NAME:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 571R MECH 563 MECH 464: Introduction to Robotics
Make-up Final Examination, Thursday December 14, 2017, 09:00-11:30.
Closed Book. Cheat sheet at the end of the assignment.
There are two problems. Maximum - 80 marks.
You will be graded out of 70.
Graduate students have an additional small problem at the end.
Reason your solutions and pay attention to your notation.

Problem 1. *(40 points)*

(1) (6 marks) Draw a schematic representation of one of the ABB arms from the figure below in the configuration shown or in a convenient nominal position (ignore the degrees of freedom of the gripper).
How many degrees of freedom does this robot have? Why? Describe two of its singular configurations.

(2) (2 marks) Consider the solution $Q(t)$ of the linear differential equation

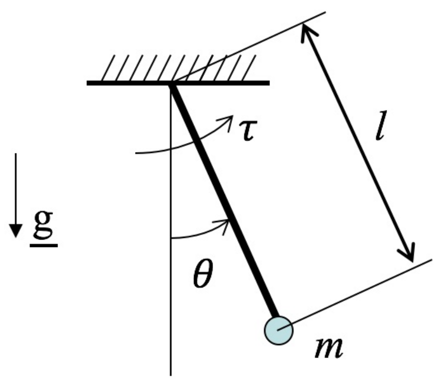
$$\dot{Q}(t) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} Q(t) \qquad (1)$$

with initial condition $Q(0) = I$, a 3×3 identity matrix. What is $Q(t)$ for $t = \pi/2$?
(3) (3 marks) The time-varying coordinates of a point $\underset{\sim}{y}$ that moves with respect to a rigid body B are given by $y(t) = d(t) + Q(t)x(t)$, where $Q(t)$ and $d(t)$ are the entries of the homogenous transformation describing the position/orientation of B with respect to a fixed base coordinate system and $x(t)$ are the coordinates of the point with respect to B. Find the acceleration coordinates $\ddot{y}(t)$ with respect to the base coordinate system

as a function of $\ddot{d}, x, \dot{x}, \ddot{x}, Q$ and the angular velocity ω and angular acceleration $\dot{\omega}$ of B with respect to the base.

(4) (2 marks). Write down the kinetic energy T of a rigid body that is translating and rotating with respect to a fixed coordinate system, as a function of its mass, its inertia tensor/inertia matrix with respect to its centre of mass, and its linear and angular velocity. Explain your choice of fixed or body-attached coordinate systems.

(5) (4 marks). Derive the equation of motion of the pendulum shown in the following figure using Lagrange's equation:



(6) (6 marks) Draw the controller block diagrams for computed torque, stiffness and PD+gravity position controllers for robots. Mark which computations need to be done on-line (in real-time, not at the path planning level).

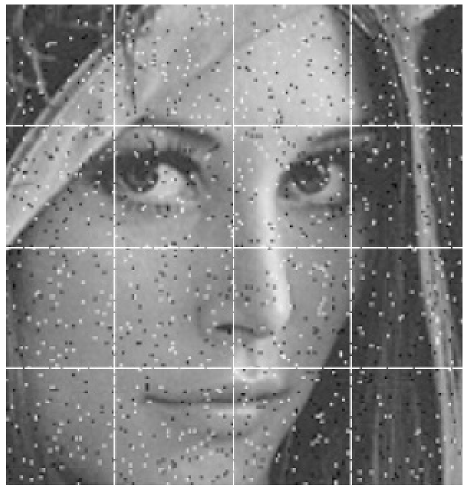
(7) (3 marks) Use a Lyapunov function argument to show that if the control law $u = k_p(x_d - x) - k_v\dot{x}$ is applied to the system $m\ddot{x} = u$, then x converges to x_d, a constant.

(8) (2 marks) We often see quadcopters and hexacopters but less often "tri-copters". Why?

(vii) (12 marks) Provide *concise* answers to the following:

- What was the purpose and the result of the colour matching experiments. Give an example of the use of colour processing that may be useful in robotics.
- What does it mean to "calibrate a camera"? What do the *intrinsic* and *extrinsic* parameters refer to? What does the process typically involve?
- What does it mean to estimate the pose of an object with a calibrated camera? What would you say are the minimum number of targets necessary to estimate pose?
- Which of the following is true: A Gaussian kernel is convolved with an image to (a) find local minima, (b) low-pass filter the image before sub-sampling, (c) obtain a gradient image, or (d) find corners in the image.

- What is the
- How would you try to clean up the salt & pepper noise such as the one you see in Lena's photo below?



- What does a morphological operation of "opening" typically involve? What about the morphological operation of "closing"?

Problem 2. Consider the manipulator of the figure below.

(1) *(10 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 5$ to links 1 through 5, using the Denavit-Hartenberg convention. Note that the *directed* axes (see arrows) that you need to use to assign joint variables are provided in the figure. Complete the table of Denavit-Hartenberg parameters. Find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem.

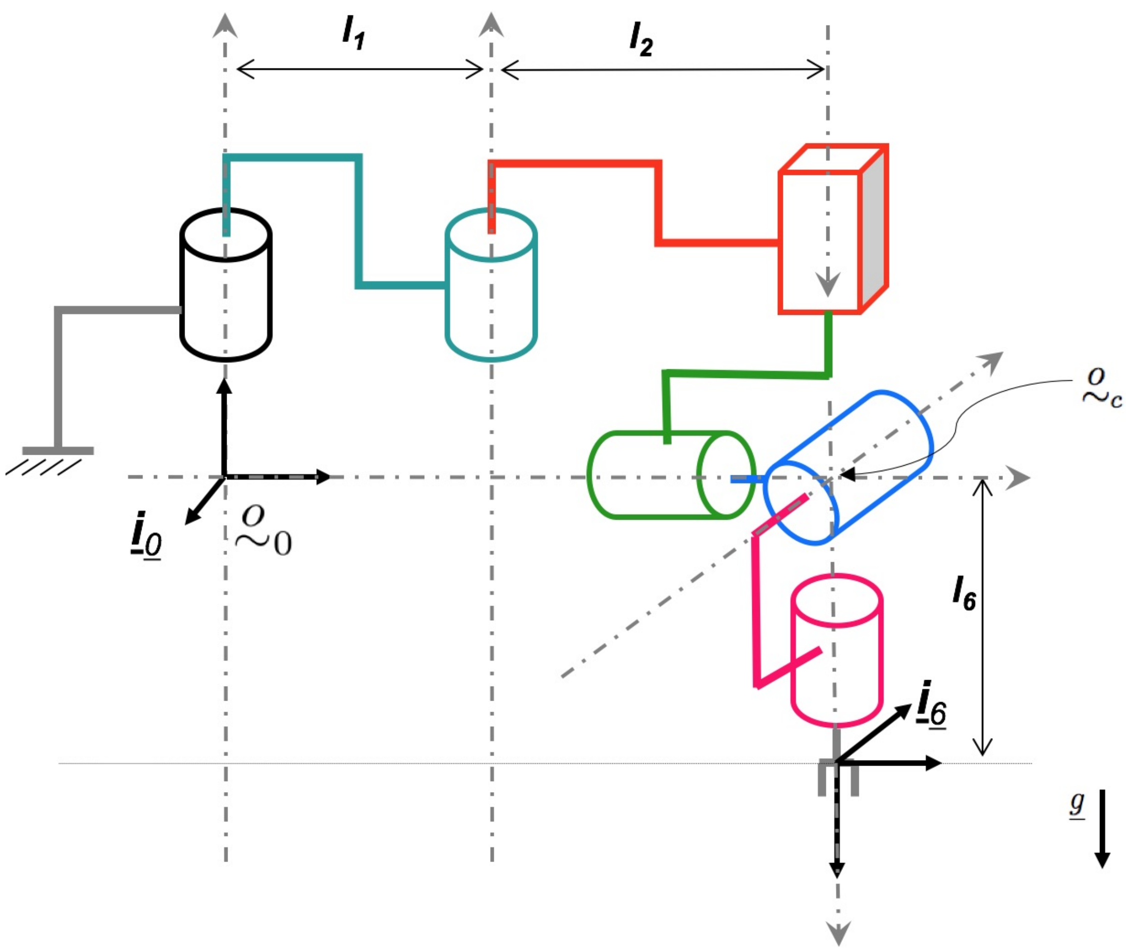
(2) *(10 marks)* Find the robot Jacobian and the robot singular configurations.
Is the robot suitable to perform tasks in a plane parallel to $\underset{\sim}{o}_0 \underline{i}_0 \underline{j}_0$, e.g. for PCB assembly? Is the robot suitable to work on a cylindrical surface (cylinder axis is $\underline{k}_0$) surrounding it?
Ignoring interference between the robot links, what are the reachable and dextrous workspaces of this manipulator?

(3) *(10 marks)* Solve the inverse kinematics for this manipulator, i.e. for a gripper location $\{\underset{\sim}{o}_d, \underline{C}_d\}$, find all joint angles such that the manipulator gripper coordinate system $\{\underset{\sim}{o}_6, \underline{C}_6\}$ coincides with a desired coordinate system $\{\underset{\sim}{o}_d, \underline{C}_d\}$. Carefully discuss the case of multiple solutions or lack of solutions. If you use Kahan's problems P1-P4 (see

attached cheat sheet), you must clearly specify the input data and provide a solution for that particular case.

(4) *(10 marks)* For certain contact tasks (e.g. following a contour with a grinding tool), the robot stiffness must be programmed in Cartesian space. For simplicity, we ignore orientation (assume that the gripper orientation stays constant), and control the robot *wrist center*. Using the 3×3 robot Jacobian mapping from $\dot{\theta}_1, \dot{\theta}_2, \dot{d}_3$ to robot Cartesian velocity coordinates (translation only), in $\underline{C}_0$, find a stiffness controller (torque u_1, torque u_2, force u_3) whose static response to a deflection of the gripper of $\underline{C}_0{}^0\delta x$ is a force $\underline{C}_0{}^0K_p{}^0\delta x$, where ${}^0K_p$ is specified as ${}^0K_p = \begin{bmatrix} k_a & 0 & 0 \\ 0 & k_b & 0 \\ 0 & 0 & k_c \end{bmatrix}$. Assume that the mass of the wrist, gripper and payload is equal to m kg.
How would you modify the controller if the stiffness were to be defined in a different coordinate system $\underline{C}_s$, i.e. deflection of the gripper of $\underline{C}_s{}^s\delta x$ is a force $\underline{C}_s{}^sK_p{}^s\delta x$?

