

FINAL EXAMINATION FOR MECH 364 MECHANICAL VIBRATIONS

8TH DECEMBER 2011

Time: 2 hrs. 30 mts. Max. Available Mark: 60

READ THE FOLLOWING INSTRUCTIONS CAREFULLY

- 1. This exam consists of 5 pages including this page
- 2. Please write your name and student number on the answer sheets
- 3. ANSWER ALL QUESTIONS
- 4. One letter-sized formula sheet is allowed

The space below is intentionally left blank. Continue onto the next page for the exam questions.

MARKS ALLOCATED FOR EACH SEGMENT ARE IN THE MAR GINS. PAGE NUMBERS ARE ON TOP RIGHT.

Question 1 Concepts tested: FBD, Initial Conditions, Equivalent Systems, Free Vibration Response.

(a) A container of mass m is being lowered from a helicopter at a constant downward velocity of V when the helicopter is hovering above the target point of delivery. To avoid a mishap the weight is suddenly stopped by the operator at point O in Figure(1). Assuming that all cables have an area of cross-section A, formulate the equations of motion for the vertical oscillations of the mass. Note that the axial stiffness of a cable made from a material of modulus E, of length L and area of cross-section A is $k = \frac{AE}{L}$.

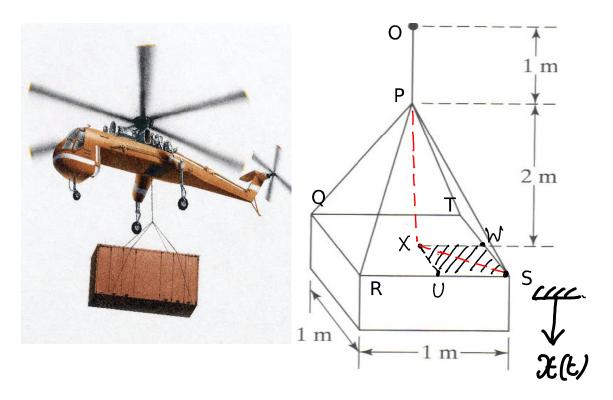


Figure 1: Figure for question 1. Point O originates at the helicopter.

- (b) Find the maximum displacement amplitude of vertical oscillations of the **(6 marks)** container for the geometric parameters shown in the left side of Figure(1). Assume E = 210 GPa, M = 200 kg, V = 1 m/s and the diameter of wires is 5 cm.
- (b) List at least two limitations of the above modelling approach. (2 marks)

QUESTION 1

MODEL EACH CABLE AS A SPRING OF CONSTANT K= AE

WE CAN SEE THAT THE SPRINGS ASSOCIATED WITH

THE CABLES PQ, PR, PS AND PT ARE IN PARALLEL

AS THEY ALL UNDERGO SAME RELATIVE DISPLACEMENT.

THE RESULTANT EQUIVALENT SPRING IS CONNECTED

IN SERIES WITH THE SPRING ASSOCIATED WITH

THE CABLE OP. WE MAKE USE OF THESE FACTS.

WE SELECT VERTICALLY DOWNWARD DISPLACEMENT

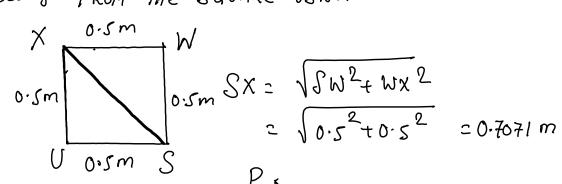
OF SEE FIG. 1) AS OUR CO-ORDINATE.

