Slide 20:

Transfer function $H(j\omega) = \left[\frac{\omega_n^2}{\omega_n^2 - \omega^2 + 2j\zeta\omega_n\omega} \right]$

Substitute $\omega = \omega_n$ → $H(j\omega) = \left[\frac{\omega_n^2}{2j\zeta\omega_n^2} \right] = \left[\frac{1}{2j\zeta} \right]$

→ $\text{Gain} = \frac{1}{2\zeta};\ \text{Phase lead} = -\frac{\pi}{2}$

Resonant frequency is where the magnitude of $H(j\omega)$ is a maximum

We need $|H(j\omega)| = \left[\frac{\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}} \right]$ to be maximum

→ Denominator squared: $\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2$ to be a minimum

Differentiate this wrt ω and equate to zero: $2\left(\omega_n^2 - \omega^2\right)(-2\omega) + 2\left(2\zeta\omega_n\omega\right)(2\zeta\omega_n) = 0$

→ $\left(-\omega_n^2 + \omega^2\right) + 2(\zeta\omega_n)^2 = 0$

→ $\omega_r = \sqrt{1 - 2\zeta^2}\,\omega_n$