< Operational Amplifier - Dynamic Model >

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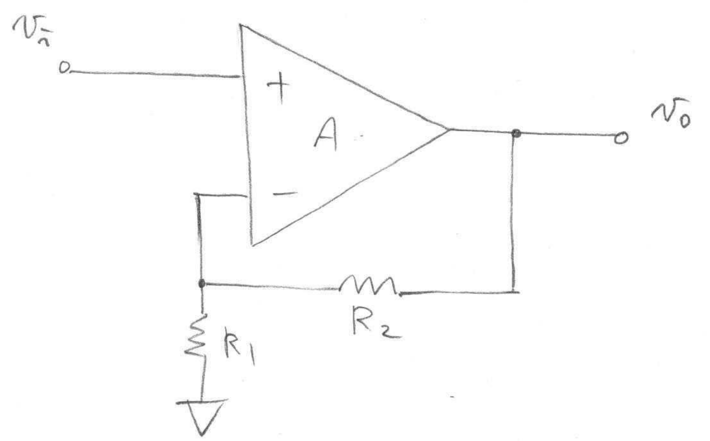
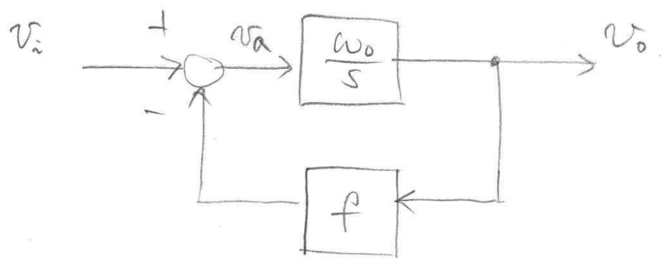


Assume :
$\begin{cases} Z_i \to \infty \\ Z_o \to 0 \end{cases}$

① $A(s) = \dfrac{\omega_0}{s}$.



- Note that $|A| \to \infty$ as $\omega \to 0$.
- ω_0 is a characteristic frequency of an op-amp.
- For op27. $|A(f)|_{f=10\,kHz} \simeq 60\ dB = 10^3$

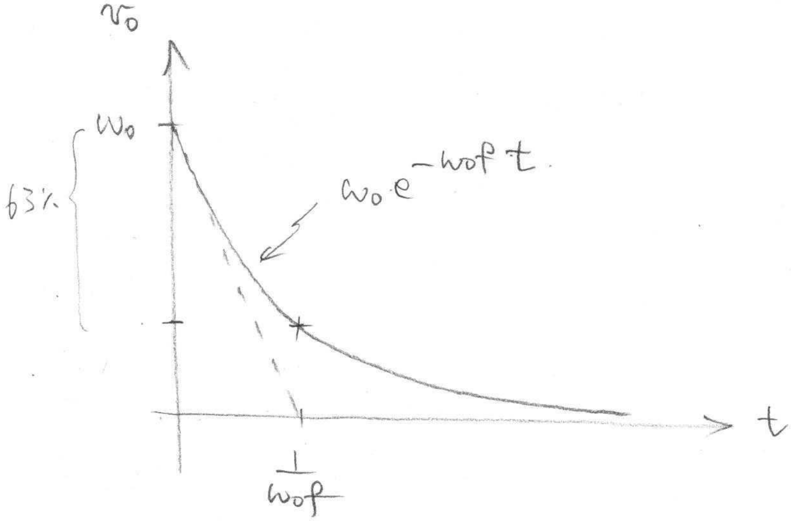
$$\left| \frac{\omega_0}{2\pi \cdot 10^4} \right| = 10^3 \qquad \therefore \omega_0 \simeq 2\pi \cdot 10^7\ [rad/s].$$
$$f_0 \simeq 10\ MHz.$$

$$G(s) = \frac{\frac{\omega_0}{s}}{1 + \frac{\omega_0}{s} f}$$

$$= \frac{\omega_0}{s + \omega_0 f}$$ "Evans Form" → Directly shows pole location

