a place of mind

UBC

MECH468 : Modern Control Engineering
MECH509 : Controls

L22 : Servo control

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Zoom lecture to be recorded and posted on Canvas

Course plan

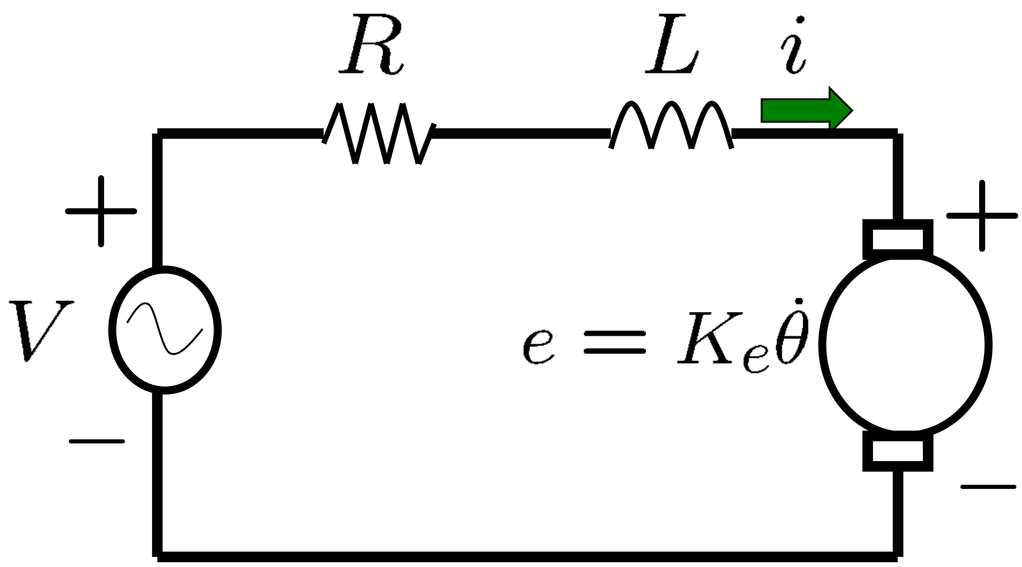
Topics	CT	DT
Modeling	✓	✓
Stability	✓	✓
Controllability/observability	✓	✓
Realization	✓	✓
→ State feedback/observer		
LQR/Kalman filter		

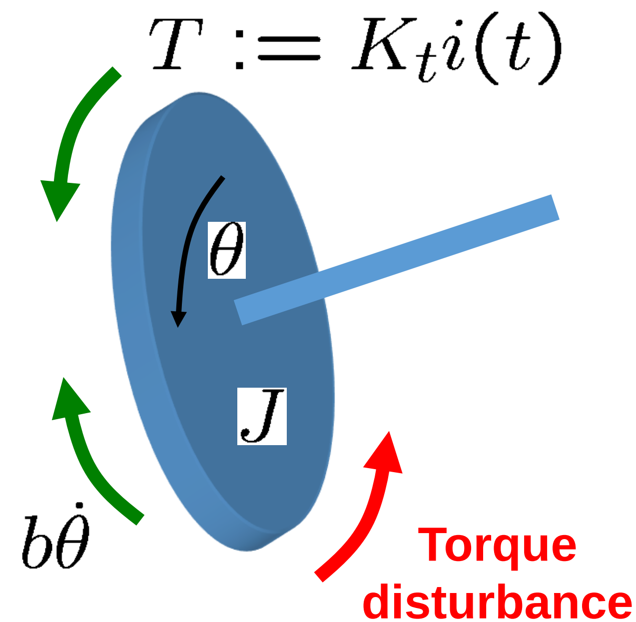
Review & today's topic

- In the last three lectures
  - State feedback
  - Pole placement theorem
  - Methods to compute state feedback gain K
  - Regulation problem, i.e., $r(t)=0$
- Today
  - Servo (tracking) problem, i.e., nonzero $r(t)$
  - We also consider disturbance rejection.
  - For simplicity, we only deal with SISO cases (but there are similar results for MIMO cases.)

