a place of mind

UBC

MECH468 : Modern Control Engineering
MECH509 : Controls

L2 : Model Classifications

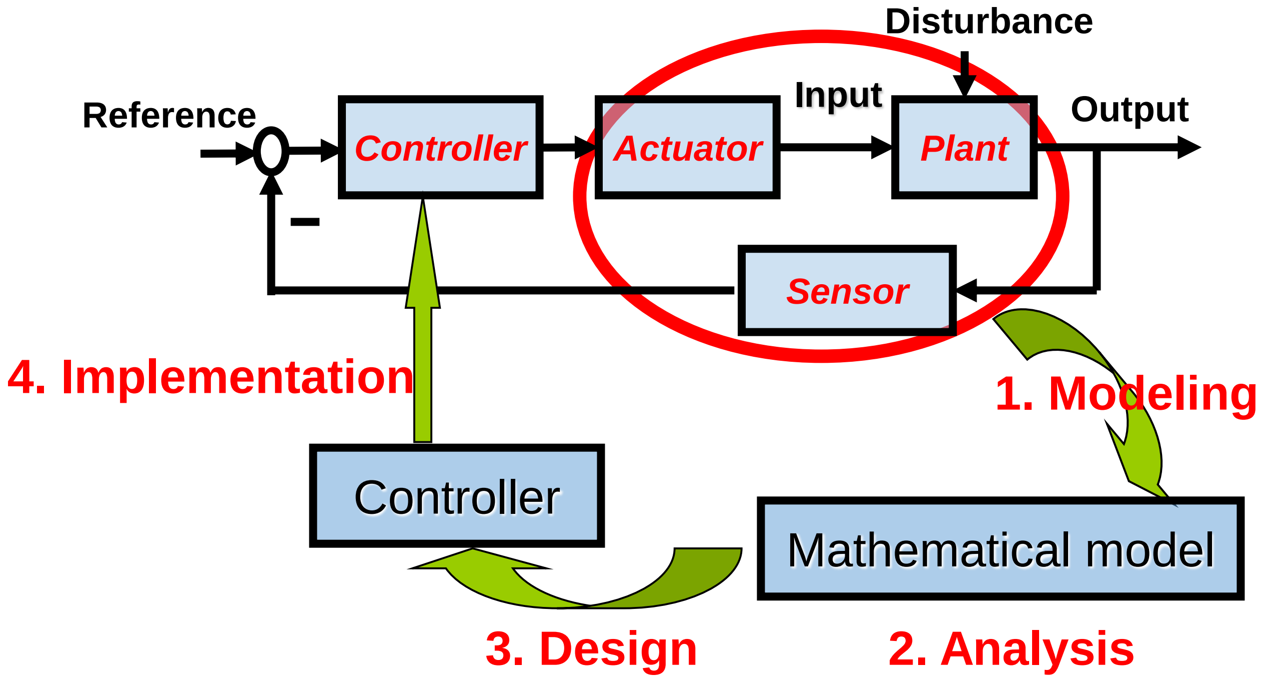
Dr. Ryozo Nagamune

Department of Mechanical Engineering

University of British Columbia

Zoom lecture to be recorded and posted on Canvas

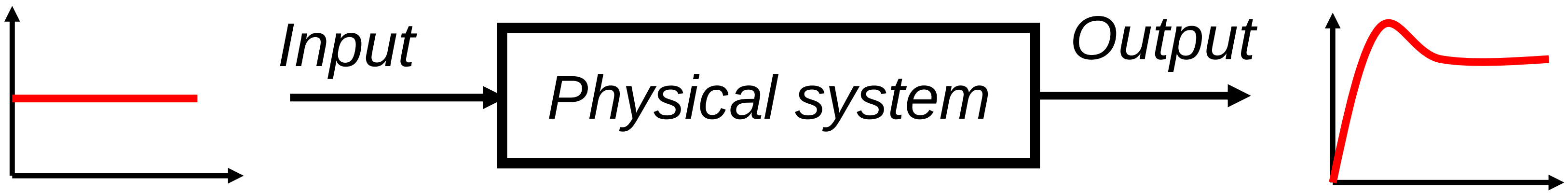
Model-based controller design (from 1st lecture)



