Bode Diagram Review

$$G(s)=\frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)} \qquad n>m$$

Input Frequency

$$\boxed{\boldsymbol{s=j\omega}}$$

$$G(j\omega)=\frac{(j\omega+z_1)(j\omega+z_2)\ldots(j\omega+z_m)}{(j\omega+p_1)(j\omega+p_2)\ldots(j\omega+p_n)}$$

$$|G(j\omega)|=\frac{\sqrt{\omega^2+z_1^2}\times\sqrt{\omega^2+z_2^2}\times\cdots\times\sqrt{\omega^2+z_m^2}}{\sqrt{\omega^2+p_1^2}\times\sqrt{\omega^2+p_2^2}\times\cdots\times\sqrt{\omega^2+p_n^2}}$$

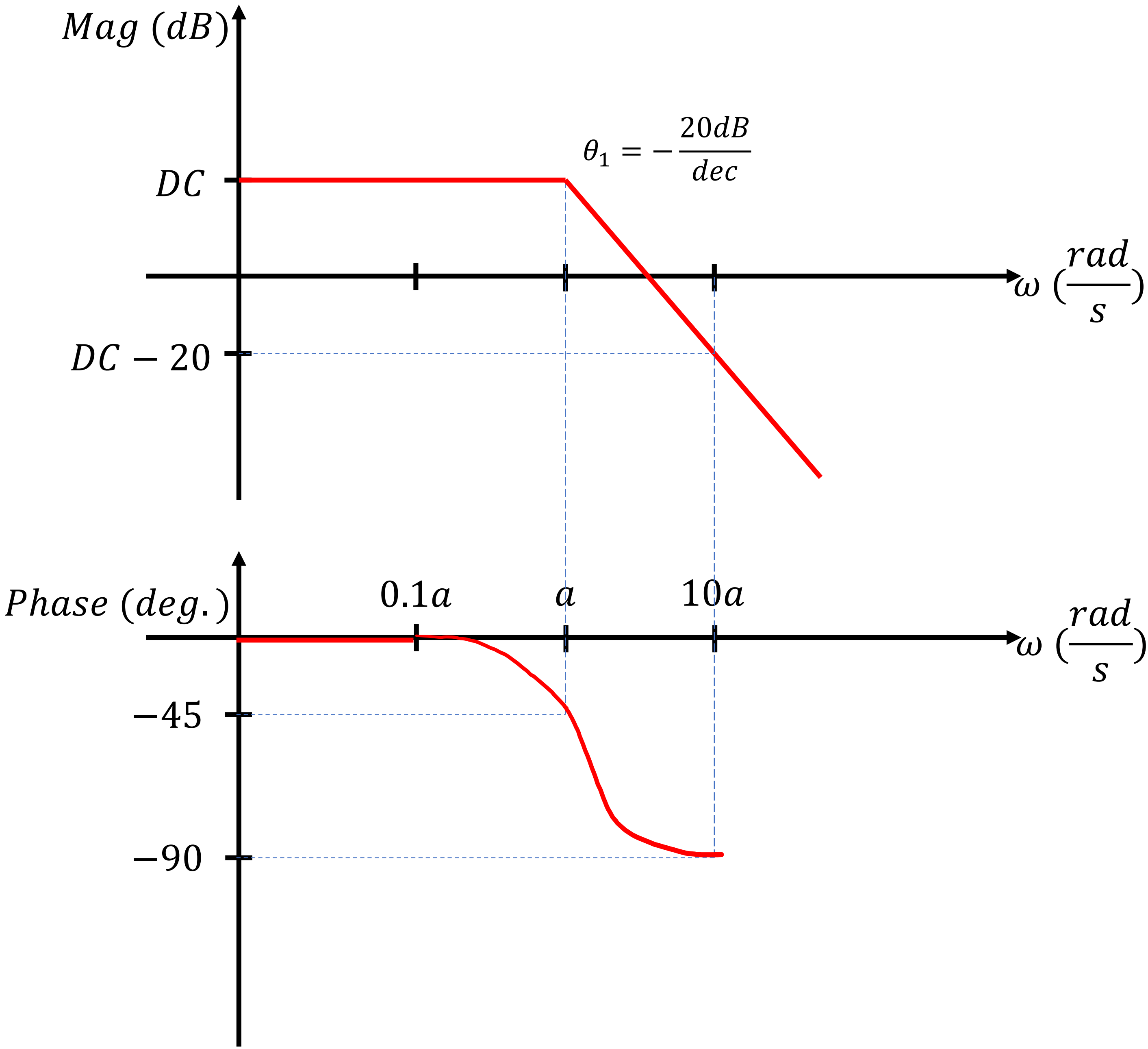
$$\angle G(j\omega)=\left(\tan^{-1}\frac{\omega}{z_1}+\tan^{-1}\frac{\omega}{z_2}+\cdots+\tan^{-1}\frac{\omega}{z_m}\right)$$

$$-\left(\tan^{-1}\frac{\omega}{p_1}+\tan^{-1}\frac{\omega}{p_2}+\cdots+\tan^{-1}\frac{\omega}{p_n}\right)$$

