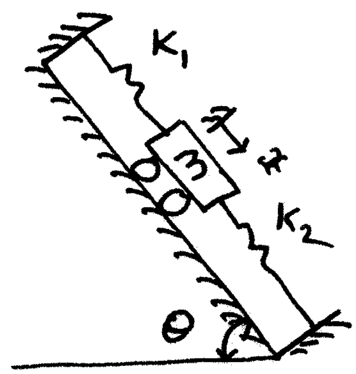
ASSIGNMENT 2 : SOLUTIONS

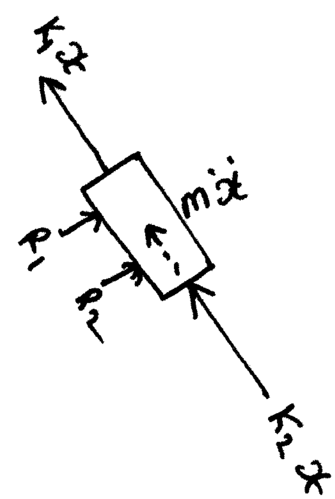
Q1 TEXT BOOK EXERCISE PROBLEM 2.9



NOTE THAT IN STATIC EQUILIBRIUM POSITION THERE ARE FORCES IN THE SPRINGS K_1 & K_2. THESE FORCES BALANCE THE GRAVITATIONAL WEIGHT OF THE MASS.

IF WE CONSIDER x AS THE DISPLACEMENT MEASURED FROM THE STATIC EQUILIBRIUM POSITION, THEN GRAVITY CAN BE IGNORED. BECAUSE, SPRING FORCES CANCEL GRAVITY AT EQUILIBRIUM!

FBD ABOUT STATIC EQUILIBRIUM



$$\underset{+ve}{\searrow \sum} F_x = 0 \quad (\text{D'ALEMBERT})$$

$$\Rightarrow -m\ddot{x} - k_2 x - k_1 x = 0$$

$$\Rightarrow \boxed{m\ddot{x} + (k_1 + k_2)x = 0}$$

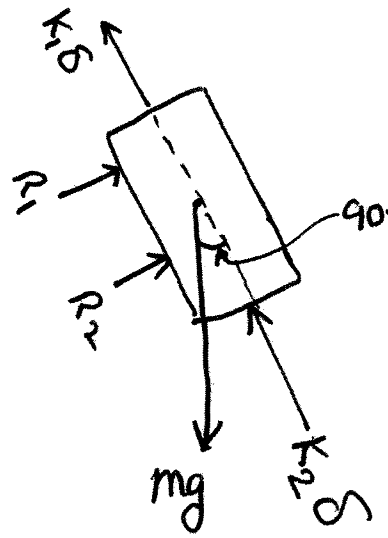
NOTE THAT THE REACTIONS FROM THE ROLLERS, R_1 & R_2, DO NOT APPEAR AS WE ARE SUMMING FORCES ALONG x-DIRECTION WHICH IS PERPENDICULAR TO R_1 & R_2.

HOW DOES GRAVITY CANCEL? EXTRA MATERIAL ↓

Q1) WE CAN INCLUDE GRAVITY AND SHOW THAT IT CANCELS, AS WE SAW IN THE TUTORIAL.

LET US FIRST ESTABLISH STATIC EQUILIBRIUM CONDITION. IF THE SPRINGS DISPLACE BY δ_{st} FROM THEIR UNSTRETCHED CONFIGURATION, WE HAVE THE FOLLOWING FBD IN STATICS.

STATICS FBD

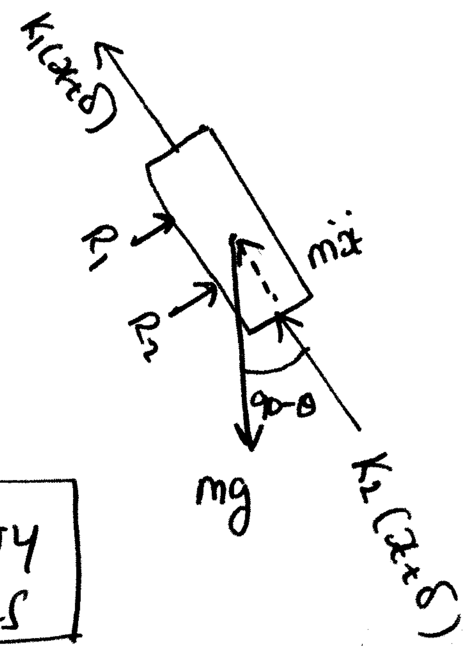


$\searrow \sum_{+ve} F_x = 0$

$\Rightarrow -k_1\delta - k_2\delta + mg \underbrace{\sin\theta}_{\cos(90-\theta)} = 0$

