L3 – LTI System Review

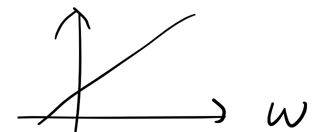
- Filter = LTI system.

① Freq-shaping : change the "shape" of the spectrum

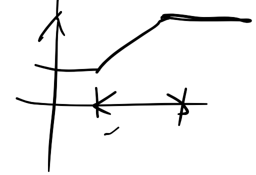
$\mathcal{F}\{\text{signal}\}$.

e.g., Equilizer.

· Differentiator "s"

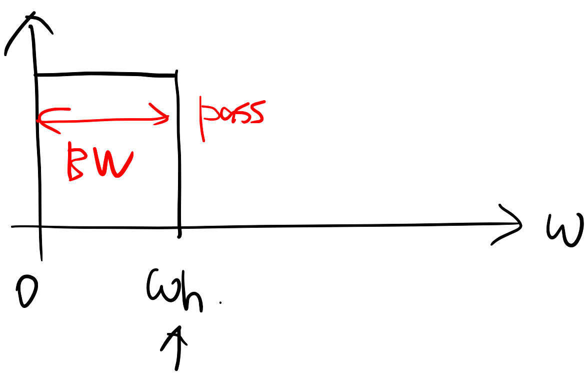


· Lead Comp.



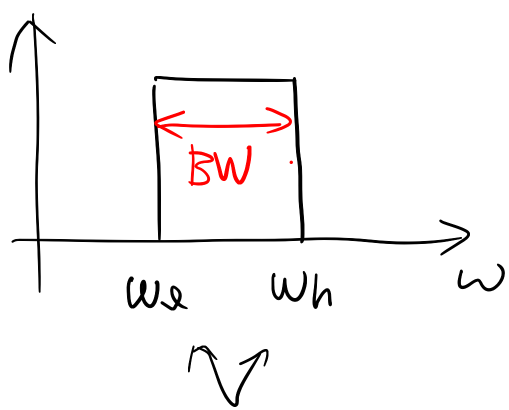
② Freq-selective

< Low pass >



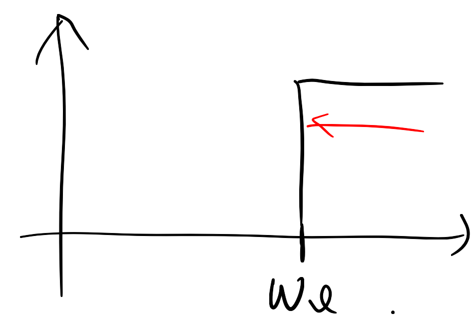
cut-off

< Band pass >



cut-off

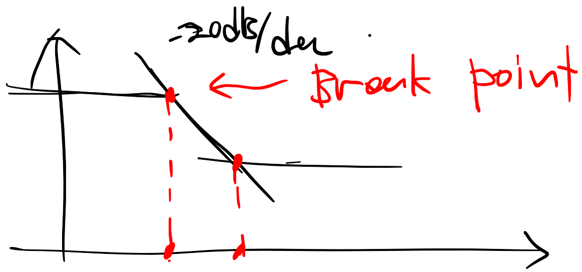
< High pass >



Lowpass : $BW = \omega_h - 0 = \omega_h$

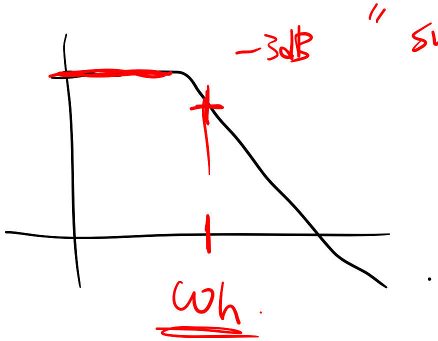
Bandpass : $BW = \omega_h - \omega_l$

- Corner freq ? Break freq.



- Cutoff.

"sufficient attenuation"



< LTI - Math Rep. >

In summary.

① Impulse Resp

② Transfer func.

