Lecture 15

Energy Relationships

V=0 @ equilibrium

$$V = \frac{1}{2}\vec{q}^{\,T}[K]\vec{q} > 0 \quad \text{for } \vec{q} \neq 0$$

$$T = \frac{1}{2}\dot{\vec{q}}^{\,T}[M]\dot{\vec{q}} > 0 \quad \text{for } \dot{\vec{q}} \neq 0$$

"Positive definite"

$$R = \frac{1}{2}\dot{\vec{q}}^{\,T}[C]\dot{\vec{q}} \geq 0 \quad \text{for } \dot{\vec{q}} \neq 0$$

$\frac{1}{2}\vec{q}^{\,T}[K]\vec{q} > 0$ for stable system

≥ 0 for neutral system — $[K]$ is positive semi-definite

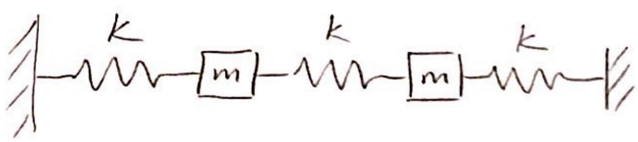
$\frac{1}{2}\vec{q}^{\,T}[K]\vec{q}$ can be anything

Test for positive definite

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

Require at diagonal determinants > 0

Ex:



$$[K] = \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}$$

minors = $2k$, $3k^2$

Both > 0 → positive definite

$$[k] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

minors = k, 0

→ Positive semi-definite

Orthogonality of Mode shapes

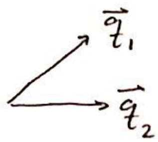
Recall $([K]-\omega^2[M])\vec{u}=\vec{0}$ where $\vec{u}$ is mode shape

$\Rightarrow$ $\omega^2[M]\vec{u}=[K]\vec{u}$ generalized Eigenvalue problem

↳ $\vec{u}$ = mode shape = eigenvector

↳ ω^2 = (natural frequency)^2 = eigenvalue

Note :



Dot product $\vec{q}_1 \bullet \vec{q}_2 \iff \vec{q}_1^{\,T}\vec{q}_2$

For eigenvector "r": $\omega_r^2[M]\vec{u}_r=[K]\vec{u}_r$

eigenvector "s": $\omega_s^2[M]\vec{u}_s=[K]\vec{u}_s$

Premultiply $\vec{u}_s^{\,T}$: $\omega_r^2\vec{u}_s^{\,T}[M]\vec{u}_r=\vec{u}_s^{\,T}[K]\vec{u}_r$

$\vec{u}_r^{\,T}$: $\omega_s^2\vec{u}_r^{\,T}[M]\vec{u}_s=\vec{u}_r^{\,T}[K]\vec{u}_s$

Consider $\vec{u}_s^{\,T}[M]\vec{u}_r=\sum_i\sum_j u_{si}M_{ij}u_{rj}$

$=\sum_i\sum_j u_{rj}M_{ij}u_{si}$ ← Rearrange

$=\sum_j\sum_i u_{rj}M_{ij}u_{si}$ ← Reverse sum order

$=\sum_i\sum_j u_{ri}M_{ji}u_{sj}$ ← Rename $i\leftrightarrow j$

$=\sum_i\sum_j u_{ri}M_{ij}u_{sj}$ ← Note $[M]$ is symmetric

$=\vec{u}_r^{\,T}[M]\vec{u}_s$

$\Rightarrow$ $\vec{u}_s^{\,T}[M]\vec{u}_r=\vec{u}_r^{\,T}[M]\vec{u}_s$ Similarly for $[K]$

Sub into $\omega_r^2 \vec{u}_s^T [M] \vec{u}_r = \vec{u}_s^T [K] \vec{u}_r$

$$\omega_r^2 \vec{u}_r^T [M] \vec{u}_s = \vec{u}_r^T [K] \vec{u}_s$$

Also $\omega_s^2 \vec{u}_s^T [M] \vec{u}_s = \vec{u}_r^T [K] \vec{u}_s$

Subtract: $(\omega_r^2 - \omega_s^2) \vec{u}_r^T [M] \vec{u}_s = \vec{0}$

$$\Rightarrow \vec{u}_r^T [M] \vec{u}_s = 0 \quad \text{for } \omega_r \neq \omega_s$$