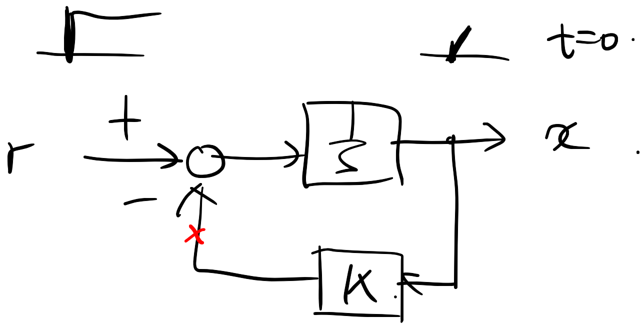
L2 – Feedback Systems (cont'd)

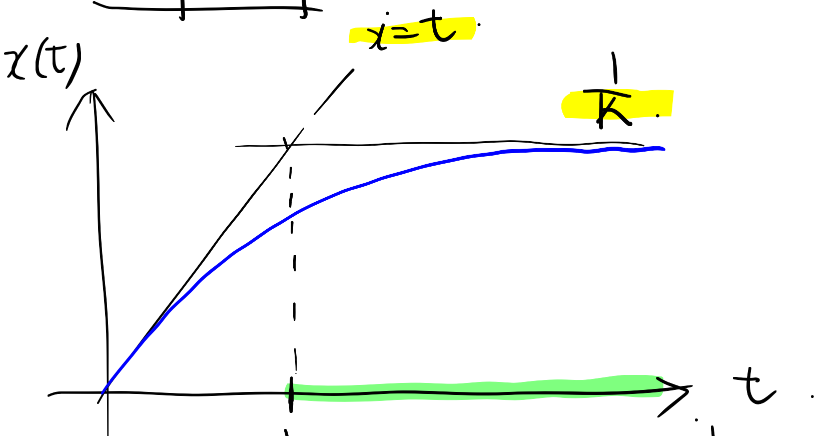


$$\frac{X}{R} = \frac{\frac{1}{s}}{1-(-1\cdot\frac{1}{s}\cdot K)} = \frac{\frac{1}{s}}{1+\frac{K}{s}} = \frac{1}{s+K}$$

① $s \to \infty$. $\frac{1}{s+K} \to \frac{1}{s}$.

② $s \to 0$. $\frac{1}{s+K} \to \frac{1}{K}$.

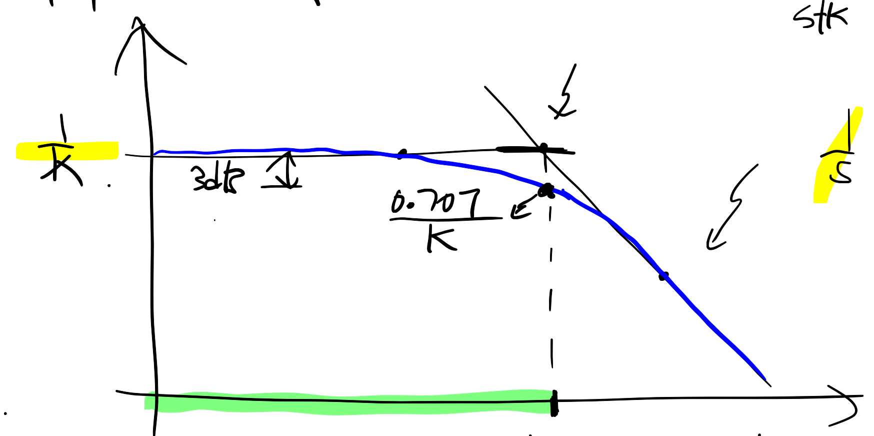
Step Resp



$\frac{1}{K} = \tau$: time const

$t \gg \tau$ effective

$|\cdot|$ Bode plot



$K = \omega_h$: Bandwidth

$dB = 20 \cdot \log_{10}(\text{absolute})$

0dB ← ×1

20dB ← ×10

Duality : Step ⟷ Bode

- Time const $\frac{1}{K}$.
- Initial resp. $x(t) = t$.
- Final $\frac{1}{K}$.

