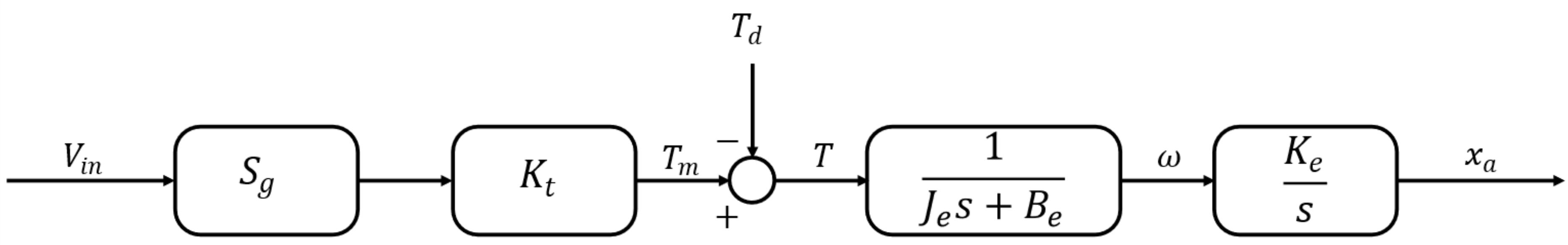
# State-space Model – Open-loop System




$$\frac{\omega(s)}{T(s)} = \frac{1}{Js+B} \longrightarrow Js\omega + B\omega = T \Rightarrow$$

$$J\dot{\omega} + B\omega = T, \quad T = T_m - T_d, \quad T_m = S_g K_t V_{in}$$

$$\dot{\omega} = -\frac{B}{J}\omega + \frac{S_g K_t}{J} V_{in} - \frac{1}{J} T_d \qquad (1)$$

# State-space Model – Open-loop System (Cont.)

$$\frac{x_a(s)}{w(s)} = \frac{K_e}{s} \rightarrow s x_a = K_e w \rightarrow \dot{x}_a = K_e w \quad (2)$$

$$(1)\ (2) \longrightarrow \underbrace{\begin{Bmatrix} \dot{\omega}(t) \\ \dot{x}_a(t) \end{Bmatrix}}_{\{\dot{x}\}} = \underbrace{\begin{bmatrix} -\frac{B}{J} & 0 \\ K_e & 0 \end{bmatrix}}_{[A]} \underbrace{\begin{Bmatrix} \omega(t) \\ x_a(t) \end{Bmatrix}}_{\{x\}} + \underbrace{\begin{bmatrix} \frac{K_t}{J} & -\frac{1}{J} \\ 0 & 0 \end{bmatrix}}_{[B]} \underbrace{\begin{Bmatrix} V_{in} \\ T_d \end{Bmatrix}}_{u_c}$$

$$\dot{x}(t) = A x(t) + B u_c(t)$$

$$x(k+1) = \phi(T) x(k) + H(T) u_c(k)$$

# State-space Model – Open-loop System (Cont.)

$$\phi(T) = e^{AT} = [I] + [A]T$$ 3

$$\phi(T) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -\frac{BT}{J} & 0 \\ K_e T & 0 \end{bmatrix} = \begin{bmatrix} 1-\frac{BT}{J} & 0 \\ K_e T & 1 \end{bmatrix}$$

$$H(T) = \left( \int_0^T e^{At}\, dt \right) \cdot B = \left( T[I] + \frac{1}{2} A T^2 \right) \times B$$ 4

$$\begin{bmatrix} T - \frac{BT^2}{2J} & 0 \\ \frac{K_e T^2}{2} & T \end{bmatrix} \times \begin{bmatrix} \frac{S_g K_T}{J} & -\frac{1}{J} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{S_g K_T T}{J}\left(1 - \frac{BT}{2J}\right) & -\frac{T}{J}\left( \right. \\ \frac{S_g K_T T^2 K_e}{2J} & - \end{bmatrix}$$

