< PI Controller & Low pass Filter >

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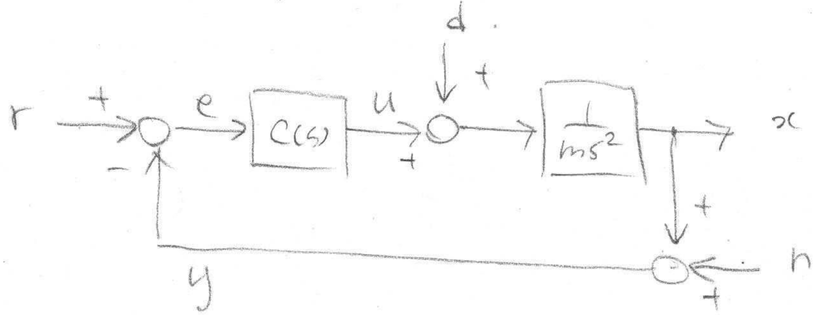
- Objectives
  - Integrator for dist. reject
  - LPF for noise attenuation.
  - Integrator anti-windup
  - PID physical meaning.

- Free mass position control



- Gang of Four :

$$\frac{E}{R} = \frac{1}{1+Cp} \triangleq S : \text{Sensitivity}$$

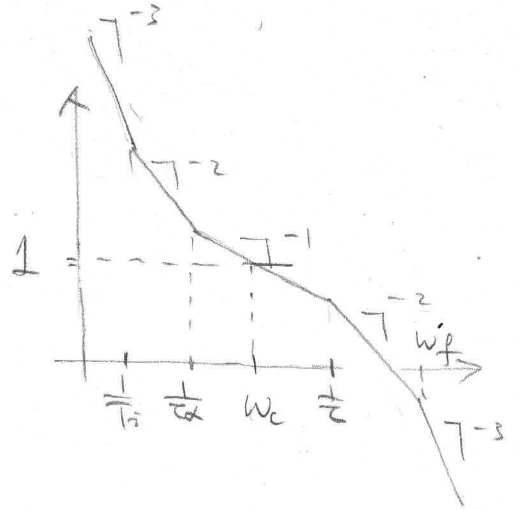
$$\frac{X}{R} = \frac{Cp}{1+Cp} \triangleq T : \text{Complementary Sensitivity}$$

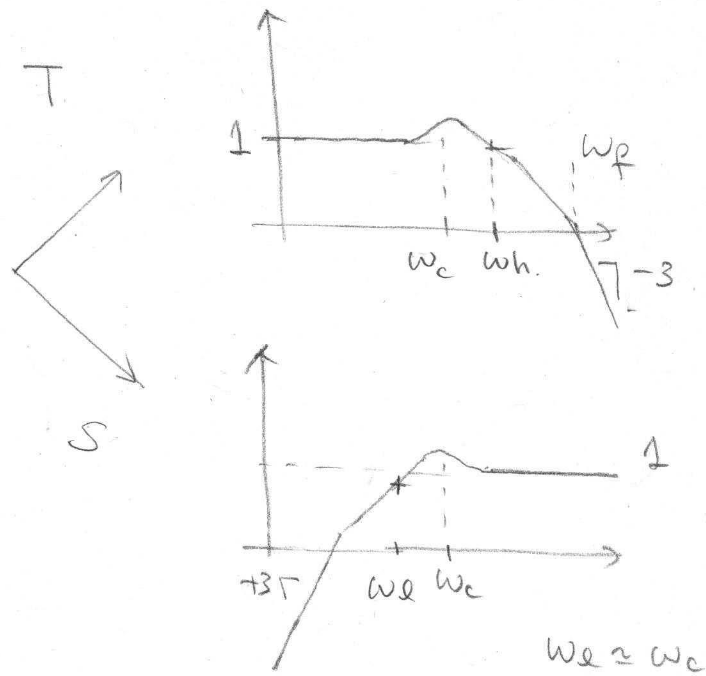
$$\frac{X}{D} = \frac{p}{1+Cp} = pS : \text{Load sensitivity.}$$

$$\frac{U}{N} = \frac{C}{1+Cp} = CS : \text{Noise sensitivity}$$

- Integrator : ↓ Load Sensitivity
- Low pass filter : ↓ Noise Sensitivity

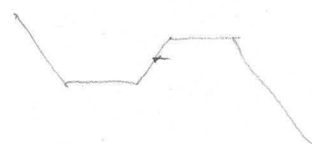
- $L(s)$ with $C(s) = K_p \left(1 + \frac{1}{T_i s}\right) \frac{\alpha \tau s + 1}{\tau s + 1} \frac{\omega_f}{s + \omega_f}$



