a place of mind

MECH468 : Modern Control Engineering
MECH509 : Controls

L24 : Observer-based control

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Zoom lecture to be recorded and posted on Canvas

Course plan

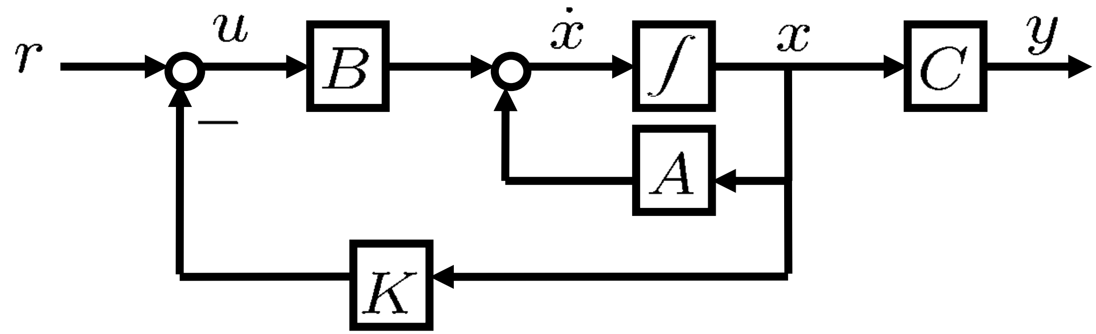
Topics	CT	DT
Modeling	✓	✓
Stability	✓	✓
Controllability/observability	✓	✓
Realization	✓	✓
→ State feedback/observer		
LQR/Kalman filter		

Review & today's topic

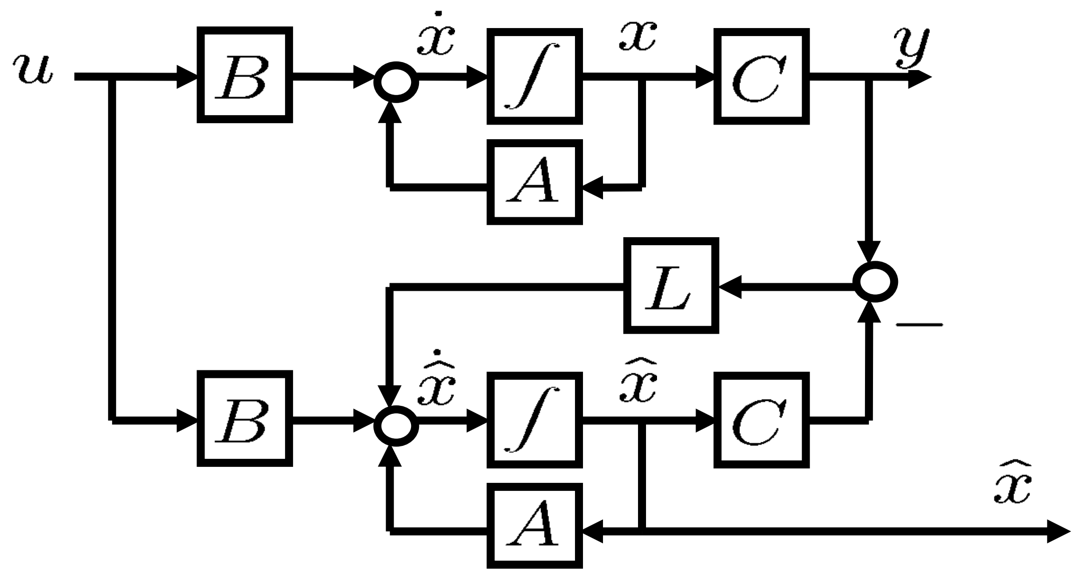
- During the last two weeks
  - State feedback
  - Observer
- Today
  - Observer-based control (i.e., combination of state feedback and observer)
  - Separation principle

State feedback & observer (review)

