MECH 420 SENSORS AND ACTUATORS
Solutions to Assignment 6

Sol-Problem 1 (Problem 6.3 from Textbook)

Pulse Counting Method

Speed resolution corresponds to the speed change for a single count. Hence it is given by

$$\Delta\omega_{res} = \frac{2\pi n}{NT} - \frac{2\pi(n-1)}{NT} = \frac{2\pi}{NT} \qquad \text{(i)}$$

Note that the resolution does not depend on n (hence ω). The fractional error in the measurement is given by

$$\frac{\Delta\omega}{\omega} = \frac{\frac{2\pi n}{NT} - \frac{2\pi(n-1)}{NT}}{\frac{2\pi n}{NT}} = \frac{1}{n} = \frac{2\pi}{\omega NT} \qquad \text{(ii)}$$

This decreases as n increases (i.e., when ω increases). Hence accuracy increases as ω increases.

Pulse Timing Method

Speed resolution corresponds to the speed change for a unit change in the frequency count. Hence it is given by,

$$(\Delta\omega)_{res} = \frac{2\pi f}{Nm} - \frac{2\pi f}{N(m+1)} = \frac{2\pi f}{Nm(m+1)} \cong \frac{2\pi f}{Nm^2} = \frac{\omega^2 N}{2\pi f} \quad \text{for large } m \qquad \text{(iii)}$$

Hence the resolution depends <u>quadratically</u> on m (and hence on the speed).

The fractional error is given by
$$\frac{\Delta\omega}{\omega} = \frac{\frac{2\pi f}{Nm} - \frac{2\pi f}{N(m+1)}}{\frac{2\pi f}{Nm}} = \frac{1}{m+1} \cong \frac{1}{m} = \frac{\omega N}{2\pi f} \qquad \text{(iv)}$$

Hence the error depends on m (and hence ω). The accuracy decreases as ω increases because m decreases as ω increases.

Other Factors: From equation (i) it is clear that in the first method, resolution improves as N (the number of windows in disk) and T (the timing duration) increase. From eq.(ii) it is clear that percentage accuracy depends primarily on speed, N and T. From equation (iii) notice that in the second method the resolution depends on N and f (the clock frequency) as well. From equation (iv) we see that percentage accuracy depends primarily on speed, N, and f.

Accuracy depends on production tolerances, mounting features and environmental factors as well. The resolution would depend on word size (which boils down to correctly reading the count n in the first method or m in the second method).

Speed, in the example, is increased by a factor of 10.

(a) Resolution does not change.

% Accuracy increases by a factor of 10 (because n increases by a factor of 10).

(b) Resolution becomes poorer by a factor of 100 (because m decreases by a factor of 10).

% Accuracy becomes poorer by a factor of 10.

Sol-Problem 2 (Problem 6.5 from Textbook)

Let p = step-up gear ratio. Using the usual notation, speed computation can be done using the following equations:

(a) Pulse-count method: $\omega_c = \dfrac{2\pi n}{NTp}$

(b) Pulse-timing method: $\omega_t = \dfrac{2\pi f}{Nmp}$

These relations are obtained simply by dividing the encoder disk speed by the gear ratio, which gives the object speed.

The speed resolution is the change in speed corresponding to a unity change in the count. Hence, for the pulse-count method: $\Delta\omega_c = \dfrac{2\pi(n+1)}{NTp} - \dfrac{2\pi n}{NTp} = \dfrac{2\pi}{NTp}$

Note: p improves the speed resolution in the pulse-count method.

For the pulse-timing method, the speed resolution is

$$\Delta\omega_t = \frac{2\pi f}{Nmp} - \frac{2\pi f}{N(m+1)p} = \frac{2\pi f}{Npm(m+1)} \approx \frac{Np}{2\pi f}\omega^2$$

Note: In this case, for a given speed, the resolution degrades with increasing *p*.

Sol-Problem 3 (Problem 6.20 from Textbook)

Advantages of an optical encoder:

Noncontact (no friction); measurement is in terms of pulses (no quantization error); in digital control ADC will not be needed for reading the sensor signal for feedback into the controller; speed as well as position can be obtained directly from the same encoder; encoders integral with motors (encoders precisely mounted on the motor shaft by the manufacturer) are available.

