Solutions to examples to do

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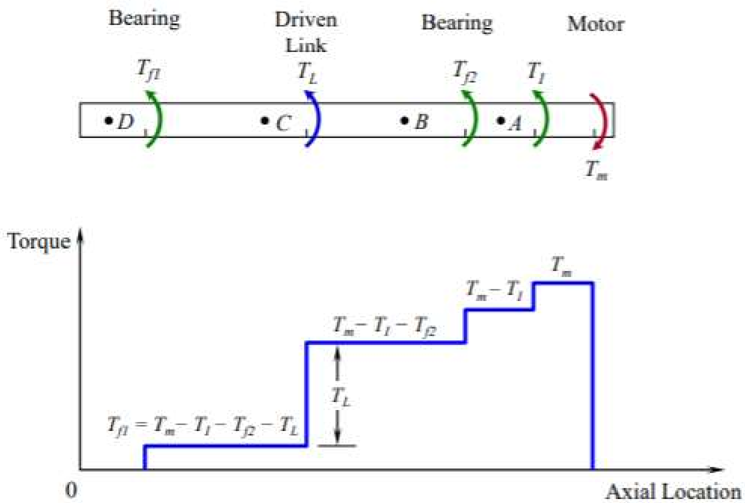


- **For accurate results two strain gages at locations B and C should be installed**
- **A single sensor at B is also a good approximation since the bearing friction is small**
- **Motor torque T_m is also approximately equal to transmitted torque when inertia and friction are small**

Sensing Bandwidth Example

Rigid load of inertia J_L driven by motor with rigid rotor of inertia J_m.

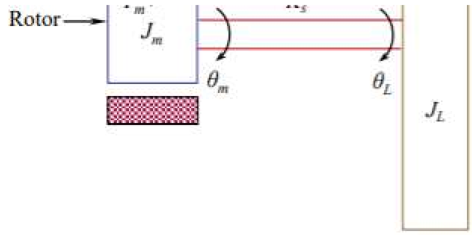
Torque sensing member: Stiffness K_s between rotor and load

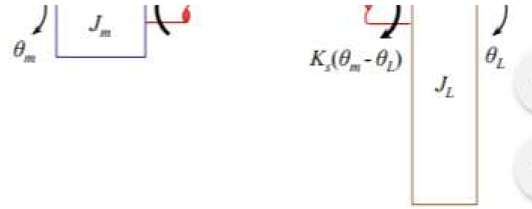
(a) Determine transfer function between motor torque T_m and twist angle of torsion member

(b) What is the torsional natural frequency ω_n of the system?

(c) Discuss why system bandwidth depends on ω_n. Show that the bandwidth can be improved by increasing K_s, decreasing J_m, or decreasing J_L

(d) Mention advantages and disadvantages of introducing a gearbox at motor output.





Example: Solution

For Motor: $T_m = J_m\ddot{\theta}_m + K_s(\theta_m - \theta_L)$; **For Load:** $K_s(\theta_m - \theta_L) = J_L\ddot{\theta}_L$

➔ $$\ddot{\theta}_m - \ddot{\theta}_L = -K_s\left(\frac{1}{J_m} + \frac{1}{J_L}\right)(\theta_m - \theta_L) + \frac{T_m}{J_m}$$

Let: $\theta = \theta_m - \theta_L$

➔ $$\ddot{\theta} + K_s\left(\frac{1}{J_m} + \frac{1}{J_L}\right)\theta = \frac{T_m}{J_m} \quad ; \quad G(s) = \frac{\theta(s)}{T_m(s)}$$

➔ $$G(s) = \frac{1/J_m}{s^2 + K_s(1/J_m + 1/J_L)}$$ ➔ $$\omega_n = \sqrt{K_s\left(\frac{1}{J_m} + \frac{1}{J_L}\right)}$$

When gears are added, equivalent inertia increases and equivalent stiffness decreases ➔ Reduction in BW

BW can be increased by increasing K_s and by decreasing J_m and J_L

Example (cont'd)

- Obtain an expression relating applied acceleration a (in units of g) to bridge output (bridge balanced at zero acceleration) in terms of the following parameters:
  $W = Mg$ = weight of seismic mass at free end of cantilever; E = Young's modulus of cantilever;
  ℓ = length of cantilever; b = X-section width of cantilever; h = X-section height of cantilever; S_s = gauge factor (sensitivity) of each strain gage; v_{ref} = supply voltage to the bridge.

- For M = 5 gm, $E = 5\text{x}10^{10}$ N/m^2, ℓ = 1 cm, b = 1 mm, h = 0.5 mm, S_s = 200, and v_{ref} = 20 V, determine sensitivity of accelerometer in mV/g.
- If yield strength of cantilever element is $5\text{x}10^7$ N/m2, what is the max acceleration that could be measured?
- If the ADC from bridge to computer has the range 0 to 10 V, how much amplification (bridge amplifier gain) would be needed so that this maximum acceleration corresponds to the upper limit of ADC (10 V)?
- Is the cross-sensitivity (i.e., sensitivity for tension and other direction of bending small with this arrangement? Explain.
- *Hint*: For a cantilever subjected to force F at free end, max stress at root =

$$\sigma = \frac{6F\ell}{bh^2}$$

SG Torque Sensor Design Example

Design a tubular torsion element.

