< Nyquist Test >

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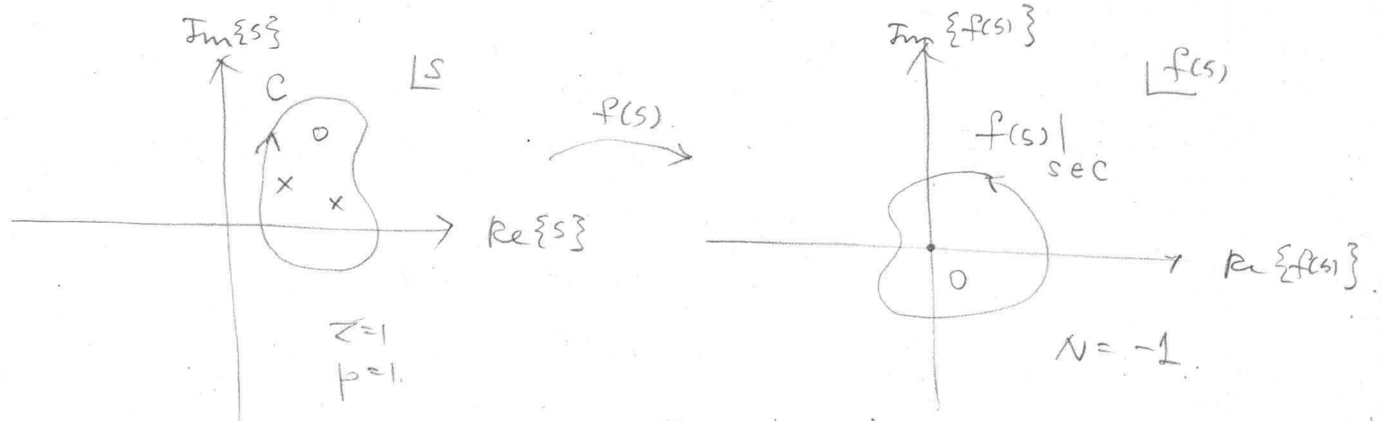
- Argument principle.

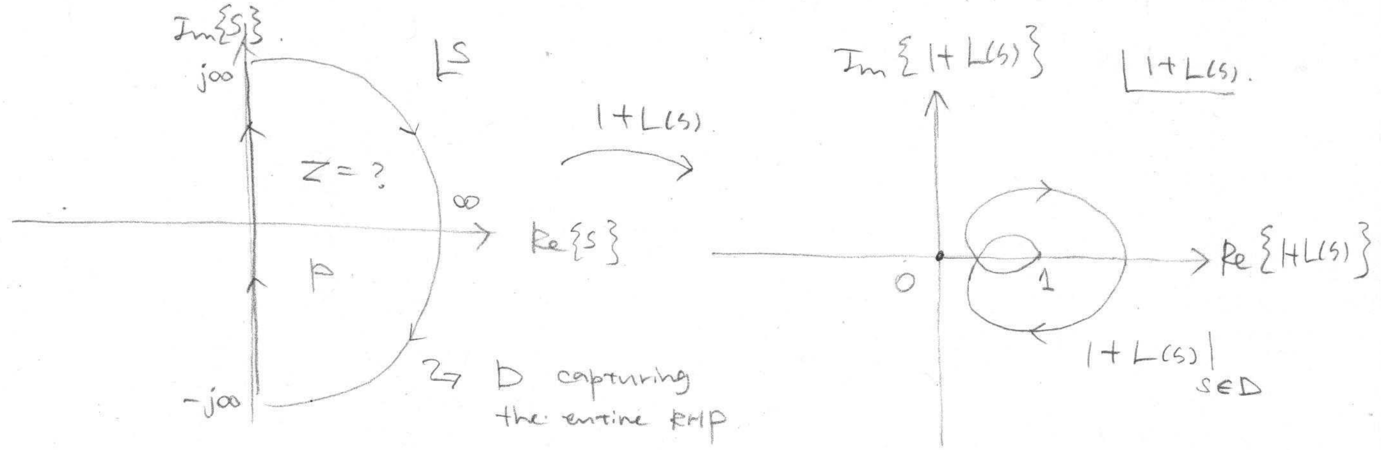
  - Consider a complex function $f: \mathbb{C} \to \mathbb{C}$
    (analytic on and inside C except at poles & zeros)



$N = Z - p \Rightarrow Z = p + N$. (Inference)

- Argument principle applied to $f(s) = 1 + L(s)$.

