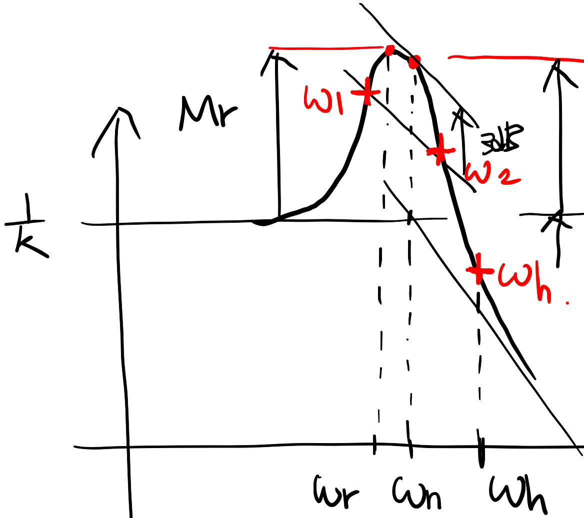
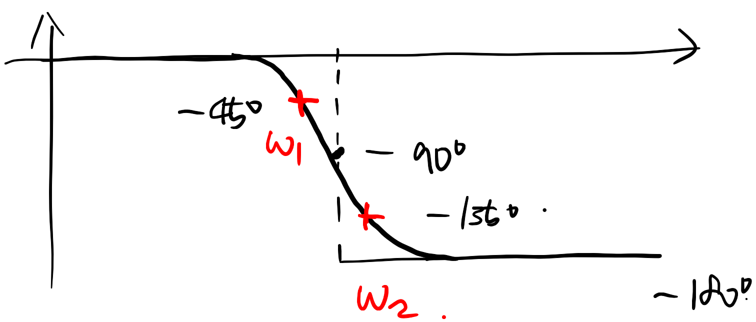
L14 – Motion Control Design ~~vis~~ via Loop Shaping

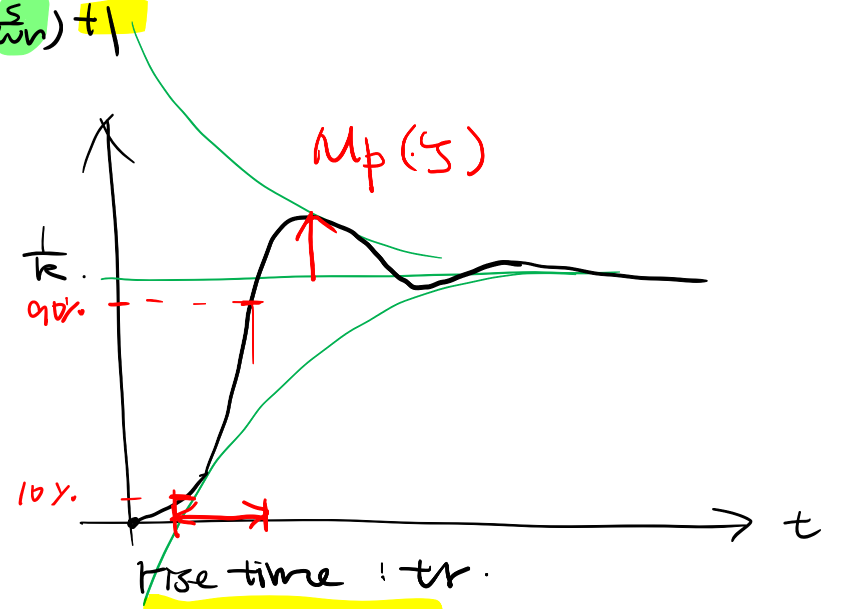
$$P(s) = \frac{1}{ms^2+bs+k} = \frac{1}{k}\,\frac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}$$