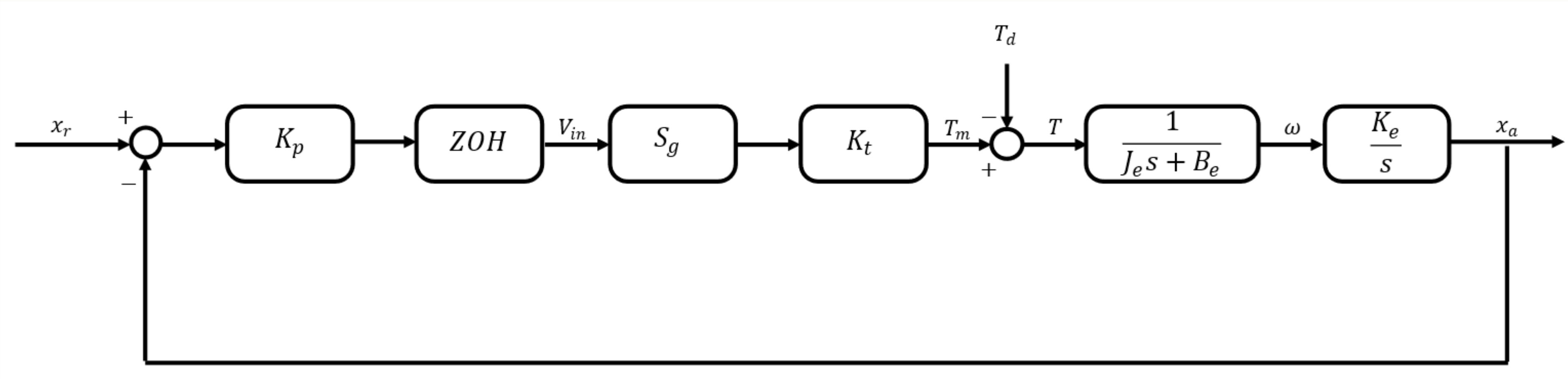
# State-space Model – Open-loop System (Cont.)

$$x(k+1) = \phi(T)\underline{x(k)} + H(T)\underline{u_c(k)}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\begin{Bmatrix} \omega(k) \\ x_a(k) \end{Bmatrix} \qquad \begin{Bmatrix} V_{in}(k) \\ T_d(k) \end{Bmatrix}$$

# State-space Model – Closed-loop System




$$V_{in}(k) = K_p\big(x_r(k) - x_a(k)\big)$$

$$y(k) = \begin{bmatrix} V_{in}(k) \\ \omega(k) \\ x_a(k) \end{bmatrix}$$

# State-space Model – Closed-loop System (Cont.)

$$y(k) = C\,x(k) + D\,u(k)$$

$$\underbrace{\begin{Bmatrix} v_{in}(k) \\ w(k) \\ x_a(k) \end{Bmatrix}}_{y(k)} = \underbrace{\begin{bmatrix} 0 & -K_p \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{Bmatrix} w(k) \\ x_a(k) \end{Bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} K_p & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} \underbrace{\begin{Bmatrix} x_r(k) \\ T_d(k) \end{Bmatrix}}_{u(k)} \qquad (5)$$

$$\begin{cases} x(k+1) = \phi(T)\,x(k) + H(T)\,u_c(k) \\ y(k) = C\,x(k) + D\,u(k) \end{cases} \qquad (6)$$

# State-space Model – Closed-loop System (Cont.)

$$u_c(k) = \begin{Bmatrix} V_{in}(k) \\ T_d(k) \end{Bmatrix} = \begin{bmatrix} K_p x_r(k) - K_p x_a(k) \\ T_d(k) \end{bmatrix} = \quad 7$$

$$\begin{bmatrix} 0 & -K_p \\ 0 & 0 \end{bmatrix} \underbrace{\begin{Bmatrix} \omega(k) \\ x_a(k) \end{Bmatrix}}_{x(k)} + \begin{bmatrix} K_p & 0 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{Bmatrix} x_r(k) \\ T_d(k) \end{Bmatrix}}_{u(k)}$$

$$6, 7 \longrightarrow x(k+1) = \varphi(T)\, x(k) + H(T) \left( \begin{bmatrix} 0 & -K_p \\ 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} K_p & 0 \\ 0 & 1 \end{bmatrix} u(k) \right)$$

# State-space Model – Closed-loop System (Cont.)

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$$x(k+1) = \underbrace{\left(\phi(T) + H(T) * \begin{bmatrix} 0 & -K_p \\ 0 & 0 \end{bmatrix}\right)}_{\phi'(T)} x(k) + \underbrace{\left(H(T) * \begin{bmatrix} K_p & 0 \\ 0 & 1 \end{bmatrix}\right)}_{H'(T)} u(k)$$

$$\begin{cases} x(k+1) = \phi'(T)\, x(k) + H'(T)\, u(k) \\ y(k) = C\, x(k) + D\, u(k) \end{cases}$$

# State-space Model – Closed-loop System (Cont.)

$\phi', H'$ from 8

$\phi, H$ from 3, 4

$C, D$ from 5

Extra Examples: Lecture Notes: Ex. 42, 44