General Case $G(s) = \dfrac{K}{s+a}$

$s = 0 \quad \rightarrow \quad DC = \dfrac{K}{a} = 20\log(\dfrac{K}{a})$

$P_1 = -a$

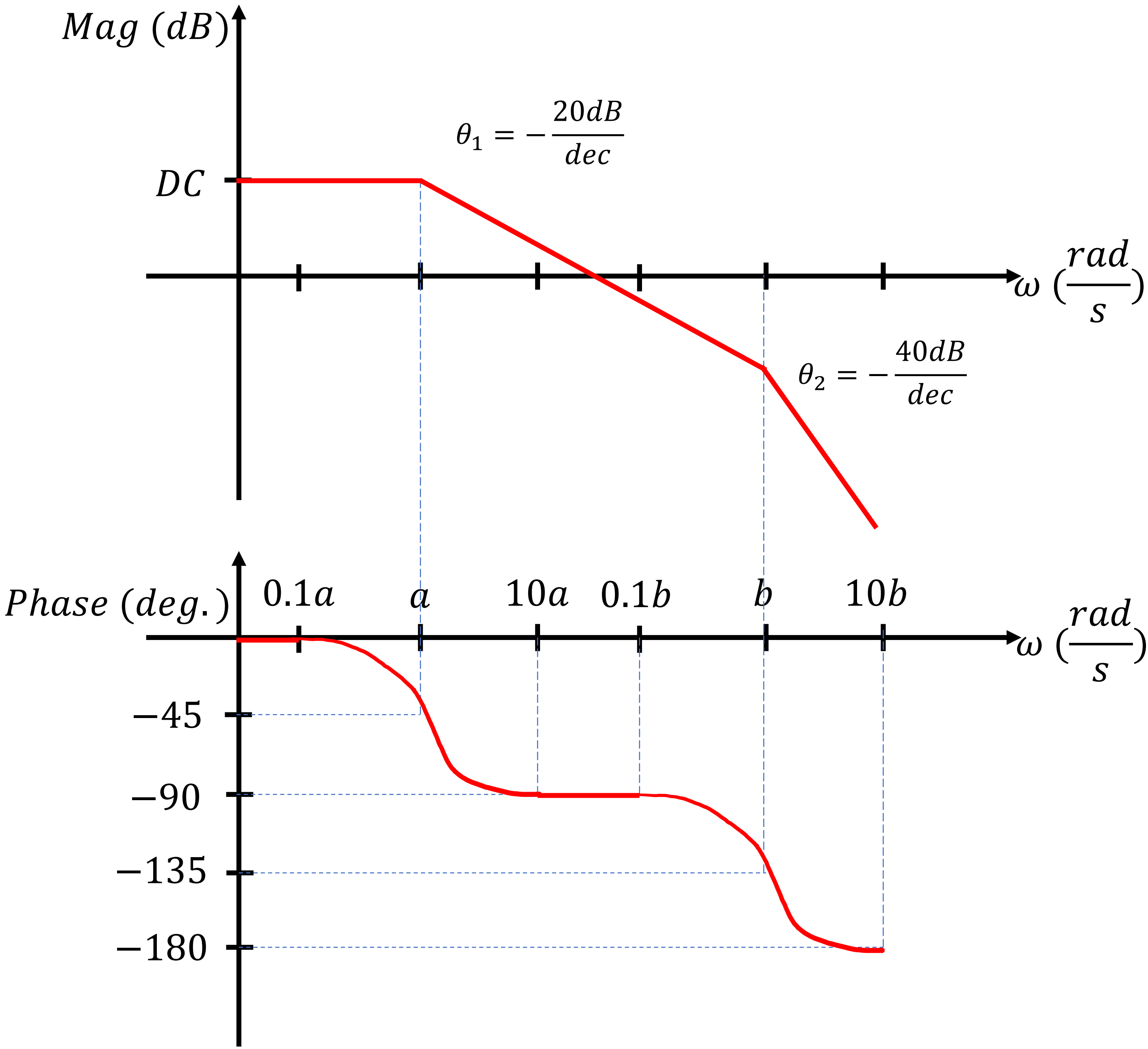


General Case $G(s) = \dfrac{K}{(s+a)(s+b)}$

$s = 0 \quad \rightarrow \quad DC = \dfrac{K}{ab} = 20\log(\dfrac{K}{ab})$

