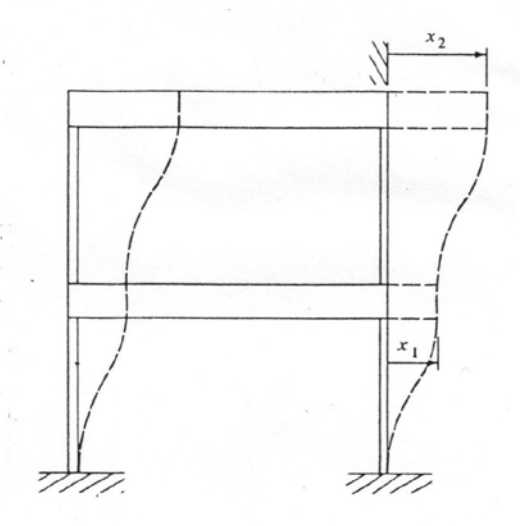
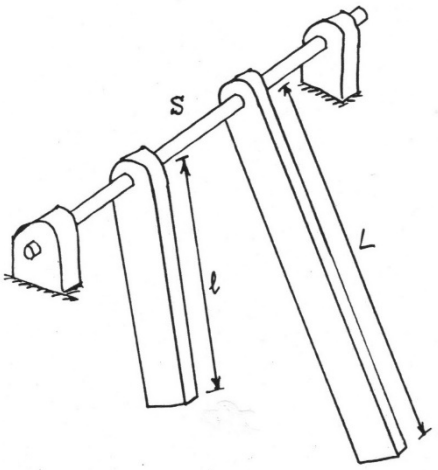
MECH 463 -- Homework 6

1. A 2-storey building is constructed of two rectangular concrete slabs, the lower one of mass 20,000 kg, and the upper of mass 10,000 kg, both supported by steel columns at the corners. After construction, the stiffness of the building is tested by applying horizontal loads on each of the two concrete slabs. When a horizontal force of 1000 N is applied to the lower slab, the displacement of that slab is 1mm. When a horizontal force of 1000 N is applied to the upper slab, the displacement of that slab is 3mm. Formulate the equations of motion in terms of the Principal Coordinates. Identify the natural frequencies and mode shapes.



2. Part of an old-fashioned clock mechanism consists of two uniform pendulum rods secured at their upper ends on a shaft. The shaft is mounted between two bearings that allow it to rotate freely. The shaft is rigid except for the section between the two rods, which has a torsional stiffness S. The two rods each have the same lineal density ρ per unit length. One rod is of length ℓ and the other is of length L. In the undisplaced condition, both rods hang vertically.



Draw a diagram of the system in its displaced condition and choose rotational displacement coordinates. Then draw the free body diagrams and determine the matrix equation of motion. Express your equation in dimensionless form using the length ratio $\mu = \ell/L$, dimensionless stiffness $\beta = S/(\rho g \ell^2)$, and dimensionless frequency $\Omega^2 = \omega^2(\ell/g)$. Derive the characteristic equation for the natural frequencies.

Program your characteristic equation into Matlab and use the software to investigate how the dimensionless frequency Ω^2 varies with length ratios μ in the range 0 to 2. Do this calculation for values of the dimensionless stiffness β = 0, 0.05, 0.10, 0.015 and 0.20. Plot some typical graphs and comment on your results.