University of British Columbia
Department of Mechanical Engineering

MECH468 Modern Control Engineering
MECH509 Controls
Final exam

Examiner: Dr. Ryozo Nagamune
April 26 (Monday), 2021, noon-2:30pm (PST)

Exam policies

- Allowed: Open-book. Any distributed material and any textbook. You can see course materials on your computer.
- Not-allowed: Matlab. Calculators. Web-browsing.
- Write all your answers on **your own sheets**.
- Motivate your answers properly. (No chance to defend your answers orally.)
- **No questions are allowed.**
- 100 points in total. (It will be scaled later.)

Before you start ...

- Turn off your mobile phone, or make it in the airplane mode.
- Make sure that your above-neck is fully captured by the webcam.

After you finish the exam ...

- You will get enough time to scan and upload your answer sheets on Canvas "Assignments" **AFTER** the exam writing time (150 minutes).
- If you finish the exam early, you can upload answer sheets and leave Zoom room.
- **Make sure that you have uploaded all your answer sheets.** You cannot add some sheets later even if you somehow missed uploading them.

Marking scheme

Question #	Expected duration	Full mark
Q1	about 30 min	20 %
Q2	about 30 min	20 %
Q3	about 45 min	30 %
Q4	about 45 min	30 %
Total	about 150 min	100 %

1. Select **only one** correct statement for the following sentences. There is no need to motivate your answers. (2pt each)

   (a) If we place the poles of the closed-loop system with state-feedback at $s = -0.04, -0.4, -4$, it is roughly estimated that the 2% settling time for the step response would be about:

      i. 0.01 second.
      ii. 0.1 second.
      iii. 1 second.
      iv. None of i, ii, iii is correct.

   (b) A series connection $G_1(s)G_2(s)$ of the two systems $G_1(s) = \dfrac{s+1}{s+2}$ and $G_2(s) = \dfrac{s+2}{s+1}$ is:

      i. asymptotically stable but not BIBO stable.
      ii. BIBO stable but not asymptotically stable.
      iii. both BIBO stable and asymptotically stable.
      iv. None of i, ii, iii is correct.

   (c) If a linear time-invariant system is:

      i. asymptotically stable, then it is always observable.
      ii. observable, then it is always stabilizable.
      iii. stabilizable, then it is always detectable.
      iv. None of i, ii, iii is correct.

   (d) In infinite-horizon LQR control, if we want to regulate $y = Cx + Du$ ($C \neq 0$, $D \neq 0$) around zero, then the cost function to be minimized will be in the form of:

      i. $\int_0^\infty (x^T Q x + u^T R u) dt$
      ii. $\int_0^\infty x^T Q x dt$
      iii. $\int_0^\infty u^T R u dt$
      iv. None of i, ii, iii is correct.

   (e) If a symmetric matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$ is:

      i. positive definite.
      ii. positive semidefinite.
      iii. negative definite.
      iv. None of i, ii, iii is correct.

(f) McMillan degree of the transfer matrix $G(s) = \frac{1}{s+1}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is:

i. 1
ii. 2
iii. 3
iv. None of i, ii, iii is correct.

(g) The solution to a state equation $\dot{x}(t) = Ax(t)$ with condition $x(1) = 2$ is:

i. $x(t) = 2e^{At}$
ii. $x(t) = 2e^{A(t-1)}$
iii. $x(t) = 2e^{A(t+1)}$
iv. None of i, ii, iii is correct.

(h) A system $\dot{x}(t) = \begin{bmatrix} \alpha & 1 \\ -1 & \alpha \end{bmatrix} x(t)$ with a constant α is:

i. asymptotically stable for any α.
ii. marginally stable for any α.
iii. unstable for any α.
iv. None of i, ii, iii is correct.

(i) In the state-feedback controller design using pole-placement technique, if the overshoot of the step response for a closed-loop system needs be reduced, then we move the closed-loop poles to the locations:

i. far from the real axis of the complex plane.
ii. close to the real axis of the complex plane.
iii. close to the imaginary axis of the complex plane.
iv. None of i, ii, iii is correct.

