Ratthamnoon Prakitpong #63205165

1. $J_{sum} = J_m + J_l$

$= 10^{-4} + .005$

$= .0051$

$T_{peak} = (R\ddot{X}_{max}) J_{sum} + (R\dot{X})B + d_b F_r M_b$

speed reduction

$= 2(5) \cdot .0051 + 2(.5) \cdot .01 + .05(200) \cdot .004 = .101$ Nm

$T_{cont} = (2\dot{X}) B + d_b F_r M_b$

$= .05$ Nm

PG1

s^2+s

2. $K_e = 10000$

$J_e = .001$

$B = .005$

$K_a K_t = 15 = 30(.5)$

$$\frac{X_a(s)}{u(s)} = \frac{K_a K_t K_e}{s(J_e s + B)} = \frac{1.5\times 10^8}{s^2 + 5s}$$

PG2

3. $G_{ol}(s) = \dfrac{K_e K_a K_p K_t}{s(J_e s + B)} = \dfrac{K}{s(J_e s + B)}$

$G_{cl}(s) = \dfrac{G_{ol}}{1+G_{ol}} = \dfrac{K}{K + s(J_e s + B)}$

$= \dfrac{K/J_e}{s^2 + \frac{B}{J_e}s + \frac{K}{J_e}}$

$\rightarrow \omega_n^2 = K/J_e \rightarrow \omega_n = \sqrt{K/J_e}$

$2\xi\omega_n = \dfrac{B}{J_e} \rightarrow \xi = 1.1 \rightarrow K = \dfrac{B^2}{4\xi^2 J_e} = \dfrac{.005^2}{4(1.1^2).001} = .0052$

$K = K_e K_a K_p K_t \rightarrow K_p = K/(K_e K_p K_t) = \dfrac{.0052}{(10000(15))} = 3.44 \times 10^{-8}$

PG 3

4. $G_{cl}(s) = \dfrac{K}{K + s(J_e s + B)}$

$$= \frac{K/J_e}{s^2 + \frac{B}{J_e}s + \frac{K}{J_e}}$$

$K = K_a K_e K_p K_t = (3.44\times10^{-8})(10000)(15) = .0052$

$K/J_e = .0052/.001 = 5.17$

$B/J_e = .005/.001 = 5$

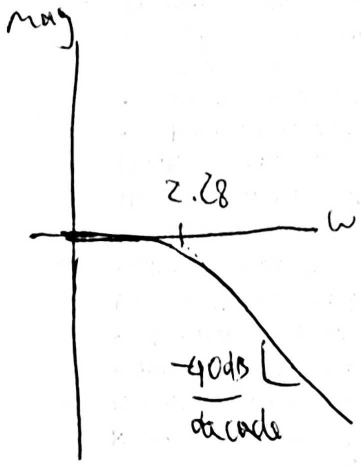
$$G_{cl}(s) = \frac{5.17}{s^2 + 5s + 5.17}$$

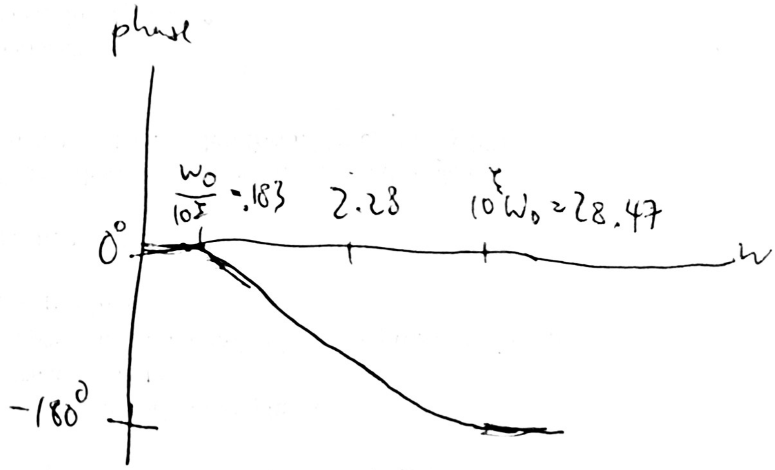
$$= \frac{1}{\frac{s^2}{5.17} + \frac{5}{5.2}s + 1}$$

PG 4

$\omega_0 = 2.28$

$\dfrac{2\zeta}{\omega_0} = \dfrac{5}{5.2} \Rightarrow \zeta = \dfrac{2.28}{2}\left(\dfrac{5}{5.2}\right) = 1.096 < 0.5$





5.

