Vibration Due to Rotating Unbalance

Mech 364: Mechanical Vibrations

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11/29/2010

Abstract

Vibrations are throughout the physical world. From guitars to cars, and in daily activities like washing your clothes to riding your bike or listening to your favourite track through your headphones. Some vibrations are pleasant, such as the harmonics produced by a skilled guitarist. However, some are unpleasant, such as turbulence during flight, or a bumpy road. This experiment was conducted to further understanding of mechanical vibrations. In the experiment, we modelled a car engine supported by four mounts using a mass supported by four springs and a viscous damper. Vibrations in this system were caused by rotating eccentric masses.

During the experiment we used an onboard accelerometer to record the acceleration felt by the supported mass through a range of rotational speeds. The vibratory response was measured and analyzed for two cases: the In Phase case, where the rotating eccentric masses are in synchrony, and the Out of Phase case where the masses are asynchronous. These two cases produced different vibratory responses from the system.

Analysis of the responses shows that they follow closely with the theoretical predictions, deviating only on the low end of the frequency range where the noise in the signal was quite large. The measured natural frequency(11.1 Hz for In Phase Case) of the system was very close to the predicted natural frequency (11 Hz for In Phase Case), differing in that it was slightly larger than the theoretical value. This slight difference is likely because of the damping present in the system. Analysis of the damping present in the system showed that the Out of Phase case had a significantly smaller damping ratio than that of the In Phase case. This difference is likely due to the types of motion the apparatus experienced in the two cases as well as the location of the viscous damper.

Introduction

During this experiment, vibrations are caused by rotating eccentric masses. An example of similar vibrations occurs due to spinning clothes in a washing machine. Outside of the laboratory, vibrations must be studied by engineers as they have the potential to cripple structures, destroy mechanisms and cause machines to function improperly. This experiment was designed to expose young engineers such as myself to vibrations so that I can learn to analyze their effects, and learn to mitigate their effects in future designs. The apparatus consisted of a mass supported by four springs and a viscous damper. Rotating eccentric masses could be placed such that they rotate in phase and out of phase from each other. We examined how the system responded in each of those cases, as well as when the systems were at their resonant frequencies. This report will outline the findings of the experiment while contrasting theoretical predictions to the experimental data.

Apparatus

The experimental apparatus was designed to be an approximate model of an automobile engine secured by engine mounts. Roughly, the apparatus (“the Shaky Table”) consists of a box-shaped structure supported by 4 identical springs and a damper. Out of balance forces are created in the system by mounting two counter-rotating shafts which have eccentric masses secured to them.
Specifically, the “Shaky Table” consists of:

-Disks, with attached eccentric masses which are fixed on parallel shafts
-A motor, to drive the shafts
-A metal enclosure to house the components
-Four identical springs to mount the system
-A dashpot to provide a damping force in the system
-A tachometer to measure the speed of the motor
-A speed controller
-A Power supply
-An accelerometer mounted to the top surface of the device
-A Data Acquisition device to collect data from the accelerometer (DAQ module)
-A charge amplifier to amplify the data signal

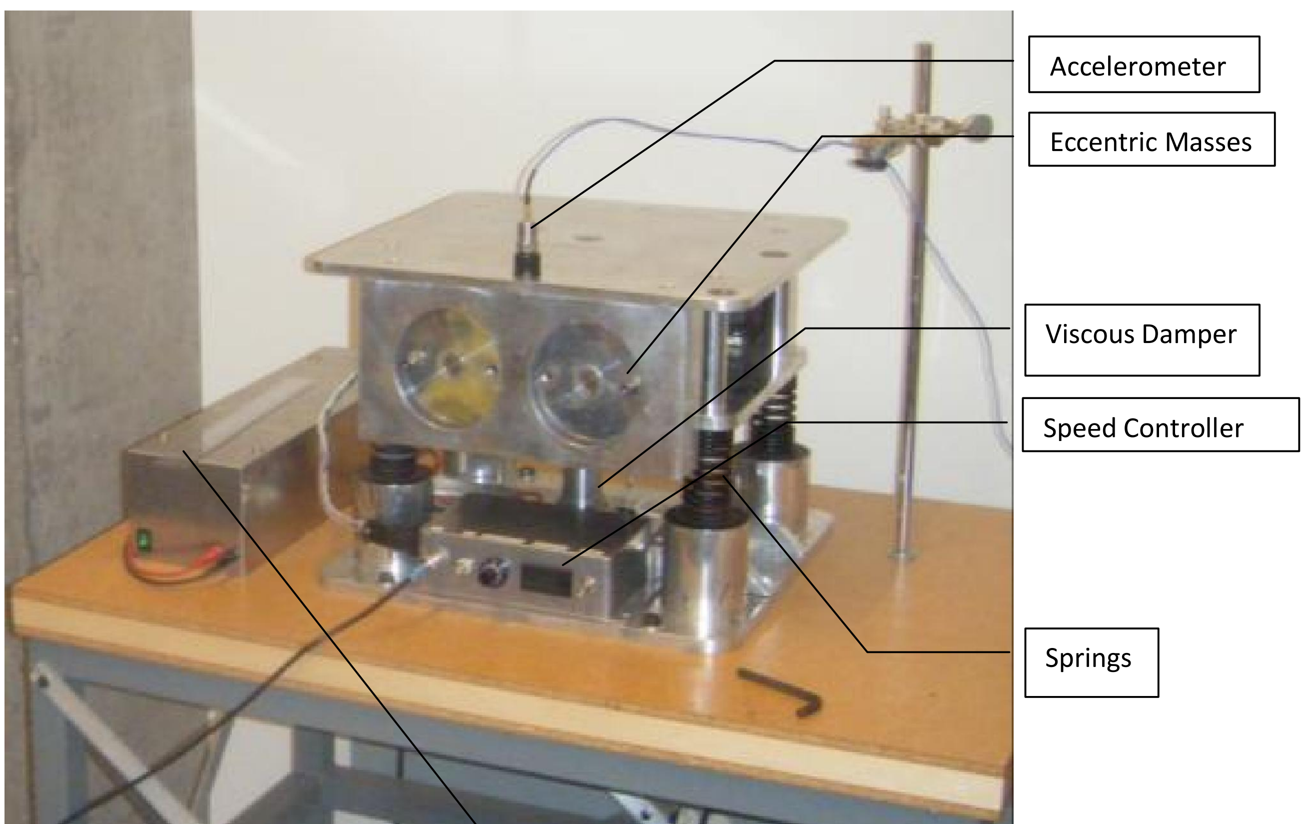


Figure 1 - Photo of the Shaky Table Apparatus

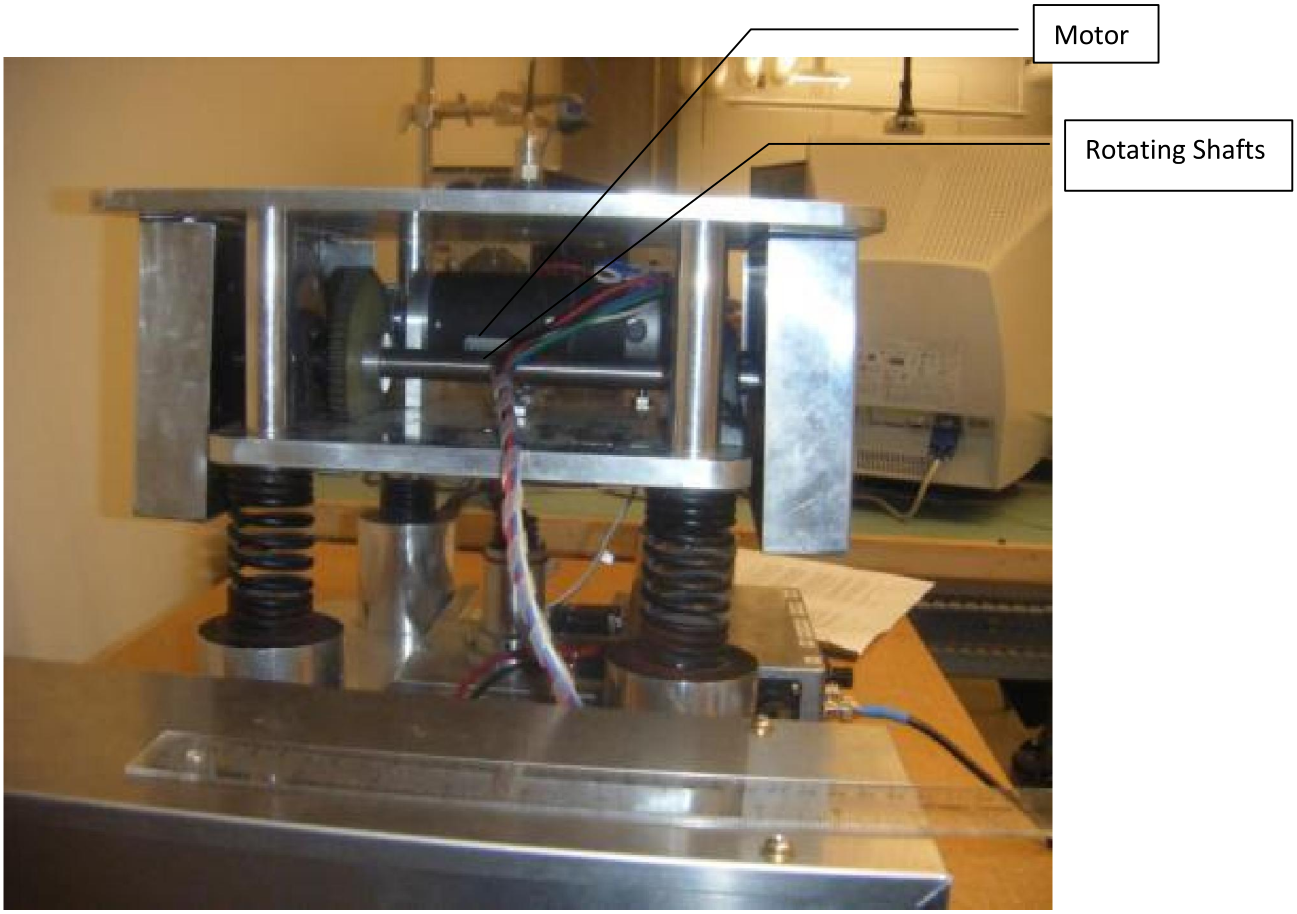
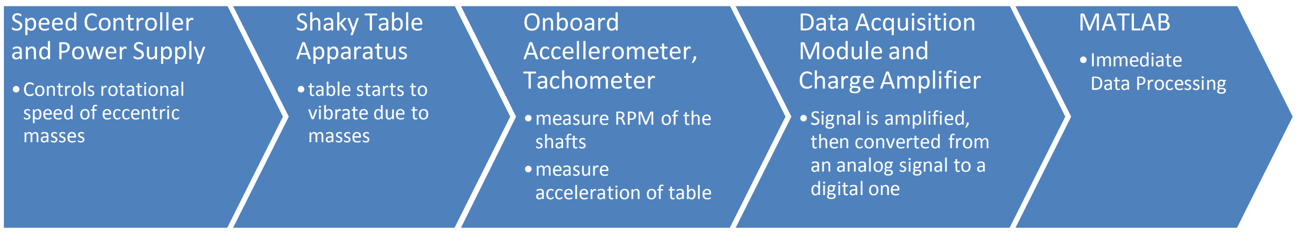