For Graduate students only: *(10 marks)*
(1) *(2 marks)* Which of the following are passive?
(a) the parallel connection of passive systems;
(b) the series connection of passive systems;
(b) the feedback connection of passive systems.
(2) *(2 marks)* True or False?: "A small delay between the input and output does not affect passivity".
(3) *(2 marks)* True or False?: "The real part of the transfer function of a linear system determines whether a linear system is passive".
(4) *(2 marks)* True or False?: "The adaptive method of Slotine and Li guarantees exponential convergence of the joint angles to the desired trajectory."
(5) *(2 marks)* True or False?: "Lyapunov stability guarantees convergence of the parameters in Slotine and Li's control scheme."



l1
l2
o
c
i0
o
0
l6
i6
g

NAME:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2012-2013): Introduction to Robotics
Final Examination, April 12, 2013, 15:30pm - 18:30pm
Closed Book
Maximum - 80 marks

Problem 1.

(i) (2 marks)
A vector product function is specified as $\underline{\underline{f}}(\underline{x}) = (\underline{i}_0 - \underline{j}_0 + 2\underline{k}_0) \times \underline{x}$. What is the matrix representation of $\underline{\underline{f}}$ in $\underline{C}_0 = [\underline{i}_0 \ \underline{j}_0 \ \underline{k}_0]$?

(ii) (2 marks)
The frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{i}_0$ by $\theta(t)$. What is the matrix representation ${}^0C_1$ of this rotation in $\underline{C}_0$?

(iii) (2 marks)
(iii) What is the matrix representation of the function $\underline{\underline{f}}$ from (i) above in $\underline{C}_1$ from (ii)? (you do not need to multiply out the matrices).

(iv) (2 marks)
If a vector $\underline{x}$ has coordinates ${}^0x$ in $\underline{C}_0$, what are its coordinates ${}^1x$ in $\underline{C}_1$?

(v) (2 marks)
What is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?

(vi)(5 marks)
Write the homogeneous transformation ${}^{i-1}T_i$ relating the coordinate systems $\{\underset{\sim}{o}_{i-1}, \underline{C}_{i-1}\}$ and $\{\underset{\sim}{o}_i, \underline{C}_i\}$ attached to link $i-1$ and i, respectively, given that the Denavit-Hartenberg parmeters of link i are θ, d, a and α.

(vii)(5 marks)
Assume that both θ and α in (vi) above are functions of time $\theta(t)$ and $\alpha(t)$. Find the angular velocity of $\underline{C}_i$ with respect to $\underline{C}_{i-1}$.

Problem 2.

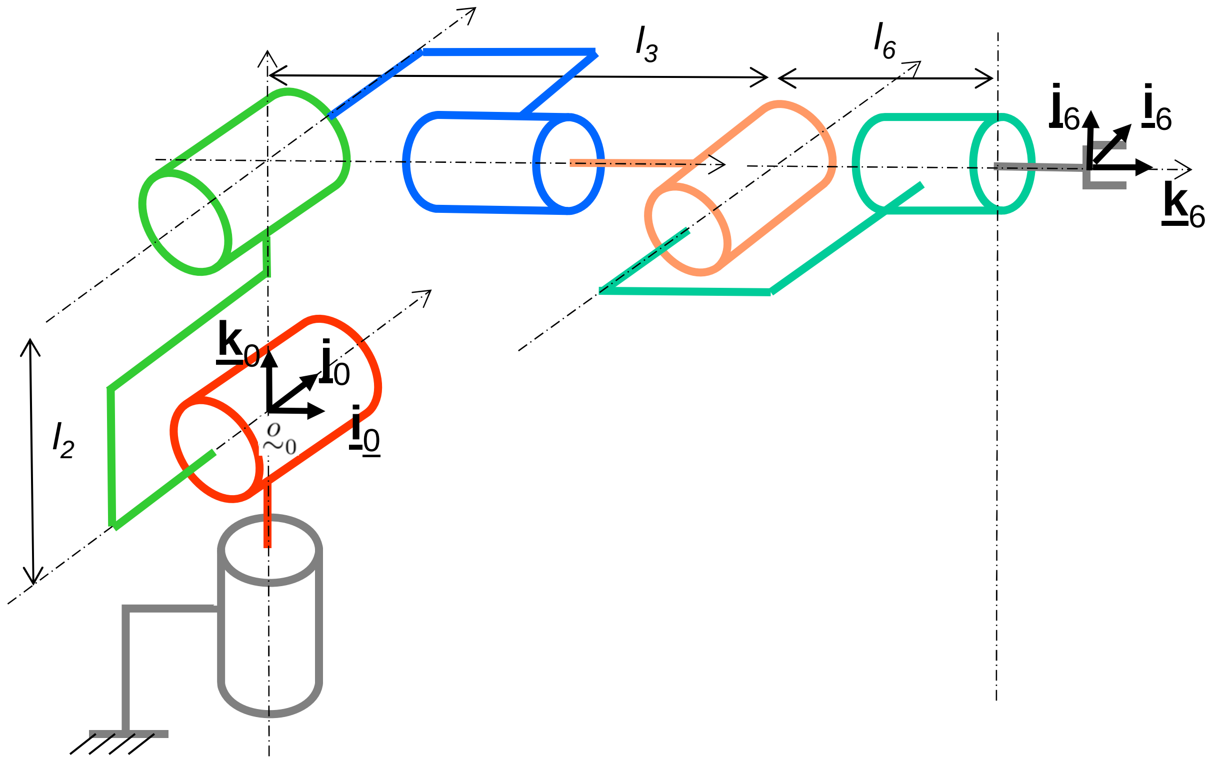
Consider the manipulator of Figure P2. (a) *(10 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 5$ to to links 1 through 5, using the Denavit-Hartenberg convention, and find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Complete the table of Denavit-Hartenberg parameters.
(b) *(10 marks)* Find the manipulator Jacobian and use it to find and discuss the manipulator singular configurations.
(c) *(10 marks)* Solve the inverse kinematics for this manipulator. Carefully discuss the case of multiple solutions or lack of solutions. If you use Kahan's problems P1-P4 (see attached sheets), you must clearly specify the input data and provide a solution for that particular case.

Problem 3.

Consider the 2 DOF planar manipulator shown in Figure P3. The mass of the first link is M. The second link has mass m and its center of mass is at a distance l from the link axis.
(a) *(15 marks)* Show that the equations of motion of this manipulator are:

$$\begin{aligned} (M+m)\ddot{y} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta &= u \qquad (1) \\ ml\cos\theta\ddot{y} + ml^2\ddot{\theta} - mgl\sin\theta &= \tau\ . \qquad (2) \end{aligned}$$

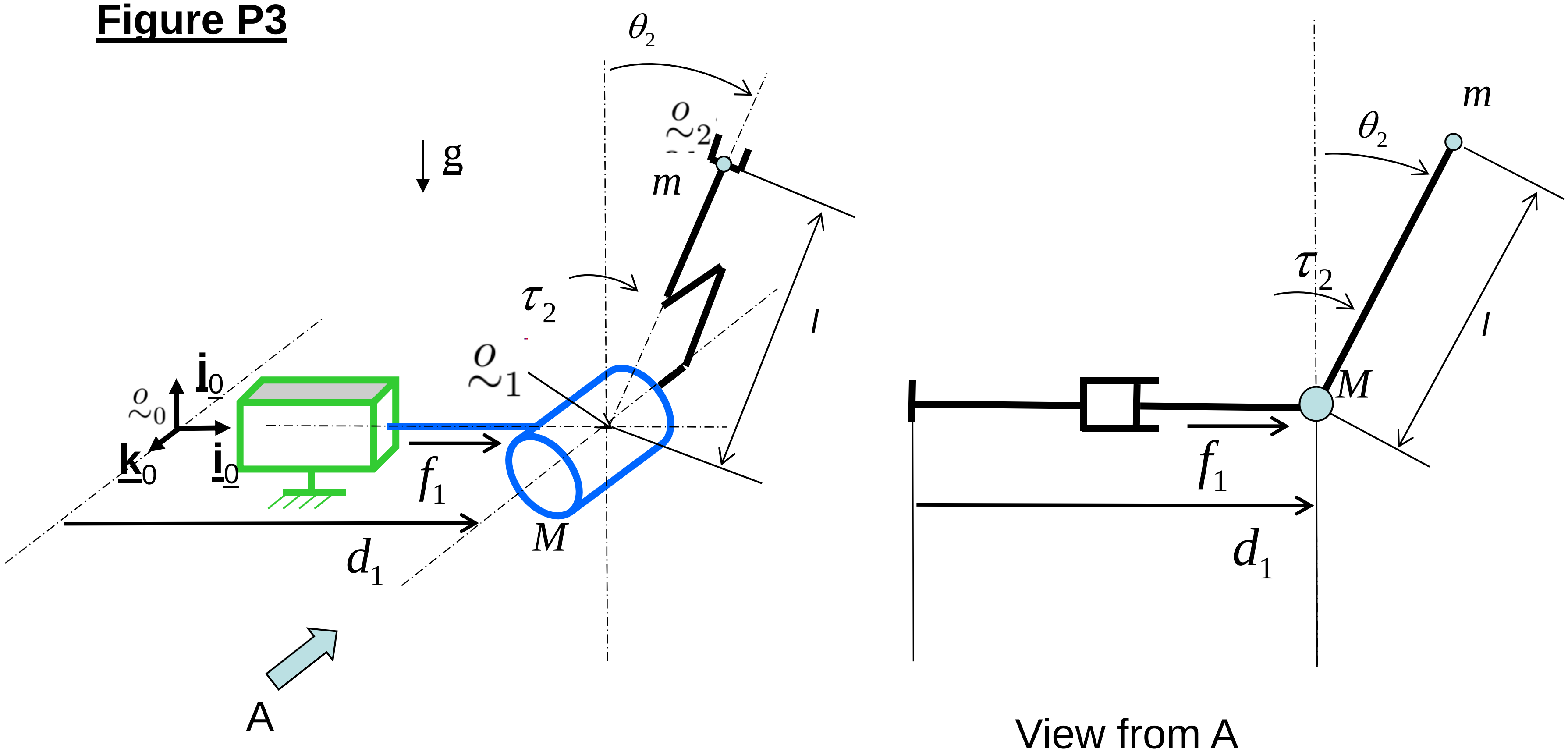
(b) *(5 marks)* In (a), identify the gravity term and the mass matrix. Using the equations of motion derived in (a), write down the control u for a feedforward controller and for a PD + gravity controller.
(c) *(10 marks)* Use the 2×2 Jacobian J to obtain a stiffness + gravity controller with Cartesian space proportional gain matrix in $\underline{C}_0$ given by $K_p = \begin{bmatrix} 100 & 0 \\ 0 & 0 \end{bmatrix}$ and with derivative gain matrix in $\underline{C}_0$ given by $K_v = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$. The above controller makes the robot behave as a damped slider in the $\underline{j}_0$ direction. What would the gain K_p be if the slider were to be in the $\frac{1}{2}(\underline{i}_0 + \underline{j}_0)$ direction?



