Question 1a.

Subject:
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1a. $G(s) = \dfrac{3}{s+3}$, $y(0) = 4$, $r(t) = 2t$

$$y(s) = G(s)\,R(s) \Rightarrow \frac{y(s)}{R(s)} = G(s) = \frac{3}{s+3} \Rightarrow$$

$$\dot{y}(t) + 3y(t) = 3r(t) \xrightarrow{L\{\}} s\,y(s) - y(0) + 3y(s) = 3R(s)$$

$$\Rightarrow y(s)\left[s+3\right] - 4 = \frac{6}{s^2} \Rightarrow y(s) = \frac{4s^2+6}{s^2(s+3)} \xrightarrow{*}$$

*) Partial Fractions with Repeated Roots

$$y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3}$$

$$A = \lim_{s\to 0} \frac{d}{ds}\left[s^2 \times G(s)\right] = \lim_{s\to 0} \frac{(8s)(s+3) - (1)(4s^2+6)}{(s+3)^2} = -\frac{2}{3}$$

$$B = \lim_{s\to 0}\left[s^2\,G(s)\right] = \lim_{s\to 0} \frac{4s^2+6}{s+3} = 2$$

$$C = \lim_{s\to -3}\left[(s+3)\,G(s)\right] = \lim_{s\to -3} \frac{4s^2+6}{s^2} = \frac{42}{9} = 4.67$$

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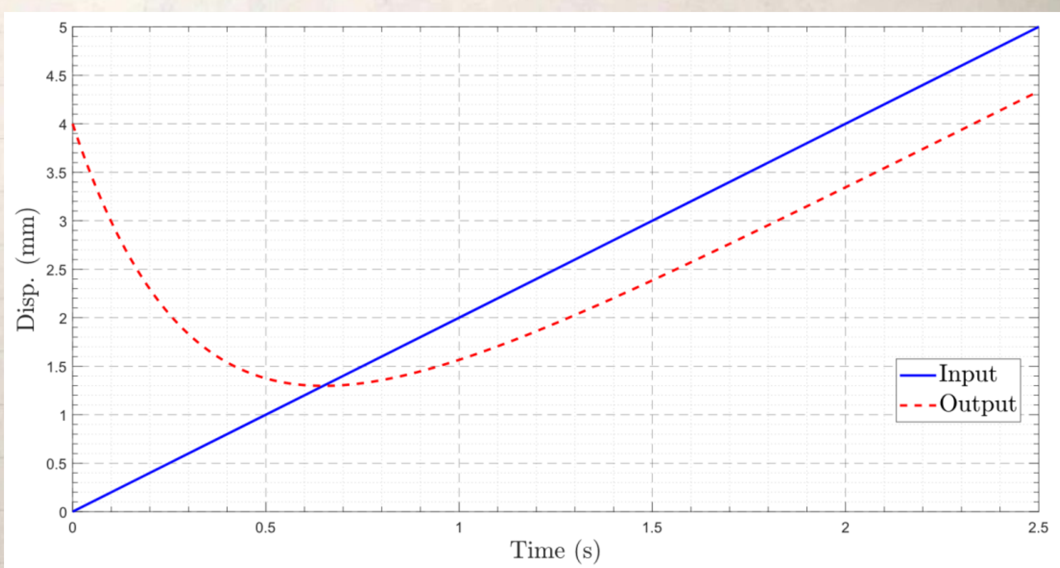
$$\Rightarrow y(s) = \frac{-2}{3s} + \frac{2}{s^2} + \frac{4.67}{s+3} \Rightarrow$$

$$\boxed{y(t) = -\frac{2}{3} + 2t + 4.67e^{-3t}}$$



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Question 1b.