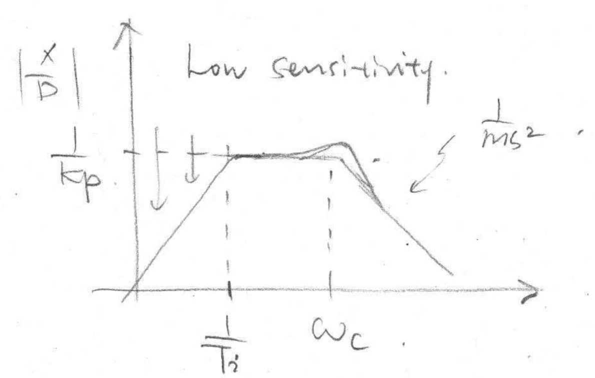


$\omega_l \simeq \omega_c \simeq \omega_h$

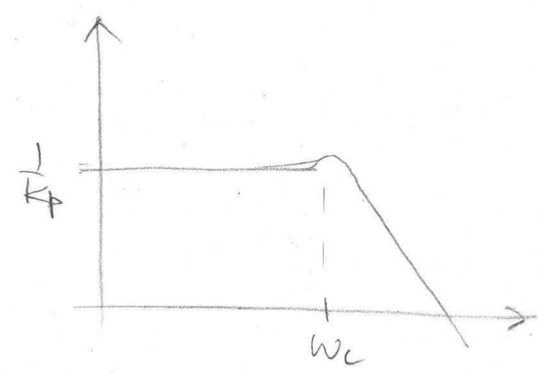
- Load Sensitivity.



$$\frac{X}{D} = \frac{P}{1 + CP} = \begin{cases} \frac{1}{C} & \text{when } |L| \gg 1 \\ P & \text{when } |L| \ll 1 \end{cases}$$



with Integrator.

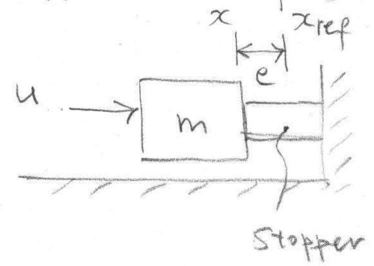


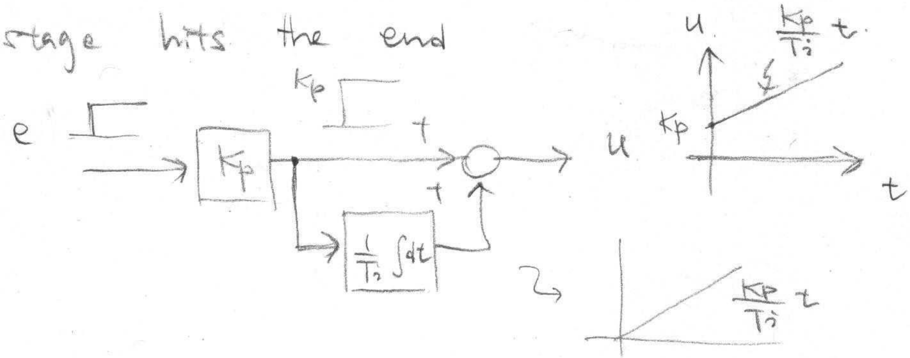
without integrator.

- Integrator reduces low-frequency load sensitivity.
(disturbance rejection)

- Integrator Anti-windup.

- Suppose the position stage hits the end





- Control effort u will keep increasing, which can damage the hardware (e.g. amplifier)

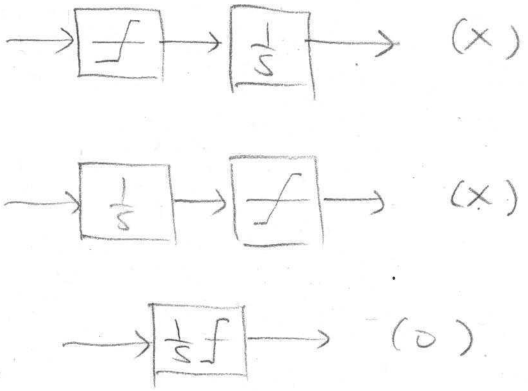
- Limiting u is important. (e.g. saturation block)

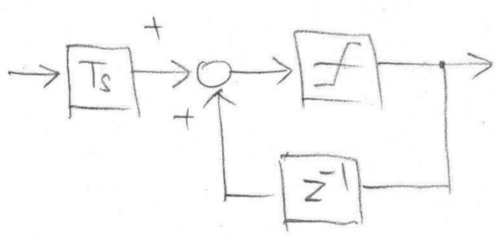
- However, it does not stop the integrator from accumulating the error.

- This can make the control effort u saturated for quite a while (Controller not working) even if x_{ref} is reversed.

