MECH468 : Modern Control Engineering
MECH509 : Controls

L19 : State feedback

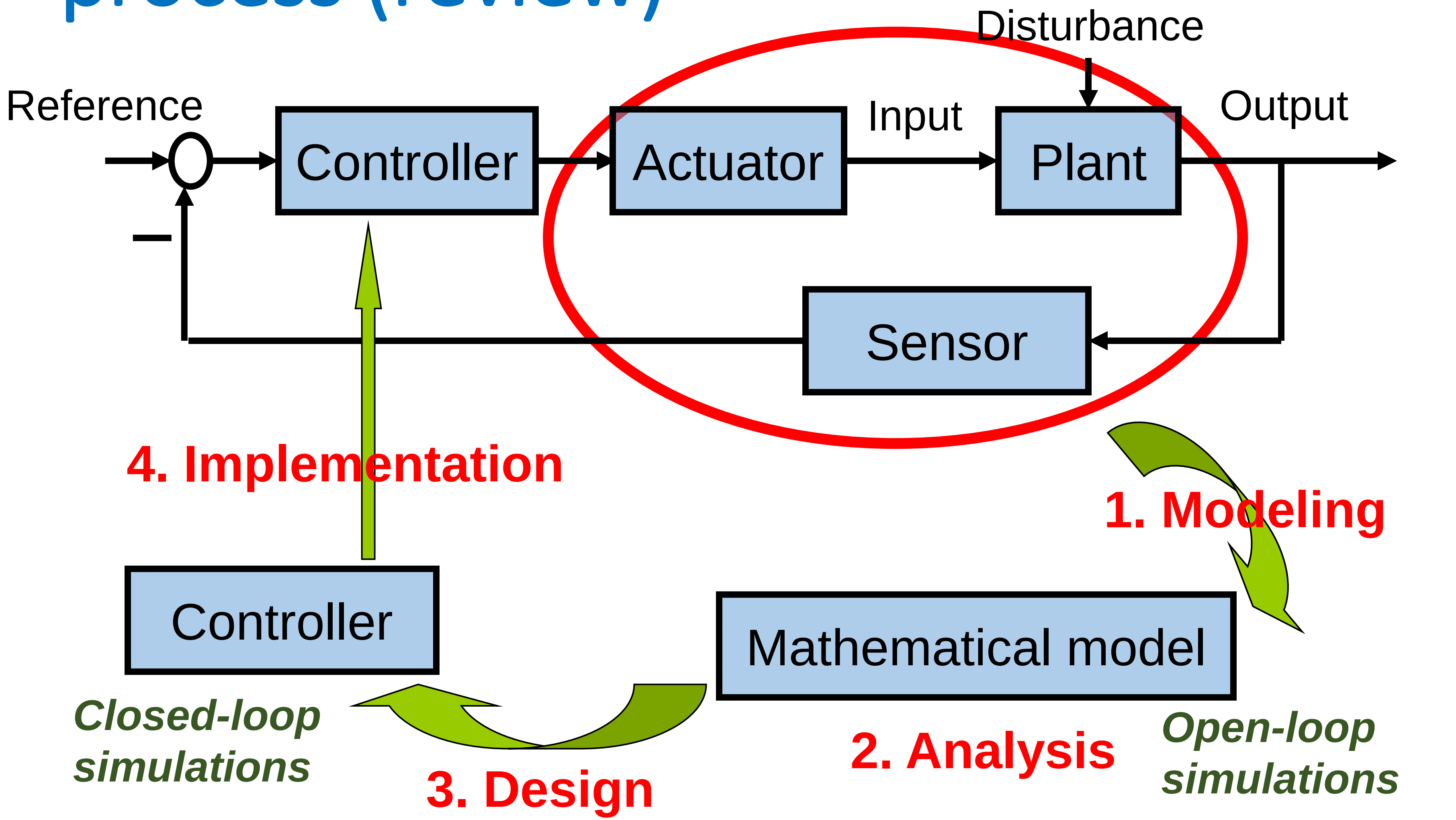
Dr. Ryozo Nagamune
Department of Mechanical Engineering
University of British Columbia

