Minkyun Noh
2021/01/18.

< Filtering >.

Oppenheim, Signals & Systems, 2nd Ed. (p.232 - 239).

- Filter = LTI system.

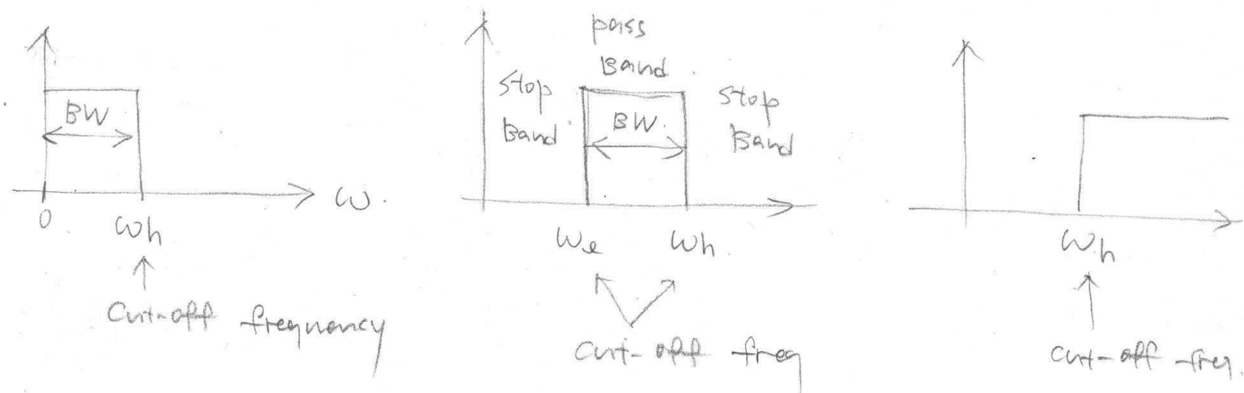
"Fourier transform"

$$\text{signal} \overset{\mathcal{F}}{\longleftrightarrow} \text{spectrum}$$

- Frequency-shaping filters.
  : change the shape of the <u>spectrum</u>. e.g., Eqalizer, Differenciator, Lead compensator.

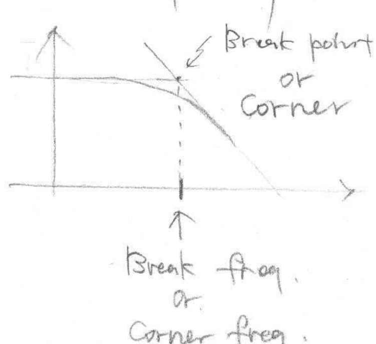
- Frequency-selective filters.
  : pass some frequencies undistorted and significantly attenuate other frequencies.

<Low pass> <Band pass> <High pass>



- For low-pass filters, Band width $= \omega_h - 0 = \omega_h$.
- For band-pass filters, Band width $= \omega_h - \omega_\ell$

- Corner frequency = Break (point) frequency



For frequency-selective filters,
We say < -3dB is sufficient attenuation (cut-off).

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< LTI System Mathematical Representations >

- Formal review is referred to the LaTex note.
- In summary, LTI systems can be represented with

① Impulse Response — time domain.
② Transfer function } Frequency domain
③ Frequency response

- The three mathematical representations are related as

$$\begin{tikzcd}
 & h(t) \arrow[dl, leftrightarrow, "\mathcal{L}"'] \arrow[dr, leftrightarrow, "\mathcal{F}"] & \\
H(s) \arrow[rr, leftrightarrow, "s=j\omega"'] & & H(j\omega)
\end{tikzcd}$$

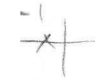
- Fourier transform is well-defined for both ways.
- Note the "duality" in the Fourier transform pair

$$\begin{cases} \text{Analysis}: & H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt \\ \text{Synthesis}: & h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega t)\, e^{j\omega t}\, d\omega . \end{cases}$$

- This is why we can infer the step resp. from the Bode plots and vice versa.

< LTI system Graphical Representations >.