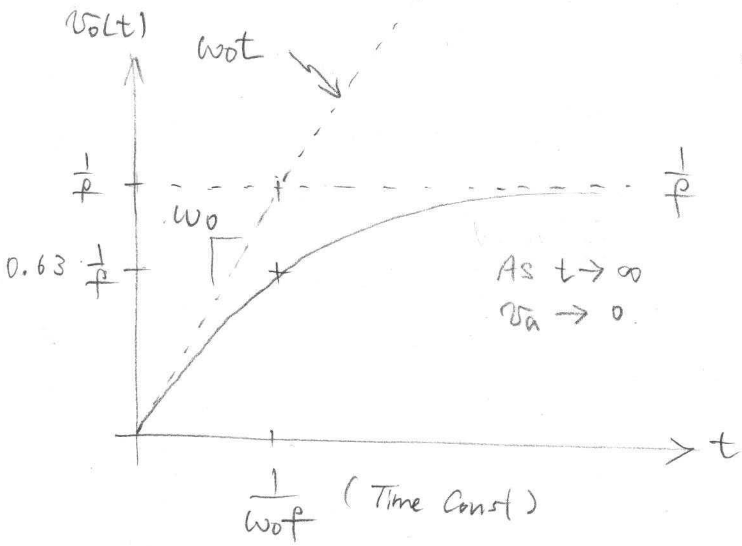
$$= \frac{1}{f} \left(\frac{1}{\frac{1}{\omega_0 f} s + 1} \right)$$ "Bode Form"
→ Directly shows DC gain & time constant.

∘ Impulse Response. ($v_i = \delta(t)$)

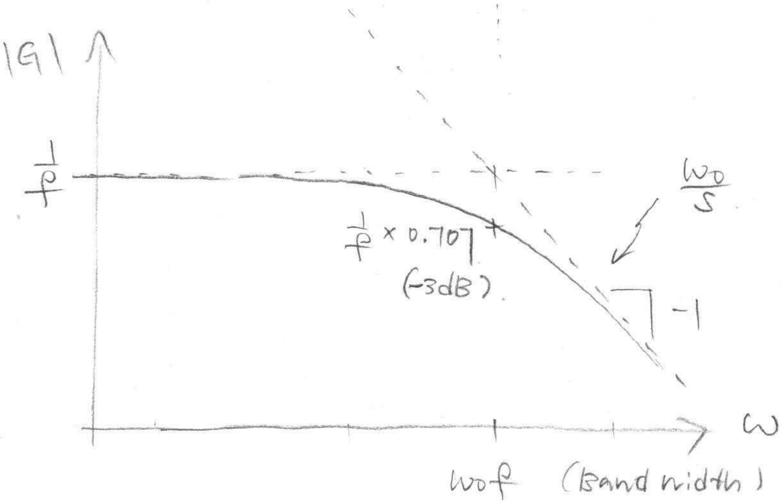


- The block diagram helps us understand the initial resp. e.g. Impulse, step.

∘ Step Response ($v_i = u(t)$)



∘ Bode Magnitude Plot of $G(s)$



- Note the "duality" between the step resp. & Freq. resp.

Time constant : $\frac{1}{w_o f}$	Bandwidth : $w_o f$
Initial response : $w_o t$	High-freq response : $\frac{w_o}{s}$
Final value : $\frac{1}{f}$	DC gain : $\frac{1}{f}$

- Note the trade-off between DC gain & Bandwidth

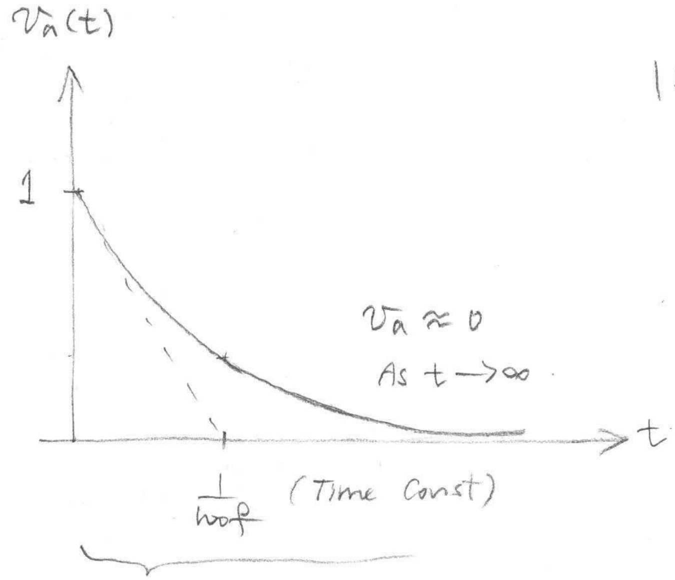
Gain-bandwidth product $= \left(\frac{1}{f}\right) \cdot (w_o f) = w_o$ (for non-invt amplifier)

