MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

# Design Problem – Magnetic Levitation Control

Assigned: Apr 6, 2020
Due: Apr 19, 2020

## 1 Introduction

This problem is about designing a controller for a bearingless motor—a special type of electric motor where the rotor is levitated and rotated by the stator. You will focus on the design of levitation controller only. The work you submit must be entirely based on your own efforts. You can use computational tools, such as MATLAB.

## 2 Hardware Description

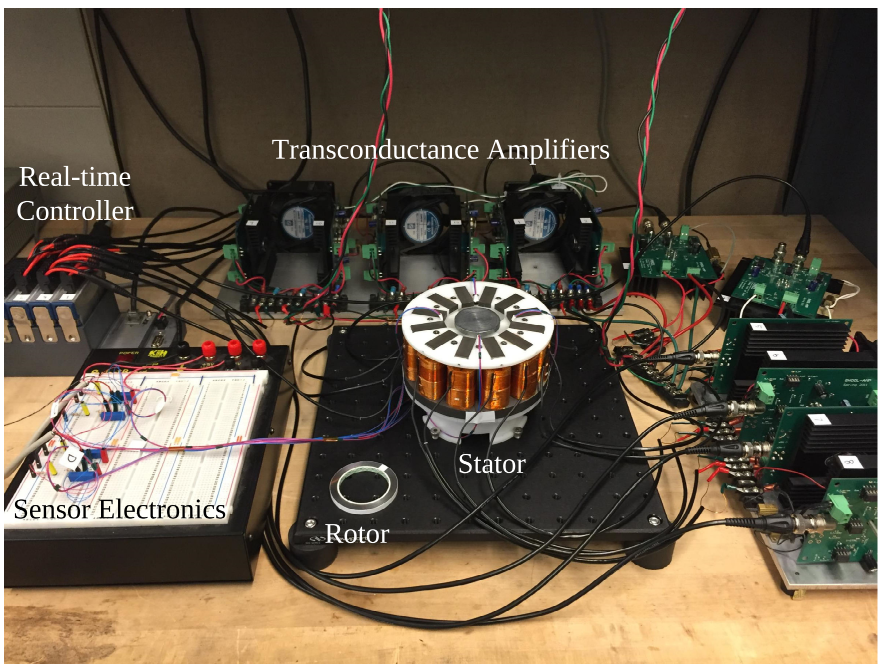


Figure 1: Prototype bearingless motor and control system.

Figure 1 shows a picture of the bearingless motor and control system. The motor stator is shown at the center of an optical bread board, and the rotor (steel ring) is placed on the corner. The rotor will be inserted into an annular groove on the stator top. There are four optical sensors around the groove to measure the rotor's radial displacements. The sensor outputs are processed through sensor electronics and fed back to a real-time controller. The controller generates current command signals (control efforts) and send those to transconductance amplifiers that control the currents through the stator windings. Then, the stator winding generates magnetic fields around the rotor, thereby applying a suspension force on the rotor. More about this research can be found in [1]. The video of levitation is available at: `https://youtu.be/bLD9GxWVy48`

Figure 1 schematically shows a cross-section of the bearingless motor. There is a permanent magnet at the center of the stator, which generates *bias magnetic flux* (blue dashed line). It starts from the permanent magnet's north pole, passes through the rotor radially outwards, and returns to the magnet's south pole through the stator core. The bias flux makes the rotor passively stable in three degrees of freedom. These include the translation along the $z$-axis, and tilts about the $x$- and $y$-axes. This is because the bias flux generates magnetic forces that put the rotor in alignment with the stator teeth. However, two radial displacements, i.e., translations along the $x$- and $y$-axes, are open-loop unstable. This is because the bias flux also generates magnetic forces that attract the rotor to the stator teeth. So, we need to implement feedback control to stabilize the radial translations, so as to fully levitate the rotor.

