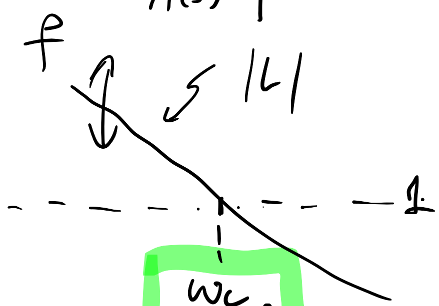
L7 – Brushed DC Motor

Review.

op-amp circuit – via loop shaping $L(s)$.

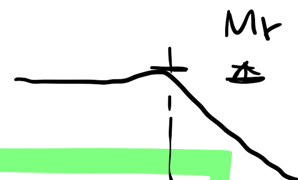
"Key Idea" to shape $L(s) = A(s)\cdot f$ $\longrightarrow$ $\left\{ \begin{matrix} G(s) \\ S(s) \\ T(s) \end{matrix} \right\}$ $\longrightarrow$ desired time-domain behaviors.

closed-loop tf.



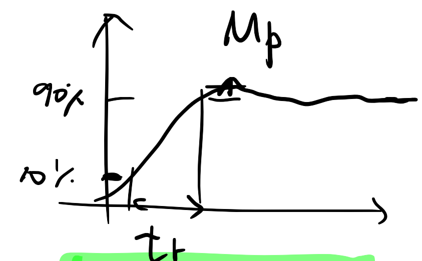
$\boxed{\omega_c,\ \phi_m}$

$\Rightarrow$



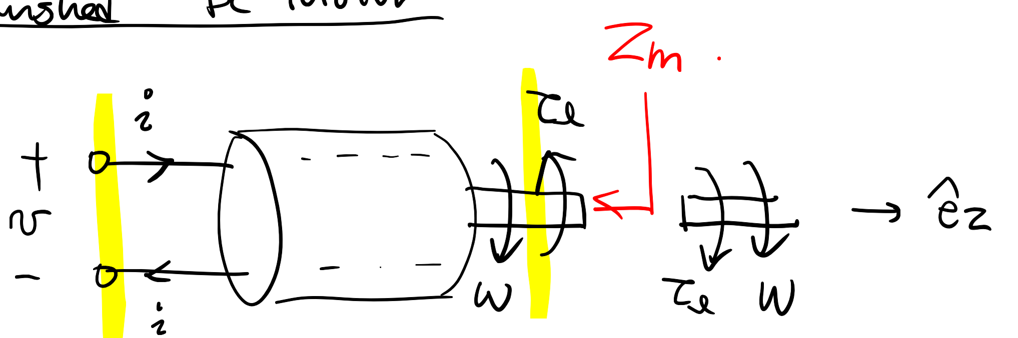
$\boxed{\begin{matrix} \omega_c \simeq \omega_b \\ M_r \simeq \frac{1}{\phi_m} \end{matrix}}$

