34929091

FINAL EXAMINATION FOR
MECH 463
MECHANICAL VIBRATIONS

13TH DECEMBER 2013

Time: 2 hrs. 30 mts. Max. Available Mark: 60

READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Please write your **name and student number** on the answer booklet(s).
2. This exam booklet has 5 pages including this page.
3. ANSWER ALL QUESTIONS.
4. Your mark in this exam must be **AT LEAST 30 OUT OF 60 to pass.**
5. One letter-sized formula sheet, written/typed on both sides, is allowed.
6. ONLY non-programmable calculators are allowed.
7. Your answer sheets will be evaluated both for the CORRECTNESS of the procedure and the numerical ACCURACY. See the SOLUTION KEY ON CONNECT FOR MARKING SCHEME.

This space is intentionally left blank. Continue onto the next page for the exam questions.

Question 1 **Concepts tested:** FBD, Forced Vibration, Isolation System Design

Consider a motorcycle shown in Figure.(1). The total mass, including the rider, is 250 kg. The motorcycle travels **with constant horizontal velocity** v over a terrain, approximately sinusoidal with a distance between peaks of 10 m and the distance from peak to valley is 10 cm.

Figure 1: Figure for question 1.

(a) Write the equations of motion governing the displacement of the rider using a *clearly labelled* free body diagram. If the vibration experienced by the rider is to be minimized, which of the two transmissibilities: force or displacement is appropriate to be used in design? **(6 marks)**

?

(b) Design the isolation system (spring k and viscous damping coefficient c of the suspension) such that the maximum transmissibility is restricted to 4 and the transmissibility in the velocity range 40 km/hr $\leq v \leq$ 100 km/hr does not exceed 0.8. **(8 marks)**

(c) Is the value of k you found in part (b) acceptable with regard to the static deflection? Which operating speed would you choose to reduce the static settlement while meeting the isolation requirements in part (b)? An index of ride comfort is the rate of change of acceleration or jerk, that is, jerk $= \frac{d^3x}{dt^3}$, where x is displacement and t is time. What is the relation between jerk amplitude and displacement amplitude in the steady state? **(6 marks)**

