MECH 467 – Computer Control of Mechatronics Systems

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Project 1: Modeling and Identification of Motion Control Mechanism

The first step in designing a digital controller is to model the open loop transfer function of the physical system which needs to be controlled. The system can be modeled mathematically by using the laws of solid mechanics, fluid mechanics, dynamics and electric circuits. Mathematical modeling is usually performed at the design stage, before the machine is built. Once the machine is built, additional effort is made to improve the accuracy of the open loop transfer function model by applying experimental identification techniques.

The aim of this project is to model a single axis, ball screw driven positioning table. Both mathematical modeling and experimental identification of open loop system dynamics will be covered in this laboratory-supported engineering project.

Identification of a Ball Screw Feed Drive System

The single-axis ball screw table shown in Figure 1 is a high speed feed drive capable of 1g acceleration, and 42 m/min feed speed.

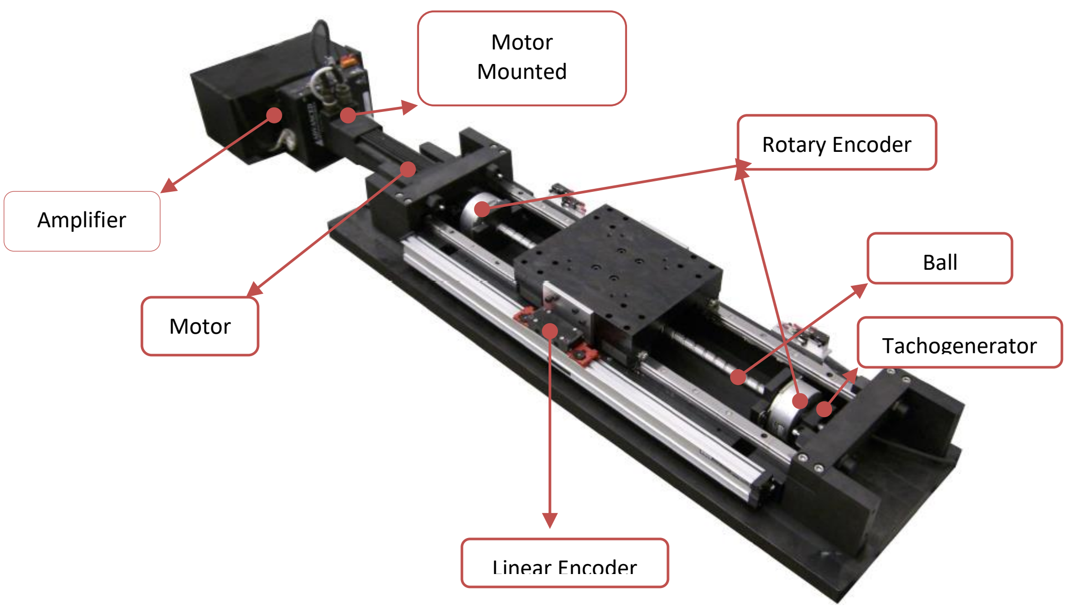

Figure 1: Ball Screw Feed Drive System

The amplifier can be operated in two modes: torque mode and velocity mode. In this project our focus is on the torque mode of the amplifier. When the drive is operated in torque mode, voltage commands sent to the amplifier are scaled proportionally into current that create torque in the motor. The block diagram that is equivalent to the physical system in torque mode is depicted in Figure2.

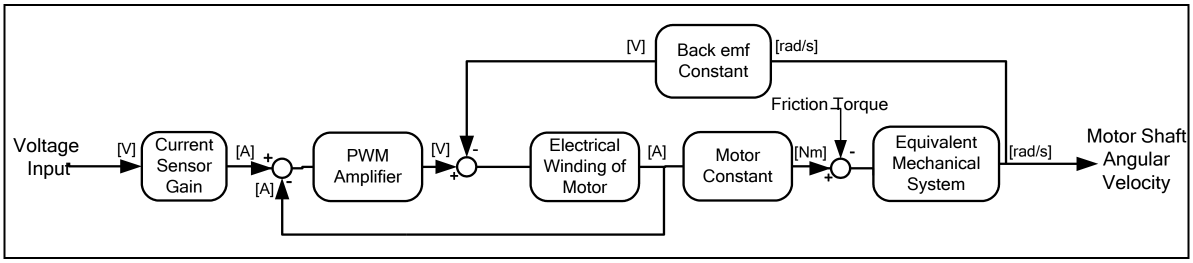

Figure 2: Ball Screw Feed Drive Block Diagram in Torque Mode

Pre Lab

This pre-lab must be completed and submitted to the TA before the lab. The pre-lab constitutes 40% of your final grade for this lab, and you will not be allowed to perform the lab without completing it. The pre-lab can be **neatly** handwritten or typed but the step response and frequency response plots must be printed out.