Subject:
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1b. $G(s) = \dfrac{3s}{s+3}$, $y(0) = 1$, $r(t) = 2t$

$Y(s) = G(s)\,R(s) \Rightarrow \dfrac{Y(s)}{R(s)} = G(s) = \dfrac{3s}{s+3} \Rightarrow$

$(s+3)\,Y(s) = 3s\,(R(s)) \xrightarrow{\mathcal{L}^{-1}\{\}} \dot{y}(t) + 3y(t) = 3\dot{r}(t)$

$\Rightarrow \dot{y}(t) + 3y(t) = 3\dfrac{d}{dt}(2t) = 6 \xrightarrow{\mathcal{L}\{\}} sY(s) - y(0) + 3Y(s) = \dfrac{6}{s}$

$\Rightarrow Y(s)\,(s+3) = \dfrac{6+s}{s} \Rightarrow Y(s) = \dfrac{6+s}{s(s+3)} \rightarrow$

$Y(s) = \dfrac{A}{s} + \dfrac{B}{s+3}$

$A = \lim_{s\to 0} (s\,G(s)) = \lim_{s\to 0} \left(\dfrac{6+s}{s+3}\right) = 2$

$B = \lim_{s\to -3} ((s+3)\,G(s)) = \lim_{s\to -3} \left(\dfrac{6+s}{s}\right) = -1$

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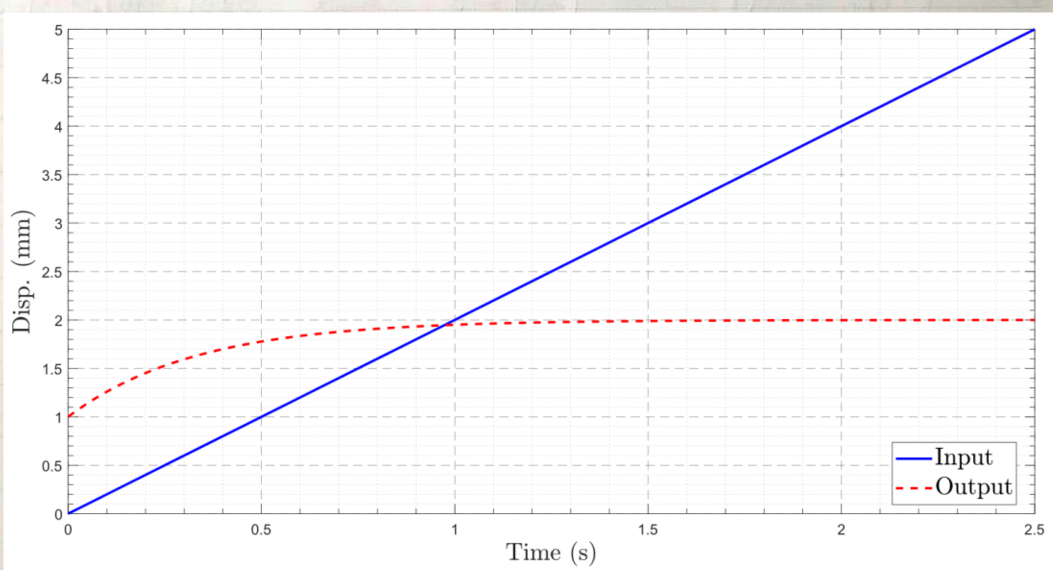
$$\Rightarrow Y(s) = \frac{2}{s} - \frac{1}{s+3} \Rightarrow$$

$$\boxed{y(t) = 2 - e^{-3t}}$$



Question 1c.

Subject:
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1c. $G(s) = \dfrac{1}{0.1s+1}$, $y(0) = 0$, $r(t) = 20 + 10\sin(5t)$

$$y(s) = G(s)\,R(s) = \frac{1}{0.1s+1} \times \left[\frac{20}{s} + \frac{50}{s^2+25}\right] \Rightarrow$$

$$y(s) = \underbrace{\frac{20}{s(0.1s+1)}}_{G_1} + \underbrace{\frac{50}{(0.1s+1)(s^2+25)}}_{G_2} \Rightarrow$$

$$y(s) = \left(\frac{A}{s} + \frac{B}{0.1s+1}\right) + \left(\frac{C}{0.1s+1} + \frac{Ds+E}{s^2+25}\right) \Rightarrow$$

$$A = \lim_{s\to 0}\left(s\,G_1(s)\right) = \lim_{s\to 0}\frac{20}{0.1s+1} = 20$$

$$B = \lim_{s\to -10}\left((0.1s+1)\,G_1(s)\right) = \lim_{s\to -10}\frac{20}{s} = -2$$

$$C = \lim_{s\to -10}\left((0.1s+1)\,G_2(s)\right) = \lim_{s\to -10}\frac{50}{s^2+25} = 0.4$$

