$$
{}^{i-1}T_i = \begin{cases} \begin{bmatrix} e^{\theta_i k\times} R_i & e^{\theta_i k\times}\delta_i \\ 0^T & 1 \end{bmatrix} & \text{if joint } i \text{ is revolute} \\ \begin{bmatrix} R_i & \delta_i + d_i k \\ 0^T & 1 \end{bmatrix} & \text{if joint } i \text{ is prismatic.} \end{cases}
$$

$$(s\times)^2 = ss^T - s^T s I$$

$$Q = I + \sin\theta(s\times) + (1-\cos\theta)(s\times)^2$$

$$L = T - V \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \left(\frac{\partial L}{\partial q}\right) = u$$

$$D(q)\ddot q + C(q,\dot q)\dot q + G(q) = u + \underline{J}_n^T \begin{bmatrix} \underline{f}_n \\ \underline{\tau}_n \end{bmatrix}$$

1. Mass matrix is:
   - symmetric
   - positive definite
   - depends only on q
   - kinetic energy is $T(q,\dot q) = \frac{1}{2}\dot q^T D(q)\dot q$
2. *Christoffel* form of $C(q,\dot q)\dot q$ is $C(q,\dot q)\dot q = \frac{1}{2}[\dot D(q) - N(q,\dot q)]\dot q$

where $N(q,\dot q)$ is *skew-symmetric*. $C(q,\dot q)\dot q$ *is not unique.*

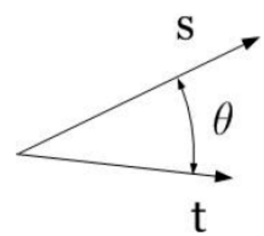
$$C(q,\dot q)\dot q = \frac{1}{2}\dot D(q)\dot q - \frac{1}{2}\{[\sum_{i=1}^{n} e_i \dot q^T \frac{\partial}{\partial q_i} D(q)] - [\sum_{i=1}^{n} e_i \dot q^T \frac{\partial}{\partial q_i} D(q)]^T\}\dot q \qquad (176)$$

$$= \frac{1}{2}[\dot D(q) - N(q,\dot q)]\dot q \; , \qquad (177)$$

3. Gravitational forces are the gradient of the potential energy

$$G(q) = \frac{\partial V(q)^T}{\partial q} \qquad V(q) = -\sum_{i=1}^{n} M_i \underline{g}^T(\underset{\sim}{c}_i(q) - \underset{\sim}{o}_0)$$

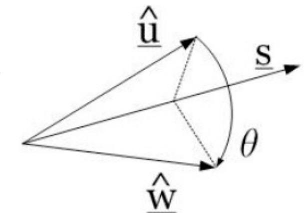
Kahan P1: Given **s** and **t**, find θ.



$$|\theta| = 2\arctan\frac{\|\underline{\hat s} - \underline{\hat t}\|}{\|\underline{\hat s} + \underline{\hat t}\|}$$

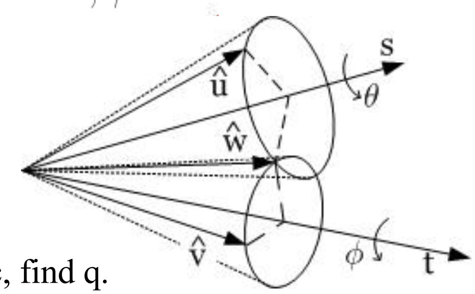
$$\underline{\hat v} = \frac{1}{\|\underline{v}\|}\underline{v}$$

Kahan P2: . Solve $e^{\theta\underline{\hat s}\times}\underline{\hat u} = \underline{\hat w}$ for



Kahan P3: Given the **s, t, u** and **v**, find θ and ϕ

Solve $e^{\theta\underline{\hat s}\times}\underline{\hat u} = e^{\phi\underline{\hat t}\times}\underline{\hat v}$ for θ, ϕ



Kahan P4: Given the a, b and c, find q.

$$|\theta| = 2\arctan\sqrt{\frac{(a+b)^2 - c^2}{c^2 - (a-b)^2}} \quad \text{if} \quad a+b \geq c \geq |a-b|$$