  - We are concerned whether $1 + L(s)$ has RHP zeros.



Z : # of zeros of $1+L(s)$ inside D.

p : # of poles of $1+L(s)$ inside D.

N : # of CW encirclement of $1+L(s)\big|_{s\in D}$ about the origin.

$Z = p + N$ tells us $\begin{cases} \text{\# of RHP zeros of } 1+L(s) \\ \text{\# of RHP poles of } \frac{1}{1+L(s)} \end{cases}$ "Stability"

- Nyquist test.

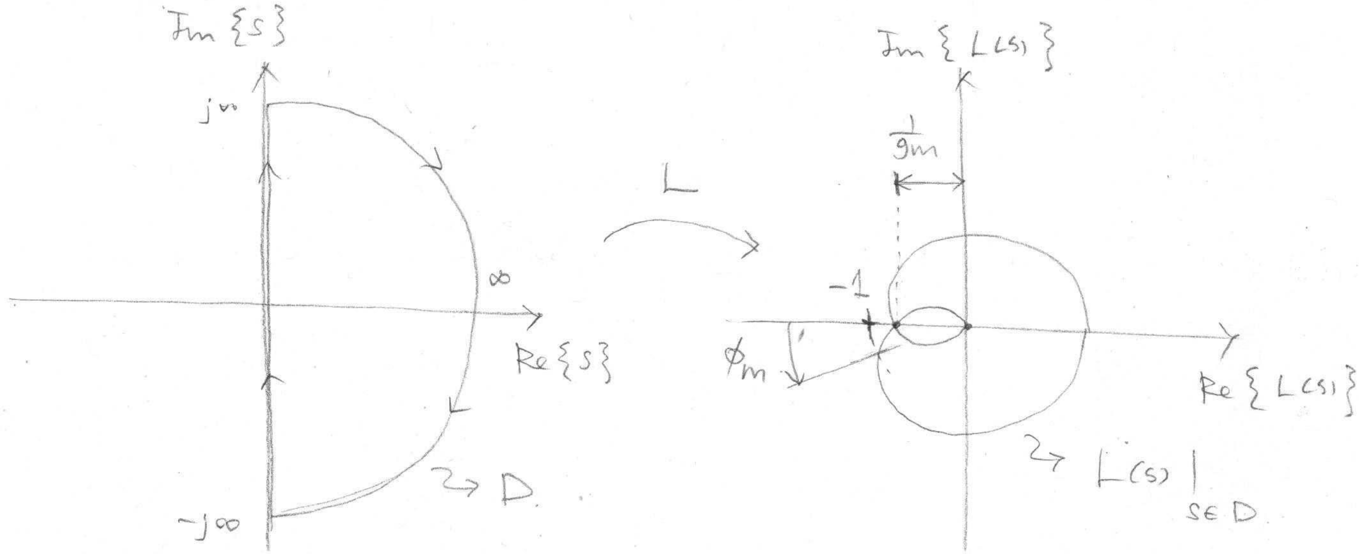
- P & N can be alternatively obtained from $L(s)$

  P : # of RHP poles of $1+L(s)$ $\left(L(s_0) \to \infty \iff 1+L(s_0) \to \infty \right)$

  $=$ # of RHP poles of $L(s)$.

  N : # of CW encirclements of $1+L(s)\big|_{s\in D}$ about "0"

  $=$ # of CW encirclements of $L(s)\big|_{s\in D}$ about "-1"



"Nyquist plot"

- In summary,

$$Z = P + N$$

Z : # of RHP zeros of $1+L(s)$

$=$ # of RHP poles of $\dfrac{1}{1+L(s)}$

$Z=0$ for stability.

P : # of RHP poles of $L(s)$.

N : # of the CW encirclements of the Nyquist plot about -1.

- Nyquist plot vs. Loop Bode plot.