$P_1 = -a$

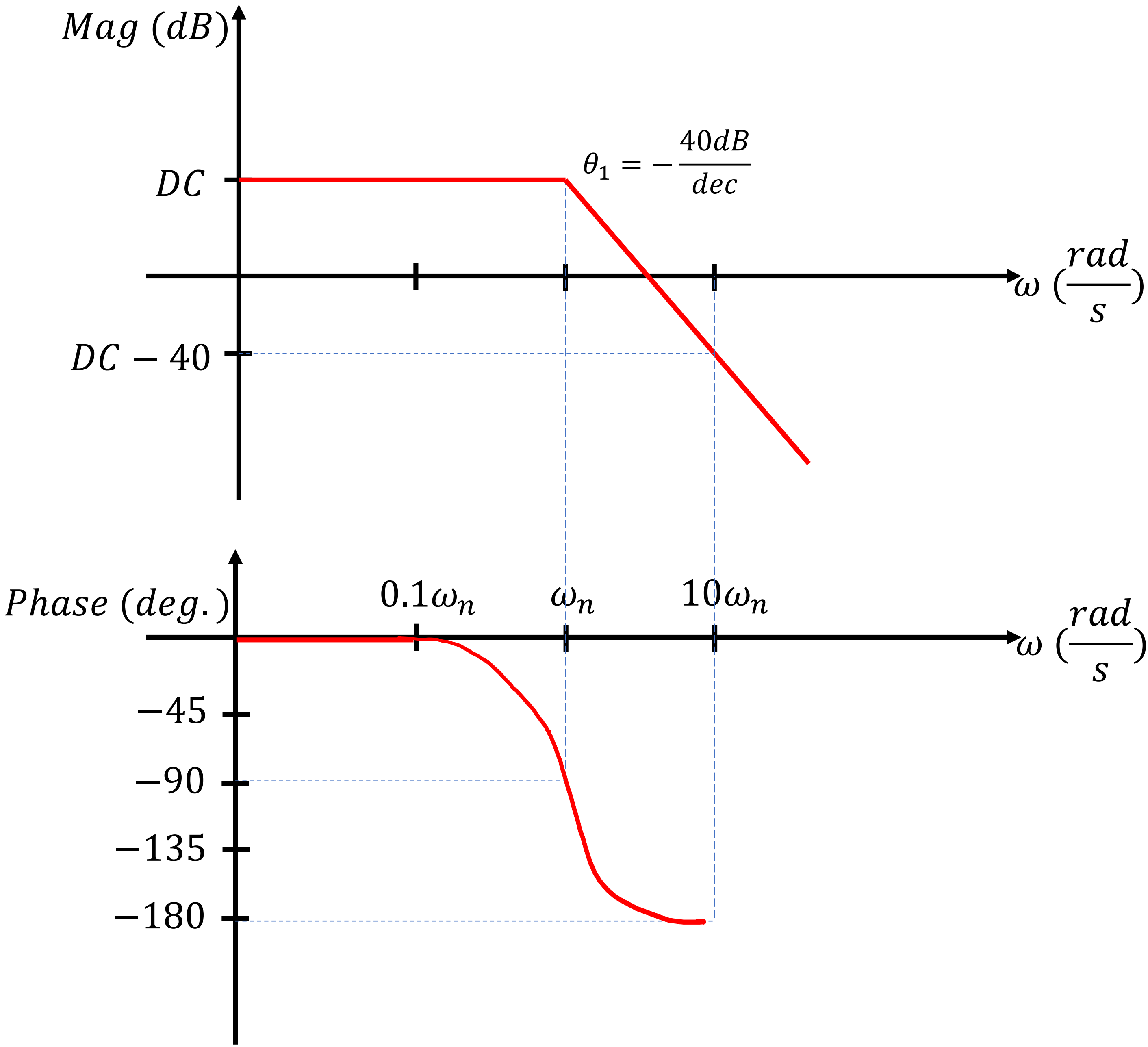
$P_2 = -b$



General Case $G(s) = \dfrac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$s = 0 \quad \rightarrow \quad DC = \frac{K}{\omega_n^2} = 20\log(\frac{K}{\omega_n^2})$$

