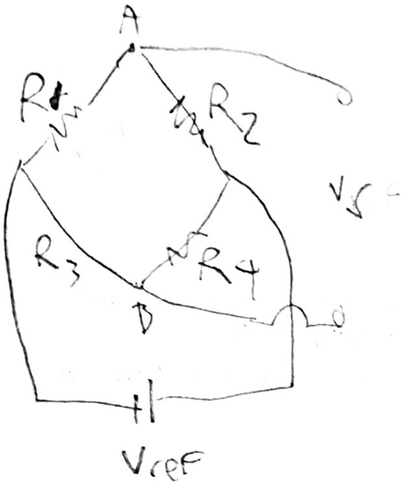
1 i



$R_1 = R_T = R(1+\alpha\Delta T)$

$R_2 = R_3 = R_4 = R$

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Wheatstone bridge equation:

$$V_S = V_A - V_B = \frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)} V_{ref}$$

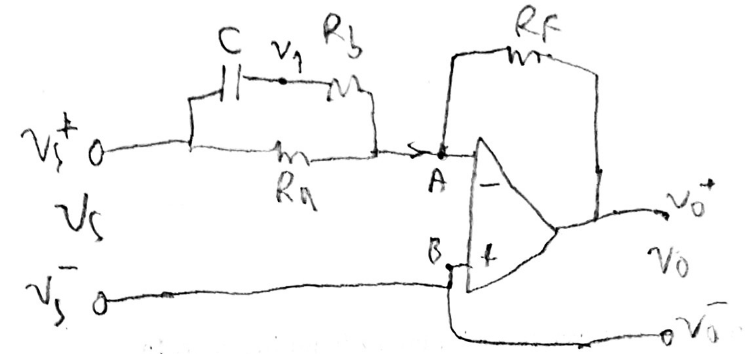
Substitute variables and simplify:

$$V_S = \frac{R(1+\alpha\Delta T)R - R^2}{(R(1+\alpha\Delta T)+R)(2R)} V_{ref}$$

$$= \left(\frac{R^2}{R^2}\right)\left(\frac{(1+\alpha\Delta T)-1}{((1+\alpha\Delta T)+1)(2)}\right) V_{ref}$$

$$= \frac{\alpha\Delta T}{4+2\alpha\Delta T} V_{ref}$$

ii



$v_s = v_s^+ - v_s^-$

$v_o = v_o^+ - v_o^-$

$v_o^- = v_s^-$

Properties of ideal op-amp:

$V_A = V_B = v_s^- = v_o^-$ ← potential at lead are equal

$i_A = i_B = 0$ ← current at lead = 0

Apply rules & substitute & simplify:

$\frac{v_1^+ - V_A}{R_b} + \frac{v_s^+ - V_A}{R_a} + \frac{v_o^+ - V_A}{R_f} = 0$ ← junction rule at A

$V_A = V_B = v_o^- = v_s^-$

$\frac{v_1 - v_s^-}{R_b} + \frac{v_s}{R_a} + \frac{v_o}{R_f} = 0 \Rightarrow \overbrace{\frac{v_1 - v_s^-}{R_b} = \frac{-v_s}{R_a} - \frac{v_o}{R_f}}^{(i)} \Rightarrow \overbrace{v_1 = +v_s^- - \frac{R_b v_s}{R_a} - \frac{R_b v_o}{R_f}}^{(ii)}$

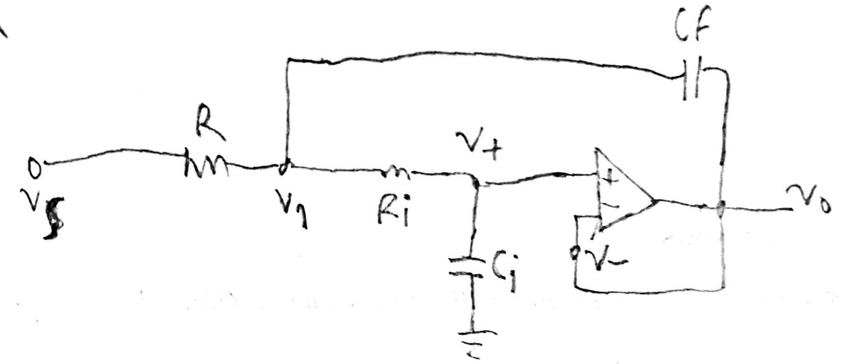
$C\frac{d(v_s^+ - v_1)}{dt} = \frac{v_1 - v_s^-}{R_b}$ ← current series is same