l_3
l_6
$\underline{\mathbf{j}}_6$
$\underline{\mathbf{i}}_6$
$\underline{\mathbf{k}}_6$
$\underline{\mathbf{k}}_0$
$\underline{\mathbf{j}}_0$
$\underline{\mathbf{i}}_0$
$\underset{\sim}{o}_0$
l_2

Figure P2

Figure P3



View from A

NAME:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2013-2014): Introduction to Robotics
Final Examination, December 9th 2013, 12:00pm - 3:00pm
Closed Book - Maximum - 100 marks

Problem 1.

(a) (3 marks)
A vector product function is specified as $\underline{\underline{f}}(\underline{x}) = (-3\underline{i}_0 + 4\underline{j}_0 + 5\underline{k}_0) \times \underline{x}$. What is the matrix representation of $\underline{\underline{f}}$ in $\underline{C}_0 = [\underline{i}_0 \ \ \underline{j}_0 \ \ \underline{k}_0]$?

(b) (3 marks)
The frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by $\theta(t)$. What is the matrix representation ${}^0C_1$ of this rotation in $\underline{C}_0$?

(c) (3 marks)
(iii) What is the matrix representation of the function $\underline{\underline{f}}$ from (a) above in $\underline{C}_1$ from (b)? (you do not need to multiply out the matrices).

(d) (3 marks)
If a vector $\underline{x}$ has coordinates ${}^0x$ in $\underline{C}_0$, what are its coordinates ${}^1x$ in $\underline{C}_1$? Write out the coordination transformation matrix in exponential form or specify each of its entries.

(e) (3 marks)
What is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?

(f)(3 marks)
Write the homogeneous transformation ${}^{i-1}T_i$ relating the coordinate systems $\{\underset{\sim}{o}_{i-1}, \underline{C}_{i-1}\}$ and $\{\underset{\sim}{o}_i, \underline{C}_i\}$ attached to link $i-1$ and i, respectively, given that the Denavit-Hartenberg parmeters of link i are θ, d, a and α.

(g)(3 marks)
Assume that both θ and α in (f) above are functions of time $\theta(t)$ and $\alpha(t)$. Find the angular velocity of $\underline{C}_i$ with respect to $\underline{C}_{i-1}$.

(h)(3 marks)
Define the center of mass, the inertia tensor and the kinetic energy of a rigid body. What is the kinetic energy of a rigid body in terms of the velocity of its center-of-mass and its angular velocity?

(i)(6 marks)
Consider the Stewart platform of Figure P1. With $\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1$ and $\underline{C}_1 = \underline{C}_0\,{}^0C_1$, find the leg lengths q_i as a function of ${}^0d_1$, ${}^0C_1$ and the coordinates ${}^0b_i$, ${}^0p_i$ of the leg joints with respect to the base and platform coordinate systems, respectively.

Problem 2.

Consider the manipulator of Figure P2.

(a) *(10 marks)*

Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 5$ to to links 1 through 5, using the Denavit-Hartenberg convention, and find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Complete the table of Denavit-Hartenberg parameters.

(b) *(10 marks)*

Find the manipulator Jacobian and use it to find and discuss the manipulator singular configurations.

(c) *(10 marks)*

Solve the inverse kinematics for this manipulator, i.e. for a gripper location $\{\underset{\sim}{o}_d, \underline{C}_d\}$, find all joint angles such that the manipulator gripper coordinate system $\{\underset{\sim}{o}_6, \underline{C}_6\}$ coincides with $\{\underset{\sim}{o}_d, \underline{C}_d\}$. Carefully discuss the case of multiple solutions or lack of solutions. If you use Kahan's problems P1-P4 (see attached sheets), you must clearly specify the input data and provide a solution for that particular case.

(d) *(10 marks)*

The inverse kinematics for this manipulator can be solved iteratively. Let $\underset{\sim}{o}_d = \underset{\sim}{o}_0 + \underline{C}_0 \, {}^0d$ and $\underline{C}_d = \underline{C}_0 e^{\theta k\times} e^{\phi j\times} e^{\psi i\times}$, and $x = \begin{bmatrix} {}^0d \\ \theta \\ \phi \\ \psi \end{bmatrix}$. Derive the Newton's iteration that allows you to find the robot joint angles q_d that correspond to a desired end-effector location x_d, starting from an approximation q_0 of q_d.

Problem 3.

Consider the 2 DOF planar manipulator shown in Figure P3. The mass of the first link is M. The second link has mass m and its center of mass is at a distance l from the link axis.

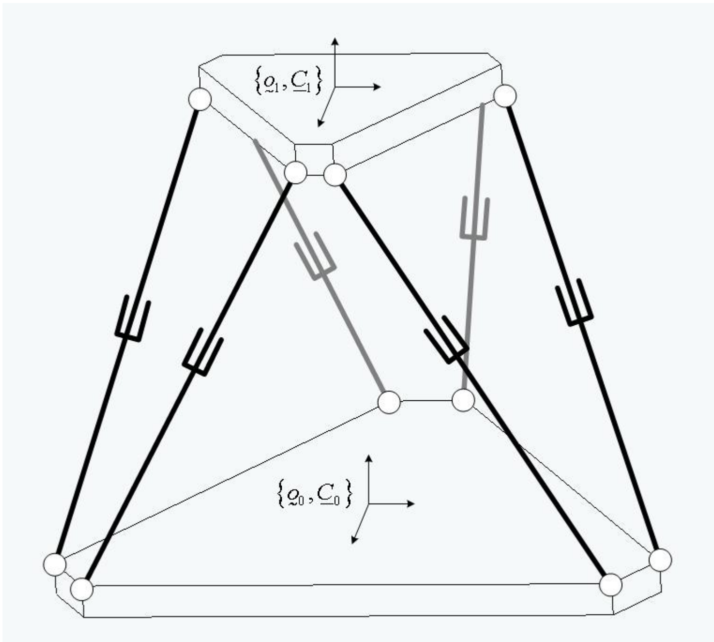
(a) *(15 marks)* Show that the equations of motion of this manipulator are:

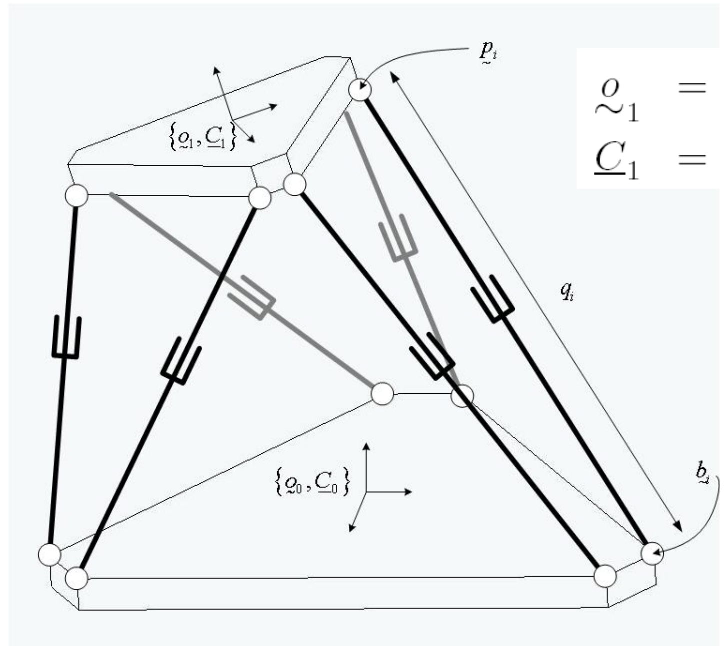
$$\begin{aligned} (M+m)\ddot{y} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta &= u \qquad (1) \\ ml\cos\theta\ddot{y} + ml^2\ddot{\theta} - mgl\sin\theta &= \tau\,. \qquad (2) \end{aligned}$$

(b) *(5 marks)* In (a), identify the gravity term and the mass matrix. Using the equations of motion derived in (a), write down the control u for a feedforward controller and for a PD + gravity controller.

(c) *(10 marks)* Use the 2×2 Jacobian J to obtain a stiffness + gravity controller with Cartesian space proportional gain matrix in $\underline{C}_0$ given by $K_p = \begin{bmatrix} 100 & 0 \\ 0 & 0 \end{bmatrix}$ and with derivative gain matrix in $\underline{C}_0$ given by $K_v = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$. The above controller makes the robot behave as a damped slider in the $\underline{j}_0$ direction. What would the gain K_p be if the slider were to be in the $\frac{1}{2}(\underline{i}_0 + \underline{j}_0)$ direction?

