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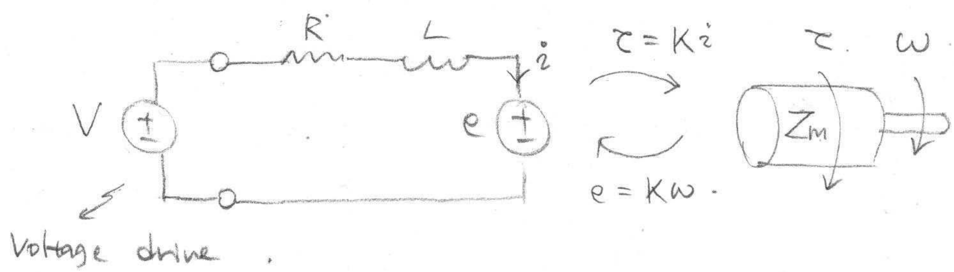
< Voltage - controlled Brushed DC Motor >

Objective

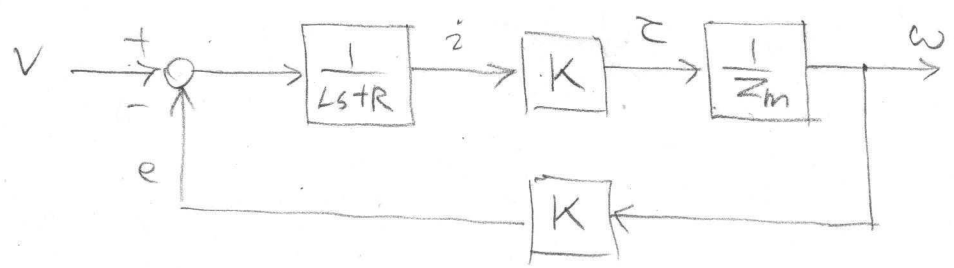
- Understand the dynamics of Voltage - controlled dc motors
- Open-loop speed & torque response.

① Lumped-parameter Model.

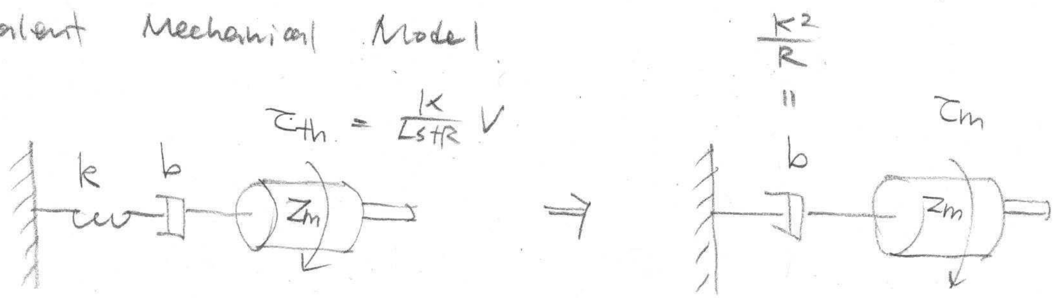


	Through	Across
Flow	i	v, ω
~~Effort~~	f, τ	V

② Block Diagram.



③ Equivalent Mechanical Model



Let's study the dynamics of voltage - controlled dc motors

$$L(s) = \frac{K^2}{Z_m Z_e} \Rightarrow \begin{cases} \dfrac{\omega}{V} = \dfrac{\frac{K}{Z_m Z_e}}{1 + \frac{K^2}{Z_m Z_e}} & \text{speed resp.} \\[2ex] \dfrac{\tau}{V} = \dfrac{\frac{K}{Z_e}}{1 + \frac{K^2}{Z_m Z_e}} & \text{torque resp.} \end{cases}$$

- Speed Response

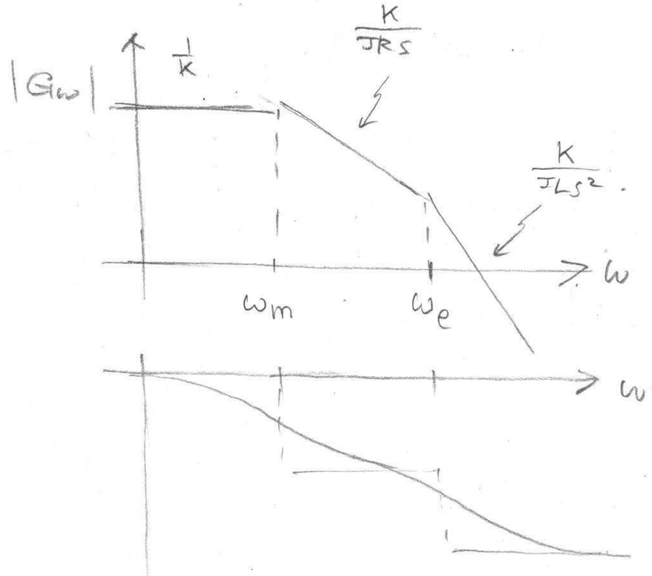
$$G_\omega \triangleq \frac{\omega}{V} = \frac{\frac{K}{Z_m(Ls+R)}}{1+\frac{K^2}{Z_m(Ls+R)}} = \frac{K}{Z_m(Ls+R)+K^2}$$

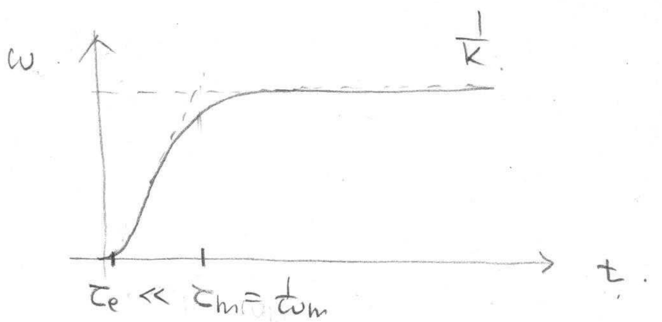
When $Z_m = Js$ (Free Inertia)

$$\frac{\omega}{V} = \frac{K}{Js(Ls+R)+K^2} = \frac{K}{\underbrace{JLs^2 + JRs}_{\omega_e = \frac{R}{L}} \underbrace{+\, JRs + K^2}_{\omega_m = \frac{K^2}{JR}}}$$

$\omega_m \ll \omega_e$ in many practical cases.

