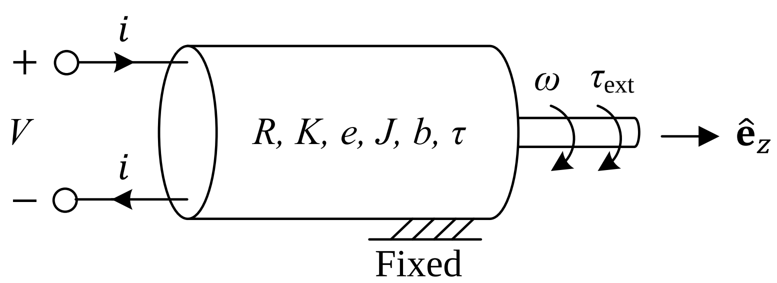
Problem 1 (25 points)

Figure 1 shows a brushed dc motor whose axis of rotation is aligned with a unit vector $\hat{\mathbf{e}}_z$.

<u>External to the motor:</u>
V is the terminal voltage, i is the terminal current, $\omega\,\hat{\mathbf{e}}_z$ is the rotor angular velocity, and $\tau_{\text{ext}}\,\hat{\mathbf{e}}_z$ is the external torque applied to the rotor.

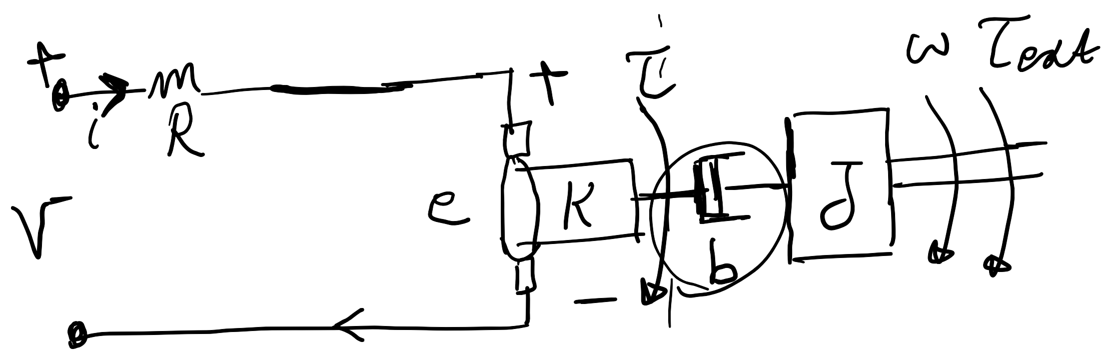
<u>Internal to the motor:</u>
R is the winding resistance, K is the torque constant, e is the back-emf, J is the rotor inertia, b is the mechanical damping between the stator and the rotor, and $\tau\,\hat{\mathbf{e}}_z$ is the torque transmitted from the stator to the rotor. The winding inductance is assumed to be zero ($L = 0$).

