2 + k) - k^2} \begin{bmatrix} m_2s^2 + k & k \\ k & m_1s^2 + k \end{bmatrix}$$

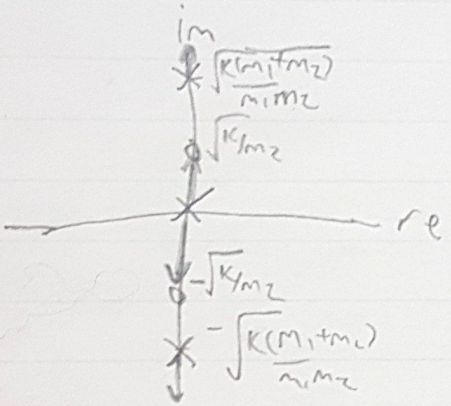
$$= \begin{bmatrix} \dfrac{m_2s^2 + k}{m_1m_2s^4 + k(m_1 + m_2)s^2} & \dfrac{k}{m_1m_2s^4 + k(m_1 + m_2)s^2} \\ \dfrac{k}{m_1m_2s^4 + k(m_1 + m_2)s^2} & \dfrac{m_1s^2 + k}{m_1m_2s^4 + k(m_1 + m_2)s^2} \end{bmatrix}$$

2c. $H_{11} = \dfrac{M_2 s^2 + K}{(M_1 M_2 s^2 + (M_1 + M_2)K)\, s^2}$

zero $\rightarrow 0 = M_2 s^2 + K \rightarrow s = \pm i\sqrt{\frac{K}{M_2}}$

pole $\rightarrow 0 = s^2 (M_1 M_2 s^2 + (M_1 + M_2)K) \rightarrow s = 0, \pm i\sqrt{\frac{K(M_1+M_2)}{M_1 M_2}}$

PZ map

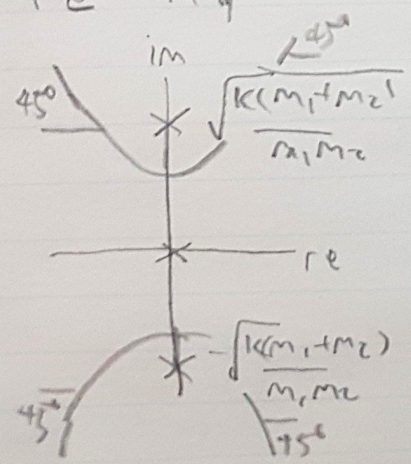


$H_{21} = \dfrac{K}{(M_1 M_2 s^2 + (M_1 + M_2)K)\, s^2}$

zero $\rightarrow$ none

pole $\rightarrow s = 0, \pm i\sqrt{\frac{K(M_1+M_2)}{M_1 M_2}}$

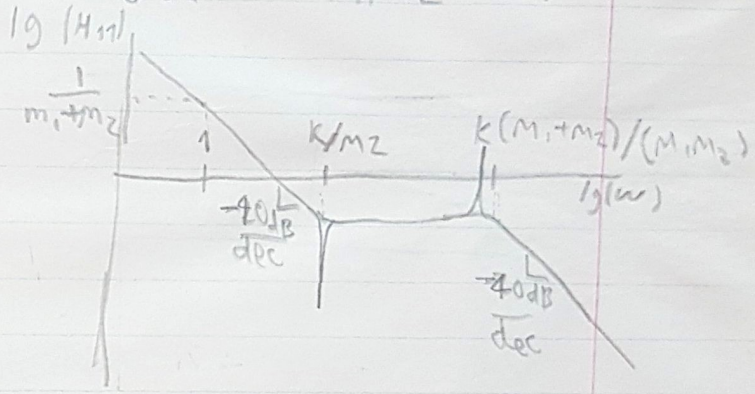
PZ map

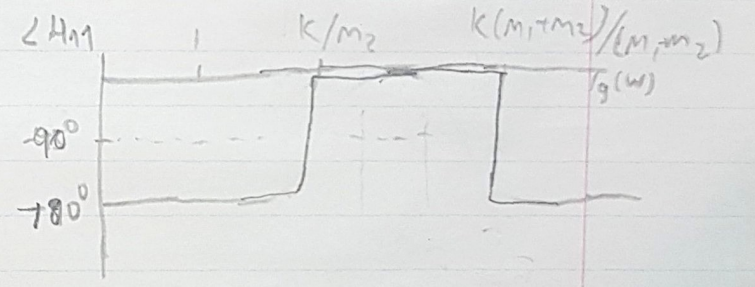


1c. $H_{11} = \dfrac{M_2 s^2 + K}{M_1 M_2 s^4 + K(M_1+M_2)s^2}$

$= \dfrac{M_2 s^2 + K}{s^2(M_1 M_2 s^2 + K(M_1+M_2))}$ $= \left(\dfrac{1}{M_1+M_2}\right) \dfrac{\left(\dfrac{s^2}{K/M_2} + 1\right)}{\left(\dfrac{s^2}{\frac{K(M_1+M_2)}{M_1 M_2}} + 1\right)} \left(\dfrac{1}{s^2}\right)$