Bode plot





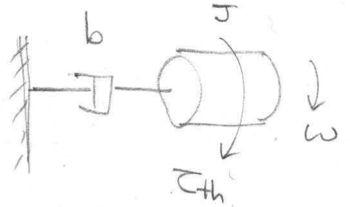
↳ See the datasheet.

- Example motor parameters

$L = 1mH$, $R = 6\Omega$, $K = 200mNm/A$. $J = 2kg\cdot cm^2$

$$\begin{cases} \omega_e \simeq 1kHz & \tau_e = 1ms \\ \omega_m \simeq 5Hz & \tau_m \simeq 200ms \end{cases}$$

- Note that the approx. mechanical model leads to Almost the same,



$$\omega = \frac{1}{Js+b}\cdot\tau_{th} = \frac{1}{Js+\frac{K^2}{R}}\cdot\frac{K}{Ls+R}V$$

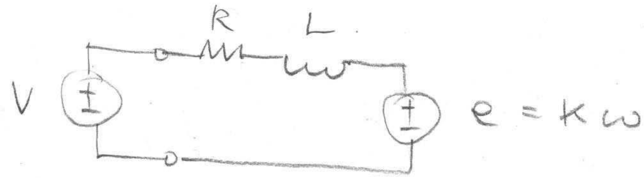
$$= \frac{K}{JLs^2 + (JR + \cancel{\tfrac{K^2}{R}L})s + K^2}V$$

$$\left(\begin{array}{l} \text{If } \frac{K^2}{R}L \ll JR \\ \Leftrightarrow \omega_m \ll \omega_e \end{array}\right) \simeq \frac{K}{JLs^2 + JRs + K^2}V$$

- Here, the steady-state speed can be understood as the terminal velocity due to the apparent damping $b = \frac{K^2}{R}$.

- The step-response is dominated (or bottlenecked) by $\tau_m \ll \tau_e$.

- From the lumped-parameter model :



At the steady-state, $\omega = \text{const}$

$\rightarrow \tau = 0 \rightarrow \dot{i} = 0$

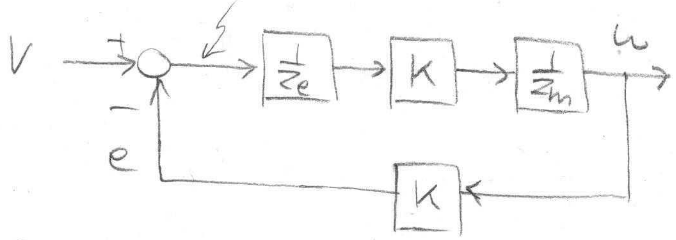
$\rightarrow$ Voltage drop across $Z_e = 0$

$\rightarrow V = e = K\omega$

$\therefore \underline{\underline{\omega = \frac{V}{K}}}$.

- From the block diagram.

$V - e = 0$



At the steady-state

the "error" signal is zero

$\Rightarrow V - e = 0 \rightarrow V = K \cdot \omega$.

- Open-loop speed control with a "voltage drive" is common for simple tasks. (e.g. cooling fans).

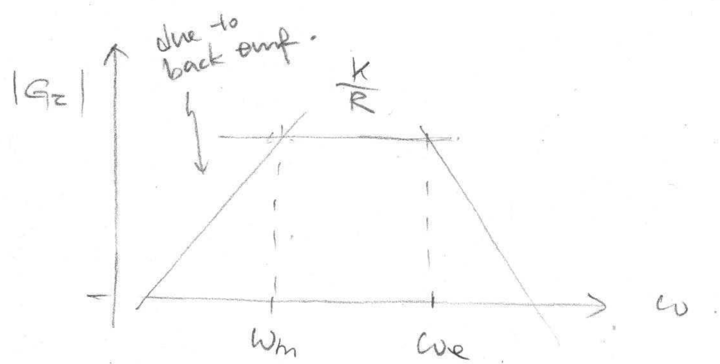
○ Torque Response

· For the analysis & control of mechanical systems (e.g. robots) we usually treat torques and forces as the driving variables.

· High-bandwidth torque control is important in robotics.
  (e.g. MIT Mini Cheetah - very light limbs)

· Voltage-controlled dc motor has some issues here.

$$G_\tau \triangleq \frac{\tau}{V} = \frac{\frac{K}{Ls+R}}{1+\frac{K^2}{(Ls+R)Js}} = \frac{K}{Ls+R+\frac{K^2}{Js}}$$



$\omega_m = \frac{K^2}{JR}$. as $J\downarrow$ $\omega_m\uparrow$
even worse.

Issues ① Slow response to V

Even if we use high-bandwidth voltage amplifier, (e.g. 300 kHz), the torque response is bottlenecked by ω_e (e.g. 1kHz)

② Back-emf effect

Back-emf decreases the low-freq torques.

{ Mechanical view : apparent damping drags the rotor.
  Electrical view : external low-freq disturbance voltage.

⇒ Current-controlled dc motor can address these issues.