Example 5.1:
Consider the light intensity sensor circuit shown in Figure 5.1(a).

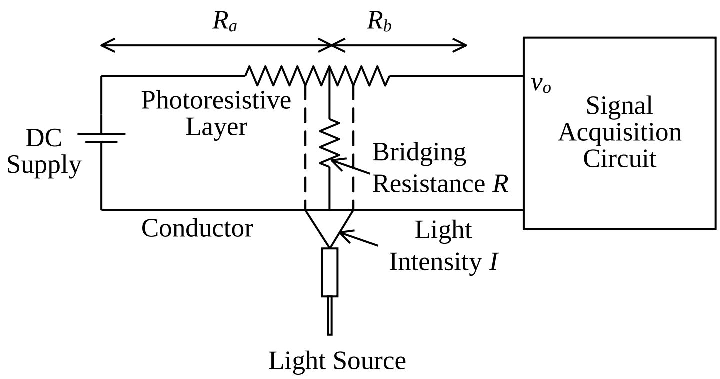

Figure 5.1(a): Light intensity sensor.

One arm of the circuit has a uniform resistance and the other arm is a perfect conductor of electricity. A photoresistive layer is sandwiched between these two arms. A light source directs a beam of light whose intensity is I, on to a narrow rectangular region of the photoresistive layer. As a result, this region becomes conductive with resistance R, which bridges the resistor arm and the conductor arm, as shown. This bridge resistor divides the uniform resistor arm into two segments of resistance R_a and R_b. The output voltage v_o of the resistor arm is acquired as the output of the sensor, which depends on the bridging resistance R (and hence the light intensity I).

In the present example, treat the resistance R as the output of the device. An empirical relation between R and I was found to be $\ln\left(\frac{R}{R_o}\right) = \left(\frac{I_o}{I}\right)^{1/4}$, where the resistance R is in kΩ and the light intensity I is expressed in watts per square meter (W/m^2). The parameters R_o and I_o are empirical constants having the same units as R and I, respectively. The curve of R vs. I is sketched in Figure 5.1(b). The significance of the parameters R_o and I_o is clear from this curve. Specifically, since R goes to infinity when I is zero, I_o provides a reasonable lower limit for I in the operating range of the sensor. Also, R_o is the minimum bridging resistance that is possible (which is reached when I becomes infinity). Typically, the upper limit for I in the operating range of the sensor would be $10I_o$.

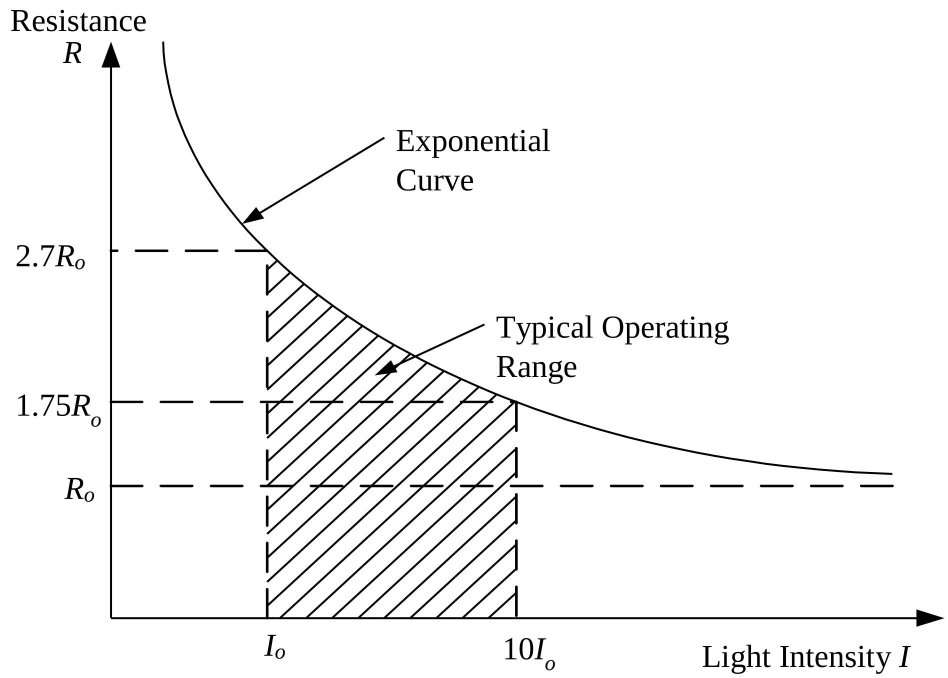

Figure 5.1(b): Characteristic curve of the sensor.

(a) Determine an expression for the sensitivity of the sensor, in terms of the given quantities.
(b) Appropriately nondimensionalize the sensitivity expression.

Solution
(a)
Static transfer relationship of the sensor is:

$$\ln\left(\frac{R}{R_o}\right) = \left(\frac{I_o}{I}\right)^{1/4} \qquad \text{(i)}$$

This may be expressed as

$$R = R_o e^{\left(\frac{I_o}{I}\right)^{1/4}} \qquad \text{(ii)}$$

Differentiate (ii) wrt I:

$$\frac{\partial R}{\partial I} = R_o e^{\left(\frac{I_o}{I}\right)^{1/4}} \times \frac{1}{4}\left(\frac{I_o}{I}\right)^{-3/4} \times I_o \times (-1) I^{-2}$$

Or,

$$\frac{\partial R}{\partial I} = -\frac{R_o}{4}\frac{I_o^{1/4}}{I^{5/4}} e^{\left(\frac{I_o}{I}\right)^{1/4}} \qquad \text{(iii)}$$

Alternatively, we can directly differentiate the ln form (i) to get:

$$\frac{1}{R}\frac{\partial R}{\partial I} = \frac{1}{4}\left(\frac{I_o}{I}\right)^{-3/4} \times I_o \times (-1) I^{-2} = -\frac{1}{4}\frac{I_o^{1/4}}{I^{5/4}}$$

Now substitute (ii) for R, and we get the result (iii).

(b)

The proper nondimensionalization of the sensitivity would be,

$$S = \frac{I_o}{R_o}\frac{\partial R}{\partial I} = -\frac{1}{4}\frac{I_o^{5/4}}{I^{5/4}} e^{\left(\frac{I_o}{I}\right)^{1/4}}$$

Now define the nondimensional light intensity as $i = \dfrac{I}{I_o}$. We get the nondimensional sensitivity,

$$S = -\frac{e^{i^{-1/4}}}{4i^{5/4}}$$