$\zeta \ \rightarrow$ *Causes Bump*

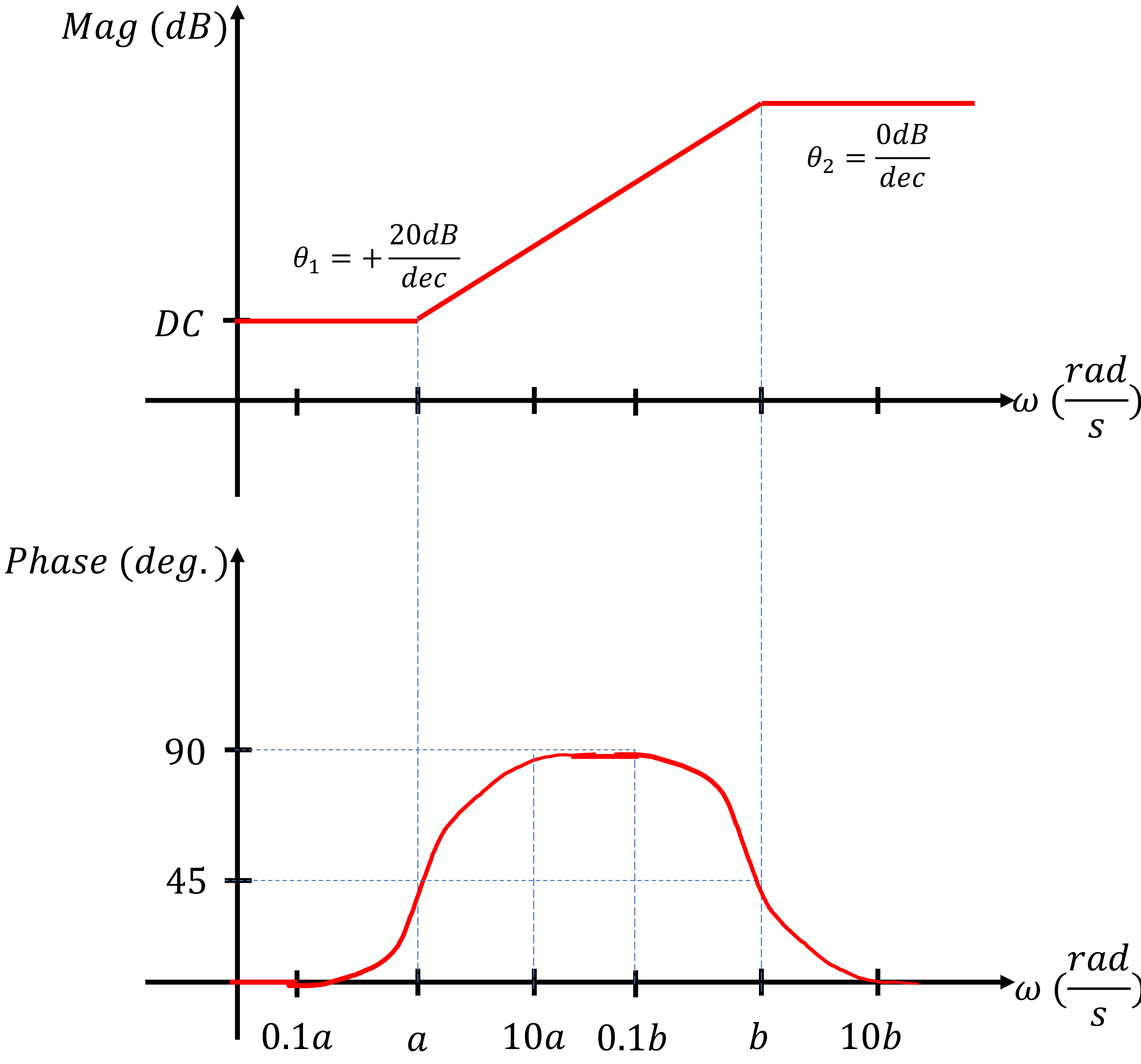


General Case $G(s) = \mathrm{K}\frac{\mathrm{s+a}}{s+b}$

$$s = 0 \quad \rightarrow \quad DC = \frac{Ka}{b} = 20\log(\frac{Ka}{b})$$

$$Z_1 = -a$$

$$P_1 = -b$$



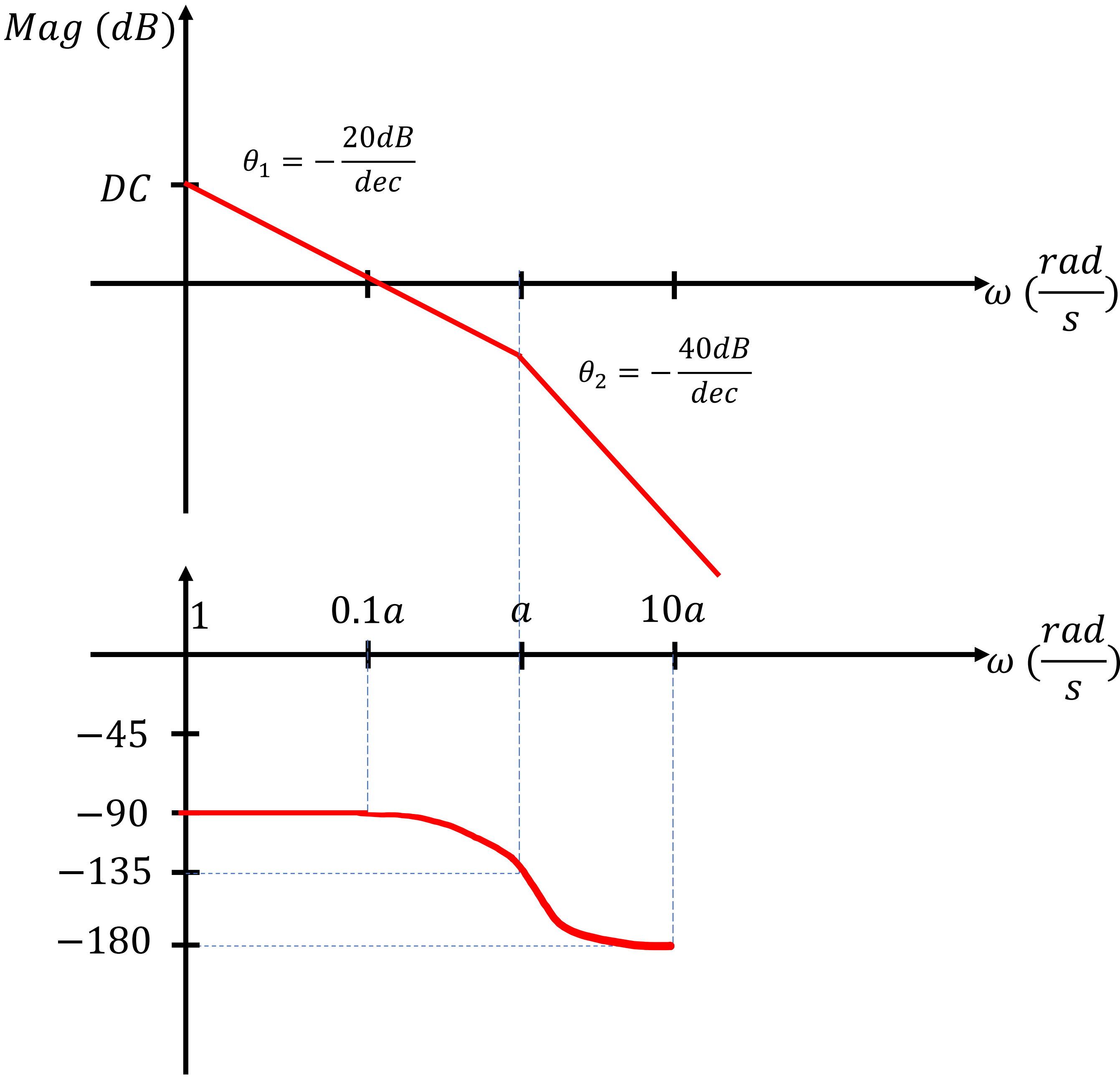
General Case $G(s) = \dfrac{K}{s(s+a)}$