- State feedback

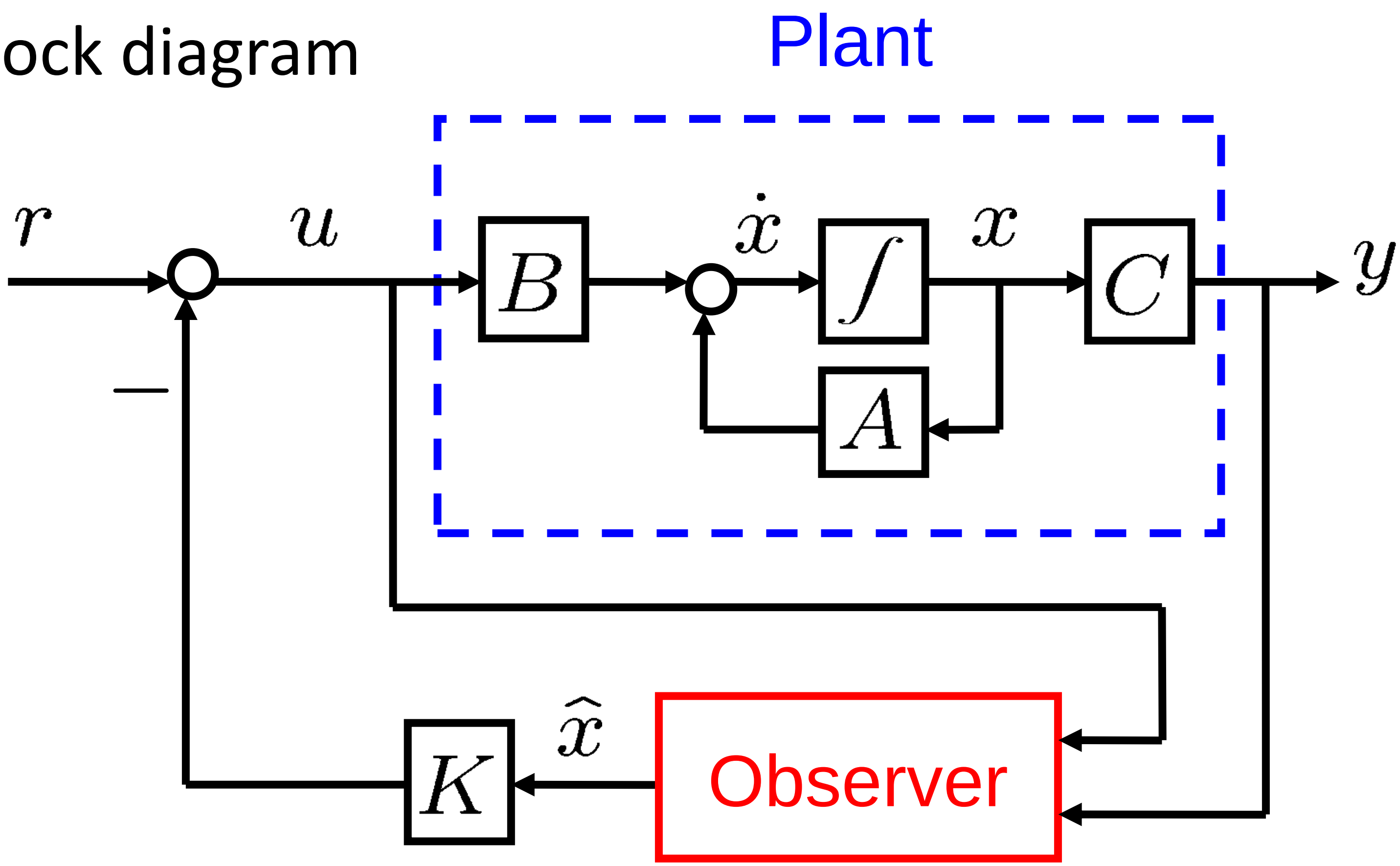


- Observer



Observer-based control

- Block diagram



Observer-based control

- LTI plant $\Sigma : \begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{cases}$
- Observer $\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t))$
- Feedback $u(t) = r(t) - K\hat{x}(t)$
- Closed-loop system

$$\begin{cases} \dot{\begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}} &= \begin{bmatrix} A & -BK \\ LC & A - LC - BK \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} r(t) \\ y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \end{cases}$$

Observer-based control (cont'd)

- Take a new state vector $\begin{bmatrix} x(t) \\ e(t) \end{bmatrix} := \underbrace{\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}}_{T} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}$ $(T^{-1} = T)$

$$\Rightarrow \left\{ \begin{array}{ll} T \begin{bmatrix} A & -BK \\ LC & A-LC-BK \end{bmatrix} T^{-1} = \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix} \\ T \begin{bmatrix} B \\ B \end{bmatrix} = \begin{bmatrix} B \\ 0 \end{bmatrix} & \begin{bmatrix} C & 0 \end{bmatrix} T^{-1} = \begin{bmatrix} C & 0 \end{bmatrix} \end{array} \right\}$$

$$\Rightarrow \left\{ \begin{array}{lcl} \begin{bmatrix} \dot{x}(t) \\ e(t) \end{bmatrix} & = & \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r(t) \\ y(t) & = & \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \end{array} \right.$$

