



MECH 420 **Sensors and Actuators**

Presentation Part 8

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Part 8

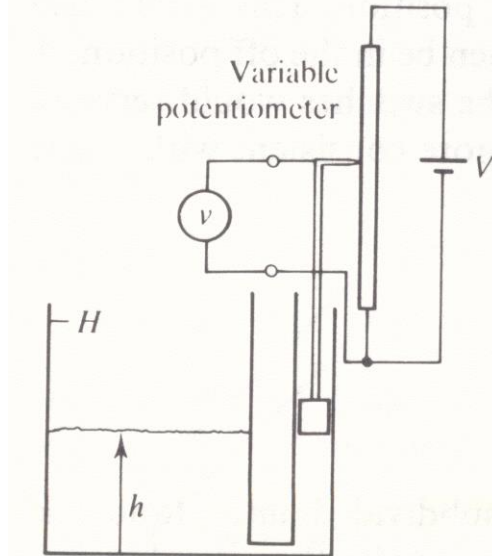
Digital and Other Transducers

Plan

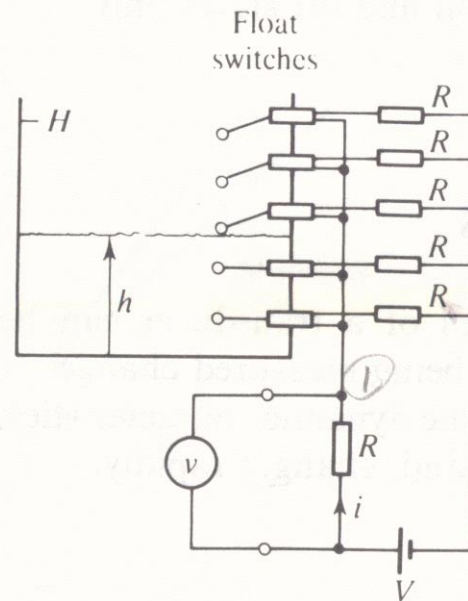
- **Advantages of Digital Transducers**
- **Terminology**
- **Applications**
- **Types and Examples of Digital Transducers**
- **Implementation of Digital Transducers
(Hardware, Computation)**
- **Other Transducers**

Example: Liquid Level Sensor

(a) Analog Sensor Method:
Float Analog Sensor + 2-bit ADC



(b) Digital Transducer Method:
4 Fixed Float Switches



Compare the two methods: Accuracy, complexity, cost, usefulness, robustness, etc.

Example: Liquid Level Sensor (Cont'd)

(a) Analog Sensor Method: Float, Analog Sensor, 2-bit ADC

1. Data accuracy is lost in sampling and digitization (**aliasing and quantization**), and cannot be recovered
2. Signal noise directly affects the results
3. Resolution can be improved by using more bits in ADC (**say, 4-bit**)
4. Less robust (**due to reasons 1 and 2**)
5. Direct and simple; but data acquisition in computer is more complex and costly
6. Increasing the digital resolution is simple and inexpensive (**see 3**)
7. Entirely fails if the sensor fails
8. **Anything else?**

(b) Digital Transducer Method: 4 Fixed Float Switches (**two-state**)

1. The 2-bit accuracy is precisely retained even if the float switch signal has high noise (**up to $\frac{1}{2}$ bit**)
2. The resolution is fixed by the number of float switches
3. More robust (**due to reason 1**)
4. More components (**less reliable**), but failure would be partial, not total
5. Easier to acquire data into computer
6. Increasing the resolution is more expensive (**need more float switches; use other digital choice**)
7. Operates even if a float switch fails
8. **Anything else?**

What are: aliasing, quantization? Does a digital transducer introduce these? Give another digital option, and compare with analog.

Digital Transducer

- Produces a **discrete or digital output** without using an **ADC**

Possible Outputs:

- **Pulse signal or pulse count**
- **Frequency** (→ count of cycles or pulse rate)

Note:

- When output is pulse signal or frequency: Use a counter to get signal value (e.g., displacement); use pulse rate or frequency to get signal rate (e.g., speed)
- For slow pulse rate, **count clock cycles** over a pulse duration or frequency period
- Transducer output can be converted to **coded form** – Binary, BCD (Binary Coded Decimal), ASCII

Sensing and Transducer Stages

- Physical systems are typically continuous time systems
→ Sensing stage is analog (and integral with the physical system, e.g., encoder plate is attached to the moving object)
- Transducer stage generates a discrete output (pulse train, count, frequency, encoded data, etc.). This is the key stage (hence the term “digital transducer” not digital sensor; the transducer stage is the key here, and defines the sensing device)

Note: In analog sensors (transducers) both sensing and transducer stages are analog and defining

Advantages of Digital Transducers

Mentioned before

- Do not introduce quantization error
- Less susceptible to noise, disturbances, parameter variation etc. (data is generated/represented/transmitted/processed as bits: two state)—Robust
- Complex signal processing with very high accuracy and speed (Hardware implementation is faster than software implementation)
- High reliability: minimal analog hardware components
- Large amounts of data may be stored using compact, high-density methods (e.g., Cloud)
- Data can be stored/maintained for very long time periods without drift, environmental effects, etc.
- Fast data transmission (communication means commonly available) over long distances without significant time delay (unlike analog systems) or accuracy degradation (data loss)
- Use low voltages (e.g., 0-12 V DC) and low power
- Typically have low overall cost

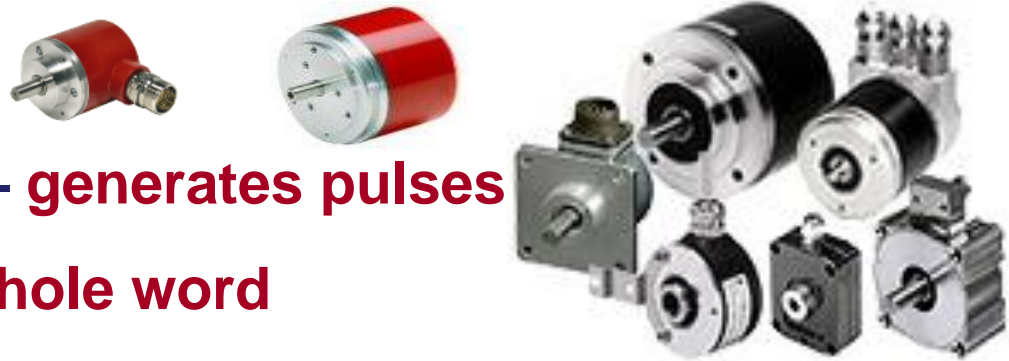
Shaft Encoders

Shaft Encoders

- These **digital transducers** are used for measuring **displacements and velocities**
- **Applications:** Robotic manipulators, machine tools, data storage systems, plotters, Battlebots, Mars Rover, large construction vehicles, semiconductor manufacturing equipment, printers and other rotating machinery, virtually any “**motion monitoring**” and control applications.
- **Advantages:** High resolution (**word size**), high accuracy (**noise immunity**), high robustness, high reliability, relative ease of adaption in digital systems (**associated reduction in cost**)

- **Two types**

- Incremental encoders – **generates pulses**
- Absolute encoders – **whole word**



Shaft Encoders

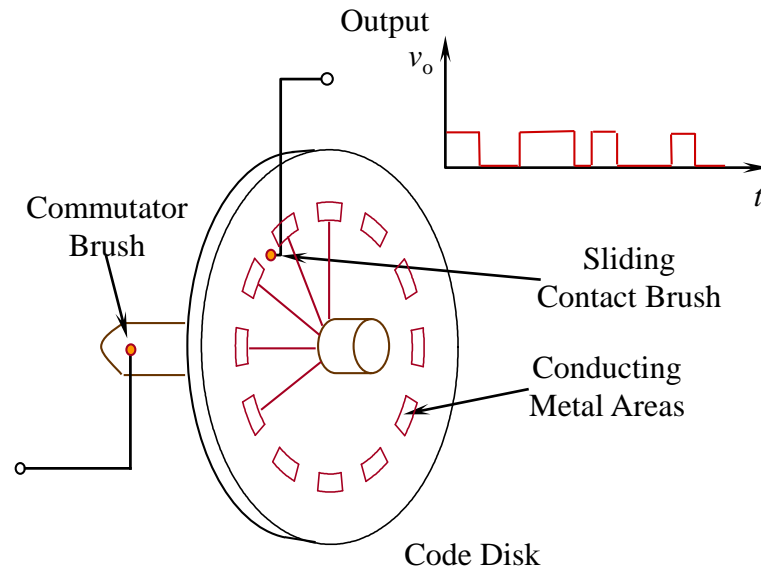
Techniques of transducer signal generation

- Optical (photosensor) method
- Sliding contact (electrical conducting) method
- Magnetic saturation (reluctance) method
- Proximity sensor method (studied before)

Optical encoder is the most popular

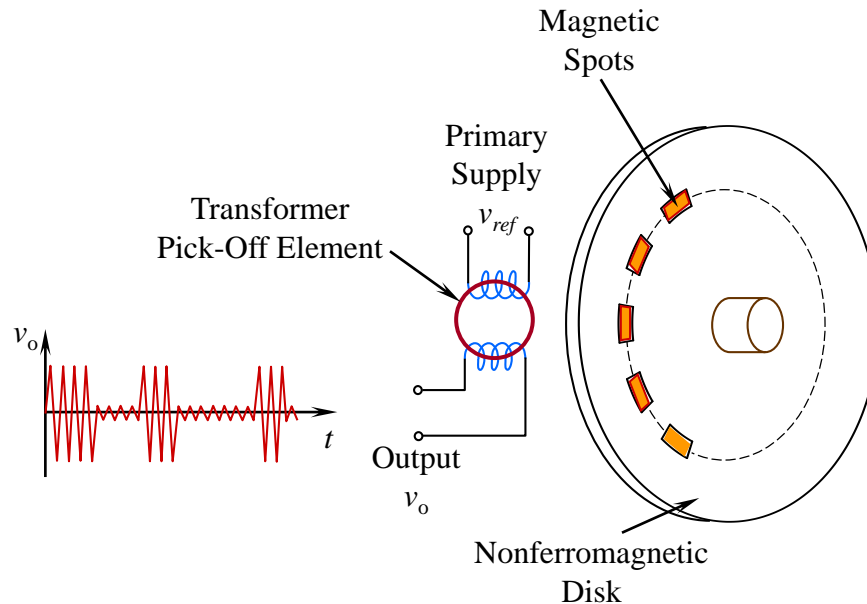


Sliding Contact Encoder (Self-study)



