Lesson 7-4 – Annuities Due

Special Acknowledgment to Dr Ron Mackinnon and Dr Tamara Etmannski who helped with the development of this material.

Reminder

- Up to now we have been considering all cashflows to occur at the *end* of a given period.
- The end of one period coincides with the start of the next period.

- For example, you take out a loan of $5,000 at 12% annual interest, compounded monthly.
  - You receive the cash at the end of period zero.
  - Immediately after, period 1 (month 1) begins
  - At the end of period 1 (month 1), 1% interest accrues on the loan amount (I_1 = $5000*1% = $50). Loan balance is now $5050.
  - Any payments you make on the loan are then subtracted from the loan balance.
  - Period 2 (month 2) begins, and the process repeats.

Annuities Due

- Alternatively, payments can accrue at the beginning of a period. Our example would then look like

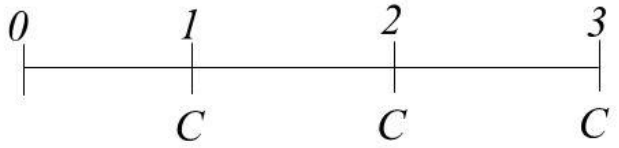
- You take out a loan of $5,000 at 12% annual interest, compounded monthly.
  - You receive the cash at the end of period zero.
  - Immediately after, period 1 (month 1) begins and
  - 1% interest accrues on the loan amount (I_1 = $5000*1% = $50). Loan balance is now $5050.
  - Any payments you make on the loan are then subtracted from the loan balance.
  - Period 2 (month 2) begins, and the process repeats.

End of Year (period) Convention

- Image p. 156 in text

Annuities

- An annuity is a series of equal payments, starting next period, and made each period for a specified number (3) of periods.



  - If payments occur at the end of each period (the first is one period from now) it is an ordinary annuity or an annuity in arrears.
  - If the payments occur at the beginning of each period (the first occurs now) it is an annuity in advance or an annuity due.

1. Payments at the Beginning of the Period

- For some series, the payments occur at the beginning of each payment period. These are called ‘annuities due’.
- The formulas are:

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right](1+i)$$

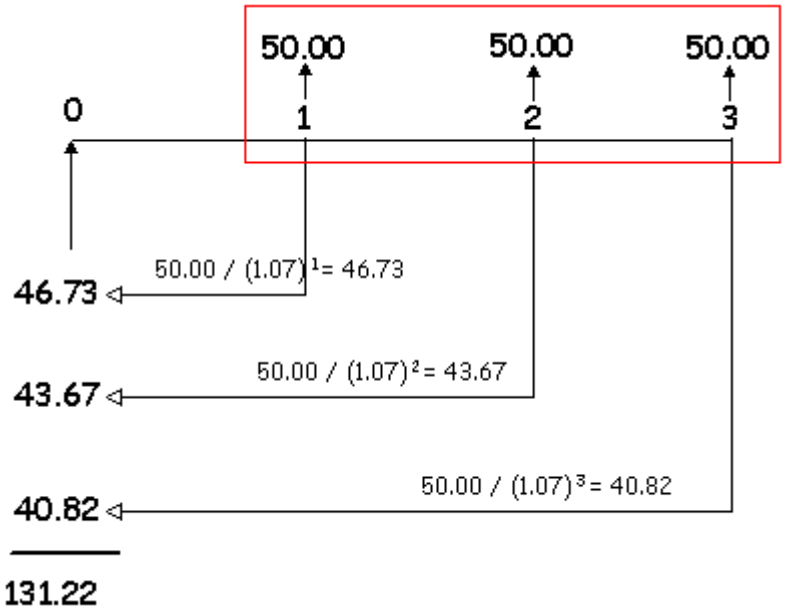
$$F = A\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

- In general, to get the formula for an annuity due, multiply the formula for the corresponding ordinary annuity by (1+ i).