ON PAGE # 2)

THE ANGLE MADE BY SPRINGS PQ, PR, PS, PT WITH

RESPECT TO THE PISPLACEMENT CO-DROINATE, Z, IS OBTAINED

AS FOLLOWS. FROM THE SQUARE USWX



FROM \triangle PXS $ton \Theta = \frac{XS}{PX} = \frac{0.7071}{2}$ 2m

=> Coso=0.9428 & PS= PX/coso=2.1213 m

 α)

NOW, THE EQUIVALENT SPRING ASSOCIATED WITH THE 4 INCLINED CABLES CAN BE OBTAINED AS

USING THE FOLLOWING FIGURE WILL

P. E. = $\frac{1}{2}$ Kap $x^2 = \frac{1}{2}$ K (xCUSO) x $x = \frac{1}{2}$ K COS 2 O x = x Keg x KCOS 2 O x

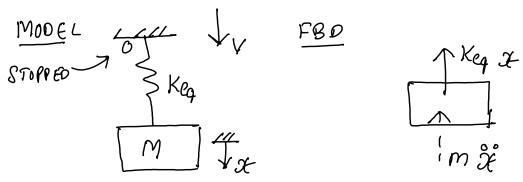
POR EACH INCLINED CABLE MODELLED AS A SPRING OF GNSTANT K = AE/L = AE = AE N/m

NOW THE RESULTANT SPRING ASSOCIATED WITH THE 4 INCLINED CABLES IS IN SERIES WITH KNETTICAL WHERE $K_{VERTICAL} = \frac{AE}{OP} = \frac{AE}{1} = AE$ N/m

THE EQUIVALENT SPRING OF 4 INCLINED SPRINGS IN SERIES WITH KVERTICAL IS

$$= \frac{1}{160} = \frac{1}{4 \times 60^{2}0} + \frac{1}{160} = \frac{1}{4 \times 60^{2}0} + \frac{1}{4 \times 60^{2}0}$$

WITH THE ABOVE WE CAN SET-UP THE MODEL.



I IS MEASURED WITH RESPECT TO EQUILIBRIUM WHERE GRAVITATIONAL WEIGHT MG CANCELS STATIC PEFLECTION IN SPRING " Keg.

EQUATION OF MOTION: 15 Fx =0 (DIALEMBERT)

$$= \int -m x^{2} - ke_{g} x^{2} = 0$$

$$= \int m x^{2} + ke_{g} x^{2} = 0.6263 A \in 0$$

$$= \int -m x^{2} + ke_{g} x^{2} = 0.6263 A \in 0$$

4 MARKS

b) | WHEN '0' 18 STOPPED

2 MARKS THE VIBRATION RESPONSE IS OBTAINED BY SOLVING 1 X= In+Xp= In = AGSWAT+BSINWAT-9

2 MARKS is
$$\mathcal{X} = \mathcal{X}_h = \frac{V}{\omega_n} \sin \omega_n t$$
; $\omega_{n=1} = \sqrt{\frac{\kappa_{eq}}{M}} = \sqrt{\frac{0.6863}{M}} \in \mathcal{X}_h = \frac{V}{M}$

NOW MAXIMUM DISPLACEMENT AMPLITUDE IS

$$\frac{V}{\omega_n}$$

GIVEN
$$V=1mlS$$
, $E=210\times10^9$ Pa,
 $d=0iAmeter$ of cable = 5×10^{-2} m
 $A=Area$ of cls of cable = $TTd^2=TTx(5\times10^{-2})^2$
 $=6.002m^2$

$$K_{qq} = 0.6269 \text{ AC} = 6.6263 \times 0.002 \times 2.10 \times 109$$

= 2.5824 x 108 N/m

$$W_n = \sqrt{\frac{k_{eq}}{M}} = \sqrt{\frac{2.5824 \times 108}{200}} = 1.1363 \times 10^3 \text{ rad/s}$$

$$2 man = \frac{\sqrt{3}}{4m} = \frac{1}{1.1363 \times 10^3} = 8.8 \times 10^{-4} m = 0.88 mm$$

2 MARKS

C)

2 marks

LIMITATIONS

- (1) CONTAINER IS A POINT MASS I.E. NO MASS MOMENT OF INERTIA
- (2) MOTION OF HELICOPTER IGNORED
- (3) WIND RESISTANCE/ EXCITATION I GNORED
- (4) CABLES ARE RIGIOLY ATTACHED TO GACH OTHER AND TOTHE MASS.



