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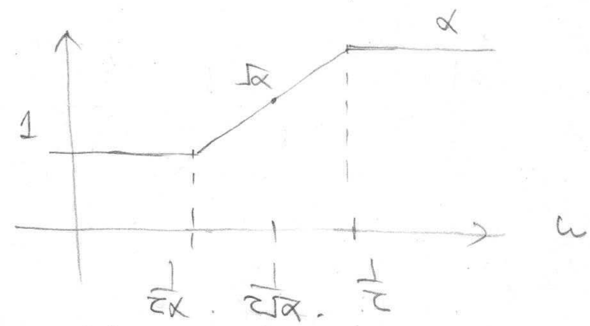
2021/3/12.

< PID Controller Design >

- Objective.
  - Phase compensation : Lead compensator
  - Magnitude compensation : PI controller & low-pass filter.
    (Lag)

- Lead Compensator.

$$H(s) = \frac{\alpha \tau s + 1}{\tau s + 1} \quad (\alpha > 1)$$



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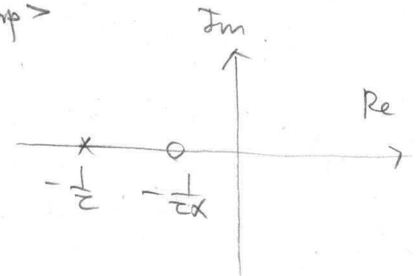

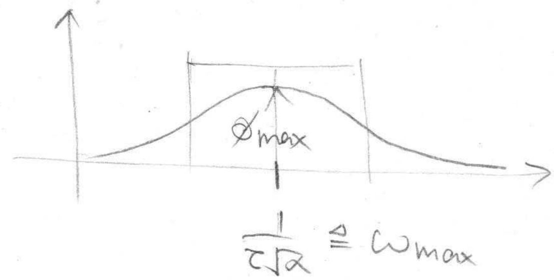

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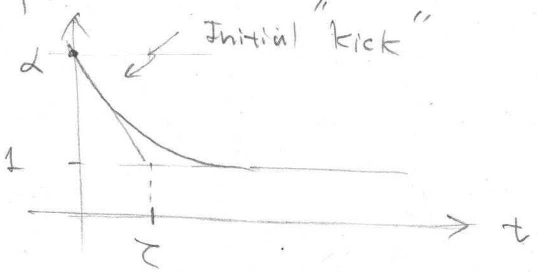


$$H(s) = \frac{\alpha \tau s + \alpha + 1 - \alpha}{\tau s + 1} = \alpha - \frac{\alpha - 1}{\tau s + 1}.$$

- It can be understood as a frequency selective "differentiator"
- Used to compensate the loop for phase around ω_{max}.

$$\phi_{max} = \sin^{-1}\left(\frac{\alpha - 1}{\alpha + 1}\right).$$

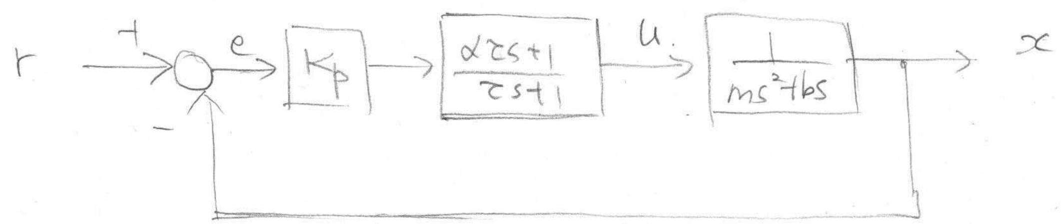
for $\alpha = 10$, $\phi_{max} \approx 55°$.

$$\omega_{max} = \sqrt{\omega_p \cdot \omega_z} = \frac{1}{\tau\sqrt{\alpha}}.$$

Typically implemented such that $\omega_c = \omega_{max}$.

Then, K_p is adjusted to set $|L(j\omega_{max})| = 1$.

<u>Example</u> : $f = ms^2 + bs$.



∘ Design & "Implementation" steps .

① Decide on the target ω_c^*, by looking at the Bode plot.

