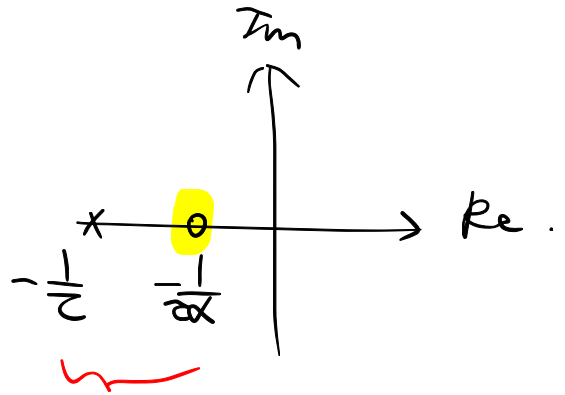
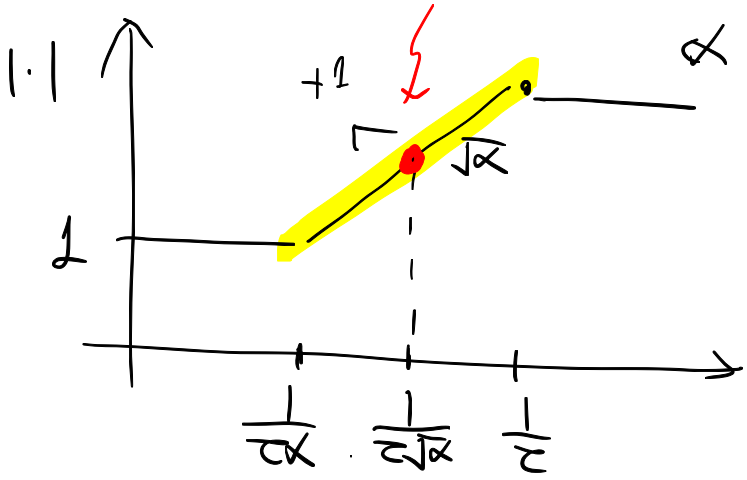
L15 – PID Control Design

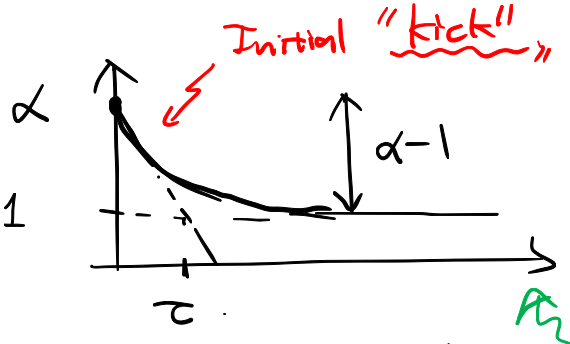
- Lead Comp.

$$H(s) = \frac{\alpha\tau s + 1}{\tau s + 1} \qquad (\alpha > 1) \quad (\tau > 0)$$



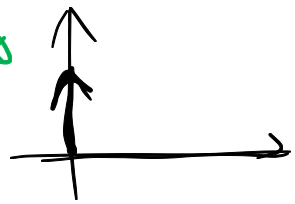
doublet.

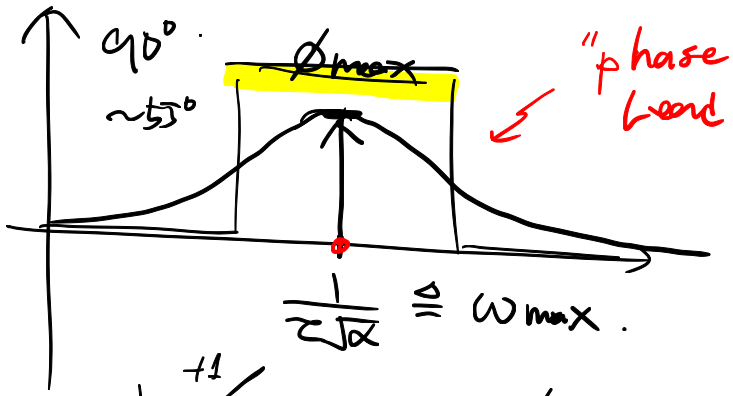


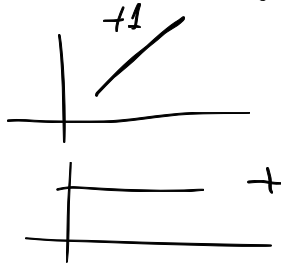


$$\frac{\alpha\tau s+1}{\tau s+1} = \frac{\alpha\tau s+\alpha-\alpha+1}{\tau s+1}$$

$$= \alpha - \frac{\alpha-1}{\tau s+1}$$

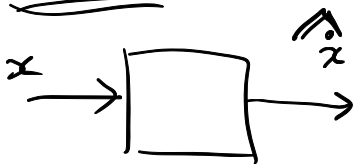






$\alpha = 10$.

