THE UNIVERSITY OF BRITISH COLUMBIA
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 420 SENSORS AND ACTUATORS

Mid-Term Examination
30 October 2019

- **Duration: 50 minutes**
- **Closed Book/Notes**
- **An 8.5”×11” Fact Sheet (two-sided) is allowed**
- **Calculators are allowed, only in the arithmetic (numerical computation) mode**
- **Fully answer both questions for full credit**
- **Clearly state all your assumptions and give all your steps of the derivations**
- **Define any new variables or parameters that you may use**
- **This question paper contains four (4) pages including this cover sheet.**

Question 1

Consider the circuit shown in Figure 1, which consists of two stages A and B. The input signal to the circuit is v_i and the output signal is v_o.

(a) What is the function of the circuit segment A? What is the function of the circuit segment B? **(5%)**

(b) Derive the time-domain differential equation of the entire circuit, relating v_i and v_o, in terms of the resistance and capacitance parameters shown in Figure 1. Determine the corresponding transfer function $\frac{v_o}{v_i}$ in the Laplace domain. Also, determine the DC gain k and the time constant τ of the overall circuit in terms of the given circuit parameters. **(30%)**

(c) Suppose that $R_1 = 100\ \text{k}\Omega$, $R_2 = 200\ \text{k}\Omega$, $R = 100\ \text{k}\Omega$, and $C = 0.2\ \mu\text{F}$. Compute the half-power bandwidth of the entire circuit in rad/s. **(5%)**

(d) Suppose that the signal v_i is generated by a sensor. With proper justification, suggest a suitable (maximum) time constant for the sensor. **(10%)**

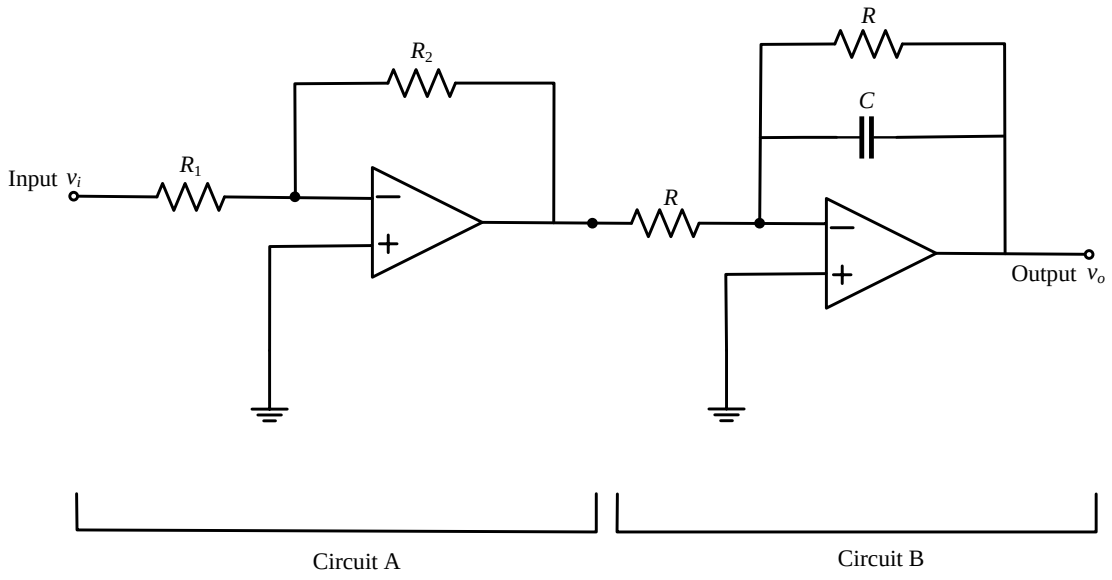

Figure 1: An analog circuit consisting of two stages.

Question 2

(i)

The sensitivities of several sensors, as found in their commercial data sheets, are given in Table 2. Carefully explain the meaning of each of these sensitivity values.

Table 2: Sensitivities of some practical sensors.

