University of British Columbia
Department of Mechanical Engineering

MECH 463. Midterm 1, October 2, 2020

Allowed Time: 50 min

Materials admitted: Pen, pencil, eraser, straightedge, simple scientific calculator without programming or communication capabilities, personal hand-written notes within one letter-size sheet of paper (one side), timer and document copier apps on your phone (all other phone functionalities are **not** allowed).

There are 2 questions in this exam. You are asked to answer all questions.

The purpose of this test is to evaluate your knowledge of the course material. Orderly presentation demonstrates your knowledge most clearly. Marks are assigned accordingly. A bonus of up to 2 marks will be given for exemplary presentation.

Honour Code: You are asked to behave honourably during this exam and to obey all instructions carefully. Please write and sign the following promise in the space below: "I promise to work honestly on this exam, to obey all instructions carefully, and not to have any unfair advantage over any other students."

Promise:

Signed: Name:

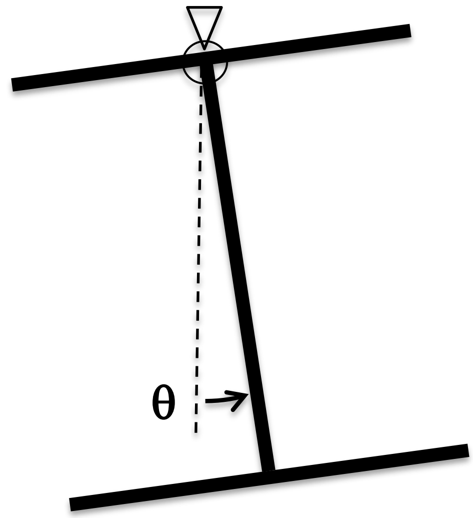
	Mark Received	**Maximum Mark**
1		10
2		10
Presentation		2 bonus
Total		20+2

Start Time running

Finish Time running

Name: ____

1. An anchor has the shape of a horizontal letter "H". It hangs from a frictionless pivot at its upper connection. Each of the three main sections of the anchor are made of uniform rods of length L and mass m, with centroidal moment of inertia $I_0 = mL^2/12$.



   (a) Draw a labeled free-body diagram of the vibrating system.

   (b) Use your free-body diagram to formulate the equation of motion.

   (c) Solve your equation of motion to determine the natural frequency of vibration of the anchor. Show the needed steps in detail.

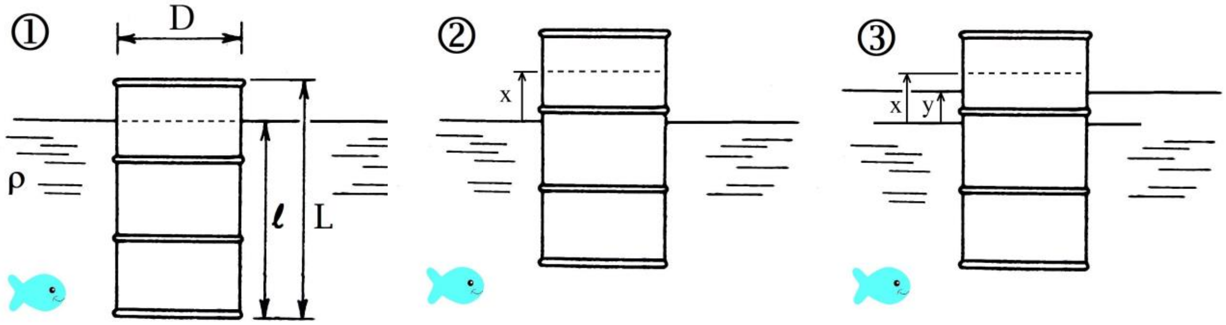
   (d) The anchor was found to be too heavy, so the lower horizontal section was cut off to make a vertical letter "T". Determine the new natural frequency of the anchor.

   (e) Comment on and explain your result.

Name: ______________________________________

2. A cylindrical oil barrel of length L and diameter D floats upright in the sea, where the water has density ρ. In calm conditions, the barrel floats at equilibrium with a submerged length ℓ, as shown in diagram ①. When disturbed, the barrel bobs up and down with vertical displacement x, as shown in diagram ②. In windy conditions, waves form that cause the sea surface to go up and down with vertical displacement $y = Y \cos \omega_f t$, as shown in diagram ③. It is observed that the water viscosity causes the barrel to have a damping rate c. As a mathematical simplification, you may assume that the damping force depends on the absolute velocity $\dot{x}$ rather than the relative velocity $\dot{x} - \dot{y}$.

(Archimedes' Principle: The upward buoyant force on a body immersed in a fluid equals the weight of the displaced fluid.)



(a) Draw a fully labeled free-body diagram for the situation shown in diagram ③. Use it to derive an equation of motion for the barrel.

(b) Solve your equation to determine the undamped natural frequency.

(c) Solve your equation to determine a compact formula for the steady-state amplitude of vibration caused by the water waves. Show the needed steps in detail. Sketch and label the corresponding magnification factor curve.