Design specifications: $\varepsilon_{max} = 3,000\mu\varepsilon; N_p = 5\%; v_o = 10\,\text{V};$

System bandwidth = 50 Hz, K = 2.5x10^3 N.m/rad.

Use a bridge with 4 active SGs I

Given parameter values:

1. For strain gages: $S_s = S_1$ = 115, S_2 = 3500

2. For the torsion element: Outer radius r = 2 cm, Shear modulus G = 3x10^{10} N/m2, Length L = 2 cm

3. For bridge circuit: v_{ref} = 20 V, K_a = 100

Expected max torque T_{max} = 10 N.m

Compute operating parameter limits for the designed sensor.

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Solution:

From triangular speed profile: $d = \frac{1}{2} v_{max} T$

Substituting numerical values: $0.1 = \frac{1}{2} v_{max} 0.2 \rightarrow v_{max} = 1.0 \text{ m/s}$

Acceleration/deceleration of system: $a = \frac{v_{max}}{T/2} = \frac{1.0}{0.2/2} \text{ m/s}^2 = 10.0 \text{ m/s}^2$

Corresponding angular acceleration/deceleration of motor: $\alpha = \frac{pa}{r}$.

With efficiency η, motor torque T_m needed to accelerate/decelerate the system:

$$\eta T_m = J_e \alpha = J_e \frac{pa}{r} = \left[J_m + J_{g1} + \frac{1}{p^2}(J_{g2} + J_d + J_s) + \frac{r^2}{p^2}(m_c + m_L) \right] \frac{pa}{r}$$

Maximum speed of motor: $\omega_{max} = \frac{p v_{max}}{r}$

Without gears (p = 1): $\eta T_m = [J_m + J_d + J_s + r^2(m_c + m_L)] \frac{a}{r}$

Stepper Motor Selection (Example 8.9, Cont'd)

Case 1: Without Gears

For η = 0.8 (i.e., 80% efficient): $0.8 T_m = [J_m + 2\times10^{-3} + 2\times10^{-3} + 0.1^2(5+5)] \frac{10}{0.1} \text{ N.m}$

$\rightarrow \quad T_m = 125.0[J_m + 0.104] \text{ N.m}$ **and** $\omega_{max} = \frac{1.0}{0.1} \text{ rad/s} = 10 \times \frac{60}{2\pi} \text{ rpm} = 95.5 \text{ rpm}$

Operating speed range: 0–95.5 rpm.

***Note*: Torque at 95.5 rpm < starting torque, for first two motors, not so for other two (See speed–torque curves). In motor selection, use the weakest point (i.e., lowest torque) in the operating speed range**

Without gears, motors cannot meet system requirements (see Table)

Data for Selecting a Motor Without a Gear Unit.

Motor Model	Available Torque at ω_{max} (N.m)	Motor–Rotor Inertia ($\times 10^{-6}$ kg.m²)	Required Torque (N.m)
50 SM	0.26	11.8	13.0
101 SM	0.60	35.0	13.0
310 SM	2.58	187.0	13.0
1010 SM	7.41	805.0	13.1

Stepper Motor Selection (Example 8.9, Cont'd)

Case 2: With Gears

***Note*: Usually system efficiency drops when gears. Ignore this here.**

With 80% efficiency (η = 0.8):

$$0.8T_m = \left[J_m + 50\times10^{-6} + \frac{1}{p^2}(200\times10^{-6} + 2\times10^{-3} + 2\times10^{-3}) + \frac{0.1^2}{p^2}(5+5) \right] p \times \frac{10}{0.1} \text{ N.m}$$

and $\omega_{max} = \frac{1.0\,p}{0.1}$ rad/s $= 10\,p \times \frac{60}{2\pi}$ rpm

➔ $T_m = 125.0 \left[J_m + 50\times10^{-6} + \frac{1}{p^2}\times104.2\times10^{-3} \right] p$ N.m **and** $\omega_{max} = 95.5\,p$ rpm

For p = 2: ω_{max} = 191.0 rpm

With p = 2, model 1010 SM satisfies the requirement (see Table)

With full stepping, step angle = 1.8°. Corresponding step in conveyor motion = positioning resolution. With p = 2 and r = 0.1 m, positioning resolution =

$$\frac{1.8°}{2} \times \frac{\pi}{180°} \times 0.1 = 1.57\times10^{-3} \text{ m.}$$

Data for Selecting a Motor With Gear.

Motor Model	Available Torque at ω_{max} (N.m)	Motor–Rotor Inertia (× 10^{-6} kg.m^2)	Required Torque (N.m)
50 SM	0.25	11.8	6.53
101 SM	0.58	35.0	6.53
310 SM	2.63	187.0	6.57
1010 SM	7.41	805.0	6.73