University of British Columbia
Department of Mechanical Engineering

MECH 463. Midterm 1, October 2, 2018

Suggested Time: 60 min **Allowed Time:** 70 min

Materials admitted: Pen, pencil, eraser, straightedge, simple scientific calculator without programming or communication capabilities, personal hand-written notes within one letter-size sheet of paper (one side).

There are 3 questions in this exam. You are asked to answer all three questions.

The purpose of this test is to evaluate your knowledge of the course material. Orderly presentation demonstrates your knowledge most clearly, while disorganized and unprofessional work creates serious doubt. Marks are assigned accordingly. A bonus of up to 2 marks will be given for exemplary presentation.

Complete the section below **during** the examination time **only.**

NAME:____

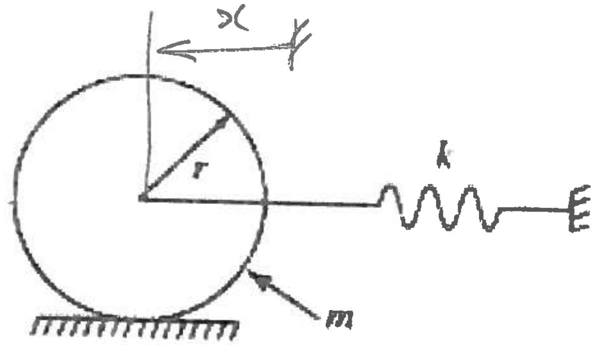
SIGNATURE:____

STUDENT NUMBER:___________________________________

	Mark Received	**Maximum Mark**
1		7
2		6
3		7
Presentation		2 bonus
Total		20+2

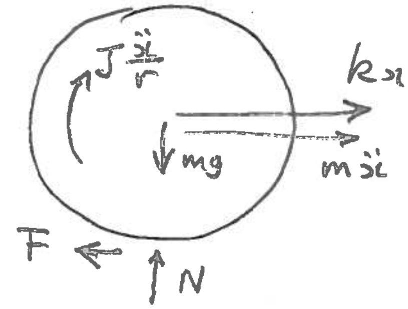
Name: ______________________________

1. A cylinder of radius r and mass m is attached to a rigid wall through a spring of stiffness k attached at its centre. The cylinder can vibrate by rolling on a rough surface.



   (a) Draw a free-body diagram of the vibrating cylinder and hence derive the equation of motion. *(Hint: $J_O = \frac{1}{2} m r^2$)*
   (b) Determine the natural frequency of vibration.
   (c) If the coefficient of friction between the cylinder and the rough surface is μ, derive a formula for the maximum amplitude of the free (natural frequency) vibration that can occur without slipping.

(a) Take moments around contact point



$$m\ddot{x}\cdot r + J_0\frac{\ddot{x}}{r} + kx\cdot r = 0$$

$$\div r \quad \left(m + \tfrac{1}{2}mr^2/r^2\right)\ddot{x} + kx = 0$$

$$\rightarrow \frac{3}{2}m\ddot{x} + kx = 0$$

(b) Try solution $x = C\cos(\omega t+\phi)$

$$\rightarrow \left(-\tfrac{3}{2}m\omega^2 + k\right)C\cos(\omega t+\phi) = 0$$

For non-trivial solution valid for all t $\quad C\cos(\omega t+\phi) \neq 0$

$$\rightarrow -\tfrac{3}{2}m\omega^2 + k = 0 \quad \rightarrow \quad \omega = \sqrt{\frac{2k}{3m}}$$

(c) $\Sigma F_{vertical} = N - mg = 0 \quad \rightarrow N = mg$

$\Sigma M_0 = J\frac{\ddot{x}}{r} + Fr = 0 \quad \rightarrow F = -J\frac{\ddot{x}}{r^2} = -\tfrac{1}{2}m\ddot{x}$

