Tutorial 1

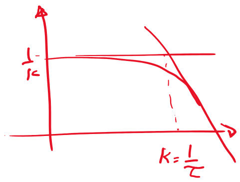
January 19, 2021 11:07 PM

Bode's form

$$G(s) = \frac{1}{k} \cdot \frac{1}{\frac{1}{k}s + 1}$$

DC gain

time constant



- Laplace transform

$$G(s) = \frac{1}{s+k}$$

$R(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$

$$Y(s) = G(s) \times R(s)$$

Frequency domain $\Leftrightarrow$ time domain

$$f(t) = 1 \xrightarrow{\mathcal{L}} F(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s} \times \frac{1}{s+k} = \frac{1}{s^2 + ks} \xRightarrow{\mathcal{L}^{-1}} y(t)$$

time domain

$$\frac{1}{s(s+k)} = \frac{A}{s} + \frac{B}{s+k} \rightarrow \begin{cases} A = \frac{1}{k} \\ B = -1/k \end{cases}$$

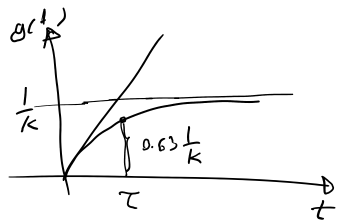
$$Y(s) = \frac{1}{k}\left(\frac{1}{s} - \frac{1}{s+k}\right) \xrightarrow{\mathcal{L}^{-1}}$$

$$\boxed{y(t) = \frac{1}{k}\left(1 - e^{-kt}\right)}$$

empilicit

$t \to \infty$, $e^{-kt} \to 0$

$$y(t) = \frac{1}{k}$$



- $$F(s) = \frac{5s - 1}{s(s^2+1)(s-1)} \xrightarrow{\mathcal{L}^{-1}} f(t) = 1 + 2e^{t} - 3\cos t - 2\sin t$$

- $$G(s) = \frac{1}{ms^2 + k} \longrightarrow Y(s) = \frac{1}{s} \times \frac{1}{ms^2 + k} \xrightarrow{\mathcal{L}^{-1}}$$

$$y(t) = \frac{1}{k} - \frac{1}{k}\cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

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ex 2.

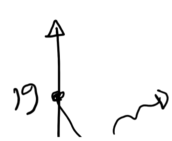
$$H(s) = \frac{20s + 1}{s + 1}$$

Bode plot
Step response

$$Y(s) = \frac{20s+1}{s(s+1)} = \frac{A}{s} + \frac{Bs + C}{s+1} \rightarrow \begin{cases} A = 1 \\ B = 0 \\ C = 19 \end{cases}$$

