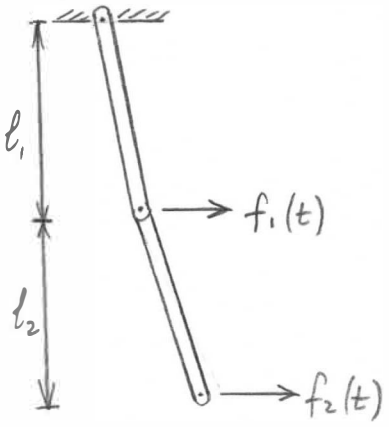
MECH 463 - Tutorial 9

1. A double compound pendulum consists of two uniform rods of lengths ℓ_1 and ℓ_2 and masses m_1 and m_2. Horizontal forces $f_1(t)$ and $f_2(t)$ act at the lower ends of the rods, as shown. Assume small motions.

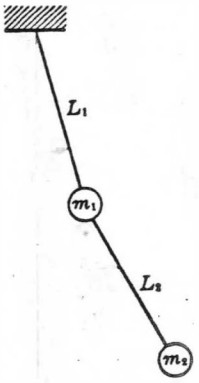


   (a) Draw the free body diagrams of the system and then determine the equations of motion. Express your equations in symmetric matrix form.

   (b) Determine the kinetic and potential energies of the system. Use Lagrange's equations to formulate the matrix equation of motion. Verify that the matrices are symmetric and are the same as found in (a).

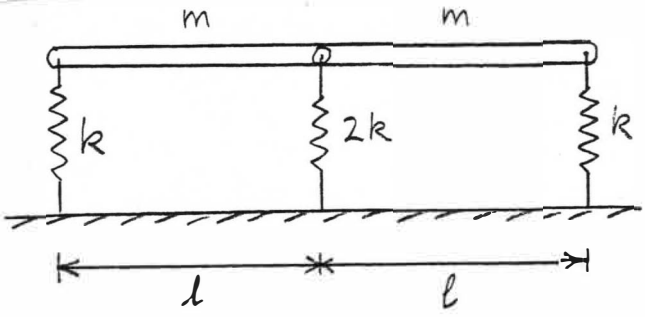
   (Only asked to formulate the equations, not to solve them.)

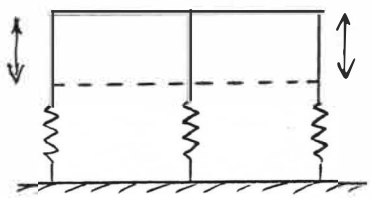
2. The diagram shows the double pendulum considered in Homework 1, question 1. Use Lagrange's equations to formulate the full non-linear equations of motion. Linearize your result and verify that it is the same as found before.



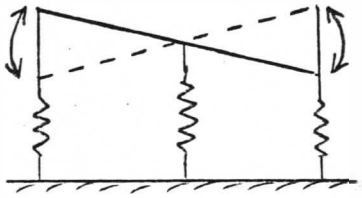
   (Only asked to formulate the equations, not to solve them.)

3. The diagram shows the vibrating system considered previously. Choose a convenient coordinate system and use Lagrange's equation to formulate the equations of motion in matrix form. Express the illustrated mode shapes in terms of your coordinate system. Verify that these mode shapes are orthogonal. Transform your equations of motion into the principal coordinates, with diagonal mass and stiffness matrices $\underset{\sim}{M}^*$ and $\underset{\sim}{K}^*$.

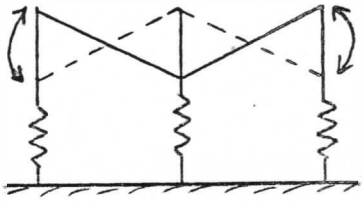




mode 1, $\omega^2 = \frac{2k}{m}$



mode 2, $\omega^2 = \frac{3k}{m}$



mode 3, $\omega^2 = \frac{6k}{m}$