- Lead : freq selective differentiator.



Estimator for derivatives.

- Use H to compensate the loop for ϕ around ω_{max}

- $\phi_{max} = \sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)$. $\alpha \to 1$. $\phi_{max} = 90^\circ$

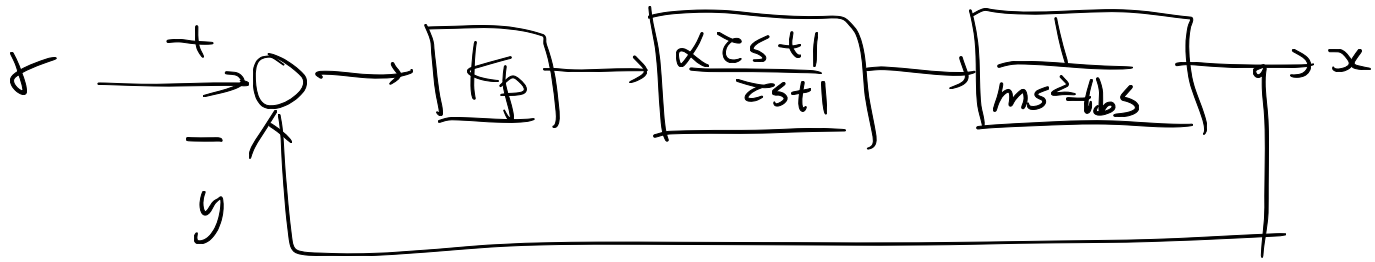
  $\alpha = 10$ $\phi_{max} \simeq 55^\circ$

- $\omega_{max} = \sqrt{\omega_p \cdot \omega_z} = \frac{1}{\tau\sqrt{\alpha}}$.

(1) $\omega_c = \omega_{max}$. → (2) Adjust K_p. $|L(j\omega_{max})| = 1$

Example. 

$f = ms^2 + bs$.



- Design steps.

① Decide on the target ω_c^* by looking at the Bode plot

② Implement a lead such that.

$$\omega_c^* \simeq \omega_{max}.$$

③ Set K_p such that. $\omega_c = \omega_c^*$.

$$\Rightarrow |L(j\omega_c^*)| = 1$$

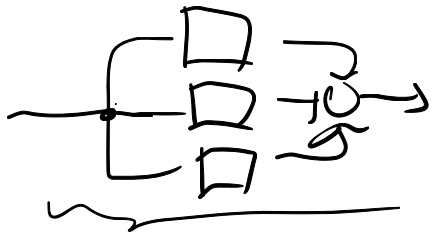
$K_p \rightarrow K_p''$

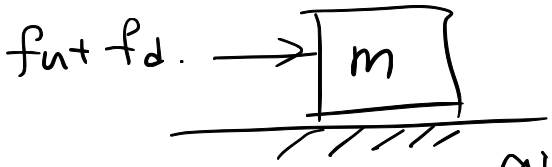
- Trade-off. (+) $\phi_{max} \uparrow$. $(\phi_{max} < 90^\circ)$

$\alpha \uparrow$ (−) $\downarrow$ low-freq gain / $\uparrow$ high-freq gain.

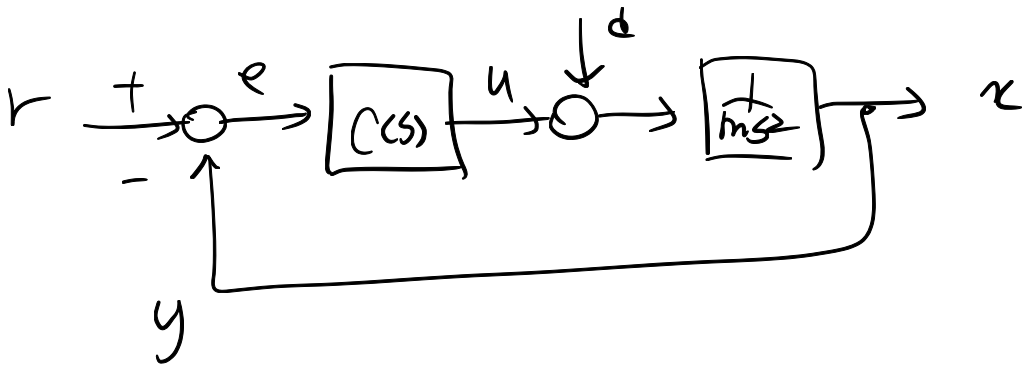
< PID Controller Design >.