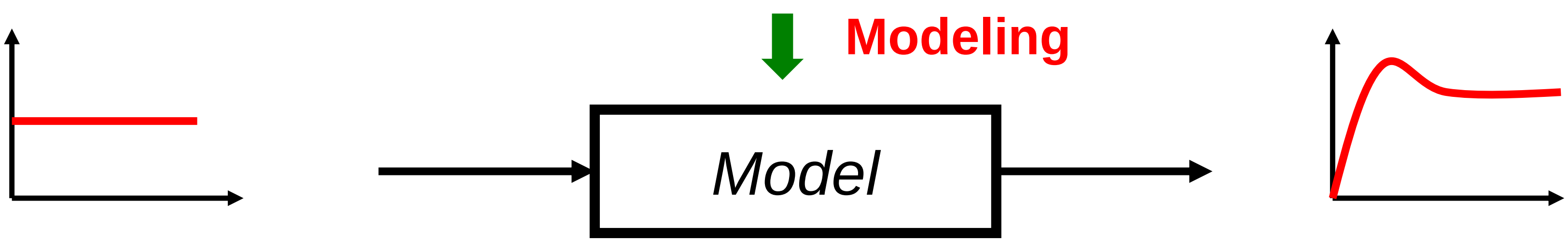
- Today's topics
  - What is the mathematical model?
  - Model classifications

Mathematical model

- Representation of input-output signal relation of a physical system



acceleration	car	position
voltage	circuit	current
deposit	bank account	balance



Important remarks on models

- No math model exactly represents a physical system.

$$\text{Math model} \neq \text{Physical system}$$
$$\text{Math model} \approx \text{Physical system}$$

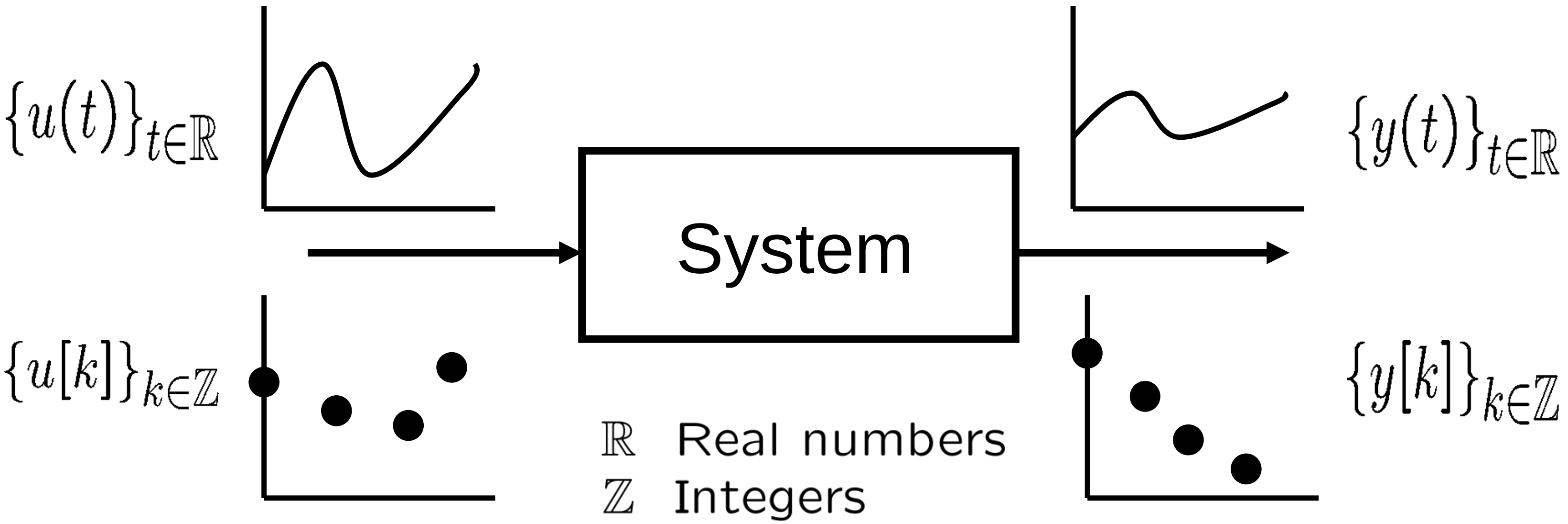
- Do not confuse models with physical systems!
- Constructing a math model close enough to a physical system and yet simple enough to be studied analytically is the most important and difficult task in control system design.
- In this course, we may use the term "system" to mean a mathematical model.