$v = 90 Km/h$

$\omega_n = 4.62$

$K = 5335.8$

$C = 298$

$\omega_2 = 17.45$

$r = 1.511$

$\frac{\pi V}{5} =$

Question 2 **Concepts tested:** Kinematics, FBD, Shaky Table Lab

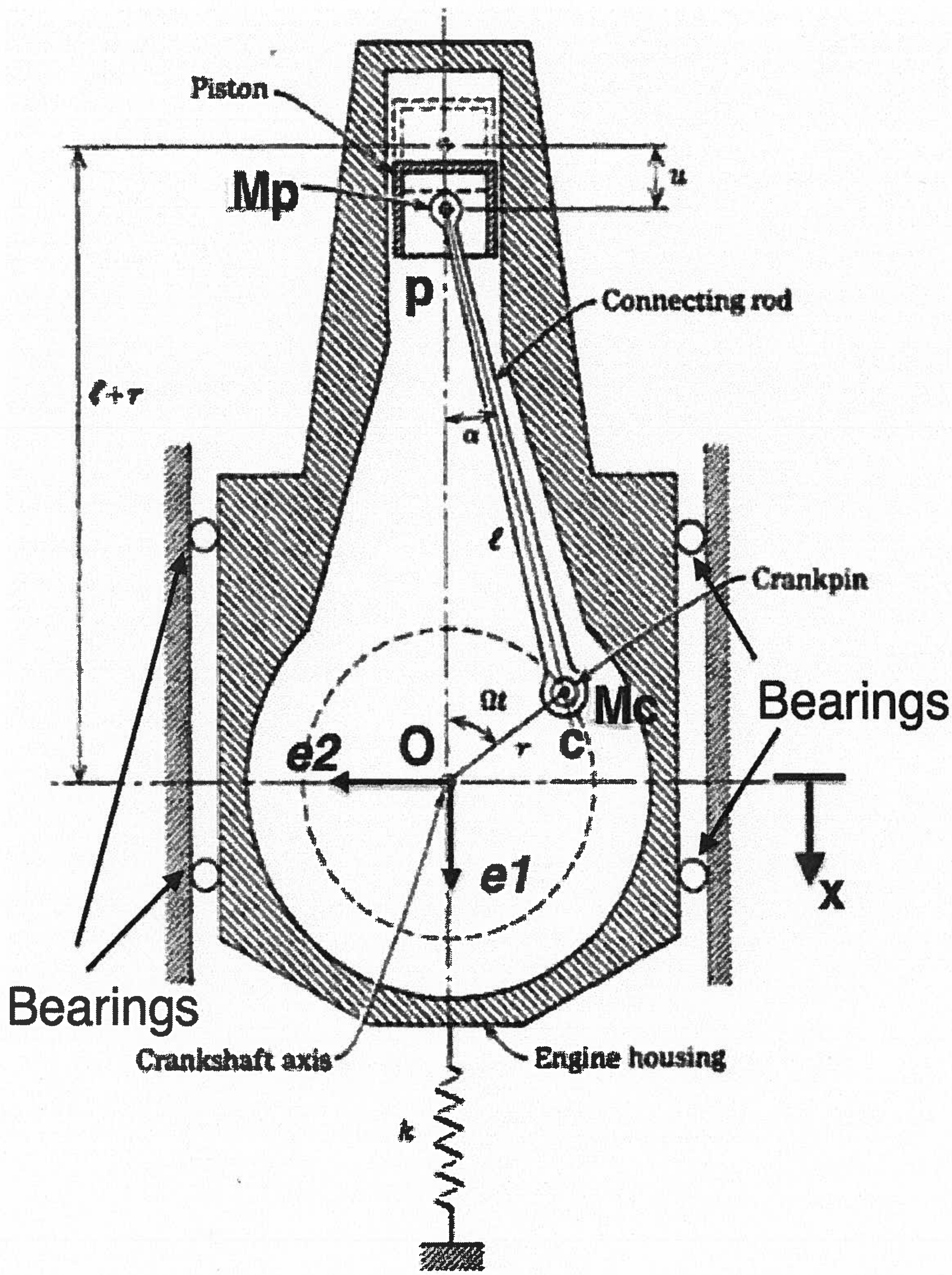

Figure 2: Figure for Question 2, parts (a) and (b). Note the distances: $OC = r$, $CP = l$. All links are rigid and Ω is constant.

(a) A single cylinder engine with unbalanced masses lumped at the crank pin (M_c) and piston head (M_p) is shown in Figure.(2). Using kinematics show that the *absolute inertial* acceleration of M_p is $\boldsymbol{a_p} \approx \ddot{x}\boldsymbol{e}_1 + \Omega^2 r\left[\cos\Omega t + \frac{r}{l}\cos 2\Omega t\right]\boldsymbol{e}_1$ and that of M_c is $\boldsymbol{a_c} = \ddot{x}\boldsymbol{e}_1 + r\Omega^2\cos\Omega t\boldsymbol{e}_1 + r\Omega^2\sin\Omega t\boldsymbol{e}_2$, Ω is **constant** and $+ve$ in clockwise direction. The unit vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are as shown in Figure.(2). **You may find the identity $\sqrt{1-x} \approx 1 - \frac{1}{2}x$ useful in approximating the acceleration $\boldsymbol{a_P}$. You may also find the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$ useful.** **(8 marks)**

(b) Using the accelerations from part (a) construct the Free Body Diagram (FBD) for the engine housing (of mass M_e) and show that the governing equation of motion for vertical vibrations is $M\ddot{x} + kx = -(M_p + M_c)r\Omega^2\cos\Omega t - M_p\frac{r^2}{l}\Omega^2\cos 2\Omega t$ where $M = M_e + M_c + M_p$. Compute the steady state displacement of the forced vibration response, **ignoring the homogeneous part** for the parameters: $r = 0.2$ m, $l = 0.6$ m, $\Omega = 600$ rpm, $M_p = 3.2$ kg, $M_c = 0.9$ kg, $M = 227$ kg and $k = 2 \times 10^6$ N/m. What is the influence of the ratio $\frac{r}{l}$ on the forces exerted by the unbalanced masses? **(8 marks)**

(c) Having seen the vibrations due to mass unbalance, what is the purpose of the added weight in the cut away of a reciprocating hand saw shown in Figure.(3)? How does it improve the performance of the saw? What is the source of unbalance? Be specific in your answers. **(4 marks)**