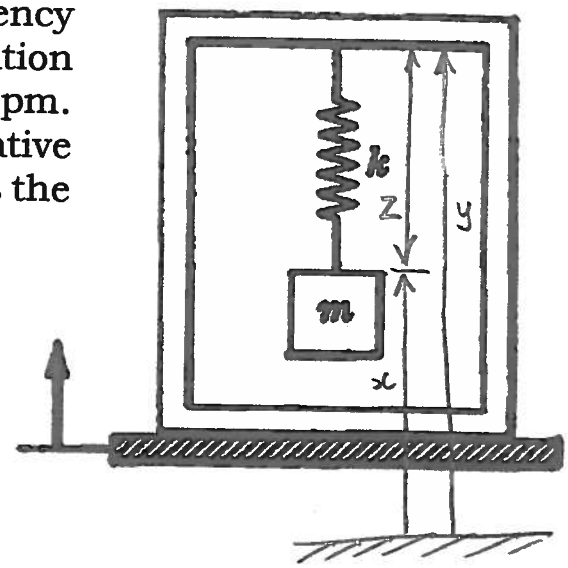
For $x = C\cos(\omega t+\phi) \rightarrow \ddot{x} - \omega^2 C\cos(\omega t+\phi)$

$\rightarrow F = \tfrac{1}{2}m\omega^2 C\cos(\omega t+\phi) \quad \rightarrow F_{max} = \tfrac{1}{2}m\omega^2 C < \mu N$ for no slip

$$\rightarrow C < \frac{\mu N}{\tfrac{1}{2}m\omega^2} = \frac{\mu mg}{\tfrac{1}{2}m\frac{2k}{3m}} = \frac{3\mu mg}{k}$$

Name: ______________________________

2. An undamped vibrometer of natural frequency 15Hz is used to measure the vibration amplitude of a machine running at 1500 rpm. The vibrometer registers a relative displacement amplitude of 0.1mm, what is the vibration amplitude of the machine ?

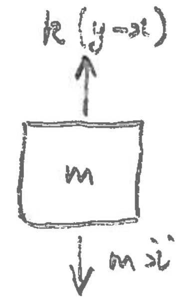


x = displacement of mass

y = displacement of frame

$z = y - x$ = stretching of spring

From FBD: $k(y-x) - m\ddot{x} = 0$



$$\rightarrow m(\ddot{z} - \ddot{y}) + kz = 0$$

$$\rightarrow m\ddot{z} + kz = m\ddot{y}$$

Base vibration $y = Y \cos \omega_f t$

$$\rightarrow \ddot{y} = -\omega_f^2 Y \cos \omega_f t$$

Try solution $z = Z \cos \omega_f t$

$$\rightarrow (-\omega_f^2 m + k) Z \cos \omega_f t = -m\omega_f^2 Y \cos \omega_f t$$

Solution valid for all $t \rightarrow \cos \omega_f t \neq 0$

$$\rightarrow Y = \frac{k - \omega_f^2 m}{-m\omega_f^2} Z = \frac{1 - \omega_f^2 \frac{m}{k}}{-\frac{m}{k}\omega_f^2} Z = \frac{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}{-\left(\frac{\omega_f}{\omega_n}\right)^2} Z$$

$$Y = \frac{1 - r^2}{-r^2} Z \quad \text{where } r = \frac{\omega_f}{\omega_n} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

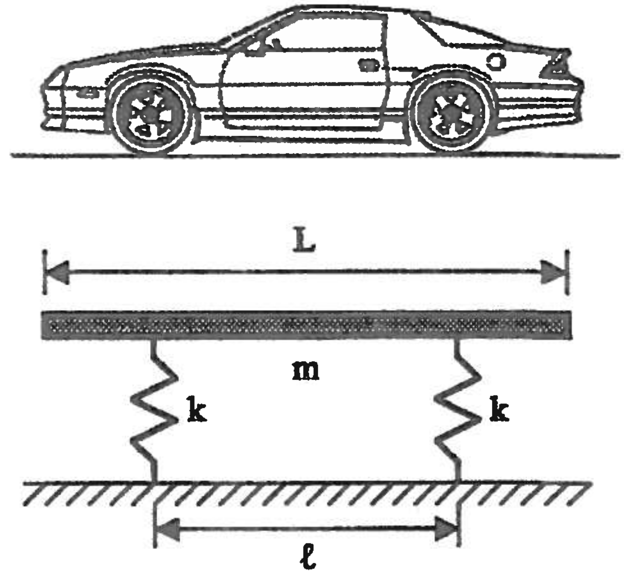
Here $\omega_n = 15 \times 2\pi = 30\pi$ rad/s $\qquad \omega_f = \frac{1500}{60} \times 2\pi = 50\pi$ rad/s