Figure 2 - Photo of the Shaky Table Apparatus

The progression of from inputs to outputs is as follows:



The key dimensions and quantities of the Shaky Table are listed below.

Table 1 - Shaky Table Key values

Shaky Table	**Quantity**
Distance between disks – front to back	35.5 cm
Distance between springs – front to back	21.5 cm
Distance between springs – left to right	32 cm
Radius of eccentric mass	4 cm
Spring center-to-center distance	30.5 cm
Total mass	14.993 kg
Mass of accelerometer	0.1 kg
Mass of damper piston	0.1134 kg
Spring constant	14365.3 N/m
Eccentric mass	0.0152 kg

Method

The procedure for this experiment is as follows:

In Phase configuration response measurements:

1) Ensure the accelerometer is placed in the centre of the Shaky Table (Location A)
2) Ensure that the eccentric masses on both the front and back of the Shaky Table are positioned in the same location, and securely fastened to the rotating disks. (i.e.: The eccentric masses on the front of the machine are both at their highest location and the masses on the back of the machine are also at their highest location)
3) Prepare MATLAB for data acquisition:
    a. Set the number of channels to 2
    b. Set the sampling frequency to 5 kHz
4) Start the Shaky Table and set the rotational speed
5) Start the logging of data in MATLAB
6) Use the Calculate Amplitude and Phase function in the MATLAB code provided
    a. Visually check the plots produced and compare to expected response
7) Save the plots and data that MATLAB has produced, and note the output from the command dialogue.

Repeat steps 3 through 7 for rotational speeds of 20 Hz through to 1 Hz at 1 Hz intervals. Additional measurements should be taken near the resonance frequency, which in the In Phase configuration is near 11 Hz.
To estimate the damping ratio in this configuration, first set the rotational speed of the shafts to resonance (approximately 11 Hz), next start the data logging in MATLAB, then shortly (5-10 seconds) after turn off the power to the shaky table and let the machine come to rest. Save the plots and data from MATLAB. The damping ratio can then be estimated by reading the first three consecutive peaks and the logarithmic decrement.

For the Out of Phase configuration response measurements:

1) Ensure the accelerometer is placed at Location B on the surface of the Shaky Table
2) Ensure that the eccentric masses on both the front and back of the Shaky Table are positioned in opposite locations from each other (i.e.: The eccentric masses in the front of the device are at their lowest position and the masses on the back of the device are at their highest)
3) Prepare MATLAB for data acquisition:
    a. Set the number of channels to 2
    b. Set the sampling frequency to 5 kHz
4) Start the Shaky Table and set the rotational speed
5) Start the logging of data in MATLAB
6) Use the Calculate Amplitude and Phase function in the MATLAB code provided
    a. Visually check the plots produced and compare to expected response

7) Save the plots and data that MATLAB has produced, and note the output from the command dialogue.

Repeat steps 3 through 7 for rotational speeds of 20 Hz through to 1 Hz at 1 Hz intervals. Additional measurements should be taken near the resonance frequencies, which in the Out of Phase configuration are near 15 Hz and 6 Hz.

To estimate the damping ratio in this configuration, first set the rotational speed of the shafts to the second resonance frequency (approximately 6 Hz), next start the data logging in MATLAB, then shortly (5-10 seconds) after turn off the power to the shaky table and let the machine come to rest. Save the plots and data from MATLAB. The damping ratio can then be estimated by reading the first three consecutive peaks and the logarithmic decrement.

Results

This section will present the results obtained through MATLAB and Excel analysis of the data. MATLAB was used to log and filter the data directly from the apparatus, and Excel was used to analyze the data as well as plot the experimental data against the theoretical predictions.

In Phase Vibration Response

Typical Signals

Below is the typical signal as recorded during the experiment. This specific data set is from the 12.04 Hz trial. In the figure, the blue signal is that of the accelerometer. This signal is clearly very noisy. Shown below in red, is the tachometer signal.

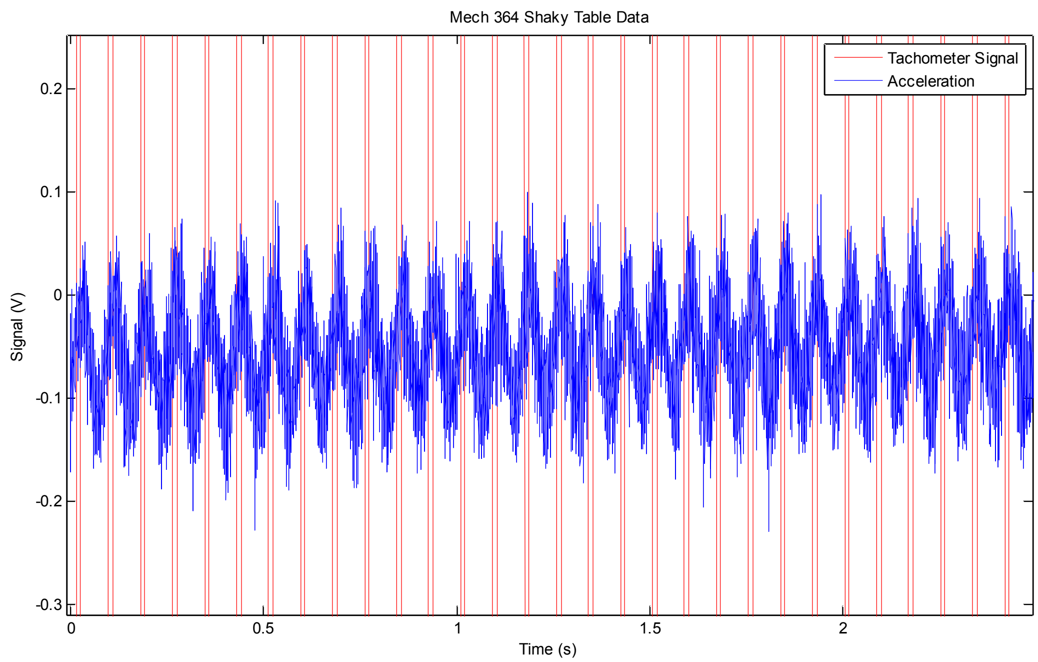


Figure 1 - Typical unfiltered signal, 12.04 Hz trial

After applying a filter to the signal to reduce the amount of noise and make the data ready for analysis, a much cleaner plot is produced. The very messy and difficult to read acceleration curve of figure 4 is replaced by a single relatively smooth line.