"Series form".





air bearing.



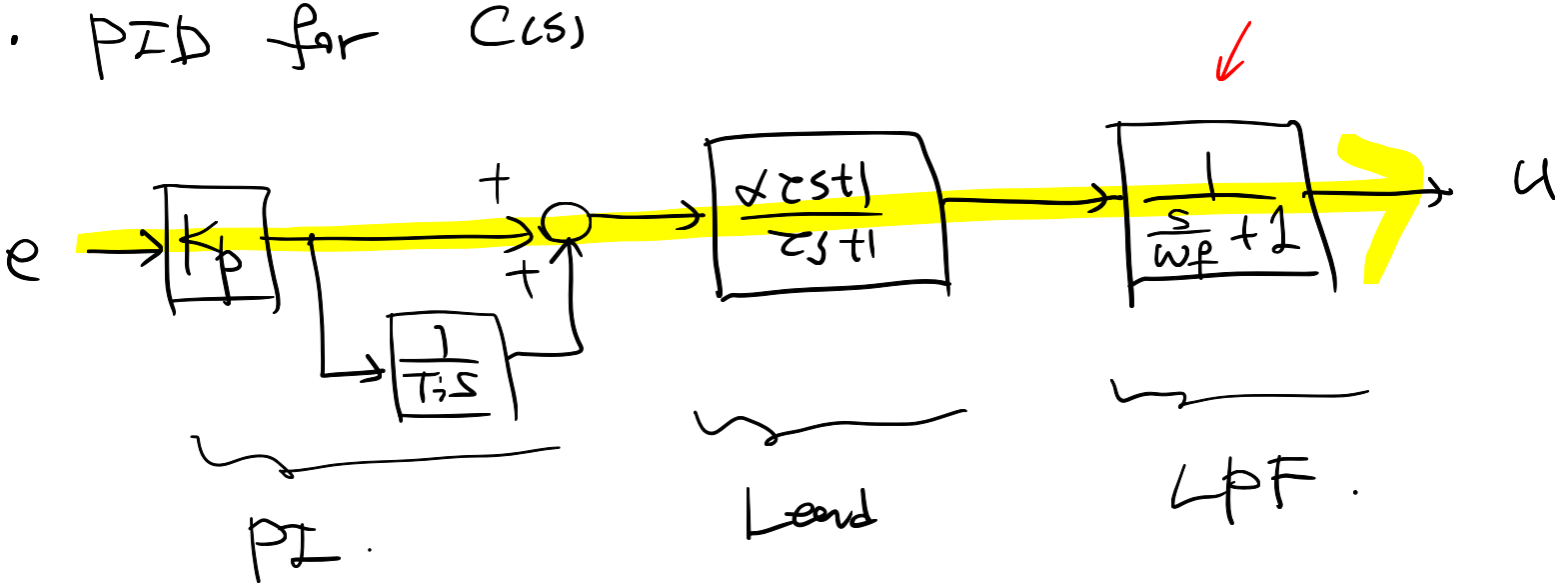
$P = \frac{1}{ms^2}$.

① $\frac{X}{R} = \frac{CP}{1+CP}$ "Tracking"

② $\frac{X}{D} = \frac{P}{1+CP}$. "Load sensitivity" → Dist rejection.

$$= \frac{\frac{1}{ms^2}}{1 + C\cdot\frac{1}{ms^2}} = \frac{1}{ms^2 + C(s)}$$ "Dynamic stiffness"

