MECH 463
Sept 24

Lecture 6

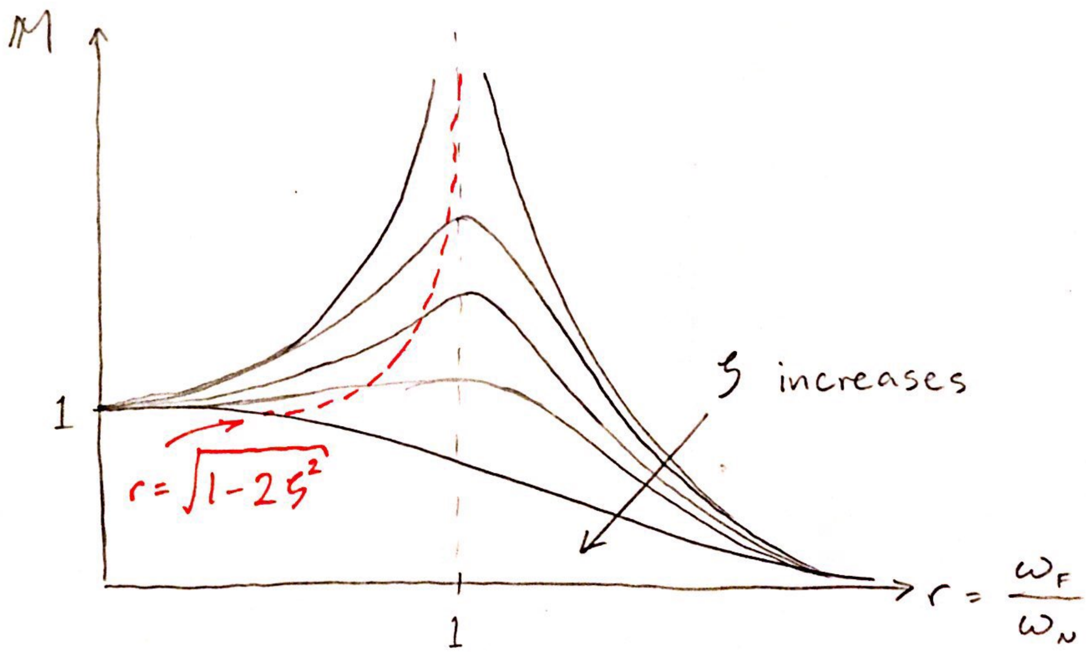
Damped Forced Vibration

$$x = \frac{(F/k)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \cos(\omega_F t + \phi_F)$$

where $\tan \phi_F = -\dfrac{2\zeta r}{1-r^2}$ (the (-) represents phase lag)

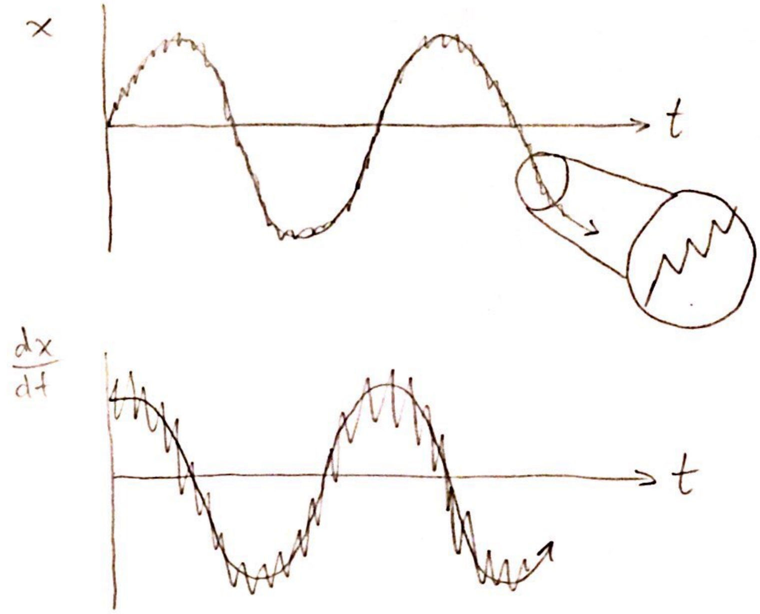
Also, $F/k = X_0$, static deflection

<u>Magnification factor</u> $M = \dfrac{X}{X_0} = \dfrac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$.



