THE UNIVERSITY OF BRITISH COLUMBIA
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 420 SENSORS AND ACTUATORS

Final Examination
09 December 2020, 8:30 am (Vancouver time)
Required Time: 90 minutes
Allowed Time: Until 12:00 noon on 09 December (Vancouver time)

Please submit your answer script as three separate pdf files, one pdf file for each question/answer. Please submit the two files containing the answers to Question 1 and Question 2 to both TAs (jing@ece.ubc.ca; willyangcy@hotmail.com). Please submit the file containing the answer to Question 3 to the Course Instructor (desilva@mech.ubc.ca). We must receive these three files no later than 12:00 noon (Vancouver time) today (09 November 2020)
Please write your name and the student number. Also, number the pages of the three files, separately. Check your pages. Corrections are not allowed after submission.

Further Instructions:

- **Open Book/Notes**
- **The submitted answer scripts should be absolutely your own individual contribution. Do not copy and paste from elsewhere**
- **Do not help others to do their exam. Do not get help from others to do your exam. Both are academic misconduct, which have serious consequences**
- **If any part of your submitted material has issues after we grade it, I will arrange a personal Zoom meeting with webcam at both ends. Your final grade for the exam will depend on that discussion**
- **Calculators are allowed**
- **Fully answer all three questions for full credit**
- **Clearly state all your assumptions and give all your steps of any derivations**
- **Define any new variables or parameters that you may use**
- **This question paper contains seven (7) pages including this cover sheet**
- **Extra time is provided to answer the exam, due to Covid-19. Late submissions are not accepted**
- **Your handwriting should be very clear and clean. If not, the unclear parts will be disregarded.**

Question 1

Consider the sensing and data acquisition arrangement shown in Figure 1. The temperature T of an object is monitored by using an RTD (resistance temperature detector). The resistance R_r of the RTD forms one branch of the Wheatstone bridge, and it changes with temperature. The other three branches of the bridge have identical resistance R, which does not change with temperature. The bridge output (voltage) v_s is modified by an analog circuit. The voltage output from this circuit is sampled and digitized by an analog-to-digital converter (ADC), and read into a digital computer to process that data (for further action such as performance assessment, fault diagnosis, control).

The resistance of the RTD obeys the equation $R_r = R_{r0}(1+\alpha\Delta T)$

where,

R_r = resistance of the RTD, in ohms (Ω), at temperature T (in °K)

R_{r0} = resistance of the RTD at temperature T_0 (the starting temperature, in °K)

$\Delta T = T - T_0$ = temperature rise (in °C)

α = temperature coefficient of RTD resistance (in /°C)

Assume: The starting temperature (the ambient temperature) T_0 is known and fixed. The corresponding RTD resistance R_{r0} equals R, the bridge completion resistance, which is also fixed.

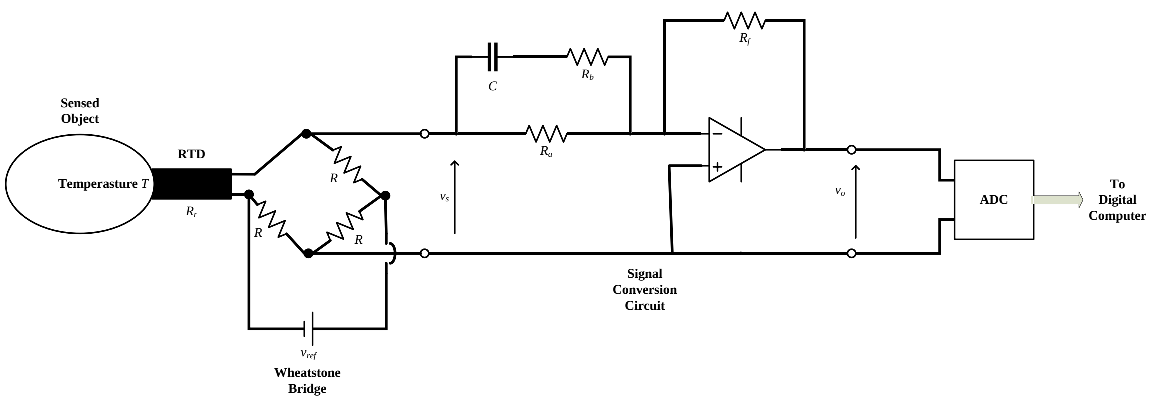

Figure 1: Temperature monitoring system for an object.

(i)

Giving the details of all the necessary steps, determine a "linear" equation relating the bridge output voltage v_s to the temperature change ΔT of the object (in terms of the bridge supply voltage (dc) v_{ref} and α.