Analysis of CL system

- Stability: CL system is internally stable if and only if both (*A*-*BK*) and (*A*-*LC*) are stable.
- Separation principle: Eigenvalues of (*A*-*BK*) and (*A*-*LC*) does not affect each other. Namely, design of state feedback and observer can be carried out separately!
- Transfer function: as if there were no observer.

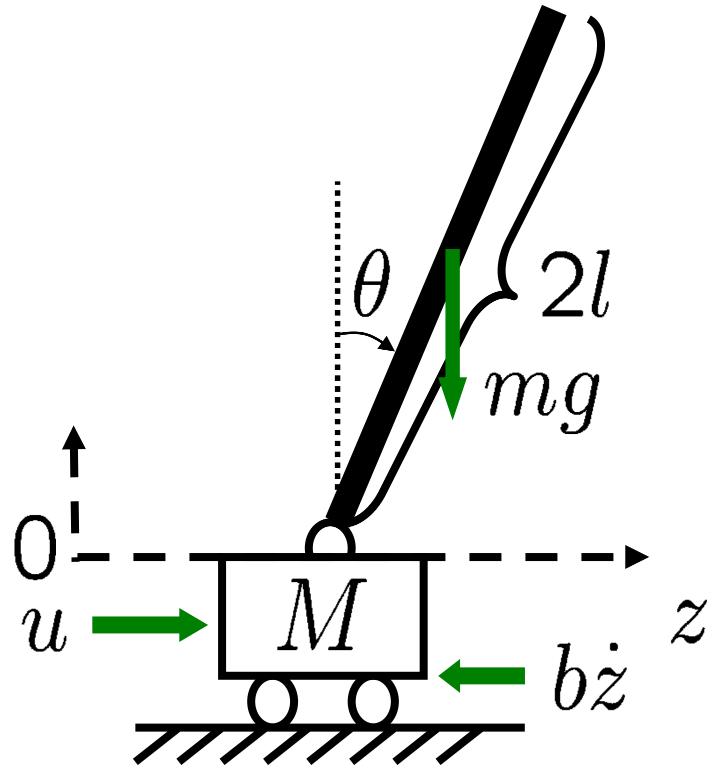
$$G_{yr}(s) = C(sI - A + BK)^{-1}B$$

- Rule of thumb: eig(*A*-*LC*) should be far left to eig(*A*-*BK*), but such eigs may amplify initial *e*. *Trade-off!*

Example: Inverted pendulum

- State vector $x := \left[z, \dot{z}, \theta, \dot{\theta}\right]^T$
- SS model around x=0

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{(I+ml^2)b}{d} & -\frac{m^2l^2g}{d} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mlb}{d} & \frac{(M+m)mgl}{d} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{I+ml^2}{d} \\ 0 \\ -\frac{ml}{d} \end{bmatrix} u \\ d := I(M+m) + Mml^2 \\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x \end{cases}$$