Sensor	**Sensitivity**
Blood Pressure Sensor	10 mV/V/mm Hg
Capacitive Displacement Sensor	10.0 V/mm
DC Tachometer	7 VDC±3% for 1000 rpm
Light Sensor (digital output with ADC)	50 counts/lux
Strain Gauge (gauge factor)	150 ΔR/R/strain (dimensionless)

(15%)

(ii)

A Wheatstone bridge circuit is shown in Figure 2. The resistance of the four arms are as indicated in the figure. The supply voltage to the bridge is v_{ref} and the output voltage of the bridge is v_o.

(a) Giving sufficient details of the derivation, show that the output voltage of the bridge is given by $v_o = \frac{R_1 v_{ref}}{(R_1 + R_2)} - \frac{R_3 v_{ref}}{(R_3 + R_4)}$. If $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ what is the output voltage? **(10%)**

(b) In sufficient detail derive the sensitivity of the bridge output to a change in the arm resistance R_1. Specifically, derive an expression for $\frac{\partial v_o}{\partial R_1}$ in terms of R_1, R_2, and v_{ref}. Using suitable normalizing quantities, determine an expression for the corresponding normalized (i.e., non-dimensional) sensitivity. If $R_1 = R_2$, evaluate the non-dimensional sensitivity. **(20%)**

(c) Suppose that R_1 is the only active element in the bridge circuit of Figure 1, and this active element is a semiconductor strain gauge. Also, suppose that initially, the bridge is balanced with $R_1 = R_2 = R_3 = R_4 = R$. For a strain ε, the strain gauge

changes its resistance by δR according to the relation $\frac{\delta R}{R} = S_1\varepsilon + S_2\varepsilon^2$, where S_1 and S_2 are parameters of the strain gauge, which may be obtained from the data sheet of the strain gauge. Derive an expression for the corresponding bridge output δv_o in terms of the measurand ε and the quantities S_1, S_2, and v_{ref}. **(5%)**

(d) Suggest a suitable way to express the sensitivity of the overall device (stain gauge and bridge combination) in Part (c), where the output is in mV and the input is in microstrains (μ-strain). Give an approximate (linearized) expression for this sensitivity. What is the maximum possible sensitivity error? **(Bonus 5%)**

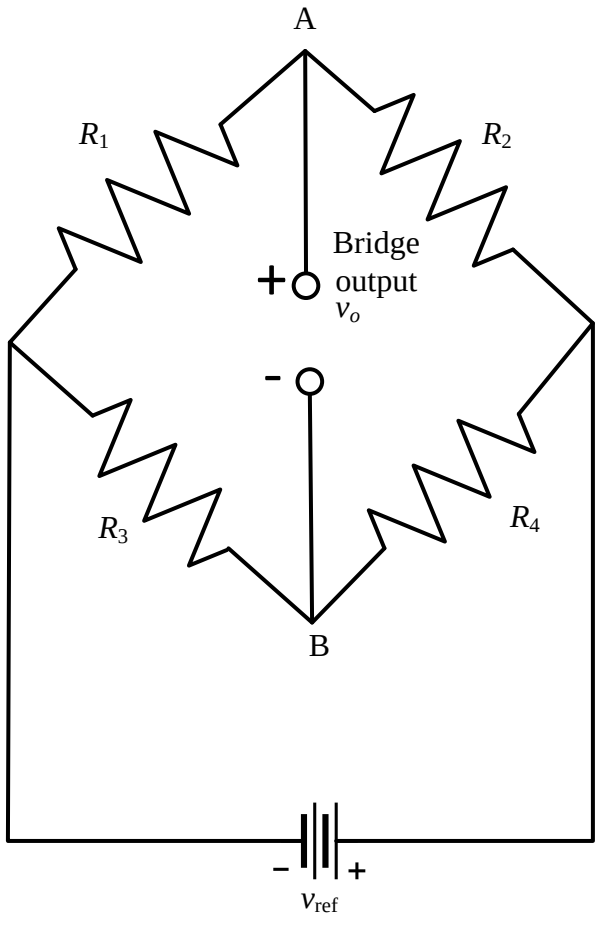

Figure 2: A Wheatstone bridge circuit.