Disadvantages of an optical encoder:

Delicate; expensive (high-resolution units); bi-directional operation will require additional electronics; cannot be simply replaced by a different model.

Now consider the servo loop shown in Figure P6.20. The system components are identified below.

Control Processor: Receives the desired motion command from the robot controller and the feedback signals from the encoder of the joint motor, and computes the necessary drive signal (digital) for the motor.

DAC: The digital-to-analog converter converts the digital drive signal that is generated by the control processor into an analog voltage signal (maximum level ± 2.5 V).

PWM: The pulse width modulator is powered by a dc power supply (10 V and 2 A capacity). It has an internal carrier ac signal of 20 kHz. A pulse signal of this frequency whose pulse width is modulated by the analog drive signal from the controller is generated by the PWM. The voltage level of the PWM signal is ± 20V and this signal energizes the windings of the permanent-magnet (PM) dc motor.

DC Motor:

The motor is driven by the PWM signal, which should ideally match the motion command. The particular motor may have a permanent-magnet stator and a wound rotor with a split ring and brushes for commutation.

Encoder:

The incremental optical encoder generates two pulse signals one 1/4 of a pitch out of phase with the other. The internal electronics of the encoder are powered by a 5 V dc supply. The two pulse signals determine the direction of rotation of the motor by one of various means (e.g., sign of the phase difference, timing of the consecutive rising edges). The encoder pulse count is stored in a register within the controller and is read at fixed intervals (say, 5 ms). The net count gives the joint position, and the difference in count at a fixed time increment gives joint speed.

When the 6 pulses/rev encoder is replaced by the 720 pulses/rev encoder, the pulse count will increase by a factor of $k = 720/6 = 120$. This is equivalent to introducing a feedback gain of k in both position and velocity paths of the servo loop. Since the feedback position information is now inconsistent with the motion command, an erratic behavior will result. Also, the increased loop gain can result in an unstable servo loop. To overcome this problem, reduce the encoder count by a factor of k simply by using the controller software. Alternatively, use an encoder that is identical to the original unit.

Sol-Problem 4 (Problem 6.21 from Textbook)

(a) Resolution, accuracy, loading (both mechanical and electrical), robustness, and influence of environmental factors (humidity, dirt, vibration, etc.), and easy cleaning will be important. For digital control, convenient interfacing with the controller will be an advantage.

Optical encoder has the advantages of easy digital interfacing, high accuracy, compact electronics, non-susceptibility to electrical noise, and availability as an assembled package with a servomotor.

It has the disadvantages of delicate construction, being prone to damage by washing down and cleaning operations (necessary in meat processing), influence of environmental conditions (dirt, moisture, etc.), relatively high cost including that of the electronics of signal generation.

(b) The arrangement shown is used to obtain a low reference signal for comparison, so that environmental light, power supply variation, etc. will not affect the reading. A difference amplifier may be used, as in Figure 6.5 of the book, to obtain a reliable pulse signal. But this arrangement does not provide the direction of motion.

For direction of motion determination, quadrature signals will be needed. Here, another pulse signal that is obtained by placing the probe at a 1/4 pitch out of phase will be used, and phase measurement or edge detection may be employed.

Sol-Problem 5 (Problem 8.9 from Textbook)

This is a PM stepper, having a permanent magnet rotor with 6 poles or teeth and a stator with four poles. Four separate phases are not necessary because only requirement for proper operation is that that the two diametrically opposite stator poles must have opposite polarities. Hence, phase 1 and phase 3 may be connected to a single phase supply (phase A), and phase 2 and phase 4 may be connected to another single phase (phase B).

In the detent position shown in Figure P8.9, phase A is on, with phase 1 producing an S pole and phase 3 producing an N pole, while phase B is off.

In the next full step, for CW rotation, phase A is turned off, and phase B is turned on with phase 2 producing an N pole and phase 4 producing an S pole.

In the next full step for CW rotation, phase B is turned off and phase A is turned on in reverse polarity (denoted by A^-), with phase 1 producing an N pole and phase 3 producing an S pole.