- PID for $C(s)$

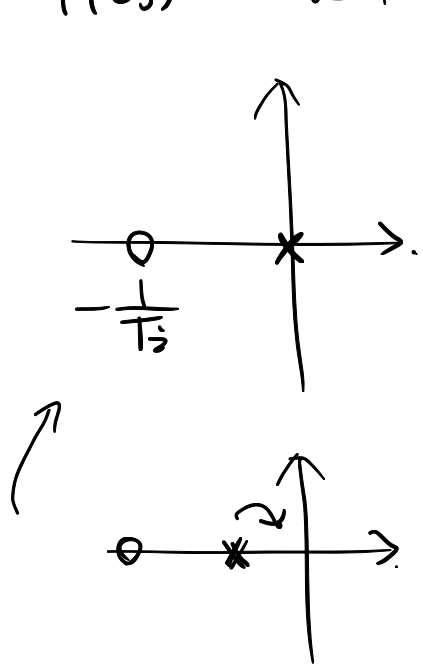


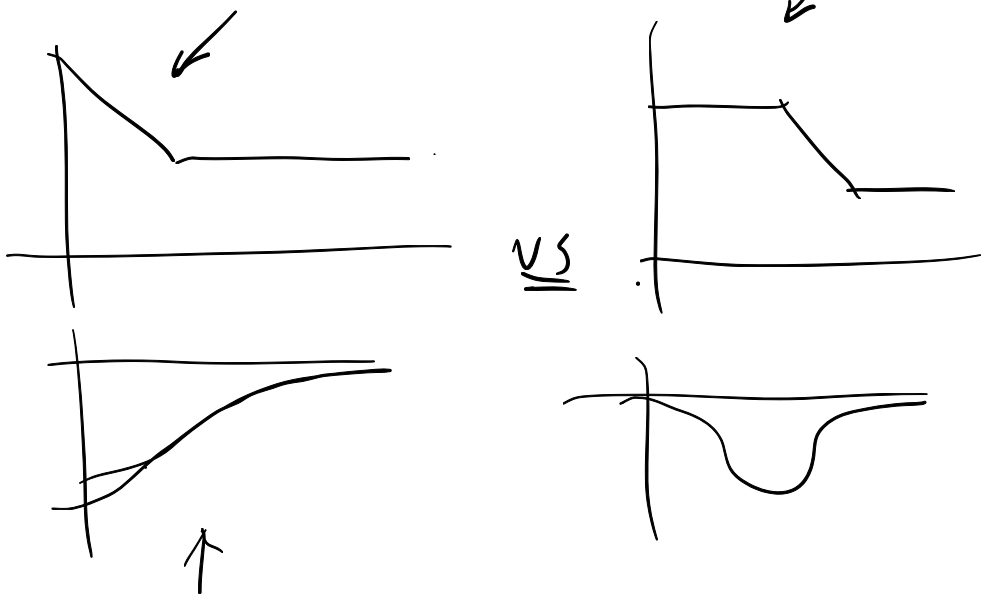
- Better for loop shaping.

$$L(s) = C(s)\,P(s)$$

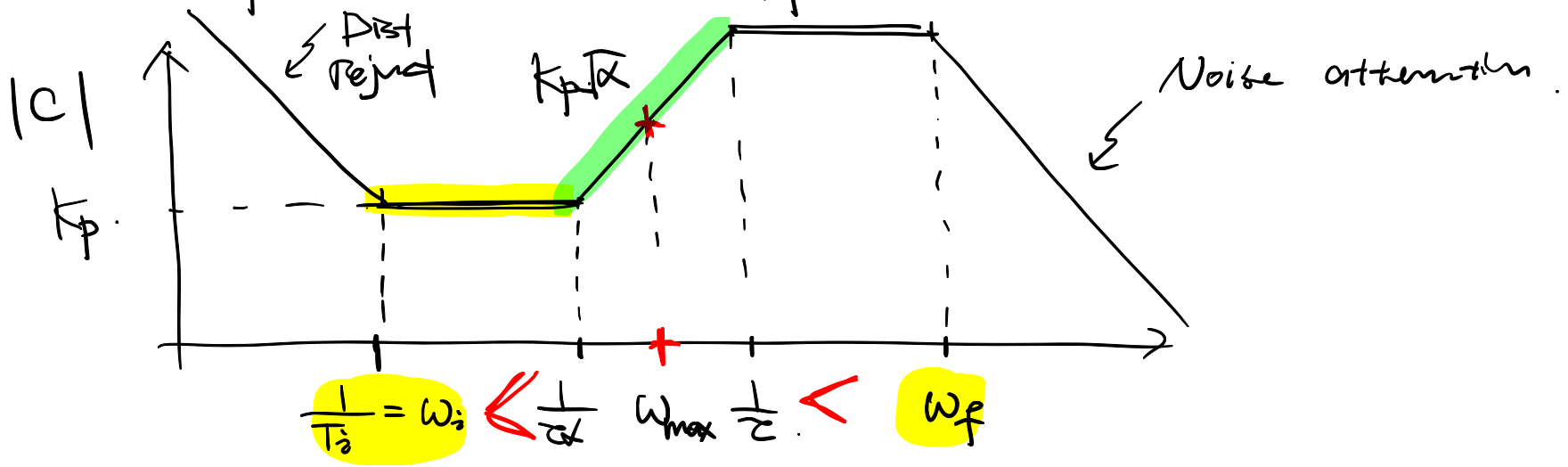
- PI. is a special case of Lag comp.