Example: DC motor position control

ctms.engin.umich.edu





$$
\begin{aligned}
J\ddot{\theta}(t) &= K_t i(t) - b\dot{\theta}(t) + w(t) \\
V(t) &= Ri(t) + L\frac{d}{dt}i(t) + K_e\dot{\theta}(t)
\end{aligned}
$$

DC motor position control (cont'd)

- State-space model

$$
\begin{cases}
\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 \\ 0 & -b/J & K_t/J \\ 0 & -K_e/L & -R/L\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ 1/L\end{bmatrix} V(t) + \begin{bmatrix}0 \\ 1/J \\ 0\end{bmatrix} w(t) \\
\theta(t) = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix}
\end{cases}
$$

J	moment of inertia	$3.2284 \cdot 10^{-6}$	kgm^2/s^2
b	damping coefficient	$3.0577 \cdot 10^{-6}$	Nms
$K_t = K_e$	emf constant	$2.74 \cdot 10^{-2}$	Nm/Amp

$$R = 4\Omega \quad L = 2.75 \cdot 10^{-6} H$$

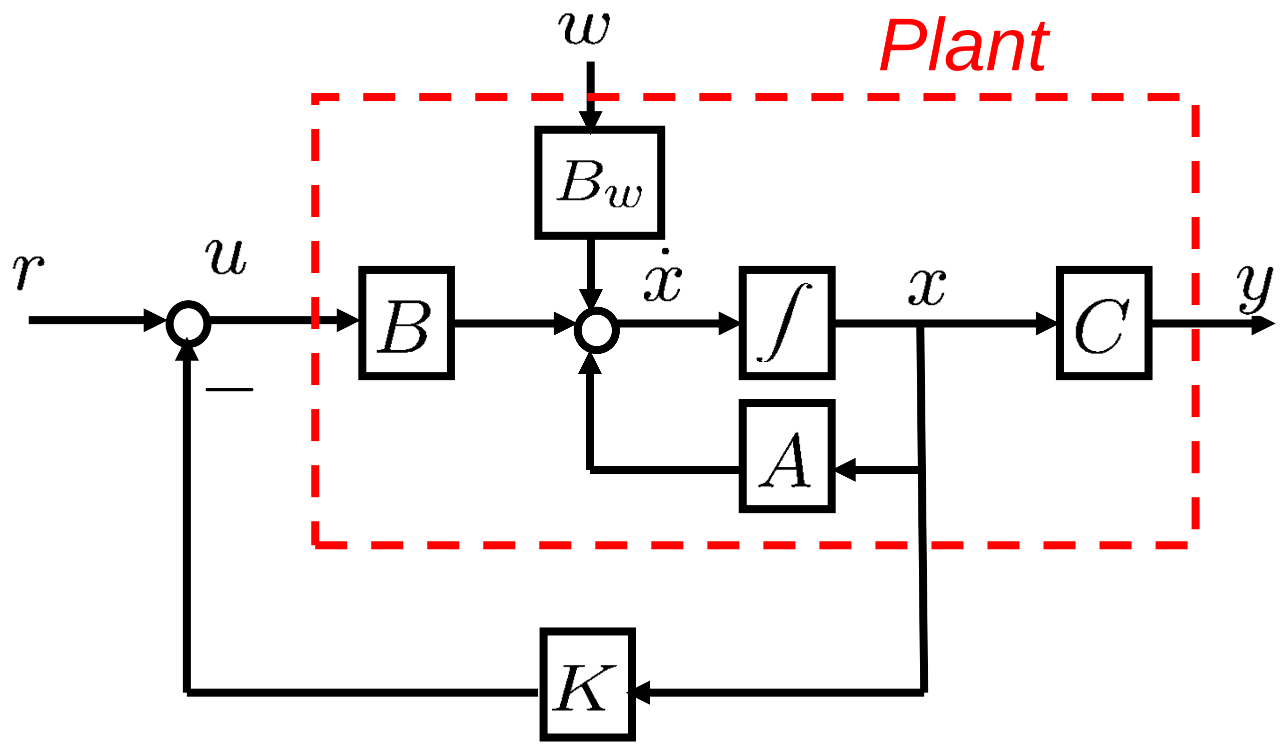
DC motor position control (cont'd)

- Specifications: For zero initial condition
  - $r(t)=1$ rad
  - Settling time < 40 ms
  - Overshoot < 16%

    (Same as Lecture 21, Slide 15)
  - Zero steady state error for
    - Step reference
    - Step disturbance
- Open-loop system
  - Poles = 0, -59.226, -1.4545E+6
  - Not asymptotically stable!