$\Rightarrow (k_1+k_2)\delta = mg\sin\theta$

NOW FOR A DYNAMIC DISPLACEMENT OF 'x' ABOUT STATIC EQUILIBRIUM THE FBD IS AS FOLLOWS.



$\searrow \sum_{+ve} F_x = 0$ (D'ALEMBERT)

$-m\ddot{x} - k_1(x+\delta_{st}) - k_2(x+\delta_{st}) + mg\cos(90-\theta) = 0$

$\Rightarrow m\ddot{x} + (k_1+k_2)x + (k_1+k_2)\delta - mg\sin\theta = 0$

$\Rightarrow \boxed{m\ddot{x} + (k_1+k_2)x = 0}$

$\therefore (k_1+k_2)\delta = mg\sin\theta$ FROM STATICS!

$\boxed{\text{GRAVITY CANCELS}}$

Q1) TEXT BOOK EXERCISE 2.10

WE CAN MODEL THE WIRE ROPE AS A LINEAR ELASTIC SPRING. FROM PAGE 46 OF NOTES:

SPRING CONSTANT FOR WIRE OF LENGTH 30':

$$K_1 = \frac{A_1 E_1}{L_1}$$

A_1 = AREA OF C/S = $\frac{\pi}{4}(0.05)^2$ GIVEN DIAMETER = 0.05"

E_1 = YOUNG'S MODULUS OF STEEL = 30×10^6 lb/in^2

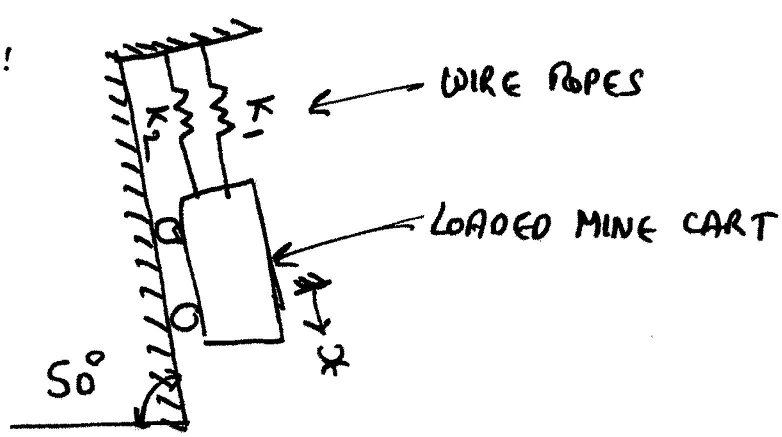
$L_1 = 30' = 30 \times 12'$

$$\therefore K_1 = \frac{\frac{\pi}{4}(0.05)^2 \times 30 \times 10^6}{30 \times 12} = 163.625 \text{ lb/in}$$

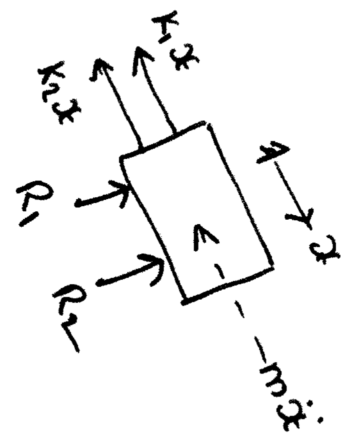
SIMILARLY SPRING CONSTANT FOR WIRE OF LENGTH 25':

$$K_2 = \frac{A_2 E_2}{L_2} = \frac{\frac{\pi}{4}(0.05)^2 \times 30 \times 10^6}{25 \times 12} = 196.35 \text{ lb/in}$$

SO OUR MODEL IS:



Q1) WE CAN FORMULATE THE EQUATION OF MOTION FROM THE FOLLOWING FBD (VERY SIMILAR TO PROBLEM 2.9 SOLVED BEFORE)



GRAVITY CANCELS AND HENCE IGNORED.