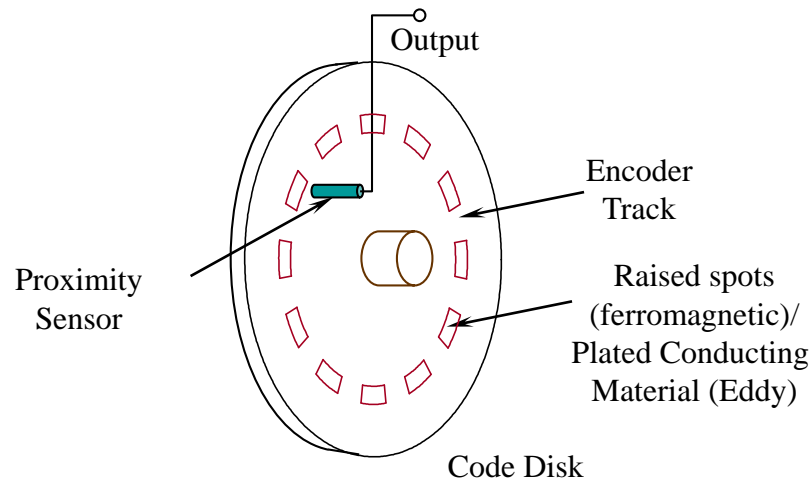
- **Advantages:** High sensitivity, simplicity of construction, low cost
- **Disadvantages:** Friction, wear, contact bounce, metal oxidation
- **Accuracy and resolution** depend on the number and printing of conducting patterns - electroplating

Magnetic Encoders (Self-study)



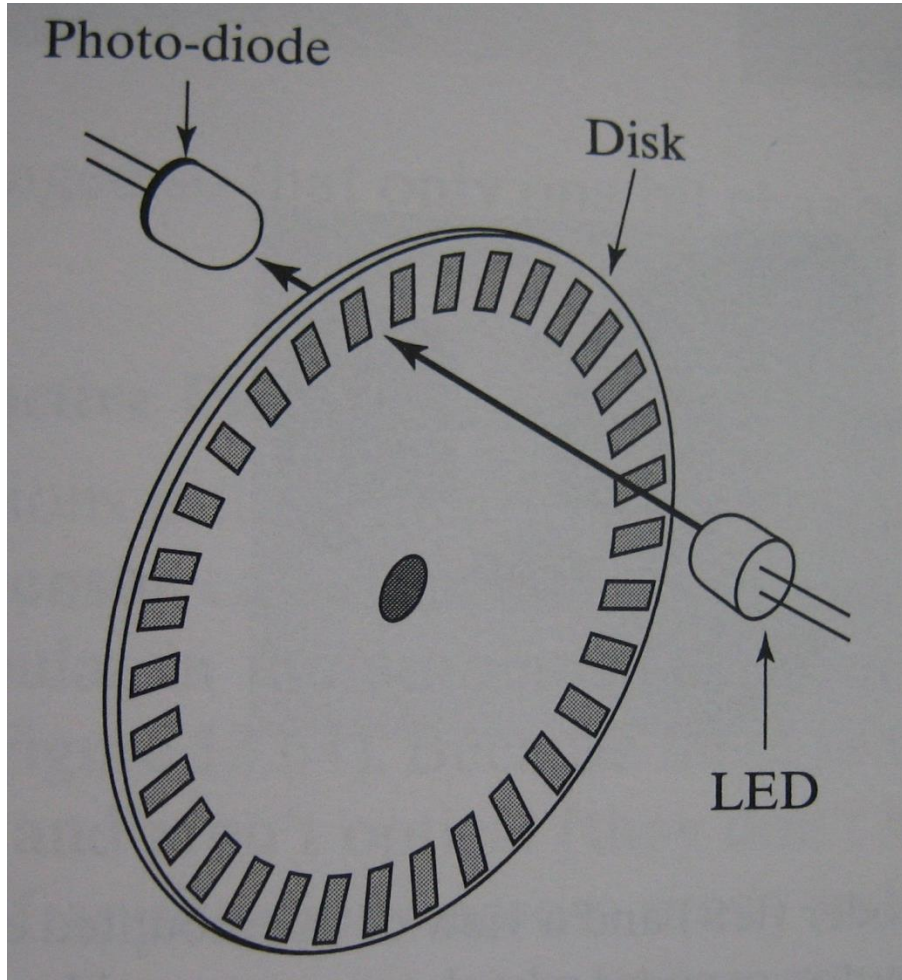
- **High strength magnetic areas are imprinted on the encoder disk – etching, stamping, recording**
- **Signal pick-off element is a micro-transformer**
- **Magnetic areas saturate the core of the transformer increasing the reluctance.**
- **This causes the induced voltage to drop**

Proximity Sensor Encoder (Self-study)

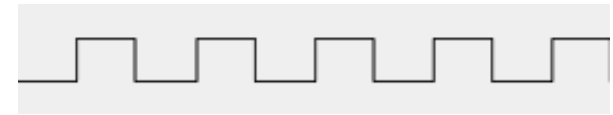
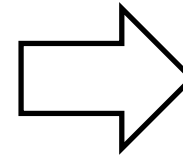


- Magnetic induction probe or Eddy current probe can be used
- **Magnetic induction** – disk is ferromagnetic, raised areas change the flux linkage
 - Reluctance decreases – flux linkage increases – induced voltage increases
- **Eddy current** – disk is plated with conducting material

Optical Encoders (Incremental): Most Common



Almost a pulse sequence



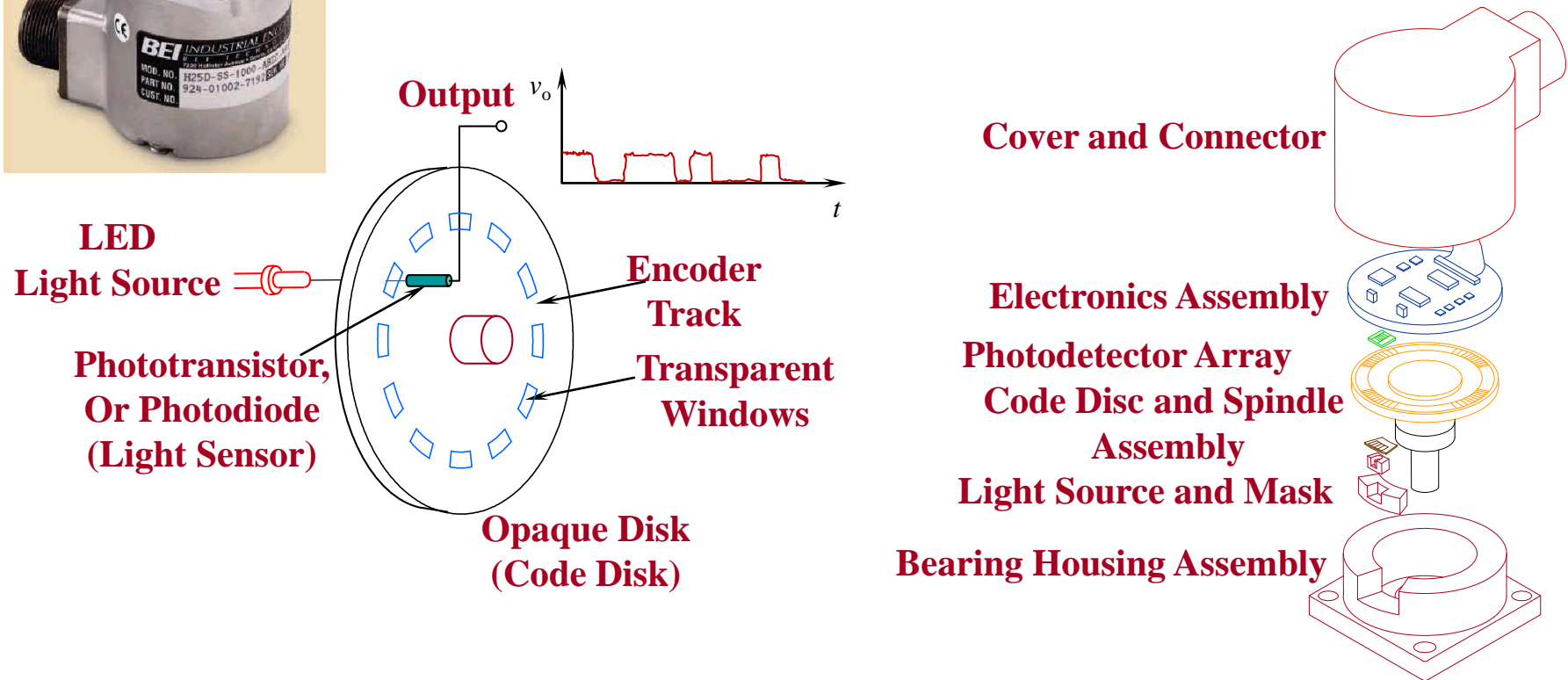
Clockwise (**CW**) rotation →
“Forward” rotation

Counter-clockwise (**CCW**)
rotation → “Backward”
rotation

Incremental Optical Encoders



Optical Encoder Components



- Accuracy and resolution depend on the number and the precision of the transparent windows on opaque background

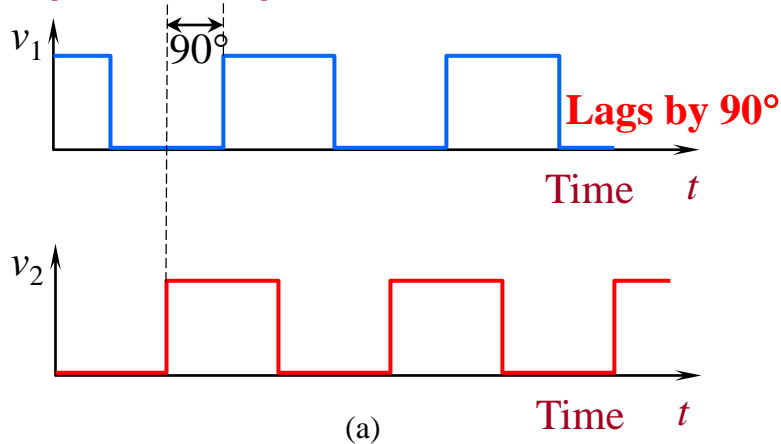
Incremental Optical Encoder Signals

Two Types: Which one do you prefer?