(05%)

(ii)

Giving the details of all the necessary steps, derive the input-output differential equation of the analog circuit (signal conversion circuit) in Figure 1, in terms of the circuit parameters: resistances R_a, R_b, and R_f, and capacitance C.

v_s = input voltage to the circuit (i.e., output of the sensor)
v_o = output voltage of the circuit

Notes:

1. The + lead of the op-amp is not grounded. Do not assume that potential there to be zero.
2. Your derivation has to be entirely in the time domain (not Laplace domain).

(20%)

Question 2

(a)

Consider the analog circuit that is connected to an analog sensor, as shown in Figure Q2. Systematically, giving all the details, obtain the input-output differential equation for the analog circuit in terms of the shown parameters (capacitances and resistances) of the circuit.

v_s = input voltage to the circuit (output of the sensor)
v_o = output voltage of the circuit

Note: Work entirely in the time domain, not in the Laplace domain.

(15%)

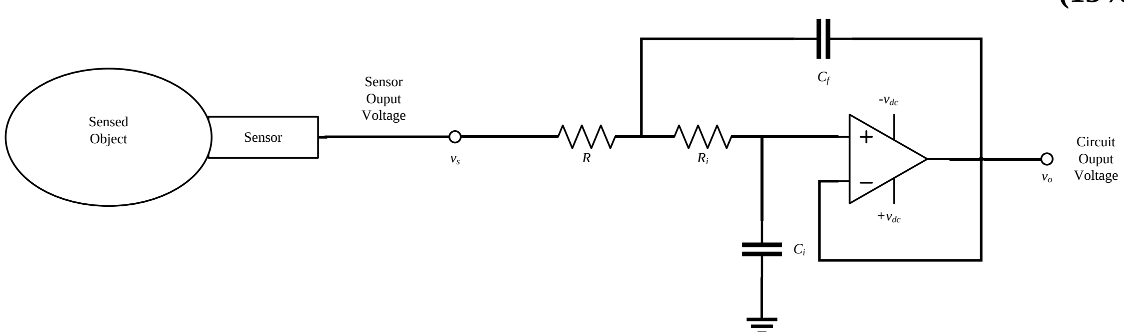

Figure Q2: An analog circuit connected to an analog sensor.

(b)

Convert the differential equation in Part (a) into a transfer function.

Next, assume that $(1+\frac{R}{R_i})^2 R_i C_i - 4RC_f = 0$

Or, if that is too difficult for you to do, assume that the transfer function has all repeated (i.e., identical) poles.

Write an expression for the bandwidth (i.e., half-power bandwidth, corner frequency, or break frequency) of the circuit in terms of the indicated circuit parameters. Use this result for all subsequent work in this question.

(10%)

(c)

Suppose that the time constant of the analog sensor is τ_s. In order to ideally match the sensor to the given analog circuit, determine an equation relating τ_s to the indicated circuit parameters.

(05%)

(d)

Now use the numerical values: $\tau_s = 2.0$ ms, $R = 1.0$ kΩ, and $R_i = 10.0$ kΩ.

In order to achieve the ideal component matching, as obtained in Part (c), determine a suitable numerical value for the capacitance C_f.

What should be a suitable sampling frequency for the data (output) from the analog circuit into a digital device?

(05%)

<u>Question 3</u>

A bucket full of material is raised from the ground level to the worksite, as schematically shown in Figure Q3(a). A dc motor with a step-down gear transmission and a pulley unit connected to the gear are used in the system. The loaded bucket starts from rest, is steadily accelerated to the top speed $v_{\max}$ (at constant acceleration) in the fixed time period T, that speed is maintained through much of the height, and is steadily decelerated to rest at the elevated worksite at the same rate as the acceleration. The corresponding speed profile is shown in Figure Q3(b). Once the material is unloaded, the empty bucket is lowered to the ground using an identical speed profile (*Hint*: The lowering of the bucket does not particularly concern the present question, and may be ignored).