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$$G_2 = \frac{50}{(0.1s+1)(s^2+25)} = \frac{C}{0.1s+1} + \frac{Ds+E}{s^2+25} = \frac{C(s^2+25)+(0.1s+1)(Ds+E)}{(0.1s+1)(s^2+25)}$$

$$\Rightarrow \begin{cases} C + 0.1D = 0 \\ D + 0.1E = 0 \\ 25C + E = 50 \end{cases} \Rightarrow D = -4,\ E = 40$$

$$\Rightarrow y(s) = \frac{20}{s} + \frac{-2+0.4}{0.1s+1} + \frac{-4s+40}{s^2+25} \Rightarrow$$

$$y(s) = \frac{20}{s} + \frac{-1.6}{0.1s+1} + \frac{-4s+40}{s^2+25} \Rightarrow$$

$$y(s) = \frac{20}{s} + \frac{-16}{s+10} + \frac{-4s+40}{s^2+25} \Rightarrow$$

$$y(s) = \frac{20}{s} - \frac{16}{s+10} - \frac{4s}{s^2+25} + \frac{8\times5}{s^2+25} \Rightarrow$$

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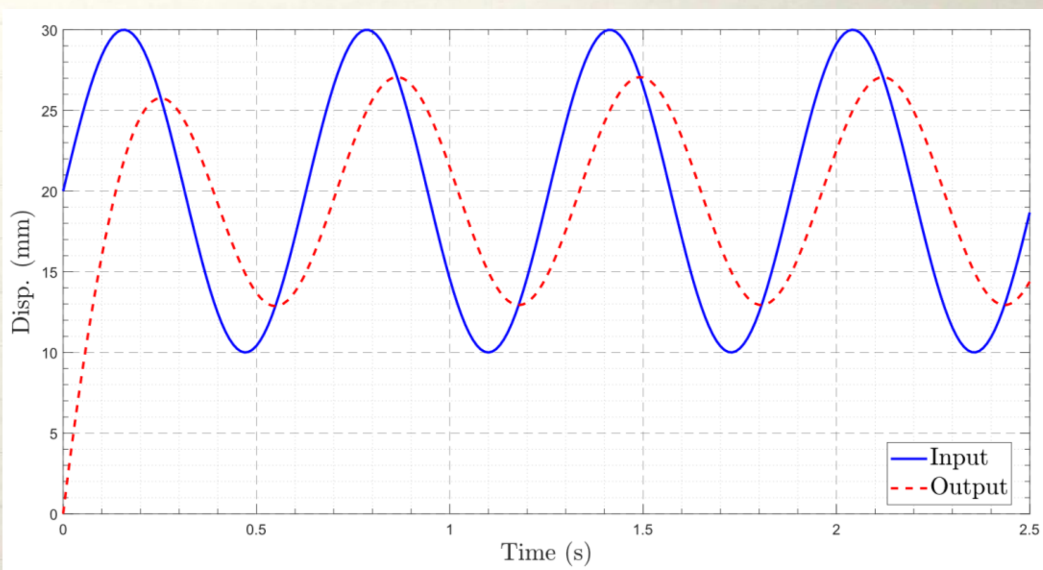
$$y(t) = 20 - 16e^{-10t} - 4\cos(5t) + 8\sin(5t)$$

$$y(t) = 20 - 16e^{-10t} + \sqrt{(-4)^2 + 8^2}\,\cos\left(5t - tg^{-1}\left(\frac{8}{-4}\right)\right)$$

$$\boxed{y(t) = 20 - 16e^{-10t} + 8.94\cos(5t + 2.03\cancel{4})}$$



Question 1d.

