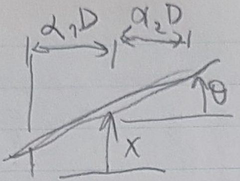
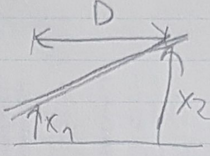
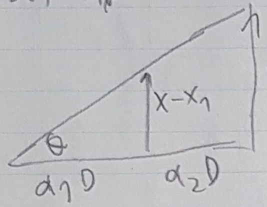
relab (1):

[I did this before the lectures so I didn't use the "redraw FBD" method.]

→ 

$\Rightarrow$ 

→ Set up relationship:



$\rightarrow$

$$\tan\theta = \frac{x_2 - x_1}{D} \quad (1)$$

$$\frac{x_2 - x_1}{D} = \frac{x - x_1}{\alpha_1 D} \quad (2)$$

→ (1) At small θ, $\tan\theta \approx \theta$

$$\tan\theta \approx \theta = \frac{x_2 - x_1}{D} \rightarrow D\theta = x_2 - x_1$$

$$\ddot{\theta} = \frac{\ddot{x}_2 - \ddot{x}_1}{D} \rightarrow D\ddot{\theta} = \ddot{x}_2 - \ddot{x}_1$$

(2)

$$\frac{x_2 - x_1}{D} = \frac{x - x_1}{\alpha_1 D}$$

$$\alpha_1 (x_2 - x_1) = x - x_1$$

$$x = \alpha_1 x_2 + (1 - \alpha_1) x_1$$

$$= \alpha_1 x_2 + \alpha_2 x_1$$

$$\ddot{x} = \alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1$$

→ Sub these equations into EQ 6 from lab manual

→ $$0 = m(\alpha_1\ddot{x}_2+\alpha_2\ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2}\left(\underbrace{\left(\frac{L_1}{\alpha_2}+\frac{L_2}{\alpha_1}\right)}_{\downarrow\ \frac{\alpha_1L_1+\alpha_2L_2}{\alpha_2\alpha_1}}(\alpha_1x_2+\alpha_2x_1)+(L_1-L_2)(x_2-x_1)\right)$$
(3)

$$0 = \frac{mR^2}{D^2}(\ddot{x}_2-\ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2}\left((L_1-L_2)(\alpha_1x_2+\alpha_2x_1)+(\alpha_2L_1+\alpha_1L_2)(x_2-x_1)\right)$$
(4)

→ Simplify

(3) $$0 = \cancel{m}(\alpha_1\ddot{x}_2+\alpha_2\ddot{x}_1) + \frac{\cancel{m}g\alpha_1\alpha_2}{L_1L_2}\left(\frac{\alpha_1L_1+\alpha_2L_2}{\alpha_2}x_2 + \frac{\alpha_1L_1+\alpha_2L_2}{\alpha_1}x_1 + L_1x_2 - L_2x_2 - L_1x_1 + L_2x_1\right)$$

$$0 = \alpha_1\ddot{x}_2+\alpha_2\ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_1L_2}\left(\frac{\alpha_1L_1}{\alpha_2}x_2 + \cancel{L_2x_2} + \cancel{L_1x_1} + \frac{\alpha_2L_2}{\alpha_1}x_1 + L_1x_2 - \cancel{L_2x_2} - \cancel{L_1x_1} + L_2x_1\right)$$

$$0 = \alpha_1\ddot{x}_2+\alpha_2\ddot{x}_1 + \frac{g\alpha_1\cancel{\alpha_2}}{\cancel{L_1}L_2}\left(\frac{\alpha_1}{\cancel{\alpha_2}}+\frac{\cancel{\alpha_2}}{\cancel{\alpha_2}}\right)\cancel{L_1}x_2 + \frac{g\cancel{\alpha_1}\alpha_2}{L_1\cancel{L_2}}\left(\frac{\alpha_2}{\cancel{\alpha_1}}+\frac{\cancel{\alpha_1}}{\cancel{\alpha_1}}\right)\cancel{L_2}x_1$$

$$0 = \alpha_1\ddot{x}_2+\alpha_2\ddot{x}_1 + g\frac{\alpha_1}{L_2}\underbrace{(\alpha_1+\alpha_2)}_{1}x_2 + g\frac{\alpha_2}{L_1}\underbrace{(\alpha_2+\alpha_1)}_{1}x_1$$

$$0 = \alpha_1\ddot{x}_2+\alpha_2\ddot{x}_1 + g\frac{\alpha_1}{L_2}x_2 + g\frac{\alpha_2}{L_1}x_1$$

