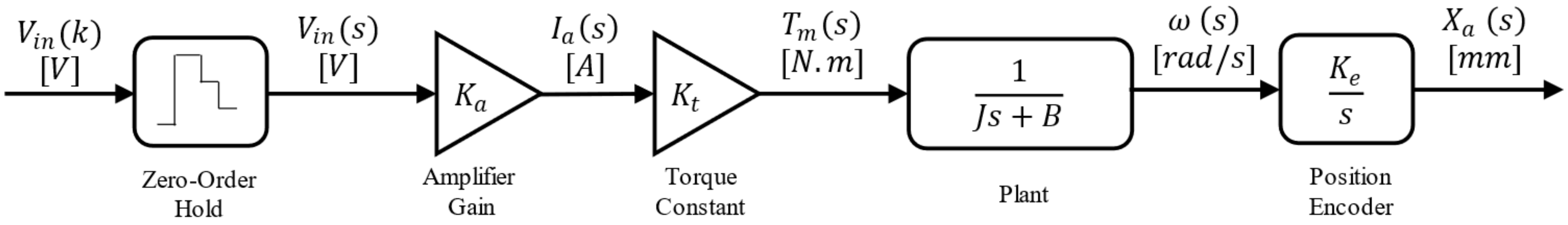
Question 1. Bode Diagram



F_s = 1000 [Hz], K_a = 0.887 [A/V], K_t = 0.72 [Nm/A], J = 7×10^{-4} [kgm^2], B = 0.006 [Nm/rad/s], K_e = 3.18 [mm/rad]

$$G_{OL}(s) = \frac{K_a K_e K_t}{Js^2 + Bs} = \boxed{\frac{2.031}{0.0007s^2 + 0.006s}}$$

$$T_s = \frac{1}{F_s} = \boxed{0.001 \text{ sec}}$$

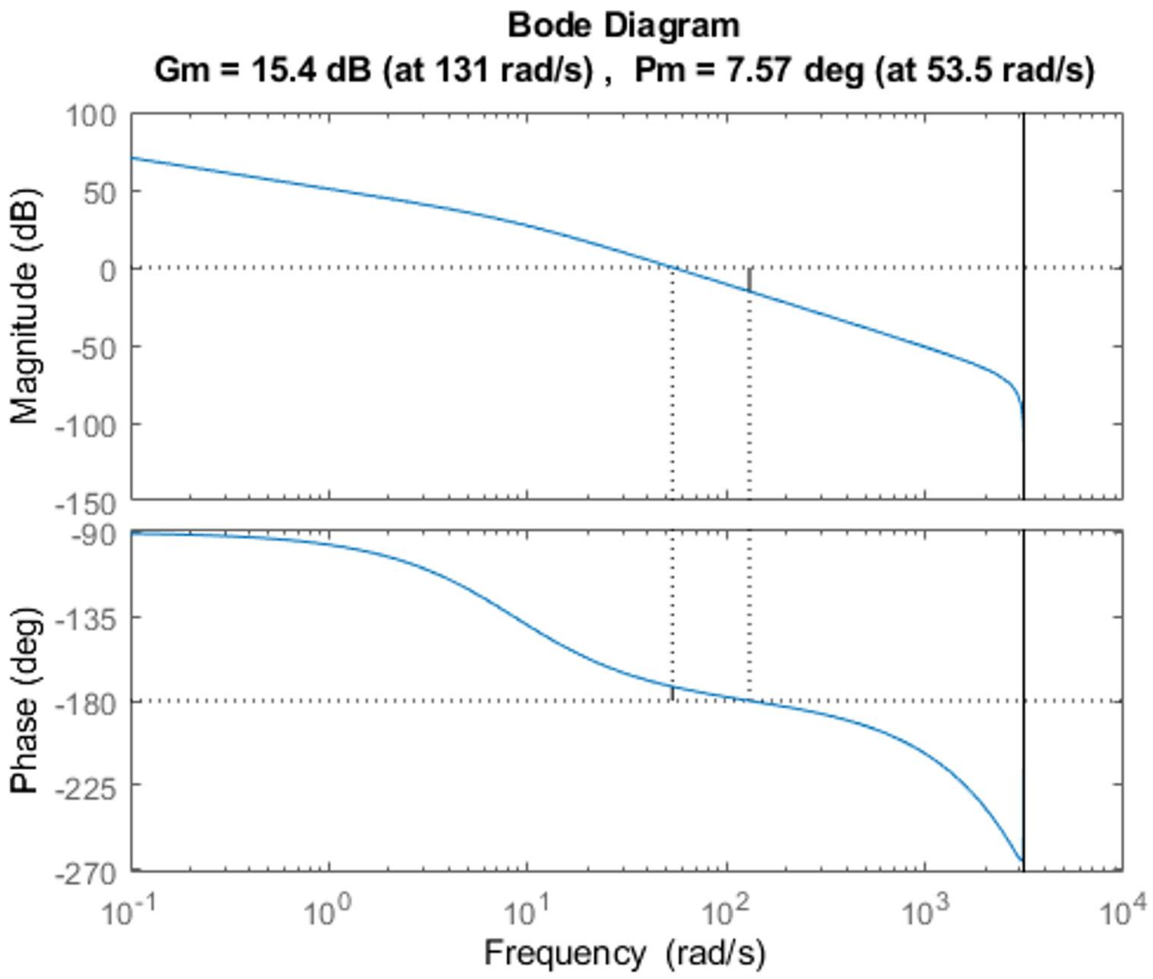
MATLAB Command: Gz = c2d(Gs, Ts, 'zoh')