1. Mathematical Modeling:

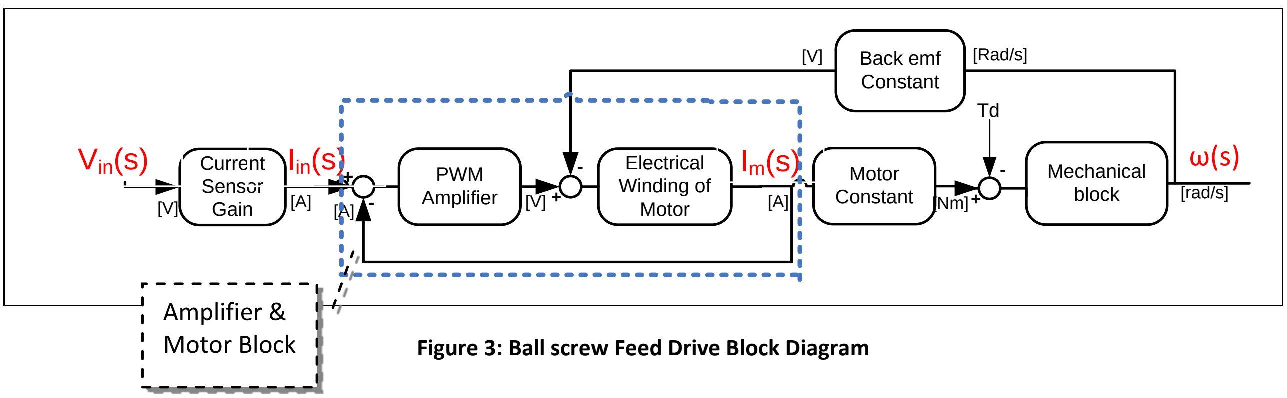

Figure 3: Ball screw Feed Drive Block Diagram

a) Find the equivalent inertia of the system, using the law of mechanics and provided data in Table 1.
b) Find the transfer functions from $I_{in}(s)$ to $I_m(s)$, from $V_{in}(s)$ to $I_m(s)$ and from $I_m(s)$ to $\omega(s)$. Assume T_d=0. You may name the blocks and carry out the derivations in terms of block names, but you are recommended to expand them so you see the order of the transfer functions. Make sure to cancel the common factors in the numerator and denominator.
c) By using the two transfer functions found from Part (a), find a transfer function from the voltage command input to the drive's angular velocity (i.e. $V_{in}(s)$ to $\omega(s)$)
d) Draw the block diagram with respect to disturbance input only (i.e. Vin=0). Rearrange the block diagram such that T_d appears as the input on the far left of the diagram. Keep the original blocks without combining them.
e) Find the transfer function from disturbance input ($T_d(s)$) to angular velocity ($\omega(s)$) (i.e. assume $V_{in}(s)$=0)

Note: The PWM amplifier block transfer function is given as: $\dfrac{V_p}{I_p} = K_{vp} + \dfrac{K_{vi}}{s}$

Motor Torque Constant	K_t	0.72	[Nm/A]
Motor Back EMF Constant	K_b	0.4173	[V/rad/s]
Amplifier Proportional Gain	K_{vp}	111.55	[V/A]
Amplifier Integral Gain	K_{vi}	$3.0019 \text{ X}10^5$	[V/A.s]
Current Sensor Gain	Sg	0.887	[A/v]
Motor Armature Resistance	R_a	6.5	[Ω]
Motor Armature Inductance	L_a	0.0375	[h]
Motor Rotor Inertia	J_m	$0.765\text{X}10^{-4}$	[kg.m²]
Table mass	m_t	20	[kg]
Ball screw pitch	h_p	0.02	[m]
Ball screw length	L_s	820	[mm]
Ball screw Diameter	d_s	20	[mm]
Steel density	ρ_s	7800	[kg/m³]
Inertia of two encoders	J_{enc}	$1.7 \text{ X}10^{-4}$	[kg.m²]
Inertia of Couplings	J_{coup}	$4 \text{ X}10^{-5}$	[kg.m²]
Inertia of tachogenerator	J_{tach}	$9.3212\text{x}10^{-7}$	[kg.m²]

