Question 1

(a) Indicate one type of sensor that may be suitable to measure the displacement of each of the following objects:

(i) A block of wood
(ii) A block of soft iron
(iii) An object with an exterior that is electrically conductive.

(b) The details of a capacitive sensor system for measuring rectilinear displacements are shown in Figure Q1. The sensor consists of three parallel, rectangular capacitor plates of identical size, with area A, facing each other. As shown in Figure Q1(a), the two end plates are fixed (do not move). The middle plate is attached to the moving object whose displacement x needs to be measured. In the beginning, the middle plate is equally spaced with the two end plates at distance d, as measured along the direction of motion.

Note: This is a "differential capacitor."

The dielectric constant of the material (air) between the capacitor plates is k.

The capacitor plates form the impedances Z_1 and Z_2 of the impedance bridge shown in Figure Q1(b), which is excited by a sinusoidal (AC) voltage v_{ref} of frequency ω rad/s. The other two arms of the bridge have equal impedances (i.e., $Z_3 = Z_4$).

In the beginning (at $x = 0$), $Z_1 = Z_2$ and hence the bridge is balanced.

(i) Suppose that the measured object (and hence the middle plate) is moved through x to the right, from the initial balanced position, as shown in Figure Q1(a). In terms of the given geometric parameters and k, give expressions for the capacitances C_1 and C_2. These two capacitances correspond to the impedances Z_1 and Z_2, respectively, in the bridge. In terms of these two capacitances, and v_{ref}, derive an expression for the bridge output v_o.

(ii) Derive an equation relating x to the bridge output v_o. Indicate the procedure of obtaining the displacement x from this bridge output.

(10%)

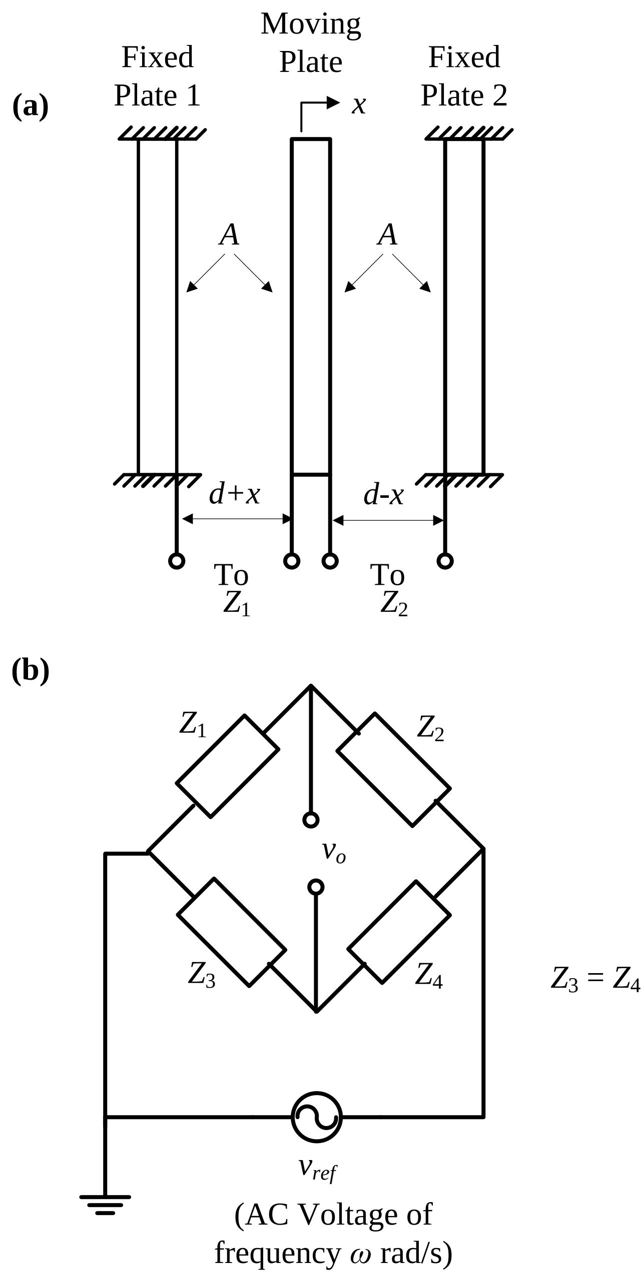

Figure Q1: A capacitive sensor for measuring rectilinear displacements. (a) Capacitor plates; (b) The impedance bridge to which the capacitor plates are connected.

Question 2

(a) What is aliasing error? Give two ways to reduce it in a digital feedback control system.