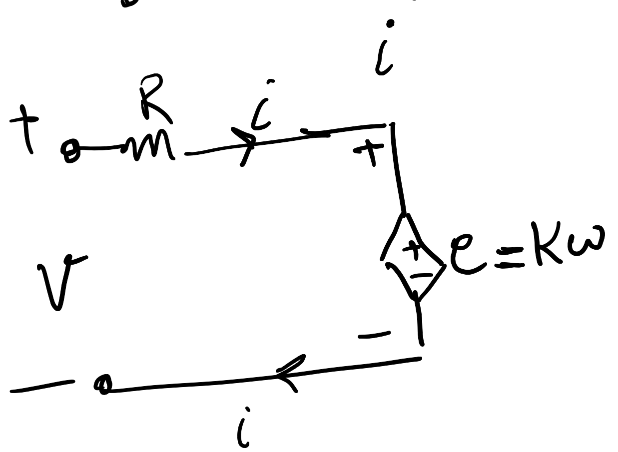


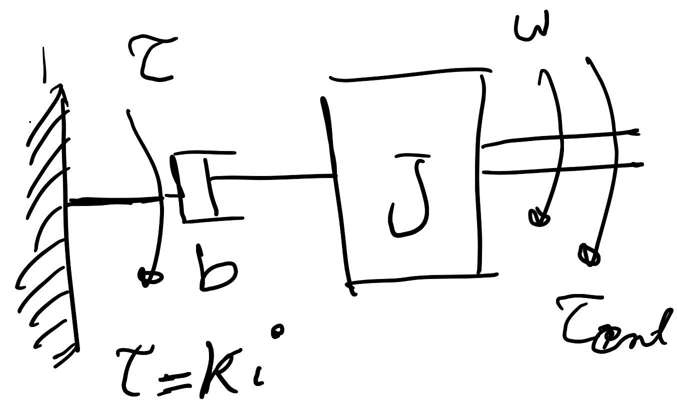
$e = K\omega$

Figure 1: Brushed dc motor for the problem.

(a) (5 pt.) Draw a lumped-parameter model of the motor, where the electrical domain is modeled as a circuit diagram and the mechanical domain is modeled as a free-body diagram. The model should include all the variables and parameters shown in Figure 1.