(4) $$0 = \frac{\cancel{m}R^2}{D^2}(\ddot{x}_2-\ddot{x}_1) + \frac{\cancel{m}g\alpha_1\alpha_2}{L_1L_2}\left(L_1\alpha_1x_2 + L_1\cancel{\alpha_2x_1} - L_2\cancel{\alpha_1x_2} - L_2\alpha_2x_1 + L_1\alpha_2x_2 - L_1\cancel{\alpha_2x_1} + L_2\cancel{\alpha_1x_2} - L_2\alpha_1x_1\right)$$

$$0 = \frac{R^2}{D^2}(\ddot{x}_2-\ddot{x}_1) + \frac{g\alpha_1\alpha_2}{L_1L_2}\left(L_1\underbrace{(\alpha_1+\alpha_2)}_{1}x_2 - L_2\underbrace{(\alpha_2+\alpha_1)}_{1}x_1\right)$$

$$0 = \frac{R^2}{D^2}\ddot{x}_2 - \frac{R^2}{D^2}\ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_2}x_2 - \frac{g\alpha_1\alpha_2}{L_1}x_1$$

→ To take simplified (3) & (4) and get diagonal matrix, we need to combine (3) & (4) in these ways to get eq. (5) & (6):

$$(5) = \alpha_1 (3) + (4)$$
$$(6) = -\alpha_2 (3) + 4$$

$$(5)\ 0 = \alpha_1^2 \ddot{x}_2 + \alpha_1\alpha_2\ddot{x}_1 + \frac{g\alpha_1^2}{L_2} x_2 + \cancel{\frac{g\alpha_2\alpha_1}{L_1} x_1}$$
$$+ \frac{R^2}{D^2}\ddot{x}_2 - \frac{R^2}{D^2}\ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_2} x_2 - \cancel{\frac{g\alpha_1\alpha_2}{L_1} x_1}$$

$$0 = \left(\frac{R^2}{D^2} + \alpha_1^2\right)\ddot{x}_2 + \left(\frac{-R^2}{D^2} + \alpha_1\alpha_2\right)\ddot{x}_1 + \left(\frac{g\alpha_1}{L_2}\right)x_2$$

$$(6)\ 0 = -\alpha_1\alpha_2\ddot{x}_2 - \alpha_2^2\ddot{x}_1 - \cancel{\frac{g\alpha_1\alpha_2 x_2}{L_2}} - \frac{g\alpha_2^2}{L_1} x_1$$
$$+ \frac{R^2}{D^2}\ddot{x}_2 - \frac{R^2}{D^2}\ddot{x}_1 + \cancel{\frac{g\alpha_1\alpha_2}{L_2} x_2} - \frac{g\alpha_1\alpha_2}{L_1} x_1$$

$$0 = \left(\frac{R^2}{D^2} - \alpha_1\alpha_2\right)\ddot{x}_2 + \left(\frac{-R^2}{D^2} - \alpha_2^2\right)\ddot{x}_1 + \left(\frac{-g\alpha_2}{L_1}\right)x_1$$

→ combining (5) & (6) into matrixes, we get:

$$\begin{bmatrix} \frac{R^2}{D^2} + \alpha_1^2 & \frac{-R^2}{D^2} + \alpha_1\alpha_2 \\ \frac{-R^2}{D^2} + \alpha_1\alpha_2 & \frac{R^2}{D^2} + \alpha_2^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_2 \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} \frac{g\alpha_1}{L_2} & 0 \\ 0 & \frac{g\alpha_2}{L_1} \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = 0$$

→ To get the same plot as mass coords, we have to flip the equations order. Matrixes become:

$$\begin{bmatrix} \frac{R^2}{D^2} + \alpha_2^2 & \frac{-R^2}{D^2} + \alpha_1\alpha_2 \\ \frac{-R^2}{D^2} + \alpha_1\alpha_2 & \frac{R^2}{D^2} + \alpha_1^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} g\frac{\alpha_2}{L_1} & 0 \\ 0 & g\frac{\alpha_1}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

For my plots, I replaced:

```
M = [[m 0]' [0 m*R^2/D^2]'];
K = m*g*a1*a2/L1/L2 * [[L1/a2+L2/a1 L1-L2]' ...
                        [L1-L2 a2*L1+a1*L2]'];
```

with:

```
M = [[R^2/D^2+a2^2 -R^2/D^2+a1*a2]' [-R^2/D^2+a1*a2
R^2/D^2+a1^2]'];
K = [[g*a2/L1 0]' [0 g*a1/L2]'];
```