Rotor tooth pitch = 360°/6 = 60°

Full step angle = 60°/2 = 30°

Accordingly, it should be clear that the switching sequences are as given below.

(a) CW full-stepping: A, B, A^-, B^-, A; step angle = 30°

CCW full-stepping: A, B^-, A^-, B, A; step angle = 30°

(b) CW half-stepping: A, AB, B, A^-B, A^-, A^-B^-, B^-, AB^-, A; step angle = 15°

CCW half-stepping: A, AB^-, B^-, A^-B^-, A^-, A^-B, B, AB, A; step angle = 15°

Sol-Problem 6 (Problem 8.35 from Textbook)

Note that the gravity forces are supported by the bellows; hence, the system can be assumed horizontal.

From equation (7.12) in the text,

$$T = \left(J + \frac{mr^2}{e} \right) \frac{a}{r} + \frac{r}{e} \cdot F_r \qquad \text{(i)}$$

We are given that F_r = 5 kg and m = 50 kg.

Since the motion through 1 m can be carried out using a constant acceleration for 2.5 s followed by a constant deceleration for 2.5 s (a triangular velocity profile), the maximum acceleration is estimated as

$$\begin{aligned} a &= \frac{2 \times \text{distance}}{(\text{time})^2} = \frac{2 \times 50}{(2.5)^2} \text{ cm.s}^{-2} \\ &= 16 \text{ cm.s}^{-2} \\ &= \ 0.16 \text{ m.s}^{-2} \end{aligned}$$

Maximum speed $= a \times 2.5 = 40.0 \text{ cm.s}^{-1}$

Motor speed $= \dfrac{40 \text{ cm.s}^{-1}}{0.5 \text{ cm / step}} = 80$ steps / sec

Use a rack-and-pinion device with 90% efficiency, hence e = 0.9. Consider the four stepper motors given in the data sheets (Table 8.2 and Figure 8.44).

Step angle = 1.8° ⇒ 200 steps/rev

Therefore, maximum motor speed = $80 \times \dfrac{1..8^\circ}{360} \times 60 \text{ rpm} = 24.0 \text{ rpm}$

$$r = \frac{0.5 \text{ cm/step} \times 200 \text{ steps/rev}}{2\pi \text{ rad/rev}} = 15.9 \text{ cm/rad} = 0.159 \text{ m/rad}$$

Substitute in (i):

$$\begin{aligned} T &= \left[J_m + \frac{50 \times 0.159^2}{0.9} \right] \frac{0.16}{0.159} + \frac{0.159}{0.9} \times 5 \times 9.81 \quad \text{N.m} \\ &= [J_m + 1.4045] \times 1.026 + 8.666 \text{ N.m} \end{aligned}$$

Table S8.35(a) is prepared using this result and the given motor data.

Table S8.35(a) Data for motor selection.

Motor Model	$J_m \left(\text{kg.m}^2\right)$	Required Torque at 24 rpm (N.m.)	Available Torque at 24 rpm (N.m)
50 SM	11.8×10^{-6}	10.11	0.27
101 SM	35.0×10^{-6}	10.11	0.60
310 SM	187×10^{-6}	10.11	2.60
1010 SM	805×10^{-6}	10.11	7.5

None of the motors meets the specification.

Hence reduce the *r* by specifying 0.1 cm/step resolution.

Then,

$$r = 0.0318 \text{ m/rad, and}$$

$$T = \left[J_m + 0.0562\right] \times 5.0314 + 1.7331 \text{ N.m}$$

Using this result, Table 8.35(a) is modified as in Table 8.35(b).

Table S8.35(b): Modified data for motor selection.

Motor Model	Required Torque at 24 rpm (N.m.)	Available Torque at 24 rpm (N.m)
50 SM	2.02	0.27
101 SM	2.02	0.60
310 SM	2.02	2.60
1010 SM	2.02	7.5

According to this, pick the motor 310 SM. Also use a rack-and-pinion with $r \leq 0.0318$ m/rad. .