Subject:
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1 d. $G(s) = \dfrac{s+4}{s^2+5s+6}$, $y(0) = y'(0) = 0$, $r(t) = 1$

$y(s) = R(s)\, G(s) \Rightarrow y(s) = \dfrac{s+4}{\underbrace{s^2+5s+6}_{CE}} \times \dfrac{1}{s}$

Δ for CE $= \sqrt{b^2 - 4ac} = \sqrt{5^2 - 4\times 6} > 0 \rightarrow$ real roots

$s_1 = -2$, $s_2 = -3$

$\Rightarrow y(s) = \dfrac{A}{(s+2)} + \dfrac{B}{(s+3)} + \dfrac{C}{s}$

$A = \lim\limits_{s \to -2} \left((s+2)\, G(s)\right) = \lim\limits_{s \to -2} \dfrac{s+4}{(s+3)\, s} = -1$

$B = \lim\limits_{s \to -3} \left((s+3)\, G(s)\right) = \lim\limits_{s \to -3} \dfrac{s+4}{(s+2)\, s} = \dfrac{1}{3}$

$C = \lim\limits_{s \to 0} \left(s\, G(s)\right) = \lim\limits_{s \to 0} \dfrac{s+4}{(s+2)(s+3)} = \dfrac{2}{3}$

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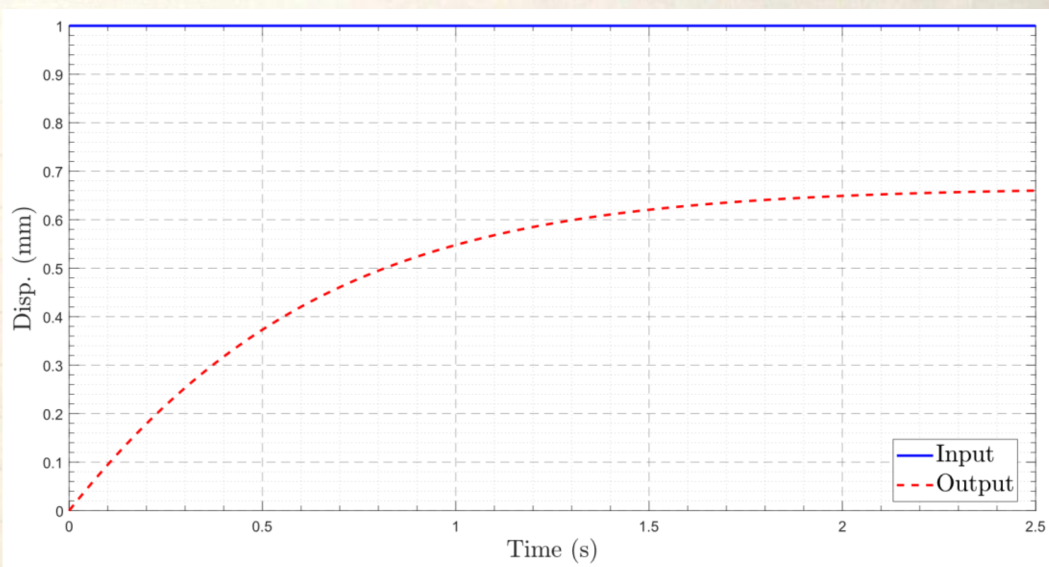
$$\Rightarrow \quad y(s) = \frac{-1}{s+2} + \frac{1}{3(s+3)} + \frac{2}{3s} \quad \Rightarrow$$

$$\boxed{y(t) = -e^{-2t} + \frac{1}{3}e^{-3t} + \frac{2}{3}}$$



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Question 1e.

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1e. $G(s) = \frac{2}{s^2+2s+5}$, $y(0) = y'(0) = 0$, $r(t) = 1$

$$y(s) = G(s)\,R(s) \Rightarrow y(s) = \underbrace{\frac{2}{s^2+2s+5}}_{CE} \times \frac{1}{s}$$

Δ for CE : $\sqrt{b^2-4ac} = \sqrt{4-4\times5} < 0 \rightarrow$ Imaginary Roots!

$$\Rightarrow y(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$A = \lim_{s\to0}\left(s\,G(s)\right) = \lim_{s\to0}\frac{2}{s^2+2s+5} = 0.4$$

$$G(s) = \frac{2}{s(s^2+2s+5)} \equiv \frac{A}{s} + \frac{Bs+C}{s^2+2s+5} \equiv \frac{A(s^2+2s+5)+s(Bs+C)}{s(s^2+2s+5)}$$