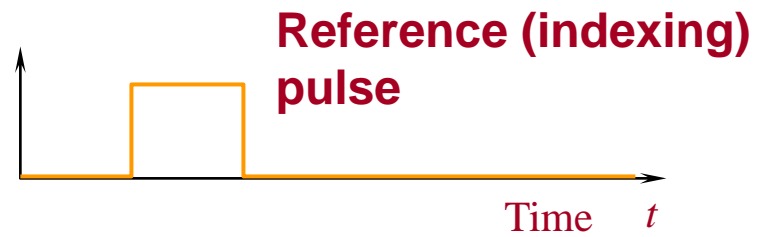
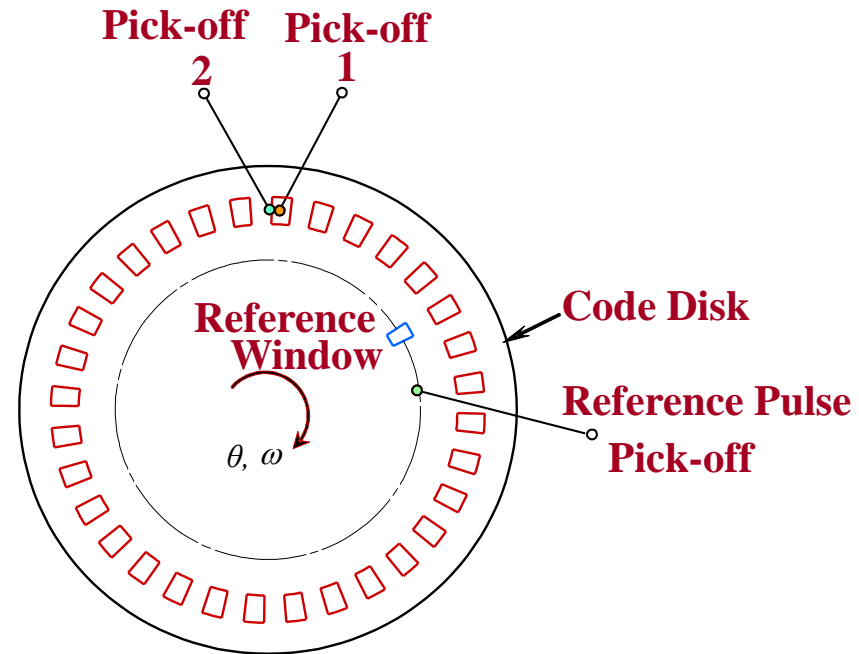
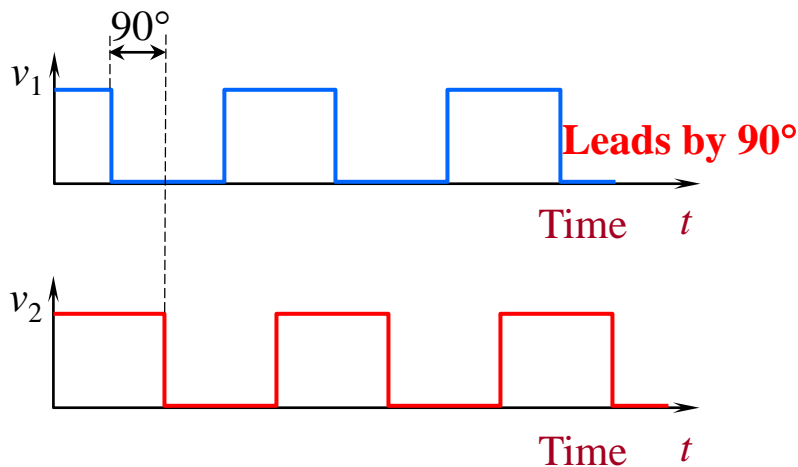
1. Offset sensor configuration
(2 sensors)

2. Offset track configuration
(2 tracks)

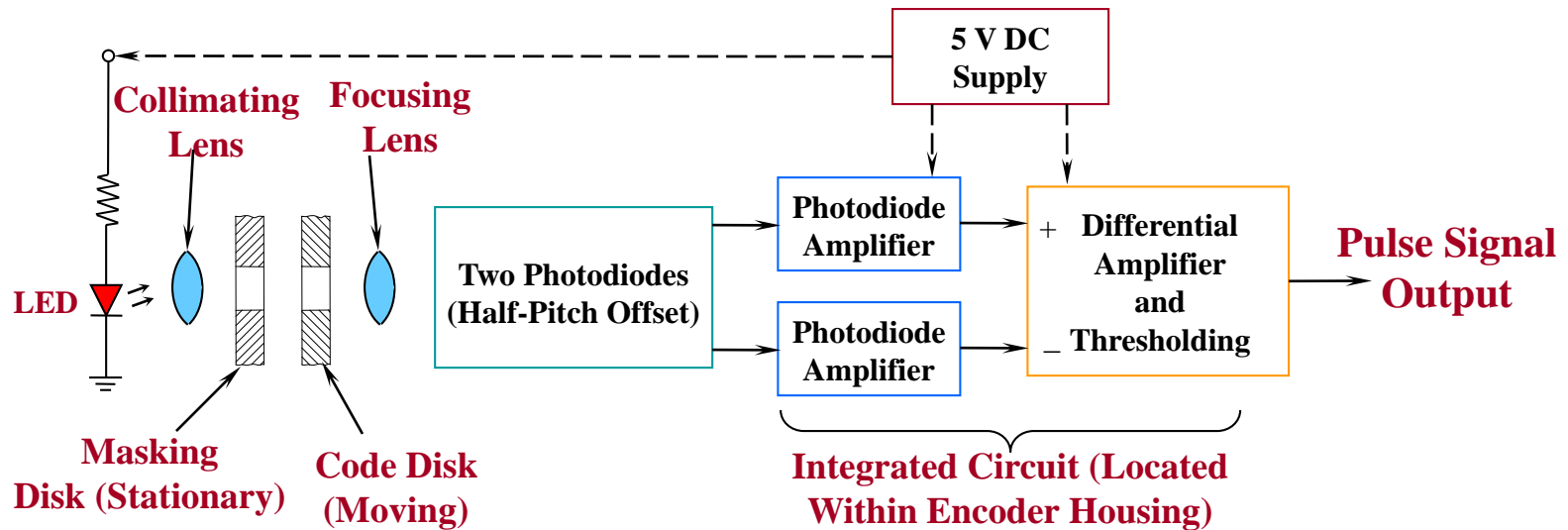
CW:



CCW:



Encoder Hardware Features (Further)

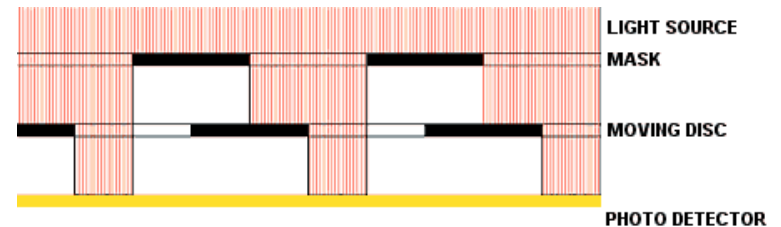
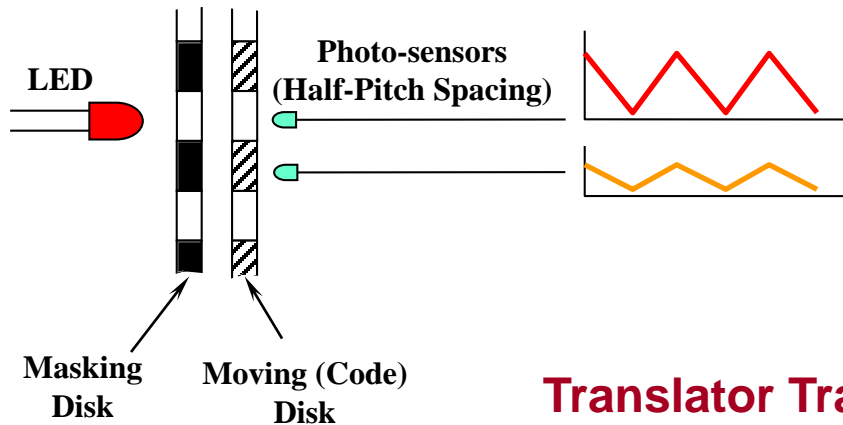


- **Collimating lens produces a pencil (parallel beam) of light**
- **Pencil of light passes through rotating (code) disk**
- **Masking disk (stationary) is identical to code disk**
- **When disks are aligned: More light passes through (because, goes through several windows depending on the size of light beam) → significantly improves pulse signal**
- **When misaligned (transparent to opaque): Nearly no light passes**

Encoder Hardware Features (Cont'd)