Damped frequency $\omega_D = \sqrt{1-\zeta^2}\,\omega_N$

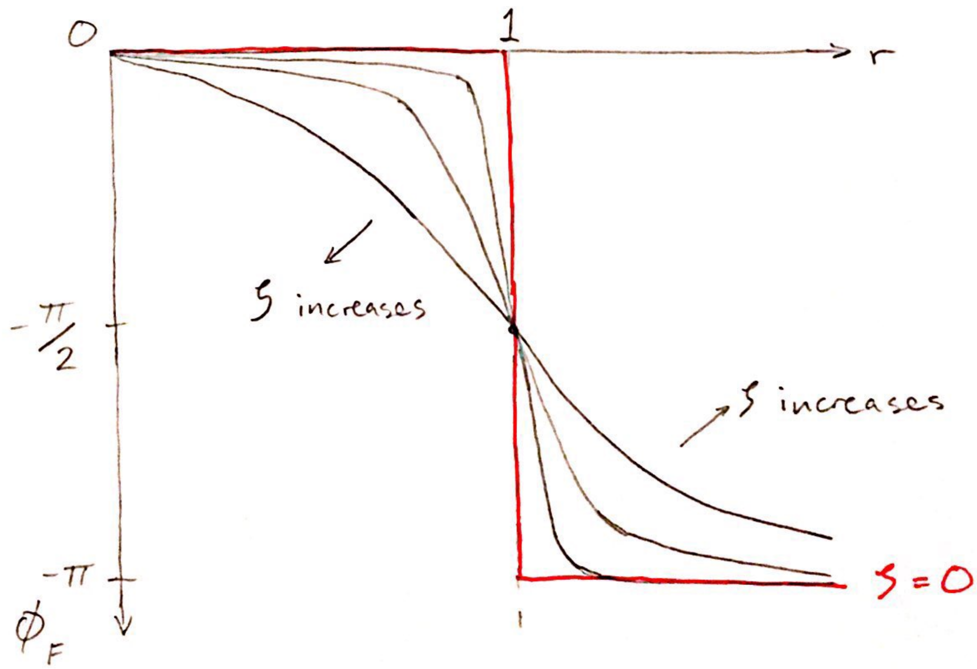
An aside about noise:

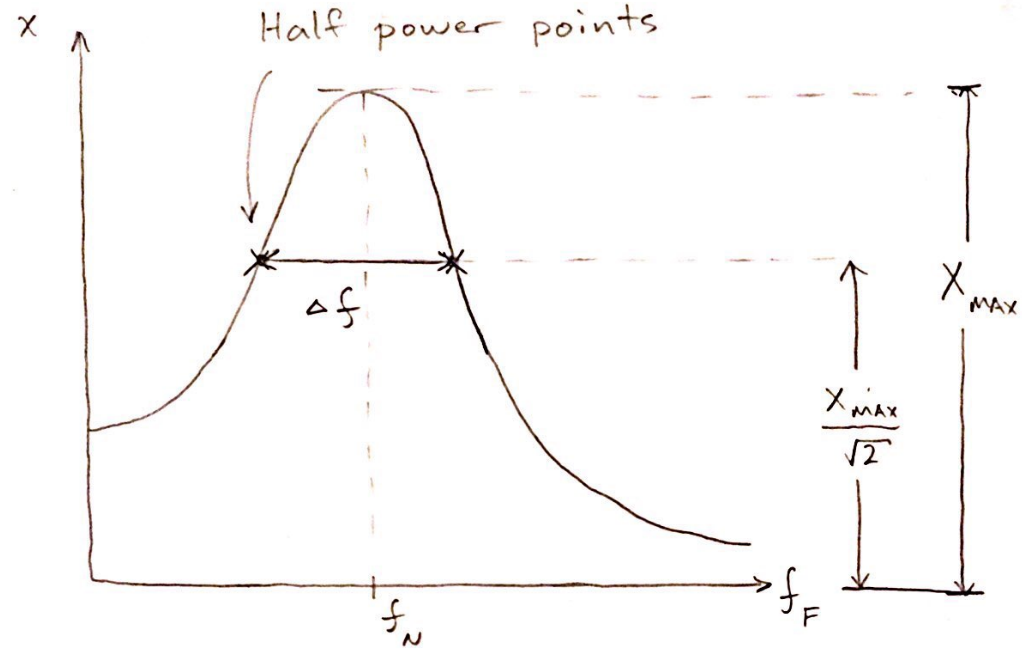


Real signals have noise. When this is differentiated the noise is amplified.