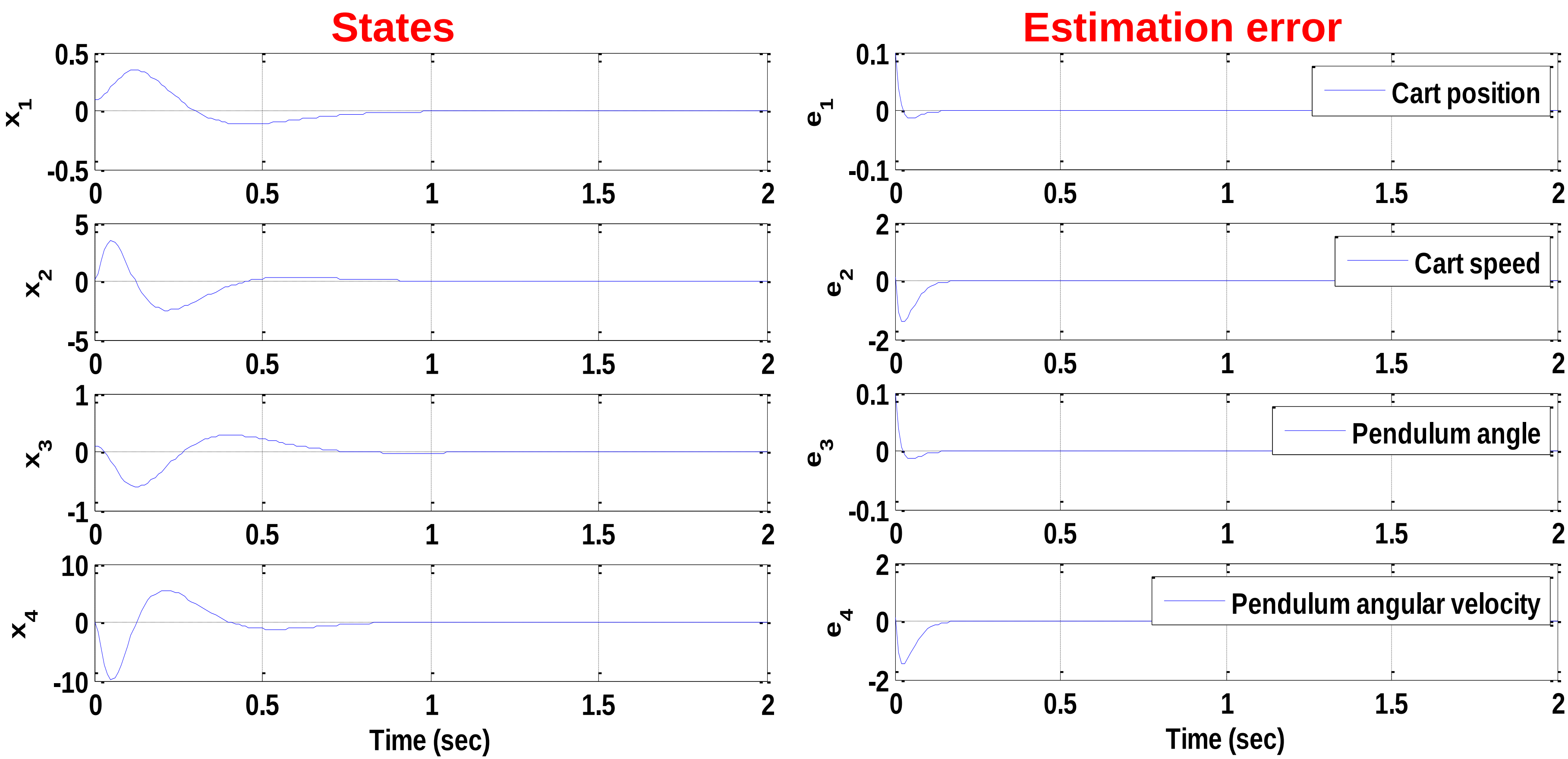
Use the estimate of the state for state feedback.
Then, what will happen?

Inverted pendulum example

$$\sigma(A-LC)=\{-40,-41,-42,-43\}$$

$$\sigma(A-BK)=\{-8\pm 7j,-5\pm j\}$$

$$\begin{bmatrix} x(0) \\ e(0) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ \vdots \\ 0.1 \end{bmatrix} \qquad r=0$$