- To avoid errors (reading light as no light) due to supply voltage variation, another photosensor is placed **half pitch** away
- This sensor faces opaque region of masking (fixed) disk → produces a low signal (**provides a reference signal level**)
- A differential amplifier is used to detect the correct signal level
- Signal conditioning circuitry integrated within transducer housing to improve the pulse shape

Note: A similar setup is used to obtain the other quadrature signal



Translator Transducer is Similar to Rotatory Transducer

Half-pitch Arrangement

Direction Sensing with Encoders

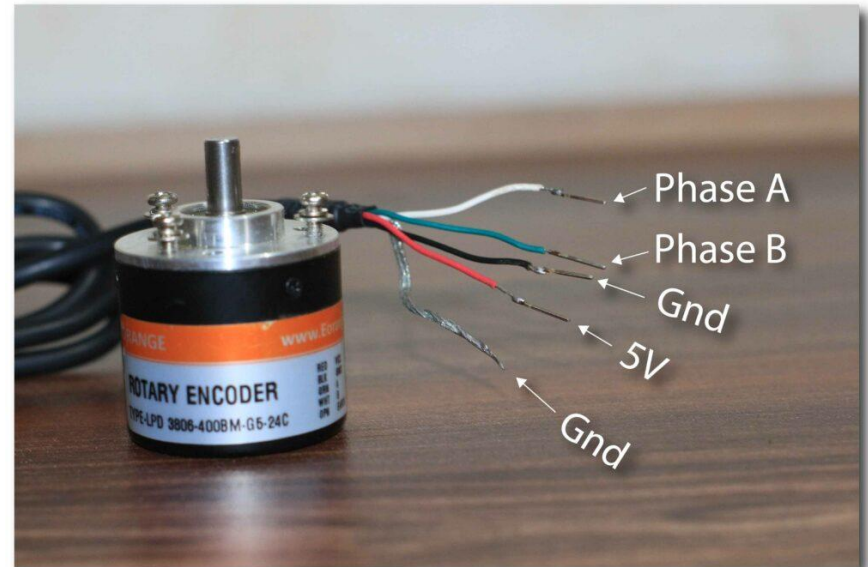
Optical Encoder Pins

5 pins (typical):

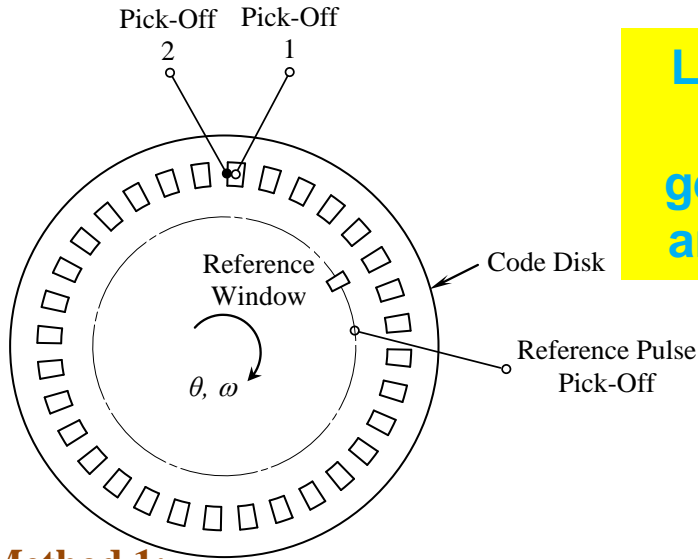
1. Ground
2. Index pulse
3. A Channel
4. +5V DC power supply
5. B Channel



Note: This 2nd encoder does not provide an index pulse (i.e., no full Rotation count)



Direction Sensing



Learn why these pulses are generated in ccw and cw rotations

Method 1:

Counting (timing) begins when the v_1 signal begins to rise (i.e., when a rising edge is detected).

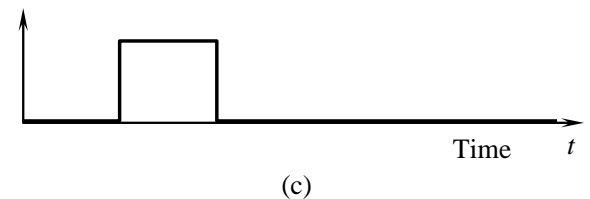
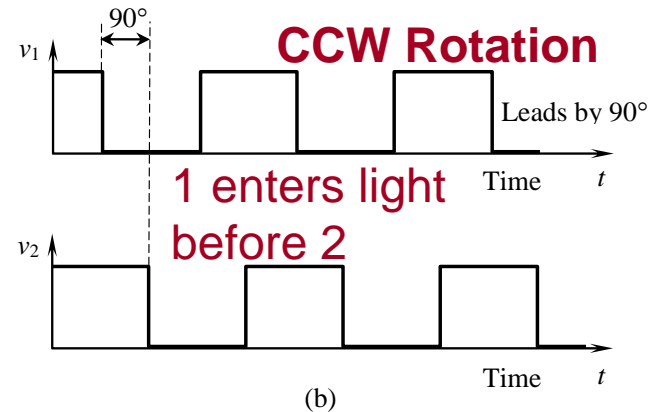
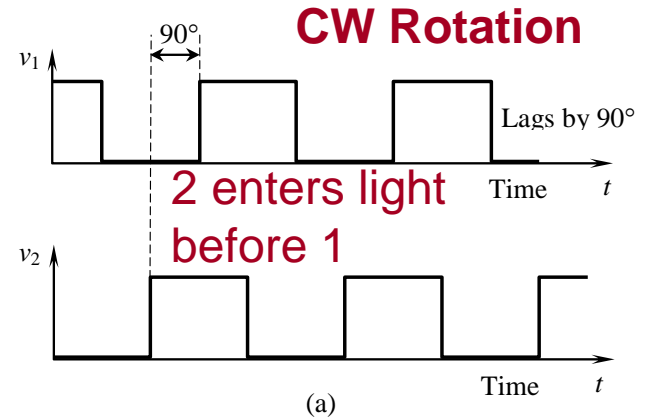
n_1 = number of clock cycles (time) up to time when v_2 begins to rise

n_2 = number of clock cycles up to time when v_1 begins to rise again.

If $n_1 > n_2 - n_1 \Rightarrow$ cw rotation

If $n_1 < n_2 - n_1 \Rightarrow$ ccw rotation.

Verify this



Other Methods for Direction Sensing

Method 2 (Direct Method):

Check phase angle between the two pulse trains:

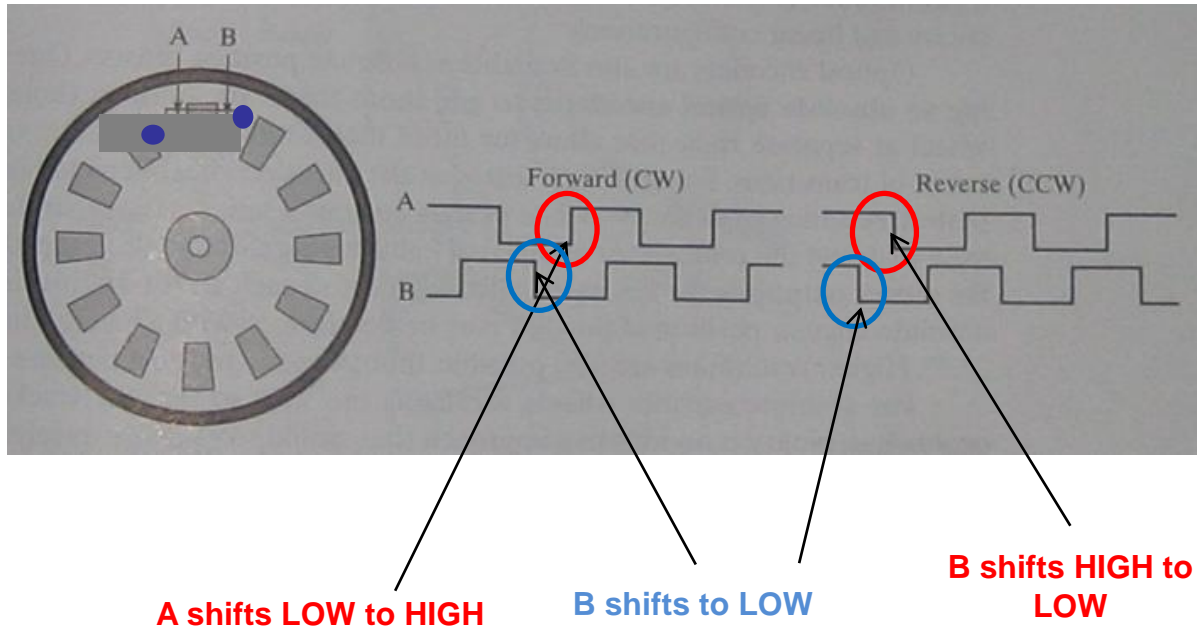
- CW rotation: v_1 lags v_2 by 90° **Verify this**
- CCW rotation: v_1 leads v_2 by 90°

Method 3:

Check edge of v_1 when v_2 is high:

- | | | | |
|---------------------|---------------|--------------|--------------------|
| If v_1 is rising | \Rightarrow | CW rotation | Verify this |
| if v_1 is falling | \Rightarrow | CCW rotation | |

Another Method (Method 4)



Logic:

1. Detect a High to Low transition in B-channel
2. If next transition in A-channel is Low to High → Forward (CW) rotation
If next transition in A-channel is High to Low → Backward (CCW) rotation

Angle and Speed Computation from Encoder Data

Motion Computation

Range of encoder = $\pm\theta_{\max}$; Maximum pulse count = M

For pulse count = n : Displacement $\theta = \frac{n}{M} \theta_{\max}$

Algorithm for Displacement and Speed Computation (Pulse Timing Method):

Off Line: Set time $T = 0$; Set rotation angle $A = 0$; Compute rotation angle during one pulse: $A = 2\pi / (4N)$; Have quadrature signals; N = number of windows in encoder disk track (known); ΔT = clock pulse period (in seconds)

On-line:

Step 1: Count clock pulses m from A-channel transition (High to Low or Low to High) to very next A-channel transition (High to Low or Low to High)

Step 2: Update, Displacement and Time: $D = D + A$; $T = T + m \times \Delta T$

Speed: $\omega = A / (m \times \Delta T)$

Speed Computation

Two Methods

- **Pulse counting method** – Count pulses over a sampling period; not good for low speeds
- **Pulse timing method** – Time one encoder cycle (using clock counts); suitable for low speeds

(a) Pulse Counting Method

Sampling period = T ; Corresponding pulse count = n

Average time for one pulse = T/n

Windows on disk track = N

Angle for one pulse (**if full pitch is used**) = $2\pi / N$

→ Speed $\omega = \frac{2\pi / N}{T / n} = \frac{2\pi n}{NT}$

Speed Computation (Cont'd)

(b) Pulse Timing Method

Clock frequency = f

Clock pulse (cycle) count for encoder period (between two windows) = m

→ Time between two windows = m/f

Angle between two windows = $2\pi / N$

→ Speed $\omega = \frac{2\pi / N}{m / f} = \frac{2\pi f}{Nm}$

Note 1: A single incremental encoder can serve as both position sensor and a speed sensor

Note 2: The resolution can be improved by a factor of 4 by using transitions of both channels (A and B) which are $\frac{1}{4}$ pitch apart.

