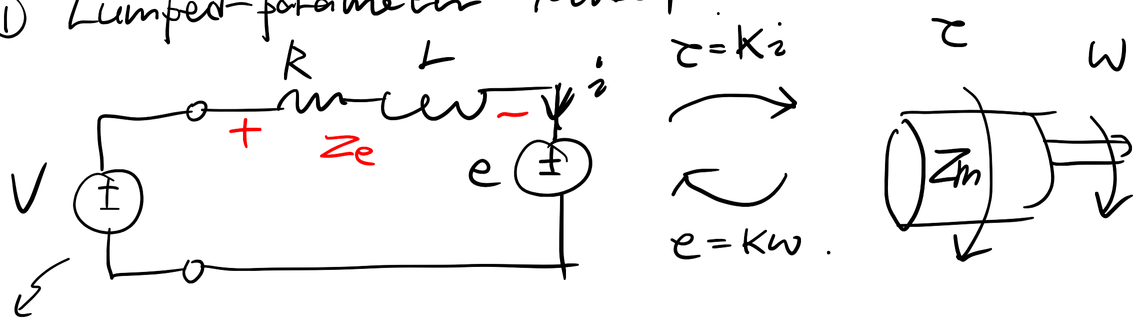
L8 – Voltage-controlled DC Motor

Modeling

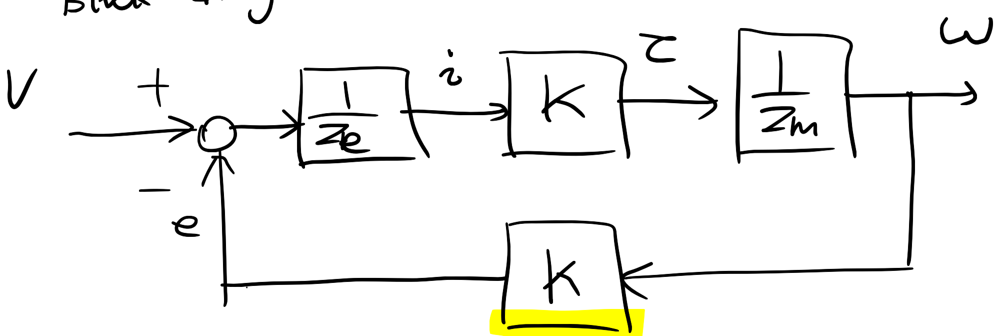
① Lumped-parameter Model



Voltage drive

$$L(s) = \frac{K^2}{Z_e \cdot Z_m}$$

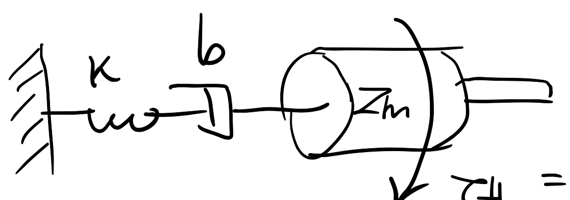
② Block diagram



"Feedback Loop"

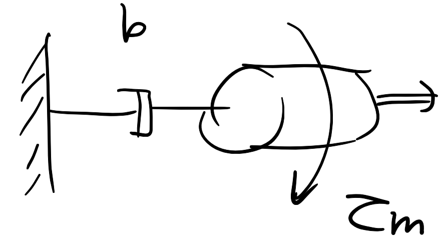
$$\frac{K^2}{R} = b$$

③ Equivalent Mech Model



$$\tau_{th} = \frac{K}{Ls+R} \cdot V$$

⇒



	Through	Across
Flow	i	v, ω
Effort	f, τ	V

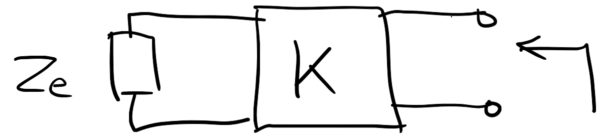
↑

Maxwell Variable

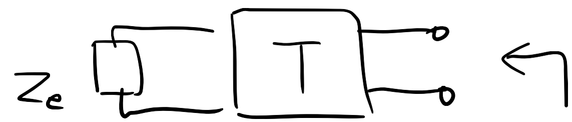
Gyrator

flow → effort

effort ← flow



$Z_m \overset{?}{=} \frac{K^2}{Z_e}$

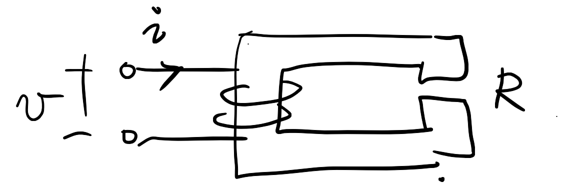


$Z_e' = T^2 \cdot Z_e$

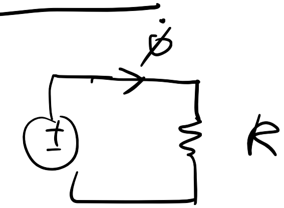
366

< Voltage-controlled dc motor. >

- $G_\omega \triangleq \frac{\omega}{V} = \dfrac{\frac{K}{Z_e Z_m}}{1 + L}$
- $G_\tau \triangleq \frac{\tau}{V} = \dfrac{\frac{K}{Z_e}}{1 + L}$



