MECH 421: Mechatronics System Instrumentation
2020/2021 Winter Session – Term 2

Homework 3

Assigned: Feb 5, 2021
Due: Feb 12, 2021

Problem 1

Let us consider an op-amp circuit in Figure 1. We assume that the op-amp input impedance is infinite, the output impedance is R_o, and the open-loop gain is $A(s)$. Note that the output impedance R_o is pulled out of the op-amp.

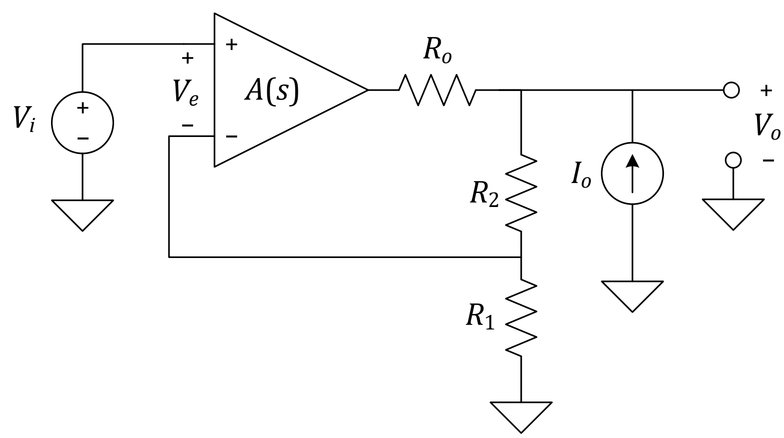

Figure 1: Schematic of a non-inverting amplifier.

(a) Draw a block diagram that shows the relation between the input voltage V_i, disturbance current I_o, and the output voltage V_o. The block diagram should show a feedback loop around $A(s)$.

(b) Find the expression for the loop transfer function $L(s)$. Also, find the expression for the output impedance $Z_o(s) = V_o/I_o$ in terms of $L(s)$, R_o, R_1, and R_2.

(c) Assuming the op-amp high-frequency open-loop gain is zero, i.e., $A(j\omega)|_{\omega\to\infty} = 0$, find the expression for the high-frequency output impedance $Z_o(j\omega)|_{\omega\to\infty}$ in terms of R_o, R_1, and R_2. Also, find the expression for the output impedance normalized to the high-frequency value, i.e.,

$$\hat{Z}(s) \equiv \frac{Z_o(s)}{Z_o(j\omega)|_{\omega\to\infty}}$$

in terms of $L(s)$.

(d) Using MATLAB, draw the Bode plots of $L(s)$ and $\hat{Z}(s)$ for

$$R_o = 50\,\Omega \qquad R_1 = 1\,\mathrm{k}\Omega \qquad R_2 = 1\,\mathrm{k}\Omega \qquad A(s) = \frac{10^4}{(s+1)(0.001s+1)}$$

on the same graph. Find the gain crossover frequency ω_c and phase margin ϕ_m of $L(s)$, and also find the cutoff frequency ω_l where $|\hat{Z}(j\omega_l)| = 0.7071(-3\,\mathrm{dB})$. Discuss how the shapes of $|L(j\omega)|$ and $|\hat{Z}(j\omega)|$ are related at frequencies below ω_c.

(e) Find the output dc impedance $Z_o(j\omega)|_{\omega=0}$

Problem 2

Let us consider a brushed dc motor driven by a voltage amplifier shown in Figure 2. Here, L_m is the winding inductance, R_m is the winding resistance, J_m is the rotor rotational inertia, and K_t is the motor torque constant. There is a wheel mounted on the motor shaft, whose rotational inertia is J_w. On the current return path, there is a shunt resistor R_s to measure the current through the motor winding.

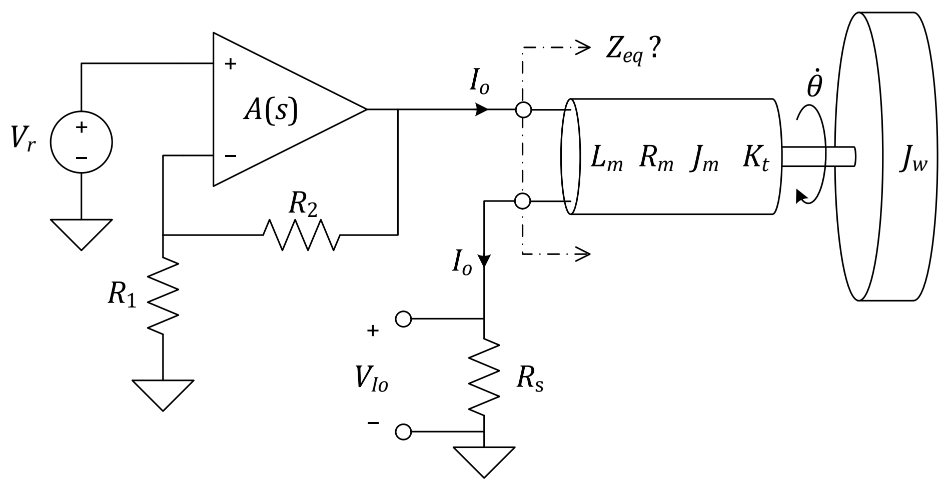

Figure 2: Brushed dc motor driven by an op-amp circuit.

(a) Find the electrical impedance $Z_{eq}(s)$ looking into the electrical port of the motor.

(b) Draw an equivalent circuit diagram where all mechanical elements are referred to the electrical domain as passive electrical elements. Find the parameter(s) of the equivalent circuit element(s) in terms of L_m, R_m, J_m, K_t, and J_w.

(c) Find the analytic expression for the transconductance from $V_r(s)$ to $I_o(s)$. Then, draw the Bode plot of $I_o(s)/V_r(s)$ using MATLAB for

$$
\begin{array}{lll}
A(s) = \dfrac{10^7}{s} & R_1 = 1\,\mathrm{k\Omega} & R_2 = 9\,\mathrm{k\Omega} \\
R_m = 4.8\,\Omega & L_m = 1\,\mathrm{mH} & R_s = 0.2\,\Omega \\
K_t = 250\,\mathrm{mNm/A} & J_m = 1\,\mathrm{kg\cdot cm^2} & J_w = 9\,\mathrm{kg\cdot cm^2}
\end{array}
$$

(d) Let us assume that the shunt resistor R_s is rated for 1 W power. That is, the resistor fails when it dissipates more than 1 W. What is the maximum rms current $I_{o,\mathrm{rms}}$ allowed for the shunt resistor?

(e) Let the input voltage V_r be sinusoidal at 50 Hz. What is the maximum rms voltage $V_{r,\mathrm{rms}}$ that the shut resistor can accommodate for its power rating?