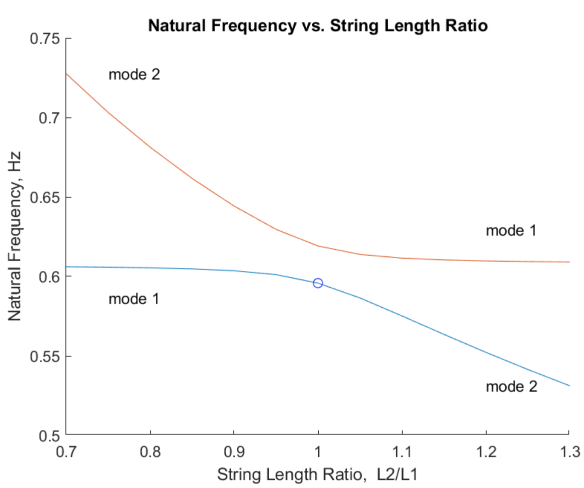
and replaced:

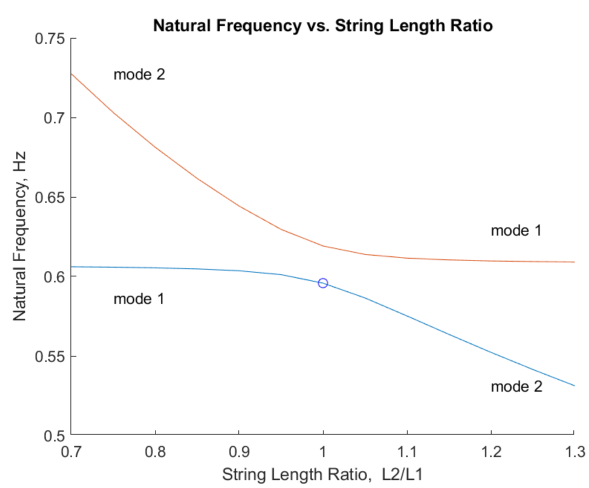
```
Vx = [[1 1]' [-a1 a2]'] * V;
```

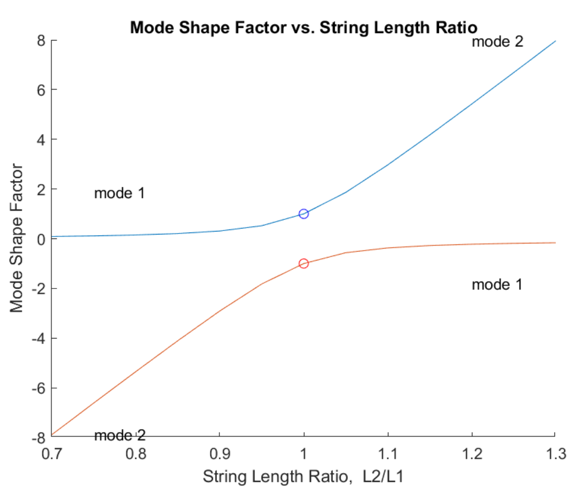
with:

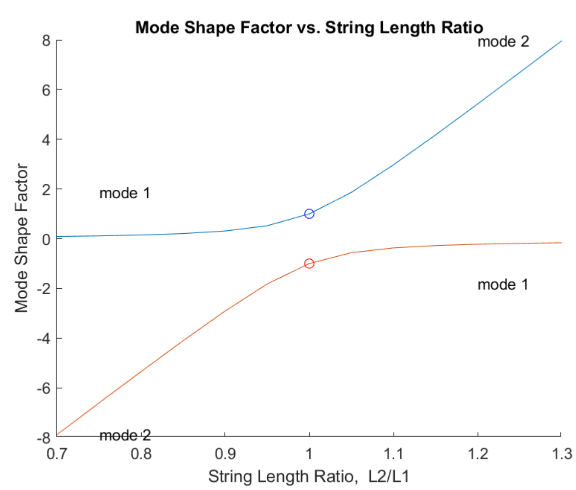
```
Vx = V;
```

These are the plots comparison:

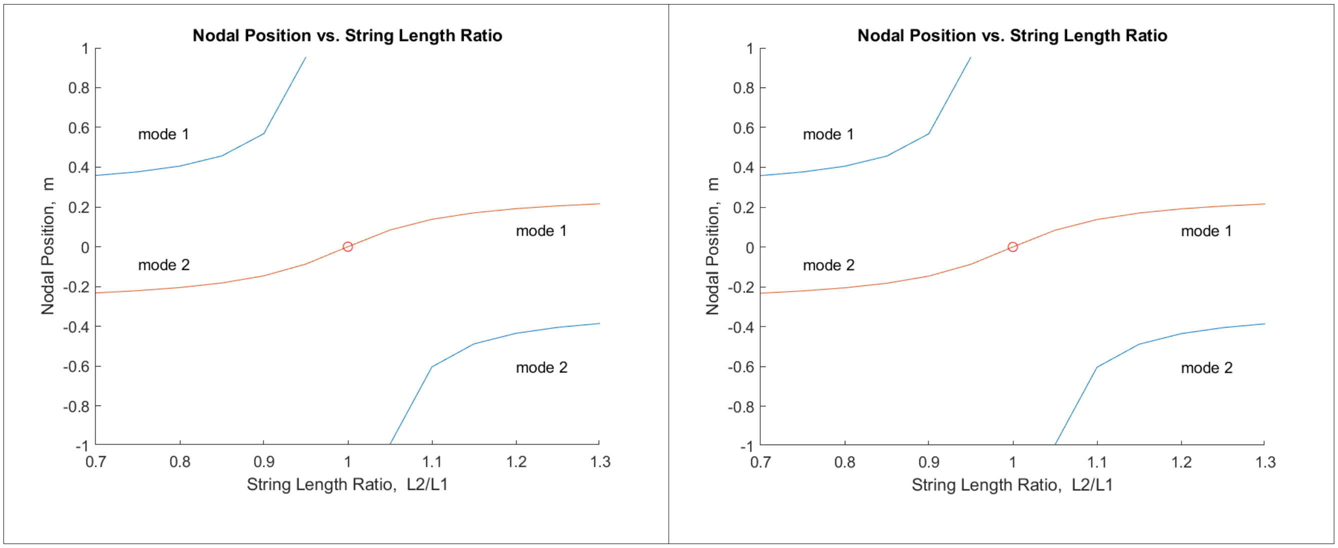
<table>
<tr><th>Mass</th><th>String</th></tr>
<tr><td>


</td><td>


</td></tr>
<tr><td>


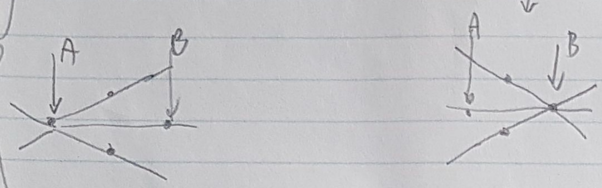
</td><td>


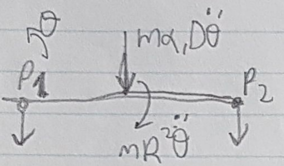
</td></tr>
</table>


Nodal Position vs. String Length Ratio
Nodal Position, m
String Length Ratio, L2/L1
mode 1
mode 2
mode 1
mode 2
Nodal Position vs. String Length Ratio
Nodal Position, m
String Length Ratio, L2/L1
mode 1
mode 2
mode 1
mode 2

(2): Theory in picture:

If ⤵ Then ⤵





From lab manual, we know that:

$$\Sigma M_2 = m\alpha_1 D\ddot{\theta}\alpha_2 D - mR^2\ddot{\theta} + P_1 D = 0$$

and for $P_1 = 0$, $D^2 = R^2/\alpha_1\alpha_2$

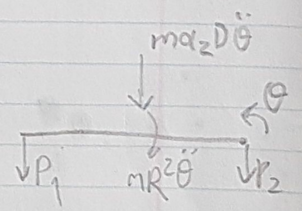
when rotation is around pt 1 & center of percussion is pt 2

and if theory is true, then conversely the bat will rotate around pt 2, and P_1 will be center of percussion ($P_2 = 0$), and equation should still satisfy $D^2 = R^2/\alpha_1\alpha_2$ if they're equivalent.

To prove this we take moment around pt 1:

$$\Sigma M_1 = 0 = m\alpha_2 D\ddot{\theta}\alpha_1 D + mR^2\ddot{\theta} + P_2 D$$

New FBD:



and when $P_2 = 0$:

$$0 = \cancel{m}\alpha_1\alpha_2 D\cancel{\ddot{\theta}}D + \cancel{m}R^2\cancel{\ddot{\theta}}$$

$$0 = \alpha_1\alpha_2 D^2 + R^2$$

$$D^2 = \frac{-R^2}{\alpha_1\alpha_2} \rightarrow -\alpha_1^2$$

Therefore, the proposed theory is true.


Wal
28 29 30 31
antech