$\searrow \sum_{+ve} F_x = 0$ (D'ALEMBERT) $\Rightarrow -m\ddot{x} - K_1 x - K_2 x = 0$

$\Rightarrow m\ddot{x} + (K_1 + K_2)x = 0$

GIVEN $m = 5000$ lb

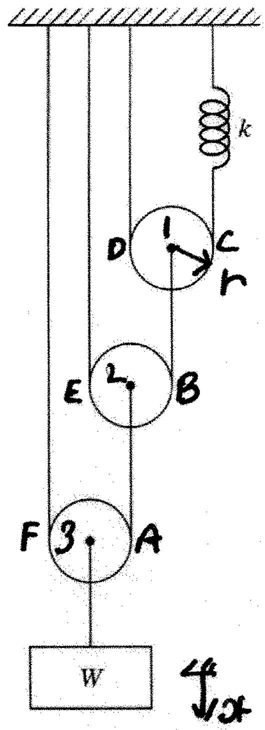
KNOWN $K_1 = 163.625$ lb/in

$K_2 = 196.35$ lb/in

$\therefore \boxed{5000\ddot{x} + 360x = 0}$

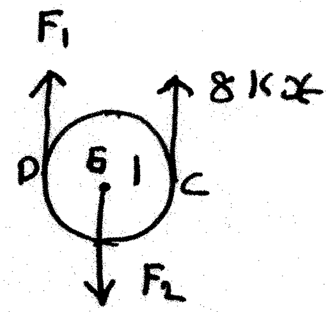
<u>NOTE</u> THAT THE ABOVE PROBLEM IS VERY SIMILAR TO THE PROBLEM 2.9 SOLVED EARLIER. CONVINCE YOURSELF THAT GRAVITY DOES CANCEL BY FOLLOWING THE SAME PROCEDURE AS WE DID IN PROBLEM 2.9.

Q1) TEXT BOOK EXERCISE PROBLEM 2.14



IF THE WEIGHT MOVES DOWN BY A DISTANCE x, THEN A MOVES BY $2x$, B MOVES BY $4x$, AND C MOVES BY $8x$. THIS IS FROM KINEMATICS OF NO-SLIP (TOPIC 1, PAGE 8 IN YOUR NOTES)

FBD'S: PULLEY 1



$$\circlearrowleft \sum_{+ve} M_G = 0$$

$$(8Kx)r - F_1(r) = 0 \Rightarrow F_1 = 8Kx$$

NOTE!

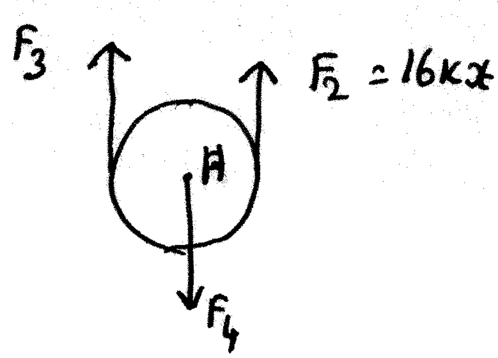
r = RADIUS OF PULLEYS

PULLEY INERTIA IGNORED

$$\uparrow \sum_{+ve} F_y = 0 \Rightarrow F_1 + 8Kx - F_2 = 0$$

$$\Rightarrow F_2 = F_1 + 8Kx = 16Kx$$

PULLEY 2:



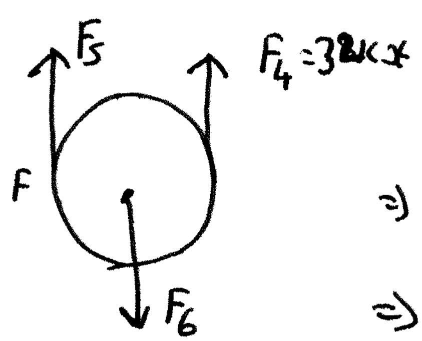
$$\circlearrowleft \sum_{+ve} M_H = 0$$

$$F_2 r - F_3 r = 0 \Rightarrow F_3 = 16Kx$$

$$\uparrow \sum_{+ve} F_y = 0 \Rightarrow F_3 + F_2 - F_4 = 0$$

$$\Rightarrow F_4 = F_3 + F_2 = 32Kx$$

Q1) PULLEY 8:



$\overset{\curvearrowleft}{\underset{+ve}{\sum}} M_F = 0$

$\Rightarrow \quad F_4(r) - F_5(r) = 0$

$\Rightarrow \quad F_5 = F_4 = \cancel{32kx} \; 32kx$

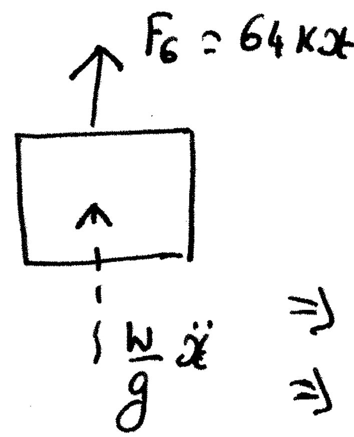
$\uparrow \underset{+ve}{\sum} F_y = 0 \Rightarrow F_5 + F_4 - F_6 = 0$

$\Rightarrow F_6 = F_5 + F_4$

$\Rightarrow F_6 = 64kx$

FBD OF WEIGHT:

(D'ALEMBERT)



$\downarrow \underset{+ve}{\sum} F_x = 0$

$\Rightarrow \; -F_6 - \frac{W}{g}\ddot{x} = 0$

$\Rightarrow \; \frac{W}{g}\ddot{x} + 64kx = 0$

GRAVITATIONAL WEIGHT BALANCES INITIAL TENSIONS/SPRING FORCE. SO GRAVITY IGNORED IN THE ABOVE FBD. NOTE THE INERTIA OF WEIGHT CAN'T BE IGNORED!!

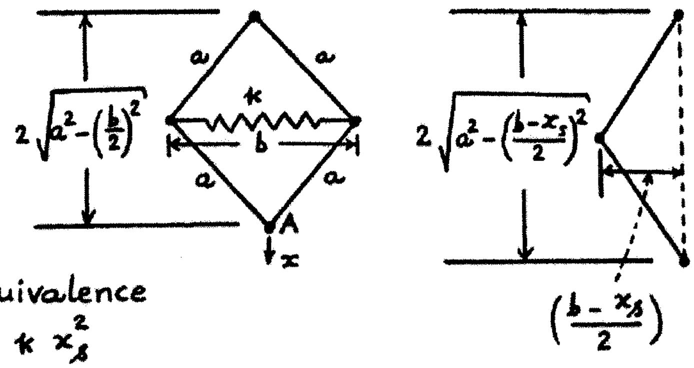
$\therefore$ EQUATION OF MOTION IS : $\boxed{\frac{W}{g}\ddot{x} + 64kx = 0}$

g = ACCELERATION DUE TO GRAVITY.

NOTE: A CAREFUL CONSTRUCTION OF FBDS IS A MUST TO GET THE ABOVE RIGHT. PRACTISE WILL HELP YOU.

Q2) TEXTBOOK EXERCISE PROBLEM 1.11

(a) x = downward deflection of point A,
x_s = resulting deformation of spring



Potential energy equivalence gives $\frac{1}{2} k_{eq} x^2 = \frac{1}{2} k x_s^2$

$$k_{eq} = k \left(\frac{x_s}{x}\right)^2$$

But
$$x = 2\left[\sqrt{a^2 - \left(\frac{b - x_s}{2}\right)^2} - \sqrt{a^2 - \left(\frac{b}{2}\right)^2}\right]$$

$$= 2\sqrt{a^2 - \left(\frac{b}{2}\right)^2}\left[\left\{\frac{a^2 - \left\{\frac{b}{2}\left(1 - \frac{x_s}{b}\right)\right\}^2}{a^2 - \left(\frac{b}{2}\right)^2}\right\}^{1/2} - 1\right]$$

$$= 2\sqrt{a^2 - \frac{b^2}{4}}\left[\left\{\frac{\left(a^2 - \frac{b^2}{4} - \frac{x_s^2}{4} + \frac{b\, x_s}{2}\right)}{\left(a^2 - \frac{b^2}{4}\right)}\right\}^{1/2} - 1\right]$$

$$= 2\sqrt{a^2 - \frac{b^2}{4}}\left[\left\{1 - \frac{\overbrace{x_s^2}^{\approx 0}}{\left(a^2 - \frac{b^2}{4}\right)} + \frac{b\, x_s}{2\left(a^2 - \frac{b^2}{4}\right)}\right\}^{1/2} - 1\right]$$

