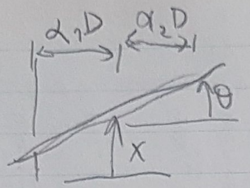
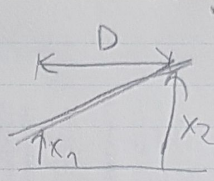
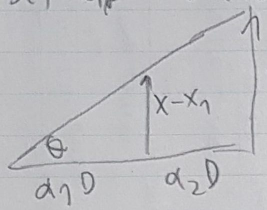
Prelab (1): [I did this before the lectures so I didn't use the "redraw FBD" method.]

→ 

⇒ 

→ Set up relationship:



→ $\tan\theta = \dfrac{x_2-x_1}{D}$ (1)

$\dfrac{x_2-x_1}{D} = \dfrac{x-x_1}{\alpha_1 D}$ (2)

→ (1) At small θ, $\tan\theta \approx \theta$

$\tan\theta \approx \theta = \dfrac{x_2-x_1}{D}$ → $D\theta = x_2-x_1$

$\ddot{\theta} = \dfrac{\ddot{x}_2-\ddot{x}_1}{D}$ → $D\ddot{\theta} = \ddot{x}_2-\ddot{x}_1$

(2)

$\dfrac{x_2-x_1}{D} = \dfrac{x-x_1}{\alpha_1 D}$

$\alpha_1(x_2-x_1) = x-x_1$

$x = \alpha_1 x_2 + (1-\alpha_1)x_1$

$= \alpha_1 x_2 + \alpha_2 x_1$

$\ddot{x} = \alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1$

→ Sub these equations into EQ6 from lab manual

$$\rightarrow \quad 0 = m(\alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2}\left(\left(\frac{L_1}{\alpha_2}+\frac{L_2}{\alpha_1}\right)(\alpha_1 x_2 + \alpha_2 x_1) + (L_1 - L_2)(x_2 - x_1)\right)$$

(3)

$$\frac{L_1}{\alpha_2}+\frac{L_2}{\alpha_1} \downarrow \frac{\alpha_1 L_1 + \alpha_2 L_2}{\alpha_2\alpha_1}$$

$$0 = \frac{mR^2}{D^2}(\ddot{x}_2 - \ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2}\left((L_1-L_2)(\alpha_1 x_2 + \alpha_2 x_1) + (\alpha_2 L_1 + \alpha_1 L_2)(x_2 - x_1)\right)$$

(4)

→ Simplify

(3) $$0 = \cancel{m}(\alpha_1\ddot{x}_2 + \alpha_2\ddot{x}_1) + \frac{\cancel{m}g\alpha_1\alpha_2}{L_1L_2}\left(\frac{\alpha_1L_1+\alpha_2L_2}{\alpha_2}x_2 + \frac{\alpha_1L_1+\alpha_2L_2}{\alpha_1}x_1 + L_1x_2 - L_2x_2 - L_1x_1 + L_2x_1\right)$$

$$0 = \alpha_1\ddot{x}_2 + \alpha_2\ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_1L_2}\left(\frac{\alpha_1L_1}{\alpha_2}x_2 + \cancel{L_2x_2} + \cancel{L_1x_1} + \frac{\alpha_2L_2}{\alpha_1}x_1 + L_1x_2 - \cancel{L_2x_2} - \cancel{L_1x_1} + L_2x_1\right)$$

$$0 = \alpha_1\ddot{x}_2 + \alpha_2\ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_1L_2}\left(\frac{\alpha_1}{\alpha_2}+\frac{\alpha_2}{\alpha_2}\right)L_1x_2 + \frac{g\alpha_1\alpha_2}{L_1L_2}\left(\frac{\alpha_2}{\alpha_1}+\frac{\alpha_1}{\alpha_1}\right)L_2x_1$$

$$0 = \alpha_1\ddot{x}_2 + \alpha_2\ddot{x}_1 + g\frac{\alpha_1}{L_2}\underbrace{(\alpha_1+\alpha_2)}_{1}x_2 + g\frac{\alpha_2}{L_1}\underbrace{(\alpha_2+\alpha_1)}_{1}x_1$$

$$0 = \alpha_1\ddot{x}_2 + \alpha_2\ddot{x}_1 + g\frac{\alpha_1}{L_2}x_2 + g\frac{\alpha_2}{L_1}x_1$$

