University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2013): Introduction to Robotics
Make-up Midterm Examination#1, November 14, 2013
Closed Book - 80 Minutes
Maximum - 30 marks

Problem 1.

You are given three coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$ with right-handed orthonormal frames.
$\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by an angle θ_1.
$\underline{C}_2$ is obtained from $\underline{C}_1$ by rotating $\underline{C}_1$ about $\underline{i}_0$ by an angle θ_2.
$\underset{\sim}{o}_2$ is obtained from $\underset{\sim}{o}_0$ by displacing $\underset{\sim}{o}_0$ by $a\underline{i}_0 + b\underline{j}_0 + c\underline{k}_0$.

(a)(5 marks)
Find the homogeneous transformation ${}^0T_2$ that relates the coordinates ${}^2x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_2, \underline{C}_2\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$. Specify ${}^0T_2$ in terms of θ_1, θ_2 and ${}^0d_2 = [a\ b\ c]^T$. You may use matrix exponential notation.

Problem 2.

(i) (2 marks)
A vector product function is specified as $\underline{\underline{f}}(\underline{x}) = (-\underline{i}_0 + 3\underline{j}_0 - 2\underline{k}_0) \times \underline{x}$. What is the matrix representation of $\underline{\underline{f}}$ in $\underline{C}_0 = [\underline{i}_0\ \ \underline{j}_0\ \ \underline{k}_0]$?

(v) (8 marks)
Write the homogeneous transformation ${}^{i-1}T_i$ relating the coordinate systems $\{\underset{\sim}{o}_{i-1}, \underline{C}_{i-1}\}$ and $\{\underset{\sim}{o}_i, \underline{C}_i\}$ attached to link $i-1$ and i, respectively, given that the Denavit-Hartenberg parmeters of link i are θ, d, a and α.
What is the inverse of this transformation?

Problem 3.

Consider the manipulator shown in the attached figure.
(a) (15 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, ...6$ to links 1 through 6 according to the Denavit-Hartenberg convention and find the homogeneous transformations required for the direct kinematics problem.

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Department of Electrical and Computer Engineering
EECE 487 (Winter 2013-2014): Introduction to Robotics
Midterm Examination, October 15, 2013, 9:30am - 10:45am
Closed Book
Maximum - 35 marks

Problem 1.

(i) (2 marks)
A vector product function is specified as $\underline{\underline{f}}(\underline{x}) = (2\underline{i}_0 + 1\underline{j}_0 - 3\underline{k}_0) \times \underline{x}$. What is the matrix representation of $\underline{\underline{f}}$ in $\underline{C}_0 = [\underline{i}_0 \;\; \underline{j}_0 \;\; \underline{k}_0]$?

(ii) (1 mark)
The frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by θ. What is the matrix representation ${}^0C_1$ of this rotation in $\underline{C}_0$?

(iv) (1 mark)
If a vector $\underline{x}$ has coordinates ${}^0x$ in $\underline{C}_0$, what are its coordinates ${}^1x$ in $\underline{C}_1$?

(iii) (3 marks)
(iii) What is the matrix representation of the function $\underline{\underline{f}}$ from (i) above in $\underline{C}_1$ from (ii)? (you do not need to multiply out the matrices).

(v) (5 marks)
Write the homogeneous transformation ${}^{i-1}T_i$ relating the coordinate systems $\{\underset{\sim}{o}_{i-1}, \underline{C}_{i-1}\}$ and $\{\underset{\sim}{o}_i, \underline{C}_i\}$ attached to link $i-1$ and i, respectively, given that the Denavit-Hartenberg parmeters of link i are θ, d, a and α.

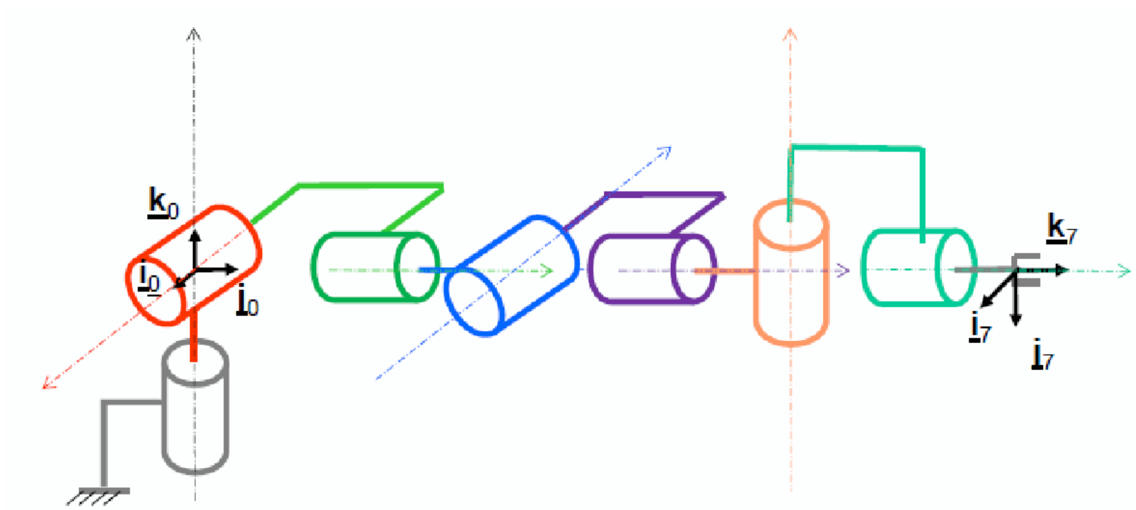
(vi)(5 marks)
Clearly explain the steps required to find the axis and angle of rotation given a rotation matrix Q.

(vii)(3 marks)
What are the axis and angle of rotation of the following rotation matrix:

$$Q = \exp(\begin{bmatrix} 0 & -\pi/\sqrt{2} & 0 \\ \pi/\sqrt{2} & 0 & \pi/\sqrt{2} \\ 0 & -\pi/\sqrt{2} & 0 \end{bmatrix})$$

Problem 2. *(15 marks)*

Consider the manipulator below. Assign joint variables in a manner consistent with the axes shown (positive angle by right hand rule), and coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, to all links, using the Denavit-Hartenberg convention. Complete the table of Denavit Hartenberg parameters. Write, as a function of ${}^{i-1}T_i$, $i = 0, \ldots, 6$, the coordinate transformation relating the coordinates ${}^{7}x$ in the gripper frame with the coordinates ${}^{0}x$ in the base frame. You DO NOT need to fill in the details for every homogeneous matrix as they will all look as in Problem 1. (v).



University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2012-2013): Introduction to Robotics
Midterm Examination#1, February 28, 2013
Closed Book - 9:30-10:45 am (75 Minutes)
Maximum - 60 marks

Problem 1.

i) *(5 marks)*What is the coordinate representation ${}^0Q$ in $\underline{C}_0$ of a rotation $\underline{y} = \underline{\underline{Q}}(\underline{j}_0, -\pi/4)\underline{x}$ about the axis $\underline{j}_0$ of angle $-\pi/4$?
(ii) *(5 marks)* What is the coordinate representation ${}^0R$ in $\underline{C}_0$ of a rotation $\underline{y} = \underline{\underline{R}}(\underline{k}_0, \pi/3)\underline{x}$ about the axis $\underline{k}_0$ of angle $\pi/3$?
(iii) *(5 marks)* What is the coordinate representation of the composition of the two rotations $\underline{y} = \underline{\underline{R}}(\underline{k}_0, \pi/3)\underline{\underline{Q}}(\underline{j}_0, -\pi/4)\underline{x}$ in $\underline{C}_0$?
(iv) *(10 marks)* Let $\underline{C}_1$ be the rotated frame $\underline{C}_1 = \underline{\underline{R}}(\underline{k}_0, \pi/3)\underline{\underline{Q}}(\underline{j}_0, -\pi/4)\underline{C}_0$. What is the coordinate representation of the composition of the two rotations $\underline{y} = \underline{\underline{R}}(\underline{k}_0, \pi/3)\underline{\underline{Q}}(\underline{j}_0, -\pi/4)\underline{x}$ in $\underline{C}_1$?

Problem 2.

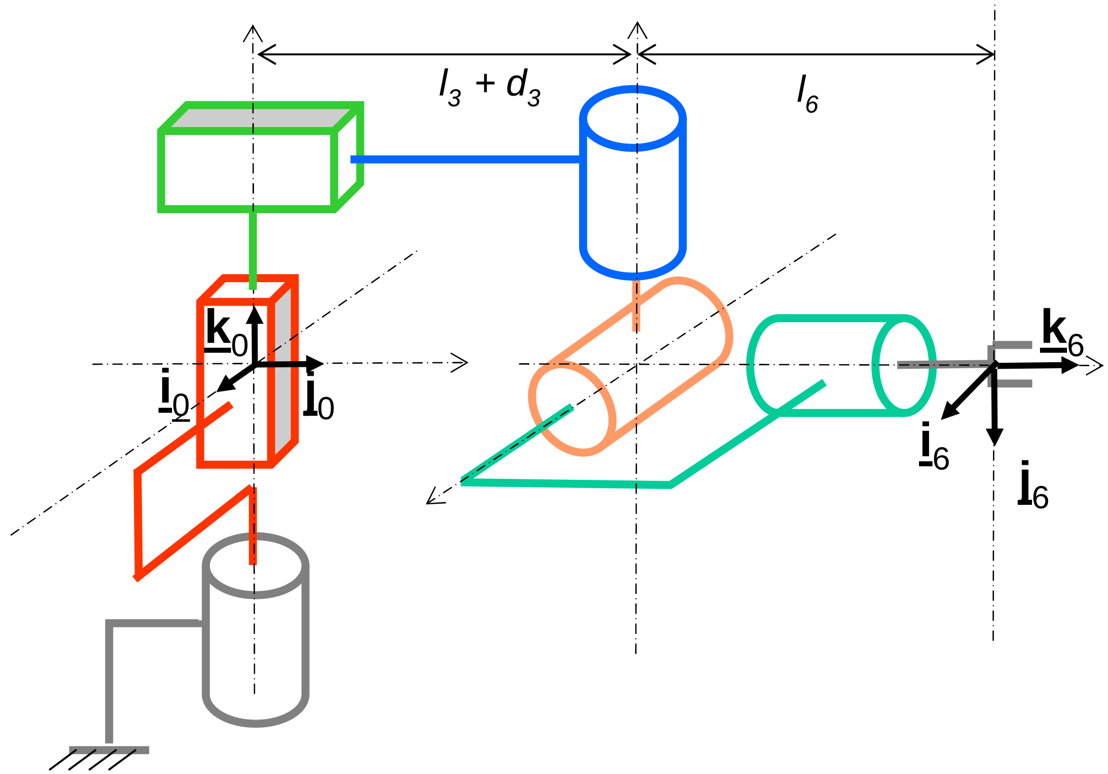
(5 marks) What is the axis and angle of rotation of the following rotation matrix:

$$Q = \exp(\begin{bmatrix} 0 & -3\pi & 0 \\ 3\pi & 0 & 4\pi \\ 0 & -4\pi & 0 \end{bmatrix})$$

Problem 3.

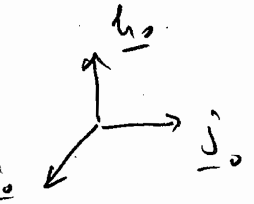
Consider the manipulator shown on the next page.
(10 marks) Assign joint variables (note directed axes on the figure) and coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, ...6$ to links 1 through 6 according to the Denavit-Hartenberg convention. List the Denavit Hartenberg parameters and find the homogeneous transformations required to solve the direct kinematics problem.

(10 marks) Write down the manipulator Jacobian in terms of the joint axes and use elementary row operations to decouple the wrist and arm singularities. Under what condition will the manipulator be in a singular configuration?
(10 marks) What are the coordinates of the angular velocity of the gripper with respect to the base frame $\underline{C}_0$ at a robot configuration $(\theta_1, d_2, d_3, \theta_4, \theta_5, \theta_6)$ for joint rates $(\dot{\theta}_1, \dot{d}_2, \dot{d}_3, \dot{\theta}_4, \dot{\theta}_5, \dot{\theta}_6)$? What are the coordinates of the angular velocity of the gripper with respect to the rotated frame $e^{\theta_1 \underline{k}_0 \times}\underline{C}_0$?

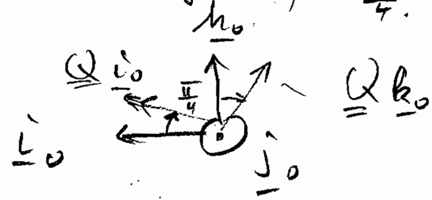


$l_3 + d_3$
l_6
$\underline{\mathbf{k}}_0$
$\underline{\mathbf{i}}_0$
$\underline{\mathbf{j}}_0$
$\underline{\mathbf{k}}_6$
$\underline{\mathbf{i}}_6$
$\underline{\mathbf{j}}_6$

#1 (i)

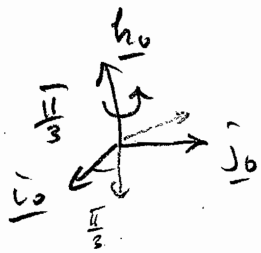


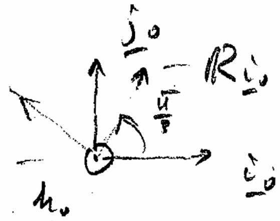
$\underline{Q}$ rotates about $\underline{j}_0$ an angle of $-\frac{\pi}{4}$.



$$^0Q = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$$

(ii)





$$^0R = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) $^0A = {}^0R\,{}^0Q$ because it is in base frame.

(iv)

$$\begin{tikzcd} \{\text{coords}\}\ {}^0x \arrow[r, "^0A"] & \{\text{coords}\}\ {}^0y \\ \{\text{coords}\}\ {}^1x \arrow[u, "^0C_1"] \arrow[r, "^1A"] & \{\text{coords}\}\ {}^1y \arrow[u, "^0C_1"'] \end{tikzcd}$$

$$\underline{C}_1 = \underline{C}_0\,{}^0C_1 = \underline{\underline{C}_0\,{}^0A}$$

$$^1A = \underbrace{{}^0C_1^T}_{{}^0A^T}\,{}^0A\,\underbrace{{}^0C_1}_{{}^0A} = \underline{{}^0A^T\,{}^0A}\,{}^0A = {}^0A$$

$(a\underline{i} + b\underline{j} + c\underline{k}) \times \underline{i} = -b\underline{k} + c\underline{j}$

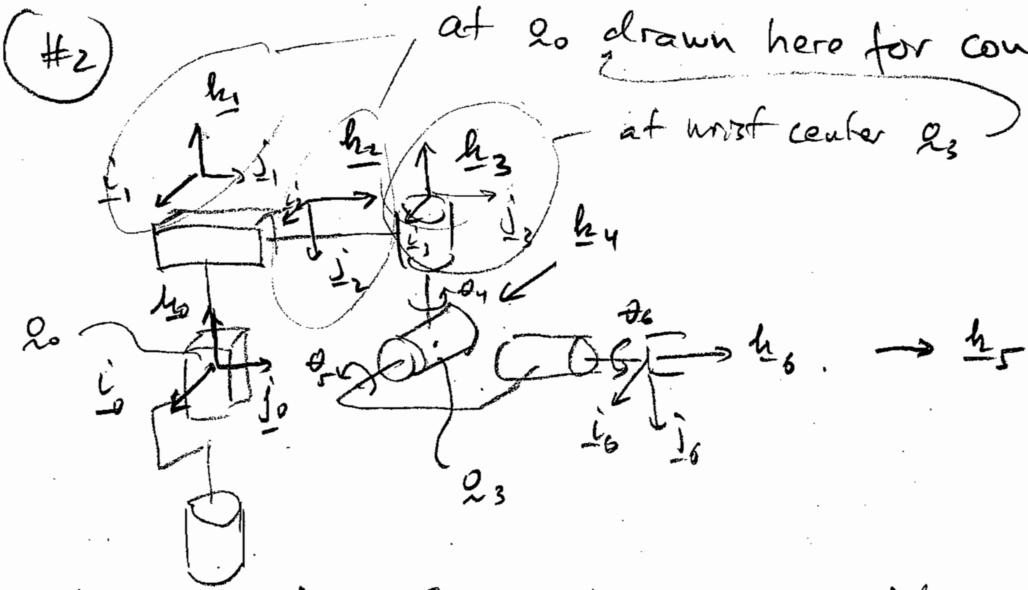
#2) $\exp\Big(\theta \underbrace{\begin{bmatrix} 0 & -c_s & b_s \\ c_s & 0 & -a_s \\ -b_s & a_s & 0 \end{bmatrix}}_{(s\times)}\Big) = Q$ $\qquad s = \begin{bmatrix} a_s \\ b_s \\ c_s \end{bmatrix}$

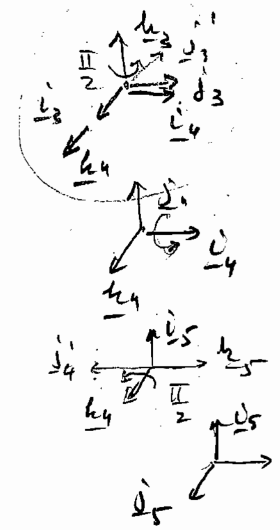
$\therefore$ $\theta c_s = 3\pi \qquad \theta a_s = -4\pi$

$$\theta \begin{bmatrix} a_s \\ b_s \\ c_s \end{bmatrix} = \begin{bmatrix} -4\pi \\ 0 \\ 3\pi \end{bmatrix} \qquad \|\theta s\| = \sqrt{4^2 + 3^2}\,\pi = 5\pi \qquad s = \begin{bmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{bmatrix}$$

$$\theta = 5\pi \cong \pi$$

#2





	θ	d	a	α
θ_1	(θ_1)	0	0	0
d_2	0	(d_2)	0	$-\frac{\pi}{2}$
d_3	0	$(d_3)+l_3$	0	$\frac{\pi}{2}$
θ_4	$\frac{\pi}{2}+(\theta_4)$	0	0	$\frac{\pi}{2}$
θ_5	$\frac{\pi}{2}+(\theta_5)$	0	0	$\frac{\pi}{2}$
θ_6	$\frac{\pi}{2}+(\theta_6)$	0	0	0

$$^0T_1 = \begin{bmatrix} e^{\theta_1 k\times} & 0 \\ 0^T & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} e^{-\frac{\pi}{2} i\times} & dk \\ 0^T & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} e^{\frac{\pi}{2} i\times} & (d_3+l_3)k \\ 0^T & 1 \end{bmatrix}$$

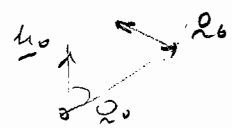
$$^3T_4 = \begin{bmatrix} e^{(\frac{\pi}{2}+\theta_4)k\times} & 0 \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} e^{\frac{\pi}{2} i\times} & 0 \\ 0^T & 1 \end{bmatrix}$$

$$^4T_5 = \begin{bmatrix} e^{(\frac{\pi}{2}+\theta_5)k\times} & 0 \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} e^{\frac{\pi}{2} i\times} & 0 \\ 0^T & 1 \end{bmatrix}$$

$$^5T_6 = \begin{bmatrix} e^{(\frac{\pi}{2}+\theta_6)k\times} & 0 \\ 0^T & 1 \end{bmatrix}$$

$$^0T_6 = {}^0T_1\,{}^1T_2 \cdots {}^5T_6$$

#3



The Jacobian is

$$\begin{bmatrix} h_0\times(\rho_6-\rho_0) & h_1 & h_2 & h_3\times(\rho_6-\rho_3) & h_4\times(\rho_6-\rho_3) & h_5\times(\rho_6-\rho_3) \\ h_0 & 0 & 0 & h_3 & h_4 & h_5 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} h_0\times(\rho_3-\rho_0) & h_1 & h_2 & 0 & 0 & 0 \\ h_0 & 0 & 0 & h_3 & h_4 & h_5 \end{bmatrix}$$

wrist singularities when $h_3 \parallel h_5$.