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$$\Rightarrow \begin{cases} A + B = 0 \\ 2A + C = 0 \\ 5A = 2 \end{cases} \Rightarrow \quad B = -0.4 \,, \; C = -0.8$$

$$\Rightarrow \quad y(s) = \frac{0.4}{s} + \frac{-0.4s - 0.8}{s^2 + 2s + 5}$$

Completing $\rightarrow$ square

$$y(s) = \frac{0.4}{s} - \left(\frac{0.4s + 0.8}{(s+1)^2 + 3} \right) \Rightarrow$$

$$y(s) = \frac{0.4}{s} - \frac{0.4(s+1) - 0.4 + 0.8}{(s+1)^2 + 3} = \frac{0.4}{s} - \frac{0.4(s+1)}{(s+1)^2 + 3} -$$

$$\frac{0.4}{(s+1)^2 + 3} \Rightarrow$$

$$y(s) = \frac{0.4}{s} - \frac{0.4(s+1)}{(s+1)^2 + 3} - \frac{0.231 \times \sqrt{3}}{(s+1)^2 + 3}$$

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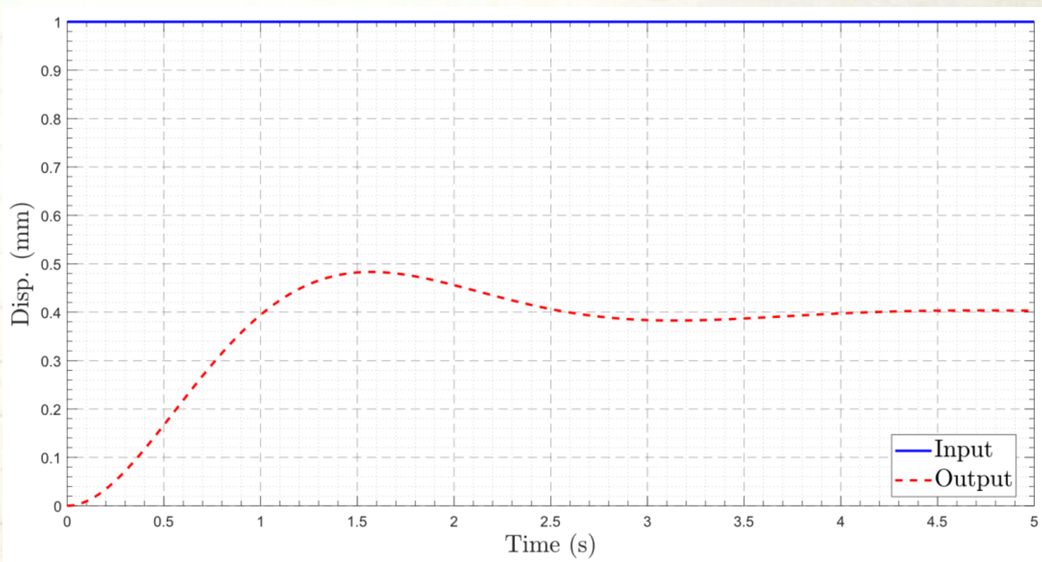
$$\Rightarrow \quad y(t) = 0.4 - 0.4e^{-t}\cos(3t) - 0.231e^{-t}\sin(3t)$$

$$\boxed{y(t) = 0.4 - 0.462\cos(3t - 0.524)}$$



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Question 1f.

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1f. $G(s) = \frac{12}{s^2+8s+12}$, $y(0)=1$, $y'(0)=-1$, $r(t)=3e^{-t}$

$$y(s) = G(s)R(s) \Rightarrow \frac{y(s)}{R(s)} = G(s) = \frac{12}{s^2+8s+12} \xRightarrow{L^{-1}\{\}}$$

$$\ddot{y}(t)+8\dot{y}(t)+12y = 12r(t) \xRightarrow{L\{\}} s^2y(s)-y'(0)-sy(0)+8sy(s)$$

$$-8y(0)+12y(s) = 12\times 3\times\frac{1}{s+1} \Rightarrow$$

$$(s^2+8s+12)\,y(s) - 2s - 15 = \frac{36}{s+1} \Rightarrow$$

$$y(s)\,(s^2+8s+12) = \frac{2s^2+17s+51}{s+1} \Rightarrow y(s) = \frac{2s^2+17s+51}{(s+1)\underbrace{(s^2+8s+12)}_{CE}}$$