DC motor position control (cont'd)

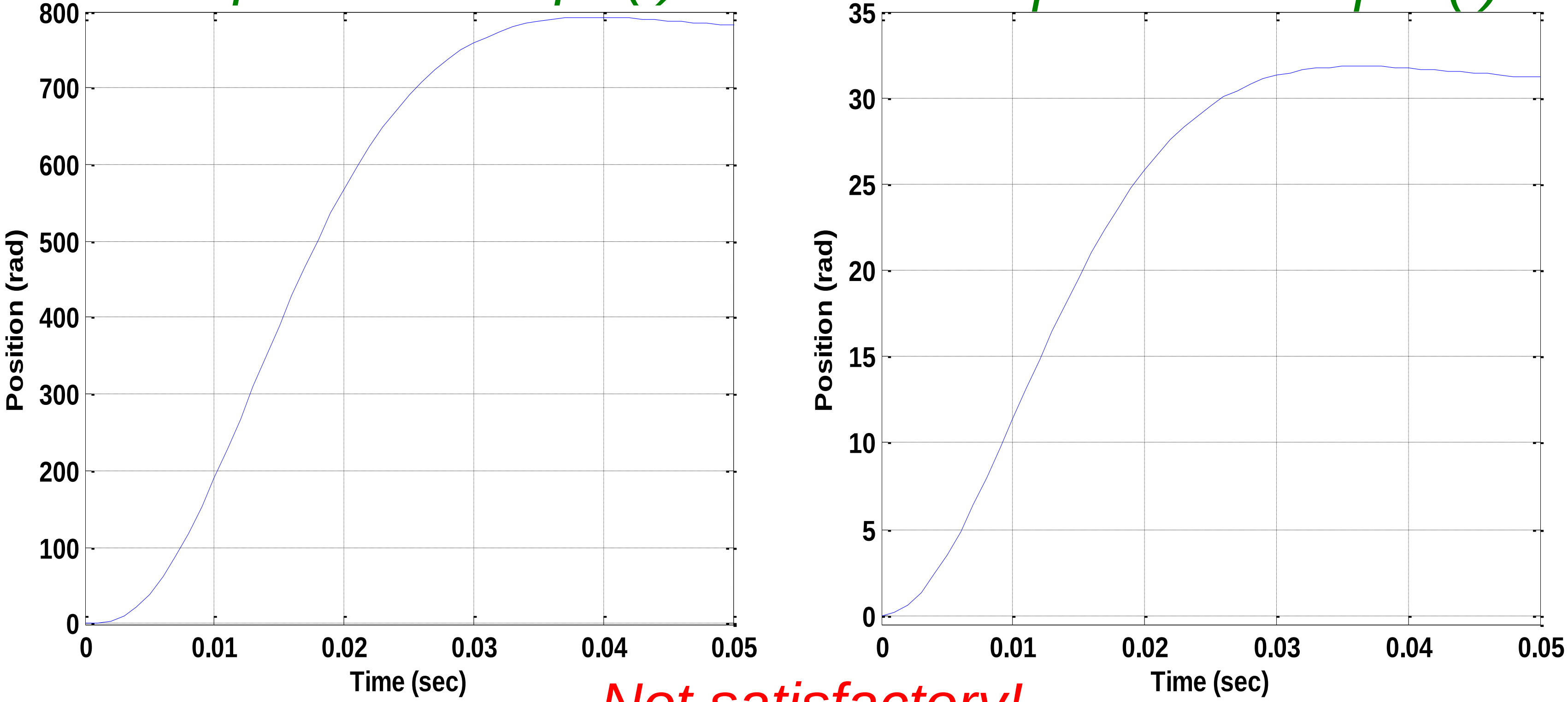
- Standard state feedback structure does not work, as shown in the next slide.



