2.4. Damped SDOF Response–2

MECH 463: Mechanical Vibrations

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Suggested Readings:

1. Topic 2.4 from notes package for detailed derivations.
2. Sections 2.6 and 3.4 from the course textbook.

Learning Objectives

1. Determine forced vibration response of a viscously damped SDOF system.

2. Apply the rotating vector technique to identify three regimes of steady forced vibration response.

3. Deduce design guidelines to mitigate vibration response.

2.16 Harmonically Forced Damped SDOF Response (NP 2.16, T 3.4+Notes)

We restrict this discussion to harmonic forcing of the form $f(t) = f_0 \cos \omega t$. Our interest lies in the particular solution of the second oder ODE:

$$m\ddot{x}_p + c\dot{x}_p + kx_p = f = F_0 \cos \omega t. \tag{1}$$

We can determine the amplitude X and phase lag ϕ of the particular solution, taken to be

$$x_p(t) = X \cos(\omega t - \phi) \tag{2}$$

using the algebraic or graphical methods. Let us solve for the two unknowns using the rotating vector diagram methods.

Example 17 — # 1

p. 124 in NP

Example 17 : Use the rotating vector method to determine X and ϕ of the steady state or particular solution response of a system governed by Eq.(**??**)

Fill in the class

Example 17 — # 2

Example 17 — # 3

Example 17 — # 4

Example 17 — # 5

Forced Damped Response — # 1

Thus we have the following result:

$$
\boxed{
\begin{aligned}
x_p(t) &= X\cos(\omega t - \phi) \\
X &= \frac{F_0}{\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2}}, \tan\phi = \frac{\omega c}{k - m\omega^2}
\end{aligned}
\qquad (3)
}
$$

The total response is given by

$$
\boxed{
\begin{aligned}
x(t) &= e^{-\zeta\omega_n t}\left[C_1\cos\omega_d t + C_2\sin\omega_d t\right] + X\cos(\omega t - \phi), \\
X &= \frac{F_0}{\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2}}, \tan\phi = \frac{\omega c}{k - m\omega^2}
\end{aligned}
\qquad (4)
}
$$

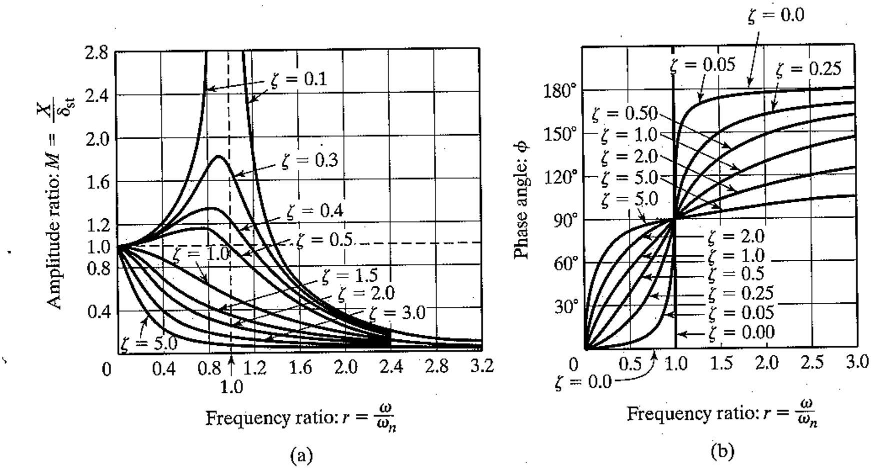
Forced Damped Response — # 2

The two unknown constants C_1 and C_2 are to be determined from the initial conditions.

We can define the Dynamic Magnification Factor (DMF) for the damped system, similar to the undamped system, in the steady state as follows:

$$
\begin{aligned}
|DMF| = M = \left|\frac{X}{\delta_{st}}\right| &= \frac{F_0}{\delta_{st}\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \\
&= \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}, \quad r = \frac{\omega}{\omega_n} \\
\phi &= \tan^{-1}\left[\frac{2\zeta r}{1 - r^2}\right]
\end{aligned}
\tag{5}
$$