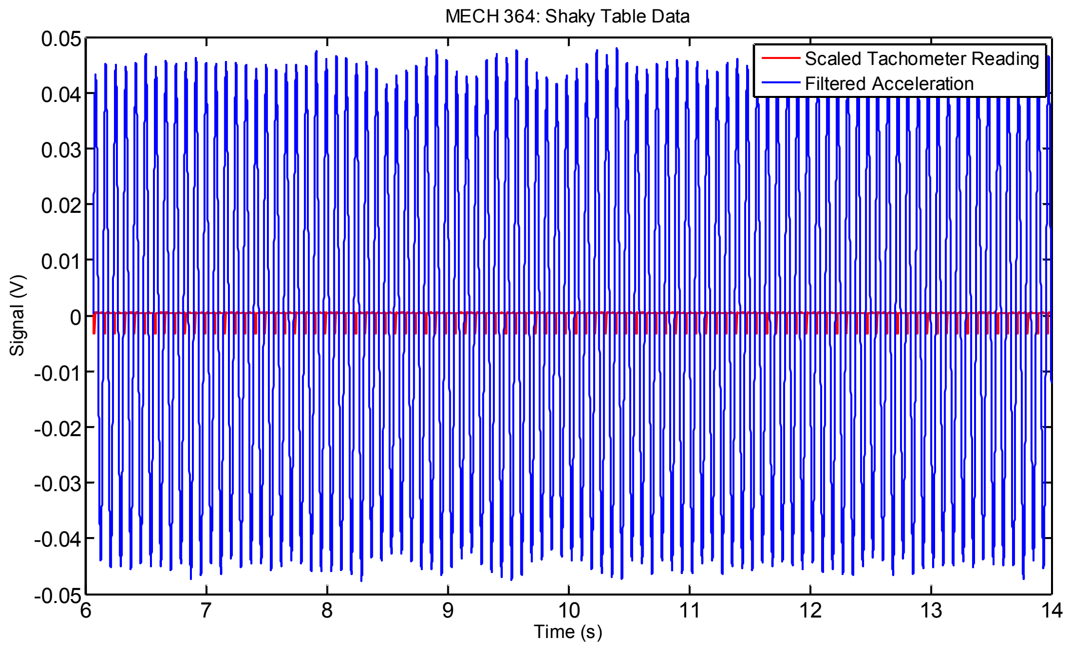


Figure 2 - Typical Filtered Signal, 12.04 Hz trial

From this much cleaner data, we can zoom in on the plot to produce a very clear and detailed signal.

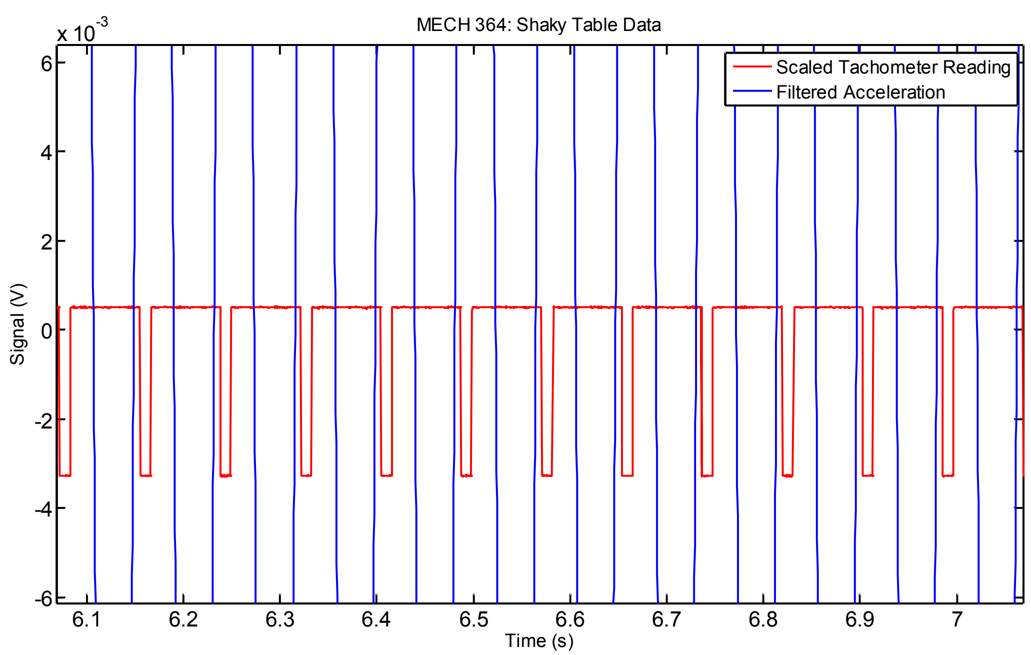


Figure 3 - Zoomed in Plot of a Typical Filtered Signal, 12.04 Hz trial

Measured Responses

After recording data across the frequency range suggested in the procedure, we can look to analyze the results. For the complete numerical results, please see Appendix A.

Since the accelerometer outputs a signal to the DAQ as a voltage, the first step in analyzing the data is to convert this voltage signal to one in m/s^2. This is accomplished using the calibration constant 99 mV/g. To convert to m/s^2, we use the relationship:

$$Acceleration\ Amplitude\ (\frac{m}{s^2}) = \frac{Acceleration\ Amplitude(V)}{Calibration(\frac{V}{g})} * g(\frac{m}{s^2})$$

To take the acceleration amplitude and convert it to the displacement amplitude, we use the relationship:

$$Displacement\ Amplitude = \frac{Acceleration\ Amplitude\ (\frac{m}{s^2})}{\omega^2(\frac{rad}{s})}$$

where ω is the frequency in radians/second.

Now, with both the displacement and acceleration amplitudes, we utilize the following relationships between displacement, velocity, and acceleration to analyze the data.

$$x(t) = Acos(\omega t)$$

$$v(t) = -\omega Asin(\omega t) = -\omega Acos(\omega t + 90°)$$

$$a(t) = -\omega^2 Acos(\omega t) = \omega^2 Acos(\omega t + 180°)$$

By plotting the experimental data we can draw conclusions and match them to the theoretical expectations. Theory predicts that near the resonant frequency of approximately 11 Hz we should see a large increase in the displacement and acceleration of the system. This prediction is confirmed by the figures below.

Figure 6 shows the displacement amplitude versus frequency across the whole range of measured values. As noted earlier, this response fits well with predictions. Near the system's resonance frequency (approximately 11 Hz) we can see a dramatic increase in displacement amplitude, peaking near 1.8 mm.

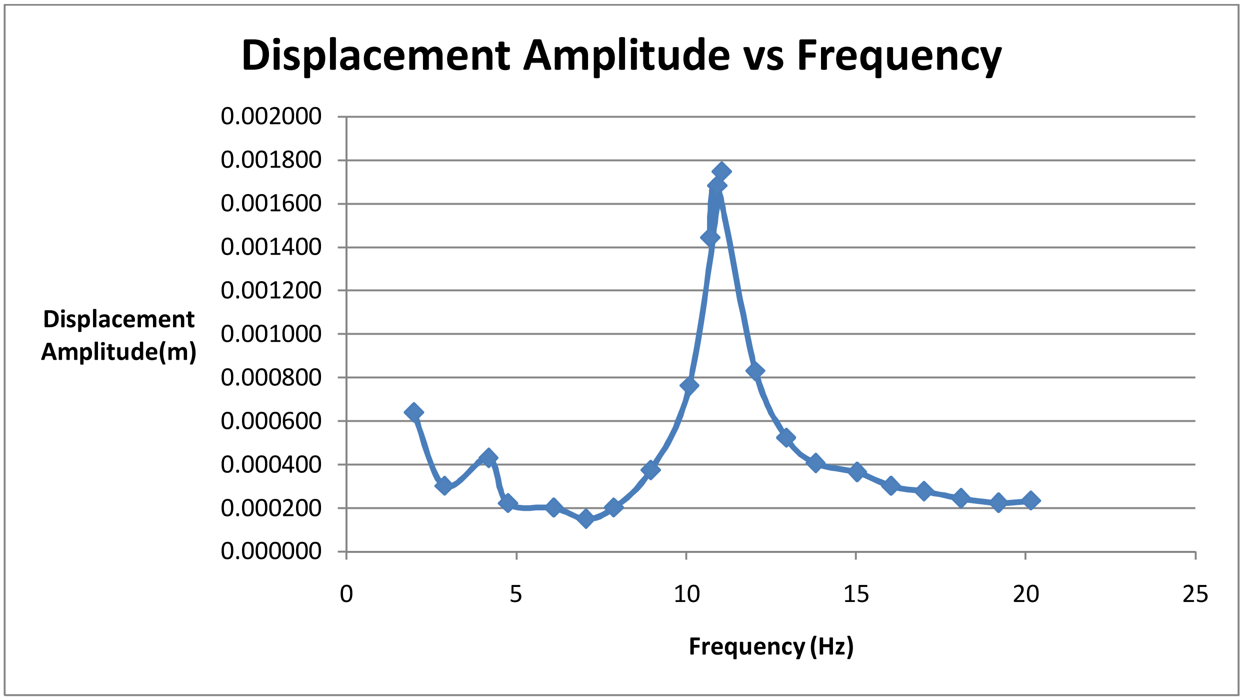


Figure 4 - Displacement Amplitude vs. Frequency for In Phase configuration

Figure 7 confirms that near resonance the phase angle of the response decreases drastically.

