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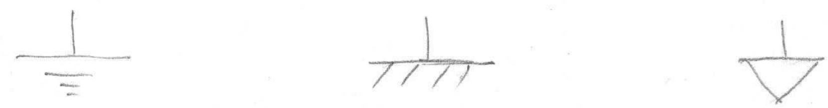
2021 / 2 / 22

< Differential Measurement >

- Objective
  - Understand grounds and common-mode noise
  - Differential amplifier
  - Instrumentation amplifier.

- Grounds : a particular "conductor" in electric circuits.

  - Symbols



Earth ground.
Mains ground

Chassis ground

PCB ground
IC ground.

  - Grounds can be categorized into two groups. p.124

① Safety grounds (Ott. p.112)

- Not conducting currents during normal operations
  e.g.) Earth ground, Mains ground.
- Role : protect people from electric shock.

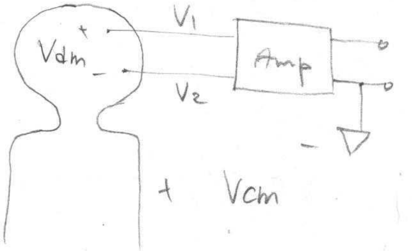
② Signal / power grounds (Ott. p.129)

- Conducting currents during normal operations.
  e.g.) PCB ground plane.
- Role : { Voltage : common reference potential for node voltages.
  Current : common low-impedance path for return currents.

< Common-mode Voltage >

- When we connect two electric systems together, the potential difference between the two grounds is not always zero.

Example Bio-potential measurement



In general, $V_{cm} \neq 0$

- Let's design a differential-input single-ended output amplifier

· V_1, V_2, and V_o are referenced to $\bigtriangledown$

$$\begin{cases} \text{Common-mode voltage} : V_{cm} \triangleq \dfrac{V_1+V_2}{2} \\ \text{Differential voltage} : V_{dm} \triangleq V_1 - V_2 \end{cases}$$

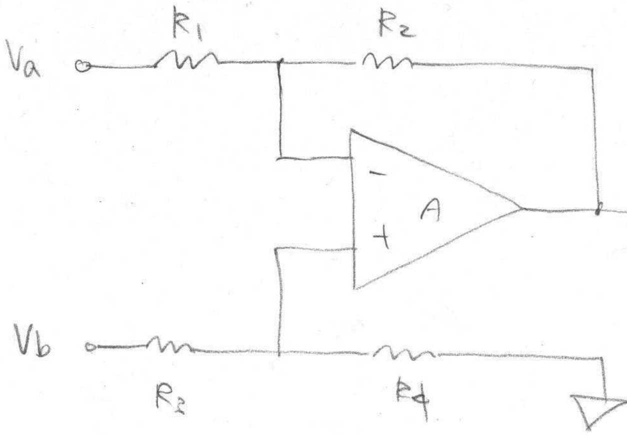
· In many cases, V_{cm} is unknown and unwanted, (e.g., EMI, ground noise, etc.)
and we want to reject it.

· On the other hand, V_{dm} contains useful information (measurement)
and we want to amplify it.

· Common-mode Rejection Ratio (CMRR) is the figure of merit

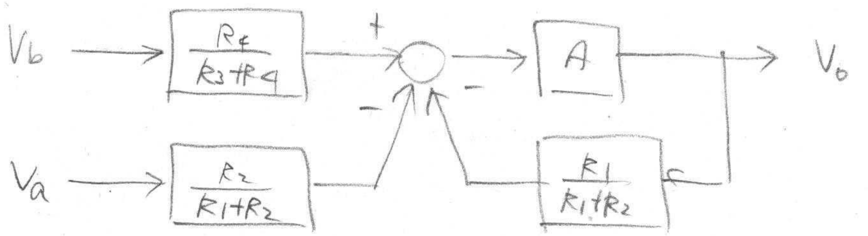
$$CMRR \triangleq \frac{A_{dm}}{A_{cm}}.$$

° Differential (difference) Amplifier. (Roberge p.449.)

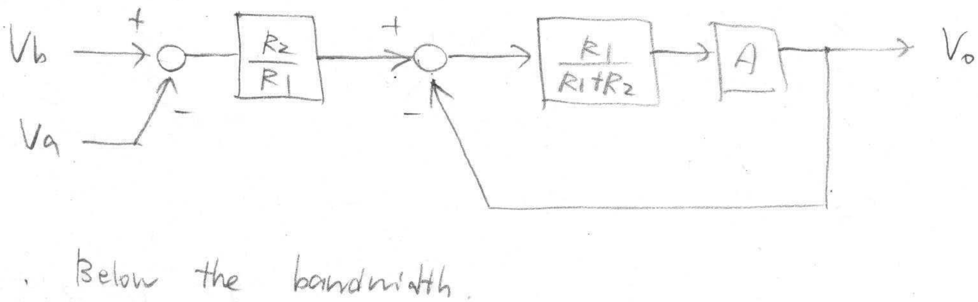


$$V_+ = \frac{R_4}{R_3+R_4} \cdot V_b \qquad\qquad V_o = A(V_+ - V_-)$$

$$V_- = \frac{R_2}{R_1+R_2} V_a + \frac{R_1}{R_1+R_2} V_o$$



If $\frac{R_4}{R_3} = \frac{R_2}{R_1} \Rightarrow \frac{R_4}{R_3+R_4} = \frac{R_2}{R_1+R_2}$.