Encoder Resolution

Factors Affecting Resolution

- **Position**

- Number of windows N
- Gear Ratio
- Word size

- **Speed (Velocity)**

- Number of windows N
- Sampling period T
- Clock frequency f
- Speed ω
- Gear Ratio

Displacement Resolution

Displacement: $\theta = \frac{n}{M} \theta_{\max}$

Physical Resolution (1 count, 1 pitch): $\Delta\theta = \frac{\theta_{\max}}{M}$

with θ_{\max} = maximum reading; M = # pulses at max reading

Digital Resolution: Change in displacement for “one bit change”

Data size = r bits ; Maximum count $M = 2^r$

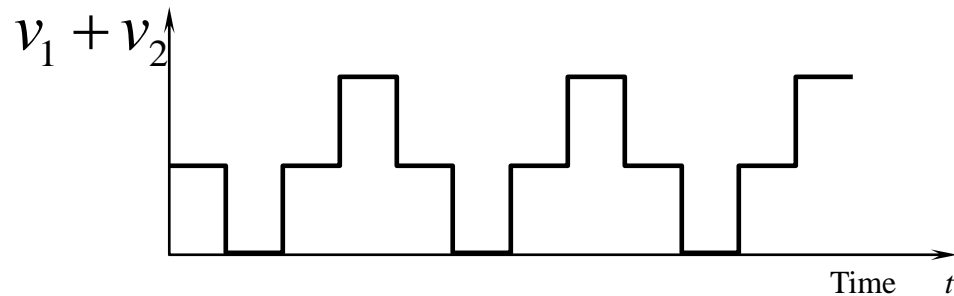
Digital resolution: $\Delta\theta_d = \frac{\theta_{\max}}{2^r} = \frac{\theta_{\max} - \theta_{\min}}{2^r - 1}$ **with** $\theta_{\min} = \frac{\theta_{\max}}{2^r}$

Physical Resolution When $\theta_{\max} = 2\pi$

1. Using one pulse signal and full pitch (full period):

Physical resolution $\Delta\theta_p = \frac{360^\circ}{N}$, where N = number of windows

2. Using both (quadrature) pulse signals (considering both rising and falling transitions): $\Delta\theta_p = \frac{360^\circ}{4N}$



Note: Gearing up is advantageous for physical resolution

Note: Digital resolution $\Delta\theta_d = \frac{(\theta_{\max} - \theta_{\min})}{(2^r - 1)}$

What is the effect of gearing on digital resolution?

- The larger of physical and digital resolutions governs the displacement resolution

Speed Resolution

- Both pulse-counting and pulse-timing methods are based on counting →
Speed resolution: speed change corresponding to count change by one

1. Pulse Counting Method

Speed $\omega_c = \frac{2\pi n}{NT}$ → **Resolution** $\Delta\omega_c = \frac{2\pi}{NT}$

- Resolution improves with increased N (# widows) and T (pulse collected period)

2. Pulse Timing Method (clock frequency f)

$$\Delta\omega_t = \frac{2\pi f}{Nm} - \frac{2\pi f}{N(m+1)} = \frac{2\pi f}{Nm(m+1)} \quad \Delta\omega_t \approx \frac{2\pi f}{Nm^2} = \frac{N\omega^2}{2\pi f}$$

- Speed resolution degrades quadratically with speed, and linearly with N (for a given speed). Resolution can be improved by improving f
- Gearing up has an adverse effect on the speed resolution of pulse timing but has favorable effect in the pulse counting method

Why?

Example

For an ideal design of an incremental encoder, obtain an equation relating the parameters d , w , and r , where

d = diameter of encoder disk

w = number of windows per unit diameter of disk

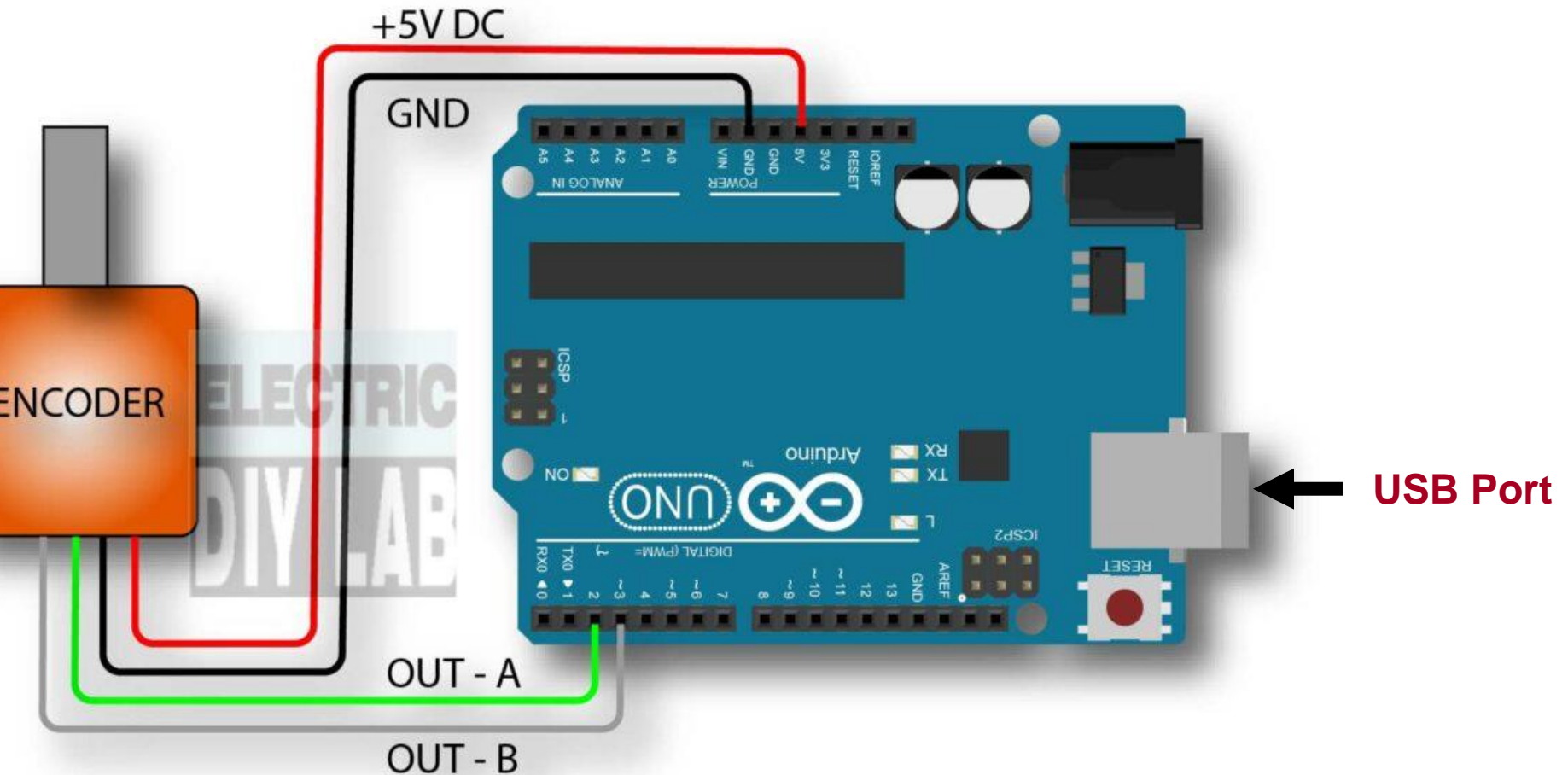
r = word size (bits) of the angle measurement

Assume that quadrature signals are available. If $r = 12$ and $w = 500/\text{cm}$, determine a suitable disk diameter.

