MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

Homework 7

Assigned: Apr 6, 2021
Due: Apr 13, 2021

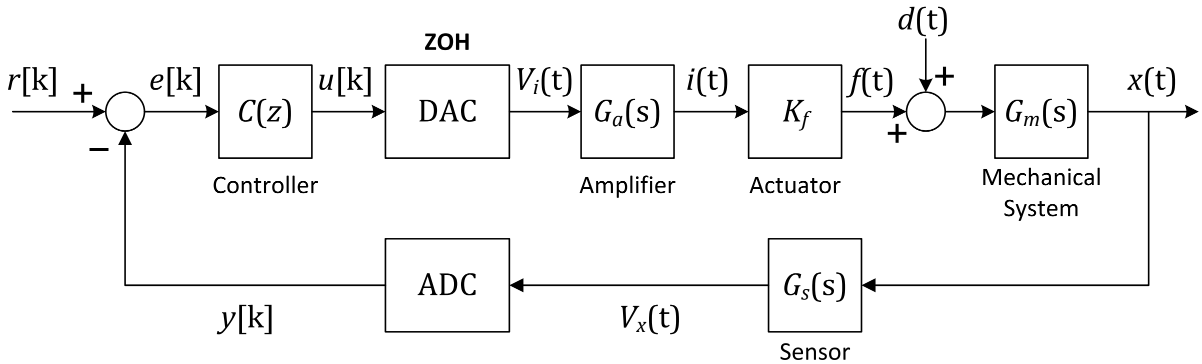

Figure 1: Block diagram of a position control system.

Figure 1 shows a block diagram of a position control system. Here, $G_a(s)$ is the transconductance amplifier, $K_f = 1\,\mathrm{N/A}$ is the actuator force constant, $G_m(s)$ is the mechanical system, and $G_s(s)$ is the sensor. The transfer functions and parameters are given as follows.

$$G_a(s) = \frac{1}{s/\omega_a + 1} \qquad \omega_a = 2\pi \times 10^3\,\mathrm{rad/s}$$

$$G_s(s) = \frac{1}{s/\omega_s + 1} \qquad \omega_s = 10\pi \times 10^3\,\mathrm{rad/s}$$

$$G_m(s) = \frac{1}{ms^2} \qquad m = 1\,\mathrm{kg}$$

The controller $C(z)$ is implemented in a real-time computer at a sampling rate $f_s = 10\,\mathrm{kHz}$ ($T = 100\,\mathrm{\mu s}$). The real-time computer interfaces with the sensor via an ADC and with the amplifier via a DAC. The ADC generates discrete-time signal $y[k]$ by scaling and sampling the sensor output signal $V_x(t)$ such that

$$y[k] = 0.1\ V_x(t)|_{t=kT} = 0.1V_x(kT).$$

The DAC generates the amplifier input signal $V_i(t)$ by scaling and zero-order holding the discrete-time control effort $u[k]$ such that

$$V_i(t) = 10\ u[k] \quad \text{for} \quad kT \le t < kT + T.$$

Use MATLAB to answer the following questions.

(a) Draw the Bode plot of the plant

$$P(s) = \frac{V_x(s)}{V_i(s)}$$

(b) Draw the Bode plot of the plant including the half-sample delay, i.e.,

$$P_{\text{delay}}(s) = P(s)e^{-s\frac{T}{2}}$$

and the Bode plot of the ZOH equivalent of the plant, i.e.,

$$P_{\text{zoh}}(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{V_x(s)}{V_i(s)}\frac{1}{s}\right\}.$$

in the same figure. (Tip: use `c2d` command with $T = 100\,\mu\text{s}$ and 'zoh' option.)

(c) In Homework 6 (b), we designed a continuous-time controller

$$C(s) = K_p\frac{\alpha\tau s + 1}{\tau s + 1}$$

for $P_{\text{delay}}(s)$. Find a discrete-time controller $C(z)$ that approximates $C(s)$ via Tustin method, i.e.,

$$C(z) = K_p\frac{\alpha\tau s + 1}{\tau s + 1}\bigg|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

and draw the Bode plot of $C(s)$ and $C(z)$ in the same figure. (Tip: use `c2d` command with $T = 100\,\mu\text{s}$ and 'tustin' option.)

(d) In Homework 6 (d), we designed a continuous-time controller that additionally implements PI control, i.e.,

$$C(s) = K_p\underbrace{\left(1 + \frac{1}{T_i s}\right)}_{PI(s)}\frac{\alpha\tau s + 1}{\tau s + 1}$$

Find a discrete-time transfer function $PI(z)$ that approximates $PI(s)$ via backward rectangular method, i.e.,

$$PI(z) = PI(s)\big|_{\frac{1}{s}=\frac{T}{1-z^{-1}}}$$

Draw the block diagram of $PI(z)$ only in terms of constant gains, summation, and delay block z^{-1}. Include a saturation block in the block diagram to properly implement anti-windup that limits the state of the discrete-time integrator within ± 0.5.