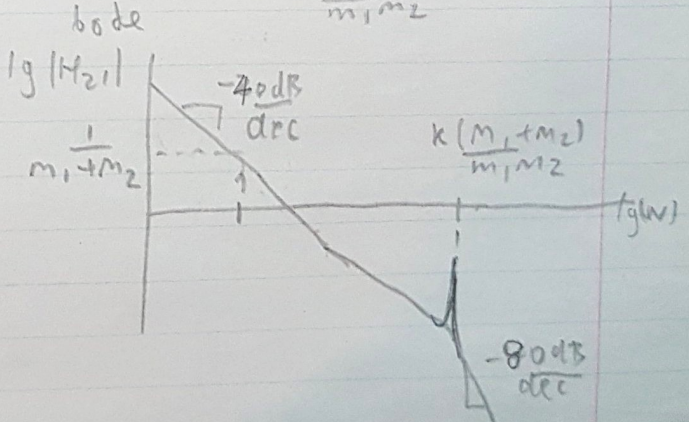
bode

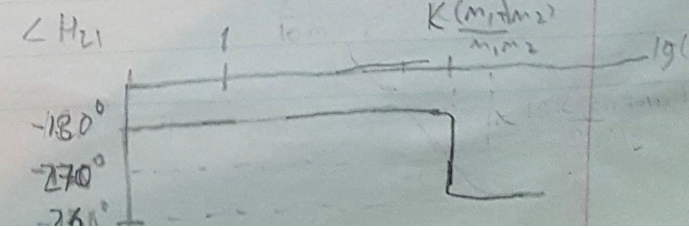




$H_{21} = \dfrac{K}{(M_1 M_2 s^2 + K(M_1+M_2))s^2} = \left(\dfrac{1}{m_1+m_2}\right)\left(\dfrac{1}{\frac{s^2}{\frac{K(M_1+M_2)}{M_1 M_2}} + 1}\right)\dfrac{1}{s^2}$

bode





2a. $H_a(s) = \frac{10}{s^2+101s+100}$

$= \frac{1}{10} \frac{1}{\frac{s^2}{100} + \frac{101}{100}s + 1} = \frac{1}{\frac{s^2}{10} + \frac{101}{10} + 10}$

zero → none

pole → $s = -1, -100$

$m = 1/10$ $\quad \omega_n = 10$

$b = 101/10$ $\quad \zeta = (101/100)(10/2)$

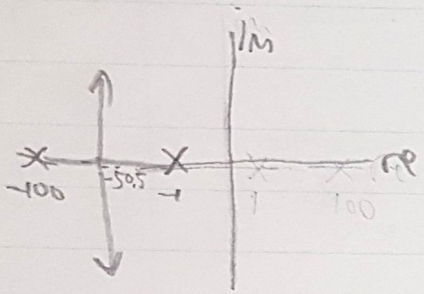
$K = 10$ $\quad = 5.05$

$> 1/\sqrt{2}$

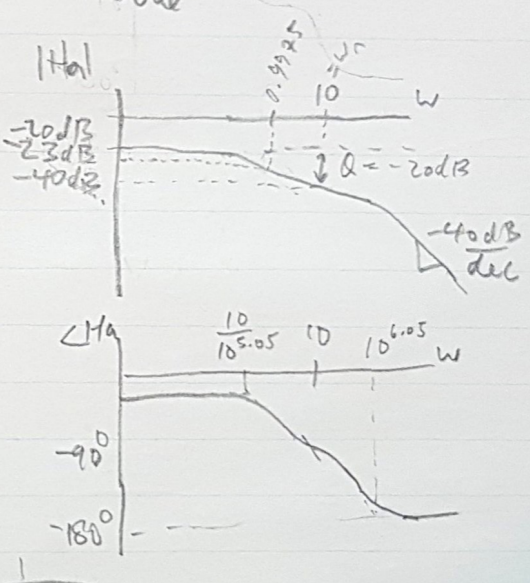
$M_r = \frac{1}{2(5.05)} \frac{1}{\sqrt{1-5.05^2}} = 0$

