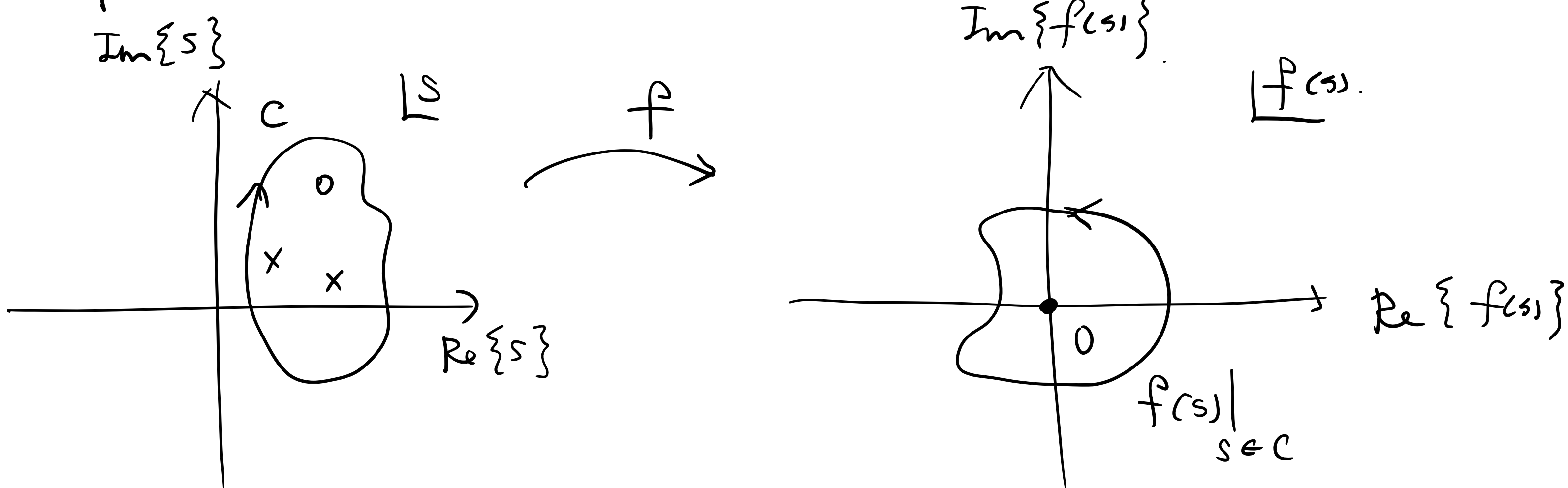
L21 – Nyquist Test 2

- Argument principle.
  - Complex function $f(s)$. $f: \mathbb{C} \to \mathbb{C}$