  - Nyquist test is more powerful because

    ① It covers a wider class of $L(s)$,

    ② It requires less information on $L(s)$: P and $L(j\omega)$

  - Nyquist plot $L(s)\big|_{s\in D}$ consists of $\begin{cases} L(s)\big|_{s\in\Gamma_1} = L(j\omega) \\ L(s)\big|_{s\in\Gamma_2} = 0 \quad \text{for physical syst.} \end{cases}$

  - $L(j\omega)$ for $\omega > 0$ can be drawn from Bode plot.

  - $L(j\omega)$ for $\omega < 0$ can be drawn from conjugate symmetry

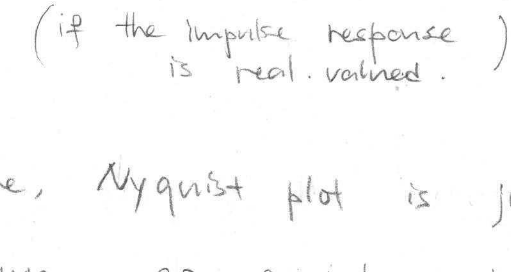
    i.e.) $L(-j\omega) = L(j\omega)^*$ (if the impulse response is real-valued.)

    $Re\{L(-j\omega)\} = Re\{L(j\omega)\}$

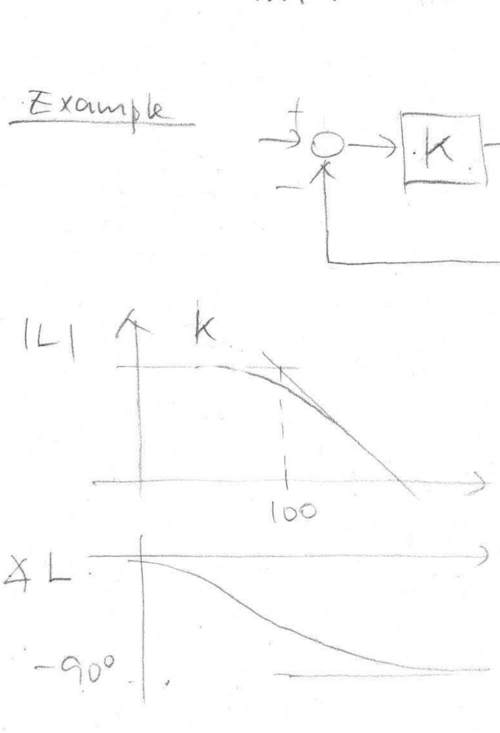
    $Im\{L(-j\omega)\} = -Im\{L(j\omega)\}$

  - Therefore, Nyquist plot is just the Loop Bode plot re-drawn as a polar plot.