Question 2 Concepts tested: Isolation System Design, Vibration Absorbers

- a) Explain in two sentences or less how isolation systems and absorber systems (4 marks) work.
- b) An electronic control unit of mass m=2 kg is located in the stores pod of a space mission shown below in Figure(2). It needs to be isolated from vibration inputs originating at the base in the form of acceleration \ddot{x} as shown. The acceleration input \ddot{x} is a broad band random vibration within the range 10 Hz and 1 kHz. It is required to limit the maximum displacement transmission ratio around resonance to 2, and the transmitted displacement ratio not to exceed 0.2 in the entire input frequency range. Design the isolator stiffness and damping. Which other design criterion would you consider in addition to the above?

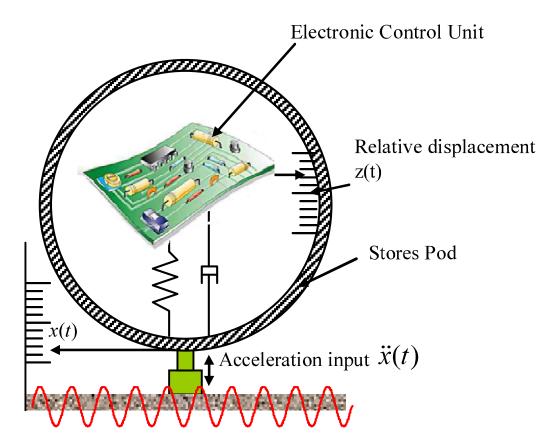


Figure 2: Figure for Question 2.

b) What is the relation between displacement, velocity, and acceleration transmission ratios at a *steady* operating frequency ω ?

ISOLATION SYSTEMS COMPRISE A SPRING-DAMPER ARRANGEMENT (RESILIENT MATERIAL SUCH AS RUBBER) INSERTED IN THE VIBRATION TRANSMISSION PATH SUCH THAT THE NATURAL PREQUENCY OF THE SYSTEM WITH ISOLATOR IS WELL BELOW_ THE FORCING FREQUENCY. THIS LEADS TO REDUCED TRANSMITTED VIBRATION OR ISOLATION. THE DYNAMICS OF THE MAIN JYSTET IS SCOWED DOWN IN RELATION TO THE EXCITATION FREQUENCY AT THE EXPENSE OF STIFFNESS REDUCTION.

A VIBRATION ABSORBER IS AN AUXILARY SYSTEM ATTACHED TO THE MAIN SYSTEM SUCH THAT AT TUNED FREQUENCIES, THE ADSORBER ENERTS A LARGE COUNTER VIBRATING FORCE 2 MARKS AT THE POINT OF ATTACHMENT. VIBRATION REPUCTION IN
THE MAIN SYSTEM IS ACHIEVED AT THE EXPENSE OF (1) INTRODUCING TWO APOLTIONAL RESUMANCES, AND (Si) SUBJECTING THE ABSORBER SPRING-MASS UNIT TO UNDERGO SUSTAINED RESONANT OSEILLA TIONS-

GIVEN m = 2 kg; 10 HZ < W < 1000 HZ => 20TT < W < 2000 TT rad/s

TR = TR_d =
$$\sqrt{1+(2\pi r)^2}$$
 $r = \frac{\omega}{\omega_n}$

$$[TR]_{man} = [TR]_{r \approx 1} = \sqrt{\frac{1+4\pi^2}{4\pi^2}} = 2$$

2 MARKS

$$=) 1+44=4 (44) \Rightarrow 4=\frac{1}{\sqrt{12}}$$

TR = 0.2 IN THE RANGE 20TT $\angle \omega \angle 2000$ Tr rad/s START ω 1774 $\omega = 20$ Tr rad/s

TR = 0.2 =
$$\frac{1 + (2\pi r)^2}{(1-r^2)^2 + (2\pi r)^2}$$
; CAU $r^2 = 24$

SQUARING BOTTH SIDES & RE-ARRANGING

=)
$$\left[\left(1 - \chi \right)^2 + 4 \zeta^2 \chi \right] \left(TR \right)^2 = 1 + 4 \zeta^2 \chi$$