$v = N\dot{\phi}$



$F = Ni$

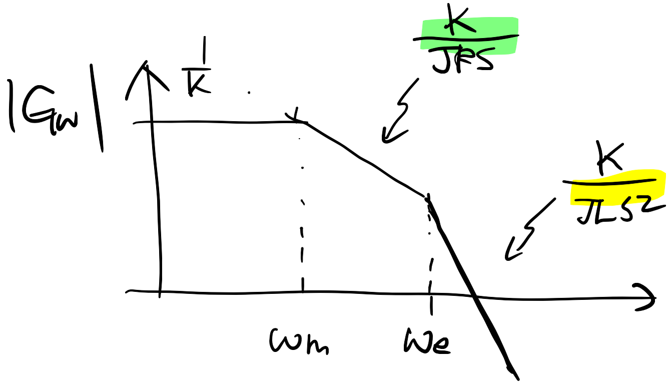
① Speed Resp : $G_\omega = \dfrac{\frac{K}{(Ls+R)Z_m}}{1 + \frac{K^2}{(Ls+R)Z_m}} = \dfrac{K}{Z_m(Ls+R) + K^2}$

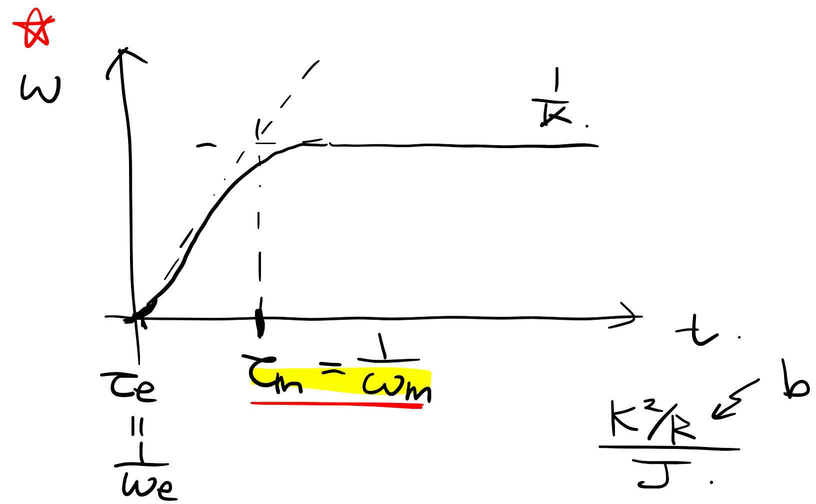
$Z_e = Ls + R$

when $Z_m = Js$. (free Inertia)

$$G_\omega = \frac{K}{Js(Ls+R)+K^2} = \frac{K}{JLs^2 + JRs + K^2}$$

$\omega_e = \frac{R}{L}$. $\omega_m = \frac{K^2}{JR}$. , $(\omega_m \ll \omega_e)$



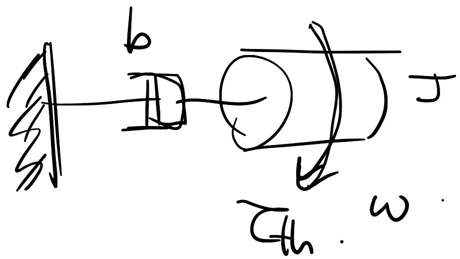


Example parameters.

$L = 1mH$ $R = 6\Omega$ $K = 200\ mNm/A$. $J = 2kg \cdot cm^2$.

$\omega_e \simeq 1kHz$ $\Downarrow$ $\omega_m \simeq 5Hz$ $\tau_e \simeq 1ms$. $\tau_m \simeq 200\ ms$.

