MECH 420
Sensors and Actuators

Presentation Part 3

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Part 3: System Integration

- Component Interconnection
- Impedance Matching

Plan

- Component Interconnection
- Impedance Matching (Max power transfer; max efficiency, reflection prevention in transmission, loading reduction)
- Mechanical Systems (isolation, transmission)

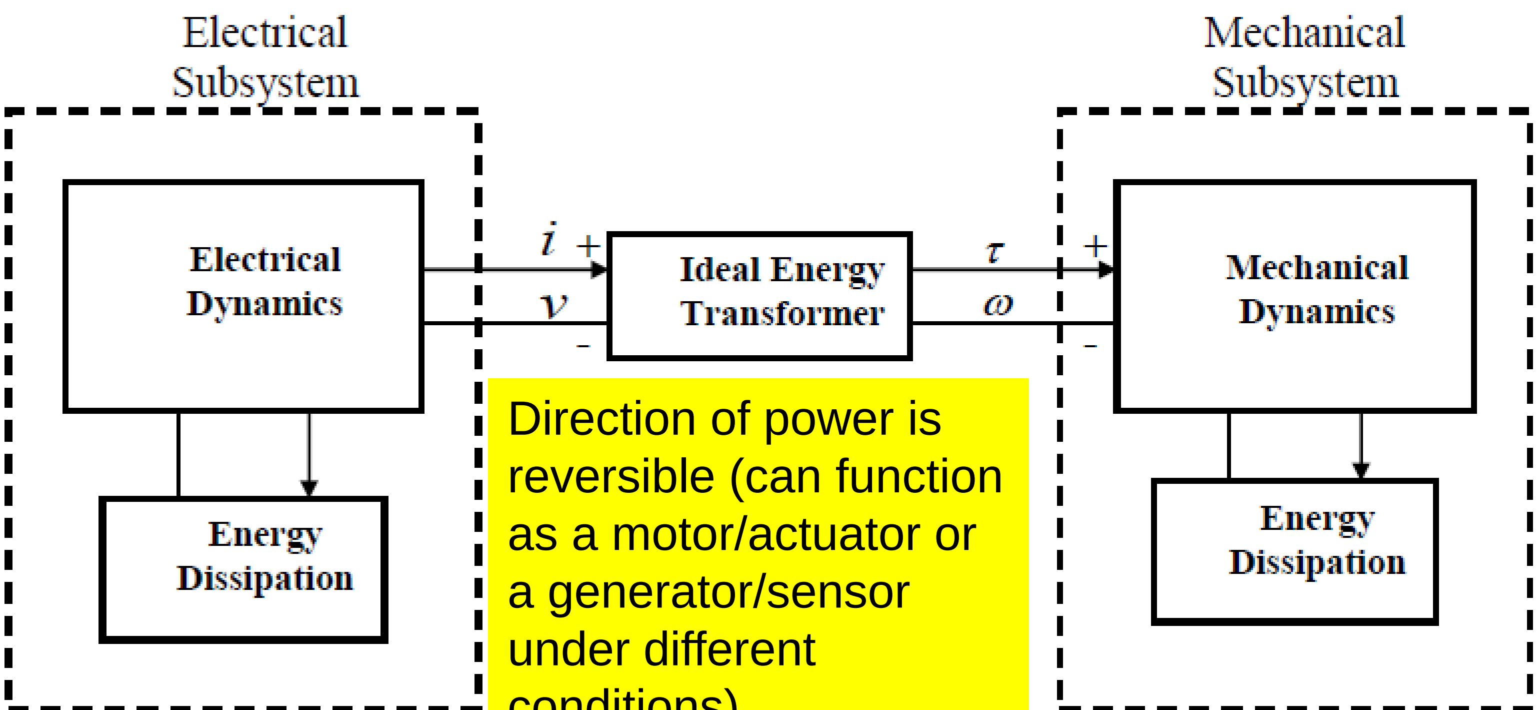
Rationale for Interconnection Study

- **When two components are interconnected, their signals are altered (due to dynamic coupling/interaction)**

New Components are Connected Because:

- **For matching of interconnected components, their operating signals may have to be modified (power, type, etc.)**
- **For application requirements, signal type and characteristics may have to be modified (e.g., power, analog-digital conversion, modulation, demodulation)**
- **In view of noise and other errors, and system requirements, the signals have to be conditioned (e.g., filtering, amplification)—studied in Part 4**

Component Interconnection



Direction of power is reversible (can function as a motor/actuator or a generator/sensor under different conditions)

A model for mixed-domain (electro-mechanical) component interconnection

Questions on System Integration & Mechatronics

- **What is system integration?**

- **Why is the mechatronic approach suitable in system integration?**

Impedance Matching

$$\text{Generalized Impedance} = \frac{\text{Across Variable}}{\text{Through Variable}}$$

Why is this called "generalized" impedance?

Across-variables: Voltage, velocity, pressure, temperature

Through-variables: Force, current, fluid flow rate, heat transfer rate

Four Key Goals of Impedance Matching:

(a) Source and Load Matching for Maximum Power Transfer

(b) Power Transfer at Maximum Efficiency

(c) Reflection Prevention in Signal Transmission

(d) Loading Reduction

Can all these goals be achieved simultaneously?

Through- and Across-variables in Four Domains

System Type	Through-variable	Across-variable
Hydraulic/Pneumatic	Flow Rate	Pressure
Electrical	Current	Voltage
Mechanical	Force	Velocity
Thermal	Heat Transfer	Temperature

Analogies and Constitutive Relations

Recall: "Unified" approach in Mechatronics.