② Implement a lead compensator such that

$$\omega_c^* = \omega_{max}$$

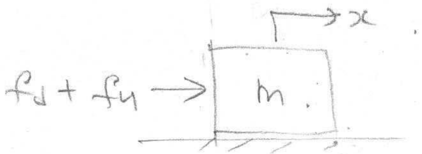
③ Set K_p such that $\omega_c = \omega_c^*$, $\left(|L(j\omega_c^*)| = 1\right)$.
(Raise from 0).

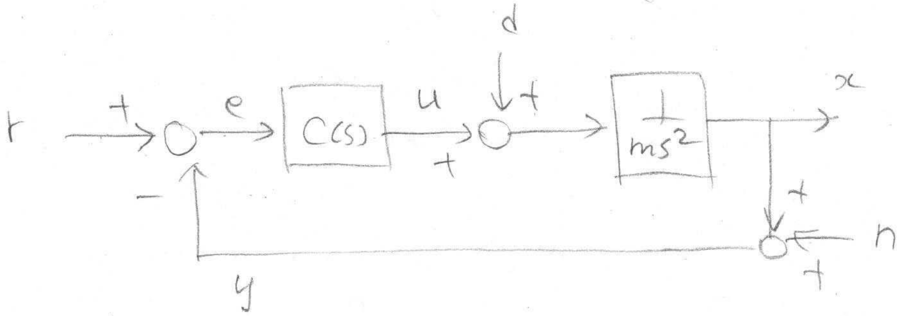
∘ Trade-off.

Large α $\begin{cases} (+) & \phi_{max} \uparrow \text{ (but } \phi_{max} < 90°) \\ (-) & \text{Low-freq gain} \downarrow, \text{ High-freq gain} \uparrow \end{cases}$

∘ PID Controller Design (Series form).

Example : free mass.





· We will consider two CL trans. functions. to evaluate the control system performance.

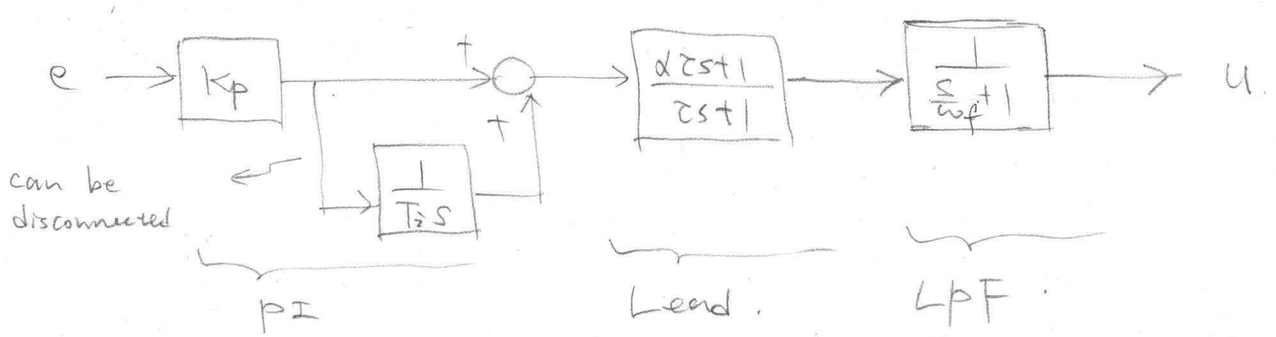
① $\frac{X}{R} = \frac{CP}{1+CP}$ " Complementary Sensitivity "

→ Tracking.

② $\frac{X}{D} = \frac{P}{1+CP}$ " Load Sensitivity "

→ Disturbance rejection.

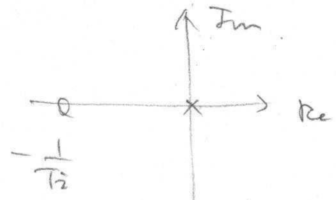
· PID in series form.

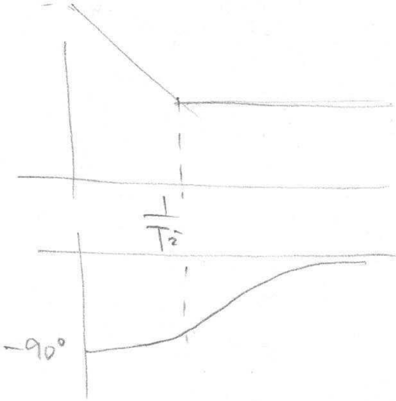


· This is better than the parallel form for loop shaping design.

