MECH 460 SENSORS AND ACTUATORS

Solution Guidelines for Mid-Term Examination, 30 October 2019

Solution 1

(a)

The circuit stage A is a voltage amplifier, and the circuit stage B is a low-pass filter.

(b)

Consider the given circuit, shown in Figure S1, where an intermediate voltage v_1 is indicated.

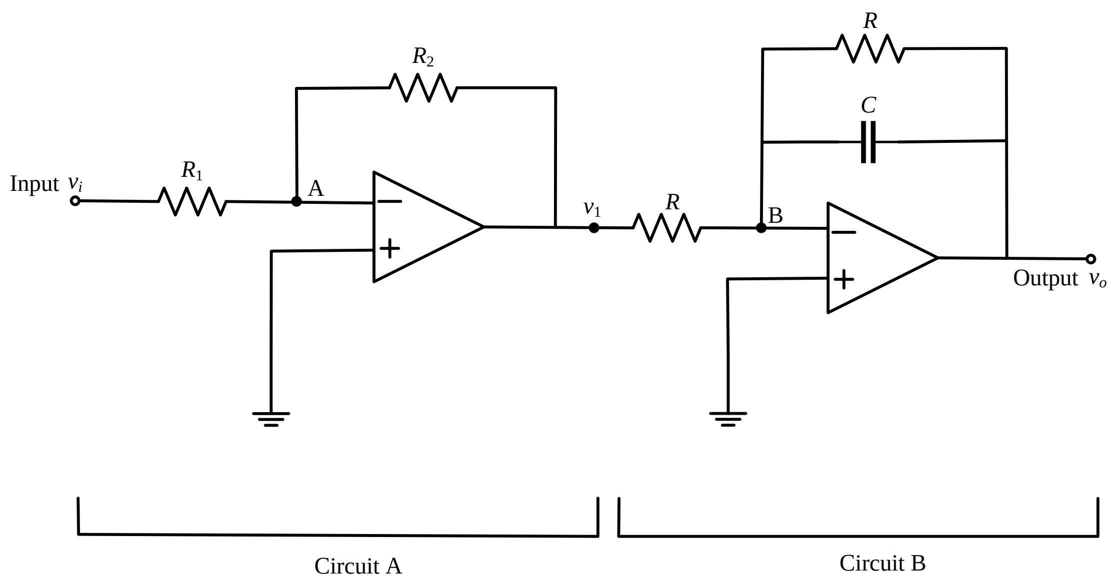

Figure S1: Analog circuit consisting of two stages.

For Stage A: Note that the potential at Node A is zero (property of an op-amp)

Current summation at Node A: $\frac{v_i}{R_1} + \frac{v_1}{R_2} = 0 \rightarrow v_1 = -\frac{R_2}{R_1} v_i$ (i)

For Stage B: Note that the potential at Node B is zero (property of an op-amp)

Current summation at Node B: $\frac{v_1}{R} + \frac{v_o}{R} + C\frac{dv_o}{dt} = 0 \rightarrow RC\frac{dv_o}{dt} + v_o = -v_1$ (ii)

Substitute (i) into (ii): The time-domain model of the circuit is,

$$RC\frac{dv_o}{dt} + v_o = \frac{R_2}{R_1} v_i$$

The corresponding transfer function is $G(s) = \dfrac{k}{(\tau s +)}$ (iii)

Where, the dc gain $k = \dfrac{R_2}{R_1}$

and the time constant $\tau = RC$

(c)

From the transfer function (iii) of the entire circuit, its half-power bandwidth is

$$\omega_h = \frac{1}{\tau} = \frac{1}{RC}$$

Substitute the given numerical values: $\omega_h = \dfrac{1}{100 \times 10^3 \times 0.2 \times 10^{-6}}$ rad/s $= 50.0$ rad/s

(d)

The operating (useful) frequency range of the circuit is indeed represented by the half-power bandwidth of the circuit. Hence, the useful frequency range of the measured signal (v_i) is also equal to this bandwidth.

The sensor dynamics should not distort the measured signal in this required frequency range. In other words, this frequency range should correspond to the DC segment of the sensor transfer function. As a rule of thumb, this condition is more than adequately satisfied if the time constant of the sensor is 10 times smaller than the time constant of the circuit. Hence, a suitable time constant for the sensor is,

$$\tau_s = 0.1\tau = 0.1RC = 0.1 \times 100 \times 10^3 \times 0.2 \times 10^{-6} = 0.002 \text{ s} = 2.0 \text{ ms}$$

Note: This is a very conservative estimate. For ordinary applications, even a two times smaller time constant would be adequate for the sensor → $\tau_s = 10.0$ ms

<u>Solution 2</u>

(i)

Blood Pressure Sensor: It generates an output of 10.0 mV for 1.00 mm Hg of measured pressure, for a supply voltage of 1.0 V to the device.

Capacitive Displacement Sensor: It generates an output of 10.0 V for 1.0 mm of measured displacement.

DC Tachometer: It generates an output of 7.0 VDC for a measured speed of 1000.0 rpm. The sensitivity error is ±3%.

Light Sensor: It generates a digital output of 50 (bit changes) for light intensity (illuminance) of 1.0 lux. *Note*: 1 lux = 1 lumen/m^2.

Strain Gauge: Its resistance changes by 150×10^{-6} Ω when subjected to a strain of 1 micro-strain (*Note*: Strain = extension/original length), if the original resistance of the strain gauge was 1.0 Ω.