Δ for CE: $\sqrt{b^2-4ac} = \sqrt{8^2-4\times 12} > 0 \rightarrow$ Real Roots ✓

$$y(s) = \frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+6}$$

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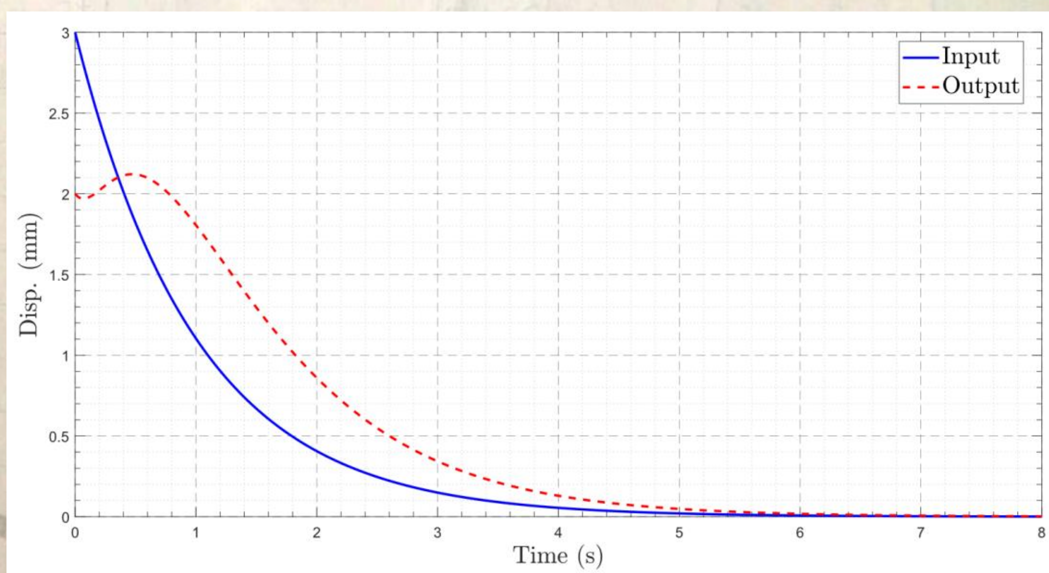
$$A = \lim_{s \to -1} \left((s+1)\, G(s) \right) = \lim_{s \to -1} \left[\frac{2s^2+17s+51}{(s+2)(s+6)} \right] = \frac{36}{5} = 7.2$$

$$B = \lim_{s \to 2} (s+2)\, G(s) = \lim_{s \to -2} \frac{2s^2+17s+51}{(s+1)(s+6)} = \frac{25}{-4} = -6.25$$

$$C = \lim_{s \to -6} (s+6)\, G(s) = \lim_{s \to -6} \frac{2s^2+17s+51}{(s+2)(s+1)} = \frac{21}{20} = 1.05$$

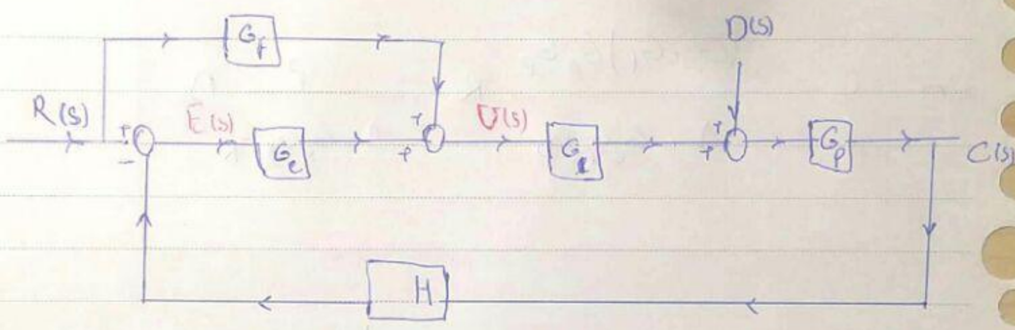
$$\Rightarrow \quad y(s) = \frac{7.2}{s+1} + \frac{1.05}{s+6} - \frac{6.25}{s+2} \quad \Rightarrow$$

$$\boxed{y(t) = 7.2\, e^{-t} + 1.05\, e^{-6t} - 6.25\, e^{-2t}}$$



Question 2a.

