MECH 463: Mechanical Vibrations
Midterm Examination 1

Time: 45 minutes 26th September 2013 Maximum Available Mark: 20

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RZ

Student Name: René Rinfret Student Number: 34929091

Write your answers on this sheet (4 pages in total). Do not remove pages.

Q1. Unbalanced masses are a common source of vibrations in rotating systems. Consider an idealized Single Degree of Freedom (SDOF) model shown in Fig.(1). The rotating mass m_2 is unbalanced, since it lies at a distance r from the centre of rotation, which moves with the mass m_1. m_1 rests on guided supports and is free to move in the vertical direction. Because of forces exerted by rotating unbalanced mass m_2, m_1 oscillates in the vertical direction. Take y the vertical displacement of mass m_1, positive downwards, as the displacement co-ordinate from the stretched spring (static equilibrium) position p. θ is the **given** angular displacement of the rotating unbalance mass m_2, measured from the vertical, positive counter clockwise. $\dot{\theta}$ is **given** angular velocity of the rotating mass.

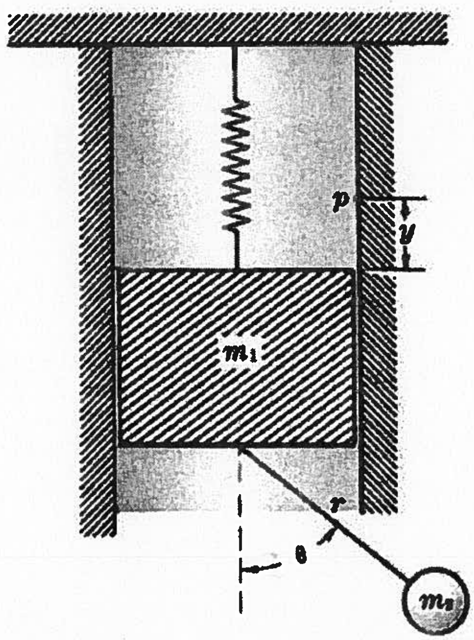

Figure 1: Figure for midterm question. A rotating unbalance m_2 causes vertical vibrations of m_1. p is the static equilibrium position and y measured with respect to p, positive downwards, is the dynamic displacement. m_1 is supported on guides and can move in the vertical direction.

a) Determine the acceleration of m_1 and m_2 with respect to a fixed observer in terms of y, r, and θ. **(4 marks)**

Answer: "continued work from pg 3"

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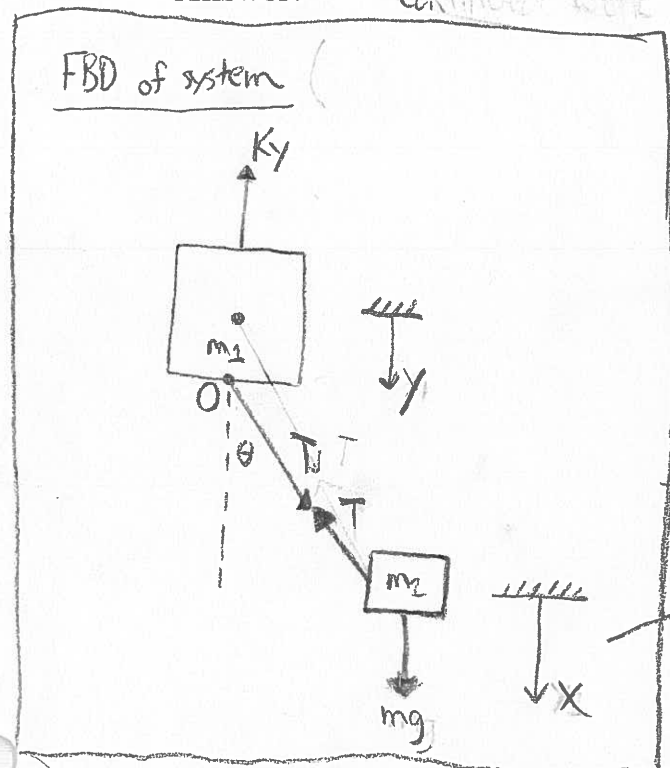

Assumptions:

- $m_1 \rightarrow$ gravity ignored due to static eqi
- assume m_2 is a point mass $\therefore J_0 = 0$

$\rightarrow$ called it something ~~other~~ than y, since acceleration is not the same!