. Below the bandwidth

$$V_o = \frac{R_2}{R_1}(V_b - V_a).$$

. Let $V_{cm} = \frac{1}{2}(V_b + V_a)$. $\Rightarrow$ $V_o = \underbrace{\frac{R_2}{R_1}}_{A_{dm}} V_{dm} + \underbrace{\emptyset}_{A_{cm}} \cdot V_{cm}$

$V_{dm} = V_b - V_a$

$$CMRR \triangleq \frac{A_{dm}}{A_{cm}} \rightarrow \infty$$

- In practice, A_{cm} to and therefore CMRR is finite.

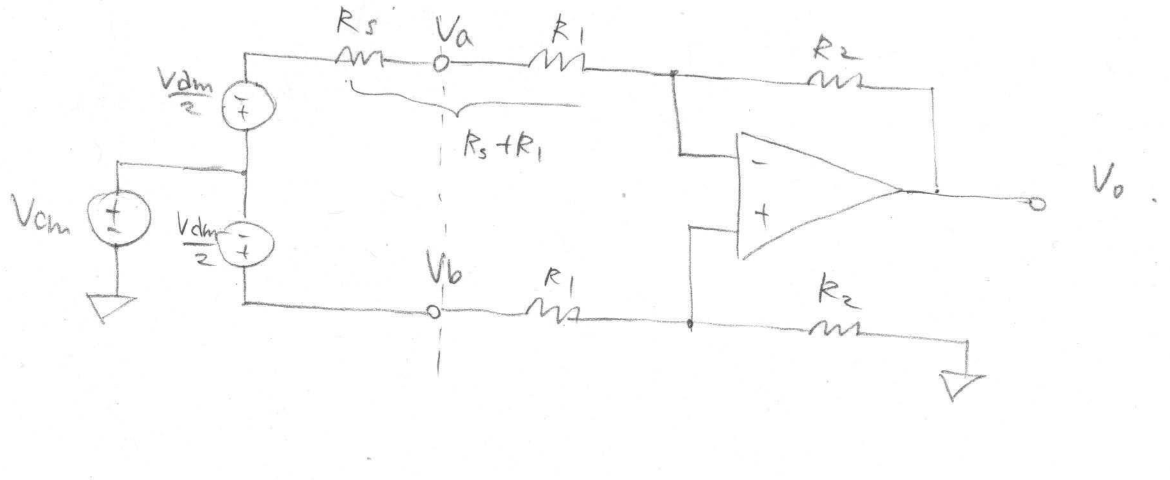
- CMRR is mainly limited by the resistor tolerance (ε). $\left(CMRR \gtrsim \frac{R_2/R_1 + 1}{4\varepsilon} \right)$

- There exist monolithic diff-amps with laser-trimmed resistors.

- Particularly useful when { the source impedance is small, and the common-mode voltage is large. ($V_{cm} > V_s$ possible. e.g. AC motor current measurement)

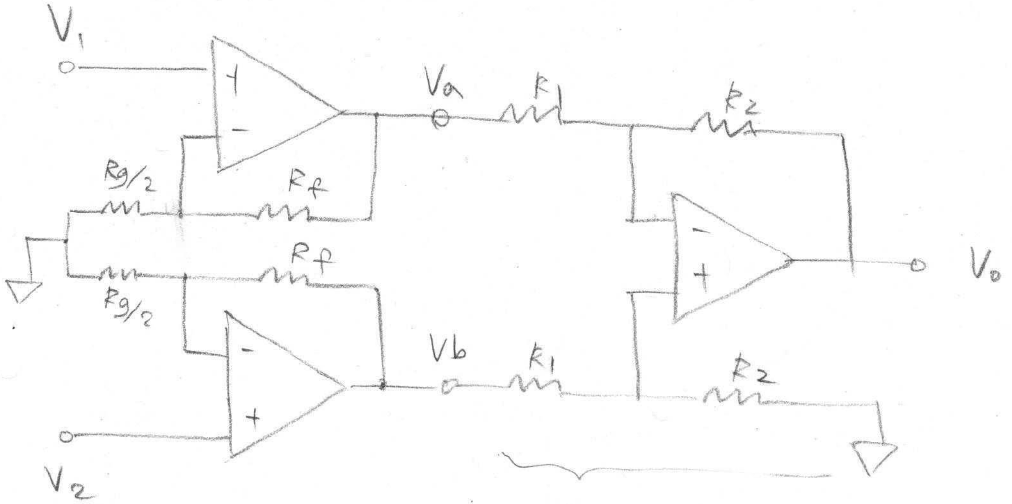
- Limitations

  - Finite input impedance : even if the resistors are matched well, the diff amp cannot reject V_{cm} well if the source impedance is not balanced.