$h_1 \perp h_2$ always $h_0\times(\rho_3-\rho_0)$ = always orthogonal to h_1, h_2, unless $\boxed{\rho_3=\rho_0}$, ρ_3 on the axis of h_0.

Coords of the angular velocity ...

$$\underline{\omega}_{6,0} = \dot{\theta}_1 \underline{k}_0 + \dot{\theta}_4 \underline{k}_3 + \dot{\theta}_5 \underline{k}_4 + \dot{\theta}_6 \underline{k}_5$$

$$\underline{C}_3 = \underline{C}_0 \; e^{\theta_1 k\times} e^{-\frac{\pi}{2} i\times} e^{\frac{\pi}{2} i\times} = \underline{C}_0 \underbrace{e^{\theta_1 k\times}}_{{}^0C_3}$$

$$\underline{C}_4 = \underline{C}_3 \; e^{(\frac{\pi}{2}+\theta_4) k\times} e^{\frac{\pi}{2} i\times}$$

$$= \underline{C}_0 \underbrace{e^{\theta_1 k\times} e^{(\frac{\pi}{2}+\theta_4) k\times} e^{\frac{\pi}{2} i\times}}_{{}^0C_4}$$

$$\underline{C}_5 = \underline{C}_0 \underbrace{e^{\theta_1 k\times} e^{(\frac{\pi}{2}+\theta_4) k\times} e^{\frac{\pi}{2} i\times} e^{(\frac{\pi}{2}+\theta_5) k\times} e^{\frac{\pi}{2} i\times}}_{{}^0C_5}$$

$$\underline{C}_6 = \underline{C}_0 \underbrace{{}^0C_5 \, e^{(\frac{\pi}{2}+\theta_6) k\times}}_{{}^0C_6}$$

$${}^0\omega_{6,0} = \dot{\theta}_1 k + \dot{\theta}_4 \, {}^0C_3 k + \dot{\theta}_5 \, {}^0C_4 k + \dot{\theta}_6 \, {}^0C_5 k.$$

$${}^1\omega_{6,0} = {}^1C_0 \, {}^0\omega_{6,0} = {}^0C_1^T \, {}^0\omega_{6,0} = e^{-\theta_1 k\times} \, {}^0\omega_{6,0}$$

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Problem 1.

(i) (2 marks)
A vector product function is specified as $\underline{\underline{f}}(\underline{x}) = (2\underline{i}_0 + 1\underline{j}_0 - 3\underline{k}_0) \times \underline{x}$. What is the matrix representation of $\underline{\underline{f}}$ in $\underline{C}_0 = [\underline{i}_0 \ \ \underline{j}_0 \ \ \underline{k}_0]$?

(ii) (1 mark)
The frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by θ. What is the matrix representation ${}^0C_1$ of this rotation in $\underline{C}_0$?

(iv) (1 mark)
If a vector $\underline{x}$ has coordinates ${}^0x$ in $\underline{C}_0$, what are its coordinates ${}^1x$ in $\underline{C}_1$?

(iii) (3 marks)
(iii) What is the matrix representation of the function $\underline{\underline{f}}$ from (i) above in $\underline{C}_1$ from (ii)? (you do not need to multiply out the matrices).

(v) (5 marks)
Write the homogeneous transformation ${}^{i-1}T_i$ relating the coordinate systems $\{\underset{\sim}{o}_{i-1}, \underline{C}_{i-1}\}$ and $\{\underset{\sim}{o}_i, \underline{C}_i\}$ attached to link $i-1$ and i, respectively, given that the Denavit-Hartenberg parmeters of link i are θ, d, a and α.

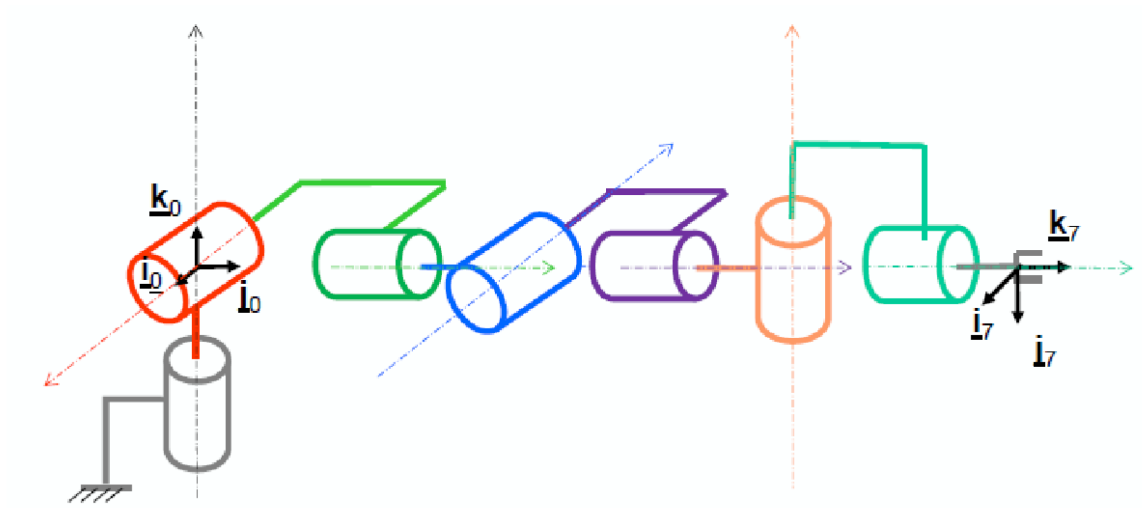
(vi)(5 marks)
Clearly explain the steps required to find the axis and angle of rotation given a rotation matrix Q.

(vii)(3 marks)
What are the axis and angle of rotation of the following rotation matrix:

$$Q = \exp(\begin{bmatrix} 0 & -\pi/\sqrt{2} & 0 \\ \pi/\sqrt{2} & 0 & \pi/\sqrt{2} \\ 0 & -\pi/\sqrt{2} & 0 \end{bmatrix})$$

Problem 2. *(15 marks)*

Consider the manipulator below. Assign joint variables in a manner consistent with the axes shown (positive angle by right hand rule), and coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, to all links, using the Denavit-Hartenberg convention. Complete the table of Denavit Hartenberg parameters. Write, as a function of ${}^{i-1}T_i$, $i = 0, \ldots, 6$, the coordinate transformation relating the coordinates ${}^{7}x$ in the gripper frame with the coordinates ${}^{0}x$ in the base frame. You DO NOT need to fill in the details for every homogeneous matrix as they will all look as in Problem 1. (v).



NAME:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2010-2011): Introduction to Robotics
Midterm Examination#1, February 10, 2011
Closed Book - 80 Minutes
Maximum - 40 marks

Problem 1. *(5 marks)*
(i) A vector product function is specified as $\underline{\underline{f}}(\underline{x}) = (2\underline{i}_0 - 3\underline{k}_0) \times \underline{x}$. What is the matrix representation of $\underline{\underline{f}}$ in $\underline{C}_0 = [\underline{i}_0 \;\; \underline{j}_0 \;\; \underline{k}_0]$?
(ii) The frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by $\pi/2$. What is the matrix representation ${}^0C_1$ of this rotation in $\underline{C}_0$?
(iii) What is the matrix representation of the function $\underline{\underline{f}}$ from (i) above in $\underline{C}_1$?
(iv) If the frame $\underline{C}_1(t) = \underline{C}_0\, {}^0C_1(t)$, how do you find the angular velocity $\underline{\omega}_{1,0}$ of $\underline{C}_1$ with respect to $\underline{C}_0$? What are the coordinates of the angular velocity $\underline{\omega}_{1,0}$ in $\underline{C}_1$?
(v) Outline a method to find the rotation axis and rotation angle from a rotation matrix R.

Problem 2.
You are given three coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$ with right-handed orthonormal frames:
- $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{i}_0$ by an angle $-\pi/3$, then by rotating about $\underline{k}_0$ by an angle θ_1. $\underset{\sim}{o}_1$ is obtained from $\underset{\sim}{o}_0$ by displacing $\underset{\sim}{o}_0$ by $(-1)\underline{j}_0 + \underline{k}_0$
- $\underline{C}_2$ is obtained from $\underline{C}_1$ by rotating $\underline{C}_1$ about $\underline{k}_1$ by an angle θ_2. $\underset{\sim}{o}_2$ is obtained from $\underset{\sim}{o}_1$ by displacing $\underset{\sim}{o}_0$ by $2\underline{i}_0$.

(a)(5 marks)
(i) Find the homogeneous transformation ${}^0T_1$ that relates the coordinates ${}^1x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$.
(ii) Find the homogeneous transformation ${}^0T_2$ that relates the coordinates ${}^2x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_2, \underline{C}_2\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$.
(iii) What is the inverse of ${}^0T_2$ from *(i)* above?
(iv) What is the coordinate representation of the rotation $e^{\theta_2 \underline{k}_1 \times}$ in frame $\underline{C}_0$.

(b) (5 marks)
Suppose θ_1 and θ_2 in *(a)* are functions of time $\theta_1(t)$ and $\theta_2(t)$.
(i) What are the coordinates ${}^0\omega_{1,0}$, in $\underline{C}_0$, of the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?
(ii) What are the coordinates ${}^0\omega_{2,0}$, in $\underline{C}_0$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_0$?
(iii) What are the coordinates ${}^1\omega_{2,0}$, in $\underline{C}_1$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_0$?

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Problem 1. *(5 marks)*
(i) A vector product function is specified as $\underline{\underline{f}}(\underline{x}) = (2\underline{i}_0 - 3\underline{k}_0) \times \underline{x}$. What is the matrix representation of $\underline{\underline{f}}$ in $\underline{C}_0 = [\underline{i}_0 \ \underline{j}_0 \ \underline{k}_0]$?

$$\underline{\underline{f}}(\underline{i}_0) = (2\underline{i}_0 - 3\underline{k}_0) \times \underline{i}_0 = -3\underline{j}_0 \tag{1}$$
$$\underline{\underline{f}}(\underline{j}_0) = (2\underline{i}_0 - 3\underline{k}_0) \times \underline{j}_0 = 2\underline{k}_0 + 3\underline{i}_0 \tag{2}$$
$$\underline{\underline{f}}(\underline{k}_0) = (2\underline{i}_0 - 3\underline{k}_0) \times \underline{k}_0 = -2\underline{j}_0 \tag{3}$$

therefore the matrix representation of $\underline{\underline{f}}(\underline{x}) = (2\underline{i}_0 - 3\underline{k}_0) \times \underline{x}$ is

$$^0A = \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix} \tag{4}$$

(ii) The frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by $\pi/2$. What is the matrix representation $^0C_1$ of this rotation in $\underline{C}_0$?

$$^0C_1 = e^{\frac{\pi}{2} j\times} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{5}$$

(iii) What is the matrix representation of the function $\underline{\underline{f}}$ from (i) above in $\underline{C}_1$?
From commutative diagram:

$$^1A = {}^0C_1^T \, {}^0A \, {}^0C_1 \tag{6}$$

(iv) If the frame $\underline{C}_1(t) = \underline{C}_0 \, {}^0C_1(t)$, how do you find the angular velocity $\underline{\omega}_{1,0}$ of $\underline{C}_1$ with respect to $\underline{C}_0$? What are the coordinates of the angular velocity $\underline{\omega}_{1,0}$ in $\underline{C}_1$?

$$^0\omega_{1,0}\times = {}^0\dot{C}_1 \, {}^0C_1^T \tag{7}$$
$$\underline{\omega}_{1,0} = \underline{C}_0 \, {}^0\omega_{1,0} \tag{8}$$
$$^1\omega_{1,0} = {}^1C_0 \, {}^0\omega_{1,0} = {}^0C_1^T \, {}^0\omega_{1,0} \tag{9}$$

(v) Outline a method to find the rotation axis and rotation angle from a rotation matrix R.
Axis = eigenvector e corresponding to eigenvalues 1
For angle, find a vector u orthogonal to the axis, let $v = e \times u$. Find Ru. The projection of Ru onto u is the cosine of the angle, the projection of Ru onto v the sine.

Problem 2.
You are given three coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$ with right-handed orthonormal frames:
- $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{i}_0$ by an angle $-\pi/3$, then by rotating about $\underline{k}_0$ by an angle θ_1.
$\underset{\sim}{o}_1$ is obtained from $\underset{\sim}{o}_0$ by displacing $\underset{\sim}{o}_0$ by $(-1)\underline{j}_0 + \underline{k}_0$
- $\underline{C}_2$ is obtained from $\underline{C}_1$ by rotating $\underline{C}_1$ about $\underline{k}_1$ by an angle θ_2. $\underset{\sim}{o}_2$ is obtained from $\underset{\sim}{o}_1$ by displacing $\underset{\sim}{o}_1$ by $2\underline{i}_0$.
(a)(5 marks)
(i) Find the homogeneous transformation ${}^0T_1$ that relates the coordinates ${}^1x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$.

$$
{}^0T_1 = \begin{bmatrix} e^{\theta_1 k\times} e^{-\frac{\pi}{3} i\times} & \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \\ 0^T & 1 \end{bmatrix} \tag{10}
$$

(ii) Find the homogeneous transformation ${}^0T_2$ that relates the coordinates ${}^2x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_2, \underline{C}_2\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$.

$$
{}^1T_2 = \begin{bmatrix} e^{\theta_2 k\times} & \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \\ 0^T & 1 \end{bmatrix} \tag{11}
$$

$$
{}^0T_2 = {}^0T_1\, {}^1T_2 \tag{12}
$$

$$
\tag{13}
$$

$$
{}^0T_2 = \begin{bmatrix} e^{\theta_1 k\times} e^{-\frac{\pi}{3} i\times} e^{\theta_2 k\times} & e^{\theta_1 k\times} e^{-\frac{\pi}{3} i\times} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} Q & d \\ 0^T & 1 \end{bmatrix} \tag{14}
$$

(iii) What is the inverse of ${}^0T_2$ from *(i)* above?

$$
{}^0T_2^{-1} = \begin{bmatrix} Q^{-1} & -Q^{-1}d \\ 0^T & 1 \end{bmatrix} \tag{15}
$$

(iv) What is the coordinate representation of the rotation $e^{\theta_2 \underline{k}_1 \times}$ in frame $\underline{C}_0$.
${}^0C_1 e^{\theta_2 k\times}\, {}^0C_1^T$.

(b) (5 marks)

Suppose θ_1 and θ_2 in *(a)* are functions of time $\theta_1(t)$ and $\theta_2(t)$.

(i) What are the coordinates ${}^0\omega_{1,0}$, in $\underline{C}_0$, of the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?
${}^0\omega_{1,0} = \dot{\theta}_1 k$

(ii) What are the coordinates ${}^0\omega_{2,0}$, in $\underline{C}_0$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_0$?

$$
\begin{aligned}
{}^1\omega_{2,1} &= \dot{\theta}_2 k && (16)\\
{}^0\omega_{2,0} &= {}^0\omega_{1,0} + {}^0C_1 \dot{\theta}_2 k = {}^0\omega_{1,0} + e^{\theta_1 k\times} e^{-\frac{\pi}{3} i\times} \dot{\theta}_2 k && (17)
\end{aligned}
$$

(iii) What are the coordinates ${}^1\omega_{2,0}$, in $\underline{C}_1$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_0$?
${}^1\omega_{2,0} = {}^1C_0\, {}^0\omega_{1,0} = e^{\frac{\pi}{3} i\times} e^{-\theta_1 k\times}\, {}^0\omega_{1,0}$

Problem 3.

Consider the manipulator shown below.

(20 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, \ldots 6$ to links 1 through 6 according to the Denavit-Hartenberg convention and find the homogeneous transformations required to solve the direct kinematics problem.

