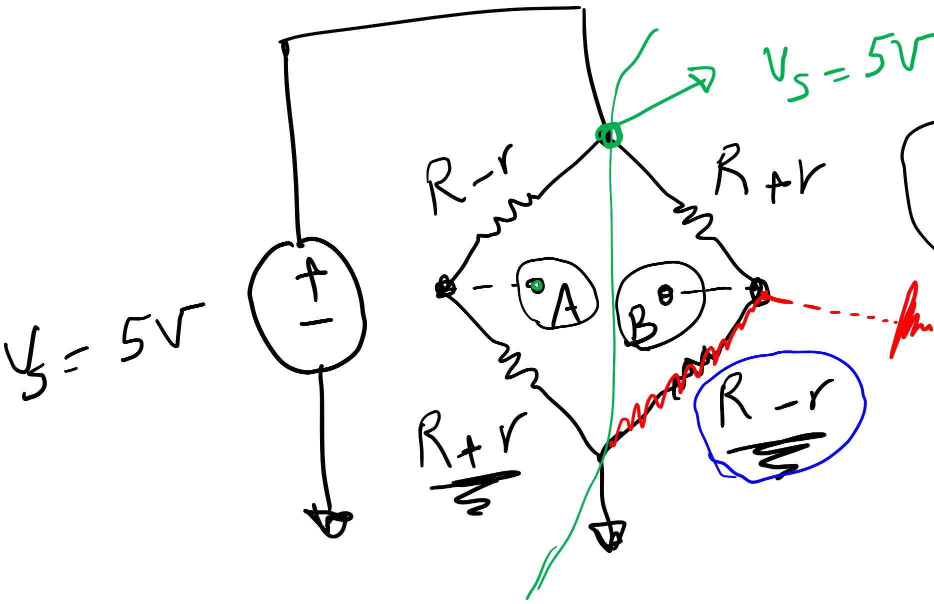
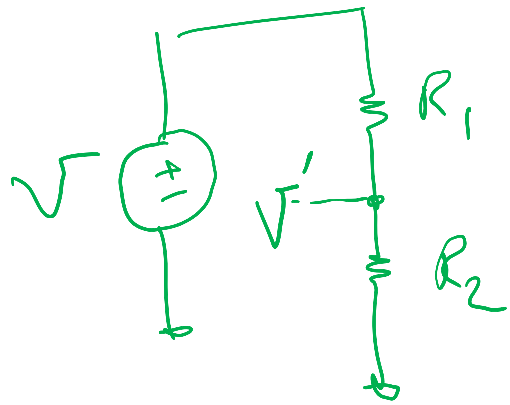
Let us consider a full-bridge strain gauge circuit in Figure 4. Here, $V_s = 5\,\mathrm{V}$ is the supply voltage, R is the nominal resistance, and r is the resistance change due to the strain.

(a) (10 pt.) Find the expression for the voltage between the output terminals A and B in terms of R and r.