- To prevent this, we should implement "anti-windup"

- The correct implementation is to limit the "state" of the integrator.

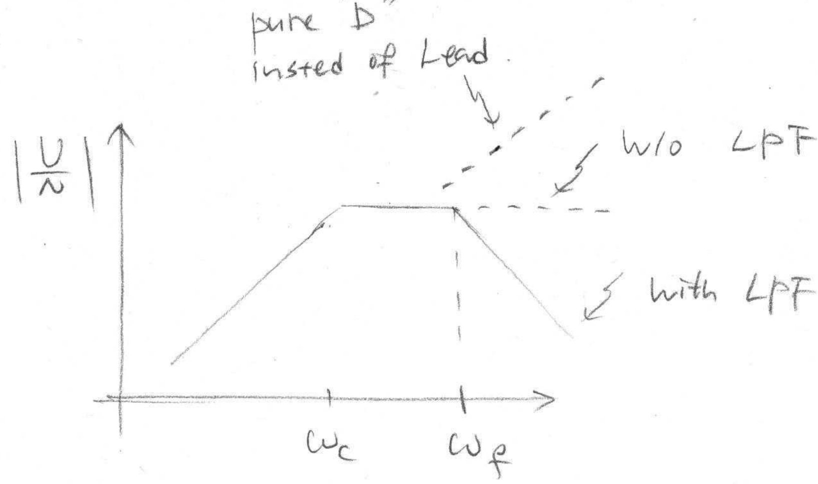




"Discrete-time Implementation"

∘ Noise & Low pass Filter.

$$\frac{U}{N} = \frac{C}{1+CP} = \begin{cases} \frac{1}{P} & \text{when } |L| \gg 1 \\ C & \text{when } |L| \ll 1 \end{cases}$$



Noise $\begin{cases} \text{Sensor Noise} \\ \text{ADC Noise} \\ \vdots \end{cases}$

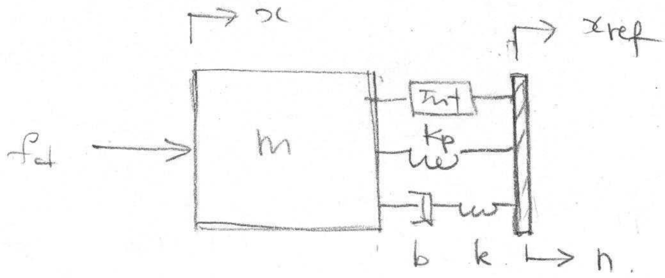
High-frequency contents.

· High-frequency componants in u is not desirable because the energy converts to

- Heat - Amplifier & Motor winding
- Vibration - High-freq structural resonance.

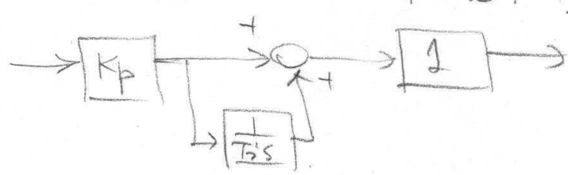
and gradually damage the hardware.

- Physical meaning of PID position control action.

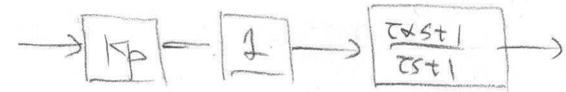


- $\omega < \frac{1}{T_i}$

Lead ≈ 1



- $\omega > \frac{1}{T_i}$



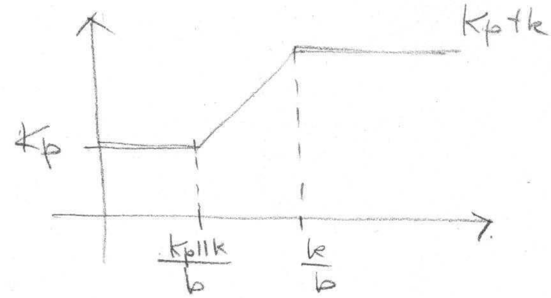
$$Z_m = K_p + k \parallel bs$$

$$= K_p + \frac{kbs}{k+bs}$$

$$= \frac{K_p k + (K_p + k)bs}{k+bs}$$

$$\therefore \omega_p = \frac{k}{b}, \quad \omega_z = \frac{K_p k}{(K_p+k)b}$$

$$\omega_z = \frac{K_p k}{(K_p+k)\,b} = \frac{K_p \parallel k}{b}$$



- without integrator.

  f_d is carried by K_p at dc

  $\therefore \quad x = \frac{f_d}{K_p}$ at dc.

- with integrator.

  f_d is carried by the integrator, and the spring (K_p) carries zero force

  $\therefore \quad x = 0$ at dc.

- High-frequency n doesn't transmit to x much due to the finite stiffness "k"