Bandwidth. K.

High-freq : $\frac{1}{s}$

DC gain : $\frac{1}{K}$.

① Can "infer" Bode $\longrightarrow$ Step.

② When feedback effective : $G(s) \simeq \frac{1}{K}$.

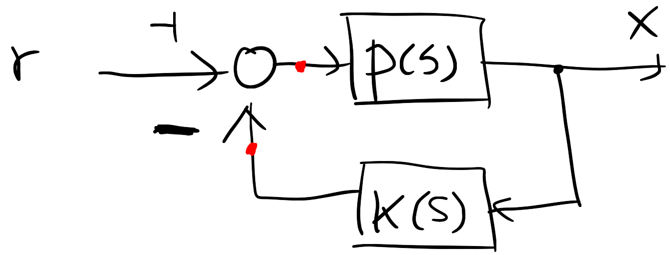
If not : $G(s) \simeq \frac{1}{s}$.

< General $L(j\omega)$ > $\frac{1}{s} \cdot K = L(s)$.

- Feedback effective. $|L(j\omega)| \gg 1$.

e.g.) $\left|\frac{K}{j\omega}\right| = \frac{K}{\omega} \gg 1$

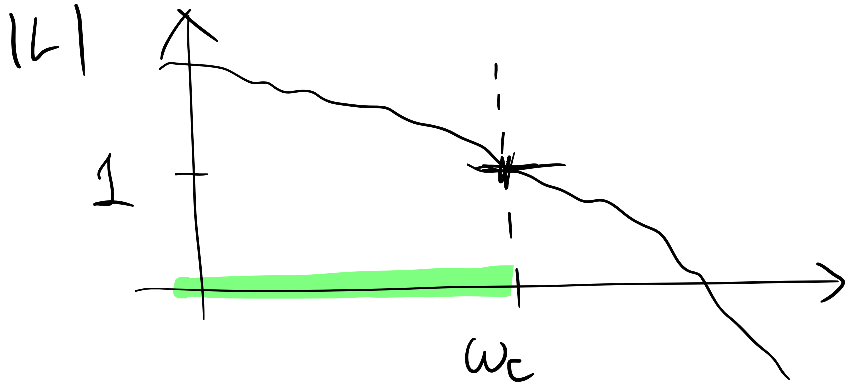
$\Rightarrow \omega \ll K$.



$L(s) = P(s)\,K(s)$.

(unity-gain)

ω_c : cross-over freq.



$|L(j\omega_c)| = 1$

$\left| \, |L(j\omega)| \Big|_{\omega=\omega_c} \right| = 1$.

- Feedback effective $\begin{cases} |L| \gg 1 . \\ \omega \ll \omega_c \leftrightarrow \omega_h . \end{cases}$

- $G(s) = \dfrac{P(s)}{1 + L(s)}$.

  $L(s)$: $P(s)\cdot K(s)$.

① $|L| \gg 1$. $(\omega \ll \omega_c)$

$G(s) \simeq \dfrac{P}{L} = \dfrac{P}{P \cdot K} = \dfrac{1}{K(s)}$ "

② $|L| \ll 1$ $(\omega \gg \omega_c)$

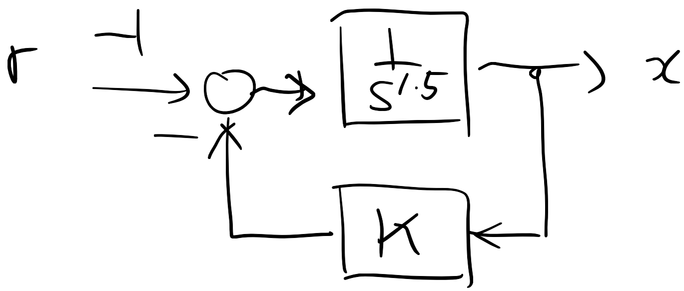
$G(s) \simeq P(s)$.

< Fractional-order Integrator >.