Table 1: Ball Screw Feed Drive Parameters

2. Model Order Reduction

In this question use the approximate damping value of 0.006 [Nm/rad/s] until the true damping and friction values found by testing the actual machine.

a) Substitute the parameters from Table 1 into the transfer function derived in Parts 1(b) and 1(c) and plot the step response and frequency response of the transfer function from $I_m(s)$ to $\omega(s)$. Find the bandwidth, Dc gain and the rise time.

b) Find the frequency response of the transfer function from $I_{in}(s)$ to $I_m(s)$. Find the bandwidth and Dc gain.

c) Plot the unit step response, unit ramp response and frequency response of transfer function from $V_{in}(s)$ to ω (s). Find the bandwidth, Dc gain and rise time. It is better to scale the velocity response with the DC gain of the transfer function to plot both the input and output on the same scale. Compare the bandwidths and rise times obtained in 2(a) and 2(c). What do you notice? Also compare the Dc gains obtained in 2(a) and 2(c). Find the steady state error for a ramp input response analytically, and compare against the simulation result. Note that the input V_{in} is in volt and the output ω is in radian/sec. In order to find the steady state error of the ramp input, you need to scale the transfer function by the DC gain of the system.

d) Based on your observations, draw a simplified (reduced) block diagram of the ball screw feed drive system (which reasonably represents frequencies up to 1000 [rad/s]).

Note: If you derive the transfer functions in terms of blocks in your MATLAB code without cancelling the common factors in numerator and denominator, you may get very large rise time in your step response (in versions before 2015). You can resolve this issue by obtaining the minimal realization (pole-zero cancellation) using the command “minreal”.

Experiment procedure

A. Identification of Friction Parameters

1. To identify the Coulomb and viscous friction parameters, move the table at constant speeds and measure the voltage supplied to the amplifier (i.e. the current supplied to the motor to overcome the frictional torque). Use the following table speeds: 15, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 120, 140, 160 [mm/s]. Convert the measured table speeds [mm/s] to angular velocity of the motor [rad/s] using the pitch of the ballscrew. Also calculate the corresponding motor current [A] and motor torque [Nm] from the measured voltage using the amplifier and motor gains. Plot the friction curve, i.e. torque [Nm] vs angular velocity [rad/s], for the positive and negative directions on the same plot.

2. Find the equivalent viscous damping constant (B_e) in [Nm/rad/s] by fitting straight lines to each direction of the friction curves and averaging the values of the slopes obtained for each of the lines.

3. Obtain the Coulomb friction constant in each direction (i.e. μ_{k+} and μ_{k-}) from the y-intercept of the fitted lines. Do you think it makes sense that μ_{k+} and μ_{k-} could have different magnitudes? Explain briefly. For simplicity average the magnitudes of μ_{k+} and μ_{k-} and obtain a single Coulomb friction constant (μ_k).

4. Simple friction curves are usually described using the **sign** function as: $T_d = \mu_k \text{sign}(\omega)$. However, the **sign** function does not give an accurate description of the friction behavior at low speed region, i.e. ω~0. Can you briefly explain why? Roughly sketch the following curves on top of each other in one plot: a) Friction curve using **sign** function, b) Friction curve with more accurate modeling of friction at low speeds, c) total friction curve (viscous+Coulomb) assuming friction is modeled as **sign** function, d) total friction curve with more accurate modeling at low speeds. (hint: search Stribeck friction on the internet)

   Note: Use the simple model $T_d = \mu_k \text{sign}(\omega)$ in the following analyses.

B. Identification of Equivalent Inertia

From the pre-lab, you observed that the open loop behavior of the ball screw drive can be reduced to a simpler form described by the equation:

$$S_g K_t V_{in}(t) - B_e \omega(t) - T_d(t) = J_e \dot{\omega}(t) \qquad (1)$$

Equation (1) is simply Newton's 2nd Law. In this section, you will apply a given $V_{in}(t)$ to the drive and measure $\omega(t)$ and $\dot{\omega}(t)$. Since B_e and $T_d(t)$ have already been identified in the previous section, the correct value of J_e is the one that makes the right hand side and left hand side of Equation (1) equal.