DC motor position control (cont'd)

- Closed-loop poles $-200, -100 \pm 100j$

Response to step r(t) *Response to step w(t)*

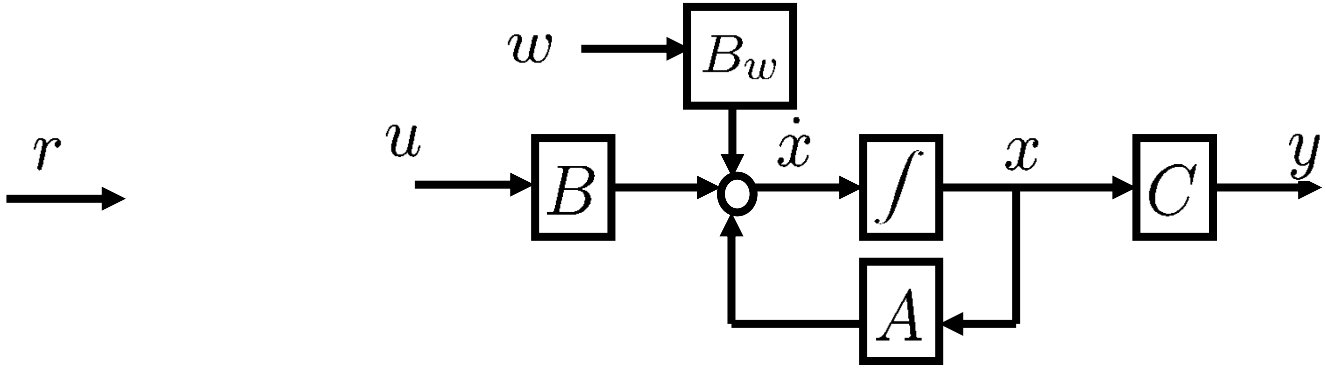


Not satisfactory!