Model classifications

- Continuous-time and discrete-time
- Memoryless, causal and noncausal
- Lumped and distributed
- Time-invariant and time-varying
- Linear and nonlinear

Continuous-time & discrete-time systems

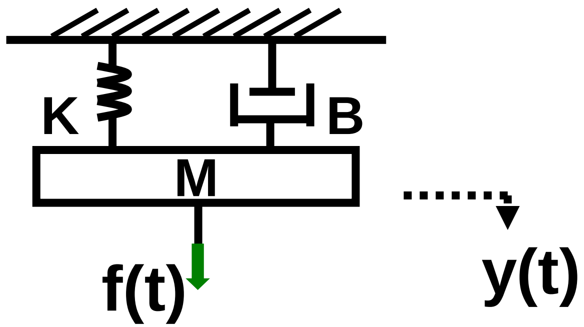
- Continuous-time (discrete-time) system has input and output vectors as continuous-time (discrete-time) signals.



Examples

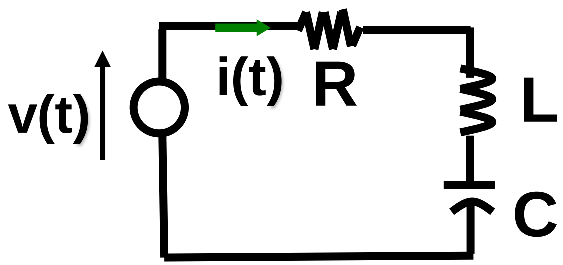
- Continuous-time system
  - Mass-spring-damper system

$$M\ddot{y}(t) = f(t) - B\dot{y}(t) - Ky(t)$$



  - RLC circuit

$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$



- Discrete-time system
  - Digital computer (Simulation, microcontroller)
  - Daily balance of a bank account

$$y[k+1] = (1+a)y[k] + u[k]$$

y[*k*] : balance at k-th day
u[*k*] : deposit/withdrawal
a : fixed interest rate

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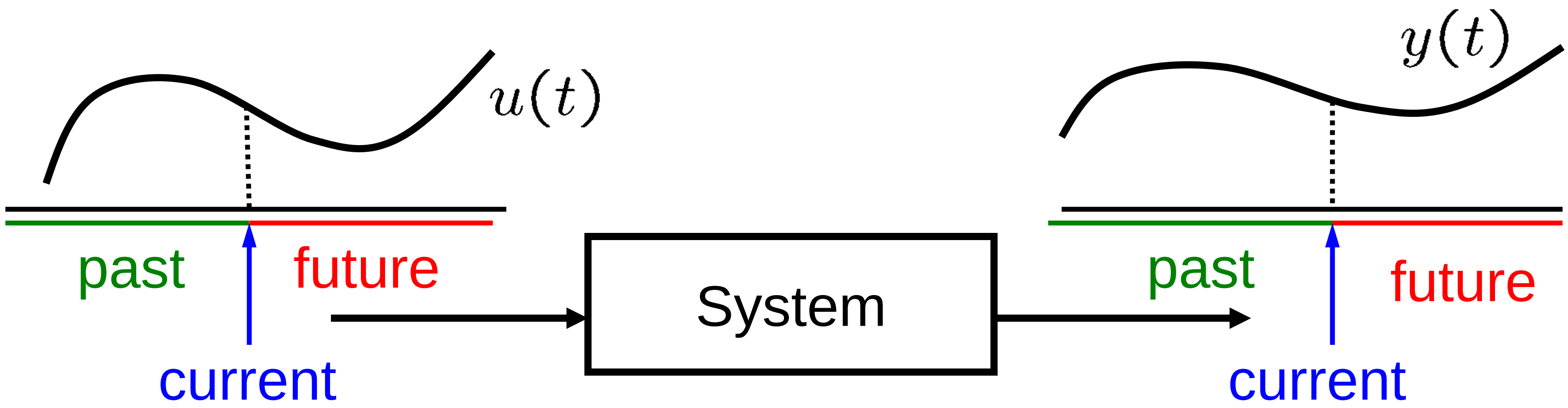
Memoryless, causal and noncausal systems

- Memoryless (static) system: Current output depends on only current input.
- Causal system: Current output depends on past input, and possibly on current input.
- Noncausal system: Current output depends on future input.



