MECH 463: Mechanical Vibrations
Self Assessment Tutorial

20/20

Group Name: TO BE DECIDED

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1. The main objective of this self-assessment exercise is to discover, on your own, your capacity to integrate the concepts learned thus far during your education at UBC. The questions are based on prerequisite knowledge for MECH 463.
2. The purpose of these questions is to demonstrate how different things learned in different courses fit together.
3. Solve the starred questions in the tutorial with your group and submit your solutions to the TA. Use the space provided.
4. Attempt the other (unstarred) questions on your own and discuss your solutions with your TA.
5. Answers for Q1 and Q3 are given at the end of this document. For other questions, you can verify them on your own and discuss with TA.
6. No worked-out solutions will be provided for this self-assessment Tutorial.
7. You can try implementing the matrix form of Mohr's stress transformations (see Q6) in a MATLAB code and see how it complements the graphical circle method, by trying out some problems you already solved in MECH2, MECH 260, or MECH 360.

MECH 463: Tutorial 3

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Problems from course text book: Mechanical Vibrations by S.S. Rao (4th Edition)
Solve both questions.

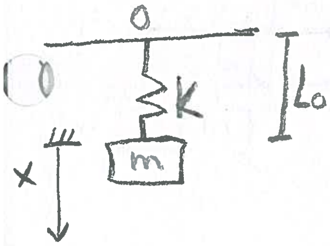
~~10/20~~ 11/20

BC

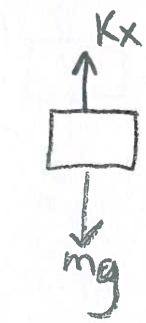
Question 1: Determine the equations of motion of a spring-mass system in which the mass is hanging vertically downwards for both *unstretched* spring configuration and the *stretched* spring configuration (static equilibrium) taken as the reference position for displacement. Note that static equilibrium configuration corresponds to the stretched spring under the influence of weight mg of mass m. For each configuration use (a) Newton's second law (b) D'Alembert's principle for each reference. Show all free body diagrams that you need.

Solution:

Newton's Second Law (Unstretched Spring)



FBD



∴ eq. of motion (Newton)

$$\downarrow \sum F = m\ddot{x} = mg - Kx$$

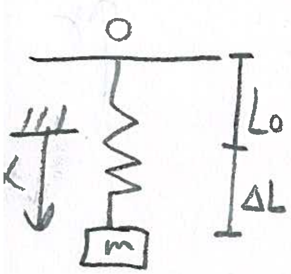
use this EOM

★Note: at $x=0$ (unstretched) where $L=L_o$

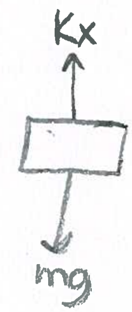
$$\boxed{m\ddot{x} = mg}$$

Yes, at that particular instance. However when it vibrates, you will have kx term.

Newton's Second Law (Stretched Spring)



FBD



∴ eq. of motion (Newton)

$$\downarrow \sum F = m\ddot{x} = mg - Kx$$

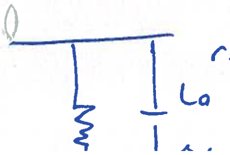
★Note: at $L = L_o + \Delta L$ (static eq.):

$$\boxed{mg = Kx}$$

$mg = k\Delta L$

$\downarrow \sum F_x = m\ddot{x}$

$\cancel{mg} - kx - \cancel{k\Delta L} = m\ddot{x}$



reference position is at static equilibrium!

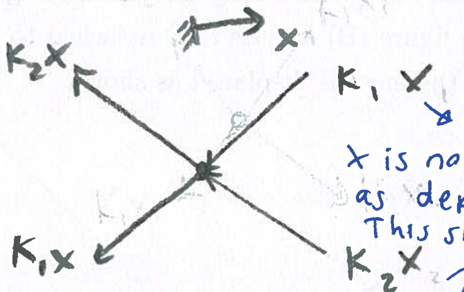
spring force = gravity force, and mass is

* Assume ...

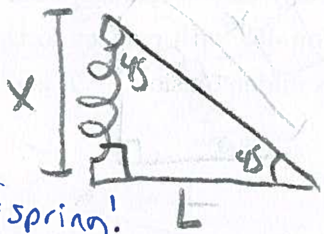
A) FBD

Assume mass deflects to the right

trig.



x is not the same as deflection of spring! This should be $x\cos 45°$ or $x \sin 45°$?



$\therefore X = L$

So using Newton:

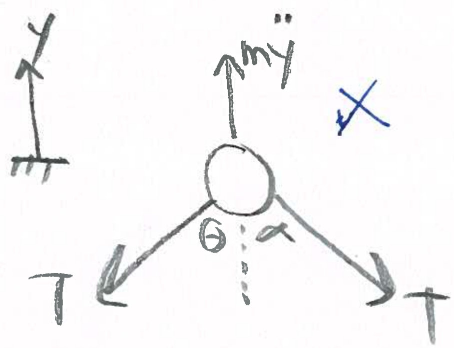
$$\sum F_x = m\ddot{x}_x = K_1 x\cos(45°) + K_2 x \cos(-45°) + K_2 x\cos(45°) + K_1 x \cos(45°)$$

$$\boxed{m\ddot{x} = -2k_1 x\cos(45) - 2k_2 x \cos 45}$$

*y-axis eq. of motion follows the same ideology, so we didn't do it.

