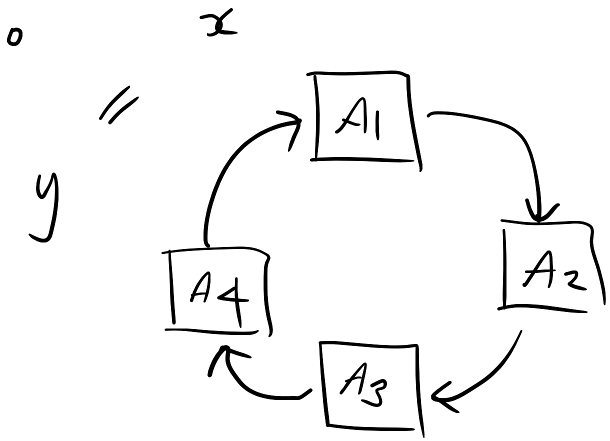
L17 – Feedback & Stability

1. The conditions for "Marginal stability"

2. Time delay $\xrightarrow{?}$ Stability



$L.T. \triangleq A_1 A_2 A_3 A_4$

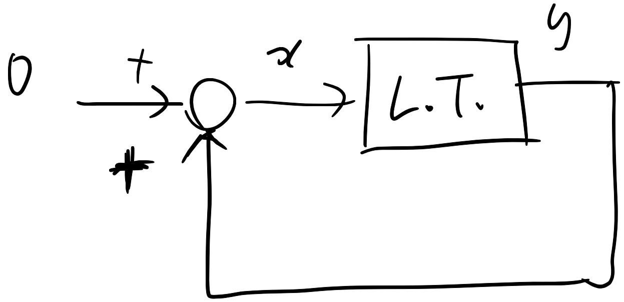
- Let $x = \cos(\omega_0 t)$.

  If $L.T.(j\omega_0) = 1 \Rightarrow y = \cos(\omega_0 t)$.

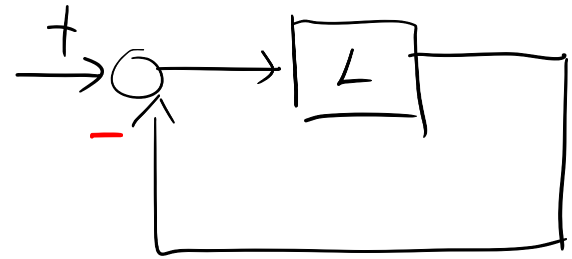
- Now, connet the arrows togeth. "keep oscillating"

- If $L.T.(j\omega_0) = 1 \Rightarrow$ "Loop" can maintain
  e.g. $\underline{\cos \omega_0 t}$.

$\boxed{L = -L.T.}$



$\Longleftrightarrow$



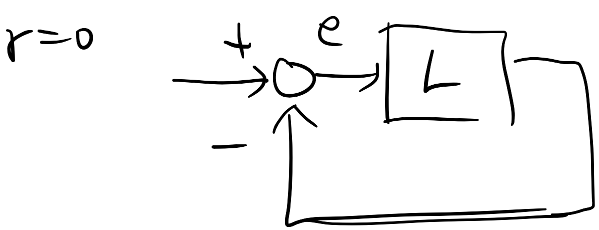
$L.T.(j\omega_0) = 1$.

for marginal stability.

$L(j\omega_0) = -1$.

$\begin{cases} |L| = 1 . \\ \angle L = -180^\circ \end{cases}$

° Sensitivity function.

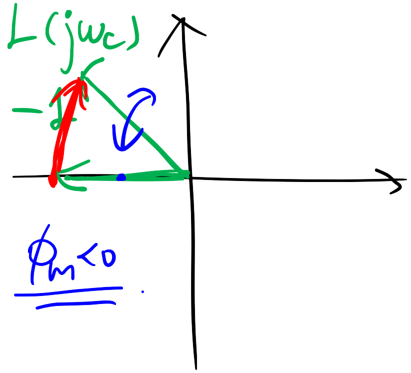


$\frac{E}{R} = \frac{1}{1+L} \triangleq S$

Almost unstable. if $r \simeq 0 \Rightarrow e \neq 0$.

• If $L(j\omega_0) = -1$. $S(j\omega_0) = \frac{1}{1+L} \to \infty$.