DH parameters:

DH	θ	**d**	**a**	α
Link 1	0	d_1	0	0
Link 2	θ_2	0	0	$-\frac{\pi}{2}$
Link 3	0	$l_3 + d_3$	0	$\frac{\pi}{2}$
Link 4	$\theta_4 + \frac{\pi}{2}$	0	0	$\frac{\pi}{2}$
Link 5	$\theta_5 - \frac{\pi}{2}$	0	0	$-\frac{\pi}{2}$
Link 6	$\theta_6 - \frac{\pi}{2}$	l_6	0	0

$$
{}^0T_1 = \begin{bmatrix} 0 & d_1 \underline{k} \\ 0^T & 1 \end{bmatrix} \tag{18}
$$

$$
{}^1T_2 = \begin{bmatrix} e^{\theta_2 k\times} e^{-\frac{\pi}{2} i\times} & 0 \\ 0^T & 1 \end{bmatrix} \tag{19}
$$

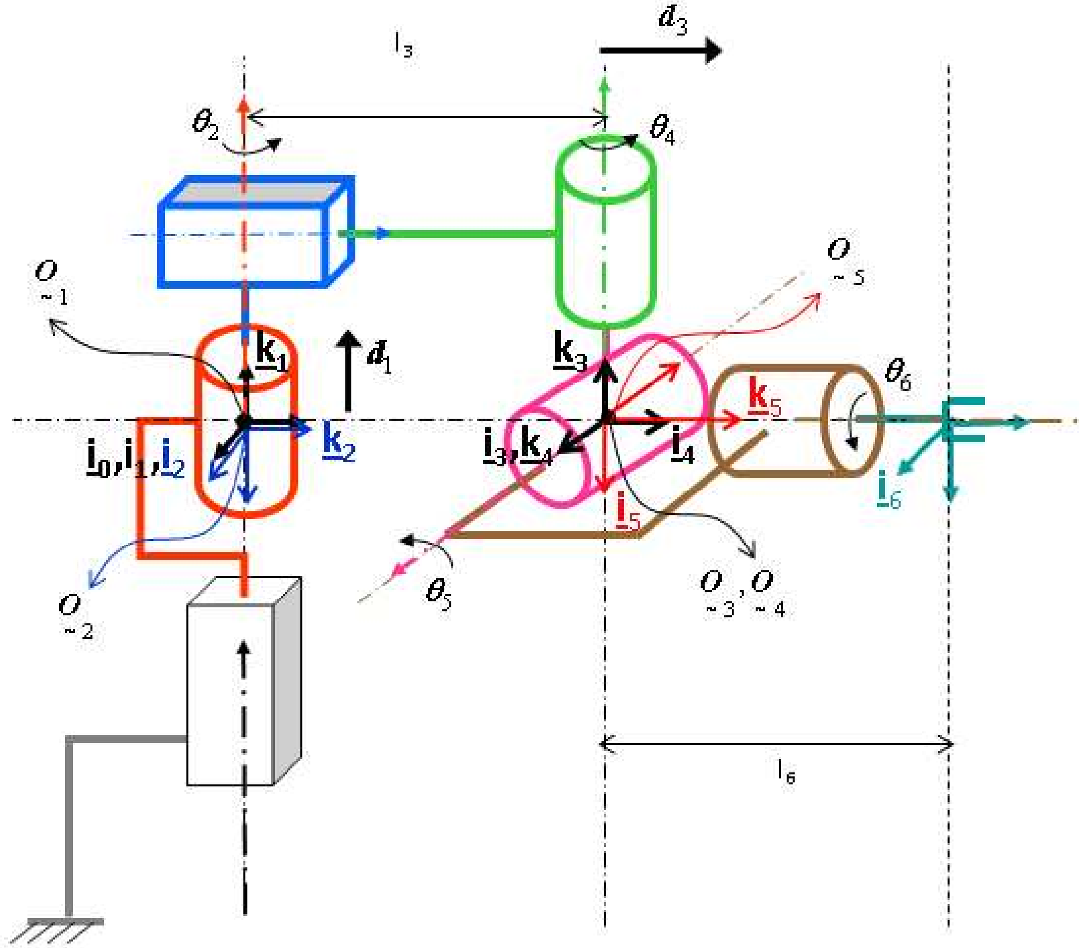
$$
{}^2T_3 = \begin{bmatrix} e^{\frac{\pi}{2} i\times} & (l_3 + d_3)\underline{k} \\ 0^T & 1 \end{bmatrix} \tag{20}
$$

$$
{}^3T_4 = \begin{bmatrix} e^{(\theta_4 + \frac{\pi}{2}) k\times} e^{\frac{\pi}{2} i\times} & 0 \\ 0^T & 1 \end{bmatrix} \tag{21}
$$

$$
{}^{4}T_{5} = \begin{bmatrix} e^{(\theta_5 - \frac{\pi}{2})k\times} e^{-\frac{\pi}{2} i\times} & 0 \\ 0^T & 1 \end{bmatrix} \tag{22}
$$

$$
{}^{5}T_{6} = \begin{bmatrix} e^{(\theta_6 - \frac{\pi}{2})k\times} & l_6 \underline{k} \\ 0^T & 1 \end{bmatrix} \tag{23}
$$

$$
{}^{0}T_{6}(d_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6) = {}^{0}T_{1}\, {}^{1}T_{2}\, {}^{2}T_{3}\, {}^{3}T_{4}\, {}^{4}T_{5}\, {}^{5}T_{6} \tag{24}
$$



Salcudean

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2010): Introduction to Robotics
Midterm Examination#1, February 11
Closed Book - 80 Minutes
Maximum - 30 marks

Problem 1.

You are given three coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$ with right-handed orthonormal frames.
$\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{i}_0$ by an angle $\pi/4$, then by rotating about $\underline{k}_0$ by an angle θ_1.
$\underline{C}_2$ is obtained from $\underline{C}_1$ by rotating $\underline{C}_1$ about $\underline{k}_1$ by an angle θ_2.
$\underset{\sim}{o}_1$ is obtained from $\underset{\sim}{o}_0$ by displacing $\underset{\sim}{o}_0$ by $10\underline{j}_0$, and $\underset{\sim}{o}_2$ and $\underset{\sim}{o}_1$ coincide.

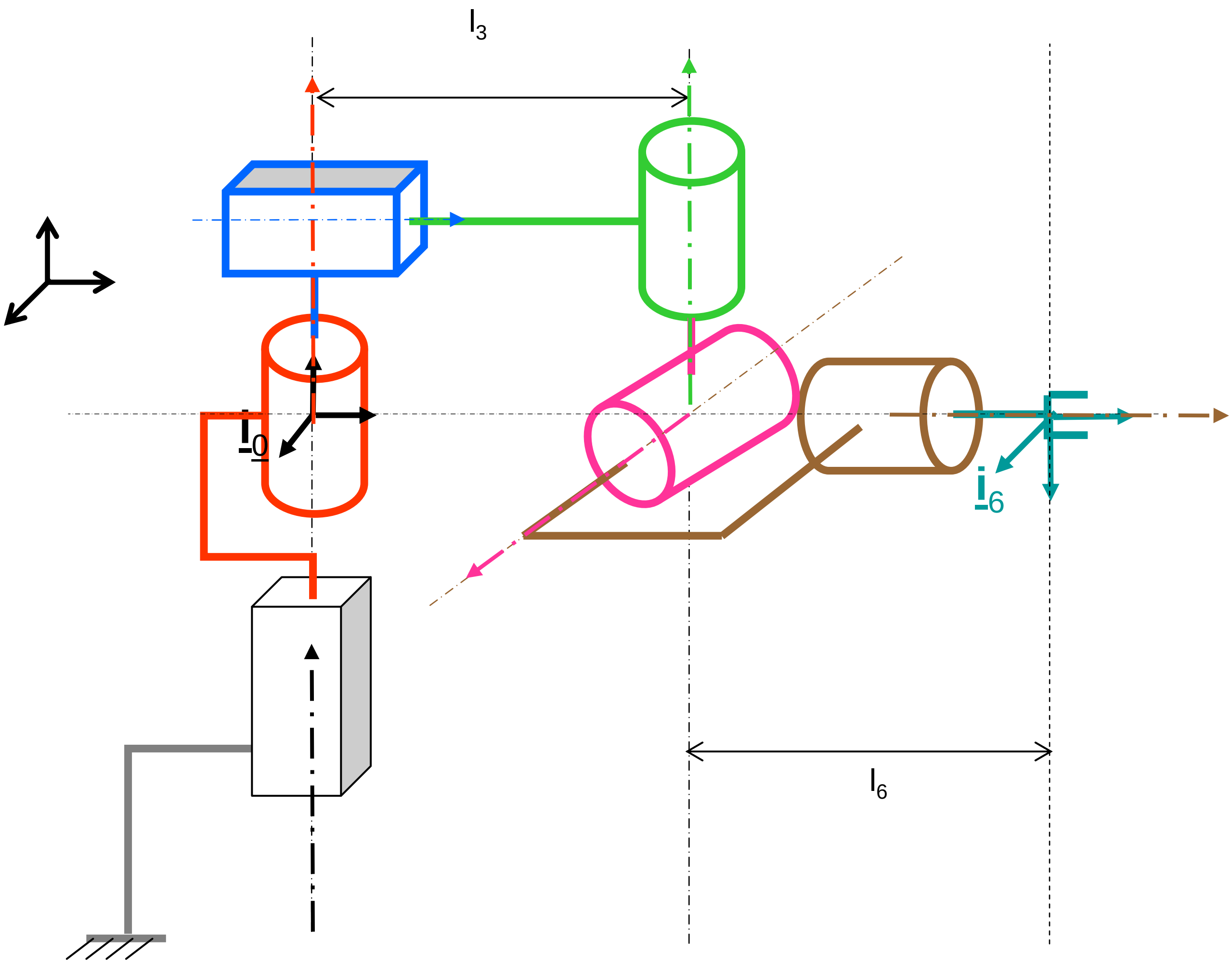
(a)(8 marks)
(i) Find the homogeneous transformation ${}^0T_1$ that relates the coordinates ${}^1x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$.
(ii) Find the homogeneous transformation ${}^0T_2$ that relates the coordinates ${}^2x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_2, \underline{C}_2\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$.
(iii) What is the inverse of ${}^0T_1$ from *(i)* above?
(iv) What is the coordinate representation of the rotation $e^{\theta_2 \underline{k}_1 \times}$ in frame $\underline{C}_0$.

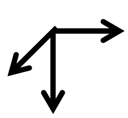
(b) (6 marks)
Suppose θ_1 and θ_2 in *(a)* are functions of time $\theta_1(t)$ and $\theta_2(t)$.
(i) What are the coordinates ${}^0\omega_{1,0}$, in frame $\underline{C}_0$, of the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?
(ii) What are the coordinates ${}^1\omega_{2,1}$, in frame $\underline{C}_1$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_1$?
(iii) What are the coordinates ${}^0\omega_{2,0}$, in frame $\underline{C}_0$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_0$?

Problem 2.

Consider the manipulator shown on the next page.
(16 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, ...6$ to links 1 through 6 according to the Denavit-Hartenberg convention and find the homogeneous transformations required to solve the direct kinematics problem.

Problem 3.
Consider the manipulator shown below.
(20 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, \ldots 6$ to links 1 through 6 according to the Denavit-Hartenberg convention and find the homogeneous transformations required to solve the direct kinematics problem.

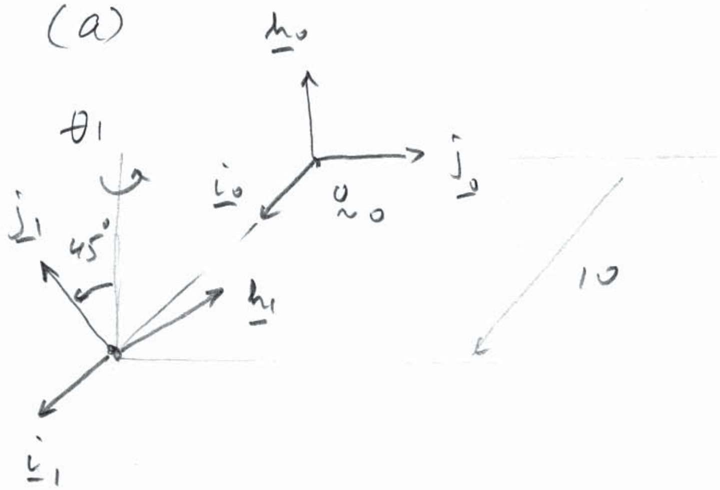




Right-to-left, rotations specified in base frame.

Problem 1

(a)



$$\underline{C}_1 = \underline{C}_0\, e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times} = \underline{C}_0\, {}^0C_1$$

$$\underline{C}_2 = \underline{C}_1\, e^{\theta_2 k\times}$$

$$\underset{\sim}{o}_2 = \underset{\sim}{o}_1 + \underline{C}_1 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0 \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

(i) $${}^0T_1 = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times}} & \begin{matrix} 0 \\ 10 \\ 0 \end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

(ii) $${}^1T_2 = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{e^{\theta_2 k\times}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$${}^0T_2 = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times}} & \begin{matrix} 0 \\ 10 \\ 0 \end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{e^{\theta_2 k\times}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times} e^{\theta_2 k\times}} & \begin{matrix} 0 \\ 10 \\ 0 \end{matrix} \\ \hline 0 & 0 & 1 & 1 \end{array}\right]$$

(iii) $${}^0x = e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times}\, {}^1x + \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$e^{-\frac{\pi}{4} i\times} e^{-\theta_1 k\times}\, {}^0x = {}^1x + e^{-\frac{\pi}{4} i\times} e^{-\theta_1 k\times} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$ {}^{1}x = e^{-\frac{\pi}{4} i x} e^{-\theta_1 h x}\, {}^{0}x - e^{-\frac{\pi}{4} i x} e^{-\theta_1 h x} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} $$

$$ {}^{1}T_{0} = \left[\begin{array}{c|c} e^{-\frac{\pi}{4} i x} e^{-\theta_1 h x} & -e^{-\frac{\pi}{4} i x} e^{-\theta_1 h x} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} \\ \hline 0 & 1 \end{array}\right] $$

(iv)

$$ \begin{array}{ccc} \overset{{}^{1}x}{{}^{1}\{\text{coords}\}} & \xrightarrow{e^{\theta_2 h x}} & \overset{{}^{1}y}{{}^{1}\{\text{coords}\}} \\ {\scriptstyle {}^{1}C_0}\uparrow & & \uparrow{\scriptstyle {}^{1}C_0} \\ \underset{{}^{0}x}{{}^{0}\{\text{coords}\}} & \xrightarrow{Q} & \underset{{}^{0}y}{{}^{0}\{\text{coords}\}} \end{array} $$

$$ Q = {}^{1}C_0^{T}\, e^{\theta_2 h x}\, {}^{1}C_0 = $$

$$ = {}^{0}C_1\, e^{\theta_2 h x}\, {}^{0}C_1^{T} = $$

$$ = e^{\theta_1 h x} e^{\frac{\pi}{4} i x} e^{\theta_2 h x} e^{-\frac{\pi}{4} i x} e^{-\theta_1 h x}. $$

$$ \Big[\ Q = e^{\theta_2 ({}^{0}C_1 h) x} = e^{\theta_2\, {}^{0}C_1 (h x)\, {}^{0}C_1^{T}} = $$

$$ = {}^{0}C_1\, e^{\theta_2 (h x)}\, {}^{1}C_0^{T} \ \Big] $$

b)

(i) ${}^0\omega_{1,0}\times = {}^0\dot{C}_1\ {}^1C_0^T = \dot{\theta}_1 k\times \underbrace{e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times} \left(e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times}\right)^T}_{I}$

$\boxed{{}^0\omega_{1,0} = \dot{\theta}_1 k}$

(ii) ${}^1\omega_{2,1} = \dot{\theta}_2 k$

(iii) ${}^0\omega_{2,0} = {}^0\omega_{1,0} + {}^0C_1\ {}^1\omega_{2,1} = \dot{\theta}_1 k + e^{\theta_1 k\times} e^{\frac{\pi}{4} i\times}\ \dot{\theta}_2 k$

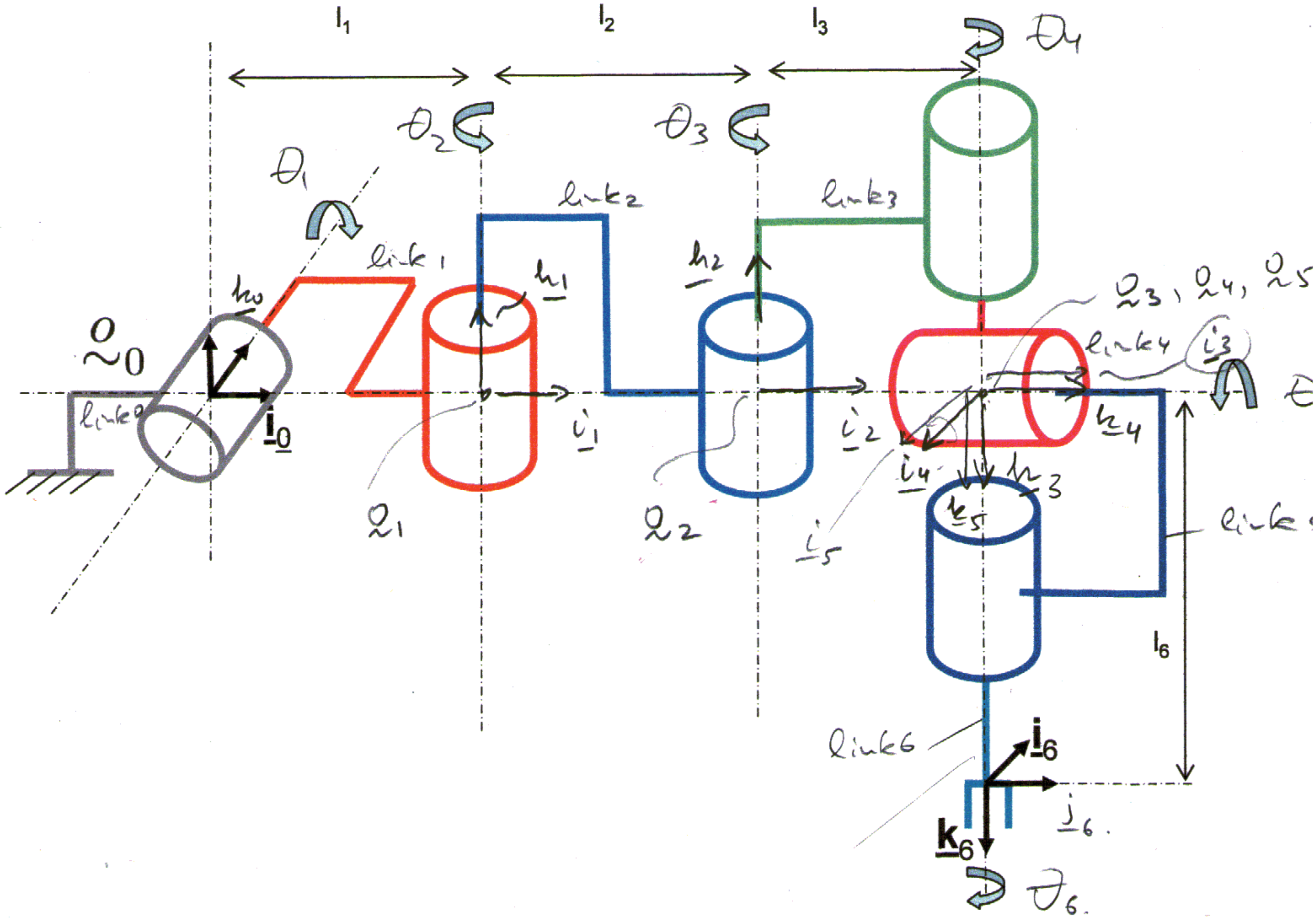
Problem 2.

