Lesson 10-3 – Modified Internal Rate of Return

Multiple Solutions for Rate

- Cash flows that contain more than one change in sign can also have more than one solution for the IRR.
- Equation for PW = 0, given some cash flows:
  - $0 = CF_0 + CF_1(1 + i)^{-1} + CF_2(1 + i)^{-2} + . . . + CF_n(1 + i)^{-n}$
- More generally:
  - $0 = CF_0 + CF_1x^1 + CF_2x^2 + . . . + CF_nx^n$
- This is a *n*th order polynomial.

Descartes Rule

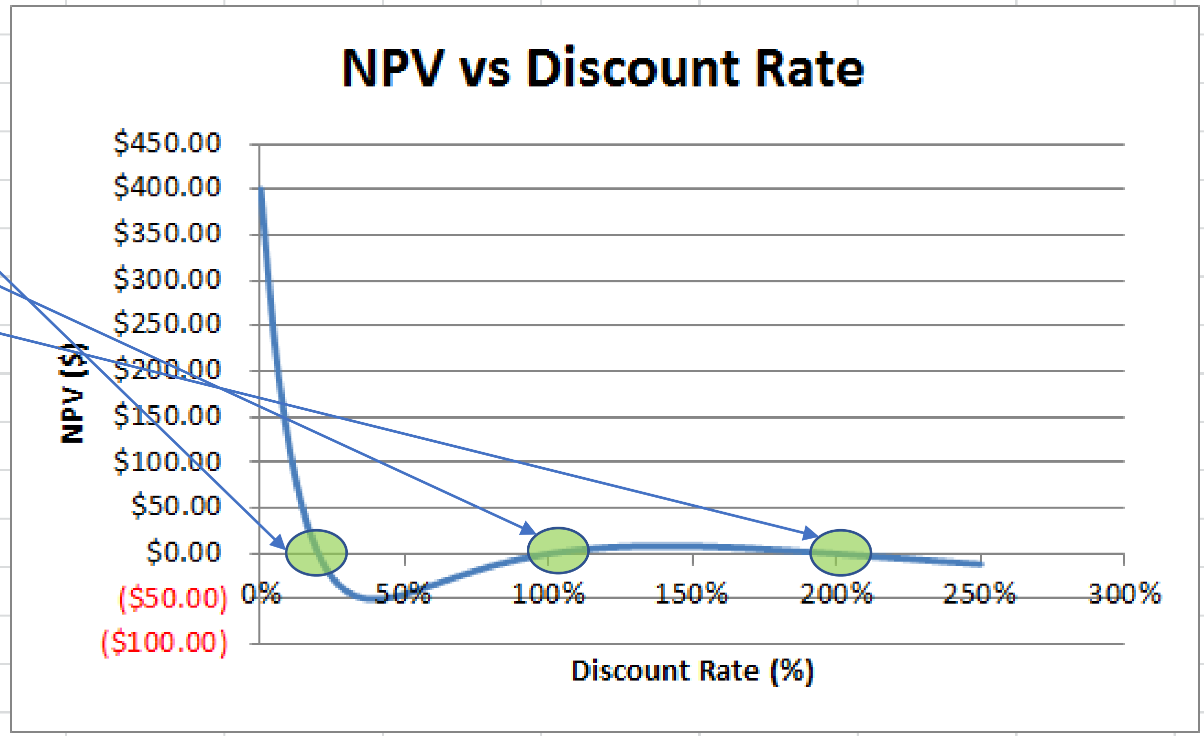
- "If a polynomial with real coefficients has m sign changes, then the number of positive roots will be $m-2k$, where k is an integer between 0 and $m/2$."
  - This means the calculation of IRR for cash flows with more than one sign change results in the possibility that multiple IRR are possible.
  - Projects often have a variety of cash flows that include more than one sign change.

