Question 1

(a) What is an LVDT?

What type of signal condition/conversion does it need in its general usage? (Indicate each type of signal conditioning/conversion and its purpose in an LVDT). **(10%)**

(b) Two op-amp circuits are shown in Figure Q1. Derive the input-output equation for each circuit in terms of the indicated parameters (resistances). **(20%)**

Note: v_i = input voltage

v_o = output voltage

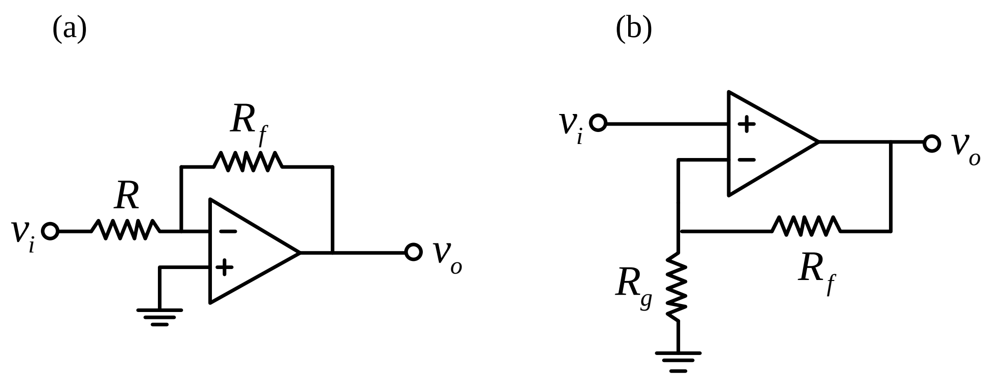


Figure Q1: Two op-amp circuits.

Indicate some uses of these two circuits. **(5%)**

Give one advantage and one disadvantage for each of the two circuits. **(5%)**

Question 2

Consider the optical potentiometer circuit shown in Figure Q2.

v_o = output voltage of the potentiometer (potentiometer reading)

x = measured displacement

L = length of the potentiometer element (assumed uniform)

v_{ref} = supply voltage (constant)

R_c = resistance of the potentiometer element (uniform)

R_p = photoresistance (constant)

R_l = load resistance (constant)

(a) Derive an equation relating only the following variables and parameters:

$u = \dfrac{x}{L}$ = nondimensional measurand

$y = \dfrac{v_o}{v_{ref}}$ = nondimensional potentiometer reading

$a = \dfrac{R_l}{R_c}$ = nondimensional load resistance

$b = \dfrac{R_p}{R_c}$ = nondimensional photoresistance

Note: Your equation must contain u, y, a, and b only. It is <u>not necessary</u> to express the equation in the form " $y =$ ". **(20%)**

(b) Determine the direct sensitivity (nondimensional) $S = \dfrac{\partial y}{\partial u}$ of this potentiometer. **(20%)**

Using this result investigate how the parameters a and b affect the sensitivity S of the potentiometer, as follows:

1. Determine S (in terms of a and b) at the three measurand values: u = 0, 0.5, 1
2. For each of these three cases, determine S (in terms of b only) at a = 0, and as $a \to \infty$. Based on these results, comment on the effect of a on the sensitivity S.
3. Using the expressions for S in terms of b, as obtained in Step 2 above, comment on the effect of b on the sensitivity S. At what value of b do we get the maximum sensitivity as $a \to \infty$, in these three cases?

(20%)

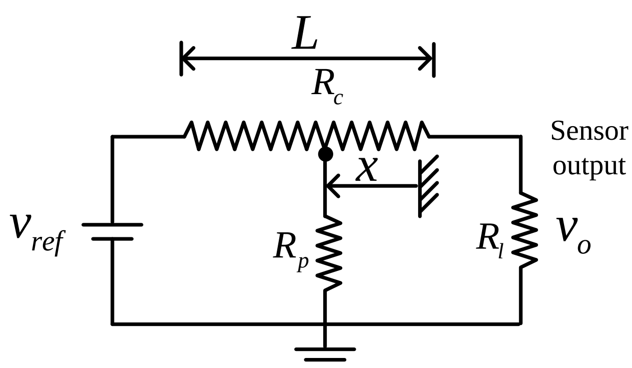


Figure Q2: Circuit of an optical potentiometer.