- Error Dynamics

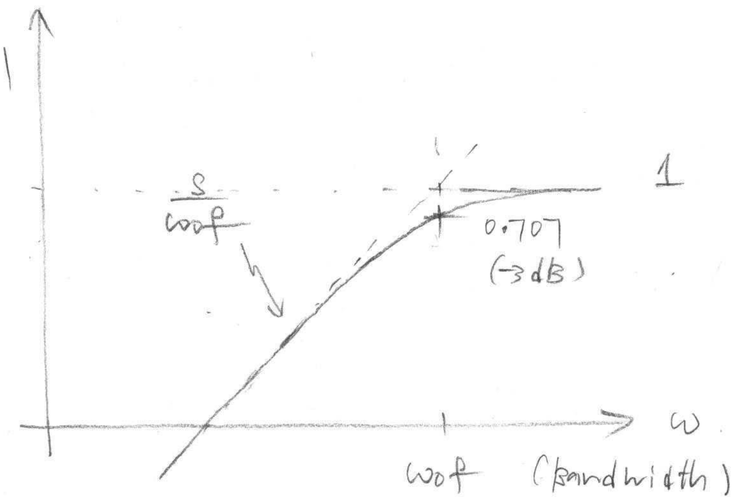
$$\frac{V_a}{V_i} = S(s) = \frac{1}{1+\frac{\omega_0}{s}\cdot f} = \frac{s}{s+\omega_0 f}$$

The same pole location
∴ the same time const.

- Step Response ($v_i = u(t)$) — Magnitude Bode Plot of $S(s)$



Transient : $v_a \neq 0$.

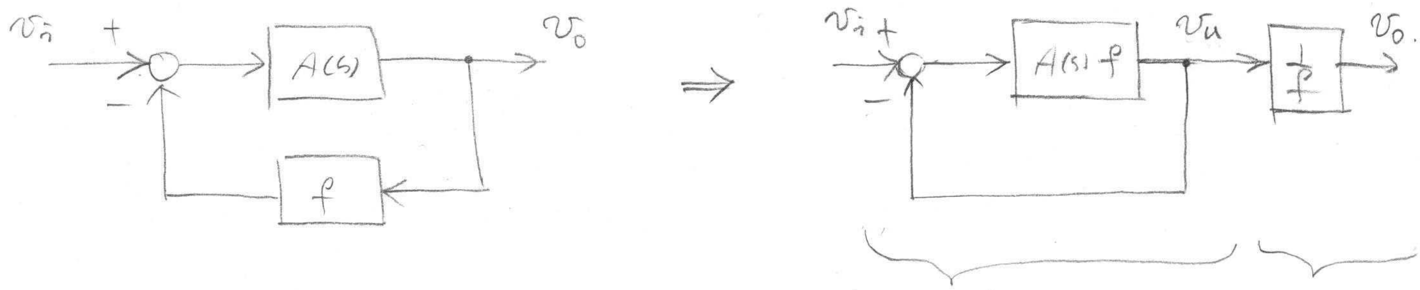


DC Gain = 0.

- "Virtual Short" approximation is acceptable when we can ignore this transient.
- For example, when the time constant of the external system is much large.
- Bandwidth
  - i) The frequency upto which a closed-loop system can track reference signals.
  - ii) The frequency upto which a closed-loop system can reject disturbance signals.