$$Y(s) = \frac{1}{s} + \frac{19}{s+1}$$

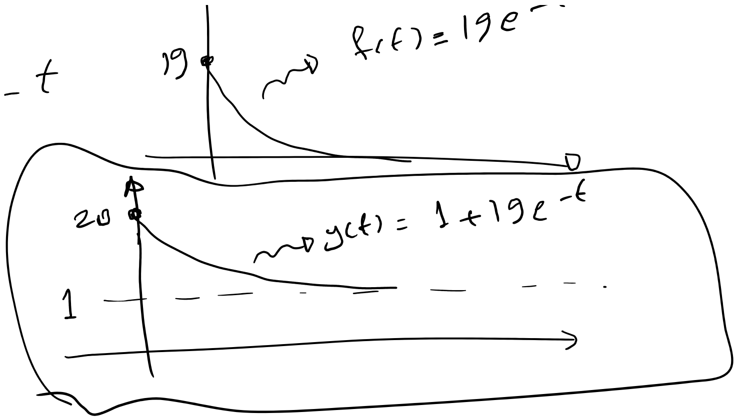
$\mathcal{L}^{-1}$ ... $-t$



$\rightsquigarrow f(t) = 19e^{-t}$

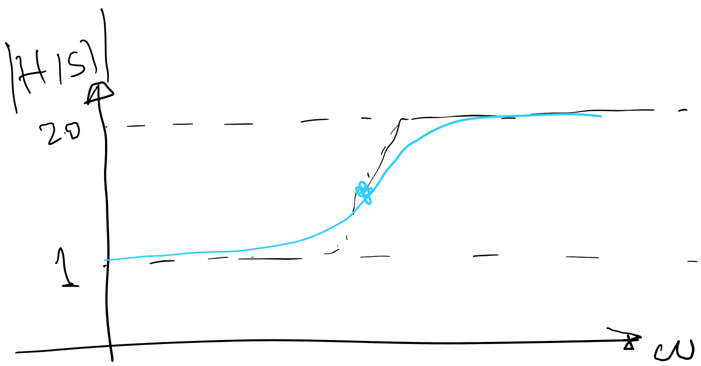
$H(s) = \dots \quad s+1$

$\mathcal{L}^{-1} \rightarrow y(t) = 1 + 19e^{-t}$



$$H(s) = \frac{20s+1}{s+1}$$

$$\begin{cases} s \to \infty : & H(s) = 20 \\ s \to 0 : & H(s) = 1 \end{cases}$$



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ex. 3 $\quad H(s) = \dfrac{5}{12s+3}$

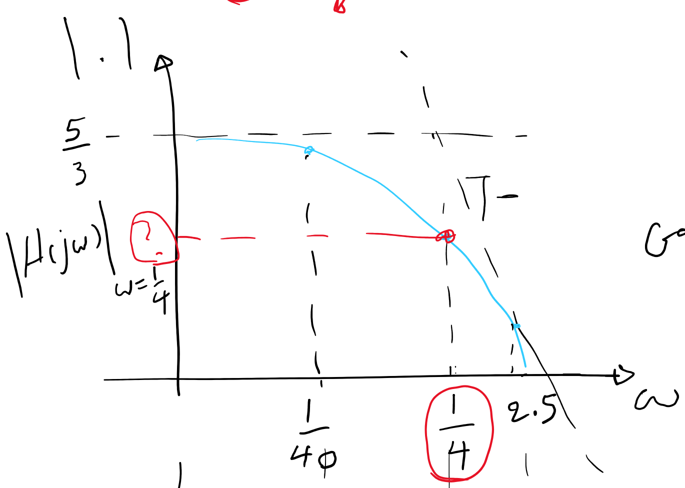
$$= \frac{5}{3} \cdot \frac{1}{4s+1}$$

$\rightarrow x(t) = 5\sin\left(\frac{1}{4}t + \frac{\pi}{4}\right)$

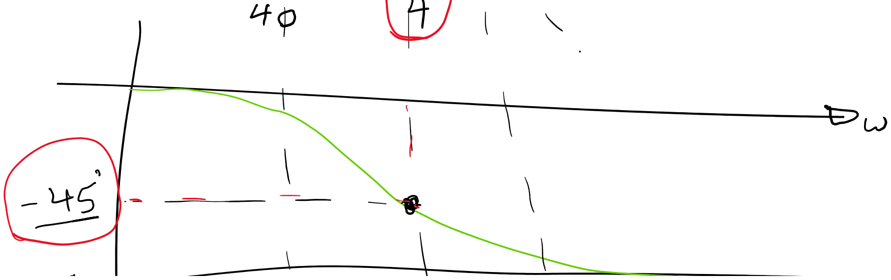
$\mathcal{L} \rightarrow X(s) \rightarrow Y(s) = X(s) \times H(s) \xrightarrow{\mathcal{L}^{-1}} y(t)$

Ans.

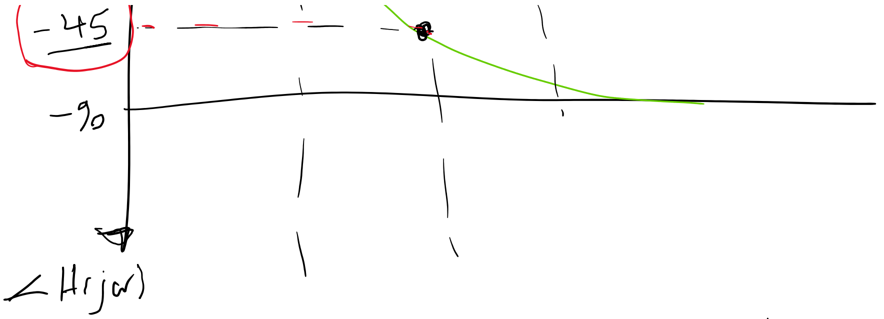
$\rightarrow y(t) = \left| H(j\omega) \right|_{\omega = \frac{1}{4}} \times 5 \sin\left(\frac{1}{4}t + \frac{\pi}{4} - \frac{\pi}{4}\right)$



