University of British Columbia
Department of Mechanical Engineering

MECH468 Modern Control Engineering
MECH522 Foundations in Control Engineering
Midterm exam

Examiner: Dr. Ryozo Nagamune
October 19 (Monday), 2015, 9:00am-9:50am

Last name, First name

Name: Student #:

Signature:

Exam policies

- Allowed: Lecture note print-outs, hand-written notes, homework, books.
- Not-allowed: PC, calculators.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 20 points in total. Mark will be scaled later.

Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

If you finish early ...

- Please stay at your seat until the end of exam, i.e., 9:50am. (You are not allowed to leave the room before the end of exam, except going to washroom.)

To be filled in by the instructor/marker

Problem #	Mark	Full mark
1		5
2		5
3		10
Total		20

Extra page. Write the problem number before writing your answer.

1. Answer the following true-or-false questions. Write (T) (meaning *true*) or (F) (meaning *false*). **No need to motivate your answers.** (0.5pt each)

   Below, x, u and y denote respectively state, input and output vectors.

   (a) The system $y(t) = \sin(t) \cdot u(t)$ is a nonlinear system.

   (b) The system $y(t) = \sin(t) \cdot u(t)$ is a memoryless system.

   (c) The system $y(t) = \sin(t) \cdot u(t)$ is a time-varying system.

   (d) Kernel space of a matrix $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \end{bmatrix}$ is of one-dimensional (i.e., the basis of the kernel space consists of one vector).

   (e) An uncontrollable and unobservable system is always unstable.

   (f) For a discrete-time system $x[k+1] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x[k] + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u[k]$, it is possible to transfer state from $x[0] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to $x[2] = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.

   (g) It is possible to asymptotically stabilize an unstable system without feedback control.

   (h) If all the element of a symmetric matrix is positive, then the matrix is always positive definite.

   (i) If we apply a state coordinate transformation ($z = Tx$) to an unstable system, then the resulting new state-space model is always unstable.

   (j) If a state-space model is asymptotically stable, then it is always BIBO stable.

Question	**Write your answer here**
(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	
(i)	
(j)	

2. For the continuous-time system $\dot{x} = Ax$ and the discrete-time system $x[k+1] = Ax[k]$ with the following A matrices, determine if it is asymptotically stable, marginally stable, or unstable. Fill out the following table, with abbreviations:

- "AS" meaning "asymptotically stable",
- "MS" meaning "marginally stable", or
- "UN" meaning "unstable".

No need to motivate your answers. (0.5pt each)

A	Continuous-time $\dot{x} = Ax$	Discrete-time $x[k+1] = Ax[k]$
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$		
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$		
$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$		
$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$		

3. Consider the following continuous-time state-space model:

$$\begin{cases} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x + \begin{bmatrix} 2 \\ -3 \end{bmatrix} u, \\ y &= \begin{bmatrix} 1 & 1 \end{bmatrix} x. \end{cases}$$

Below, you need to motivate your answers. Do not just write "Yes" or "No".

(a) Verify that the system is asymptotically stable, using:
- eigenvalue criteria (1pt)
- Lyapunov theorem (2pt)

(b) Is the system BIBO stable? (1pt)

(c) Is the system controllable? (1pt)

(d) Is the system observable? (1pt)

(e) Obtain Kalman decomposition. Indicate which state is controllable / uncontrollable and observable / unobservable. (2pt)

(f) Compute the A-matrix of the zero-order-hold discretized system, with the sampling period $T > 0$. (2pt)

Hint: You may want to use the Laplace transform:

$$\mathcal{L}\left\{e^{-t}\right\} = \frac{1}{s+1}, \ \mathcal{L}\left\{te^{-t}\right\} = \frac{1}{(s+1)^2}.$$

Extra page. Write the problem number before writing your answer.