Zoom lecture to be recorded and posted on Canvas

Course plan

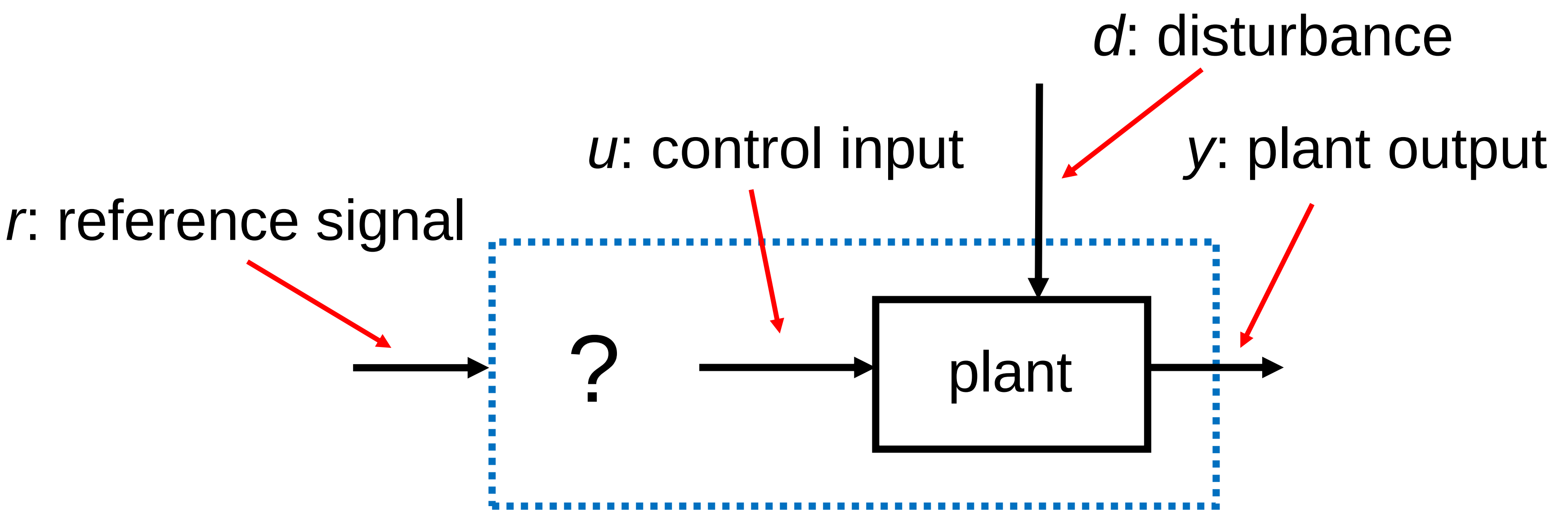
Topics	CT	DT
Modeling	✓	✓
Stability	✓	✓
Controllability/observability	✓	✓
Realization	✓	✓
→ State feedback/observer		
LQR/Kalman filter		

Model-based controller design process (review)



