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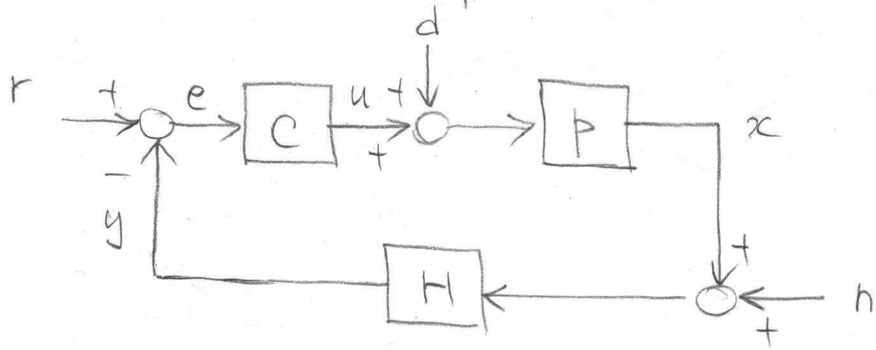
< Feedback systems >

Consider a closed-loop system.



Inputs
- r : reference
- d : disturbance
- n : noise

$C(s)$: Controller
$P(s)$: plant
$H(s)$: Sensor

Outputs
- x : plant output
- y : measurement
- e : error
- u : control effort

Transfer function "Matrix" is

$$\begin{bmatrix} X \\ Y \\ E \\ U \end{bmatrix} = \begin{bmatrix} \frac{CP}{1+CPH} & \frac{P}{1+CPH} & \frac{-CPH}{1+CPH} \\ \frac{CPH}{1+CPH} & \frac{PH}{1+CPH} & \frac{H}{1+CPH} \\ \frac{1}{1+CPH} & \frac{-PH}{1+CPH} & \frac{-H}{1+CPH} \\ \frac{C}{1+CPH} & \frac{-CPH}{1+CPH} & \frac{-HC}{1+CPH} \end{bmatrix} \begin{bmatrix} R \\ D \\ N \end{bmatrix}$$

Note that $\frac{1}{1+CPH}$ is the common factor.

$\triangleq S(s)$

- Loop Transmission
  - Loop Transmission (L.T.) of a feedback loop is the product of all gains through the loop including the signs of summing junctions.

    e.g.) $L.T. = -CPH$

- Black's Formula.

$$G = \frac{\text{Forward-path Gain}}{1 - L.T.}$$

e.g

$$G_{er} = \frac{1}{1 - L.T.} = \frac{1}{1 + CPH} = \frac{1}{1 + L(s)} \triangleq S(s)$$

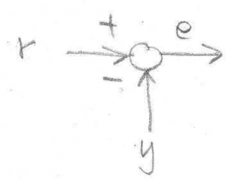
$$G_{yr} = \frac{CPH}{1 - L.T.} = \frac{CPH}{1 + CPH} = \frac{L(s)}{1 + L(s)} \triangleq T(s).$$

$$G_{xr} = \frac{CP}{1 - L.T.} = \frac{1}{H} \frac{CPH}{1 + CPH} = \frac{1}{H} \cdot T(s).$$

- Loop Return Ratio (a.k.a Loop Transfer function)
  - Loop return ratio $L(s)$ is defined as

    $L(s) \triangleq -L.T.$

  - It mean how much of the error signal returns back to the negative summing junction.
  - That is, for a negative feedback loop, $L(s)$ is the product of all gains except the negative sign of the summing junction
  - In this example, $L(s) = CPH$