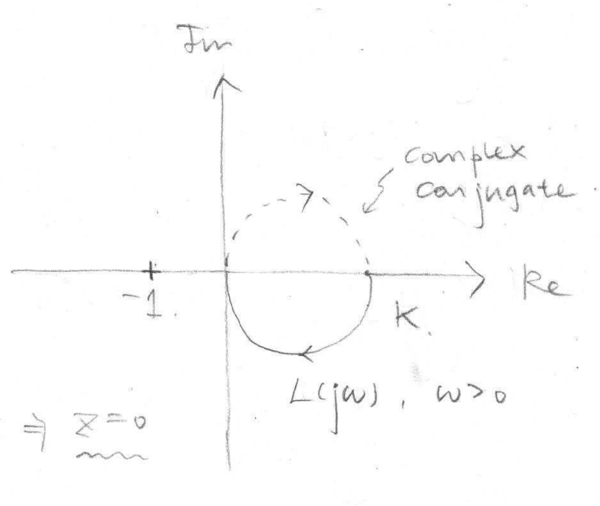
Example



$L(s) = K \cdot \frac{100}{s+100}$

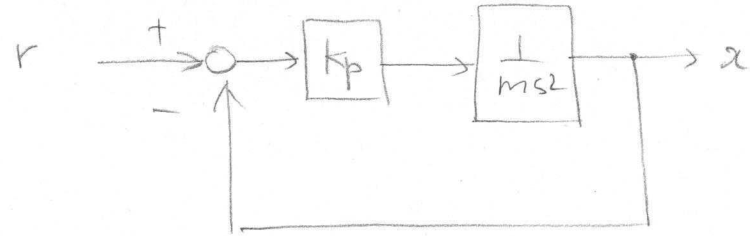


polar plot $\longrightarrow$

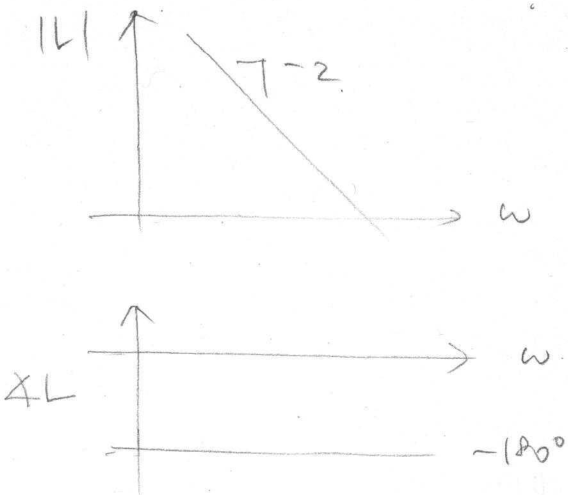


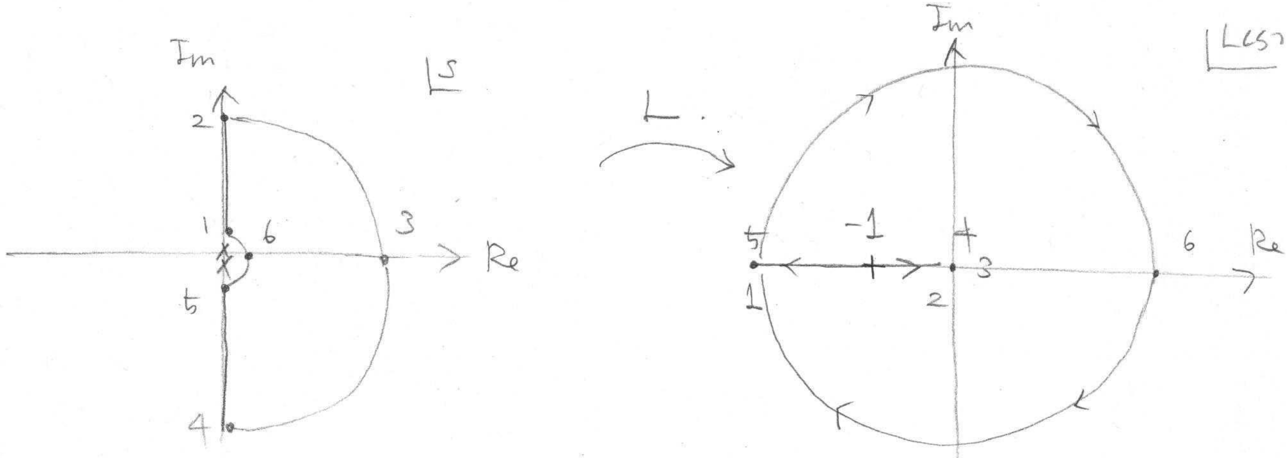
$P=0$, $N=0$ $\Rightarrow$ $\underline{Z=0}$

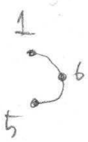
Example   Free mass + K_p.



$$L(s) = \frac{K_p}{ms^2}$$





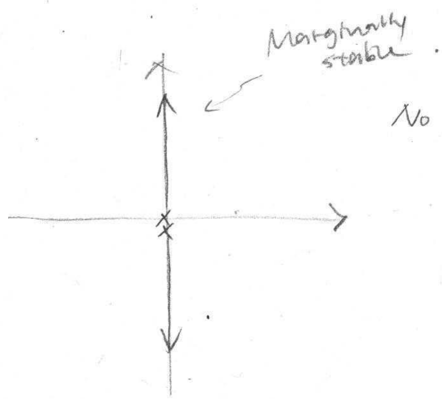


$s = re^{j\theta}$, $\theta = -\frac{\pi}{2} \to 0 \to \frac{\pi}{2}$

$$L(s)\Big|_{s=re^{j\theta}} = \frac{K_p}{mr^2 e^{j2\theta}} = \frac{K_p}{mr^2} \cdot e^{-j2\theta} \quad \therefore \angle L = \pi \to 0 \to -\pi$$