Using the relation $(1+\theta)^{1/2} \simeq 1 + \frac{\theta}{2}$, we obtain

$$x = 2\left(a^2 - \frac{b^2}{4}\right)^{1/2}\left[1 + \frac{b\, x_s}{4\left(a^2 - \frac{b^2}{4}\right)} - 1\right] = \frac{b\, x_s}{2\left(a^2 - \frac{b^2}{4}\right)^{1/2}}$$

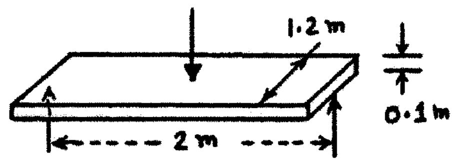
$$\therefore\ k_{eq} = k\left(\frac{x_s}{x}\right)^2 = 4k\left(\frac{a^2 - \frac{b^2}{4}}{b^2}\right) = k\left(\frac{4a^2 - b^2}{b^2}\right)$$

(b) Here $x = x_s$ (spring deflection)

$$\therefore\ k_{eq} = k$$

Q2) TEXT BOOK EXERCISE PROBLEM 1.10

For simply supported beam,
for load at middle,



$$k_1 = \frac{48\,EI}{l^3} = \frac{48(2.06\times10^{11})(10^{-4})}{8}$$

$= 12.36 \times 10^7$ N/m where $I = \frac{1}{12}(1.2)(0.1)^3 = 10^{-4}\ m^4$.

δ_1 = original deflection $= \frac{mg}{k_1} = \frac{500 \times 9.81}{12.36 \times 10^7} = 396.8447 \times 10^{-7}$ m

When spring k is added, $k_{eq} = k + k_1$.

(a) New deflection $= \frac{mg}{k_{eq}} = \frac{\delta_1}{4}$; $k_{eq} = \frac{4\,mg}{\delta_1} = 4\,k_1$
$= k + k_1$

$\therefore\ k = 3\,k_1 = 37.08 \times 10^7$ N/m

(b) New deflection $= \frac{mg}{k_{eq}} = \frac{\delta_1}{2}$; $k_{eq} = \frac{2\,mg}{\delta_1} = 2\,k_1$

$\therefore\ k = k_1 = 12.36 \times 10^7$ N/m $\qquad = k + k_1$

(c) New deflection $= \frac{mg}{k_{eq}} = \frac{3}{4}\,\delta_1$; $k_{eq} = \frac{4\,mg}{3\,\delta_1} = \frac{4}{3}\,k_1$

$\therefore\ k = \frac{1}{3}\,k_1 = 4.12 \times 10^7$ N/m $\qquad = k + k_1$

Q2) TEXTBOOK EXERCISE PROBLEM 1.29

Assume small angles θ_1 and θ_2; $\theta_2 = \left(\frac{p_1}{p_2}\right)\theta_1$

x_1 = horizontal displacement of C.G. of mass $m_1 = \theta_1 r_1$

x_2 = vertical displacement of C.G. of mass $m_2 = \theta_2 r_2 = p_1 \theta_1 r_2 / p_2$

y_1 = horizontal displacement of springs k_1 and $k_2 = \theta_1 (r_1 + l_1)$

y_2 = vertical displacement of springs k_3 and $k_4 = \theta_2 l_2 = p_1 l_2 \theta_1 / p_2$

Equivalence of kinetic energies gives

$$\frac{1}{2} J_{eq} (\dot{\theta}_1)^2 = \frac{1}{2} J_1 (\dot{\theta}_1)^2 + \frac{1}{2} J_2 (\dot{\theta}_2)^2 + \frac{1}{2} m_1 (\dot{x}_1)^2 + \frac{1}{2} m_2 (\dot{x}_2)^2$$

$$\therefore J_{eq} = J_1 + J_2 (p_1/p_2)^2 + m_1 r_1^2 + m_2 r_2^2 (p_1/p_2)^2$$