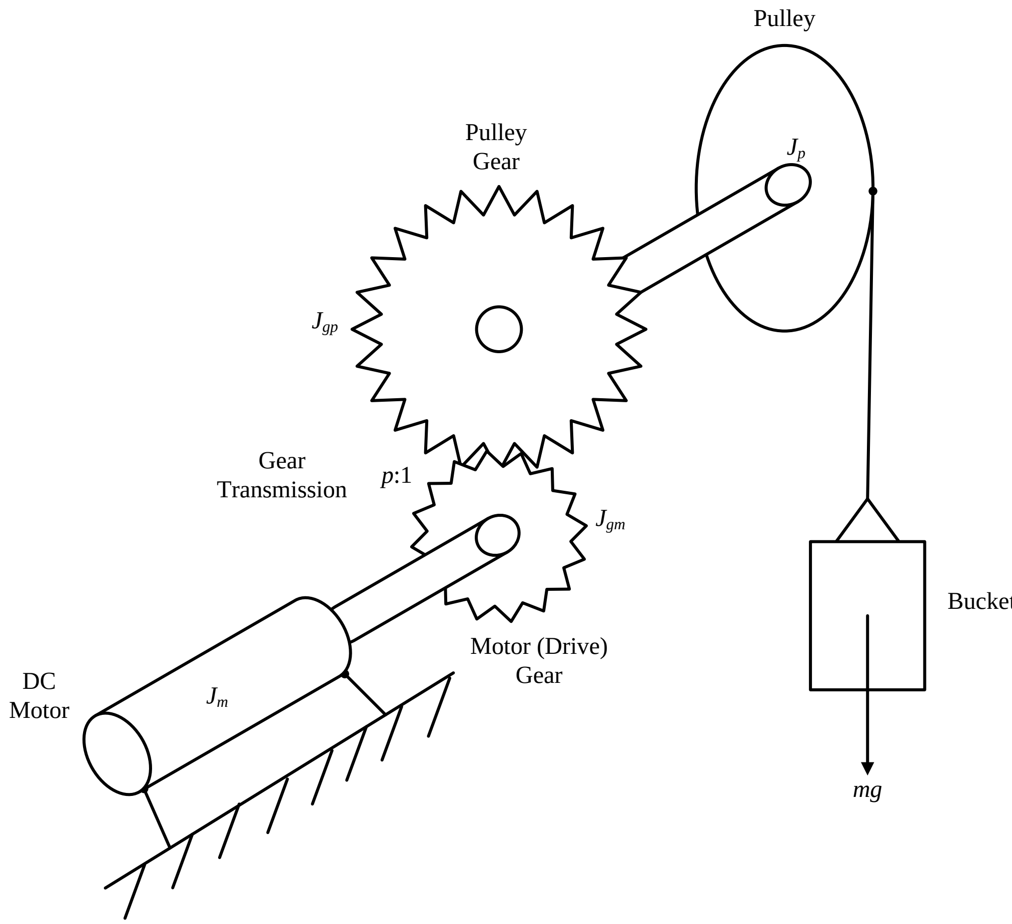

Figure Q3(a): Material transfer system for an elevated worksite.

Note that a step-down (speed reducing) gear unit and a dc motor both are used in the system, and we cannot make alternative types of choices. The following system parameters are defined:

J_m = moment of inertia of the motor rotor
J_{gm} = moment of inertia of the gear wheel connected to the motor shaft
J_{gp} = moment of inertia of the gear wheel connected to the pulley shaft
J_p = moment of inertia of the pulley (assumed constant)
m = overall mass of the bucket with a full load of material (the load)
r_p = effective radius of the pulley (assumed constant)
p = stepdown speed ratio of the gear (i.e., p = motor gear speed/pulley gear speed)

(a) In terms of the defined parameters, derive an expression for the equivalent overall moment of inertia, J_e that is felt at the motor shaft, as the entire system operates.

(05%)

(b) Giving all necessary details, determine the torque equation (using Newton's 2nd law) for the system, with reference to the motor, which is all that is needed to determine the overall torque felt at the motor shaft, for selecting a matching motor.

(25%)

(c) The speed profile of the conveyor has a steadily accelerating start-up segment, a constant-speed segment, and a steadily decelerating segment for stopping the load, as shown in Figure Q3(b). Numerical values for the key parameters of the speed profile are also shown in the figure.

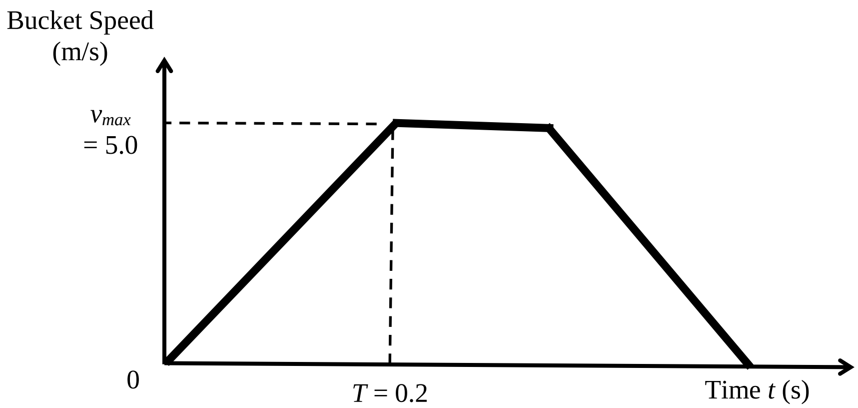

Figure Q3(b): Speed profile of the bucket.

