University of British Columbia
Department of Mechanical Engineering

MECH468 Modern Control Engineering
MECH522 Foundations in Control Engineering
Midterm exam

Examiner: Dr. Ryozo Nagamune
February 26 (Wednesday), 2020, 1-1:50pm

Last name, First name

Name: Student #:

Signature:

Exam policies

- Allowed: One-page letter-size hand-written cheat sheet (both front side and back side). Do not hand in your cheat sheet.
- Not-allowed: PC, calculators.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 30 points in total.

Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

If you finish early ...

- Please stay at your seat until the end of exam, i.e., 1:50pm. (You are not allowed to leave the room before the end of exam, except going to washroom.)

To be filled in by the instructor/marker

Problem #	Expected duration	Mark	Full mark
1	about 10 min		10
2	about 5 min		6
3	about 5 min		4
4	about 25 min		10
Total	about 45 min		30

Extra page. Write the problem number before writing your answer.

1. Answer the following 'true' or 'false' questions. **You don't need to motivate your answers**; Mark will be given solely based on your answer 'true' or 'false'. (1pt each)

   (a) A system $y(t+1) = \sqrt{2} \cdot u(t)$ is a linear system.
   (b) A system $y(t+1) = \sqrt{2} \cdot u(t)$ is a causal system.
   (c) A system $y(t+1) = \sqrt{2} \cdot u(t)$ is a lumped system.
   (d) Any linear time-invariant (LTI) state-space model with order $n \geq 1$ has at least one state which is both controllable and observable.
   (e) If an LTI system is not BIBO stable, then the system is not asymptotically stable.
   (f) For single-input-single-output (SISO) transfer function $G(s)$, it is always the case that the controllable canonical realization and the observable canonical realization have the same number of states.
   (g) A nonlinear model $\dot{x} = -x^3 - xu$ can be linearized around the equilibrium point $(x_0, u_0) = (1, 1)$.
   (h) If all the elements of a symmetric matrix are positive, then the matrix is positive definite.
   (i) For a controllable discrete-time LTI system with order 3, 2-step state transfer from any initial state to any final state is always possible.
   (j) If a continuous-time LTI system is observable, then the corresponding zero-order-hold discretized system is always observable.

Question	'True' or 'False'
(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	
(i)	
(j)	

2. For the **continuous-time** linear time-invariant systems $\dot{x} = Ax$ with the following A-matrices, judge whether the system is asymptotically stable ('AS'), marginally stable ('MS'), or unstable ('US').

   You don't need to motivate your answers; Mark will be given solely based on your answer 'AS', 'MS' or 'US'. (2pt each)

   (a) $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

Question	'AS', 'MS' or 'US'
(a)	
(b)	
(c)	

3. For transfer matrices $G(s)$ below, obtain its realization (in any form, i.e., you can use either controllable, or observable canonical form, or any other form).

   You will get full-mark if your final answers (A, B, C, D)-matrices are correct. If they are incorrect, partial mark may be given based on the derivation process. (2pt each)

   (a) $G(s) = \dfrac{1}{s} + \dfrac{1}{s+1}$ (b) $G(s) = \begin{bmatrix} \dfrac{s+2}{s+1} \\ 2 \end{bmatrix}$

   <u>Write your answer here.</u>

4. Let us consider the following **discrete-time** system.

$$\begin{cases} x[k+1] &= \underbrace{\begin{bmatrix} 1/2 & 1 \\ 0 & -1 \end{bmatrix}}_{A} x[k] + \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{B} u[k] \\ y[k] &= \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} x[k] \end{cases}$$

(a) Check the controllability and the observability. (2pt)

(b) Obtain the Kalman decomposition. Explain which state is controllable or uncontrollable, and which state is observable or unobservable. (2pt)

— (Continued to Page 8.) —

<u>**Write your answer here.**</u>

Write your answer here.

(c) For the **discrete-time** system above, let us consider the minimum energy control which solves the following optimization problem.

$$\min_{u[\cdot]} \sum_{k=0}^{2} (u[k])^2 \quad \text{subject to} \quad \begin{cases} x[k+1] = \underbrace{\begin{bmatrix} 1/2 & 1 \\ 0 & -1 \end{bmatrix}}_{A} x[k] + \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{B} u[k] \\ x[0] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; x[3] = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \end{cases}$$

i. Compute the **discrete-time** minimum energy control $u^*[0]$, $u^*[1]$, $u^*[2]$, and corresponding states $x[1]$ and $x[2]$. (2pt)

ii. Briefly explain the reason why the minimum energy control is not practically useful. (2pt)

iii. Obtain another (non-minimum energy) control input sequence $u[0]$, $u[1]$, $u[2]$ which achieves the state transfer from $x[0]$ to $x[3]$ given above, and confirm that the following inequality is satisfied by calculating values of left-hand side and right-hand side: (2pt)

$$\sum_{k=0}^{2} (u^*[k])^2 \leq \sum_{k=0}^{2} (u[k])^2.$$

— (END OF MIDTERM EXAM) —

Write your answer here.

Write your answer here.

Extra page. Write the problem number before writing your answer.