



UNIVERSITY OF BRITISH COLUMBIA
DEPARTMENT OF MECHANICAL ENGINEERING
FINAL EXAMINATION, December 2019
MECH 463 – Mechanical Vibrations

Duration: 2.5 hours. Answer all 4 questions.

Materials admitted: Non-communicating, non-programmable calculator, personal handwritten notes within one side of one 8.5x11 sheet.

The purpose of this exam is to evaluate your knowledge of the course material. Orderly presentation demonstrates your knowledge most clearly, while disorganized and unprofessional work creates confusion and doubt (both for the marker and you!)

Write your name on each page during the examination time.

NAME: _____ SIGNATURE: _____

SECTION: _____ STUDENT NUMBER: _____

Student Conduct During Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
6.
 - i. speaking or communicating with other examination candidates, unless otherwise authorized;
 - ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
 - iii. purposely viewing the written papers of other examination candidates;
 - iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) —

(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

7. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
8. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
9. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

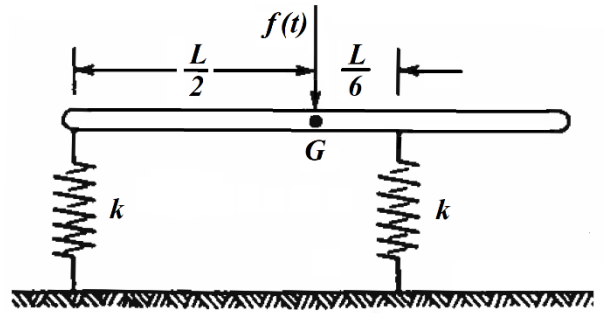
**CANDIDATES MUST IMMEDIATELY STOP
WRITING WHEN THE INVIGILATOR
ANNOUNCES THE EXAM IS OVER.**

| # | Mark | Max. |
|--------------|------|-----------|
| 1 | | 10 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| Total | | 40 |

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1. A part of a machine contains a uniform slender rod of length L , mass m , and centroidal $J = mL^2/12$. The rod is supported by two springs, each of stiffness k . One spring secures the rod at its left end, and the other spring secures the rod one third the way from the right end. An harmonic vertical force $f(t) = F \cos \omega_f t$ acts at the centre of mass G .

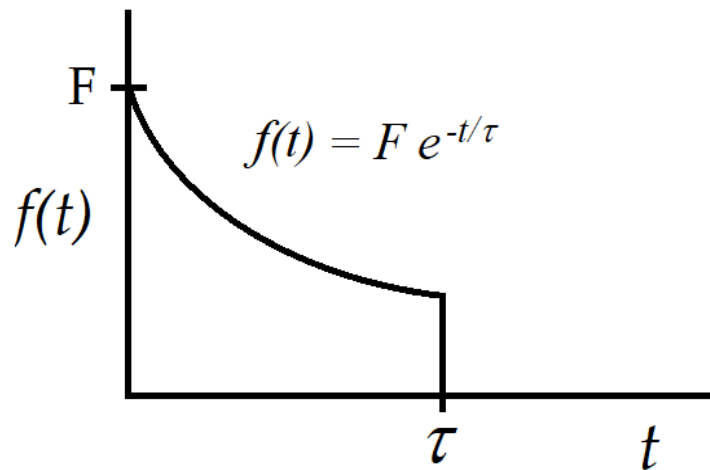
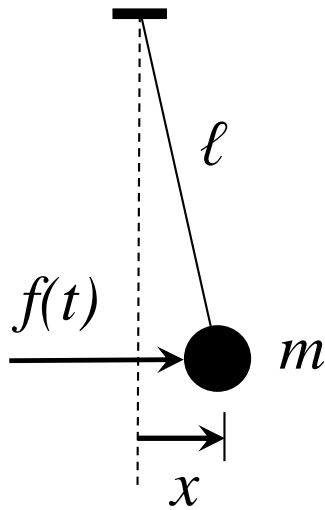


- Choose coordinates that will give no static coupling. Formulate the equations of motion in matrix format. Add/subtract rows as needed to make the matrices symmetrical.
- Determine the natural frequencies and corresponding mode shapes. Sketch the mode shapes and comment on any interesting features of the system.
- Determine and sketch the response of the beam at the excitation point G vs. frequency due the force $f(t) = F \cos \omega_f t$.

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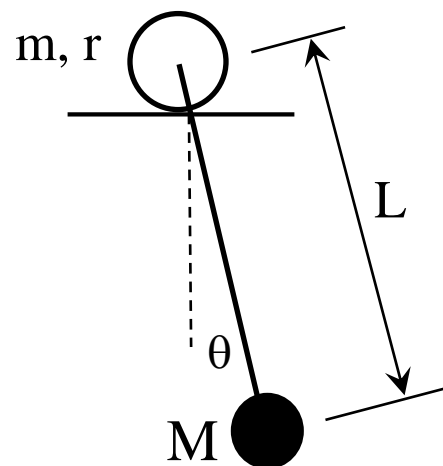
2. A horizontal force $f = f(t)$ acts on a pendulum of mass m and length ℓ .
 - a. Determine the equivalent spring constant “ k ” for the pendulum.
 - b. Determine the horizontal displacement response x of the system for zero initial conditions, given $f(t) = F e^{-t/\tau}$. Assume small vibrations.
 - c. Starting from the solution to (a), determine the response of the pendulum to the pulse excitation shown in the diagram for times $t > \tau$.
 - d. Is it possible to choose the time constant τ so that the oscillation becomes zero for $t > \tau$? If so, what value of τ should be chosen? If not, why not?