$s = 0 \quad \rightarrow \quad DC = \infty$

$P_1 = -a$

$P_2 = 0$

$\omega = 1 \quad \rightarrow \quad DC = \dfrac{K}{\sqrt{a^2+1}} = 20\log\left(\dfrac{K}{\sqrt{a^2+1}}\right)$



Question 1. Bode Diagram

$$G(s) = \frac{28900}{s(s^2 + 240s + 28900)}$$

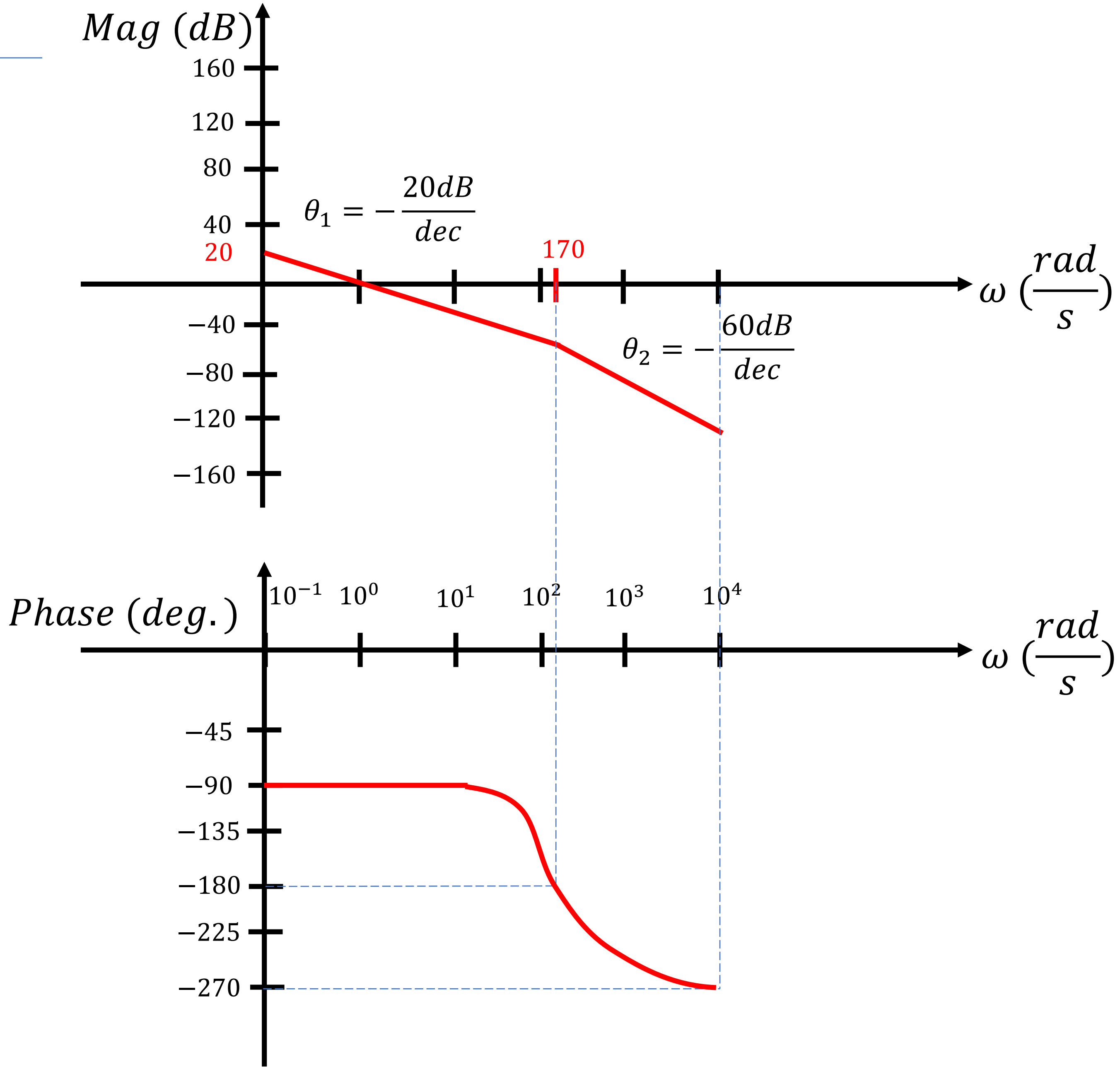
$\Delta = b^2 - 4ac = 240^2 - 4 \times 28900 = -58000 < 0$

