



Step response = Complementary function + Particular integral (as for the 1<sup>st</sup> order system)

Mechanical oscillator (mass-spring-damper) example (see figure):

$$m\ddot{y} + b\dot{y} + ky = f(t) = ku(t)$$

$$\rightarrow \ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \omega_n^2 u(t)$$

$$\rightarrow \omega_n^2 = \frac{k}{m}, 2\zeta\omega_n = \frac{b}{m}$$

$$\rightarrow \zeta = \frac{b}{2\sqrt{km}}$$

$$\omega_d = \sqrt{1 - \zeta^2} \omega_n$$

$$\omega_r = \sqrt{1 - 2\zeta^2} \omega_n$$

$\zeta$  = damping ratio

$\omega_n$  = undamped natural frequency

$\omega_d$  = damped natural frequency

$\omega_r$  = resonant frequency

$$\tau = \frac{1}{\zeta\omega_n} = \text{time constant}$$