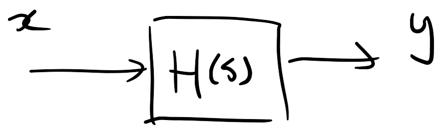
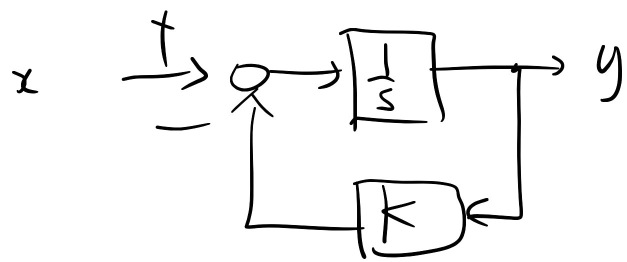
L20 – Nyquist Test

- Stability of LTI sys



Stable $\longleftrightarrow$ $\mathrm{Re}\{p_i\} < 0$ for all i.

- ① Directly finding CL poles.



$$\frac{Y}{X} = \frac{\frac{1}{s}}{1+\frac{K}{s}} = \frac{1}{s+K}$$ stable iff $K > 0$.

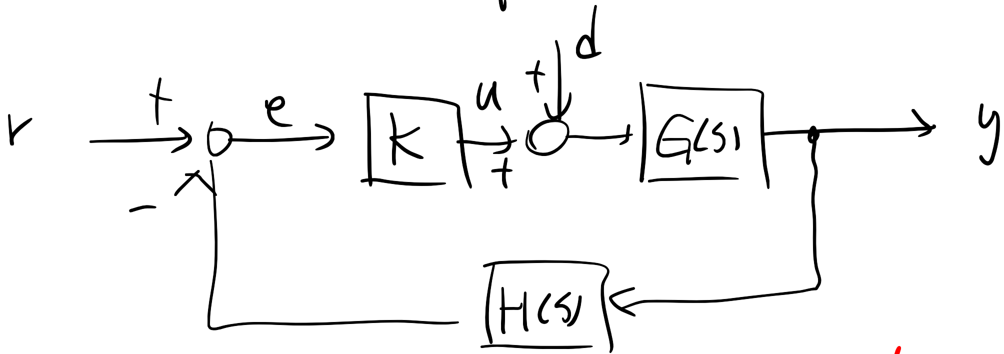
② Infer the CL poles from $L(s)$.

i) Root Locus: the locations of CL poles (explicit).

ii) Nyquist test: the number of CL poles in RHP (implicit)

→ "Loop Analysis". $L(s)$

• Characteristic eqn : $f(s) = 0$.



$$\begin{bmatrix} y \\ u \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{KG}{1+KGH} & \frac{G}{1+KGH} \\ \frac{K}{1+KGH} & -\frac{KGH}{1+KGH} \end{bmatrix}}_{\text{"Matrix"}} \begin{bmatrix} R \\ D \end{bmatrix}.$$

$$= \frac{1}{1+KGH} \begin{bmatrix} KG & G \\ K & -KGH \end{bmatrix} \begin{bmatrix} R \\ D \end{bmatrix}.$$

• CL system is stable iff $\frac{1}{1+KGH} = S(s)$

No poles in RHP.

Q. What if $G(s)$ has RHP poles? $s = p_0$

$$\left.\frac{G}{1+KGH}\right|_{s=p_0} = \frac{G(p_0)}{1+KG\,H} = \left|\frac{1}{KH(p_0)}\right| < \infty$$

$\triangle$. p_0 is not the pole of $\frac{G}{1+KGH}$ anymore.

- $\boxed{\text{poles of } \frac{1}{1+KGH}}$ $\Longleftrightarrow$ zeros of $1+KGH$.
  $= 1+L(s) \triangleq f(s)$

- For stability, $f(s)=0$ No roots on RHP.
  "Char Eqn"

Root Locus.