$s^2 + \underline{240}s + \underline{28900} \equiv s^2 + \underline{2\zeta\omega_n}s + \underline{\omega_n^2} \rightarrow \begin{cases} \omega_n^2 = 28900 \rightarrow \omega_n = 170\ \frac{rad}{s} \\ 2\zeta\omega_n = 240 \rightarrow \zeta = 0.71 \end{cases}$

$\omega = 0.1 \rightarrow |G(j\omega)|_{\omega=0.1} = \dfrac{28900}{(0.1) \times \underbrace{|(28900 - \omega^2) + j240\omega|}_{= 28900}} = 10 = 20\ dB$

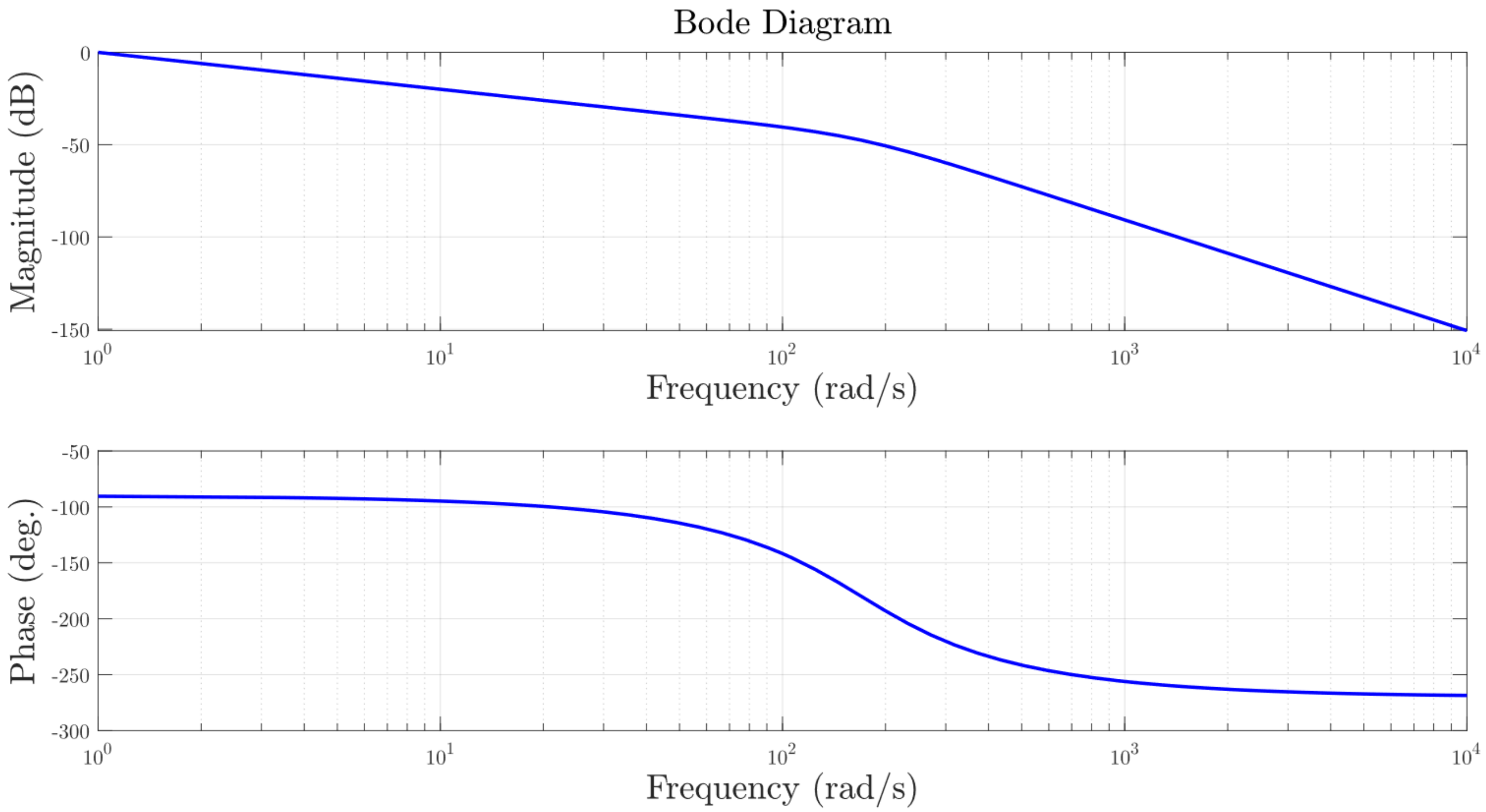
Question 1. Bode Diagram (Cont.)