Solutions

Solution 1

(a) LVDT is a non-contact displacement sensor. It uses the principle of mutual induction; incorporating a primary coil activated by a carrier AC, two segments of secondary coil connected in series opposition, and a ferromagnetic core (connected to the moving object whose displacement is measured).

Signal condition/Conversion:

1. **Modulation:** The carrier AC is modulated by the movement of the core, which induces an AC in the secondary coil.
   Amplification: The induced voltage in the secondary coil is amplified to a practical level.

Multiplier: The amplified secondary-coil signal is multiplied by the carrier AC (This is the first stage of demodulation).
Low-pass Filtering: The product signal is filtered to remove the AC component (whose frequency is double the carrier frequency). This is the final stage of demodulation. The result is proportional to the displacement of the core.

(b) Circuit (a): See Figure S1(a)

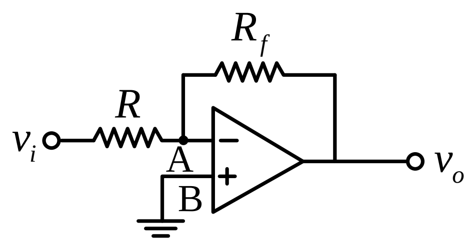


Figure S1(a): Circuit of a resistor-feedback op-amp.

Voltage at A = Voltage at B = 0 (ground; by op-amp property)

Current balance at Node A: $\frac{v_i}{R}+\frac{v_o}{R_f}=0$

Note: Current into op-amp = 0 (by op-amp property)

$\rightarrow \quad v_o = -\frac{R_f}{R}v_i$

This is an *inverting amplifier* (voltage amplifier)

Amplifier gain $=\frac{R_f}{R}$

Circuit (b): See Figure S1(b)

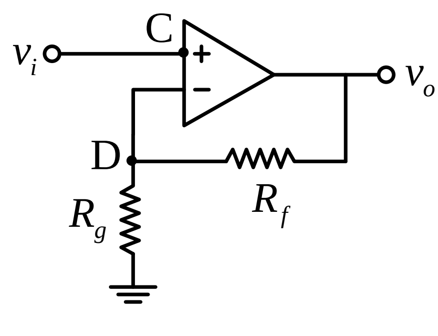


Figure S1(b): Circuit of a resistor feedback op-amp.

Voltage at C = Voltage at D = v_i (by op-amp property)

Current balance at Node D: $\frac{v_o - v_i}{R_f} = \frac{v_i}{R_g}$

Note: Current into op-amp = 0 (by op-amp property)

$$\rightarrow \quad v_o = [1+\frac{R_f}{R_g}]v_i$$

This is a *non-inverting amplifier* (voltage amplifier)

Amplifier gain $=1+\dfrac{R_f}{R_g}$

Uses: Both devices can be used as voltage amplifiers. Circuit (a) is particularly suitable when the sign of the input signal needs to be changed. This is not an absolute necessity or disadvantage, because the sign can be changed by switching the output terminals as will.

Advantage: Both circuits provide high input impedance and low output impedance. Hence electrical loading problem will be greatly reduced.

Disadvantage: Neither of the circuits is a difference amplifier. Hence any noise at the input will be amplified and transmitted to the output.

Solution 2

(a) *Note*: x is measured from the low-voltage end of the potentiometer element (R_c).

Current through load: $$i_l = \frac{v_o}{R_l} \qquad \text{(i)}$$

Voltage at point x of the coil: $$v_p = (uR_c + R_l)i_l = \frac{(uR_c + R_l)v_o}{R_l} \qquad \text{(ii)}$$

Current balance at x: $$\frac{v_{ref} - v_p}{(1-u)R_c} = \frac{v_p}{R_p} + i_l$$

Substitute (i) and (ii): $$\frac{v_{ref} - \dfrac{(uR_c + R_l)v_o}{R_l}}{(1-u)R_c} = \frac{\dfrac{(uR_c + R_l)v_o}{R_l}}{R_p} + \frac{v_o}{R_l}$$