1. Set the parameters of the varying square wave signal as:
   Amplitude: 3
   Time step: 0.1 s (period: 0.2 s)
2. Apply the provided lowpass filter commands below on the measured angular position of the drive, θ(t). Lowpass filtering removes the high frequency content (larger than 100 [Hz] in the code below) of the signals thereby eliminating the effect of noise on the identification.
   [B,A]=butter(4,100*2*Ts);
   filtfilt(B,A,theta);
   Ts is the sampling time.
3. Calculate $\omega(t)$ and $\dot{\omega}(t)$ by differentiating the measured angular position using the *deriv* Matlab script provided to you.
4. On the same graph, plot the LHS and RHS of Equation (1). Use the J_e value you calculated in the pre-lab as a starting value
5. Identify the correct inertia by adjusting the J_e value until the two plots are roughly coincident. Provide the graph in your report. What are the reasons for the difference between the calculated and identified inertias?
6. Analyze your saved data in **VCNC**, and compare the results with your identified parameters (J_e, B_e, μ_k). Provide the **VCNC** results in your report. Pay attention to units.

C. Measurement of Open Loop Frequency Response Function

In this section you will measure the open loop frequency response.

1. Apply the given V_{in} voltage command to the drive at the following frequencies and amplitudes:
   freq: 1,2,3,4,5 [Hz], amp: 0.2 V
   freq: 6,7,8,9,10 [Hz], amp: 0.5 V
   freq: 20,30,40,50,60,70 [Hz], amp: 0.9 V
   Why is it necessary to move the table (with constant velocity) before applying the sine waves?
   Discussion: If we assume that the system has no Coulomb or viscous friction, and we give a pure Sine wave as V_{in}, would the table move back and forward as pure Sine wave? Discuss.
   (Hint: start with the equation of motion, and assuming $V_{in}(t)=A\sin(\omega t)$ obtain θ(t)).
2. Calculate the ω from the measured θ using the *deriv* function.
3. For each of the frequencies, calculate the magnitude ratio ($|\omega_m|/|V_{in}|$) by measuring the peak-to-peak values, and calculate the phase difference (phase (ω)-phase (V_{in})) by measuring the time delay between the corresponding peaks in the input and response. Make sure you use the third or fourth cycle for the calculations (if V_{in} and ω_m have enough cycles). Why is this important?
   Note: Check MATLAB command “ginput”. You can write a code to significantly reduce the amount of visual measurements.
4. Plot the measured frequency response function. Use same units as default MATLAB bode plots.
5. On the same graph, plot the frequency response of the transfer function obtained from 2(d). However, this time use the B_e value obtained from A.2 and the J_e value obtained from B.4. Use

the same frequency range used in C.4 for this plot. Comment on the similarity/difference of the two plots.

6. Analyze your saved data in **VCNC**, and compare the frequency response given by **VCNC** against yours. Provide the VCNC results in your report. Pay attention to units.

Report Requirements

The report must be typed and orderly. It accounts for 60% of your lab grade. Poorly presented reports will not be accepted. Equations may be handwritten into the report, but they must be written very neatly and numbered. Please be as concise as possible. Do not provide unnecessary information. Credit will however be given for concise comments which demonstrate a good understanding of the material covered in the lab.

The following are required of you.

1. **Title Page:** Your name, student ID, email, etc
2. **Abstract:** 3-4 lines describing the objective of the laboratory as you understand it.
3. **Introduction**: Describe the application of ball screw drives in industry and the importance of modeling and identification (maximum 5 to 6 lines)
4. **Body**: In this Part you need to answer the questions in Sections A, B and C of the lab procedures clearly (but concisely). Other useful comments are welcome (as long as they are good and concise). Following figures are required:
   - Measured friction Curve (with both positive and negative directions on the same plot) together with the straight lines fitted to each curve.
   - Sketch of the friction curves in part A.4. This can be done by hand (as long as it is neat).
   - Plot of RHS and LHS of Equation (1) using the final (correct) value of J_e.
   - Measured and simulated frequency response function of ball screw drive (i.e. from V_{in} to ω). Simulated frequency response should be based on the experimentally identified B_e and J_e
   - Make sure to answer all the discussion questions.

5. **Conclusion:** Explain what you have understood from the lab in terms of the merits and drawbacks of mathematical modeling, model-order reduction and experimental parameter identification. Briefly describe any other important observation you had in this project (5-8 lines).

6. **Appendix:** MATLAB scripts and Simulink block diagram.