$P(s) = \frac{1}{s^{1.5}}$

Q. Exist ? practical ? **PA13** $A(s) \simeq \frac{1}{s^{1.2}}$

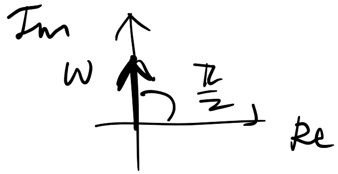
Q. How handle ? → Freq. Resp.



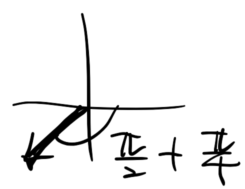
$$L(s)\Big|_{s=j\omega} = \frac{K}{s^{1.5}}$$

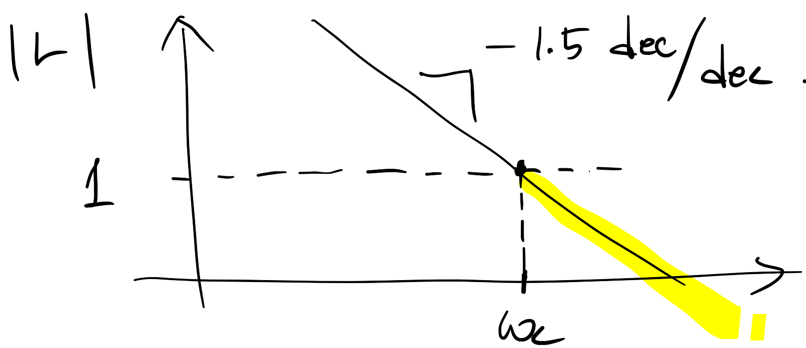
$$L(j\omega) = \frac{K}{(j\omega)^{1.5}} = \frac{K}{\omega^{1.5}} \frac{1}{(e^{j\frac{\pi}{2}})^{1.5}}$$

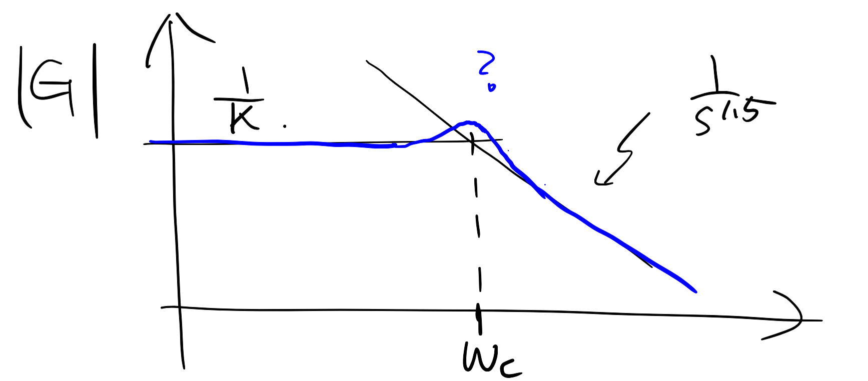
$$= \frac{K}{\omega^{1.5}} e^{-j(\frac{\pi}{2}\times 1.5)}$$