Remarks & Examples

- Non-static system is called dynamic(al) system.
- Memoryless (static) system
  - Spring: input *f*(t), output *x*(t) $f(t) = kx(t)$
  - Resistor: input *v*(t), output *i*(t) $v(t) = Ri(t)$
- Causal system
  - Input: acceleration, output: position of a car
    Current position depends on all the past accelerations.
- Noncausal system does not exist in real world; It exists only mathematically. (From now on, we consider only causal systems.)

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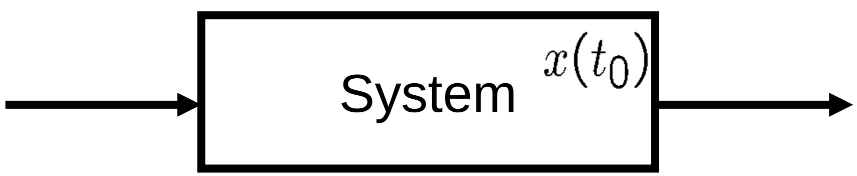
State

- For a causal system,

$$\left.\begin{array}{l}\text{(current/future input)} \\ \underline{\text{(past input)}}\end{array}\right\} \Rightarrow \text{(current/future output)}$$

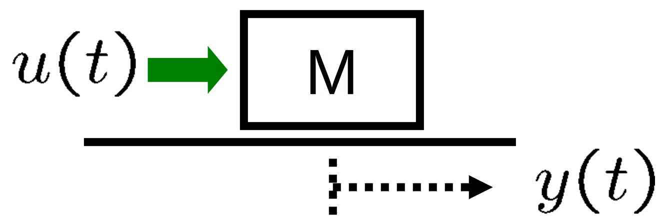
- To memorize the information of (past input), we use a **state vector** $x(t_0)$ $\quad t_0$:current time



- *State is a memory of the system.*

Lumped and distributed systems

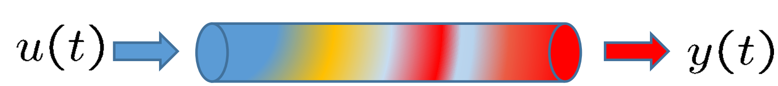
- Lumped system: State vector is finite-dimensional.
  - Example: Input: force *u*, Output: displacement *y*



- Distributed system: State vector is infinite-dim.
  - Example: Unit delay system $y(t)=u(t-1)$

For this system, to determine future output $\{y(t), t>t_0\}$, we need $\{u(t), t_0-1<t<t_0\}$.

Ex: fluid temperature in a tube



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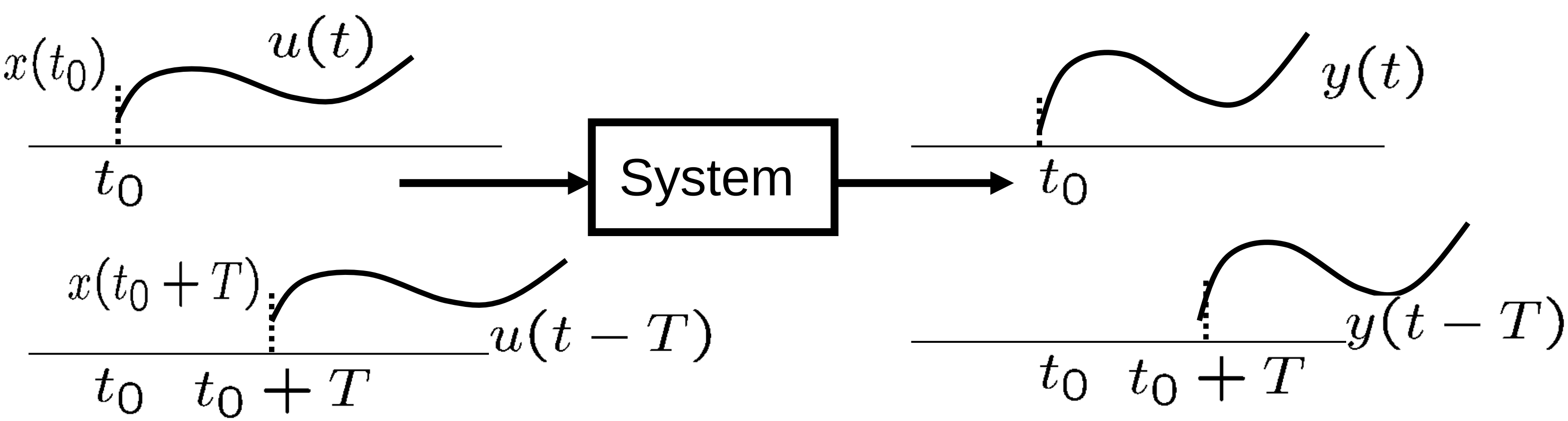
Time-invariant and time-varying systems