Encoder Sensing Hardware

Encoder Wiring to a Microcontroller (Arduino)

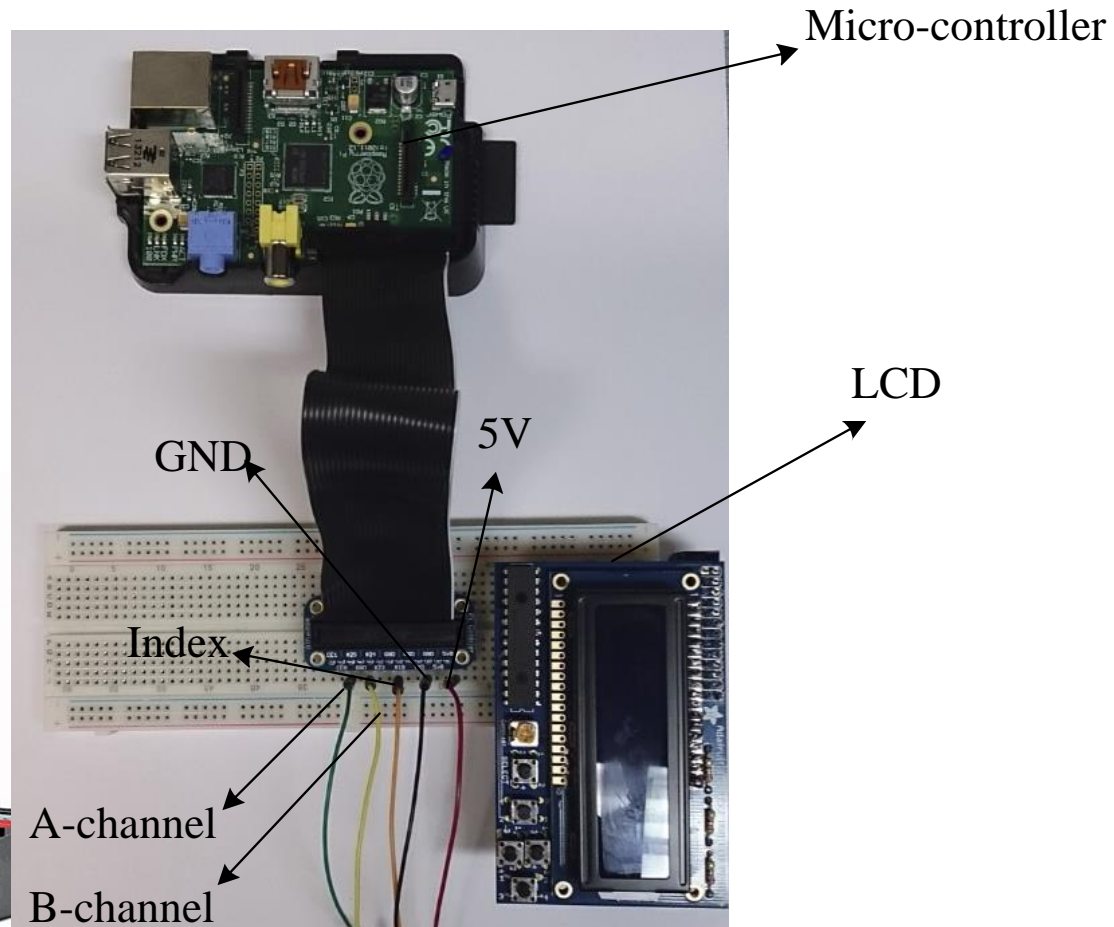
The microcontroller may be programmed and also the encoder readings may be acquired and recorded using a host computer, though USB port.



Example Hardware Setup

DC motor with Optical Encoder; Microcontroller with digital input channels; LCD display (**Note:** Power for the encoder may come from a separate dc power supply or from the microcontroller)

A dc motor with
an optical
encoder



Direction, Angular Rotation and Speed Computations

Direction Determination Scheme:

- Detect a High to Low transition in B-channel
- If next transition in A-channel is Low to High → Forward rotation;
If next transition in A-channel is High to Low → Backward rotation

Pseudocode for Angle and Angular Speed Computation (Pulse-Timing Method):

- **Off the Loop: Set:** time $T = 0$, rotation angle $A = 0$; **Compute** rotation angle during one pulse $A = 2\pi / (4N)$; **Known:** N = number of windows on disk track; Quadrature signals & clock period ΔT
- **Loop:**
- Count clock pulses from A-channel transition (High to Low or Low to High) to the very next A-channel transition (High to Low or Low to High). **Call it m**
- **Update Time and Displacement:** $T = T + m \times \Delta T$; $D = D + A$
- **Compute Speed:** $W = A / (m \times \Delta T)$

Modify this algorithm for pulse timing method.

Questions

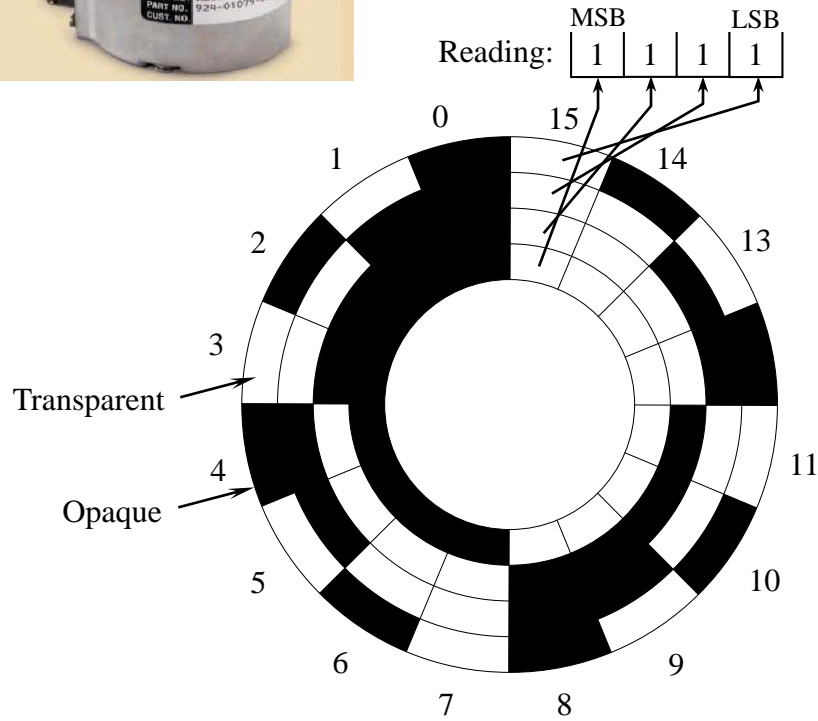
3 advantages of an optical encoder over a rotary pot:

1 disadvantage of an optical encoder over a rotary pot:

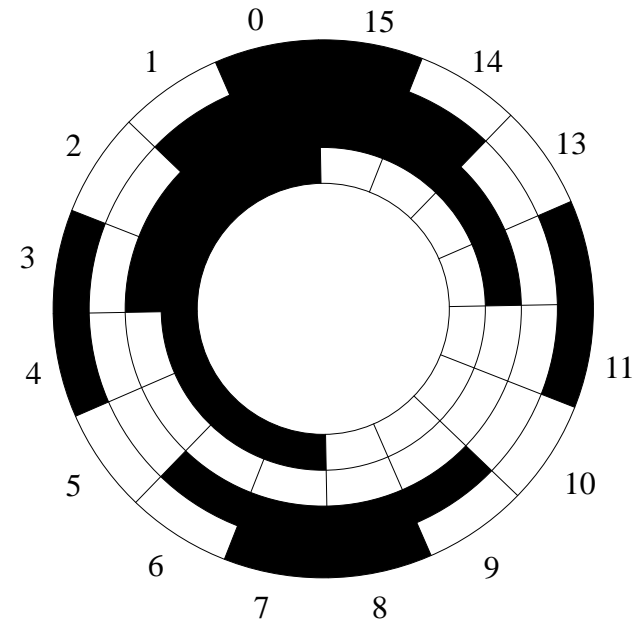
Absolute Encoder



Absolute Optical Encoder



Binary Code



Gray Code

For n tracks, there are n pick-off elements; Disk is divided into 2^n sectors.

Example: If $n = 4$ (as in figure) $\Delta\theta = \frac{360^\circ}{2^4} = 22.5^\circ$

Data word uniquely determines the position (absolute) at the time

Gray Coding

- A disadvantage with binary coding: Many bits can switch in one transition

→ Errors and ambiguity, if pick-off elements are not properly aligned or if the codes not properly printed (**bit switching won't happen simultaneously**)

Consider transition from 0011 – 0100. If 3rd bit switches first → 0001
→ indicates a rotation in opposite direction

- In gray code:

Each transition involves
only one bit change

Sector Number	Straight Binary Code (MSB → LSB)	A Gray Code (MSB → LSB)
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 0	0 0 1 1
3	0 0 1 1	0 0 1 0
4	0 1 0 0	0 1 1 0
5	0 1 0 1	0 1 1 1
6	0 1 1 0	0 1 0 1
7	0 1 1 1	0 1 0 0
8	1 0 0 0	1 1 0 0
9	1 0 0 1	1 1 0 1
10	1 0 1 0	1 1 1 1
11	1 0 1 1	1 1 1 0
12	1 1 0 0	1 0 1 0
13	1 1 0 1	1 0 1 1
14	1 1 1 0	1 0 0 1
15	1 1 1 1	1 0 0 0

Absolute Encoder

Advantages and Disadvantages

- Measurement is in coded form—does not require digital counters or buffers
- If a reading is missed (e.g., power loss) it will not affect the subsequent readings
- More expensive than incremental encoder (Complex tracks on code disk, number of sensors, complex electronics, complex I/O)

Encoder Error (Self-study)

Encoder Error

- Shaft encoder errors can arise from several factors
 - Assembly error (eccentricity of rotation, etc.)
 - Coupling error (gear backlash, belt slippage, loose fit, etc.)
 - Structural deformations (disk deformation and shaft deformation due to loading)
 - Manufacturing tolerances (errors from inaccurately imprinted code patterns, inexact positioning of the pick-off sensors)
 - Ambient effects (vibration, temperature, light noise, humidity, dirt, smoke, etc.)