(b) A schematic diagram of a feedback control system is shown in Figure Q2. The plant is approximated by a mass-spring-damper system that is pushed using a hydraulic piston-cylinder device. The mass of the moving component is m, the stiffness of the resisting spring is k, and the viscous damping constant of the motion damper (linear) is b. In the hydraulic cylinder there are two moving pistons, one connected to a linear electric actuator that provides a required displacement u, and the other connected to the plant, which moves through y. The areas of the two pistons are equal, at A. The displacement y of the mass is measured by an analog displacement sensor (e.g., LVDT) of time constant τ_s and provided to the analog-to-digital converter (ADC) of the digital controller where it is sampled at a fixed rate, and digitized. The digital controller computes the necessary control action, according to a specific control law, using the position feedback and other pertinent information. The digital control action is converted into an analog signal by means of a digital-to-analog converter (DAC). This signal is passed through analog signal-conditioning hardware (e.g., amplifier and filter), as shown. The resulting conditioned signal is used to drive the linear actuator, which moves the left piston through u.
Note: Assume that in the beginning, $u = 0$ and $y = 0$, and in that position the pressure of the hydraulic oil in the cylinder is equal to the atmospheric pressure. The volume of the hydraulic oil then (nominal volume) is V_o.

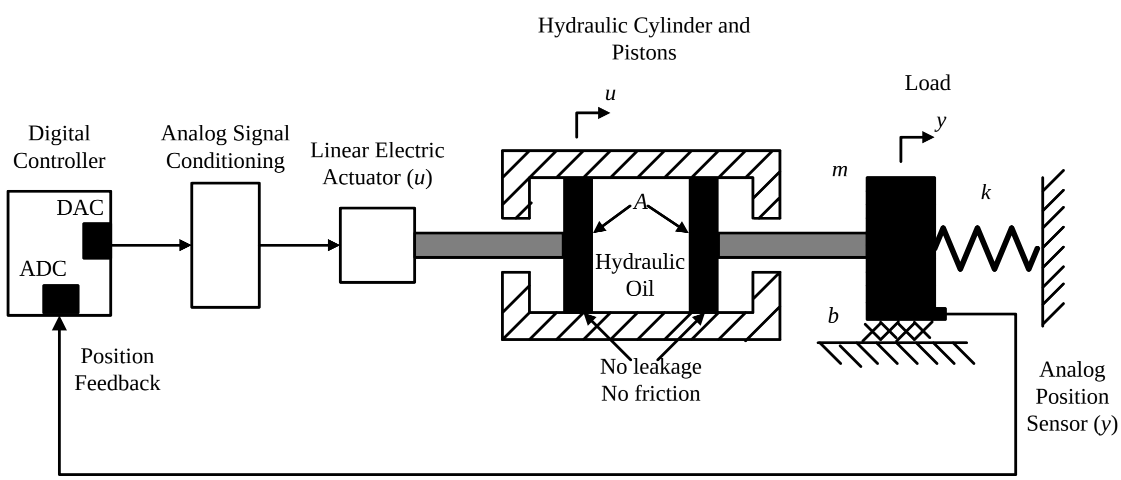

Figure Q2: A position feedback control system.

(i) It is given that the nominal volume (initial) of the hydraulic oil in the cylinder is V_o and its bulk modulus is β. Obtain an expression for the gauge pressure P of the oil inside the cylinder, in a general condition of displacement, in terms of the displacements u and y and the parameters A, V_o and β. Using this and other given parameters (m, b, k) obtain a differential equation of motion of the plant (where the input is the displacement u and the output is the displacement y). From this equation, write the expression for the undamped natural frequency ω_n of the plant (in rad/s). Indicate five ways to increase this natural frequency.

Hint: Bulk modulus $\beta = \dfrac{P}{(-\Delta V / V_o)}$ where P is the oil pressure (gauge) and ΔV is the "increase" in volume of the oil.

(ii) Given: A = 100.0 cm^2; V_o = 5000.0 cm^3; $\beta = 1.0\times10^9$ Pa; m = 30.0 kg, and $k = 1.0\times10^7$ N/m.

Note: 1 Pa = 1 N/m^2.

Calculate the undamped natural frequency of the plant (ω_n) in rad/s. Assuming very low damping, the undamped natural frequency is approximately equal to the resonant frequency ω_r of the plant. Suppose that the operating frequency range of the plant (i.e., operating bandwidth or practical frequency range of operation of the plant, or the frequency range within which the plant should be controlled) is 0 to $2\omega_r$ rad/s. Estimate suitable numerical values for the following:

- Time constant τ_h of the analog signal conditioning hardware
- Minimum rate in Hz at which the digital control actions should be generated by the control computer
- Minimum rate in Hz at which the sensor signal should be sampled by the ADC
- An upper limit for the time constant τ_s of the displacement sensor.

