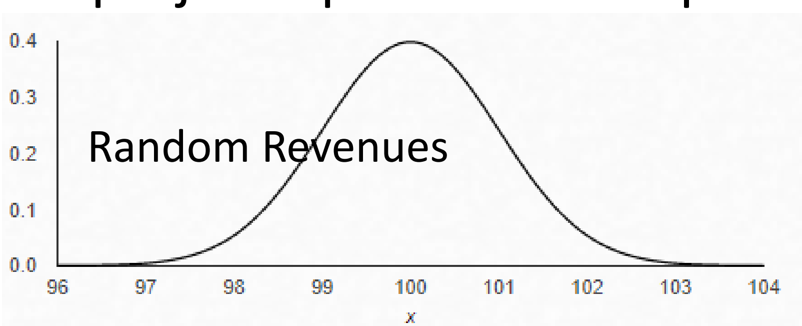
Lesson 15-2 - Simulations

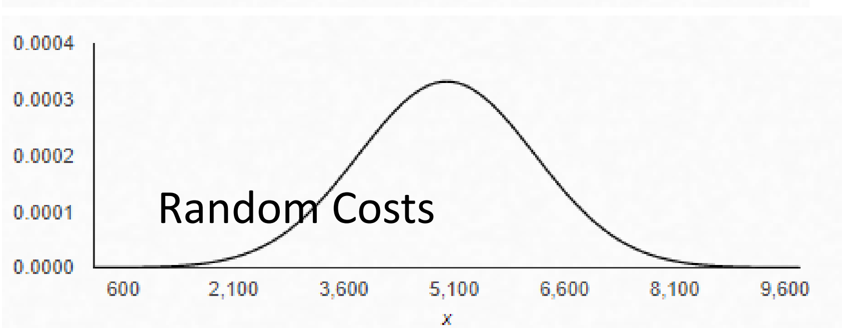
Simulation

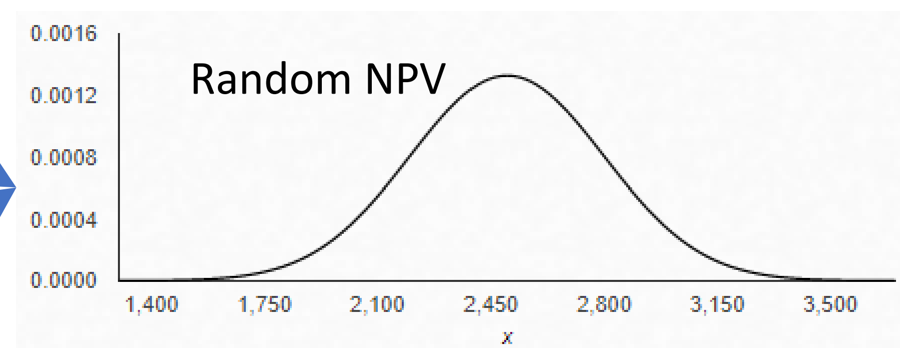
- Simulation is a more advanced approach where random sampling is used from the probability distributions of one or more variables for analysis.
- Simulation can be done by hand using a table of random numbers and only a few iterations.
- More commonly, it is done with a computer using a spreadsheet random function or some other simulation package.
- Using a computer allows for larger numbers of iterations to be calculated and/or more variables to be randomized.
- As well, using a computer allows for the evaluation of a number of different types of probability distributions.

Simulation

- When any of the project components is a random variable, the outcome of the project, e.g. the NPV, is also a random variable.
- If we want to assess a project with uncertain parameters, we estimate the probability distribution of the outcome using the relative frequency approach.
- We can do this by repeatedly sampling from the distributions of the project's parameters. Spreadsheets are helpful in this procedure.







Random inputs lead to random output

Simulation Process

- Monte Carlo simulation procedure:
  1. Formulate the model for determining the project outcome from the project components.
  2. Determine the probability distributions of all project components that are random variables.
  3. Use a random number generator to produce values for the project components that are random variables and calculate the project outcome using the model.
  4. Repeat step 3 until a large enough sample has been taken (250 is usually a sufficient number as a rule of thumb, but look at your output histogram)
  5. Produce a frequency distribution and a histogram to estimate the probability distribution of the project outcome.
  6. Produce summary statistics of the project outcome, e.g. mean, median, standard deviation, range, minimum, maximum, …

Step 1 – Develop the model

- What is the output variable of interest?
  - NPV
  - IRR
  - Benefit/Cost Ratio
  - Future Value
  - Other?

- Develop a model that gives us this output variable, based on our known (but potentially random) inputs

Step 1 Example

A business wants to invest in a production line that will run for three years. The initial investment will cost $100,000 and the firm has an interest rate of 14%.

The annual benefits are estimated to be $73,000 on average, with a standard deviation of $5000.

The annual costs are variable as well, and will be uniformly distributed between $18000 and $26000.

After three years there is a 25% chance the line will be needed for a fourth year. The costs and benefits would be the same.

Use a simulation to evaluate the NPV of this investment.