2a.



① $U = E \times G_c + R\,G_f$

② $E = R - C \times H$

③ $C = U \times G_l \times G_p + D \times G_p$

①, ② $\rightarrow$ $U = (R - CH)\,G_c + R\,G_f$ ④

③, ④ $\rightarrow$ $C = \left((R - CH)\,G_c + R\,G_p\right) G_l\,G_p + D\,G_p \Rightarrow$

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$$C \times (1 + G_c G_l G_p) = (G_c + G_f) G_l G_p R + D G_p$$

$$\Rightarrow \boxed{C = \frac{(G_c + G_f) G_l G_p}{1 + G_c G_l G_p} R + \frac{G_p}{1 + G_c G_l G_p} D}$$

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Question 2b.

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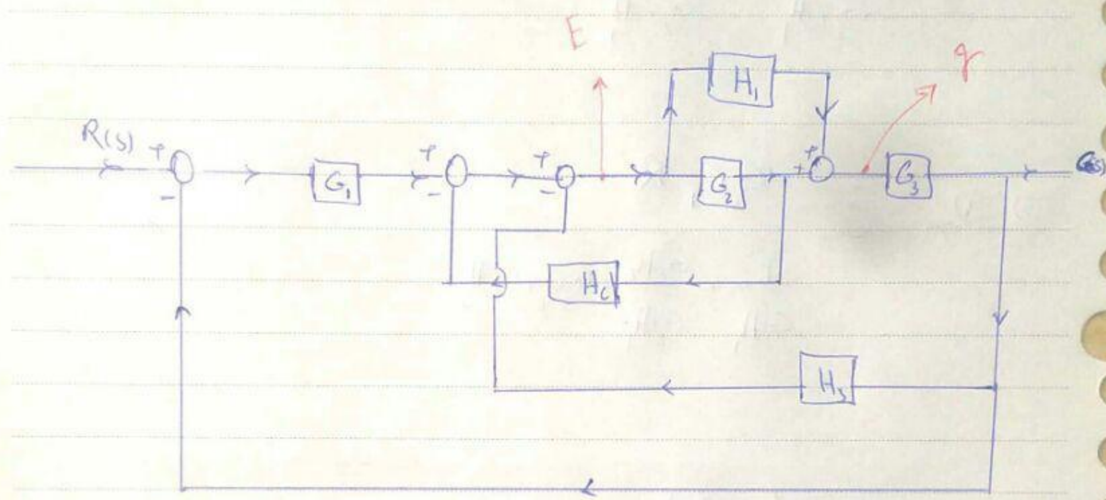
2b.



① $q = E\,(G_2 + H_1)$

② $E = G_1 R - G_2 H_2 E - q G_3 G_1 - q G_3 H_3$

③ $C = G_3\, q$

①,② $\longrightarrow$ $E = \dfrac{q}{G_2 + H_1}$ or $\dfrac{q}{G_2 + H_1} = G_1 R - G_2 H_2 \dfrac{q}{G_2 + H_1} - q G_3 G_1 - q G_3 H_3$

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$$\Rightarrow q\left(\frac{1}{G_2+H_1}+\frac{G_2H_2}{G_2+H_1}+G_3G_1+G_3H_3\right)=G_1R \qquad (4)$$

$$(3),(4) \rightarrow C=\frac{G_3G_1R}{\frac{1}{G_2+H_1}+\frac{G_2H_2}{G_2+H_1}+G_3G_1+G_3H_3} \Rightarrow$$

$$\boxed{C=\frac{G_3G_1(G_2+H_1)}{1+G_2H_2+G_3G_1(G_2+H_1)+G_3H_3(G_2+H_1)}\,R}$$

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