③ Freq. Resp.

$$\begin{tikzcd}
 & h(t) \arrow[dl, leftrightarrow, "\mathcal{L}"'] \arrow[dr, leftrightarrow, "\mathcal{F}"] & \\
H(s) \arrow[rr, leftrightarrow, "s=j\omega"'] & & H(j\omega).
\end{tikzcd}$$

- $\mathcal{F}$ is well defined for both ways.

$$h(t) \underset{\mathcal{F}}{\overset{\mathcal{F}}{\rightleftarrows}} H(j\omega)$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt.$$

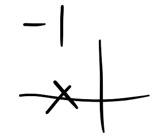
$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega t)\, e^{j\omega t}\, d\omega$$

"Duality"

- We can "infer" Bode $\rightleftarrows$ Step

< Graphical Rep >

$H(s)$ {
1. Block
2. p-z map → × dc gain
3. Bode plot.

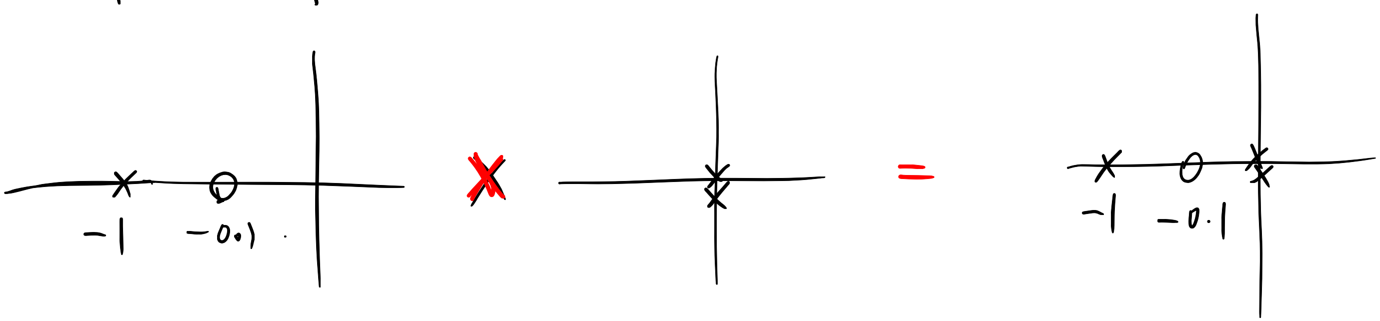


{ $\frac{1}{s+1}$? ; $\frac{10}{s+1}$?

I. Series : $H(s) = H_1 H_2$.

$\boxed{H_1 = \frac{10s+1}{s+1}}$ $\boxed{H_2 = \frac{1}{s^2}}$

- Block. $\to \boxed{H_1} \to \boxed{H_2} \to$
- p-z map.



{ poles remain the same.
zeros remain the same.

∘ Bode plot, (skip).

Convinient when "shaping" FRF.

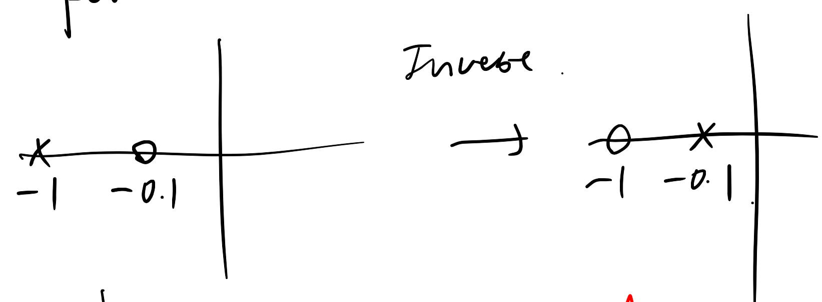
II. Inverse.