- Infer the locations of CL poles : $\frac{1}{1+L(s)}$
  $\Leftrightarrow$ zeros of $1+L(s) \triangleq f(s)$
- Shows how the zeros of $f(s)$ move.

$$1+KG(s)H(s)=0 \Rightarrow G(s)H(s) = -\frac{1}{K}.$$

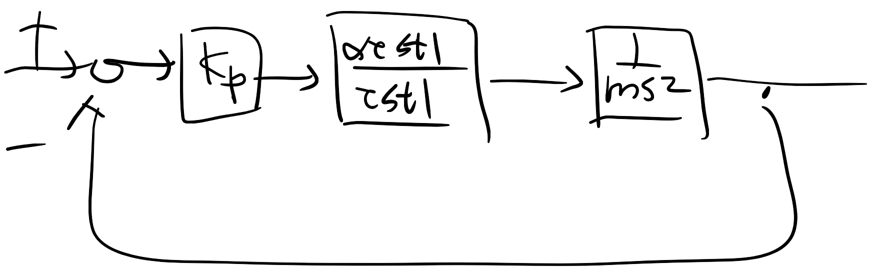
0.

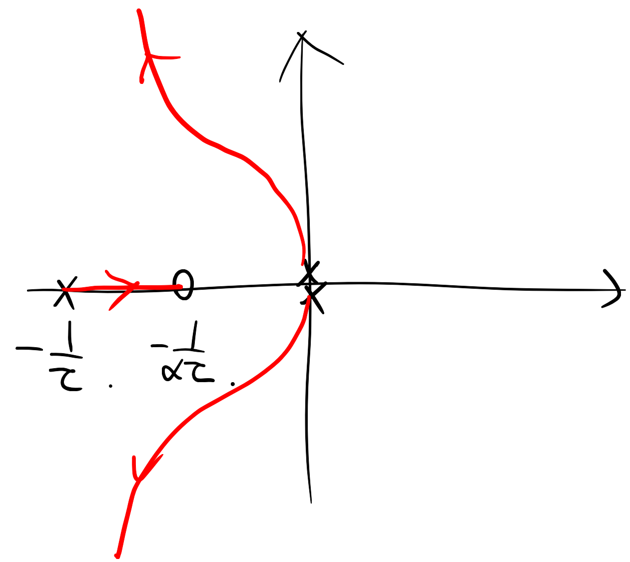
① when $k \to \infty$. roots of $f(s)=0$ $\to$ zeros of $L(s)$.

② when $k \to 0$. root of $f(s)=0$ $\to$ poles of $L(s)$.

③ when $0<k<\infty$, // $\to$ ★ $\measuredangle G(s_0)H(s_0) = 180°$

<u>Example</u>.





$$L(s) = K_p \cdot \frac{\alpha\tau s+1}{\tau s+1} \frac{1}{ms^2}.$$

<u>Remark</u>.

- $L(s) = \frac{a(s)}{b(s)}$. "Rational function"
- Can't handle e^{-sT}, $\sqrt{s}$.
- Difficult when many poles & zeros.

Motivations

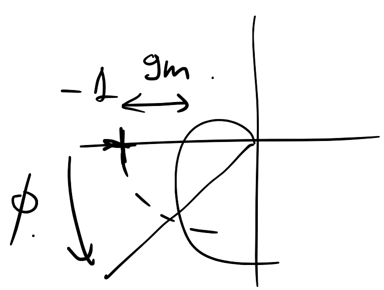
- No need for exact CL pole locations
- Just need to check "existence" of CL poles on RHP.

- Nyquist test.
  - Infer the # of unstable CL poles from $L(s)$.

$$\left(\frac{1}{1+L}, \quad 1+L \triangleq f(s), \quad f(s) = 0 \right)$$

  - Requires less info on $L(s)$.
    - ① $L(j\omega)$
    - ② No of unstable poles of $L(s)$.