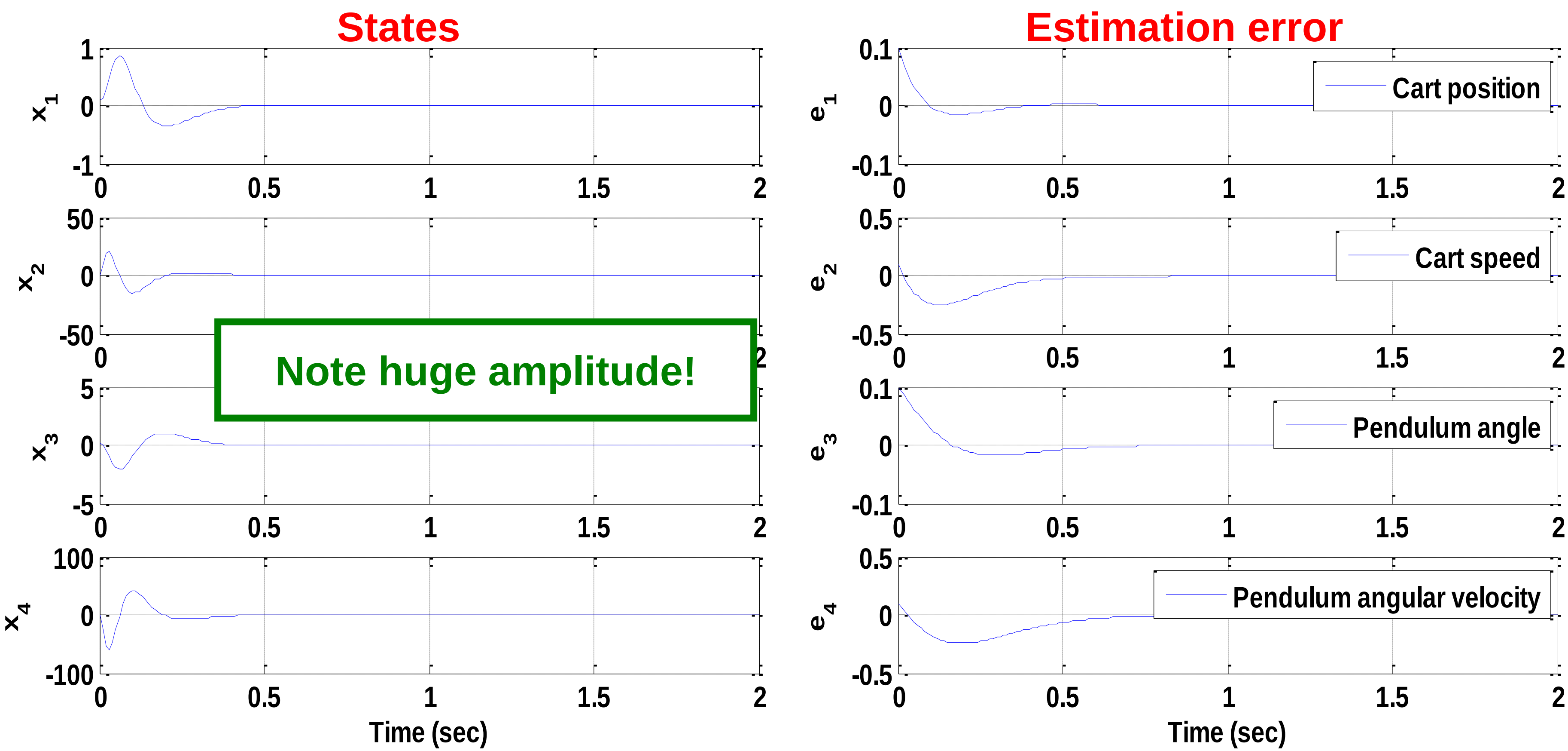


Inv. pendulum example (cont'd)