Note: Assume that the linear electric actuator has the full capability to accurately provide the necessary piston movement u, without distorting it with the actuator dynamics.

Question 3

A sensing mechanism for measuring the rectilinear speed of an object is shown in Figure Q3. The object whose speed (v) has to be measured, is directly connected to the rigid rod of the sensing device. The rod passes through smooth and leak-proof bearings of the cylindrical casing containing hydraulic oil, whose effective mass (including the "added mass" of the hydraulic oil in it, and any moving attachments) is m. The casing moves on rollers, which do not apply a resisting (frictional) force on the casing. The rod has a cylindrical piston at the other end, which moves in the viscous liquid inside the casing, and the associated linear viscous damping constant is b. This generates a viscous damping force on the casing (due to the relative velocity between the piston and the casing) in the direction of motion. The resulting displacement of the casing is y. This motion of the casing is resisted by a linear spring of stiffness k. Firmly attached to the casing is the ferromagnetic core of an LVDT. The modulated output voltage v_o of the LVDT determines y and hence v. The carrier voltage signal of the LVDT is v_{ref}.

(a) Explain the principle of operation of an LVDT and the procedural steps for determining the displacement y from the LVDT output v_o.

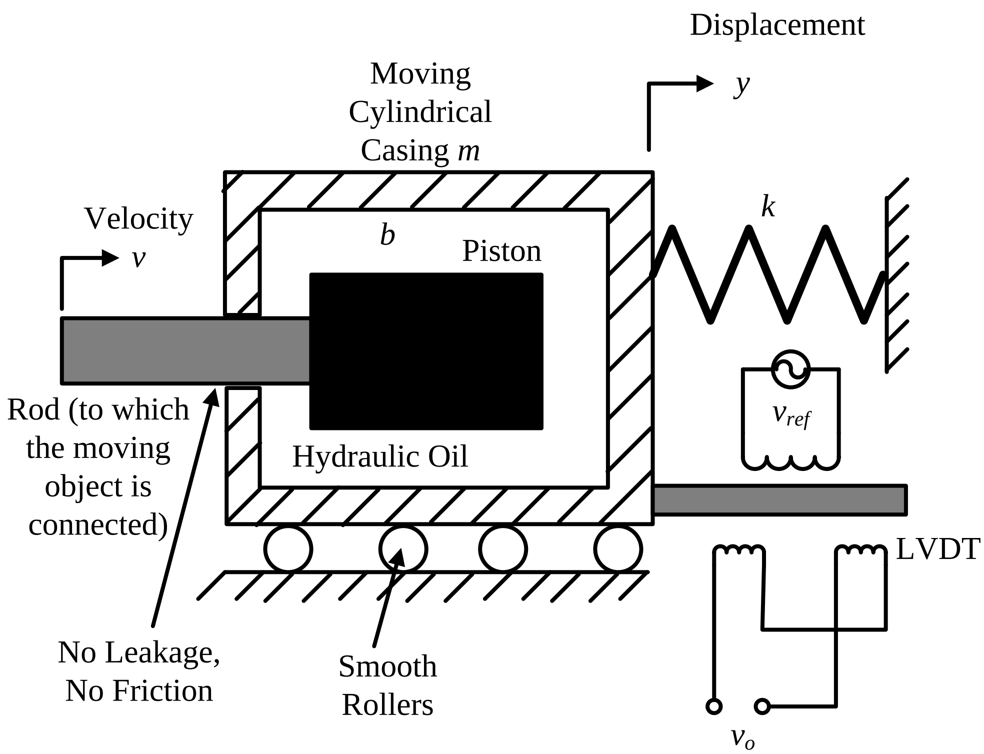

Figure Q3: A sensor for velocity measurement.

(b) In terms of the indicated parameters and variables, determine the differential equation of motion (according to Newton's 2^{nd} law) for the casing. From that equation obtain the transfer function of the device, relating the output displacement y and the input velocity v.

(c) Given: $m = 1.0\ \text{kg}$. Suppose that the operating bandwidth of the sensor should be 50.0 Hz, which is taken as one-fifth of its undamped natural frequency. Also, it is required that the damping ratio of the measuring device be 0.2. Determine the required values for the spring constant k (N/m) and the viscous damping constant b (N/m/s), in order to meet the indicated specifications. What is the corresponding static gain of the measuring device, and what are its units?
Suggest, giving reasons, a suitable value for the carrier frequency of the LVDT.