Divide the numerators throughout by v_{ref}, divide the denominators throughout by R_c, and substitute $\dfrac{v_o}{v_{ref}} = y$; $\dfrac{R_l}{R_c} = a$; and $\dfrac{R_p}{R_c} = b$

$$\rightarrow \quad \frac{1-(\frac{u}{a}+1)y}{1-u} = \frac{(\frac{u}{a}+1)y}{b} + \frac{y}{a}$$

$$\rightarrow \quad ab[1-(\frac{u}{a}+1)y] = a(1-u)(\frac{u}{a}+1)y + b(1-u)y$$

$$\rightarrow \quad ab - buy - aby = uy + ay - u^2 y - auy + by - buy$$

$$\rightarrow \quad (u - au - u^2 + a + b + ab)y = ab \qquad \text{(iii)}$$

(b) Differentiate (iii) wrt y:

$$(u - au - u^2 + a + b + ab)\frac{\partial y}{\partial u} + (1 - a - 2u)y = 0$$

Substitute (iii):

$$S = \frac{\partial y}{\partial u} = -\frac{(1 - a - 2u)ab}{(u - au - u^2 + a + b + ab)^2} \qquad \text{(iv)}$$

Note: $a = 0 \Rightarrow$ Closed-circuit output; $a \to \infty \Rightarrow$ open-circuit output.

The nature of the sensitivity as a function of the sensor displacement u for a typical value of a = 0.5 and b = 0.5 is shown below. It is seen that sensitivity increases with displacement, more rapidly when the displacement u gets closer to 1.0.

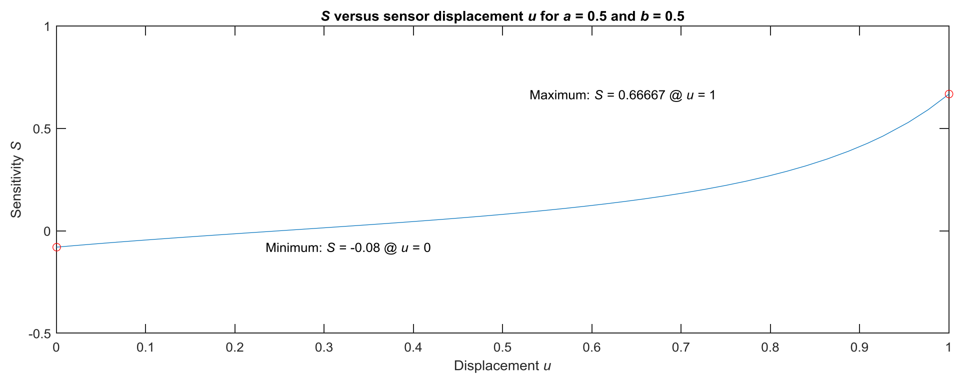


Figure S2(0): Variation of sensitivity with sensor displacement at *a* = 0.5 and *b* = 0.5.

1. <u>At $u = 0$:</u> $\qquad S = -\dfrac{(1-a)ab}{(a+b+ab)^2} \qquad$ (v)

   At $a = 0$: $\qquad S = 0$

   At $a \to \infty$: $\qquad S = -\dfrac{b}{(1+b)^2}$

Overall, the shape of the sensitivity expression (v) as a (load) changes from 0 to ∞ is shown in Figure S2(a).

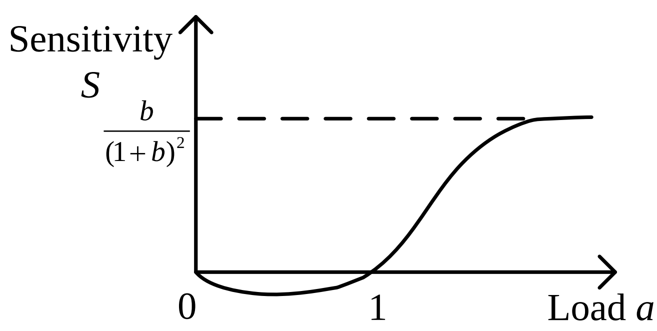
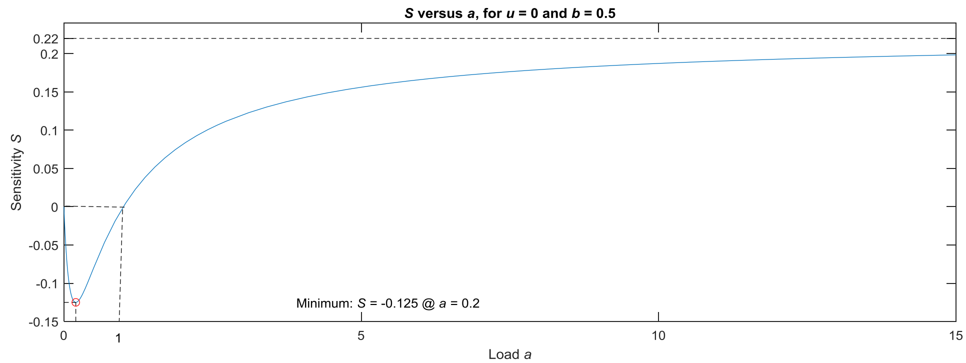