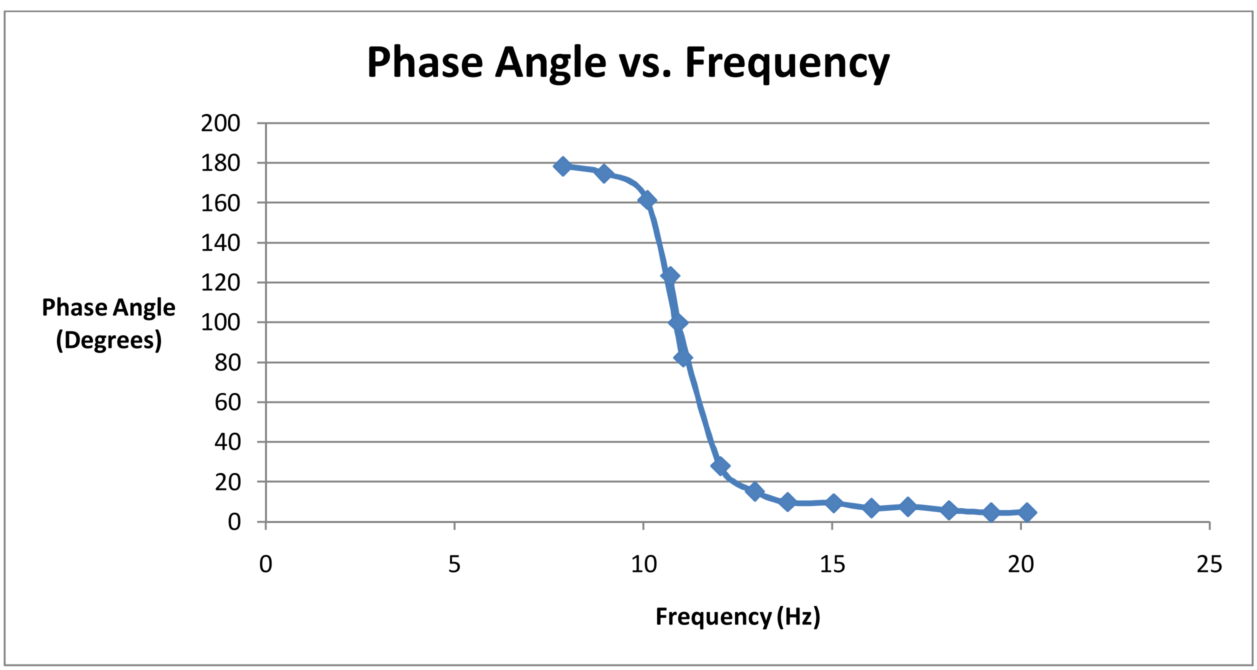


Figure 5 - Phase Angle vs. Frequency for In Phase Configuration

Figure 8 shows the acceleration of the system plotted against the frequency for this In Phase configuration. From the plot, we notice that the system's acceleration spikes near resonance and levels off on both sides of this resonant frequency.

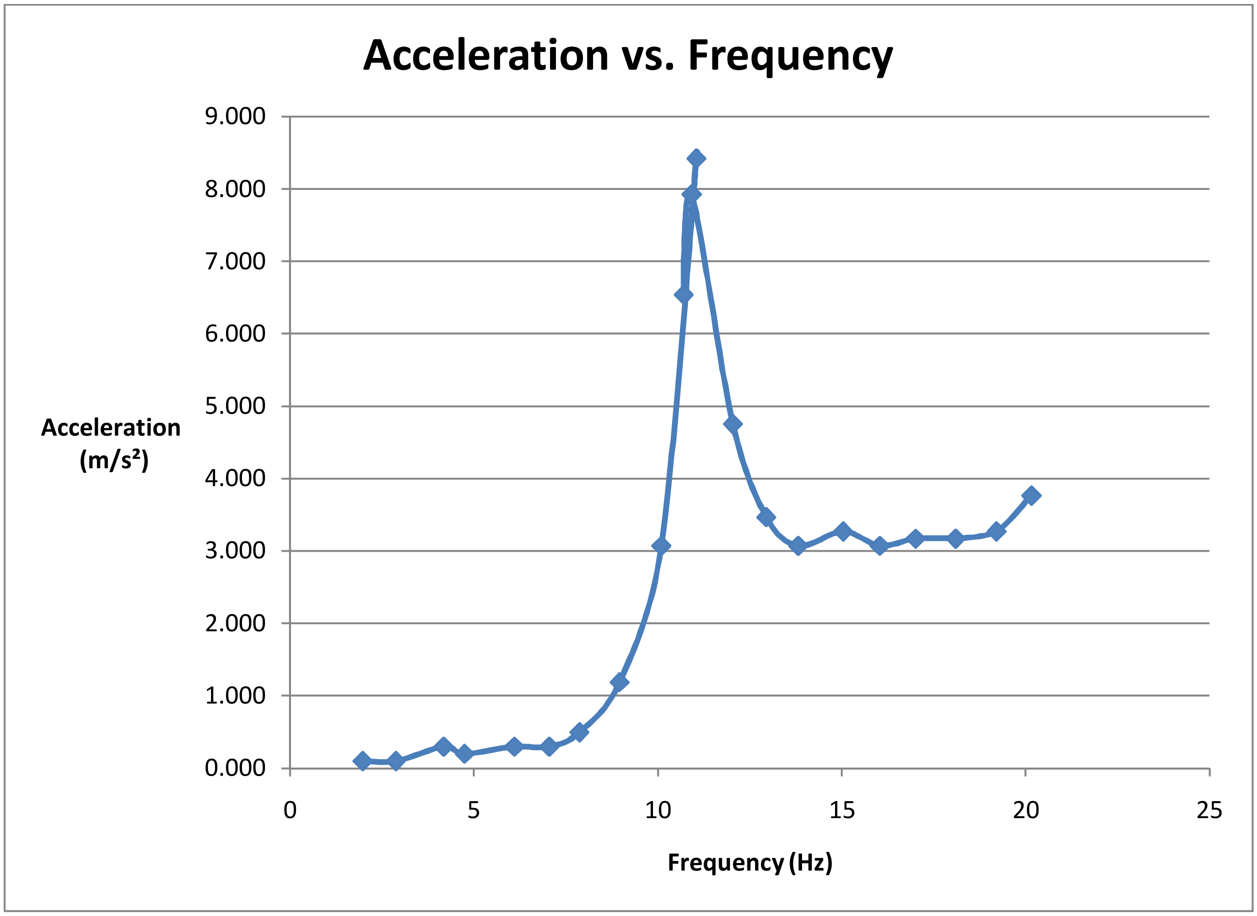


Figure 6 -Acceleration vs. Frequency for In Phase Configuration

Estimation of Damping from In Phase Configuration

To estimate the damping coefficient, we utilize the logarithmic decrement. First, we need a filtered response to analyze. Shown below is the filtered plot for the 11.05 Hz trial, which occurs very near to resonance.

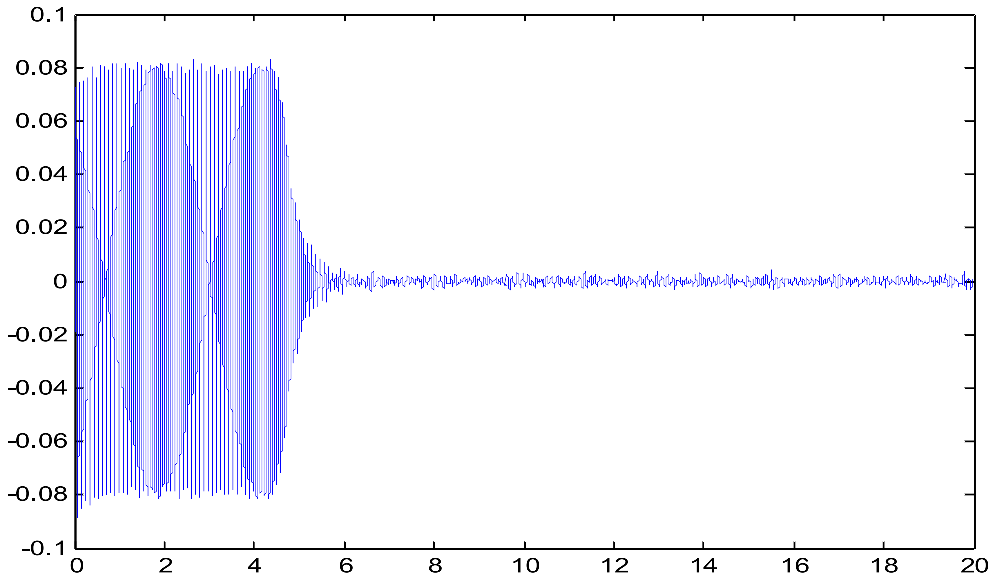


Figure 7 - Filtered Plot for 11.05 Hz

Zooming in on the filtered plot, so that the first few clean peaks are visible so that the logarithmic decrement can be estimated:

Peak 1(4.648, 0.06721) X: 4.648 Y: 0.06721 , Peak 2 (4.741,0.05139) X: 4.741 Y: 0.05139 , Peak 3(4.833,0.03458) X: 4.833 Y: 0.03458

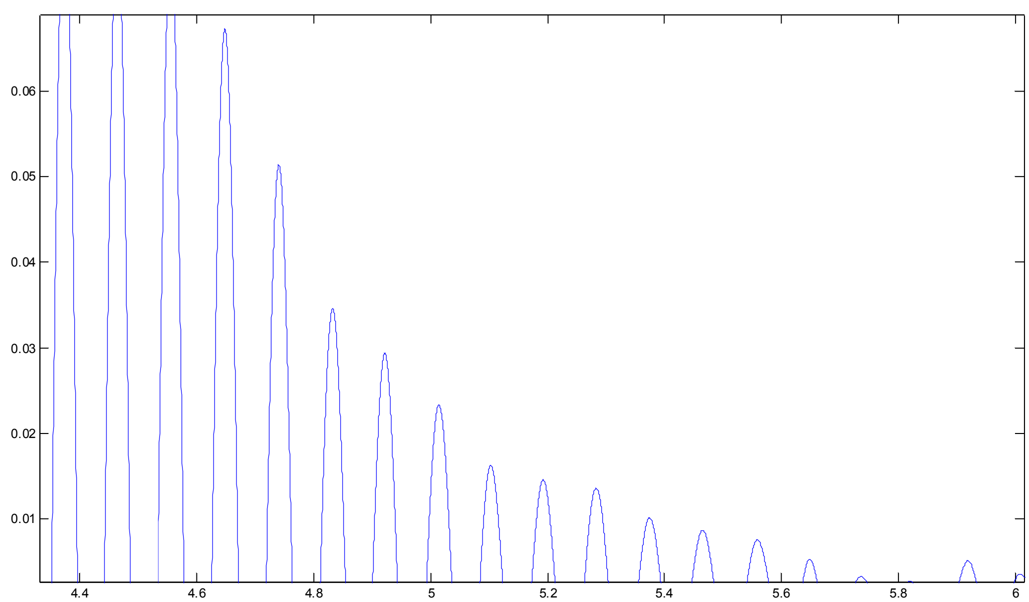


Figure 8 - Zoomed in Filtered Response for 11.05 Hz trial

We use the Logarithmic Decrement:

$\delta = \left(\frac{1}{r}\right)\ln\left(\frac{A_i}{A_{i+r}}\right)$ and Displacement Amplitude, $A(m) = \frac{\frac{Acceleration\ Amplitude(V)}{Calibration(\frac{V}{g})} * g(\frac{m}{s^2})}{Frequency(\frac{rad}{s})^2}$

$$\delta = \left(\frac{1}{3}\right)\ln\left(\frac{\frac{\frac{0.06721 * 9.81}{0.099}}{69.43^2}}{\frac{\frac{0.03458 * 9.81}{0.099}}{69.43^2}}\right)$$

$$\delta = 0.221$$

Next, using:

$$\zeta \approx \frac{\delta}{2\pi}$$

and substituting with known values:

$$\delta = 0.221$$

$$\zeta \approx \frac{0.221}{2\pi}$$

We arrive with an estimate of the damping ratio for the In Phase Case of:

$$\zeta \approx 0.0352$$

Comparison to Theoretical Responses

Figures 9 and 10 show the experimental data plotted against their theoretical counterpart. In both cases we notice that the experimental data matches reasonably well against the theoretical values. In Figure 9, we observe a very tight match between the two curves throughout the range. However, in Figure 10, the Frequency Response Function, we note deviance from the theoretical values at the low end of the range. This low end correlates to a small rotational speed of the eccentric masses compared to the natural frequency of the system. These lower rotational speeds lie approximately in the range of 1-8 Hz. In this range, noise in the signal is much more prevalent and can explain this deviance from theory.

Theoretical values were calculated using the following relationships:

$$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}}$$

$$r = \frac{\omega}{\omega_n}$$

$$u = \frac{4me}{M}$$

$$\frac{Y}{u} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\tan(\varphi) = \frac{2\zeta r}{1-r^2}$$

Where:

K_{eq} = Equivalent Spring Constant

M_{eq} = Equivalent Mass

m = Mass of rotating eccentric mass

M = Mass of System

r = frequency ratio

φ = Phase angle

ζ = Damping ratio

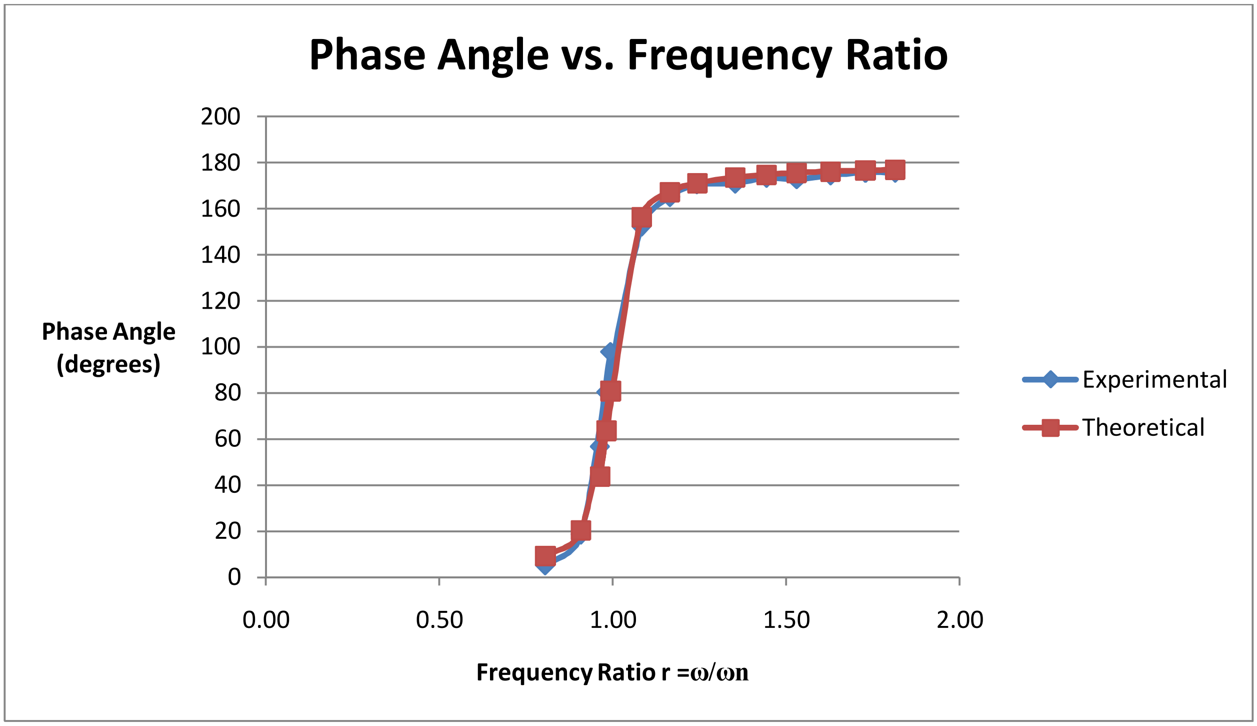


Figure 9 - Phase Angle vs. Frequency Ratio for In Phase Configuration

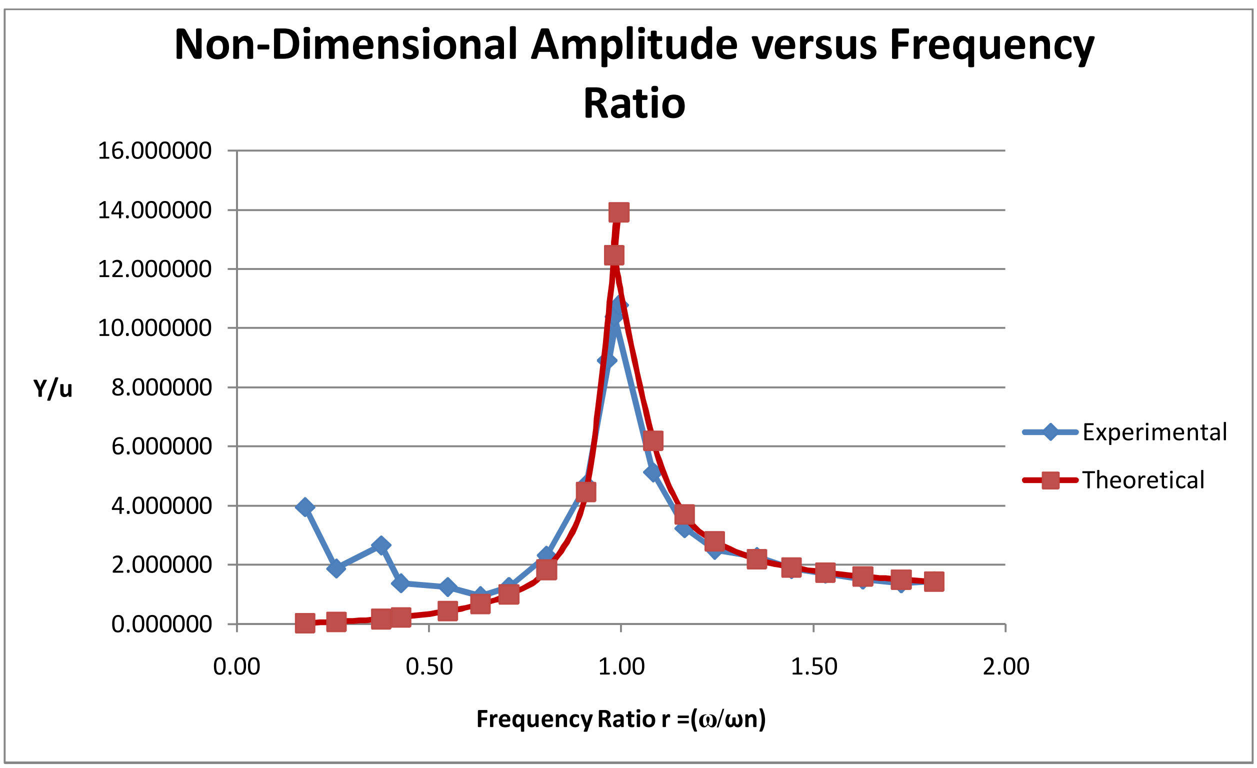


Figure 10 - Non-Dimensional Amplitude vs. Frequency Ratio for In Phase Configuration

Out of Phase Vibration Response

Typical Signals

Below is the typical signal as recorded during the experiment. This specific data set is from the 7.95 Hz trial. In the figure, the blue signal is that of the accelerometer. This signal is clearly very noisy. Shown below in red, is the tachometer signal.

