MECH467 Pre-lab 1
Name: Ratthamnoon Prakitpong
Student ID: 63205165
Email: rprakitpong@gmail.com

467 pre lab.

1a. $J_{sum} = J_{enc} + J_{coup} + J_{tach} + J_m + \underbrace{J_{rod}}$

$J_{rod} = \frac{mr^2}{2}$

$= \frac{\left(\rho_s L_s \pi \left(\frac{d_s}{2}\right)^2\right)\left(\frac{d_s}{2}\right)^2}{2}$

$= 1.7\times10^{-4} + 4\times10^{-5} + 9.3212\times10^{-7} + .765\times10^{-4} + \frac{7800}{2}(.82)\pi\left(\frac{.02}{2}\right)$

$= 3.879\times10^{-4}\ kgm^2$

b. $\rightarrow I_{in}(s) \rightarrow I_m(s)$

$I_p = I_{in} - I_m$

$V_p = I_p\left(K_{vp} + \frac{K_{vi}}{s}\right)$ } PWM Amp

$I_m = \frac{V_P - \omega K_b}{L_a s + R_a}$ } elec winding of motor

$I_m(s) = \frac{(I_{in}(s) - I_m(s))\left(K_{vp} + \frac{K_{vi}}{s}\right) - \omega(s) K_b}{L_a s + R_a}$

$I_m(s)\left(1 + \frac{\left(K_{vp} + \frac{K_{vi}}{s}\right)}{L_a s + R_a}\right) = \frac{I_{in}(s)\left(K_{vp} + \frac{K_{vi}}{s}\right)}{L_a s + R_a} - \frac{\omega(s) K_b}{L_a s + R_a}$

$I_m(s) = \frac{I_{in}(s)(K_{vp} + K_{vi}/s) - \omega(s) K_b}{L_a s + R_a + K_{vp} + \frac{K_{vi}}{s}}$

sub $\omega(s)/I_m(s)$ in

$I_m(s)\left(1 + \frac{K_t K_b}{(J_m s + B)(L_a s + R_a + K_{vp} + K_{vi}/s)}\right) = \frac{(K_{vp} + K_{vi}/s)}{(L_a s + R_a + K_{vp} + K_{vi}/s)} I_{in}($

$I_m(s) = \frac{(J_m s + B)(K_{vp} + K_{vi}/s)}{((J_m s + B)(L_a s + R_a + K_{vp} + K_{vi}/s) + K_t K_b)} I_{in}(s)$

$\rightarrow V_{in}(s) \rightarrow I_{in}(s)$

$I_{in} = S_g V_{in}$ } current sensor gain

Just sub $S_g V_{in}(s)$ in $I_{in}(s)$

$\rightarrow I_m(s) \rightarrow \omega(s)$

$T_m = K_t I_m$ } motor constant

$\omega = \left(\frac{1}{J_m s+B}\right)(T_m - \cancel{T_d})$ $T_d = 0$

$$\omega(s) = \frac{K_t I_m(s)}{J_m s + B}$$

c- $\rightarrow V_{in}(s) \rightarrow \omega(s)$

$\omega = \left(\frac{K_t}{J_m s+B}\right)\left(S_g V_{in}(K_{vp} + K_{vi}/s) - \omega K_b\right) / (L_a s + R_a + K_{vp} + K_{vi}/s)$

$a = L_a s + R_a + K_{vp} + K_{vi}/s$ } arbitrarily defined
$b = K_t / J_m s + B$

$\omega\left(\frac{a + K_b b}{a}\right) = \left(\frac{b}{a}\right) S_g V_{in} (K_{vp} + K_{vi}/s)$

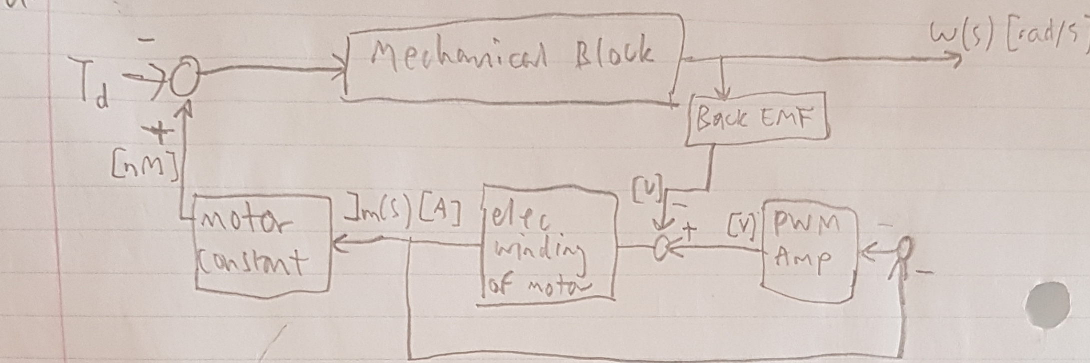
$\omega = \left(\frac{b}{a+K_b b}\right)(S_g)\left(K_{vp} + \frac{K_{vi}}{s}\right) V_{in}$

