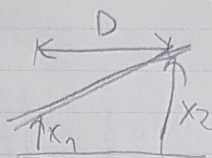
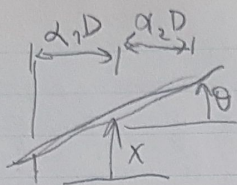
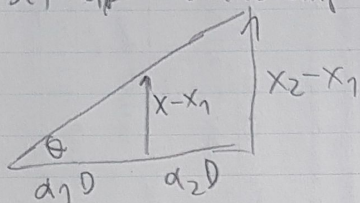


relab (1):

[I did this before the lectures so I didn't use the "redraw FBD" method.]



→ Set up relationship:



$$\tan \theta \approx \frac{X_2 - X_1}{D} \quad (1)$$

$$\frac{X_2 - X_1}{D} = \frac{X - X_1}{\alpha_1 D} \quad (2)$$

→ (1) At small  $\theta$ ,  $\tan \theta \approx \theta$

$$\tan \theta \approx \theta \approx \frac{X_2 - X_1}{D} \rightarrow D\theta = X_2 - X_1$$

$$\ddot{\theta} = \frac{\ddot{X}_2 - \ddot{X}_1}{D} \rightarrow D\ddot{\theta} = \ddot{X}_2 - \ddot{X}_1$$

(2)

$$\frac{X_2 - X_1}{D} = \frac{X - X_1}{\alpha_1 D}$$

$$\alpha_1 (X_2 - X_1) = X - X_1$$

$$X = \alpha_1 X_2 + (1 - \alpha_1) X_1$$

$$= \alpha_1 X_2 + \alpha_2 X_1$$

$$\ddot{X} = \alpha_1 \ddot{X}_2 + \alpha_2 \ddot{X}_1$$

→ Sub these equations into Eq 6 from lab manual



$$\rightarrow 0 = m(\alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2} \left( \left( \frac{L_1+L_2}{\alpha_2\alpha_1} \right) (\alpha_1x_2 + \alpha_2x_1) + (L_1-L_2)(x_2-x_1) \right)$$

(3)

$$0 = \frac{mR^2}{D^2} (\ddot{x}_2 - \ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2} \left( (L_1-L_2)(\alpha_1x_2 + \alpha_2x_1) + (\alpha_2L_1 + \alpha_1L_2)(x_2-x_1) \right)$$

(4)

→ Simplify

$$(3) \quad 0 = m(\alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2} \left( \frac{\alpha_1L_1 + \alpha_2L_2}{\alpha_2} x_2 + \frac{\alpha_1L_1 + \alpha_2L_2}{\alpha_1} x_1 + L_1x_2 - L_2x_2 - L_1x_1 + L_2x_1 \right)$$

$$0 = \alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_1L_2} \left( \frac{\alpha_1L_1}{\alpha_2} x_2 + L_2x_2 + L_1x_1 + \frac{\alpha_2L_2}{\alpha_1} x_1 + L_1x_2 - L_2x_2 - L_1x_1 + L_2x_1 \right)$$

$$0 = \alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_1L_2} \left( \frac{\alpha_1}{\alpha_2} + \frac{\alpha_2}{\alpha_1} \right) L_1 x_2 + \frac{g\alpha_1\alpha_2}{L_1L_2} \left( \frac{\alpha_2}{\alpha_1} + \frac{\alpha_1}{\alpha_2} \right) L_2 x_1$$

$$0 = \alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1 + \frac{g\alpha_1}{L_2} (\alpha_1 + \alpha_2) x_2 + \frac{g\alpha_2}{L_1} (\alpha_2 + \alpha_1) x_1$$

$$(4) \quad 0 = \alpha_1 \ddot{x}_2 + \alpha_2 \ddot{x}_1 + \frac{g\alpha_1}{L_2} x_2 + \frac{g\alpha_2}{L_1} x_1$$