$Z = 1$

$p = 2$

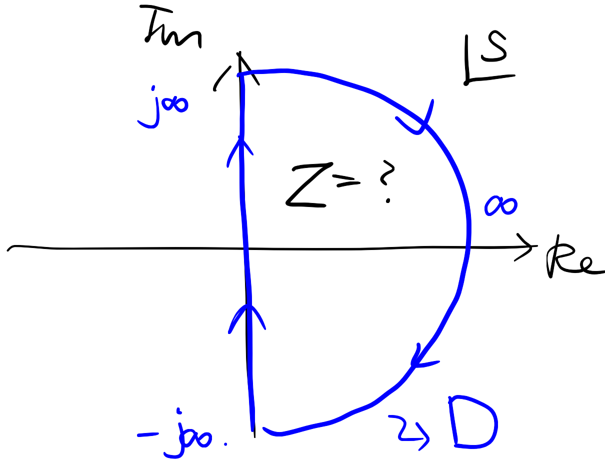
$N = Z - P$

$= -1$

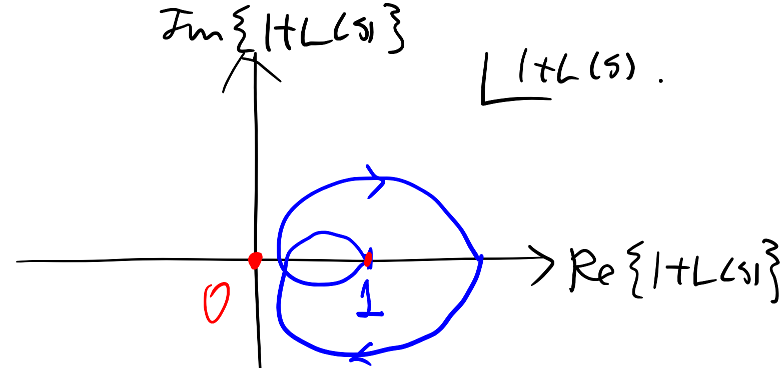
$\frac{1}{1+L(s)}$ "Z" Zeros of $1+L(s)$ $\Rightarrow$ "Infer" $Z = P + N$

- Apply it to $f(s) = 1 + L(s)$.

$\left(\frac{1}{1+L(s)}\right)$



$\xrightarrow{1+L(s)}$



$1+L(s)|_{s\in D}$

$\Downarrow$

N

Z : # of zeros of $1+L(s)$ inside D.

P : # of poles of $1+L(s)$ "

N : # of CW encirclement of $1+L(s)|_{s\in D}$ about the origin.

$$\underline{Z = P + N}.$$

$\#$ of RHP zeros of $\boxed{1+L(s)}$

$=$ # of RHP poles of $\frac{1}{1+L}$.

- Nyquist test.
  - P & N can be obtained from $\underline{L(s)}$.

P : # of RHP poles of $\underline{1+L(s)}$.

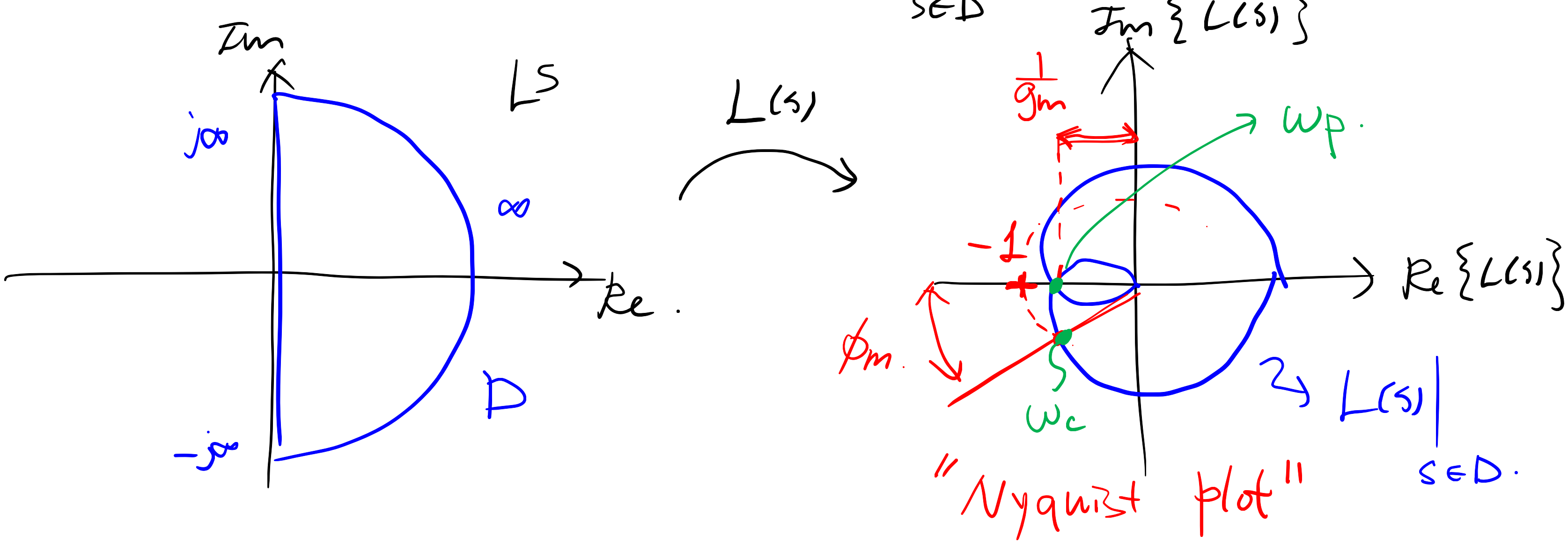
$= $ # of RHP poles of $\underline{L(s)}$.

