MECH 364: MECHANICAL VIBRATIONS
MIDTERM EXAMINATION 2

Time: 45 minutes 20th October 2010 Maximum Available Mark: 20

Q1.

a) A driver of 70 kg mass is seated inside a car as shown below. He is not wearing a seat belt. When the car encounters a speed bump, he is thrown upwards and freely drops through a height of 7.5 cm onto the unpadded seat and does not rebound. Determine the maximum acceleration transmitted by his spinal cord by formulating the equations of motion and initial conditions using a Free Body Diagram (FBD), and solving for the response. Use the spring mass model shown. From extensive biomechanical tests, the spinal stiffness of a typical human is found to be around 81000 N/m, for the spring k in the model. **(14 marks)**

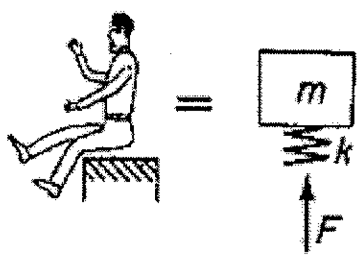


Figure 1: Driver in a car (left) and the spring-mass model (right). F is the force exerted by the seat on the driver, acting through the spine of stiffness $k = 81000$ N/m.

b) Using the FBD in part a), formulate the equations of motion and initial conditions if the seat has a low density foam padding, modelled by a linear elastic spring of constant $k_s = 5000 N/m$? **You need not find the response. Just give the equations of motion and initial conditions for this new case.** **(4 marks)**

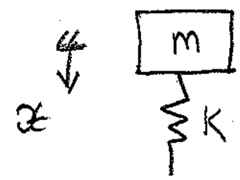
c) Without solving the problem in part b), indicate whether the force transmitted through the spine is reduced by a padded seat or not? Justify your answer in one or two sentences using the model you developed in parts a) and b). **(2 marks)**

ALL THE BEST!

MECH 364

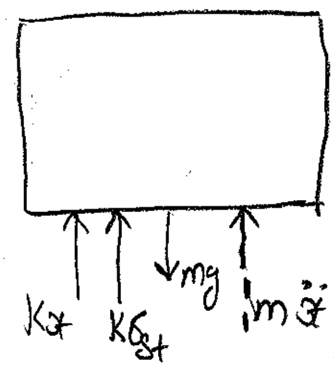
MIDTERM EXAMINATION 2 : SOLUTION

Q1 a) <u>MODEL:</u>



<u>REFERENCE:</u> STATIC EQUILIBRIUM POSITION IN WHICH THE SPINE HAS COMPRESSED BY $\delta_{st} = \frac{mg}{K}$

<u>FREE BODY DIAGRAM:</u>
(AT STATIC EQUILIBRIUM)



$\downarrow \sum_{+ve} F_x = 0$

$\Rightarrow mg - K\delta_{st} - Kx - m\ddot{x} = 0$

$\Rightarrow \underline{\underline{m\ddot{x} + Kx = 0}}$ $\left(\because mg - K\delta_{st} = 0 \text{ FROM STATICS} \right)$

<u>INITIAL CONDITIONS:</u> $x(0) = -\delta_{st}$

$\dot{x}(0) = \sqrt{2gh}$ $\quad$ h: height of fall

* <u>SAME AS BUNGEE JUMPER / CUSHION PROBLEM!!</u>

$x = A_1 \cos\omega_n t + A_2 \sin\omega_n t = A\cos(\omega_n t - \phi_0)$; $A = \sqrt{A_1^2 + A_2^2}$

$\tan\phi_0 = \frac{A_2}{A_1}$

$x(0) = -\delta_{st} \Rightarrow A_1 = -\delta_{st}$

$\dot{x}(0) = \sqrt{2gh} \Rightarrow A_2 = \frac{\sqrt{2gh}}{\omega_n}$

$\therefore x = \sqrt{\delta_{st}^2 + \frac{2gh}{\omega_n^2}} \cos\left(\omega_n t - \phi_0\right)$; $\tan\phi_0 = -\frac{\sqrt{2gh}}{\omega_n \delta_{st}}$

MAXIMUM DISPLACEMENT: $x_{max} = A = \sqrt{\delta_{st}^2 + \frac{2gh}{\omega_n^2}}$

MAXIMUM ACCELERATION: $\ddot{x}_{max} = -\omega_n^2 A = -\omega_n^2 \sqrt{\delta_{st}^2 + \frac{2gh}{\omega_n^2}}$

USE $\omega_n^2 = \frac{-g}{\delta_{st}}$ TO GET

$$\ddot{x}_{max} = -\frac{g}{\delta_{st}} \sqrt{\delta_{st}^2 + \frac{2gh}{g}\delta_{st}} = -\frac{g}{\delta_{st}} \sqrt{\delta_{st}^2 + 2h\delta_{st}}$$

$$= -g\sqrt{1 + \frac{2h}{\delta_{st}}}$$

$$= -4.32\, g$$

$\delta_{st} = \frac{mg}{K} = \frac{70 \times 9.81}{81000}$

$= 8.48 \times 10^{-3}$ m

$h = 7.5$ cm $= 7.5 \times 10^{-2}$ m

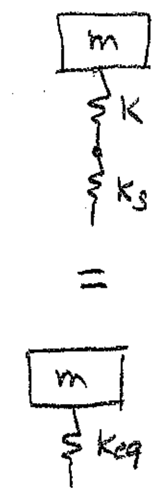
$$\ddot{x}_{max} = -42.41 \text{ m/s}^2$$

b) WE HAVE TWO SPRINGS K_s & K IN SERIES.

$$K_{eq} = \frac{K_s K}{K_s + K} \quad \text{(SPRINGS IN SERIES FORMULA)}$$

$$= \frac{5000 \times 81000}{81000 + 5000} = 4709.3 \text{ N/m}$$

$$\delta_{st} = \frac{mg}{K_{eq}} = \frac{70 \times 9.81}{4709.3} = 0.1458 \text{ m}$$



INITIAL CONDITIONS: $\dot{x}(0) = \sqrt{2gh} = 1.213$ m/s

$x(0) = -\delta_{st} = -0.1458$ m

EQUATION OF MOTION: $m\ddot{x} + K_{eq}x = 0$ Replace 'K' WITH K_{eq}

c) FOR THE SAME HEIGHT OF FALL $\ddot{x}_{max}$ WILL DECREASE BECAUSE δ_{st} IS MORE IN $\ddot{x}_{max} = -g\sqrt{1 + \frac{2h}{\delta_{st}}}$. FORCE TRANSMITTED WILL ALSO DECREASE.