$Q = \frac{1}{2(5.05)} = \frac{1}{10.1} = -20\,dB$

PZ MAP



bode



2b. $H_b(s) = \frac{1}{1/10} \frac{1}{s^2+20s+100} = \frac{1}{10} \frac{1}{\frac{s^2}{100} + \frac{s}{5} + 1}$

zero → none

pole → $s = -10 \pm \sqrt{\frac{20^2}{4} - 100}$

$= -10$

$m = 1/10$

$K = 10$

$b = 2$

$\omega_n = 10$

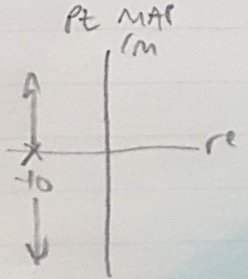
$\zeta = (1/5)(10/2)$

$= 1$

$M_r = \frac{1}{2} \frac{1}{\sqrt{1-1^2}} = \infty$

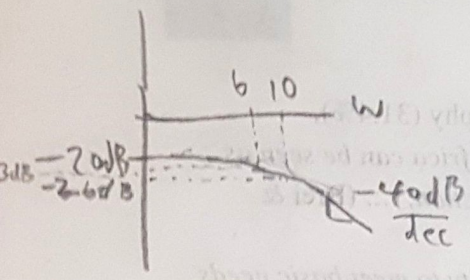
$Q = \frac{1}{2} = -6\,dB$

PZ MAP

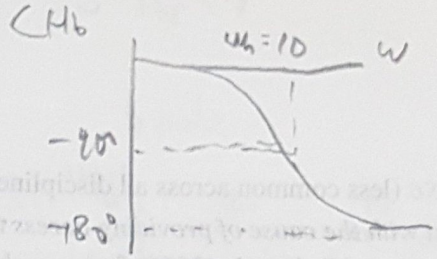


bode

$|H_b|$



$\angle H_b$



2c. $H_c = \frac{1}{1/10} \frac{1}{s^2 + 2\zeta\omega_n + \omega_n^2} = \frac{1}{10} \frac{1}{\frac{s^2}{100} + \frac{s}{50} + 1}$

zero → none

pole →

$m = 1/10$

$k = 1/5$

$K = 10$

$\omega_n = 10$

$\zeta = (1/50)(10/2)$

$= 1/10$

$< 1/\sqrt{2}$

$s = -\frac{2}{2} \pm \sqrt{\frac{2^2}{4} - 10^2}$

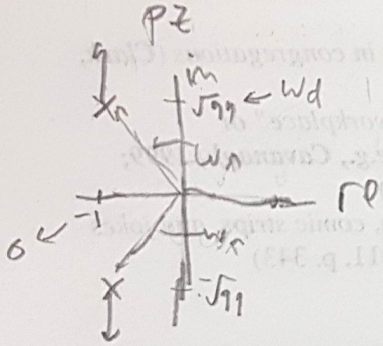
$= -1 \pm i\sqrt{99}$

$\omega_r = 10\sqrt{1 - 2/100} = 9.9 \approx 10$

$M_r \approx Q = \frac{1}{2(1/10)} = 5 = 14\,dB$

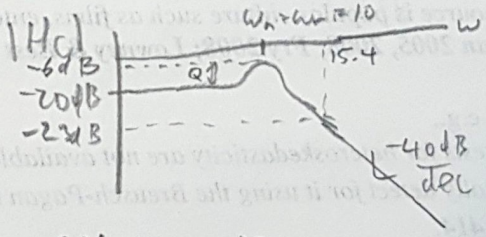
$\omega_d = 10\sqrt{1 - 1/10^2} = 9.95$

$\sigma = (\frac{1}{10})(10) = 1$

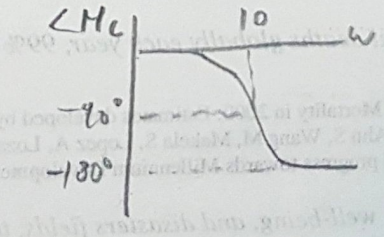


bode

$|H_c|$



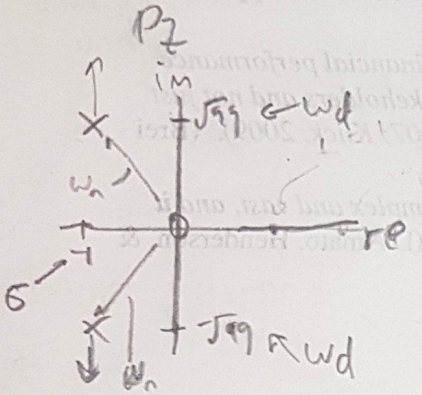
$\angle H_c$



2d $H(s) = \frac{s}{s^2+2s+100} = s\,H_c(s)$

Zero → 0

Pole → $s = -1 \pm i\sqrt{99}$



bode