System Type	Constitutive Relation for Energy Storage Elements: *A*-Type (Across) Element	Constitutive Relation for Energy Storage Elements: *T*-Type (Through) Element	Energy Dissipating Elements: *D*-Type (Dissipative) Element
Translatory-Mechanical v = velocity f = force	Mass $m\frac{dv}{dt} = f$ (Newton's 2nd Law) m = mass	Spring $\frac{df}{dt} = kv$ (Hooke's Law) k = stiffness	Viscous Damper $f = bv$ b = damping constant
Electrical v = voltage i = current	Capacitor $C\frac{dv}{dt} = i$ C = capacitance	Inductor $L\frac{di}{dt} = v$ L = inductance	Resistor $Ri = v$ R = resistance
Thermal T = temperature difference Q = heat transfer rate	Thermal Capacitor $C_t\frac{dT}{dt} = Q$ C_t = thermal capacitance	None What are the implications of this empty cell?	Thermal Resistor $R_t Q = T$ R_t = thermal resistance
Fluid P = pressure difference Q = volume flow rate	Fluid Capacitor $C_f\frac{dP}{dt} = Q$ C_f = fluid capacitance	Fluid Inertor $I_f\frac{dQ}{dt} = P$ I_f = inertance	Fluid Resistor $R_f Q = P$ R_f = fluid resistance

Further Question on System Integration & Mechatronics

Unified approach enables:

Impedance Matching

Impedance Matching

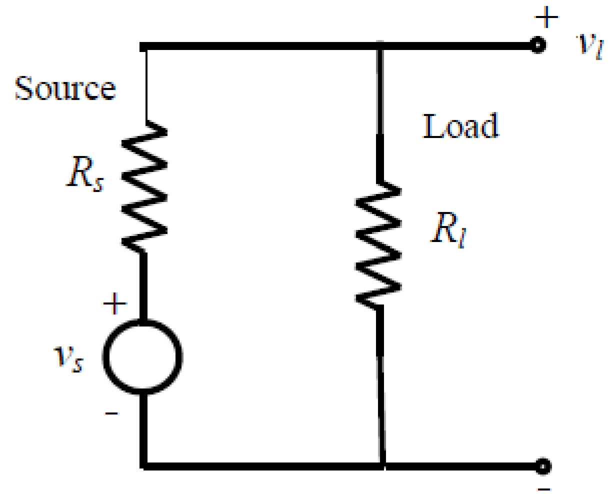
- **Instrumentation of Engineering Systems (in any domain) involves component interconnection**
- **Components: Sensors, signal conditioning circuitry, actuators, cables, gears, mounts, support structures, etc.**
- **When Components are interconnected, signals will flow between them, and conditions will change (coupling)**
  - **Motor and Load: Mechanical power flow – involves angular velocity and torque**

What kind of loading are these two?

  - **Electrical Sensor and Signal Conditioning Hardware: Electrical power flow—involves voltage and current**
- **Impedance matching may be required to achieve the desired operating conditions**
- **The type of impedance matching depends on the purpose/objective of the system**

Impedance Matching for Maximum Power Transfer

DC (Pure Resistance) Example



What is the adjustable parameter here?
What does R_s represent?
If R_s is the variable, what does that mean? How would you maximize the power transfer then?

Current through circuit: $$i = \frac{v_s}{R_l + R_s}$$

Voltage across the load: $$v_l = iR_l = \frac{v_s R_l}{R_l + R_s}$$

Power absorbed by the load: $$p_l = iv_l = \frac{v_s^{\,2} R_l}{[R_l + R_s]^2}$$

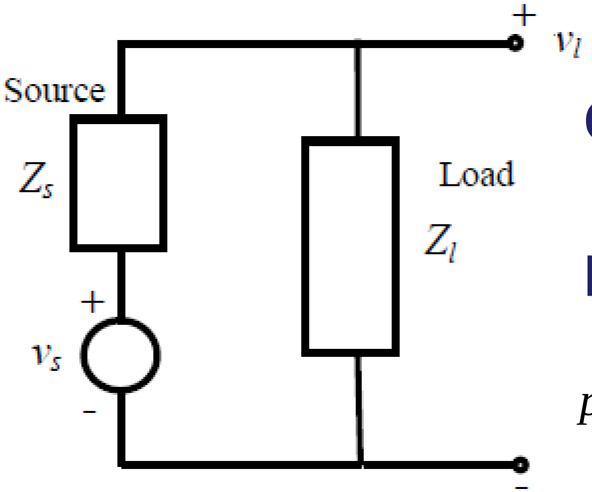
Note: The rest is in open circuit. What happens if another component is connected there?

Maximum power ➔ $$\frac{dp_l}{dR_l} = 0$$

➔ $$R_l = R_s$$

Prove

General Impedance



Current through circuit: $$|I| = \frac{|V_s|}{|Z_l + Z_s|}$$

Power absorbed by load (resistive power):

$$p_l = I_{rms}^2 R_l = \frac{1}{2}|I|^2 R_l = \frac{1}{2}\frac{|V_s|^2}{|Z_l + Z_s|^2} R_l = \frac{1}{2}\frac{|V_s|^2}{(R_l + R_s)^2 + (X_l + X_s)^2} R_l$$

For max power (see denominator): $X_l = -X_s$

Why can't we do this for resistance?

Note: Resistive and reactive parts can be adjusted independently

➔ **Problem becomes "pure resistive" ➔** $R_l = R_s$ **(from before)**

Overall Requirement: $$Z_l = Z_s^*$$

(Conjugate matching)

Max Power: $$p_{l\max} = \frac{|V_s|^2}{8R_s} = \frac{|V_s|^2}{8R_l}$$

Impedance Matching for Power Transfer at Maximum Efficiency

Power Transfer at Maximum Efficiency

Efficiency of power absorption by load = [Absorbed power]/ [Total power]:

$$\eta = \frac{1/2 \times |I|^2 R_l}{1/2 \times |I|^2 (R_l + R_s)} = \frac{R_l}{(R_l + R_s)}$$

➔ **To increase efficiency, increase load resistance**

When is the best efficiency achieved? Is this practical?

