Lesson 14-1 – Risk Analysis – Cases and Joint Probability

A Range of Estimates

- We can’t foretell precisely the costs and benefits for future years.
- It is more realistic to describe a range of possible values.
  - For example a range could include:
    - “Optimistic,” “most likely,” and “pessimistic” estimates
    - The analysis could then determine whether the decision is sensitive to the range of values.
- All estimates are inherently uncertain.
  - Far-term estimates are generally more uncertain than near-term estimates.
- Changes in any estimate(s) can alter the results of an economic analysis.
- Using breakeven and sensitivity analysis yields an understanding of how changes in variables will affect the economic analysis.

Decision-making and Uncertainty of Future Outcomes

- It is good practice to examine the effects on outcomes of variability in the estimates.
  - By how much and in what direction will a measure of merit (e.g., NPV, EACF, IRR) be affected by variability in the estimates?
- But, this does not take into account the inherent variability of parameters in an economic analysis.
- We need to consider a range of estimates

Uncertainty of Future Outcomes

- In the left box, one cash flow in Project B is uncertain and $NPV_A > NPV_B$ (i = 10%).
- In the right box, the cash flow estimate has changed by a small amount and now $NPV_A < NPV_B$.

Year	Project A	Project B
0	–$1000	–$2000
1	$400	$700
2	$400	$700
3	$400	$700
4	$400	$700
NPV	$267.95	$218.91

Year	Project A	Project B
0	–$1000	–$2000
1	$400	$700
2	$400	$700
3	$400	$700
4	$400	$800
NPV	$267.95	$287.21

Probabilities of Future Events

- Probabilities of future events can be based on theory, empirical data, judgement, or a combination
  - games of chance
  - weather and climate data
  - expert judgement on events
- Most data has some level of uncertainty
  - Small uncertainties are often ignored.
- Variables can be known with certainty (deterministic) or with uncertainty (random).

Probability

- Probability of flipping a coin:
  - Head (50%), Tail (50%)

  (This is the long range frequency and the single trial likelihood.)
- Mathematically: Between 0 and 1
  - 0 (can never happen) – 0%
  - 1 (will always happen) – 100%
  - 0.5 (half the time—as in the above example of flipping a coin—50%)
  - The sum of probabilities for all possible outcomes = 1 (or 100%)

Probability, cont'd

- The sum of all probabilities must equal 1.
  - Head (0.5), Tail (0.5)
  - $0 \leq$ Probability ≤ 1
  - $\sum P(\text{outcome}_j) = 1$, where there are K outcomes

- As opposed to large distributions, it is more common in economic analysis to use 2 to 5 discrete possibilities.
  - This is because:
    - Each outcome requires more analysis.
    - Expert judgement should be limited for accuracy.

Probability Continued…

- It is usual in engineering economics to use between two and five discrete outcomes with their probabilities.
  - Expert judgement limits the number of outcomes.
  - Each additional outcome requires more analysis.
- An outcome's probability can be determined as the long-run relative frequency of its occurrence

Example

- An oil exploration company is drilling a new wells in an established field. Based on the previous data from that field, they estimate
  - A 70% chance of a well being dry
  - A 25% chance of a well being productive
  - A 5% chance of the well being highly productive

- If they plan to drill 40 wells over the next two years, how many of each kind of well should they expect?
  - 40*70% = 28 dry wells
  - 40*25% = 10 productive wells
  - 40*5% = 2 very productive wells

Joint Probability Distributions

- Random variables are assumed to be statistically independent.
  - For instance, the project lifetime and the annual benefit are assumed to be independent (unrelated).
- Project criteria (eg. NPV & IRR), depend on the probability distributions of input variables.
- We need to determine the joint probability distributions of different combinations of input parameters.

Joint Probability Distributions Continued...

- If A and B are independent, P(A and B) = P(A) $\times$ P(B).
- Example: Flipping a coin and rolling a die
  - Turning up a head and rolling a 4
    - 1/2 x 1/6 = 1/12 is the joint probability
- Suppose there are three values for the annual benefit and two values for the lifetime. This leads to six possible combinations that represent the full set of outcomes and probabilities
- Joint probabilities can increase the number of possibilities and become arithmetically burdensome.

Example (cont'd)

- Remember the oil exploration company has a
  - A 70% chance of a well being dry
  - A 25% chance of a well being productive
  - A 5% chance of the well being highly productive

- Additionally, there is a 60% probability oil prices will rise in the next three months, and a 40% probability oil prices will fall. If the company can drill one well in the next three months, what are the likelihoods of the potential outcomes?

Example (cont’d)

- Remember the oil exploration company has a
  - A 70% chance of a well being dry
  - A 25% chance of a well being productive
  - A 5% chance of the well being highly productive

Case	P(Well Type) x	P(Oil Price) =	P(case)
Dry Well, Low Price	70%	40%	28%
Dry Well, High Price	70%	60%	42%
Productive Well, Low Price	25%	40%	10%
Productive Well, High Price	25%	60%	15%
Very Productive Well, Low Price	5%	40%	2%
Very Productive Well, High Price	5%	60%	3%

Expected Value

Each outcome is weighted by its probability and the results are summed:

Expected value = $\text{Outcome}_A\ (P(\text{A})) + \text{Outcome}_B\ (P(\text{B})) + \ldots$

Where outcome is our measure of worth or other value, e.g. NPV, Rate of Return, Future Value…

A weighted average of the values of each case based on the probability of those cases occurring

Expected Value: Problem

The proposed projects have the potential uniform annual benefits and associated probability at occurrence shown below. Which project is more desirable based on these data?

Project A		Project B	
EUAB	**Probability**	**EUAB**	**Probability**
$1000	0.10	$1500	0.20
$2000	0.30	$2500	0.40
$3000	0.40	$3500	0.30
$4000	0.20	$4500	0.10

Expected Value: Solution

Determine which project looks better based on expected values.

Expected $\text{Value}_A = 1000(0.1) + 2000(0.3) + 3000(0.4) + 4000(0.2)$

$= \$2700$

Expected $\text{Value}_B = 1500(0.2) + 2500(0.4) + 3500(0.3) + 4500(0.1)$

$= \$2800$

Project B has the greatest expected value (benefit) and should be selected.

Oil exploration example

- Suppose drilling a well costs $700,000. A dry well provides zero benefits. A productive well provides $150000 in net annual benefits for 15 years if oil prices are low, $200,000 if high.
- A very productive well provides $300,000 in annual benefits for 20 years if oil prices are low, $400,000 if high. What is the expected value of drilling a single well?

Case	Cost	Annual Benefits	NPV (@15% interest)
Dry Well, Low Price	$700000	$0	-$700,000
Dry Well, High Price	$700000	$0	-$700,000
Productive Well, Low Price	$700000	$150000	$177,105
Productive Well, High Price	$700000	$200000	$469,474
Very Productive Well, Low Price	$700000	$300000	$1,177,800
Very Productive Well, High Price	$700000	$400000	$1,803,733

Oil exploration example

- Combining the probability for each case and the NPV for each case, we can calculate the expected value for each case, and the expected value for drilling a well overall

Case	P(case) x	NPV(case) =	Expected NPV
Dry Well, Low Price	28%	-$700,000	-$196,000
Dry Well, High Price	42%	-$700,000	-$294,000
Productive Well, Low Price	10%	$177,105	$17,711
Productive Well, High Price	15%	$469,474	$70,421
Very Productive Well, Low Price	2%	$1,177,800	$23,556
Very Productive Well, High Price	3%	$1,803,733	$54,112
Total (sum of all cases)	100%	N/A	-$324,200