Question 4

(a) Giving reasons, indicate a situation (i.e., nature of the error) where the "Absolute (ABS)" method of error combination is preferred over the "Square-root of Sum of Squares (SRSS)" method, and a situation where the SRSS method is preferred over the ABS method.

(b) The block diagram of a control system that uses a *model-based controller* is shown in Figure Q4. Specifically, the velocity of the plant is measured using a velocity sensor and used together with a damping model to compute the corresponding damping force in the plant. This computed damping force value is used in the controller to determine the control command for the actuator, according to an appropriate control logic. The actuator drives the plant using this control command.

The damping model that is used is given by,

$$F_d = (F_c + F_s e^{-\alpha|v|}) \operatorname{sgn} v + bv$$

where,

v = measured velocity

F_d = overall damping force (in the direction opposite to the motion)

F_c = Coulomb frictional force

F_s = maximum Stribeck damping force

α = Stribeck coefficient

b = linear viscous damping constant

Also, $|v|$ denotes the absolute value of velocity v, and "sgn" denotes the signum (sign) function.

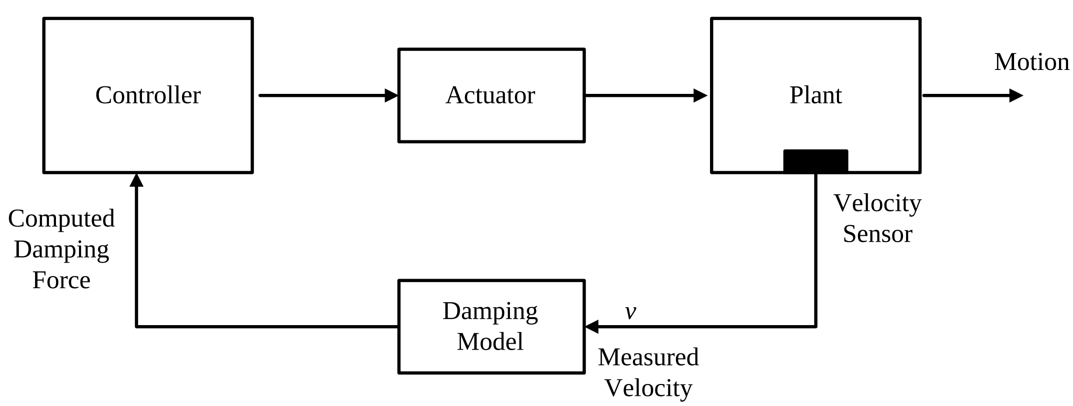

Figure Q4: A control system that uses model-based control.

(i) Sketch the shape of the Stribeck damping function $F_s e^{-\alpha|v|} \operatorname{sgn} v$ as the velocity v varies from -∞ to +∞. On the graph (the y-axis is the damping force and the x-axis is the velocity) indicate some key values (e.g., maximum, minimum, 0) of the damping force.

(ii) It is known that there are *model errors* in the terms of Coulomb friction, Stribeck damping and viscous damping, and also there is *measurement error* in v. Assume that the parameter α is error free. Using the absolute method of error combination, express the combined fractional error e_d in the computed overall damping force F_d in terms of $e_c, e_s,\ e_v$, and e_b, which are the fractional errors in F_c, F_s, v, and b, respectively. Completely express the non-dimensional sensitivity function of each fractional error term, in terms of the quantities given in the question.
Note: Ignore any computational errors. Consider only the model errors and the measurement error.

(iii) At some operating condition of the system, the following numerical values are known: F_c = 80.0 N, F_s = 100.0 N, b = 100.0 N/m/s, v = 2.0 m/s, and α = 0.5 s/m. Compute the corresponding non-dimensional sensitivities $S_c, S_s,\ S_v$, and S_b of the fractional errors $e_c, e_s,\ e_v$, and e_b, on the combined fractional error e_d. In order for the individual contributions to the overall error from the 4 error sources to be equal, which of the four quantities F_c, F_s, v, and b, should be made the most accurate and which should be made the least accurate?

Solution 1

(a)

(i) Capacitive displacement sensor where one of the capacitor plates is attached to the moving object.

(ii) An LVDT whose moving core is formed by the soft iron block.

(iii) An eddy current sensor may be used. Only small displacements (or proximities) may be measured through this sensor.