· Mech Model



$$\omega = \frac{1}{Js+b}\,\tau_{th} = \left(\frac{1}{Js+\frac{K^2}{R}}\right)\left(\frac{K}{Ls+R}V\right)$$

$$= \frac{K}{JLs^2 + \left(JR + \cancel{\frac{K^2}{R}\cdot L}\right)s + K^2}\,V$$

$$\frac{K}{\underbrace{JLs^2 + JRs}\ \underbrace{+\,K^2}}$$

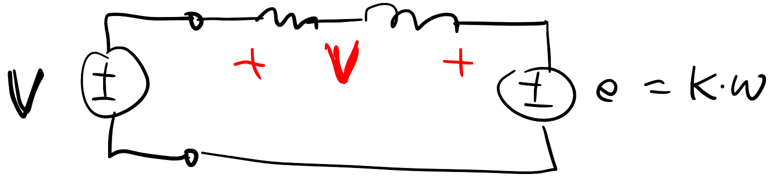
$$\frac{K^2}{R}\cdot L \ll JR \iff \omega_m \ll \omega_e$$

· Steady-state speed: ω = terminal velocity

①

↓

· From ①.



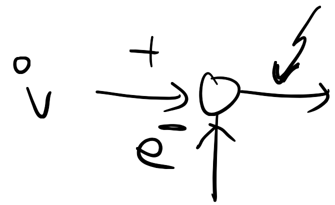
At the ss.

$\omega = \text{const} \rightarrow \tau = 0 \rightarrow i = 0.$

$\rightarrow$ Voltage across $Z_e = 0$.

$\rightarrow V = e = K\omega.$

$\Rightarrow \omega = \frac{V}{K}$,



$V - e = 0$.

⊙ Open-loop speed control with "voltage drive (source)" is common for simple tasks. (e.g. cooling fans).

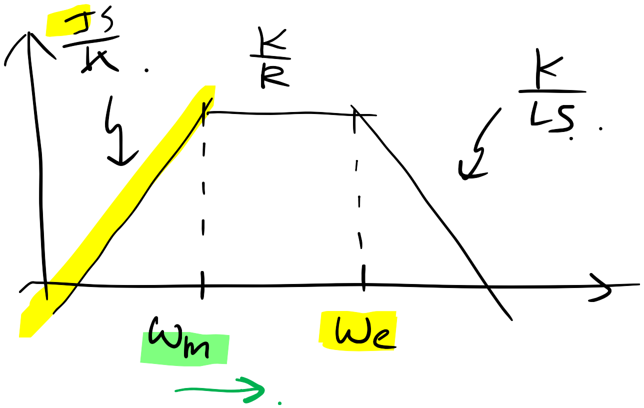
< Torque Response >.

- Torque = driving variable.
- High BW torque control, important for robotics.

\+

Very light limbs.

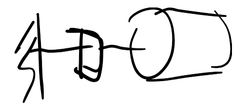
$$G_\tau = \frac{\tau}{V} = \frac{\frac{K}{Ls+R}}{1 + \frac{K^2}{(Ls+R)Js}} = \frac{K}{Ls+R+\frac{K^2}{Js}}.$$



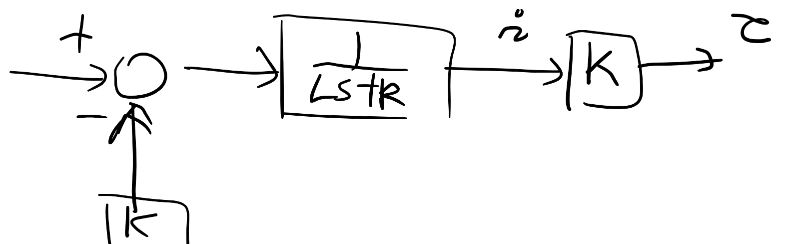
$\omega_m = \frac{K^2/R}{J}$. as $J \downarrow$. $\omega_m \uparrow$.

① Back-emf effect : ↓ low frq torques.

Mech : "apparent damping"



Elec : V



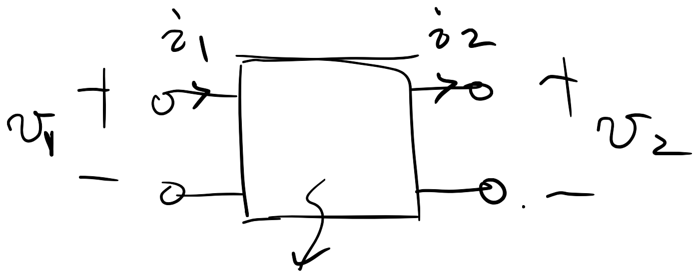
ω low frq. disturbance.

② . $\omega_e \simeq$ 1kHz

300kHz.

⇒ Voltage-controlled dc motor, not good for torque resp.

⇒ Current-controlled dc motor ⇒



$P_{in} = P_{out}$.

Lossless converter. { ① Trans.
② Gyra.

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \begin{bmatrix} N & \\ & Y \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & K \\ \frac{1}{K} & 0 \end{bmatrix}$$