Open loop block diagram of a feed drive system powered by a linear motor is given in Fig. 1 where $u$ [V] is the voltage command to the current amplifier with a gain of $K_a$ = 1 [A/V]. The current $i$ [A] supplied to the linear motor which has force constant of $K_t$ = 20 [N/A]. The force produced by the motor is $F_m$ [N], and disturbance force is given as $F_d$[N].

**Important Note:** Derive all answers symbolically first before using any numeric values. Otherwise, the possible numerical errors will propagate through the solution.

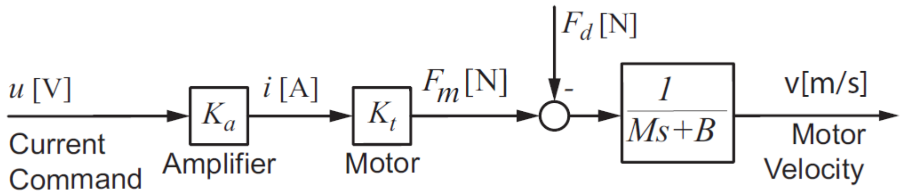


**Fig. 1.** Open-loop block diagram of linear feed drive table

**1.** Identify the equivalent mass ($M$ [kg]) and viscous damping ($B$ [Ns/m)]) from the disturbance frequency response function (FRF) measurement ($v/F_d$) obtained between the table velocity $v$ [m/s] and disturbance force ($F_d$ [N]) as shown in Fig. 2.

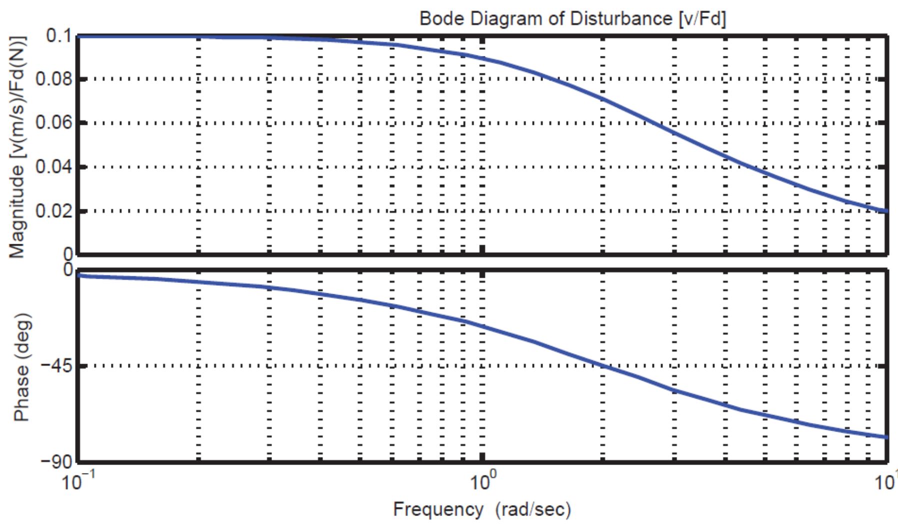


**Fig. 2.** FRF between the output linear velocity $v$ [m/s] and the disturbance force $F_d$ [N]

**2.** Assume that the drive is used to accelerate a table mass of $M$ = 5 [kg] with a viscous damping of $B$ = 10[Ns/m]. The drive has a Coulomb Friction of $F_s$ = 0.3 [N] and the maximum expected cutting load is $F_c$

= 100 [N]. If the maximum acceleration and velocity are $A$ = 10 [m/s$^2$] and $v_{max}$ = 1 [m/s], respectively, what are the required peak and continuous motor current for this drive? (Compute symbolically first before using numerical values)

**3.** As shown in Fig. 3, a linear encoder with a gain of $K_e$ = 1 [m/m] is used to measure the actual position of the table. Plot the root locus of the system with a proportional controller ($D(s) = K_p$) and tune $K_p$ to lead a closed loop system with a damping ratio of $\zeta$ = 0.8. Use the values given to you so far.