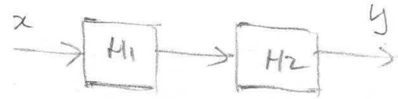
$H(s)$ $\begin{cases} \text{Block diagrams} \\ \text{pole-zero maps (doesn't show dc gain).} \\ \text{Bode plots.} \end{cases}$



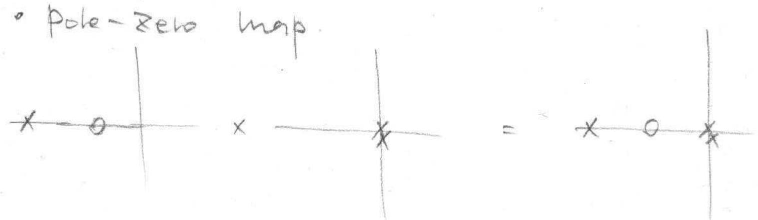
$\begin{cases} \frac{1}{s+1} \; ? \\ \frac{10}{s+1} \; ? \end{cases}$

① Series : $H = H_1 H_2$. $H_1 = \frac{10s+1}{s+1}$ $H_2 = \frac{1}{s^2}$

• Block diagram

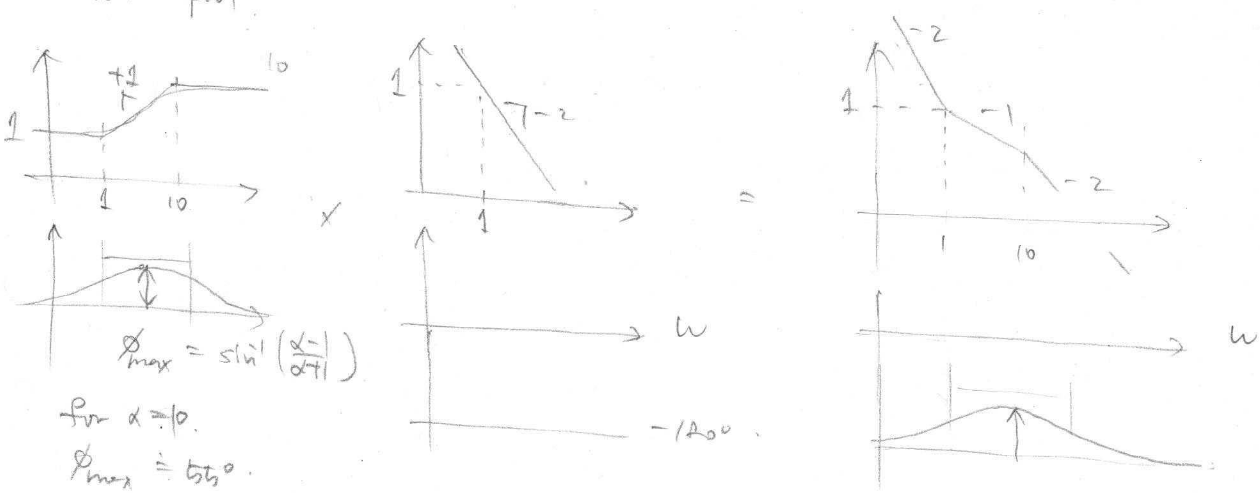


• pole-zero map



$\begin{cases} \text{poles remain the same.} \\ \text{zeros remain the same} \end{cases}$

• Bode plot



$\phi_{max} = \sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)$

for $\alpha = 10$.

$\phi_{max} \doteq 55°$.

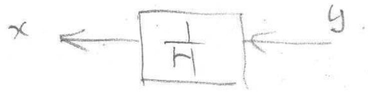
• Series compensation is convenient when shaping FRF.

e.g. Frequency-shaping filter

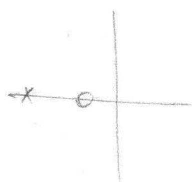
② Inverse $\quad H = \frac{10s+1}{s+1} \rightarrow H^{-1} = \frac{s+1}{10s+1}$

· Block diagram.

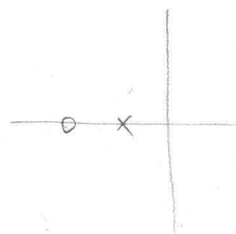
Reverse the signal flow.



