University of British Columbia
Department of Mechanical Engineering
MECH 463. Midterm 1, October 1, 2019

Allowed Time: 70 min

Materials admitted: Pen, pencil, eraser, straightedge, simple scientific calculator without programming or communication capabilities, personal hand-written notes within one letter-size sheet of paper (one side).

There are 3 questions in this exam. You are asked to answer all three questions.

The purpose of this test is to evaluate your knowledge of the course material. Orderly presentation demonstrates your knowledge most clearly, while disorganized and unprofessional work creates serious doubt. Marks are assigned accordingly. A bonus of up to 2 marks will be given for exemplary presentation.

Complete the section below **during** the examination time **only.**

NAME:____

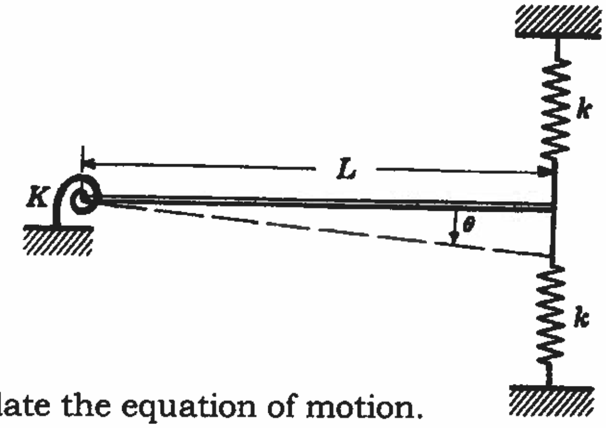
SIGNATURE:______________________________________

STUDENT NUMBER:________________________________

	Mark Received	**Maximum Mark**
1		6
2		7
3		7
Presentation		2 bonus
Total		20+2

Name: ____

1. The diagram shows a uniform rod of length L and mass m secured at its left side by a rotational spring of torsional stiffness K and on its right side by two springs of stiffness k. (Hint: the centroidal moment of inertia of a uniform rod is $I_0 = mL^2/12$).



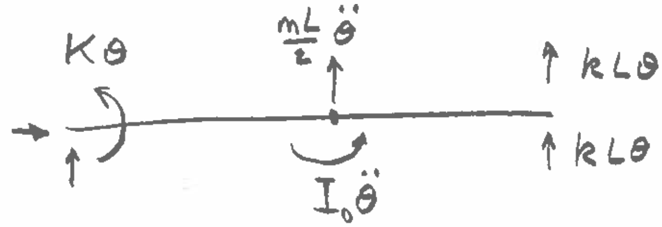
   (a) Draw a labeled free-body diagram of the vibrating system.

   (b) Use your free-body diagram to formulate the equation of motion.

   (c) Solve your equation of motion to show the vibrational motion in time, then determine the natural frequency of vibration of the rod. Show the needed steps in detail.

(a) Centre displ. $= \frac{L}{2}\theta$

ight displ. $= L\theta$



(b) Moments about left end (to eliminate reaction forces)

$$\Sigma M = K\theta + I_0\ddot{\theta} + \frac{L}{2}\cdot\frac{mL}{2}\ddot{\theta} + L\cdot 2kL\theta = 0$$

$$\rightarrow \left(\frac{mL^2}{12} + \frac{mL^2}{4}\right)\ddot{\theta} + \left(K + 2kL^2\right)\theta = 0$$

$$= \frac{mL^2}{3}\ddot{\theta} + \left(K + 2kL^2\right)\theta = 0$$

(c) Try solution $\theta = C\cos(\omega t + \phi)$

$$\rightarrow \ddot{\theta} = -\omega^2 C\cos(\omega t + \phi)$$

Sub. $\left(-\omega^2\frac{mL^2}{3} + \left(K + 2kL^2\right)\right) C\cos(\omega t + \phi) = 0$

For a non-trivial solution valid for all time

$\rightarrow C\cos(\omega t + \phi) \neq 0 \rightarrow \left(-\omega^2\frac{mL^2}{3} + \left(K + 2kL^2\right)\right) = 0$