−2

B) FBD (D'Alembert)

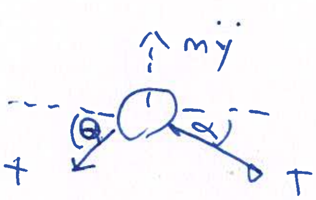


$\theta = \tan^{-1}(a/y)$
$\alpha = \tan^{-1}(b/y)$ when Δy is slightly

at small angle, $\tan\theta \approx \theta$
$\tan\alpha \approx \alpha$
$\sin\theta \approx \theta$
$\sin\alpha \approx \alpha$

$$\boxed{\uparrow \Sigma F_y = 0 = m\ddot{y} - T\cos\theta - T\cos\alpha}$$

Remember that T is constant even when the mass is displaced. It will only happen under this condition:



α and θ is very small!

→3 −2

$\frac{20}{20}$ LM

MECH 463: TUTORIAL 4

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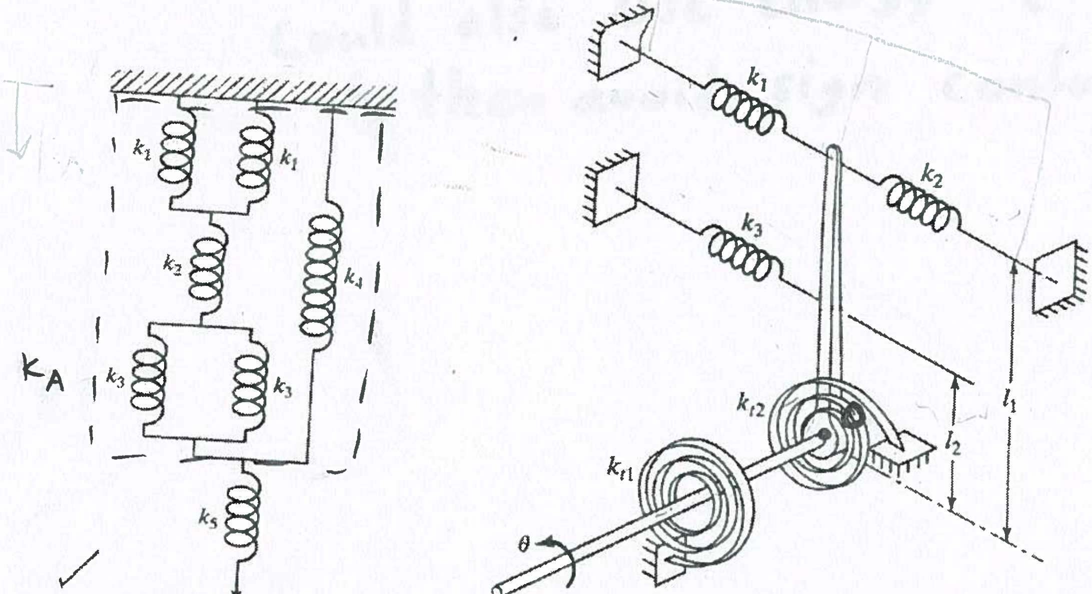
PROBLEMS FROM TOPIC 2.2 (NOTES PACKAGE)

Question 1: Example 5

Determine the equivalent spring constant for the system in (A) at the end point of spring k_5. Find the and equivalent torsional spring constant of the system in (B) in terms of θ co-ordinate. Ignore the flexibilities of the bars in (B). **Show detailed calculations**.



(A) (B)

Figure 1: Figure for example 5.

Solution:

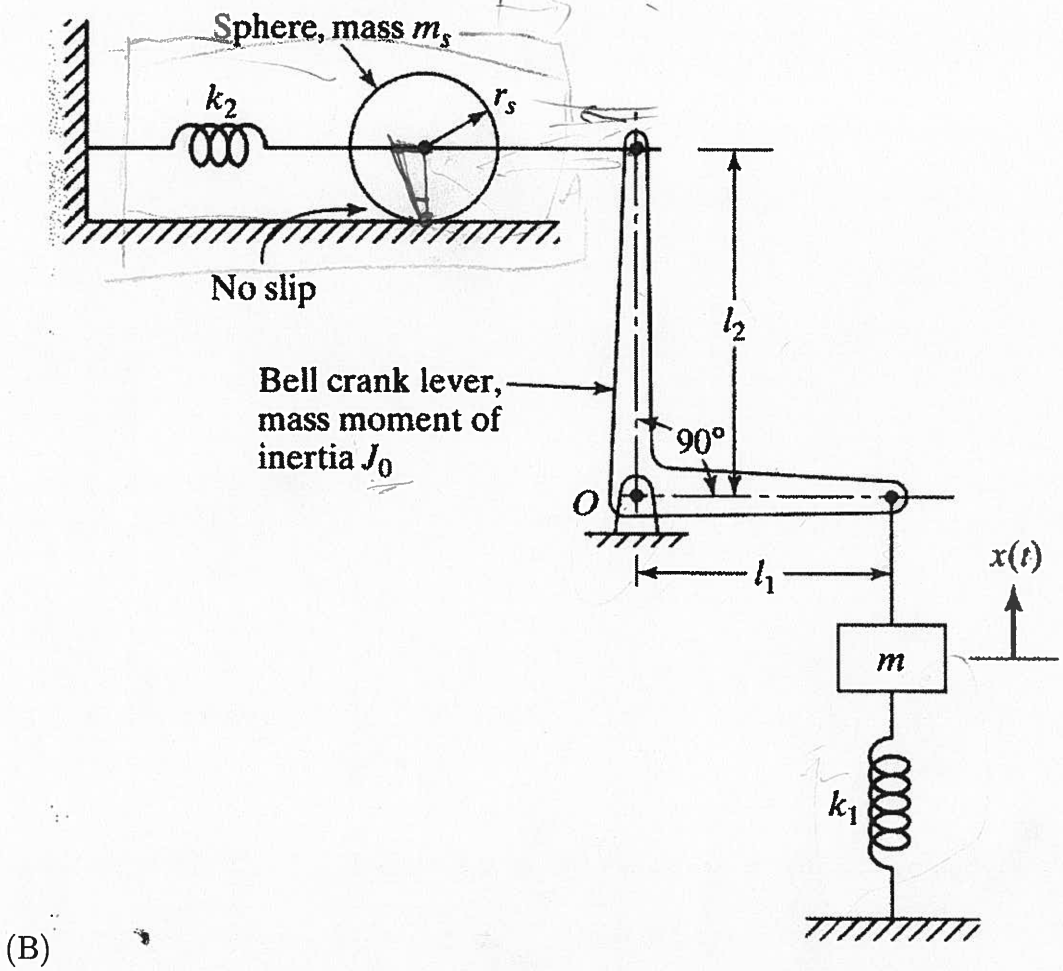
System in (A)

$$\left(\frac{1}{2K_1} + \frac{1}{K_2} + \frac{1}{2K_3}\right)^{-1} + K_4 = K_A$$

$$\frac{1}{K_{eq}} = \frac{1}{K_A} + \frac{1}{K_5}$$

$$\boxed{K_{eq} = \left[\frac{1}{\left[\frac{1}{\cdots}\right]^{-1}} + \frac{1}{\cdots}\right]^{-1}}$$ ANSWER

Question 2: Example 6(B) Determine the equivalent mass and spring constant for the system shown below at the mass m in terms of displacement co-ordinate x. Ignore gravity. first: m_{eq}, K_{eq}



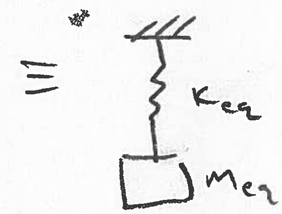


Figure 2: Figure for example 6(B).

Solution:

$$[KE]_{sys} = [K.e.]_{sphere} + [K.e]_{crank} + [K.e]_{mass} = \frac{1}{2} M_{eq} \dot{x}^2 \quad (1)$$

$$[PE]_{sys} = \frac{1}{2} K_2 \left(\frac{x(t) \cdot L_1}{L_2}\right)^2 + \frac{1}{2} K_1 x(t)^2 = \frac{1}{2} K_{eq} x(t)^2 \quad (2)$$

$\rightarrow$ From (2), $\boxed{K_{eq} = K_2 \left(\frac{L_1}{L_2}\right)^2 + K_1}$ ANSWER

From ①, $[KE]_{sys} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_o\dot{\theta}^2 + \frac{1}{2}m_s\dot{x}_2^2 + \frac{1}{2}J_s\dot{\theta}_s^2 = \frac{1}{2}m_{eq}\dot{x}^2$

→ J_s = mom. inertia at center of sphere

- From earlier: $x_2 = \frac{x(t)L_1}{L_2}$ ∴ $\dot{x}_2 = \frac{\dot{x}(t)L_1}{L_2}$ ✓

- $\dot{x}(t) = L_1\dot{\theta}$

- $\dot{\theta}_s = \frac{\dot{x}_2}{r_s}$ and since $\dot{x}_2 = \frac{\dot{x}(t)L_1}{L_2}$, $\dot{\theta}_s = \frac{\dot{x}(t)L_1}{L_2 r_s}$

∴ $\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_o\left(\frac{\dot{x}}{L_1}\right)^2 + \frac{1}{2}m_s\left(\frac{\dot{x}(t)L_1}{L_2}\right)^2 + \frac{1}{2}J_s\left(\frac{\dot{x}(t)L_1}{L_2 r_s}\right)^2$ ✓

$$\boxed{M_{eq} = m + \frac{J_o}{L_1^2} + m_s\left(\frac{L_1}{L_2}\right)^2 + J_s\left(\frac{L_1}{L_2 r_s}\right)^2}$$ ✓

You could equally write the equivalent mass moment of inertia and torsional spring constant in terms of rotations about point O. Do this as an exercise on your own.

