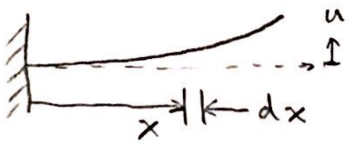
MECH 463
Nov 26

Lecture 21

Beam Vibration



not wave speed

$$\Rightarrow \quad \frac{\partial^2 u}{\partial t^2} + c^2 \frac{\partial^4 u}{\partial x^4} = 0 \quad \text{and} \quad c = \sqrt{\frac{EI}{\rho A}}$$

Solution: $u(x,t) = X(x)\,T(t)$

mode shape — vibration

$$\Rightarrow X(x)\ddot{T}(t) + c^2 X''''(x)\,T(t) = 0$$

$$\Rightarrow \frac{X^{(4)}}{X} = -\frac{1}{c^2}\frac{\ddot{T}}{T} = \beta^4$$

Each side: $X^{(4)} - \beta^4 X = 0$

$T + \omega^2 T = 0$ where $\omega = \beta^2 c$

hyperbolic

Solutions: $T = A\cos(\omega t) - B\sin(\omega t)$

$$X = C\cos(\beta x) - D\sin(\beta x) + G\cosh(\beta x) + H\sinh(\beta x)$$

Boundary conditions:

Geometric:

1) $u(0,t) = 0$

2) $\frac{\partial u}{\partial x}(0,t) = 0$

Force:

3) $V = EI\frac{\partial^3 u}{\partial x^3}(L,t) = 0$

4) $M = EI\frac{\partial^2 u}{\partial x^2}(L,t) = 0$

Characteristic equation: $\cos(\beta L)\cosh(\beta L) = -1$

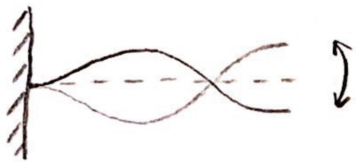
$$\Rightarrow \beta L = 1.875,\ 4.694,\ 7.855,\ \ldots$$

Natural frequency: $\omega = \beta^2 c$

$$\Rightarrow \omega = (\beta L)^2 \frac{1}{L^2} \sqrt{\frac{EI}{\rho A}}$$

Mode shapes:

1) 

2) 

3)