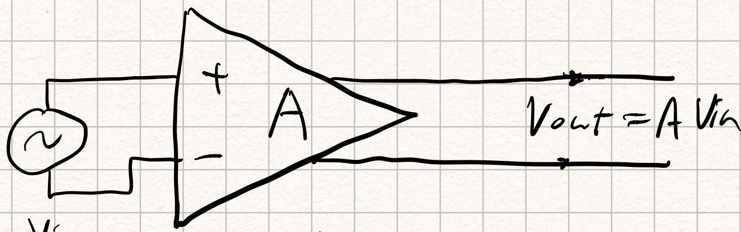
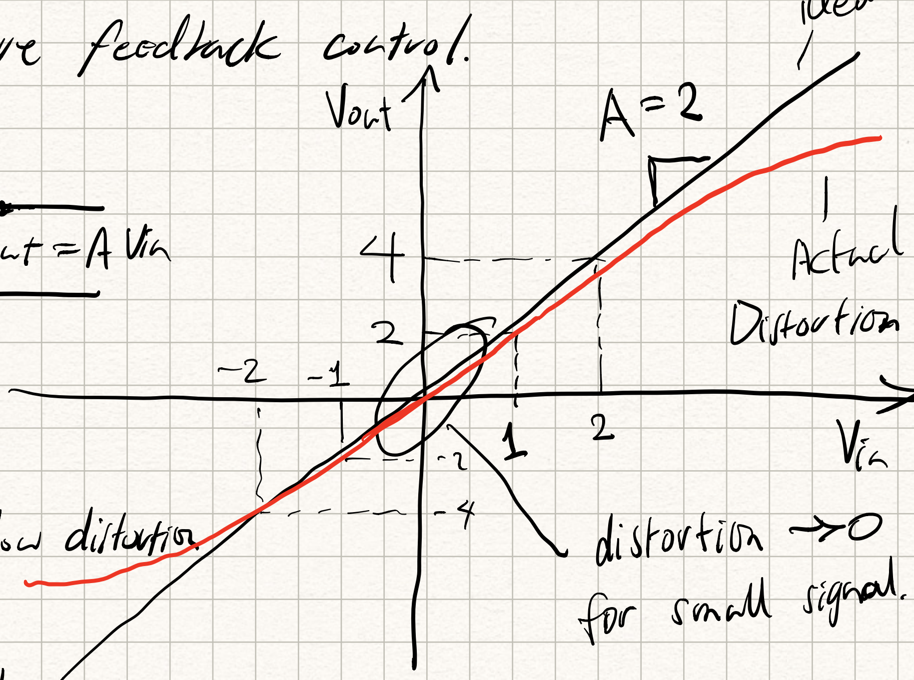
Origins of negative feedback control.