· pole - zero map

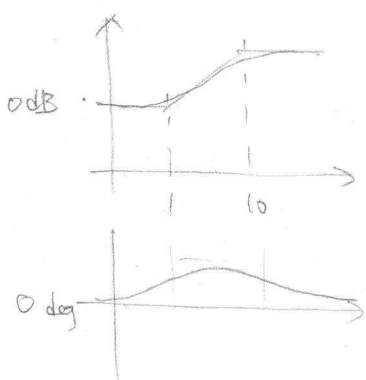


Inverse →

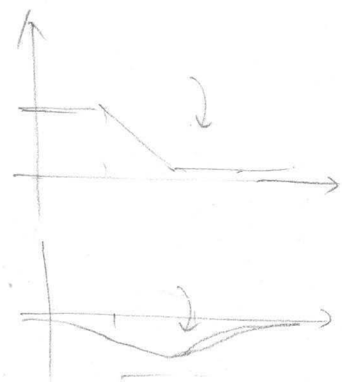


swap pole ↔ zero

· Bode plot



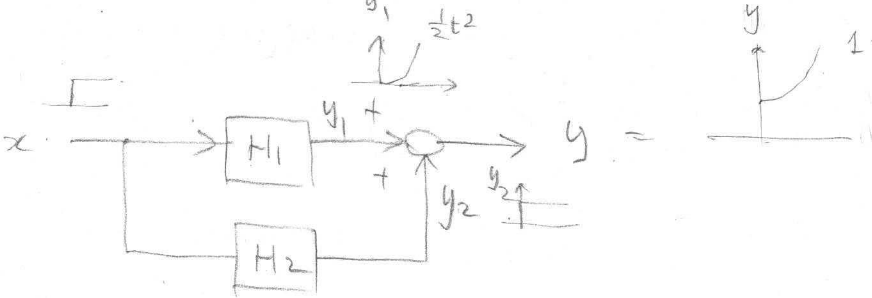
Inverse →



Mirroring w.r.t 0dB & 0 deg

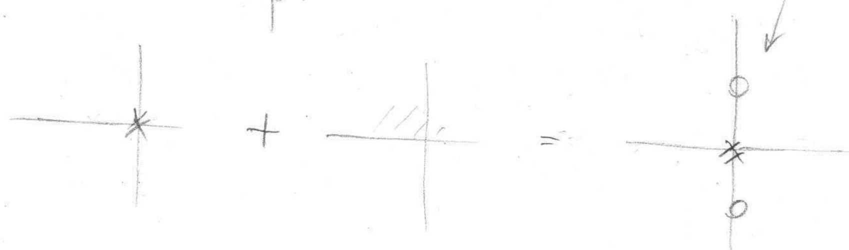
③ Parallel : $H = H_1 + H_2$. $H_1 = \frac{1}{s^2}$ $H_2 = 1$

- Block diagram.



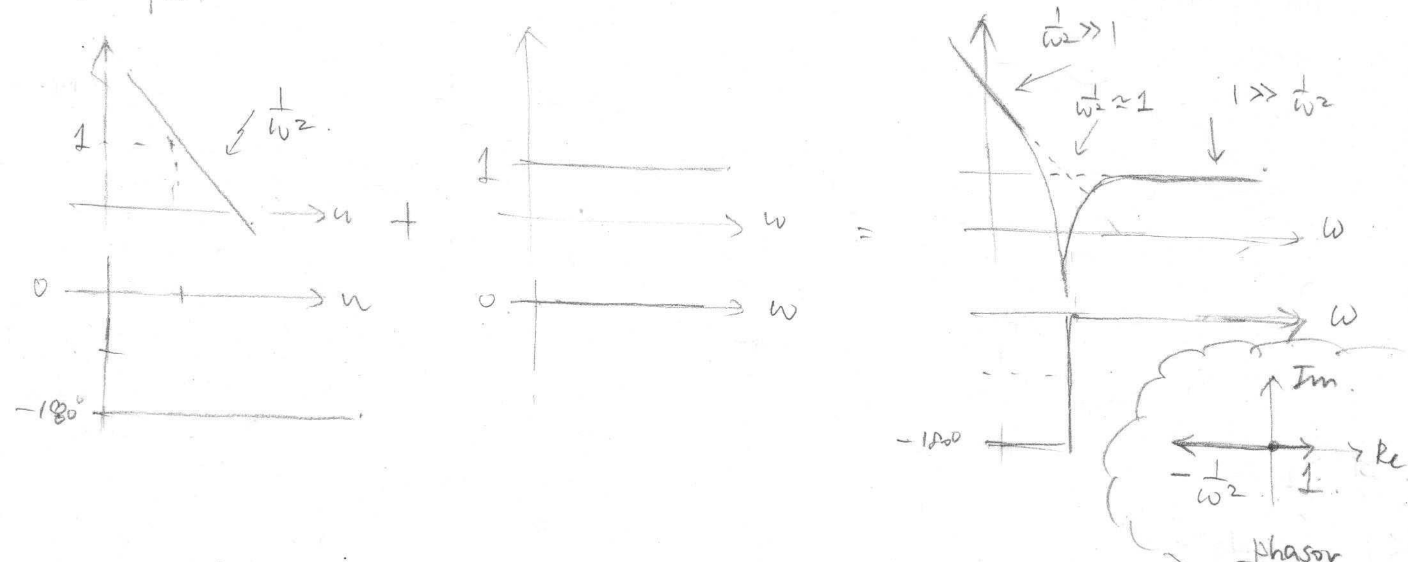
zeros on the imaginary axis
"Anti-resonance"
→ y_1 & y_2 cancel out at the zero frequency