***Note*: Condition for max efficiency ≠ condition for maximum power**

Specifically, at maximum power ($R_l = R_s$), efficiency = 50%

Impedance Matching for Reflection Prevention in Signal Transmission

Reflection in Signal Transmission

When an electric signal encounters an abrupt change in impedance, part of the signal will be reflected back at the location of impedance change.

Undesirable Results of Signal Reflection:

- **Signal deterioration (both magnitude and phase angle)**
- **Dissipation (power loss)**

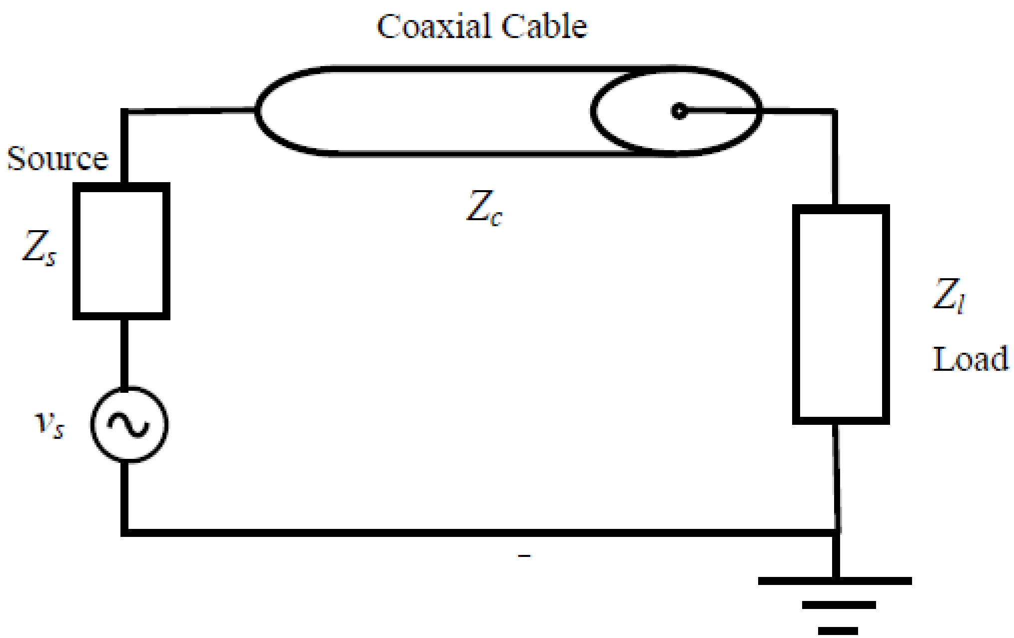
Both these are undesirable

***Note*: This is particularly critical in high-frequency (e.g., radio frequency—RF) systems.**

Similar signal reflections occur in other domains: E.g., Optical (e.g., fiber optics), acoustic, magnetic, elastic

So, can we develop “analogous” approaches for impedance matching in different domains?

Reflection Prevention in Signal Transmission



Reflection coefficient: $\Gamma = \dfrac{v_r}{v_i}$

Incident signal voltage = v_i

Reflected signal voltage = v_r

What is a sensing application of signal reflection?

➔ **Reflection coefficient:** $$\Gamma = \left| \frac{Z_l - Z_c}{Z_l + Z_c} \right|$$

Transmission (cable characteristic) impedance: Z_c

Terminating impedance: Z_l

Ideally, we want $\Gamma = 0$ ➔ $Z_s = Z_c = Z_l$

(for both directions)

Cable Fault Location

Time-domain Reflectometry (TDR) is used to measure the fault location: $$L_f = \frac{1}{2} T \times v$$

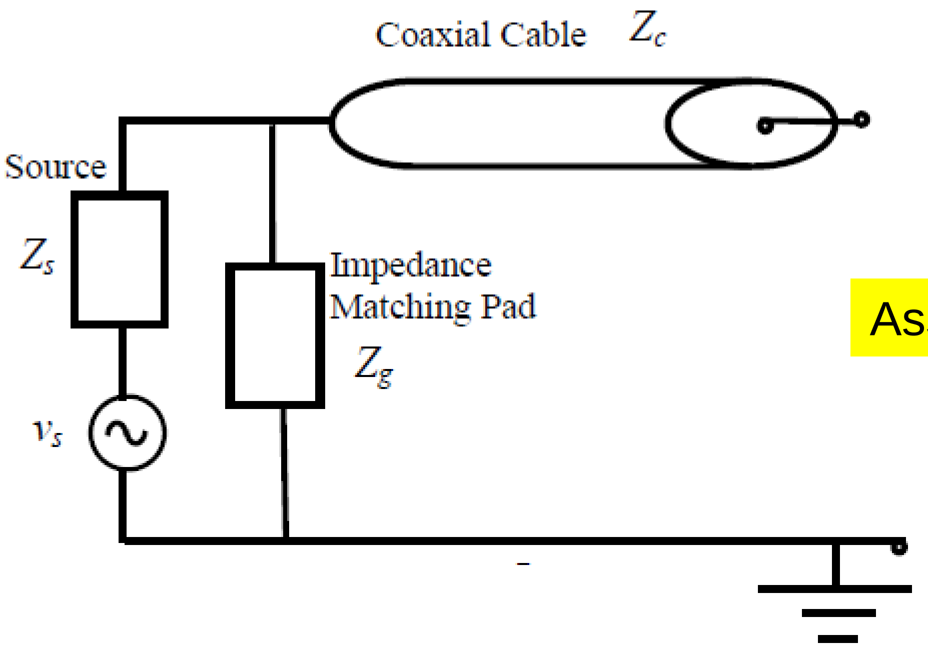
Where, *T* = time taken for voltage pulse to return (on reflecting at the fault)