$$G_{OL}(z) = \boxed{\frac{0.001446z + 0.001442}{z^2 - 1.991z + 0.9915}}$$

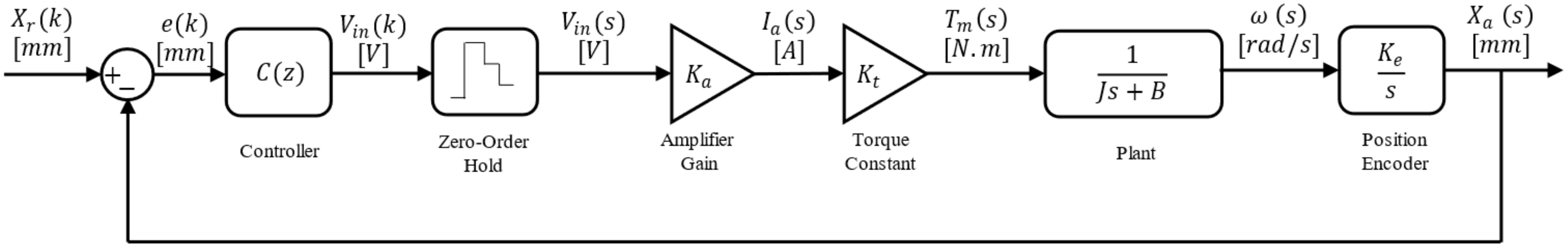
Question 1. Bode Diagram

MATLAB Command:

margin(Gz)



