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1.1a $G(s) = \frac{1}{s^2(s+1)} \begin{bmatrix} s & s+1 \end{bmatrix}$

$= \frac{1}{s^3+s^2} \begin{bmatrix} [1 \ \ 1]s & [0 \ \ 1] \end{bmatrix}$ $\quad N \to \underset{\underset{q}{\uparrow}}{1} \times \underset{\underset{p}{\uparrow}}{2}$

$r = 3$

$\alpha_1 = 1 \qquad N_3 = (0 \ \ 1)$

$\alpha_2 = 0 \qquad N_2 = (1 \ \ 1)$

$\alpha_3 = 0 \qquad N_1 = (0 \ \ 0)$

$$\dot{x} = \left(\begin{array}{cc|cc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right) x + \left(\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ \hline 1 & 0 \\ 0 & 1 \end{array}\right) u$$

$$y = \left[\begin{array}{cc|cc|cc} 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right] x$$

1-1b $G(s) = \begin{pmatrix} \frac{1}{s^2+s} \\ \frac{1}{s^2} \end{pmatrix} = \frac{1}{s^3+s^2} \begin{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} s & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{bmatrix}$ $\quad N = \underset{\underset{q}{\uparrow}}{2} \times \underset{\underset{p}{\uparrow}}{1}$

$r = 3$

$\alpha_1 = 1 \qquad N_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\alpha_2 = 0 \qquad N_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\alpha_3 = 0 \qquad N_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\dot{x} = \left(\begin{array}{c|c|c} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{array}\right) x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} x$$

1.1c $G(s) = \dfrac{1}{s^2(s+1)}\begin{bmatrix} s^2 & s \\ s & s+1 \end{bmatrix} = \dfrac{1}{s^3+s^2}\left[\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}s^2 + \begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}s + \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\right]$

$\alpha_1 = 1 \qquad N_3 = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$

$\alpha_2 = 0 \qquad N_2 = \begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}$

$\alpha_3 = 0 \qquad N_1 = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$

$N \rightarrow \underset{q}{\underset{\uparrow}{2}} \times \underset{p}{\underset{\uparrow}{2}} \qquad r = 3$

$$\dot{x} = \left[\begin{array}{cc|cc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right] x + \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ \hline 1 & 0 \\ 0 & 1 \end{array}\right] u$$

$$y = \left[\begin{array}{cc|cc|cc} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right] x$$

1.2a $\dot{x} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} x + \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{pmatrix} u$

$y = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x$

1.2b $\dot{x} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} u$

$y = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} x$

1.2c $\dot{x} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} u$

$y = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} x$

1.3a. $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}$ $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

$$\mathcal{C} = \begin{pmatrix} B & AB & A^2B \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{pmatrix}$$

$$\mathrm{Im}(\mathcal{C}) = \left\{ x_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + x_5 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, x_i \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\mathcal{O} = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\ker(\mathcal{O}) = \left\{ x \in \mathbb{R}^3, \begin{array}{l} x_3 = 0 \\ x_2 - x_3 = 0 \\ x_1 - x_2 + x_3 = 0 \end{array} \right\} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$T_{c\bar{o}} = \mathrm{Im}(\mathcal{C}) \cap \ker(\mathcal{O}) = 0$$

$$T_{co} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$T_{\bar{c}\bar{o}} = 0$$

$$T_{\bar{c}o} = 0$$

$$T^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad T = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$TAT^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \qquad TB = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & 0 \end{pmatrix} \qquad CT^{-1} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

136 $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $C = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$

$$\mathcal{C} = (B \quad AB \quad A^2B) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\operatorname{Im}(\mathcal{C}) = \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\mathcal{O} = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\ker(\mathcal{O}) = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$T_{c\bar{o}} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$T_{\bar{c}\bar{o}} = T_{\bar{c}o} = T_{c\bar{o}} = 0$$

$$T^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$TAT^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad TB = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad CT^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

13c $A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$

$$\mathcal{C} = \begin{pmatrix} B & AB & A^2B & A^3B & A^4B & A^5B \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 \end{pmatrix}$$

$\mathrm{Im}(\mathcal{C}) = \left\{ \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix} \cdots \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \right\}$ (6 unit vectors) colspace

$$\mathcal{O} = \begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ '' & '' & '' & '' & 1 & 0 \\ '' & '' & '' & '' & -1 & 1 \\ '' & '' & '' & '' & 1 & 0 \\ '' & '' & '' & '' & 1 & -1 \\ & & & & -1 & 0 \\ & & & & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\ker(\mathcal{O}) = \left\{ \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$ null

$$T_{C\bar{O}} = \operatorname{im}(C) \cap \ker(O) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$T_{CO} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix} \cdots \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

$$T_{\bar{C}\bar{O}} = 0$$

$$T_{\bar{C}O} = 0$$

$$T^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/\sqrt{2} \\ 1 & 0 & 0 & 0 & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{2} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$TAT^{-1} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad TB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad CT^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Q2

```
mr = 0.095;
r = 0.085;
br = 0.001;
mp = 0.024;
Lp = .129;
bp = .00005;
g = 9.81;
Jr = mr*r*r/3;
Jp = mp*Lp*Lp/3;
l = Lp/2;
Jt = Jr*Jp - (mp*r*l)^2;
disp("non inv pendulum");
a = [0 1 0 0; 0 -Jp*br/Jt ((mp*l)^2)*r*g/Jt mp*r*l*bp/Jt; 0 0 0 1; 0 mp*r*l*br/Jt -Jr*mp*g*l/Jt -Jr*bp/Jt];
b = [0; Jp/Jt; 0; -mp*r*l/Jt];
c = [1 0 0 0; 0 0 1 0];
d = [0; 0];
Co = ctrb(a, b);
unco = length(a) - rank(Co);
disp("uncontrollable states:");
disp(unco);
Ob = obsv(a, c);
unob = length(a) - rank(Ob);
disp("unobserveable states:");
disp(unob);
disp("inv pendulum");
a = [0 1 0 0; 0 -Jp*br/Jt ((mp*l)^2)*r*g/Jt -mp*r*l*bp/Jt; 0 0 0 1; 0 -mp*r*l*br/Jt Jr*mp*g*l/Jt -Jr*bp/Jt];
b = [0; Jp/Jt; 0; mp*r*l/Jt];
c = [1 0 0 0; 0 0 1 0];
d = [0; 0];
Co = ctrb(a, b);
unco = length(a) - rank(Co);
disp("uncontrollable states:");
disp(unco);
Ob = obsv(a, c);
unob = length(a) - rank(Ob);
disp("unobserveable states:");
disp(unob);
disp("both pendulums are controllable and observeable, therefore they are minimal");
```

```
non inv pendulum
uncontrollable states:
     0

unobserveable states:
     0

inv pendulum
uncontrollable states:
     0

unobserveable states:
     0

both pendulums are controllable and observeable, therefore they are minimal
```

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