2.2. SDOF– Equivalent Systems

MECH 463: Mechanical Vibrations

A. Srikantha Phani
srikanth@mech.ubc.ca

Suggested Readings:

1. Topic 2.2 from notes package.
2. Sections 1.7–1.8 in the course textbook.

Learning Objectives (NP 2.5)

1. Determine equivalent spring (translational and rotational) and mass (translational) or mass moment of inertia (rotational) of a multi-component mechanical system at a point of interest.

2. Apply the force and energy methods learned in Topic 2.1.

3. Recognize the advantages and limitations of equivalent systems.

Fill in the class

2.5. Equivalent Spring– Force Method (NP 2.6, T1.7) — # 1

The key idea here is to obtain force-displacement relation at the point of interest.

Parallel Configuration (p.46 of NP)

Fill in the class

2.5. Equivalent Spring– Force Method (NP 2.6, T1.7) — # 2

Series Configuration

Fill in the class

2.5. Equivalent Spring– Energy Method (NP 2.6, T1.7) — # 1

The key idea here is to obtain potential energy in terms of the displacement co-ordinate <u>at the point of interest</u>.

<u>Parallel Configuration</u>

Fill in the class

2.5. Equivalent Spring– Energy Method (NP 2.6, T1.7) — # 2

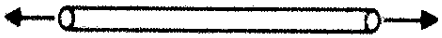
Series Configuration

Fill in the class

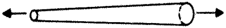
2.5. Equivalent Spring– Energy Method (NP 2.6, T1.7) — # 3

Fill in the class

2.5. Equivalent Spring– Energy Method (NP 2.6, T1.7) — # 4

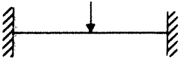


Rod under axial load
(l = length, A = cross sectional area) $\quad k_{eq} = \dfrac{EA}{l}$



Tapered rod under axial load
(D, d = end diameters) $\quad k_{eq} = \dfrac{\pi EDd}{4l}$

Helical spring under axial load
(d = wire diameter, D = mean coil diameter, n = number of active turns) $\quad k_{eq} = \dfrac{Gd^4}{8nD^3}$



Fixed-fixed beam with load at the middle $\quad k_{eq} = \dfrac{192\,EI}{l^3}$

Cantilever beam with end load $\quad k_{eq} = \dfrac{3EI}{l^3}$

2.5. Equivalent Spring– Energy Method (NP 2.6, T1.7) — # 5

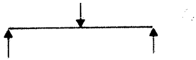
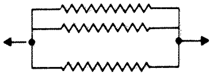
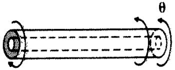
	Simply supported beam with load at the middle	$k_{eq} = \frac{48EI}{l^3}$
	Springs in series	$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}$
	Springs in parallel	$k_{eq} = k_1 + k_2 + \cdots + k_n$
	Hollow shaft under torsion (l = length, D = outer diameter, d = inner diameter)	$k_{eq} = \frac{\pi G}{32l}(D^4 - d^4)$

Figure: Equivalent spring constants.

2.5. Equivalent Mass or Equivalent Mass Moment of Inertia (NP 2.7, T1.7) — # 1

The key idea here is to obtain kinetic energy in terms of the velocity co-ordinate at the point of interest.

Translation masses connected by a rigid bar (p.50 of NP)

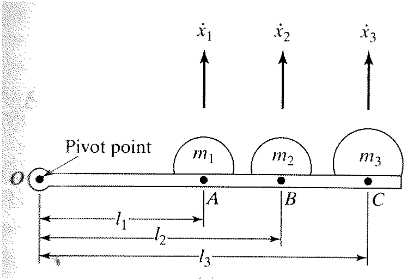
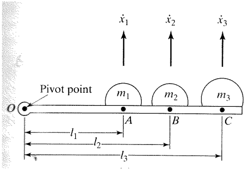


Figure: Translation masses connected by a rigid bar.

2.5. Equivalent Mass or Equivalent Mass Moment of Inertia (NP 2.7, T1.7) — # 2



Fill in the class

2.5. Equivalent Mass or Equivalent Mass Moment of Inertia (NP 2.7, T1.7) — # 3

Q: Can you see how you can find equivalent mass moment of inertia of the above system? (p.51 of NP)

Fill in the class

2.5. Equivalent Mass or Equivalent Mass Moment of Inertia (NP 2.7, T1.7) — # 4

Translational and rotational masses coupled

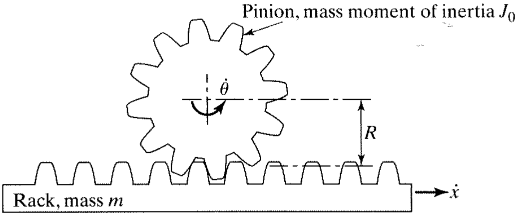
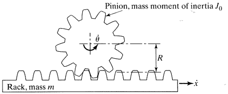


Figure: Rack and Pinion mechanism.