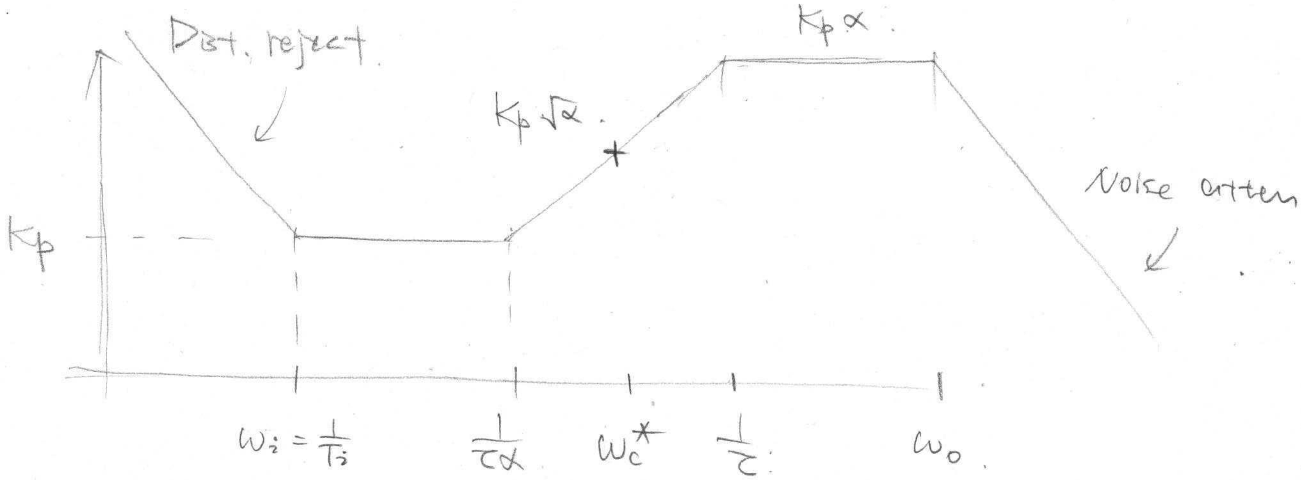
◦ PI is a special case of Lag.

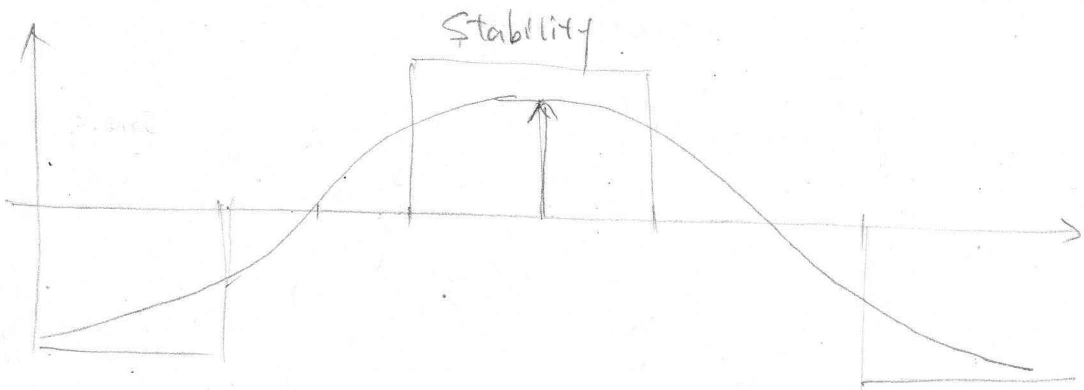
$$H(s) = 1 + \frac{1}{T_i s} = \frac{T_i s + 1}{T_i s}$$





◦ Bode plot of $C(s)$





Note that the "unit" of $C(s)$ is $[N/m]$ "stiffness"

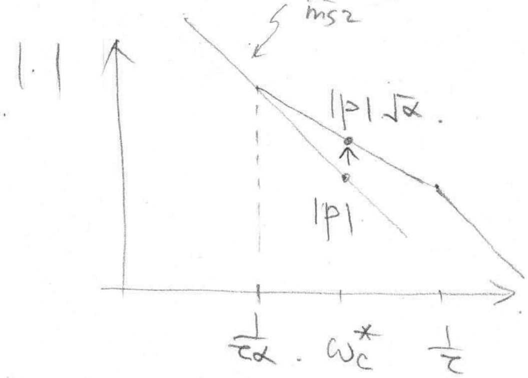
$C(s)$ is an additional "Dynamic stiffness"