< Buffered Diff-amp >

Let's place a buffer stage to increase the input impedance



$A_{cm} \neq 0$, $A_{dm} \simeq \frac{R_2}{R_1}$

$$V_a = \frac{R_f + R_g/2}{R_g/2} \cdot V_1$$

$$V_b = \frac{R_f + R_g/2}{R_g/2} V_2$$

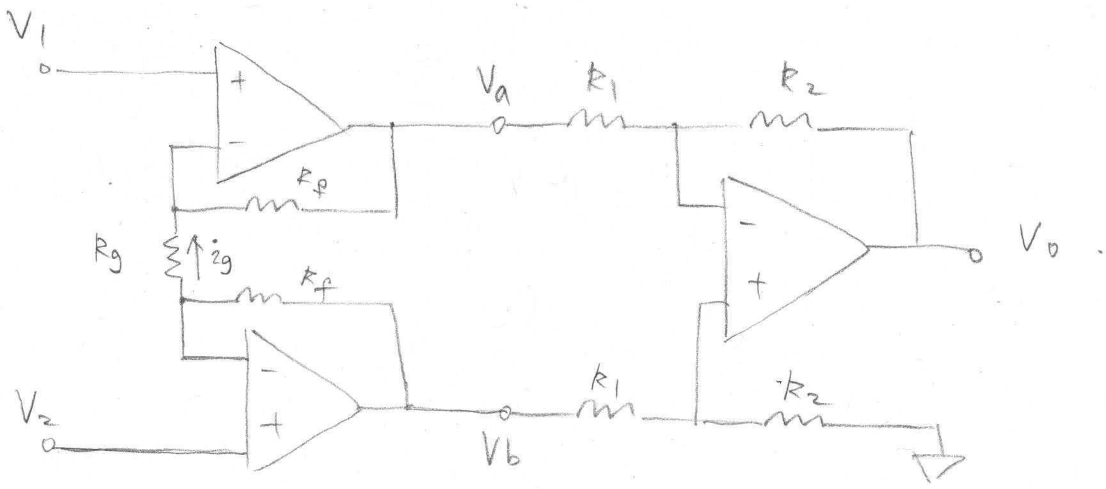
$$\therefore V_o = \underbrace{A_{dm} \left(\frac{R_f + R_g/2}{R_g/2} \right)}_{A_{dm}'} (V_2 - V_1) + \underbrace{A_{cm} \left(\frac{R_f + R_g/2}{R_g/2} \right)}_{A_{cm}'} \left(\frac{V_2 + V_1}{2} \right)$$

$$CMRR' = \frac{A_{dm}'}{A_{cm}'} = \frac{A_{dm}}{A_{cm}}$$ No improvement in CMRR

$$A_{dm}' = A_{dm} \cdot \left(\frac{2R_f + R_g}{R_g} \right).$$ $\therefore$ can be increased by decreasing R_g (gain resistor)

< Instrumentation Amplifier (In-amp) >

· Let the gain resistor R_g float !



$$i_g = \frac{V_2 - V_1}{R_g}$$

$$V_a = V_1 - R_f\, i_g = V_1 - \frac{R_f}{R_g}(V_2 - V_1) = \left(\frac{R_g + R_f}{R_g}\right) V_1 - \frac{R_f}{R_g} V_2$$

$$V_b = V_2 + R_f\, i_g = V_2 + \frac{R_f}{R_g}(V_2 - V_1) = -\frac{R_f}{R_g} V_1 + \left(\frac{R_g + R_f}{R_g}\right) V_2$$

$$\Rightarrow V_o = A_{dm} \cdot (V_b - V_a) + A_{cm}\left(\frac{V_b + V_a}{2}\right)$$

$$= \underbrace{A_{dm} \cdot \left(\frac{R_g + 2R_f}{R_g}\right)}_{A_{dm}'} (V_2 - V_1) + \underbrace{A_{cm}\left(\cancelto{1}{\frac{R_g}{R_g}}\right)}_{A_{cm}'} \left(\frac{V_2 + V_1}{2}\right)$$

$$CMRR' = \frac{A_{dm}'}{A_{cm}'} = \frac{A_{dm}}{A_{cm}} \cdot \left(\frac{R_g + 2R_f}{R_g}\right).$$ Improved by "$\frac{R_g + 2R_f}{R_g}$"

· This is because the buffer stage only amplifies V_{dm} and doesn't amplify V_{cm}.

- There exist monolithic In-amps with laser-trimmed resistors (R_1, R_2, and R_f). Typically $R_1 = R_2$.

- One external register R_g is needed for gain tuning.

- As opposed to diff-amps, V_{cm} cannot exceed the rail voltages.

- Extremely useful building blocks for measurement & instrumentation.