Question 5

A schematic diagram of an automated conveyor system for grading fruits is shown in Figure Q5(a). At the start-up, the conveyor reaches the top speed at constant acceleration, and that speed is maintained subsequently. The fruits on the conveyor pass under a sensing and grading unit (4). There, each fruit is illuminated using multiple stripes of laser lighting, and an image of the fruit is acquired using a CCD camera. Three-dimensional information of the fruit is captured in this manner, and a grade (one of possible three, not including the rejects) is determined for the fruit. The graded fruit is carried forward by the conveyor, and the control logic activates the pneumatic jet (1, 2, or 3) corresponding to the grade of that fruit as it reaches the jet. The graded fruits are collected by the proper bins on the side of the conveyor in this manner. The rejects are dropped into the bin at the end of the conveyor. The controller of the system, which receives the actual speed of the conveyor (from the encoder attached to the motor shaft), is able to adjust the conveyor speed, through the drive unit (11) of the motor.

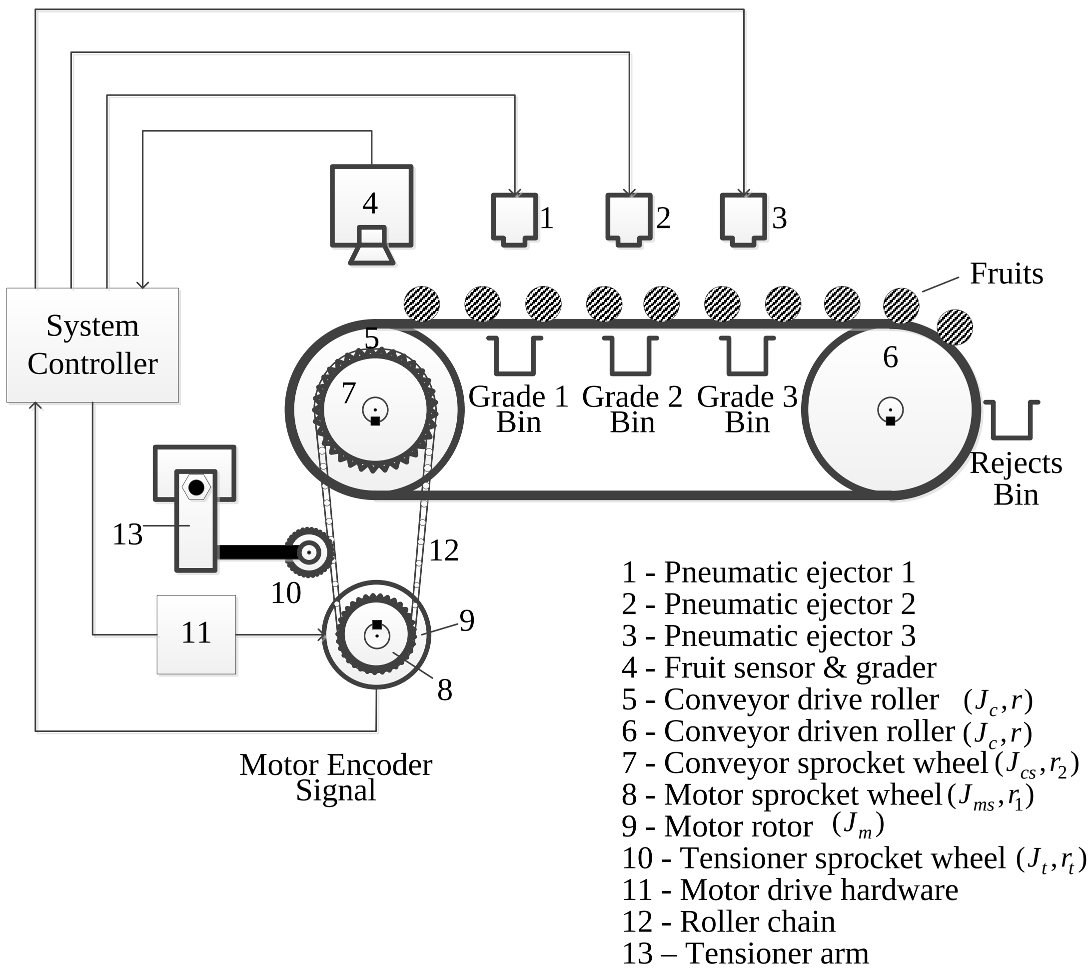

Figure Q5(a): An automated conveyor system for fruit grading.

