Slide 30:

Hysteresis Loop:

Note the input and output axes of hysteresis curve, and also the direction (arrow) of the curve.

If input = force and output = displacement, work done = projection on the *y*-axis (displacement axis).

Net work done in a cycle = area of hysteresis loop = energy dissipation in a damped system.

Even "linear" damping generates a loop in the force vs displacement plane.

Example: Linear Viscous Damping (speed is proportional to force)

$$f = f_0 \sin \omega t; \quad \dot{x} = v_0 \sin \omega t$$
$$\rightarrow x = x_0 \cos \omega t$$

$$\left(\frac{f}{f_0}\right)^2 + \left(\frac{x}{x_0}\right)^2 = \sin^2 \omega t + \cos^2 \omega t = 1$$
$$\rightarrow \text{ Ellipse with two axis: } f_0 \text{ and } x_0$$

We get an ellipse (loop) for the *f*-*x* plot. But this is a linear system. The area of the loop gives energy loss per cycle.

Jump Phenomenon:

Linear natural frequency $\omega_n = \sqrt{\frac{k}{m}}$

Softening (lower *k*): Decreased resonant (and natural) frequency;

Hardening (higher *k*): Increased resonant (and natural) frequency