$$\rightarrow \omega = \sqrt{\frac{3K + 6kL^2}{mL^2}}$$

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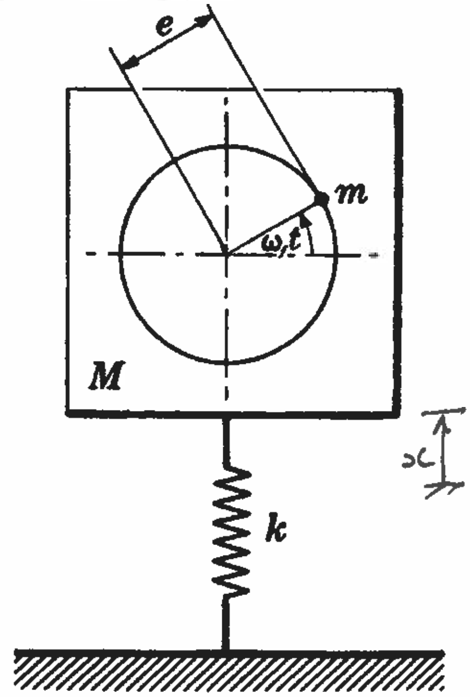
2. A production machine of mass M and supported on a spring mount of stiffness k, contains a rotor that has an unbalanced mass m that rotates with an eccentricity e. The total mass of the machine plus rotor is M+m. The rotor turns with angular frequency ω_f.

   (a) Draw a free body diagram and formulate an equation of motion for the resulting vibrational displacement x of the machine.

   (b) Using the method discussed in class, solve for the steady-state vibration of the machine as a function of frequency ratio. For simplicity, you need consider only vertical motions.

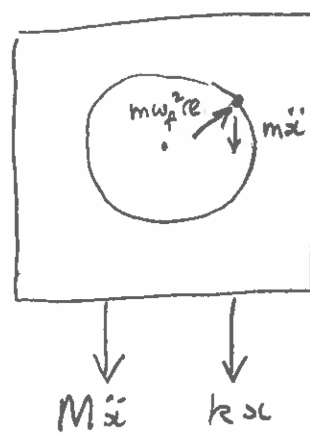
   (c) Sketch the magnification factor vs. rotation speed response of the machine. The curve is somewhat different from the one considered in class, so be sure to explain your reasoning.



(a) Vertical force equilibrium: (downwards)

$$M\ddot{x} + kx + m\ddot{x} - m\omega_f^2 e \sin\omega_f t = 0$$

vertical component of centripetal force



(Alternatively, can set $y = x + e\sin\omega_f t$ as vertical component of m displacement and then consider $m\ddot{y} = m\ddot{x} - m\omega_f^2 e \sin\omega t$)

(b) For steady state vibration, need only consider the particular solution to $(M+m)\ddot{x} + kx = m\omega_f^2 e \sin\omega_f t$

Try $x = C\sin\omega_f t \rightarrow \left(-\omega_f^2 (M+m) + k\right) C\sin\omega_f t = m\omega_f^2 e \sin\omega_f t$

Vibration amplitude $$C = \frac{m\omega_f^2 e}{k - \omega_f^2 (M+m)} = \frac{\frac{m}{M+m}\cdot\frac{M+m}{k}\,\omega_f^2 e}{1 - \omega_f^2\frac{M+m}{k}} = \frac{\frac{m}{M+m}\, e\, r^2}{1 - r^2}$$

where $\omega_n^2 = \dfrac{k}{M+m}$ $\quad r = \dfrac{\omega_f}{\omega_n}$

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(c)