(b)

(i) Using the well-known equation for the capacitance of a capacitor with two parallel plates, we can write:

$$C_1 = \frac{kA}{d+x} \quad \text{and} \quad C_2 = \frac{kA}{d-x}$$

Also, $Z_1 = \dfrac{1}{j\omega C_1}$ and $Z_2 = \dfrac{1}{j\omega C_2}$

Through straightforward circuit analysis (specifically, voltage division between series impedances), and assuming that the output of the bridge is in open-circuit, the output of the bridge may be expressed as

$$v_o = \frac{Z_1}{(Z_1+Z_2)} v_{ref} - \frac{Z_3}{(Z_3+Z_4)} v_{ref}$$

Since $Z_3 = Z_4$, we have,

$$v_o = \left[\frac{Z_1}{(Z_1+Z_2)} - \frac{1}{2}\right]v_{ref} = \left[\frac{1/Z_2}{(1/Z_2+Z_1)} - \frac{1}{2}\right]v_{ref}$$

Note: The second expression was obtained by dividing both numerator and denominator by the product Z_1Z_2.
Now substitute the previous expressions for Z_1 and Z_2. We get

$$v_o = \left[\frac{j\omega C_2}{(j\omega C_2 + j\omega C_1)} - \frac{1}{2}\right]v_{ref} = \left[\frac{C_2}{(C_2+C_1)} - \frac{1}{2}\right]v_{ref} = \left[\frac{2C_2-(C_2+C_1)}{2(C_2+C_1)}\right]v_{ref}$$

Or, $$v_o = \left[\frac{(C_2-C_1)}{2(C_2+C_1)}\right]v_{ref}$$

(ii) By dividing both numerator and denominator by the product C_1C_2, the above result may be written as,

$$v_o = \left[\frac{(1/C_1 - 1/C_2)}{2(1/C_1+1/C_2)}\right]v_{ref}$$

Now, substitute the previous expressions for the two capacitances. We get,

$$v_o = \left[\frac{\frac{d+x}{kA} - \frac{d-x}{kA}}{2\left(\frac{d+x}{kA} + \frac{d-x}{kA}\right)}\right]v_{ref} = \left[\frac{2x}{2(2d)}\right]v_{ref}$$

Or, $$v_o = \frac{x}{2d}v_{ref}$$

It is clear from this result that the bridge output is in fact the carrier signal v_{ref} modulated by the displacement term $\frac{x}{2d}$. We need to demodulate this output in order to recover the modulating term $\frac{x}{2d}$, which gives the displacement x (on multiplying by the scaling factor $2d$).

Demodulation Process: 1. Multiply the bridge output $\frac{x}{2d}v_{ref}$ by the carrier voltage v_{ref}. 2. Low-pass filter the resulting signal at a cut-off frequency less than 2ω, where ω is the frequency of the carrier voltage v_{ref}. This will generate $\frac{x}{2d}$.

Solution 2

(a) When an analog signal is sampled, discrete-time data samples are produced. Information between the sampled data is lost in this process. The resulting error is called the aliasing error. In the frequency domain, the aliasing error is the error that distorts the frequency spectrum of an analog signal due to sampling of that signal. If the sampling frequency is f_s, due to the data sampling, the frequency spectrum of the original analog signal beyond half of this frequency (i.e., the Nyquist frequency $f_c = 0.5\, f_s$) will be effectively folded into the low-frequency segment of the spectrum. This follows from Shannon's sampling theorem.

In a digital feedback control system, an analog output signal has to be sampled when feeding back into the controller. Aliasing will occur then. The aliasing error can be reduced by the following two methods.

Method 1 (Preferred): Pass the analog signal through an anti-aliasing filter before it is sampled. This filter is a low-pass filter whose cut-off frequency is, ideally, half the sampling frequency. Since the spectral segment beyond f_c is filtered out by the anti-aliasing filter, the folding of the spectrum beyond this frequency cannot occur, and aliasing will not occur.

Method 2: Increase the sampling frequency. This increases the Nyquist frequency, and hence reduces the portion of the spectrum that is folded, thereby reducing the aliasing.

(b)

(i)

Increase in volume of the hydraulic oil, from the initial condition, is

$$\Delta V = -A \times (u - y)$$

Substitute: Bulk modulus $\beta = \dfrac{P}{(-\Delta V / V_o)} = \dfrac{V_o P}{A \times (u - y)}$

Hence, the gauge pressure of the hydraulic oil,

$$P = \frac{\beta A \times (u - y)}{V_o} \qquad \text{(i)}$$

Apply Newton's 2nd law to mass m (Inertial force = sum of external forces):

$$m\ddot{y} = PA - ky - b\dot{y} \qquad \text{(ii)}$$

Substitute (i) into (ii). This gives the differential equation of motion of the plant (i.e., the time-domain model):

$$m\ddot{y} = A \frac{\beta A \times (u - y)}{V_o} - ky - b\dot{y}$$

Or,

$$m\ddot{y} + b\dot{y} + \left[\frac{A^2 \beta}{V_o} + k \right] y = \frac{A^2 \beta}{V_o} u \qquad \text{(iii)}$$

From this 2nd order model (damped oscillator), the undamped natural frequency of the plant may be expressed as,

$$\omega_n = \sqrt{\frac{\frac{A^2\beta}{V_o} + k}{m}} \qquad \text{(iv)}$$

From (iv) it is clear that the undamped natural frequency of the plant can be increased by:

1. Increasing stiffness of the spring
2. Decreasing the mass of the moving parts
3. Increasing the bulk modulus of the hydraulic oil
4. Increasing the cross-sectional area of the cylinder (i.e., of the moving pistons)
5. Decreasing the amount of hydraulic oil in the cylinder.

