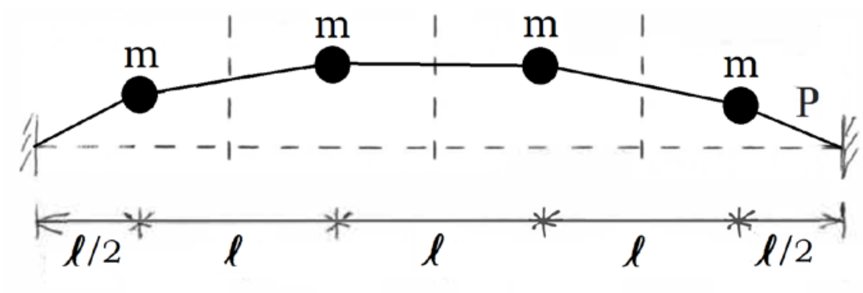
MECH 463 -- Homework 12

1. A uniform string of length L, mass density ρ and cross-section area A is stretched to a tension P. It is desired to model the string as a sequence of n equal segments, each of length $\ell = L/n$ and mass $m = \rho A \ell$. The mass of each segment is centred within the segment, so the distances of the first and last masses from the fixed ends are $\ell/2$, while the distances between all the interior masses are ℓ. Consider the case where n = 4, draw a free-body diagram and formulate the matrix equation of motion. Examine the structure of your matrices and then generalize them for larger n. Program your equations into Matlab and compute the first three natural frequencies and plot the corresponding mode shapes for n = 10, 20, 40, 80. Compare your results with the theoretical solution of a vibrating string.



2. A uniform rod of length L, cross-section area A, Young's modulus E and mass density ρ is rigidly fixed at its left end and connected to a spring of stiffness k at its right end. Solve for the natural frequencies and mode shapes of the system starting from the wave equation for longitudinal vibrations:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where u(x,t) is the longitudinal vibrational displacement, and $c = \sqrt{(E/\rho A)}$ is the wave speed. Leave your equations in symbolic form, but indicate how the roots could be evaluated if numerical answers were required. (*Hint: The boundary condition at the right end is $\partial u/\partial x(L) = -(k/EA)\, u(L)$.*)