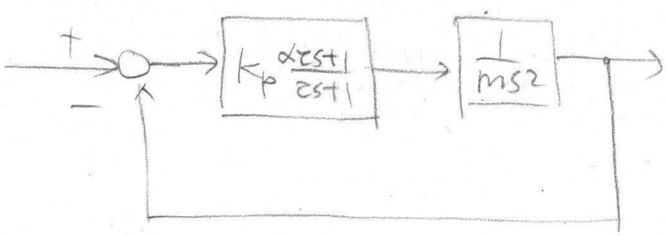
$P = 0$, $N = 0$ $\rightarrow$ $Z = 0$. No RHP poles.

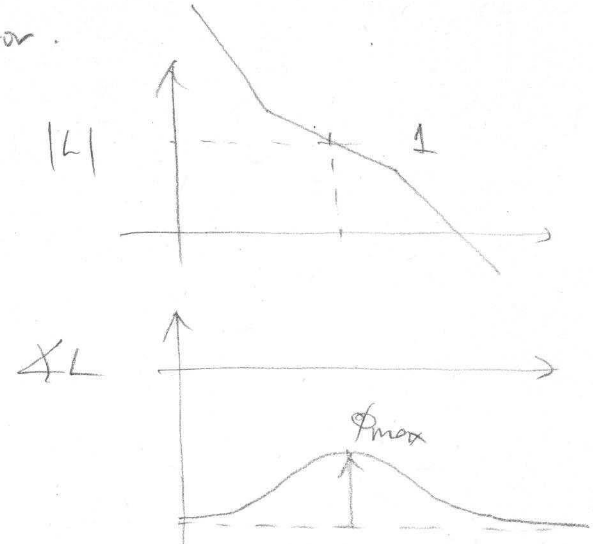
Root Locus



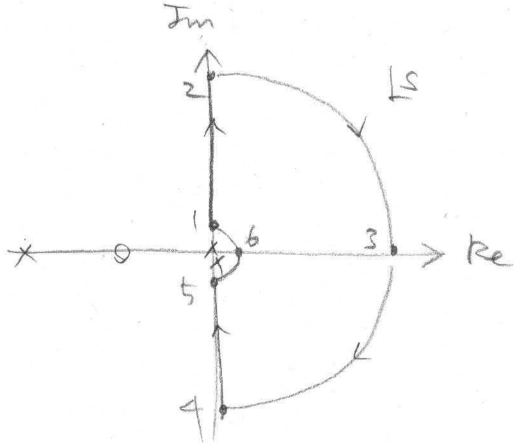
No RHP poles

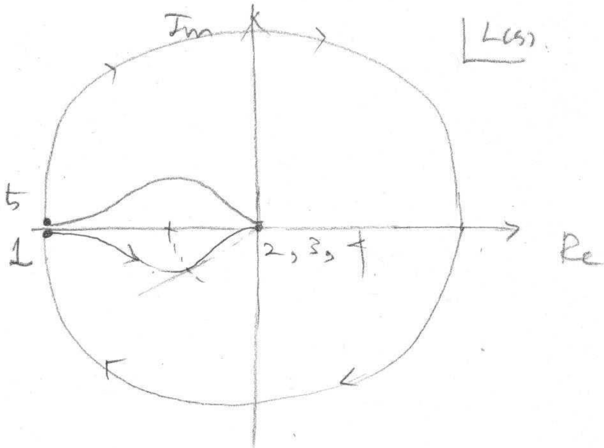
Example. Free mass + Lead compensator.