(ii)

Substitute the given numerical values:

$$\frac{A^2\beta}{V_o} = \frac{(100.0\times10^{-4})^2 \times 1\times10^9}{(5000.0\times10^{-6})} \text{ N/m} = 2.0\times10^7 \text{ N/m}$$

The undamped natural frequency of the plant is:

$$\omega_n = \sqrt{\frac{(2.0+1.0)\times10^7}{30.0}} \text{ rad/s} = 1000.0 \text{ rad/s}$$

In view of low damping, the resonant frequency is approximately equal to this. Hence,

$$\omega_r \approx 1000.0 \text{ rad/s}$$

It is given that the operating bandwidth of the plant is $2\omega_r \approx 2000.0$ rad/s $\approx$ 320.0 Hz

Hence, the DC range of the analog signal conditioning hardware should span at least up to 2000.0 rad/s so that its dynamics would not affect the plant performance. Hence, this should nominally be the *half-power bandwidth* of the hardware. However, to be even more accurate, as a rule of thumb (with a factor of safety), the DC range of this hardware may be taken as half of the half-power bandwidth of the circuit. Hence,

$$\frac{1}{2}\times\frac{1}{\tau_h} = 2000.0 \;\rightarrow\; \tau_h = 0.25 \text{ ms}$$

The digital control action should be generated at least at twice the operating bandwidth of the plant, in order for the frequency spectrum of the digital control signal to cover the operating bandwidth of the plant (according to Shannon's sampling theorem). Hence, the required minimum digital control action rate is,

$$f_c = 2\times\frac{2000.0}{2\pi} \text{ Hz} = 640.0 \text{ Hz}$$

In order to generate the control action in the frequency range of the digital control action, the frequency spectrum of the plant signal (which is used in the computation of the control action) should fall within at least the frequency range of the digital control action. Hence, according to Shannon's sampling theorem, the feedback signal from the plant should be sampled at least at double the control action frequency. The corresponding minimum sampling frequency of the feedback (sensor) signal is,

$$f_s = 2 \times 640.0 \text{ Hz} = 1280.0 \text{ Hz}$$

The DC range of the sensor should cover at least this frequency range, in order to measure the signal accurately in this frequency range (without distorting it by sensor dynamics). Hence, this should nominally be the *half-power bandwidth* of the sensor. However, to be even more accurate, as a rule of thumb (with a factor of safety), this DC range of the sensor is taken as half of the half-power bandwidth of the sensor. Hence, we need,

$$\frac{1}{2} \times \frac{1}{\tau_s} = 1280.0 \times 2\pi \text{ rad/s} = 8000.0 \text{ rad/s}.$$

This gives the upper limit for the sensor time constant as,

$$\tau_s = \frac{1}{2} \times \frac{1}{8000.0} \text{ s} = 0.0625 \times 10^{-3} \text{ s} = 62.5 \text{ μs}$$

Question 3

(a)

The ferromagnetic core of the LVDT is firmly attached to the object whose rectilinear displacement needs to be measured. In the home position of the sensor (i.e., at zero displacement) the primary coil, which is excited by a carrier voltage of suitable frequency, is placed centrally around the core. The secondary coil has two segments, which are placed symmetrically around the two end segments of the core, in this home position, and connected in series opposition. In this manner, in the home position, the overall induced voltage in the secondary coil will be zero (the voltages in the two segments are equal and opposite). As the core moves with the object, an imbalance voltage will be induced in the secondary coil. It is demodulated by multiplying by the carrier signal and then low-pass filtering at a cut-off frequency sufficiently below twice the carrier frequency. In the linear range of the LVDT, this demodulation process provides the displacement, with a scaling factor (which is known either by calibration or from the physics of the device).

(b)

The free-body diagram of the casing, which is moving with displacement *y*, is shown in Figure S3. The mass of the LVDT core may be neglected or included in *m*. The added mass from the hydraulic oil is included in *m*.

