FRF Review

$$G(s) = \frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)}$$

$$n > m$$

$$\downarrow$$

Input Frequency

$$\boxed{\boldsymbol{s = j\omega}}$$

$$\downarrow$$

$$G(j\omega) = \frac{(j\omega+z_1)(j\omega+z_2)\ldots(j\omega+z_m)}{(j\omega+p_1)(j\omega+p_2)\ldots(j\omega+p_n)}$$

$$|G(j\omega)| = \frac{\sqrt{\omega^2+z_1^2}\times\sqrt{\omega^2+z_2^2}\times\cdots\times\sqrt{\omega^2+z_m^2}}{\sqrt{\omega^2+p_1^2}\times\sqrt{\omega^2+p_2^2}\times\cdots\times\sqrt{\omega^2+p_n^2}}$$

$$\angle G(j\omega) = \left(\tan^{-1}\frac{\omega}{z_1}+\tan^{-1}\frac{\omega}{z_2}+\cdots+\tan^{-1}\frac{\omega}{z_m}\right)$$

$$-\left(\tan^{-1}\frac{\omega}{p_1}+\tan^{-1}\frac{\omega}{p_2}+\cdots+\tan^{-1}\frac{\omega}{p_n}\right)$$

Problem 1. $G(s) = \dfrac{50}{s+10}$

$s = j\omega$

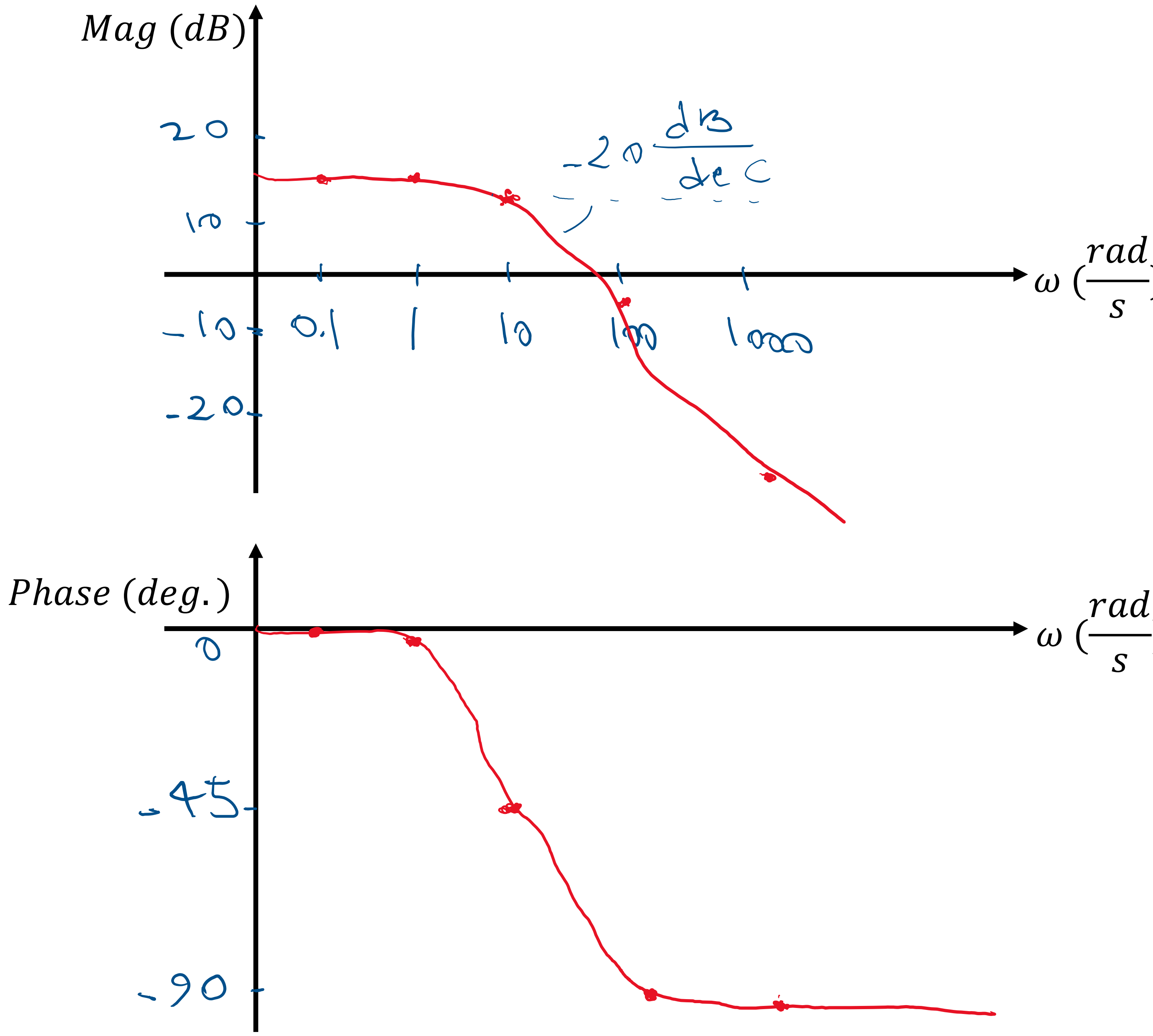
$$G(j\omega) = \frac{50}{j\omega + 10}$$

