P.9a

$v_e = v^+ - v^- = v_i - v_1$

$v_2 = v_o$

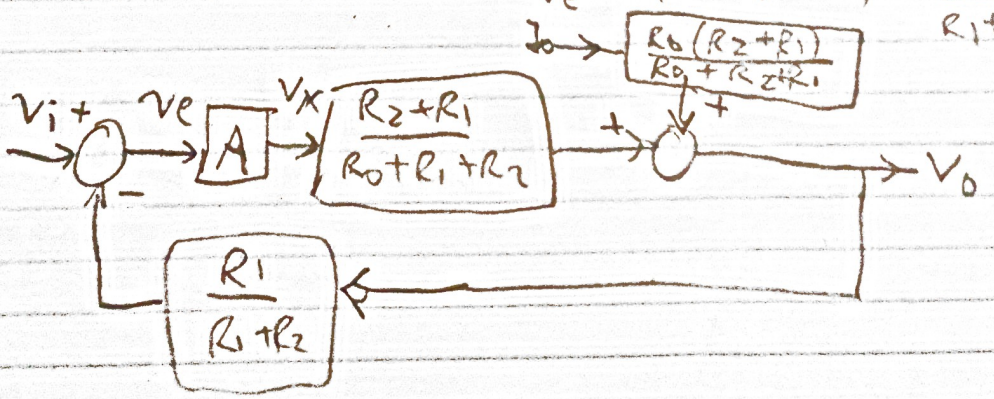
Node current at v_2: $\underbrace{v_x}$

$$\frac{v_o - Av_e}{R_o} + \frac{v_o}{R_2+R_1} - I_o = 0$$

$$v_o = \left(\frac{R_o(R_2+R_1)}{R_o+R_1+R_2}\right)\left(\frac{v_x}{R_o} + I_o\right)$$

$$v_1 = \frac{R_1}{R_1+R_2} v_2 = \frac{R_1}{R_1+R_2} v_o$$

$$v_e = v_i - v_1 = v_i - \frac{R_1}{R_1+R_2} v_o$$



b) $L(s)$ = loop return ratio

$$= -A\left(\frac{R_2+R_1}{R_o+R_1+R_2}\right)\left(\frac{R_1}{R_1+R_2}\right)$$

$$= -A\left(\frac{R_1}{R_o+R_1+R_2}\right)$$

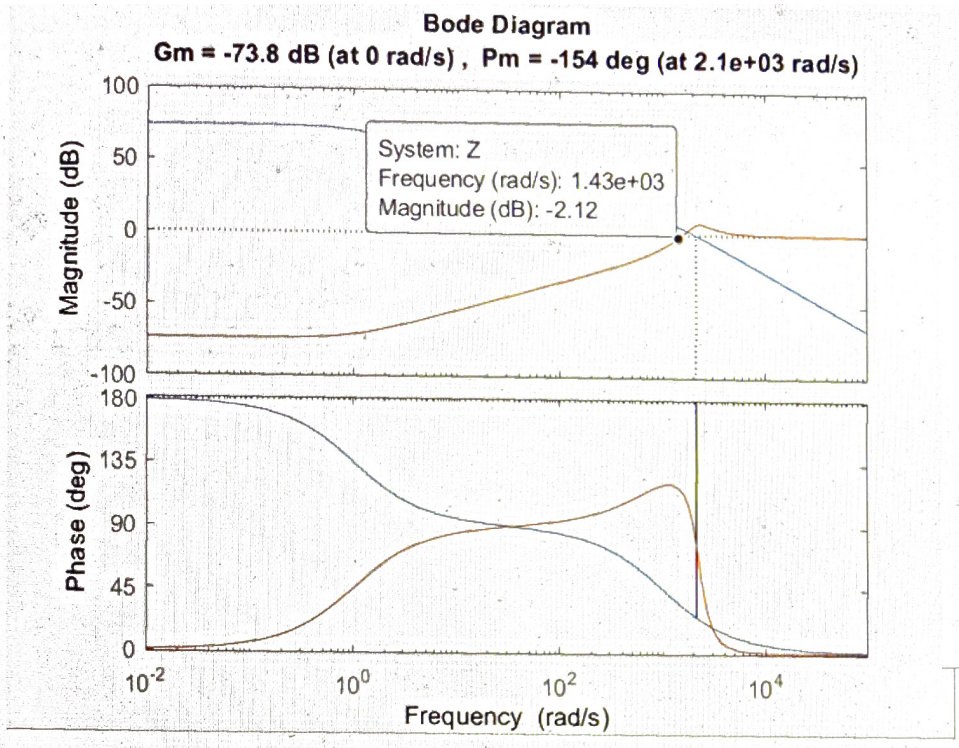
$$Z_0 = \frac{V_0}{I_0} = \frac{\text{forward gain}}{1 - \text{loop ratio}} = \frac{\frac{R_0(R_1+R_2)}{R_0+R_1+R_2}}{1-L}$$