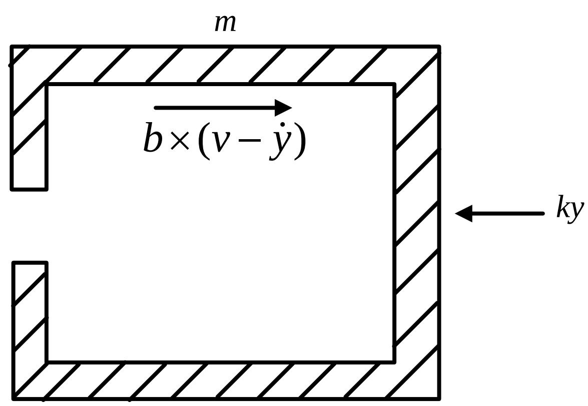

Figure S3: Free-body diagram of the casing.

The linear viscous damping force is proportional to the relative velocity between the piston and the cylinder, and is given by $b \times (v - \dot{y})$. The spring force is given by ky.

Newton's 2nd law (Inertial force = sum of external forces) for the casing:

$$m\ddot{y} = b(v - \dot{y}) - ky$$

Or, $\quad m\ddot{y} + b\dot{y} + ky = bv$

This is the time-domain model of the sensor. To get the transfer function of the sensor, simply replace the time derivative by the Laplace variable *s*. We get the corresponding transfer function,

$$G(s) = \frac{y}{v} = \frac{b}{ms^2 + bs + k} \qquad \text{(i)}$$

(b)

The denominator polynomial (characteristic polynomial) of the transfer function is

$$ms^2 + bs + k \rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$$

where, $\omega_n^2 = \frac{k}{m}$ and $2\zeta\omega_n = \frac{b}{m}$. Hence,

Undamped natural frequency $\omega_n = \sqrt{\frac{k}{m}}$

→ $\quad k = m \times (\omega_n)^2 \qquad$ (ii)

Damping ratio $\zeta = \frac{1}{2\omega_n}\frac{b}{m} = \frac{b}{2\sqrt{km}}$

→ $\quad b = 2\zeta\sqrt{km} \qquad$ (iii)

Given, the natural frequency of the sensor = 5×50.0 Hz = 250.0 Hz = 2π×250.0 rad/s

With m = 1.0 kg, we have from (ii):

$$k = 1.0 \times (2\pi \times 250)^2 \text{ N/m} = 2.47 \times 10^6 \text{ N/m} = 2.47 \text{ MN/m}$$

From (iii), damping constant $b = 2 \times 0.2 \times \sqrt{2.47 \times 10^6 \times 1}$ N/m/s $= 628.0$ N/m/s

From (i): Static gain = $\dfrac{b}{k} = \dfrac{628.0\text{ (N/m/s)}}{2.47\times10^{6}\text{ (N/m)}} = 2.54\times10^{-4}$ s

As a rule of thumb, a suitable carrier frequency for the LVDT would be 10 times the operating bandwidth (50 Hz), which is 500 Hz. This would provide high measurement accuracy (carrier would not affect the displacement reading, which could be accurately recovered through demodulation) and reduce rate errors in the LVDT.

<u>Solution 4</u>

(a) The ABS method provides a conservative upper bound for the combined error. Hence, this method is preferred when the errors are rather systematic and their behavior is fairly clear. Since the absolute values of the errors are simply added together, this method is accurate if all the errors have the same sign (either all positive or all negative).

The method of error combination in SRSS is the same as the method of combining standard deviations of independent random variables that are identically distributed. Hence, this method is particularly suitable if the individual errors are stochastic, independent, and identically distributed (i.e., they have similar probability distributions).

(b) The damping model is

$$F_d = (F_c + F_s e^{-\alpha|v|})\operatorname{sgn} v + bv \qquad \text{(i)}$$

.

(i) The Stribeck component of damping is

$$F_s e^{-\alpha|v|}\operatorname{sgn} v$$

This function can be expressed as:

$$F_s e^{-\alpha v} \text{ for } v > 0 \qquad \text{(ii)}$$

$$-F_s e^{\alpha v} \text{ for } v < 0 \qquad \text{(iii)}$$

It is seen that the function (ii) is always +ve and the function (iii) is always –ve. Also,

At v = 0, (ii) has the maximum value F_s
At v = 0, (iii) has the minimum value -F_s
As $v \rightarrow \infty$, (ii) goes to zero from the +ve side
As $v \rightarrow -\infty$, (iii) goes to zero from the -ve side.

Using this information, Stribeck damping component is sketched in Figure S4, with velocity v as the independent variable (x-axis).