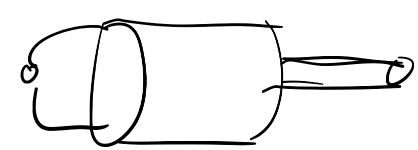
$\Rightarrow$

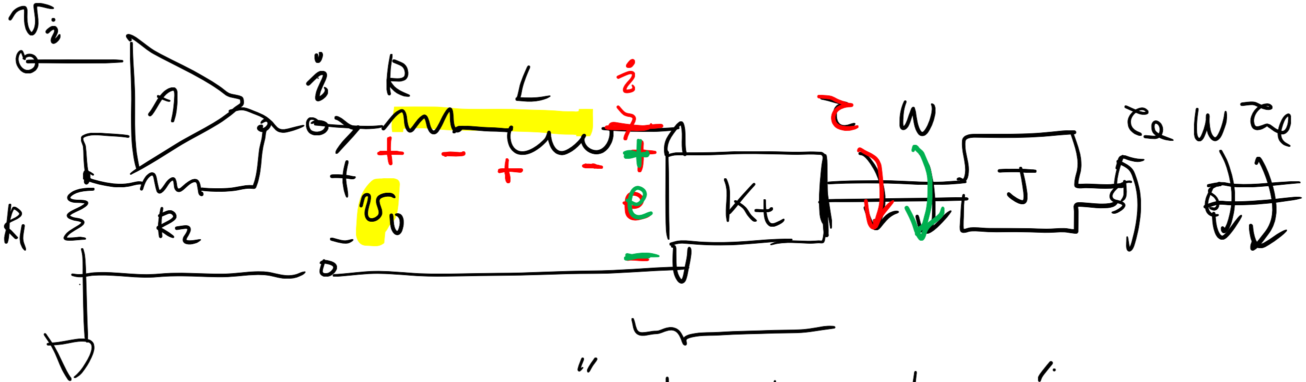


$\boxed{\begin{matrix} t_r \sim \omega_b \\ M_p \sim M_r \end{matrix}}$

Brushed DC Motor







"Ideal transducer"

< Terminal variables >

elec : v . i

Mech : τ_e w

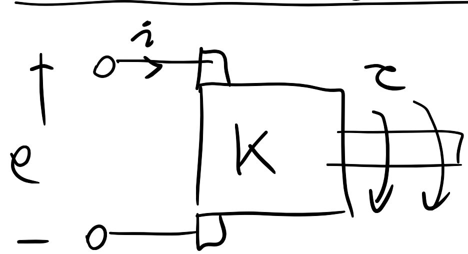
< Internal variables >

$\begin{cases} \text{Rotor torque} : \tau \; (\neq \tau_e) \\ \text{Back emf} : e . \end{cases}$

< Terminal Relation >.

$$\begin{cases} \tau = K_t i \\ e = K_t w \end{cases}$$

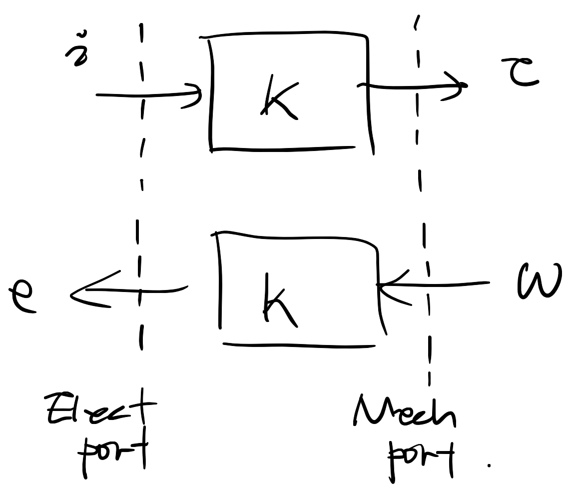
"Transducer relation" .

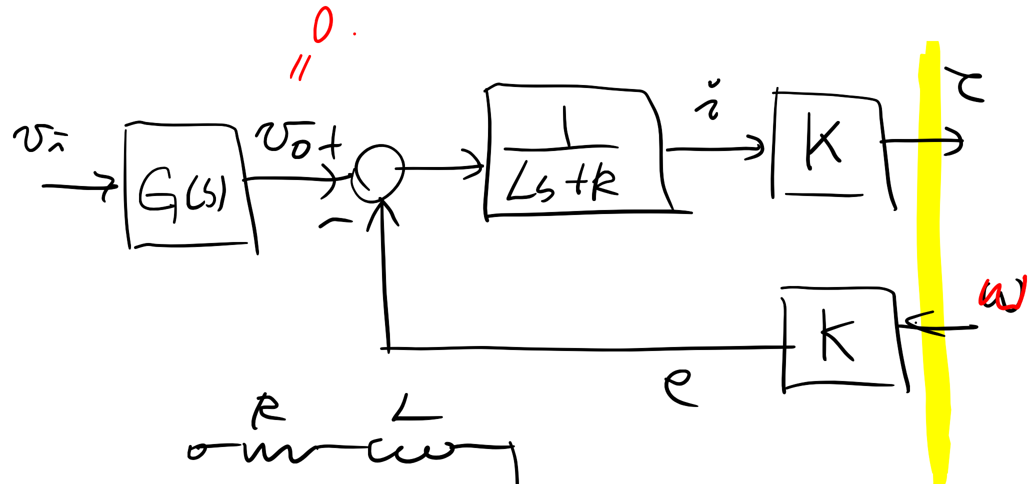


· Lossless power converter.

