(1)

MECH 364: ASSIGNMENT 4

Requires course text book: MECHANICAL VIBRATIONS BY S.S. RAO (4TH EDITION). Solutions will appear approximately ten days after the assignment is posted on VISTA.

Q1. A simplified model of a washing machine is illustrated in Figure below. A bundle of wet clothes forms a mass of 10 kg (m_b) in the machine and causes a rotating unbalance. The rotating mass is 20 kg (including the mass of the wet clothes) and the diameter of the washer basket (2e) is 50 cm. Assume that the spin cycle is set at 300 rpm.

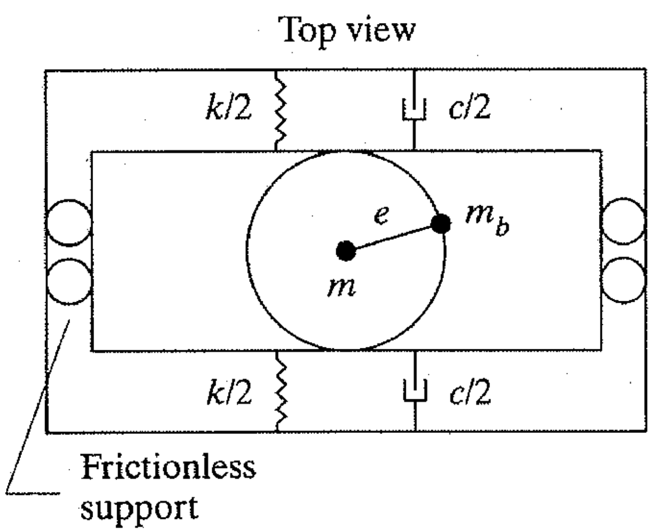


Figure A3.1: Figure for Question 1.

a) Let k be 1000 N/m and damping ratio $\zeta = 0.01$. Calculate the force transmitted to the sides of the washing machine through the springs and damper. Clearly state any assumptions you made using your knowledge on how washing machines work.

b) It is desired to design an isolation system (choose k and c) such that the force transmitted to the sides of the washing machine is less than 100 N at the operating speed, while the maximum transmitted force possible during the operation should not exceed twice the force generated. Find the spring constant k and dashpot coefficient c.

c) What is the role of frictionless supports? Find the maximum reaction exerted by the guides on the wall.

MECH 364

ASSIGNMENT #4 : SOLUTION

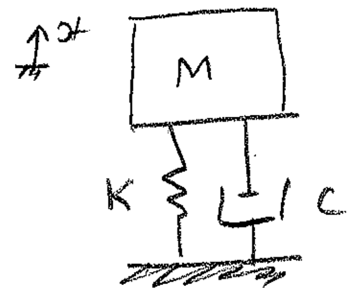
Q1)

$m_b = 10\ kg$

$M = 20\ Kg$

$2e = 50\ cm = 0.5\ m \Rightarrow e = 0.25\ m$

$\omega = 300\ rpm = 300 \times \frac{2\pi}{60} = 31.4159\ rad/s$



a) $K = 1000\ N/m$, $\zeta = 0.01$

EFFECTIVE STIFFNESS $= \frac{K}{2} \times 2 = K$

EFFECTIVE DAMPING $= \frac{C}{2} \times 2 = C$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{1000}{20}} = 7.07\ rad/s$$

$$r = \frac{\omega}{\omega_n} = \frac{31.4159}{7.07} = 4.4436$$

$$TR = \left|\frac{F_T}{F}\right| = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \sqrt{\frac{1+(2\times 0.01\times 4.4436)^2}{[1-(4.4436)^2]^2+(2\times 0.01\times 4.4436)^2}} = 0.0536$$

$\Rightarrow$ FORCE TRANSMITTED $= F_T = 0.0536 \times F = 0.0536 \times m_b\, e\, \omega^2$

$$= 0.0536 \times 10 \times 0.25 \times (31.4159)^2$$

$\therefore\ F_T = 132.2\ N$

b) DESIGN OF ISOLATION SYSTEM

$$[TR]_{max} = \left[\frac{F_T}{F}\right]_{max} = 2 \quad @\ r=1$$

$$\Rightarrow \sqrt{\frac{1+(2\zeta)^2}{(1-1^2)^2+(2\zeta)^2}} = 2 \Rightarrow \sqrt{\frac{1+4\zeta^2}{4\zeta^2}} = 2$$

$$\Rightarrow 1+4\zeta^2 = 4\times 4\zeta^2 \Rightarrow 12\zeta^2 = 1 \Rightarrow \zeta = \frac{1}{\sqrt{12}} = 0.2887$$

WE ALSO WANT $TR = \frac{100}{F}$ @ $\omega = 300\ rpm$

USING $\zeta = 0.2887$ AND $F = m_b e\omega^2 = 2467.4\ N$

$$\frac{100}{2467.4} = \sqrt{\frac{1+(2\times 0.2887\times r)^2}{(1-r^2)^2+(2\times 0.2887\times r)^2}}$$

$$\Rightarrow 0.0405 = \sqrt{\frac{1+0.3333r^2}{(1-r^2)^2+0.3333r^2}}$$ SQUARE BOTH SIDES & RE-ARRANGE

$$\Rightarrow (1-r^2)^2+0.3333r^2 = \frac{1+0.3333r^2}{(0.0405)^2} = 609.66+203.2r^2$$

Let $r^2 = x$

$$\Rightarrow (1-x)^2 + 0.3333x = 609.66 + 203.2x$$ QUADRATIC EQUATION IN x

$$\Rightarrow x^2 - 204.8667x - 608.66 = 0 \Rightarrow x = r^2 = 207.7958 \Rightarrow r = 14.4151$$

$$\Rightarrow \frac{\omega}{\omega_n} = 14.4151 \Rightarrow \omega_n = \frac{\omega}{14.4151} = 2.1794 \Rightarrow \sqrt{\frac{K}{M}} = 2.1794$$

$$\Rightarrow K = (2.1794)^2 \times 20 = 94.9933\ N/m$$

SINCE $\zeta = \frac{c}{2m\omega_n} \Rightarrow c = 2\zeta m \omega_n$

$$= 2 \times 0.2887 \times 20 \times 2.1794$$

$$= 25.167 \ \frac{N\text{-}S}{m}$$

THEREFORE WE WANT TO CHOOSE THE ISOLATION SYSTEM SUCH THAT

$$K = 94.9933 \ N/m$$

$$C = 25.167 \ \frac{N\text{-}S}{m}$$

NOTE: $ax^2 + bx + c = 0$ HAS ROOTS $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

c) THE ROLE OF FRICTIONLESS SUPPORTS IS TO RESIST THE HORIZONTAL COMPONENT OF UNBALANCE FORCE AND ENSURE THAT THE MOTION OF THE MACHINE IS ALONG THE GUIDES.

MAXIMUM REACTION = R_{max} (Total for 4 SUPPORTS)

$$= m_b e \omega^2 = 10 \times 0.25 \times (31.4159)^2$$

$$= 2467.4 \ N$$