Let us consider an op-amp circuit in Figure 1. We assume that the op-amp has infinite input impedance, zero output impedance, and open-loop transfer function $A(s)$. Figure 3 shows the Bode plot of $A(s)$.

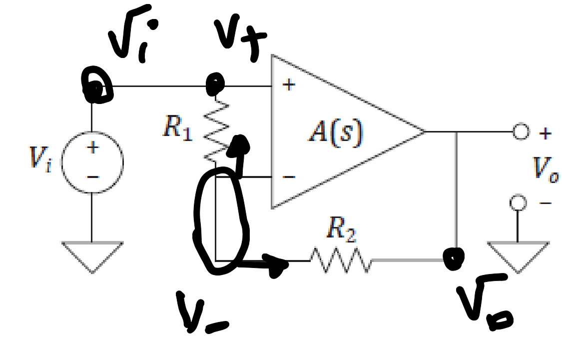


Figure 1: Op-amp circuit for Problem1.

(a) (20 pt.) Draw a block diagram that shows the feedback relation between the input voltage $V_i(s)$ and output voltage $V_o(s)$.

$V_+ = V_i$

KCL: V_-

$$\frac{V_- - V_+}{R_1} + \frac{V_- - V_o}{R_2} = 0$$

$\xrightarrow{R_1 R_2}$ $R_2(V_- - V_i) + R_1(V_- - V_o) = 0$

$$R_2 V_- - R_2 V_i + R_1 V_- - R_1 V_o = 0$$

$$\Rightarrow (R_2 + R_1) V_- = R_2 V_i + R_1 V_o$$

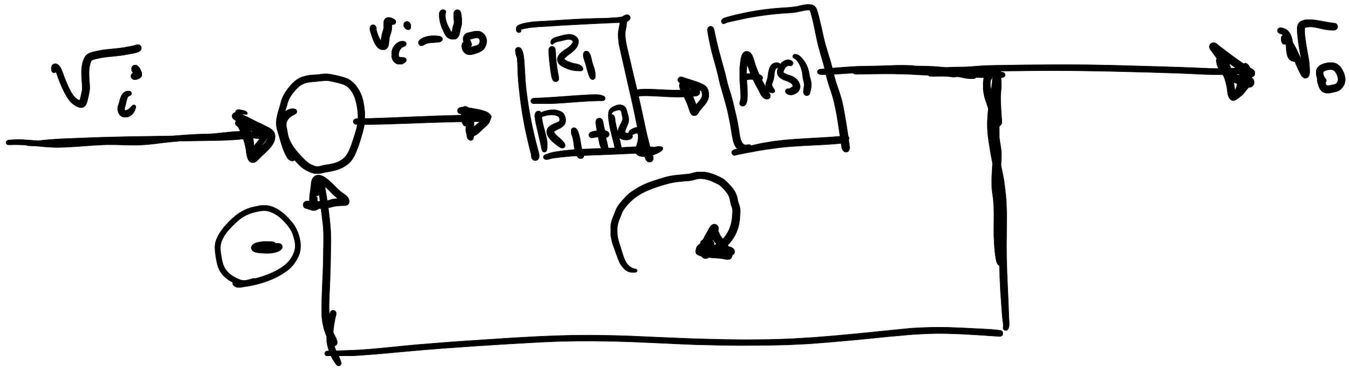
$$V_- = \frac{R_2}{R_2 + R_1} V_i + \frac{R_1}{R_1 + R_2} V_o$$

$$V_0 = A(s)\,(V_+ - V_-)$$

$$V_0 = A(s)\left(\underbrace{V_i - \frac{R_2}{R_2+R_1}V_i}\; - \frac{R_1}{R_1+R_2}V_0\right)$$

$$V_0 = A(s)\left(\frac{R_1}{R_1+R_2}V_i - \frac{R_1}{R_1+R_2}V_0\right)$$

$$\boxed{V_0 = A(s)\left(\frac{R_1}{R_1+R_2}\right)(V_i - V_0)}$$



(b) (10 pt.) Find the expression for the loop transfer function $L(s)$ in terms of R_1, R_2, and $A(s)$.