- pole - zero map.

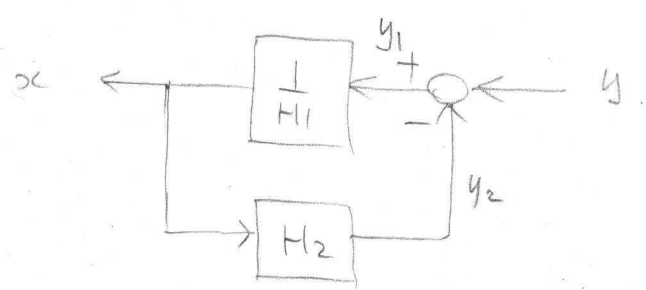


{ poles remain the same
zeros are newly created!

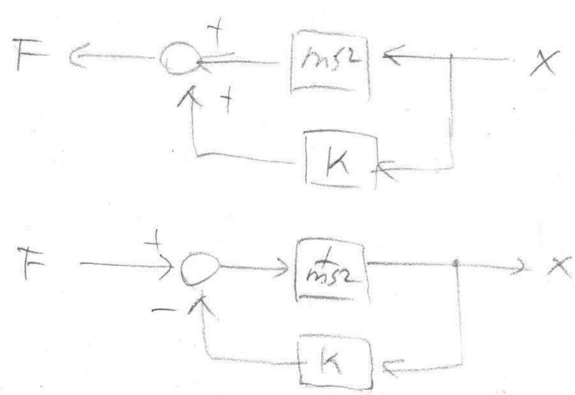
- Bode plot



(Optional) Inverse of Parallel



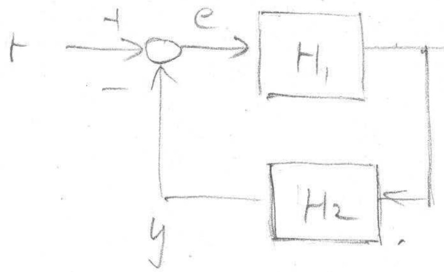
example : mechanical impedance.



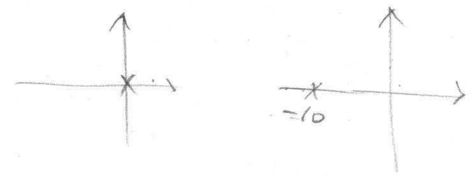
- Flip the signal flow
- Change the summing junction:
  $y = y_1 + y_2 \rightarrow y_1 = y - y_2$
- Reverse H_1, preserve H_2

☺ This skill is useful when handling 2x2 TF. Matrix

④ Feedback



$H_1 = \frac{1}{s}$. $H_2 = \frac{10}{s+10}$.



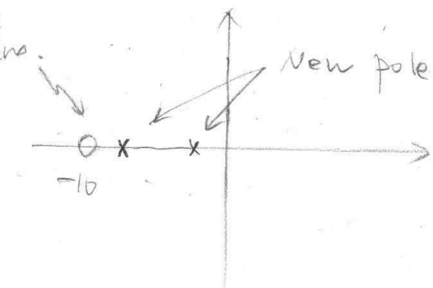
· pole-zero map.

$H_1 = \frac{a_1(s)}{b_1(s)}$ $H_2 = \frac{a_2(s)}{b_2(s)}$.

$$G = \frac{H_1}{1+H_1H_2} = \frac{\frac{a_1}{b_1}}{1+\frac{a_1a_2}{b_1b_2}} = \frac{a_1b_2}{a_1a_2+b_1b_2} = \frac{s+10}{s^2+10s+10}$$

pole of H_2 becomes zero.

New pole location : $s = -5 \pm \sqrt{15}$.



· poles : the roots of $a_1a_2+b_1b_2 = 0$.

these are different from the original poles, which are the roots of $b_1b_2 = 0$.