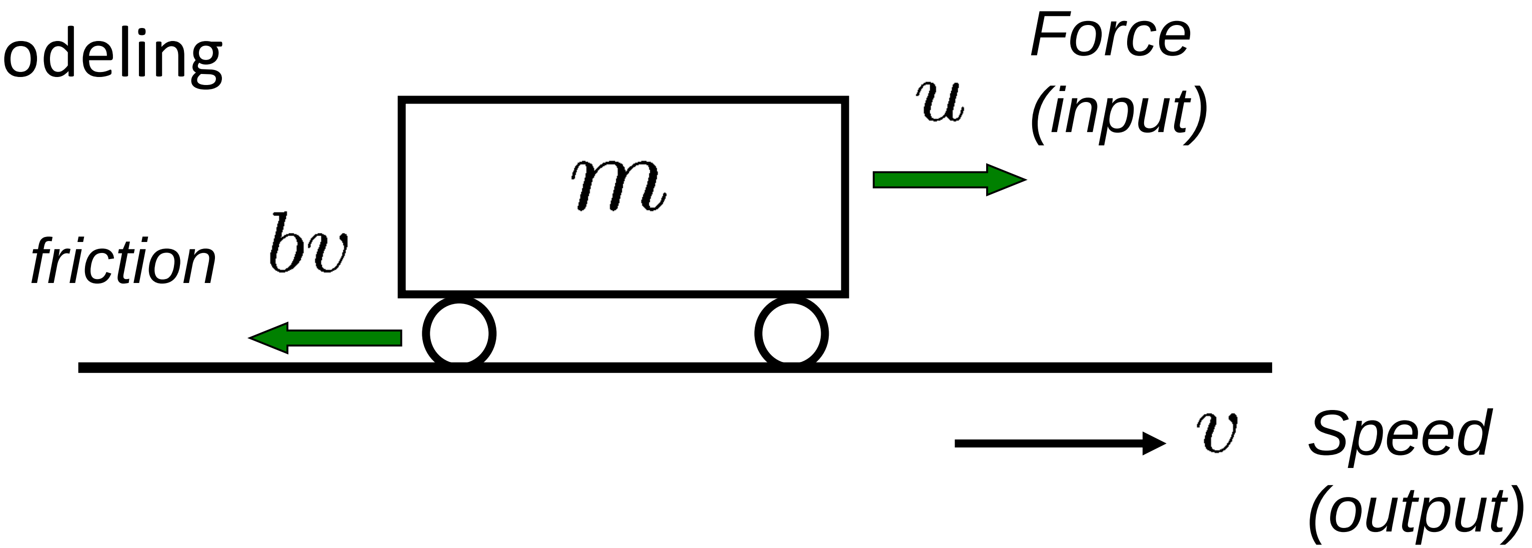
Design of control systems

- Objective: Find a feedback control law such that
  - The feedback system is internally stable, and
  - The plant output *y(t)* follows as closely as possible to the reference *r(t)* against disturbances *d(t).*



Example: Cruise control
ctms.engin.umich.edu

- Modeling

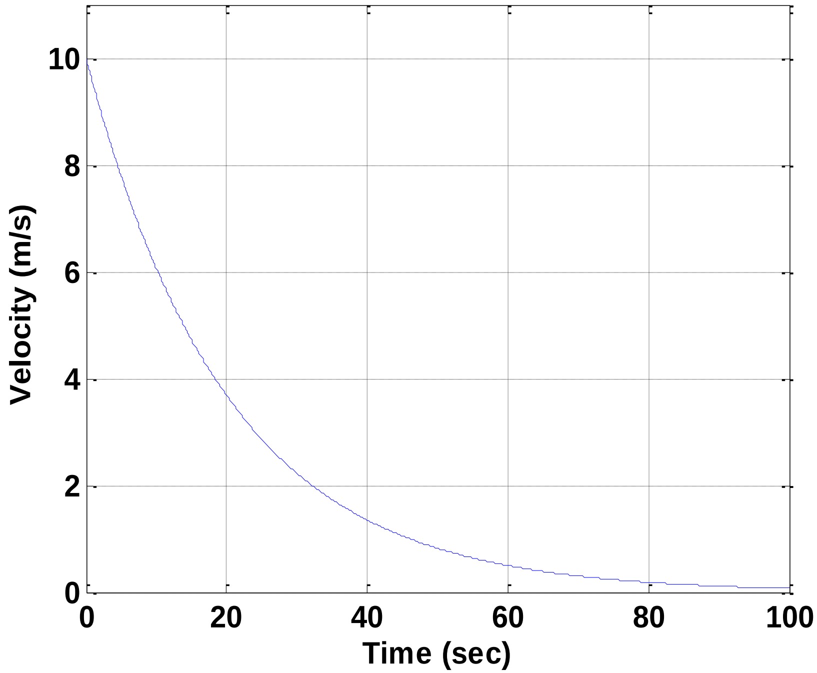


$$\begin{cases} \dot{v}(t) &= -\dfrac{b}{m}v(t) + \dfrac{1}{m}u(t) \\ y(t) &= v(t) \end{cases}$$

$$\begin{aligned} m &= 1000\text{kg} \\ b &= 50\text{Nsec/m} \end{aligned}$$

Cruise control (cont'd)

- Specifications: For initial velocity 10m/s,
  - $r(t)=0$
  - Rise time < 5sec
  - Overshoot < 10%
  - Steady state error < 2%
- Open-loop system
  - Pole: -0.05
  - Takes too much time.