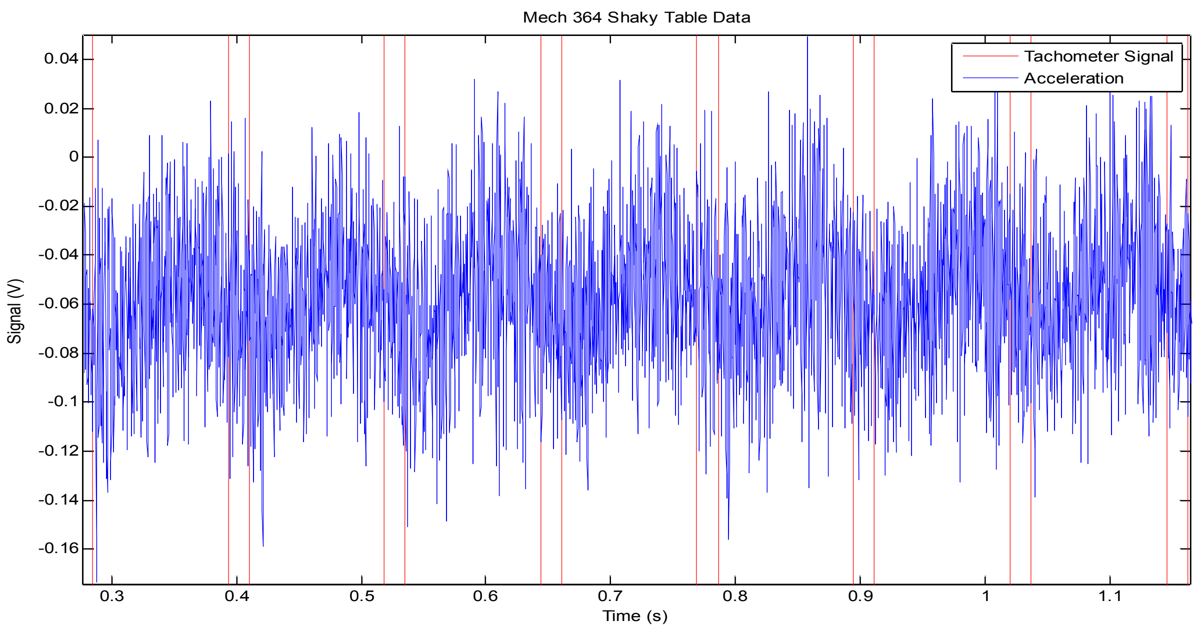


Figure 11 - Typical Unfiltered Signal, 7.95 Hz trial

After applying a filter to the signal to reduce the amount of noise and make the data ready for analysis, a much cleaner plot is produced. The very messy and difficult to read acceleration curve of Figure 11 is replaced by a single relatively smooth line in Figure 12.

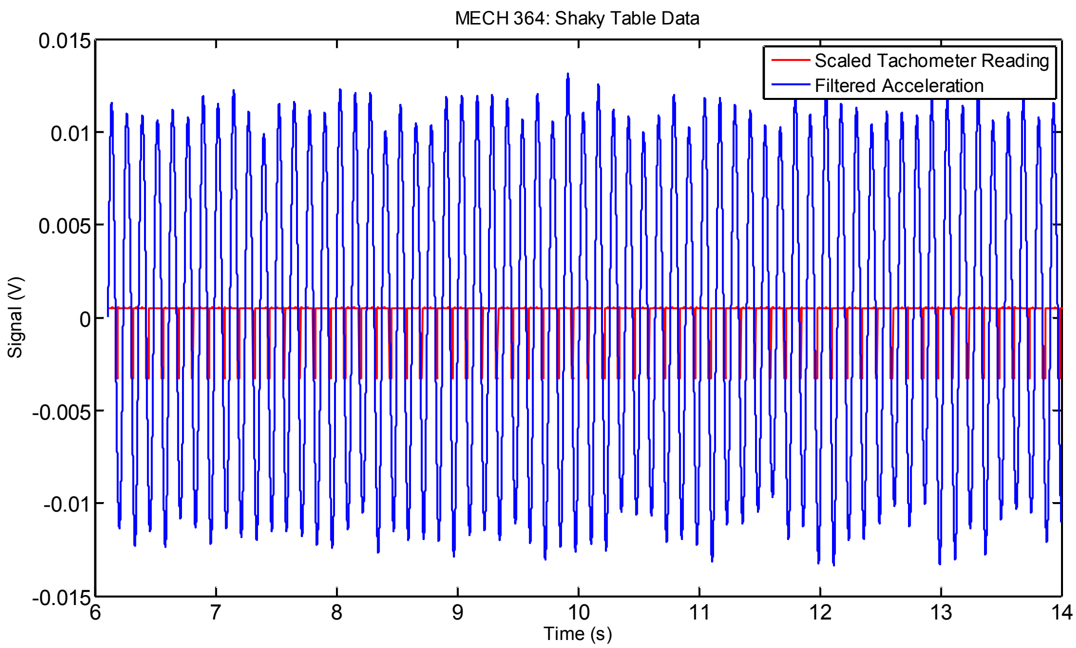


Figure 12 -Typical Filtered Signal, 7.95 Hz

From this much cleaner data, we can zoom in on the plot to produce a very clear and detailed signal (Figure 13).

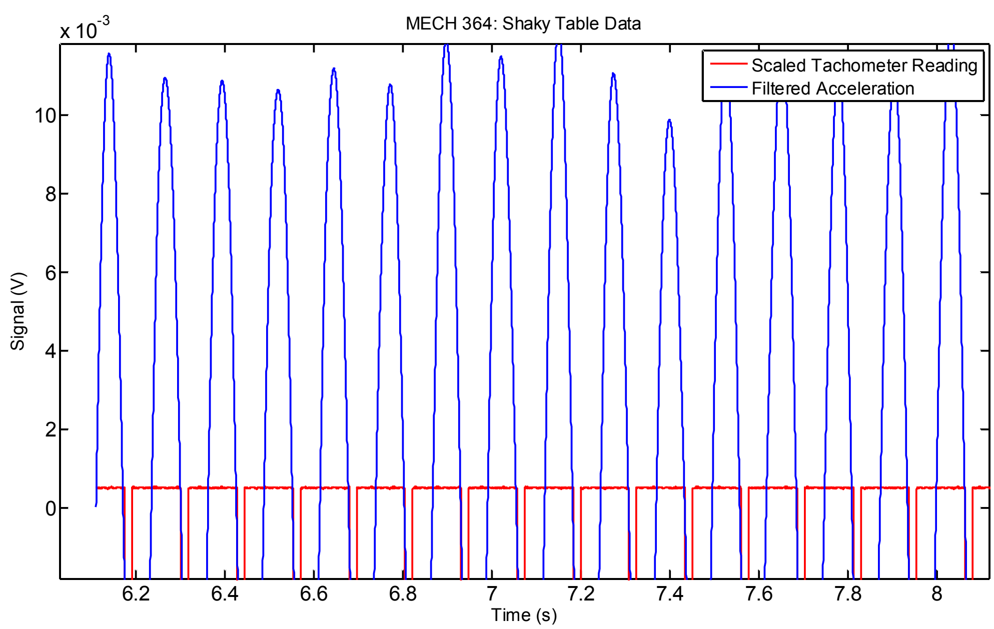


Figure 13 - Zoomed in Typical Filtered Signal, 7.95 Hz

Measured Responses

Responses from the out of phase configuration will be presented here. Similar to the In Phase configuration, theory predicts that near resonance we should see dramatic changes to the acceleration amplitude. This is indeed the case, as shown in Figure 14.

In the Out of Phase case we notice two peaks, as opposed to the one peak shown in the In Phase case. This is because in the 1-20 Hz range that was used, there are two resonant frequencies for this system. The first resonant frequency occurred at around 15 Hz, and the second around 6 Hz. We note that the Shaky table was relatively calm between the two resonant peaks, showing very small acceleration values.

As noted by Figure 15, the phase angle changes as the rotational speed approaches resonance. The two peak values indicate the resonant frequencies. Data below 6.5 Hz was too noisy and was omitted from Figure 15.