Link	θ	d	a	α
1	(θ_1)	0	l_1	$\frac{\pi}{2}$
2	(θ_2)	0	l_2	0
3	(θ_3)	0	l_3	π
4	$(\theta_4 + \frac{\pi}{2})$	0	0	$\frac{\pi}{2}$
5	(θ_5)	0	0	$-\frac{\pi}{2}$
6	$(\theta_6 + \pi)$	l_6	0	0

$${}^0T_1 = \begin{bmatrix} e^{\theta_1 k\times} & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} e^{\frac{\pi}{2} i\times} & l_1 i \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{\theta_1 k\times} e^{\frac{\pi}{2} i\times} & e^{\theta_1 k\times} l_1 i \\ 0^T & 1 \end{bmatrix}$$

etc



l1
l2
l3
θ1
θ2
θ3
θ4
θ6.
link0
link1
link2
link3
link4
link6
o0
o1
o2
o3, o4, o5
i0
i1
i2
i3
i4
i5
i6
j6.
k6
k1
k2
k3
k4
k5
l6

NAME:

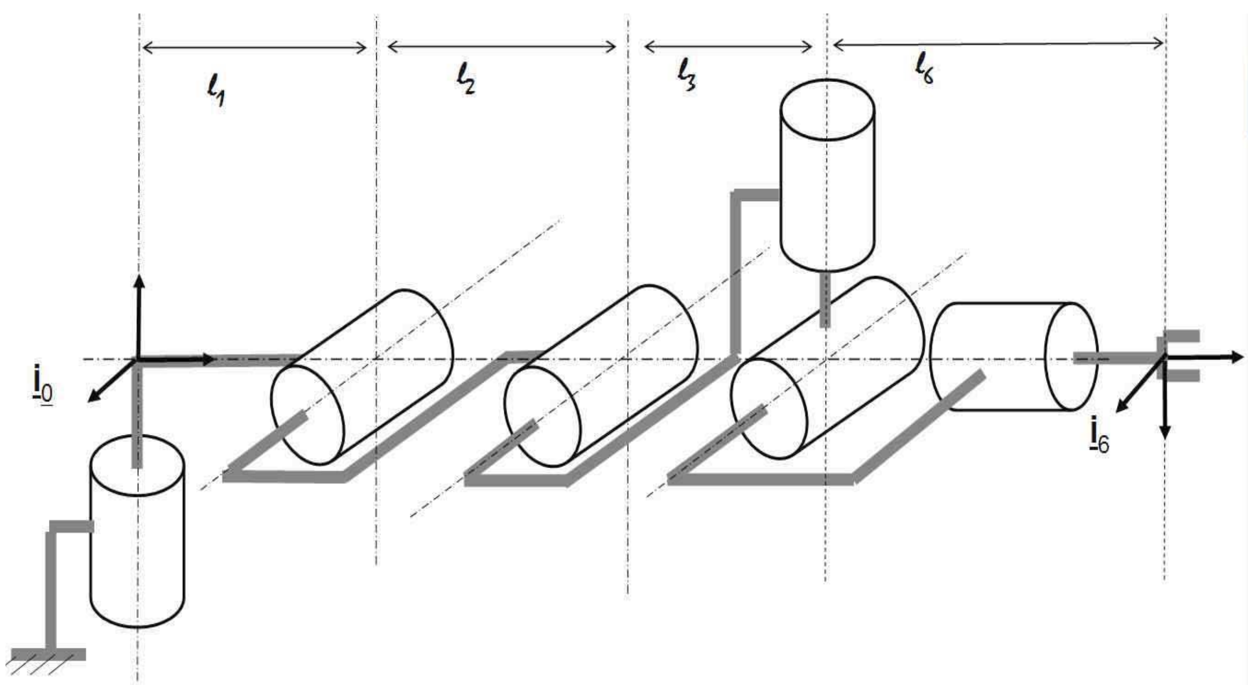
University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2009): Introduction to Robotics
Midterm Examination #1, February 12, 2008
Closed Book - 60 Minutes
Maximum - 30 marks

Consider the manipulator shown below.

(a) (15 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, ..., 6$ to links 1 through 6 according to the Denavit-Hartenberg convention. Write down the table of Denavit-Hartenberg parameters for the robot. Find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Assuming you have obtained the homogeneous transformation ${}^0T_6 = \begin{bmatrix} {}^0C_6 & {}^0d_6 \\ 0^T & 1 \end{bmatrix}$, what is its inverse ${}^6T_0$ in terms of ${}^0C_6$ and ${}^0d_6$?

(b) (8 marks) Write down the manipulator Jacobian in terms of the joint axes and use elementary row operations to decouple the wrist and arm singularities. Under what condition will the manipulator be in a singular configuration?

(c) (7 marks) Assuming that the first three joint angles are fixed at their nominal position ($\theta_1 = \theta_2 = \theta_3 \equiv 0$), find the coordinates of the angular velocity vector $\underline{\omega}_{6,0}$ of the gripper frame $\underline{C}_6$ with respect to $\underline{C}_0$, with respect to frame $\underline{C}_0$, as a function of $\theta_4, \theta_5, \theta_6$.

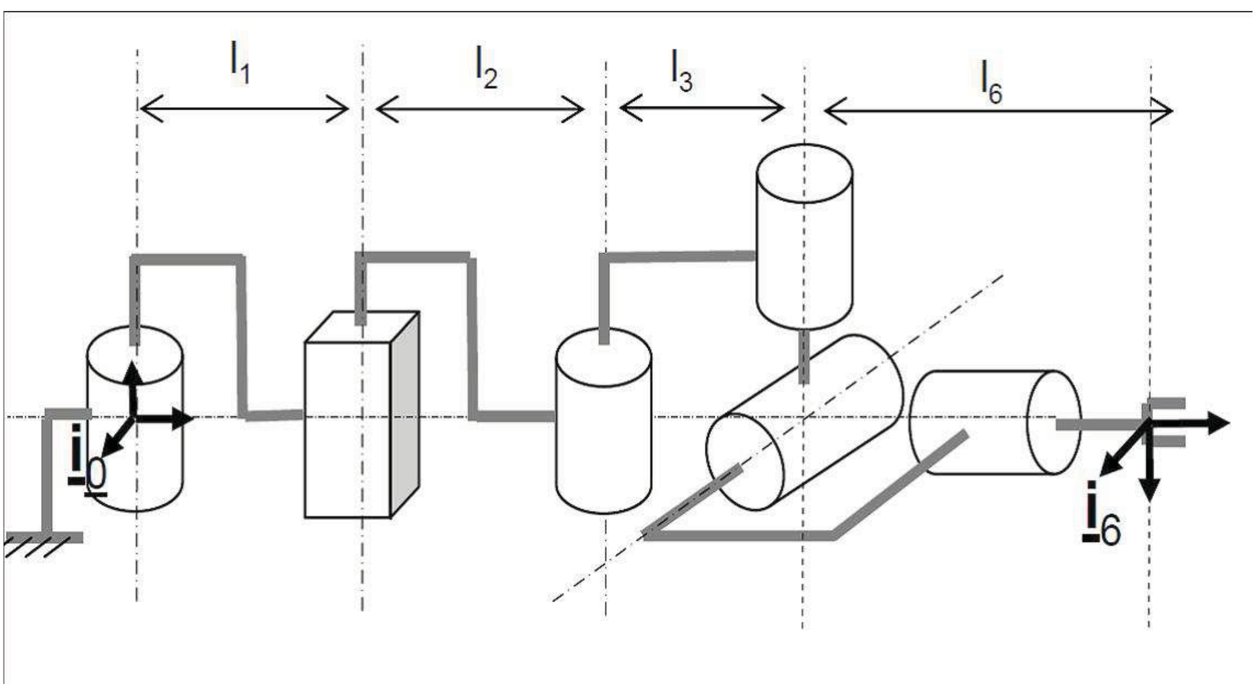


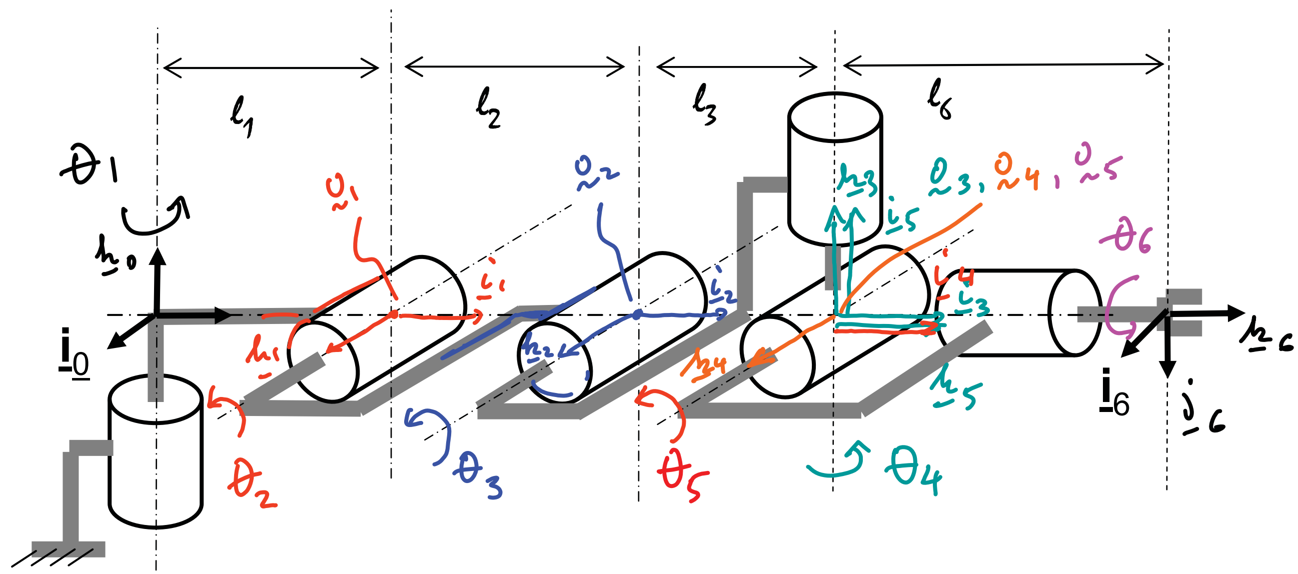
University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2009): Introduction to Robotics
Midterm Examination #1 - Make-up Exam, March6^{th}, 2009
Closed Book - 80 Minutes
Maximum - 30 marks

Consider the manipulator shown below.
(a) (15 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, ..., 6$ to links 1 through 6 according to the Denavit-Hartenberg convention. Write down the table of Denavit-Hartenberg parameters for the robot. Find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Assuming you have obtained the homogeneous transformation ${}^0T_6 = \begin{bmatrix} {}^0C_6 & {}^0d_6 \\ 0^T & 1 \end{bmatrix}$, what is its inverse ${}^6T_0$ in terms of ${}^0C_6$ and ${}^0d_6$?

(b) (8 marks) Write down the manipulator Jacobian in terms of the joint axes and use elementary row operations to decouple the wrist and arm singularities. Under what condition will the manipulator be in a singular configuration?

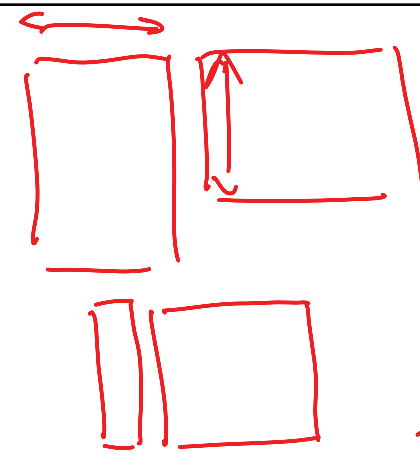
(c) (7 marks) Assuming that the first three joint angles are fixed at their nominal position ($\theta_1 = \theta_2 = \theta_3 \equiv 0$), find the coordinates of the angular velocity vector $\underline{\omega}_{6,0}$ of the gripper frame $\underline{C}_6$ with respect to $\underline{C}_0$, with respect to frame $\underline{C}_0$, as a function of $\theta_4, \theta_5, \theta_6$.





	Angle	Offset	Length	Twist
link 1	$(\theta_1 + 90^\circ)$	0	l_1	90°
2	(θ_2)	0	l_2	0°
3	(θ_3)	0	l_3	-90°
4	(θ_4)	0	0	90°
5	$(\theta_5 + 90^\circ)$	0	0	90°
6	$(\theta_6 + 90^\circ)$	l_6	0	0°

	Angle	Offset	Length	Twist
link 1	(θ_1+90°)	0	l_1	90°
2	(θ_2)	0	l_2	0°
3	(θ_3)	0	l_3	-90°
4	(θ_4)	0	0	90°
5	(θ_5+90°)	0	0	90°
6	(θ_6+90°)	l_6	0	0°



$AB \neq BA$

$\cancel{AB = BA}$

$$
{}^0T_1 = \underbrace{\left[\begin{array}{c|c} e^{(\theta_1+\frac{\pi}{2})\hat{k}\times} & 0 \\ \hline 0^T & 1 \end{array}\right]}_{\text{angle}}
\underbrace{\left[\begin{array}{c|c} I & 0 \\ \hline 0^T & 1 \end{array}\right]}_{\text{offset}}
\underbrace{\left[\begin{array}{c|c} I & l_1 i \\ \hline 0^T & 1 \end{array}\right]}_{\text{length}}
\underbrace{\left[\begin{array}{c|c} e^{\frac{\pi}{2} i\times} & 0 \\ \hline 0^T & 1 \end{array}\right]}_{\text{twist}}
$$

$$
= \left[\begin{array}{c|c} e^{(\theta_1+\frac{\pi}{2})\hat{k}\times} & 0 \\ \hline 0^T & 1 \end{array}\right]
\left[\begin{array}{c|c} e^{\frac{\pi}{2} i\times} & l_1 i \\ \hline 0^T & 1 \end{array}\right]
$$

$$
= \left[\begin{array}{c|c} e^{(\theta_1+\frac{\pi}{2})\hat{k}\times} e^{\frac{\pi}{2} i\times} & e^{(\theta_1+\frac{\pi}{2})\hat{k}\times} l_1 i \\ \hline 0^T & 1 \end{array}\right]
$$

	Angle	Offset	Length	Twist
link 1	$(\theta_1 + 90^\circ)$	0	l_1	90°
2	(θ_2)	0	l_2	0°
3	(θ_3)	0	l_3	-90°
4	(θ_4)	0	0	90°
5	$(\theta_5 + 90^\circ)$	0	0	90°
6	$(\theta_6 + 90^\circ)$	l_6	0	0°

$$^{1}T_2 = \left[\begin{array}{c|c} e^{\theta_2 \hat{x}} & e^{\theta_2 \hat{x}} l_2 i \\ \hline 0^T & 1 \end{array}\right] \qquad ^{2}T_3 = \left[\begin{array}{c|c} e^{\theta_3 \hat{x}} e^{-\frac{\pi}{2} i \times} & e^{\theta_3 \hat{x}} l_3 i \\ \hline 0^T & 1 \end{array}\right]$$

$$^{3}T_4 = \left[\begin{array}{c|c} e^{\theta_4 \hat{x}} e^{\frac{\pi}{2} i \times} & 0 \\ \hline 0^T & 1 \end{array}\right] \qquad ^{4}T_5 = \left[\begin{array}{c|c} e^{(\theta_5 + \frac{\pi}{2}) \hat{x}} e^{\frac{\pi}{2} i \times} & 0 \\ \hline 0^T & 1 \end{array}\right]$$

$$^{5}T_6 = \left[\begin{array}{c|c} e^{(\theta_6 + \frac{\pi}{2}) \hat{x}} & l_6 k \\ \hline 0^T & 1 \end{array}\right]$$

$$^{0}T_6 = {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_3 \, {}^{3}T_4 \, {}^{4}T_5 \, {}^{5}T_6$$

$$\begin{bmatrix} {}^0x \\ 1 \end{bmatrix} = {}^0T_6 \begin{bmatrix} {}^6x \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} {}^0C_6 & {}^0d_6 \\ \hline 0^T & 1 \end{array}\right] \begin{bmatrix} {}^6x \\ 1 \end{bmatrix}$$

$$\boxed{{}^0x = {}^0C_6\,{}^6x + {}^0d_6}$$

$${}^0C_6^T\,{}^0x = \underbrace{{}^0C_6^T\,{}^0C_6}_{I}\,{}^6x + {}^0C_6^T\,{}^0d_6$$

$$\therefore \quad \boxed{{}^6x = {}^0C_6^T\,{}^0x - {}^0C_6^T\,{}^0d_6}$$

$$\therefore \quad {}^0T_6^{-1} = \left[\begin{array}{c|c} {}^0C_6^T & -{}^0C_6^T\,{}^0d_6 \\ \hline 0^T & 1 \end{array}\right]$$

$h_0 \times (o_6 - o_0)$

(b)

$$\underline{J} = \begin{bmatrix} \underline{h}_0 \times (o_6 - o_0) & \underline{h}_1 \times (o_6 - o_1) & \underline{h}_2 \times (o_6 - o_2) & \underline{h}_3 \times (o_6 - o_3) & \underline{h}_4 \times (o_6 - o_4) & \underline{h}_5 \times (o_6 - o_5) \\ \underline{h}_0 & \underline{h}_1 & \underline{h}_2 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{bmatrix}$$

$$o_3 = o_4 = o_5$$

$$\underline{J} \sim \left[\begin{array}{ccc|ccc} \underline{h}_0 \times (o_3 - o_0) & \underline{h}_1 \times (o_3 - o_0) & \underline{h}_2 \times (o_3 - o_0) & 0 & 0 & 0 \\ \hline \underline{h}_0 & \underline{h}_1 & \underline{h}_2 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{array}\right]$$

Wrist singularities: $\underline{h}_3 \parallel \underline{h}_5$ – axes aligned.