Feedback control for performance improvement!

State feedback

- Given an LTI system $\Sigma : \begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{cases}$

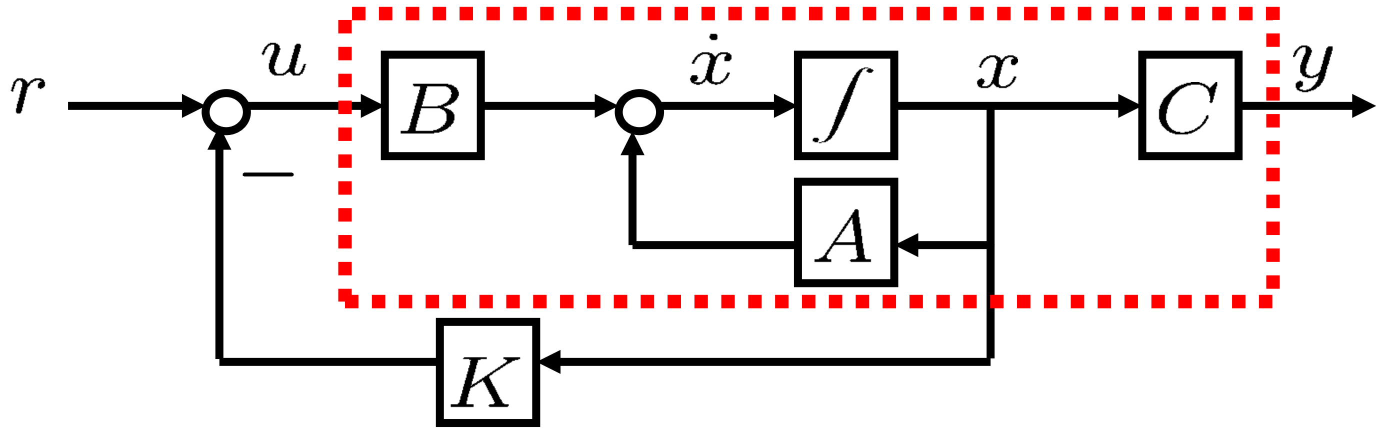
  design a state feedback control law

$$u(t) = -Kx(t) + r(t)$$

  so that all the specifications are satisfied.
  - Internal stability
  - Time domain specs (steady state, transient)
  - Frequency domain specs (bandwidth, gain/phase property etc.)

State feedback (cont'd)

- Block diagram



- Open-loop and closed-loop systems

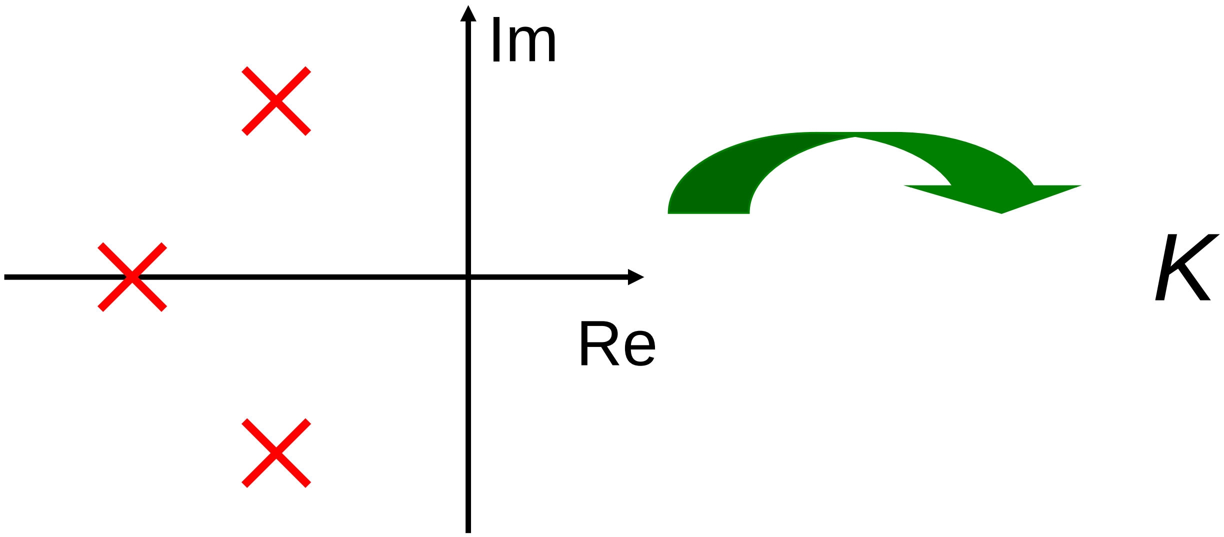
$$\left.\begin{array}{l}\left\{\begin{array}{rcl}\dot{x}(t) & = & Ax(t)+Bu(t)\\ y(t) & = & Cx(t)\end{array}\right.\\[2ex] u(t) = -Kx(t)+r(t)\end{array}\right\} \Rightarrow \left\{\begin{array}{rcl}\dot{x}(t) & = & (A-BK)x(t)+Br(t)\\ y(t) & = & Cx(t)\end{array}\right.$$