$L(s_0) \to \infty$.

$\Leftrightarrow 1+L(s_0) \to \infty$

N : # of CW enc. of $1+L(s)\big|_{s\in D}$ about "0"

$=$ # of CW enc of $L(s)\big|_{s\in D}$ about "-1"



$$Z = P + N.$$

Z: # of RHP zeros of $1+L(s)$.

\# of RHP poles of $\frac{1}{1+L(s)}$.

$Z=0 \Rightarrow$ "stable"

P: # of RHP poles of $L(s)$.

N: # of the CW enc. of the Nyquist plot about -1.

- Nyquist plot vs Loop Bode plot. $L(j\omega)$.

$$L(s)\Big|_{s\in D}. \quad \begin{cases} L(s)\Big|_{s\in \uparrow} = L(j\omega) \\ L(s)\Big|_{s\in)} = 0 \text{ for most syst.} \end{cases}$$

$$\begin{cases} L(j\omega) \text{ for } \omega>0. \leftarrow \text{Bode plot.} \\ L(j\omega) \text{ for } \omega<0. \leftarrow \text{"Conjugate Symmetriy"} \end{cases}$$

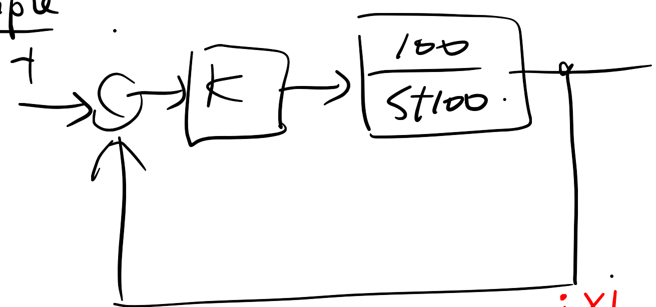
"Real" signal

$$L(-j\omega) = L(j\omega)^*$$

"Conjugate Sym"

$$\begin{cases} \mathrm{Re}\{L(-j\omega)\} = \mathrm{Re}\{L(j\omega)\} \\ \mathrm{Im}\{L(-j\omega)\} = -\mathrm{Im}\{L(j\omega)\} \end{cases}$$