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MECH 463: TUTORIAL 5

20/20 LM

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PROBLEM FROM TOPIC 2.3 (NOTES PACKAGE)

Example 12: (Modelling a drop test in packaging industry) A 5-kg fragile glass vase is packed in chopped sponge rubber and placed in a cardboard box that has negligible mass. It is then dropped from a height of 1m in a drop test. This particular sponge rubber exhibits the force-deflection curve sketched below. Determine the maximum acceleration experienced by the vase. Why is acceleration a design criterion for packaging selection?

y =

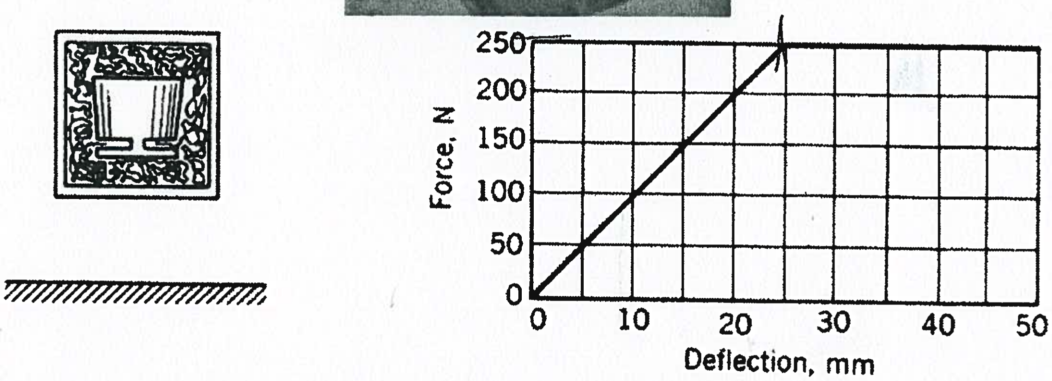
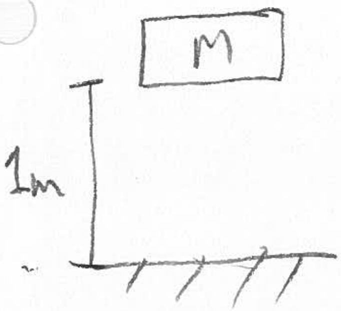


Figure 1: Drop test in packaging industry (top) and figure for example 12 (bottom).

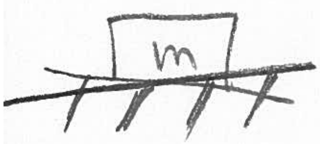
• state 1 : Mass is dropped



$$\text{Energy} = PE + \cancel{KE}^{\,0} = PE = mgh = 5(9.81)(1) = 49J$$

✓

• state 2: Mass is about to hit ground



Energy at ② = Energy at ① ✓

$$49J = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2\times 49}{5}} = \boxed{4.427\ m/s}$$

• For free vibrations, homogeneous has the form:

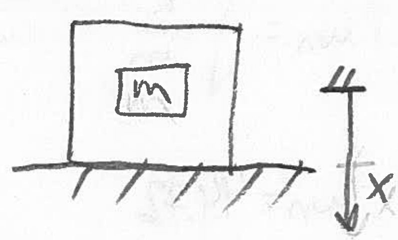
$$x(t) = A_1\cos(\omega_n t) + A_2\sin(\omega_n t)$$

★ Let t=0 be the time right before impact ✓

$$\dot{x}(0) = v = 4.427$$

$$x(0) = mg/k = 5\times 9.81/10{,}000 = -4.9\ mm$$

t=0:



Next page: solve A_1, A_2

$x(0) = -0.0049 = A_1 + 0$

$\therefore \boxed{A_1 = -0.0049}$

$\dot{x}(t) = -\omega_n A_1 \sin(\omega_n t) + \omega_n A_2 \cos(\omega_n t)$ ✓

$\sin(\omega_n \cdot 0) = 0$ so term drops

$\dot{x}(0) = 4.427$

$\therefore \boxed{A_2 = \dfrac{4.427}{\omega_n}}$

So:

$\dot{x}(t) = 4.427 \cos(\omega_n t) + \omega_n (0.0049) \sin(\omega_n t)$

$\boxed{\ddot{x}(t) = -\omega_n (4.427) \sin(\omega_n t) + 0.0049\, \omega_n^2 \cos(\omega_n t)}$

$\omega_n = \sqrt{\dfrac{K}{m}} \rightarrow m = 5,\ K = \dfrac{F}{\delta} = \dfrac{25}{0.025} = 10{,}000\ N/m$ ✓

$\omega_n = 44.72$

$\boxed{\ddot{x}(t)_{MAX} = 198.22\ m/s^2 \text{ UPWARDS}}$ ✓ ANSWER

*Note: Solved for t on next page

Acceleration is an impotant design factor as exemplified by a solution. High accelerations can damage products, even if take $\dddot{x}(t) = 0$, set sides equal

$$-8853.46 \cos(\phi) = 438.23 \sin(\phi)$$

$$-20.2 \cos(\phi) = \sin(\phi)$$

$$-20.2 = \tan(\phi)$$

$$\phi = -1.52 \qquad t = \phi / w_n$$

$$t = -0.034 \text{ seconds}$$

(unreal, but there is another t in the positive domain that will give same magnitude. It doesn't really matter for finding max accel amplitude)

✓

true, careful o midterm/exams about this statement though.