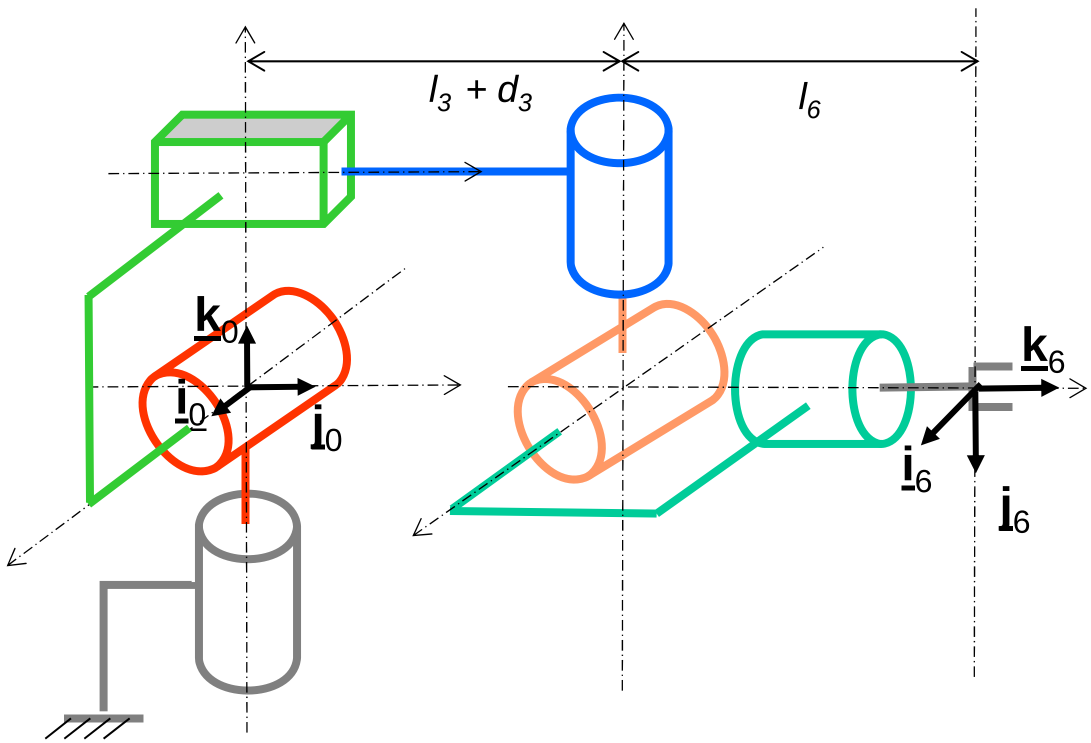
<u>Figure P1</u>





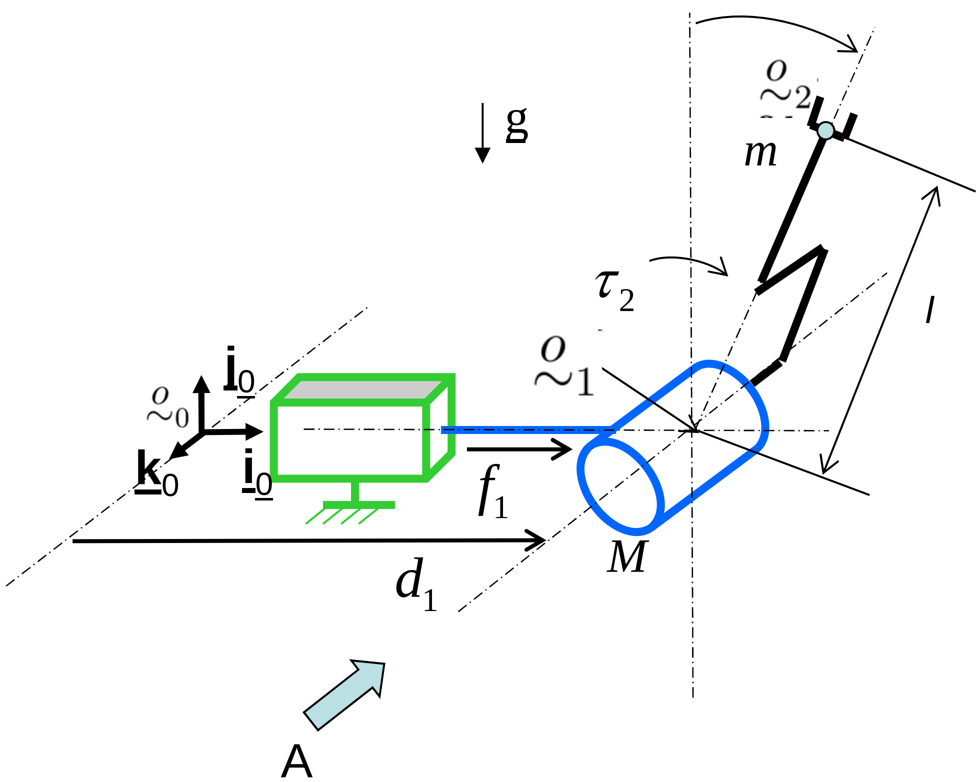
$$\begin{aligned} \underset{\sim}{o}_1 &= \underset{\sim}{o}_0 + \underline{C}_0 {}^0 d_1 \\ \underline{C}_1 &= \underline{C}_0 {}^0 C_1 \end{aligned}$$

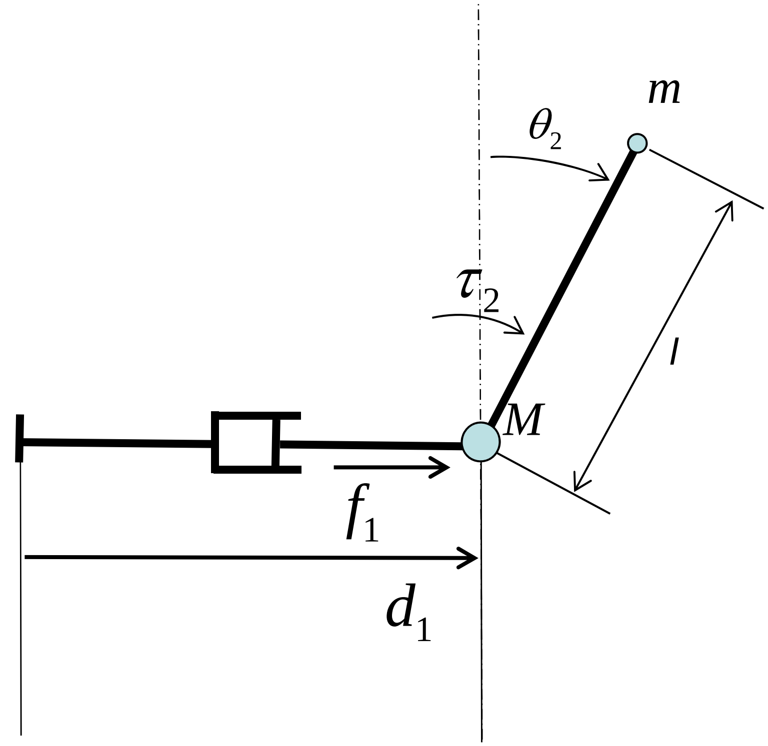
Figure P2



$l_3 + d_3$
l_6
$\underline{\mathbf{k}}_0$
$\underline{\mathbf{i}}_0$
$\underline{\mathbf{j}}_0$
$\underline{\mathbf{k}}_6$
$\underline{\mathbf{i}}_6$
$\underline{\mathbf{j}}_6$

Figure P3





View from A

NAME:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2010-2011): Introduction to Robotics
Final Examination, April 18, 2011, 12:00pm - 14:30pm
Closed Book
Maximum - 65 marks

Problem 1.

(a) (2 marks)
You are given two right handed orthonormal frames, $\underline{C}_0 = [\underline{i}_0\ \underline{j}_0\ \underline{k}_0]$ and $\underline{C}_1 = [\underline{i}_1\ \underline{j}_1\ \underline{k}_1]$. If a vector $\underline{x}$ has coordinates ${}^0x$ in $\underline{C}_0$ and ${}^1x$ in $\underline{C}_1$, what are the entries of the coordinate transformation matrix ${}^0C_1$ specifying ${}^0x$ in terms of ${}^1x$? What is ${}^1C_0$?

(b)(2 marks)
If ${}^0C_1 = e^{\theta s\times}R$, where R is a constant rotation matrix, s is a constant unit rotation axis, and $\theta = \theta(t)$ is a function of time, what is the angular velocity of $\underline{C}_1(t)$ with respect to $\underline{C}_0$?

(c)(2 marks)
If $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{i}_0$ by an angle θ, and $\underset{\sim}{o}_1$ is obtained by displacing $\underset{\sim}{o}_0$ by the vector $\underline{C}_0\,{}^0d$, find the homogeneous transformation ${}^1T_0$ that transforms the coordinates ${}^0x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$ to its coordinates ${}^1x$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$.

(d)(2 marks)
Write the homogeneous transformation ${}^{i-1}T_i$ relating the coordinate systems $\{\underset{\sim}{o}_{i-1}, \underline{C}_{i-1}\}$ and $\{\underset{\sim}{o}_i, \underline{C}_i\}$ attached to link $i-1$ and i, respectively, given that the Denavit-Hartenberg parmeters of link i are θ, d, a and α.

(e)(2 marks)
Consider a translating and rotating rigid body with a time-varying coordinate system $\{\underset{\sim}{c}(t), \underline{C}(t)\}$ attached at its center of mass $\underset{\sim}{c}$. With respect to a fixed frame, $\underset{\sim}{c}(t) = \underset{\sim}{o}_0 + \underline{C}_0 c(t)$ and $\underline{C}(t) = \underline{C}_0 Q(t)$. If the body has mass m and inertia matrix $J_{\underset{\sim}{c}}$ with respect to $\{\underset{\sim}{c}, \underline{C}\}$, what is the kinetic energy of the rigid body?