$$H(s) = \frac{10s+1}{s+1} \qquad H^{-1}(s) \overset{?}{=} \frac{s+1}{10s+1}.$$

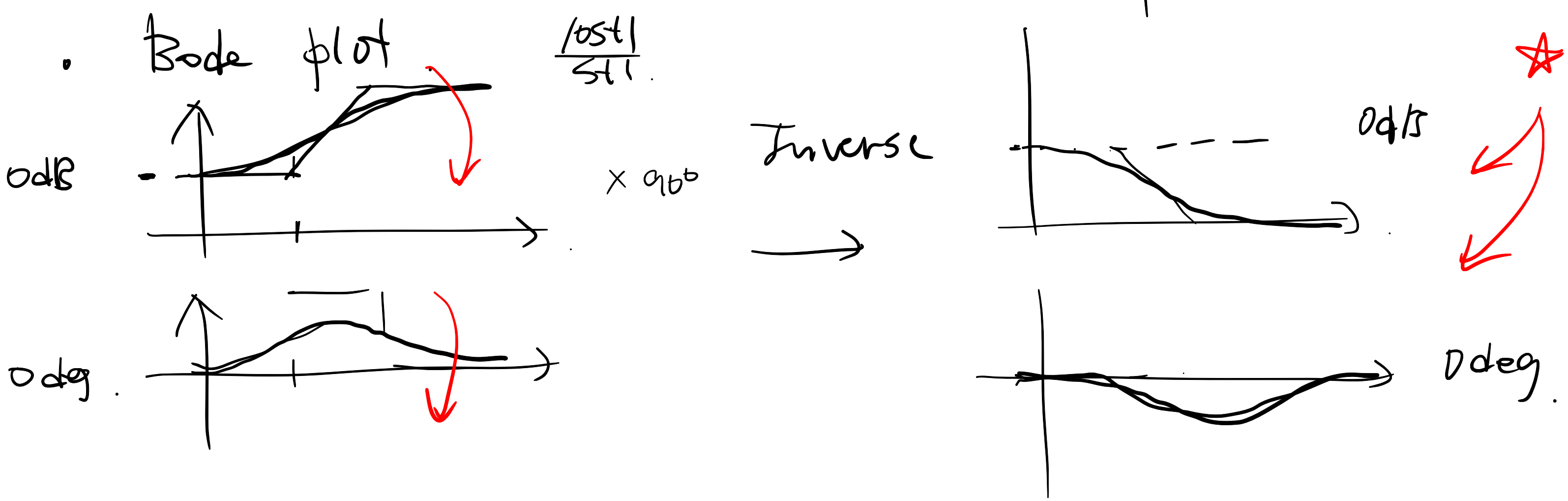
∘ Block.

$$x \leftarrow \boxed{\frac{1}{H(s)}} \leftarrow y.$$

∘ pole-zero.



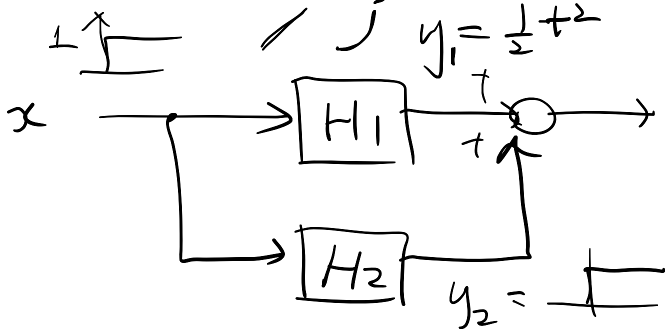
∘ Bode plot $\frac{10s+1}{s+1}$.