$$P_{in} = e\,i = (\cancel{K}\omega)\left(\frac{\tau}{\cancel{K}}\right) = \tau\omega.$$

$$P_{out} = \tau \cdot \omega$$





$e = K\omega.$

<Equivalent Mechanical Model>

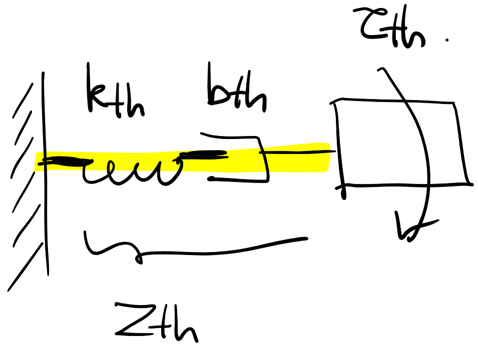
① Thevenin torque.

$$\tau_{th} = \frac{K}{Ls+R} \cdot v_o.$$

"Stall" torque

② Thev impedance.

$$Z_{th} = \frac{\tau}{\omega} = \left(\frac{K^2}{Ls+R}\right) = \left(\frac{\frac{K^2}{Ls}\cdot\frac{K^2}{R}}{\frac{K^2}{Ls}+\frac{K^2}{R}}\right) = \frac{K^2}{Ls} \,\|\, \frac{K^2}{R}.$$



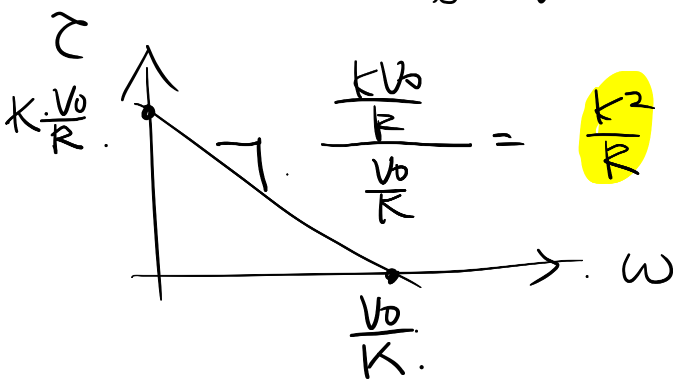
$$\begin{cases} \tau_{th} = \dfrac{K}{Ls+R} \cdot v_0 \\ k_{th} = \dfrac{K^2}{L} \\ b_{th} = \dfrac{K^2}{R} \end{cases}$$

- **Apparent damping**

$$Z_{th} = \frac{K^2}{Ls+R} \xrightarrow[\text{As } \omega \to 0]{} \boxed{\frac{K^2}{R}}$$

A measure of goodness.

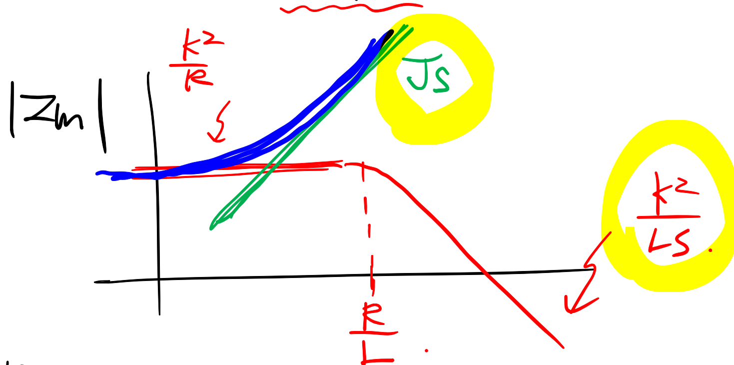
"Gradient" or "Steepness"