Figure S2(a): The effect of load at *u* = 0.

MATLAB Figure:



The variation of the sensitivity with respect to b at u = 0 and $a \to \infty$ is shown in Figure S2(b).

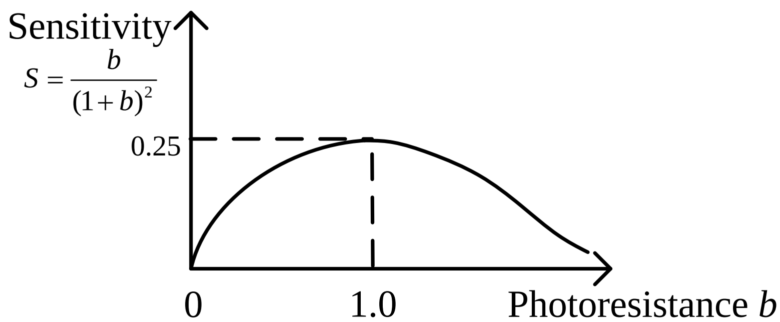
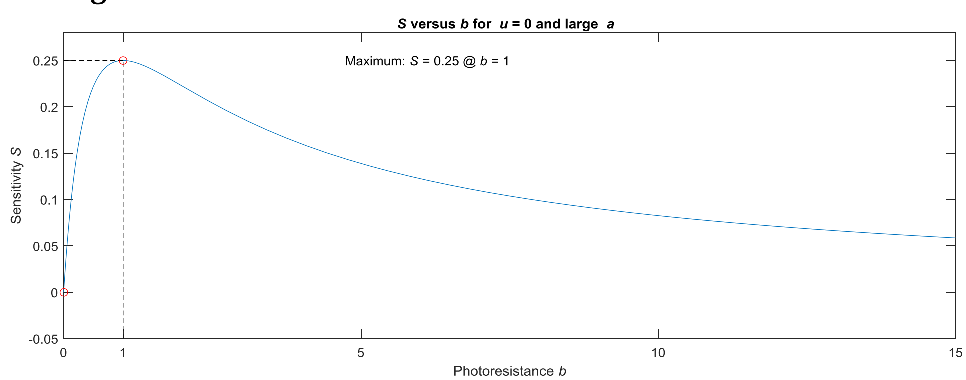


Figure S2(b): The effect of photoresistance at *u* = 0 with large loads.

MATLAB Figure:



2. <u>At u = 1:</u> $$S = \frac{a}{b(1+a)} \qquad \text{(vi)}$$

At a = 0: $S = 0$

At $a \to \infty$: $S \to \frac{1}{b}$

Overall, the shape of the sensitivity expression (vi) as a (load) changes from 0 to ∞ is shown in Figure S2(c).

