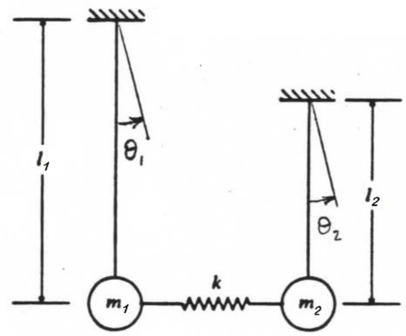
MECH 463 -- Homework 11

1. Two pendulums of lengths ℓ_1 and ℓ_2, and masses m_1 and m_2, are coupled together by a spring of stiffness k. In the particular case considered here $\ell_1 = 2\ell$, $\ell_2 = \ell$, $m_1 = m_2 = m$ and $mg/\ell = 3k$. The matrix equation of motion for the coupled pendulum system is:



$$\begin{bmatrix} m_1\ell_1^2 & 0 \\ 0 & m_2\ell_2^2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} mg\ell_1+k\ell_1^2 & -k\ell_1\ell_2 \\ -k\ell_1\ell_2 & mg\ell_2+k\ell_2^2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Use the Raleigh Quotient to estimate the lowest natural frequency of the pendulum system. To get a good natural frequency estimate, use a guessed mode shape $\underline{v} = [\ 1\ v_2\ ]^T$, where v_2 is a variable. Find a value of v_2 to give a good natural frequency result. Justify your procedure.

2. A pulsed square wave force f(t) = F for 0 < t < T, f(t) = -F for T < t < 2T and f(t) = 0 for t > 2T is applied to a 1-DOF vibrating system. Starting from the equation $m\ddot{x} + kx = f(t)$, calculate the response of the system for t > 2T, i.e., after completion of the pulse force. Assume the system is at rest before the force application. Confirm that if $T = 2n\pi/\omega$, where n is a positive integer, then the response after completion of the pulse force is zero.