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MECH 463: TUTORIAL 6

20/20 LM

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PROBLEM FROM TOPIC 2.3 (NOTES PACKAGE)

Example 13: (Vibration due to rotating unbalance) Unbalanced masses are a common source of vibration problems in many rotating systems: turbo machinery; washing machines; and shaky table, to name a few. The simplest SDOF system that models the vibrations of these systems is sketched below. A mass m_u is mounted by a shaft and bearings to the mass m. The unbalance mass m_u follows a circular path of radius e (unbalance). *Ignoring damping*, determine the equations that govern the motion of the main mass m. Draw the appropriate FBD. Discuss the possible motions of the mass m by considering equilibrium in the horizontal and vertical directions.

*gravity ignored due to static equilibrium

cos

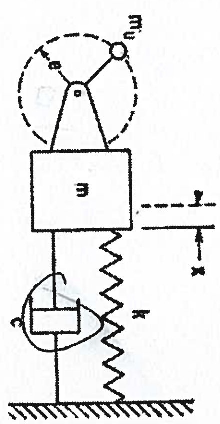
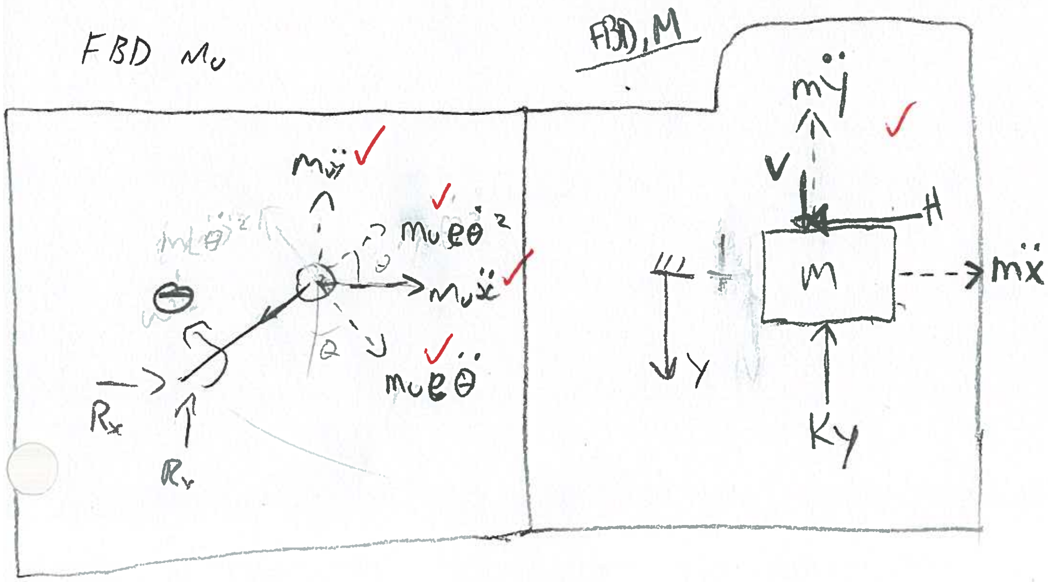


Figure 1: Figure for example 13. Ignore damping.

FBD m_u

FBD, M



$R_y = V$
$R_x = H$

Now, combine both FBDs to write eqns. of motion

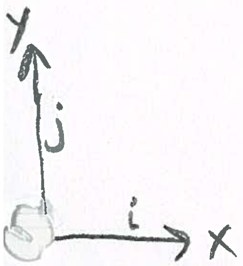
$$\overset{+}{\uparrow}\ \Sigma F_y = 0 = -m_u\ddot{y} - m\ddot{y} - Ky - m_u e\omega^2 \sin(\omega t) - \cancel{R_y} + m_u e\ddot{\theta}\cos(\omega t) + \cancel{R_y}$$

$$0 = m_u e\dot{\omega}\cos(\omega t) - m_u e\omega^2 \sin(\omega t) - Ky - m\ddot{y} - m_u\ddot{y} \quad [1]$$

$$\overset{+}{\rightarrow}\ \Sigma F_x = 0 = \cancel{R_x} - \cancel{R_x} + m_u\ddot{x} + m\ddot{x} + m_u e\omega^2\cos(\omega t) + m_u e\dot{\omega}\sin(\omega t) \quad [2]$$

$$[1] \quad 0 = m_u\left[e\dot{\omega}\cos(\omega t) - e\omega^2\sin(\omega t) - \ddot{y}\right] - m\ddot{y} - Ky$$

$$[2] \quad 0 = m\ddot{x} + m_u\left[\ddot{x} + e\omega^2\cos(\omega t) + e\dot{\omega}\sin(\omega t)\right]$$



$$\boxed{\text{So, } \Sigma F = \Big[m_u\left[e\dot{\omega}\cos(\omega t) - e\omega^2\sin(\omega t) - \ddot{y}\right] - m\ddot{y} - Ky\Big]\hat{j} + \Big[m\ddot{x} + m_u\left[\ddot{x} + e\omega^2\cos(\omega t) + e\dot{\omega}\sin(\omega t)\right]\Big]\hat{i} = 0}$$

what is the possible motion of mass?