$$\omega(s) = \frac{S_g(K_{vp} + K_{vi}/s)}{\frac{a}{b} + K_b} V_{in}(s)$$

$$= \frac{S_g(K_{vp} + K_{vi}/s)}{\frac{(L_a s + R_a + K_{vp} + K_{vi}/s)(J_m s + B)}{K_t} + K_b} V_{in}(s)$$

$$= \frac{K_t S_g (K_{vp} s + K_{vi})}{\left((L_a s^2 + (R_a + K_{vp})s + K_{vi}\right)(J_m s + B) + K_t K_b s)} V_{in}(s)$$

d.



e. $\omega(s) = \frac{1}{J_m s + B}(T_m - T_d)$ } mech block

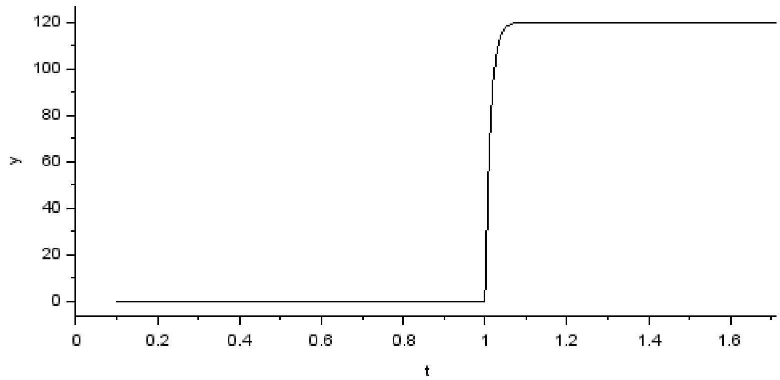
$T_m = K_t I_m$ } motor const

$I_m = \frac{-\omega K_b}{L_a s + R_a + K_{vp} + K_{vi}/s}$ } elec winding & PWM Amp as derived in 1b

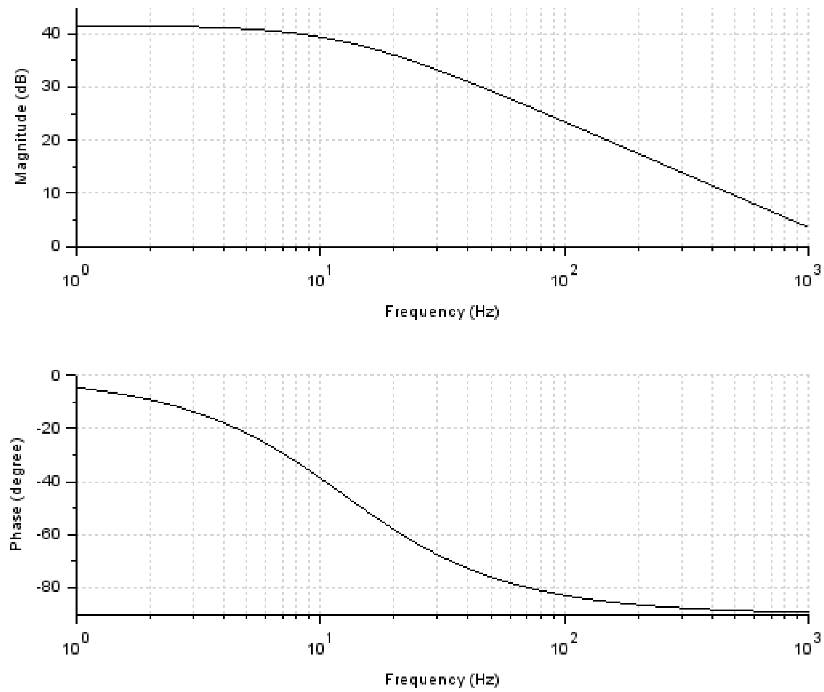
$$\omega(s) = \frac{1}{J_m s + B}\left(\left(\frac{-K_b K_t \omega(s)}{L_a s + R_a + K_{vp} + K_{vi}/s}\right) - T_d(s)\right)$$

$$\omega(s) = \left(\frac{-(L_a s + R_a + K_{vp} + K_{vi}/s)}{(J_m s + B)(L_a s + R_a + K_{vp} + K_{vi}/s) + 1}\right) T_d(s)$$

2a. w(s)/Im(s)
Step response:



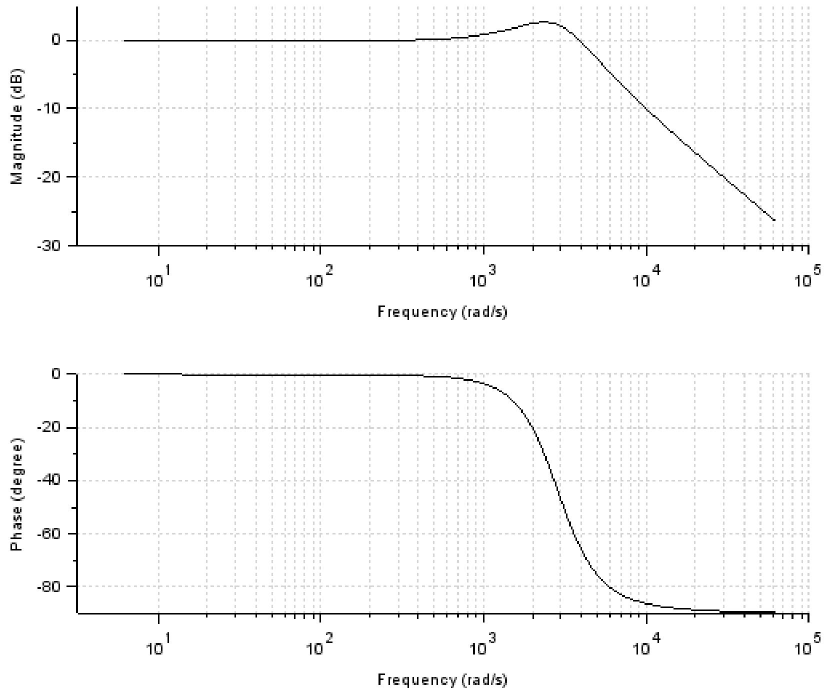
Frequency response:



DC Gain = 120 rad/As
Bandwidth = 77.96 rad/s or 12.4 Hz at ~38.6 dB
Rise time: 0.028 s

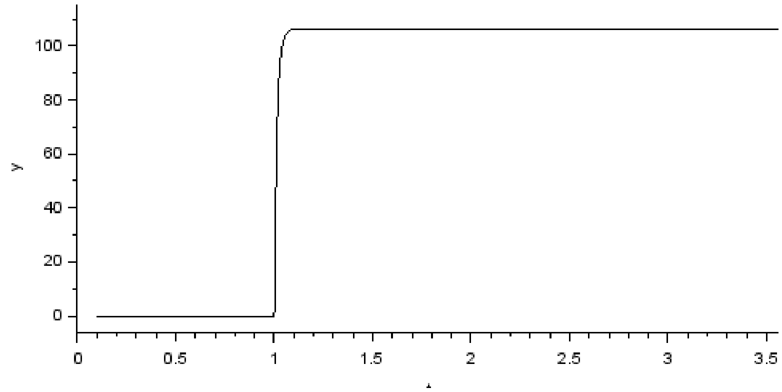
t_90% = ~1.0295 s and t_10% = ~1.0014 s

2b. Im(s)/Iin(s)
Frequency response:

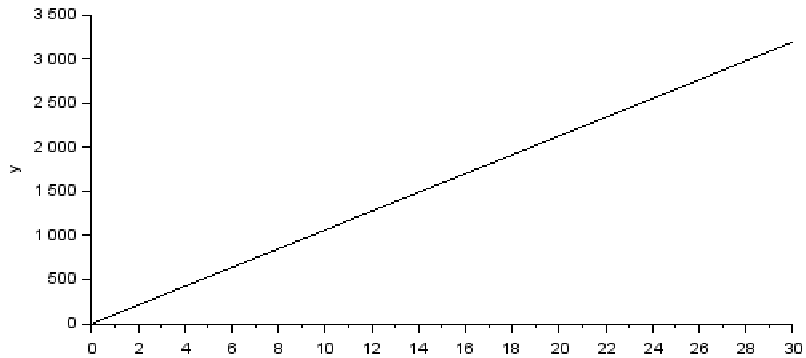


DC gain = 1 A/A
Bandwidth: 5131.0 rad/s
Linear interpolation from (5082.9227, -2.8969316) and (5239.749, -3.2331028)