(f)(2 marks)
Write down the form of the equations of motion for a serial manipulator and a computed torque controller. Choose the gains so that the closed-loop dynamics for every joint angle has characteristic equation $s^2 + 2s + 1 = 0$ (poles at $s = -1$).

(g) (3 marks)
Consider the pendulum of Figure 1. Find the equation of motion relating θ and τ using Lagrange's equation $\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} - \frac{\partial L}{\partial \theta} = \tau$.

Problem 2.

Consider the manipulator shown in Figure 2.
(a) *(10 marks)* Assign angles θ_1 through θ_6 to the robot axes such that with the robot as drawn, positive angles correspond to positive rotations about the base frame vectors. Assign coordinate systems to the links and solve the manipulator direct kinematics. What are the Denavit-Hartenberg parameters of this robot?
(b) *(10 marks)* Find the manipulator Jacobian and specify all the manipulator singular configurations.
(c) *5 marks* Find the coordinates of the angular velocity of link 3 with respect to link 0 and in frame 0. what are the coordinates of this angular velocity in link 3? (c) *(10 marks)* Solve the manipulator inverse kinematics. Specify all solutions. You may make use of Kahan's problems (see attached), but you must clearly specify the inputs and outputs.

Problem 3.

Consider the 2 DOF planar manipulator shown in Figure 3. Assume that the mass of the first link is zero, and that the entire mass m of the second link is concentrated at the gripper center $\underset{\sim}{o}_2$.
(a) *(10 marks)* Obtain the equations of motion of this manipulator. What is the manipulator mass matrix? Write a simulation diagram that takes the force and torque and initial conditions as inputs and produces joint rates and joint angles as outputs.
(b) *(5 marks)* Write the block diagram of a joint-space PD + gravity set-point controller. For what gain matrices K_p and K_v is this controller stable?
(d) *(5 marks)* Write down the block diagram and the expressions for τ_1 and f_2 to implement a cartesian space stiffness + gravity controller with cartesian proportional gain matrix in $\underline{C}_0$ given by $K_P = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$ and $K_v = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$.

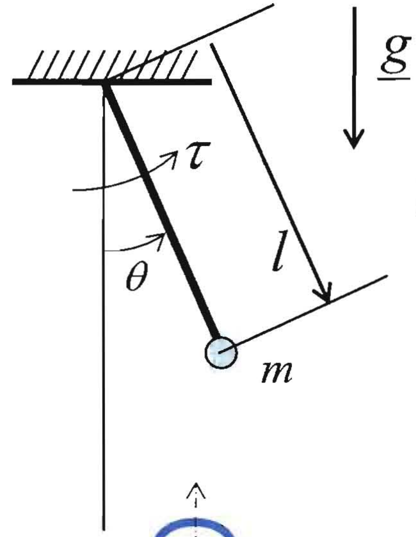


Figure 1

Figure 2

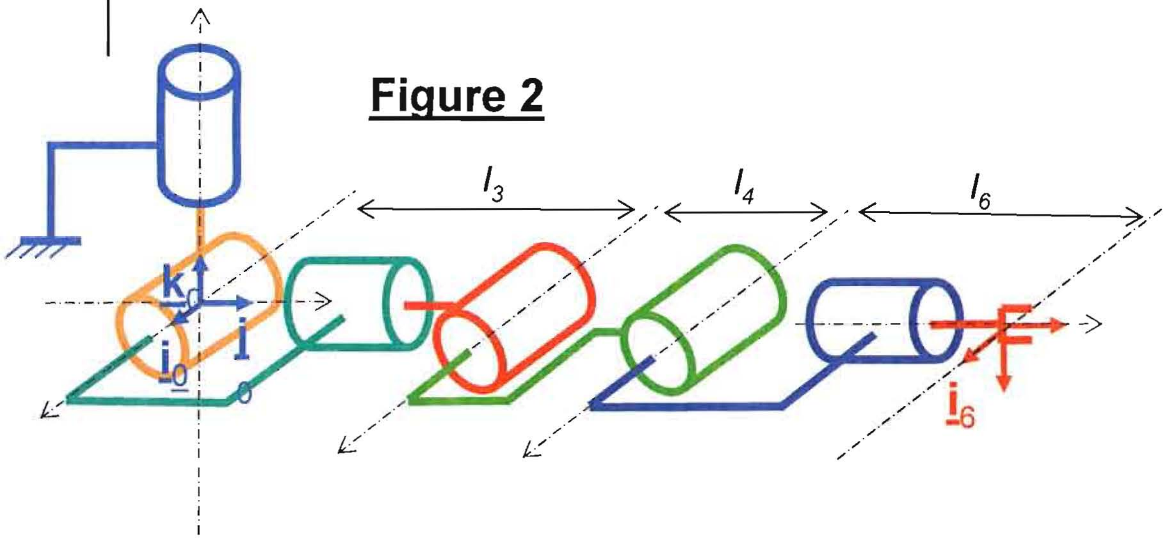
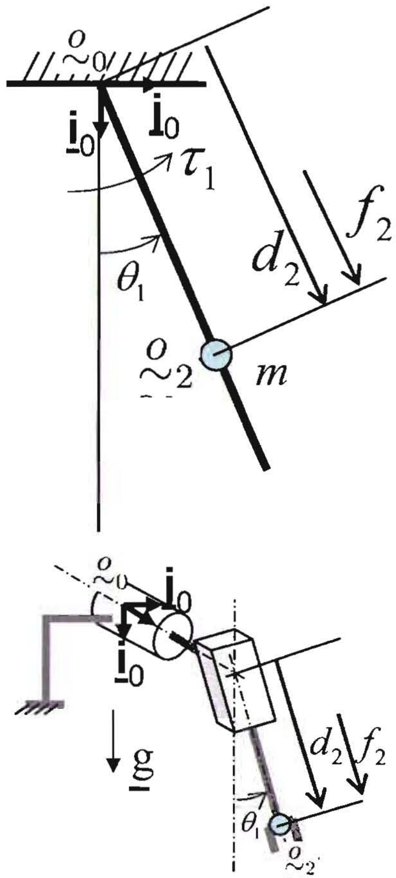
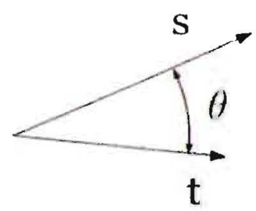


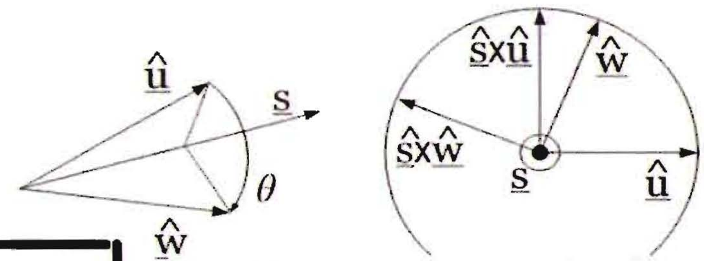
Figure 3



P1: Given **s** and **t**, find θ

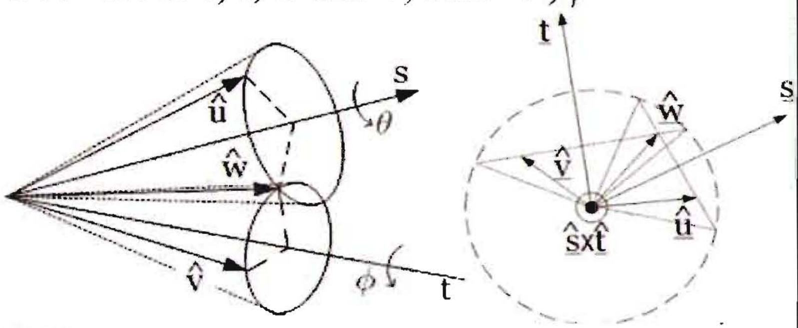


P2: Given **u** and **w**, find θ



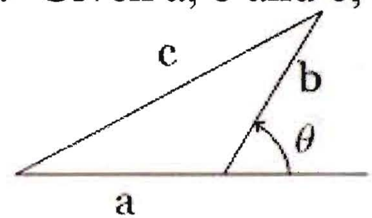
$\boxed{\textbf{P2}}$ Solve $e^{\theta \hat{\underline{s}}\times}\underline{\hat{u}} = \underline{\hat{w}}$ for θ

P3: Given **s, t, u** and **v**, find θ, ϕ



$\boxed{\textbf{P3}}$ Solve $e^{\theta \hat{\underline{s}}\times}\underline{\hat{u}} = e^{\phi \hat{\underline{t}}\times}\underline{\hat{v}}$ for θ, ϕ

P4: Given a, b and c, find θ



NAME:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2010): Introduction to Robotics
Final Examination, April 27, 2010, 19:00 - 21:00
Closed Book
Maximum - 60 marks

Problem 1.

Consider the planar Stewart platform shown in Figure P1.

(a) *(5 marks)* Solve its inverse kinematics, i.e., find l_1, l_2 and l_3 given the coordinates x, y and the angle θ of the platform with respect to the base ($\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{i}_0 x + \underline{j}_0 y$).
(b) *(5 marks)* Find the platform Jacobian giving the leg extension rates $[\dot{l}_1 \;\; \dot{l}_2 \;\; \dot{l}_3]^T$ as a function of platform linear and angular velocity $v = [\dot{x} \;\; \dot{y} \;\; \dot{\theta}]^T$.

Problem 2.

Consider the manipulator shown in Figure P2.
(a) *(10 marks)* Assign angles θ_1 through θ_6 to the robot axes such that with the robot as drawn, positive angles correspond to positive rotations about the base frame vectors. Assign coordinate systems to the links and solve the manipulator direct kinematics. What are the Denavit-Hartenberg coordinates of this robot?
(b) *(5 marks)* Find the manipulator Jacobian and specify all the manipulator singular configurations.
(c) *(15 marks)* Solve the manipulator inverse kinematics. Specify all solutions.

