

Slide 39:

Input sensitivity $S = \frac{\Delta y}{\Delta u} \rightarrow \frac{\partial y}{\partial u}$ (in the limit)

Nondimensionalize (for many reasons):

$$S = \frac{u}{y} \frac{\Delta y}{\Delta u} \rightarrow \frac{u}{y} \frac{\partial y}{\partial u}$$

Vector case:

$$\text{Sensitivity matrix } S = \frac{\partial \mathbf{y}}{\partial \mathbf{u}} = \begin{bmatrix} \frac{\partial y_1}{\partial u_1} & \frac{\partial y_1}{\partial u_2} & \cdot & \cdot & \frac{\partial y_1}{\partial u_r} \\ \frac{\partial y_2}{\partial u_1} & \frac{\partial y_2}{\partial u_2} & \cdot & \cdot & \frac{\partial y_2}{\partial u_r} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial y_m}{\partial u_1} & \frac{\partial y_m}{\partial u_2} & \cdot & \cdot & \frac{\partial y_m}{\partial u_r} \end{bmatrix}_{m \times r}$$

Reasons for nondimensionalization:

1. Fair comparison of different sensitivity values
2. Physical units (dimensions; e.g., changing from m to mm) won't affect the value
3. Typically simplifies the expression (fewer affecting quantities are involved)
4. The affecting quantities are also usually nondimensionalized
5. Some important effects may be masked (a different dimension or scale will change a sensitivity value for the same condition)

Note: Sensitivity given in a device data sheet is not nondimensionalized because it deals with a specific device. But, if we are comparing sensitivities of different types of devices, use nondimensional sensitivities.

Note: "Sensitivity" may be treated (analytically) the same way as "error." Will discuss this under error.