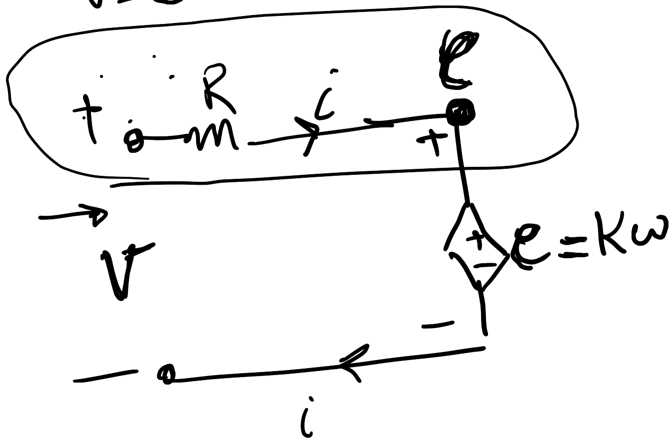


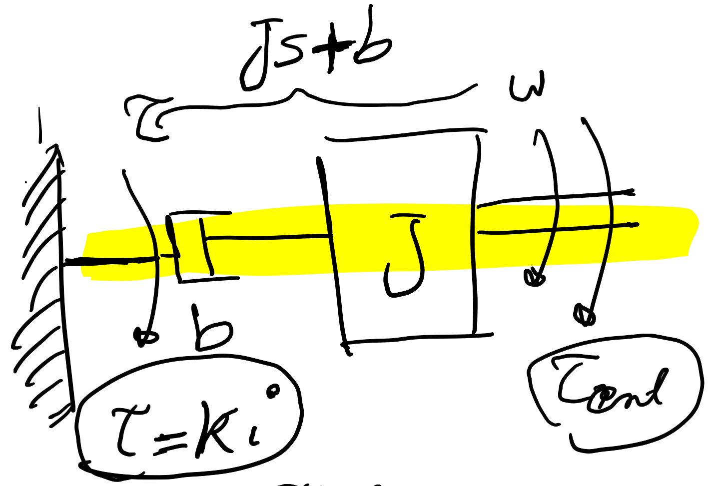


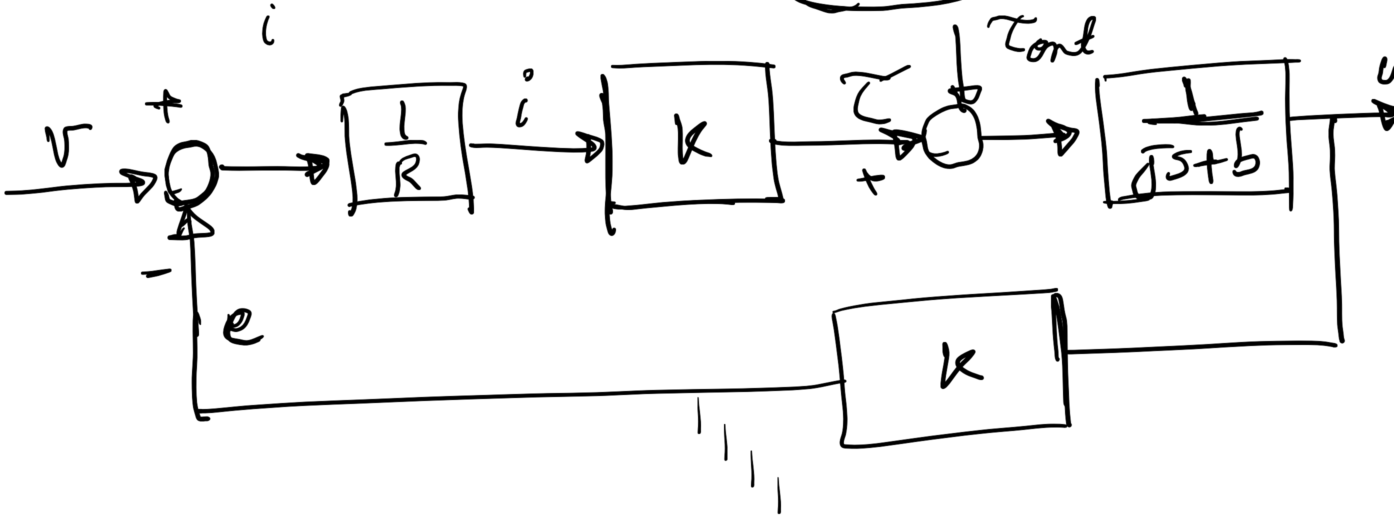
$$b_{TH} = \frac{k^2}{R} \quad \begin{cases} \text{If short-circuit} \rightarrow \text{feel } b + \dfrac{k^2}{R} \\ \text{If open-circuit} \rightarrow \text{feel } b \end{cases}$$

(b) (6 pt.) Suppose a unit-step voltage V is applied across the electrical terminals. Draw the response of the rotor angular speed $\omega(t)$.

$V - e = Ri$







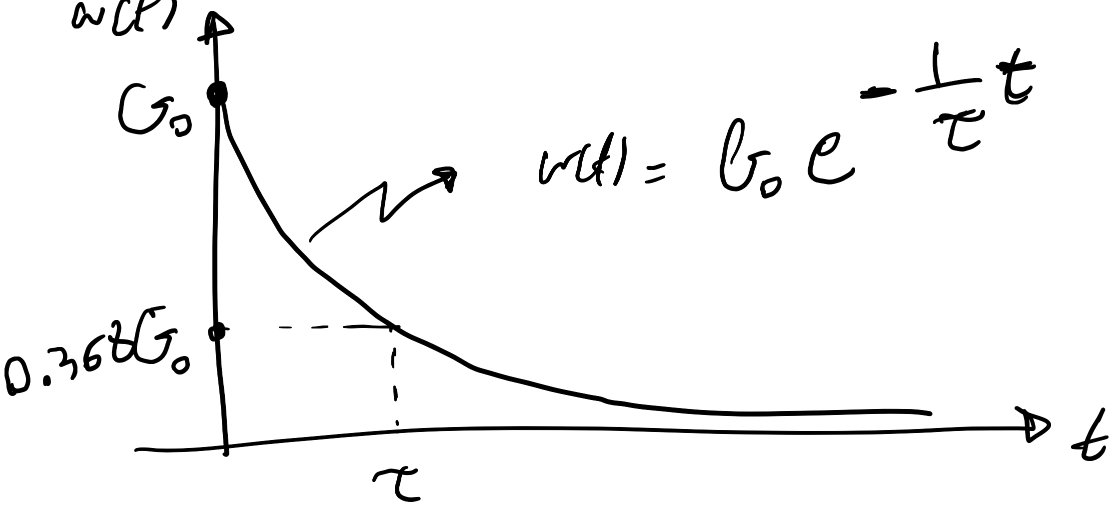
$\tau + \tau_{ond} = (Js+b)\,\omega$

$$\frac{\omega}{V} = \frac{\frac{K}{R}\,\frac{1}{Js+b}}{1 + \frac{K^2}{R}\,\frac{1}{Js+b}} = \frac{\frac{K}{R}}{Js + b + \frac{K^2}{R}}$$