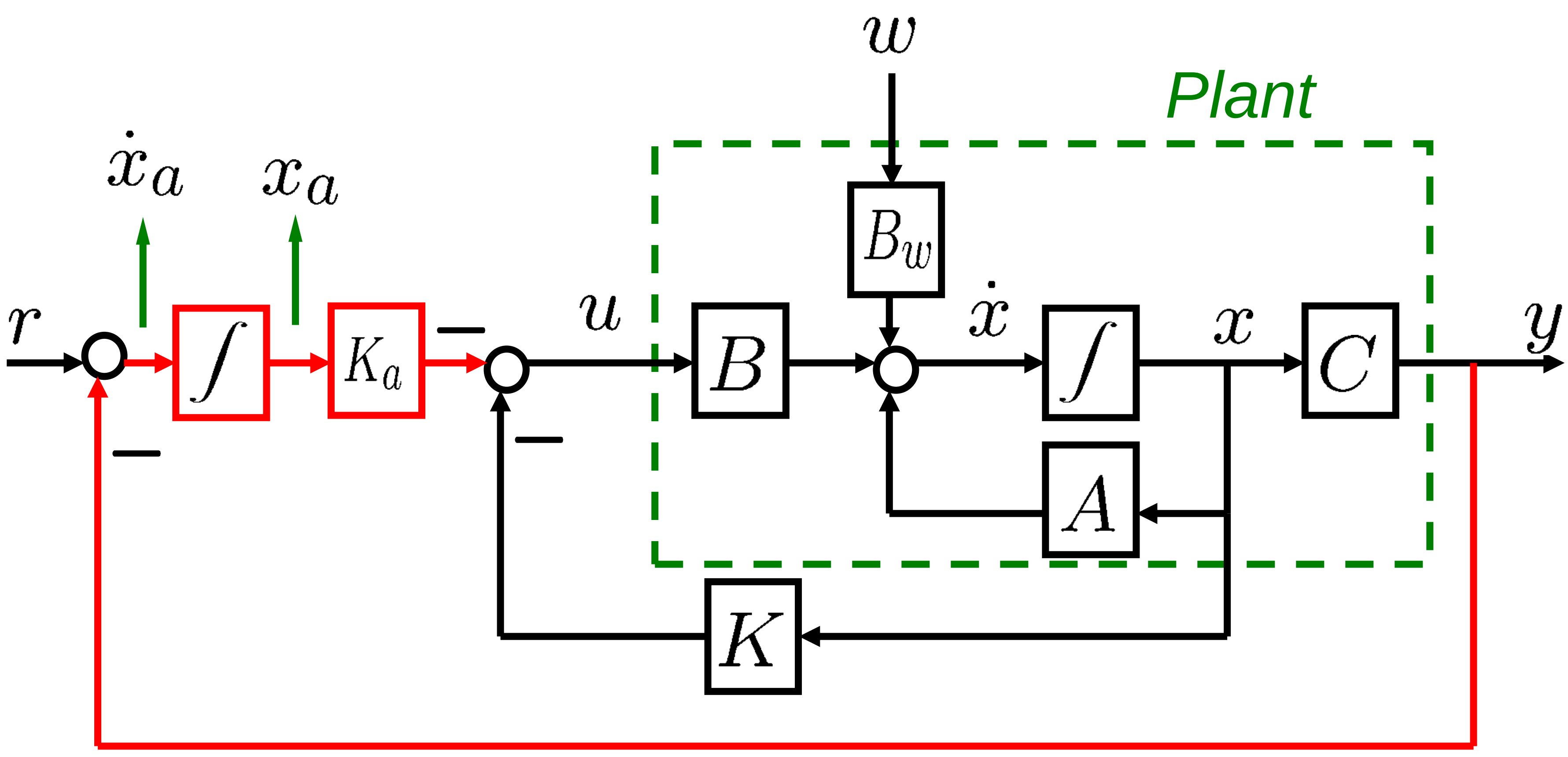
Robust tracking

- Given $\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \\ y(t) = Cx(t) \end{cases}$

  design a feedback control law s.t.
  - feedback system is internally stable
  - *y(t)* will track asymptotically any step *r(t)* even with:
    - small plant parameter variations
    - step disturbance *w(t)* of unknown magnitude



State feedback with an integrator
Block diagram



State feedback with an integrator
Closed-loop system

- SS model $$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A - BK & -BK_a \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) + \begin{bmatrix} B_w \\ 0 \end{bmatrix} w(t) \\ y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \end{bmatrix} \end{cases}$$
- Closed-loop A-matrix

$$\begin{bmatrix} A - BK & -BK_a \\ -C & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}}_{A_{aug}} - \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_{aug}} \begin{bmatrix} K & K_a \end{bmatrix}$$

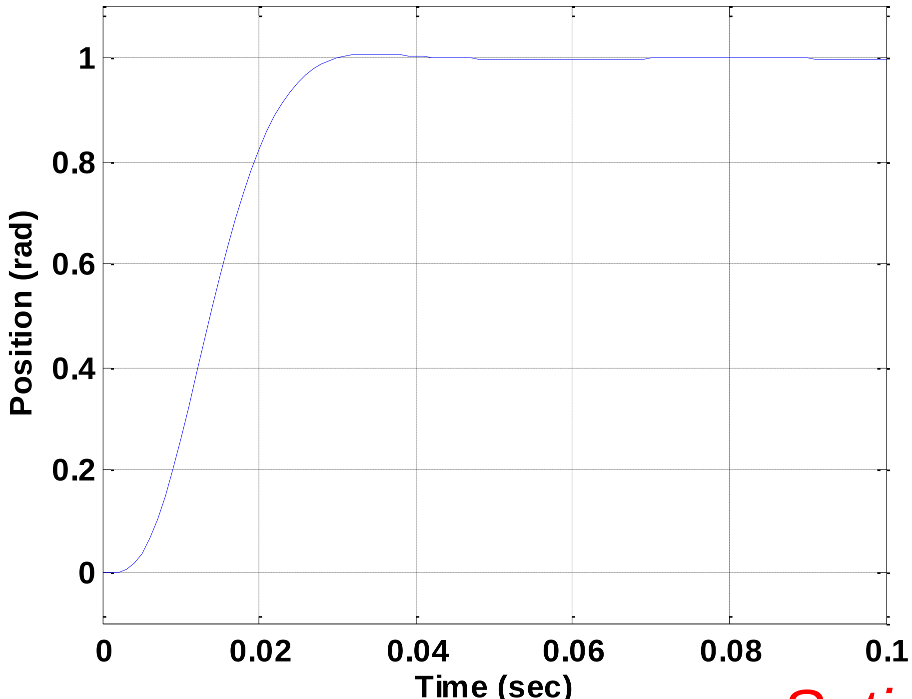
$$\begin{cases} (A, B) \text{ is controllable} \\ C(sI - A)^{-1}B \text{ has no zero at } s = 0 \end{cases} \Rightarrow (A_{aug}, B_{aug})\text{:controllable}$$

Any pole placement possible!

