MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

Homework 5

Assigned: Mar 5, 2021
Due: Mar 12, 2021

Problem 1

Let us consider the two-degrees-of-freedom system in Figure 1. Here, two masses m_1 and m_2 are connected via a spring k and excited by external forces f_1 and f_2. The displacements of the masses are denoted as x_1 and x_2.

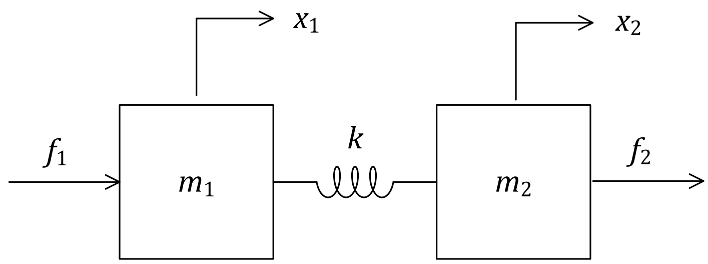

Figure 1: Mass-spring-mass system.

(a) Derive the equations of motion and organize them in the following form

$$M \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + K \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix},$$

where $M \in \mathbb{R}^{2\times 2}$ is the mass matrix and $K \in \mathbb{R}^{2\times 2}$ is the stiffness matrix.

(b) Find the transfer function matrix $H(s) \in \mathbb{C}^{2\times 2}$ that relates the displacements to the forces, i.e.,

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \underbrace{\begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix}}_{H(s)} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}.$$

(Tip: take the Laplace transform of the equations of motion and then matrix inversion.)

(c) Manually draw the pole-zero maps and Bode plots of $H_{11}(s)$ and $H_{21}(s)$. Show the pole-zero locations and break frequencies in terms of m_1, m_2, and k.

Problem 2

Manually draw the pole-zero maps and Bode plots of the following second-order systems. Clearly show the break frequencies, asymptotes, and $-3\,\mathrm{dB}$ bandwidth ω_h in the Bode plots.

For the systems exhibiting a resonance – clearly show in the **pole-zero map** the natural frequency ω_n, damped natural frequency ω_d, and decay rate σ ; and in the **Bode plot** the natural frequency ω_n, quality factor Q, resonance frequency ω_r, and resonance peak M_r.

(a) $H_a(s) = \dfrac{10}{s^2 + 101s + 100}$

(b) $H_b(s) = \dfrac{10}{s^2 + 20s + 100}$

(c) $H_c(s) = \dfrac{10}{s^2 + 2s + 100}$

(d) $H_d(s) = \dfrac{s}{s^2 + 2s + 100}$