Multiple Solutions for Rate

Year	Cashflow
0	$ (1,000.00)
1	$ 6,200.00
2	$ (12,000.00)
3	$ 7,200.00

IRR	20%
	100%
	200%

i	NPV
0%	$400.00
5%	$228.61
10%	$116.80
15%	$44.95
20%	$0.00
25%	($26.88)
30%	($41.67)
35%	($48.43)
40%	($49.98)
45%	($48.21)
50%	($44.44)
55%	($39.57)
60%	($34.18)

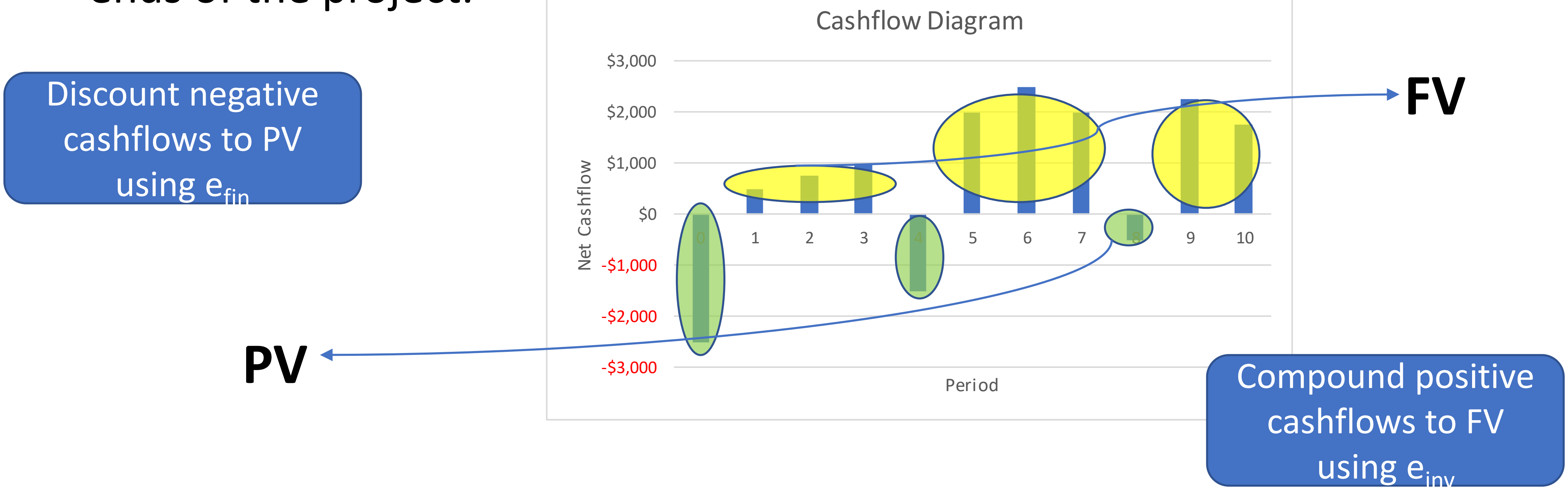


Modified Internal Rate of Return (MIRR)

- Two of the problems with the IRR can be solved by calculating the modified internal rate of return (MIRR):
  - Multiple IRR values
  - Financing and reinvesting at the IRR rate of return
- The MIRR technique requires two *external* rates of return:
  - e_{inv} for investing
  - e_{fin} for financing

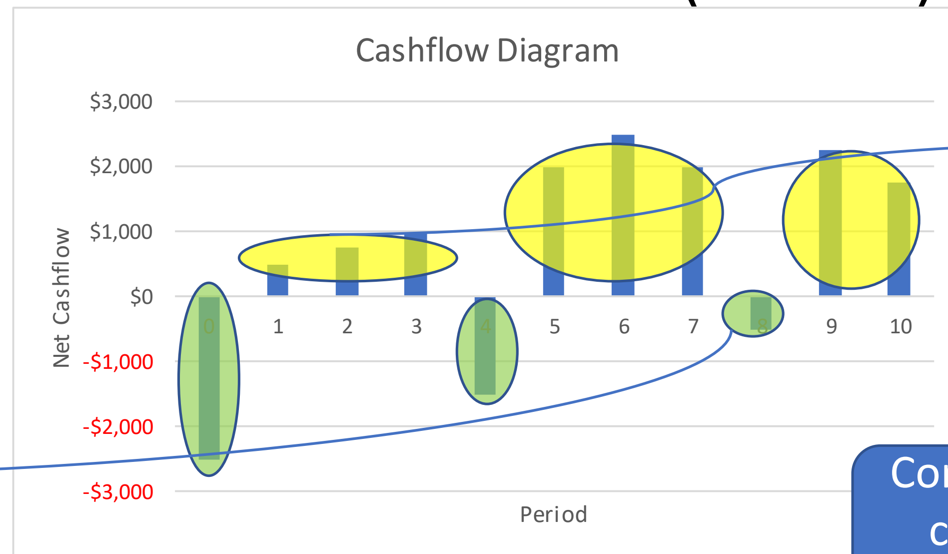
Modified Internal Rate of Return (MIRR)