$|L+1|$



$L \to -1$

$|L+1| \to 0$

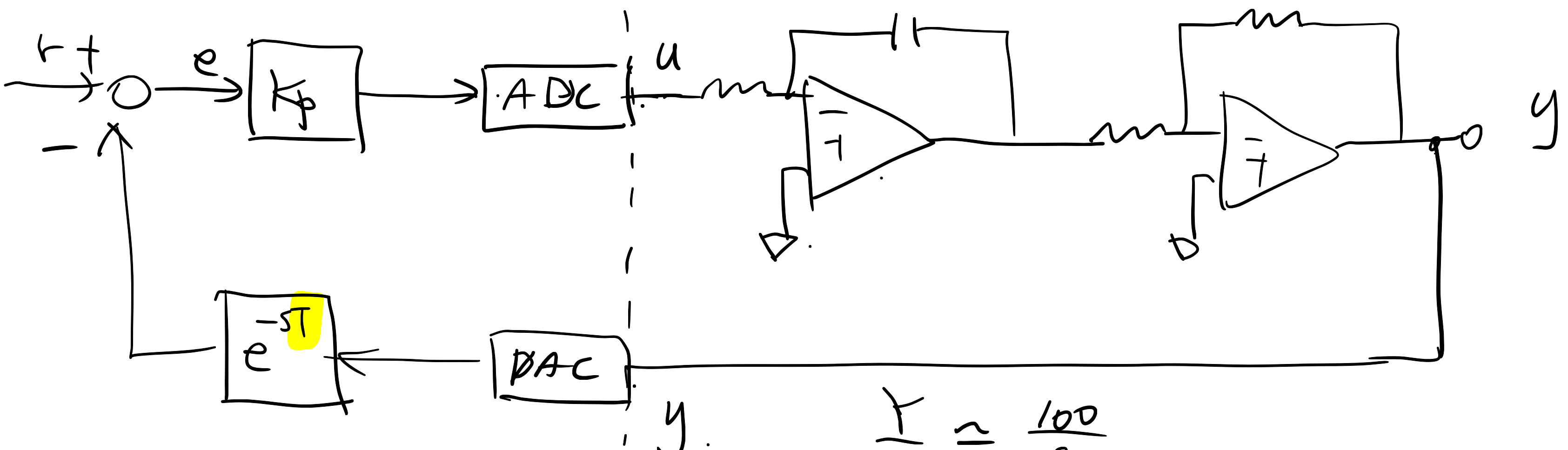
$|s| \to \infty$

$\measuredangle L - (-180^\circ)$

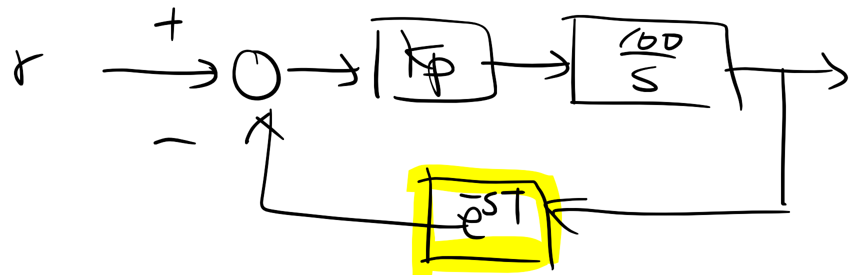
Q. What if $\phi_m < 0$. $\dfrac{1}{|1+L|} < \infty$?

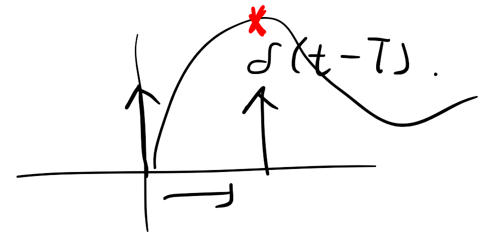
A. We need to rely on the "Nyquist test"

<Demo>.

