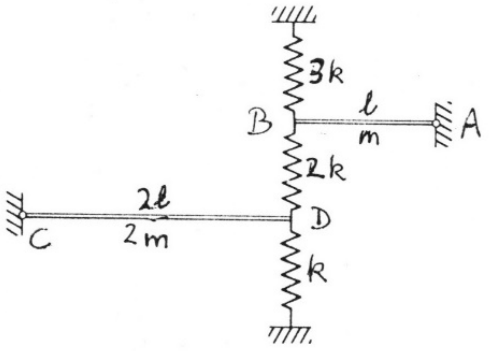
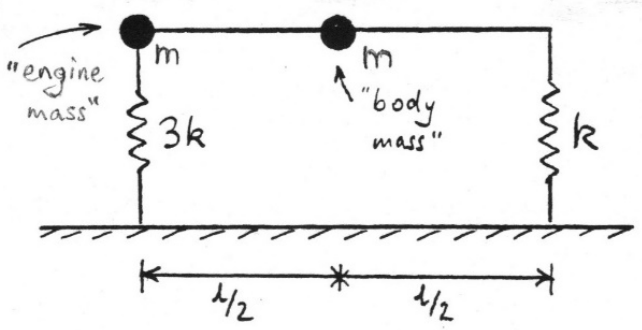
MECH 463 -- Homework 5

1. The diagram shows an idealization of a system of machine parts. Slender bars AB (length ℓ, mass m) and CD (length 2ℓ and mass 2m) pivot freely at A and C respectively. The bars are fastened to the springs by pins at B and D. The three springs have stiffnesses 3k, 2k and k, as shown. Determine the natural frequencies and mode shapes for small vibrations. Comment on any interesting features of the system.



Ans. $\omega^2 = 3k/m,\ 16.5\ k/m \qquad u = 2,\ -0.25$

2. (a) The diagram show a very idealized 2-DOF model of an automobile. A beam of length ℓ, whose mass m is concentrated at its centre, represents the body of the vehicle. A concentrated mass m at one end of the beam represents the engine. Springs of stiffness 3k and k represent the front and rear suspensions respectively. Choose a coordinate system that will give no static coupling, formulate the equations of motion, and solve for the two natural frequencies.



(b) Repeat part (a) using a coordinate system that will give no dynamic coupling. Formulate the equations of motion and solve for the two natural frequencies.

(c) Repeat part (a) using a coordinate system based on the displacement and rotation of the midpoint between the two masses, i.e., the overall mass centre. Formulate the equations of motion for this coordinate system. What type of coupling is present ? Solve for the natural frequencies in the most direct and simple way, and confirm that the results are the same as found in part (a).

(d) Determine and sketch the vibration mode shapes using each of the above formulations. Confirm that the results are geometrically consistent. Comment on what you find.