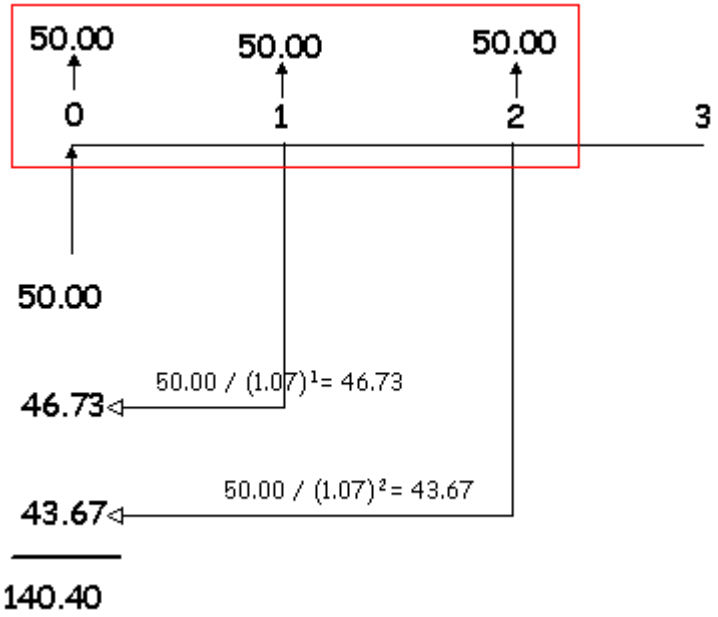
Annuity in arrears (ordinary annuity) vs. Annuity Due

Annuity: $50 per year for three years at 7% interest. Find the PV of each

In Arrears



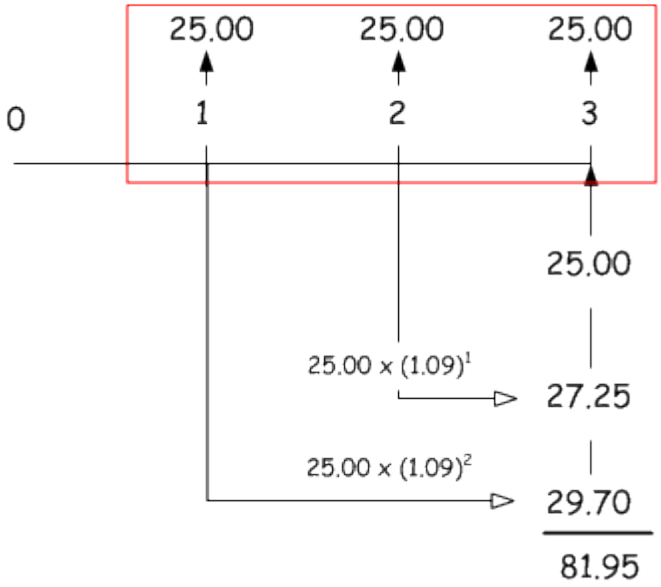
Annuity Due



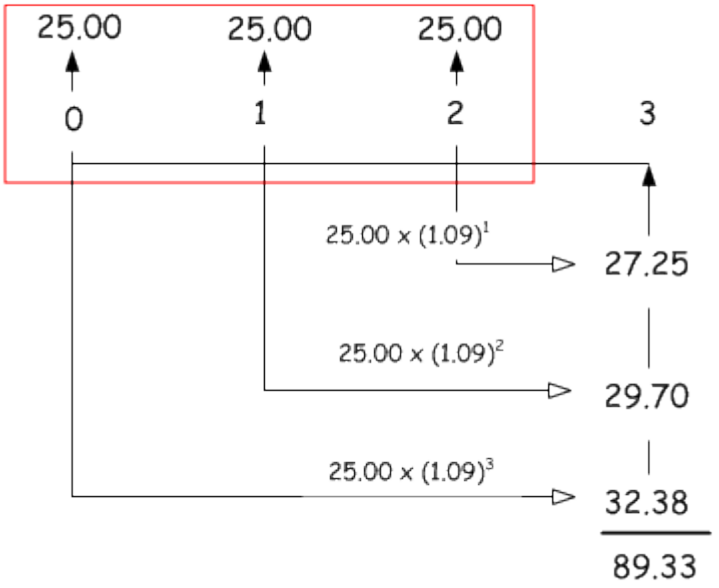
Annuity in arrears (ordinary annuity) vs. Annuity Due

Annuity: $25 per year for three years at 9% interest. Find the FV of each

In Arrears



Annuity Due



Diagrams by David Frick: http://www.frickcpa.com/tvom/tvom_annuity_due.asp

Annuities Due Example

- You want to lease a vehicle with a leased value of $42,500 by making monthly payments in advance for four years at an interest rate of 2.75% compounded monthly. Calculate the payment required.

$$42500 = A\left[\frac{(1+\frac{0.0275}{12})^{48}-1}{\frac{0.0275}{12}(1+\frac{0.0275}{12})48}\right](1+\frac{0.0275}{12})$$

$$A = \$933.88$$