Eccentricity Error

- Shaft eccentricity (e_s) – axis of rotation is different from geometric axis
- Assembly eccentricity (e_a) – center of code disk is not on the shaft axis
- Track eccentricity (e_t) – center of the track is not the center of the disk
- Radial play (e_p) – looseness in the assembly in radial direction

Encoder Error Combination

Consider 4 error parameters: Structural, assembly, ambient (thermal), and manufacturing (production)

Suppose random with, Mean: $\mu_s, \mu_a, \mu_t, \mu_p$; Std: $\sigma_s, \sigma_a, \sigma_t, \sigma_p$

Upper bound (SRSS) for mean value for overall eccentricity:

$$\mu = \sqrt{\mu_s^2 + \mu_a^2 + \mu_t^2 + \mu_p^2}$$

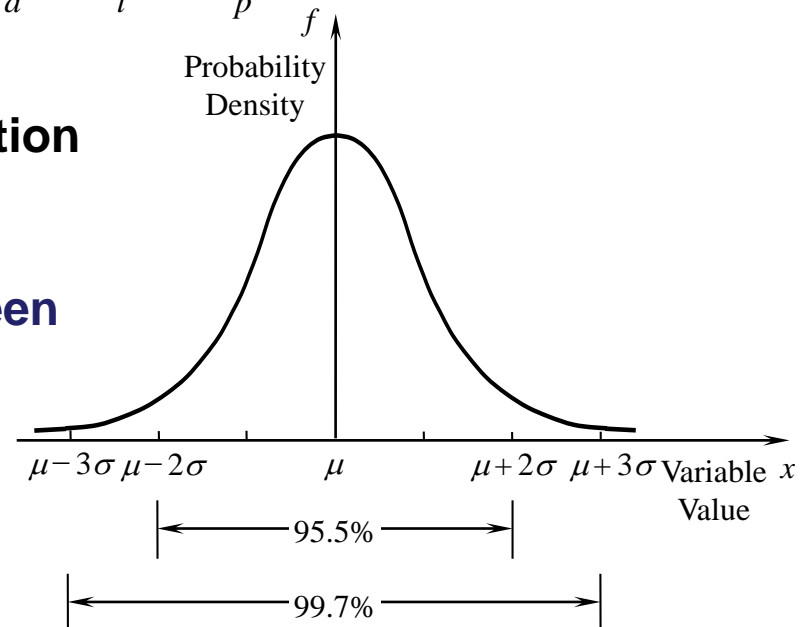
If they are independent: $\sigma = \sqrt{\sigma_s^2 + \sigma_a^2 + \sigma_t^2 + \sigma_p^2}$

Assume: Eccentricity has Gaussian distribution

Probability that eccentricity would fall between

$\mu - 2\sigma$ and $\mu + 2\sigma$ is 95.5%

$\mu - 3\sigma$ and $\mu + 3\sigma$ is 99.7%



Eccentricity Error

True angle = θ ; Measured angle = θ_m \rightarrow Eccentricity error = $\Delta\theta = \theta_m - \theta$

Maximum error occurs when the line of eccentricity is symmetrically located within angle of rotation:

$$\frac{\sin(\Delta\theta/2)}{e} = \frac{\sin(\theta/2)}{r}$$

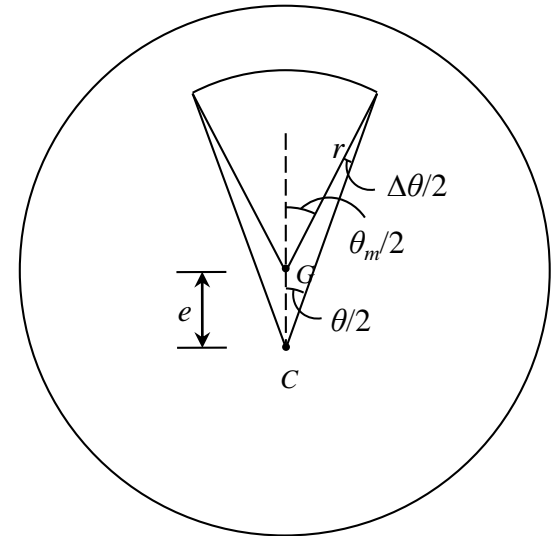
$$\Delta\theta = 2 \sin^{-1} \left(\frac{e}{r} \sin \frac{\theta}{2} \right)$$

For small angles

$$\Delta\theta = \frac{2e}{r} \sin \frac{\theta}{2}$$

Maximum error occurs when $\theta = \pi$

$$\Delta\theta_{\max} = 2 \sin^{-1} \frac{e}{r} \quad \longrightarrow \quad \Delta\theta_{\max} = \frac{2e}{r}$$



Example

The mean values and the standard deviations of the four primary contributions to eccentricity in a shaft encoder are as follows (in millimeters):

Shaft eccentricity = (0.1, 0.01)

Assembly eccentricity = (0.2, 0.05)

Track eccentricity = (0.05, 0.001)

Radial play = (0.1, 0.02)

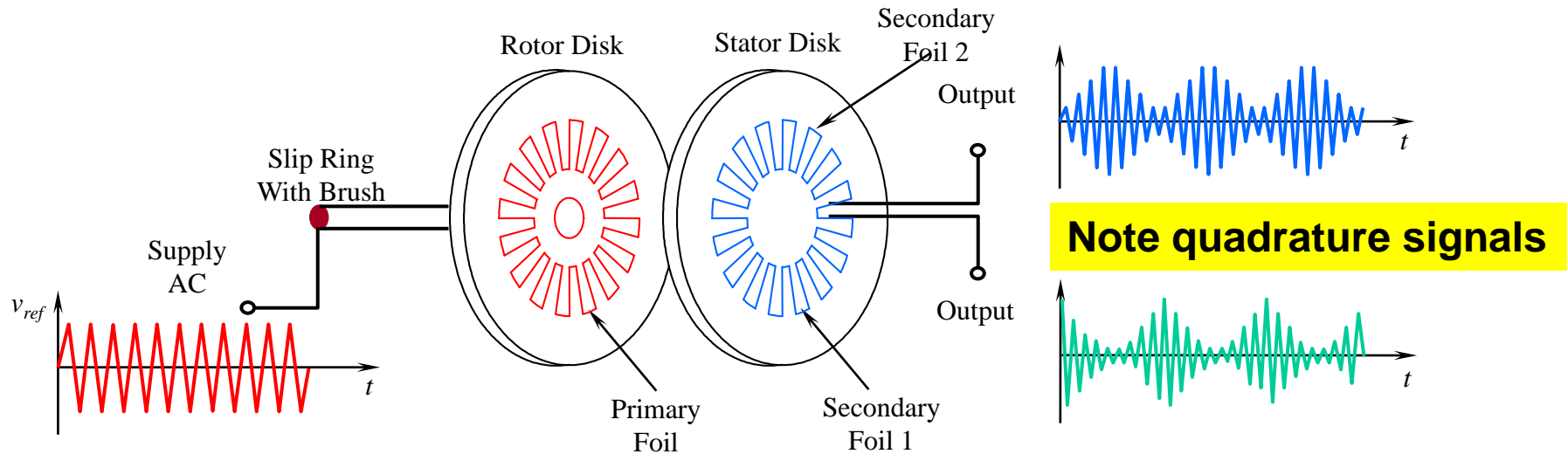
Estimate the overall eccentricity at a confidence level of 96 percent.

Example

Suppose that in the previous example, the radius of the code disk is 5 cm. Estimate the maximum error due to eccentricity. If each track has 1,000 windows, determine whether the eccentricity error is significant.