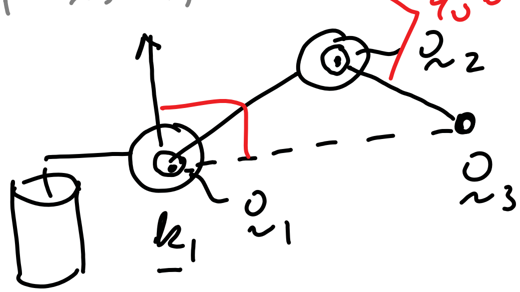
$\underline{h}_0 \quad k$

$\underline{h}_0 \times (o_6 - o_0)$

Arm singularities:

$\underline{h}_2 \times (o_3 - o_2) = \underline{h}_1 \times (o_3 - o_2)$

$\underline{h}_1 \times (o_3 - o_1)$

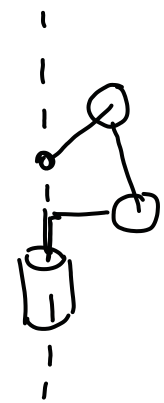


$\underline{h}_0 \times (o_3 - o_0)$

$\underline{h}_0 \times (o_3 - o_0)$

(i) $(o_3 - o_2) \parallel (o_3 - o_1)$

(ii) $(o_3 - o_0) \parallel \underline{h}_0$



(c) Addition rule of angular velocities.

$$\underline{\omega}_{6,0} = \underline{\omega}_{6,3} = \underline{k}_3 \dot{\theta}_4 + \underline{k}_4 \dot{\theta}_5 + \underline{k}_5 \dot{\theta}_6$$

$\uparrow$

$$\theta_1 = \theta_2 = \theta_3 \equiv 0$$

$$\underline{k}_5 = \underline{C}_5 k$$

$$\underline{k}_4 = \underline{C}_4 k$$

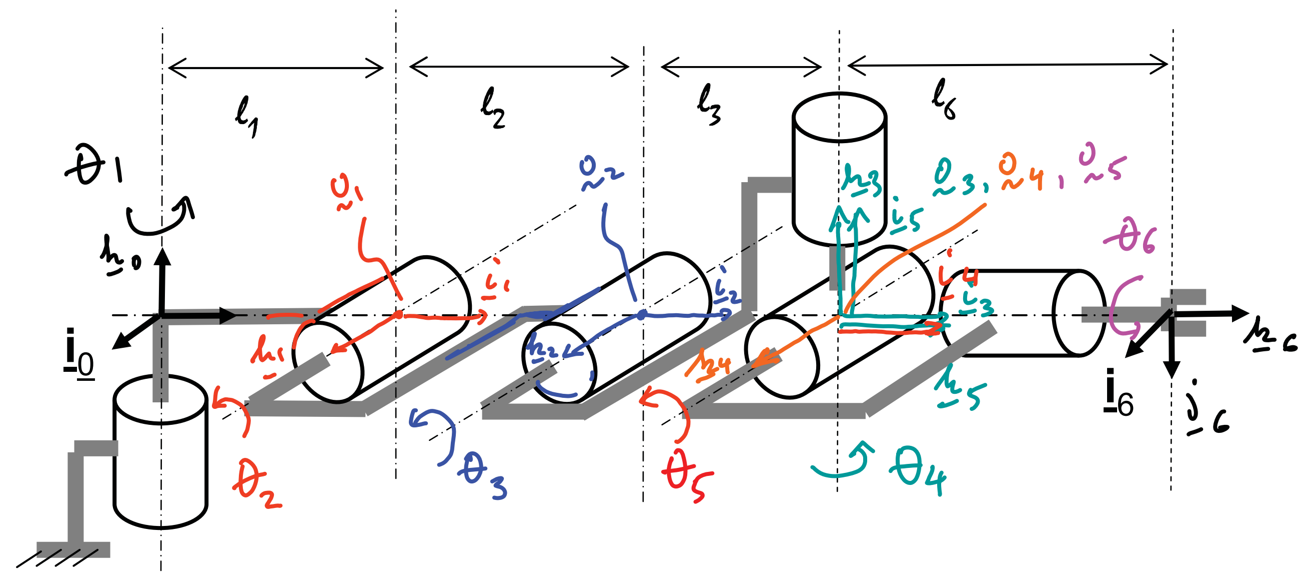
$$\underline{k}_3 = \underline{C}_3 k.$$

$$\underline{C}_4 = \underline{C}_3 \, {}^3C_4$$

$$\underline{C}_5 = \underline{C}_4 \, {}^4C_5 = \left(\underline{C}_3 \, {}^3C_4 \, {}^4C_5 \right.$$

from ${}^3T_4$, ${}^4T_5$, part (a).

$$\underline{C}_3 = \underline{C}_0 \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\underline{\omega}_{6,3} = \underline{h}_3 \dot{\theta}_4 + \underline{h}_4 \dot{\theta}_5 + \underline{h}_5 \dot{\theta}_6$$

$$\underline{\omega}_{6,0} = \dot{\theta}_4 \underline{h}_0 + \dot{\theta}_5 e^{\theta_4 \underline{h}_0 \times} \underline{i}_0 + \dot{\theta}_6 e^{\theta_4 \underline{h}_0 \times} e^{\theta_5 \underline{i}_0 \times} \underline{j}_0$$

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2008): Introduction to Robotics
Midterm Examination#1, February 14, 2008
Closed Book - 80 Minutes
Maximum - 30 marks

Problem 1.

You are given three coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$ with right-handed orthonormal frames.
$\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by an angle θ_1.
$\underline{C}_2$ is obtained from $\underline{C}_1$ by rotating $\underline{C}_1$ about $\underline{i}_0$ by an angle θ_2.
$\underset{\sim}{o}_2$ is obtained from $\underset{\sim}{o}_0$ by displacing $\underset{\sim}{o}_0$ by $a\underline{i}_0 + b\underline{j}_0 + c\underline{k}_0$.

(a)(5 marks)
Find the homogeneous transformation ${}^0T_2$ that relates the coordinates ${}^2x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_2, \underline{C}_2\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$. Specify ${}^0T_2$ in terms of θ_1, θ_2 and ${}^0d_2 = [a\ b\ c]^T$. You may use matrix exponential notation.

(b)(5 marks)
What is the inverse of ${}^0T_2$ from *(a)* above? Specify it in terms of θ_1, θ_2 and ${}^0d_2 = [a\ b\ c]^T$. You may use matrix exponential notation.

(c) (5 marks)
If θ_1 and θ_2 in *(a)* are functions of time $\theta_1(t)$ and $\theta_2(t)$, what are the coordinates ${}^0\omega_{1,0}$, in frame $\underline{C}_0$, of the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?
What are the coordinates ${}^1\omega_{2,1}$, in frame $\underline{C}_1$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_1$?
What are the coordinates ${}^0\omega_{2,0}$, in frame $\underline{C}_0$, of the angular velocity of $\underline{C}_2$ with respect to $\underline{C}_0$?

(a) $\underline{C}_1 = e^{\theta_1 \underline{j}_0 \times} \underline{C}_0 = \underline{C}_0 e^{\theta_1 j \times}$

$$\underline{C}_2 = e^{\theta_2 \underline{i}_0 \times} \underline{C}_1 = e^{\theta_2 \underline{i}_0 \times} \underline{C}_0 e^{\theta_1 j \times}$$

$$= \underline{C}_0 \underbrace{e^{\theta_2 i \times} e^{\theta_1 j \times}}$$

rotations about base frame axes, not right-to-left order

$$\underset{\sim}{o}_2 = \underset{\sim}{o}_0 + \underline{C}_0 \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \underset{\sim}{o}_0 + \underline{C}_0 \, {}^0 d_2$$

$$\begin{bmatrix} \underline{C}_2 & \underset{\sim}{o}_2 \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} \underline{C}_0 & \underset{\sim}{o}_0 \\ 0^T & 1 \end{bmatrix} \underbrace{\left[\begin{array}{ccc|c} & e^{\theta_2 i \times} e^{\theta_1 j \times} & & {}^0 d_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]}_{{}^0 T_2}$$

$$\begin{bmatrix} {}^0 x \\ 1 \end{bmatrix} = {}^0 T_2 \begin{bmatrix} {}^2 x \\ 1 \end{bmatrix}$$

(b) $${}^0 x = e^{\theta_2 i \times} e^{\theta_1 j \times} \, {}^2 x + {}^0 d_2$$

$$e^{-\theta_2 i \times} \, {}^0 x = e^{\theta_1 j \times} \, {}^2 x + e^{-\theta_2 i \times} \, {}^0 d_2$$

$$e^{-\theta_1 j \times} e^{-\theta_2 i \times} \, {}^0 x = {}^2 x + e^{-\theta_1 j \times} e^{-\theta_2 i \times} \, {}^0 d_2$$

$$\therefore \quad {}^2 x = e^{-\theta_1 j \times} e^{-\theta_2 i \times} \, {}^0 x - e^{-\theta_1 j \times} e^{-\theta_2 i \times} \, {}^0 d_2$$

$$\therefore \quad {}^{2}T_{0} = {}^{0}T_{2}^{-1} = \left[\begin{array}{c|c} e^{-\theta_1 j\times} e^{-\theta_2 i\times} & -e^{-\theta_1 j\times} e^{-\theta_2 i\times}\, {}^{0}d_2 \\ \hline 0^T & 1 \end{array}\right]$$

(c) $$\underline{C}_1 = \underline{C}_0\, e^{\theta_1 j\times}$$

$$\therefore \quad {}^{0}\omega_{1,0} = \dot{\theta}_1 j$$

$$\underline{C}_2 = \underline{C}_0\, e^{\theta_2 i\times} e^{\theta_1 j\times}$$ sequence of two rotation

$${}^{0}\omega_{2,0} = \dot{\theta}_2 i + e^{\theta_2 i\times} \dot{\theta}_1 j$$

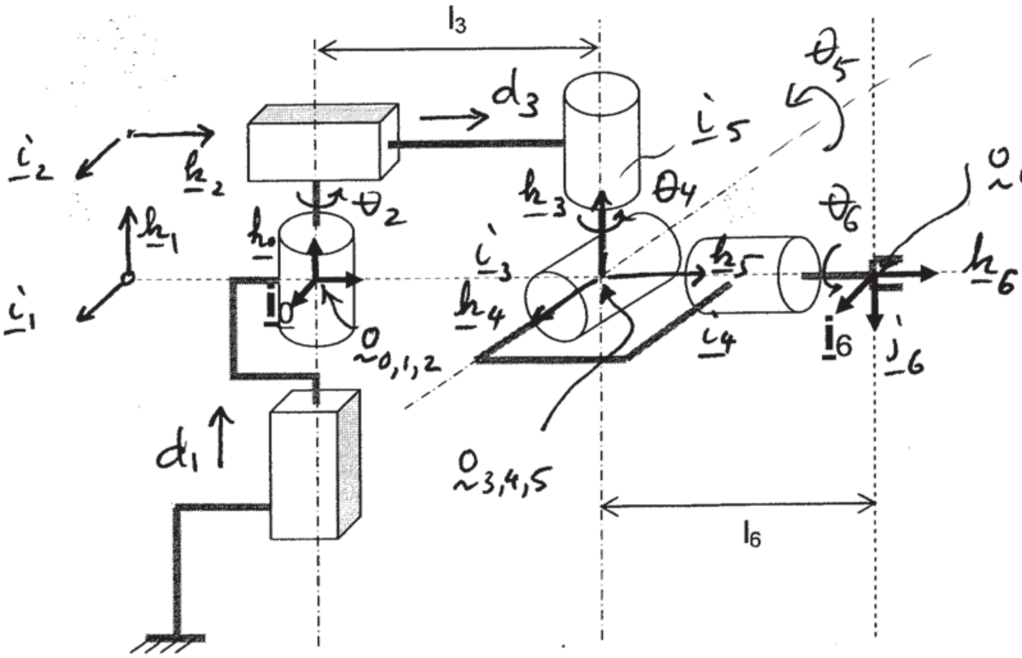
$$\underline{C}_2 = \underline{C}_0\, e^{\theta_2 i\times} e^{\theta_1 j\times} = \underline{C}_1 \underbrace{e^{-\theta_1 j\times} e^{\theta_2 i\times} e^{\theta_1 j\times}}$$

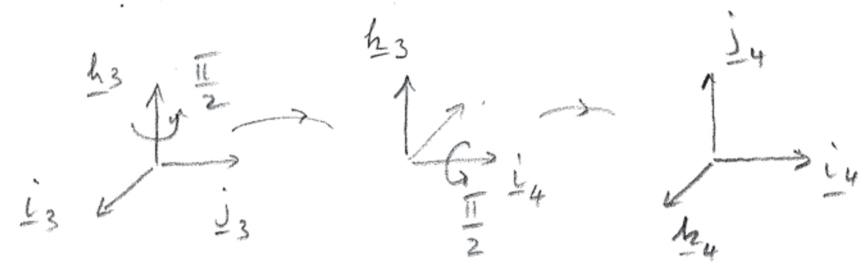
sequence of three rotation.

$${}^{1}\omega_{2,1} = -\dot{\theta}_1 j + e^{-\theta_1 j\times} \dot{\theta}_2 i + e^{-\theta_1 j\times} e^{\theta_2 i\times} \dot{\theta}_1 j$$

Problem 2.

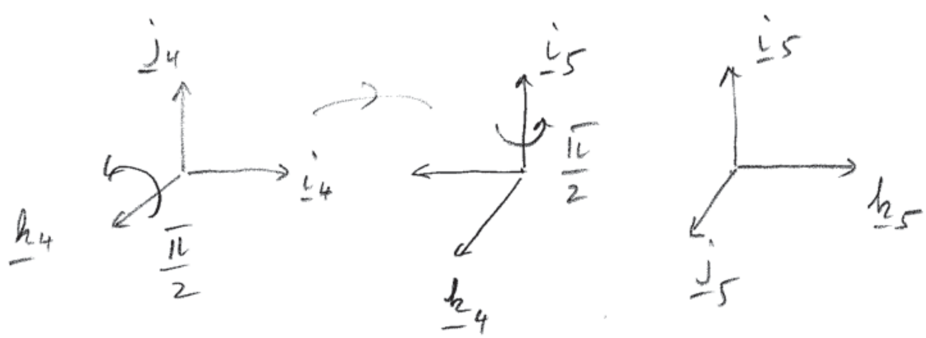
Consider the cylindrical manipulator shown in the following figure:





(a) (10 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, \ldots 6$ to links 1 through 6 according to the Denavit-Hartenberg convention and find the homogeneous transformations required for the direct kinematics problem.

(b) (5 marks) Write down the manipulator Jacobian in terms of the joint axes and use elementary row operations to decouple the wrist and arm singularities. Under what condition will the manipulator be in a singular configuration?



$$\begin{cases} \underline{C}_1 = \underline{C}_0 \\ \underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0 d_1 k \end{cases} \qquad {}^0T_1 = \begin{bmatrix} I & d_1 k \\ 0^T & 1 \end{bmatrix}$$

$$\begin{cases} \underline{C}_2 = \underline{C}_1 e^{\theta_2 k\times} e^{-\frac{\pi}{2} i\times} \\ \underset{\sim}{o}_2 = \underset{\sim}{o}_1 \end{cases} \qquad {}^1T_2 = \left[\begin{array}{c|c} e^{\theta_2 k\times} e^{-\frac{\pi}{2} i\times} & 0 \\ \hline 0^T & 1 \end{array}\right]$$

$$\begin{cases} \underline{C}_3 = \underline{C}_2 e^{\frac{\pi}{2} i\times} \\ \underset{\sim}{o}_3 = \underset{\sim}{o}_2 + \underline{C}_2 (d_3 + \ell_3) k \end{cases} \qquad {}^2T_3 = \left[\begin{array}{c|c} e^{\frac{\pi}{2} i\times} & (d_3 + \ell_3) k \\ \hline 0^T & 1 \end{array}\right]$$

$$\begin{cases} \underline{C}_4 = \underline{C}_3 e^{(\theta_4 + \frac{\pi}{2}) k\times} e^{\frac{\pi}{2} i\times} \\ \underset{\sim}{o}_4 = \underset{\sim}{o}_3 \end{cases} \qquad {}^3T_4 = \left[\begin{array}{c|c} e^{(\theta_4 + \frac{\pi}{2}) k\times} e^{\frac{\pi}{2} i\times} & 0 \\ \hline 0^T & 1 \end{array}\right]$$

$$\begin{cases} \underline{C}_5 = \underline{C}_4 e^{(\theta_5 + \frac{\pi}{2}) k\times} e^{\frac{\pi}{2} i\times} \\ \underset{\sim}{o}_5 = \underset{\sim}{o}_4 \end{cases} \qquad {}^4T_5 = \left[\begin{array}{c|c} e^{(\theta_5 + \frac{\pi}{2}) k\times} e^{\frac{\pi}{2} i\times} & 0 \\ \hline 0^T & 1 \end{array}\right]$$

$$\begin{cases} \underline{C}_6 = \underline{C}_5 e^{(\theta_6 + \frac{\pi}{2}) k} \\ \underset{\sim}{o}_6 = \underset{\sim}{o}_5 + \underline{C}_5 \ell_6 k \end{cases} \qquad {}^5T_6 = \left[\begin{array}{c|c} e^{(\theta_6 + \frac{\pi}{2}) k\times} & \ell_6 k \\ \hline 0^T & 1 \end{array}\right]$$

$${}^0T_6(\underbrace{d_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6}_{q}) = {}^0T_1\, {}^1T_2\, {}^2T_3\, {}^3T_4\, {}^4T_5\, {}^5T_6 \; (q)$$

$$J = \begin{bmatrix} \underline{h}_0 & \underline{h}_1 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_1) & \underline{h}_2 & \underline{h}_3 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_3) & \underline{h}_4 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_3) & \underline{h}_5 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_3) \\ \underline{0} & \underline{h}_1 & 0 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|ccc} \underline{h}_0 & \underline{h}_1 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_1) & \underline{h}_2 & 0 & 0 & 0 \\ \hline 0 & \underline{h}_1 & 0 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{array}\right]$$

- Wrist singularities when $\underline{h}_3, \underline{h}_5$ aligned.
- Arm singularities when

  $\underline{h}_0 \quad \underline{h}_1 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_1) \quad \underline{h}_2$ in the same plane.

$\underline{h}_0, \underline{h}_2$ always orthogonal.

Singularity when $\underset{\sim}{p}_6 - \underset{\sim}{p}_1 = \underline{0}$, i.e. wrist center coincides with waist axis, in which case motion in the $\underline{i}_0$ axis not possible; otherwise, $\underline{h}_1 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_1) =$
$= \underline{h}_0 \times (\underset{\sim}{p}_6 - \underset{\sim}{p}_1)$ has component $\perp$ on $\underline{h}_0, \underline{h}_2$.

NAME:
Student #:

University of British Columbia
Department of Electrical and Computer Engineering
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Midterm Examination #1, February 15, 2007
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Problem 1.

Consider two coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and $\{\underset{\sim}{o}_1, \underline{C}_1\}$ such that the $\underset{\sim}{o}_1$ is obtained from $\underset{\sim}{o}_0$ by a translation of $d(t)$ meters along $\underline{k}_0$, and $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating an angle $\theta(t)$ degrees about $\underline{i}_0$.

(a)(4 marks) Find the homogeneous transformation ${}^0T_1$ that relates the coordinates ${}^1x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$. Specify every entry of the matrix ${}^0T_1$.

(b) (4 marks) What is the inverse of ${}^0T_1$?

(c)(2 marks) What are the coordinates of the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$ in frame $\underline{C}_0$ and in frame $\underline{C}_1$?