=)
$$(TR)^2 x^2 + x [4 a^2 (TR)^2 - 2(TR)^2 - 4a^2] + (TR)^2 - 1 = 0$$

$$=) \times = 0.4 \pm \sqrt{(0.4)^2 + 4 \times 0.04 \times 0.96} = 12 \text{ or } -2$$

$$2 \times 0.04$$
|GNORE

$$\mathcal{X}=f^2=12=)$$
 $f=\frac{\omega}{\omega_n}=\sqrt{12}=\omega_n=\frac{\omega}{\sqrt{12}}$

$$\Rightarrow) \quad \omega_{n2} \quad \int \frac{16}{m} = \frac{\omega}{r} = \frac{20\pi}{\sqrt{12}} \quad \text{ bad/s}$$

$$\Rightarrow K: m \left(\frac{20\pi}{\sqrt{12}}\right)^2 = 2 \left(\frac{20\pi}{\sqrt{12}}\right)^2$$

$$68 \text{ K} = 687-9736 \text{ N/m}$$

$$61 \text{ VEN } 6 = \frac{C}{2 \text{ MWn}} = \frac{1}{\sqrt{12}}$$

$$=\frac{2}{\sqrt{12}}\sqrt{657-9736\times2}=\frac{20.944}{m}$$

2MARKS

CHECK: FOR
$$\omega = 2000\pi$$
 rad/s & $\mathcal{L} = \frac{1}{\sqrt{12}}$, $\omega_n = \frac{20\pi}{\sqrt{12}}$ rad/s
$$V = \frac{\omega}{\omega_n} = \frac{2000\pi}{20\pi} \sqrt{12} = 346.41$$

$$TR_{2} \sqrt{\frac{1+(2\pi r)^{2}}{(1-r^{2})^{2}+(2\pi r)^{2}}} = 0.0017 < 0.2 0.4$$

2 MARKS PERMITTED RELATIVE DISPLACEMENT Z(t).

C) IN A STEADY HARMONIC MOTION AT EREQUENCY W rad/s

XVEC = WX; Xaccin = - W2 X

YUEL = WY; Yaccin = - W2Y

[RECALL "SHAKYTABLE" LAB]

TRd = /Y/x/

$$TR_{VEC} = \left| \frac{\omega y}{\omega x} \right| = \left| \frac{y}{x} \right| = TR_{el}$$

TR_{VEC} =
$$\left|\frac{\omega y}{\omega x}\right| = \left|\frac{y}{x}\right| = TR_{el}$$

TR_{accur} = $\left|\frac{-\omega^2 y}{-\omega^2 x}\right| = \left|\frac{y}{x}\right| = TR_{el}$

Question 3 Concepts tested: Kinematics, FBD, Forced Vibration, DMF, Shaky Table Laboratory

Helpful hint: If you are unable to find the accelerations within 15 minutes in part a) you may proceed to part b).

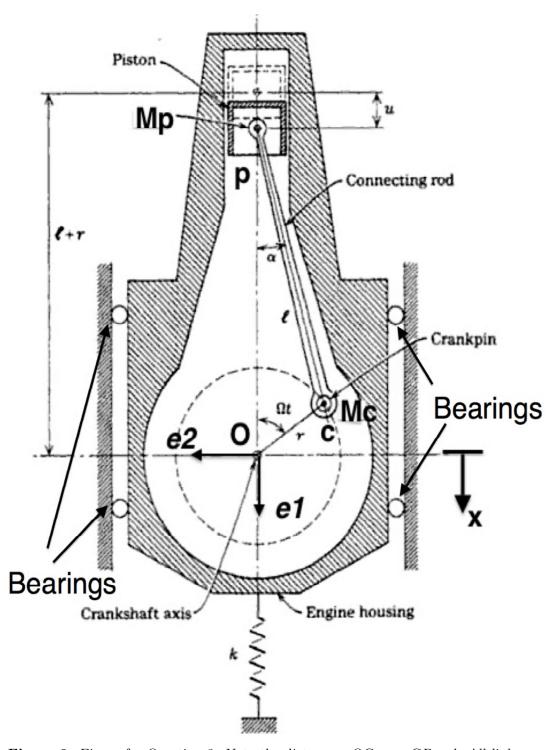


Figure 3: Figure for Question 3. Note the distances: OC = r, CP = l. All links are rigid and Ω is constant.