  - Gm, Pm ⟵ Nyquist plot.



- Key Ideas

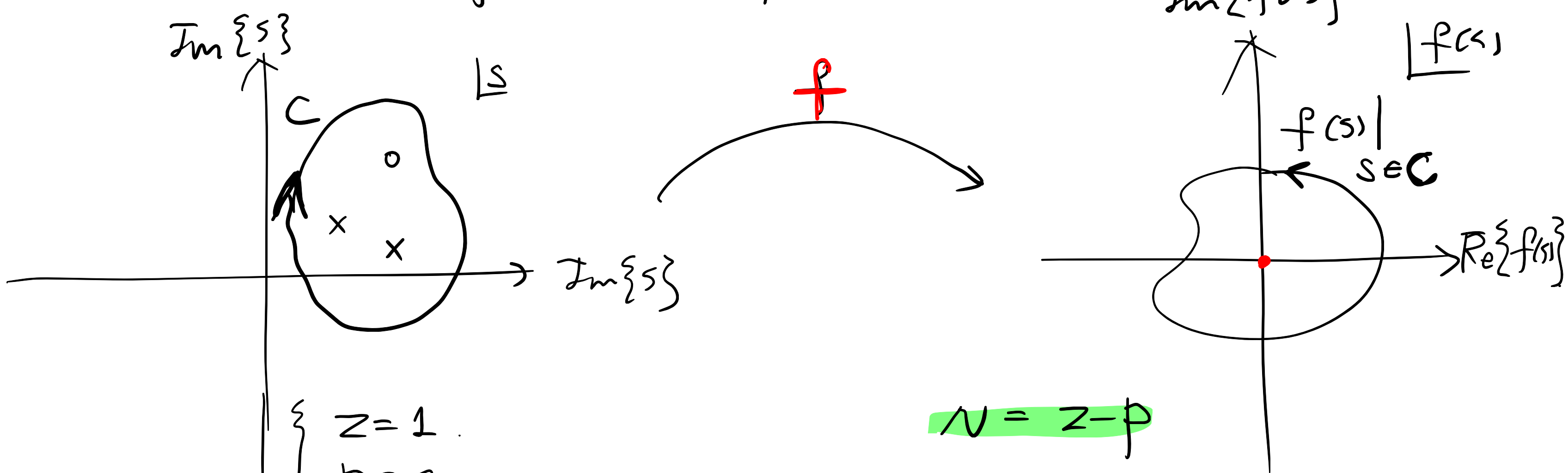
  - CL poles $\Longleftrightarrow$ zeros of $f(s) \triangleq 1 + L(s)$.

    "Z".

  -

- Cauchy's Argument principle. : $f(s)$.

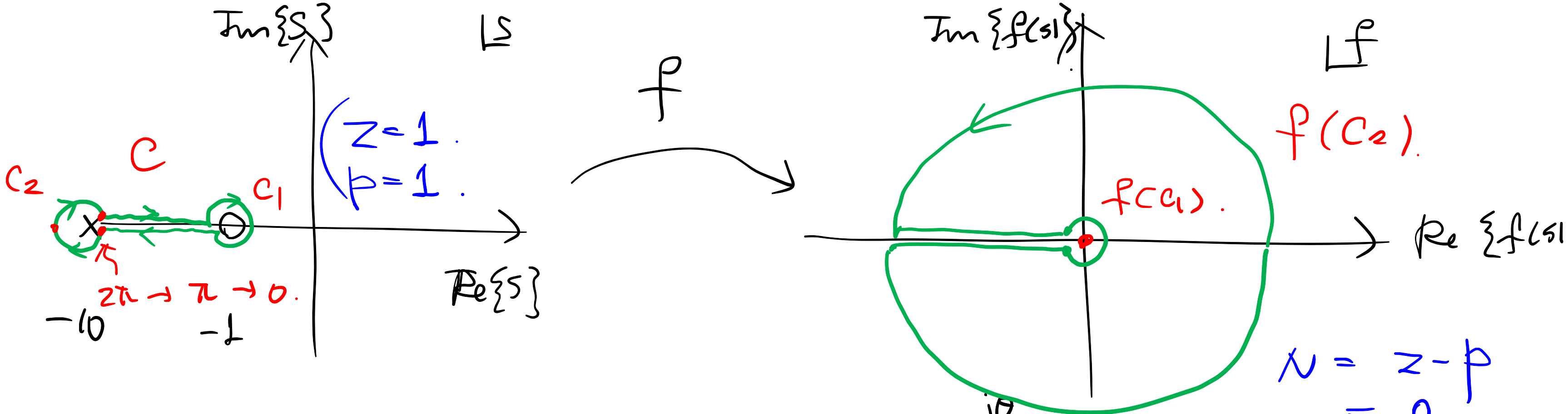
  - Consider a complex ftn $f: \mathbb{C} \to \mathbb{C}$.