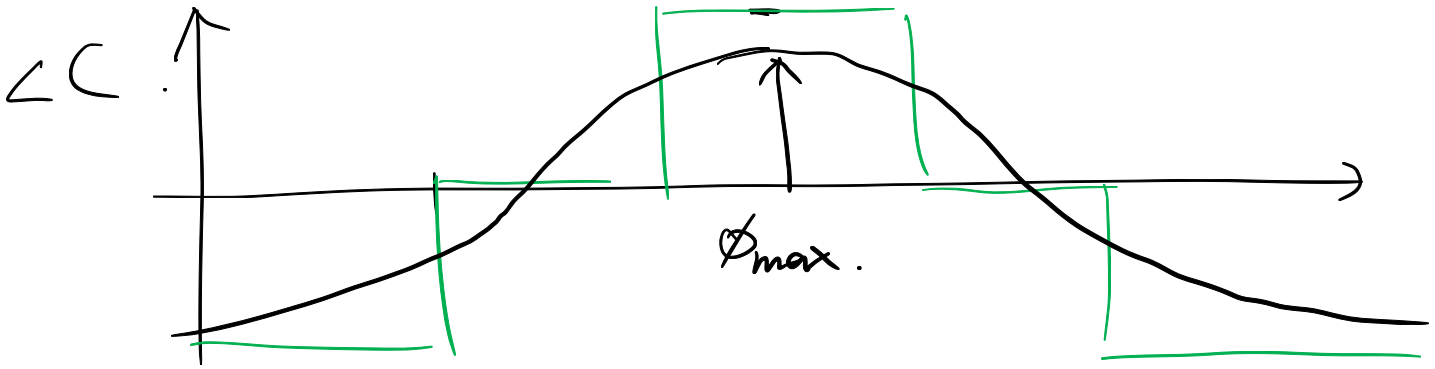
$$H(s) = 1 + \frac{1}{T_i s} = \frac{T_i s + 1}{T_i s}$$





- Bode plot of $C(s)$.

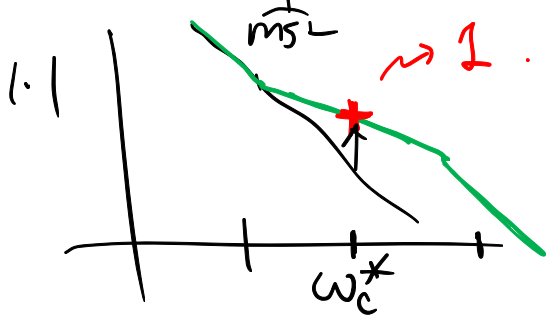




- PID tuning. $P(s) = \frac{1}{ms^2}$

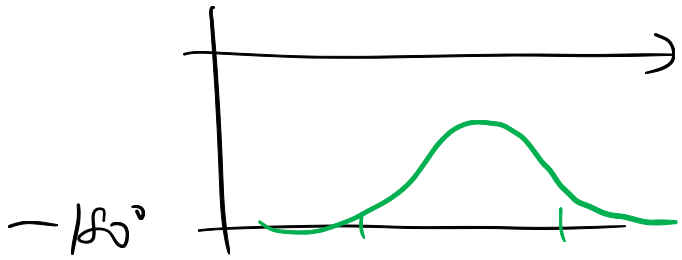
① Decide ω_c^* { Sensor BW. power amp BW. * Delay in digital cont. High-freq res.

② Implement Lead.



Set K_p. $\Rightarrow$ $K_p \cdot |P| \cdot \sqrt{\alpha} = 1$.

$\Rightarrow K_p = \frac{1}{|P|\sqrt{\alpha}}$.



③ Set K_p such that $\omega_c = \omega_c^*$

(Raise K_p 0 → K_p^*)

④. Introduce Integrator "Lag"

* $\omega_i < \omega_c$. $\left(\omega_i = \frac{1}{10}\omega_c\right)$

⑤ Introduce LPF. $\left(\omega_p = 10\,\omega_c\right)$

$\omega_p > \omega_c$