Question 2. Proportional Controller Design



$$\omega_g = 100 \text{ rad/s}$$

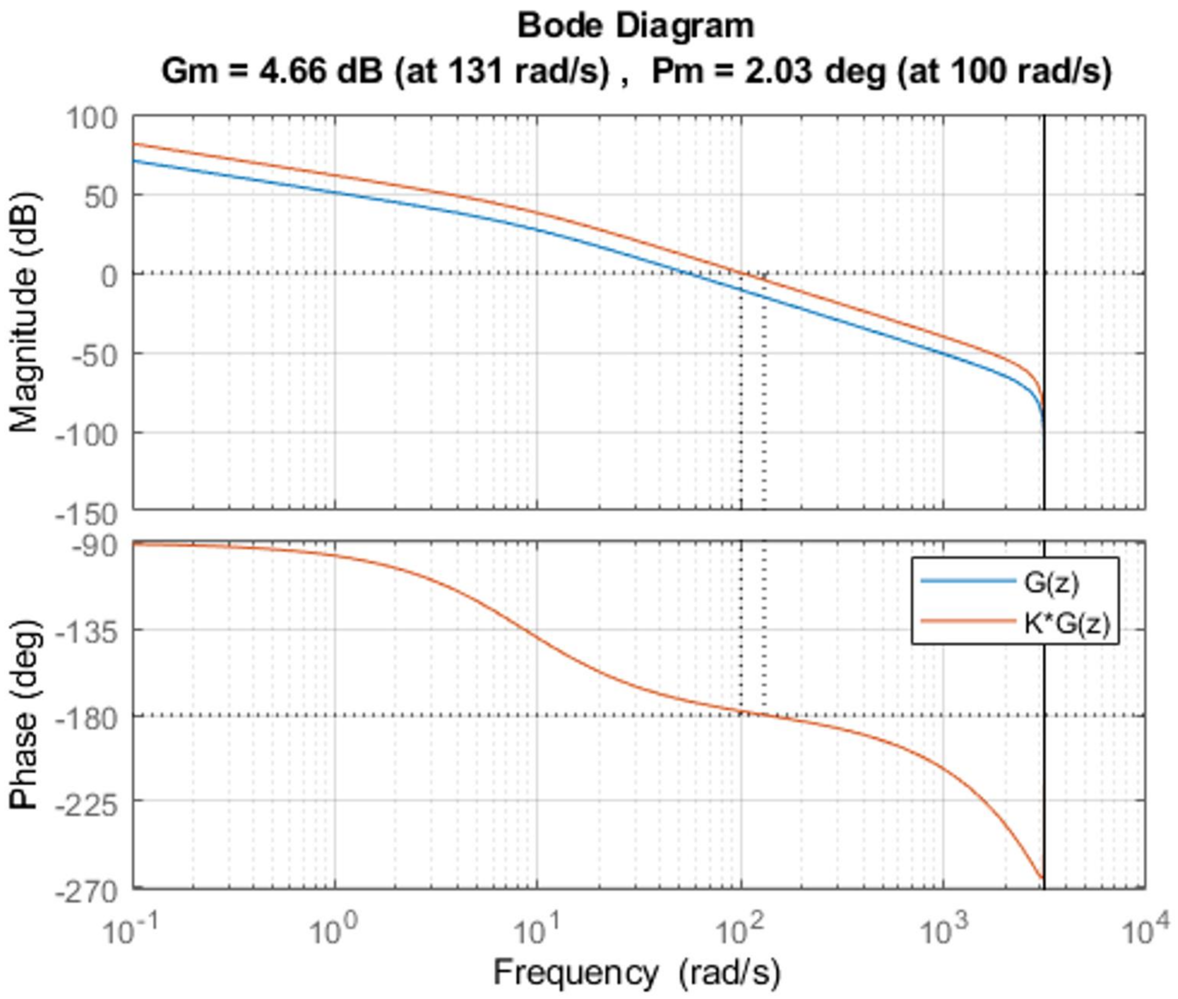
$$C(z) = K_p$$

$$\left| G_{ol}(z) \right|_{\omega = 100 \frac{rad}{s}} = 0.2889 = -10.785 \text{ dB}$$

MATLAB Command: $[mag, \sim] = bode(Gz, \omega_g)$

$$mag = 0.2889 \rightarrow K_p = \frac{1}{mag} = \boxed{3.4809}$$

Question 2. Proportional Controller Design



Question 3. Lead-Lag Controller Design

$$PM = 60° \quad \text{at} \quad \omega_g = 300 \text{ rad/s}$$

$$C(s) = K \frac{1 + aTs}{1 + Ts} = K \times LL(s)$$

MATLAB Command: [~ , phase] = bode(Gz, 300)