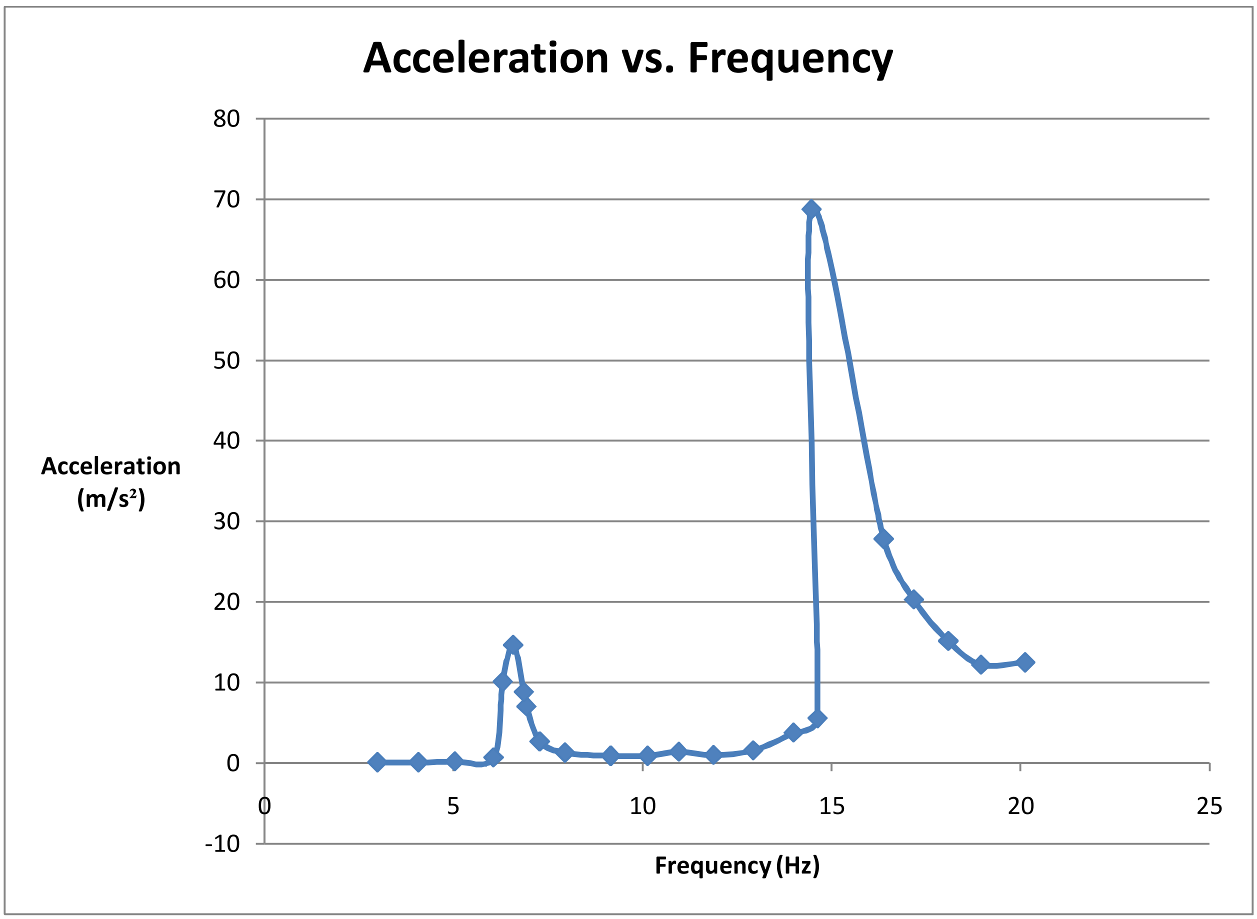


Figure 14 - Acceleration vs. Frequency for Out of Phase configuration

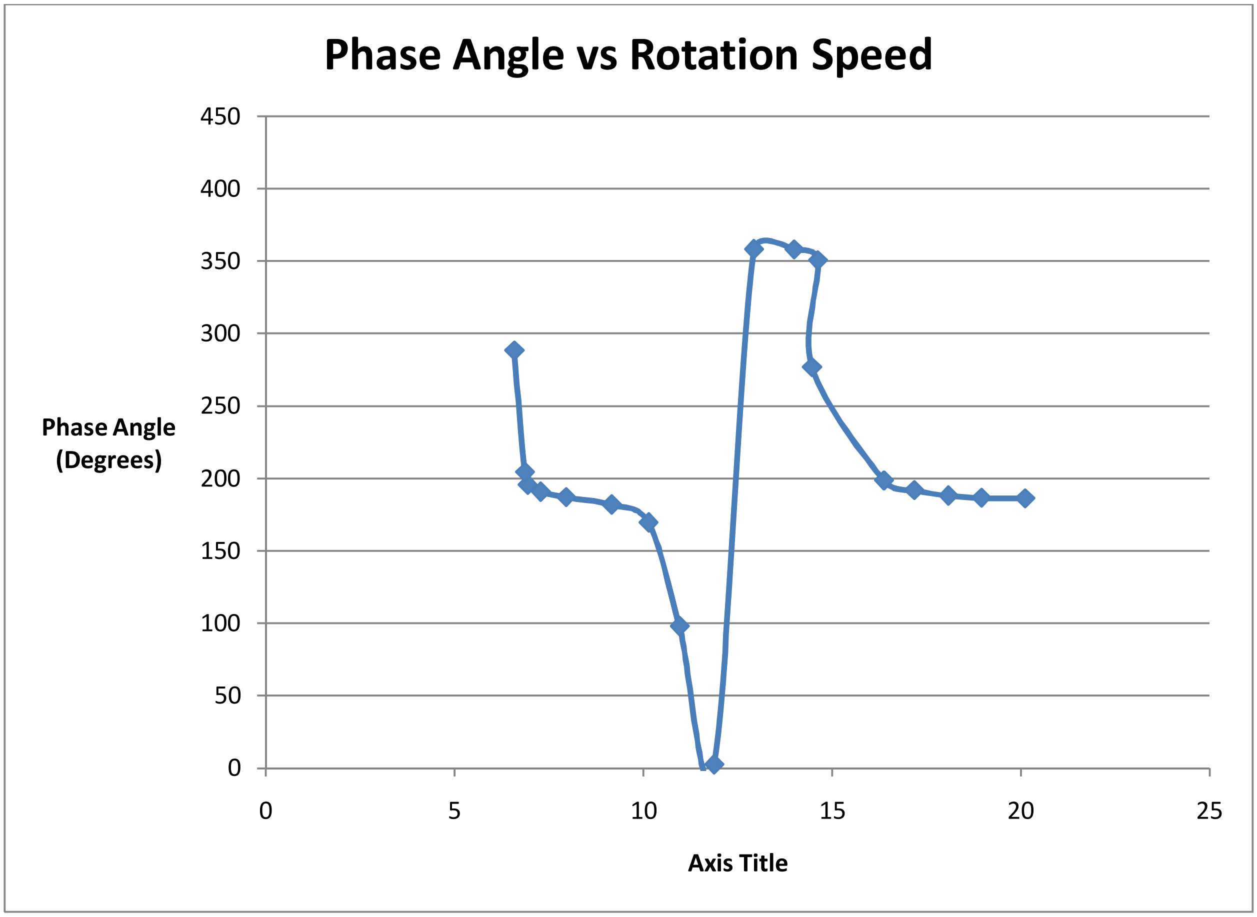


Figure 15 - Phase Angle vs Rotation Speed

Estimation of Damping Coefficient from Out of Phase Resonant Response, 6.3hz

To estimate the damping coefficient, we utilize the logarithmic decrement. First, we need a filtered response to analyze. Shown below is the filtered plot for the 6.3 Hz trial, which occurs very near to the second resonance frequency.

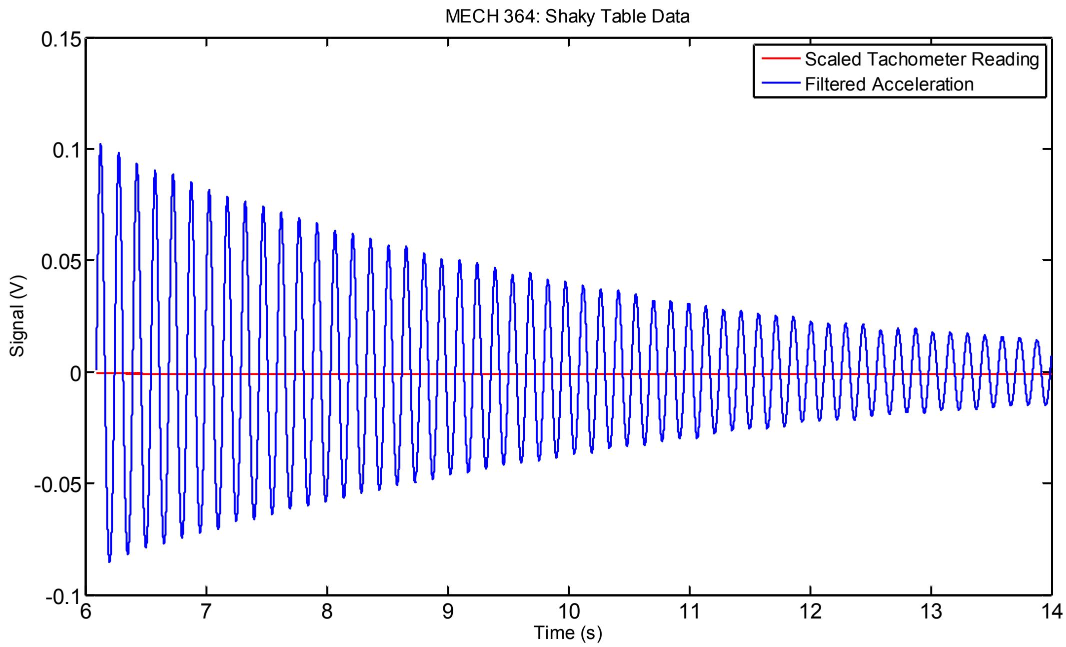


Figure 16 - Filtered Plot for 6.3 Hz trial

Zoomed in on the First 3 waves:

-Pick the 1st peak(Signal Voltage = 0.102) , and the 4th peak(Signal Voltage = 0.09)