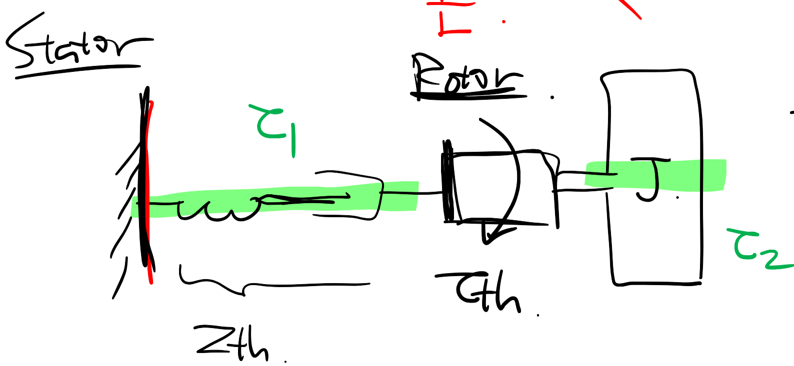


$$\frac{\frac{KV_0}{R}}{\frac{V_0}{R}} = \frac{K^2}{R}$$

- **Apparent stiffness** $\frac{K^2}{L}$ No significant meaning.

Bode $Z_m = \frac{K^2}{Ls+R} + Js$





$J = J_{rotor} + J_{load}$

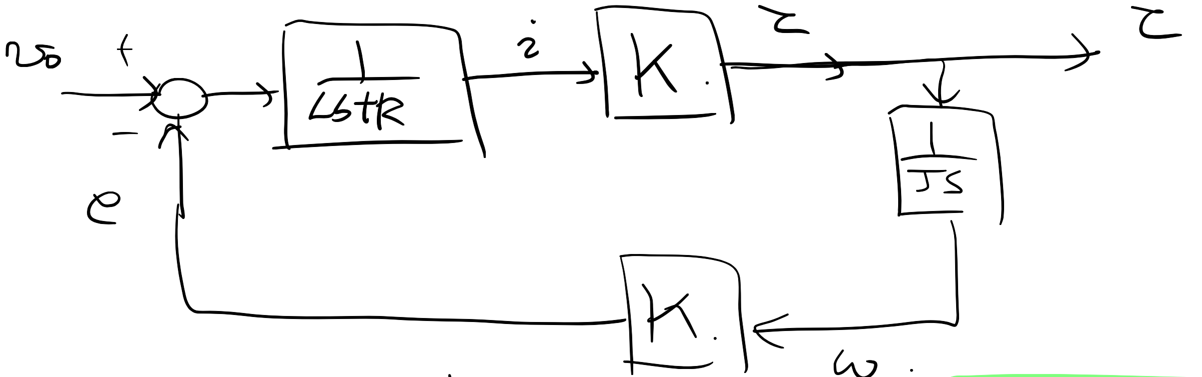
$$\tau_{th} = \frac{K}{Ls+R} v_0$$

$$\tau_2 = \frac{Js}{Z_{th} + Js} \tau_{th}$$

$$= \frac{Js}{\frac{K^2}{\cancel{Ls+R}} + Js} \cdot \frac{K}{\cancel{Ls+R}} v_0$$

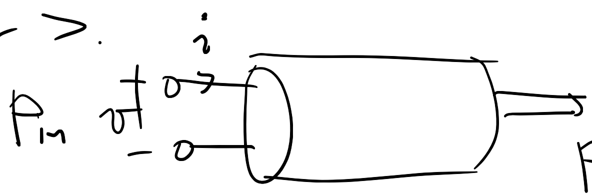
$$= \frac{Js \cdot K}{K^2 + Js(Ls+R)}$$

$$= \boxed{\frac{K}{Ls + R + \frac{K^2}{Js}} \cdot v_0}$$



$$\frac{\tau}{v_0} = \frac{\frac{K}{Ls+R}}{1 + K^2 \frac{1}{Js(Ls+R)}} = \boxed{\frac{K}{Ls+R + \frac{K^2}{Js}}}$$

<power>



two-port

$$P_{in} = v i.$$

$$v = R i + L \frac{di}{dt} + e.$$

$$P_{out} = \tau \omega$$

$$P_{in} = \underbrace{R i^2}_{P_{loss}.} + L \frac{di}{dt} i + \underbrace{e i}_{P_{out} = \tau \omega.}$$

$$L \frac{di}{dt} i = \frac{d}{dt} \underbrace{\left(\frac{1}{2} L i^2 \right)}_{W_m}.$$