MECH 467/541 -Final Exam 8:30am-11:00am (December 7, 2020) Prof. Y. Altintas

Open Book. Computers/calculator/MATLAB are allowed. Cameras: ON. No communication with others during the exam. Write your answers legible, and put box around the final answers.

1. (10) A ball screw drive system with the following parameters need to be designed: Table mass= $500kg$; Maximum Workpiece mass: 500kg; Maximum Feed cutting force: $4000N$, Maximum normal cutting force acting perpendicular to the table: $1000N$. Ball screw with a mean diameter of $d_m = 50mm$, pitch length of $10mm$, and its inertia with a mounted gear is $J_l = 0.005kgm^2$. Coulomb Friction coefficient between the table and guides - $\mu_{gf} = 0.05$. The thrust bearing with a mean diameter $d_b = 50mm$ and preload $F_p = 200N$ with a friction coefficient of $\mu_b = 0.004$ is used. The equivalent Viscous friction is $B = 0.01Nm/(rad/s)$. Use a speed reduction with a gear ratio of 2:1. The motor shaft and pinion has a combined inertia of $J_m = 10^{-4}kgm^2$. The motor must deliver a maximum translational acceleration of $\ddot{x}_{\max} = 5m/s^2$ and a velocity of $\dot{x}_{\max} = 0.5(m/s)$ at the table. Identify the peak and continuous torque requirements of the servo motor.

2.(10) Construct the open loop transfer function of the physical servo drive system by considering the mechanical drive shown in Figure 1. The motor shaft mounted encoder gives $10,000$ impulses per revolution. Assume that the equivalent inertia is $J_e = 0.001kgm^2$ and $B = 0.005Nm/(rad/s)$. The voltage command ($u[V]$) received from the computer generates current on the armature of the motor (I). The combined electrical (current-voltage amplifier and electrical winding of the motor) has a gain of $K_a = 30[A/V]$, and the torque constant of the motor is $K_t = 0.5[Nm/A]$.

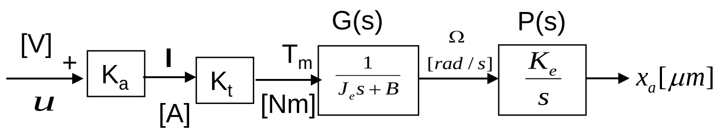

Figure 1: Open loop dynamics of a ball screw system with an encoder.

3. (15) Design an over-damped proportional position loop controller (K_p) with a damping ratio of $\varsigma = 1.1$ (Figure 2). Neglect the D/A controller between the computer and amplifier, hence treat the complete servo drive as a continuous system. (Derive it symbolically first and use the parameters from question 2 to obtain numerical results).

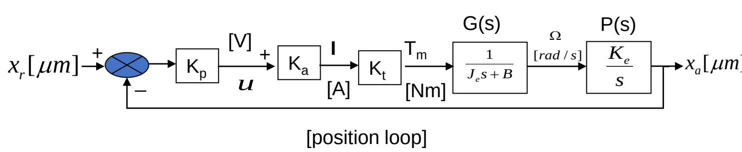

Figure 2: Proportional position controller.

4. (10) Express the FRF of the closed loop system with the proportional position controller (Figure 2) gain K_p. (Derive it symbolically first and use the parameters from questions 2 and 3 to obtain numerical results).

5. (20) Show the discrete time state space model of the **continuous** closed loop position controller (Figure 2) system with a sampling time interval of T_s. Just show the derivations symbolically without numerical calculations.

6. a) (20) Design a pole placement controller in z domain (Figure 3) using a desired damping ratio of $\varsigma_m = 1.1$ and natural frequency $\omega_m = 10[rad/s]$. (Note: be careful, it is an over-damped system). The sampling time interval is $T_s = 1ms$ and the digital to analog converter has 12 bits with $\pm 10V$ range. You can use MATLAB's command (c2d) to obtain ZOH equivalent of the open loop transfer function of the machine. (Note: c2d in MATLAB is equivalent to $(1 - z^{-1})Z\frac{G_0(s)}{s}$). Express the control command $u(k)$. Use symbolic parameters during derivation of the pole placement controller, and put the numbers at the end.

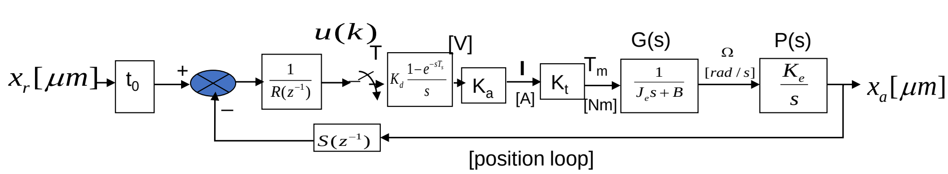

Figure 3: Pole placement position controller.

6. b) (15) Evaluate the steady state error of the pole placement controller designed in question 6.a (Figure 3) for a step input of $x_r(t) = U$. (Evaluate it symbolically).