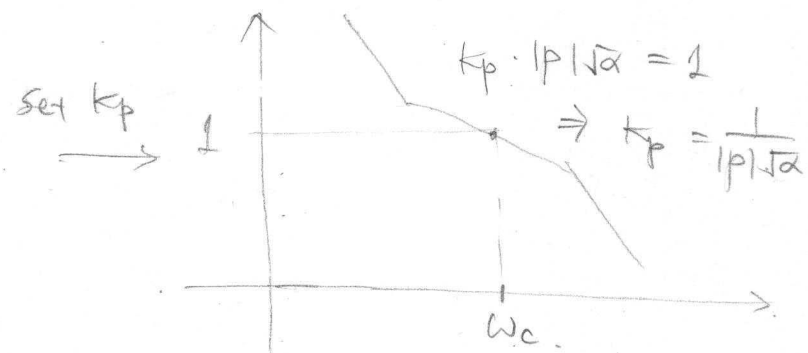
- PID tuning steps. $P(s) = \frac{1}{ms^2}$

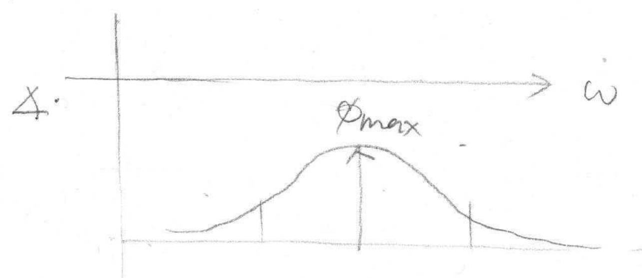
① Decide on the target ω_c^* by looking at the Bode plot.

Commonly, it is limited by
- Sensor BW.
- power amp BW.
- Delay in digital controller.
- High-freq resonance.

② Implement a lead comp. such that $\omega_{max} = \omega_c^*$.







③ Set K_p such that $\omega_c = \omega_c^*$ $|L(j\omega_c^*)| = 1$.
(Raise from 0)

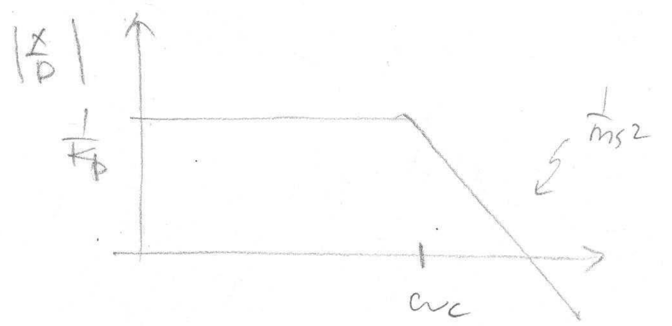
④ Introduce Integrator such that $\omega_i < \omega_c$ (e.g. $\omega_i = \frac{1}{10}\omega_c$)

⑤ Introduce LPF such that $\omega_f > \omega_c$ (e.g. $\omega_f = 10\,\omega_c$)

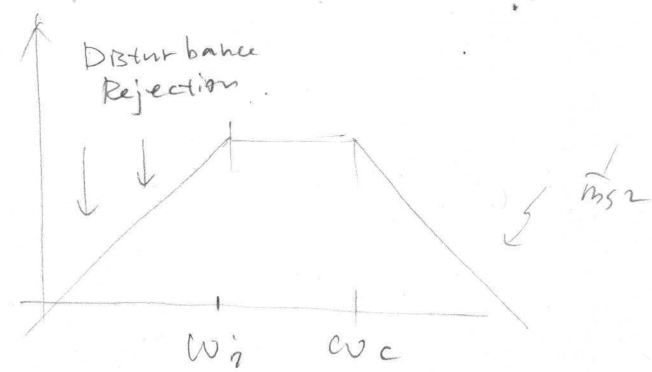
○ Disturbance Rejection

$$\frac{X}{D} = \frac{P}{1+CP} = \begin{cases} \frac{1}{C} & \text{when } |L| \gg 1 \\ P & \text{when } |L| \ll 1 \end{cases}$$

< W/o Integrator >



< With Integrator >



· Integrator improves dc disturbance rejection.