$$G(s) = \frac{28900}{s(s^2 + 240s + 28900)}$$



Question 1. Bode Diagram (MATLAB)

$$G(s) = \frac{28900}{s(s^2 + 240s + 28900)}$$



Question 2. Gain and Phase Margins

$$G(s) = \frac{28900}{s(s^2 + 240s + 28900)}$$

$$G(jw) = \frac{28900}{(jw)(-w^2 + 28900 + 240jw)} = \frac{28900}{(-240w^2) + j(-w^3 + 28900w)} =$$

$$\frac{-6936000w^2}{57600w^4 + (-w^3 + 28900w)^2} + j\frac{-28900(-w^3 + 28900w)}{57600w^4 + (-w^3 + 28900w)^2}$$

$$|G(jw)| = \frac{28900}{\sqrt{(-240w^2)^2 + (-w^3 + 28900w)^2}}, \quad \phi = -\frac{\pi}{2} - tg^{-1}\left(\frac{240w}{-w^2 + 28900}\right)$$

Question 2. Gain and Phase Margins (Cont.)

$$Im\left[G(j\omega)\right] = 0 \rightarrow \frac{-28900(-\omega^3+28900\omega)}{57600\omega^4+(-\omega^3+28900\omega)^2} = 0 \rightarrow -\omega^3+28900\omega = 0$$

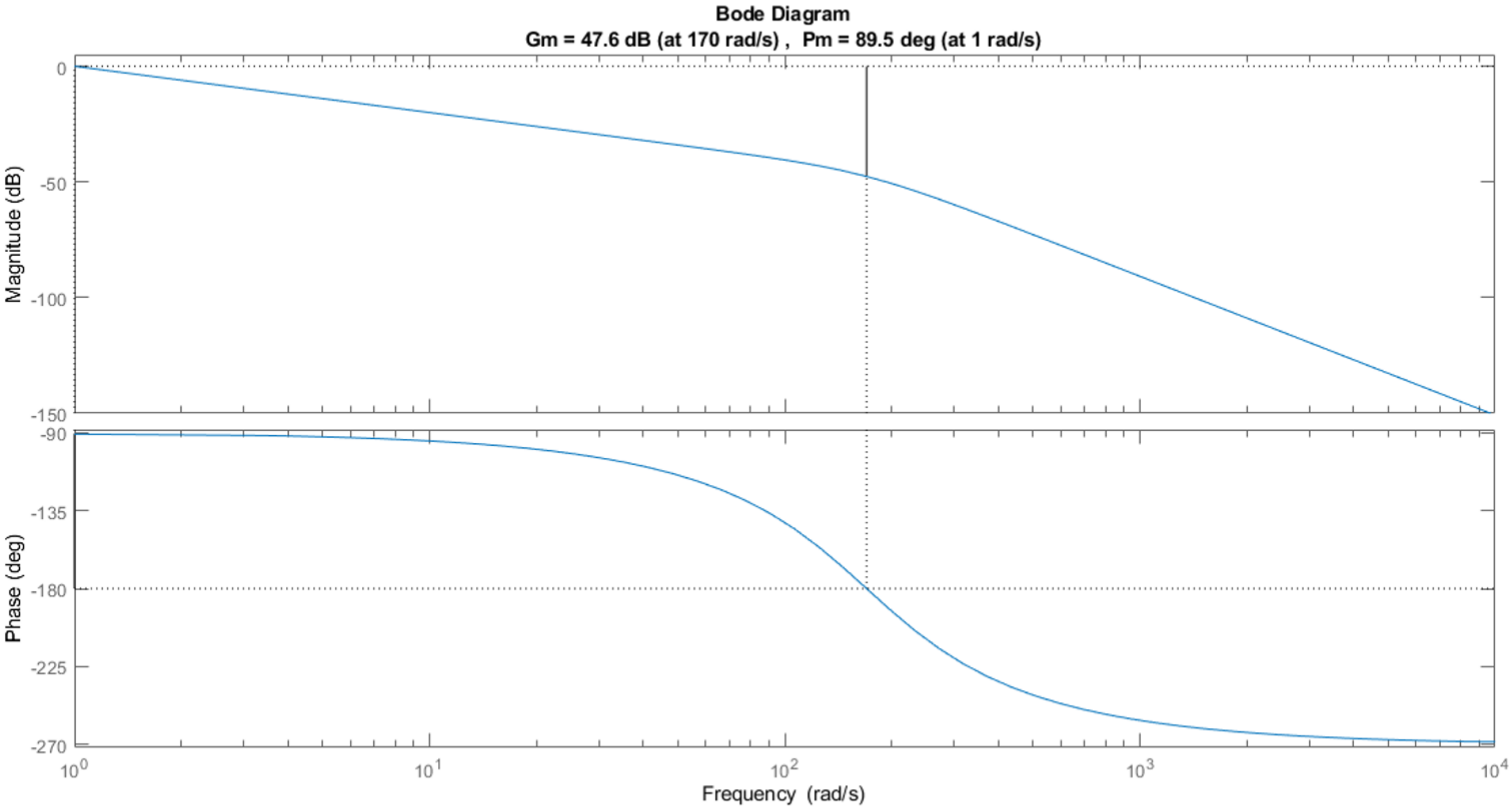
$$\omega_p = 170\ \frac{rad}{s} \rightarrow GM = \frac{1}{\left|Re(G(j\omega))\right|_{\omega=170}} = 240 = \boxed{47.6\ dB}$$

$$|G(j\omega)| = 1 \rightarrow (240\omega^2)^2 + (-\omega^3+28900\omega)^2 = 28900^2 \rightarrow \omega_g = 1\ \frac{rad}{s}$$

$$PM = 180 + \underbrace{\phi\left(\omega = 1\ \frac{rad}{s}\right)}_{-90.5} \rightarrow \boxed{PM = 89.5^\circ}$$

$$\boxed{K_{cr} = GM = 240}$$

Question 2. Gain and Phase Margins (MATLAB)