Assumptions

- $D=0$
  - Just for simplicity
  - This assumption holds in most practical problems.
- State vector x is available.
  - This assumption does not hold in many applications, and will be removed later.
  - State estimator (observer and Kalman filter)
- Reference r
  - Regulation: $r=0$ (will be considered for 3 lectures)
  - Tracking or servo: nonzero r (will be considered later)

Pole placement theorem (Eigenvalue assignment theorem)

- *If (A,B) is controllable, the eigenvalues of (A-BK) can be placed arbitrarily (provided that they are symmetric with respect to the real axis).*



 : Closed-loop poles (design parameters)

Three questions

- For a specified set of closed-loop poles, how to design the feedback gain *K*?
  - Direct method (today)
  - Canonical form method (“place.m”: next lecture)
  - Ackermann’s formula (“acker.m”)
  - Lyapunov method (in two lectures)
- How to select desired closed-loop pole locations?
  - Rules of thumb (in two lectures)
- Proof of pole placement theorem
  - Not covered in this course

State feedback design: Direct method

- Ex.

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} u(t)$$

Note that (*A*,*B*) is controllable. Hence, any pole placement is possible. Note also that eigenvalues of *A* are 4, -2, i.e., unstable!

Let us stabilize the system by placing poles at

$$-1 \pm 2j$$

Direct method (cont'd)

- Desired characteristic polynomial

$$(s-(-1+2j))(s-(-1-2j)) = s^2 + 2s + 5$$

- Characteristic polynomial of CL system $\det[sI-(A-BK)]$

$$A-BK = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 1-k_1 & 3-k_2 \\ 3 & 1 \end{bmatrix}$$

$$\Rightarrow \det[sI-(A-BK)] = (s-(1-k_1))(s-1) - 3(3-k_2) = s^2 + (k_1-2)s + (3k_2 - k_1 - 8)$$