Problem 3.

Consider the 2 DOF planar manipulator shown in Figure P3. The mass of the first link is M. The second link has mass m and its center of mass is at a distance l from the link axis.
(a) *(10 marks)* Obtain the equations of motion of this manipulator.
(b) *(5 marks)* Using the equations of motion derived in (a), obtain a computed torque controller such that the closed-loop system dynamics (in d_1, θ_2) are linear, decoupled and have poles at $s = -1$ and $s = -1$.
(c) *(5 marks)* Find the manipulator 2×2 Jacobian J and use it to obtain a stiffness controller with Cartesian space proportional gain matrix in $\underline{C}_0$ given by $K_p = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$.

NAME:
Student #:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2009): Introduction to Robotics
Final Examination - version 2, April 24, 2009
Closed Book - 150 Minutes
Maximum - 70 marks

Solve all the problems, reason your solutions, and follow our notational convention.

Problem 1.

(a) *(5 marks)* You are given two coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$, with $\underline{i}_1 = c_{11}\underline{i}_0 + c_{21}\underline{j}_0 + c_{31}\underline{k}_0$, $\underline{j}_1 = c_{12}\underline{i}_0 + c_{22}\underline{j}_0 + c_{32}\underline{k}_0$, and $\underline{k}_1 = c_{13}\underline{i}_0 + c_{23}\underline{j}_0 + c_{33}\underline{k}_0$, and $\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + d_1\underline{i}_0 + d_2\underline{j}_0 + d_3\underline{k}_0$.
If a point $\underset{\sim}{x}$ has coordinates ${}^0x$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and ${}^1x$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$, write the homogeneous matrices ${}^0T_1$ expressing ${}^0x$ in terms of ${}^1x$ and ${}^1T_0$ expressing ${}^1x$ in terms of ${}^0x$.

(b) *(5 marks)* If $\underline{C}_1 = \underline{C}_0 e^{\theta(t)s\times}$, where the unit length rotation axis s is constant, what is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$? What are the coordinates of this angular velocity in $\underline{C}_1$?

(c) *(5 marks)* Suppose $\{\underset{\sim}{o}_0, \underline{C}_0\}$ is fixed and $\{\underset{\sim}{o}_1, \underline{C}_1\}$ is attached to a translating and rotating rigid body, with $\underset{\sim}{o}_1$ at the center of mass. The velocity of $\underset{\sim}{o}_1$ with respect to $\{\underset{\sim}{o}_0, \underline{C}_0\}$ is $\underline{v}$ and the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$ is $\underline{\omega}$.
Write down an expression for the kinetic energy of the rigid body assuming its mass is M and its inertia matrix with respect to $\{\underset{\sim}{o}_1, \underline{C}_1\}$ is J.

(d) *(5 marks)* Consider the pendulum shown in Figure P1. Obtain the Lagrangian, the equations of motion and a computed torque controlled leading to a closed-loop poles at $s = -1$.

(e) *(5 marks)* What is the form of the dynamics equations of motion for a serial manipulator? What are the motor torques/forces for a PD+gravity controller? Draw a block diagram of the closed-loop system consisting of the robot controlled by a PD+gravity controller.
Intuitively, why is this closed loop system stable?
Draw a block diagram of a stiffness(+gravity) controller. How does it relate to the PD controller? What woudl be its uses?

Problem 2.

Consider the manipulator shown in Figure P2. All joint parameters are zero in the nominal configuration shown in the figure.
(a) *(10 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 5$ to to links 0 through 5, using the Denavit-Hartenberg convention, and find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Complete the table of Denavit-Hartenberg parameters.
(b) *(5 marks)* Find the manipulator Jacobian and use it to find and discuss the manipulator singular configurations.
(c) *(10 marks)* Solve the inverse kinematics for this manipulator. Carefully discuss the case of multiple solutions or lack of solutions. If you use Kahan's problems P1-P4 (see attached sheets), you must clearly specify the input data and provide a solution for that particular case.

Problem 3.

Consider the 2 DOF planar manipulator shown in Figure P3. The mass of the first link is m. The second link has mass M and its center of mass is at a distance l m from the link axis.

(a) *(10 marks)* Show that the equations of motion of this manipulator are given by:

$$\begin{aligned}(M+m)\ddot{d} - Ml\ddot{\theta}\sin\theta - Ml\dot{\theta}^2\cos\theta + Mg + mg &= f && (1)\\ Ml^2\ddot{\theta} - Ml\ddot{d}\sin\theta + Mgl\sin\theta &= \tau\,. && (2)\end{aligned}$$

What is the manipulator mass matrix? What is the gravitational vector? Can you write a form of the equations in which $\dot{D} - 2C$ is skew symmetric?
(b) *(5 marks)* Using the equations of motion derived in (a), obtain a computed torque controller such that the closed-loop system dynamics (in d, θ) are linear, decoupled and have both poles at $s = -2$.
(c) *(5 marks)* Obtain a joint-space PD + gravity controller, with the same gains as used in (b). Discuss the trade-offs between the PD+gravity and computed torque controller. Under what conditions do these controllers behave similarly?
(d) *(5 bonus marks)* Obtain a cartesian space stiffness + gravity controller with cartesian proportional gain matrix in $\underline{C}_0$ given by $K_P = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$. Explain why such a controller might be needed.

Figure P1

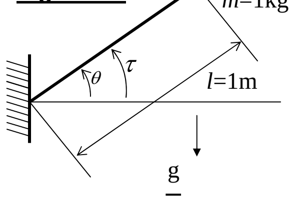


Figure P2

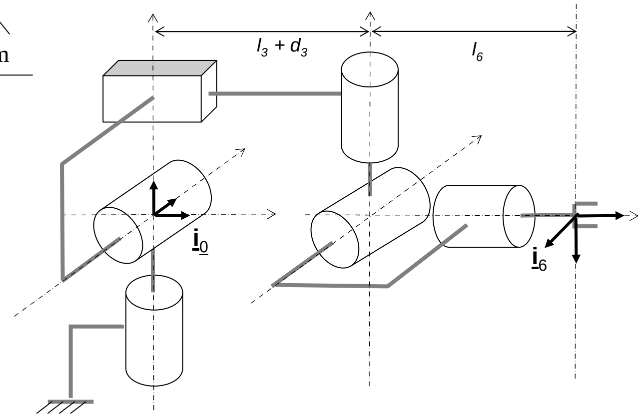
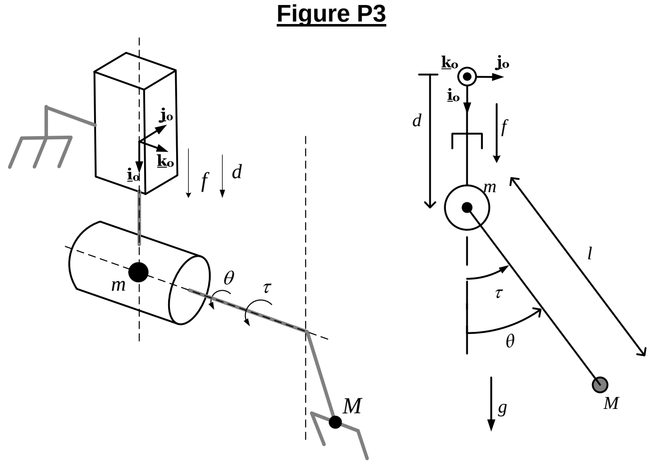


Figure P3



NAME:
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University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2008): Introduction to Robotics
Final Examination, April 15, 2008
Closed Book - 150 Minutes
Maximum - 70 marks

Solve all the problems, reason your solutions, and follow our notational convention.

Problem 1.

(a) *(10 marks)* Sketch a manipulator that is described by the table of DH parameters below. Joint variables are enclosed in parantheses. Start with a base coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$ as shown in Figure P1, show and label the coordinate systems $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$, $\{\underset{\sim}{o}_3, \underline{C}_3\}$. Label the dimensions d_i and a_i.

DH Parameter	θ_i	d_i	a_i	α_i
Link 1	$-\pi/2 + (\theta_1)$	d_1	0	$-\pi/2$
Link 2	$\pi/2$	(d_2)	a_2	0
Link 3	(θ_3)	d_3	0	0

(b) *(5 marks)* Find the angular velocity of the frame $\underline{C}_3$ with respect to $\underline{C}_0$ as a function of the joint rates.
(c) *(5 marks)* Find the acceleration of $\underset{\sim}{o}_2$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ as a function of the joint rates and accelerations.

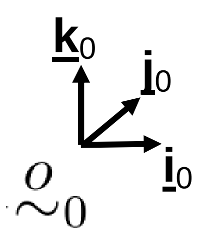


Figure P1

Problem 2.

Consider the manipulator shown in Figure P2. All joint parameters are zero in the nominal configuration shown in the figure.
(a) *(10 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 5$ to to links 1 through 5, using the Denavit-Hartenberg convention, and find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Complete the table of Denavit-Hartenberg parameters.
(b) *(5 marks)* Find the manipulator Jacobian and use it to find and discuss the manipulator singular configurations.
(c) *(15 marks)* Solve the inverse kinematics for this manipulator. Carefully discuss the case of multiple solutions or lack of solutions. If you use Kahan's problems P1-P4 (see attached sheets), you must clearly specify the input data and provide a solution for that particular case.

