University of British Columbia
Department of Mechanical Engineering

MECH 463. Final Exam, December 16, 2020

Allowed Time: 2hrs 30min

Materials admitted: Pen, pencil, eraser, straightedge, simple scientific calculator without programming or communication capabilities, personal hand-written notes within one letter-size sheet of paper (one side), timer and document copier apps on your phone (all other phone functionalities are **not** allowed). Matlab is **not** allowed.

There are 4 questions in this exam. You are asked to answer all questions.

The purpose of this test is to evaluate your knowledge of the course material. Orderly presentation demonstrates your knowledge most clearly. Marks are assigned accordingly. A bonus of up to 4 marks will be given for exemplary presentation.

Honour Code: You are asked to behave honourably during this exam and to obey all instructions carefully. Please write and sign the following promise in the space below: "I promise to work honestly on this exam, to obey all instructions carefully, and not to have any unfair advantage over any other students."

Promise:

Signed: Name:

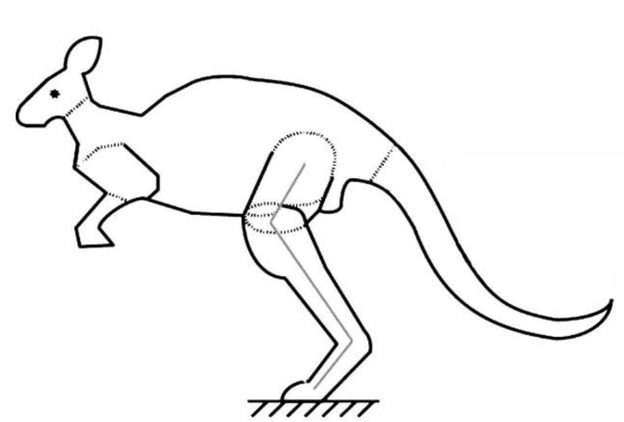
	Mark Received	**Maximum Mark**
1		8
2		12
3		9
4		11
Presentation		4 bonus
Total		40+4

Start Time

Finish Time

Name: ____

1. An engineer wishes to model the jumping behaviour of a kangaroo. As a first approximation, the kangaroo is modeled as a simple mass-spring system, where the body has a mass m and the legs have stiffness k. The kangaroo jumps once. While in the air its legs are relaxed, corresponding to an unstretched spring. The kangaroo lands vertically on the ground with velocity v. Unfortunately for the kangaroo, it had jumped into some sticky mud and its feet got firmly stuck, so preventing it from rebounding.



   (a) Draw a labeled free-body diagram of the "mass" part of the mass-spring model of the kangaroo after contact with the ground. *(Hint: Choose a coordinate system with zero datum corresponding to the unstretched length of the spring. This is not the equilibrium position, so consider accordingly.)*

   (b) Use your free-body diagram to formulate the equation of motion.

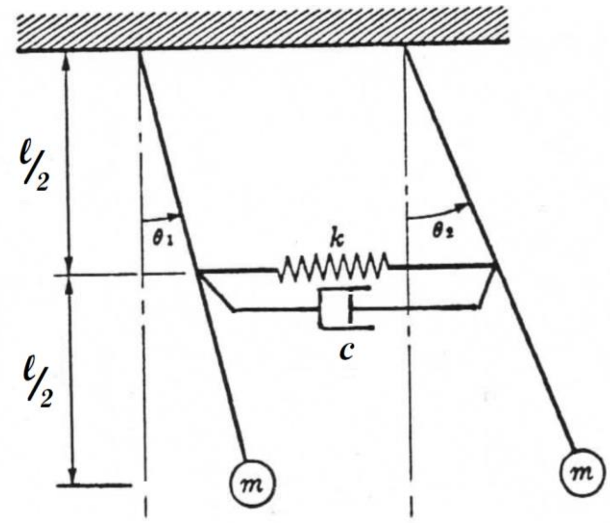
   (c) Solve for the vibration response of the kangaroo, assuming that it is passive and does not use its leg muscles to control its motion.

   (d) Derive a formula for the amplitude of the subsequent vibration. Sketch a graph of the motion and fully label all significant features.