Sol-Problem 7 (Problem 9.23 from Textbook)

Characteristic	DC Motor	Torque Motor	Stepper Motor	Induction Motor	AC Synchronous Motor
Power Capability	Low to Average	Average	Low	High	High
Speed Controllability	Good	Good	Good	Average	Average
Speed Regulation	Average to Good	Average to Good	Average	Good	Excellent
Linearity	Average to Good	Average	Poor	Poor	Poor
Operating Bandwidth	Good	Good	Low	Good with Frequency Control	Good with Frequency Control
Starting Torque	Good	Good	Good	Average	Nil
Power Supply Requirements	DC	DC	DC and AC	AC	AC
Commutation Requirements	Split-Ring & Brushes	None	None	None	Slip-Ring & Brushes
Power Dissipation	Average	Average	Average to High	Low	Low

Reversing:

For a conventional dc motor, change the direction of the armature voltage or reverse the field direction. For a brushless dc torque motor, reverse the direction of the field current or reverse the sequence of phase activations. In a stepper motor, the sequence of phase activation determines the direction of rotation. In a three-phase induction motor or ac synchronous motor, reverse the direction of the rotating field by interchanging phase 2 and phase 3.

Sol-Problem 8 (Problem 9.46 from Textbook)

(a) This is a flow control valve. The flow to the load depends on the pressure difference at the fixed area orifice. If the load pressure P_3 drops from the steady-state value, the pressure difference at the orifice increases and the flow rate to the load will increase. The net force to the right on the valve (due to the pressure difference $P_2 - P_3$) increases, the valve moves to the right, and x decreases as a result. This reduces the flow into the while increasing the available volume for oil inside the valve. As a result, pressure P_2 decreases. This decreases the pressure difference $P_2 - P_3$, and hence reverses the process, leading to self-correction.

(b) Refer to Figure S9.46(a). Flow through the valve opening increases with x and decreases with P_2, for a given supply pressure Ps. Hence for the linearized case we have, $Q_1 = K_1 x + K_2(P_s - P_2)$ [K_1 and K_2 are positive]

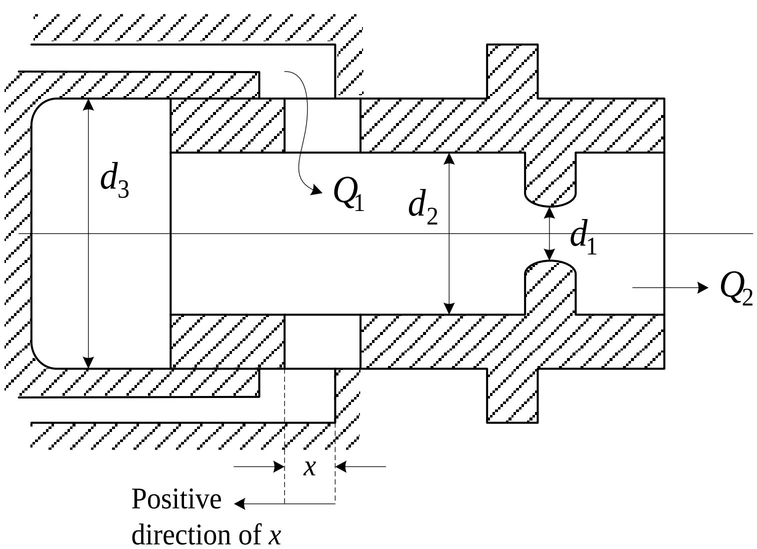

Figure S9.46(a): Valve parameters and variables.

Flow through the fixed area orifice increases with the pressure difference $P_2 - P_3$.

Hence for the linearized case we have,

$$Q_2 = K_3\left(P_2 - P_3\right) \qquad [K_3 \text{ is positive}] \qquad \text{(i)}$$

Net inflow: $Q_1 - Q_2 =$ rate of increase of valve volume due to movement x + rate of decrease of oil volume due to compression.