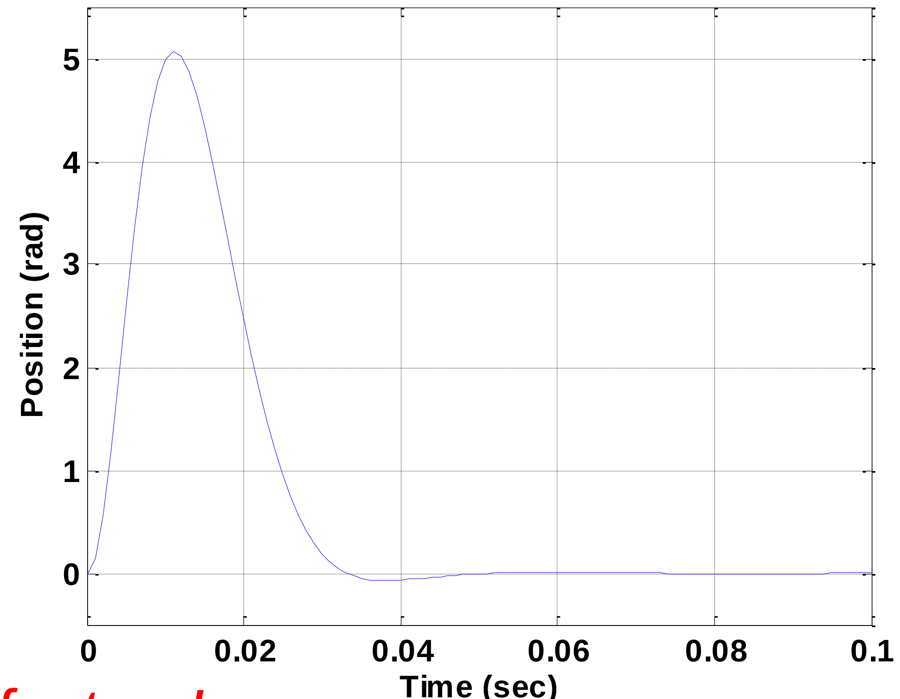
DC motor position control (revisited)

- Closed-loop poles: $-200, -400, -150 \pm 150j$

Response to step r(t)



Response to step w(t)



Satisfactory!

Why no steady state error to any step disturbance? (optional)

- Transfer function from *w* to *y*:

$$G_{yw}(s) = \frac{C(sI - A + BK)^{-1}B_w}{1 - \frac{K_a}{s}C(sI - A + BK)^{-1}B}$$

- When *w(t)=w*, the steady state value of the output y is obtained by final value theorem:

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sy(s)$$

$$= \lim_{s \to 0} \frac{sC(sI - A + BK)^{-1}B_w}{1 - \frac{K_a}{s}C(sI - A + BK)^{-1}B} \cdot \frac{w}{s} = 0$$

- This holds robustly against (*A,B,C*) variations as long as the closed-loop system is stable.

Why no steady state error to any step reference? (optional)

- Transfer function from *r* to *y*:

$$G_{yr}(s) = \frac{-\frac{K_a}{s}C(sI - A + BK)^{-1}B}{1 - \frac{K_a}{s}C(sI - A + BK)^{-1}B}$$

- When *r(t)=r*, the steady state value of the output y is obtained by final value theorem:

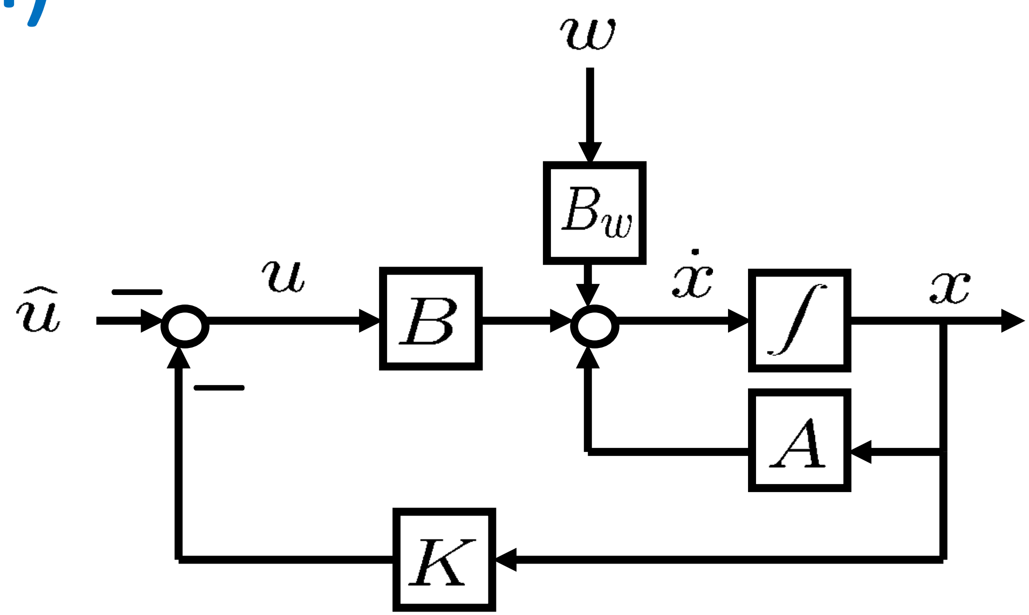
$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sy(s)$$

$$= \lim_{s \to 0} \frac{-s\frac{K_a}{s}C(sI - A + BK)^{-1}B}{1 - \frac{K_a}{s}C(sI - A + BK)^{-1}B} \cdot \frac{r}{s} = r$$

- This holds robustly against (*A,B,C*) variations as long as the closed-loop system is stable.

Transfer function derivation (optional)