(a) $\underset{\sim}{x} = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0x = \underset{\sim}{o}_1 + \underline{C}_1\, {}^1x$

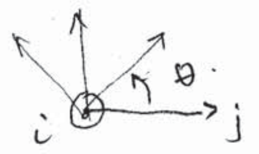
$\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0d_1 \quad ; \quad \underline{C}_1 = \underline{C}_0\, {}^0C_1$

${}^0d_1 = d(t)\, k \quad ; \quad {}^0C_1 = e^{\theta(t)\, i^{\times}}$

$$\underset{\sim}{o}_0 + \underline{C}_0\, {}^0x = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0d_1 + \underline{C}_0\, {}^0C_1\, {}^1x \Rightarrow$$

$$\Rightarrow \quad {}^0x = {}^0d_1 + {}^0C_1\, {}^1x$$

$$\Rightarrow \quad \begin{bmatrix} {}^0x \\ 1 \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} {}^0C_1 & {}^0d_1 \\ \hline 0^T & 1 \end{array}\right]}_{{}^0T_1} \begin{bmatrix} {}^1x \\ 1 \end{bmatrix}$$



$${}^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta(t) & -\sin\theta(t) & 0 \\ 0 & \sin\theta(t) & \cos\theta(t) & d(t) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $\quad {}^0x = {}^0d_1 + {}^0C_1\,{}^1x \Rightarrow$

$${}^0C_1^T\,{}^0x = {}^0C_1^T\,{}^0d_1 + {}^1x \Rightarrow \quad {}^1x = -{}^0C_1^T\,{}^0d_1 + {}^0C_1^T\,{}^0x$$

$$\therefore \quad {}^0T_1^{-1} = \left[\begin{array}{c|c} {}^0C_1^T & -{}^0C_1^T\,{}^0d_1 \\ \hline 0^T & 1 \end{array}\right]$$

$$-{}^0C_1^T\,{}^0d_1 = -\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta(t) & \sin\theta(t) \\ 0 & -\sin\theta(t) & \cos\theta(t) \end{bmatrix}}_{{}^0C_1^T}\begin{bmatrix} 0 \\ 0 \\ d(t) \end{bmatrix} = -\begin{bmatrix} 0 \\ d(t)\sin\theta(t) \\ d(t)\cos\theta(t) \end{bmatrix}$$

(c) $\quad \underline{C}_1 = \underline{C}_0\, e^{\theta(t)\,i^{\times}} = \underline{C}_0\,{}^0C_1(t)$

$${}^0\omega_{1,0} = \dot\theta(t)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$${}^1\omega_{1,0} = {}^1C_0(t)\,{}^0\omega_{1,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cdots & \\ 0 & & \end{bmatrix}\dot\theta(t)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \dot\theta(t)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(does not change as ${}^1C_0$ leaves $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ unchanged...).

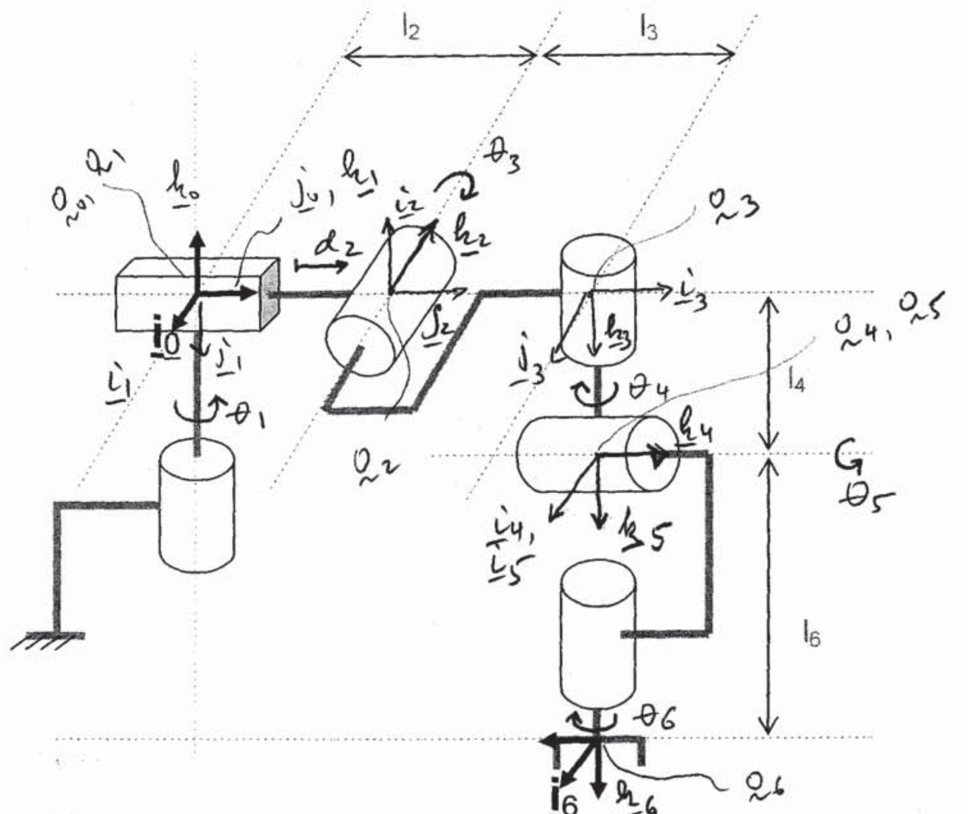
Problem 2.

Consider the manipulator shown below.

(a) *(9 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ...5$ to links 1 through 5, using the Denavit-Hartenberg convention. Complete the table of Denavit-Hartenberg parameters. Find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem.

(b) *(7 marks)* Find the manipulator Jacobian. Find all the singular configurations of the manipulator using the Jacobian.

(c) *(4 marks)* Find the angular velocity of link 3 as a function of the joint rates in the base frame coordinates.



link \ Param	θ	d	a	α
1	(θ_1)	0	0	$-\pi/2$
2	$-\pi/2$	$l_2 + (d_2)$	0	$\pi/2$
3	$(\theta_3) + \pi/2$	0	l_3	$-\pi/2$
4	$(\theta_4) + \pi/2$	l_4	0	$\pi/2$
5	(θ_5)	0	0	$-\pi/2$
6	(θ_6)	l_6	0	0

DH Table

$$
{}^{i-1}T_i = \left[\begin{array}{c|c} e^{\theta_i h x} & 0 \\ \hline 0^T & 1 \end{array}\right]\left[\begin{array}{c|c} I & d_i k \\ \hline 0^T & 1 \end{array}\right]\left[\begin{array}{c|c} I & a_i i \\ \hline 0^T & 1 \end{array}\right]\left[\begin{array}{c|c} e^{\alpha_i i x} & 0 \\ \hline 0^T & 1 \end{array}\right]
$$

$$
= \left[\begin{array}{c|c} e^{\theta_i h x} & d_i k \\ \hline 0^T & 1 \end{array}\right]\left[\begin{array}{c|c} e^{\alpha_i i x} & a_i i \\ \hline 0^T & 1 \end{array}\right] = \left[\begin{array}{c|c} e^{\theta_i h x} e^{\alpha_i i x} & e^{\theta_i h x} a_i i + d_i k \\ \hline 0^T & 1 \end{array}\right]
$$

$$
{}^0T_1 = \left[\begin{array}{c|c} e^{\theta_1 h x} e^{-\frac{\pi}{2} i x} & 0 \\ \hline 0^T & 1 \end{array}\right]
$$

$$
{}^1T_2 = \left[\begin{array}{c|c} e^{-\frac{\pi}{2} h x} e^{\frac{\pi}{2} i x} & (l_2 + d_2) k \\ \hline 0^T & 1 \end{array}\right]
$$

$$
{}^2T_3 = \left[\begin{array}{c|c} e^{(\theta_3 + \frac{\pi}{2}) h x} e^{-\frac{\pi}{2} i x} & e^{(\theta_3 + \frac{\pi}{2}) h x} l_3 i \\ \hline 0^T & 1 \end{array}\right]
$$

$$
{}^3T_4 = \left[\begin{array}{c|c} e^{(\theta_4 + \frac{\pi}{2}) h x} e^{\frac{\pi}{2} i x} & l_4 k \\ \hline 0^T & 1 \end{array}\right]
$$

$$
{}^4T_5 = \left[\begin{array}{c|c} e^{\theta_5 h x} e^{-\frac{\pi}{2} i x} & 0 \\ \hline 0^T & 1 \end{array}\right]
$$

$$
{}^5T_6 = \left[\begin{array}{c|c} e^{\theta_6 h x} & l_6 k \\ \hline 0^T & 1 \end{array}\right]
$$

(b)

$$\underline{J} = \begin{bmatrix} \underline{h}_0 \times (\underline{o}_6 - \underline{o}_0) & \underline{h}_1 & \underline{h}_2 \times (\underline{o}_6 - \underline{o}_2) & \underline{h}_3 \times (\underline{o}_6 - \underline{o}_3) & \underline{h}_4 \times (\underline{o}_6 - \underline{o}_4) & \underline{h}_5 \times (\underline{o}_6 - \underline{o}_5) \\ \underline{h}_0 & 0 & \underline{h}_2 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{bmatrix}$$

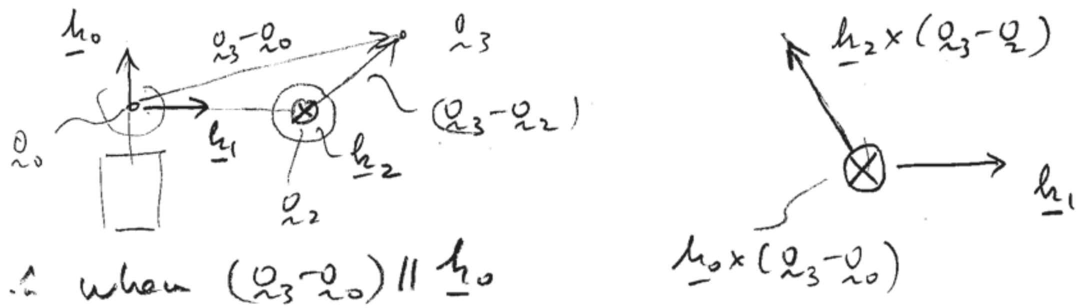
- move $\underline{o}_3$ to coincide with $\underline{o}_4$, $\underline{o}_5$ (can do this because

$$\underline{h}_3 \times (\underline{o}_6 - \underline{o}_3) = \underline{h}_3 \times (\underline{o}_6 - \underline{o}_3 + \underbrace{\underline{o}_3 - \underline{o}_4}_{\parallel \text{ to } \underline{h}_3}) = \underline{h}_3 \times (\underline{o}_6 - \underline{o}_4)$$

- use row operations ($(\underline{o}_6 - \underline{o}_3) \times$ second row added to 1st row) to obtain that

$$\underline{J} \sim \left[\begin{array}{ccc|ccc} \underline{h}_0 \times (\underline{o}_3 - \underline{o}_0) & \underline{h}_1 & \underline{h}_2 \times (\underline{o}_3 - \underline{o}_2) & 0 & 0 & 0 \\ \hline \underline{h}_0 & 0 & \underline{h}_2 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{array}\right]$$

- wrist singularity when $\underline{h}_3 \parallel \underline{h}_5$
- arm singularity when $\underline{h}_0 \times (\underline{o}_3 - \underline{o}_0)$, $\underline{h}_1$, $\underline{h}_2 \times (\underline{o}_3 - \underline{o}_2)$ lie in the same plane.



$\therefore$ when $(\underline{o}_3 - \underline{o}_0) \parallel \underline{h}_0$

$\therefore$ when $(\underline{o}_3 - \underline{o}_2)$ is aligned with $\underline{h}_0$, $\underline{h}_1$ and $\underline{h}_2 \times (\underline{o}_3 - \underline{o}_2)$ are aligned.

$\therefore$ Arm singularities when elbow is up or down.

(c) $${}^0\omega_{3,0} = {}^0\omega_{1,0} + {}^0C_1 \, {}^1\omega_{2,1} + {}^0C_2 \, {}^2\omega_{3,2}$$

$${}^0\omega_{1,0} = \dot{\theta}_1 k \ ; \quad {}^1\omega_{2,1} = 0 \ ; \quad {}^2\omega_{3,2} = \dot{\theta}_3 k$$

$${}^0C_2 = {}^0C_1 \, {}^1C_2 = e^{\theta_1 k \times} e^{-\frac{\pi}{2} i \times} e^{-\frac{\pi}{2} k \times} e^{\frac{\pi}{2} i \times}$$

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2006): Introduction to Robotics
Midterm Examination #1, February 9, 2006
Closed Book - 60 Minutes
Maximum - 30 marks

Problem 1.

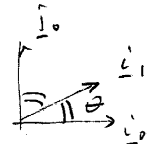
You are given two coordinate systems with orthonormal frames $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$ related by $\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1$ and $\underline{C}_1 = \underline{C}_0\,{}^0C_1$.
(a) *(2 marks)* Suppose that

$$\underline{C}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \underline{C}_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

What is the axis and angle of rotation between $\underline{C}_0$ and $\underline{C}_1$?
(b) *(3 marks)* If a point $\underset{\sim}{x}$ has coordinates ${}^0x$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and ${}^1x$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$, what are the homogeneous transformations expressing ${}^1x$ in terms of ${}^0x$ and ${}^0x$ in terms of ${}^1x$?
(c) *(2 marks)* If ${}^0C_1 = {}^0C_1(t)$ is a function of time, what is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$, as a function of ${}^0C_1(t)$?
(d) *(3 marks)* Suppose $\{\underset{\sim}{o}_1, \underline{C}_1\}$ is attached to link 1 of a manipulator and $\{\underset{\sim}{o}_0, \underline{C}_0\}$ to the base, and the Denavit-Hartenberg convention has been followed in assigning $\{\underset{\sim}{o}_1, \underline{C}_1\}$, with angle, offset, length and twist given by θ, d, a, α. What is the homogeneous transformation ${}^0T_1$ in terms of θ, d, a, α.

a) axis is $\underline{k}_0$; coords of $\underline{i}_1$ in $\underline{C}_0$ are $\begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$, so

$\underline{i}_1 = \frac{\sqrt{2}}{2}\underline{i}_0 + \frac{\sqrt{2}}{2}\underline{j}_0 \Rightarrow \theta = \pi/4$



b) $\underset{\sim}{x} = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0x = \underset{\sim}{o}_1 + \underline{C}_1\,{}^1x = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1 + \underline{C}_0\,{}^0C_1\,{}^1x$

$\therefore\ {}^0x = {}^0d_1 + {}^0C_1\,{}^1x$

${}^1x = {}^0C_1^{-1}({}^0x - {}^0d_1) = {}^0C_1^T({}^0x - {}^0d_1)$

c) ${}^0\dot{C}_1 = ({}^0\omega_{1,0}\times)\,{}^0C_1 \quad \therefore\ {}^0\omega_{1,0}\times = {}^0\dot{C}_1(t)\,{}^0C_1(t)^T$

d) $${}^0T_1 = \underbrace{\begin{bmatrix} e^{\theta k\times} & 0 \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} I & dk \\ 0^T & 1 \end{bmatrix}}_{\begin{bmatrix} e^{\theta k\times} & dk \\ 0^T & 1 \end{bmatrix}}\underbrace{\begin{bmatrix} I & ai \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} e^{\alpha i\times} & 0 \\ 0^T & 1 \end{bmatrix}}_{\begin{bmatrix} e^{\alpha i\times} & ia \\ 0^T & 1 \end{bmatrix}} = \begin{bmatrix} e^{\theta k\times} e^{\alpha i\times} & e^{\theta k\times} ai + dk \\ 0^T & 1 \end{bmatrix}$$

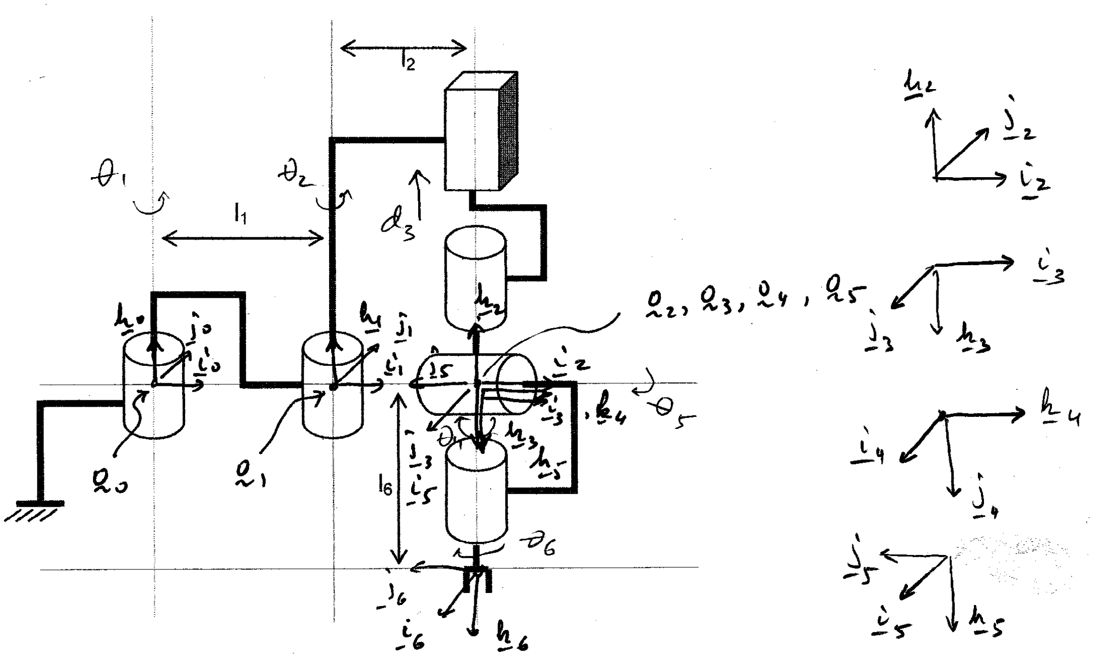
Problem 2.

Consider the SCARA manipulator shown below.

(a) *(10 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 0, \ldots 6$ to the base and to links 1 through 6, using the Denavit-Hartenberg convention, and find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Complete the table of Denavit-Hartenberg parameters.

(b) *(5 marks)* Find manipulator Jacobian and use it to find the manipulator singular configurations. Suppose that somebody suggests that the manipulator should be used to place surface-mount packages on printed circuit boards horizontal to the robot's first axis $\underline{k}_0$. Can you foresee a problem with this suggestion? Explain.

(c) *(5 marks)* Find the gripper angular velocity as a function of the joint rates, in base frame coordinates.