Note: It is not appropriate here to say "a resistance change of 150 Ω when subject to 1 unit of strain" because that level of strain is entirely not practical (and certainly will not be linear!).

(ii)

(a)

Since no current is passing to the output terminals, using the voltage divider equation, the voltage at Node A is: $v_A = \dfrac{R_1 v_{ref}}{(R_1 + R_2)}$

Similarly using the voltage divider equation, the voltage at Node B is: $v_B = \dfrac{R_3 v_{ref}}{(R_3 + R_4)}$

Hence,
$$v_o = v_A - v_B = \frac{R_1 v_{ref}}{(R_1 + R_2)} - \frac{R_3 v_{ref}}{(R_3 + R_4)} \qquad \text{(i)}$$

This can be written as $v_o = \dfrac{v_{ref}}{(1 + R_2 / R_1)} - \dfrac{v_{ref}}{(1 + R_4 / R_3)}$

It is clear that, if $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$, the output voltage $v_o = 0$. This condition is called a "balanced bridge."

(b)

Differentiate (i) with respect to R_1:

$$S = \frac{\partial v_o}{\partial R_1} = \frac{(R_1 + R_2) - R_1}{(R_1 + R_2)^2} v_{\text{ref}} = \frac{R_2}{(R_1 + R_2)^2} v_{\text{ref}} \qquad \text{(ii)}$$

Note: The 2nd term on the RHS of (i) does not have any R_1 terms.

Clearly, the proper quantity to normalize the change in R_1 is R_1 itself. Also, the proper quantity to normalize the change in the output voltage is the supply voltage v_{ref}. Hence the normalized sensitivity is

$$S_n = \frac{R_1}{v_{\text{ref}}} \frac{\partial v_o}{\partial R_1} = \frac{R_1 R_2}{(R_1 + R_2)^2} \qquad \text{(iii)}$$

If $R_1 = R_2$, the non-dimensional sensitivity is $S_n = \dfrac{1}{4}$

(c)

If only R_1 changes, by δR_1, the corresponding change in the bridge output is,

$$\delta v_o = \frac{\partial v_o}{\partial R_1} \delta R_1 = \frac{R_2}{(R_1 + R_2)^2} v_{\text{ref}} \delta R_1 \qquad \text{(From (ii))} \qquad \text{(iv)}$$

Note: If, $R_1 = R_2 = R_3 = R_4 = R$, the bridge is initially balanced, and hence the change in the output (δv_o) is the output voltage (v_o) itself.

Then, from (iv) we have the bridge output voltage,

$$\delta v_o = \frac{R}{(R + R)^2} v_{\text{ref}} \delta R = \frac{\delta R}{4R} v_{\text{ref}} \qquad \text{(v)}$$

For the semiconductor strain gauge, it is given that

$$\frac{\delta R}{R} = S_1\varepsilon + S_2\varepsilon^2 \qquad \text{(vi)}$$

Substitute (vi) into (v). The input-output relation of the entire device is:

$$\delta v_o = \frac{v_{\mathrm{ref}}}{4}(S_1\varepsilon + S_2\varepsilon^2) \qquad \text{(vii)}$$

(d)

Since the bridge is initially balanced, the change in the output (δv_o) is the output voltage (v_o) itself. Now, from (vii), the sensitivity of the overall system is given by,

$$\frac{1}{v_{\mathrm{ref}}}\frac{\partial \delta v_o}{\partial \varepsilon} = \frac{1}{4}(S_1 + 2S_2\varepsilon) \text{ V/V/strain} \qquad \text{(viii)}$$

Note: More appropriately, the sensitivity should be expressed in "mV/V/μ-strain."

From (vii) it is seen that, the "linearized" sensitivity of the system is

$$\frac{S_1}{4} \text{ V/V/strain} = \frac{S_1}{4}\times 10^{-3} \text{ mV/V/μ-strain}$$

Also, from (viii) it is seen that, the maximum sensitivity error is $\frac{S_2\varepsilon_{\max}}{2}$ V/V/strain, where $\varepsilon_{\max}$ is the maximum strain in the dynamic range (upper limit in the operating range) of the device. *Note*: This is in fact = Max sensitivity – Min sensitivity = $\left(\frac{1}{4}S_1 + \frac{1}{2}S_2\varepsilon_{\max}\right) - \left(\frac{1}{4}S_1\right)$.

Hence, the "% sensitivity error" is $\frac{S_2\varepsilon_{\max}}{2\times S_1/4}\times 100\% = \frac{200S_2\varepsilon_{\max}}{S_1}\%$ (ix)

Alternatively, from (vii), the maximum "linear" output is $\frac{v_{\mathrm{ref}}S_1}{4}\varepsilon_{\max}$

The corresponding, maximum nonlinearity is $\frac{v_{\mathrm{ref}}S_2}{4}\varepsilon_{\max}^2$

Hence, the "% nonlinearity" may be expressed as $\frac{v_{\mathrm{ref}}S_2\varepsilon_{\max}^2/4}{v_{\mathrm{ref}}S_1\varepsilon_{\max}/4}\times 100\% = \frac{100S_2\varepsilon_{\max}}{S_1}\%$ (x)

From (ix) and (x) it is seen that there is a close relation between the "sensitivity error" and the "nonlinearity" of a device.