A chain and sprocket mechanism is used to drive the conveyor through a dc motor. A tensioner arm can be adjusted to press an idler sprocket wheel (10) onto the chain (12) so as to remove any slack in the chain, which may occur after prolonged operation. The following system parameters are defined:

J_m = moment of inertia of the motor rotor
J_{ms} = moment of inertia of the motor sprocket wheel
J_t = moment of inertia of the tensioner sprocket wheel
J_{cs} = moment of inertia of the conveyor sprocket wheel
J_c = moment of inertia of each of the two cylinders of the conveyor
m_c = overall mass of the conveyor belt and the moved fruits (load)
r = radius of each of the two conveyor cylinders
r_1 = effective radius of the motor sprocket wheel
r_2 = effective radius of the conveyor sprocket wheel
r_t = effective radius of the tensioner sprocket wheel

(a) In terms of the above parameters, derive an expression for the equivalent overall moment of inertia, J_e that is felt at the motor shaft, as the entire conveyor system operates.

(10%)

(b) The speed profile of the conveyor has a steadily accelerating start-up segment and a constant-speed continuous segment, as shown in Figure Q5(b).

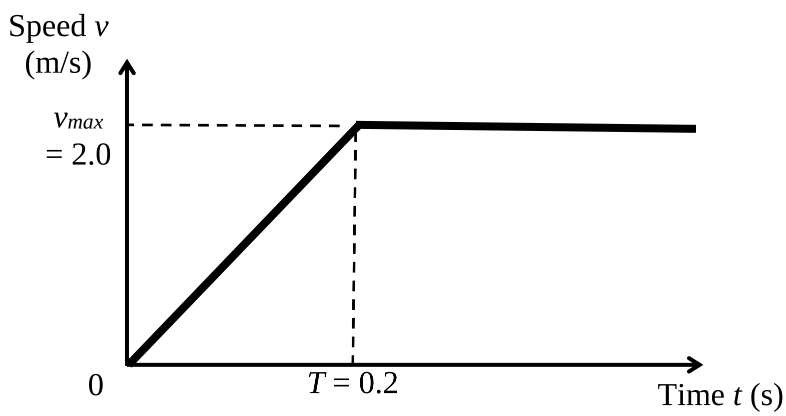

Figure Q5(b): Speed profile of the conveyor.

Three models of dc motor are available for this application. Their pull-out curves (for continuous operation) are shown in Figure Q5(c). The moments of inertia of the rotors of these three motors are: $J_{m1} = 6.0 \times 10^{-2}$ kg. m^2, $J_{m2} = 8.0 \times 10^{-2}$ kg. m^2, and $J_{m3} = 11.0 \times 10^{-2}$ kg. m^2, for the models 1, 2, and 3, respectively. Also, the following parameter values are known for the system: $r = 0.40$ m, $r_1 = 0.10$ m, $r_2 = 0.25$ m, $r_t = 0.08$ m, $J_{ms} = 4.0 \times 10^{-2}$ kg.m^2, $J_t = 3.0 \times 10^{-2}$ kg.m^2, $J_{cs} = 7.0 \times 10^{-2}$ kg.m^2, $J_c = 15.0 \times 10^{-2}$ kg.m^2, and $m_c = 10.0$ kg.
Giving all necessary details, select a suitable motor from the three models, for the present application.
Note: Assume an overall efficiency of 80% for the entire system (including the motor), regardless of the model of the motor. **(15%)**

If none of the three motor models are suitable for the present application, you have to give the reasons for that, in detail.