$$M = \frac{50}{\sqrt{\omega^2 + 100}} = 20\log_{10}(M)$$

$$\phi = 0 - \left(\tan^{-1}\frac{\omega}{10}\right)$$

ω (rad/s)	M	M (dB)	Ø (deg.)
0.1	4.99	13.98	-0.573
1	4.97	13.94	-5.71
10	3.53	10.97	-45
100	0.49	-6.06	-84.3
1000	0.05	-26.02	-89.43

$$G(s) = \frac{50}{s+10}$$



General Case $G(s) = \dfrac{K}{s+a}$

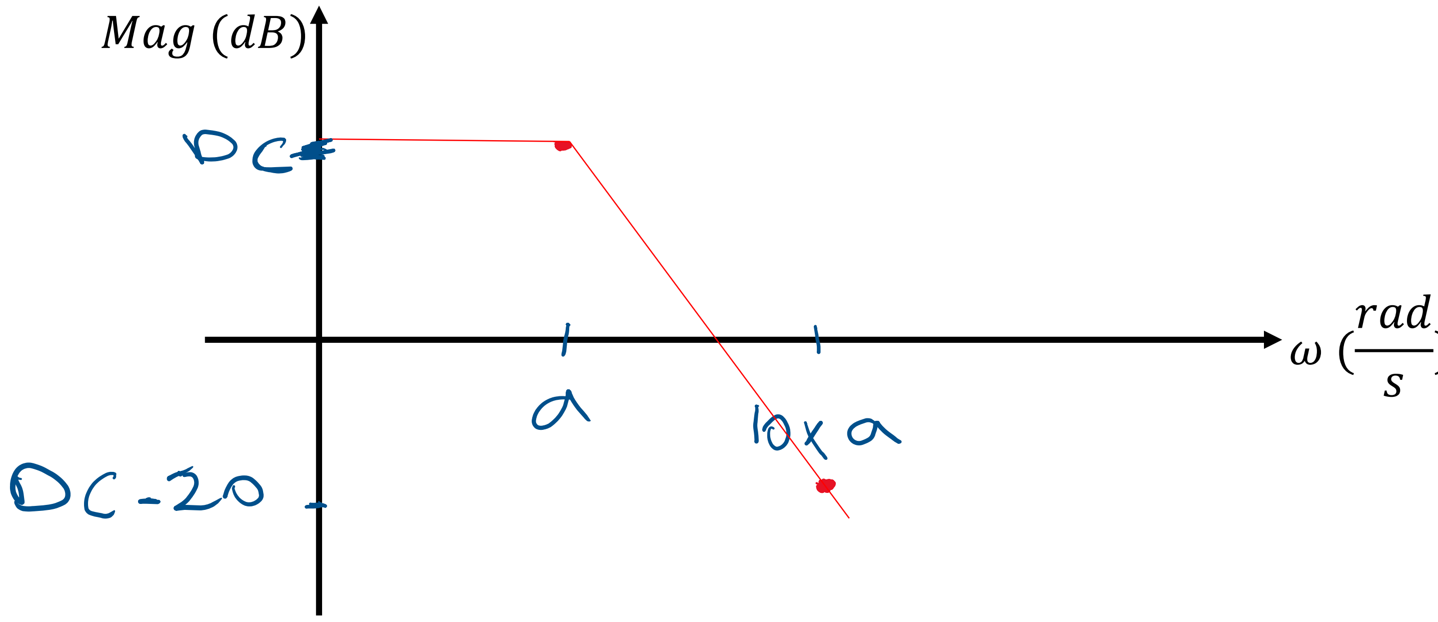
$s = j\omega$