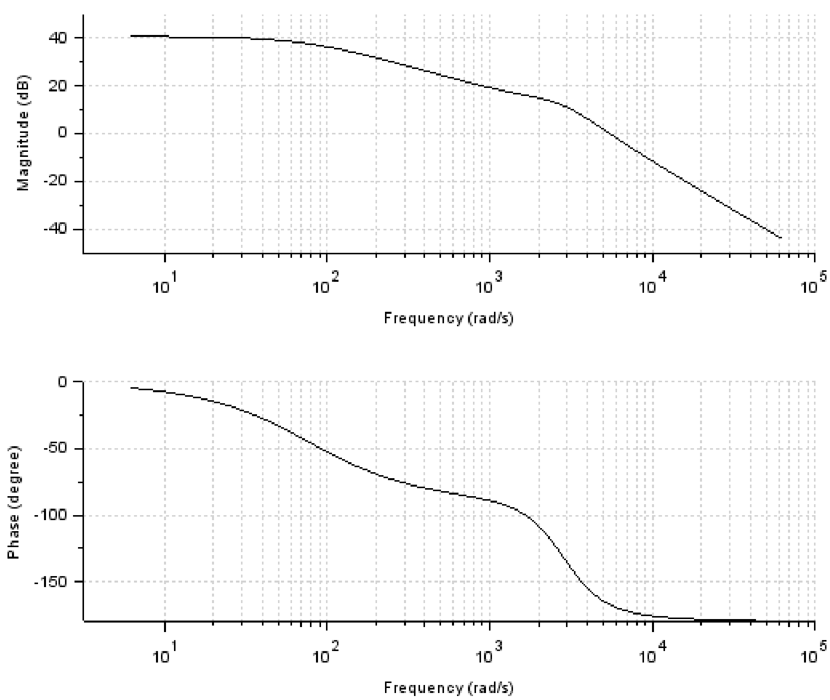
2c. w(s)/Vin(s)
Step response:



Ramp response:



Frequency response:

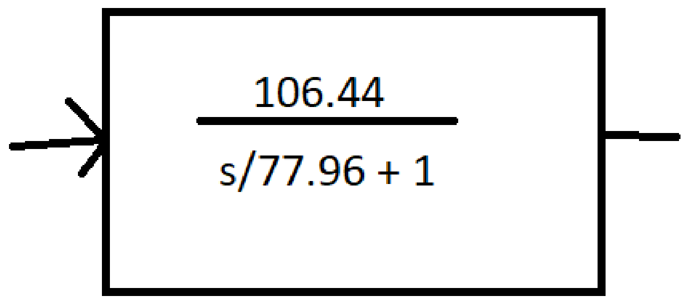


DC gain = 106.44 rad/Vs
Bandwidth = 77.96 rad/s or 12.4 Hz @ ~37.5 dB
Rise time: 0.029 s
t_90% = ~1.0305 s and t_10% = ~1.0015 s

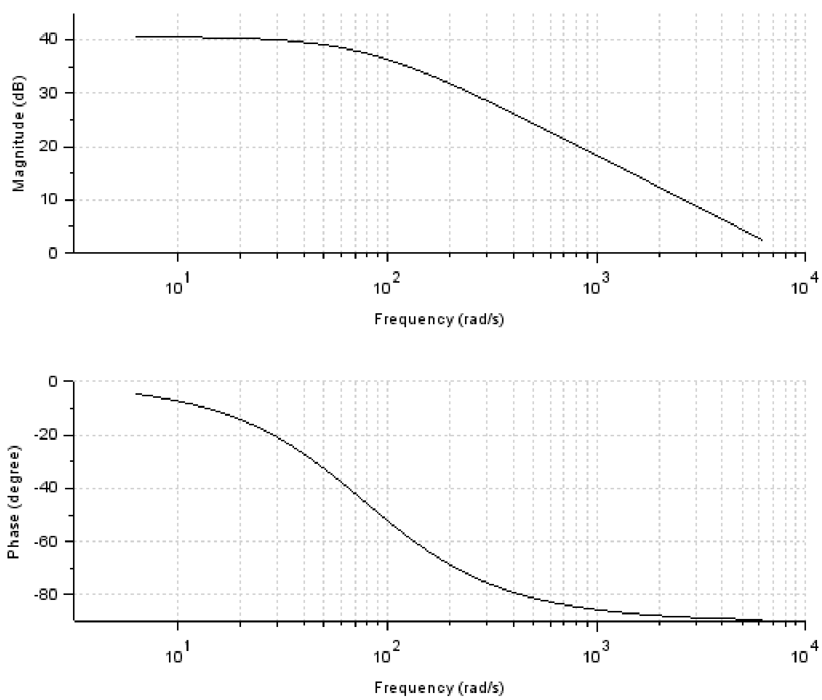
Compare 2a [w(s)/Im(s)] and 2c [w(s)/Vin(s)]:
DC gain is 120 dB and 106.44 dB, where 106.44/120 = 0.887, matching current sensor gain Sg that we used to transform Vin to Iin.
Bandwidth and rise time is the same since there's no added component that would change them.

Steady state error for a ramp input response:
At steady state, all terms with s will disappear, leaving us with w(s)/Vin(s) = KtSgKvi/KviB, which simplifies to w(s)/Vin(s) = KtSg/B = 106.44 rad/Vs, same as simulated DC gain.

2d.
Simplified block diagram:



Frequency response:



Gain and bandwidth are the same up to 10^3 rad/s, and then it starts to differ.