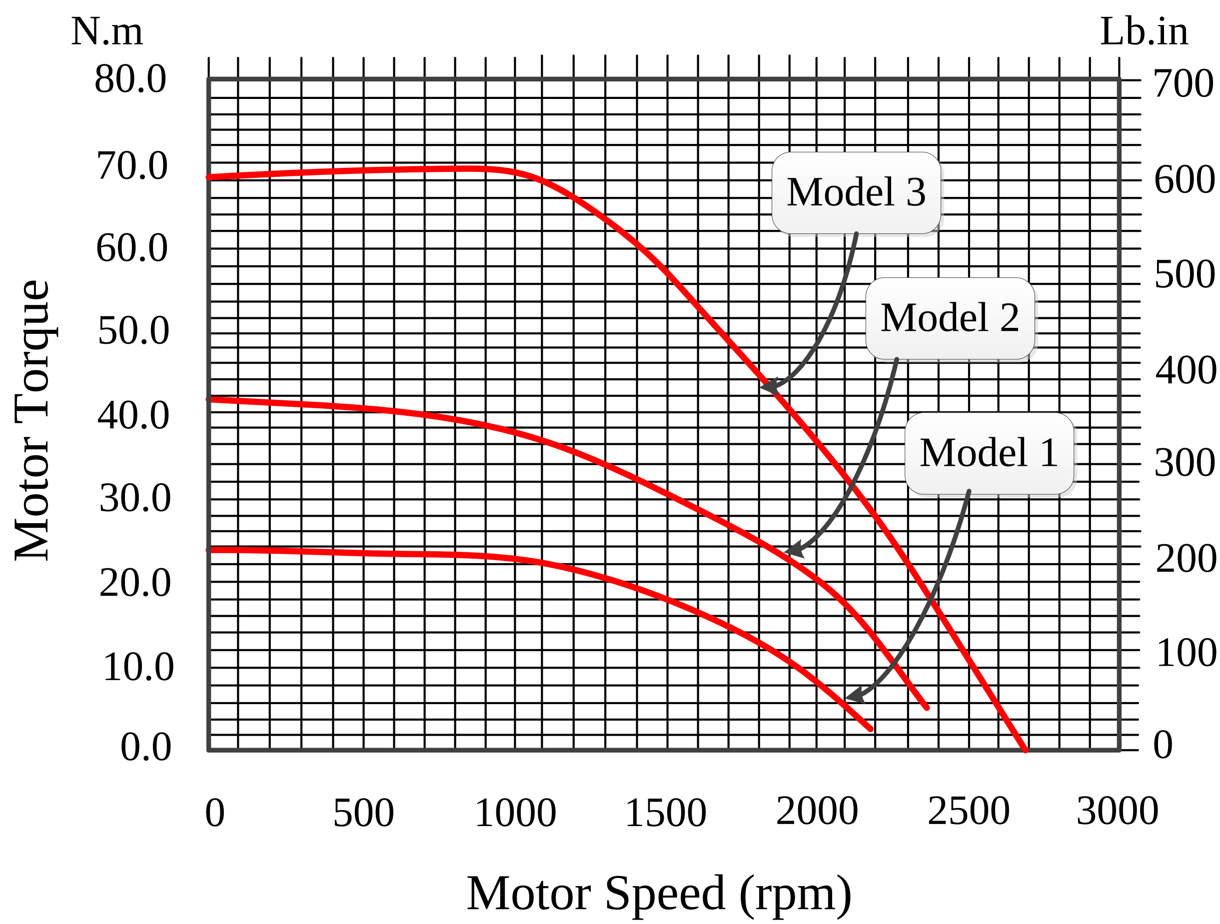

Figure Q5(c): Pull-out curves of the three motors for continuous operation.

Solution 5

(a)

Angular speed of the motor and drive sprocket = ω_m.