$$= \frac{\frac{R_0(R_1+R_2)}{R_0+R_1+R_2}}{1+A\left(\frac{R_1}{R_0+R_1+R_2}\right)}$$

1c $\quad A\big|_{w\to\infty} = 0 \rightarrow L\big|_{w\to\infty} = A(\cdots) = 0(\cdots) = 0$

$$\rightarrow Z_0\big|_{w\to 0} = \frac{\frac{R_0(R_1+R_2)}{R_0+R_1+R_2}}{1-L} = \frac{\frac{R_0(R_1+R_2)}{R_0+R_1+R_2}}{1-0} = \frac{R_0(R_1+R_2)}{R_0+R_1+R_2}$$

$$\hat{z}(1) = \frac{z_0(1)}{z_0(jw)\big|_{w\to 0}} = \frac{\frac{R_0(R_1+R_2)}{R_0+R_1+R_2}/1-L}{\frac{R_0(R_1+R_2)}{R_0+R_1+R_2}} = \frac{1}{1-L(s)}$$




```
r0 = 50;
r1 = 1000;
r2 = 1000;
A = tf([10000], [.001 1.001 1]);
L = -(r1/(r0+r1+r2))*A;
Z = 1/(1-L);
figure(1);
hold on;
margin(L);
bode(Z);
hold off;
[Gm,Pm,Wcg,Wcp] = margin(L);
cutoffFreq = bandwidth(Z);
disp(Wcg);
disp(Pm);
disp(L);
disp(Z);
```

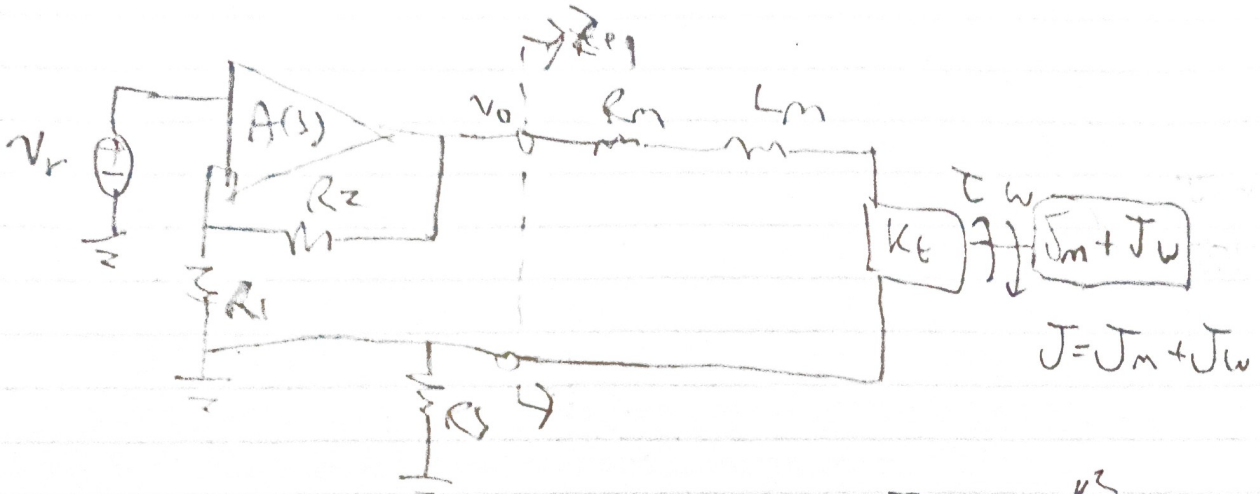
w_c = 0
pm = -154 deg
w_t = 1.43e3 rad/s

