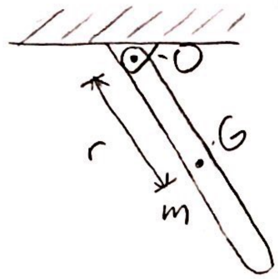
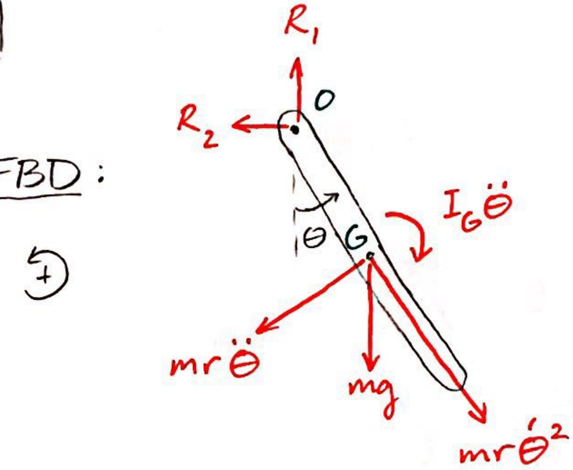
Lecture 3

Compound Pendulum



FBD:



O - center rotation
G - center mass

Sum of moments about O:

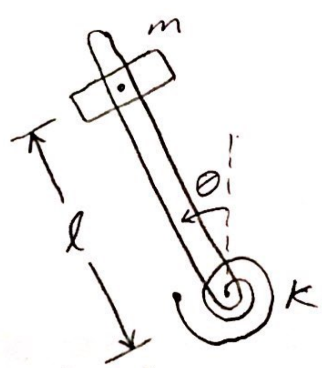
$$I_G\ddot{\theta} + mr^2\ddot{\theta} + mgr\sin\theta = 0$$

$$\underbrace{(I_G + mr^2)}_{I_o}\ddot{\theta} + mgr\theta = 0 \qquad (\text{Small angle: } \sin\theta \simeq \theta)$$

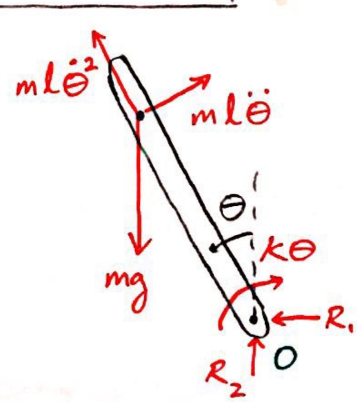
$$\Rightarrow \ddot{\theta} + \frac{mgr}{I_o}\theta = 0 \qquad (I_o = \tfrac{4}{3}mr^2\text{, given})$$

$$\Rightarrow \omega^2 = \frac{mgr}{I_o} = \sqrt{\frac{3}{4}\frac{g}{r}}$$

Metronome (Inverted Pendulum)



FBD:



*mass of rod << m, assume zero

Sum of moments about O:

$$m l^2 \ddot{\theta} + k\theta - mgl\sin\theta = 0 \quad \text{(No } I_G \text{ term, concentrated mass)}$$

$$m l^2 \ddot{\theta} + (k - mgl)\theta = 0 \quad \text{(small angle)}$$

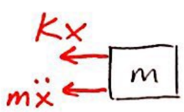
$$\ddot{\theta} + \left(\frac{k - mgl}{ml^2}\right)\theta = 0$$

$$\Rightarrow \omega = \sqrt{\frac{k}{ml^2} - \frac{g}{l}}$$

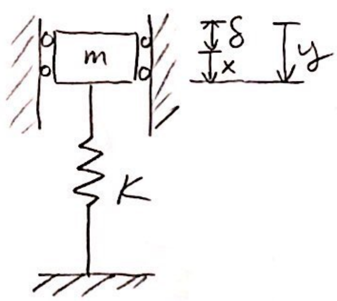
Metronome buckles when $k - mgl < 0$.

Measurements from Equilibrium Position

FBD:



vs:



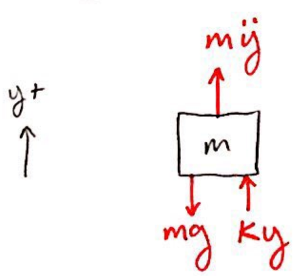
y = disp. from unstretched

x = disp. from stretched (equilibrium)

δ = initial stretch

$\delta = mg/k \iff k = \dfrac{mg}{\delta}$

FBD of vertical system:



$$+\uparrow \sum F_Y = m\ddot{y} + ky - mg = 0$$

$$m\ddot{y} + ky = mg$$

$\Rightarrow$ complementary and particular solutions.

Guess particular solution:

$y = \not{c}$ (constant)

$\Rightarrow \ddot{y} = 0$

$\Rightarrow m\ddot{y} + ky = 0 + k\not{c} = mg$

$\Rightarrow \not{c} = \frac{mg}{k} = \delta$

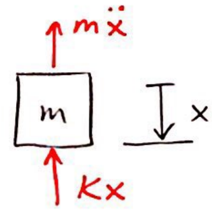
Full solution $\boxed{y = A\cos(\omega t) - B\sin(\omega t) + \delta}$

Choose coordinate $x = y - \delta$ measured from equilibrium position

$\Rightarrow y = x + \delta$

$\Rightarrow x = A\cos(\omega t) - B\sin(\omega t)$

FBD:



Typically, do calculations from equilibrium position. If you are unsure of the role of gravity, include it. Always include it for pendulum-type elements.