$$\text{Phase} = -188.9577° \longrightarrow \phi = -6.9577°$$

$$\phi_m = PM - \phi = 60 - (-6.9577) = 66.9577°$$

Question 3. Lead-Lag Controller Design

$$\alpha = \frac{1+\sin(\phi_m)}{1-\sin(\phi_m)} = \frac{1+\sin(66.9577)}{1-\sin(66.9577)} = 24.0678$$

$$T = \frac{1}{\omega_g\sqrt{\alpha}} = \frac{1}{300\sqrt{24.0678}} = 6.794\times 10^{-4}$$

$$LL(s) = \frac{1+0.01635\,s}{1+0.0006795\,s}$$

MATLAB Command: LLz = c2d(LLs, Ts, 'tustin')

$$\boxed{LLz = \frac{90.12 - 84.76}{z - 0.1522}}$$

Question 3. Lead-Lag Controller Design

MATLAB Command:

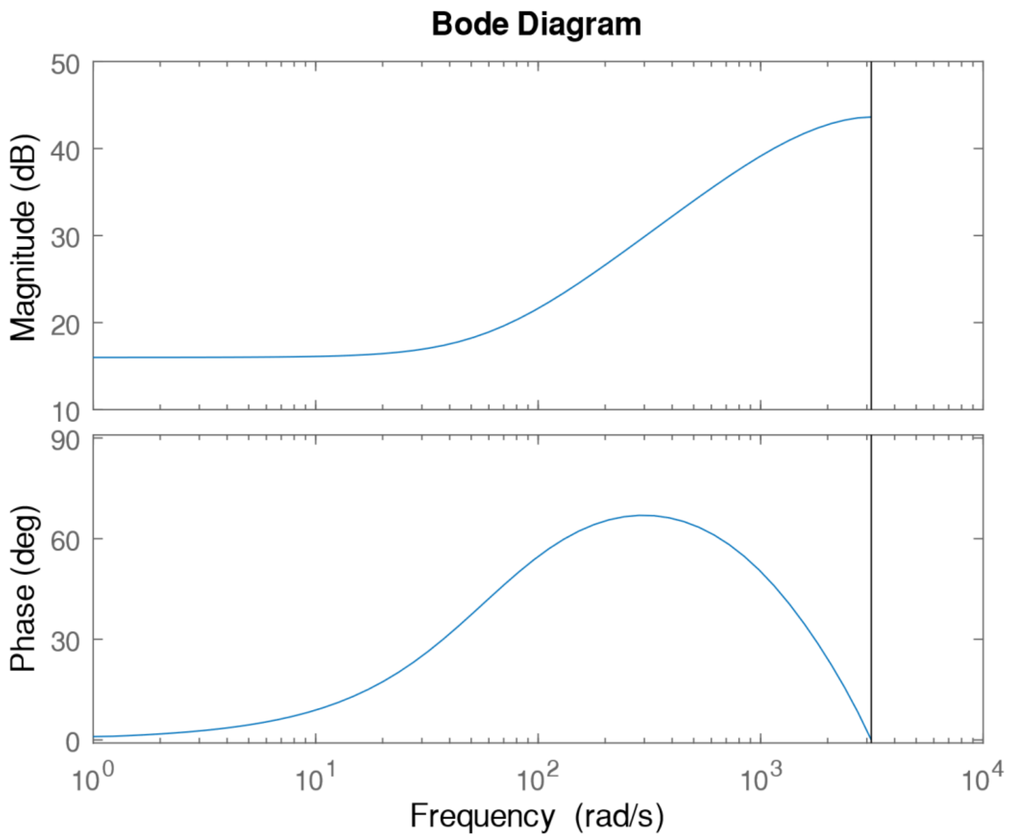
$$\left| LL(z) * G(z) \right|_{\omega = 300 \frac{rad}{s}}$$