$$Q = \frac{1}{2\zeta} = \frac{\omega_n}{\omega_2-\omega_1}$$

$3dB = \sqrt{2}$



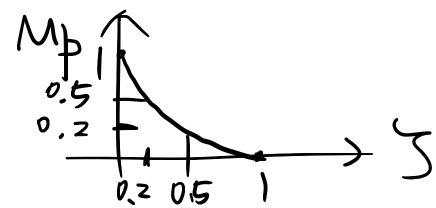


rise time : t_r

$$t_r = \frac{2.2}{\omega_h}$$

For 2nd order as well

$$t_r \simeq \frac{2.2}{\omega_h} < \frac{2.2}{\omega_n}$$



Nat freq : $\omega_n = \sqrt{\frac{k}{m}}$

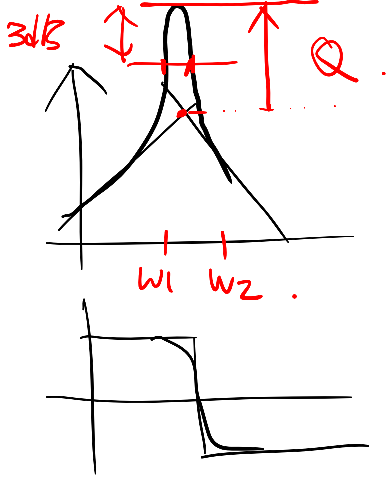
Quality factor : $Q = \frac{1}{2\zeta}$

Res freq : $\omega_r = \omega_n\sqrt{1-2\zeta^2} \simeq \omega_n$ $\quad \zeta^2 \ll 1$

Resonant peak : $M_r = \frac{1}{2\zeta}\frac{1}{\sqrt{1-\zeta^2}} \simeq \frac{1}{2\zeta} = Q$

$0.3 \rightarrow 0.09$

$\frac{X}{F}$ $\frac{\dot{X}}{F}$



- **Remark**

$\{ G_0, \omega_n, \zeta \}$

- Step Freq.

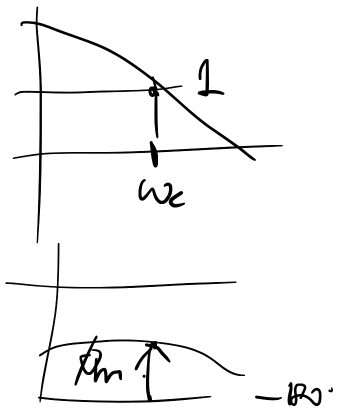
$M_p = M_p(\zeta) \longleftrightarrow M_r \simeq \frac{1}{2\zeta}$

$t_r \simeq \frac{2.2}{\omega_h} < \frac{2.2}{\omega_n} \longleftrightarrow \omega_h > \omega_n$

- This is our "Template" → Motion control desing through "Loop shaping"

- $L(s)$. $T(s) = \dfrac{L}{1+L}$.

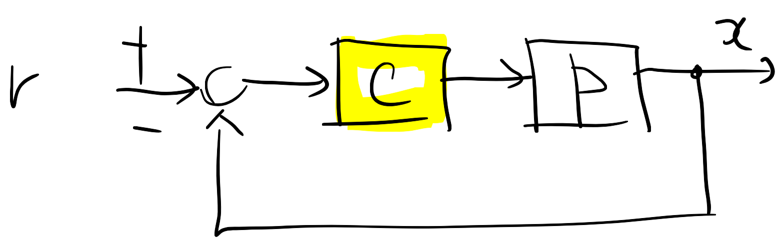
★ $\left\{ \begin{matrix} \omega_c \\ \phi_m \end{matrix} \right\} \longleftrightarrow$ ★ $\left\{ \begin{matrix} \omega_n \approx \omega_r \\ \zeta \end{matrix} \right\}$.



- $\omega_n \simeq \omega_c$
- $\zeta \simeq \dfrac{\phi_m\,[\deg]}{100}$.

$5^\circ \rightarrow \zeta = 0.05$

- Why do we care $L(s)$? rather than $T(s)$?



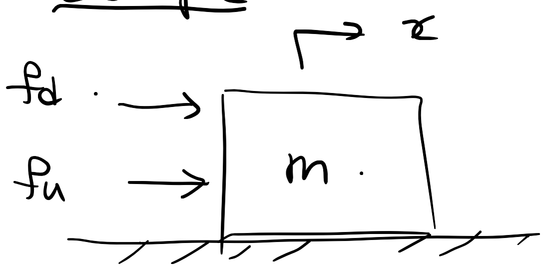
$\left\{ \begin{array}{ll} T(s) = \dfrac{cp}{1+cp}. & \text{Nonlinear with } c. \\ L(s) = cp & \text{Linear with } c. \end{array} \right.$

- It is much easier to shape $L(s)$ with $C(s)$
- ω_c & ϕ_m $\Rightarrow$ ω_n & ζ.

$$\begin{cases} t_r \simeq \dfrac{2.2}{\omega_n} & \text{"1st order"} \\ M_r \simeq \dfrac{1}{2\zeta} & \text{"2nd order"} \end{cases}$$

$\Rightarrow$ Extrapolate to general syst.