- Net out cashflows for each period.
- Find the **present value** at the start of the project of all *negative* net cash flows using e_{fin} as the discount rate.
- Find the **future value** at the end of the project of all *positive* net cash flows using e_{inv} as the discount rate.
- Find the rate of return (MIRR) that balances these two values at both ends of the project.



Modified Internal Rate of Return (MIRR)

Discount negative cashflows to PV using e_{fin}

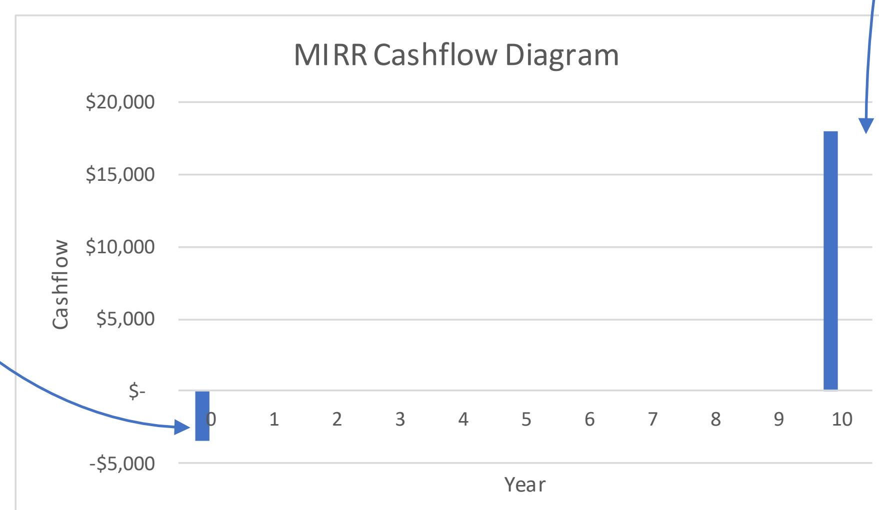


FV

PV

Compound positive cashflows to FV using e_{inv}

- We know PV, FV and n: balance the two values and solve for i
- $P = F(1+i)^{-n}$
- $i = (P/F)^{-1/n} - 1$

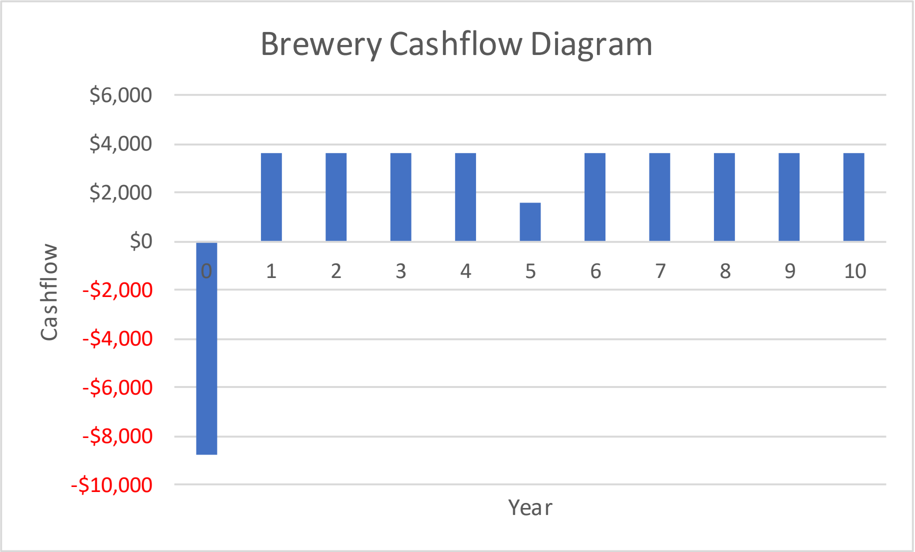


Simple example:

- A brewery is considering changing can suppliers. The current supplier provides cans at \$0.08 per can. The new supplier provides cans at \$0.065 per can.
- The can feeding machinery will need to be modified to accept the new cans. This will cost \$8,800 and require a \$2000 overhaul in five years
- Production volumes are ~240,000 cans per year. Changing the can is not expected to affect production speed or volumes.
- The brewery has a line of credit that currently charges 8% interest annually
- Any savings from the project will be set aside in retained earnings, and earmarked for additional projects that offer a rate of return of at least 15%
- The analysis period for the project is 10 years

Brewery Example

- Costs: $8,800 plus $2,000 overhaul in year five
- Benefits: $0.015 per can * 240,000 cans per year = $3,600 per year for 10 years
- Finance rate: 8% Investment rate: 15%

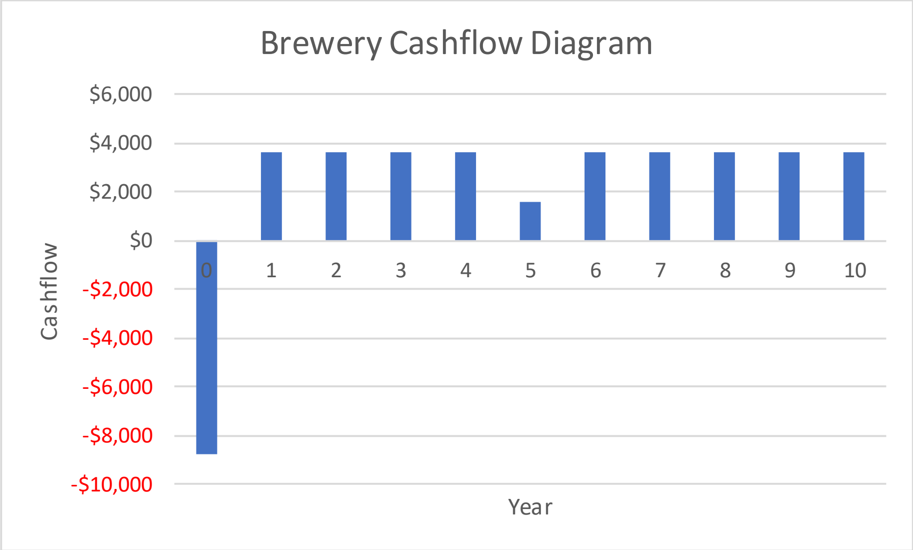


- PV of costs = $8,800
- FV of benefits = $3,600(F/A, 15%, 10) - $2000(F/P, 15%, 5) = $69070

Brewery Example

- PV of costs: $8800
- FV of benefits: $69070 (at year 10)

- $P = F(1+i)^{-n}$
- $\$8800 = \$69070(1+i)^{-10}$
- Solve for i = 23%



Modified Internal Rate of Return (MIRR) Summary

- Use of external rates:
  - Investing rate (e_{inv}) is used for positive cash flows.
  - Financing rate (e_{fin}) is used for negative cash flows.
  - The positive cash flows are moved to the end of the project's time period using (F/P, e_{inv}, n).
  - The negative cash flows are moved to the beginning of the project's time period using (P/F, e_{fin}, n).
    - Boils down the cashflows into a single PV for costs, and a single FV of benefits.
    - $P = F(1+MIRR)^{-n}$
    - or $F = P(1+MIRR)^{n}$
- This results in a unique MIRR, which you can solve for.
- http://www.propertymetrics.com/blog/2015/10/28/how-to-use-the-modified-internal-rate-of-return-mirr/

Suggested Problems

- Chapter 8
- 1, 3, 5, 19, 28, 48, 52, 57, 67, 70, 74, 82, 86