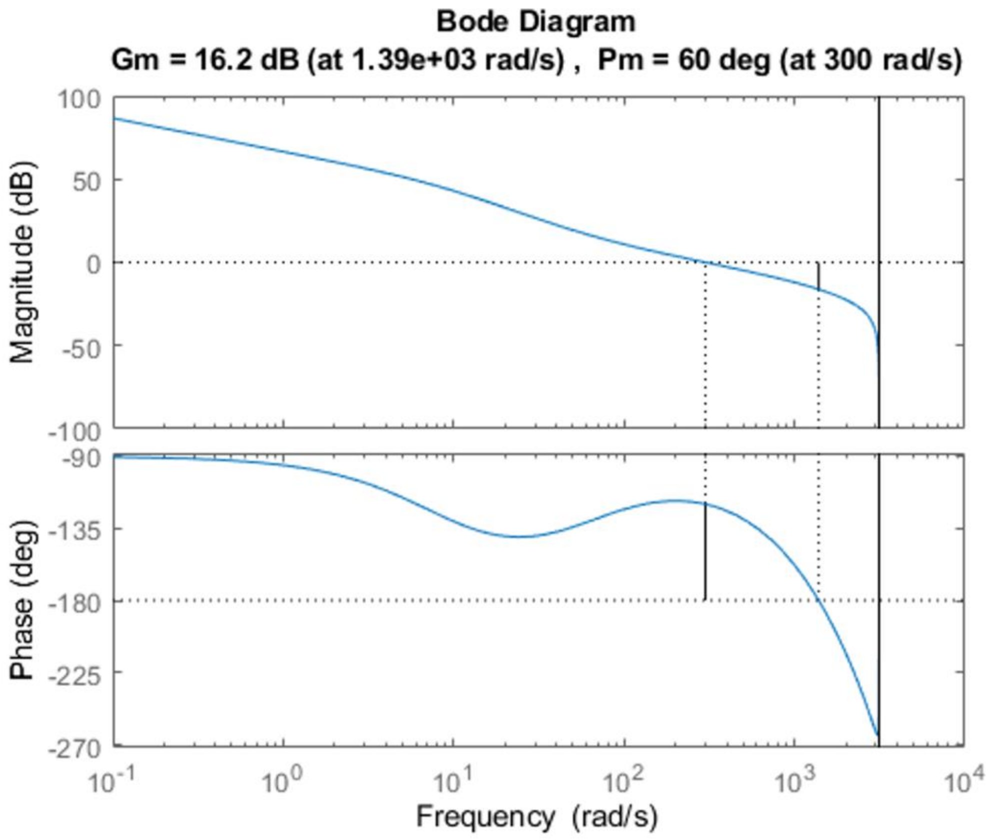
$$mag = 0.1586$$

$$\rightarrow K = \frac{1}{mag} = \boxed{6.3059}$$

>> [mag,~] = bode(LLz*Gz, 300)

Question 3. Lead-Lag Controller Design

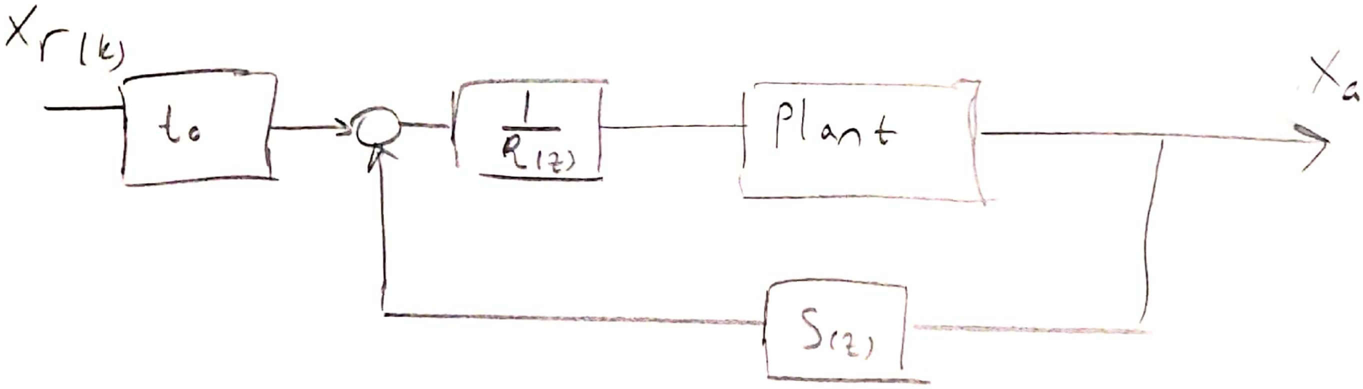




4)