Amplifier



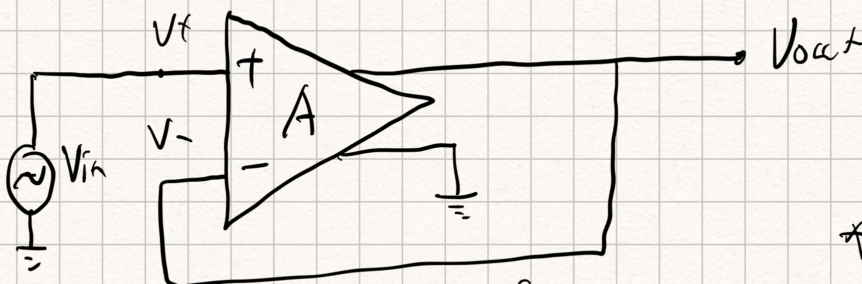
Easy: Gain

Hard: linear gain — low distortion

stable gain

1927 Harold Black

— AT&T Bell Labs



* If $V_+ > V_-$

$A(V_+ - V_-) > 0$

$V_{out} \uparrow \quad V_- \uparrow \quad V_- \to V_+$

* If $V_+ < V_-$

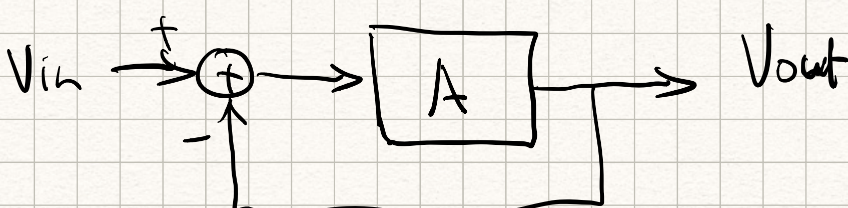
$A(V_+ - V_-) < 0$

$V_{out} \downarrow \quad V_- \downarrow \quad V_- \to V_+$

Negative feedback forces

$V_- \to V_+$

Input → 0 distortion → 0.



$$V_{out} = A(V_{in} - V_{out})$$

$$V_{out} + AV_{out} = AV_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{A+1}$$

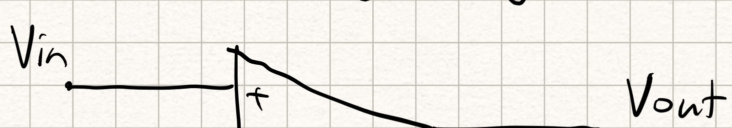
$A >> 1$

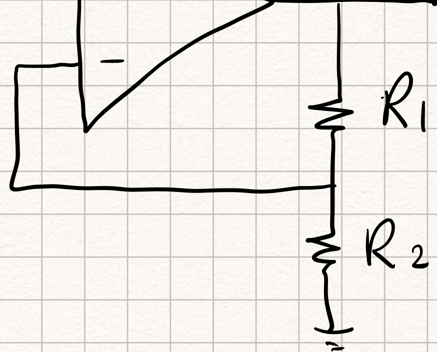
$V_{out} \approx V_{in}$

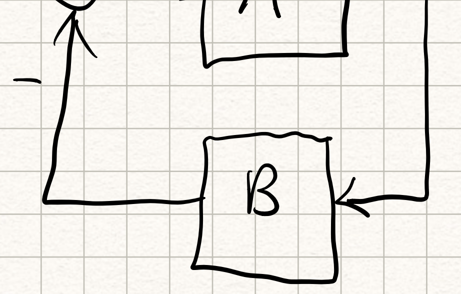
for op Amps

$A > 10^6$

How do we get gain?







$$B = \frac{R_2}{R_1 + R_2}$$

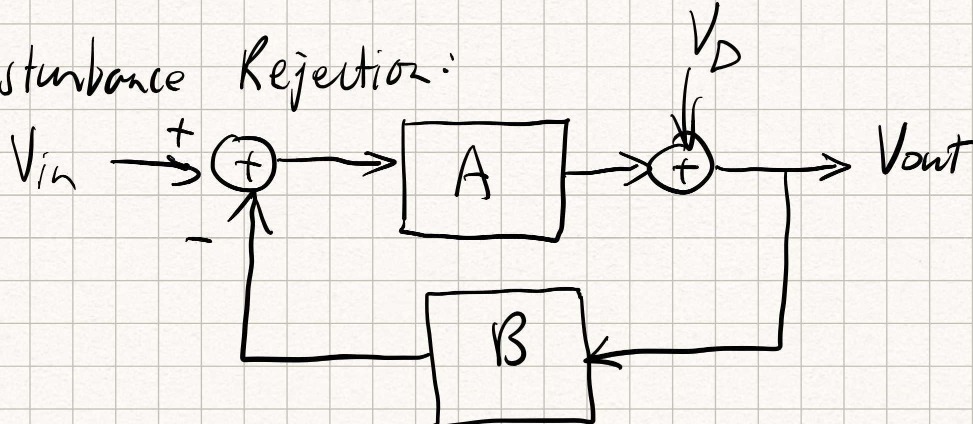
$$V_{out} = A(V_{in} - B V_{out})$$

$$V_{out}(1 + AB) = A V_{in}$$

$$V_{out} = \left(\frac{A}{1+AB}\right) V_{in}$$

If $AB \gg 1$ then $V_{out} \simeq \frac{A}{AB} V_{in} = \frac{1}{B} V_{in}$

Disturbance Rejection:



$$V_{out} = V_D + A(V_{in} - B V_{out})$$

$$V_{out}(1 + AB) = V_D + A V_{in}$$

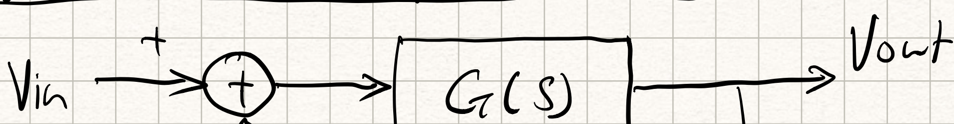
$$V_{out} = \left(\frac{A}{1+AB}\right) V_{in} + \frac{V_D}{1+AB}$$

V_D is reduced by a factor of $1 + AB$

Negative Feedback: Use gain (AB) to improve system performance:
- distortion
- error
- disturbance rejection

Notice: Our Analysis doesn't depend on units — use in elec, mech, chem, ... systems.

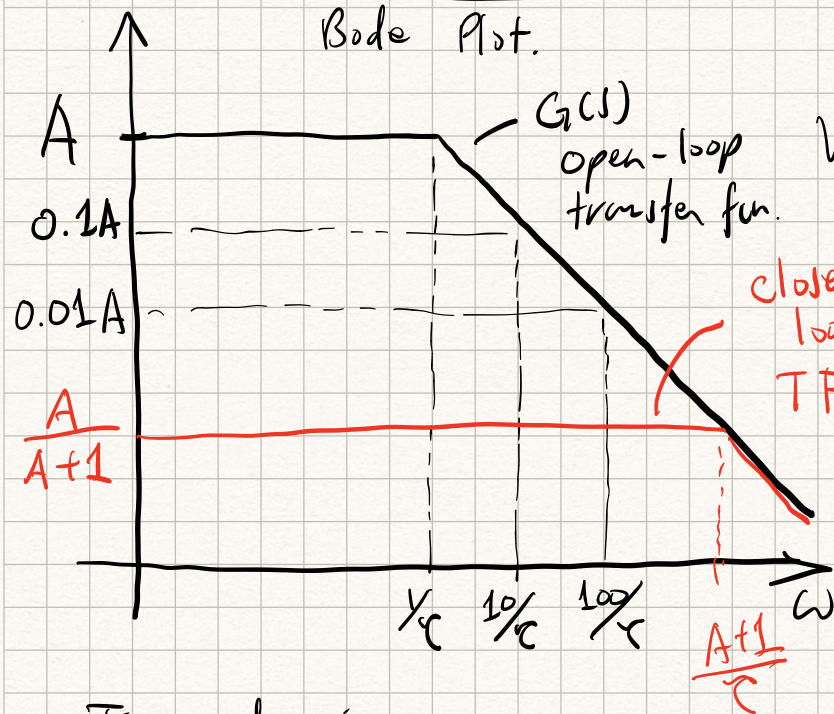
Systems with freq. response



$G(s) = \frac{A}{\ldots}$

$$\tau s + 1$$

Bode Plot.



$s = j\omega$

$$V_{out} = G(s)(V_{in} - V_{out})$$

$$V_{out}(1 + G(s)) = V_{in}(G(s))$$

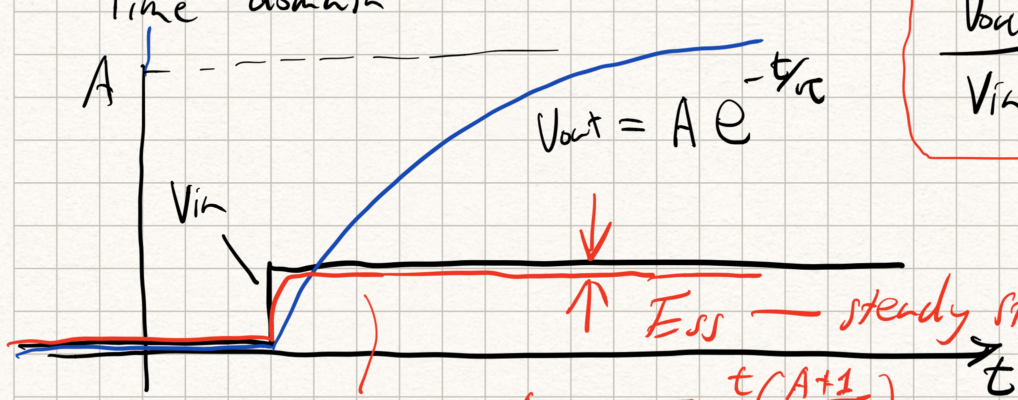
$$\frac{V_{out}}{V_{in}} = \frac{G(s)}{1 + G(s)}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{A}{\tau s + 1}}{1 + \frac{A}{\tau s + 1}}$$