Angular speed of the tensioner sprocket = $n_t\omega_m$

where $n_t = \dfrac{r_1}{r_t}$

Angular speed of the conveyor sprocket and conveyor cylinders = $n_c\omega_m$

where $n_c = \dfrac{r_1}{r_2}$

Rectilinear speed of the conveyor and objects, $v_c = rn_c\omega_m$.

Hence, kinetic energy of the overall system

$$= \frac{1}{2}(J_m + J_{ms})\omega_m^2 + \frac{1}{2}J_t n_t^2 \omega_m^2 + \frac{1}{2}(J_{cs} + 2J_c)n_c^2\omega_m^2 + \frac{1}{2}m_c r^2 n_c^2 \omega_m^2$$

$$= \frac{1}{2}[J_m + J_{ms} + J_t n_t^2 + (J_{cs} + 2J_c)n_c^2 + m_c r^2 n_c^2]\omega_m^2$$

$$= \frac{1}{2}J_e \omega_m^2$$

Hence, the equivalent moment of inertia as felt at the motor rotor is

$$J_e = J_m + J_{ms} + J_t n_t^2 + (J_{cs} + 2J_c)n_c^2 + m_c r^2 n_c^2$$

(b)

Substitute the given numerical values for the system parameters:

$$n_t = \frac{0.1}{0.08} = 1.25; \quad n_c = \frac{0.1}{0.25} = 0.4$$

$$J_e = [J_m \times 10^2 + 4.0 + 3.0 \times 1.25^2 + (7.0 + 2 \times 15.0) \times 0.4^2 + 10.0 \times 0.4^2 \times 0.4^2 \times 10^2] \times 10^{-2}$$

$$= [J_m \times 10^2 + 4.0 + 4.6875 + 5.92 + 25.6] \times 10^{-2} \text{ kg.m}^2$$

Or,

$$J_e = J_m + 40.21 \times 10^{-2} \text{ kg.m}^2 \qquad \text{(i)}$$

From the speed profile, the maximum acceleration of the conveyor is

$$a_{\max} = \frac{2.0}{0.2} = 10.0 \text{ m/s}^2$$

The corresponding maximum angular acceleration of the motor shaft is

$$\alpha_{\max} = \frac{a_{\max}}{r n_c} = \frac{10.0}{0.4 \times 0.4} = 62.5 \text{ rad/s}^2$$

Using (i), the corresponding motor torque T_m (allowing for efficiency η) may be expressed as

$$\eta T_m = J_e \alpha_{\max} = \left[J_m + 40.21 \times 10^{-2}\right]\alpha_{\max}$$

Substituting the numerical values, we have

$$T_m = \frac{\alpha_{\max}}{\eta}\left[J_m + 40.21 \times 10^{-2}\right] = 78.125 \times \left[J_m + 40.21 \times 10^{-2}\right] \text{ N.m}$$

From the speed profile, the maximum speed of the conveyor is

$$v_{\max} = 2.0 \text{ m/s}$$

The corresponding maximum angular speed of the motor shaft is

$$\omega_{\max} = \frac{v_{\max}}{r n_c} = \frac{2.0}{0.4 \times 0.4} = 12.5 \text{ rad/s} = \frac{12.5 \times 60}{2\pi} \text{ rpm} = 119.4 \text{ rpm}$$

The available torque at this speed, from the three motor models, are obtained from the pull-out curves given in Figure Q5(c). Since the motor rotor inertia of the three motors are also known, Table S is formed for the three motor models.

Table S: Data for motor selection.

It is seen from this table that the best choice from the available motors, for the present application, is Model 2.

Motor Model	$J_m\left(\text{kg.m}^2\right)$	Required Torque at 120 rpm (N.m.)	Available Torque at 120 rpm (N.m)
1	6.0×10^{-2}	36.1	23.0
2	8.0×10^{-2}	37.7	41.0
3	11.0×10^{-2}	40.0	68.0