MECH 364
MIDTERM EXAMINATION # 3 : SOLUTION

Note Title

Q1)

a) AN ISOLATOR WORKS BY REDUCING THE NATURAL FREQUENCY OF THE COMBINED SYSTEM (ISOLATOR+ORIGINAL SYSTEM) WITH RESPECT TO THE EXTERNAL FORCING FREQUENCY.

b) GIVEN: $m_u e = 1$ kg-m

$800 \text{ rpm} \le \omega \le 200 \text{ rpm}$ (OPERATING RANGE)

$\Rightarrow \dfrac{800 \times 2\pi}{60} \le \omega \le \dfrac{200 \times 2\pi}{60}$

$\Rightarrow 83.776 \text{ rad/s} \le \omega \le 209.44 \text{ rad/s}$

$\zeta = 0.08$; TOTAL MASS $m = 200$ kg

REQUIRED : $F_T \le 4000\text{N}$ IN THE OPERATING RANGE

$$TR = \frac{F_T}{F} = \frac{F_T}{m_u e \omega^2} = \sqrt{\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}} \quad \text{— (1)}$$

STEP 1: SOLVE FOR r GIVEN $\zeta = 0.08$, & CHOOSING $\omega = 83.776$ rad/s

SQUARING BOTH SIDES OF (1)

$$(TR)^2 = \frac{1 + 4\zeta^2 r^2}{(1-r^2)^2 + 4\zeta^2 r^2}$$

INTRODUCING $x = r^2$ IN THE ABOVE & RE-ARRANGING

$$(TR)^2 \left[(1-x)^2 + 4\zeta^2 x \right] = 1 + 4\zeta^2 x$$

$$\Rightarrow \quad x^2 + \left[4\zeta^2 - \frac{4\zeta^2}{(TR)^2} - 2\right]x + 1 - \frac{1}{(TR)^2} = 0 \qquad -(2)$$

WHERE WE KNOW

$$TR = \frac{F_T}{F} = \frac{4000}{m_u e \omega^2} = \frac{4000}{1\times(83.776)^2}$$

$$= 0.5699$$

$$\zeta = 0.08$$

$$4\zeta^2 - \frac{4\zeta^2}{(TR)^2} - 2 = -2.0193$$

$$1 - \frac{1}{(TR)^2} = -2.0786$$

$$\therefore (2) \Rightarrow x^2 - 2.0193x - 2.0786 = 0$$

$$\Rightarrow x = r^2 = \frac{2.0193 \pm \sqrt{(2.0193)^2 + 4\times 2.0786}}{2}$$

$$= 2.7698 \text{ (OR) } -1.5009 \text{ (DISCARD THIS ROOT)}$$

$$\Rightarrow r = 1.6643 \Rightarrow \frac{\omega}{\omega_N} = 1.6643 \Rightarrow \omega_N = \sqrt{\frac{K}{M}} = \frac{\omega}{1.6643}$$

$$\Rightarrow K = \left(\frac{\omega}{1.6643}\right)^2 \times M = \left(\frac{83.776}{1.6643}\right)^2 \times 200 = 5.0676\times10^5 \text{ N/m}$$

$$\Rightarrow \boxed{K = 506.76 \text{ KN/m}}$$

$$\omega_N = \sqrt{\frac{K}{M}} = \sqrt{\frac{5.0676\times10^5}{200}} = 50.3371 \text{ rad/s}$$

STEP 2 : CHECK AT $\omega = 209.44$ rad/s

$$r = \frac{\omega}{\omega_N} = \frac{209.44}{50.3371} = 4.1608$$

with $\zeta = 0.08$

$$TR = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}} = 0.0736 = \frac{F_T}{m_u e \omega^2}$$

$$\Rightarrow F_T = 0.0736 \times m_u e \omega^2 = 0.0736 \times 1 \times (209.44)^2$$

$$= 3227.9\ N < 4000\ N$$ SO OUR CHOICE OF 'K' IS GOOD

OK.

STEP 3: CHOOSE $K = 506.76$ kN/m

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c) SOME DAMPING IS ALWAYS DESIRED IN AN ISOLATOR TO AVOID LARGE RESONANT VIBRATIONS. DAMPING HOWEVER INCREASES TR SLIGHTLY IN THE REGION $r > \sqrt{2}$. IN THIS REGION LESS DAMPING IS DESIRED. THUS, SOME AMOUNT OF DAMPING IS ACCEPTABLE AND EVEN DESIRABLE AT RESONANCE.

— THE END —