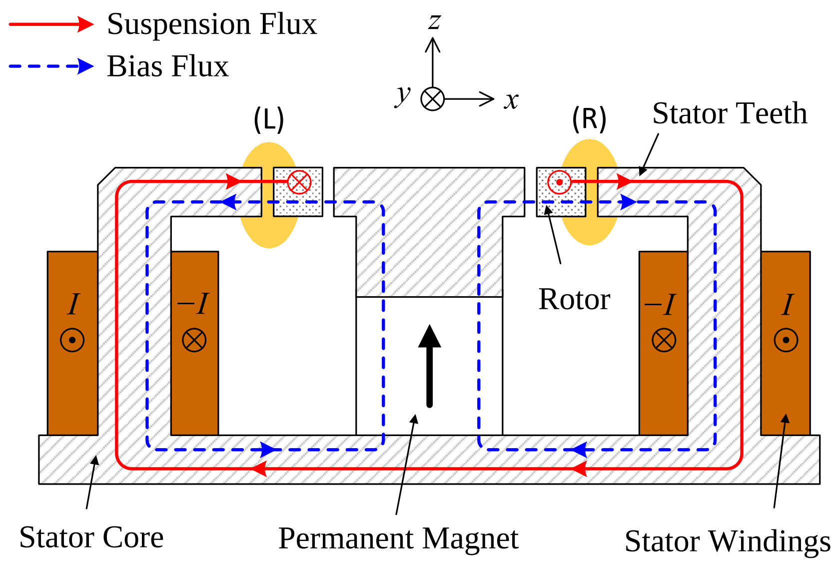


Figure 2: Schematic diagram of the lateral positioning stage and the control system.

Suppose the stator windings conduct current $I$ as shown in Figure 2. Here, the black dots and crosses show the directions of the current inside the winding. Such current excitation generates *suspension magnetic flux* (red solid line) that circulates the stator core clockwise. Note that it enters the rotor to the left and exit the rotor from the right. Now, let us think about the net flux density (flux per area) in the air gaps marked with (L) and (R). In the air gap (L), the bias flux and suspension flux are opposing each other, so the net flux density becomes lower. In the air gap (R), the two fluxes are in the same direction, so the net flux density becomes higher. Such imbalance in flux density generates a net magnetic force toward the right. When the stator windings conduct the opposite current, it generates a magnetic force toward the left. Further analysis can show that the radial magnetic force is proportional to the winding current, i.e., $f = K_f I$.

## 3 Controller Design

Figure 3 shows a simplified block diagram for $x$-axis levitation control. Here, $u$ [V] is the current command signal (control effort) and $y$ [V] is the position sensor output. Suppose you do not know the amplifier gain $G_a$, force constant $K_f$, and sensor gain $G_s$. Instead, you are given a measured frequency response of the plant shown in Figure 4. Here, the blue curves are from the measured data, whereas the red dashed curves are from a parametric model fitting the data. The data for Figure 4 is available on Canvas (`MaglevPlant.mat`). Address the following problems to design a stabilizing controller $C(s)$.

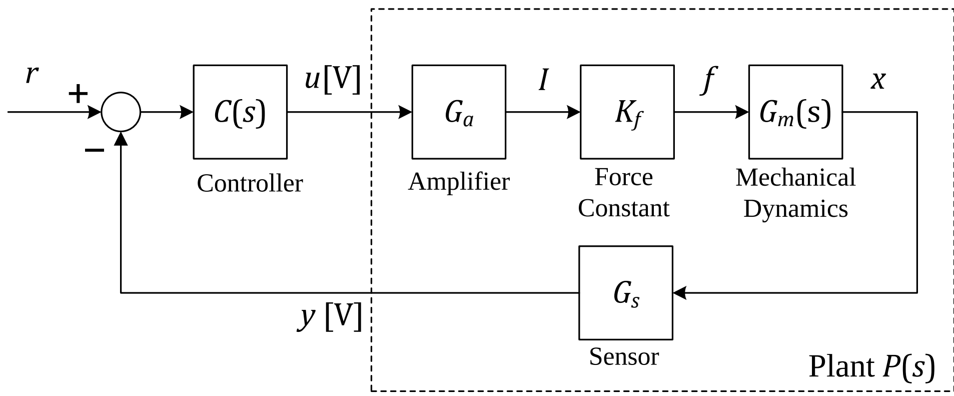


Figure 3: Simplified block diagram of the $x$-axis levitation control.