$C\frac{d}{dt}\left(v_s^+ - \left(v_s^- - \frac{R_b v_s}{R_a} - \frac{R_b v_o}{R_f}\right)\right) = \frac{-v_s}{R_a} - \frac{v_o}{R_f}$ ← substitute eq(i) & eq(ii) as derived

$C\frac{d}{dt}\left(v_s + \frac{R_b}{R_a}v_s + \frac{R_b}{R_f}v_o\right) = \frac{-v_s}{R_a} - \frac{v_o}{R_f}$

$C\left(1+\frac{R_b}{R_a}\right)\frac{dv_s}{dt} + \frac{v_s}{R_a} = -\frac{CR_b}{R_f}\frac{dv_o}{dt} - \frac{v_o}{R_f}$

-a



Op amp properties:

$v_- = v_+ = v_0$

$i_- = i_+ = 0$

Apply junction rule:

$$C_i \frac{d(0-v_+)}{dt} + \frac{v_1 - v_+}{R_i} = 0 \rightarrow v_1 = R_iC_i\frac{dv_+}{dt} + v_+ \quad (i)$$

$$\frac{v_s - v_1}{R} + C_f\frac{d}{dt}(v_0 - v_1) = \frac{v_1 - v_+}{R_i} \quad (ii)$$

substitute (i) into (ii) to get rid of v_1, & sub v_- & v_+ with v_0:

$$\frac{v_s}{R} = \frac{v_1}{R} - C_f\frac{d}{dt}(v_0 - v_1) + \frac{v_1 - v_+}{R_i}$$

$$\frac{v_s}{R} = \frac{R_iC_i}{R}\frac{dv_+}{dt} + \frac{v_+}{R} - C_f\frac{d}{dt}\left(v_0 - R_iC_i\frac{dv_+}{dt} - v_+\right) + C_i\frac{dv_+}{dt}$$

$$\frac{v_s}{R} = \frac{R_iC_i}{R}\frac{dv_0}{dt} + \frac{v_0}{R} - \cancel{C_f\frac{dv_0}{dt}} + R_iC_iC_f\frac{d^2v_0}{dt^2} - C_f\frac{dv_0}{dt} + C_i\frac{dv_0}{dt}$$

$$v_s = RR_iC_iC_f\frac{d^2v_0}{dt^2} + (R_iC_i + RC_i)\frac{dv_0}{dt} + v_0$$

2b -

convert to s domain (assume $v_s(0) = v_0'(0) = v_0(0) = 0$):

$$V_s(s) = RR_iC_iC_f s^2 V_0(s) + C_i(R_i+R)sV_0(s) + V_0(s)$$

$$\frac{V_0(s)}{V_s(s)} = \frac{1}{RR_iC_iC_f s^2 + C_i(R_i+R)s + 1} = H(s)$$

$$H(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

$$w_n^2 = \frac{1}{RR_iC_iC_f}$$

$$\xi = \frac{C_i}{2}(R+R_i)\sqrt{RR_iC_iC_f}$$

$$H(jw) = \frac{w_n^2}{(w_n^2 - w^2) + j2\xi w_n w} = \frac{1}{(1-\frac{w^2}{w_n^2}) + j2\xi\frac{w}{w_n}}$$

$$|H(jw)| = \frac{1}{\sqrt{(1-\frac{w^2}{w_n^2})^2 + (2\xi\frac{w}{w_n})^2}}$$

Band width $\rightarrow$ $|H(jw)| = \frac{|H(0)|}{\sqrt{2}}$, get w

$$|H(0)| = \frac{1}{\sqrt{1+0}} = 1$$

$$\frac{|H(0)|}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1-\frac{w^2}{w_n^2})^2 + (2\xi\frac{w}{w_n})^2}}$$

$$2 = 1 - \frac{2}{w_n^2}(w^2) + \frac{1}{w_n^4}(w^4) + \frac{2^2\xi^2}{w_n^2}(w^2)$$