$$C = \frac{\frac{m}{M+m} e r^2}{1 - r^2}$$

At very low speeds, $r \simeq 0$ $\rightarrow$ numerator $\simeq 0$

denominator $\simeq 1$

$\rightarrow C \simeq 0$

At natural frequency, $r = 1$ $\rightarrow$ numerator $= \frac{m}{M+m} e$

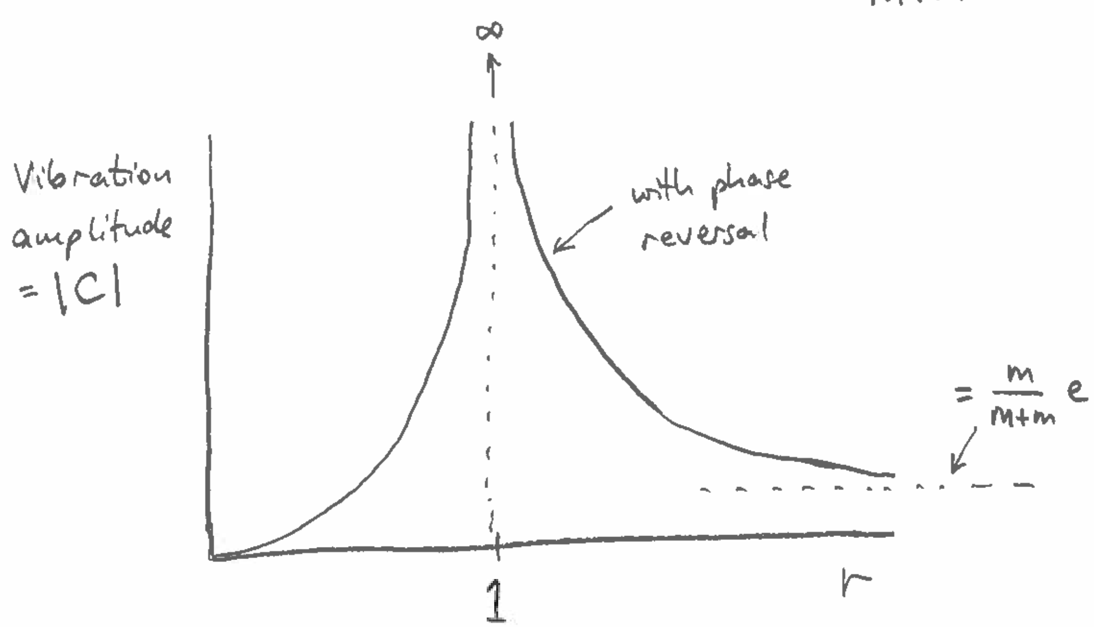
denominator $= 0$

$\rightarrow C = \infty$

At very high speeds, $r \gg 1$ $\rightarrow$ numerator $= \frac{m}{M+m} e r^2$

denominator $\simeq r^2$

$\rightarrow C = \frac{m}{M+m} e$



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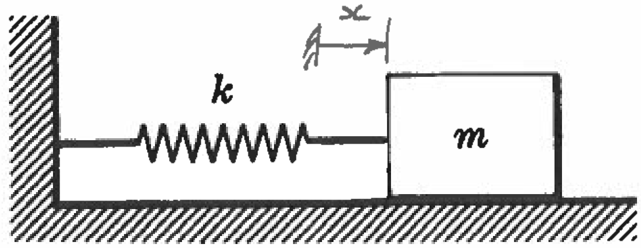
3. A mass m is attached to a fixed wall through a spring of stiffness k. The mass slides on a rough surface with friction coefficient μ.

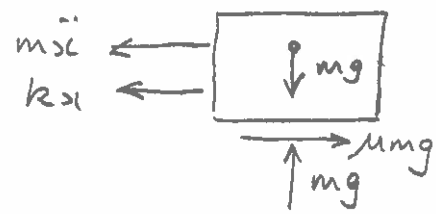


(a) Draw a free-body diagram showing the forces acting on the mass when it is at a distance x toward the right while moving towards the left.

(b) Based on your free-body diagram, determine the maximum initial displacement of the mass (with zero initial velocity) for which there is no subsequent motion. Explain your reasoning.

(c) The mass is given an initial displacement (with zero initial velocity) to the right of 4μmg/k. Determine the position x reached by the mass at the end of its subsequent motion towards the left.

(a) If the mass is moving towards the left, the opposing friction force μmg must be towards the right



(b) If there is no subsequent motion at initial displacement x, then $\ddot{x} = 0$ and the spring force kx must be insufficient to overcome friction $\rightarrow$ $kx < \mu mg$ $\rightarrow$ $x < \frac{\mu mg}{k}$

Also true for left displacements $\rightarrow$ $\underline{|x| < \frac{\mu mg}{k}}$

(c) From FBD $\quad m\ddot{x} + kx = \mu mg$

General solution = complementary solution + particular solution

Complementary solution $x_c = A\cos\omega t - B\sin\omega t$

Particular solution follows form of right side = a constant

Try $x_p = c$ $\rightarrow$ $\ddot{x}_p = 0$ $\rightarrow$ $0 + kc = \mu mg$ $\rightarrow$ $c = \frac{\mu mg}{k}$.

$\rightarrow$ $x = x_c + x_p = A\cos\omega t - B\sin\omega t + \frac{\mu mg}{k}$

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Determine integration constants from initial conditions

$$x_0 = \frac{4\mu mg}{k} \qquad \dot{x}_0 = 0$$

$$x_0 = x(0) = A\cos(\omega.0) - B\sin(\omega.0) + \frac{\mu mg}{k}$$

$$= A - 0 + \frac{\mu mg}{k} = 4\frac{\mu mg}{k}$$

$$\longrightarrow A = \frac{3\mu mg}{k}$$

$$\dot{x}_0 = \dot{x}(0) = -\omega A\sin(\omega.0) - \omega B\cos(\omega.0)$$

$$= 0 - \omega B = 0 \qquad \longrightarrow B = 0$$

$$\longrightarrow x = (3\cos\omega t + 1)\frac{\mu mg}{k}$$

$$\dot{x} = (-3\omega\sin\omega t)\frac{\mu mg}{k} = 0 \text{ when } \omega t = \pi$$

for stop at left

Position at left stop $= x(\omega t = \pi)$

$$= (3\cos\pi + 1)\frac{\mu mg}{k}$$

$$= (-3+1)\frac{\mu mg}{k}$$

$$\underline{x = -2\frac{\mu mg}{k}}$$

$|x| > \frac{\mu mg}{k}$, so the mass will then start sliding back towards the right