- For a causal system, $\left.\begin{array}{l} x(t_0) \\ u(t), t \geq t_0 \end{array}\right\} \Rightarrow y(t), t \geq t_0$
- Time-invariant system: For any time shift T>0,

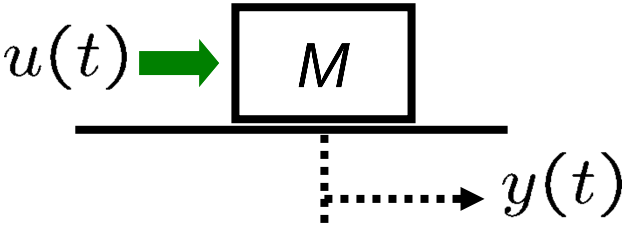
$$\left.\begin{array}{l} x(t_0+T) \\ u(t-T), t \geq t_0+T \end{array}\right\} \Rightarrow y(t-T), t \geq t_0+T$$



- Time-varying system: Not time-invariant

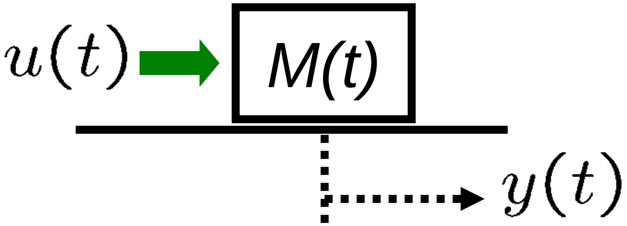
Examples

- Car, rocket etc.: If we regard mass *M* to be:
  - constant (even though *M* changes very slowly), then this system is time-invariant.
  - changing (due to fuel consumption), then this system is time-varying.



$$M\ddot{y}(t) = u(t)$$

(Laplace applicable)



$$M(t)\ddot{y}(t) = u(t)$$

(Laplace not applicable)

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Linear and nonlinear systems

- For a causal system, $\left.\begin{array}{l} x_i(t_0) \\ u_i(t), t \geq t_0 \end{array}\right\} \Rightarrow y_i(t), t \geq t_0, \quad i = 1, 2$

- Linear system: Superposition property holds.

$$\left.\begin{array}{l} \alpha_1 x_1(t_0) + \alpha_2 x_2(t_0) \\ \alpha_1 u_1(t) + \alpha_2 u_2(t), t \geq t_0 \end{array}\right\} \Rightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t), t \geq t_0$$

$$\forall \alpha_1, \alpha_2 \in \mathbb{R}$$

- Nonlinear system: Superposition property does not hold.

Remarks

- All systems in real world are nonlinear.
  - Example: Linear spring $f(t) = Ky(t)$

    (Input: force $f(t)$, Output: displacement $y(t)$)

    The linear relation holds only small f and y in reality.
- However, linear approximation is often good enough for control purposes.
- Linearization: Approximation of a nonlinear system by a linear system around some operating point (We will study this topic in Lecture 4.)

Summary

- Model classifications
  - Continuous-time and discrete-time
  - Memoryless, causal and noncausal
  - Lumped and distributed
  - Time-invariant and time-varying
  - Linear and nonlinear
- In this course, we will not consider noncausal, distributed, nonlinear systems (except linearization).
- Next, state-space models $\begin{cases} \frac{dx(t)}{dt} &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{cases}$