$$\Rightarrow k_1 = 4, \ k_2 = \frac{17}{3}$$

Feedback cruise control

- State equation

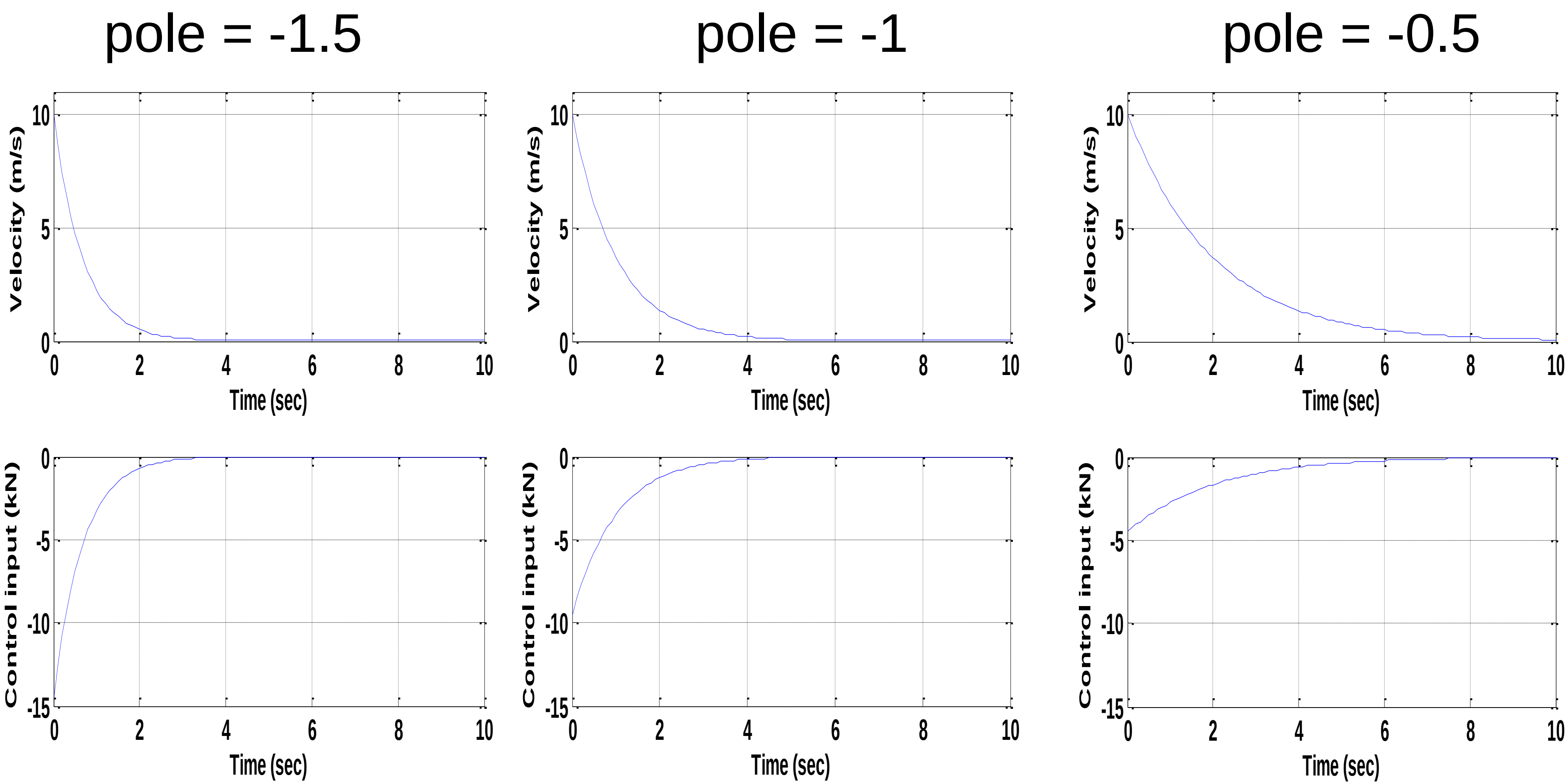
$$
\begin{aligned}
\dot{v}(t) &= -\frac{b}{m}v(t) + \frac{1}{m}u(t) \\
&= \underbrace{-0.05}_{A}v(t) + \underbrace{10^{-3}}_{B}u(t)
\end{aligned}
\qquad
\left(\begin{array}{lcl} m &=& 1000\text{kg} \\ b &=& 50\text{Nsec/m} \end{array}\right)
$$

Note that (*A*,*B*) is controllable (obviously!). Hence, any pole placement is possible.

$$\text{Pole} = A - BK$$

$$\Rightarrow \quad K = \frac{A - \text{Pole}}{B} = -1000(0.05 + \text{Pole})$$

Feedback cruise control (cont'd)



Exercise

- Try the simulation by yourselves! (Matlab code "cruise.m" is posted on Canvas.) Change the pole location, and get a feeling how responses are affected by the pole location.
- For the system below, design a state feedback control law *u=-Kx* so that the closed-loop system has *-1* and *-2* as its eigenvalues, by using the direct method.

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} u(t)$$

Summary

- State feedback
  - Pole placement theorem
  - Direct method for calculating state feedback gain
    - Suitable only for problems with $n=1,2,3$.
  - Cruise control example
    - As the pole is moved to the left, the convergence becomes faster, at the price of large control input.
- Next, canonical form method for designing the state feedback gain