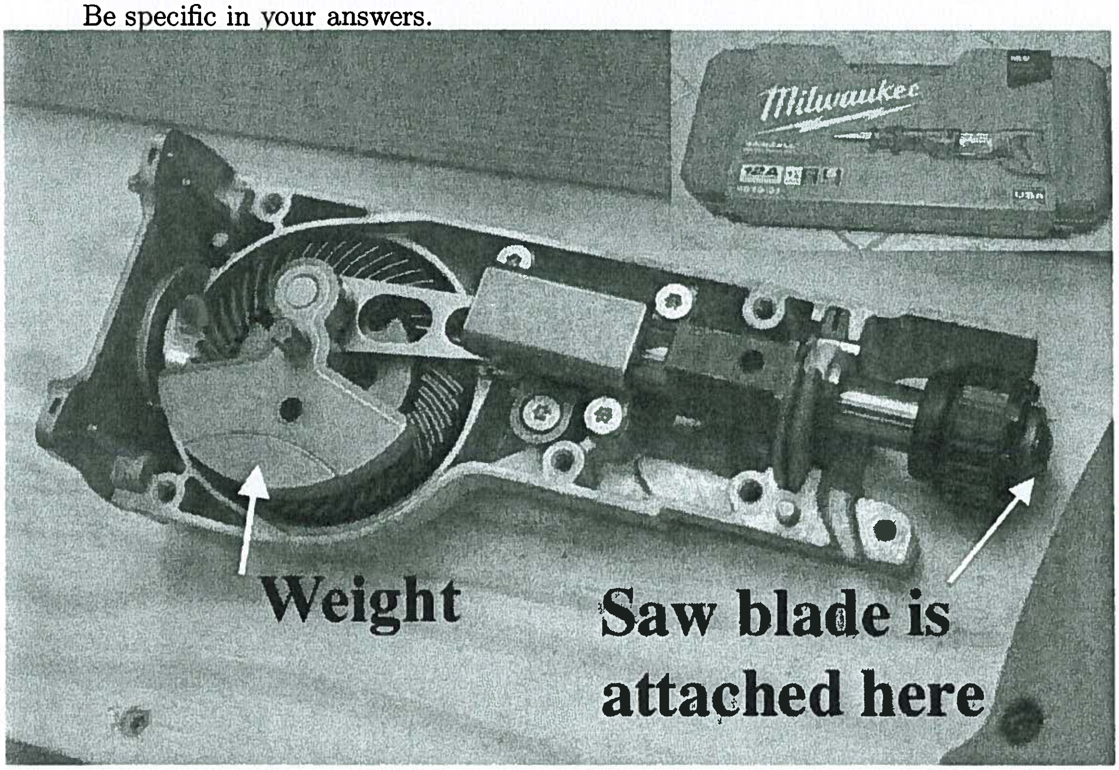

Figure 3: Figure for Question 2 part (c). See the inset for the complete hand tool.

Question 3 **Concepts tested:** Equivalent Systems, General Excitation

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(a) The propeller of a ship, of weight $10^5 N$ and polar mass moment of inertia 10,000 kg-m^2, is connected to the engine through a hollow stepped steel propeller shaft, as shown below. Assuming that water provides viscous damping ratio of $\zeta = 0.1$, determine the torsional vibratory response of the propeller when the engine induces a harmonic angular displacement of $0.05 \sin 314.16t$ rad at the base (point A) of the propeller shaft. You may find the formula $k_\theta = \frac{GI_p}{L}$ for torsional stiffness of a shaft useful here. I_p is polar area moment of inertia which depends on the cross sectional geometry. For a hollow shaft it is given by $I_p = \frac{\pi}{32}\left[d_o^4 - d_i^4\right]$, where d_o and d_i are respectively the outer and the inner diameters of the shaft. **You can ignore the homogeneous solution.** **(15 marks)**