Rate of increase in valve volume $= -\frac{\pi}{4}\left(d_3^2\right)\frac{dx}{dt} = -A_1\frac{dx}{dt}$

Bulk modulus of oil $\beta = -V\left(\frac{\partial P_2}{\partial V}\right)$ [By definition], where, V = volume of oil in the value (under pressure P_2) ➔ Decrease of oil volume due to compression $-\delta V = \frac{V}{\beta}\ \delta P_2$

Rate of decrease of oil volume: $-\left(\frac{\delta V}{\delta t}\right)_{\delta t \to 0} = \frac{V}{\beta}\left(\frac{\delta P_2}{\delta t}\right)_{\delta t \to 0} = \frac{V}{\beta}\frac{dP_2}{dt}$

The continuity of the flow rate for the valve is:

$$K_1 x + K_2(P_s - P_2) - K_3\left(P_2 - P_3\right) = -A_1\frac{dx}{dt} + \frac{V}{\beta}\frac{dP_2}{dt} \qquad \text{(ii)}$$

Note: Decrease of the oil volume inside the valve due to compressibility (due to the increase of P_2), will create room for more oil to enter the valve.

With reference to Figure S9.46(b), net pressure force on the valve, acting in the RHS direction $= \frac{\pi}{4}\left(d_3^2 - d_2^2\right)\left(P_2 - P_3\right) + \frac{\pi}{4}\left(d_2^2 - d_1^2\right)\left(P_2 - P_3\right) = \frac{\pi}{4}\left(d_3^2 - d_1^2\right)\left(P_2 - P_3\right) = A_2\left(P_2 - P_3\right)$

where, A_2 = area between the orifice and the outer diameter of the moving component (valve). Net spring force on the valve to the right $= k(x - x_0)$; where, x_0 = spring setting. The overall equation of motion of the valve is

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - k(x - x_0) - A_2(P_2 - P_3) \qquad \text{(iii)}$$

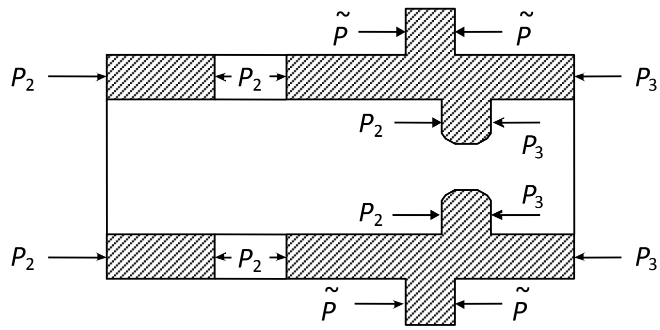

Figure S9.46(b): Pressures acting on the valve.

(c) A block diagram for the system is shown in Figure S9.46(c).

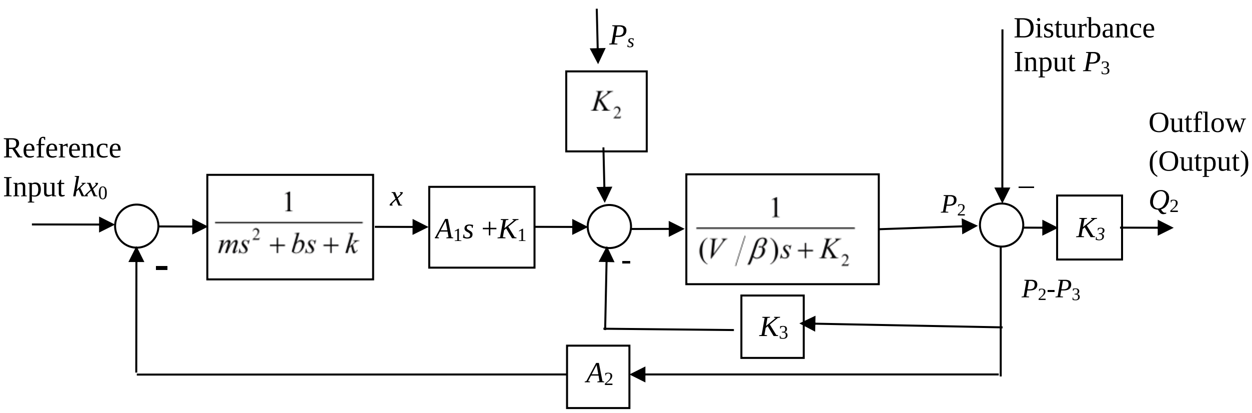

Figure S9.46(c): System block diagram.