MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

Homework 6

Assigned: Mar 19, 2021
Due: Mar 26, 2021

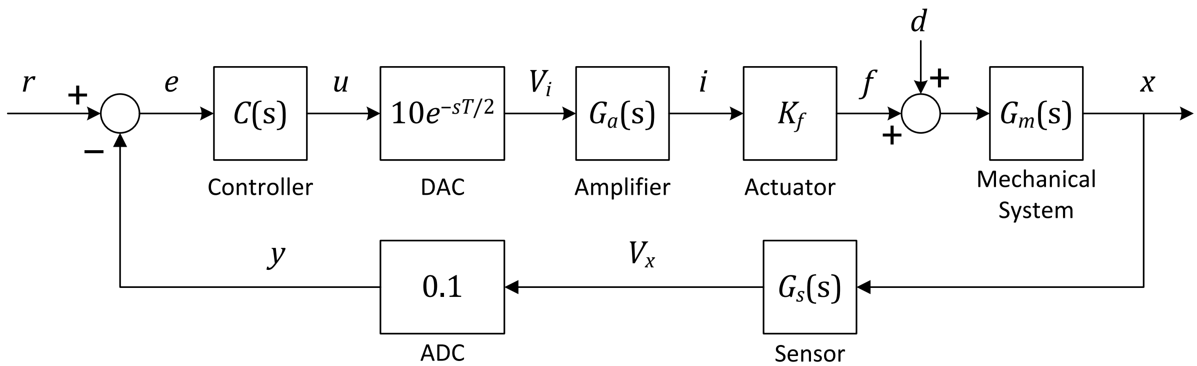

Figure 1: Block diagram of a position control system.

Figure 1 shows a block diagram of a position control system. Here, $G_a(s)$ is the transconductance amplifier, $K_f = 1\,\mathrm{N/A}$ is the actuator force constant, $G_m(s)$ is the mechanical system, and $G_s(s)$ is the sensor. The transfer functions and parameters are given as follows.

$$G_a(s) = \frac{1}{s/\omega_a + 1} \qquad \omega_a = 2\pi \times 10^3\,\mathrm{rad/s}$$
$$G_s(s) = \frac{1}{s/\omega_s + 1} \qquad \omega_s = 10\pi \times 10^3\,\mathrm{rad/s}$$
$$G_m(s) = \frac{1}{ms^2} \qquad m = 1\,\mathrm{kg}$$

The controller $C(s)$ is implemented in a real-time computer at a sampling rate $f_s = 10\,\mathrm{kHz}$. The real-time computer interfaces with the sensor via an ADC and with the amplifier via a DAC. The ADC has a gain of $0.1\,\mathrm{V}^{-1}$ and the DAC has a gain of $10\,\mathrm{V}$ with a half-sample delay $e^{-s\frac{T}{2}}$, where $T = 1/f_s = 100\,\mu\mathrm{s}$. Use MATLAB to answer the following questions.

(a) Draw the Bode plot of the plant

$$P(s) = \frac{Y(s)}{U(s)}$$

(b) Design a controller that implements a proportional gain and lead compensator

$$C(s) = K_p \frac{\alpha\tau s + 1}{\tau s + 1}$$

such that the loop transfer function $L(s) = C(s)P(s)$ achieves the gain crossover at $\omega_c = 100\,\mathrm{Hz}$ with a phase margin $\phi_m > 45°$. Select the values for K_p, α, and τ. Draw the Bode plot of $L(s)$.

(c) With $C(s)$ designed in part (b), simulate the step responses of the closed-loop system 1) from the reference r to position x and 2) from the disturbance d to position x, i.e.,

$$G_{xr}(s) = \frac{X(s)}{R(s)} \qquad\qquad G_{xd}(s) = \frac{X(s)}{D(s)}$$

(d) Design a controller that additionally implements an integral action

$$C(s) = K_p\left(1 + \frac{1}{T_i s}\right)\frac{\alpha\tau s + 1}{\tau s + 1}$$

such that the loop transfer function $L(s) = C(s)P(s)$ achieves the gain crossover at $\omega_c = 100\,\mathrm{Hz}$ with a phase margin $\phi_m > 40°$. Use the same values K_p, α, and τ from the part (b), and select the value of the integral time constant T_i. Draw the Bode plot of $L(s)$.

(e) With $C(s)$ designed in part (d), simulate the step responses of the closed-loop system $G_{xr}(s)$ and $G_{xd}(s)$. Compare the results with those from part (c).