Tutorial 3

Final Value Theorem

$$y_{ss} = \lim_{s \to 0} s\, G(s)\, R(s) \Leftrightarrow \lim_{t \to \infty} G(t) \cdot R(t)$$

$$e(t) = r(t) - y(t) \rightarrow e_{ss} = \lim_{s \to 0} s\,[1 - G(s)]\, R(s)$$

a) Find the steady state response y_{ss} or/& steady state error e_{ss} given a system $G(s)$ for given input $r(t)$

t	s
1	$\frac{1}{s}$

$$G(s) = \frac{3}{s+3} \qquad R(t) = 6$$

$$y_{ss} = \lim_{s \to 0} s\left[\frac{3}{s+3}\,\frac{6}{s}\right] = 6$$

$$e_{ss} \lim_{s \to 0} s\left[1 - \frac{3}{s+3}\right]\frac{6}{s} = 0$$

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2) $G(s) = \dfrac{-4s+20}{s+300}$ $\qquad R(t) = 10$

t	s
10	$\frac{10}{s}$

$$y_{ss} = \lim_{s \to 0} s\left[\frac{-4s+20}{s+300}\right]\frac{10}{s} = 0.67$$

$$e_{ss} = \lim_{s \to 0} s\left[1 - \frac{-4s+20}{s+300}\right]\frac{10}{s} = 9.33$$

3) $G(s) = \dfrac{3}{s^2+0,5s+4}$ $\qquad R(t) = 2$

t	S
2	$\frac{2}{s}$

$$y_{ss} = \lim_{s\to 0} s\left[\frac{3}{s^2+0,5s+4}\right]\frac{2}{s} = 1,5$$

$$e_{ss} = \lim_{s\to 0} s\left[1 - \frac{3}{s^2+0,5s+4}\right]\frac{2}{s} = 0,5$$

4) $G(s) \dfrac{3}{s^2+0,5s+4}$ $\qquad R(t) = 2t$

t	S
$2t$	$\frac{2}{s^2}$

$$y_{ss} = \lim_{s\to 0} s\left[\frac{3}{s^2+0,5s+4}\right]\frac{2}{s^2} = \infty$$

$$e_{ss} = \lim_{s\to 0} s\left[1 - \frac{3}{s^2+0,5s+4}\right]\frac{2}{s^2} = \infty$$