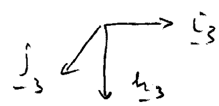


$${}^0T_1 = \left[\begin{array}{c|c} e^{\theta_1 k\times} & e^{\theta_1 k\times} l_1 i \\ \hline 0^T & 1 \end{array}\right]$$

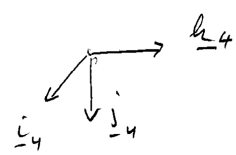
from using Problem 1.d)

$$
{}^{1}T_{2} = \left[\begin{array}{c|c} e^{\theta_2 \underline{k}\times} & e^{\theta_2 \underline{k}\times} l_2 \underline{i} \\ \hline 0^T & 1 \end{array}\right]
$$

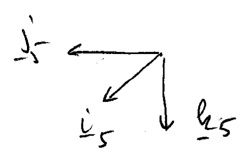
$$
{}^{2}T_{3} = \left[\begin{array}{c|c} e^{\pi \underline{i}\times} & d_3 \underline{k} \\ \hline 0^T & 1 \end{array}\right]
$$



$$
{}^{3}T_{4} = \left[\begin{array}{c|c} e^{(\theta_4 + \frac{\pi}{2})\underline{k}\times} e^{\frac{\pi}{2}\underline{i}\times} & 0 \\ 0^T & 1 \end{array}\right]
$$



$$
{}^{4}T_{5} = \left[\begin{array}{c|c} e^{\theta_5 \underline{k}\times} e^{-\frac{\pi}{2}\underline{i}\times} & 0 \\ \hline 0^T & 1 \end{array}\right]
$$



$$
{}^{5}T_{6} = \left[\begin{array}{c|c} e^{\theta_6 \underline{k}\times} & l_6 \underline{k} \\ \hline 0^T & 1 \end{array}\right]
$$

Link	θ	d	a	α
1	(θ_1)	0	l_1	0
2	(θ_2)	0	l_2	0
3	0	(d_3)	0	π
4	$(\theta_4 + \frac{\pi}{2})$	0	0	$\pi/2$
5	(θ_5)	0	0	$-\frac{\pi}{2}$
6	(θ_6)	l_6	0	0

b)

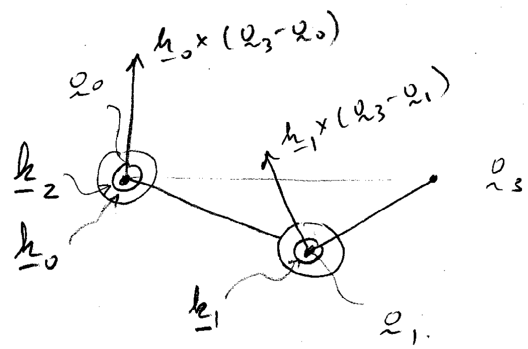
$$\overbrace{}^{\text{same}}$$

$$J = \begin{bmatrix} \underline{h}_0 \times (\underline{o}_6 - \underline{o}_0) & \underline{h}_1 \times (\underline{o}_6 - \underline{o}_1) & \underline{k}_2 & \underline{h}_3 \times (\underline{o}_6 - \underline{o}_3) & \underline{h}_4 \times (\underline{o}_6 - \underline{o}_4) & \underline{h}_5 \times (\underline{o}_6 - \underline{o}_5) \\ \underline{h}_0 & \underline{h}_1 & 0 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{bmatrix}$$

$\therefore$ singularities decouple

$$J \sim \left[\begin{array}{ccc|ccc} \underline{h}_0 \times (\underline{o}_3 - \underline{o}_0) & \underline{h}_1 \times (\underline{o}_3 - \underline{o}_1) & \underline{h}_2 & & 0 & \\ \hline \underline{h}_0 & \underline{h}_1 & 0 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{array}\right]$$

wrist singular when $\underline{h}_3 \parallel \underline{h}_5$



- only arm singularity when arm fully extended or retracted, i.e. $(\underline{o}_3 - \underline{o}_0)$ aligned to $(\underline{o}_3 - \underline{o}_1)$.

(c) $$^0\underline{\omega}_{6,0} = \dot{\theta}_1 \underline{h}_0 + \dot{\theta}_2 \underset{\underset{\underline{h}_0}{\parallel}}{\underline{h}_1} + \dot{\theta}_4 \underset{\underset{-\underline{h}_0}{\parallel}}{\underline{h}_3} + \dot{\theta}_5 \underline{h}_4 + \dot{\theta}_6 \underline{h}_5$$

$$\underline{h}_4 = \underline{C}_4 k = \underline{C}_0 \, {}^0C_4 k$$
$$\underline{h}_5 = \underline{C}_5 k = \underline{C}_0 \, {}^0C_5 k.$$

$$\left.\begin{array}{l} {}^0C_4 = {}^0C_1 \, {}^1C_2 \, {}^2C_3 \, {}^3C_4 \\ {}^0C_5 = {}^0C_4 \, {}^4C_5 \end{array}\right\} \text{from (b).}$$

$$^0\omega_{6,0} = (\dot{\theta}_1 + \dot{\theta}_2 - \dot{\theta}_4) k + \dot{\theta}_5 \, {}^0C_4 k + \dot{\theta}_6 \, {}^0C_5 k.$$

NAME:

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Department of Electrical and Computer Engineering
EECE 487 (Winter 2001): Introduction to Robotics
Midterm Examination, February 15, 2001
Closed Book - 60 Minutes
Maximum - 30 marks

Problem 1. *(10 marks)*

You are given two coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$ and a point $\underset{\sim}{x}$ that has coordinates ${}^1x$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$.
(a) If $\underline{C}_1 = \underline{C}_0\,{}^0C_1$ and $\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1$, write an expression for $\underset{\sim}{x}$ in terms of ${}^1x$, $\underset{\sim}{o}_0$, $\underline{C}_0$, ${}^0d_1$ and ${}^0C_1$ only.
(b) If frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating about $\underline{j}_0$ by an angle θ and $\underset{\sim}{o}_1$ is obtained by displacing $\underset{\sim}{o}_0$ by $a\underline{i}_0 + b\underline{j}_0 + c\underline{k}_0$, then write the homogenous transformation ${}^0T_1$ that expresses the relationship between the coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and $\{\underset{\sim}{o}_1, \underline{C}_1\}$. Specify every entry of the matrix ${}^0T_1$.
What is the inverse of ${}^0T_1$? It is enough to specify the inverse in terms of the rotation matrix of ${}^0T_1$ and ${}^0d = [a\ b\ c]^T$.

$$\begin{bmatrix} \underline{C}_1 & \underset{\sim}{o}_1 \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} \underline{C}_0 & \underset{\sim}{o}_0 \\ 0^T & 1 \end{bmatrix} \quad {}^0T_1$$

Problem 2. *(10 marks)*

Sketch a manipulator that is described by the table of DH parameters below. Joint variables are enclosed in parantheses. Start with a base coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, show and label the coordinate systems $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$, $\{\underset{\sim}{o}_3, \underline{C}_3\}$. Label the dimensions d_i and a_i.

DH Parameter	θ_i	d_i	a_i	α_i
Link 1	(θ_1)	d_1	a_1	$\pi/2$
Link 2	(θ_2)	d_2	0	$-\pi/2$
Link 3	$\pi/2$	(d_3)	0	0

Problem 3. *(10 marks)*

For a spherical wrist

$$\begin{aligned}
\underline{C}_1 &= \underline{C}_0\,{}^0C_1 = \underline{C}_0 e^{\theta_1 k\times} \\
\underline{C}_2 &= \underline{C}_1\,{}^1C_2 = \underline{C}_1 e^{\theta_2 j\times} \\
\underline{C}_3 &= \underline{C}_2\,{}^2C_3 = \underline{C}_2 e^{\theta_3 i\times}
\end{aligned}$$

(a) Find the coordinate ${}^0\omega_{3,0}$ of the angular velocity of $\underline{C}_3$ with respect to $\underline{C}_0$ from the addition rule of angular velocities.
(b) Verify your result by using direct differentiation of ${}^0C_3$.

Sample Solution

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EECE 487 (Winter 2001): Introduction to Robotics
Midterm Examination, February 15, 2001
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Problem 1. *(10 marks)*

You are given two coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$ and a point $\underset{\sim}{x}$ that has coordinates ${}^1x$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$.
(a) If $\underline{C}_1 = \underline{C}_0\,{}^0C_1$ and $\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1$, write an expression for $\underset{\sim}{x}$ in terms of ${}^1x$, $\underset{\sim}{o}_0$, $\underline{C}_0$, ${}^0d_1$ and ${}^0C_1$ only.
(b) If frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating about $\underline{j}_0$ by an angle θ and $\underset{\sim}{o}_1$ is obtained by displacing $\underset{\sim}{o}_0$ by $a\underline{i}_0 + b\underline{j}_0 + c\underline{k}_0$, then write the homogenous transformation ${}^0T_1$ that expresses the relationship between the coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and $\{\underset{\sim}{o}_1, \underline{C}_1\}$. Specify every entry of the matrix ${}^0T_1$.
What is the inverse of ${}^0T_1$? It is enough to specify the inverse in terms of the rotation matrix of ${}^0T_1$ and ${}^0d_1 = [a\ b\ c]^T$.

$$\begin{bmatrix} \underline{C}_1 & \underset{\sim}{o}_1 \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} \underline{C}_0 & \underset{\sim}{o}_0 \\ 0^T & 1 \end{bmatrix} \quad {}^0T_1$$

a) $\underset{\sim}{x} = \underset{\sim}{o}_1 + \underline{C}_1\,{}^1x = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1 + \underline{C}_1\,{}^1x =$

$= \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1 + \underline{C}_0\,{}^0C_1\,{}^1x$

$= \underset{\sim}{o}_0 + \underline{C}_0\,({}^0d_1 + {}^0C_1\,{}^1x)$.

b) $\underline{C}_1 = \underline{C}_0\, e^{\theta j \times} \qquad \underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0 \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

$${}^0T_1 = \left[\begin{array}{c|c} e^{\theta j \times} & \begin{matrix} a \\ b \\ c \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \qquad e^{\theta j \times} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.$$

$$\underline{C}_1 = \underline{C}_0\, e^{\theta \hat{j} \times} \Rightarrow \underline{C}_0 = \underbrace{\underline{C}_1\, e^{-\theta \hat{j} \times}}_{=\underline{C}_0}$$

$$\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\, {}^{0}\!d_1 \Rightarrow \underset{\sim}{o}_0 = \underset{\sim}{o}_1 - \underline{C}_0\, {}^{0}\!d_1 =$$

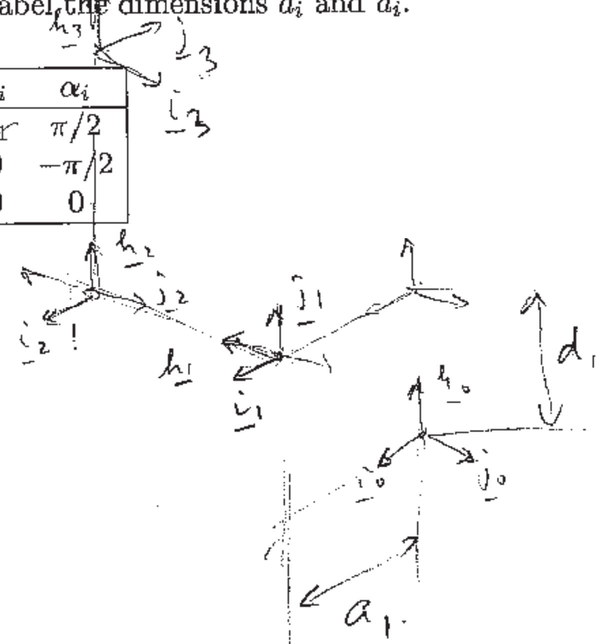
$$= \underset{\sim}{o}_1 + \underline{C}_1 \left[-e^{-\theta \hat{j} \times}\, {}^{0}\!d_1 \right]$$

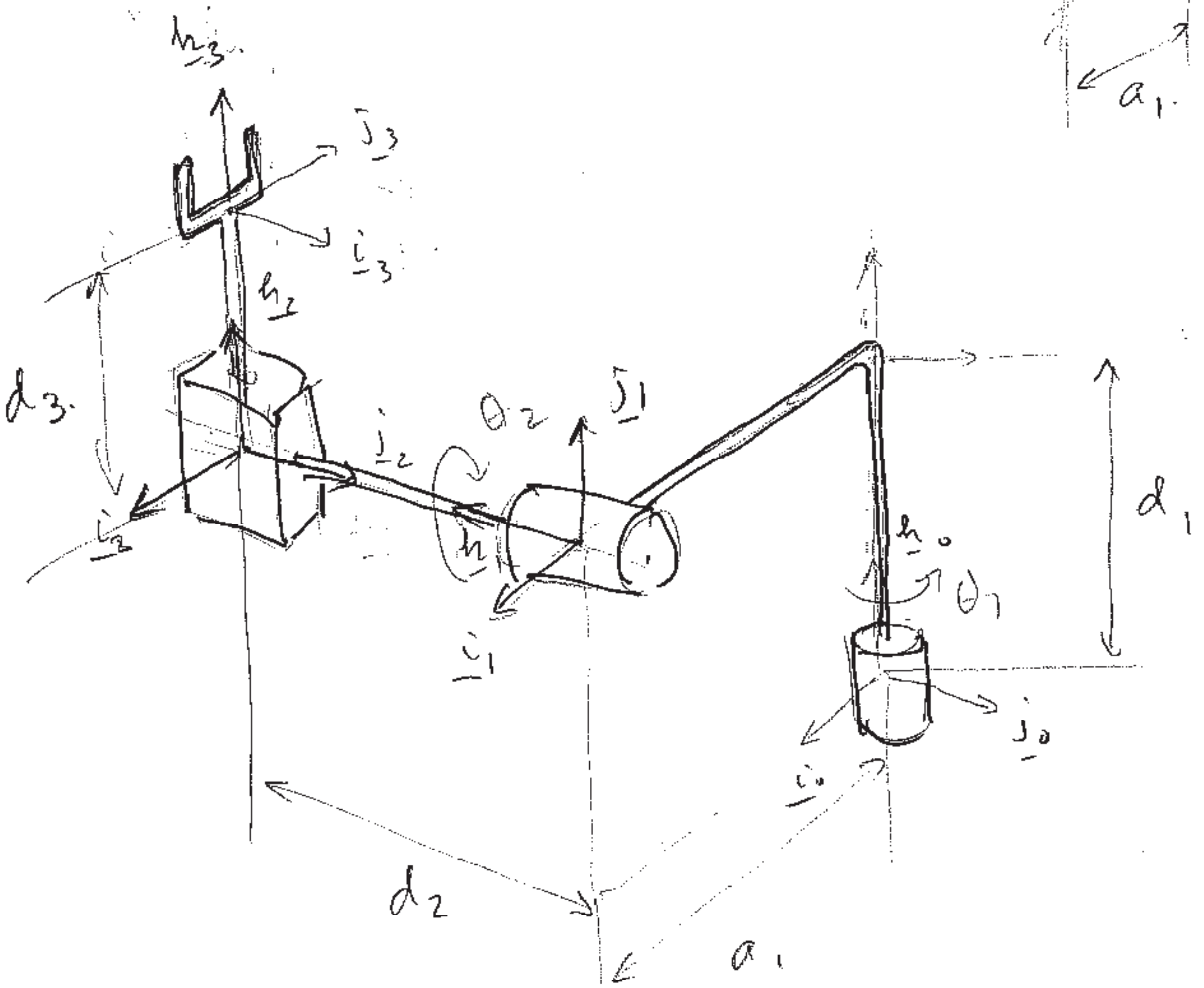
$$\therefore \quad {}^{1}T_0 = {}^{0}T_1^{-1} = \left[\begin{array}{c|c} e^{-\theta \hat{j} \times} & -e^{-\theta \hat{j} \times}\, {}^{0}\!d_1 \\ \hline 0^T & 1 \end{array} \right]$$

Problem 2. *(10 marks)*

Sketch a manipulator that is described by the table of DH parameters below. Joint variables are enclosed in parantheses. Start with a base coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, show and label the coordinate systems $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$, $\{\underset{\sim}{o}_3, \underline{C}_3\}$. Label the dimensions d_i and a_i.

DH Parameter	θ_i	d_i	a_i	α_i
Link 1	(θ_1)	d_1	a_1	$\pi/2$
Link 2	(θ_2)	d_2	0	$-\pi/2$
Link 3	$\pi/2$	(d_3)	0	0





Problem 3. *(10 marks)*

For a spherical wrist

$$\underline{C}_1 = \underline{C}_0\,{}^0C_1 = \underline{C}_0 e^{\theta_1 k\times} \qquad {}^0\omega_{1,0} = \dot\theta_1 k$$
$$\underline{C}_2 = \underline{C}_1\,{}^1C_2 = \underline{C}_1 e^{\theta_2 j\times} \qquad {}^1\omega_{2,1} = \dot\theta_2 j$$
$$\underline{C}_3 = \underline{C}_2\,{}^2C_3 = \underline{C}_2 e^{\theta_3 i\times} \qquad {}^2\omega_{3,2} = \dot\theta_3 i$$

(a) Find the coordinate ${}^0\omega_{3,0}$ of the angular velocity of $\underline{C}_3$ with respect to $\underline{C}_0$ from the addition rule of angular velocities.
(b) Verify your result by using direct differentiation of ${}^0C_3$.

$$\underline{\omega}_{3,0} = \underline{C}_0\,{}^0\omega_{3,0} = \underline{C}_0\,{}^0\omega_{1,0} + \underline{C}_1\,{}^1\omega_2 + \underline{C}_2\,{}^2\omega_3 =$$
$$= \underline{C}_0\,{}^0\omega_{1,0} + \underline{C}_0\,{}^0C_1\,{}^1\omega_{2,1} + \underline{C}_1\,{}^1C_2\,{}^2\omega_{3,2}$$
$$= \underline{C}_0\,{}^0\omega_{1,0} + \underline{C}_0\,{}^0C_1\,{}^1\omega_{2,1} + \underline{C}_0\,{}^0C_1\,{}^1C_2\,{}^2\omega_{3,2}$$
$$\therefore\ {}^0\omega_{3,0} = \dot\theta_1 k + e^{\theta_1 k\times}\dot\theta_2 j + e^{\theta_1 k\times} e^{\theta_2 j\times}\dot\theta_3 i \quad //$$

$${}^0\dot C_3 = ({}^0\omega_{3,0}\times)\,{}^0C_3 \implies {}^0\omega_{3,0}\times = {}^0\dot C_3\,{}^0C_3^T$$
$${}^0C_3 = {}^0C_1\,{}^1C_2\,{}^2C_3 = e^{\theta_1 k\times} e^{\theta_2 j\times} e^{\theta_3 i\times}, \quad \boxed{{}^0C_3^T = e^{-\theta_3 i\times} e^{-\theta_2 j\times} e^{-\theta_1}}$$
$${}^0\dot C_3 = \dot\theta_1 (k\times) e^{\theta_1 k\times} e^{\theta_2 j\times} e^{\theta_3 i\times} +$$
$$+ e^{\theta_1 k\times}(\dot\theta_2 j\times) e^{\theta_2 j\times} e^{\theta_3 i\times} + e^{\theta_1 k\times} e^{\theta_2 j\times}(\dot\theta_3 i\times) e^{\theta_3 i\times}$$
$${}^3\omega_{3,0}\times = \dot\theta_1 (k\times) + e^{\theta_1 k\times}\dot\theta_2 (j\times)\boxed{e^{-\theta_1 k\times}} + e^{\theta_1 k\times}\boxed{e^{\theta_2 j\times}\dot\theta_3 (i\times) e^{-\theta_2 j\times}} e^{-\theta_1 k\times}$$
$$= \dot\theta_1 (k\times) + \dot\theta_2 (e^{\theta_1 k\times} j)\times + \dot\theta_3\, e^{\theta_1 k\times}(e^{\theta_2 j\times} i)\times e^{-\theta_1 k\times}$$
$$= \dot\theta_1 (k\times) + \dot\theta_2 (e^{\theta_1 k\times} j)\times + \dot\theta_3 (e^{\theta_1 k\times} e^{\theta_2 j\times} i)\times \qquad (Q\,x)^{\times} = Q(x\times)Q^T$$

– remove $\times$, same as above.