$\left\{ \begin{array}{l} Z = 1 . \\ P = 2 . \end{array} \right.$

$$N = Z - P$$
$$= 1 - 2$$
$$= -1 .$$

Example $f(s) = \dfrac{s+1}{s+10}$.



$N = Z - P$
$= 0$

$$C_1: -1 + re^{j\theta} \longrightarrow f(s)\Big|_{s \in C_1} \simeq \frac{re^{j\theta}}{9 + \cancel{re^{j\theta}}}$$

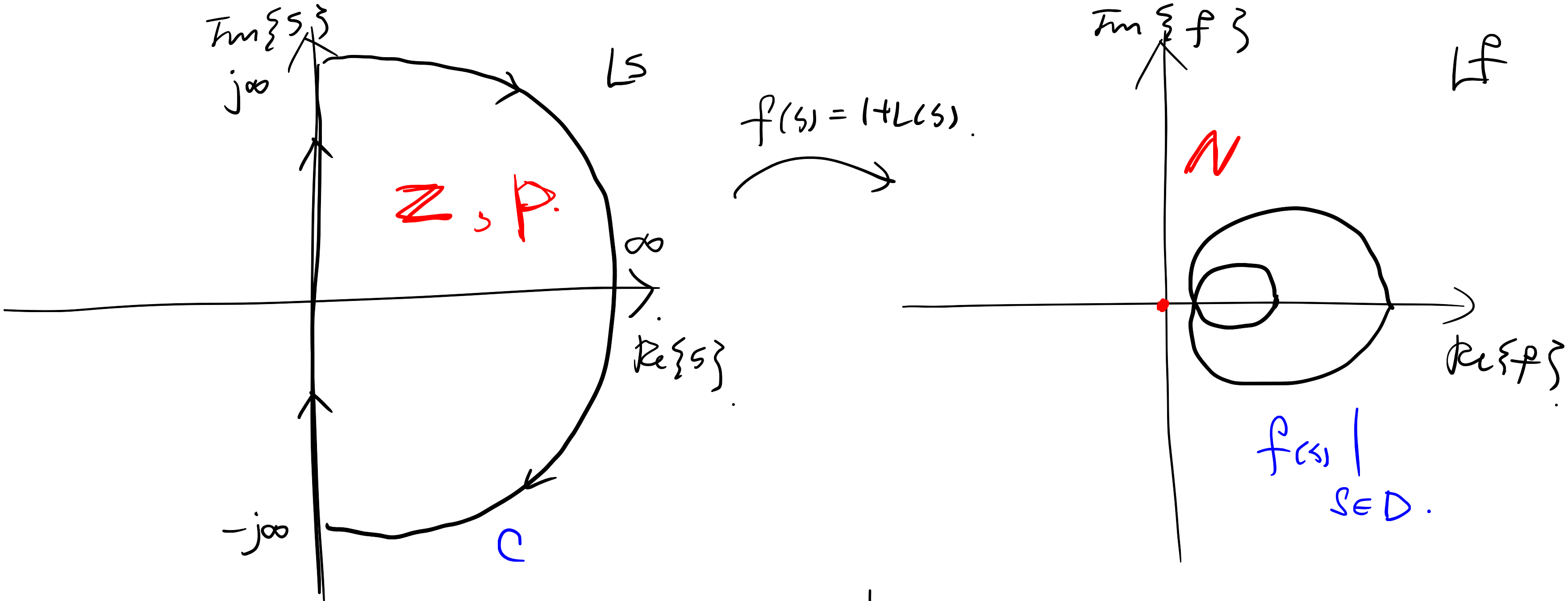
$$C_2: -10 + re^{j\theta} \longrightarrow f(s)\Big|_{s \in C_2} \simeq \frac{-9 + \cancel{re^{j\theta}}}{re^{j\theta}} = \frac{9}{r} e^{j(\pi - \theta)}$$