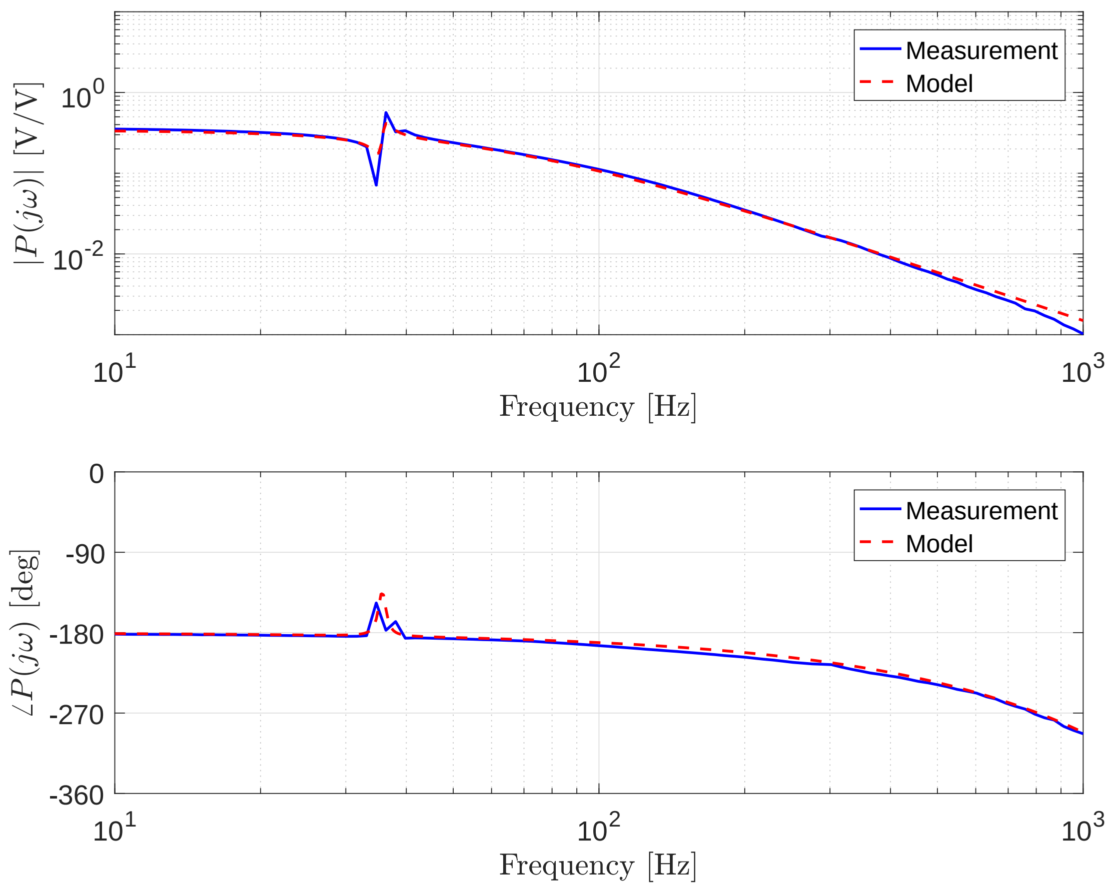


Figure 4: Experimentally measured frequency response of the plant.

1. Find a transfer function $P(s)$ that fits the measured Bode plot of the plant. A suggested form for the plant transfer function is

$$P(s) = \left( \frac{K_1}{ms^2 - k} + \frac{K_2}{Js^2 + bs + \kappa} \right) e^{-sT_d},$$

   which consists of three elements. First one in the parentheses captures the typical dynamics of a magnetic levitation system. Note that the sign in front of the stiffness $k$ is negative and makes the system open-loop **unstable**. Such a *negative stiffness* arises from attractive magnetic forces. The second one in the parentheses captures a lightly damped tilting mode dynamics. The last one outside the parentheses captures the time delay in the plant. If you feel the problem is too challenging, set $K_2 = 0$.

   Use $P(s)$ obtained here for the rest of the problems.

2. Draw the pole-zero map of $P(s)$ and check the number of unstable poles. What is the frequency of the unstable poles?

3. Design a controller $C(s)$ that satisfies the following requirements.

   - Stabilizing the closed-loop system
   - Phase margin $\phi_m > 30°$
   - No integral control

4. Draw the Bode plot of the loop transfer function $L(s)$ and mark the crossover frequency $\omega_c$ and phase margin $\phi_m$.

5. Draw the Nyquist plot of the loop transfer function $L(s)$ and assess the stability of the closed-loop system. That is, calculate $Z = N + P$, where $Z$ is the number of closed-loop unstable poles, $N$ is the number of encirclement of the Nyquist plot about $-1$ point, and $P$ is the number of unstable pole of $L(s)$.

6. Mark the phase margin $\phi_m$ on the Nyquist plot.

7. Simulate the step response of a closed-loop transfer function from $r$ to $y$.

## References

[1] M. Noh, W. Gruber, and D. L. Trumper, "Hysteresis Bearingless Slice Motors With Homopolar Flux-Biasing," *IEEE/ASME Transactions on Mechatronics*, vol. 22, no. 5, pp. 2308–2318, Oct. 2017.