$$\sigma(A - LC) = \{-8 \pm 7j, -5 \pm j\}$$

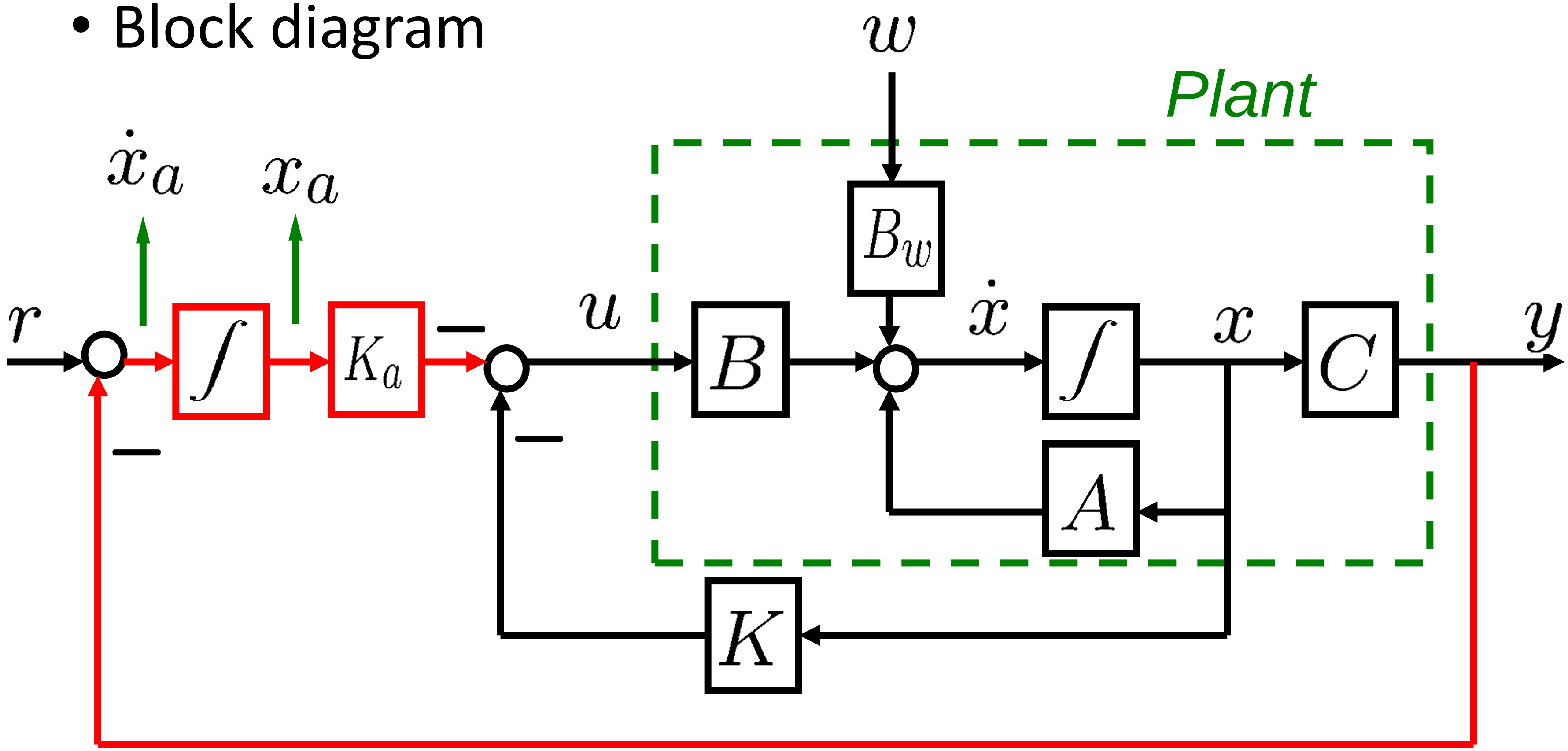
$$\sigma(A - BK) = \{-40, -41, -42, -43\}$$

$$\begin{bmatrix} x(0) \\ e(0) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ \vdots \\ 0.1 \end{bmatrix} \qquad r = 0$$



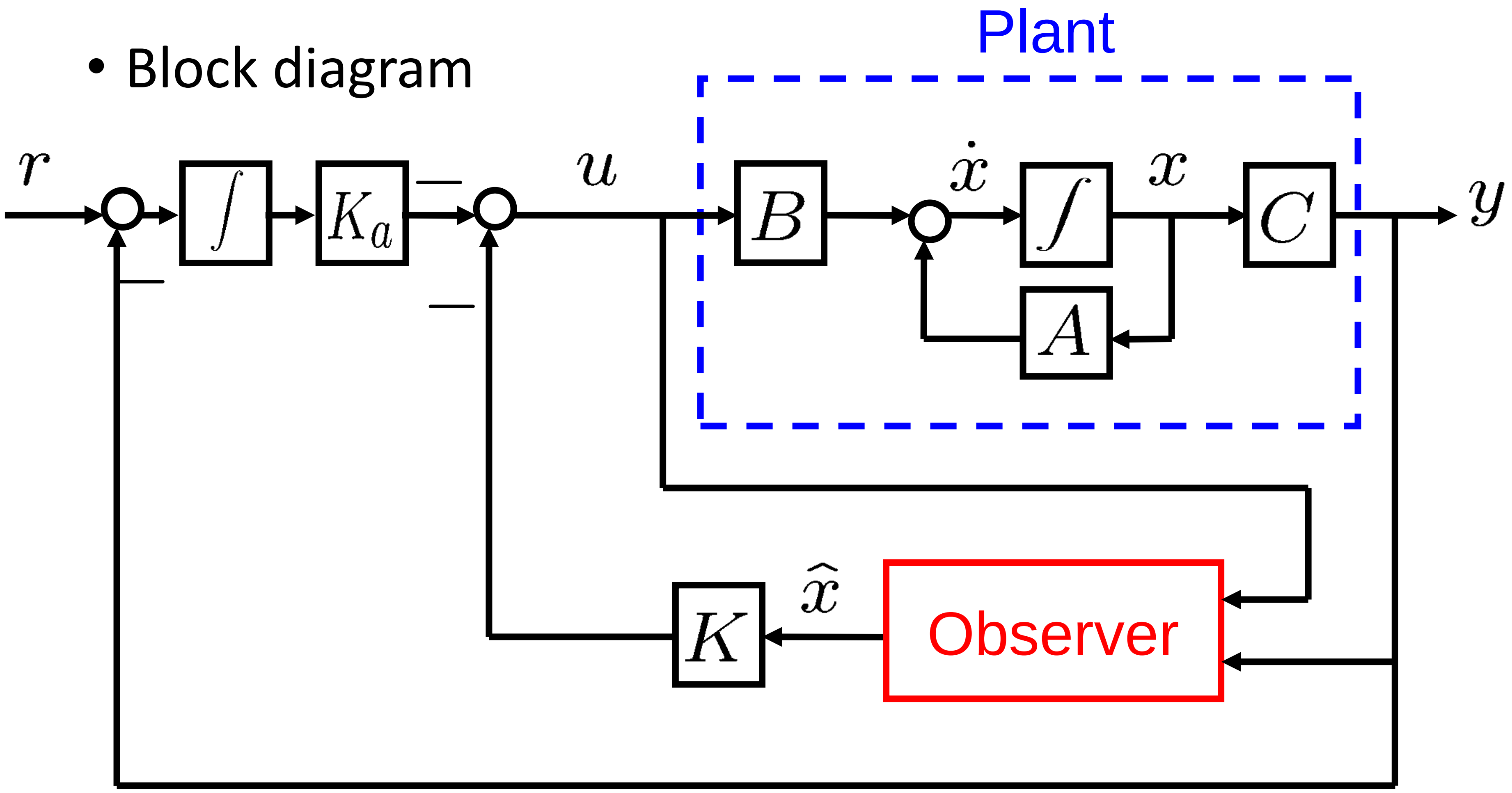
State feedback with an integrator (Review)

