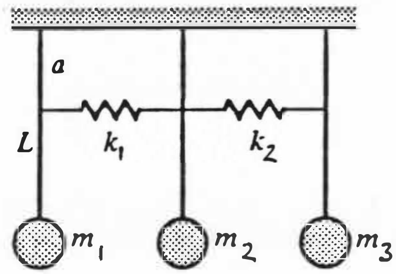
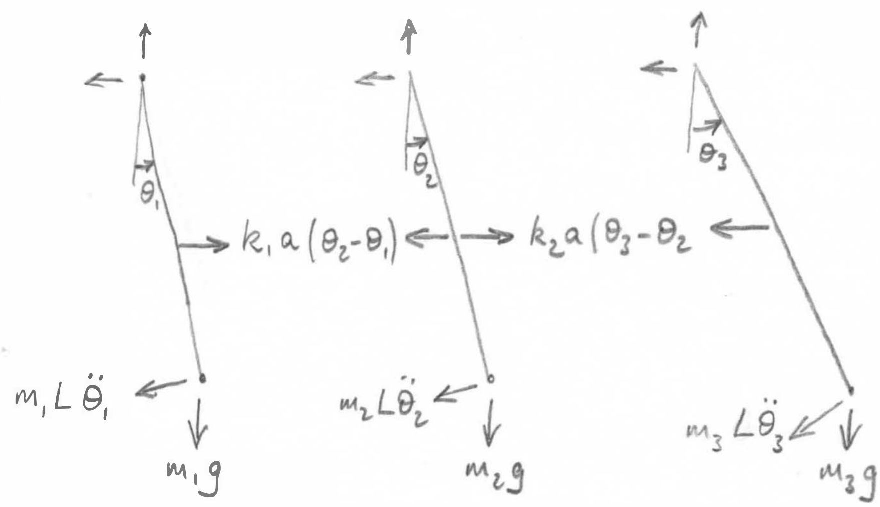
MECH 463 -- Tutorial 4

1. The vibrating system shown in the diagram consists of three pendulums of mass m_1, m_2, and m_3. They are supported on rigid massless rods of uniform length L. The pendulums are connected at distance a from their upper ends by springs of stiffness k_1 and k_2. Draw the free body diagrams of the system and formulate the equations of motion in matrix form for small amplitude vibrations. Do not proceed to solve the equations of motion or write the characteristic equation.





Choose angular coordinate system $\theta_1 - \theta_2 - \theta_3$. Coordinates describe the positions of the mass centres, so we expect no dynamic coupling.

Assume small vibrations $\longrightarrow \sin\theta \simeq \theta$

Take moments about tops of pendulums

$$m_1 L^2 \ddot{\theta}_1 + m_1 g L \theta_1 - k_1 a^2(\theta_2-\theta_1) = 0$$

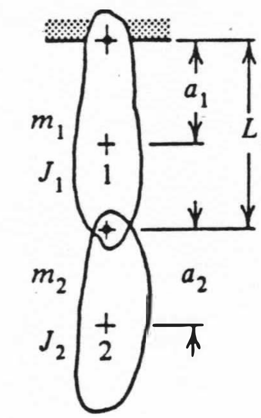
$$m_2 L^2 \ddot{\theta}_2 + m_2 g L \theta_2 + k_1 a^2(\theta_2-\theta_1) - k_2 a^2(\theta_3-\theta_2) = 0$$