$$L(s) = K_p \frac{\alpha\tau s+1}{\tau s+1} \frac{1}{ms^2}.$$



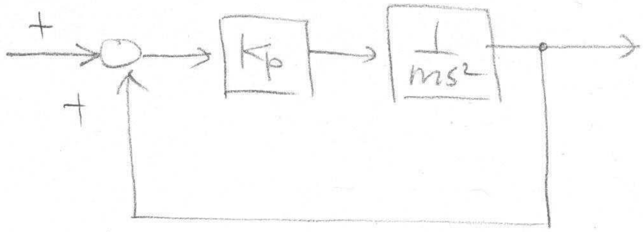


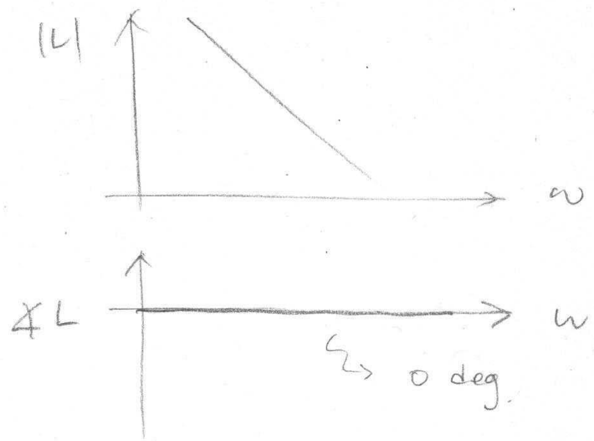
$s = re^{j\theta}$; $\theta = -\frac{\pi}{2} \to 0 \to \frac{\pi}{2}$.

$$L(s)\Big|_{s=re^{j\theta}} = \frac{K_p}{mr^2 e^{j2\theta}} = \frac{K_p}{mr^2} e^{-j2\theta} \qquad \measuredangle L = \pi \to 0 \to -\pi.$$

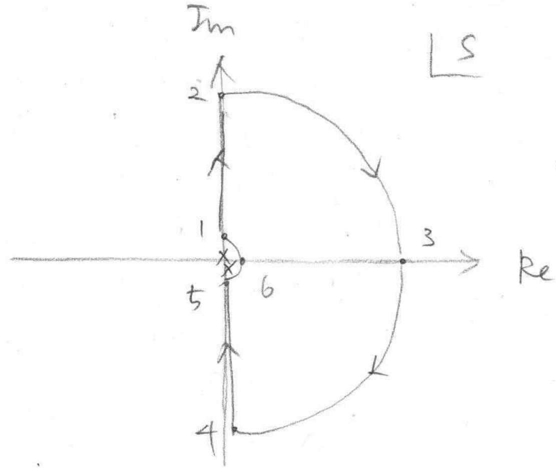
$p = 0$, $N = 0$ $\Rightarrow$ $Z = 0$ No RHP poles.

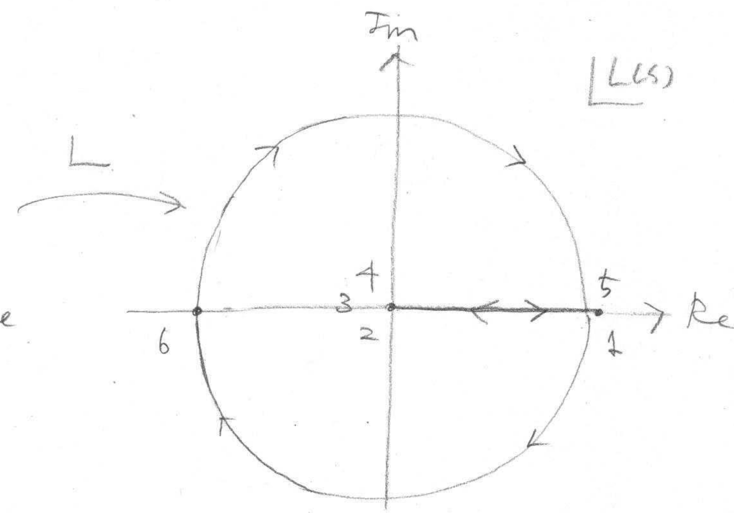
Example. Positive feedback.

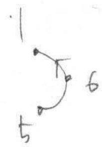




$$L(s) = -\frac{K_p}{ms^2}.$$







$s = re^{j\theta}$, $\theta = -\frac{\pi}{2} \to 0 \to \frac{\pi}{2}$.

$$L(s)\Big|_{s=re^{j\theta}} = -\frac{K_p}{mr^2 \cdot e^{j2\theta}} = \frac{K_p}{mr^2} e^{j(\pi - 2\theta)}$$

$$\measuredangle L = 2\pi \to \pi \to 0.$$

$p = 0$, $N = 1$ $\Rightarrow$ $Z = 1$. One RHP pole.

Root Locus