- Subsystem

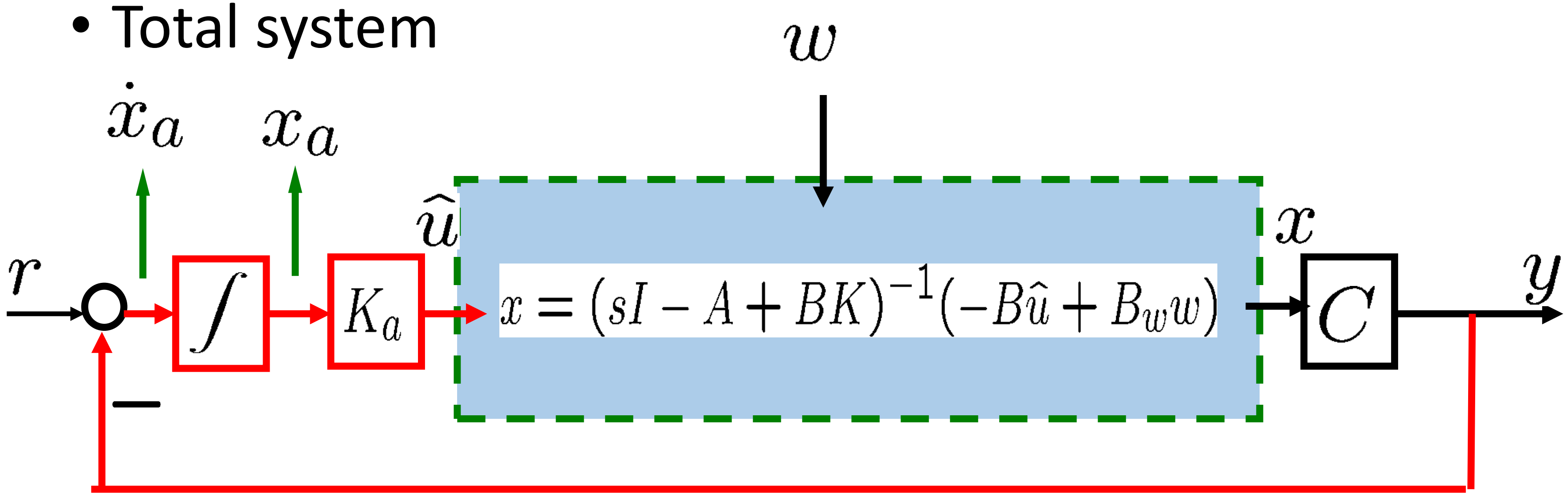


$$x = \frac{1}{s}Ax - \frac{1}{s}BKx - \frac{1}{s}B\hat{u} + \frac{1}{s}B_w w$$

$$\Rightarrow \quad x = (sI - A + BK)^{-1}(-B\hat{u} + B_w w)$$

Transfer function derivation (cont'd, optional)

- Total system



- To obtain closed-loop transfer functions, use

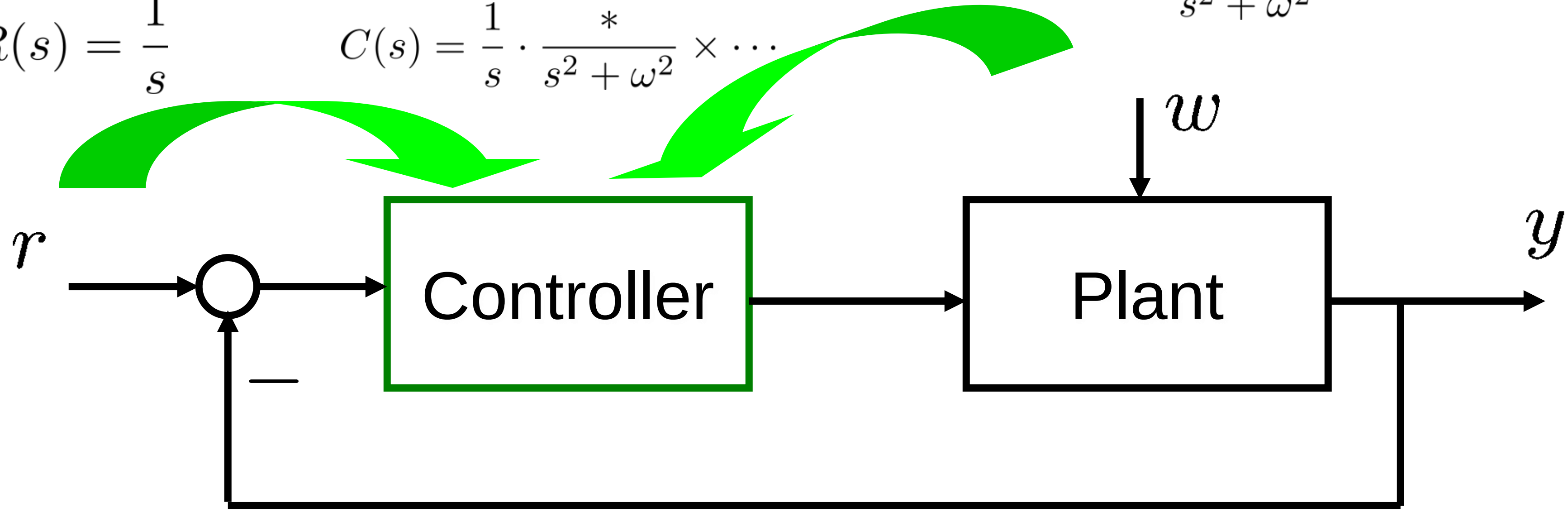
$$G_{cl}(s) = \frac{\text{Forward TF}}{1 + \text{Open-loop TF}}$$

Internal model principle

- For robust tracking (y tracks r even with w & plant perturbation), the controller must contain the dynamics of the exogenous system.

$$W(s) = \frac{*}{s^2+\omega^2}$$

$$R(s) = \frac{1}{s} \qquad C(s) = \frac{1}{s}\cdot\frac{*}{s^2+\omega^2}\times\cdots$$



Summary

- Servo control
  - State feedback with an integrator
  - Reduction to standard pole placement technique
  - Internal model principle
- Next, observer