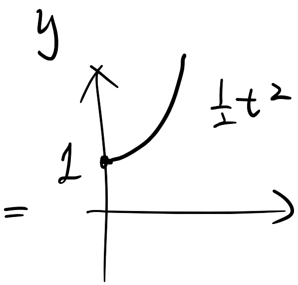
III. Parallel

$H = H_2 + H_2$. $H_1 = \frac{1}{s^2}$ $H_2 = 1$

• Block Diagram

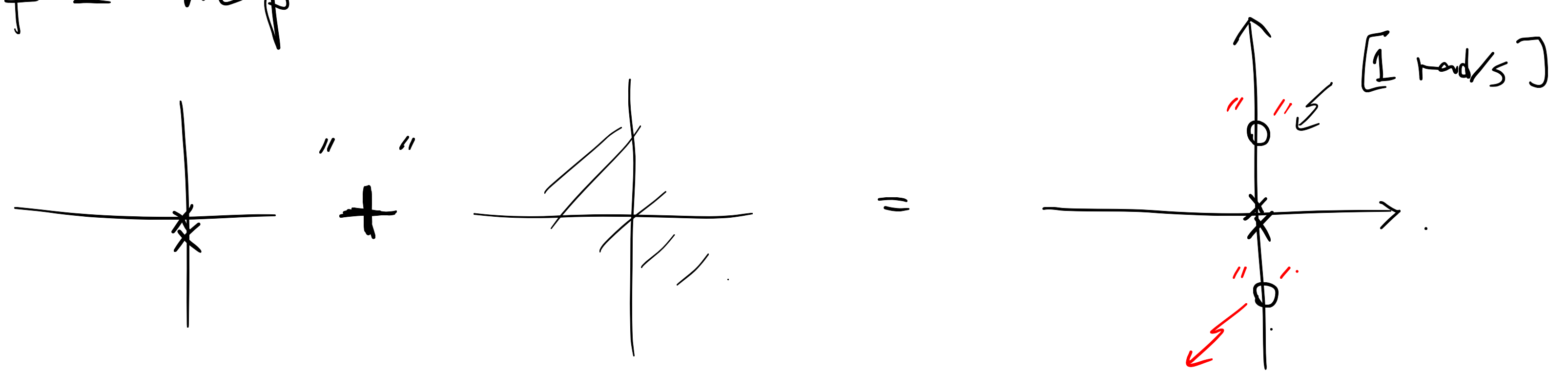


$y =$



$$H(s) = 1 + \frac{1}{s^2} = \frac{s^2+1}{s^2}.$$

• p-z map



· on Im axis

{ poles the same.

zeros are "newly" created.

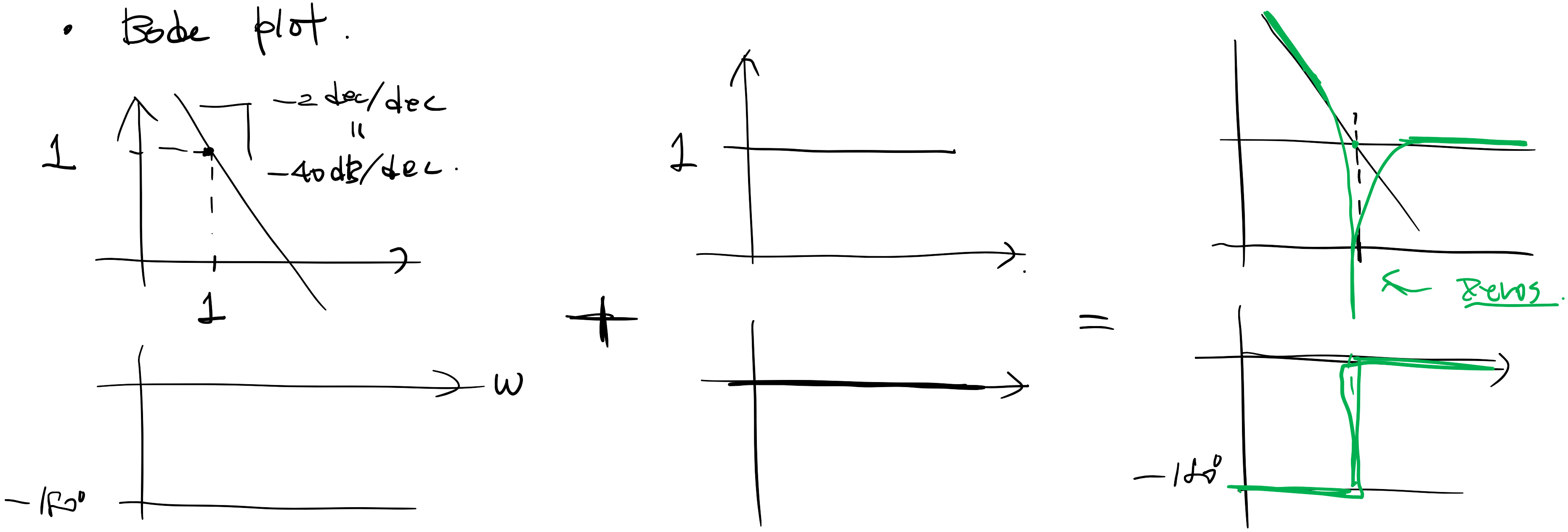
"Anti-resonance"