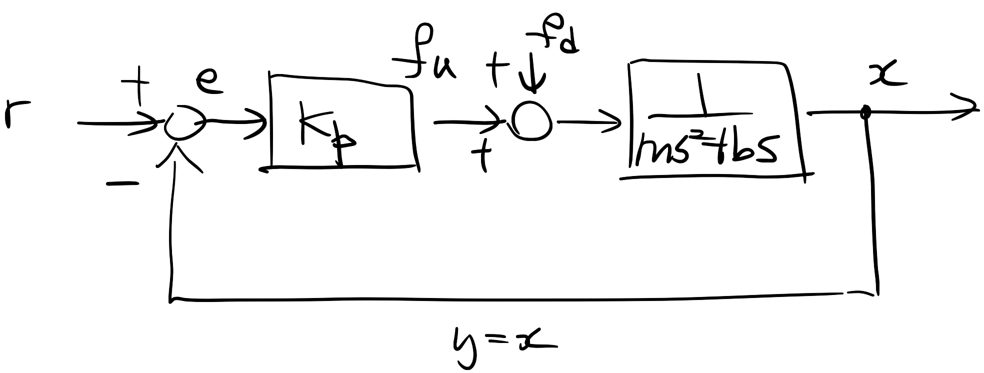
Example



Linear bearing: b.

$$m\ddot{x} = f_u + f_d - b\dot{x}$$

$$\Rightarrow (ms^2 + bs)X = f_u + f_d$$



$C(s) = K_p$.

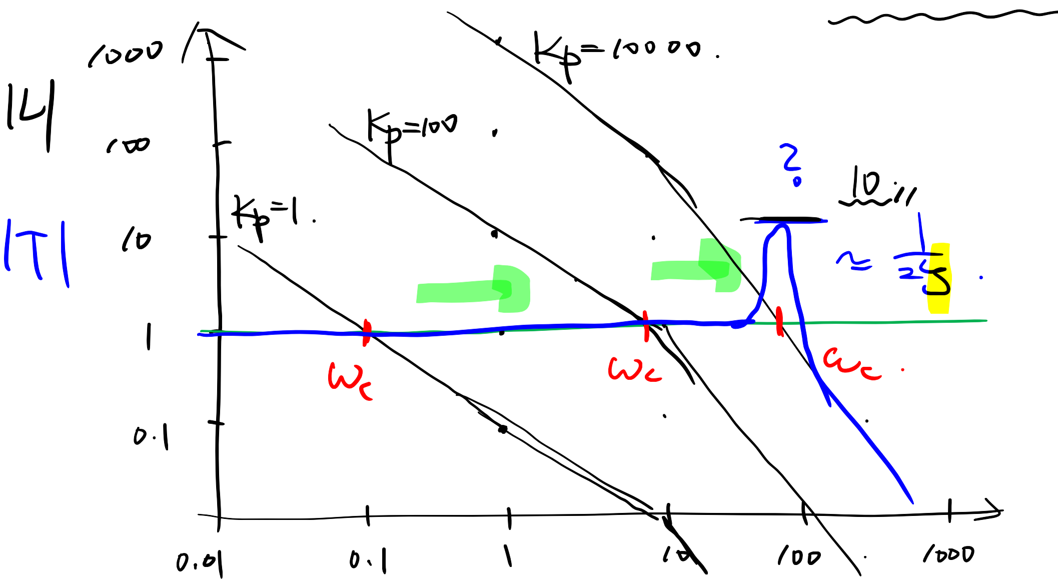
$\begin{cases} m = 1\,kg \\ b = 10\ Ns/m \end{cases} \Rightarrow P(s) = \dfrac{1}{s^2+10s}$

★ $L(s) = \dfrac{K_p}{ms^2+bs} = \boxed{\dfrac{K_p}{s^2+10s}}$

$\dfrac{x}{f_d} = \dfrac{\frac{1}{ms^2+bs}}{1+\frac{K_p}{ms^2+bs}} = \left[\dfrac{1}{ms^2+bs+K_p}\right]$