Step 1 Example

- Set up the initial model:

	Nominal	Variable	
Initial Investment	$100,000		
Annual Benefits	$73,000	$73,000	$5,000
Annual Costs	$22,000	$18,000	$26,000
Lifetime	3	25%	
Interest Rate	14%		
Nominal NPV	$18,403.23	=PV(E7, E6, -(E4-E5))-E3	

Step 2 – Determine the distribution of input variables

- Common distributions:
  - Constant – fixed output, does not change
    - E.g. MARR is fixed at 16%
  - Uniform – variable output, all outcomes equally likely
    - E.g. the number that comes up on a roulette table
  - Normal – variable output, defined by mean and standard deviation
    - Values closer to the mean are more likely
    - Less likely to occur farther from the mean
    - E.g. a small food truck might have annual revenues that are normally distributed, with a mean of $680,000 and a standard deviation of $40,000.
    - Or a production line might output a mean of 2,000 units per month, with a standard deviation of 125 units.
  - Cases – variable output, but discrete
    - E.g. coin toss – 50% chance of heads, 50% chance of tails
    - Roulette table – 47.4% chance of red, 47.4% chance of black, 5.2% chance of neither

Step 2 Example

- Our variables were given in the first example slide.
- Initial cost: Fixed, $100,000
- Annual Benefits: Variable, normal distribution (mean $73,000, std. dev. $5000)
- Annual costs: Variable, uniform distribution between $18000 and $26000
- Interest: Fixed at 14%
- Lifetime: Variable, case: 75% chance = 3 years, 25% = 4 years

Step 2 Example

- Write the formula for these
- Initial costs: $100,000
- Annual benefits: =NORM.INV(RAND(), $73000, $5000)
- Annual Costs: =RANDBETWEEN($18000, $26000)
- Cashflow per year: =Benefits – Costs

- YEAR FOUR:
- Conditional, only a 25% chance.
- =IF(RAND()<25%, NORM.INV(RAND(), $73000,$5000)-RANDBETWEEN(18000,26000), 0)

Step 3 – Run a simulation

- For each random variable, select a value from its distribution
- Typically feed a randomly generated number (=RAND()) into the distribution function.
- Uniform distribution: =RANDBETWEEN(bottom, top)
  - Can use =round() as well to get desired level of precision
- Normal distribution: =NORM.INV(probability, mean, standard dev)
  - Feed a random number, =rand(), into the probability
  - Function will then output the cumulative normal value for the given mean and standard deviation
  - E.g. if mean = 100,000, and standard deviation = 10,000, then =NORM.INV(rand(), 100000, 10000) = 99611 if rand() = 0.4844

- Use these values to calculate our output variable

Step 3 Example

	Nominal	Variable	
Initial Investment	$100,000		
Annual Benefits	$73,000	$73,000	$5,000
Annual Costs	$22,000	$18,000	$26,000
Lifetime	3	25%	
Interest Rate	14%		
Nominal NPV	$18,403.23	=PV(E7, E6, -(E4-E5))-E3	

Period Cashflow

0	**1**	**2**	**3**	**4**	**NPV**
$ (100,000)	$ 61,084	$ 53,751	$ 44,926	$ -	$ 25,266

Benefits:	=NORM.INV(RAND(), E4, F4)
Costs	=RANDBETWEEN(18000, 26000)
Periodic Cashflow:	=Benefits - Costs
Extra Year?	=IF(RAND()<F6, NORM.INV(RAND(),E4,G4)-RANDBETWEEN(18000,26000), 0)

Step 4 – Run many more simulations

- Generate new random numbers and new input variables, and record the output variable again
- Repeat ad naseum
- 250 simulations is a good starting point as a rule of thumb
- When looking at your histogram, evaluate for sufficient data points

Step 4 Example

	Nominal	Variable	
Initial Investment	$100,000		
Annual Benefits	$73,000	$73,000	$5,000
Annual Costs	$22,000	$18,000	$26,000
Lifetime	3	25%	
Interest Rate	14%		
Nominal NPV	$18,403.23	=PV(E7, E6, -(E4-E5))-E3	

	Period Cashflow					
Sim #	**0**	**1**	**2**	**3**	**4**	**NPV**
1	$ (100,000)	$ 48,576	$ 44,879	$ 55,809	$ -	$ 14,813
2	$ (100,000)	$ 46,977	$ 62,444	$ 54,723	$ -	$ 26,192
3	$ (100,000)	$ 55,314	$ 63,668	$ 57,347	$ -	$ 36,219
4	$ (100,000)	$ 64,204	$ 47,156	$ 53,003	$ -	$ 28,381
5	$ (100,000)	$ 46,691	$ 56,003	$ 49,866	$ 56,321	$ 51,054
6	$ (100,000)	$ 52,051	$ 50,327	$ 53,128	$ -	$ 20,244
7	$ (100,000)	$ 45,267	$ 46,478	$ 63,574	$ 43,725	$ 44,271
8	$ (100,000)	$ 52,549	$ 42,783	$ 52,052	$ 50,712	$ 44,175
9	$ (100,000)	$ 48,912	$ 46,214	$ 50,914	$ -	$ 12,831
10	$ (100,000)	$ 53,488	$ 50,703	$ 55,915	$ 41,100	$ 48,009
11	$ (100,000)	$ 58,346	$ 50,203	$ 54,897	$ 53,535	$ 58,562

Steps 5 and 6 – Analyze the results

- We now have a long list of randomly determined output variables
- Individual simulation results don’t tell us anything. But the distribution of many of those results does
- Create a histogram, and ensure you have completed enough simulations (look for empty bins)
- Try and fit a distribution to the histogram
- Evaluate the summary statistics

Analysis Example

Summary Statistics		
Mean	$	25,180
Median	$	21,280
Maximum	$	63,029
Minimum	$	(4,056)
St. Dev.	$	14,975

Bin	*Frequency*
$0	1
$10,000	27
$20,000	84
$30,000	70
$40,000	17
$50,000	25
$60,000	23
More	3