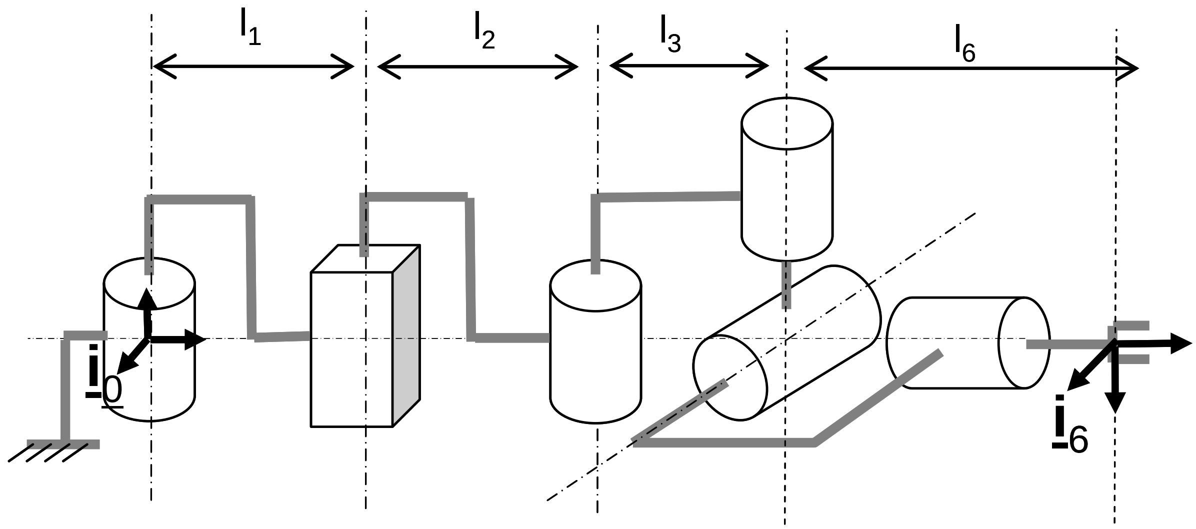


Figure P2

Problem 3.

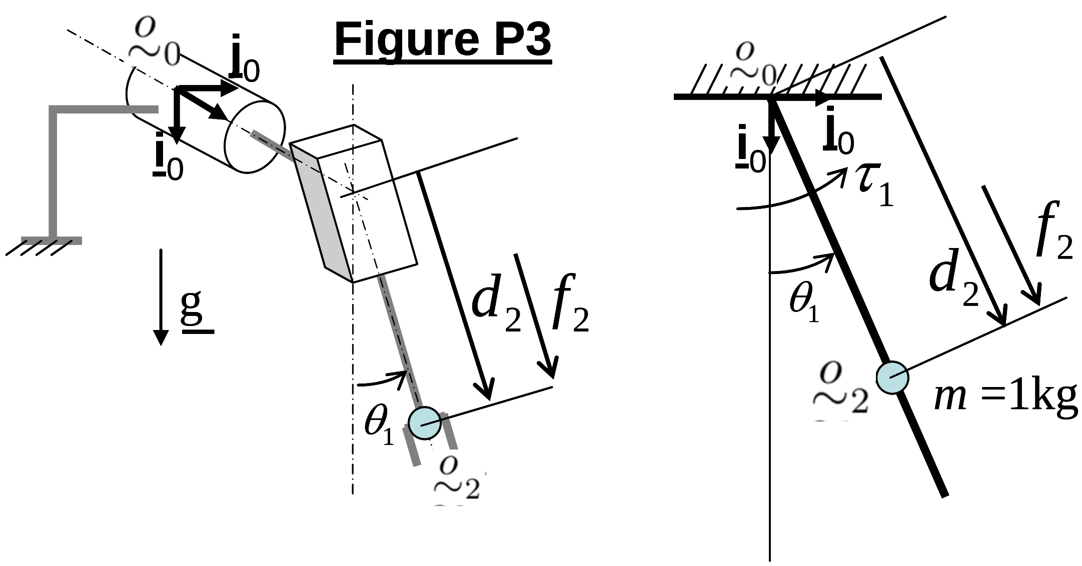
Consider the 2 DOF planar manipulator shown in Figure P3. Assume that the mass of the first link is zero, and that the entire mass m of the second link is concentrated at the gripper center $\underset{\sim}{o}_2$.

(a) *(5 marks)* Obtain the equations of motion of this manipulator.

(b) *(5 marks)* Using the equations of motion derived in (a), obtain a computed torque controller such that the closed-loop system dynamics (in θ_1, d_2) are linear, decoupled and have both poles at $s = -1$.

(c) *(5 marks)* Obtain a joint-space PD + gravity controller, with the same gains as used in (b).

(d) *(5 marks)* Obtain a cartesian space stiffness + gravity controller with cartesian proportional gain matrix in $\underline{C}_0$ given by $K_P = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}$.



NAME:
Student #:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2007): Introduction to Robotics
Final Examination, April 16, 2007
Closed Book - 150 Minutes
Maximum - 70 marks

Solve all the problems, reason your solutions, and follow our notational convention.

Problem 1.

(a) *(5 marks)* Sketch a manipulator that is described by the table of DH parameters below. Joint variables are enclosed in parantheses. Start with a base coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ as shown in Figure P1, show and label the coordinate systems $\{\underset{\sim}{o}_1, \underline{C}_1\}$, $\{\underset{\sim}{o}_2, \underline{C}_2\}$, $\{\underset{\sim}{o}_3, \underline{C}_3\}$. Label the dimensions d_i and a_i.

DH Parameter	θ_i	d_i	a_i	α_i
Link 1	(θ_1)	d_1	a_1	$\pi/2$
Link 2	(θ_2)	0	a_2	$-\pi/2$
Link 3	$-\pi/2$	(d_3)	0	0

(b) *(5 marks)* Find the coordinates ${}^1\omega_{3,0}$, in frame $\underline{C}_1$, of the angular velocity of the frame $\underline{C}_3$ with respect to $\underline{C}_0$.

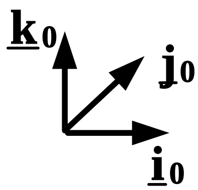
Problem 2.

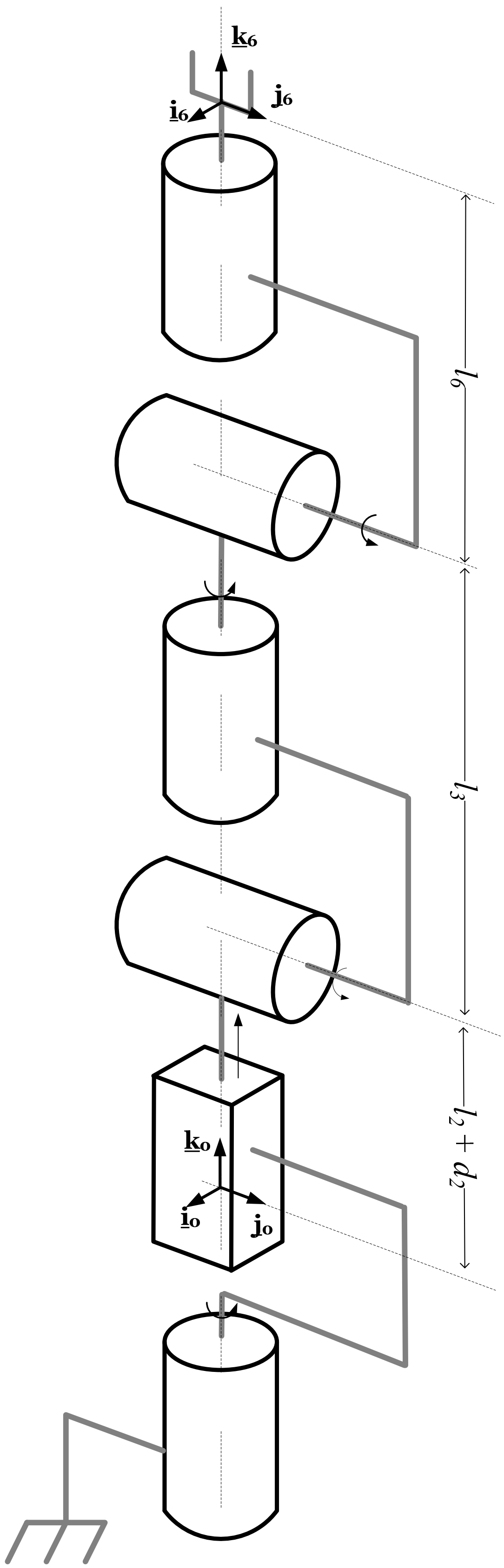
Consider the manipulator shown in Figure P2. All joint parameters are zero in the nominal configuration shown in the figure.
(a) *(10 marks)* Assign coordinate systems $\{\underset{\sim}{o}_i, \underline{C}_i\}$, $i = 1, ..., 5$ to to links 1 through 5, using the Denavit-Hartenberg convention, and find the homogeneous transformations ${}^0T_1$ through ${}^5T_6$ required for the direct kinematics problem. Complete the table of Denavit-Hartenberg parameters.
(b) *(10 marks)* Find the manipulator Jacobian and use it to find and discuss the manipulator singular configurations.
(c) *(20 marks)* Solve the inverse kinematics for this manipulator. Carefully discuss the case of multiple solutions or lack of solutions. If you use Kahan's problems P1-P3 (see attached sheet), you must clearly specify the input data and provide a solution for that particular case.

Problem 3.

Consider the 2 DOF planar manipulator shown in Figure P3. The mass of the first link is M. The second link has mass m and its center of mass is at a distance l m from the link axis.
(a) *(10 marks)* Obtain the equations of motion of this manipulator. Obtain the open loop torque $\tau_d(t)$ and force $f_d(t)$ so that $d(t) \equiv 0$ and $\theta(t) = \omega_0 t$, i.e., the revolute joint is fixed at the origin and the arm swings around with a constant angular velocity ω_0.
(b) *(5 marks)* Using the equations of motion derived in (a), obtain a computed torque controller such that the closed-loop system dynamics (in d, θ) are linear, decoupled and have both poles at $s = -1$.
(c) *(5 marks)* Show and explain the block diagram of a joint-space PD + gravity controller. with the same gains as used in (b). Discuss the trade-offs between the PD+gravity and computed torque controller. Under what conditions do these controllers behave similarly?