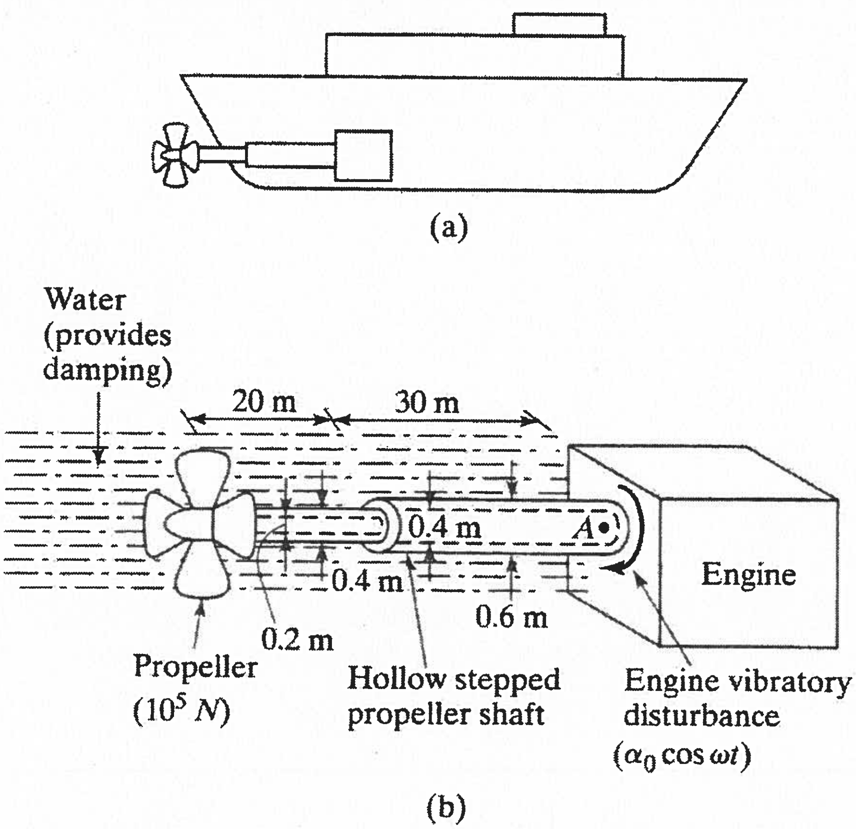


Figure 4: Figure for Question 3.

(b) Explain how the principle of superposition is used to arrive at the convolution integral $x_p(t) = \int_{t=0^+}^{t} h(t-\tau) f(\tau) d\tau$ to find the response due to arbitrary force $f(t)$. You can sketch the diagrams as appropriate to aid your explanation. **(5 marks)**

BEST OF LUCK AND HAPPY HOLIDAYS!

limitations: - point mass, no mom. of inertia - no slippage - motion of helicopter ignored - rigid attachments - wind resistance

#co-od = #points * #coord p...

MECH 463: Summary

Degrees of Freedom: Minimum number of independent co-ordinates required to completely specify a system's motion. A single degree of freedom system requires one co-ordinate, two DOF system requires 2 co-ordinates *etc.* #DOF = #co-od − #constraints

Vibration Analysis: Develop a model → Formulate equations of motion using Newton/D'Alembert (FBD) or Energy methods → Solve for response → Design calculations. **Equivalent systems are valid at a single spatial location where the response is sought.** For springs in series $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$ for springs in parallel $k_{eq} = k_1 + k_2$. In general equivalent mass is obtained from kinetic energy expression: $KE = \frac{1}{2}m_{eq}\dot{x}^2$ and equivalent spring constant is obtained from potential energy expression: $PE = \frac{1}{2}k_{eq}x^2$

Planar Kinematics: $r = re_1$ (displacement); $\frac{dr}{dt} = \dot{r} = \dot{r}e_1 + r\dot{\theta}e_2$ (velocity); $\frac{d^2r}{dt^2} = \ddot{r} = \left[\ddot{r} - r\dot{\theta}^2\right]e_1 + \left[2\dot{r}\dot{\theta} + r\ddot{\theta}\right]e_2$ (accln.) e_2 is e_1 rotated 90^0 in the direction of the angular velocity vector ω non-viscous: $\omega_n = 2\pi/t_d$

Harmonic Response of a Viscously Damped SDOF System (Underdamped): Equation of motion: $m\ddot{x}+c\dot{x}+kx = F\cos(\omega t)$; $x = x_h + x_p$ $x_h = e^{-\zeta\omega_n t}\left[C_1\cos(\omega_d t) + C_2\sin(\omega_d t)\right]$; $x_p = X\cos(\omega t - \phi)$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$, $X = \frac{F}{\sqrt{[k-m\omega^2]^2+[c\omega]^2}} = \frac{F}{k}\frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}} = \delta_{st}\frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$; $\delta_{st} \equiv \frac{F}{k}$, $\tan\phi = \frac{c\omega}{k-m\omega^2} = \frac{2\zeta r}{1-r^2}$, $r \equiv \frac{\omega}{\omega_n}$, $\zeta = \frac{c}{2m\omega_n}$

Note: (a) $F = m_u e\omega^2$ for an unbalanced eccentric mass m_u at a distance e from center of rotation; (b) Set $\zeta = 0$ and $\omega_d = \omega_n$ to obtain undamped SDOF response; (c) Initial conditions ALWAYS apply on the TOTAL response.