$\circlearrowleft \Sigma M_o = J_o\ddot{\theta} - m_2 g r \sin\theta = 0$

since $J_o = \cancel{J_A} + m_2 r^2$

$J_o = m_2 r^2$

$\therefore \Sigma M_o = 0 = m_2 r^2 - m_2 g r \sin\theta$

$\cancel{m_2} r^{\cancel{2}} = \cancel{m_2} g \cancel{r} \sin\theta$

$\boxed{r = g\sin\theta \quad [\text{Eqn. } 1]}$

eq. of motions:

$\boxed{m_1: \downarrow \Sigma F = m_1\ddot{y} = T\cos\theta - K_y}$

$m_2: \downarrow \Sigma F = m_2\ddot{x} = mg - T\cos\theta$

from Eqn. 1:

$\boxed{m_2: \downarrow \Sigma F = m_2\ddot{x} = \dfrac{m_2 r}{\sin\theta} - T\cos\theta}$

ANSWER

*All equations in terms of y, r, θ (and T...)

when asked for acceleration, just follow kinematics

Just marking for FBD

b) Consider the case when $\dot{\theta}$, the given angular velocity is constant ($\dot{\theta} = \text{constant}$). **(8 marks)** Sketch an appropriate free body diagram and formulate the equations of motion in terms of y, θ, and r. **5 marks are for FBD indicating all forces with correct location of their points of action.** You can use Newton's method or D'Alembert's principle. Ignore gravity.

Answer:

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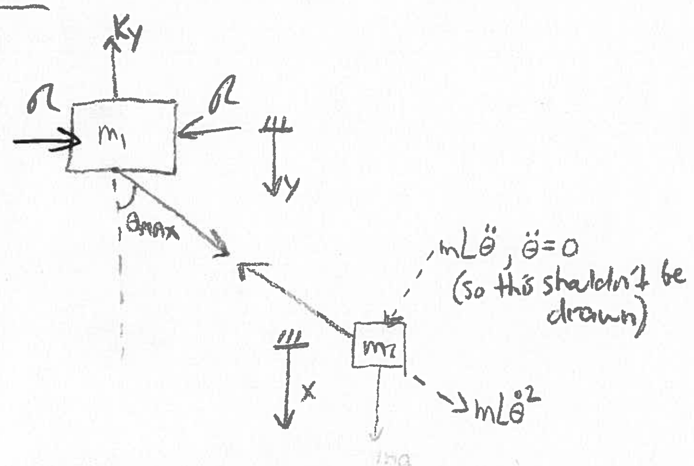
* So, $\frac{d\theta}{dt} = \text{constant} = \dot{\theta}$ $\therefore \theta = f(t)$ but also $\ddot{\theta} = 0$

* This would mean that this situation is when m_2 is furthest from m_1 in x-axis, θ_{MA}

FBD



$$\boxed{\begin{aligned} m_1 &: \downarrow \Sigma F = m_1\ddot{y} = T\cos\theta - K_y \\ m_2 &: \downarrow \Sigma F = m_2\ddot{x} = \frac{m_2 r}{\sin\theta} - T\cos\theta \end{aligned}}$$

I suspect I need to mention the equations for planar kinematics incorporating θ, but I'm not sure how to do this:

$$\begin{cases} r = r\vec{e_1} \\ \dot{r} = \dot{r}e_1 + r\dot{\theta}e_2 \\ \ddot{r} = [\ddot{r} - r\dot{\theta}^2]\vec{e_1} + [2\dot{r}\dot{\theta}]\vec{e_2} \end{cases}$$ (since $\ddot{\theta} = 0$

c) Treating $\dot{\theta}$, the given angular velocity not as a constant ($\dot{\theta} \neq$ constant). Sketch an appropriate free body diagram and formulate the equations of motion in terms of y, θ and r. **5 marks are for FBD indicating all forces with correct location of their points of action.** You can use Newton's method or D'Alembert's principle. Ignore gravity. **(8 marks)**

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Answer:

If $\dot{\theta} \neq$ constant, $\ddot{\theta}$ = constant

FBD?

→ FBD would be when

→ I suspect I need to mention planar kin:

$$r = r\vec{e}_1$$
$$\dot{r} = \dot{r}\vec{e}_1 + r\dot{\theta}\vec{e}_2$$
$$\ddot{r} = [\ddot{r} - r\dot{\theta}^2]\vec{e}_1 + [2\dot{r}\dot{\theta} + r\ddot{\theta}]\vec{e}_2$$

$$\sum F = m_1\ddot{y} = T\cos\theta - ky$$ ✓

$$\sum F = m\ddot{x} = \frac{m\ddot{r}}{\sin\theta} - T\cos\theta$$ missed something here