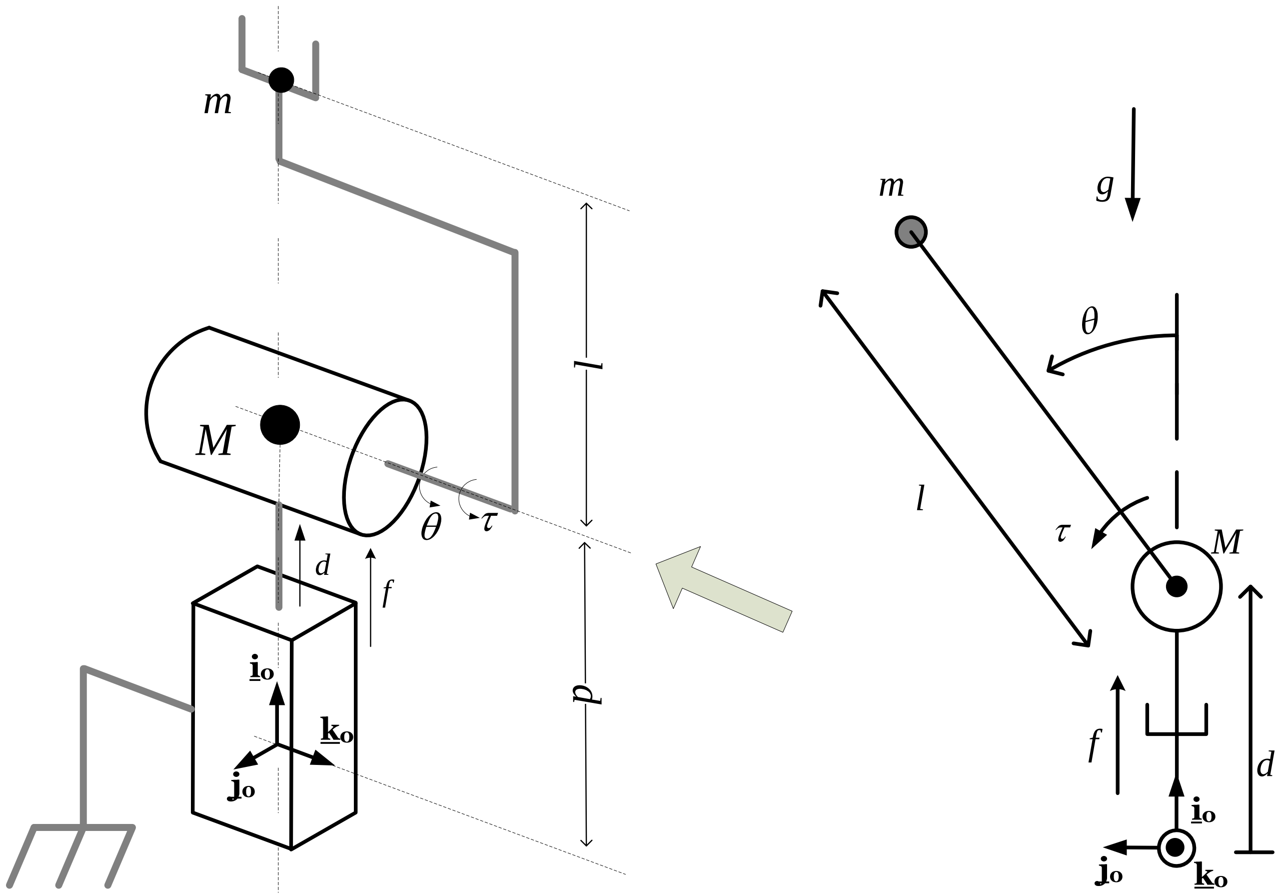
FIGURE P1





$\underline{k}_6$
$\underline{j}_6$
$\underline{i}_6$
l_6
l_3
$l_2 + d_2$
$\underline{k}_o$
$\underline{i}_o$
$\underline{j}_o$

FIGURE P2


m
M
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$\underline{\mathbf{k}}_o$

FIGURE P3

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2006): Introduction to Robotics
Final Examination #1, April 24, 2006
Closed Book - 120 Minutes
Maximum - 30 marks

Problem 1.

You are given two coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$, $\{\underset{\sim}{o}_1, \underline{C}_1\}$. The orthonormal frame $\underline{C}_1$ is obtained from $\underline{C}_0$ by rotating $\underline{C}_0$ about $\underline{j}_0$ by an angle θ, and $\underset{\sim}{o}_1$ is obtained by displacing $\underset{\sim}{o}_0$ by $a\underline{i}_0 + b\underline{j}_0 + c\underline{k}_0$.

(a)(3 marks)
Find the homogeneous transformation ${}^0T_1$ that relates the coordinates ${}^1x$ of a point $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$ to the coordinates ${}^0x$ of $\underset{\sim}{x}$ in coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$. Specify every entry of ${}^0T_1$.

(b)(2 marks)
What is the inverse of ${}^0T_1$ from *(a)* above? Specify the inverse in terms of the rotation matrix ${}^0C_1$ of ${}^0T_1$ and ${}^0d_1 = [a\ b\ c]^T$.

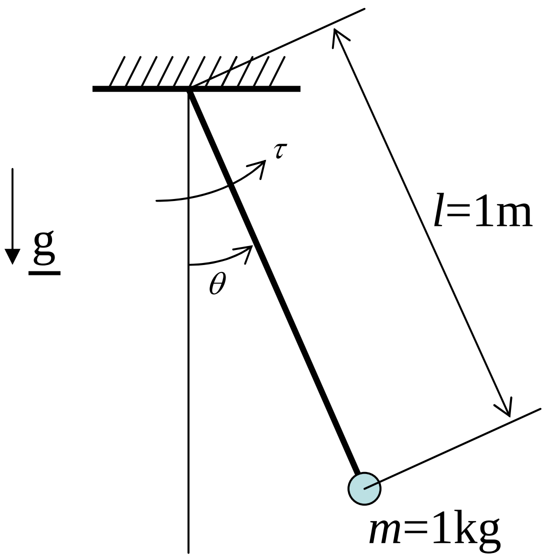
(c) (2 marks)
If θ in *(a)* is a function of time $\theta(t)$, what are the coordinates ${}^1\omega_{1,0}$, in frame $\underline{C}_1$, of the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$?

Problem 4.

(5 marks)
Show the block diagram of a cartesian space stiffness+gravity robot controller. How does it differ from a joint space PD+gravity controller? How would you select the gains to have the gripper simulate a linear slider?

Problem 2.

Consider the pendulum shown below.



(a) (3 marks)
Obtain the Lagrangian, the equations of motion and a computed torque controller leading to a closed-loop system with poles at $s = -1$.

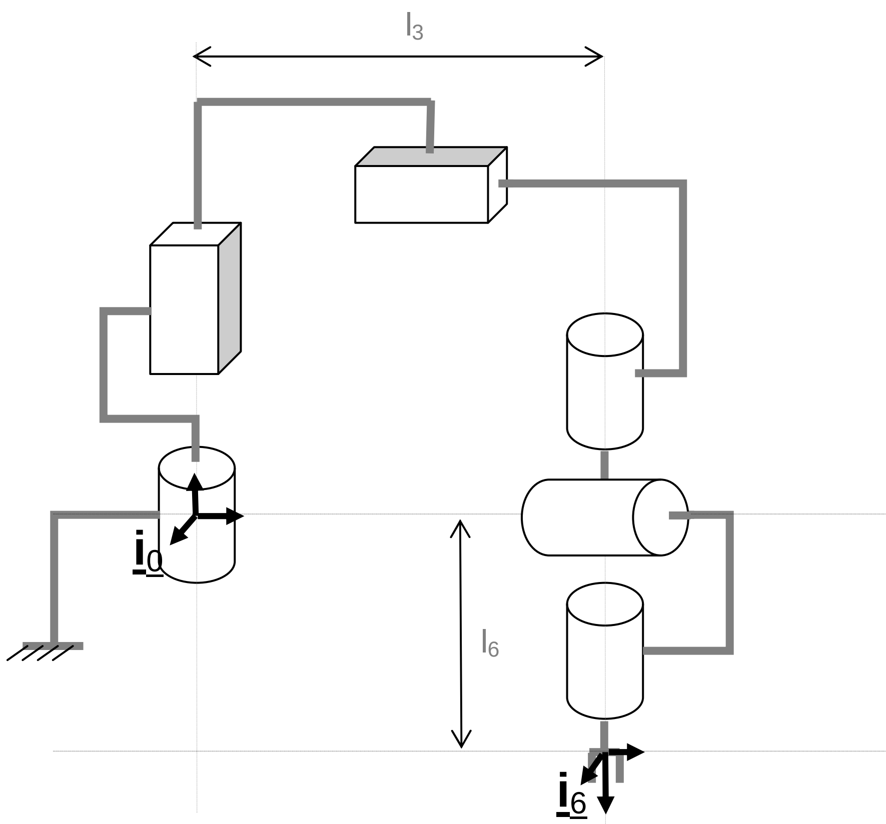
(b) (3 marks)
Obtain the torque $\tau_d(t)$ necessary to drive the pendulum along the following trajectory:

$$\theta_d(t) = \begin{cases} \frac{\pi}{2}t^2 & \text{if} \quad 0 \leq t \leq 1 \\ \pi - \frac{\pi}{2}(t-2)^2 & \text{if} \quad 1 < t \leq 2 \\ \pi & \text{if} \quad 2 < t \end{cases} \tag{1}$$

(c) (2 marks)
Carefully draw a diagram of a feedforward position controller (PD about the linearized dynamics) with the same PD gains as used in *(a)*.

Problem 3.

Consider the cylindrical manipulator shown in the following figure:



(a) (5 marks) Assign coordinate systems $\{o_i, \underline{C}_i\}$, $i = 1, ...6$ to links 1 through 6 according to the Denavit-Hartenberg convention and find the homogeneous transformations required for the direct kinematics problem.

(b) (5 marks) Write down the manipulator Jacobian in terms of the joint axes and use elementary row operations to decouple the wrist and arm singularities. Assuming that the wrist center cannot intersect the robot arm vertical axes, under what condition will the manipulator be in a singular configuration?