$x(t) = \cos(t)$

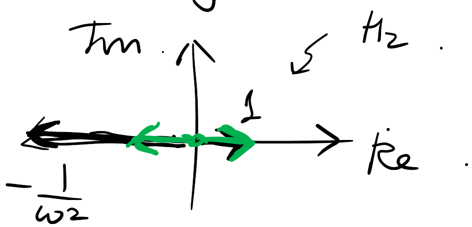
$\boxed{y_1 + y_2 = 0}$

1 rad/s

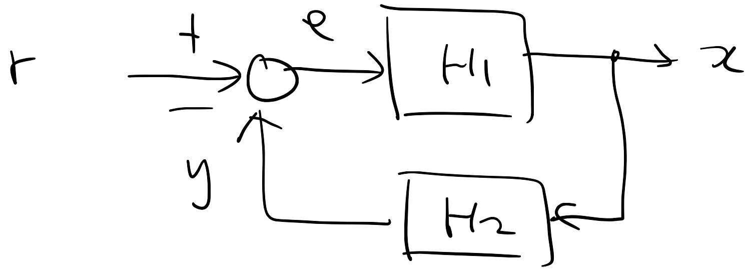
• Bode plot.



"Phase diagram"



IV. Feedback



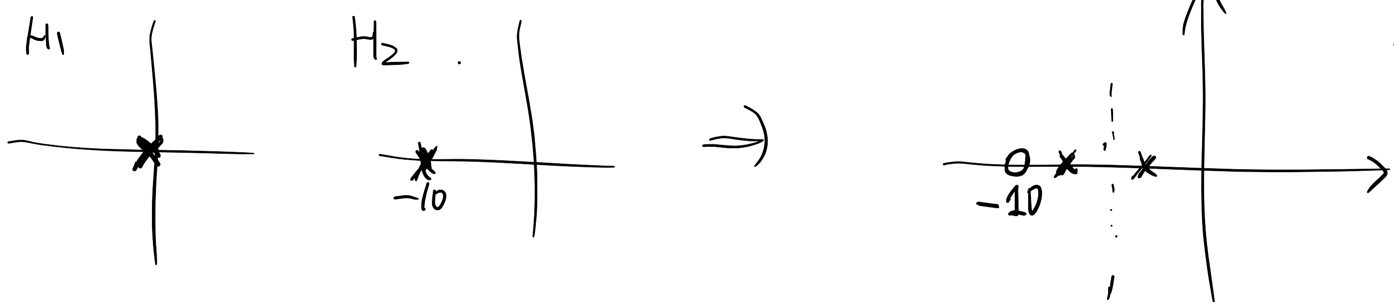
$H_1 = \frac{1}{s}$

$H_2 = \frac{10}{s+10} = \frac{1}{0.1s+1}$

$$G(s) = \frac{\frac{1}{s}}{1 + \frac{1}{s}\cdot\frac{10}{s+10}} = \frac{s+10}{s^2+10s+10}$$

$s = -5 \pm \sqrt{15}$

- pole-zero map



$H_1 = \frac{a_1(s)}{b_1(s)}$ $H_2 = \frac{a_2(s)}{b_2(s)}$ "Rational T.F."

e^{-sT}

$$G(s) = \frac{\frac{a_1}{b_1}}{1 + \frac{a_1}{b_1}\cdot\frac{a_2}{b_2}} = \frac{a_1 b_2}{a_1 a_2 + b_1 b_2}$$

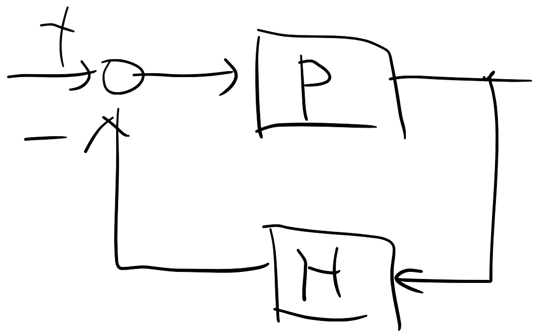
G { poles: the roots of $a_1 a_2 + b_1 b_2 = 0$. "Root Locus"

+ the roots of $b_1 b_2 = 0$.

Zeros : the roots of $a_1 b_2 = 0$.

① $a_1 = 0$. "Zeros of H_1"

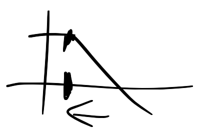
② $b_2 = 0$ "poles of H_2"



LPF on feedback.

"Light", Anti-aliasing filter.

Next



- Bode plot.

$$G = \frac{\text{Forward}}{1 + L(s)}.$$

① Draw $L(s) + 1$.