Shape of |L(jw)| and |Z(jw)| are approximately symmetric along 0dB line.

1e. $Z_o(jw)|_{w=0} = \frac{R_o(R_1+R_2)}{R_o+R_1+R_2} \approx \frac{50(1k+1k)}{50+1k+1k} = \frac{100000}{2050}$

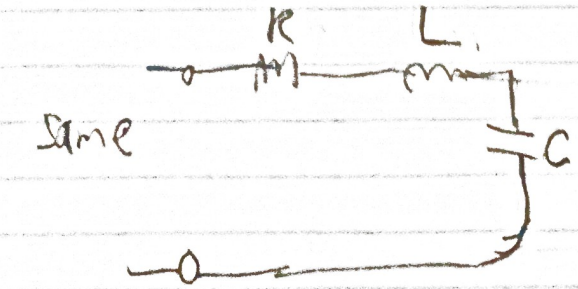
$= 48.78$

2a



$\tau = \frac{K_t}{L_m s + R_m + \frac{K_t^2}{Js}} V_o \rightarrow Z_{eq} = Z_{th} = \frac{\tau}{w_r} = \frac{K_t^2}{L_m s + R_m + \frac{K_t^2}{Js}}$

2b

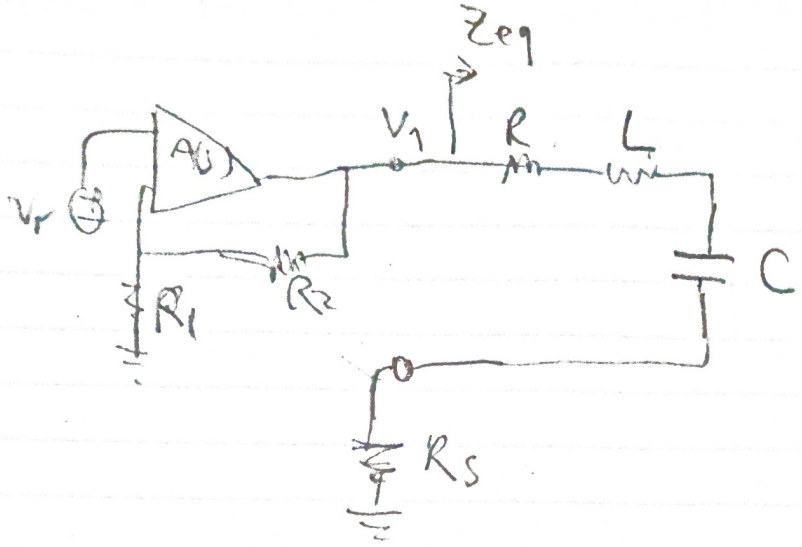


$R = R_m$

$sL = sL_m \rightarrow L = L_m$

$\frac{1}{sC} = \frac{K_t^2}{Js} \rightarrow C = \frac{J}{K_t^2} = \frac{J_m + J}{K_t^2}$