(4) $$0 = \frac{\cancel{m}R^2}{D^2}(\ddot{x}_2 - \ddot{x}_1) + \frac{\cancel{m}g\alpha_1\alpha_2}{L_1L_2}\left(L_1\alpha_1x_2 + L_1\alpha_2x_1 - L_2\alpha_1x_2 - L_2\alpha_2x_1 + L_1\alpha_2x_2 - L_1\alpha_2x_1 + L_2\alpha_1x_2 - L_2\alpha_1x_1\right)$$

$$0 = \frac{R^2}{D^2}(\ddot{x}_2 - \ddot{x}_1) + \frac{g\alpha_1\alpha_2}{L_1L_2}\left(L_1\underbrace{(\alpha_1+\alpha_2)}_{1}x_2 - L_2\underbrace{(\alpha_2+\alpha_1)}_{1}x_1\right)$$

$$0 = \frac{R^2}{D^2}\ddot{x}_2 - \frac{R^2}{D^2}\ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_2}x_2 - \frac{g\alpha_1\alpha_2}{L_1}x_1$$

→ To take simplified (3) & (4) and get diagonal matrix, we need to combine (3) & (4) in these ways to get eq. (5) & (6):

$$(5) = \alpha_1 (3) + (4)$$
$$(6) = -\alpha_2 (3) + 4$$

$$(5)\quad 0 = \alpha_1^2 \ddot{x}_2 + \alpha_1\alpha_2 \ddot{x}_1 + g\frac{\alpha_1^2}{L_2} x_2 + \cancel{g\frac{\alpha_2\alpha_1}{L_1} x_1}$$
$$+ \frac{R^2}{D^2}\ddot{x}_2 - \frac{R^2}{D^2}\ddot{x}_1 + g\frac{\alpha_1\alpha_2}{L_2} x_2 - \cancel{g\frac{\alpha_1\alpha_2}{L_1} x_1}$$

$$0 = \left(\frac{R^2}{D^2} + \alpha_1^2\right)\ddot{x}_2 + \left(\frac{-R^2}{D^2} + \alpha_1\alpha_2\right)\ddot{x}_1 + \left(\frac{g\alpha_1}{L_2}\right) x_2$$

$$(6)\quad 0 = -\alpha_1\alpha_2 \ddot{x}_2 - \alpha_2^2 \ddot{x}_1 - \cancel{g\frac{\alpha_1\alpha_2}{L_2} x_2} - g\frac{\alpha_2^2}{L_1} x_1$$
$$+ \frac{R^2}{D^2}\ddot{x}_2 - \frac{R^2}{D^2}\ddot{x}_1 + \cancel{g\frac{\alpha_1\alpha_2}{L_2} x_2} - g\frac{\alpha_1\alpha_2}{L_1} x_1$$

$$0 = \left(\frac{R^2}{D^2} - \alpha_1\alpha_2\right)\ddot{x}_2 + \left(\frac{-R^2}{D^2} - \alpha_2^2\right)\ddot{x}_1 + \left(\frac{-g\alpha_2}{L_1}\right) x_1$$

→ combining (5) & (6) into matrixes, we get:

$$\begin{bmatrix} \frac{R^2}{D^2} + \alpha_1^2 & \frac{-R^2}{D^2} + \alpha_1\alpha_2 \\ \frac{-R^2}{D^2} + \alpha_1\alpha_2 & \frac{R^2}{D^2} + \alpha_2^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_2 \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} \frac{g\alpha_1}{L_2} & 0 \\ 0 & \frac{g\alpha_2}{L_1} \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = 0$$