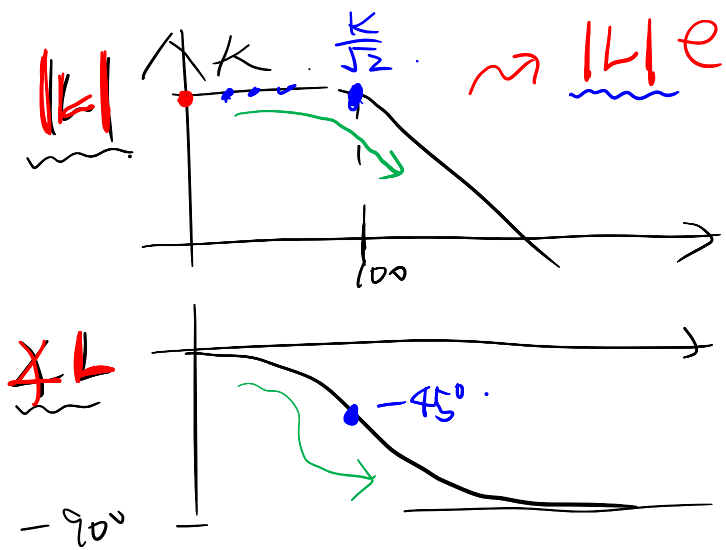
Example



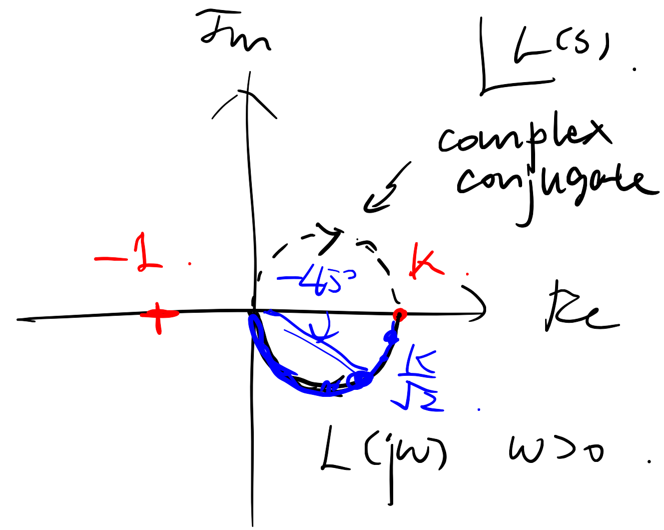
$$L(s) = K \frac{100}{s+100}.$$

$\rightarrow$ $P = 0$.

$N = 0$

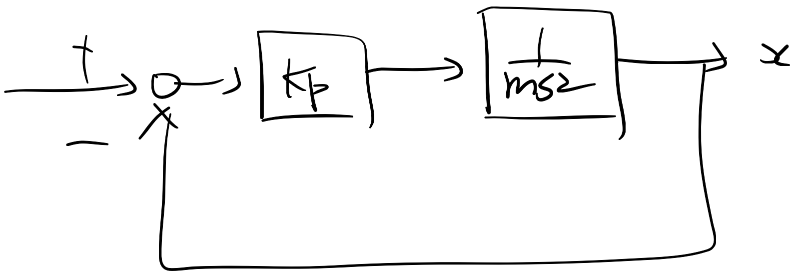


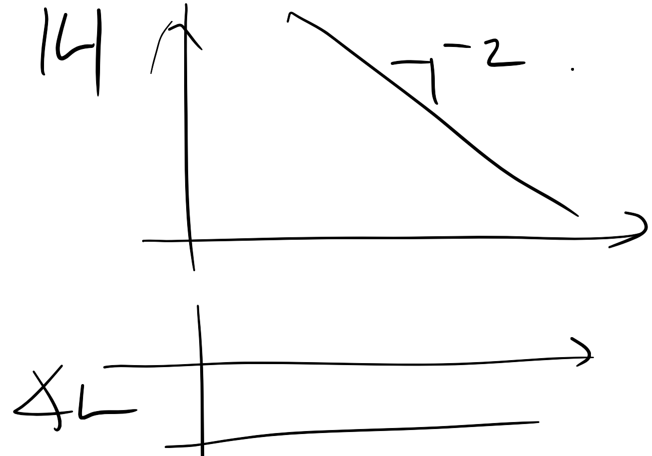
$\rightsquigarrow |L| e^{j\angle L}$ polar plot.



$\Rightarrow$ "$Z = 0$" $\Rightarrow$ stable.