Natural Frequency: Free vibration of an undamped SDOF system takes place at it's natural frequency $\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{g}{\delta_{st}}}$ for linear vibrations and $\omega_n = \sqrt{\frac{k_{eq}}{J_{eq}}} = \sqrt{\frac{g}{\theta_{st}}}$ for torsional vibrations. MDOF systems have more than one natural frequency and each natural frequency has a characteristic modeshape associated with it. They are obtained by solving the eigenvalue problem $\boldsymbol{Ku} = \omega^2\boldsymbol{Mu}$.

Damping Measures: $\delta = \frac{1}{N}\ln\left(\frac{x_1}{x_{1+N}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \approx 2\pi\zeta$, $\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}}$, $Q = \frac{\text{Dynamic displacement at } \omega=\omega_n}{\text{Static displacement, } \delta_{st}} = \frac{1}{2\zeta} = \frac{\omega_n}{\Delta\omega}$

Isolation System Design ↳ 0.5 S.S: $x_{vel} = \omega X$, $y_{vel} = \omega Y$ } $TR_D = TR_V = TR_{Acc}$ DMF $= \left|\frac{X}{\delta_{st}}\right|$

$TR = \frac{F_t}{F} = \frac{\sqrt{k^2+(c\omega)^2}}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$; and the displacement transmissibility is given by $TR_d = \frac{X}{Y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$, where $r = \frac{\omega}{\omega_n}$. Maximum value occurs at $r = 1$. $TR < 1$ above $r = \sqrt{2}$. For rotating unbalance replace $F = m_u e\omega^2$ → $X = X\cos(\omega t)$; $Y = Y\cos(\omega t)$

Forced Response (General): Use the look-up table→Use Fourier series if the force is periodic→ Use convolution integral $x_p = \int_0^t h(t-\tau)f(\tau)d\tau$, where $h(t) = \frac{1}{m\omega_d}e^{-\zeta\omega_n t}\sin(\omega_d t)$. Convolution integral is the most general method available for arbitrary forces which is suitable for *computer* implementation. Harmonic Response → Set $\omega = 0$ to get step response → Differentiate step response to get impulse response. $\Delta Work = \frac{1}{T_0}\int_0^T W(t)dt = \frac{1}{T_0}\int_0^T f\frac{dx}{dt}dt$

*Main principle in isolation is to slow down dynamics of combined system by reducing its ω_n relative to ω such that $r > \sqrt{2}$ and $TR < 1$

Fourier Series: $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)\right] = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$, $\omega_0 = \frac{2\pi}{T}$;

$a_0 = \frac{2}{T}\int_0^T f(t)dt$, $a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt$, $b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$, $c_n = \frac{a_n - jb_n}{2}$, $j = \sqrt{-1}$

Fourier Transform : $F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$, Forward Transform $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega$, Inverse Transform

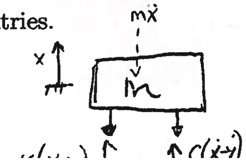
Frequency Response Function: Ratio of output to input of a linear system in frequency (Fourier) domain. Displacement FRF (Receptance)$=\frac{X(j\omega)}{F(j\omega)} = \frac{1}{k-m\omega^2+j\omega c}$. This allows coupling **systems** in frequency domain much like springs coupled in series or parallel.

Absorber Design: Select the spring constant k_a and mass m_a of the absorber such that the natural frequency of the absorber unit $\omega_a = \sqrt{\frac{k_a}{m_a}}$ is tuned to the forcing frequency ω. Given H_{sys} of ANY system, design the absorber H_a such that the combined system's frequency response $\frac{1}{H} = \frac{1}{H_{sys}} + \frac{1}{H_a}$ has the desired response characteristics. The absorber introduces two resonances which limit it's operating range. The maximum displacement of an undamped absorber is $X_a = \frac{F_0}{k_a}$ where F_0 is the amplitude of the applied harmonic force on the main system. Damping reduces absorber's effectiveness. m_a and X_a are design constraints.

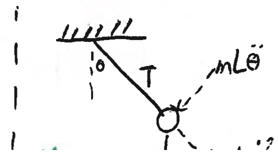
Orthogonality Conditions: The eigenvectors (modes) of the undamped system obtained from $\boldsymbol{Ku} = \lambda\boldsymbol{Mu}$, $\lambda = \omega^2$ obey the orthogonality conditions $\boldsymbol{u}_i^T\boldsymbol{Mu}_j = m_i\delta_{ij}$ $\boldsymbol{u}_i^T\boldsymbol{Ku}_j = k_i\delta_{ij}$ $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$. These conditions can be interpreted in terms of work. For stiffness orthogonality, the work done by elastic forces associated with mode j do no work on displacements u_i. These orthogonality conditions enable the transformation $\boldsymbol{x} = \boldsymbol{\Phi q}$ from physical co-ordinates to normal/principal co-ordinates which uncouple MDOF matrix equations.