It will obviously move up and down (with no damping, apparently) and the rotating mass m_u is the reason why it will also have motion in the x-axis. The exact response is dependent on $K, \omega, \dot{\omega}$, etc as outlined in our equation of motion ✓

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MECH 463: Tutorial 7

18/20

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PROBLEM FROM TOPIC 2.4 (NOTES PACKAGE)

Example 17: (T 2.91) A railroad car of mass 2000 kg travelling at a velocity of $v = 10$ m/s is stopped at the end of the tracks by a spring-damper system, as shown below. If the stiffness of the spring is $\frac{k}{2} = 40$ N/mm and the damping constant is $c = 20$ N-s/mm, determine (a) the maximum displacement of the car after engaging the springs and damper and (b) the time taken to reach the maximum displacement.

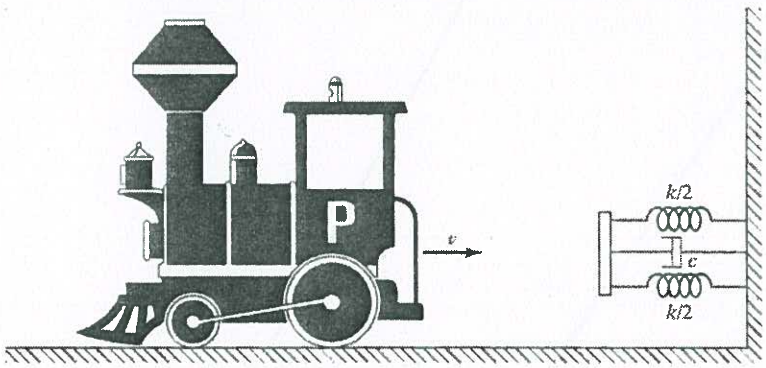

Figure 1: Figure for example 17.

Note: To find maximum use calculus, namely, maximum of $f(x)$ occurs for x satisfying $\frac{df}{dx} = 0$.

Given:

$M_{eq} = 2000 Kg$

$V = 10 m/s$

$K_{eq} = \frac{K}{2} + \frac{K}{2} = 80 N/mm$

$C = 20 Ns/mm$

Find: X_{max}, t at X_{max}

$2m\omega_n$

$\hookrightarrow \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{80{,}000 \text{ N/m}}{2000 \text{ Kg}}} = 6.325$ rad/s ✓

$\hookrightarrow \zeta = \frac{20000}{2(2000)(6.325)} = 0.791$ underdamped

→ For underdamped:

$x_h(t) = e^{-\zeta\omega_n t} A\cos(\omega_d t - \phi_0)$ [Eqn. 1]

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.325\sqrt{1-0.791^2} = 3.87$ rad/s

→ initial conditions:

$x_0(0) = 0$ at $t=0$

$\dot{x}_0(0) =$ initial velocity $= 10$ m/s ✓

→ $A = \sqrt{x_0^2 + \left[\frac{\zeta\omega_n x_0 + \dot{x}_0}{\omega_d}\right]^2} = \frac{15 \text{ m/s}}{3.87 \text{ rad/s}} = \boxed{3.88 \text{m}}$

$\phi_0 = \sin^{-1}\left(\left[\frac{\zeta\omega_n x_0 + \dot{x}_0}{\omega_d}\right]/A\right) = \sin^{-1}(1) = \frac{\pi}{2}$

$\boxed{\phi_0 = \frac{\pi}{2}}$

Using Eqn. 1 :

$$3.88 = e^{-6.325(0.791)t} \times 3.88\cos\left(3.87t - \frac{\pi}{2}\right)$$

$$1 = e^{-5t}\cos\left(3.87t - \frac{\pi}{2}\right)$$

$$1 = e^{-5t}\sin(3.87t)$$

$$\ln\left(\frac{1}{\sin(3.87t)}\right) = -5t$$

$$-\ln\left(\sin(3.87t)\right) = -5t$$

ignore

Recall: $X_h(t) = e^{-\zeta\omega_n t}A\sin(\omega_d t)$

$$\dot{X}_h(t) = -\zeta\omega_n e^{-\zeta\omega_n t}A\sin(\omega_d t) + e^{-\zeta\omega_n t}A\omega_d\cos(\omega_d t)$$

at $X_{h\,max}$, $\dot{X}_h = 0$

$\therefore$ find t: $0 = -\zeta\omega_n\sin(\omega_d t) + \omega_d\cos(\omega_d t)$

$$\zeta\omega_n\sin(\omega_d t) = \omega_d\cos(\omega_d t)$$

$$\tan(\omega_d t) = \frac{\omega_d}{\zeta\omega_n}$$

✓

$$t = \tan^{-1}\left(\frac{\omega_d}{\zeta\omega_n}\right)\Big/\omega_d = \tan^{-1}\left(\frac{3.87}{0.791(6.325)}\right)\Big/3.87 = \boxed{0.17\,\text{sec}}$$ ANS