$$m_3 L^2 \ddot{\theta}_3 + m_3 g L \theta_3 + k_2 a^2(\theta_3-\theta_2) = 0$$

Divide by L^2 and put into matrix form: where $\mu = \frac{a}{L}$

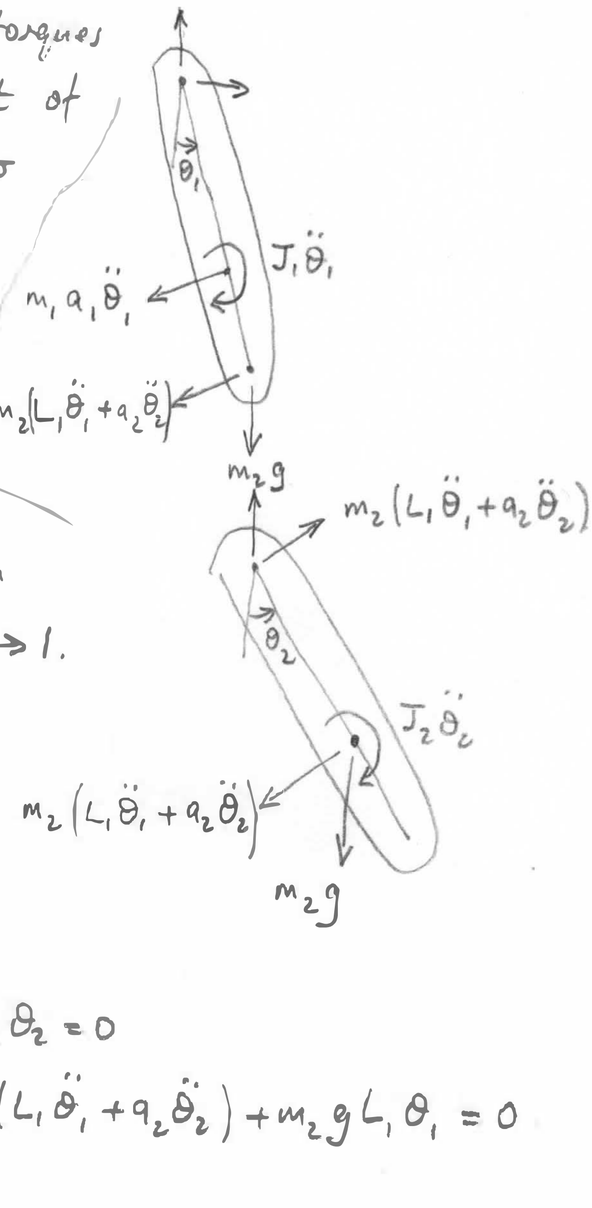
$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} m_1 g/L + k_1\mu^2 & -k_1\mu^2 & 0 \\ -k_1\mu^2 & m_2 g/L + (k_1+k_2)\mu^2 & -k_2\mu^2 \\ 0 & -k_2\mu^2 & m_3 g/L + k_2\mu^2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2. The diagram shows a double pendulum constructed of two rods of masses m_1 and m_2. Their moments of inertia about their centres of mass are J_1 and J_2. These centres of mass are at distances a_1 and a_2 from the upper ends of the rods. Draw the free body diagrams of the system and formulate the equations of motion in matrix form for small vibrations. Identify and explain the type of coupling that you observe.



Choose the $\theta_1 - \theta_2$ coordinate system shown.

These coordinates describe the restoring torques ($mg \sin\theta$) which here is the equivalent of the springs. Therefore we expect no static coupling. We expect some dynamic coupling because the coordinates do not define the position of the mass centres (m_2 in particular).



Choose to linearize the system from the outset. $\sin\theta \to \theta$ and $\cos\theta \to 1$. This greatly simplifies the acceleration term for the lower pendulum.

Take moments about the tops of the pendulums:

$$J_2\ddot{\theta}_2 + m_2 a_2 (L_1\ddot{\theta}_1 + a_2\ddot{\theta}_2) + m_2 g a_2 \theta_2 = 0$$

$$J_1\ddot{\theta}_1 + m_1 a_1^2 \ddot{\theta}_1 + m_1 g a_1 \theta_1 + m_2 L_1 (L_1\ddot{\theta}_1 + a_2\ddot{\theta}_2) + m_2 g L_1 \theta_1 = 0$$

In matrix form:

$$\begin{bmatrix} (J_1 + m_1 a_1^2 + m_2 L_1^2) & (m_2 L_1 a_2) \\ (m_2 L_1 a_2) & (J_2 + m_2 a_2^2) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} (m_1 g a_1 + m_2 g L_1) & 0 \\ 0 & (m_2 g a_2) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$