② $A(s)$ in general

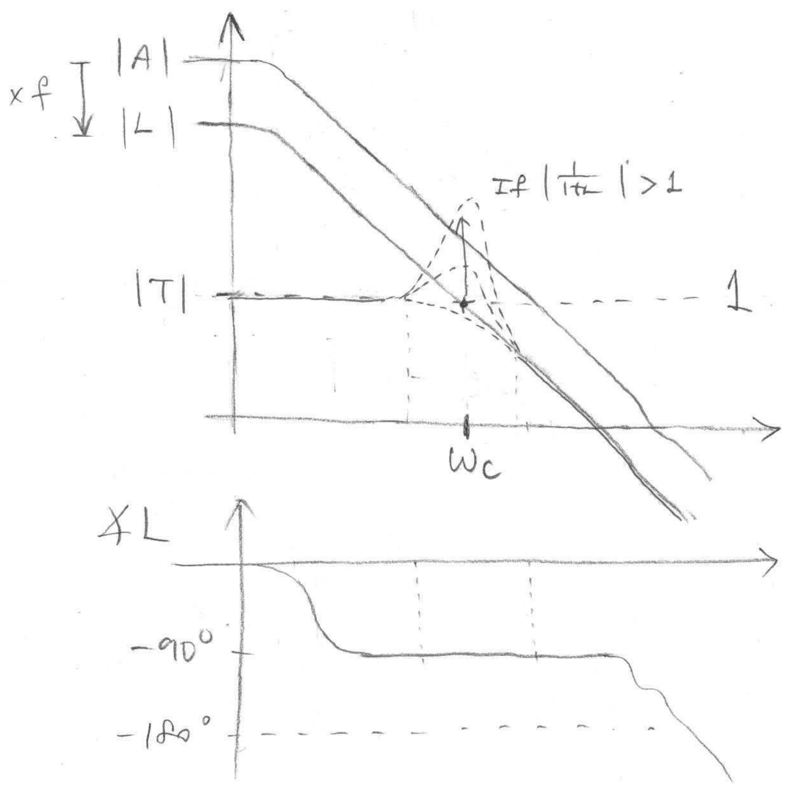
"Unity-feedback Factorization"



Dynamics. Scaling.

$L(s) = A(s)f$ "Loop"

$T(s) = \frac{V_u}{V_i} = \frac{L}{1+L}$ "Tracking"



- when $|L| \gg 1$

  $T \simeq 1$.

- when $|L| \ll 1$

  $T \simeq L$.

- ω_c : (Gain) cross-over frequency $|L(j\omega)|_{\omega=\omega_c} = 1$
  - Approximately equal to 3dB-bandwidth ω_h : $(\omega_c \simeq \omega_h)$
  - Some people even define ω_c as the bandwidth.
  - It allows us to "estimate" the rise time (10% − 90%)

$$t_r \simeq \frac{2.2}{\omega_h} \simeq \frac{2.2}{\omega_c} \quad \left(\frac{0.35}{f_c}\right)$$

ω_c [rad/s]

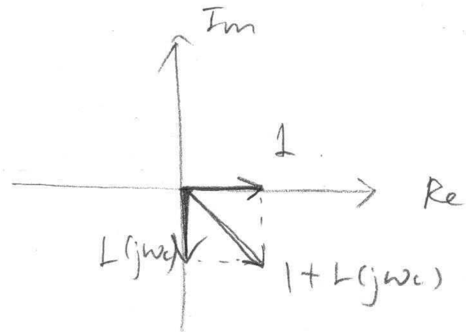
f_c [Hz].

- When $|L| = 1$

$$|T| = \underbrace{|L|}_{1}\underbrace{\left|\frac{1}{1+L}\right|}_{S}$$

The magnitude of T at $\omega = \omega_c$ is determined by the phase of L at $\omega = \omega_c$.

- Vector Diagram for $1 + L(j\omega_c)$



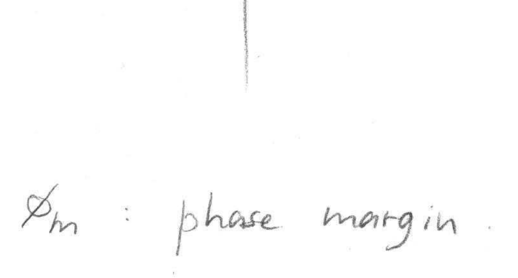
if $\measuredangle L(j\omega_c) = -90^\circ$

$$|1 + L(j\omega_c)| = \sqrt{2}$$

$$\therefore |T| = \frac{1}{\sqrt{2}} \simeq 0.707$$

$(-3dB)$

Q. What if $\measuredangle L(j\omega_c)$ approches -180° ?



$$|1+L| \simeq \phi_m$$

$$\therefore |T| \simeq \frac{1}{\phi_m} \simeq \underbrace{Q = \frac{1}{2\zeta}}_{\text{From 2nd order syst.}}$$

- ϕ_m : phase margin $\measuredangle L(j\omega)\Big|_{\omega=\omega_c} - (-180^\circ)$
  - It allows us to "estimate" the damping ratio ζ
  - $\zeta \simeq \dfrac{\phi_m[\text{rad}]}{2} = \dfrac{\phi_m[\text{deg}]\left(\frac{\pi}{180}\right)}{2} = \dfrac{\phi_m[\text{deg}]}{114.6} \simeq \dfrac{\phi_m[\text{deg}]}{100}$