University of British Columbia
Department of Mechanical Engineering

MECH468 Modern Control Engineering
MECH522 Foundations in Control Engineering
Midterm exam

Examiner: Dr. Ryozo Nagamune
October 17 (Monday), 2016, 8:50am-9:50am

Last name, First name

Name: Student #:

Signature:

Exam policies

- Allowed: Lecture note print-outs, hand-written notes, homework, books.
- Not-allowed: PC, calculators.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 20 points in total.

Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

If you finish early ...

- Please stay at your seat until the end of exam, i.e., 9:50am. (You are not allowed to leave the room before the end of exam, except going to washroom.)

To be filled in by the instructor/marker

Problem #	Mark	Full mark
1		12
2		6
3		2
Total		20

1. Consider the following continuous-time system:

$$\begin{cases} \dot{x}(t) & = & \begin{bmatrix} -1 & a \\ 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} b \\ 1 \end{bmatrix} u(t), \\ y(t) & = & \begin{bmatrix} c & 1 \end{bmatrix} x(t), \end{cases}$$

where a, b, c are constants.

(a) Obtain the condition for asymptotic stability in terms of a. (2pt)

Write your answer here.

(b) Obtain the condition for controllability in terms of a and b. (2pt)

(c) Obtain the condition for observability in terms of a and c. (2pt)

Write your answer here.

(d) Consider the case when $a = 0$ and $b = c = 1$, i.e.,

$$\begin{cases} \dot{x}(t) &= \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} 1 & 1 \end{bmatrix} x(t). \end{cases} \tag{1}$$

i. Obtain the Kalman decomposition. (2pt)

ii. Write explicitly which state is controllable / uncontrollable and observable / unobservable. (1pt)

iii. Verify that the "controllable-and-observable part" (A_{co}, B_{co}, C_{co}) is actually controllable and observable. (1pt)

Write your answer here.

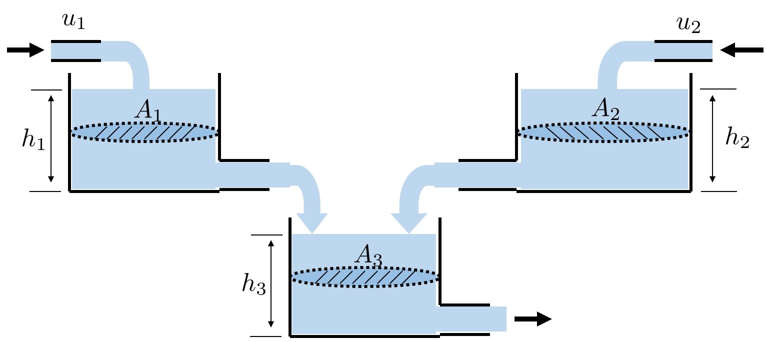
(e) For the system (1) in question (d), compute the A_d-matrix (A-matrix of a discrete-time system) of the discretized system (by zero-order-hold) with sampling period $T = 1$. (2pt)

Write your answer here.

2. Consider a three-water-tank system in the figure below. Here, A_i, $i = 1, 2, 3$, are tank section areas, h_i, $i = 1, 2, 3$, are the water heights of the tanks, and u_i, $i = 1, 2$, are input flow rates u_1 and u_2. The nonlinear state equation of this system is assumed to be expressed as

$$\begin{aligned}
\dot{h}_1(t) &= \frac{1}{\rho A_1}\left(-K\sqrt{h_1(t)} + u_1(t)\right), \\
\dot{h}_2(t) &= \frac{1}{\rho A_2}\left(-K\sqrt{h_2(t)} + u_2(t)\right), \\
\dot{h}_3(t) &= \frac{1}{\rho A_3}\left(-K\sqrt{h_3(t)} + K\sqrt{h_1(t)} + K\sqrt{h_2(t)}\right),
\end{aligned}$$

where ρ and K are given positive constants.



We would like to linearize the nonlinear state equation around the situation when we maintain the water heights at $h_1(t) = h_{10}$ and $h_2(t) = h_{20}$, where h_{10} and h_{20} are given positive constant heights.

(a) Obtain the corresponding constant input flow rates $u_1(t) = u_{10}$ and $u_2(t) = u_{20}$ in terms of given constants h_{10} and h_{20}. (2pt)

(b) Obtain the corresponding constant water height $h_3(t) = h_{30}$ in terms of given constants h_{10} and h_{20}. (2pt)

(c) Derive a linearized state equation $\dot{\delta h}(t) = A\delta h(t) + B\delta u(t)$ around the equilibrium point $(h_1, h_2, h_3) = (h_{10}, h_{20}, h_{30})$ and $(u_1, u_2) = (u_{10}, u_{20})$. To answer this question, you do not need to use solutions obtained in (a) and (b); just use (h_{10}, h_{20}, h_{30}) and (u_{10}, u_{20}). (2pt)

Write your answer here.

Write your answer here.

3. Consider the following controllable discrete-time system:

$$x[k+1] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[k],$$

Compute the minimum energy control $u[k]$, $k = 0, 1, 2$, which transfers state vector from $x[0] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to $x[3] = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$. (2pt)

——— (End of Midterm Exam) ———

Write your answer here.

Extra page. Write the problem number before writing your answer.

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