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3. The diagram shows a rolling pendulum of length L and mass M , supported by a cylindrical roller of radius r , mass m and centroidal moment of inertia $J = mr^2/2$. The pendulum is rigidly fixed to the roller by a massless rod such that the roller and pendulum move together. The roller rolls without slipping on a horizontal surface.



- i. Describe in words the motion of the pendulum mass and the roller as the pendulum vibrates.
- ii. Use Lagrange's equation to formulate the equation of motion. Assume small vibrations.
- iii. Find a formula for the natural frequency of the rolling pendulum. Check that your result corresponds to a simple pendulum when $r \rightarrow 0$.
- iv. For the finite r case, does an increase in M cause an increase or decrease in natural frequency? Comment on your result and give a physical (not just mathematical) explanation.

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4. A graduate student receives a small sample of a metal alloy from a manufacturer. For her research, she needs to know the Young's modulus of the alloy, and she decides to do this by measuring the natural frequencies of the sample. The sample is a plate, 152mm square, 3.07mm thick, and mass 557grams.

The student drills a small hole in one corner, and suspends the plate by a thin thread. She then taps the plate at each of the four points shown in the diagram, and measures the resulting vibrations using a microphone connected to a spectrum analyzer. The four graphs show the resulting frequency spectra.

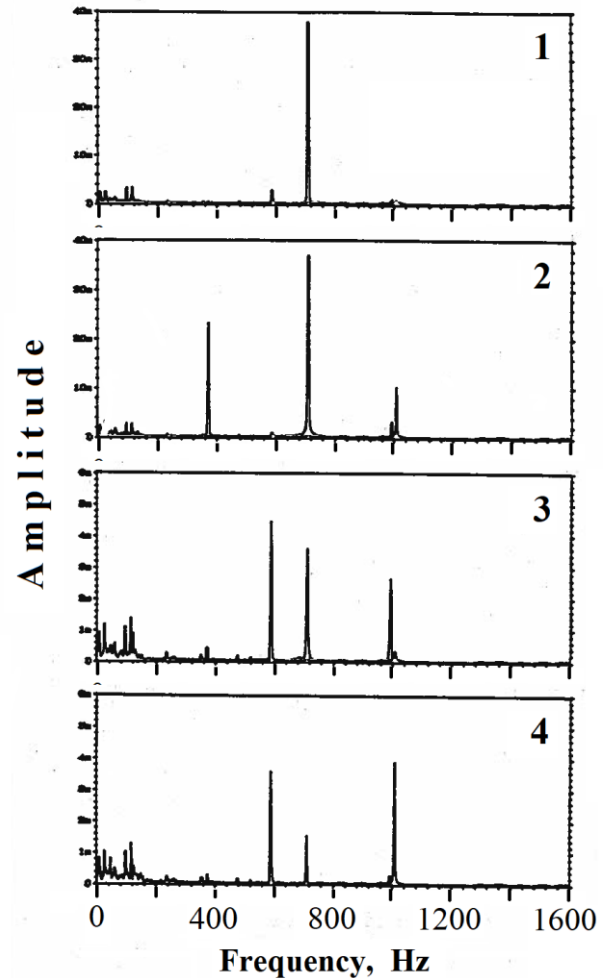
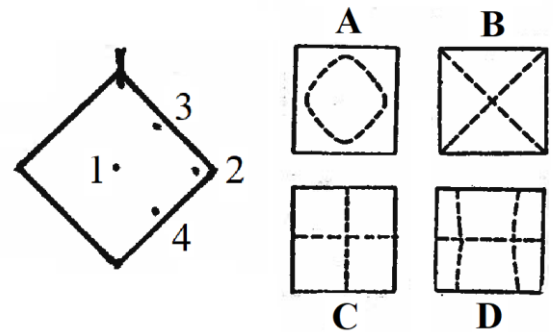
In the library, the student found a book that shows the first four vibration modes of a square plate. These are reproduced in diagrams A, B, C and D. (They are not in frequency order). The dashed lines represent the nodal lines (zero vibration positions) of the vibration modes. In another book, she found the following formula for the natural frequency f (in Hz) of mode "D" :

$$f = \frac{35.02}{2\pi a^2} \sqrt{\frac{E h^2}{12 \rho (1 - \nu^2)}}$$

where a = side length of the square, h = plate thickness, and ρ = mass density. Identify the frequencies of the four vibration modes from the measured frequency spectra. Explain your identifications. Calculate the Young's modulus of the new alloy (assume $\nu = 0.3$).

Bonus: Can you identify the direction the plate was rolled during its manufacture?

(Hint: The ideas useful for this question were also used in Lab 2.)



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