**University of British Columbia**
**Department of Mechanical Engineering**

**MECH468 Modern Control Engineering**
**MECH522 Foundations in Control Engineering**
**Midterm exam**

**Examiner: Dr. Ryozo Nagamune**
**February 9 (Friday), 2018, 1-1:50pm**

Last name, First name

Name: Student #:

Signature:

## Exam policies

- Allowed: One-page letter-size hand-written cheat sheet (both front side and back side)
- Not-allowed: PC, calculators.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 20 points in total.

## Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

## If you finish early ...

- Please stay at your seat until the end of exam, i.e., 1:50pm. (You are not allowed to leave the room before the end of exam, except going to washroom.)

## To be filled in by the instructor/marker

| Problem # | Mark | Full mark |
|---|---|---|
| 1 | | 12 |
| 2 | | 4 |
| 3 | | 4 |
| Total | | 20 |

1. Consider the following continuous-time system:

$$\begin{cases} \dot{x}(t) &= \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}$$

   (a) Check the internal stability of the system.
      i. Use the eigenvalue criterion. (2pt)
      ii. Use the Lyapunov Theorem. (2pt)
   (b) Check if the system is controllable. (1pt)
   (c) Check if the system is observable. (1pt)
   (d) Obtain the Kalman decomposition. Write explicitly which state is controllable/uncontrollable and observable/unobservable. (4pt)
   (e) Descretize the system with sampling period $T$, where $T$ is a positive constant. (Hint: In this question, to obtain the matrix exponential, diagonalization method does not work.) (2pt)
   (You can use $\mathcal{L}(\frac{1}{s+a}) = e^{-at}$. $\mathcal{L}(\frac{1}{(s+a)^2}) = te^{-at}$.)

**Write your answer here.**

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**Write your answer here.**

2. For the following controllable discrete-time system:

$$x[k+1] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x[k] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[k],$$

compute the minimum energy control which transfers the state vector from $x[0] = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ to $x[k_f] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ for the cases when:

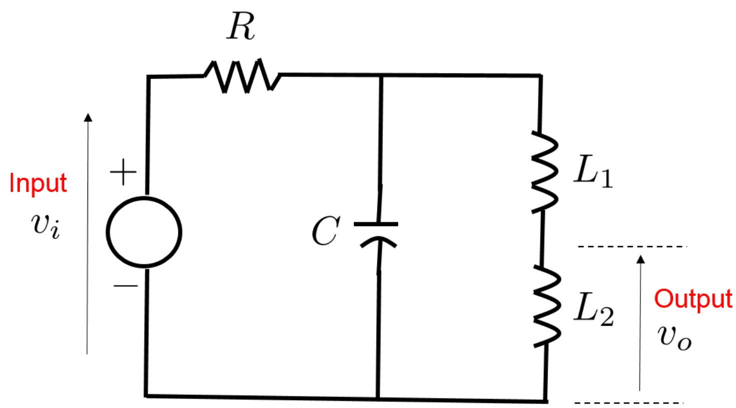
(a) the final time $k_f = 1$. (2pt)

(b) the final time $k_f = 3$. (2pt)

<u>**Write your answer here.**</u>

**Write your answer here.**

3. Derive a state-space model <u>with two states</u> for the following electric circuit. Here, $R$ is the resistance, $C$ is the capacitance, and $L_1$ and $L_2$ are the inductances. The input is the voltage $v_i$ and the output is the voltage $v_o$ (i.e., voltage across $L_2$). (4pt)




——————— (END OF EXAM) ———————

**Write your answer here.**

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