1

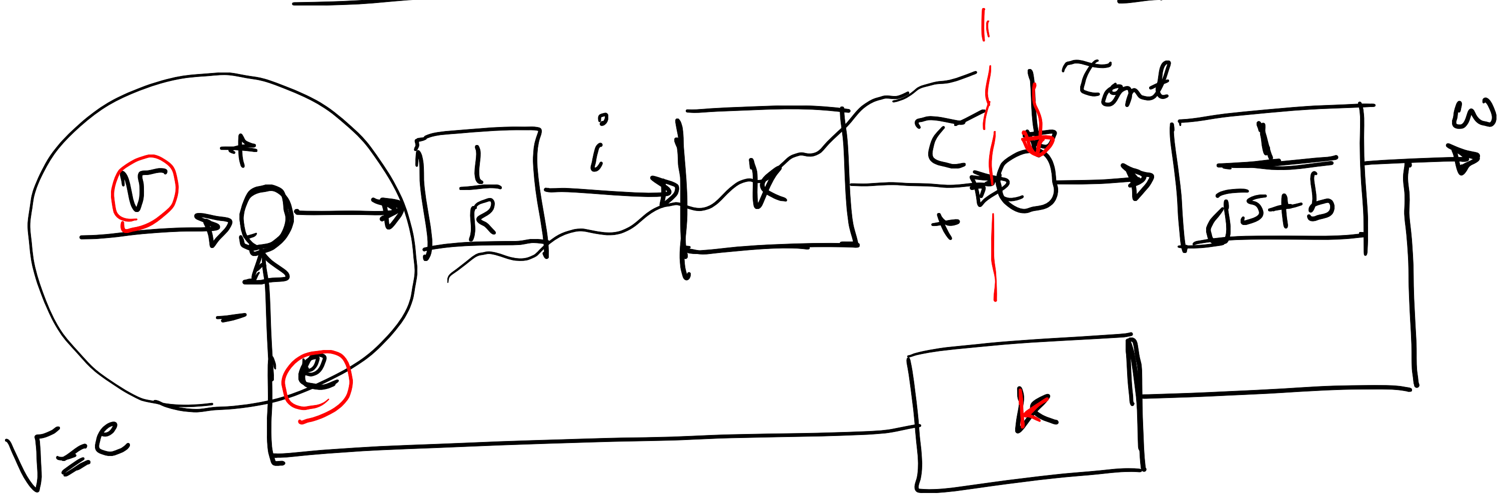
$$= \underbrace{\frac{\frac{K}{R}}{b + \frac{K^2}{R}}}_{G_0} \; \frac{1}{\underbrace{\frac{J}{b+\frac{K^2}{R}}}_{\tau} s + 1}$$