$\xi_m = 0.8$

$w_m = 200 \text{ rad/s}$



$$G_{ol(z)} = \frac{0.001446z + 0.001442}{z^2 - 1.991z + 0.9915} = \frac{z^{-1}(0.001446 + 0.001442z^{-1})}{1 - 1.991z^{-1} + 0.9915z^{-2}}$$

$$= \frac{z^{-d} B(z^{-1})}{A(z^{-1})}$$

$b_0 = 0.001446$

$b_1 = 0.001442$

$a_1 = -1.991$

$a_2 = 0.9915$

$d = 1$

$n = \deg(B) = 1$

$m = \deg(A) = 2$

$$G_{m(z)} = \frac{z^{-d} B_m(z^{-1})}{A_m(z^{-1})}$$

$$A_m(z^{-1}) = 1 + m_1 z^{-1} + m_2 z^{-2}$$

$$m_1 = -2e^{-\xi_m w_m T} \cos\left(w_m T \sqrt{1-\xi_m^2}\right) = -1.692$$

$$m_2 = e^{-2\xi_m w_m T} = 0.7261$$

$$A_m = 1 - 1.692 z^{-1} + 0.7261 z^{-2}$$

$$\deg(R) = p = d + n - 1 = 1 + 1 - 1 = 1$$

$$\Rightarrow R_{(z)} = 1 + r_1 z^{-1}$$

$$\deg(S) = f = m - 1 = 2 - 1 = 1$$

$$S_{(z^{-1})} = s_0 + s_1 z^{-1}$$

$$t_0 = \left[\frac{A_m(z^{-1})}{z^{-d} B_{(z^{-1})}}\right]_{z=1} = \left[\frac{1 + m_1 + m_2}{z^{-1}(0.001446 + 0.001442)}\right]_{z=1} = 11.8101$$

$$AR + z^{-d} BS = A_m$$

$$(1 + a_1 z^{-1} + a_2 z^{-2})(1 + r_1 z^{-1}) + z^{-1}(b_0 + b_1 z^{-1})(s_0 + s_1 z^{-1}) = 1 + m_1 z^{-1} + m_2 z^{-2}$$

$$\rightarrow \underline{(r_1 + b_0 s_0)} z^{-1} + \underline{(a_1 r_1 + b_0 s_1 + b_1 s_0)} z^{-2} + \underline{(b_1 s_1 + r_1 a_2)} z^{-3} = \underline{(m_1 - a_1)} z^{-1} + \underline{(m_2 - a_2)} z^{-2}$$

$$\begin{cases} r_1 + b_0 s_0 = m_1 - a_1 \\ a_1 r_1 + b_0 s_1 + b_1 s_0 = m_2 - a_2 \\ b_1 s_1 + r_1 a_2 = 0 \end{cases}$$

$$\begin{bmatrix} 1 & b_0 & 0 \\ a_1 & b_1 & b_0 \\ a_2 & 0 & b_1 \end{bmatrix} \begin{bmatrix} r_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} m_1 - a_1 \\ m_2 - a_2 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} r_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} 1 & b_0 & 0 \\ a_1 & b_1 & b_0 \\ a_2 & 0 & b_1 \end{bmatrix}^{-1} \begin{bmatrix} m_1 - a_1 \\ m_2 - a_2 \\ 0 \end{bmatrix}$$

$r_1 = 0.1416 \qquad s_0 = 109.1298 \qquad s_1 = -97.3197$

$\boxed{R_{(z^{-1})} = 1 + 0.1416 z^{-1}}$, $\boxed{S_{(z^{-1})} = 109.1298 - 97.3197\, z^{-1}}$

$\boxed{t_0 = 11.8101}$