$$V_A - V_B = ?$$



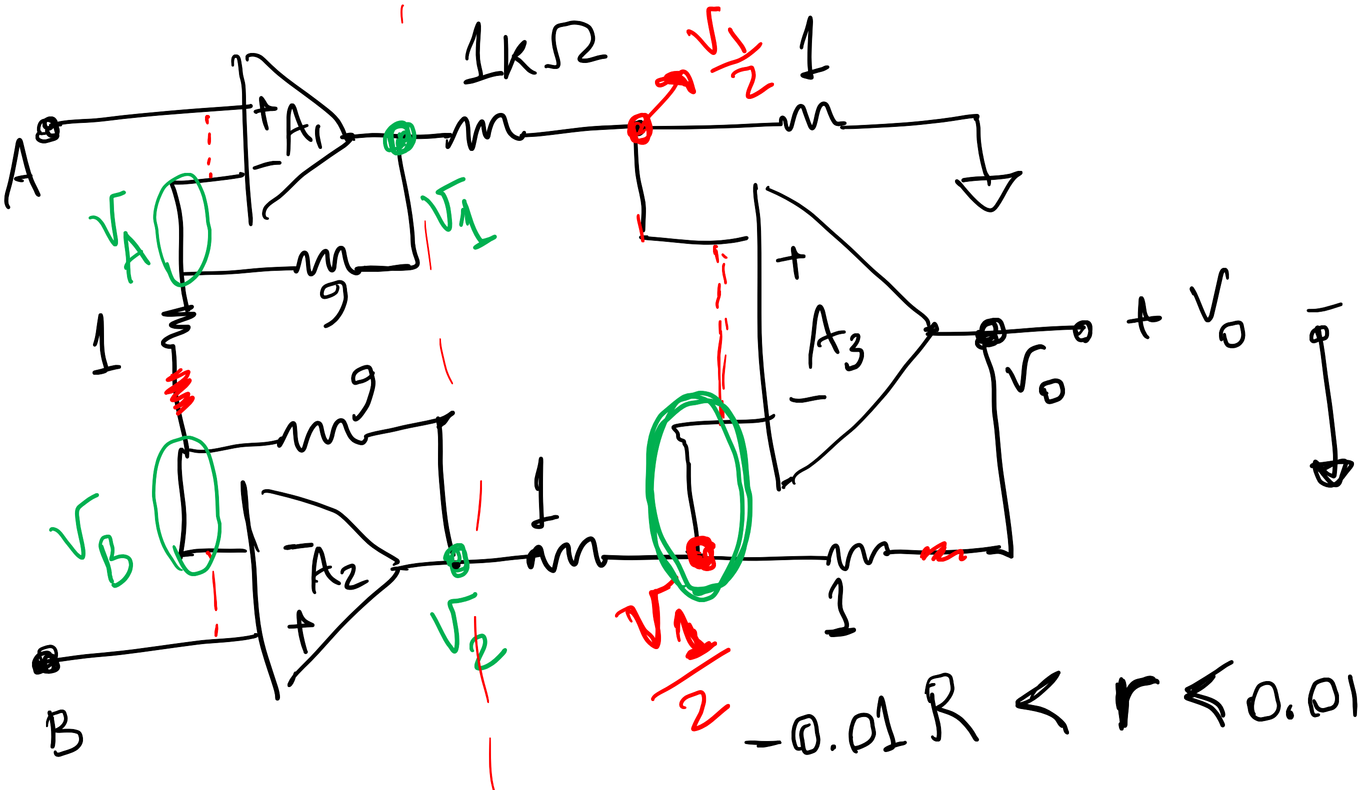
$$V' = \frac{R_2}{R_1 + R_2} \times V$$

$$V_A = \frac{R + r}{R - \cancel{r} + R + \cancel{r}} \times V_s = \frac{R + r}{2R} V_s$$

$$V_B = \frac{R - r}{R - r + R + r} = \frac{R - r}{2R} V_s$$

$$V_A - V_B = \left(\frac{\cancel{R} + r}{2R} - \frac{\cancel{R} - r}{2R} \right) V_s = \frac{r}{R} V_s = 5 \frac{r}{R}$$

(b) (30 pt.) The output terminals A and B of the bridge circuit are connected to the input terminals A and B of the op-amp circuit in Figure 5. The grounds of the two systems are also connected together. Suppose the op-amps are ideal (i.e., infinite input impedance, zero output impedance, and infinite open-loop gain) and do not saturate, and r varies such that $-0.01R < r < 0.01R$. Find the range of the output voltage V_o.