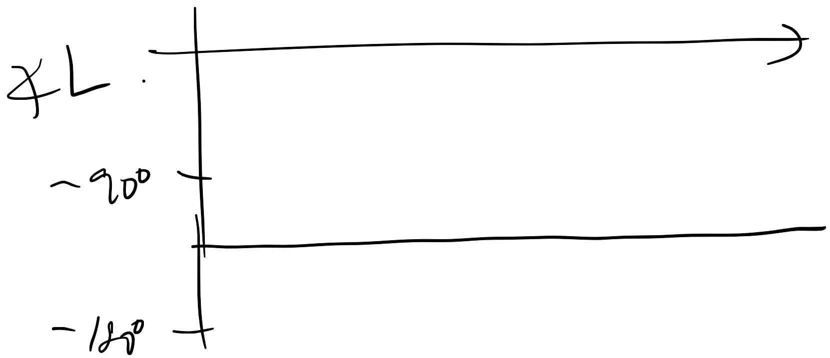


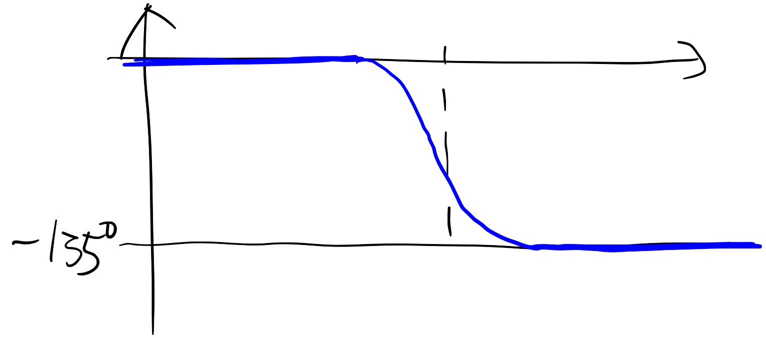
$j\omega = (e^{j\frac{\pi}{2}})\omega$



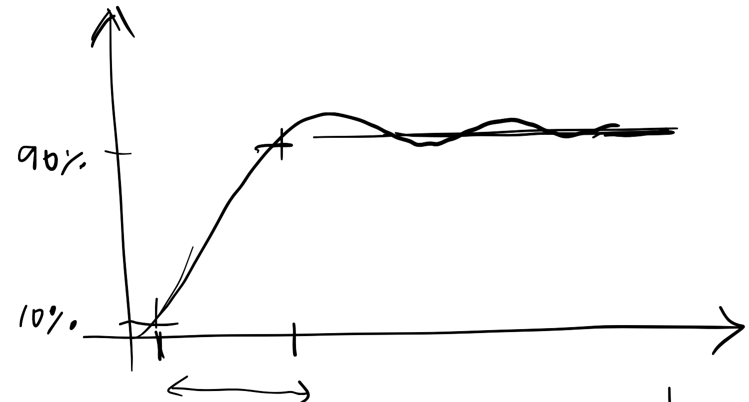








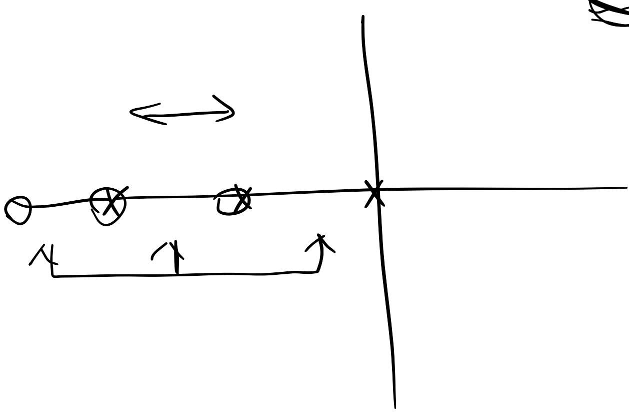
Step Response



t_r : rise time $\sim \frac{1}{\omega_c}$

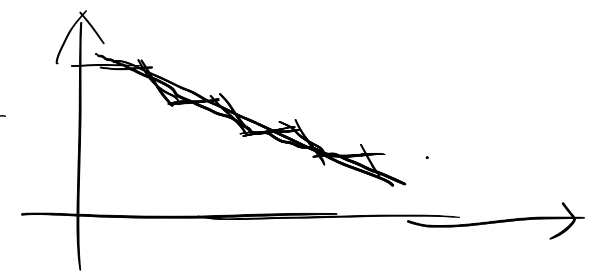
Implementation

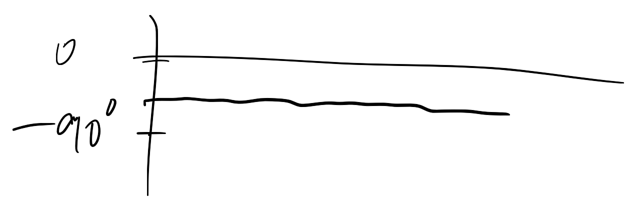
$\frac{1}{\sqrt{s}} \cdot \frac{1}{s}$



$$\frac{1}{s\sqrt{s}} = \frac{1}{s \cdot s^{\frac{1}{2}}} = \frac{1}{s^{1.5}}$$

1.





< 2nd order >