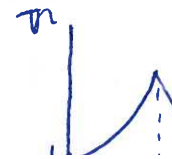
vibration absorber: auxiliary system attached to main system such that at tuned frequencies, absorber exerts a counter vibrating force at point of attachment. vibration reduction in main system is achieved at expense of introducing two additional resonances and subjecting absorber spring-mass unit to undergo sustained resonant oscillation

Principal and Normal Co-ordinates: Both co-ordinates uncouple equations of motion by transforming mass and stiffness matrices into diagonal matrices. In normal co-ordinates, mass matrix is identity matrix and stiffness matrix contains squared undamped natural frequencies as the diagonal entries.



$\Sigma F_o = K(y-x) + C(\dot{y}-\dot{x}) = 0$
$\Sigma F_M = K(x-y) + C(\dot{x}-\dot{y}) = 0$





mass w/ spring mass: ... mass w/ beam: ... mass middle w/ beam: ... rack pinion: rack→$m_{eq} = m + J_0/R^2$ ... for with axial load: EA/l ... cantilever beam, end load: $3EI/l^3$ ... hollow shaft, torsion: $\pi G(r_o^4 - r_i^4)/2l$ ... beams: $48EI/l^3$ ... shaft, torsion: ...

$W/(4a)$ ... $K_{eq} = \frac{K_1 K_2}{4(K_1 + ...)}$

$(J_G + mL^2)\ddot\theta + mgL\sin\theta = 0$

$m\ddot{x} + \frac{kx}{4} = 0$

TOPIC 2.1

$T = \frac{1}{2}m\dot{x}^2$; $T = \frac{1}{2}J_G\dot\theta^2 \rightarrow$ finds M_{eq}, J_{eq}

$U = \frac{1}{2}Kx^2 \rightarrow$ find K_{eq}

$K_{eq} = K_1\left(\frac{l_1}{l}\right)^2 + K_2\left(\frac{l_2}{l}\right)^2$

Torsion

$\Sigma M_O = 0$, write equation:

$J\ddot\theta + [\ \ ]\theta = 0$; $\omega_n = \sqrt{[\ ]/J}$

collision

$(M+m)\ddot{y} + Ky = mg$

$m\sqrt{2gl} = (M+m)\dot{y}(0)$

UNDAMPED VIBRATIONS

Free Vibrations

$m\ddot{x} + Kx = 0$

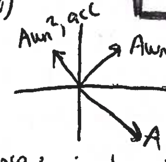
$x = A_1\cos(\omega_n t) + A_2\sin(\omega_n t)$

$x = x_0\cos(\omega_n t) + \frac{\dot{x}_0}{\omega_n}\sin(\omega_n t)$

$\rightarrow$ convert: $x(t) = A\cos(\omega_n t \pm \phi)$ — lead

$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}$

$\phi = \tan^{-1}(\dot{x}_0/x_0\omega_n)$



- $\frac{Acc}{\omega_n^2} = \frac{V}{\omega_n} = Amp.$ response: - in phase with force $\omega < \omega_n$
  - $-90°$ lag at $\omega = \omega_n$
  - $-180°$ lag at $\omega > \omega_n$

Forced vibrations

$m\ddot{x}_p + Kx_p = F_0\cos(\omega t)$ $\quad X = \frac{F_0}{K - m\omega^2}$ $\quad x(t) = x_0\cos(\omega_n t) + \frac{\dot{x}_0}{\omega_n}\sin(\omega_n t) + \frac{F_0\cos(\omega t)}{K - m\omega^2}$

$x_p = X\cos(\omega t)$ — switch to sin if need be

DAMPED VIBRATIONS

I. Free

$m\ddot{x} + c\dot{x} + Kx = f$

torsion: $c = 2\zeta\sqrt{JK_{eq}}$

$\neq x_p = F$ ... $\sqrt{(K_{eq} - J\omega^2)^2}$

$x(t) = e^{-\zeta\omega_n t}A\cos(\omega_d t - \phi)$ for $\zeta < 1$

$x(t) = e^{-\zeta\omega_n t}[x_0 + (\omega_n x_0 + \dot{x}_0)t]$ for $\zeta = 1$

$x(t) = e^{-\zeta\omega_n t}\left[\frac{\omega_d x_0 + \zeta\omega_n x_0 + \dot{x}_0}{2\omega_d}e^{\omega_d t} + \frac{\omega_d x_0 - \zeta\omega_n x_0 - \dot{x}_0}{2\omega_d}e^{-\omega_d t}\right]$