loop transfer function = − L.T.

$$= -\left(-\frac{R_1}{R_1+R_2} \times A(s)\right) \Rightarrow$$

$$\boxed{L(s) = \frac{R_1}{R_1+R_2} A(s)}$$

(c) (20 pt.) For $R_1 \to \infty$, $R_2 = 1\,\mathrm{k}\Omega$, and $A(s)$ given in Figure 3, find the gain crossover frequency ω_c and phase margin ϕ_m of $L(s)$.

$$|L(j\omega)|_{\omega=\omega_c} = 1$$

$$R_1 \to \infty \Rightarrow \frac{R_1}{R_1+R_2} \simeq 1 \Rightarrow L(s) = A(s)$$

$$|A(j\omega)|_{\omega=\omega_c} = 1 \overset{\text{Fig3}}{\Longrightarrow} \omega_c = 10^7\ \mathrm{Hz}$$

$$\phi_m = \angle L(j\omega) + 180^\circ$$

$$\angle L(j\omega_c) \underset{\text{Fig3.}}{=} -110^\circ \Rightarrow \phi_m = -110 + 180 = 70^\circ$$

(d) (30 pt.) For $R_1 = 1\,\text{k}\Omega$ and $A(s)$ given in Figure 3, find the resistance value R_2 that makes the closed-loop transfer function $G(s) = V_o(s)/V_i(s)$ achieve a -3 dB bandwidth of 100 kHz.

-3dB bandwidth ω_h $\simeq$ crossover frequency ω_c

closed-loop transfer function $G(s)$ loop transfer function $L(s)$

if $\phi_m = 90^\circ \Rightarrow \omega_h = \omega_c$

(1) $\omega_h \simeq \omega_c$

$$G(s) = \frac{\overbrace{\frac{R_1}{R_1+R_2}}^{f} A(s)}{1 + \frac{R_1}{R_1+R_2} A(s)} = \frac{f A(s)}{1 + f A(s)}$$

$R_2 \rightarrow ?$

$\omega_h = 100\,kHz = \omega_c$

$$\left| L(j\omega) \right|_{\omega = 100kHz} = 1$$

$$\left| \frac{R_1}{R_1+R_2} A(j\omega) \right|_{\omega = 100kHz} = 1$$

$$\frac{R_1}{R_1+R_2} \boxed{|A(j\omega)|_{\omega=100kHz}} = 1$$

$$|A(j\omega)|_{\omega=100kHz=10^5Hz} = 100$$

$$\Rightarrow \frac{R_1}{R_1+R_2}(100) = 1 \Rightarrow \frac{R_1}{R_1+R_2} = \frac{1}{100}$$

$$\frac{1}{1+R_2} = \frac{1}{100}$$

$$\Rightarrow \boxed{R_2 = 99k\Omega}$$

(2) $\boxed{|G(j\omega)|_{\omega=100kHz} = -3dB = \frac{1}{\sqrt{2}}}$

$$\left|\frac{f\,A(j\omega)}{1+f\,A(j\omega)}\right|_{\omega=100kHz} = \frac{1}{\sqrt{2}}$$

$$A(j\omega) = A(10^5 j) = 100\angle -90 = 100\,e^{-\frac{\pi}{2}j} = -100j$$

$-j$

$$\left| \frac{-100fj}{1-100fj} \right| = \frac{1}{\sqrt{2}}$$

$$\left| \frac{-100fj(1+100fj)}{(1-100fj)(1+100fj)} \right| = \left| \frac{-100fj + 10^4 f^2}{1+10^4 f^2} \right| = \frac{1}{\sqrt{2}}$$