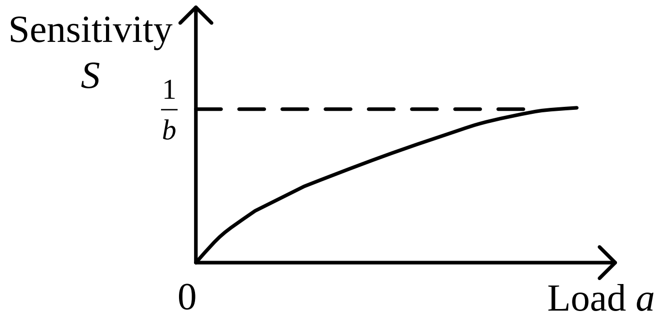
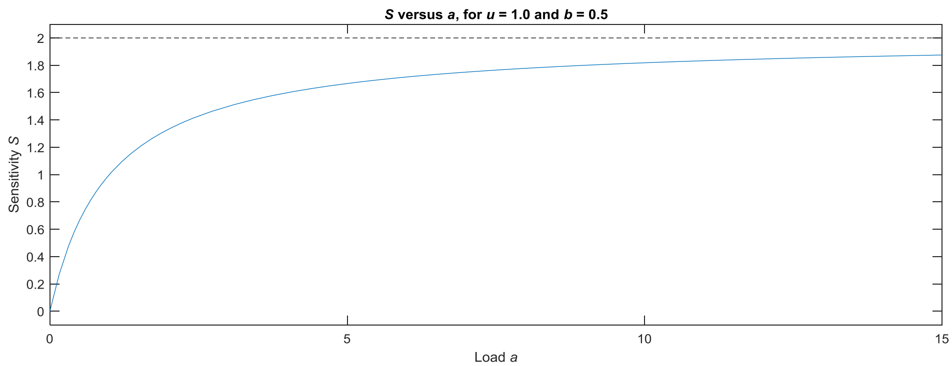


Figure S2(c): The effect of load at u = 1.

MATLAB Figure:



The variation of the sensitivity with respect to b, at u = 1 and $a \to \infty$ is shown in Figure S2(d).

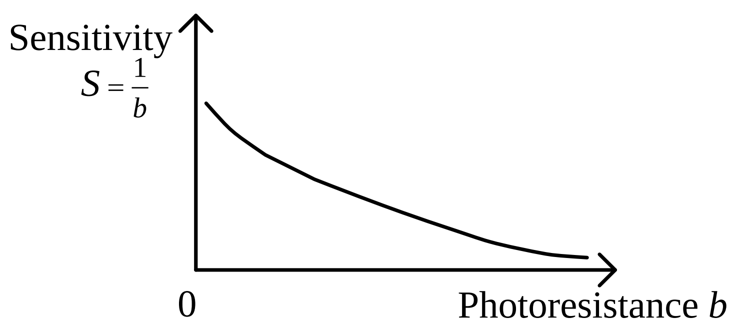
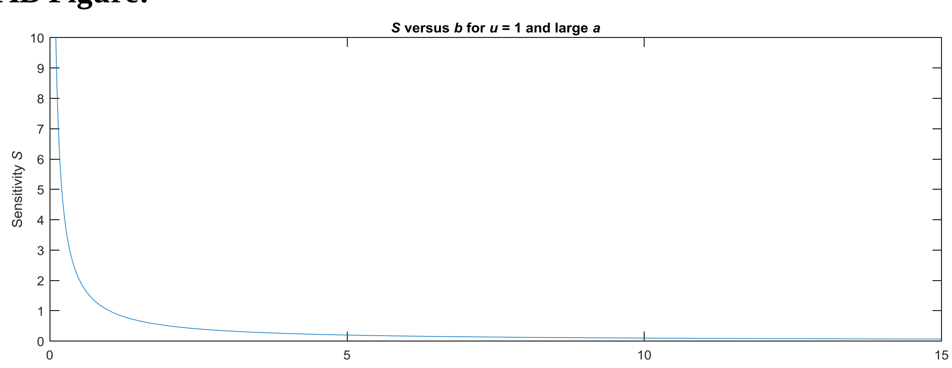


Figure S2(d): The effect of photoresistance at u = 1 for large loads.

MATLAB Figure:



3. <u>At u = 0.5:</u> $$S = \frac{16a^2 b}{(1+2a+4b+4ab)^2} \qquad \text{(vii)}$$

At a = 0: $S = 0$

At $a \to \infty$: $$S \to \frac{4b}{(1+2b)^2}$$

The overall shape of the sensitivity expression (vii) as a (load) changes from 0 to ∞ is shown in Figure S2(e).