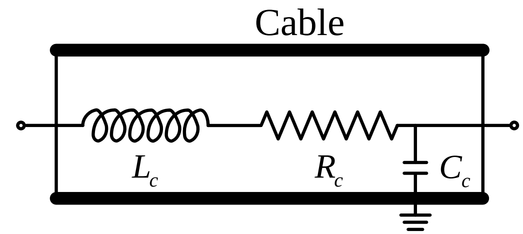
***v* = propagation velocity of pulse (or, standing wave)**

Type of Cable	Typical Propagation Velocity (*v*)
XLPE	82 – 86 m/µs (269 – 282 ft/µs)
PILC	77 – 82 m/µs (253 – 269 ft/µs)
Hybrid	83 m/µs (272 ft/µs)
Overhead wire	148 m/µs (485 ft/µs)
Telecommunication	95 – 120 m/µs (312 – 394 ft/µs)

***Note*: Speed of light in vacuum, *c* = 300 m/µs**

Example





Assume $Z_s > Z_c$

Cable characteristic impedance = 50 Ω Why?

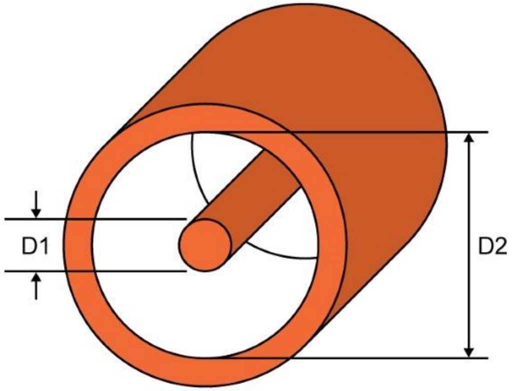
Match source impedance using impedance matching pad such that:

$$\frac{1}{Z_s} + \frac{1}{Z_g} = \frac{1}{Z_c}$$

Why?

Question on Cable Impedance in Impedance Matching

- In a coaxial cable, if the separation between core and outer shield (difference of D1 and D2 in figure) increases, what happens to:
- Cable capacitance



- Cable impedance
- What is special about 50 Ω cable impedance?

Impedance Matching for Loading Reduction

Loading

When two components are interconnected, the conditions (e.g., current, voltage) in the components will change

➔ Loading

In many applications, the output component should not load the input component.

Think about both "mechanical" loading and "electrical/electronic" loading

Examples:

- In a sensing process, the sensor should not alter the conditions of the sensed object (measuring instrument should not distort the signal that is measured)
- In a signal acquisition system of a sensor, the signal acquisition hardware should not distort the acquired signal from the sensor (*Note*: Signal acquisition system will have filtering, amplification, sampling, etc.)
- The load that is connected to the power source should not considerably change the output voltage of the power source (in a "regulated power source," this problem is minimal)

Loading Reduction

- **Loading: Device connected at signal output distorts the signal ← caused by improper impedance conditions**
- **Loading error can far exceed other errors (measurement error, sensor error, noise, input disturbances, etc.)**
- **Loading can occur in any physical domain (e.g., electrical mechanical)**

Difference between "measurement error" and "sensor error"?

Electrical loading situations: Output device (e.g., measuring device, signal acquisition hardware) that has low *input impedance* connected to an input device (e.g., signal source, sensor) with moderate to high impedance

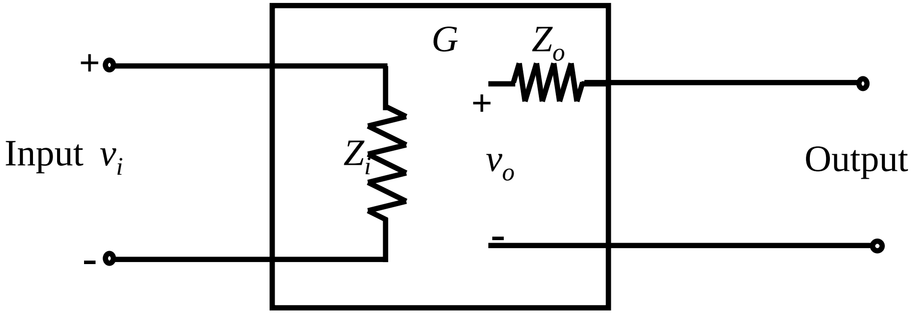
Mechanical loading situations: An output component (e.g., gear transmission, mechanical load) connected to a mechanical device (e.g., a motor) will load the device due to inertia, friction, and other resistive forces ← *System and environment*

Choose impedances properly to reduce loading effects. Such *impedance matching* is called *impedance bridging*

Give another example of "mechanical" loading.

