Problem 1

(i) Explain why component matching with respect to their impedance characteristics is important for electrical components that are interconnected in a control system.

(10 points)

(ii) Consider the mechanical system where a torque source (motor) of torque T and moment of inertia J_m is used to drive a purely inertial load of moment of inertia J_L as shown in Figure 1(a). What is the resulting angular acceleration $\ddot{\theta}$ of the system? Neglect the flexibility of the connecting shaft.

(a) (b)

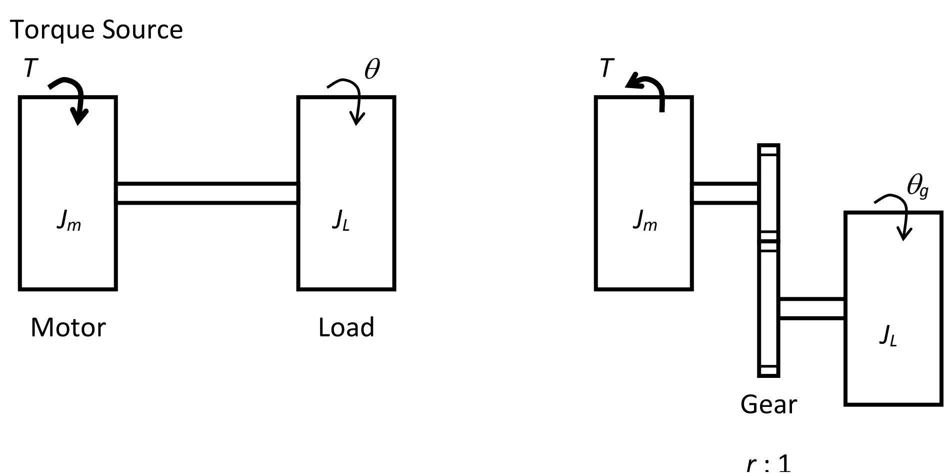

Figure 1: An Inertial Load Driven by a Motor
(a) Without Gear Transmission
(b) With a Gear Transmission.

Now suppose that the load is connected to the same torque source through an ideal (loss free) gear of motor-to-load speed ratio r : 1, as shown in Figure 1(b). What is the resulting acceleration $\ddot{\theta}_g$ of the load?

Obtain an expression for the normalized load acceleration $a = \ddot{\theta}_g / \ddot{\theta}$ in terms of r and $p = J_L / J_m$. Sketch a versus r for p = 0.1, 1.0, and 10.0. Determine the value of r in terms of p that will maximize the load acceleration a.

Comment on the results obtained in this problem.

(40 points)

Solution

Problem 1

(i) When two electrical components are interconnected, current (and energy) will flow between the two components. This will change the original (unconnected) conditions. This is known as the (electrical) loading effect, and it has to be minimized. At the same time, adequate power and current would be needed for signal communication, conditioning, display, etc. Both situations can be accommodated through proper matching of impedances when the two components are connected. Usually an impedance matching amplifier (impedance transformer) would be needed between the two components.

(ii) For the unit without the gear transmission:
Newton's 2nd law gives

$$(J_m + J_L)\ddot{\theta} = T$$

Hence

$$\ddot{\theta} = \frac{T}{J_m + J_L} \tag{i}$$

For the unit with the gear transmission:
See the free-body diagram shown in Figure 1, in the case of a loss-free (i.e., 100% efficient) gear transmission.

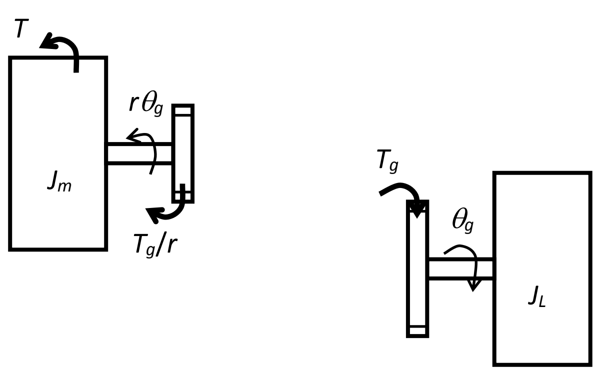

Figure 1: Free-body Diagram.

Newton's 2nd law gives

$$J_m r\ddot{\theta}_g = T - \frac{T_g}{r} \tag{ii}$$

and

$$J_L\ddot{\theta}_g = T_g \tag{iii}$$

where T_g = gear torque on the load inertia. Eliminate T_g in (ii) and (iii). We get

$$\ddot{\theta}_g = \frac{rT}{(r^2 J_m + J_L)} \tag{iv}$$

Divide (iv) by (i).

$$\frac{\ddot{\theta}_g}{\ddot{\theta}} = a = \frac{r(J_m + J_L)}{(r^2 J_m + J_L)} = \frac{r(1 + J_L / J_m)}{(r^2 + J_L / J_m)}$$

or,

$$a = \frac{r(1+p)}{(r^2 + p)} \tag{v}$$

where $p = J_L/J_m$.