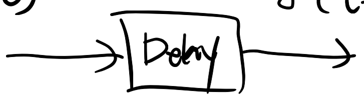


$$\frac{Y}{U} \simeq \frac{100}{s}$$



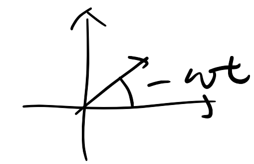


$\delta(t)$



$\delta(t-T) = \underline{h(t)}$

$$H(s) = \int_{-\infty}^{\infty} \delta(t-T)\, e^{-st}\, dt$$

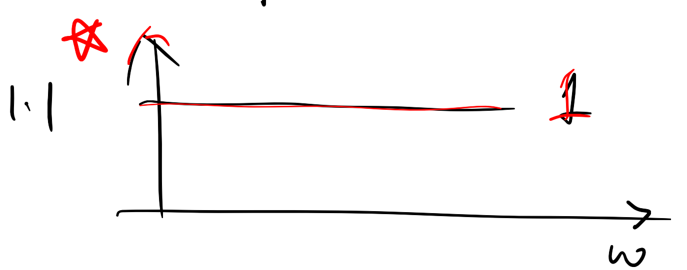


$$= \int_{-\infty}^{\infty} \delta(t-T)\, e^{-sT}\, dt$$

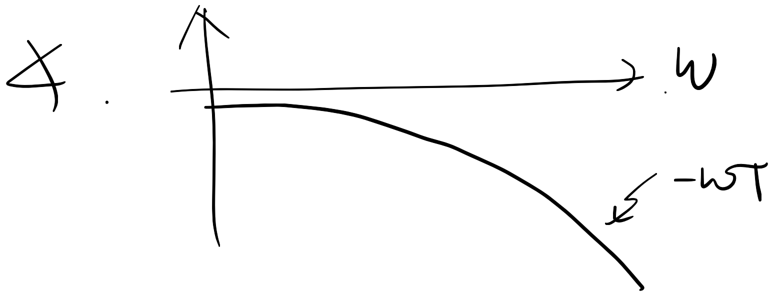
$$= e^{-sT} \underbrace{\int_{-\infty}^{\infty} \delta(t-T)\, dt}_{1} = e^{-sT} \rightarrow e^{-j\omega T}$$

$$\begin{cases} |e^{-j\omega T}| = 1 \\ \measuredangle = -\omega T \end{cases}$$

Bode plot



"All-pass"



" Linear − phase "

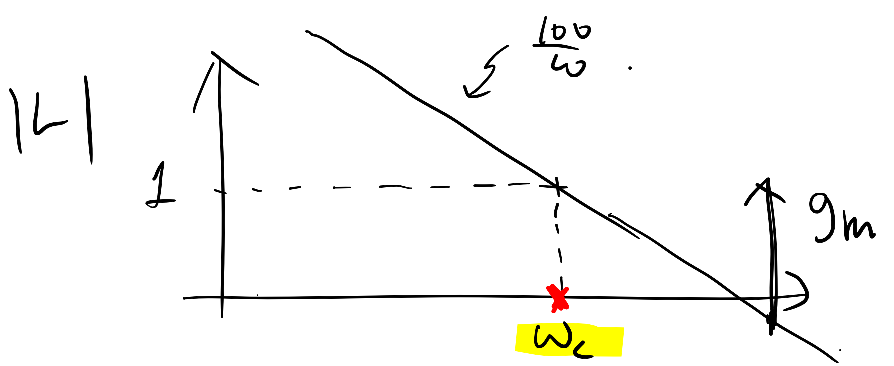
" Non − minimum phase " (lag) system

∘ $L(s) = K_p \frac{100}{s} \cdot e^{-sT}$

For $K_p = 1$, $T = 10\,ms$.

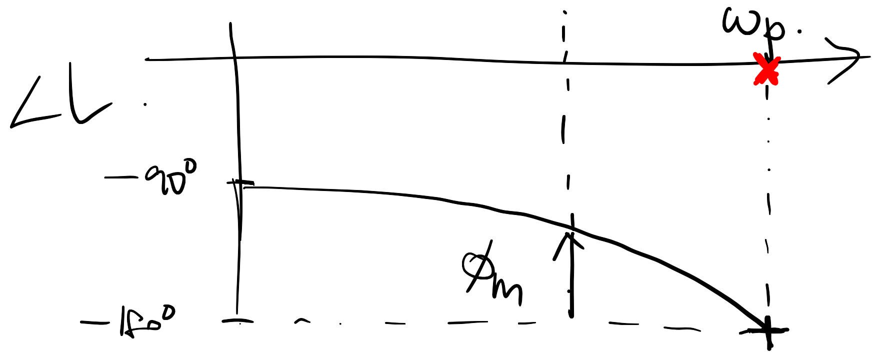
$L(s) = \frac{100}{s} \cdot e^{-0.01s}$.

$$\begin{cases} |L| = \frac{100}{\omega} \\ \measuredangle = -\frac{\pi}{2} - \frac{\omega}{100} \end{cases}$$



ω_c : (gain) —

ϕ_m :



ω_p : (phase) —

g_m : gain margin.

"Suspicious freq" for marginal stability

① ω_c : $|L| = 1$

$\measuredangle L \neq -180^\circ$.

$\left|\frac{100}{\omega_c}\right| = 1 \rightarrow \omega_c = 100$ rad/s.

$\simeq$ 16 Hz

② ω_p : $|L| \neq 1$

$\measuredangle L = -180^\circ$. $\rightarrow -\frac{\pi}{2} - \frac{\omega_p}{100} = -\pi$.

$\omega_p = \frac{\pi}{2} \times 100$.

$= 50\pi$ rad/s.

$=$ 25 Hz

- Gain margin.

$$g_m = \frac{1}{|L(j\omega_p)|} = \frac{1}{100/50\pi} = \frac{\pi}{2} \simeq \boxed{1.57}$$

1.49

g_m "How much gain increase the loop can tolerance"

- phase margin

$$\phi_m = \measuredangle L(j\omega_c) + 180^\circ.$$

$$\phi_m = 0.57 \text{ rad}. \qquad \phi \simeq 33^\circ.$$

ϕ_m "How much delay the loop can tolerance"