$$\rightarrow r = \frac{50\pi}{30\pi} = \frac{5}{3} \quad \rightarrow Y = \frac{1 - \left(\frac{5}{3}\right)^2}{-\left(\frac{5}{3}\right)^2} Z = \frac{1 - \frac{25}{9}}{-\frac{25}{9}} Z = \frac{-16}{-25} Z$$

$$\rightarrow Y = \frac{-16}{-25} \times 0.1 = \underline{0.064 \text{ mm}} \qquad \text{in-phase}$$

Name: ____

3. A car is modeled as a uniform beam (representing the body) resting of two springs (representing the suspension system. The beam has mass m, length L and centroidal moment of inertia J = $mL^2/12$. The springs each have stiffness k and are symmetrically placed, distance ℓ apart.



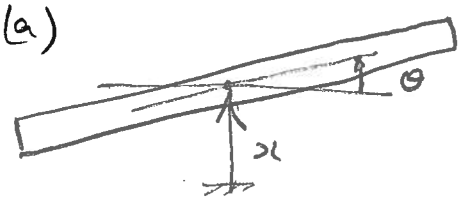
(a) Choose a coordinate system (displacement and rotation) based on the beam centroid and draw a labeled free-body diagram.

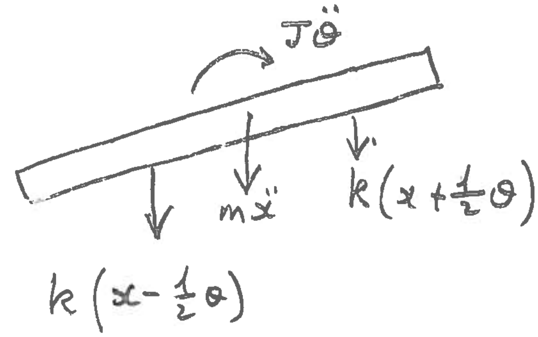
(b) Formulate the equations of motion in matrix format. What is notable about the equations?

(c) Determine the natural frequencies and mode shapes.

(d) What ratio ℓ/L is required for the two natural frequencies to be equal ?

(a)





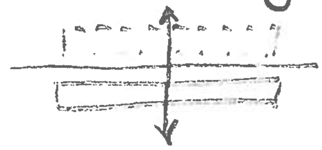
(b) FBD $\Sigma F_v = m\ddot{x} + k(x - \frac{1}{2}\theta) + k(x + \frac{1}{2}\theta) = 0$

$\rightarrow m\ddot{x} + 2kx = 0$

This equation involves x only

(c)

$\rightarrow \omega_1 = \sqrt{\frac{2k}{m}}$ mode shape

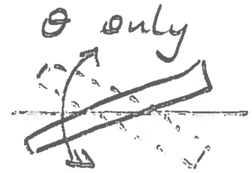


FBD $\Sigma M_0 = J\ddot{\theta} + \frac{1}{2}k(x + \frac{1}{2}\theta) - \frac{1}{2}k(x - \frac{1}{2}\theta) = 0$

$\rightarrow \frac{1}{12}mL^2\ddot{\theta} + \frac{kl^2}{2}\theta = 0$

This equation involves θ only

$\rightarrow \omega_2 = \sqrt{\frac{6kl^2}{mL^2}}$ mode shape



$$\begin{bmatrix} m & 0 \\ 0 & mL \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 2k & 0 \\ 0 & kL \end{bmatrix}\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(d) $\omega_1 = \omega_2$ when

$\frac{2k}{m} = \frac{6kl^2}{mL^2}$

$\rightarrow \frac{l}{L} = \sqrt{\frac{1}{3}}$