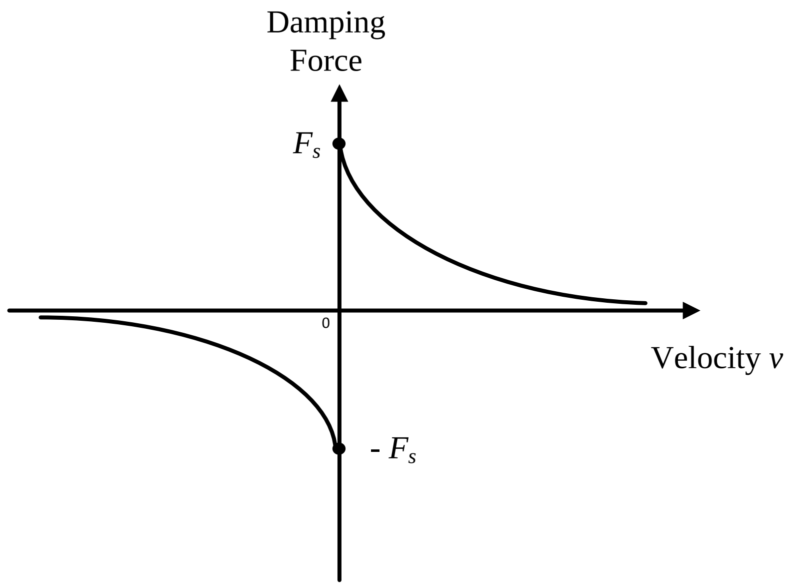

Figure S4: Behavior of the Stribeck damping component.

(ii) Take the differential of (i), assuming small changes (i.e., small errors):

$$\delta F_d = \operatorname{sgn} v \times \delta F_c + e^{-\alpha|v|} \operatorname{sgn} v \times \delta F_s + F_s e^{-\alpha|v|} \times (-\alpha) \operatorname{sgn} v . \operatorname{sgn} v \times \delta v + v.\delta b + b.\delta v$$

Note: $|v| = v \operatorname{sgn} v$; $\operatorname{sgn} v$ is a constant; hence, $\dfrac{d \operatorname{sgn} v}{dv} = 0$

These differentials denote the errors in the corresponding quantities. Normalize them by dividing by the nominal values of the corresponding quantities, to get the fractional errors. We have,

$$\frac{\delta F_d}{F_d} = \frac{F_c \operatorname{sgn} v}{F_d} \times \frac{\delta F_c}{F_c} + \frac{F_s e^{-\alpha|v|} \operatorname{sgn} v}{F_d} \times \frac{\delta F_s}{F_s} - \frac{v F_s \alpha e^{-\alpha|v|}}{F_d} \times \frac{\delta v}{v} + \frac{bv}{F_d} \times \frac{\delta b}{b} + \frac{vb}{F_d} \times \frac{\delta v}{v} \text{ Or,}$$

$$e_d = S_c e_c + S_s e_s + S_v e_v + S_b e_b \qquad \text{(iv)}$$

where, according to the ABS method, the non-dimensional sensitivities are:

$$S_c = \frac{F_c}{F_d}; \;\; S_s = \frac{F_s e^{-\alpha|v|}}{F_d}; \;\; S_v = \left[\frac{|v| F_s \alpha e^{-\alpha|v|}}{F_d} + \frac{|v| b}{F_d} \right]; \;\; S_b = \frac{b|v|}{F_d} \qquad \text{(v)}$$

Note: The signs of the terms are ignored in view of the ABS method.

(iii) Given: F_c = 80.0 N, F_s = 100.0 N, b = 100,0 N/m/s, v = 2.0 m/s, and α = 0.5 s/m.
Then, $\operatorname{sgn} v = 1$

Substitute in (i):

$$F_d = 80.0 + 100.0 e^{-0.5\times2.0} + 100.0 \times 2.0 \text{ N} = 317.0 \text{ N}$$

Substitute in (v):

$$S_c = \frac{80.0}{317.0} = 0.25$$

$$S_s = \frac{100.0e^{-0.5\times 2.0}}{317.0} = 0.12$$

$$S_v = \left[\frac{2.0\times 100.0\times 0.5e^{-0.5\times 2.0}}{317.0} + \frac{2.0\times 100.0}{317.0}\right] = 0.1161 + 0.6309 = 0.75$$

$$S_b = \frac{100.0\times 2.0}{317.0} = 0.63$$

For equal contribution into the combined error, the error with the highest sensitivity, should have the least value (i.e., should be the most accurate). In the present example, it is the velocity measurement.

Similarly, for equal contribution into the combined error, the error with the least sensitivity, should have the highest value (i.e., should be the least accurate). In the present example, it is the Stribeck damping model.