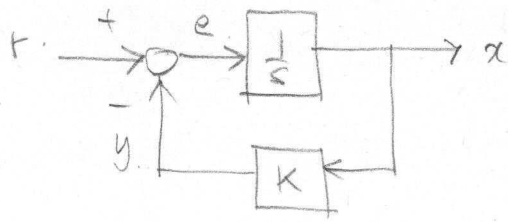


$L(s) = \frac{Y}{E}$

$S(s) = \frac{E}{R} = \frac{1}{1+L}$

$T(s) = \frac{Y}{R} = \frac{L}{1+L}$

Example : Single Integrator feedback



$L.T. = -\frac{K}{s}$

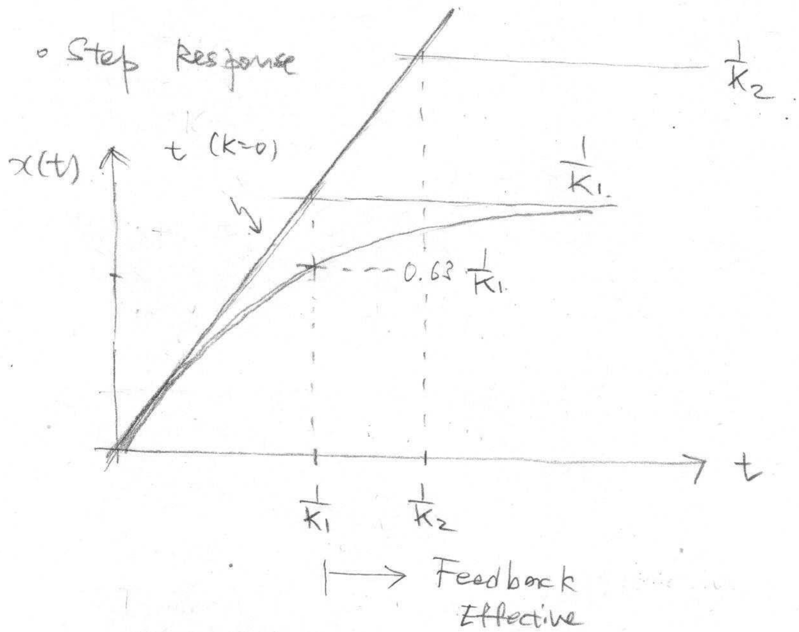
$L(s) = \frac{K}{s}$

Let's see how the feedback changes the original dynamics.

$$G \triangleq \frac{X}{R} = \frac{\frac{1}{s}}{1-\left(-\frac{K}{s}\right)} = \frac{\frac{1}{s}}{1+\frac{K}{s}} = \frac{1}{s+K} \quad \text{"Evan's Form"}$$

$$= \frac{1}{K}\,\frac{1}{\frac{1}{K}s+1} \quad \text{"Bode's Form"}$$

∘ Step Response

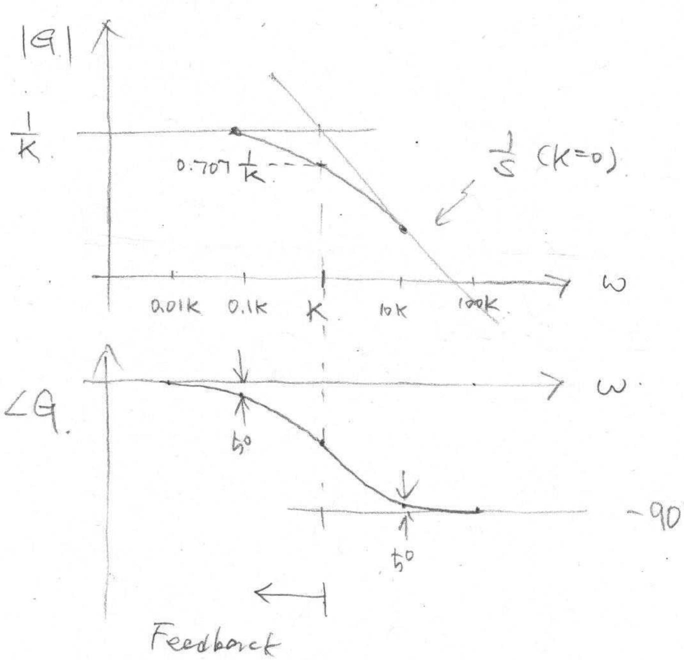


<Bode's Form>

$G_0\left(\frac{1}{\tau s+1}\right)$ unity dc gain system

dc gain time const.

∘ Bode plot



- Feedback is "effective", or changes the original dynamics

  - Time domain : for $t > \tau$ (time constant)
  - Freq. domain : for $\omega < \omega_h$ (Bandwidth.)

- When feedback is not-effective, the closed-loop system behaves like the original (forward-path) system.

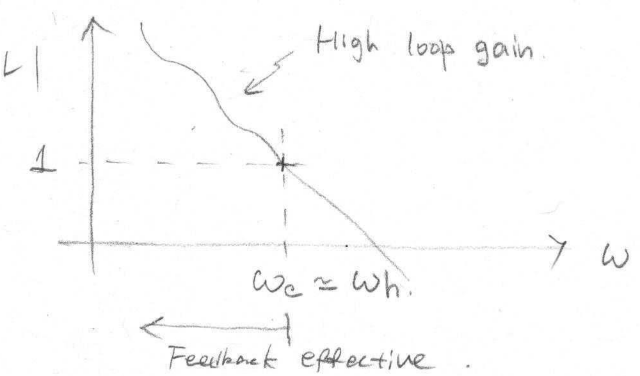
- When feedback is effective, the closed-loop system behaves like $G(s) \simeq \frac{1}{H(s)}$, where $H(s)$ is the feedback gain.

- Feedback is effective when.

$$\left|\frac{1}{s}\right| > \frac{1}{K} \Leftrightarrow \underbrace{\left|\frac{K}{s}\right|}_{L(s)} > 1.$$

In general, feedback is effective when the "loop gain" is high.

That is, $|L(j\omega)| > 1$



(Unity-gain) Cross-over frequency ω_c : $|L(j\omega_c)| = 1$

(-3dB) Bandwidth $\omega_h \simeq \omega_c$

- Many servo engineers alternatively define ω_c as the bandwidth