MECH 463: Mechanical Vibrations
Midterm Examination 2

Time: 45 minutes 31st October 2013 Maximum Available Mark: 20

Student Name: Student Number:

Write your answers on this sheet (4 pages in total). Do not remove pages.

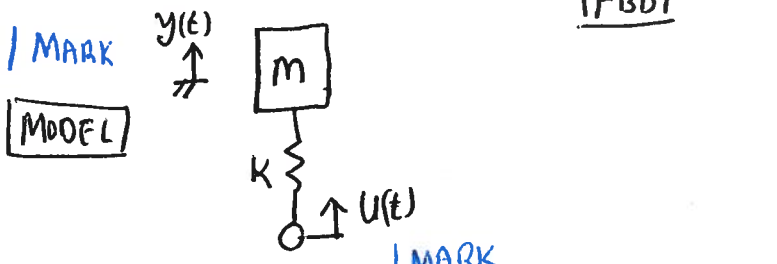
Q1. Consider a motorcycle shown in Fig.(1). The totals mass, including the rider, is 250 kg and the stiffness of the suspension is 70 kN/m. The motorcycle travels over a terrain **with constant velocity** v, approximately sinusoidal with a distance between peaks of 10 m and the distance from peak to valley is 10 cm.

Figure 1: Figure for midterm question. **Ignore damping and fee vibration response.**

a) Write a SDOF model for this problem along with the equation of motion. 4 **marks are for FBD and 2 marks for co-ordinate. Accuracy of FBD is marked not just the equation of motion.** **(10 marks)**

Answer (15 minutes or less):

FBD



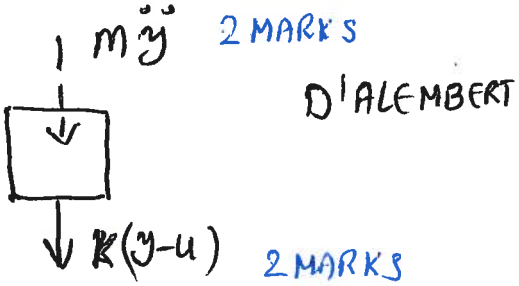
1 MARK

MODEL

1 MARK

y: DISPLACEMENT OF 'm' W.R.T. GROUND

u: DISPLACEMENT INDUCED BY GROUND ON THE WHEEL



GRAVITY CANCELS INITIAL COMPRESSIVE FORCE IN 'k'

EQUATION OF MOTION

$\uparrow \sum F_y = 0 \Rightarrow -m\ddot{y} - k(y-u) = 0$ (+ve)

$\Rightarrow m\ddot{y} + k(y-u) = 0 \quad -(1)$

INTRODUCING RELATIVE DISPLACEMENT $z = y - u$ AND $\ddot{z} = \ddot{y} - \ddot{u} \Rightarrow \ddot{y} = \ddot{z} + \ddot{u}$

IN THE EQUATION OF MOTION (1) WE GET

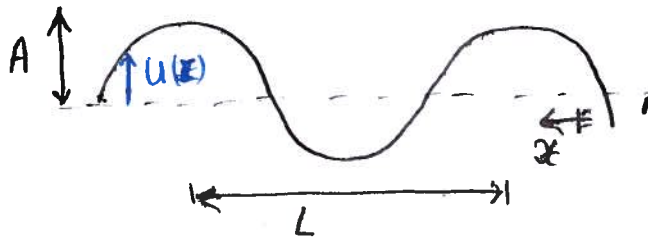
$$m(\underbrace{\ddot{z} + \ddot{u}}_{\ddot{y}}) + k\underbrace{z}_{y-u} = 0 \Rightarrow m\ddot{z} + kz = -m\ddot{u} \quad \text{— (2)}$$

b) Deduce the expression for the maximum acceleration felt by the rider due to the **forced vibration** from the rough terrain. **Ignore free vibration (homogeneous solution).** Hence find the maximum **acceleration** experienced by the rider while travelling at $v = 30$ m/s, $v = 60$ m/s and $v = 120$ m/s. **(6 marks)**

Answer (15 minutes or less):

$A = \frac{10}{2}\text{ cm} = 0.05\text{ m}$ 1 MARK

$L = 10\text{ m}$ 1 MARK



$$u(x) = A \sin \frac{2\pi x}{L}$$

x = HORIZONTAL DISTANCE TRAVELLED $= Vt$ (KINEMATICS)

$$\therefore u(t) = A \sin \frac{2\pi V t}{L} \quad \text{— (3)}$$

$$= A \sin \omega t \quad \text{WHERE} \quad \omega = \frac{2\pi V}{L}$$

(8) IN (2) GIVES THE FORCED VIBRATION PROBLEM

$$m\ddot{z} + kz = +m\omega^2 A \sin\omega t \quad \text{WHERE} \quad \omega = \frac{2\pi V}{L}$$