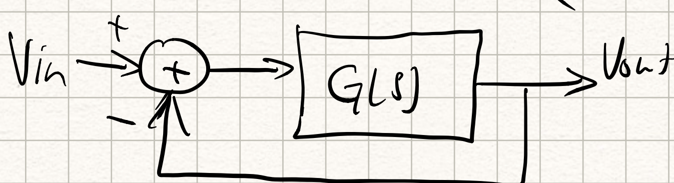
$$\frac{V_{out}}{V_{in}} = \frac{A}{\tau s + 1 + A}$$

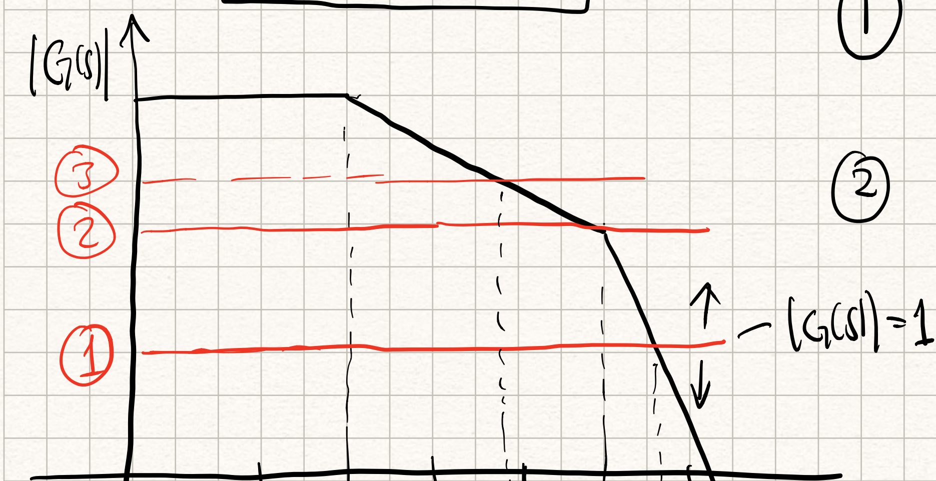
$$\boxed{\frac{V_{out}}{V_{in}} = \frac{\frac{A}{A+1}}{\left(\frac{\tau}{A+1}\right)s + 1}}$$

Time domain



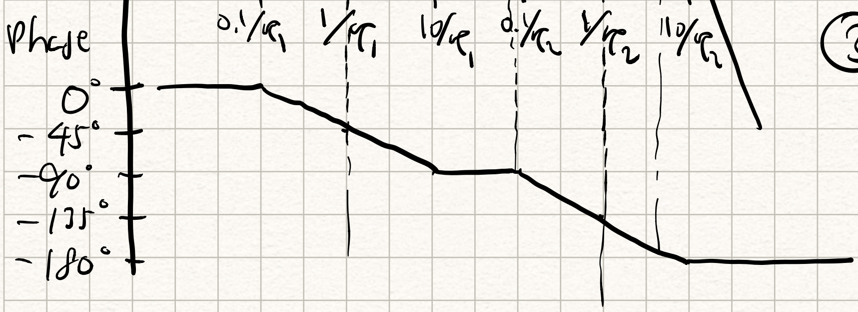
What if $G(s) = \dfrac{A}{(\tau_1 s + 1)(\tau_2 s + 1)}$





① Phase margin ≈ 0°
- unstable system

② Phase margin = 45°
- Typical minimum phase margin for a useful system



③ Phase margin > 60°

– stable enough