University of British Columbia
Department of Mechanical Engineering

MECH 463. Midterm 2, October 29, 2019

Allowed Time: 70 min

Materials admitted: Pen, pencil, eraser, straightedge, simple scientific calculator without programming or communication capabilities, personal hand-written notes within one letter-size sheet of paper (one side).

There are 3 questions in this exam. You are asked to answer all three questions.

The purpose of this test is to evaluate your knowledge of the course material. Orderly presentation demonstrates your knowledge most clearly, while disorganized and unprofessional work creates serious doubt. Marks are assigned accordingly. A bonus of up to 2 marks will be given for exemplary presentation.

Complete the section below **during** the examination time **only.**

NAME:______________________________________

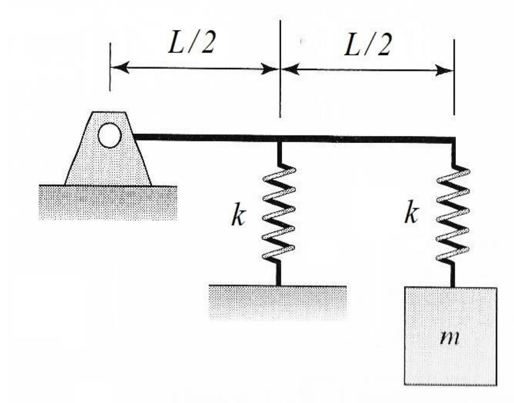
SIGNATURE:____________________________________

STUDENT NUMBER:_______________________________

	Mark Received	**Maximum Mark**
1		6
2		7
3		7
Presentation		2 bonus
Total		20+2

Name: ____

1. A massless rod of length L pinned at its left end and supported by a spring of stiffness k at its midpoint. At its right end, the rod supports a mass m through a spring, also of stiffness k.



   (a) Choose a convenient coordinate system and draw a labeled free-body diagram of the vibrating system.

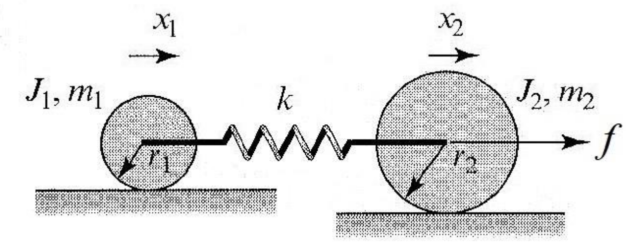
   (b) Use your free-body diagram to formulate the equation of motion. *(Hint: some careful thought is required here.)*

   (c) Solve your equation of motion for natural frequency. Show the needed steps in detail.

   (d) Comment on any notable features of the vibrating system.

Name: ____

2. Two cylindrical rollers are joined by a spring of stiffness k. Their radii, mass and moment of inertia respectively are $r_1 = r$, $m_1 = m$, $J_1 = \frac{1}{2} mr^2$ and $r_2 = 2r$, $m_2 = 2m$, $J_2 = 4mr^2$. An harmonic force $f = F \cos \omega_f t$ acts on the second roller.



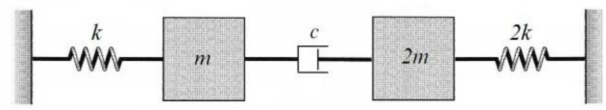
(a) Draw labeled free body diagrams of the rollers.

(b) Formulate a matrix equation of motion for the resulting vibrational displacements of the rollers.

(c) Solve for the steady-state vibration of the first roller.

(d) Sketch the vibration amplitude vs. excitation frequency response of the roller. Comment on and explain any notable or surprising features.

3. Two masses, m and 2m are supported by springs k and 2k, and joined by a damper c, as shown in the diagram.



(a) Draw labeled free-body diagrams of the vibrating system.

(b) Derive the matrix equation of motion from your free-body diagrams.

(c) Use the trial solution $\underline{x} = \underline{X}\, e^{\lambda t}$ to find the characteristic solution.

(d) Given that one root of the characteristic equation is $(m\lambda^2 + k)$, find the other root.

(e) Find the undamped and damped natural frequencies and the damping factors of the vibrating system.

(f) Comment on and explain any notable features of your results.