Accelerometers are noisy, but easy to implement. Integration reduces noise.

Phase



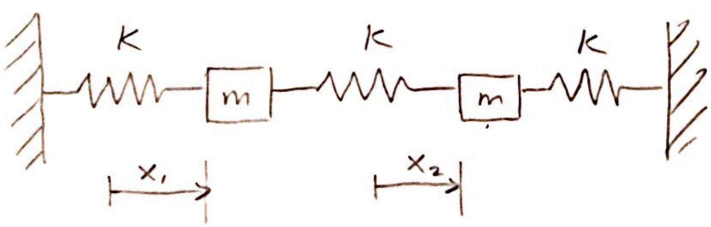


$$\zeta \simeq \frac{\Delta f}{2 f_N}$$

Quality Factor

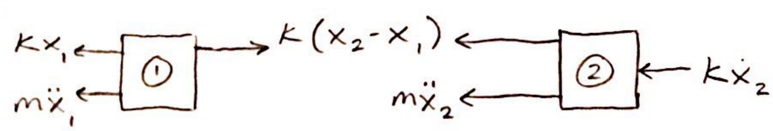
$$Q = \frac{1}{2\zeta}$$

2-DOF Vibration



Datum is equilibrium position.

<u>FBD</u>:



<u>E.O.M</u>

① $m\ddot{x}_1 + kx_1 - k(x_2 - x_1) = 0 \iff m\ddot{x}_1 + 2kx_1 - kx_2 = 0$

② $m\ddot{x}_2 + k(x_2 - x_1) + kx_2 = 0 \iff m\ddot{x}_2 - kx_1 + 2kx_2 = 0$

Solve: ① $x_2 = \frac{m}{k}\ddot{x}_1 + 2x_1$

Differentiate twice:

$\ddot{x}_2 = \frac{m}{k}\ddddot{x}_1 + 2\ddot{x}_1$ and sub into ②

② $m\left(\frac{m}{k}\ddddot{x}_1 + 2\ddot{x}_1\right) - kx_1 + 2k\left(\frac{m}{k}\ddot{x}_1 + 2x_1\right) = 0$

Multiply by k

$$\boxed{m^2\ddddot{x}_1 + 4mk\ddot{x}_1 + 3k^2 x_1 = 0}$$

Try harmonic solution (Sol. #2)

Let $x = C\cos(\omega t + \phi)$

Then $x_1 = (m^2\omega^4 - 4mk\omega^2 + 3k^2)\,C\cos(\omega t + \phi) = 0$

characteristic equation

$$m^2\omega^4 - 4mk\omega^2 + 3k^2 = 0$$

This is a quadratic equation for ω^2.

$$\omega^2 = \frac{4mk \pm \sqrt{16m^2k^2 - 12m^2k^2}}{2m^2}$$

$$\Rightarrow \boxed{\omega^2 = \frac{k}{m} \text{ or } \frac{3k}{m}}$$

Solution for x_1: $x_1 = C_1 \cos(\omega_1 t + \phi_1) + C_2 \cos(\omega_2 t + \phi_2)$

where $\omega_1 = \sqrt{\frac{k}{m}}$ and $\omega_2 = \sqrt{\frac{3k}{m}}$

Solve for x_2: $x_2 = \frac{m}{k} \ddot{x}_1 + 2x_1$

$$= C_1\left(-\omega_1^2 \frac{m}{k} + 2\right)\cos(\omega_1 t + \phi_1) + \ldots$$

$$+ C_2\left(-\omega_2^2 \frac{m}{k} + 2\right)\cos(\omega_2 t + \phi_2)$$

$$x_2 = u_1 C_1 \cos(\omega_1 t + \phi_1) + u_2 C_2 \cos(\omega_2 t + \phi_2)$$

where $u_1 = \left(-\omega_1^2 \frac{m}{k} + 2\right) = 1$

$u_2 = \left(-\omega_2^2 \frac{m}{k} + 2\right) = -1$

$$\Rightarrow x_2 = C_1 \cos(\omega_1 t + \phi_1) - C_2 \cos(\omega_2 t + \phi_2)$$