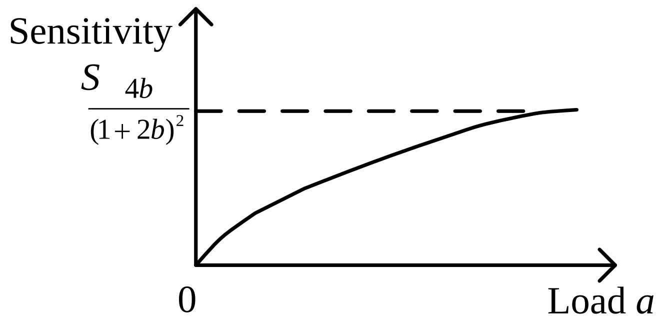
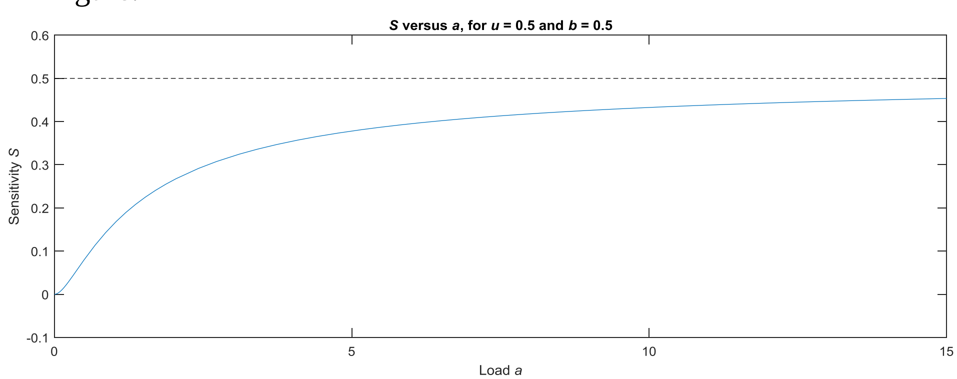


Figure S2(e): The effect of load at u = 0.5.

MATLAB Figure:



The variation of the sensitivity with respect to b, at u = 0.5 and $a \to \infty$ is shown in Figure S2(f).

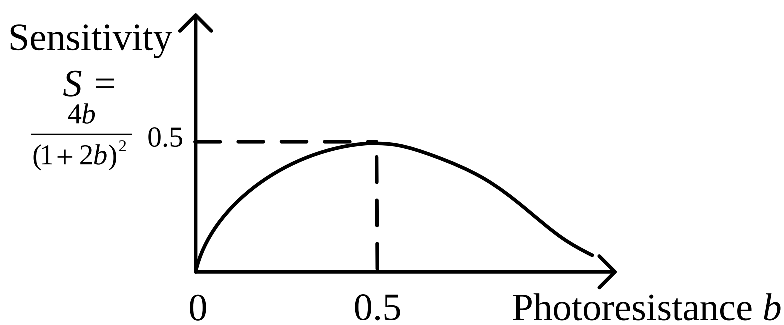
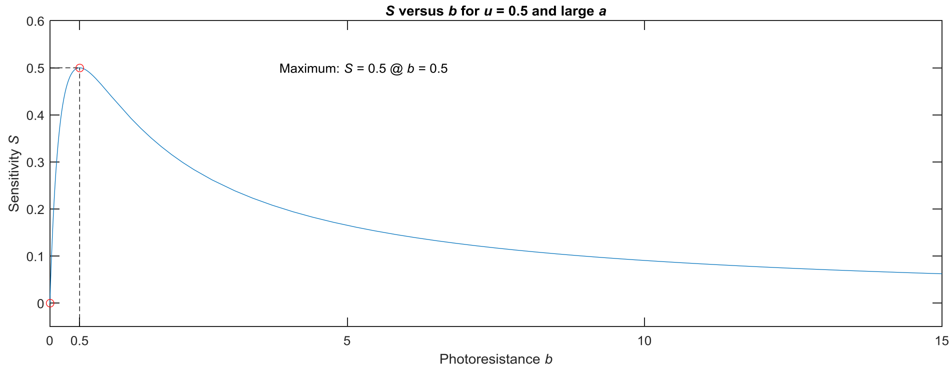


Figure S2(f): The effect of photoresistance at u = 0.5 and large loads.

MATLAB Figure



General comments from the results:

1. Better sensitivity is possible for larger loads (i.e., as $a \to \infty$).
2. The best sensitivity is possible in the neighborhood of b = 0.5 (i.e., when the photoresistance is about half the potentiometer element resistance)
3. For average values of a and b, sensitivity increases with displacement, more rapidly when the displacement u gets closer to 1.0 (.e., the best sensitivity is achieved at large displacement).