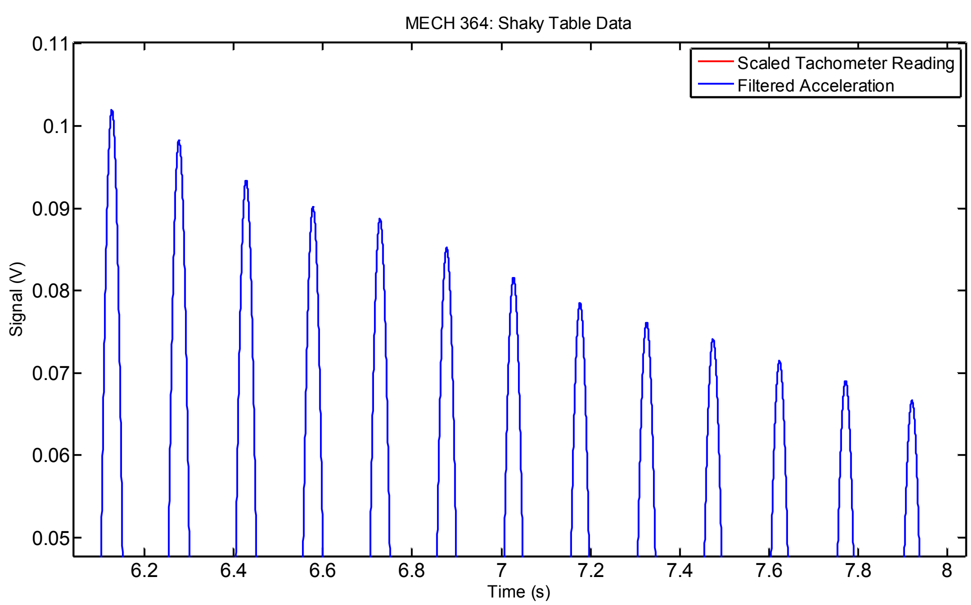


Figure 17 - Zoomed in Filtered Plot for 6.3 Hz trial

Using the Logarithmic Decrement:

$\delta = \left(\frac{1}{r}\right) \ln\left(\frac{A_i}{A_{i+r}}\right)$ and Displacement Amplitude, $A(m) = \frac{\frac{Acceleration\ Amplitude(V)}{Calibration(\frac{V}{g})} * g(\frac{m}{s^2})}{Frequency(\frac{rad}{s})^2}$

$$\delta = \left(\frac{1}{4}\right) \ln\left(\frac{\frac{\frac{0.102 * 9.81}{0.099}}{39.58^2}}{\frac{\frac{0.09 * 9.81}{0.099}}{39.58^2}}\right)$$

$$\delta = 0.0132$$

Next, using:

$$\zeta \approx \frac{\delta}{2\pi}$$

And substituting with known values:

$$\delta = 0.0132$$

$$\zeta \approx \frac{0.0132}{2\pi}$$

$$\zeta \approx 0.0208$$

Therefore, via the logarithmic decrement, the damping coefficient is estimated to be approximated 0.0208.

Moment of Inertia Estimation

Using the formula $\omega_n = \sqrt{\frac{4Kl^2}{I_c}}$ we can estimate the moment of inertia of the system. At resonance, the frequency of vibration is approaching equality with the natural frequency of the system and so we can substitute this resonant frequency in this equation for use in the estimation.

Rearranging the equation to solve for I_c yields:

$$I_c = \frac{4Kl^2}{{\omega_n}^2}$$

Substituting in the known values, K = 18277 N/m and l = 0.16 m yields an approximate moment of inertia for this system of I_c =1.09 kg m^2.

Discussion

In this experiment, we measured the acceleration and vibratory frequency of the Shaky Table as it was subjected to oscillations caused by rotating eccentric masses placed in either an In Phase or Out of Phase configuration. MATLAB was used to log the signal transmitted to a DAQ module by an onboard accelerometer and tachometer.

During the experiment we observed the violent response of the system as it was vibrating at its resonance frequency. We observed that at each resonant frequency the oscillations of the system were extremely large.

From the data plots we were also able to estimate the damping ratio of both the In Phase and Out of Phase configurations of the system. The In Phase system has an estimated damping coefficient of 0.0352 and the Out of Phase system has an estimated damping coefficient of 0.0208. The major difference in damping coefficients can be explained by noting the motion of the system in each configuration. During the In Phase vibration response, the motion of the Shaky Table is a vertical translation. However, due to the imbalance, in the Out of Phase response, the Shaky Table tends to vibrate in a rotational manner, with the front and back oscillating from high to low in opposition to each other. This difference in motion coupled with the location of the viscous damper causes the reduction in effective damping visible in the damping ratios. Since the viscous damper is mounted in the centre of the Shaky Table it can effectively dampen translational motions in the vertical plane. However, when the motion turns to rotation, since the damper is positioned at essentially the pivot point its effectiveness is drastically reduced.

Discrepancies between what the response was predicted to be from theory and what was observed in the data are likely due to the large amount of noise in the signal. As noted in the In Phase Vibration Response discussion earlier, in the low end of the frequency range (approx 1-8 Hz) the signal is flooded with noise. This noise translates to less accurate calculations and in turn the deviance noted.

Questions from Page 18 of Lab Manual

Can you list at least two possible differences between this experiment and the vibrations in a washing machine?

One difference between the vibrations in this experiment and those of a washing machine are due to the relative simplicity of our experiment compared to the washing machine. The washing machine has eccentric masses unevenly distributed as it spins, and has a large amount of fluid swirling around as well.

Another possible difference is that the washing machine is not rigidly mounted to the floor whereas the Shaky table is rigidly mounted. The rigid mounting of the Shaky Table likely dampens the vibrations considerably.

How long must you wait between recording data for two different RPMs?

You must wait until the system has reached its steady state. This takes approximately 15-20 seconds.

Is the system lightly damped? Justify

Yes, this system is lightly damped. This is evident when observing the motion of the machine. The machine does not immediately return to rest, as you would expect with a critically damped system, nor does it stop oscillating and slowly return to rest as would be expected in an over damped system. Instead, the system continues to vibrate with shrinking amplitudes as is consistent with an under damped system. This conclusion is again evident when noting the estimated damping coefficients for both configurations. In both cases, they are less than 1, consistent with under damped systems.

Explain why r=1 in Fig. A3 is not the location of the maximum response.

r =1 is not the location of maximum response due to damping in the system. Damping in the system shifts the peak to the right of r=1.

Why does the width of pulse (channel 1 data in your measurements) change with rpm in readings from optical encoder (channel 1)?

The width of the pulse changes with the RPM of the system because of the mounting of the Tachometer. During the lab, the TA demonstrated that the masses and shafts could be spun manually and that this would turn the tachometer light on as the mass passes the sensor. The sensor will remain activated for a few degrees of rotation. As the rotational speed increases, this lag is less evident. It is very similar to a bicycle computer (with wheel mounted magnet-sensors) in this sense.

Questions from Throughout Manual

Can you see a clear harmonic acceleration response in Channel 2? Why not?

No, there isn't a clear harmonic response in channel 2. This is because there is too much noise in the signal.

Comment on the phase changes near resonance and why it occurs?

The phase drops dramatically near resonance, and this is because at lower frequencies, the force and displacement are in phase. At higher frequencies, they are out of phase by 180 degrees. Near resonance this shift occurs.

It is crucial that you excite the system as close to resonance as you can. Why?

It is crucial to excite the system as close as possible to resonance for accurate estimation of the damping ratio. The closer you get to resonance, the larger amplitudes you have, and these large amplitudes are critical to start with if we want to observe a clear decay due to damping.

U is a measure of amplitude of input excitation to the system. Why? Think of the case e=0. What is the amplitude of the excitation force?

Y/u is a measure of amplitude through non-dimensionalization, where u = 4me/M. If e = 0, u would also equal zero which would yield a division by zero error. Physically, this means that the masses are rotating at the very centre of a circle, and this positioning would not cause any centripetal force.

What is the phase relation between acceleration and force? How is acceleration related to displacement in harmonic motion? Also how does the trigonometric identity tan (180+phi) = tan(phi) influence the phase relation?

At r=0, the displacement is in phase with the acceleration/force. As the rotation speed increases towards r =1, the displacement is 90 degrees behind the force, and as r >1 displacement is 180 behind the force.

The equations below indicate the relationship between acceleration/force and displacement.

$x = A\cos(\omega t)$

$v = -\omega A\sin(\omega t) = -\omega A(\omega t + 90)$

$a = -\omega 2A\cos(\omega t) = \omega 2A\cos(\omega t + 180)$

We can see that the acceleration value has a phase shift of 180 degrees in this harmonic motion. That indicates that displacement is lagging behind force. The identity tan(phi) = tan(180+phi) does not affect this relation.

Table A1 shows some practical frequency response functions and their peaks. Which one of them is useful in this experiment?

The middle column is useful in this lab because they can be compared to our results. This column is normalized resonant frequencies and corresponds to the A3 graphs.

Did you observe an exponentially decaying sinusoid of one time period in the experiment? Why not?

No, I did not observe an exponentially decaying sinusoid of one time period in the experiment. That response would correspond to a critically damped system. As noted earlier, this system is overdamped.

Learning Outcomes

Through this experiment I was able to learn a great deal about how rotating eccentric masses can cause vibrations in a system, and the response provided by the system. Specifically, I noted that systems that have an Out of Phase configuration of their rotating masses can experience violent vibrations which may be damaging. These powerful vibrations occurred at or near the resonant frequencies for the system. Therefore, we should take care when designing any system that is subject to vibrations to ensure that the frequency of vibration is not near resonance. While the vibrations at the resonant frequencies are very large in amplitude, a slight change in frequency can drastically reduce the strength of vibrations.

This experiment was also very useful in taking the theory and formulas learned through the lecture series and applying them to a real system. Vibrations are everywhere, and the ability to confidently apply the theories from the chalkboard onto these systems is empowering for an engineer. With this ability to analyze vibrations I can now design against vibrations in the future.

Appendix A