Gain



phase



$$\left|H(j\omega)\right|_{\omega=\frac{1}{4}} = \left|\frac{5}{3}\,\frac{1}{4(\frac{1}{4}j)+1}\right| = \frac{5}{3}\left|\frac{1}{j+1}\right|$$

$$\frac{\times(j-1)}{\times(j-1)} = \frac{5}{3}\left|\frac{j-1}{j^2-1}\right| = \frac{5}{3}\left|\frac{j-1}{-2}\right| = \frac{5}{3}\left|-\frac{1}{2}j+\frac{1}{2}\right|$$

$j^2=-1$

$$= \frac{5}{3}\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \frac{5}{3}\times\frac{\sqrt{2}}{2}$$

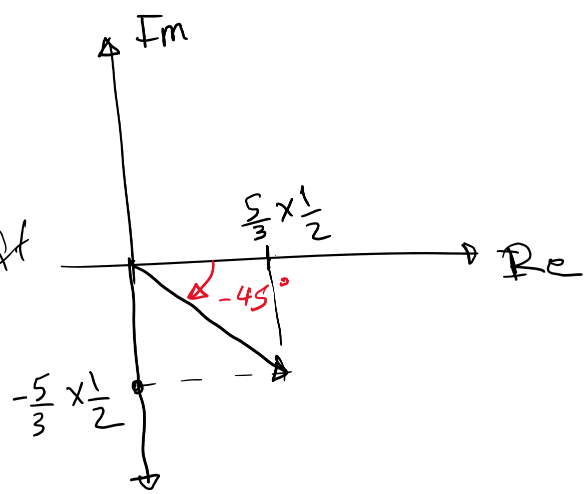
Gain

$\angle H(jw)$

$$H(jw) = \frac{5}{3}\left(-\frac{1}{2}j+\frac{1}{2}\right)$$

$$\angle H(jw) = \operatorname{atan}\left(\frac{-\frac{5}{6}}{\frac{5}{6}}\right) = -\frac{\pi}{4}$$

phase shift



$$y(t) = \frac{5}{3}\times\frac{\sqrt{2}}{2}\times 5\sin\left(\frac{1}{4}t+\frac{\pi}{4}-\frac{\pi}{4}\right)$$

mass_spring step response

- $$G(s) = \frac{1}{ms^2+k}$$

$$Y(s) = \frac{1}{\quad} = \frac{A}{s} + \frac{Bs+C}{ms^2+k}$$

$$Y(s) = \frac{1}{s(ms^2+k)} = \frac{A}{s} + \frac{Bs+C}{ms^2+k}$$

$$\xrightarrow{\times s} \quad \frac{1}{(ms^2+k)} = - \left.\frac{Bs+C}{{}^2+k}\right)s^{0}$$

$$\xrightarrow{s=0} \quad \frac{1}{k} = A$$

$$\frac{1}{s(ms^2+k)} = \frac{1/k}{s} + \frac{Bs+C}{ms^2+k} = \frac{\frac{m}{k}s^2 + 1 + Bs^2 + Cs}{s(ms^2+k)}$$

$$\begin{cases} \frac{m}{k} + B = 0 \Rightarrow B = -\frac{m}{k} \\ C = 0 \end{cases}$$

$$Y(s) = \frac{\frac{1}{k}}{s} - \frac{m}{k}\frac{s}{ms^2+k}$$

$$= \frac{1/k}{s} - \frac{\frac{m}{k} s}{ms^2+k} = \frac{\frac{1}{k}}{s} - \frac{\frac{\cancel{m}}{k} s}{\cancel{m}(s^2+\frac{k}{m})}$$

$$= \frac{\frac{1}{k}}{s} - \frac{1}{k}\frac{s}{s^2+\frac{k}{m}} \rightarrow \left(\sqrt{\frac{k}{m}}\right)^2$$

$$= \frac{\frac{1}{k}}{s} - \frac{1}{k}\frac{s}{s^2+\left(\sqrt{\frac{k}{m}}\right)^2}$$

$$\xrightarrow{L^{-1}} y(t) = \frac{1}{k}\left(1 - \cos\left(\sqrt{\frac{k}{m}}\, t\right)\right)$$