THE UNIVERSITY OF BRITISH COLUMBIA
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 420 SENSORS AND ACTUATORS

Mid-Term Examination
06 November 2020, 3:00 pm (Vancouver time)
Required Time: 50 minutes
Allowed Time: Until 5:00 pm on 06 November (Vancouver time)

Please submit your answer script (as a pdf file or scanned pages) to both TAs (jing@ece.ubc.ca; willyangcy@hotmail.com) no later than 5:00 pm (Vancouver time) today (06 November 2020)
Please write your name and the student number. Also, number the pages.

Further Instructions:

- **Open Book/Notes.**
- **The submitted answer script should be absolutely your own individual contribution. Do not copy and paste from elsewhere**
- **Do not help others to do their exam. Do not get help from others to do your exam. Both are academic misconduct, which have serious consequences**
- **If any part of your submitted material has issues, I will arrange a personal Zoom meeting with webcam at both ends. Your final grade for the exam will depend on that discussion**
- **Calculators are allowed**
- **Fully answer both questions for full credit**
- **Clearly state all your assumptions and give all your steps of any derivations**
- **Define any new variables or parameters that you may use**
- **This question paper contains four (4) pages including this cover sheet**
- **Extra time is provided to answer the exam, due to Covid-19. Late submissions are not accepted**
- **Your handwriting should be very clear and clean. If not, the unclear parts will be disregarded.**

Question 1

Consider the sensing and data acquisition arrangement shown in Figure 1. The motion of an object is monitored by using a speed sensor. The sensor output (voltage) v_s is modified by an analog circuit. The voltage output from this circuit is sampled and digitized by an analog-to-digital converter (ADC), and read into a digital computer to process that data (for further action such as performance assessment, fault diagnosis, control).

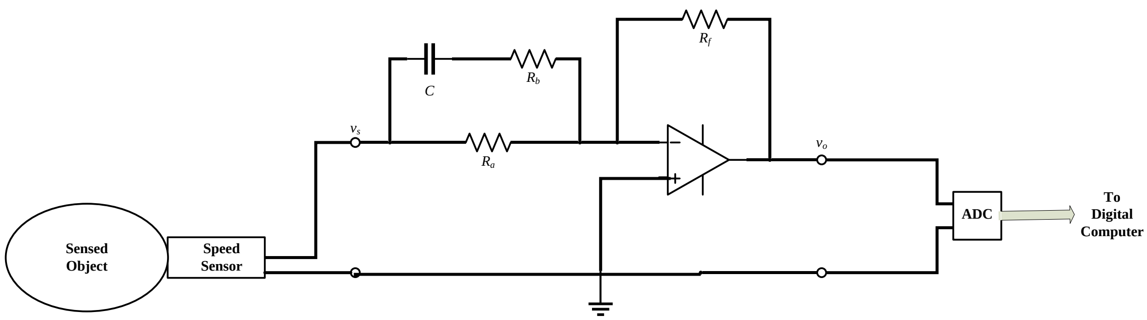

Figure 1: Speed monitoring of an object.

(i)

Giving the details of all the necessary steps, derive the input-output differential equation of the analog circuit in Figure 1, in terms of the circuit parameters: resistances R_a, R_b, and R_f, and capacitance C.

Circuit input voltage (sensor output) = v_s

Circuit output voltage (ADC input) = v_o

Obtain its transfer function $G(s)$ of the analog circuit.

Sketch, along with the asymptotes, the Bode magnitude curve and the Bode phase angle curve (not to scale, the general shape only) of this transfer function, while explaining the reasons behind the nature of the shown asymptotes lines.

(25%)

(ii)

By examining the nature of the circuit transfer function (the analytical expression and the Bode curves), determine expressions for the following quantities, in terms of the circuit parameters:

(a) Steady-state gain (amplification) provided by the circuit
(b) High-frequency gain (amplification) provided by the circuit
(c) The phase "lead" angle (in radians) provided by the circuit
(d) Break points (frequencies, in radians/s) of the Bode plot

Indicate three benefits that this circuit provides for the overall measurement system, in different frequency ranges.

(20%)

(iii)
The following numerical values are given:

The maximum useful voltage that the sensor can generate (the full-scale voltage) is 2.0 V, and this happens when the measured speed is 100.0 cm/s.

The ADC has 8 bits. It generates its maximum count for its full-scale input 6.0 V.

The analog circuit parameters: R_a = 10.0 kΩ, R_b = 2.0 kΩ and C = 2.0 μ F

(a) What should be the suitable value for the feedback resistor R_f in order to realize the best sensitivity for the overall arrangement (consisting of the sensor, analog circuit and the ADC)? What is that overall sensitivity (in counts/cm/s)?

(10%)

Note: Use this value of R_f for all subsequent computations.

(b) What would be three suitable frequency ranges of operation for the system? Under what operating conditions or requirements is each of these frequency ranges suitable? What should be a satisfactory sampling rate (of data into the computer from the ADC) in each frequency range of operation?

(15%)

<u>Question 2</u>

A thermistor (a temperature sensor) has the empirical relation, relating its resistance R (in Ω) and the measured temperature T (in ºK), given by $R = R_o \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_o}\right)\right]$.

Here, β is a positive quantity. Assume that it is known very accurately.

(i)
Sketch this characteristic curve (R versus T) for the thermistor.
Where possible, with proper justification, indicate the values (analytical expressions) at key points on the curve, in terms of the empirical parameters R_o, T_o and β .

Giving reasons, determine the relationship between R_o and T_o .

Note: β is called the “characteristic temperature” of the thermistor.

(15%)

(ii)

In a typical sensing procedure, the resistance R is measured and the above equation (thermistor model) is used to compute the corresponding temperature T. There is measurement error in R and model error in R_o. Assume that β is known accurately.

(a) Using the "absolute error" method, derive an equation for the combined fractional error e_T in the temperature (estimation) in terms of the fractional errors e_R and e_{R_o} of R and R_o, respectively. From this relation, show that a larger T will result in larger fractional error in the determined temperature.

(10%)

(b) Suppose that $e_{R_o} = \pm 0.02$ and at temperature 400 ºK, $e_R = \pm 0.01$. What is the fractional error e_T in the determined (estimated) temperature?

Given: $R_o = 5000\ \Omega$ at $T_o = 298°\text{K}$ (i.e., 25°C) and $\beta = 4200°\text{K}$.

(5%)