$A = \sqrt{x_0^2 + \left[\frac{\zeta\omega_n x_0 + \dot{x}_0}{\omega_d}\right]^2}$

$\tan\phi = \frac{\zeta\omega_n x_0 + \dot{x}_0}{\omega_d x_0}$

II. Forced

$m\ddot{x}_p + c\dot{x}_p + Kx_p = F_0\cos\omega t$

$x_p(t) = X\cos(\omega t - \phi)$

$X = \frac{F_0}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}$

$x(t) = e^{-\zeta\omega_n t}[C_1\cos(\omega_d t) + C_2\sin(\omega_d t)] + X\cos(\omega t \ldots$

GUEST SPEAKER

$PE = \frac{1}{2}K_2\left(\frac{xl_1}{l_2}\right)^2 + \frac{1}{2}K_1x^2 = \frac{1}{2}K_{eq}x^2$

Jeffcott rotor, one DOF ($\Omega = 0$)

$m\ddot{x} + Kx = m\varepsilon\Omega^2\cos\Omega t$

$m\ddot{y} + Ky = m\varepsilon\Omega^2\sin\Omega t$

$r = x + iy \rightarrow m\ddot{r} + Kr = m\varepsilon\Omega^2 e^{i\Omega t} \rightarrow r = \frac{\varepsilon\Omega^2}{\omega_n^2 - \Omega^2}e^{i\Omega t} = Ae^{i\Omega t}$

anisotropic bearings $\longrightarrow r = r_+e^{i\Omega t} + r_-e^{-i\Omega t}$; $r_+ = \frac{\Omega^2\varepsilon}{2}\frac{(\omega_x^2 + \omega_y^2 - 2\Omega^2)}{(\omega_x^2 - \Omega^2)(\omega_y^2 - \Omega^2)}$

$r_- = \frac{\Omega^2\varepsilon}{2}\frac{(\omega_y^2 - \omega_x^2)}{(\omega_x^2 - \Omega^2)(\omega_y^2 - \Omega^2)}$

- forward whirl: $r_+ > r_-$
- backward: $r_+ < r_-$

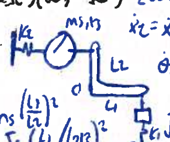
damped rotors $\longrightarrow \ddot{r} + 2\zeta\omega_n\dot{r} + \omega_n^2 r = \varepsilon\Omega^2 e^{i\Omega t}$

$r = (\varepsilon\Omega^2)/(\omega_n^2 - \Omega^2 + i2\zeta\omega_n\Omega)e^{i\Omega t}$

$e^{i\Omega t} = r \rightarrow \rho = \frac{\varepsilon\Omega^2}{\omega_n^2 - \Omega^2} + P_1e^{-i(\omega_n + \Omega)t} + P_2e^{i(\omega_n - \Omega)t}$

$[KE] = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot\theta^2 + \frac{1}{2}m_s\dot{x}_2^2 + \frac{1}{2}J_s\dot\theta_s^2 = \frac{1}{2}m_{eq}\dot{x}^2$

$x_2 = x\frac{L_1}{L_2}$ $\quad \theta_s = x_2/r_s$



$M_{eq} = m + \frac{J_0}{L_1^2} + m_s\left(\frac{L_1}{L_2}\right)^2 + J_s\left(L_1/L_2r_s\right)^2$

EXTRA

$\int u\,dv' = uv - \int u'v$ [parts]

substitution

$f(x)$	$y_p(t)$
a	$ax^n + bx^{n-1} + \ldots$
	ae^{Kx}
x	$(ax+b)e^{Kx}$
$x^n e^{Kx}$	$(ax^n + bx^{n-1} + \ldots)e^{Kx}$
...	$a\sin px + b\cos px$
$\sin px$ / $\cos px$	$e^{Kx}(a\sin px + b\cos px)$

dynamic stress

$\sigma = E\frac{\delta}{L}$

N forces!

MAX TR is at $r = 1$

let $r^2 = x$, quadratic eqn.