$$0 = \frac{mR^2}{D^2} (\ddot{x}_2 - \ddot{x}_1) + \frac{mg\alpha_1\alpha_2}{L_1L_2} \left( L_1\alpha_1x_2 + L_1\alpha_2x_1 - L_2\alpha_1x_2 - L_2\alpha_2x_1 + L_1\alpha_2x_2 + L_1\alpha_1x_1 + L_2\alpha_1x_2 + L_2\alpha_2x_1 \right)$$

$$0 = \frac{R^2}{D^2} (\ddot{x}_2 - \ddot{x}_1) + \frac{g\alpha_1\alpha_2}{L_1L_2} \left( L_1(\alpha_1 + \alpha_2)x_2 - L_2(\alpha_2 + \alpha_1)x_1 \right)$$

$$0 = \frac{R^2}{D^2} \ddot{x}_2 - \frac{R^2}{D^2} \ddot{x}_1 + \frac{g\alpha_1\alpha_2}{L_2} x_2 - \frac{g\alpha_1\alpha_2}{L_1} x_1$$



→ To take simplified (3) & (4) and get diagonal matrix,  
we need to combine (3) & (4) in these ways to get eq. (5) & (6):

$$(5) = \alpha_1(3) + (4)$$

$$(6) = -\alpha_2(3) + 4$$

$$(5) \quad 0 = \alpha_1^2 \ddot{x}_2 + \alpha_1 \alpha_2 \ddot{x}_1 + g \frac{\alpha_1^2}{L_2} x_2 + g \frac{\alpha_2}{L_1} x_1 \\ + \frac{R^2}{D^2} \ddot{x}_2 - \frac{R^2}{D^2} \ddot{x}_1 + g \frac{\alpha_1 \alpha_2}{L_2} x_2 - g \frac{\alpha_1 \alpha_2}{L_1} x_1$$

$$0 = \left( \frac{R^2}{D^2} + \alpha_1^2 \right) \ddot{x}_2 + \left( -\frac{R^2}{D^2} + \alpha_1 \alpha_2 \right) \ddot{x}_1 + \left( \frac{g \alpha_1}{L_2} \right) x_2$$

$$(6) \quad 0 = -\alpha_1 \alpha_2 \ddot{x}_2 - \alpha_2^2 \ddot{x}_1 - g \frac{\alpha_1 \alpha_2}{L_2} x_2 - g \frac{\alpha_2^2}{L_1} x_1 \\ + \frac{R^2}{D^2} \ddot{x}_2 - \frac{R^2}{D^2} \ddot{x}_1 + g \frac{\alpha_1 \alpha_2}{L_2} x_2 - g \frac{\alpha_1 \alpha_2}{L_1} x_1$$

$$0 = \left( \frac{R^2}{D^2} - \alpha_1 \alpha_2 \right) \ddot{x}_2 + \left( \frac{-R^2}{D^2} - \alpha_2^2 \right) \ddot{x}_1 + \left( -\frac{g \alpha_2}{L_1} \right) x_1$$

→ combining (5) & (6) into matrixes, we get:

$$\begin{bmatrix} \frac{R^2}{D^2} + \alpha_1^2 & -\frac{R^2}{D^2} + \alpha_1 \alpha_2 \\ -\frac{R^2}{D^2} + \alpha_1 \alpha_2 & \frac{R^2}{D^2} + \alpha_2^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_2 \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} \frac{g \alpha_1}{L_2} & 0 \\ 0 & \frac{g \alpha_2}{L_1} \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = 0$$



→ To get the same plot as mass coords, we have to flip the equations order. Matrixes become:

$$\begin{bmatrix} \frac{R^2}{D^2} + \alpha_2^2 & -\frac{R^2}{D^2} + \alpha_1 \alpha_2 \\ -\frac{R^2}{D^2} + \alpha_1 \alpha_2 & \frac{R^2}{D^2} + \alpha_1^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} g \frac{\alpha_2}{L_1} & 0 \\ 0 & g \frac{\alpha_1}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$