$$\sqrt{\left(\frac{100f}{1+10^4f^2}\right)^2 + \left(\frac{10^4f^2}{1+10^4f^2}\right)^2} = \frac{\sqrt{(100f)^2 + (10^4f^2)^2}}{1+10^4f^2} = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{10^4f^2 + 10^8f^4}}{1+10^4f^2} = \frac{\sqrt{10^4f^2(1+10^4f^2)}}{1+10^4f^2}$$

$$= \frac{10^2 f \sqrt{1+10^4f^2}}{1+10^4f^2} = ! \frac{\sqrt{2}}{2}$$

$$\Rightarrow 10^2 f = 1 \Rightarrow f = \frac{1}{100}$$

$$1+10^4f^2 = 2 \Rightarrow 10^4f^2 = 1 \Rightarrow \boxed{f = \frac{1}{100}}$$

(e) (20 pt.) What is the dc gain of $G(s)$ designed in part (d)?

$\boxed{R_1 = 1\,k\Omega, \quad R_2 = 99\,k\Omega}$, $f = \frac{R_1}{R_1+R_2} = \frac{1}{100}$

$$\left| G(j\omega) \right|_{\omega=0} = \left| \frac{f A(j\omega)}{1 + f A(j\omega)} \right|$$

$$= \left| \frac{0.01\, A(0)}{1 + 0.01\, A(0)} \right| = \frac{0.01 \times 10^6}{1 + (0.01 \times 10^6)} = \frac{10^4}{1+10^4}$$

$$\simeq 1 = 0.9999$$

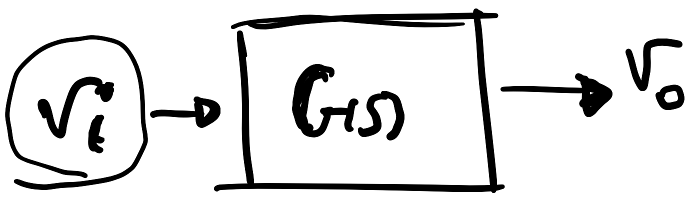
(f) (30 pt.) Suppose $G(s)$ designed in part (d) is excited with an input voltage

$$\boxed{V_i(t) = 1\cos(2\pi \times 10^5\ t),}$$

which is a persistent sinusoid defined for all time including $t < 0$. Find the magnitude M_o and phase ϕ_o of the output voltage

$$V_o(t) = M_o \cos(2\pi \times 10^5 t + \phi_o).$$

$$G(s) = \frac{0.01 A(s)}{1 + 0.01 A(s)}$$



$$A\cos(2\pi f t + \phi) \qquad A \left|G(s)\right|_{\omega = f} \cos\left(2\pi f t + \phi + \angle G(s)\big|_{\omega = f}\right)$$

$$\left|G(j\omega)\right|_{\omega = 2\pi \times 10^5 \text{ rad/s} = 10^5 \text{ Hz}}$$

$$= \left|\frac{0.01\ A(j\omega)}{1 + 0.01\ A(j\omega)}\right|_{\omega = 10^5 Hz} = \left|\frac{(0.01)(-100j)}{1 + (0.01)(-100j)}\right|$$

$$A(j\omega)\Big|_{\omega = 10^5 Hz} = 100\angle -90 = -100j$$

$$= \left|\frac{-j}{1-j}\right| = \left|\frac{-j(1+j)}{(1-j)(1+j)}\right| = \left|\frac{-j+1}{1+1}\right| = \left|\frac{-j+1}{2}\right|$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

$$\boxed{V_o = \frac{1}{\sqrt{2}} \cos(2\pi \times 10^5 t - 45^\circ)}$$

$$\angle \frac{-j}{1-j} = \arctan\left(\frac{-\frac{1}{2}}{\frac{1}{2}}\right) = -45^\circ$$