- (a) A single cylinder engine with unbalanced masses lumped at the crank pin (8 marks) (M_c) and piston head (M_p) is shown in Figure(3). Using kinematics show that the absolute inertial acceleration of M_p is $\mathbf{a}_p \approx \ddot{x}\mathbf{e}_1 + \Omega^2 r \left[\cos\Omega t + \frac{r}{l}\cos2\Omega t\right]\mathbf{e}_1$ and that of M_c is $\mathbf{a}_c = \ddot{x}\mathbf{e}_1 + r\Omega^2\cos\Omega t\mathbf{e}_1 + r\Omega^2\sin\Omega t\mathbf{e}_2$, Ω is constant and +ve in clockwise direction. The unit vectors \mathbf{e}_1 and \mathbf{e}_2 are as shown in Figure(3). You may find the identity $\sqrt{1-x} \approx 1 \frac{1}{2}x$ useful in approximating the acceleration \mathbf{a}_P . You may also find the trigonometric identity $2\sin\theta\cos\theta = \sin2\theta$ useful.
- (b) Using the accelerations from part a) construct the Free Body Diagram (10 marks) (FBD) for the engine housing (of mass M_e) and show that the governing equation of motion for vertical vibrations is $M\ddot{x}+kx=-(M_p+M_c)r\Omega^2\cos\Omega t-M_p\frac{r^2}{l}\Omega^2\cos2\Omega t$ where $M=M_e+M_c+M_p$. Show that the steady state displacement amplitude of the forced vibration response, ignoring the homogeneous part, is $x(t)=-0.0029\cos\Omega t+5.36\times 10^{-4}\cos2\Omega t$ m for the parameters: r=0.2 m, l=0.6 m, $\Omega=600$ rpm, $M_p=3.2$ kg, $M_c=0.9$ kg, M=227 kg and $k=2\times 10^6$ N/m. Can you explain the signs of displacement amplitudes in the above response? What is the influence of the ratio $\frac{r}{l}$ on the forces exerted by the unbalance masses.
- (c) What is the **total** horizontal reaction force on the bearings between the **(2 marks)** engine housing and the rest of the body.

Parting thoughts

- 1. I enjoyed explaining this challenging but practically important material to you at 8AM every week!
- 2. Notice how difficult it is for an engineer to *intuitively* guess the forcing frequencies in Question 3 to avoid resonance for single cylinder engine, unless one knows how to *apply* kinematics and *basic* mathematics. The unexpected forcing at 2Ω is counter-intuitive! So are practical *engineering vibration* problems!!

ALL THE VERY BEST IN YOUR FUTURE ENDEAVOURS!

We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.

-TS Eliot in Little Gidding.

a)

ACCELERATION OF MC

$$x = \frac{\sqrt{2}}{2} |\underline{e}_3| = |\underline{e}_4| = 1$$

DISPLACEMENT WITH RESPECT TO A FIXED OBSERVER

$$\frac{\dot{\Gamma}_{c} = \dot{\Gamma}_{0} + \dot{\Gamma}_{c/o} = \mathcal{X} \in [+ \Upsilon e_{3}]}{\dot{\Gamma}_{c} = \mathring{\mathcal{X}} \in [+ \Upsilon \mathfrak{L} \in 4]}$$

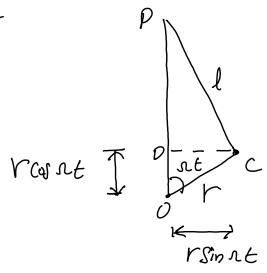
$$\frac{\dot{\Gamma}_{c} = \mathring{\mathcal{X}} \in [+ \Upsilon \mathfrak{L} \in 4]}{\dot{\Gamma}_{c} = \dot{\Gamma}_{c} = \mathring{\mathcal{X}} \in [- \Upsilon \mathfrak{L}^{2} e_{3}]}$$

3 MARKS

ACCELERATION OF Mp:

DISPLACEMENT: Ip = to+ IPlo

& C1 IS the DOMNMAKOS!



$$= r \cos nt + \sqrt{l^2 - r^2 \sin^2 nt}$$

$$= r \cos nt + l \sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2 nt}$$

USING
$$\sqrt{1-\left(\frac{r}{\lambda}\right)^2 \sin^2 nt} \approx 1-\frac{1}{2}\left(\frac{r}{\lambda}\right)^2 \sin^2 nt$$

(2) IN (1) GIVES

VELOCITY: Ip = Delt Fresin rt e, + resin rt cort

Note: 2 Sin St Cos St = Sin 2st (TRIG. 10ENTITY)

ACCELERATION: ap = ip

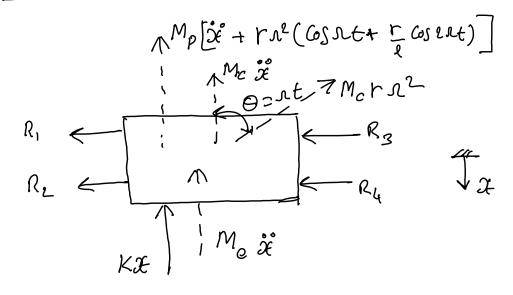
Snarks

$$\left[Q_{p} \approx \ddot{\mathcal{X}} \leq_{1} + r \mathcal{N}^{2} \left[Cosne + \frac{r}{r} Cos(ene)\right]\right]$$

NOTE: THE APPROXIMATION ARISES FROM TRUNCATING THE SERIES EXPANSION OF \[\left(\frac{r}{\tau}\right)^2 \Sin^2 r \tau \ \ FOR \ SMALL \left(\frac{r}{\tau}\right)

b)

FREE BODY DIAGRAM FOR Me



5marks

Note: GRAVITATIONAL WELGHT CANCELS STATIC

DEFLECTION IN THE SPRING AT STATIC EQUILIBRIUM.

$$-M_{c}\ddot{x} - M_{c}\ddot{x} + r n^{2} \left[\cos x + \frac{r}{2} \cos x + \frac{r}{2} \cos x + \frac{r}{2} \cos x \right] - M_{c} \ddot{x}$$

$$-M_{c} r n^{2} \cos x + - \kappa x = 0$$

=)
$$\left(M_{p+me+mc}\right)\tilde{x}_{+kx} = -\left(M_{p+mc}\right)r n^{2} \cos nt$$

 $-M_{p}\frac{r^{2}}{l}n^{2} \cos 2nt$

STEADY STATE DISPLACEMENT:

3marks

THE opposite SIGNS CAN BE EXPLAINED BASED ON THE PACT THAT
$$W_{n}$$
: $\sqrt{\frac{K}{M}} = \sqrt{\frac{2\chi_{106}}{227}} = 93.9 \text{ rad/s}$.

12 600 rpm = 600 x 211 radly = 62.8 radls

(i) [Un > 1] SO RESPONSE DUE TO - (Mp + Mc) Y 12 COS 1 t 15 IN-PHASE = J - VE SIGN FOR DISPLACEMENT, SAME AS
FORCE

2 MARKS

(ii) Wn < 2 1 So RESPONSE DUE to -Mp 1/2 COS ESTE 15 OUT-OF-PHASE => - (-) = +Ve SIGN FOR DUPLACEMENT, MDPOSITE SIGN AS FORCE!! OPPOSITE SIGN AS PORCE!!

SMALLER THE I RATIO SHALLER IS THE CONTRIBUTION OF HIGHER ORDER FORCING TERMS, UNLESS RESONANCE WITH HIGHER ORDER FORCING TERMS TAKES PLACE.

NOTE: DMF CURVE EXPLAINS WHY DISPLACEMENT AT A SLE IS MUCH SMALLER COMPARED TO THAT AT SL

TOTAL HURIZONIAL REACTION = RITRETRS + RL = MCKIlinst

2 710-6 SMILE

of MAX. HORIZONTAL REACTION = 710-6N

2 MARKS

THE GND -HAPPY HOLIDAYS! Strikanth