② Draw $\frac{1}{1+L(s)} \triangleq S(s)$

"Sensitivity"

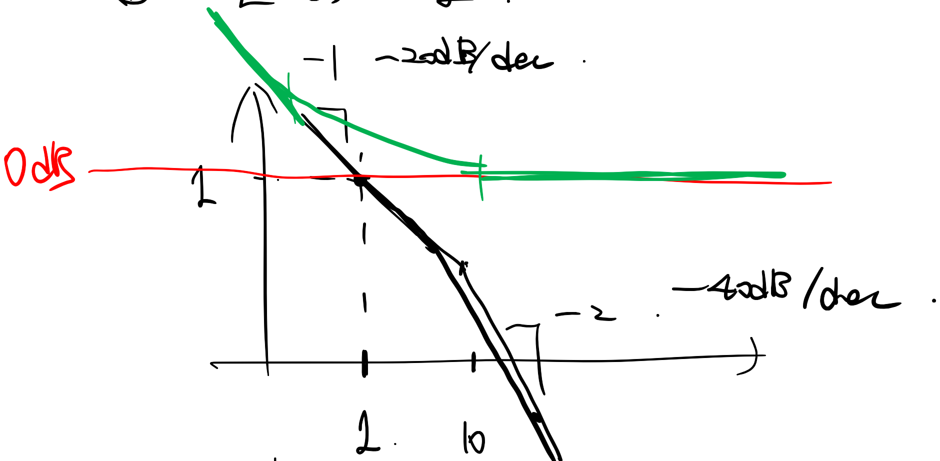
③ Draw. $S(s) \cdot \text{Forward}$.

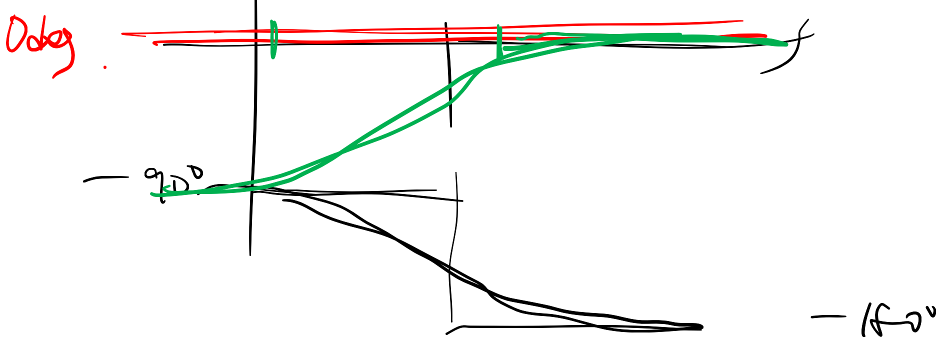
$$L(s) = \frac{1}{s}\cdot\left(\underbrace{\frac{1}{0.1s+1}}_{\text{unity upto}}\right).$$

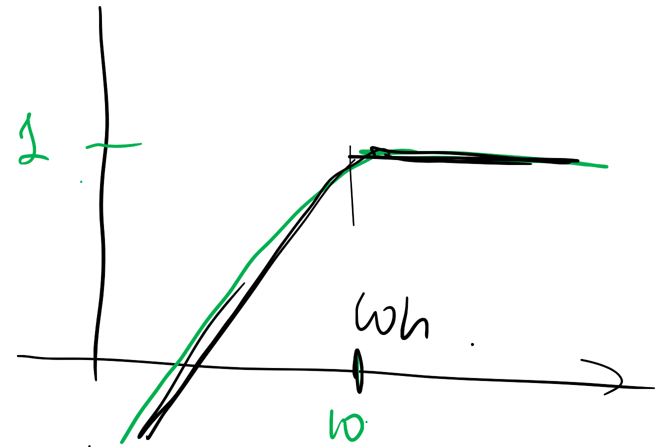
unity upto 10 rad/s.

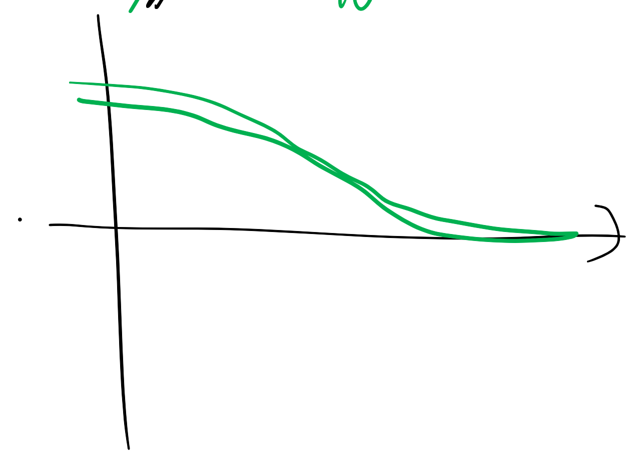
① $L(s) + 1$.