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

WORKED PROBLEMS

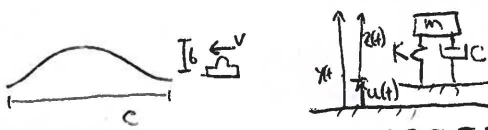
$u(t) = b\sin\left(\frac{\pi v t}{c}\right)$

m: mass of car

K: susp/tire stiffness

c: friction

u: ground disp.



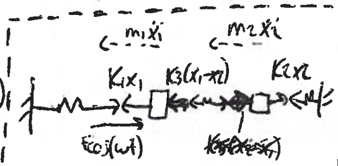
$y(t) = u(t) + z(t)$

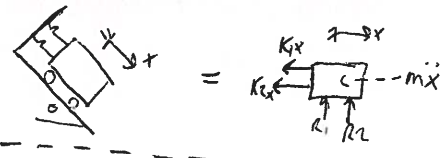
$m\ddot{z} + Kz = -m\ddot{u}$

$\rightarrow z_p(t) = A\cos\left(\frac{\pi v t}{c}\right) + B\sin\left(\frac{\pi v t}{c}\right)$

$\rightarrow$ solve for B, plug into ODE

$\rightarrow$ find $z_p(t)$

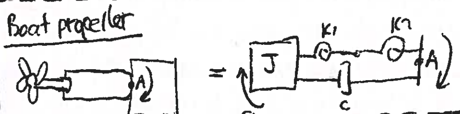




$m\ddot{x} + (K_1 + K_2)x = \ldots$

if a weight moves down x at A, then B moves down $2x$

Boat propeller



α = given

$J\ddot\theta + c(\dot\theta - \dot\alpha) + K_{eq}(\theta - \alpha) = 0$

$J\ddot\theta + c\dot\theta + K_{eq}\theta = \underbrace{c\dot\alpha + K_{eq}\alpha}_{\text{forcing function}}$

- convert force to "$A\sin(\omega t - \beta)$"
  - solve A, β using phase diag.

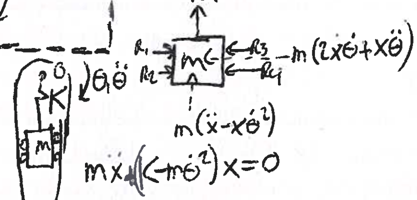
$J\ddot\theta + c\dot\theta + K_{eq} = \alpha_0\sqrt{K_{eq}^2 + c^2\omega^2}\sin(\omega t - \beta)$

$\Theta_p = F/\sqrt{(K_{eq} - J\omega^2)^2 + (c\omega)^2}$

$F = \alpha_0\sqrt{K_{eq} + c^2\omega^2}$

- solve K_{eq}
- solve $c = 2\zeta\sqrt{JK_{eq}}$
- solve F
- solve EOM

ROT + TRANS

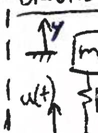


$m(\ddot{x} - x\dot\theta^2)$

$-m(2\dot{x}\dot\theta + x\ddot\theta)$

$m\ddot{x} + (K - m\dot\theta^2)x = 0$

GROUND NOT FLAT



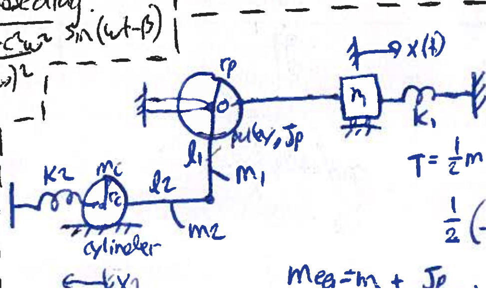
y: disp wrt ground

u: disp by ground on wheel

$y = u + x$ $\quad m\ddot{x} + K = m\ddot{u}$

~~[illegible]~~ ...

$u = A\sin\left(\frac{\pi v t}{c}\right)$



$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_p\dot\theta_p^2 + \frac{1}{2}J_1\dot\theta_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}J_c \ldots$
$+ \frac{1}{2}m_c\dot{x}_2^2$

$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_p\left(\frac{\dot{x}}{R_p}\right)^2 + \frac{1}{2}\left(\frac{m_1l_1^2}{3}\right)\left(\frac{\dot{x}}{R_p}\right)^2 + \frac{1}{2}m_2\left(\frac{\dot{x}l_1}{r_p}\right)^2 +$

$\frac{1}{2}\left(\frac{m_c r_c^2}{2}\right)\left(\frac{\dot{x}l_1}{r_pr_c}\right)^2 + \frac{1}{2}m_c\left(\frac{\dot{x}l_1}{r_p}\right)^2$

$M_{eq} = m + J_p \ldots$