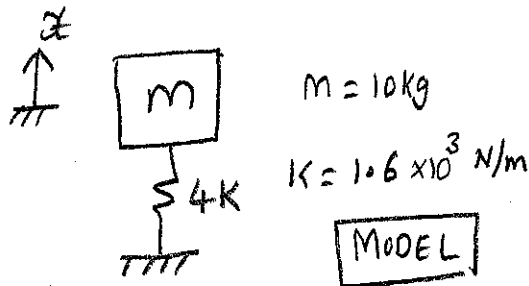


MIOTERM EXAMINATION 2: SOLUTIONS

a)

MODEL & FREE BODY DIAGRAMS

VERTICAL VIBRATION

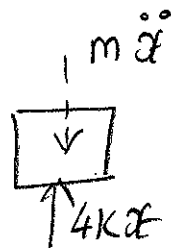


CO-ORDINATE: $x \uparrow +ve$

KEY STEP.

⇒ ALL FOUR SPRINGS ARE IN PARALLEL

FBD



EQUATION OF MOTION:

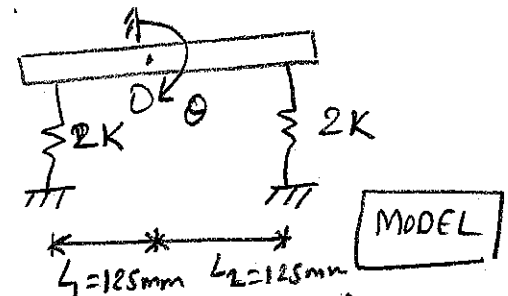
$$\uparrow \sum F_x = 0 \Rightarrow m\ddot{x} + 4Kx = 0$$

+ve

NATURAL FREQUENCY

$$\omega_n = \sqrt{\frac{4K}{m}} = \sqrt{\frac{4 \times 1.6 \times 10^3}{10}}$$

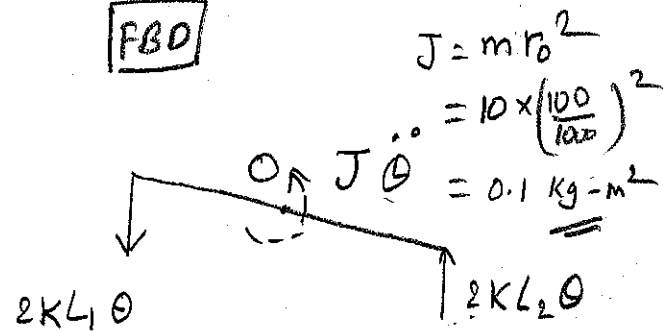
TORSIONAL (TILTING) VIBRATION



CO-ORDINATE: $\theta \downarrow +ve$

SIDE SPRINGS ARE IN PARALLEL

FBD



(SMALL ANGLE θ)

EQUATION OF MOTION

$$\uparrow \sum M_O = 0 \Rightarrow$$

+ve

$$-J\ddot{\theta} - 2KL_1\theta L_1 - 2KL_2\theta L_2 = 0$$

$$\Rightarrow J\ddot{\theta} + (2KL_1^2 + 2KL_2^2)\theta = 0$$

NATURAL FREQUENCY

$$\omega_n = \sqrt{\frac{2KL_1^2 + 2KL_2^2}{J}} = \sqrt{\frac{2 \times 1.6 \times (0.125)^2 \times 2}{0.1}}$$

$L_1 = L_2 = 125 \text{ mm}$

NATURAL FREQUENCIES ARE

$$\omega_n = \sqrt{\frac{4 \times 1.6 \times 10^3}{10}} = 25.3 \text{ rad/s} \text{ for VERTICAL VIBRATION}$$

AND

$$\omega_n = \sqrt{\frac{2 \times 1.6 \times (0.125)^2 \times 2}{0.1}} = 31.62 \text{ rad/s}$$

WE ASSUMED THAT GRAVITY IS NOT IMPORTANT AND THE CENTRE OF GRAVITY OF THE MOTOR IS IN THE MIDPLANE. OTHERWISE, 4 EQUAL SPRINGS CAN'T MAINTAIN THE MOTOR IN A PERFECTLY LEVELLED CONFIGURATION. THIS MEANS THAT BOTH VERTICAL & TORSIONAL MOTIONS ARE COUPLED.

b) PLACE ADDITIONAL MASS FARTHER FROM SHAFT AXIS, IDEALLY ANYWHERE ON THE FOUR CORNERS. THIS WOULD ENSURE THAT THE MASS CHANGE HAS MAXIMUM INFLUENCE ON KINETIC ENERGY. SIMILARLY INCREASE THE STIFFNESS OF 4 SPRINGS ON THE 4 CORNERS TO INCREASE NATURAL FREQUENCIES.

c) HERE WE WANT THE MASS AT THE CENTRE, ON THE SHAFT AXIS. THIS ENSURES THAT J REMAINS THE SAME AS BEFORE BUT M CHANGES! THUS VERTICAL VIBRATION'S NATURAL FREQUENCY WILL DECREASE WHILE TORSIONAL VIBRATION'S NATURAL FREQUENCY REMAINS UNCHANGED.

NOTE: IN b) & c) THE DESIGN CHANGE CAN CAUSE COUPLED VERTICAL-TORSIONAL MOTION.

—THE END—