$2\pi \to \pi \to 0$

$-\pi \to 0 \to \pi$

- Apply it to $f(s) = 1 + L(s)$.

? zeros in RHP.



Z : # of zeros of f(s) = 1+L(s) inside D.

P : # of poles of f(s) = 1+L(s) inside D. ⟺ # of poles of L(s) inside D.

N : # of cw encirclements of $f(s)\big|_{s\in D}$ about the origin.

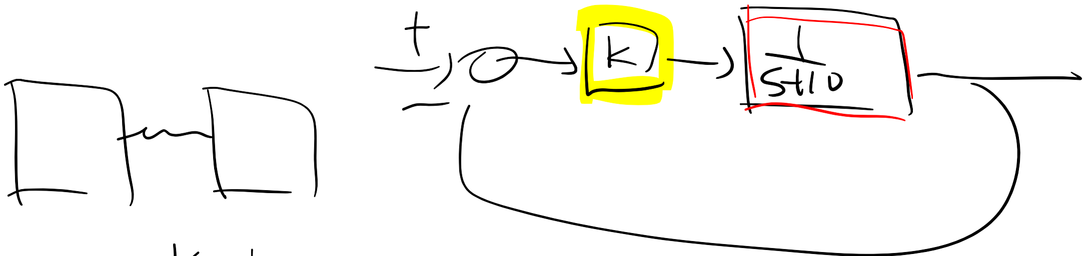
$\Rightarrow \quad N = Z - P \quad \Longleftrightarrow \quad \boxed{Z = N + P}$

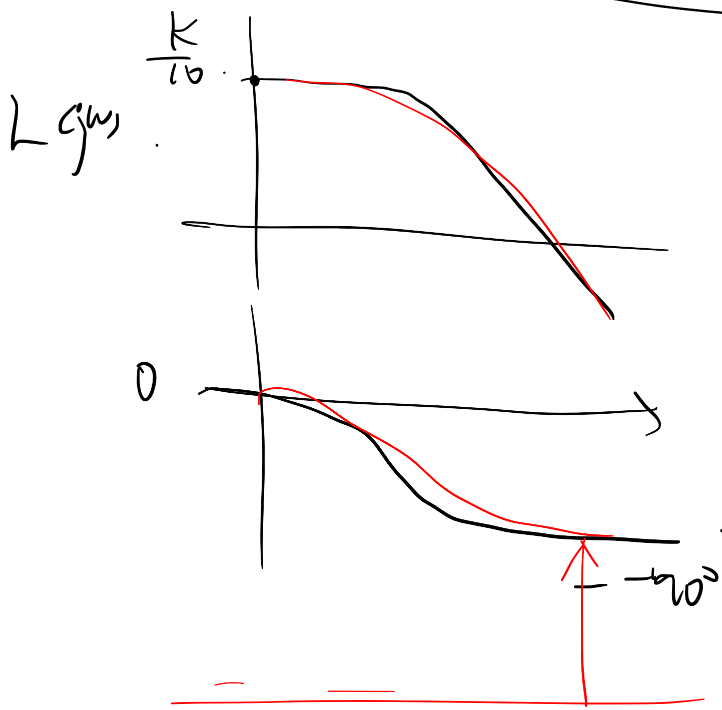
f(s)