② $\dfrac{1}{L+1} = S(s)$.





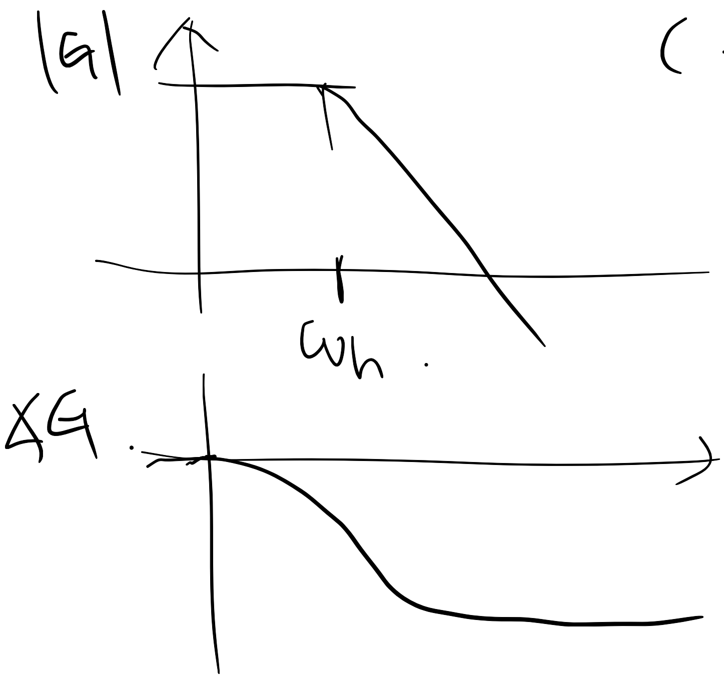




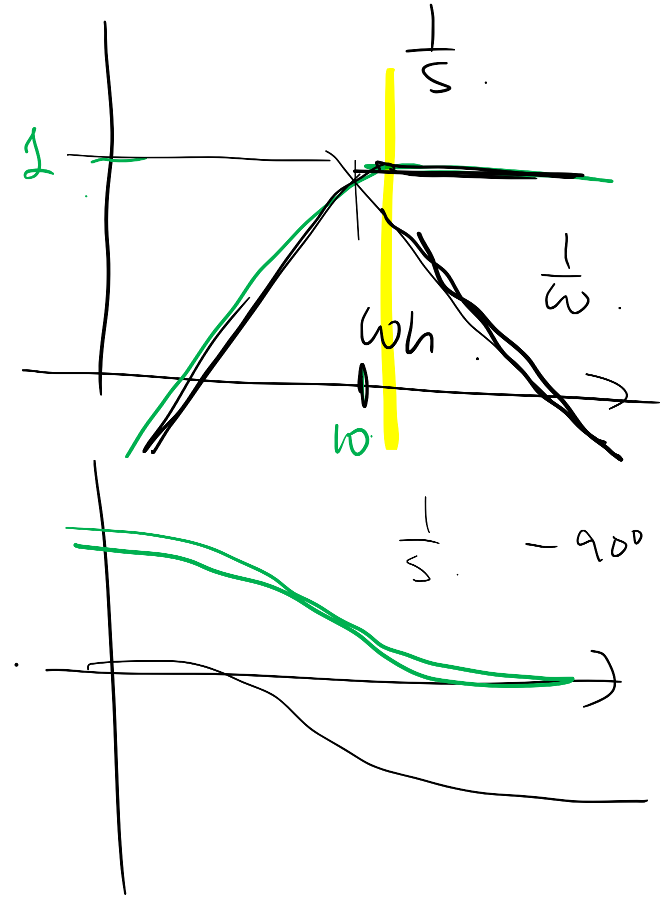
"Min-phase"

③ Multiply Forward

$\left(\frac{1}{s}\right)$



② $\frac{1}{L+1} = S(s)$.



$\times \frac{1}{s}$

$\frac{1}{s}$ "