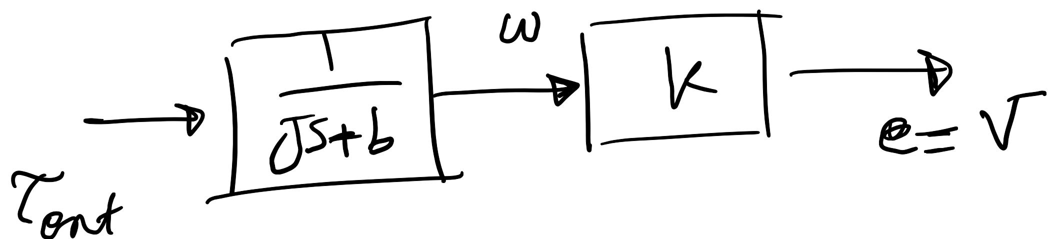
Bode form



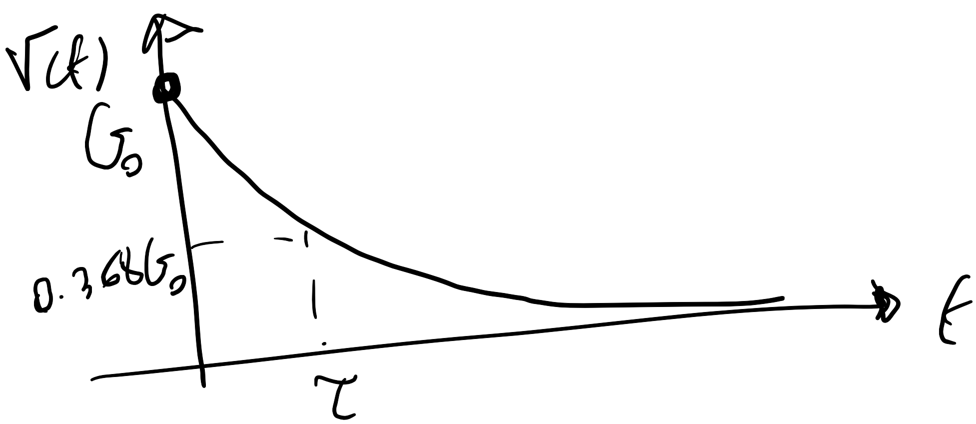
w(t)
G_0
$0.368G_0$
$w(t) = G_0 e^{-\frac{1}{\tau}t}$
τ
t

(c) (6 pt.) Suppose a unit-step external torque τ_{ext} is applied to the rotor while the electrical terminals are **open-circuited**. Draw the response of the terminal voltage $V(t)$.



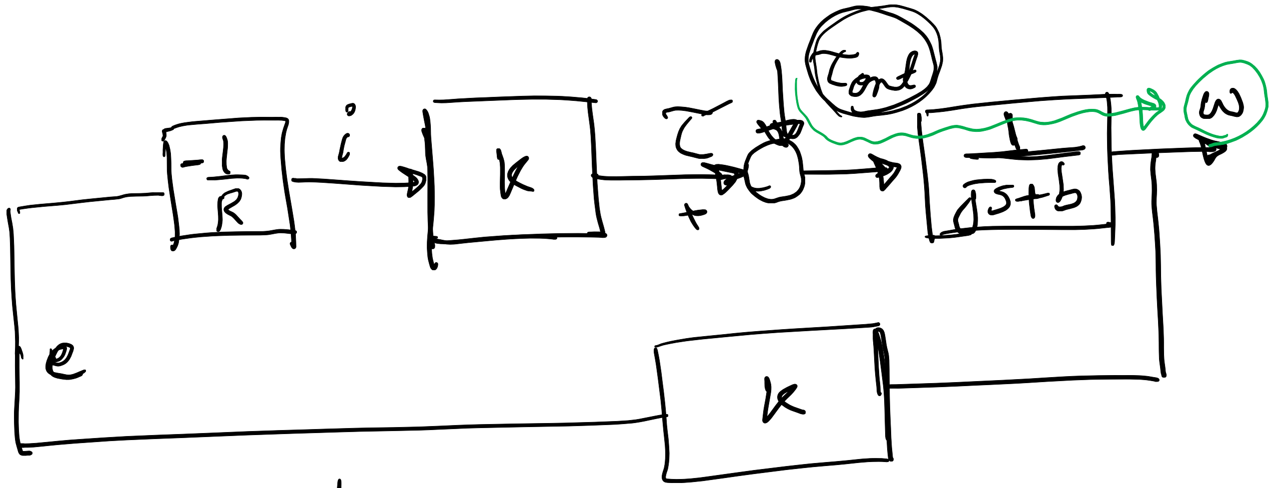


$$\frac{V}{\tau_{ent}} = \frac{K}{Js+b} = \underbrace{\frac{K}{b}}_{G_0} \frac{1}{\underbrace{\frac{J}{b}}_{\tau} s + 1}$$



(d) (8 pt.) Suppose a unit-step external torque τ_{ext} is applied to the rotor while the electrical terminals are **short-circuited**. Draw the response of the rotor angular speed $\omega(t)$.

$V = 0$



$$\frac{\omega}{\tau_{ext}} = \frac{\frac{1}{Js+b}}{1+\frac{k^2}{R}\frac{1}{Js+b}} = \frac{1}{Js+b+\frac{k^2}{R}}$$

$$= \underbrace{\frac{1}{b+\frac{k^2}{R}}}_{G_0}\;\frac{1}{\underbrace{\frac{J}{b+\frac{k^2}{R}}}_{\tau}\,s+1}$$