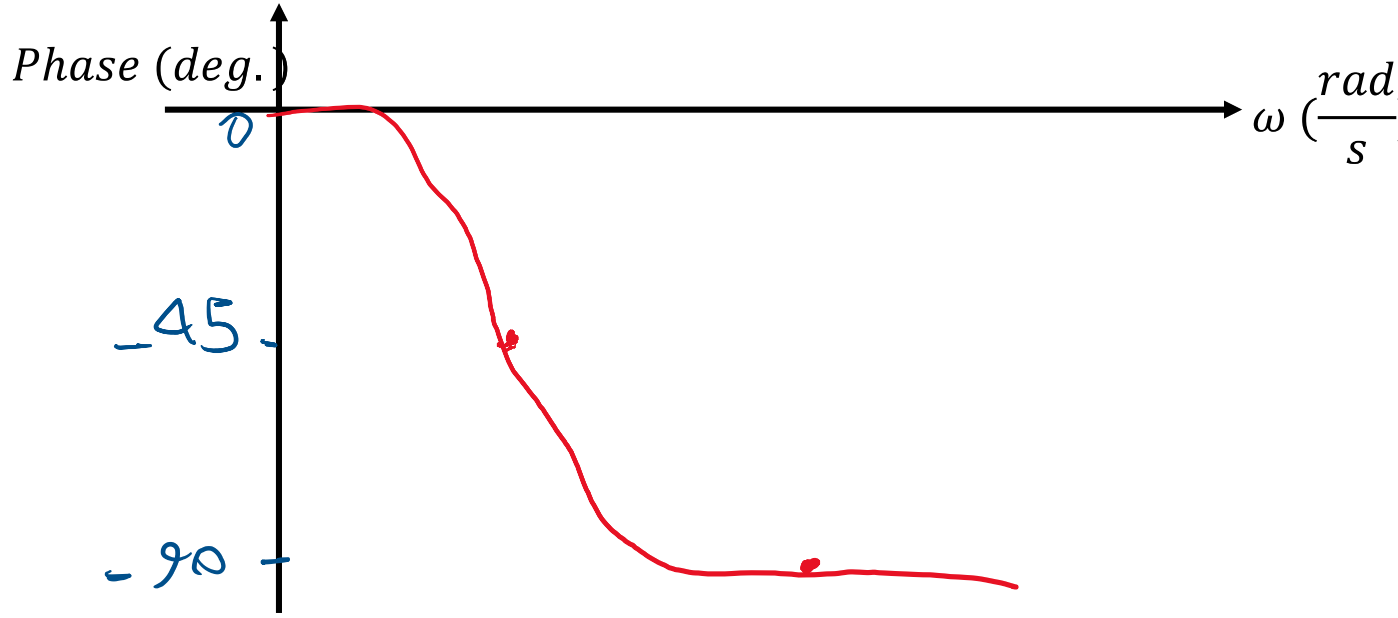
$G(j\omega) = \dfrac{K}{j\omega + a}$

$M = \dfrac{K}{\sqrt{\omega^2 + a^2}}$

$DC = 20\log\left(\dfrac{K}{a}\right)$

$\phi = -\tan^{-1}\left(\dfrac{\omega}{a}\right)$





Problem 2. $G(s) = \dfrac{10^6}{s^2 + 10100s + 10^6}$

$$\Delta = \sqrt{b^2 - 4ac} = 9900$$

$$\rightarrow r_1 = -100 \,,\quad r_2 = -10^4$$