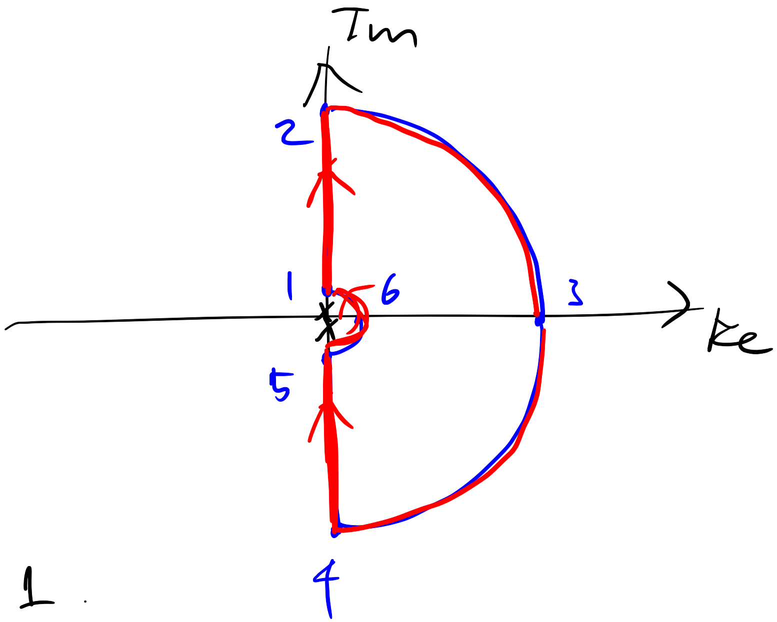
Examples

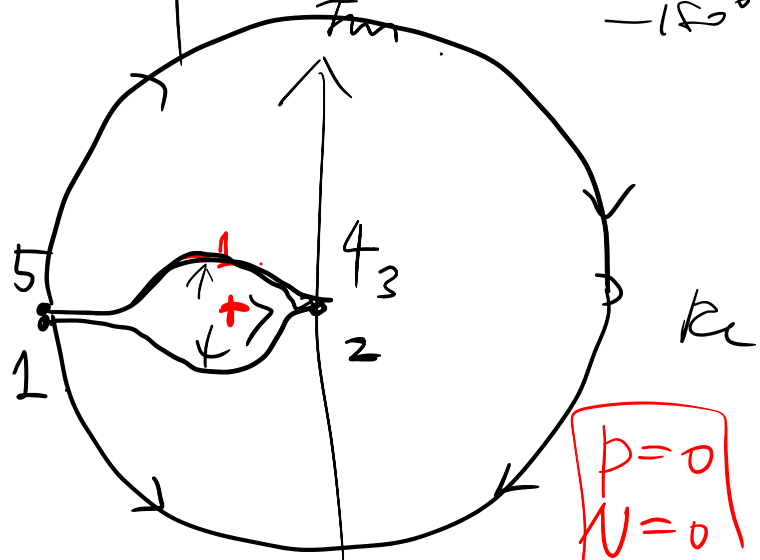




$-180°$

$$L(s) = K_p \cdot \frac{1}{ms^2}$$

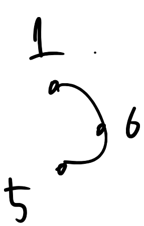




$p = 0$

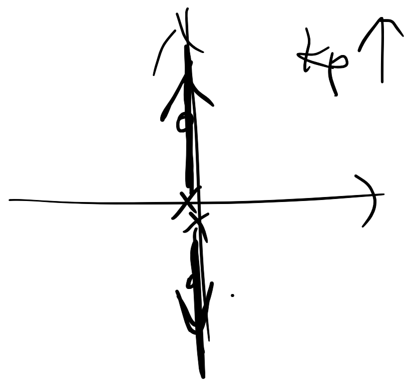
$N = 0$

$\Rightarrow Z = 0$



$s = re^{j\theta}$. $\quad \theta = -\frac{\pi}{2} \to 0 \to \frac{\pi}{2}$. $\quad \angle L: \pi \to 0 \to -\pi$

$$L(s)\big|_{s=re^{j\theta}} = \frac{K_p}{mr^2 e^{j2\theta}} = \left(\frac{K_p}{mr^2}\right) e^{-j2\theta}$$



Kp ↑