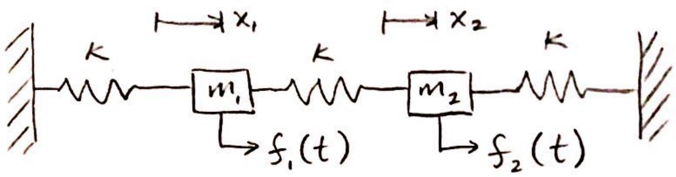
MECH 463
Oct 15

Lecture 11

2-DOF Forced Vibration



Harmonic excitation

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}\cos(\omega_F t)$$

For now, $f_1(t)$ and $f_2(t)$ have same frequency & phase

Try solution $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}\cos(\omega_F t) \Rightarrow \vec{x} = \vec{X}\cos(\omega_F t)$

Equation becomes $[M]\ddot{\vec{x}} + [K]\vec{x} = \vec{F}\cos(\omega_F t)$

$$\left(-\omega_F^2[M] + [K]\right)\vec{X}\cos(\omega_F t) = \vec{F}\cos(\omega_F t)$$

$$\Rightarrow \left(-\omega_F^2[M] + [K]\right)\vec{X} = \vec{F}$$

Expand: $\begin{bmatrix} 2k - m\omega_F^2 & -k \\ -k & 2k - m\omega_F^2 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$

Cramer's Rule:

$$X_1 = \frac{\det\begin{bmatrix} F_1 & -k \\ F_2 & 2k - m\omega_F^2 \end{bmatrix}}{\det\begin{bmatrix} 2k - m\omega_F^2 & -k \\ -k & 2k - m\omega_F^2 \end{bmatrix}}$$

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Compute det():

$$X_1 = \frac{F_1(2k - m\omega_F^2) + F_2 k}{m^2\omega_F^4 - 4mk\omega_F^2 + 3k^2}$$

Characteristic Equation

For X_2:

$$X_2 = \frac{\det\begin{bmatrix} 2k - m\omega_F^2 & F_1 \\ -k & F_2 \end{bmatrix}}{\det\begin{bmatrix} 2k - m\omega_F^2 & -k \\ -k & 2k - m\omega_F^2 \end{bmatrix}}$$

$$X_2 = \frac{F_1 K + F_2(2K - m\omega_F^2)}{m^2\omega_F^4 - 4mk\omega_F^2 + 3k^2}$$

Natural Frequencies

The denominator can be expanded to: $m^2\left(\omega_F^2 - \frac{k}{m}\right)\left(\omega_F^2 - \frac{3k}{m}\right)$

If $\omega_F = \omega_N$ then X_1 and X_2 go to infinity.

For simplicity, let $F_2 = 0$. Later, set $F_1 = 0$ and add (superposition)

Now, $X_1 = \dfrac{F_1(2k - m\omega_F^2)}{m^2\left(\omega_F^2 - \frac{k}{m}\right)\left(\omega_F^2 - \frac{3k}{m}\right)}$ $\quad X_2 = \dfrac{F_1 k}{m^2\left(\omega_F^2 - \frac{k}{m}\right)\left(\omega_F^2 - \frac{3k}{m}\right)}$

For static deflection set $\omega_F = 0$. $\omega_1^2 = \frac{k}{m}$ and $\omega_2^2 = \frac{3k}{m}$

Static deflections $X_{01} = \dfrac{2F_1 k}{m^2\omega_1^2\omega_2^2}$ and $X_{02} = \dfrac{F_1 k}{m^2\omega_1^2\omega_2^2}$

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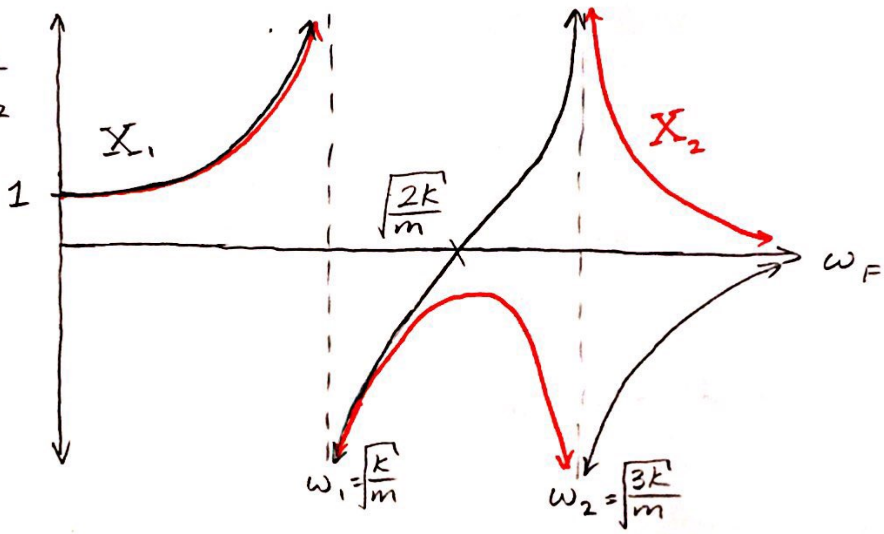
Magnification:

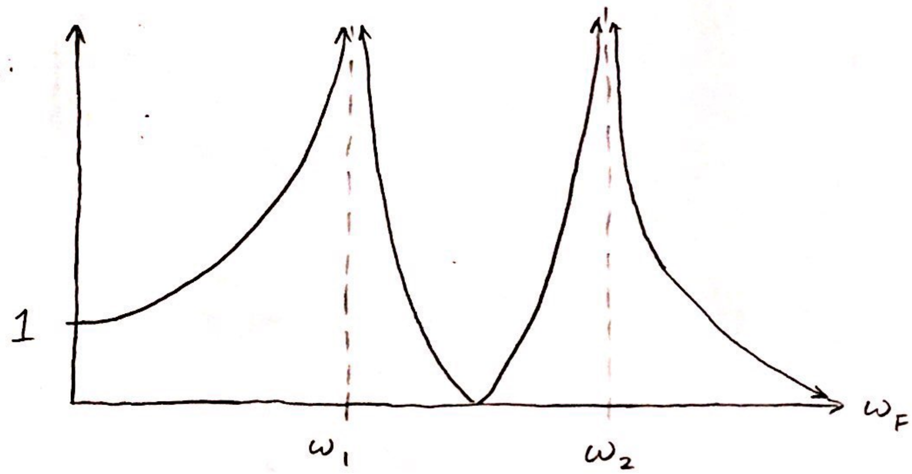
$$\frac{X_1}{X_{01}} = \frac{\omega_1^2\omega_2^2\left(1-\frac{m\omega_F^2}{2k}\right)}{(\omega_F^2-\omega_1^2)(\omega_F^2-\omega_2^2)}$$ equals 0 when $\omega_F = \frac{2k}{m}$

$$\frac{X_2}{X_{02}} = \frac{\omega_1^2\omega_2^2}{(\omega_F^2-\omega_1^2)(\omega_F^2-\omega_2^2)}$$

Plot: $\frac{X_1}{X_{01}}, \frac{X_2}{X_{02}}$



Plot magnitude: $\left|\frac{X_1}{X_{01}}\right|$



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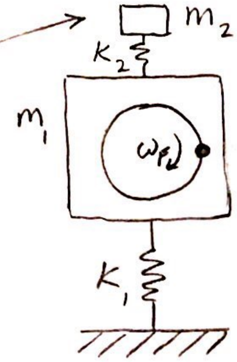
Vibration absorber



We want $X_1 = 0$

$X_{01} = X_{02} = \frac{F_1}{k}$

Add m_2, k_2 on top

Now $$X_1 = \frac{F_1 (k_2 - m_2 \omega_F^2)}{m_1 m_2 \omega_F^4 - (m_2(k_1+k_2) + m_1 k_2)\omega_F^2 + k_1 k_2}$$

$$X_2 = \frac{F_1 k}{m_1 m_2 \omega_F^4 - (m_2(k_1+k_2) + m_1 k)\omega_F^2 + k_1 k_2}$$

Want $X_1 = 0$, need $\omega_F^2 = \frac{k_2}{m_2}$

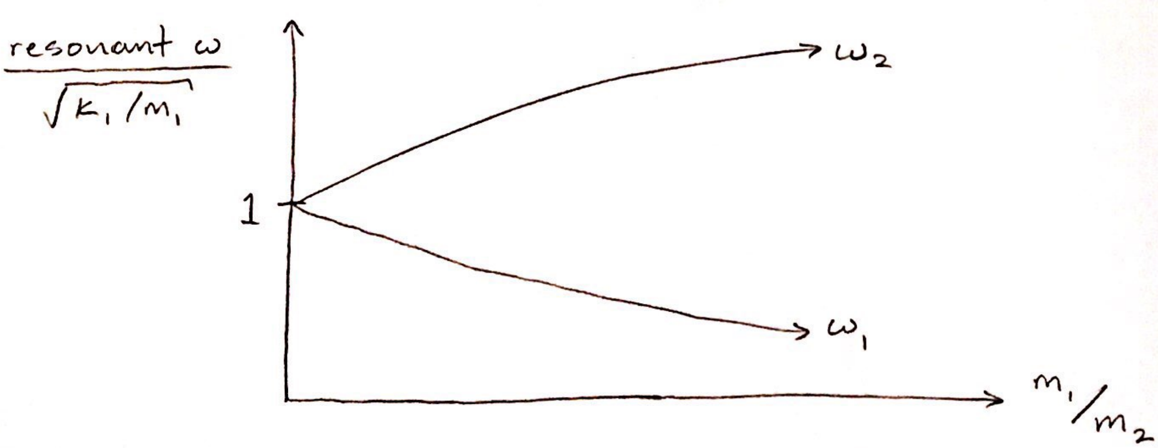
old resonant frequency

For a tuned absorber (i.e. at $\omega_F^2 = \frac{k_1}{m_1}$):

Want $X_1 = 0$

Then $X_2 = -\frac{F_1}{k_2} = -\frac{k_1}{k_2}\frac{F_1}{k_1} = -\frac{m_1}{m_2} X_0$

$\Rightarrow X_2 = -\frac{m_1}{m_2}$ (static deflection)



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