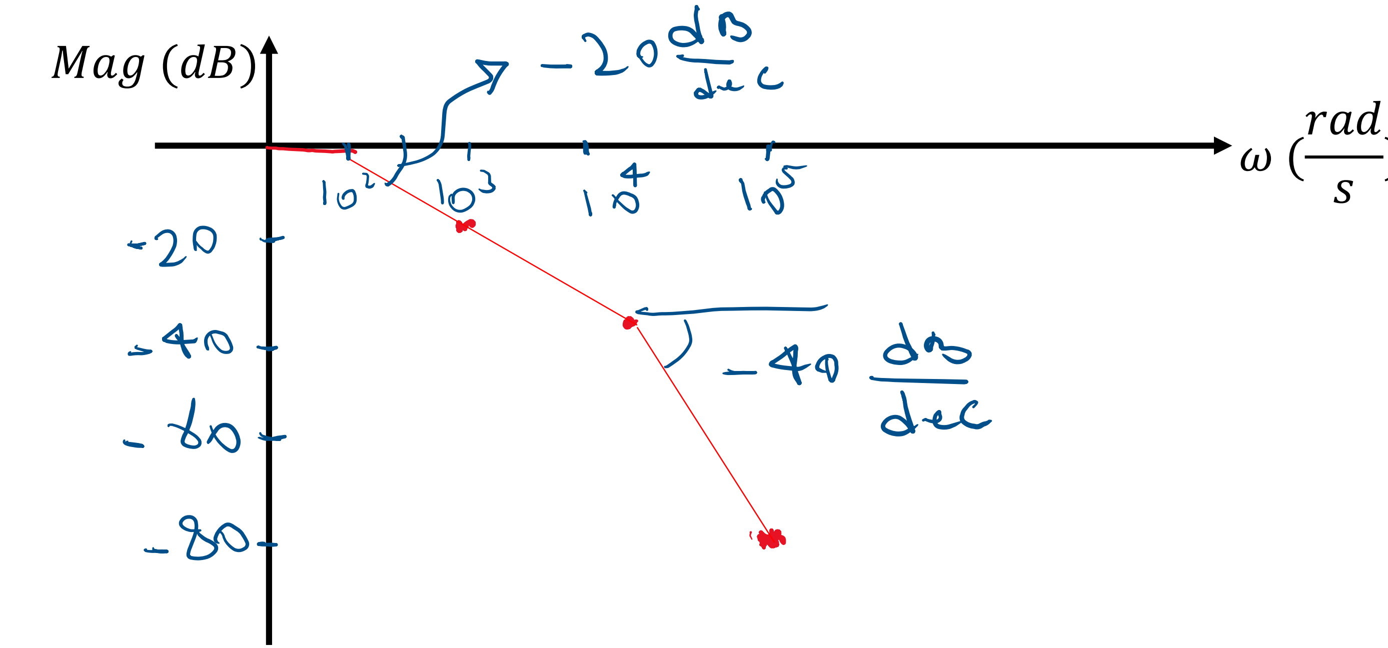
$$G(s) = \frac{10^6}{(s+100)(s+10^4)}$$

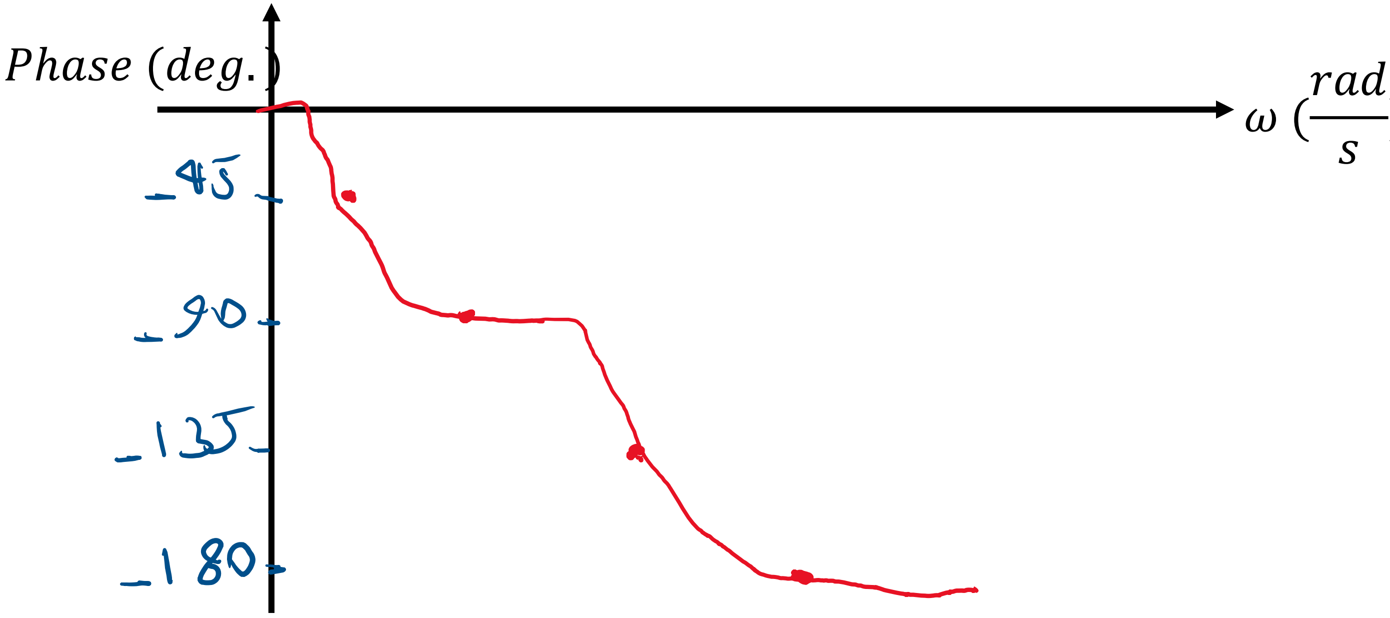
$$G(jw) = \frac{10^6}{(jw+100)(jw+10^4)}$$

$$M = \frac{10^6}{\sqrt{w^2+100^2} \times \sqrt{w^2+10^8}}$$

$$\phi = -\left(tg^{-1}\left(\frac{w}{100}\right) + tg^{-1}\left(\frac{w}{10^4}\right)\right)$$

$$G(s) = \frac{10^6}{s^2 + 10100s + 10^6}$$





Problem 3. $G(s) = \dfrac{4}{s^2 + 0.5s + 4}$

$$\Delta = \sqrt{b^2 - 4ac} = \sqrt{0.5^2 - 16} < 0$$

$$G'(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\begin{cases} 2\zeta\omega_n = 0.5 \\ \omega_n^2 = 4 \end{cases} \rightarrow \zeta = 0.125, \ \omega_n = 2$$

$$K = 4 \longrightarrow G(s=0) = 1 \rightarrow DC = 0 \text{ dB}$$

$$G(s) = \frac{4}{s^2 + 0.5s + 4}$$