2.5. Equivalent Mass or Equivalent Mass Moment of Inertia (NP 2.7, T1.7) — # 5



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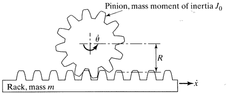
2.5. Equivalent Mass or Equivalent Mass Moment of Inertia (NP 2.7, T1.7) — # 6



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Example 5 — # 1

p.40 in course notes package

Example 5: Determine the equivalent mass and spring constant of the following systems (p.53 of NP)

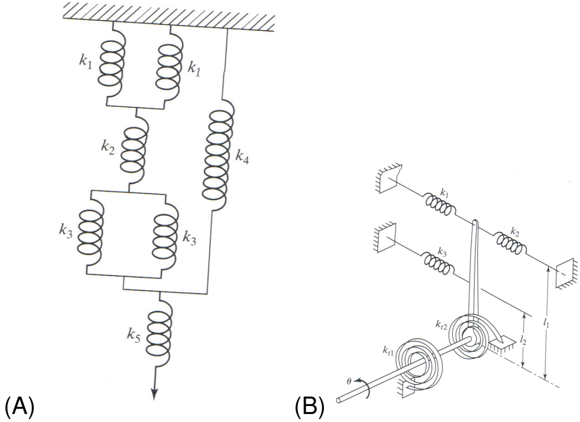
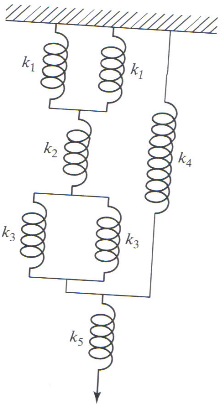


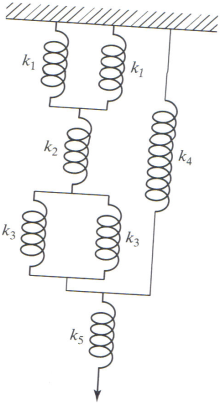
Figure: Figure for example 5.

Example 5 — # 2



Fill in the class

Example 5 — # 3



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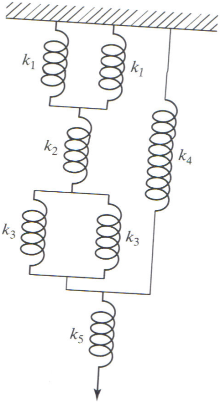
Example 5 — # 4



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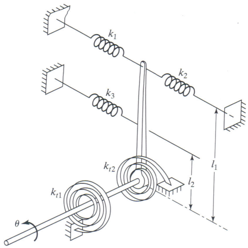
Example 5 — # 5



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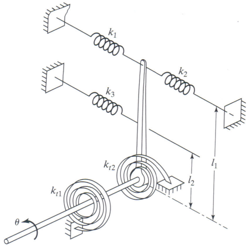
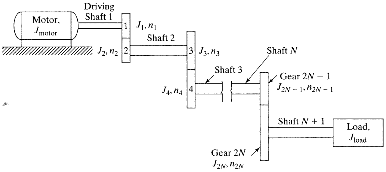
Example 5 — # 6



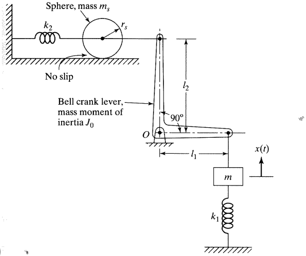
Fill in the class

Example 6 — # 1

Example 6: Determine the equivalent mass of the spring-mass system when the spring has a total mass of m_s. Determine the equivalent mass moment of inertia /mass of the systems shown below (p.56 of NP)



(A)



(B)

Figure: Figures for example 6.

Example 6 — # 2

Fill in the class

Example 6 — # 3

Fill in the class

Example 6 — # 4

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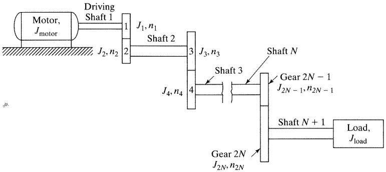
Example 6 — # 5



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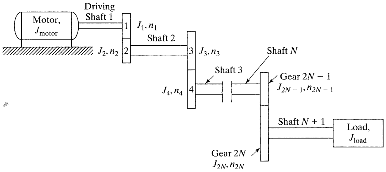
Example 6 — # 6



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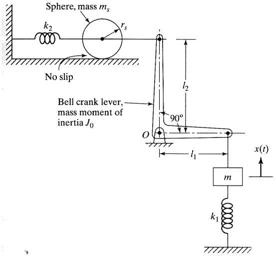
Example 6 — # 7



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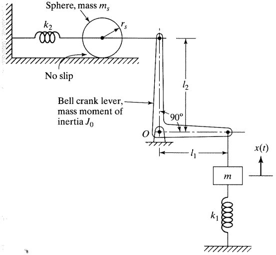
Example 6 — # 8



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Summary

1. Equivalent systems simplify the modelling of multi-component systems **at the point of interest**.
2. Equivalent system parameters change with the location of point of interest and hence can't capture entire dynamic response.
3. Equivalent spring is determined from the potential energy and equivalent mass (or mass moment of inertia) is determined from the kinetic energy expression of the system expressed in terms of a **single displacement co-ordinate**.
4. A thorough understanding of planar kinematics is essential in relating translations and rotations.