Three models of dc motor (models 1, 2, and 3) are available for this application. Their pull-out curves (for repeated operation) are shown in Figure Q3(c). The moments of inertia of the rotors of the motor models 1, 2, and 3 are: $J_{m1} = 6.0 \times 10^{-2}$ kg. m^2, $J_{m2} = 8.0 \times 10^{-2}$ kg. m^2, and $J_{m3} = 11.0 \times 10^{-2}$ kg. m^2, respectively. Also, the following parameter values are known for the system: $r_p = 0.40$ m, $p = 2$, $J_{gm} = 4.0 \times 10^{-2}$ kg.m^2, $J_{gp} = 7.0 \times 10^{-2}$ kg.m^2, $J_p = 15.0 \times 10^{-2}$ kg.m^2 (constant), and $m = 15.0$ kg.

Giving all necessary details, select a suitable motor from the provided three models, for the present application.

Note: Assume an overall efficiency of 80% for the entire system (including the motor and the gear transmission), regardless of the used model of the motor.

(10%)

Note: If none of the given three motor models are suitable for the present application, you have to give the reasons for that, in detail.

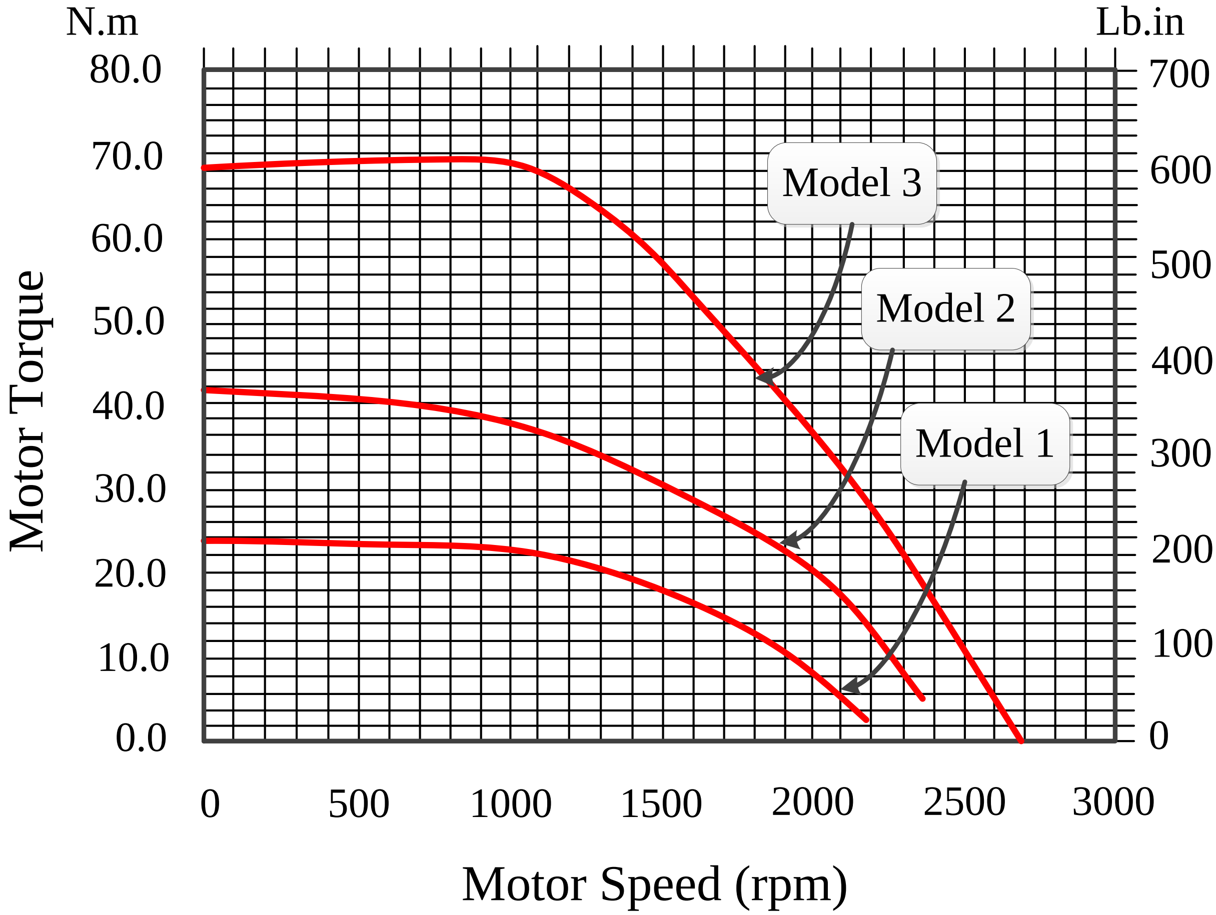

Figure Q3 (c): Pull-out curves of the three motors for repeated operation.