$$0 = w^4 + (-w_n^2 2 + w_n^2 4\xi^2)w^2 - w_n^4$$

$$w^2 = \frac{-(-w_n^2 2 + w_n^2 4\xi^2) \pm \sqrt{(-w_n^2 2 + w_n^2 4\xi^2)^2 + 4w_n^4}}{+2w_n^4}$$

$$= \frac{\left(-(-2 + 4\xi^2) \pm \sqrt{(-2+4\xi^2)^2 + 4}\right) w_n^2}{2w_n}$$

$$w^2 = \left((1-2\xi^2) \pm \sqrt{4\xi^2 - 4\xi + 2}\right) w_n^2$$

$$= \frac{1}{RR_iC_iC_F}\left(\left(1 - \frac{2}{4} C_i^2(R+R_i)^2 (R R_i C_F)\right) \pm \sqrt{\frac{4}{4} C_i^2 (R+R_i)^2 RR_iC_iC_F - \frac{4}{2} C_i(R+R_i)\sqrt{RR_iC_iC_F} + 2}\right)$$

2c. $\tau_s = \frac{1}{\xi w_n} = \frac{2}{C_i(R+R_i)}$

2d. $C_i = \frac{2}{\tau_s(R+R_i)} = \frac{1}{11} F$

$$w_n^2 = \frac{1}{C_F (k\Omega)(10k\Omega)\frac{1}{11}F}$$

$$= \frac{1.1\times10^{-6}}{C_F} F\left(\frac{rad}{s}\right)^2$$

Bandwidth should exceed resonance → $w^2 = w_n^2$

$$w^2 = w_n^2 = \frac{1.1\times10^{-6}}{C_F} = \frac{1}{RR_iC_iC_F}(\cdots)$$

$$0 = -1 + \left(1 - 4.545\times10^{11} C_F \pm \sqrt{7.513\times10^{17} C_F - 476731\sqrt{C_F} + 2}\right)$$

use solver

$x = \sqrt{C_F} = .00190, .0019 \rightarrow C_F = 3.61\times10^{-6} F$

According to Nyquist frequency theory, sampling frequency should be double operating frequency. Assume operate at bandwidth (max operable frequency).

$$W = w_n = \sqrt{\frac{1}{3.61\times10^{-6}(1000)(10000)(\frac{1}{11})}} = 3.282 \frac{rad}{s} \rightarrow f_c = .52 Hz$$

$f_s = 2f_c = 1.044$ Hz

3a. $p = 1 \rightarrow$ ignored

$J_e = J_m + J_{gm} + (J_{gp} + J_p)$

b. $T = J\alpha \rightarrow (\alpha\,[rad/s^2])\left(\frac{1\,rev}{2\pi\,rad}\right)\left(\frac{60\,s}{min}\right) = \alpha\left[\frac{rpm}{s}\right]$

$\eta T_m = J_e\alpha + m r_p^2 (\alpha + g/r_p)$

$T_m = \frac{\alpha}{\eta}(J_m + J_{gm} + J_{gp} + J_p + m r_p^2) + \underbrace{\frac{m r_p g}{\eta}}$

has to overcome gravity term, or load will yank on motor

c. $\omega = v_{max}/r_p$

$= \frac{5}{.4}\frac{rad}{s}\left(\frac{rev}{2\pi\,rad}\right)\left(\frac{60\,s}{min}\right)$

$= 119.37$ rpm ← all 3 motors can operate at this rpm

$\alpha = \frac{v_{max} - v_0}{T - T_0}$

$= \frac{(5/.4 - 0)\,rad/s}{.2s - 0}$

$= 62.5\ rad/s^2$

$T_m = \frac{62.5}{.8}(J_m + 4\times10^{-2} + 7\times10^{-2} + 15\times10^{-2} + 15\times0.4^2) + \frac{15\times0.4\times9.81}{.8}$

$= 78.125\,\frac{J_m}{s^2} + 281.4$ Nm

Motor	J_m (kgm^2)	T_m (Nm)	T at 119.5 rpm	T_m < T at 120 ?
1	6×10^{-2}	286.1	24 Nm	X
2	8×10^{-2}	287.4	42 Nm	X
3	11×10^{-2}	290.0	68 Nm	X

None suitable, load too large