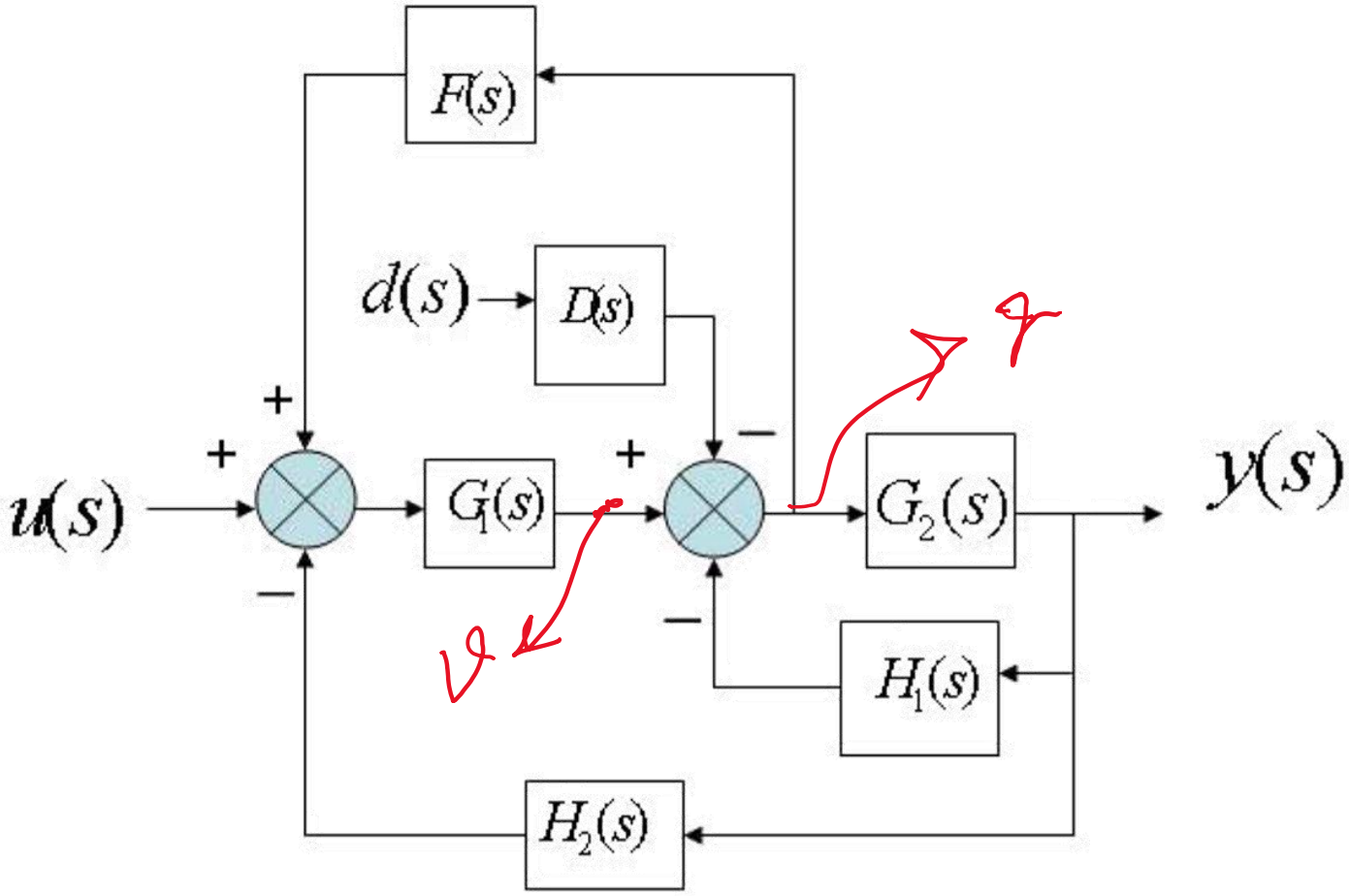
5) $G(s) = \dfrac{10}{s^2+3s+10}$ $\qquad R(t) = 2t$

$$y_{ss} = \lim_{s\to 0} s\left[\frac{10}{s^2+3s+10}\right]\frac{2}{s^2} = \infty$$

$$e_{ss} = \lim_{s\to 0} s\left[1 - \frac{10}{s^2+3s+10}\right]\frac{2}{s^2} = \lim_{s\to 0}\left[\frac{2}{s} - \frac{20}{s(s^2+3s+10)}\right]$$

$$= \lim_{s\to 0}\left[\frac{2s^2+6s+20-20}{s(s^2+3s+10)}\right] = \lim_{s\to 0}\left[\frac{2s+6}{s^2+3s+10}\right] = 0.6$$

Question 3. & 4.



Equations:

(1) $q = v - Dd - H_1 y$

(2) $v = G_1 u + F G_1 q - H_2 G_1 y$

(3) $y = G_2 q$

(1) $r = v - Dd - H_1 y$

(2) $v = G_1 u + F G_1 r - H_2 G_1 y$

(3) $y = G_2 r$

Equations
(repeated from previous page)

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2 → 1 $r = (G_1 u + F G_1 r - H_2 G_1 y) - D d - H_1 y$ ⇒

$$r(1 - F G_1) = G_1 u - H_2 G_1 y - D d - H_1 y$$

$$q(1-FG_1) = G_1 u - H_2 G_1 y - D d - H_1 y$$

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$$\xrightarrow{4 \to 3} \quad y = G_2 q = G_2 \times \frac{G_1 u_1 - H_2 G_1 y - D d - H_1 y}{1 - F G_1} \quad \Rightarrow$$

$$y(1-FG_1) = G_2 G_1 u - D G_2 d - (H_2 G_1 + H_1) G_2 y \quad \Rightarrow$$

$$y\,(1 - FG_1 + H_2 G_1 G_2 + H_1 G_2) = G_2 G_1 u - dD G_2$$

$$y = \underbrace{\frac{G_2 G_1}{1 - FG_1 + H_2 G_1 G_2 + H_1 G_2}}_{G_{yu}} u + \underbrace{\frac{-D G_2}{1 - FG_1 + H_2 G_1 G_2 + H_1 G_2}}_{G_{yd}} d$$