From (v) note that for $r = 0$, $a = 0$ and for $r \to \infty$, $a \to 0$. Peak value of a is obtained through differentiation:

$$\frac{\partial a}{\partial r} = \frac{(1+p)[(r^2 + p) - r \times 2r]}{(r^2 + p)^2} = 0$$

We get, by taking the positive root,

$$r_p = \sqrt{p} \tag{vi}$$

where r_p is the value of r corresponding to peak a. The peak value of a is obtained by substituting (vi) in (v); thus,

$$a_p = \frac{1+p}{2\sqrt{p}} \tag{vii}$$

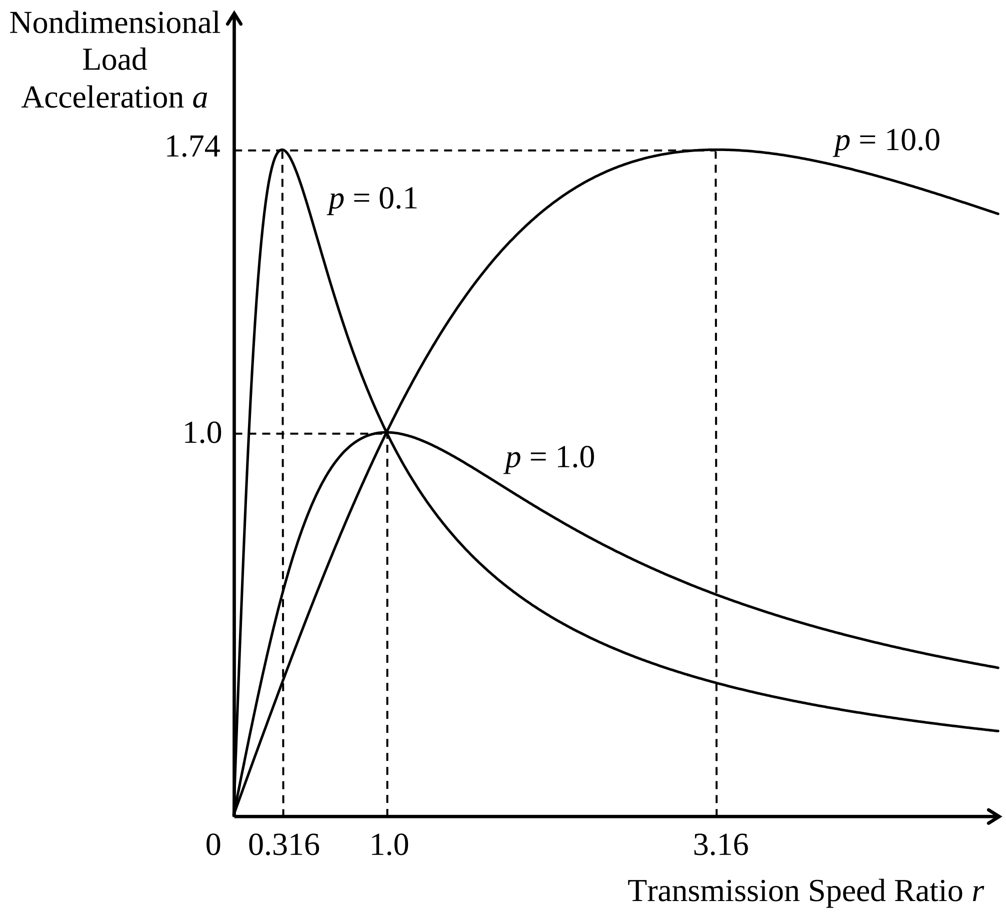

Figure 2: Normalized Acceleration versus Speed Ratio.

Also, note from (v) that when $r = 1$ we have $a = r = 1$. Hence, all curves (v) should pass through the point (1,1).
The relation (v) is sketched in Figure 2 for $p = 0.1$, 1.0, and 10.0. The peak values are tabulated below.

p	r_p	a_p
0.1	0.316	1.74
1.0	1.0	1.0
10.0	3.16	1.74

Note from Figure 2 that the transmission speed ratio can be chosen, depending on the inertia ratio, to maximize the load acceleration. In particular, we can state the following:

1. When $J_L = J_m$, pick a direct-drive system (no gear transmission; i.e., $r = 1$).
2. When $J_L < J_m$, pick a speed-up gear at the peak value of r ($=\sqrt{J_L / J_m}$).

3. When $J_L > J_m$, pick a speed-down gear at the peak value of r.