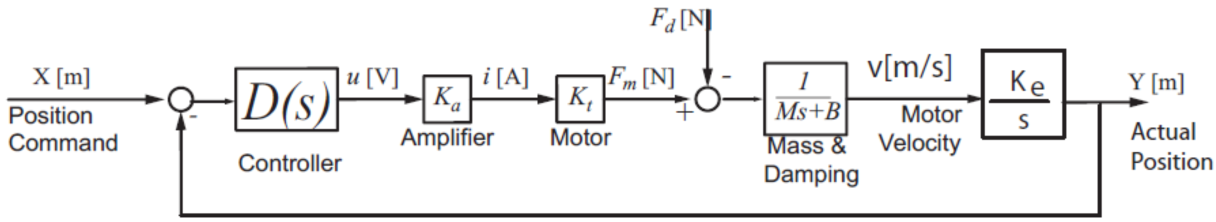


**Fig. 3.** Closed-loop block diagram of linear feed drive table

**4.** Design a lead compensator $D(s)$ which gives phase margin of 60 deg. at a desired gain cross over frequency of $\omega_c$ = 50 [rad/s].

**Note:**

$$D(s) = K\frac{1+\alpha Ts}{1+Ts}, \quad \alpha = \frac{1+\sin(\phi_m)}{1-\sin(\phi_m)}, \quad T = \frac{1}{\omega_c\sqrt{\alpha}}$$

**5.** If the position command is $x(t) = ft$ and the disturbance is constant $F_d(t) = F_0$, what is the steady state error of the closed loop system with the lead compensator designed in Q.4? (Express symbolically)

**6.** A pole placement controller is desired to be designed for the linear motor driven table as in Fig. 4. Derive the zero-order hold equivalent of the linear motor driven table for a sampling frequency of $T$.

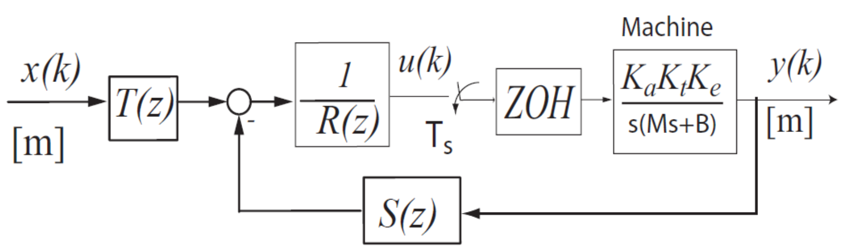


**Fig. 4.** Pole Placement control of the linear feed drive table

**Notes:**

$$ZOH\left(G_p(s)\right) = (1-z^{-1})Z\left(\frac{G_p(s)}{s}\right) = (1-z^{-1})Z\left(\frac{K_aK_tK_e}{s^2(Ms+B)}\right)$$

$$G_p(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-1}(b_0+b_1z^{-1})}{1+a_1z^{-1}+a_2z^{-2}}$$

| $\mathbf{x(t)}$ | $1$ | $e^{-at}$ | $t$ |
|---|---|---|---|
| $\mathbf{X(s)}$ | $\frac{1}{s}$ | $\frac{1}{s+a}$ | $\frac{1}{s^2}$ |
| $X(z)$ | $\frac{1}{1-z^{-1}}$ | $\frac{1}{1-e^{-aT}z^{-1}}$ | $\frac{Tz^{-1}}{(1-z^{-1})^2}$ |

**7.** Design a pole placement controller which has a desired second order underdamped characteristic equation with a damping ratio of $\zeta_m$ and natural frequency of $\omega_m$ ? (Express symbolically)

**Notes:**

$$\frac{x_a(k)}{x_r(k)} = \frac{z^{-d}BT}{AR + z^{-d}BS} = \frac{z^{-d}B_m(z^{-1})}{A_m(z^{-1})}$$

$$A_m(z^{-1}) = 1 - 2e^{-\zeta_m\omega_m T}\cos\left(\omega_m T\sqrt{1-\zeta_m^2}\right)z^{-1} + e^{-2\zeta_m\omega_m T}z^{-2} = 1 + m_1 z^{-1} + m_2 z^{-2}$$

$$\deg(R) = d + \deg(B) - 1, \quad \deg(S) = \deg(A) - 1$$

$$B_m(z^{-1}) = B(z^{-1})b_m$$

$$z^{-1}B(z^{-1})T = z^{-d}B_m(z^{-1}), \quad AR + z^{-1}BS = A_m(z^{-1})$$

$$\left[\frac{z^{-d}B_m(z^{-1})b_m}{A_m(z^{-1})}\right]_{z=1} = 1 \quad \rightarrow \quad b_m = T = t_0$$

**8.** Express the control command generated by the pole placement controller $u$ (k).

**9.** What are the steady-state error of the closed-loop system to a ramp input (i.e. $x(z) = f\frac{Tz^{-1}}{(1-z^{-1})^2}$), and a step input (i.e. $x(z) = \frac{U}{1-z^{-1}}$)?