Model of a Two-port Device

$$v_o = G\, v_i$$



- Output Impedance (Z_0) =

  Open Circuit Output Voltage/Short Circuit Output Current

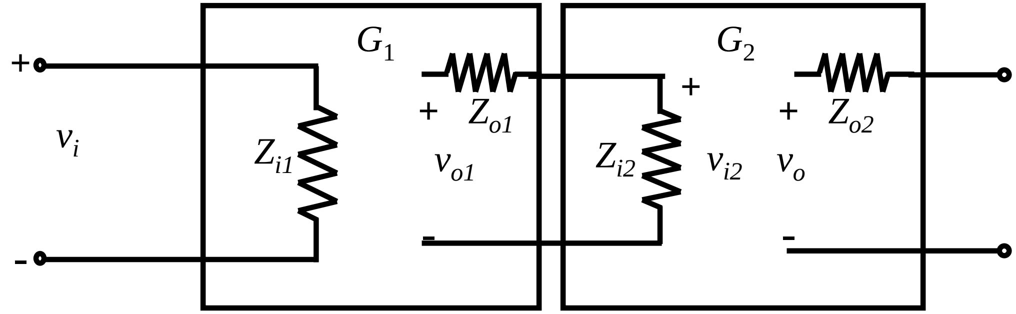
- Input Impedance (Z_i) =

  Rated Input Voltage/Input Current (under OC)

  (Output kept in open-circuit condition)

Note: The above model satisfies these definitions

Cascade Connection of Two-port Devices



$$v_{o1} = G_1 v_i \quad ; \quad v_{i2} = \frac{Z_{i2}}{Z_{o1} + Z_{i2}} v_{o1} \quad ; \quad v_o = G_2 v_{i2}$$

➔ $$v_o = \frac{Z_{i2}}{Z_{o1} + Z_{i2}} G_2 G_1 v_i$$

Note: $$\frac{Z_{i2}}{Z_{o1} + Z_{i2}} = \frac{1}{Z_{o1} / Z_{i2} + 1}$$

➔ **To make actual transfer function close to ideal G_1G_2, we need $Z_{o1}/Z_{i2} \ll 1$**

➔ **Output Impedance of device 1 has to be low and Input Impedance of device 2 has to be high (relatively)**

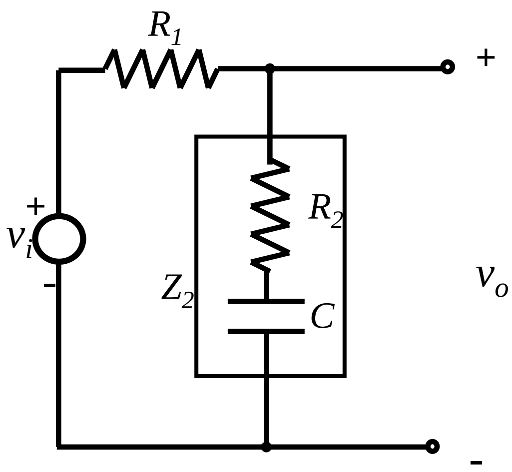
Example

A lag network (a compensator element of a control system) is shown. Determine:

(a) Transfer function for the circuit

(b) Input impedance and output impedance

Solution



(a)

Voltage division along current path for oc conditions:

$$v_o = \frac{(R_2 + (1/Cs))}{\{R_1 + R_2 + (1/Cs)\}} v_i \quad \Rightarrow \quad \frac{v_o}{v_i} = \frac{Z_2}{R_1 + Z_2}$$

with,

$$Z_2 = R_2 + (1/Cs)$$

(b)

Input current under oc: $i = v_i/(R_1 + Z_2)$

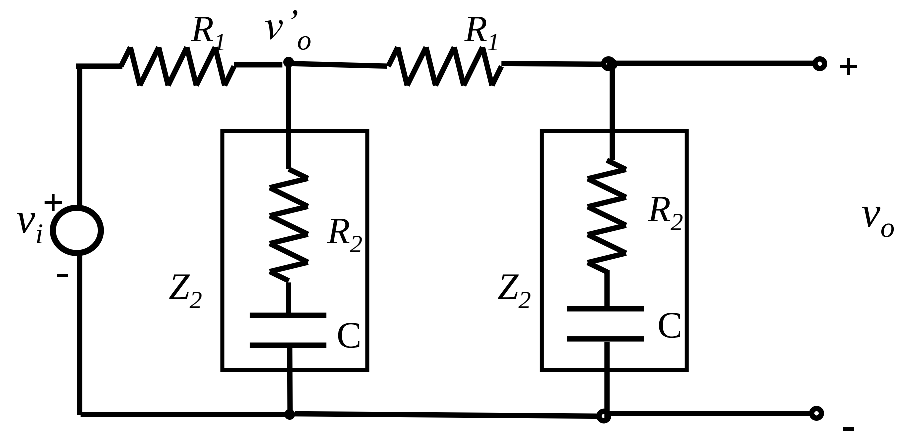
$$\Rightarrow \quad Z_i = \frac{v_i}{i} = R_1 + Z_2$$

SC current: $i_{sc} = v_i/R_1$

$$Z_o = \frac{v_o}{i_{sc}} = \frac{Z_2/(R_1 + Z_2)v_i}{v_i / R_1} = \frac{R_1 Z_2}{R_1 + Z_2}$$

How would you reduce this output impedance?

Example (Cont'd)



If two lag circuits are cascaded what is the overall transfer function?

How would you make this transfer function close to the ideal result?

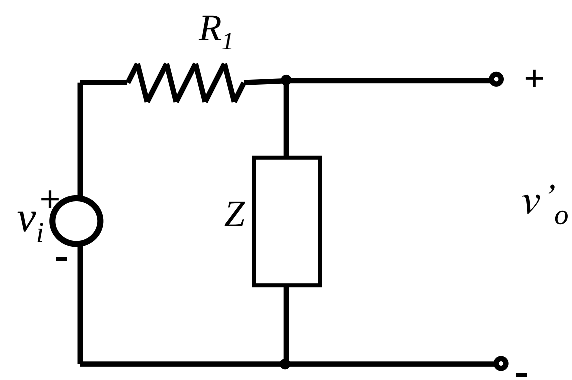
See equivalent circuit:

$$\frac{1}{Z} = \frac{1}{Z_2} + \frac{1}{R_1 + Z_2} \qquad \textbf{(i)}$$

Voltage drop across *Z:* $$v'_o = \frac{Z}{R_1 + Z} v_i$$

Substitute single-stage result:

$$v_o = \frac{Z_2}{(R_1 + Z_2)} \frac{Z}{(R_1 + Z)} v_i$$



Equivalent Circuit

➔ $$G = \frac{v_o}{v_i} = \frac{Z_2}{(R_1 + Z_2)} \frac{Z}{(R_1 + Z)} = \frac{Z_2}{(R_1 + Z_2)} \frac{1}{(R_1 / Z + 1)}$$

➔ $$G = \left[\frac{Z_2}{R_1 + Z_2}\right]^2 \frac{1}{[1 + R_1 Z_2 / (R_1 + Z_2)^2]}$$

Prove.