Now,

at $t = 0.17s$, $X_h = X_{max}$

$$X_{max} = e^{-\zeta\omega_n t} A\sin(\omega_d t) = e^{-0.791(6.325)(0.17)} \left[3.88 \sin(3.87 \times 0.17) \right]$$

Not quite

$\hookrightarrow \frac{V_0}{\omega_d} = 2.58$

something like this

(-2)

$\boxed{X_{max} = 1.013m}$ ANS

some like ~~0.64m~~ 0.67 for final Ans

END OF SUBMISSION

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MECH 463: Tutorial 8

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PROBLEM GIVING PRACTICE ON USING TR FORMULA. MARKED FOR PROCEDURE.

(T9.36, 4th Edition) An internal combustion engine has a rotating unbalance of 1·kg·m and operates between 800 and 2000 rpm. Find the stiffness of the isolator necessary to reduce the transmitted force to the floor to 6000N over the operating speed range of the engine. Assume that the damping ratio of the isolator ζ is 0.08, and the mass of the engine is 200 kg. **Follow the steps below.**

1. At the lower end of operating speed (800 rpm) find the k required using the TR formula. To find k: first find r using the TR formula, then $\omega_n = \frac{\omega}{r}$ and finally k using $k = m\omega_n^2$.

2. Check that your design in step 1 works for higher rpm, by ensuring that force transmitted is below 6000N.

1) given: $m = 1\text{Kg}$ $\quad F = 6000\text{N}$ $\quad \zeta = 0.08$ $\quad m_{eng} = 200\text{Kg}$

$$\omega = \frac{800\text{ rot}}{\text{min}} \times \frac{2\pi\text{ rad}}{\text{rot}} \times \frac{1\text{ min}}{60\text{s}} = 83.78\text{ rad/s}$$

$$TR = \frac{F_t}{F} = \frac{6000\text{N}}{me\omega^2} = \frac{6000\text{N}}{(1\text{Kg}\cdot\text{m})(83.78)^2} = 0.855$$

$$\text{Since } TR = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = 0.855 \implies 0.731 = \frac{1+0.0256r^2}{(1-r^2)^2+0.0256r^2}$$

$$= \frac{1+0.0256r^2}{1+r^4+0.0256r^2-2r^2}$$

$$\implies 0.731 - 1.44r^2 + 0.731r^4 = 1 + 0.0256r^2$$

$$0 = 0.731r^4 - 1.4656r^2 - 0.269 \qquad \implies r = 1.4745 \checkmark$$

$$\omega_n = \frac{\omega}{r} = \frac{83.78}{1.4745} = 56.82 \text{ rad/s}$$

$$K = m\omega_n^2$$

$$= 200(56.82)^2$$

$$= \boxed{645686 \text{ N/m}}$$ ✓ round off error

2. $\omega = 2000\text{rpm} \times \frac{2\pi}{60} = 209.44 \text{ rad/s}$

$$r = \frac{\omega}{\omega_n} = \frac{209.44 \text{ rad/s}}{56.82 \text{ rad/s}} = 3.686$$ ✓

$$\therefore TR = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = 0.083$$ ✓ round off error

$$F_t = TR \times F = TR \cdot me\omega^2 = 0.083(1\text{kg}\cdot\text{m})(209.44)^2$$

$$= \boxed{3{,}640.8\text{N}}$$ ANS ✓

Since $F_t < 6000\text{N}$, all is good!

MECH 463: Tutorial 9

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Resonance is here.

18/20 RR

Resonance is not always undesirable.
(A) Practical devices, known as vibration absorbers or tuned mass dampers, use resonance. To understand how they work, consider the SDOF system shown below. From the free body diagrams (FBDs) of **the point of attachment** O and the **mass** m_a write the two equilibrium equations at O and m_a in the displacement co-ordinates x_a and y.