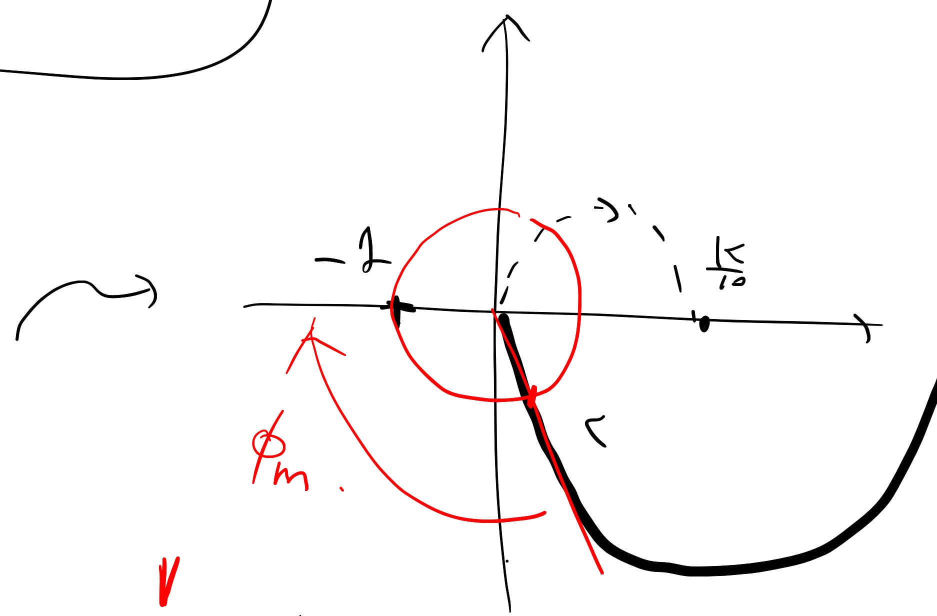
Z = 0 ⇒ Stability!

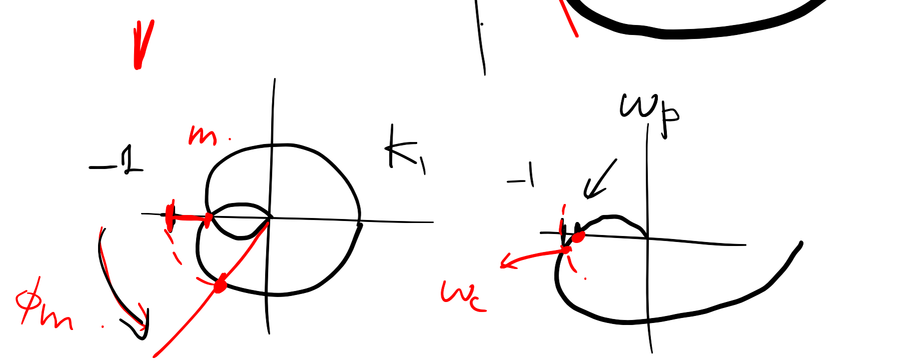
$\vee\ f(s_0) \to \infty \iff 1 + L(s_0) \to \infty$

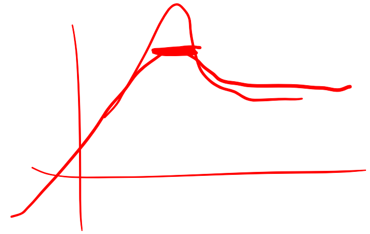
$\Rightarrow$ poles of $f(s)$ = poles of $L(s)$.











$$|L+1| = \frac{1}{|S|}$$