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Problem 1.

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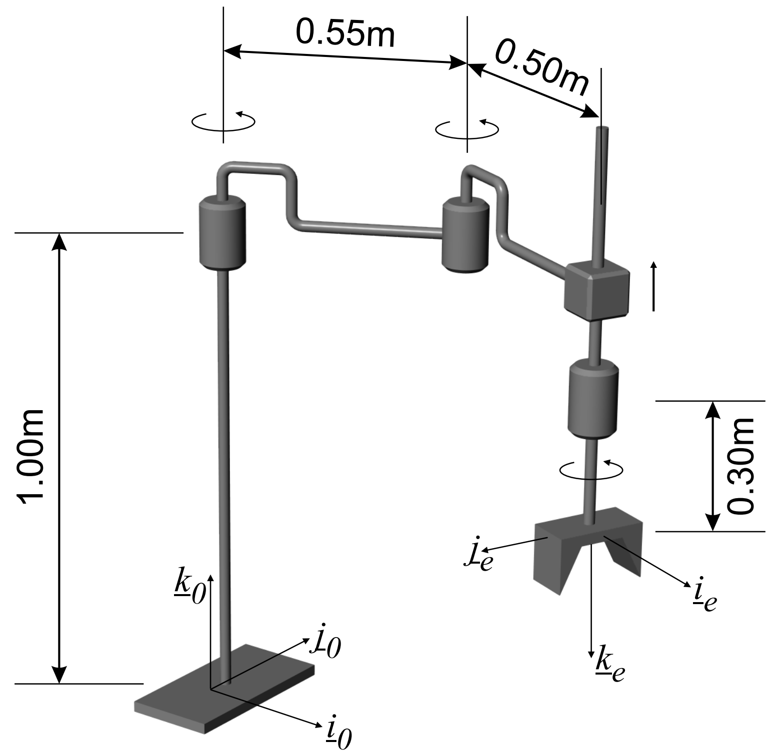
(a) *(2 marks)* What are the columns of ${}^0C_1$? When is ${}^0C_1$ a rotation?
(b) *(2 marks)* How would you find the axis and angle of the rotation ${}^0C_1$?
(c) *(2 marks)* If ${}^0C_1 = {}^0C_1(t)$ is a function of time, what is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?
(d) *(4 marks)* If a point $\underset{\sim}{x}$ has coordinates ${}^0x$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and ${}^1x$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$, what are the homogeneous transformations expressing ${}^1x$ in terms of ${}^0x$ and ${}^0x$ in terms of ${}^1x$?

Problem 2. *(10 marks)*

Consider the following SCARA manipulator:



Assign coordinate systems $\{o_i, \underline{C_i}\}$ to the links (using the Denavit-Hartenberg convention or any other convenient way) and find the homogeneous transformations required for the direct kinematics problem. What are the DH parameters of this SCARA robot?

Problem 3. *(10 marks)*

An oblique wrist has 3 intersecting axes and implements the following kinematic transformation: frame $\underline{C}_1$ is obtained from frame $\underline{C}_0$ by rotating about $\underline{k}_0$ an angle θ_1, frame $\underline{C}_2$ is obtained from frame $\underline{C}_1$ by rotating about $\underline{i}_1$ an angle θ_2 and frame $\underline{C}_3$ is obtained from frame $\underline{C}_2$ by rotating about $\frac{1}{\sqrt{2}}(\underline{k}_2 + \underline{j}_2)$ an angle θ_3. Obtain $\underline{C}_3$ in terms of $\underline{C}_0$ and the angular velocity of $\underline{C}_3$ with respect to $\underline{C}_0$ in terms of $\theta_1, \theta_2, \theta_3$ and $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$ (You do not need to multiply out matrices).

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(a) *(2 marks)* Suppose that

$$\underline{C}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \underline{C}_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

What is the axis and angle of rotation between $\underline{C}_0$ and $\underline{C}_1$?

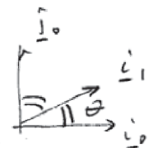
(b) *(3 marks)* If a point $\underset{\sim}{x}$ has coordinates ${}^0x$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and ${}^1x$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$, what are the homogeneous transformations expressing ${}^1x$ in terms of ${}^0x$ and ${}^0x$ in terms of ${}^1x$?

(c) *(2 marks)* If ${}^0C_1 = {}^0C_1(t)$ is a function of time, what is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$, as a function of ${}^0C_1(t)$?

(d) *(3 marks)* Suppose $\{\underset{\sim}{o}_1, \underline{C}_1\}$ is attached to link 1 of a manipulator and $\{\underset{\sim}{o}_0, \underline{C}_0\}$ to the base, and the Denavit-Hartenberg convention has been followed in assigning $\{\underset{\sim}{o}_1, \underline{C}_1\}$, with angle, offset, length and twist given by θ, d, a, α. What is the homogeneous transformation ${}^0T_1$ in terms of θ, d, a, α.

a) axis is $\underline{k}_0$; coords of $\underline{i}_1$ in $\underline{C}_0$ are $\begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$, so

$\underline{i}_1 = \frac{\sqrt{2}}{2}\underline{i}_0 + \frac{\sqrt{2}}{2}\underline{j}_0 \Rightarrow \theta = \pi/4$



b) $\underset{\sim}{x} = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0x = \underset{\sim}{o}_1 + \underline{C}_1\,{}^1x = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1 + \underline{C}_0\,{}^0C_1\,{}^1x$

$\therefore\ {}^0x = {}^0d_1 + {}^0C_1\,{}^1x$

${}^1x = {}^0C_1^{-1}({}^0x - {}^0d_1) = {}^0C_1^T({}^0x - {}^0d_1)$

c) ${}^0\dot{C}_1 = ({}^0\omega_{1,0}\times)\,{}^0C_1 \quad \therefore\ {}^0\omega_{1,0}\times = {}^0\dot{C}_1(t)\,{}^0C_1(t)^T$

d) $${}^0T_1 = \underbrace{\left[\begin{array}{c|c} e^{\theta k\times} & 0 \\ \hline 0^T & 1 \end{array}\right]\left[\begin{array}{c|c} I & dk \\ \hline 0^T & 1 \end{array}\right]}_{\left[\begin{array}{c|c} e^{\theta k\times} & dk \\ \hline 0^T & 1 \end{array}\right]}\underbrace{\left[\begin{array}{c|c} I & ai \\ \hline 0^T & 1 \end{array}\right]\left[\begin{array}{c|c} e^{\alpha i\times} & 0 \\ \hline 0^T & 1 \end{array}\right]}_{\left[\begin{array}{c|c} e^{\alpha i\times} & ia \\ \hline 0^T & 1 \end{array}\right]} = \left[\begin{array}{c|c} e^{\theta k\times}e^{\alpha i\times} & e^{\theta k\times}ai + dk \\ \hline 0^T & 1 \end{array}\right]$$

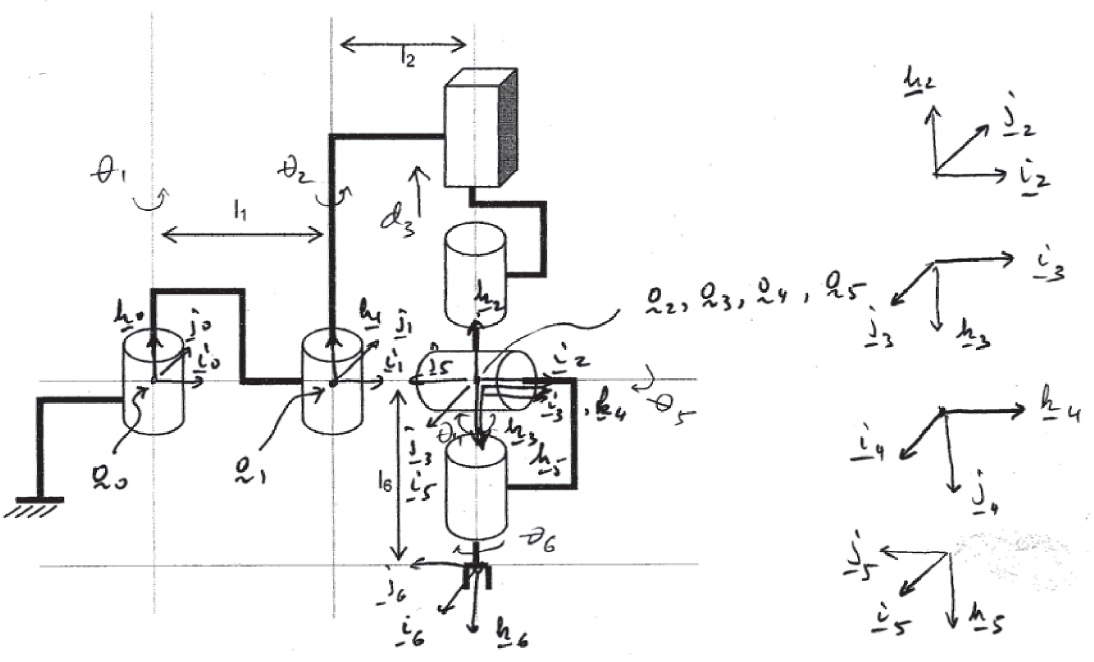
Problem 2.

Consider the SCARA manipulator shown below.

(a) *(10 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 0, \ldots 6$ to the base and to links 1 through 6, using the Denavit-Hartenberg convention, and find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Complete the table of Denavit-Hartenberg parameters.

(b) *(5 marks)* Find manipulator Jacobian and use it to find the manipulator singular configurations. Suppose that somebody suggests that the manipulator should be used to place surface-mount packages on printed circuit boards horizontal to the robot's first axis $\underline{k}_0$. Can you foresee a problem with this suggestion? Explain.

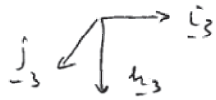
(c) *(5 marks)* Find the gripper angular velocity as a function of the joint rates, in base frame coordinates.



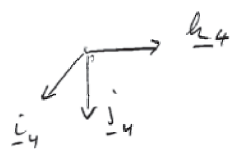
$${}^0T_1 = \begin{bmatrix} e^{\theta_1 k \times} & e^{\theta_1 k \times} l_1 i \\ \hline 0^T & 1 \end{bmatrix}$$ from using Problem 1.d)

$$
{}^{1}T_{2} = \left[\begin{array}{c|c} e^{\theta_2 \underline{k}\times} & e^{\theta_2 \underline{k}\times} l_2 \underline{i} \\ \hline 0^T & 1 \end{array}\right]
$$

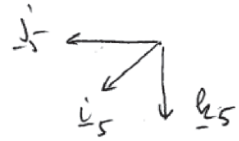
$$
{}^{2}T_{3} = \left[\begin{array}{c|c} e^{\pi \underline{i}\times} & d_3 \underline{k} \\ \hline 0^T & 1 \end{array}\right]
$$



$$
{}^{3}T_{4} = \left[\begin{array}{c|c} e^{(\theta_4+\frac{\pi}{2})\underline{k}\times} e^{\frac{\pi}{2}\underline{i}\times} & 0 \\ 0^T & 1 \end{array}\right]
$$



$$
{}^{4}T_{5} = \left[\begin{array}{c|c} e^{\theta_5 \underline{k}\times} e^{-\frac{\pi}{2}\underline{i}\times} & 0 \\ \hline 0^T & 1 \end{array}\right]
$$



$$
{}^{5}T_{6} = \left[\begin{array}{c|c} e^{\theta_6 \underline{k}\times} & l_6 \underline{k} \\ \hline 0^T & 1 \end{array}\right]
$$

Link	θ	d	a	α
1	(θ_1)	0	l_1	0
2	(θ_2)	0	l_2	0
3	0	(d_3)	0	π
4	$(\theta_4+\frac{\pi}{2})$	0	0	$\pi/2$
5	(θ_5)	0	0	$-\frac{\pi}{2}$
6	(θ_6)	l_6	0	0

b)

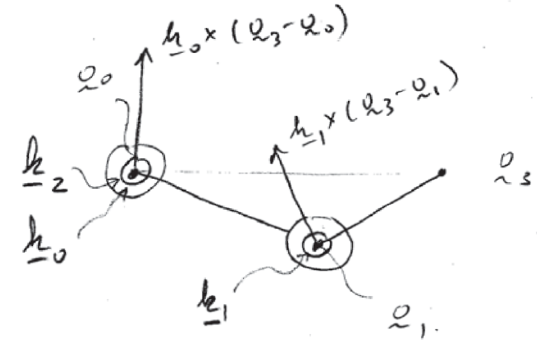
same

$$J = \begin{bmatrix} \underline{h}_0\times(\underset{\sim}{p}_6-\underset{\sim}{p}_0) & \underline{h}_1\times(\underset{\sim}{p}_6-\underset{\sim}{p}_1) & \underline{h}_2 & \underline{h}_3\times(\underset{\sim}{p}_6-\underset{\sim}{p}_3) & \underline{h}_4\times(\underset{\sim}{p}_6-\underset{\sim}{p}_4) & \underline{h}_5\times(\underset{\sim}{p}_6-\underset{\sim}{p}_5) \\ \underline{h}_0 & \underline{h}_1 & 0 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{bmatrix}$$

∴ singularities decouple

$$J \sim \left[\begin{array}{ccc|ccc} \underline{h}_0\times(\underset{\sim}{p}_3-\underset{\sim}{p}_0) & \underline{h}_1\times(\underset{\sim}{p}_3-\underset{\sim}{p}_1) & \underline{h}_2 & & 0 & \\ \hline \underline{h}_0 & \underline{h}_1 & 0 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{array}\right]$$

wrist singular when $\underline{h}_3 \parallel \underline{h}_5$



- only arm singularity when arm fully extended or retracted, i.e. $(\underset{\sim}{p}_3-\underset{\sim}{p}_0)$ aligned to $(\underset{\sim}{p}_3-\underset{\sim}{p}_1)$.

(c) $${}^0\underline{\omega}_{6,0} = \dot\theta_1\underline{h}_0 + \dot\theta_2 \underset{\overset{\parallel}{\underline{h}_0}}{\underline{h}_1} + \dot\theta_4 \underset{\overset{\parallel}{-\underline{h}_0}}{\underline{h}_3} + \dot\theta_5\underline{h}_4 + \dot\theta_6\underline{h}_5$$

$$\underline{h}_4 = \underline{C}_4 k = \underline{C}_0\,{}^0C_4 k$$

$$\underline{h}_5 = \underline{C}_5 k = \underline{C}_0\,{}^0C_5 k.$$

$$\left.\begin{aligned} {}^0C_4 &= {}^0C_1\,{}^1C_2\,{}^2C_3\,{}^3C_4 \\ {}^0C_5 &= {}^0C_4\,{}^4C_5 \end{aligned}\right\} \text{from (b).}$$

$${}^0\omega_{6,0} = (\dot\theta_1 + \dot\theta_2 - \dot\theta_4)k + \dot\theta_5\,{}^0C_4 k + \dot\theta_6\,{}^0C_5 k.$$

Name:
Student No.:

University of British Columbia
Department of Electrical Engineering
ELEC 487 (Fall 1994): Introduction to Robotics
Midterm Examination, November 10, 1994
Closed Book - 50 Minutes
Maximum - 30 marks

Consider the manipulator shown in Figure 1:

(a) Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 6$ to links 1 through 6, find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem, and write down the Denavit-Hartenberg parameters. If a point has coordinates ${}^6x$ in the $\{\underset{\sim}{o}_6, \underline{C}_6\}$ coordinate system, what are its coordinates ${}^0x$ in the $\{\underset{\sim}{o}_0, \underline{C}_0\}$ coordinate system? *(15 marks)*

(b) Write down the manipulator Jacobian and use elementary row operations to locate its singularities. How would you re-design the wrist if this robot were to be used to place components in the plane orthogonal to its prismatic joint axis? *(10 marks)*

(c) Given the gripper approach direction $\underline{k}_6$, what are the possible wrist pitch axes $\underline{k}_4$?. *(5 marks)*

FIGURE 1

Name:
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Department of Electrical Engineering
ELEC 487 (Fall 1993): Introduction to Robotics
Midterm Examination, November 1, 1993
Closed Book - No Calculators - 50 Minutes
Maximum - 30 marks

Problem 1.

(a) Suppose that *(3 marks)*

$$\underline{C}_0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } \underline{C}_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

If a vector $\underline{x}$ has coordinates x_0 in $\underline{C}_0$ and x_1 in $\underline{C}_1$, what are the coordinate transformations expressing ${}^1x$ in terms of ${}^0x$ and ${}^0x$ in terms of ${}^1x$?
(b) Let $\underline{C}_1 = \underline{C}_0 R e^{\theta(t)s\times}$, where R, s are constant. What is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$? *(3 marks)*

Problem 2. Consider the manipulator shown in Figure 1:
(a) Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 6$ to links 1 through 6, find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem, and write down the Denavit-Hartenberg parameters. If a point has coordinates ${}^6x$ in $\{\underset{\sim}{o}_6, \underline{C}_6\}$, system, what are its coordinates ${}^0x$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$? *(10 marks)*

(b) Write down the manipulator Jacobian and use elementary row operations to locate its singularities. *(7 marks)*
(c) What is the angular velocity of the prismatic link in Figure 1 as a function of $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\theta}_3$? *(7 marks)*