$m\omega^2 A$ = F

$$z = \cancel{z_h}^{\text{IGNORE}} + z_p$$

$$\therefore z = z_p = \frac{m\omega^2 A \sin\omega t}{k - m\omega^2}$$ USING FORCED VIBRATION FORMULA

$$\ddot{z} = \ddot{z}_p = \frac{-m\omega^2 A\, \omega^2 \sin\omega t}{k - m\omega^2}$$ 2 MARKS

*** NOTE THAT $\ddot{z}$ IS THE ACCELERATION FELT BY THE RIDER AS HE IS IN A MOVING FRAME NOT $\ddot{y}$

USING $m = 250$ kg; $A = 0.05$ m; $k = 70000$ N/m; $\omega = \frac{2\pi \times V}{10}$ WE GET 'V' DEPENDENT ACCELERATION AMPLITUDE $= \frac{-m\omega^2 A}{k - m\omega^2} = A_{ACCLN}$

$V = 30$ m/s	$\omega = 18.8496$ rad/s	$A_{ACCLN} = 83.82$ m/s^2
$V = 60$ m/s	$\omega = 37.6991$ rad/s	$A_{ACCLN} = 88.5$ m/s^2
$V = 120$ m/s	$\omega = 75.3982$ rad/s	$A_{ACCLN} = 2\cancel{9}9.0$ m/s^2

2 MARKS

NOTE RESONANCE $\omega_n = \sqrt{\frac{k}{m}} = 16.73$ rad/s

NOTE $\omega > \omega_n$ FOR ALL SPEEDS ABOVE

c) Based on the form of the solution above, sketch the variation of the displacement felt by the rider as a function of speed and indicate the critical (resonant) speed. **(4 marks)**

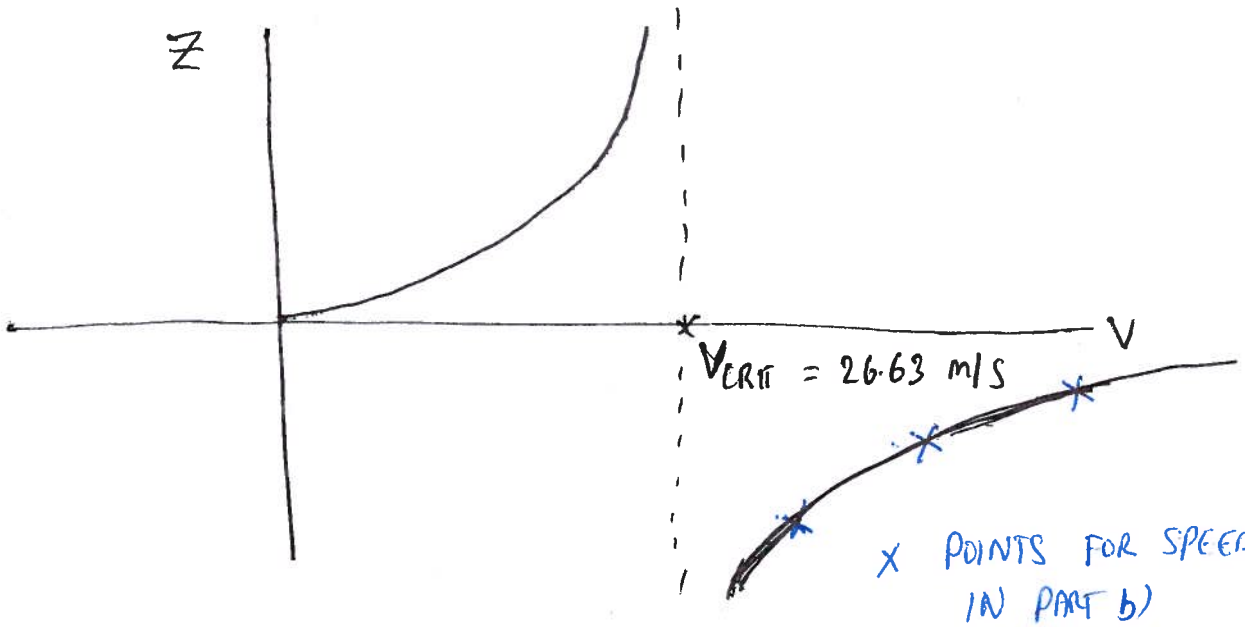
Answer (5 minutes or less):

DISPLACEMENT AMPLITUDE (FELT BY RIDER) : $\frac{m\omega^2 A}{k - m\omega^2} = Z$ FROM PART b)

CRITICAL SPEED IS V ASSOCIATED WITH $\omega_n = \sqrt{\frac{k}{m}} = \frac{2\pi V_{CRIT}}{10}$

$\Rightarrow V_{CRIT} = \frac{10\omega_n}{2\pi} = \frac{10 \times 16.73}{2 \times \pi} = 26.63$ m/s

2 MARKS



2 MARKS

X POINTS FOR SPEEDS IN PART b)

ALL THE BEST!