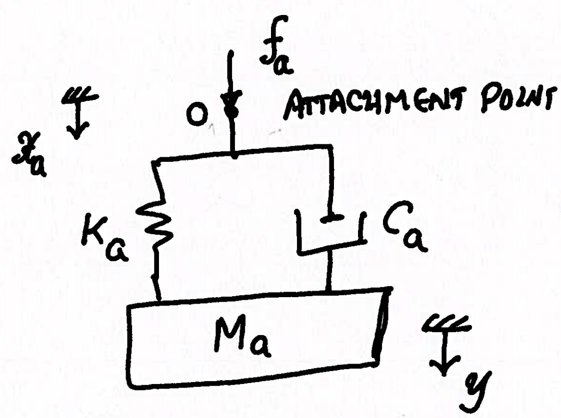
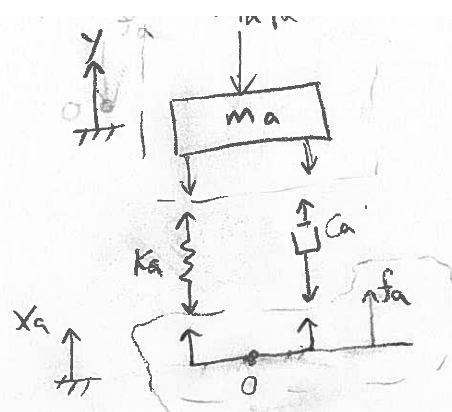


Figure 1: Figure for Question.

(B) Now assume that the force is $f_a = F_a e^{j\omega t}$ and the response is $x_a = X_a e^{j\omega t}$ at O and $y = Y e^{j\omega t}$ at m_a in the steady vibration at ω. Eliminating Y between the two equilibrium equations above, deduce an expression for the *Frequency Response Function* (FRF) defined as $H_a(\omega) = \frac{X_a}{F_a}$.

(C) What is the displacement amplitude $|X_a|$ for the case when $\omega = \omega_a = \sqrt{\frac{k_a}{m_a}}$ and $c_a = 0$. What is the corresponding displacement amplitude $|Y|$ of the mass m_a and what is the force exerted at the attachment point?

Ignore homogenous solution and focus only on steady state. We are using complex number notation in this problem to simplify algebra. X_a and Y are complex numbers.



Force on O: $\sum F_0 = Ka(Xa - y) + Ca(\dot{x}a - \dot{y}) - fa = 0$

Force on M: $\sum F_{Ma} = Ka(y - xa) + Ca(\dot{y} - \dot{x}a) + m\ddot{y} = 0$

$$\boxed{\begin{aligned} &Kaxa + Ca\dot{x}a - fa = Kay + Ca\dot{y} \\ &Kay + Ca\dot{y} + m\ddot{y} = Kaxa + Ca\dot{x}a \end{aligned}}$$ ANS

b) Eqn 1: $Ka(Xa\cancel{e^{j\omega t}}) + Ca(j\omega Xa\cancel{e^{j\omega t}}) - F_a = Ka(Y\cancel{e^{j\omega t}}) + Ca(j\omega Y\cancel{e^{j\omega t}})$

Eqn 2: $Ka(Y\cancel{e^{j\omega t}}) + Ca(j\omega Y\cancel{e^{j\omega t}}) + (j^2\omega^2 Y\cancel{e^{j\omega t}})m_a = Ka(Xa\cancel{e^{j\omega t}}) + Ca(Xaj\omega\cancel{e^{j\omega t}})$

from Eqn 2: $Y = \dfrac{Xa\cancel{e^{j\omega t}}(Ka + Ca)}{Ka\cancel{e^{j\omega t}} + Caj\omega\cancel{e^{j\omega t}} + j^2\omega^2\cancel{e^{j\omega t}}}$

plug it into Eqn 1:

$$K_aX_a + C_aX_aj\omega - F_a = \frac{K_aX_a(K_a + C_a)}{K_a + C_aj\omega + j^2\omega^2} + Ca\frac{X_a(K_a + C_a)}{K_a + C_aj\omega + j^2\omega^2}j\omega$$

$$\Rightarrow \boxed{\frac{F_a}{X_a} = -\left[\frac{K_a(K_a + C_a)}{K_a + C_aj\omega + j^2\omega^2} + \frac{C_a(K_a + C_a)}{K_a + C_aj\omega + j^2\omega^2}j\omega - K_a - C_aj\omega\right] = Ha(\omega)}$$

$\therefore$ $Ha(\omega) = KaCaj\omega - Ca^2\omega^2$ ... $Caj\omega^3 - KaCa - Ca^2j\omega$ ... ANS

c) if $\omega = \omega_a$ and $c_a = 0$:

$$|X_a| = |H_a(\omega) F_a|$$

$$= \left| -K_a \omega_a^2 F_a \right| = F_a K_a \omega_a^2 = \boxed{\frac{F_a K_a^2}{m_a}} \text{ ANS}$$

Recall: $Y = \dfrac{X_a(K_a + \cancel{c_a})}{K_a + \cancel{c_a} j\omega - \omega^2}$

$$\therefore |Y| = \frac{F_a K_a^2 (K_a)}{\left(K_a - \frac{K_a}{m_a}\right) m_a} = \boxed{\frac{F_a K_a^3}{m_a K_a - K_a}} = \boxed{\frac{F_a K_a^2}{m_a - 1}} \text{ ANS}$$

$$\frac{X}{Y} = \frac{F_t}{F_a} \quad \therefore F_t = \frac{F_a X_a}{Y}$$

$$\boxed{F_t = F_a \left(\frac{m_a - 1}{m_a} \right)} \text{ ANS}$$