$u = K_p x_r - x_a$

$x_a = \frac{K_e}{s}\Omega \rightarrow \dot{x}_a(t) = K_e \Omega(t)$

$\Omega = \frac{K_a K_t}{J_e s + B} u$ $\quad u = K_p x_r - x_a \rightarrow J_e \dot{\Omega} + B\Omega = K_a K_t (K_p x_r(t) - x_a(t))$

$\cancel{J_e}\,\dot{\Omega} = -\frac{B}{J_e}\Omega + \frac{K_a K_t K_p}{J_e} x_r - \frac{K_a K_t}{J_e} x_a$

$[\dot{y}] = [A][y] + [B][x]$

$[y] = \begin{bmatrix} x_a \\ \Omega \end{bmatrix} \qquad [x] = [x_r]$

$[A] = \begin{bmatrix} 0 & K_e \\ -\frac{K_a K_t}{J_e} & -\frac{B}{J_e} \end{bmatrix} \qquad [B] = \begin{bmatrix} 0 \\ +\frac{K_a K_t K_p}{J_e} \end{bmatrix}$

$\rightarrow [y(K+1)] = [\Phi(T)][y(K)] + [H(T)][x(K)]$

$[\Phi(T)] = e^{[A]T} \approx [I] + [A]T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & K_e T \\ -\frac{K_a K_t T}{J_e} & -\frac{BT}{J_e} \end{bmatrix}$

$= \begin{bmatrix} 1 & K_e T \\ -\frac{K_a K_t T}{J_e} & 1 - \frac{BT}{J_e} \end{bmatrix}$

$[H(T)] = \left(\int_0^T [\Phi(T)]\,dt\right) \cdot [B] = \left(T[I] + \frac{T^2}{2}[A]\right) \times [B]$

$= \left(\begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} + \begin{bmatrix} 0 & \frac{T^2}{2}K_e \\ -\frac{T^2 K_a K_t}{2J_e} & -\frac{T^2 B}{2J_e} \end{bmatrix} \right) \times \begin{bmatrix} 0 \\ \frac{K_a K_t K_p}{J_e} \end{bmatrix}$

FGJ

$= \begin{bmatrix} \frac{T^2}{2}\frac{K_a K_t K_p K_e}{J_e} \\ \left(T - \frac{T^2 B}{2J_e}\right)\left(\frac{K_a K_t K_p}{J_e}\right) \end{bmatrix}$

Where $[y(k)] = \begin{bmatrix} x_a(k) \\ \Omega(k) \end{bmatrix}$ & $[x(k)] = [x_r(k)]$

6a. $K_d = 10$

$T_s = 1\,ms = .001$

$$K_d \frac{1-e^{-sT_s}}{s} = 10\,\frac{1-e^{-s(.001)}}{s}$$

FG6

$$G_{o1}(s) = \frac{K_d K_a K_t K_e}{s^2(Js+B)}\left(1-e^{-sT_s}\right)$$

```
% MATLAB
Go11s=tf([Kd Ka Kt Ke],[J B 0 0]);
Go11z=c2d(Go11s, Ts, 'zoh');
Go12s=tf([Kd Ka Kt Ke],[J B 0 0],'Input Delay',Ts);
Go12z=c2d(Go12s,Ts, 'zoh');
Go1z=Go11z - Go12z;
```

$$\frac{X_a(z)}{U(z)} = G_{o1}(z) = z^{-1}\frac{(b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + \ldots}{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5}}$$

$b_0 = 0.2497$	$a_0 = 1$
$b_1 = -.0000$	$a_1 = -5.9900$
$b_2 = -2.2416$	$a_2 = 14.9501$
$b_3 = 3.9801$	$a_3 = -19.9003$
$b_4 = -2.2366$	$a_4 = 14.9004$
$b_5 = -.0000$	$a_5 = -5.9502$
$b_6 = .2478$	$a_6 = .9900$

~~deg~~ 6a continue.

$$G_d(z) = z^{-1}\frac{(b_0 + b_1 z^{-1} \cdots b_6 z^{-6})}{(a_0 + a_1 z^{-1} \cdots a_6 z^{-6})} = z^{-1}\frac{B(z^{-1})}{A(z^{-1})}$$