➔ **For ideal *G*, make $R_1 Z_2 / (R_1 + Z_2)^2$ small** How?

((i) is substituted)

Note: $R_1 Z_2 / (R_1 + Z_2)$ is the output impedance of original circuit

Derive the input impedance and the output impedance of the cascaded circuit

Impedance Matching in Mechanical Systems

(Self-study)

Example on Loading Reduction

Explain Electrical Impedance, Mechanical Impedance, and Mobility

- **In the shown example of vibration monitoring, are there mechanical loading considerations? Explain. If so, how would you reduce the mechanical loading effects in this situation?**
- **In the shown problem, are there electrical loading considerations? Explain. If so, how would you reduce the electrical loading effects in this situation?**

Questions on Impedance Matching in a Multi-physics Problem

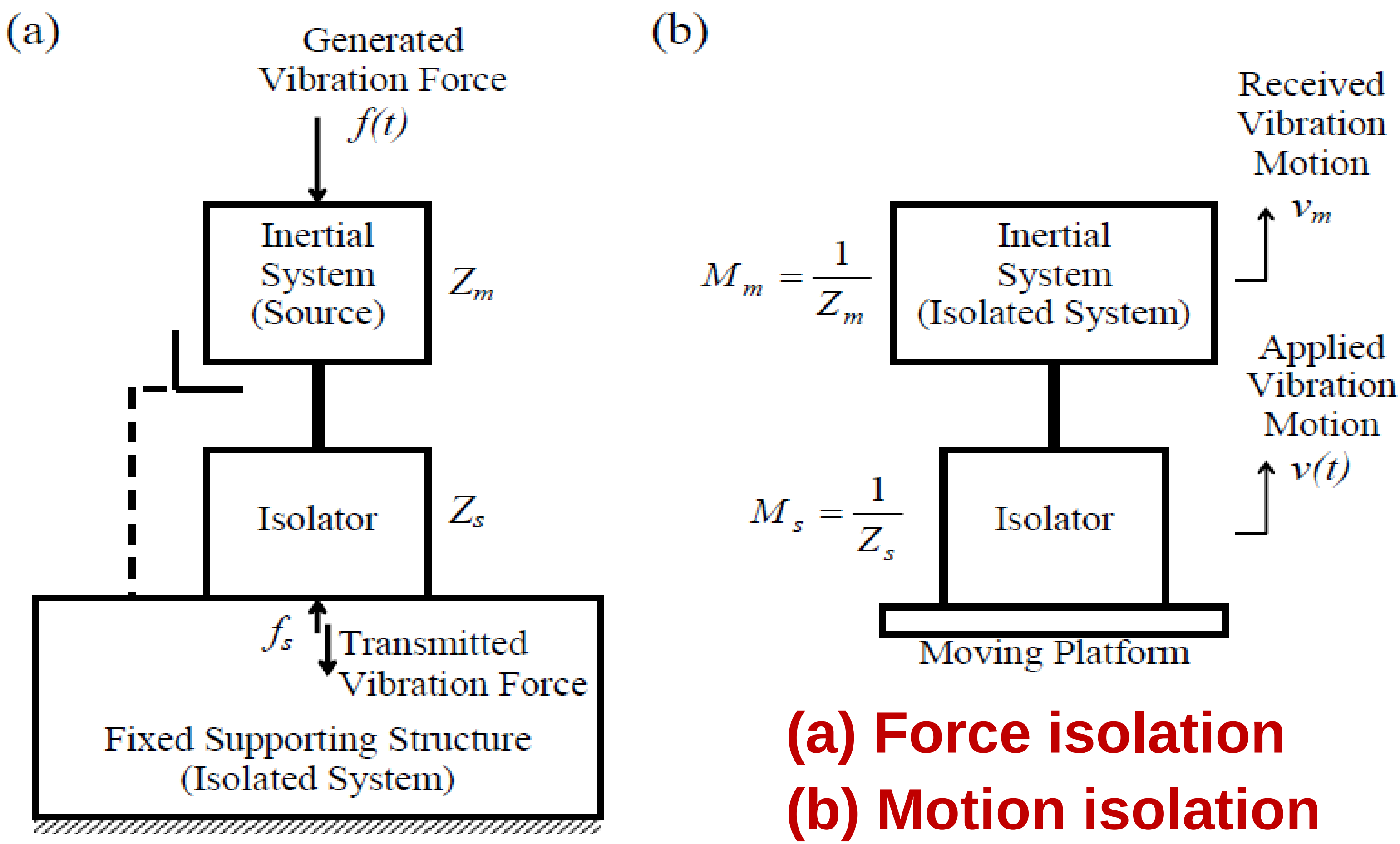
- **What is a multi-physics problem?**

- **How would you extend the concepts of impedance matching to a multi-physics problem?**

Vibration Isolation

Delicate instruments, computer hardware, machine tools, vehicles, etc. can be isolated from shock and vibration using vibration isolators (or shock mounts or suspensions)

This is impedance matching in the "mechanical domain"