   Note: *No kangaroos were harmed during the formulation of this question.* ☺

Name: ____

2. Two simple pendulums consist of light rigid rods of length ℓ, with masses m at their lower ends. The two rods are connected together at their mid-points through a spring of stiffness k and a dashpot of rate c. You may assume that the vibrations are small.



   (a) Formulate the equations of motion in matrix form using *Lagrange's Equations*.

   (b) Comment on any notable features of your equations.

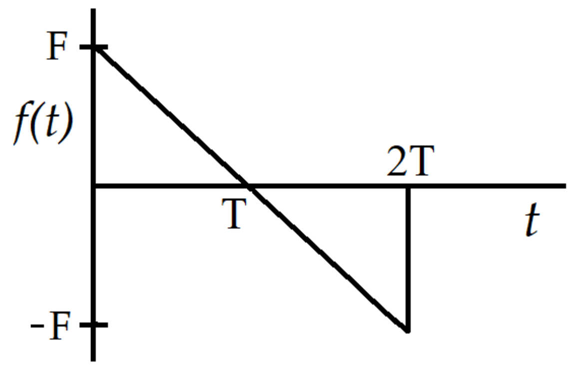
   (c) Determine the undamped natural frequencies and mode shapes.

   (d) Transform your equations into principal coordinates.

   (e) Determine the damping ratios (assume underdamped). Comment on your results.

Name: ____

3. The force history f = f(t) shown in the diagram acts on a simple mass-spring system.



   (a) Derive a formula for the vibration response x of the mass for zero initial conditions during the time interval $0 < t < 2T$.

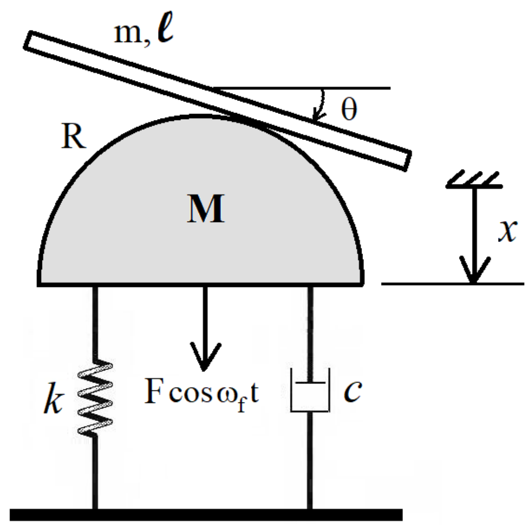
   (b) Derive a formula for the vibration response x of the mass for times $t > 2T$.

   (c) Sketch the overall response (both before and after $t = 2T$). For pictorial clarity, you may assume in your sketch that the natural frequency ω is significantly higher than π/T.

   (d) For the special case where $\omega = \pi/T$, what are the amplitudes of the oscillatory part of the response before and after $t = 2T$?

Name: ______________________________________

4. A rod of mass m, length ℓ and centroidal moment of inertia $m\ell^2/12$ rests symmetrically on the top of a half-cylinder of mass M and radius R. The rod can rock from side to side without slipping. The half-cylinder is supported by a spring of stiffness k and a damper of rate c, and is mounted on a frictionless slider (not shown in the diagram), allowing it to move vertically up and down, but not to rotate. A vertical harmonic force $f(t) = F \cos \omega_f t$ acts on the half-cylinder, as shown.



(a) Draw and label in detail a diagram of the geometry of the rod when it is displaced by an angle θ from the central position. Write formulas for the various dimensions that will be needed for subsequent calculations.

(b) Draw free-body diagrams of the rod and of the (rod + half-cylinder). (Combining the two parts avoids having to handle the contact forces). *(Hint: choose the x coordinate with zero datum corresponding to the unstretched length of the spring. This is not the equilibrium position, so consider accordingly.)*

(c) Use your FBDs to formulate the equations of motion. The vibrations are small, so you need consider only first-order terms.

(d) Determine the natural frequencies, damping factors and mode shapes of the vibrating system. Comment on these results

(e) Derive formulas for the vibration response. Comment on these results.

(f) Sketch graphs of the forced vibration amplitudes of x and θ vs. forcing frequency ω_f.