- Block diagram



Observer-based servo control

- Block diagram



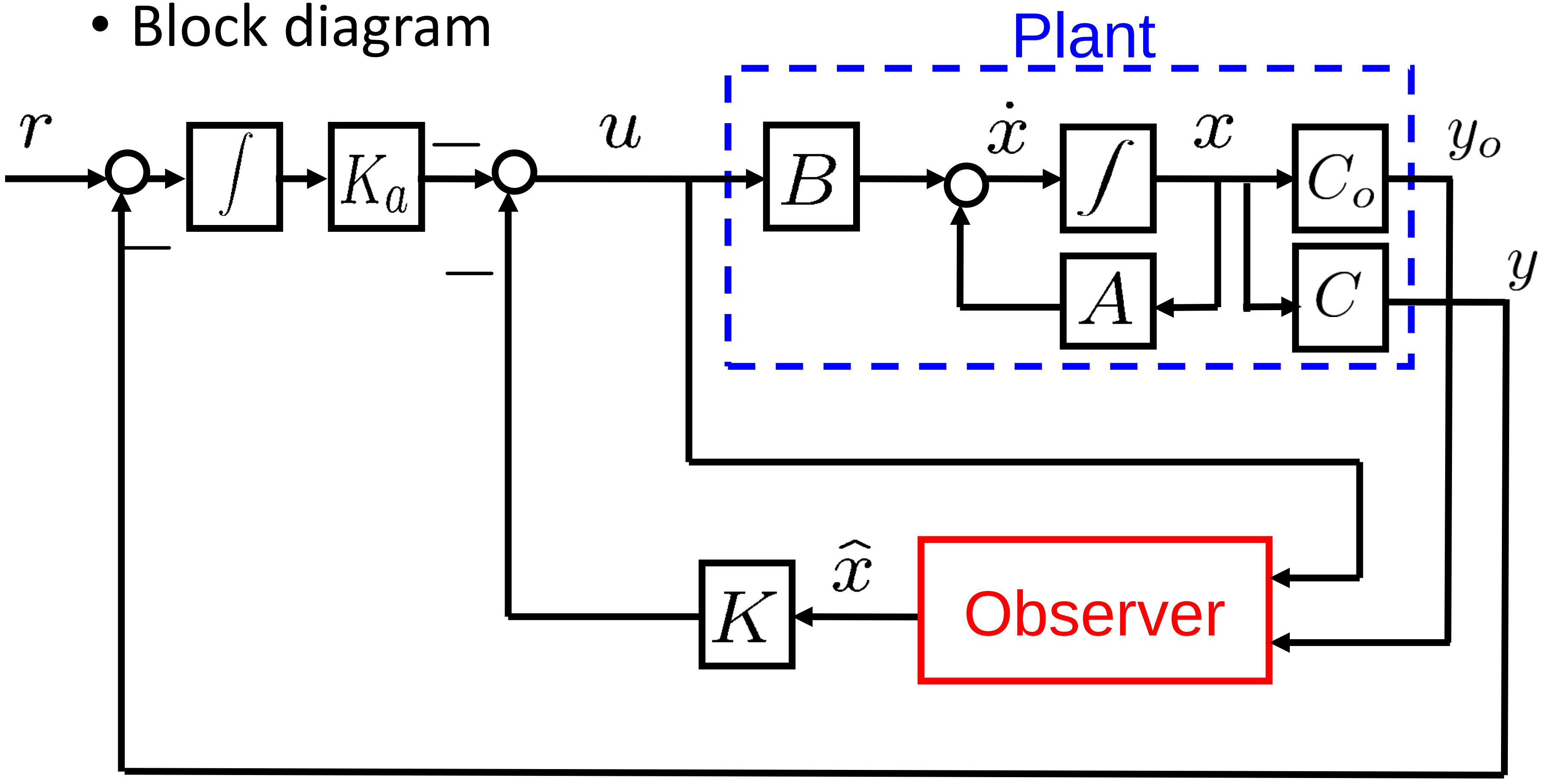
Observer-based servo control

- LTI plant $\Sigma : \begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{cases}$
- Observer $\frac{d}{dt}\widehat{x}(t) = A\widehat{x}(t) + Bu(t) + L(y(t) - C\widehat{x}(t))$
- Feedback $u(t) = -[K, K_a]\begin{bmatrix} \widehat{x}(t) \\ x_a(t) \end{bmatrix}$ $\quad \dot{x}_a(t) = r(t) - Cx(t)$
- Closed-loop system

$$\begin{cases} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_a(t) \\ \dot{e}(t) \end{bmatrix} = \left[\begin{array}{cc:c} A - BK & -BK_a & BK \\ -C & 0 & 0 \\ \hdashline 0 & 0 & A - LC \end{array}\right] \begin{bmatrix} x(t) \\ x_a(t) \\ e(t) \end{bmatrix} + \left[\begin{array}{c} 0 \\ I \\ \hdashline 0 \end{array}\right] r(t) \\ y(t) = \left[\begin{array}{cc:c} C & 0 & 0 \end{array}\right] \begin{bmatrix} x(t) \\ x_a(t) \\ e(t) \end{bmatrix} \end{cases}$$

Observer-based servo control When *y* for observer and *y* for feedback signal are different

- Block diagram



Observer-based servo control (cont'd)

- LTI plant $\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \\ y_o(t) = C_o x(t) \end{cases}$

- Observer $\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y_o(t) - C_o\hat{x}(t))$
- Feedback $u(t) = -[K, K_a]\begin{bmatrix} \widehat{x}(t) \\ x_a(t) \end{bmatrix}$ $\quad \dot{x}_a(t) = r(t) - Cx(t)$
- Closed-loop system

$$\begin{cases} \dot{\begin{bmatrix} x(t) \\ x_a(t) \\ e(t) \end{bmatrix}} = \left[\begin{array}{cc:c} A - BK & -BK_a & BK \\ -C & 0 & 0 \\ \hdashline 0 & 0 & A - LC_o \end{array}\right] \begin{bmatrix} x(t) \\ x_a(t) \\ e(t) \end{bmatrix} + \left[\begin{array}{c} 0 \\ I \\ \hdashline 0 \end{array}\right] r(t) \\ y(t) = \left[\begin{array}{cc:c} C & 0 & 0 \end{array}\right] \begin{bmatrix} x(t) \\ x_a(t) \\ e(t) \end{bmatrix} \end{cases}$$

Summary

- Observer-based control
  - Separation principle
  - Inverted pendulum example ("pendulum3.m")
- State feedback and observer for DT systems are exactly the same as those for CT systems.
- Next,
  - Linear Quadratic Regulator (optimal state feedback)
  - Kalman Filter (optimal state estimator)

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Modeling	✓	✓
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Realization	✓	✓
State feedback/observer	✓	✓
LQR/Kalman filter	6 lectures	