(a) Force isolation

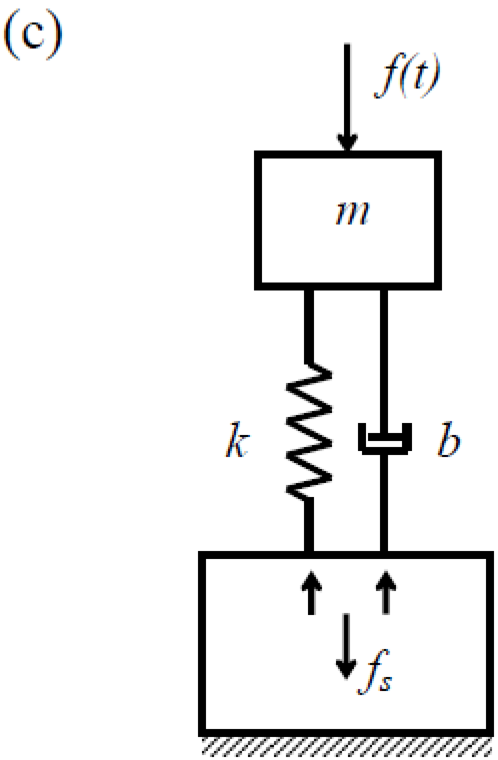
(b) Motion isolation

Impedance and Mobility Functions of Mechanical Elements

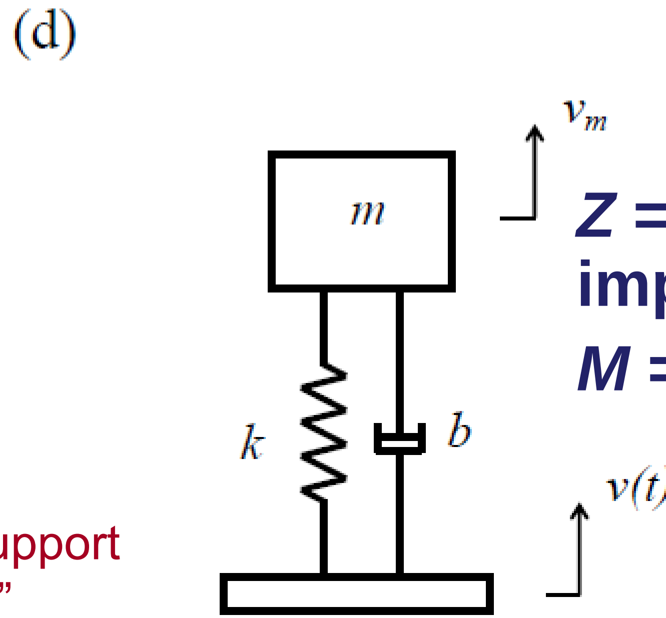
Element	Time-Domain Model	Impedance	Mobility (Generalized Impedance)
Mass m	$m\dfrac{dv}{dt} = f$	$Z_m = ms$	$M_m = \dfrac{1}{ms}$
Spring k	$\dfrac{df}{dt} = kv$	$Z_k = \dfrac{k}{s}$	$M_k = \dfrac{s}{k}$
Damper b	$f = bv$	$Z_b = b$	$M_b = \dfrac{1}{b}$

***Note*: Frequency domain is a special case of Laplace domain.**
Commonly, frequency domain is used when dealing with impedance approaches

Simplified Models



"s" → "support structure"



Z **= mechanical impedance**

M **= mobility**

Can prove: Force transmissibility in (a) = motion transmissibility in (d)

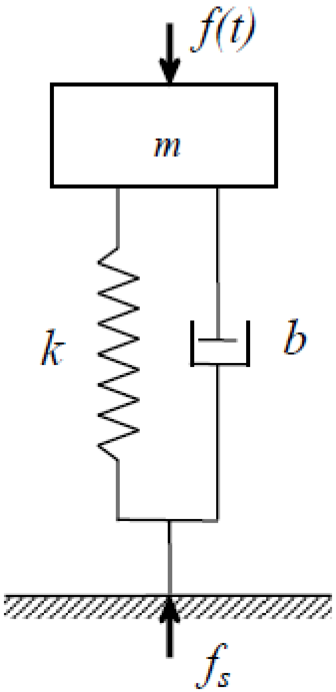
$$T_m = \frac{v_m}{v} = \frac{M_m}{M_m + M_s} = \frac{Z_s}{Z_s + Z_m} = \frac{f_s}{f} = T_f$$

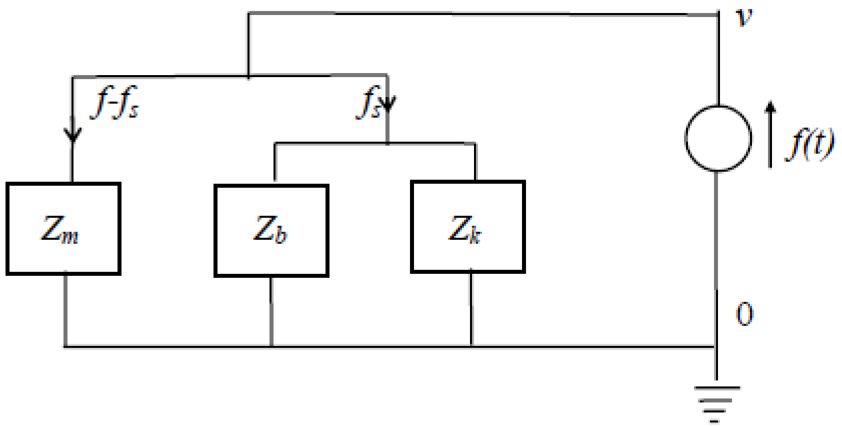
Mechanical impedances add in parallel; Mobilities add in series

$$T = \frac{k + bj\omega}{\left(k - m\omega^2 + bj\omega\right)} = \frac{\omega_n^2 + 2\zeta\omega_n\omega j}{\left(\omega_n^2 - \omega^2 + 2\zeta\omega_n\omega j\right)} = \frac{1 + 2\zeta rj}{1 - r^2 + 2\zeta rj} \quad ; \quad |T| = \sqrt{\frac{1 + 4\zeta^2 r^2}{(1 - r^2)^2 + 4\zeta^2 r^2}}$$