"Feedback moves the poles" : Root Locus.

· zeros : the roots of $a_1b_2 = 0$

these are $\begin{cases} \text{zeros of } H_1 \\ \text{poles of } H_2 \end{cases}$

· Caution : poles of feedback gain becomes the zeros of a closed-loop system.

· Common mistake : put LPF on the feedback path (x). unless it is for anti-aliasing.

∘ Bode plot

$$G = \frac{\text{Forward}}{1 + L(s)}$$

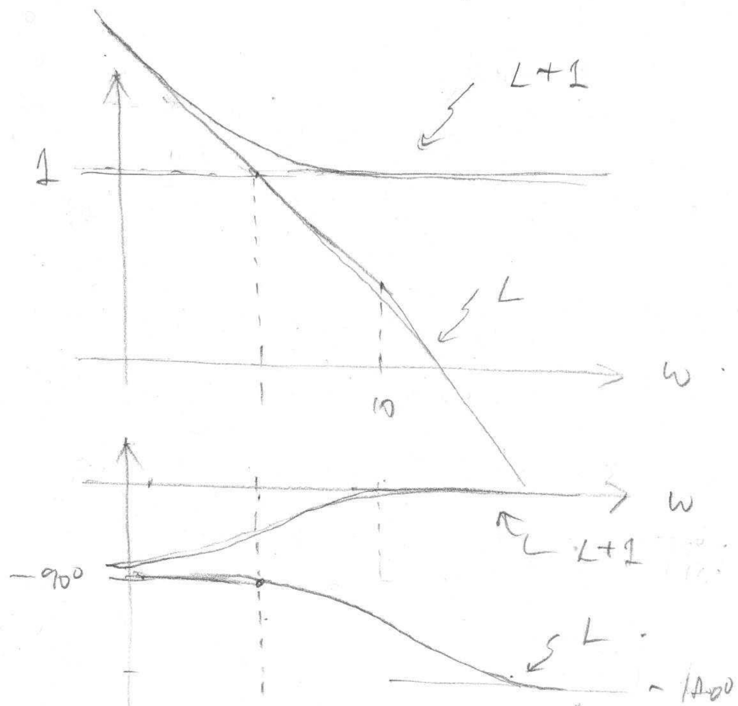
① Draw $1 + L(s)$

② Draw $\frac{1}{1+L} \triangleq S(s)$: "Sensitivity"

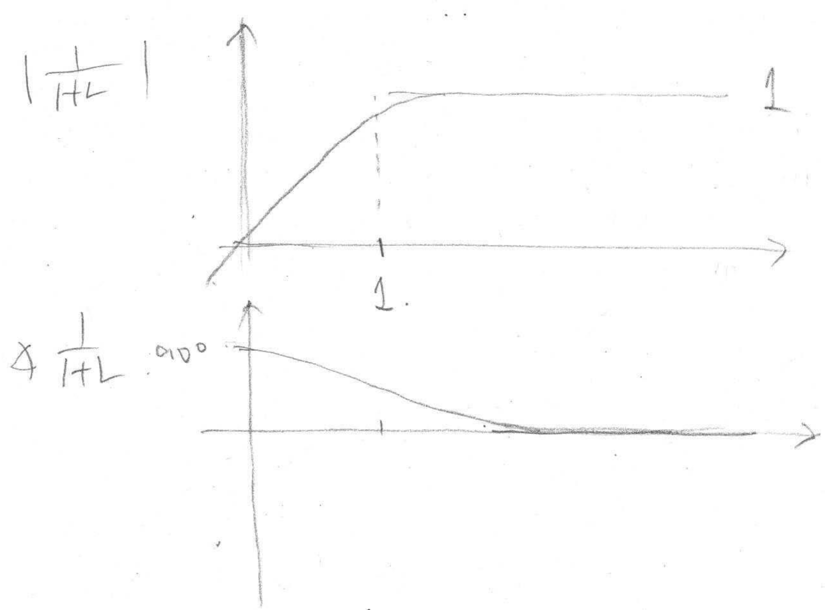
③ Draw $\frac{\text{Forward}}{1+L}$.

$$L(s) = \frac{1}{s}\left(\frac{1}{0.1s+1}\right)$$ ↪ unity upto 10 rad/s

① "Add" two Bode plots.



② Flip it around 0 dB & 0 deg



③ "Multiply" forward gain ($= \frac{1}{s}$)