Question 3. Proportional Controller Design

3) Design the proportional controller such that the gain crossover freq is 60 rad/sn

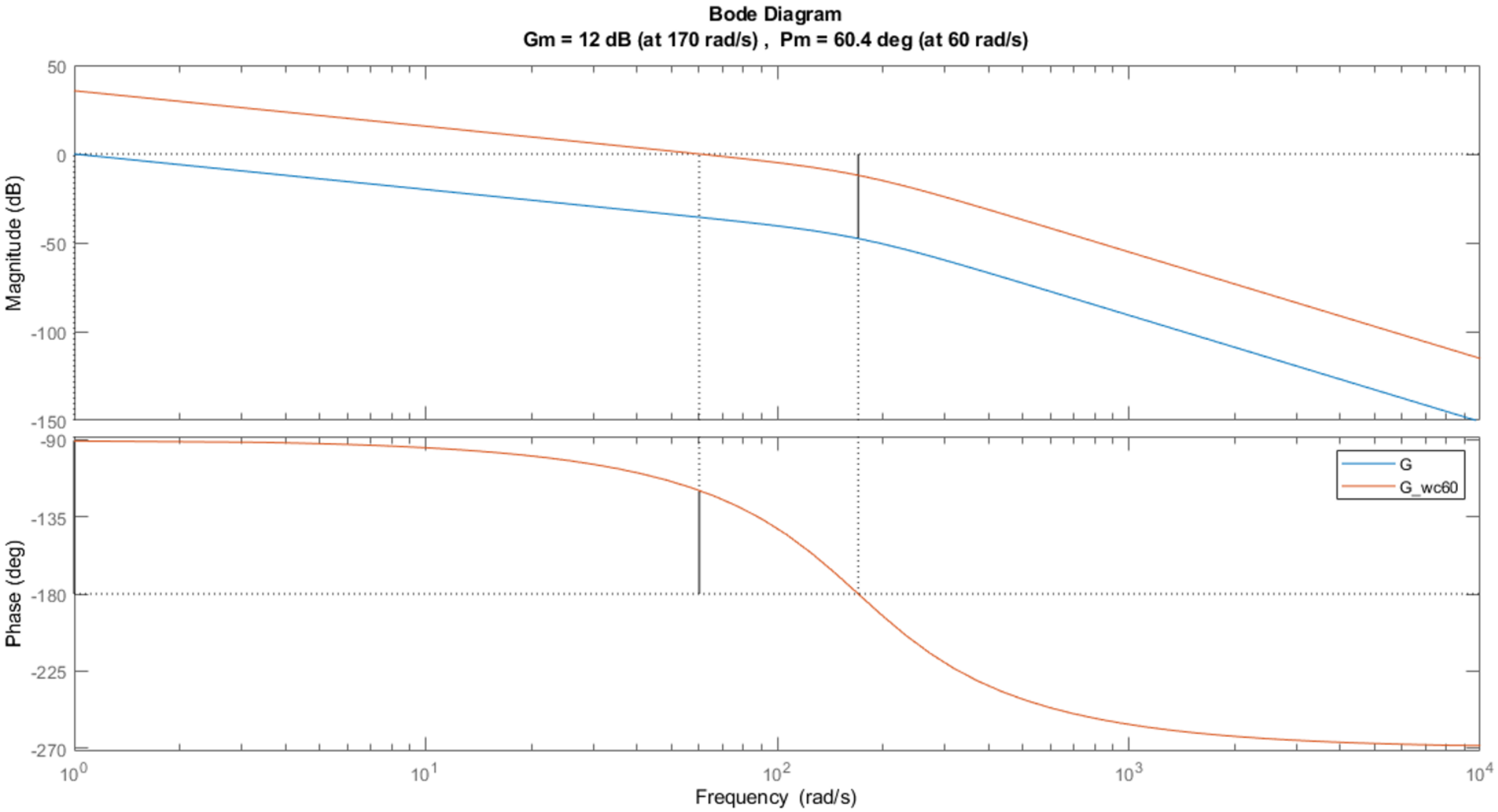
$$G(s) = \frac{28900}{s(s^2+240s+28900)}$$

$$|G(j\omega)|_{\omega=60} = 0,0165$$

$$K_p\,|G(j\omega)|_{\omega=60} = 1$$

$$K_p = \frac{1}{0,0165} = 60,4381$$

Question 3. Proportional Controller Design (MATLAB)



Question 4. Nyquist and Root Locus (MATLAB)