2c



$$\frac{V_1}{V_r} = \frac{A}{1+Af} \qquad f = \frac{R_2}{R_1+R_2}$$

$$V_1 = I_0(Z_{eq} + R_s)$$

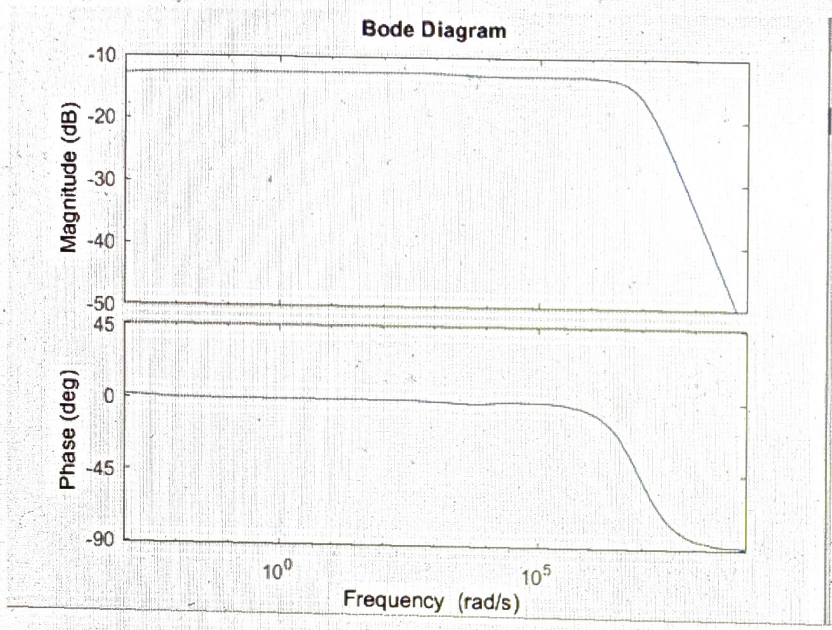
$$\frac{I_0}{V_1} = (Z_{eq} + R_s)$$

$$\frac{I_0}{V_r} = \frac{I_0}{V_1}\left(\frac{V_1}{V_r}\right) = (Z_{eq} + R_s)\left(\frac{A}{1+Af}\right)$$

$$= \left(\frac{K_t^2}{L_m s + R_m + \frac{K_t^2}{(J_m + J_w)s}} + R_s\right)\left(\frac{A}{1+A\left(\frac{R_2}{R_1+R_2}\right)}\right)$$

$$= \left(\left(\frac{K_t^2}{(L_m s + R_m)J_s + K_t^2}{J_s}\right) + R_s\right)\left(\frac{A}{1+A\left(\frac{R_2}{R_1+R_2}\right)}\right)$$

2C.



q2_vars.m | hw2_1.m | hw2_2.m

```
1 -  A = tf([10000000], [1 0]);
2 -  r1 = 1000;
3 -  r2 = 9000;
4 -  rm = 4.8;
5 -  Lm = .001;
6 -  rs = .2;
7 -  Kt = .25;
8 -  Jm = 1;
9 -  Jw = 9;
10   %Jm = 1*0.0001; % use SI
11   %Jw = 9*0.0001;
12 -  J = Jm + Jw;
13 -  Zeq = tf([Kt*Kt*J 0], [J*Lm J*rm Kt*Kt]);
14 -  I_V = (Zeq+rs)*(A/(1+A*(r2/(r1+r2))));
15 -  bode(I_V);
```

Command Window

New to MATLAB? See resources for Getting Started.

4. $P = I^2 R$

$P_{max} = I_{max} R_s$

$I_{max} = \frac{P_{max}}{R_s} = \frac{1W}{.2\Omega} = 5A$

$I_{o,rms} = \frac{I_{max}}{\sqrt{2}} = 3.536A$

5. $50Hz = 314.6 \text{ rad/s}$

$\left.\frac{I_o}{V_t}\right|_{\omega = 314.16 \text{ rad/s}} = -12.5 \text{ dB} = 0.237 \frac{A}{V}$ (From Bode plot)

$V_{t,rms} = I_{o,rms}\left(\frac{V_t}{I_o}\right) = 3.536\,(.237)^{-1} = 14.9 \text{ V}$