Example

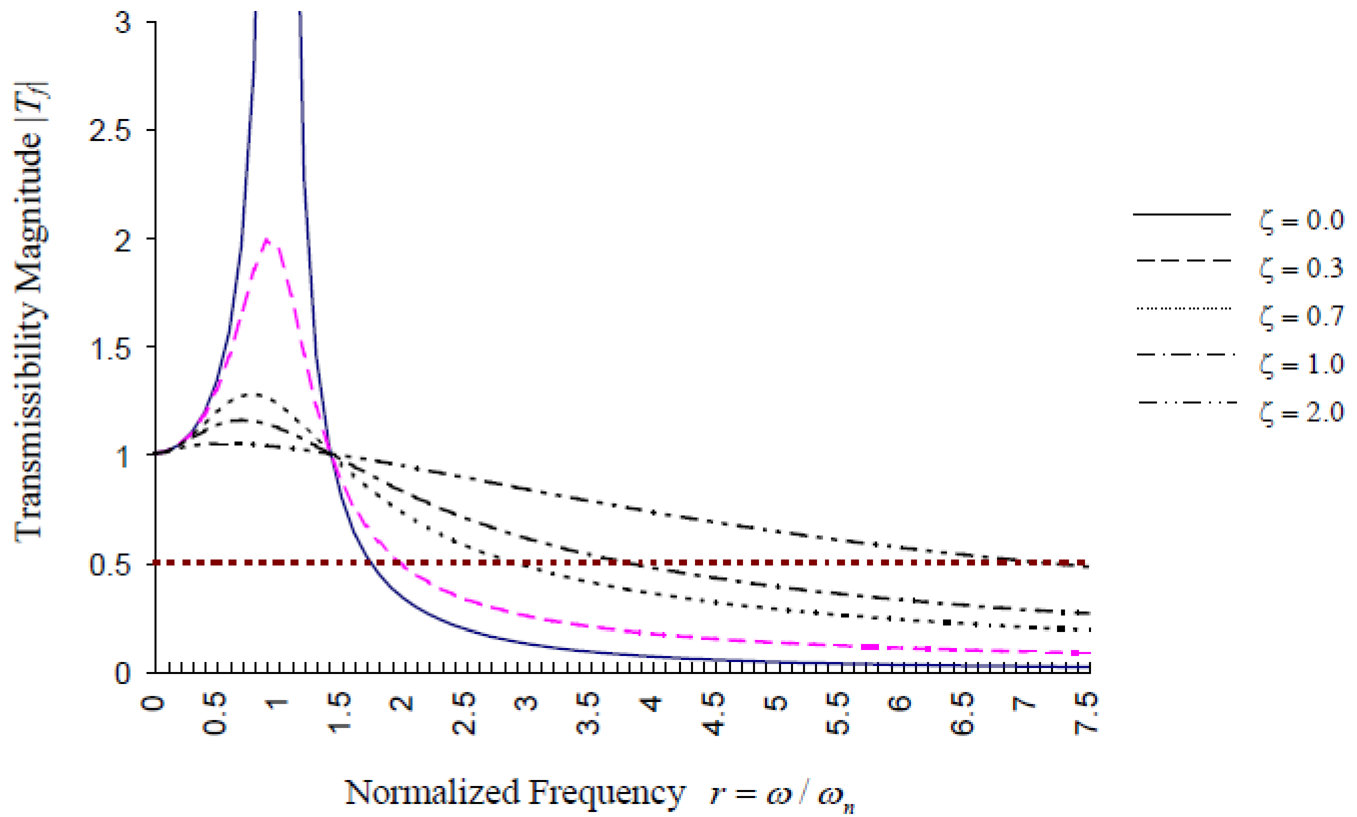
A machine tool and its supporting structure are modeled as the simple mass-spring-damper system





$$\left|T_f\right| = \left|\frac{Z_b + Z_k}{Z_m + Z_b + Z_k}\right| = \sqrt{\frac{1+4\zeta^2 r^2}{(1-r^2)^2 + 4\zeta^2 r^2}} \quad \text{with} \quad r = \frac{\omega}{\omega_n}$$

Transmissibility Curves



Transmissibility Curves

Observations:

- There is a non-zero frequency value at which the transmissibility magnitude peaks (resonance)
- For small ζ, this peak transmissibility magnitude occurs at (approx.) r =1. As ζ increases, peak point shifts to the left (to a lower frequency)
- Peak magnitude decreases as ζ increases
- All the transmissibility curves pass through magnitude value 1.0 at the same frequency $r = \sqrt{2}$
- Isolation (i.e., $|T_f| < 1$) occurs when $r > \sqrt{2}$. In this region, $|T_f|$ increases with ζ
- In the isolation region, transmissibility magnitude decreases as r increases.

How would you improve isolation?

Curves of Vibration Isolation

Used for designing vibration isolators (shock mounts)

Percentage isolation $I = [1 - |T|] \times 100\%$

➔

$$I = [1 - \sqrt{\frac{1 + 4\zeta^2 r^2}{(r^2 - 1)^2 + 4\zeta^2 r^2}}] \times 100$$

How would you design an isolator for specified isolation level?