"Apparent Stiffness"

★ $\dfrac{X}{R} = \dfrac{L}{1+L} = \dfrac{K_p}{ms^2+bs+K_p} = \boxed{\dfrac{K_p}{s^2+10s+K_p}}$

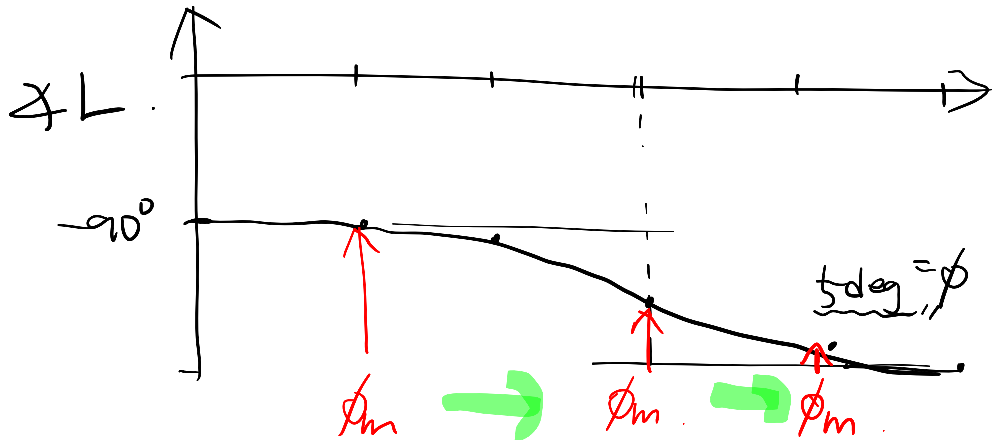


ω [rad/s]

$K_p = 1,\ 100,\ 10000$

$\dfrac{1}{s^2+10s}$ $\quad \omega = 1$

$|\cdot| \approx \frac{1}{10}$



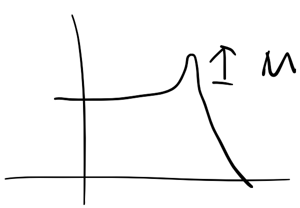
$5\text{deg} = \phi \Rightarrow \zeta \simeq \frac{5}{100} = 0.05.$

$\frac{1}{2\zeta} \simeq 10.$

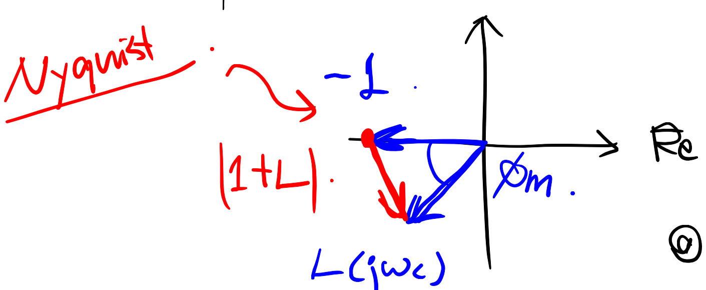
- Remarks.

  - $\boxed{\zeta \simeq \frac{\phi_m(\text{deg})}{100}.}$ (2nd) $\Rightarrow$ others as well

  - $t_r \simeq \frac{2.2}{\omega_b}.$ (1st)



$\uparrow M_r \simeq |T(j\omega)|_{\omega=\omega_c} = \left|\frac{L}{1+L}\right|_{\omega_c} = \frac{1}{|1+L|_{\omega_c}}.$



$1+L \Rightarrow L-(-1).$

@ $\omega=\omega_c$

For small ϕ_m.

$$|1+L| \simeq \phi_m \text{ [rad]}.$$

$$\Rightarrow \quad M_r \simeq \frac{1}{\phi_m} \simeq \frac{1}{2\zeta}$$

$$\Rightarrow \quad \zeta \simeq \frac{\phi_m \text{[rad]}}{2}.$$

$$= \frac{\phi_m \text{ [deg]}}{2 \cdot \frac{180}{\pi}} \quad 2 \cdot \frac{180}{\pi} \simeq \underline{\underline{100}}. !$$

$$= \frac{\phi_m \text{ [deg]}}{100}.$$