$-0.01R < r < 0.01R$

$V_o = ?$

KCL V_A:

$$\frac{V_A - V_B}{1} + \frac{V_A - V_1}{9} = 0$$

KCL V_B:

$$\frac{V_B - V_A}{1} + \frac{V_B - V_2}{9} = 0$$

(−)

$$(V_A - V_B) - (V_B - V_A) + \frac{V_A - V_1}{9} - \frac{V_B - V_2}{9} = 0$$

$$\underbrace{2(V_A - V_B) + \frac{1}{9}(V_A - V_B)} + \frac{1}{9}(V_2 - V_1) = 0$$

$$\frac{19}{9}(V_A - V_B) = \frac{1}{9}(V_1 - V_2)$$

$$\boxed{V_1 - V_2 = 19(V_A - V_B)}$$

KCL for negative port of A_3:

$$\frac{\frac{V_1}{2} - V_2}{1} + \frac{\frac{V_1}{2} - V_0}{1} = 0$$

$$\Rightarrow V_1 - V_2 - V_0 = 0 \Rightarrow \boxed{V_0 = V_1 - V_2}$$

$$V_0 = V_1 - V_2 = 19(V_A - V_B)$$

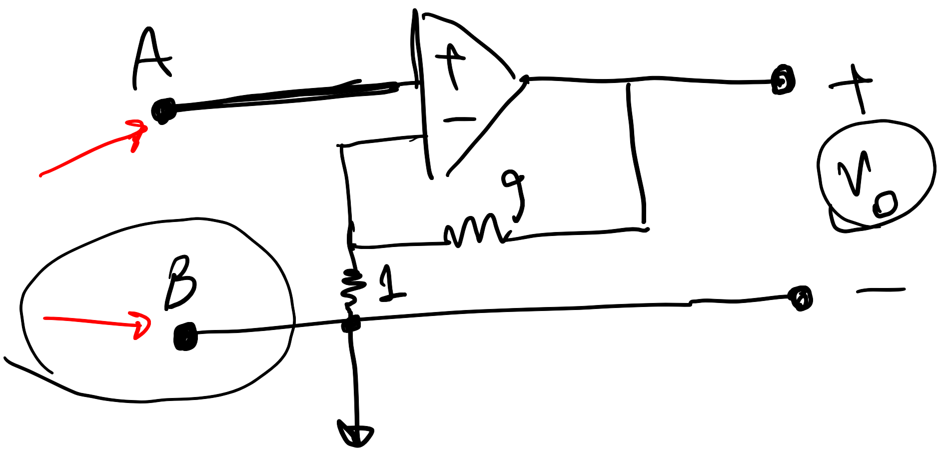
strain gauge: $V_A - V_B = 5\frac{r}{R}$

$\boxed{-0.01R < r < 0.01R} \Rightarrow -0.01 < \frac{r}{R} < 0.01 \Rightarrow$

$-0.05 < \underset{V_A - V_B}{\frac{5r}{R}} < 0.05 \underset{(\times 19)}{\Rightarrow} -0.95 < 19(V_A - V_B) < 0.95$

$\Rightarrow \boxed{-0.95 < V_0 < 0.95}$

(c) (30 pt.) The output terminals A and B of the bridge circuit are connected to the input terminals A and B of the op-amp circuit in Figure 7. The grounds of the two systems are also connected together. Suppose the op-amp is ideal (i.e., infinite input impedance, zero output impedance, and infinite open-loop gain) and does not saturate, and r varies such that $-0.01R < r < 0.01R$. Find the range of the output voltage V_o.



$$f = \frac{1}{1+9} = \frac{1}{10}$$

$$G(s) = \frac{A(s)}{1 + A(s) f} \rightarrow G = \frac{1}{f} = 10$$

$$V_o = 10 V_A \;,\; V_A = \left(\frac{R+r}{2R}\right) 5 = \left(1 + \frac{r}{R}\right) 2.5$$

$$-0.01R < r < 0.01R \Rightarrow -0.01 < \frac{r}{R} < 0.01$$

$$\Rightarrow 0.99 < \left(1 + \frac{r}{R}\right) < 1.01 \Rightarrow 2.475 < 2.5\left(1 + \frac{r}{R}\right) < 2.525$$

V_A

$$24.75 < V_o < 25.25$$