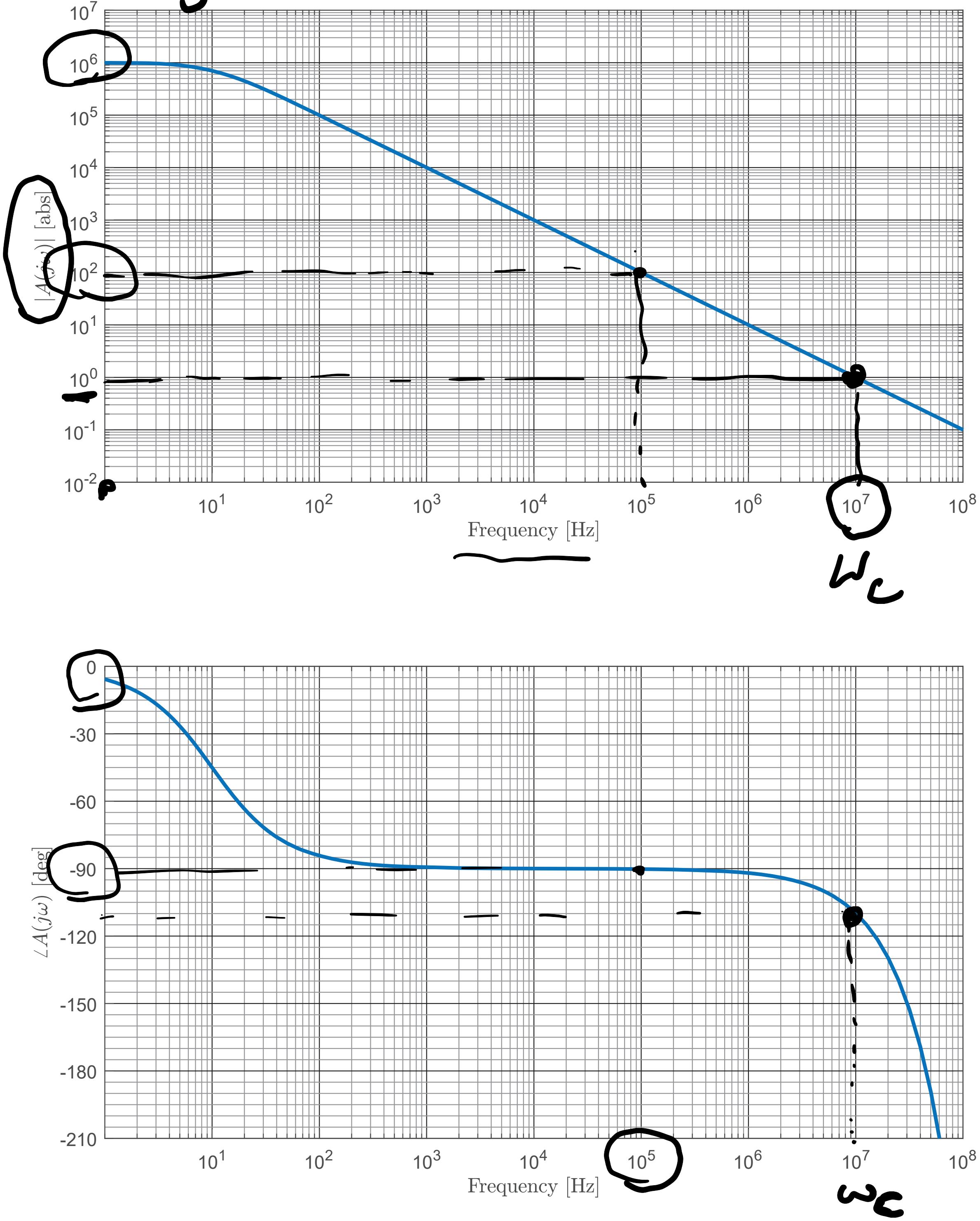


Figure 3: Bode plot of $A(s)$.