$\deg(B) = 6 \qquad d = 1$

$\deg(A) = 6$

$\deg(R) = d + \deg(B) + -1 = 6 \rightarrow R = 1 + r_1 z^{-1} \cdots r_6 z^{-6}$

$\deg(S) = \deg(A) - 1 = 5 \rightarrow S = S_0 + S_1 z^{-1} \cdots S_5 z^{-5}$

$A_m(z^{-1}) = 1 + m_1 z^{-1} + m_2 z^{-2} = 1 - 2e^{-\xi_m w_m T}\cos(w_m T\sqrt{1-\xi_m^2})z^{-1} + e^{-2\xi_m w_m T}z^{-2}$

$B_m(z^{-1}) = B(z^{-1})b_m = B(z^{-1})T_s$

$AR + z^{-1}BS = A_m(z^{-1})$:

$(a_0 + a_1 z^{-1} \cdots a_6 z^{-6})(1 + r_1 z^{-1} \cdots r_6 z^{-6}) + z^{-1}(b_0 + b_1 z^{-1} \cdots b z^{-6})(s_0 + s_1 z^{-1} \cdots s_5 z^{-5})$

$= (1 + m_1 z^{-1} + m_2 z^{-2})$

$1 = a_0$

$m_1 = a_0 r_1 + a_1 + b_0 S_0$

$m_2 =$ ⋮

⋮

so on so forth

PG7

```
% MATLAB
s1 = (-2ξm wm + sqrt(4ξm^2 wm^2 - 4wm^2))/2;
s2 = (-2ξm wm - sqrt(4ξm^2 wm^2 - 4wm^2))2;
Gz_ss = ss(Gd(z));
K = place(Gz_ss.A, Gz_ss.B, [s1 s2]);
```

pole placement controller where $\xi_m = 1.1$ & $w_m = 10$

6a.
$$u(k) = \frac{t_0 X_r(k) - S(z^{-1}) X_a(k)}{R(z^{-1})}$$

$$u(k)(1 + r_1 z^{-1} \dots r_6 z^{-6}) = t_0 X_r(k) - (s_0 + s_1 z^{-1} \dots s_5 z^{-5}) X_a(k)$$

$$u(k) = -(r_1 u(k-1) + r_2 u(k-2) \dots r_6 u(k-6)) + t_0 X_r(k)$$
$$- (s_0 X_a(k) + s_1 X_a(k-1) \dots s_5 X_a(k-5))$$

6b. $e(z) = t_0 X_r(\cancel{z}) - y(\cancel{z})$

$$= t_0 X_r(\cancel{z}) - S(z^{-1}) X_a(\cancel{z})$$

$$\rightarrow X_a(k) = G_{01}(z^{-1}) u(k) \rightarrow X_a(k) = \left(\frac{1}{G_{01}(z^{-1})} + \frac{S(z^{-1})}{R(z^{-1})}\right)^{-1} \frac{t_0}{R(z^{-1})} X_r(k)$$

$$\frac{X_a(k)}{X_r(k)} = \frac{t_0 G_{01}(z^{-1})}{R(z^{-1}) + G_{01}(z^{-1}) S(z^{-1})} \quad \cancel{X_r(k)}$$

$$e(z) = t_0 \left(1 - \frac{S(z^{-1}) G(z^{-1})}{R(z^{-1}) + G(z^{-1}) S(z^{-1})}\right) X_r(\cancel{z})$$

$$X_r(t) = U \rightarrow X_r(z) = \frac{U}{1 - z^{-1}}$$

$$e_{ss} = \lim_{z \to 1} (1 - z^{-1}) e(z) \frac{U}{(1 - z^{-1})} = \lim_{z \to 1} t_0 U \left(1 - \frac{S(z^{-1}) G(z^{-1})}{R(z^{-1}) + G(z^{-1}) S(z^{-1})}\right)$$

$$= t_0 U \left(1 - \frac{(s_0 + s_1 \dots s_5)(b_0 + b_1 \dots b_6)/(a_0 + a_1 \dots a_6)}{(1 + r_1 + \dots r_6) + ((s_0 + s_1 \dots s_5)(b_0 + b_1 \dots b_6)/(a_0 + a_1 \dots a_6))}\right)$$

Pg 8