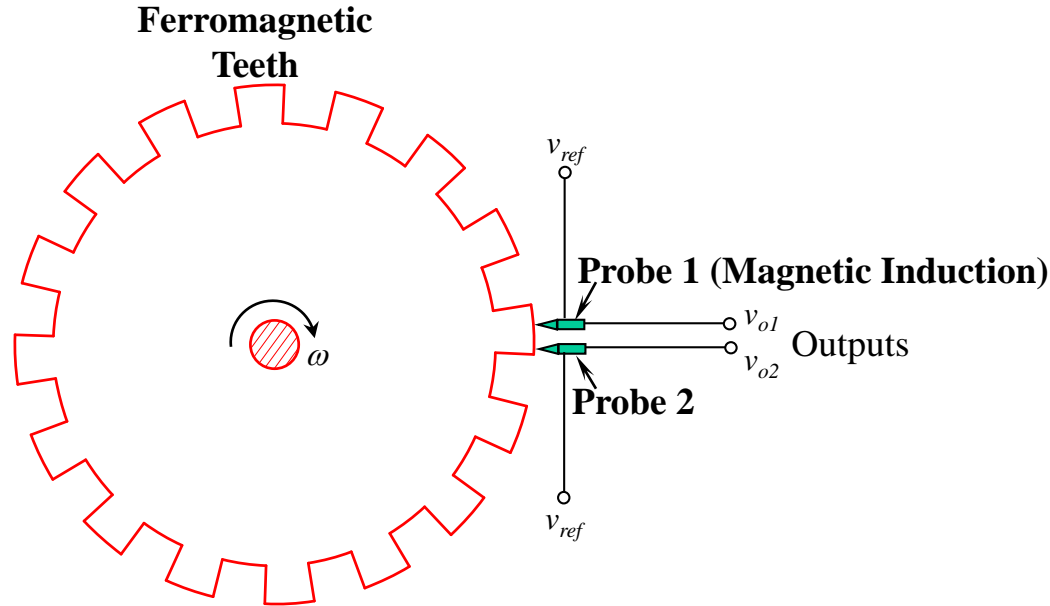
Other Transducers

Digital Resolver



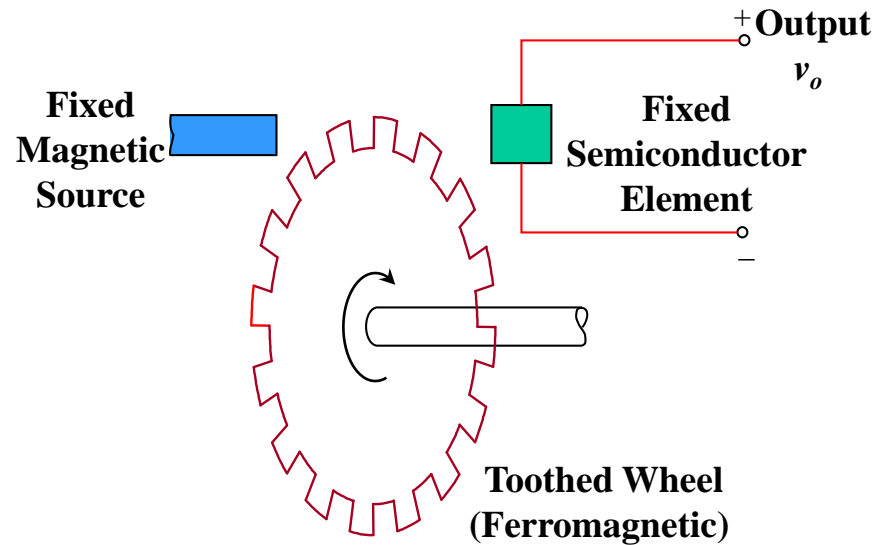
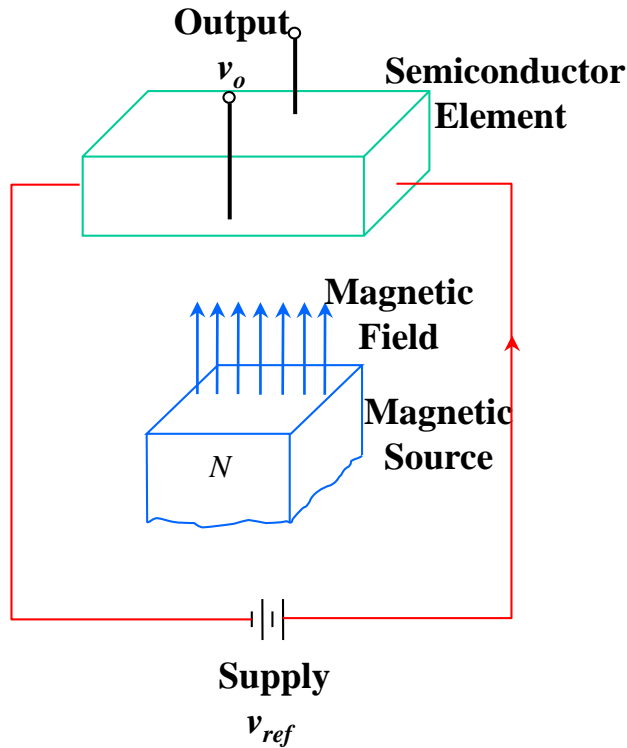
- Stator has two patterns imprinted $\frac{1}{4}$ pitch apart
- When rotor pattern aligns with stator pattern \rightarrow maximum induced voltage. When rotor pattern is half-pitch from stator pulse pattern \rightarrow minimum induced voltage \rightarrow “pulse” (or “beat”) signal
- Two stator signals have a 90° phase difference (lead/lag) \rightarrow Direction sensing
- Pulse counting or pulse timing is used for velocity measurements (see before)
- Very fine resolutions (up to 0.0005°) are possible

Digital Tachometer



- Toothed wheel made of ferromagnetic material
- Two proximity probes (**magnetic induction type**) $1/4$ pitch apart
- As wheel turns \rightarrow “reluctance” (**field strength**) at probe changes \rightarrow two pulse signals with 90° phase shift \rightarrow directional sensing (**see incremental encoder**)
- Angular velocity: Computed using pulse counting or pulse timing
- Advantages: Simplicity, robustness, low cost. Disadvantages: Poor resolution, mechanical loading (**due to wheel inertia**)

Hall Effect Sensors



Semiconductor Element; Apply voltage and magnetic field perpendicular to each other

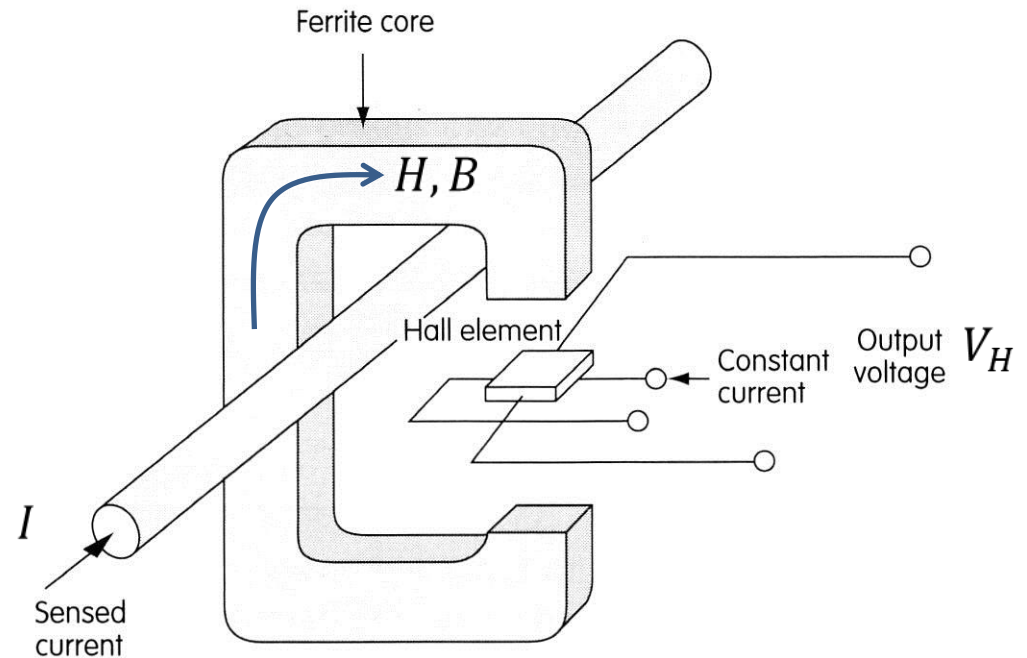
Produces a voltage orthogonal to them

Ferromagnetic wheel (rotates with sensed object) changes flux linkage

Produces pulse signal → Shaft encoder or digital tachometer

Current Measurement Using Hall Effect

- Conductor current (I) generates magnetic field, concentrated **(in vertical direction)** by ferrite core
- Current applied to Hall-effect **(semiconductor)** element in orthogonal **(lateral)** direction
- Voltage V_H generated by Hall-effect element **(in the 3rd orthogonal direction)** → a measure of current I

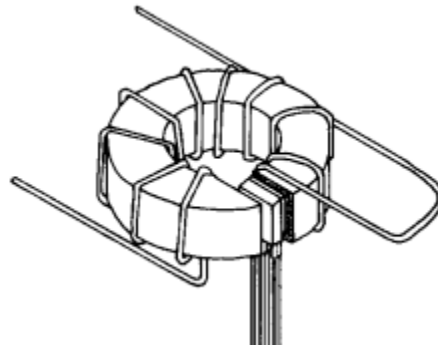


Toroidal Current Sensor



current detectors by
Honeywell
(25 A < I < 120 A)

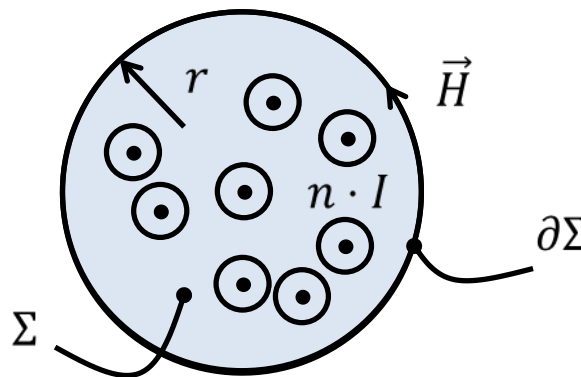
Multiple turns (n)
for measuring low
currents



**Senses the generated
magnetic field
strength**

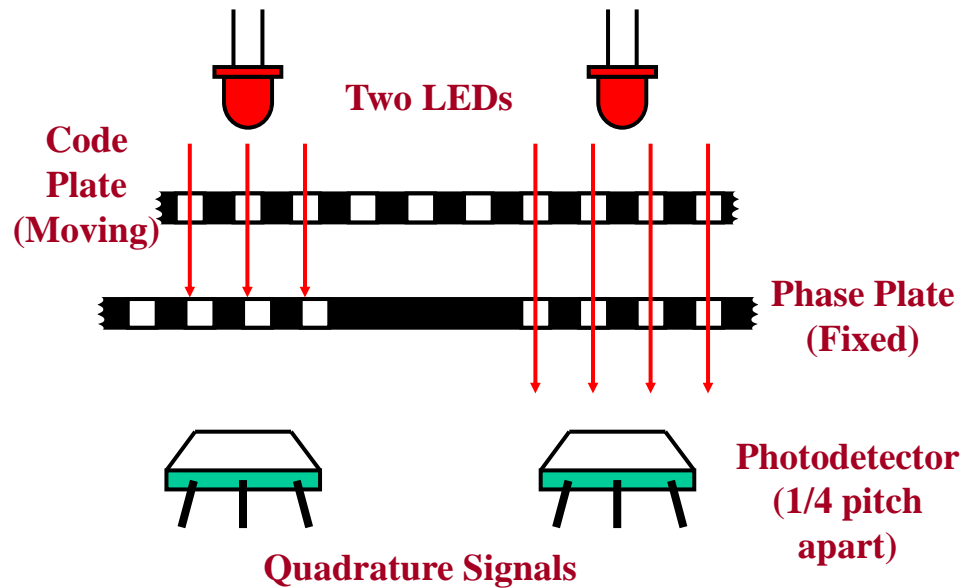
$$\oint_{\partial \Sigma} \vec{H} d\vec{s} = \iint_{\Sigma} \vec{J} d\vec{a}$$

$$n \cdot I = \iint_{\Sigma} \vec{J} d\vec{a}$$



$$B = \frac{\mu_0 \mu_r n I}{2\pi r}$$

Linear Encoders



- Operation is similar to shaft encoders
- $\frac{1}{4}$ pitch off probes produce quadrature signals → Direction sensing (as before)
- $\frac{1}{2}$ pitch probe → Improved output (pulse) signal (as before)