→ To get the same plot as mass coords, we have to flip the equations order. Matrixes become:

$$\begin{bmatrix} \frac{R^2}{D^2} + \alpha_2^2 & \frac{-R^2}{D^2} + \alpha_1\alpha_2 \\ \frac{-R^2}{D^2} + \alpha_1\alpha_2 & \frac{R^2}{D^2} + \alpha_1^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} g\frac{\alpha_2}{L_1} & 0 \\ 0 & g\frac{\alpha_1}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

For my plots, I replaced:

```
M = [[m 0]' [0 m*R^2/D^2]'];
K = m*g*a1*a2/L1/L2 * [[L1/a2+L2/a1 L1-L2]' ...
                       [L1-L2 a2*L1+a1*L2]'];
```

with:

```
M = [[R^2/D^2+a2^2 -R^2/D^2+a1*a2]' [-R^2/D^2+a1*a2
R^2/D^2+a1^2]'];
K = [[g*a2/L1 0]' [0 g*a1/L2]'];
```

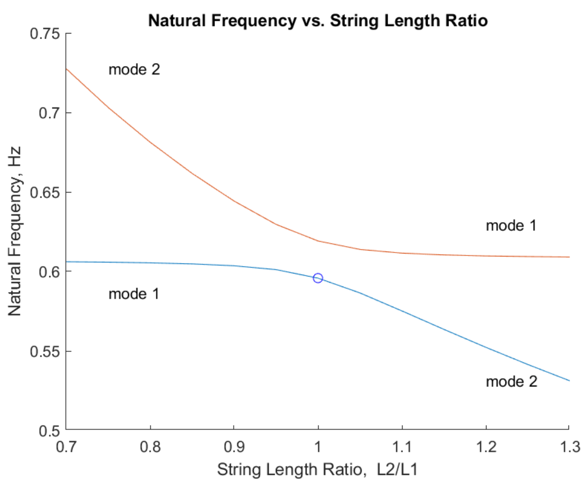
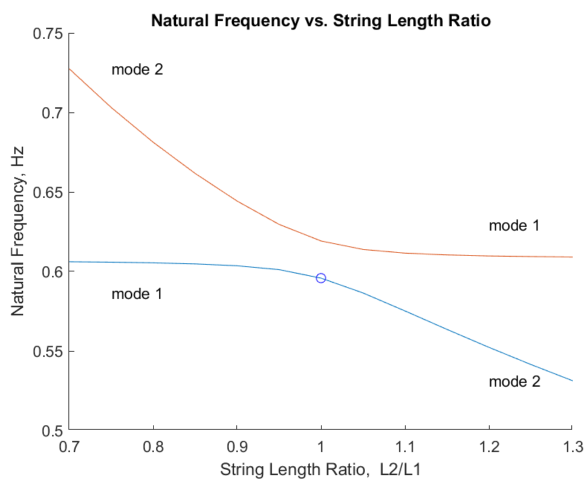
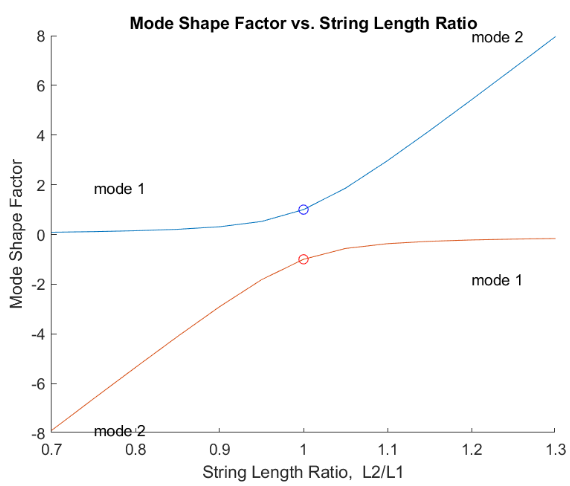
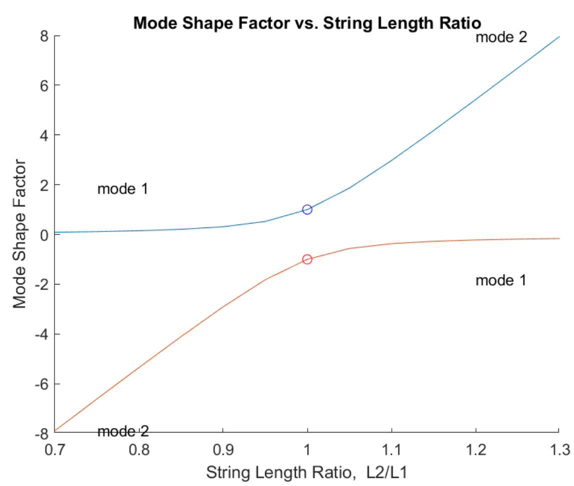
and replaced:

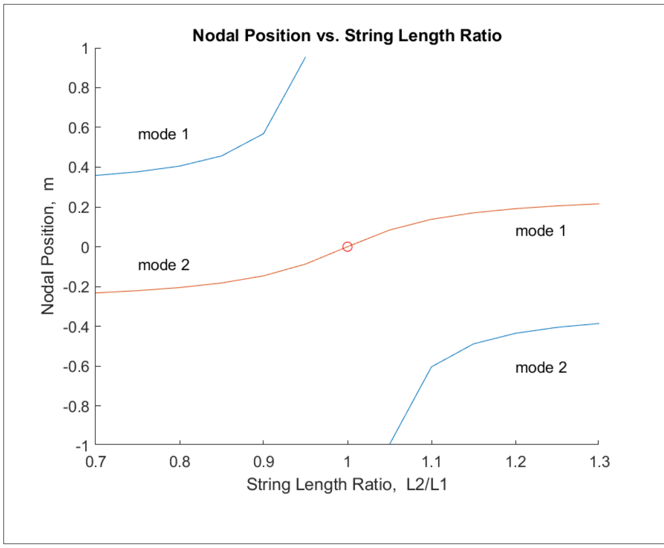
```
Vx = [[1 1]' [-a1 a2]'] * V;
```

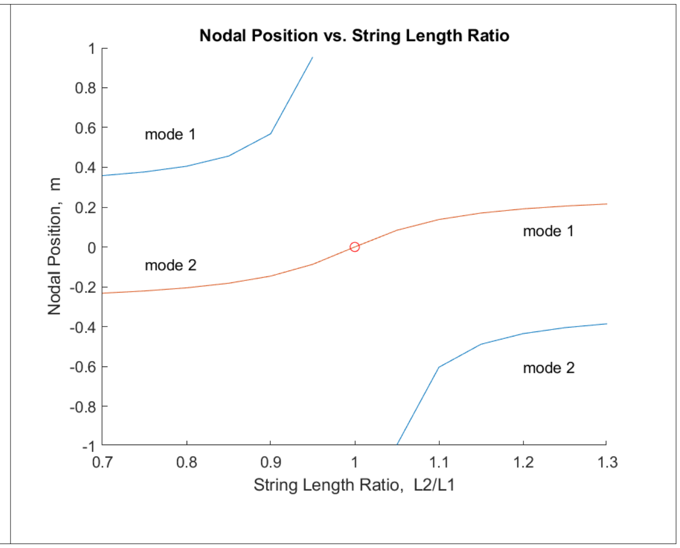
with:

```
Vx = V;
```

These are the plots comparison:

Mass	String
	
	


Nodal Position vs. String Length Ratio
Nodal Position, m
String Length Ratio, L2/L1
mode 1
mode 2
mode 1
mode 2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.7
0.8
0.9
1
1.1
1.2
1.3


Nodal Position vs. String Length Ratio
Nodal Position, m
String Length Ratio, L2/L1
mode 1
mode 2
mode 1
mode 2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.7
0.8
0.9
1
1.1
1.2
1.3