Forced Damped Response — # 3



Forced Damped Response — # 4

1. Damping has pronounced effect around resonance in reducing the value of the magnification factor M. It does decrease the response at other frequencies, but not as effectively.
2. For a constant, or static, force $r = 0$ and $M = 1$.
3. The maximum magnification occurs slightly below the undamped natural frequency $\omega < \omega_n$, or $r < 1$ at $r = \sqrt{1 - 2\zeta^2}$ and is given by $M_{max} = \left|\frac{X}{\delta_{st}}\right|_{max} = \frac{1}{2\zeta\sqrt{1-2\zeta^2}}$.
4. At the undamped natural frequency, $\left|\frac{X}{\delta_{st}}\right|_{\omega=\omega_n} = \frac{1}{2\zeta}$
5. For frequencies well above resonance the DMF curves for different levels of ζ for an underdamped system show small but insignificant differences in M, suggesting that damping is most effective around resonance.
6. For overdamped systems M decreases monotonically with r.
7. The phase lag of the response starts at 0^0 (in-phase) for $r = 0$ and gradually increases to a value of 90^0 at $r = 1$. Eventually, well above resonance $r >> 1$, the phase lag is 180^0 (out-of-phase). Notice that the phase change around $r = 1$ is gradual and not as abrupt as in the case of an undamped system $\zeta = 0$.

Forced Damped Response — # 5

Question: How do we measure DMF curve of a practical system such as a machine tool? (p. 129 in NP)

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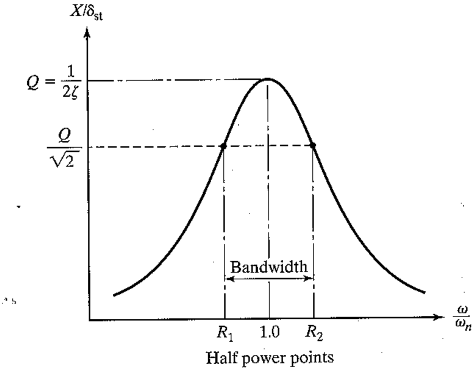
Forced Damped Response — # 6



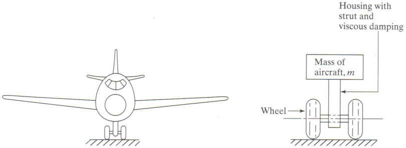
Q is defined as follows:

$$Q \approx \frac{1}{2\zeta} \approx \frac{\omega_n}{\Delta\omega}, \quad \Delta\omega = \omega_2 - \omega_1 \text{ (HPBW).} \tag{6}$$

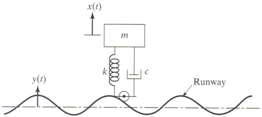
Example 18 — # 1

p. 131 in NP

The landing gear of an airplane can be idealised as the spring-mass-damper system shown below. If the runway surface is described by $y(t) = y_0 \cos \omega t$, determine the equations of motion and steady damped vibration response. What design criteria will you use to select the k and c? Open ended problem



(a)



(b)

Example 18 — # 2

Fill in the class

Example 18 — # 3

Example 18 — # 4

Example 18 — # 5

Design Guidelines for Forced Response — # 1

Summary

1. The DMF of a viscously damped system in the steady state is given by:

$$|DMF| = M = \left|\frac{X}{\delta_{st}}\right| = \frac{F_0}{\delta_{st}\sqrt{\left(k-m\omega^2\right)^2+(c\omega)^2}} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}, \quad r = \frac{\omega}{\omega_n}$$

$$\phi = \tan^{-1}\left[\frac{2\zeta r}{1-r^2}\right]$$

2. The maximum magnification occurs slightly below the undamped natural frequency $\omega < \omega_n$, or $r < 1$ at $r = \sqrt{1-2\zeta^2}$ and is given by $M_{max} = \left|\frac{X}{\delta_{st}}\right|_{max} = \frac{1}{2\zeta\sqrt{1-2\zeta^2}}$. At the undamped natural frequency, $\left|\frac{X}{\delta_{st}}\right|_{\omega=\omega_n} = \frac{1}{2\zeta}$
3. The phase lag of the response starts at 0^0 (in-phase) for $r = 0$ and gradually increases to a value of 90^0 at $r = 1$. Eventually, well above resonance $r >> 1$, the phase lag is 180^0 (out-of-phase).
4. Quality factor or Q-factor is a frequency-domain measure of damping, obtained from steady state forced vibration response. It is related to ζ via:$Q \approx \frac{1}{2\zeta} \approx \frac{\omega_n}{\Delta\omega}, \quad \Delta\omega = \omega_2 - \omega_1$ (HPBW).