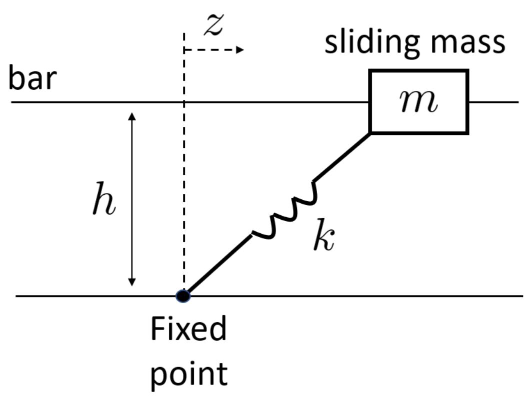
(j) For a discrete-time system $x[k+1] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u[k]$, it is possible to transfer the state from $x[0] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ to $x[k_f] = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ for:

i. any k_f satisfying $k_f \geq 1$.
ii. any k_f satisfying $k_f \geq 2$.
iii. any k_f satisfying $k_f \geq 3$.
iv. None of i, ii, iii is correct.

2. Consider a mechanical system consisting of a mass m sliding on a horizontal bar and connected to a spring with the spring constant k as shown in the figure below. When the spring is not stretched, its angle with the horizontal is equal to 45 degrees. When a force f is applied to the mass, the horizontal displacement z of the system evolves according to the following equation:

$$m\ddot{z} = k\left(\frac{\sqrt{2}h}{\sqrt{z^2+h^2}} - 1\right)z + f,$$

where h is the distance shown in the figure.



(a) Obtain the nonlinear state equation (output equation is NOT necessary) for the system when the input is the force f, by introducing the state vector $x := [z, \dot{z}]^T$. (5pt)

(b) Obtain all equilibrium states x_{eq} of the system for the zero input $f_{eq} = 0$. For each equilibrium, draw a figure which illustrates the mass location and the spring angle are physically. (10pt)

(c) Compute linearizations of the state equation around each equilibrium point. (5pt)

3. Consider to estimate a 1-dimensional position x [m] of a mass which stays at one location, using multiple position sensor measurements:

$$y_i = x + n_i, \quad i = 1, 2, 3,$$

where n_i is the noise for i-th measurement. The variances of the measurement noises n_i are assumed to be

$$E\left\{n_1^2\right\} = 1, \quad E\left\{n_2^2\right\} = \frac{1}{2}, \quad E\left\{n_3^2\right\} = \frac{1}{4},$$

and they are not correlated, i.e.,

$$E\left\{n_1 n_2\right\} = E\left\{n_2 n_3\right\} = E\left\{n_3 n_1\right\} = 0.$$

(a) First, using y_1, y_2 and y_3, and using the **batch weighted least-squares estimation**, obtain the position estimate $\hat{x}_b$ and its error variance $P_b := E\left\{(x - \hat{x}_b)^2\right\}$. (10pt)

(b) Next, in (b) and (c), we will demonstrate that x_b and P_b in (a) can be obtained by the recursive least-squares estimation. For this purpose, first, using only y_1 and y_2, and using the **batch weighted least-squares estimation**, obtain the position estimate $\hat{x}_o$ and its error variance $P_o := E\left\{(x - \hat{x}_o)^2\right\}$. (In the question (b), the subscript 'o' means that the estimate is based on "old" measurements.) (10pt)

(c) Finally, using $(\hat{x}_o, P_o)$ in (b) and y_3, and using the **recursive weighted least-squares estimation**, obtain the position estimate $\hat{x}_n$ and its error variance $P_n := E\left\{(x - \hat{x}_n)^2\right\}$. (In (c), the subscript 'n' means that the estimate is updated based on "new" measurements.) (10pt)

4. Consider a system $\dot{x}(t) = Ax(t) + Bu(t)$ with the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

(a) Determine whether this system is controllable. (10pt)

(b) Without finding a stabilizing state-feedback gain K, prove that this system is stabilizable. (10pt)

Hint: Although the coordinate transformation matrix T is 4-by-4 in this question, if you select T^{-1} in a 'clever' way, you can hand-calculate its inverse T.

(c) By selecting the closed-loop poles in the stable region by yourself, compute a matrix K such that the eigenvalues of $A - BK$ are placed at your selected closed-loop pole locations. (10pt)

——— (End of Final Exam) ———