Lesson 5-3 – Nominal and Effective Interest Rates

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Interest Rates can be confusing

- Look at some ads for financial services, and read the details on the interest rate

Ford

FordCredit

View our Rates

2 yr fixed closed	5 yr fixed closed	5 yr variable closed
		RBC Prime Rate
3.340%	3.740%	+ 0.000%
3.390% APR	3.760% APR	3.470% APR

- What do all these mean?

Compounding:
Nominal and Effective Interest

- Nominal Interest Rate (r):
  - ‘in name only’
  - Interest rate without consideration of compounding
  - Specified per year
  - E.g. A bank bond pays 1% every quarter. The nominal rate is 1% x 4 = 4% per year

Compounding:
Nominal and Effective Interest

- Effective Interest Rate (i_a):
  - Interest rate that takes compounding into consideration
  - Can be broken into different compounding periods: Semi-annually (2/yr), quarterly (4/yr), monthly (12/yr), biweekly (26/yr), weekly (52/yr), daily (365/yr)

Determining Effective Interest

- $i_a = (1+r/m)^m - 1$

- How do we get this? We can get it from our F/P formulas
  - Take a nominal interest rate of r, with a compounding period of m
  - Consider investing $1000 at 6% per year, compounded semi-annually

  - What is the future value?
  - F = P(1+i)n = $1000(1+0.06/2)^2 = $1,060.90

  - Our interest amount, I, is $60.90

  - Our effective annual interest rate is then $60.90/$1000 = 6.09%
  - Pull out the F and P and normalize to $1 and we get the above formula

Nominal & Effective Interest Continued…

If you were given a loan at 12% interest per year compounded monthly…

- The Nominal Interest Rate would be:
  - r =12%/year compounded monthly
- The Effective Monthly Rate (i) would be:
  - i = r/m where m = compounding period
  - i = 0.12/12 = 1% per month effective
- The Effective Yearly Rate (i_a) would be:
  - $\mathbf{i_a = (1+r/m)^m - 1} = (1+i)^m - 1$
  - $i_a = (1+0.12/12)^{12} - 1 = 12.68\%$/year effective

Compounding

- The number of compounding periods depends on the number of subdivisions.
- A nominal interest rate of 12%/year compounded:
  - m = 1: yearly (equals 12% effective yearly)
  - m = 2: semi-annually (equals 12.360% effective yearly)
  - m = 4: quarterly (equals 12.551% effective yearly)
  - m = 52: weekly (equals 12.734% effective yearly)
  - m = 365: daily (equals 12.747% effective yearly)
  - m = 8760: hourly (equals 12.749% effective yearly)
  - m = large: continuous (approaches ~12.750%)

Nominal & Effective Interest Example 1

EXAMPLE 3-9

If a savings bank pays 1.5% interest every three months, what are the nominal and effective interest rates per year?

SOLUTION

Nominal interest rate per year $r = 4 \times 1.5\% = 6\%$

Effective interest rate per year

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.06}{4}\right)^4 - 1 = 0.061$$

$$= 6.1\%$$

Alternatively,

Effective interest rate per year

$$i_a = (1+i)^m - 1$$

$$= (1+0.015)^4 - 1 = 0.061$$

$$= 6.1\%$$

Nominal & Effective Interest Example 2

Question:

A loan shark lends you money on these terms: “If I give you $50 on Monday, you owe me $60 on the following Monday.”

a) What is the nominal interest rate per year? (r)

b) What is the effective rate per year? (i_a)

c) If the loan shark started with $50 and kept it and all the interest he made for one year, how much would he have? (F)

Nominal & Effective Interest Example 2

Solutions:

a) i = 20% per week

r = 52 weeks (0.20) = 10.40 = 1040%

b) $$i_a = \left[1 + \frac{r}{m}\right]^m - 1 \qquad i_a = \left[1 + \frac{10.40}{52}\right]^{52} - 1$$

i_a = 13104 = 1,310,400%

[or: $i_a = (1 + i)^m - 1 = (1 + 0.20)^{52} - 1 = 13{,}104$]

c) $F = P(1 + i)^n = 50(1 + 0.20)^{52} = \$655{,}200$