Equivalence of potential energies gives

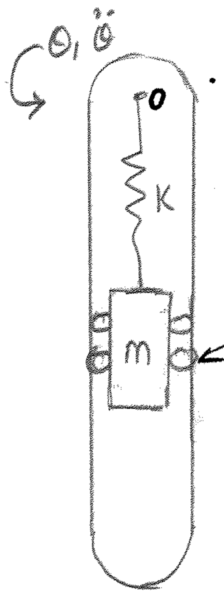
$$\frac{1}{2} k_{eq} \theta_1^2 = \frac{1}{2} k_{12} y_1^2 + \frac{1}{2} k_{34} y_2^2 + \frac{1}{2} k_{t1} \theta_1^2 + \frac{1}{2} k_{t2} \theta_2^2$$

with $k_{12} = k_1 + k_2$, $k_{34} = k_3 k_4 / (k_3 + k_4)$

$y_1 = \theta_1 (r_1 + l_1)$, $y_2 = p_1 l_2 \theta_1 / p_2$ and $\theta_2 = p_1 \theta_1 / p_2$.

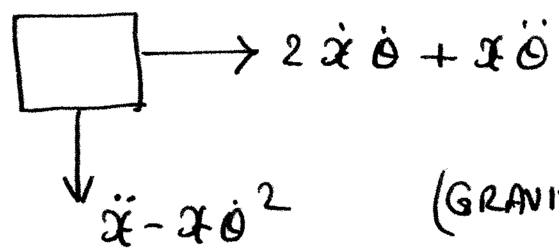
$$\therefore k_{eq} = (k_1 + k_2)(r_1 + l_1)^2 + \left(\frac{k_3 k_4}{k_3 + k_4}\right) \frac{p_1^2 l_2^2}{p_2^2} + k_{t1} + k_{t2} \frac{p_1^2}{p_2^2}.$$

Q3) TUTORIAL 1 PROBLEM (ii) IN QUESTION 1: ROTATING SPRING MASS SYSTEM



x: DISTANCE OF MASS 'm' RELATIVE TO 'O'.

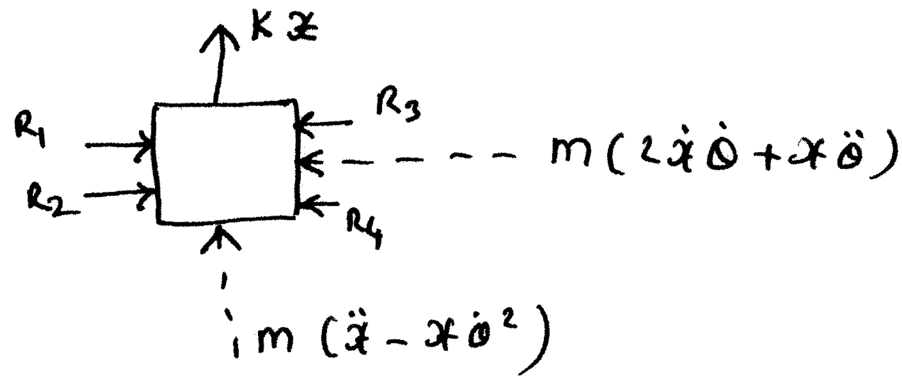
ACCELERATION DIAGRAM (SEE TUTORIAL 1)



(GRAVITY IGNORED)

FORCES IN FBD (D'ALEMBERT)

R_1, R_2, R_3, R_4 ARE REACTIONS FROM ROLLER GUIDES



$\downarrow \sum F_x = 0$
+ve

$$-Kx - m(\ddot{x} - x\dot{\theta}^2) = 0$$

$$\Rightarrow \boxed{m\ddot{x} + (K - m\dot{\theta}^2)\, x = 0}$$

NOTE EFFECTIVE STIFFNESS $K - m\dot{\theta}^2$ DEPENDS ON ROTATIONAL SPEED!

HORIZONTAL EQUILIBRIUM DETERMINES NET REACTION FROM ROLLER GUIDES!