University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2010-2011): Introduction to Robotics
Midterm Examination #2, March 24, 2011
Closed Book - 80 Minutes
Maximum - 40 marks

Problem 1.

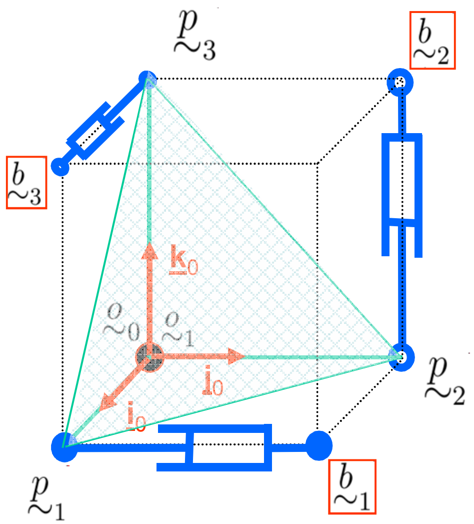
Consider the three degree of freedom rotational motion Stewart platform shown in Figure 1, which has a coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$ attached to a base (base hinge points shown in red), and a coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$ (not shown) attached to the platform shown in green. The platform is pyramidal in shape. It has a ball joint such that $\underset{\sim}{o}_1$ and $\underset{\sim}{o}_0$ always coincide ($\underset{\sim}{o}_1$ is fixed), and three mounting hinges (also ball joints) $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$, $\underset{\sim}{p}_3$ to which actuating cylinders are mounted. When the platform is at its nominal position, $\underline{C}_0$ and $\underline{C}_1$ coincide.
By extending the leg lengths q_1, q_2, q_3, an object attached to the platform, such as a camera attached to a plate formed by $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$, $\underset{\sim}{p}_3$, can be oriented with high speed. The base hinge points $\underset{\sim}{b}_1$, $\underset{\sim}{b}_2$, $\underset{\sim}{b}_3$ lie at the vertices of a cube with side length of l attached to $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and have fixed coordinates b_1, b_2, b_3 with respect to $\{\underset{\sim}{o}_0, \underline{C}_0\}$. The platform hinge points $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$, $\underset{\sim}{p}_3$ lie at the vertices of the pyramid. These coincide with the vertices of the cube when the platform is in its nominal position. These vertices have fixed coordinates p_1, p_2, p_3 with respect to $\{\underset{\sim}{o}_1, \underline{C}_1\} = \{\underset{\sim}{o}_0, \underline{C}_1\}$.
(a) *(10 marks)* Find p_1, p_2, p_3 and b_1, b_2, b_3 and solve the platform inverse kinematics, i.e., find the leg lengths q_1, q_2 and q_3 (with $q_1 = \|\underset{\sim}{p}_1 - \underset{\sim}{b}_1\|$, etc) given the coordinates Q of a platform-attached frame $\underline{C}_1$ in base frame $\underline{C}_0$, i.e. $\underline{C}_1 = \underline{C}_0 Q$.
(b) *(10 marks)* Find the 3×3 platform Jacobian giving the leg extension rates as a function of platform angular velocity in base frame ω (recall $\dot{Q} = \omega \times Q$). When will the platform be in a singular configuration? Explain.

Problem 2.

Consider the 6-DOF manipulator shown in Figure 2.
(a) *(10 marks)* Label the axes and the origins of the coordinate systems attached to each link. Find the manipulator Jacobian and use it to find the manipulator singular configurations.

(b) *(10 marks)* Solve the manipulator inverse kinematics. You may specify all solutions in terms of Kahan's problems (see attached page).

Figure P1



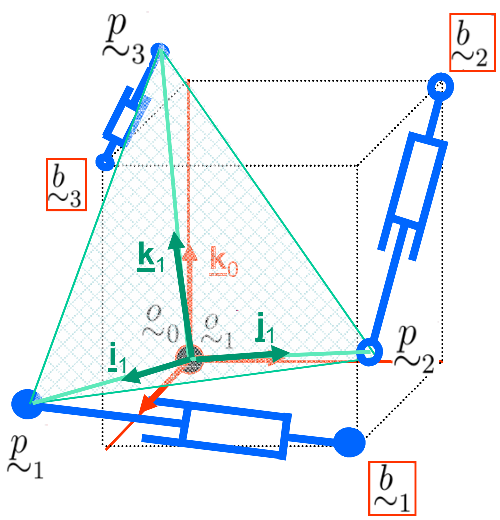
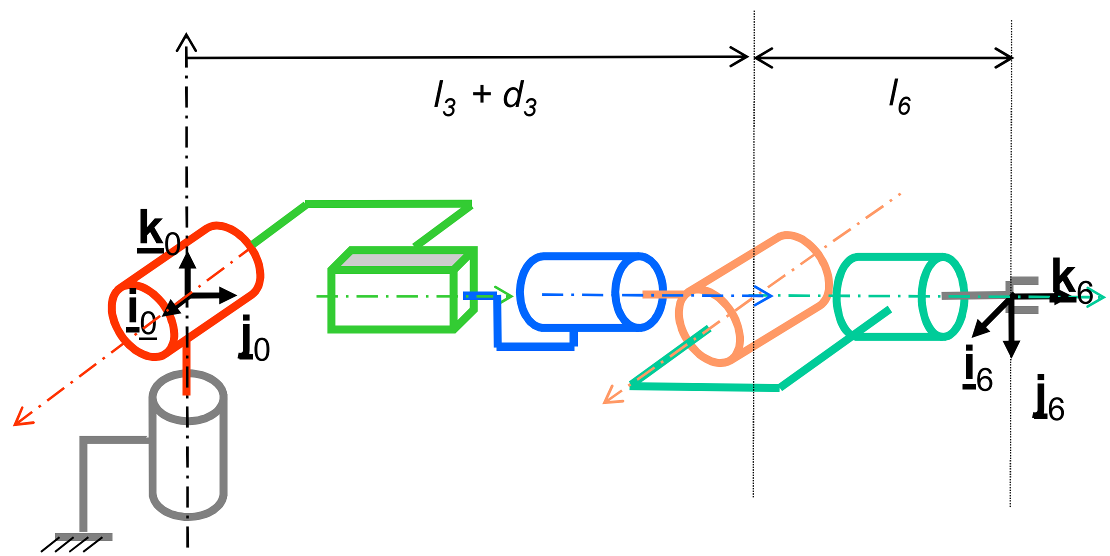
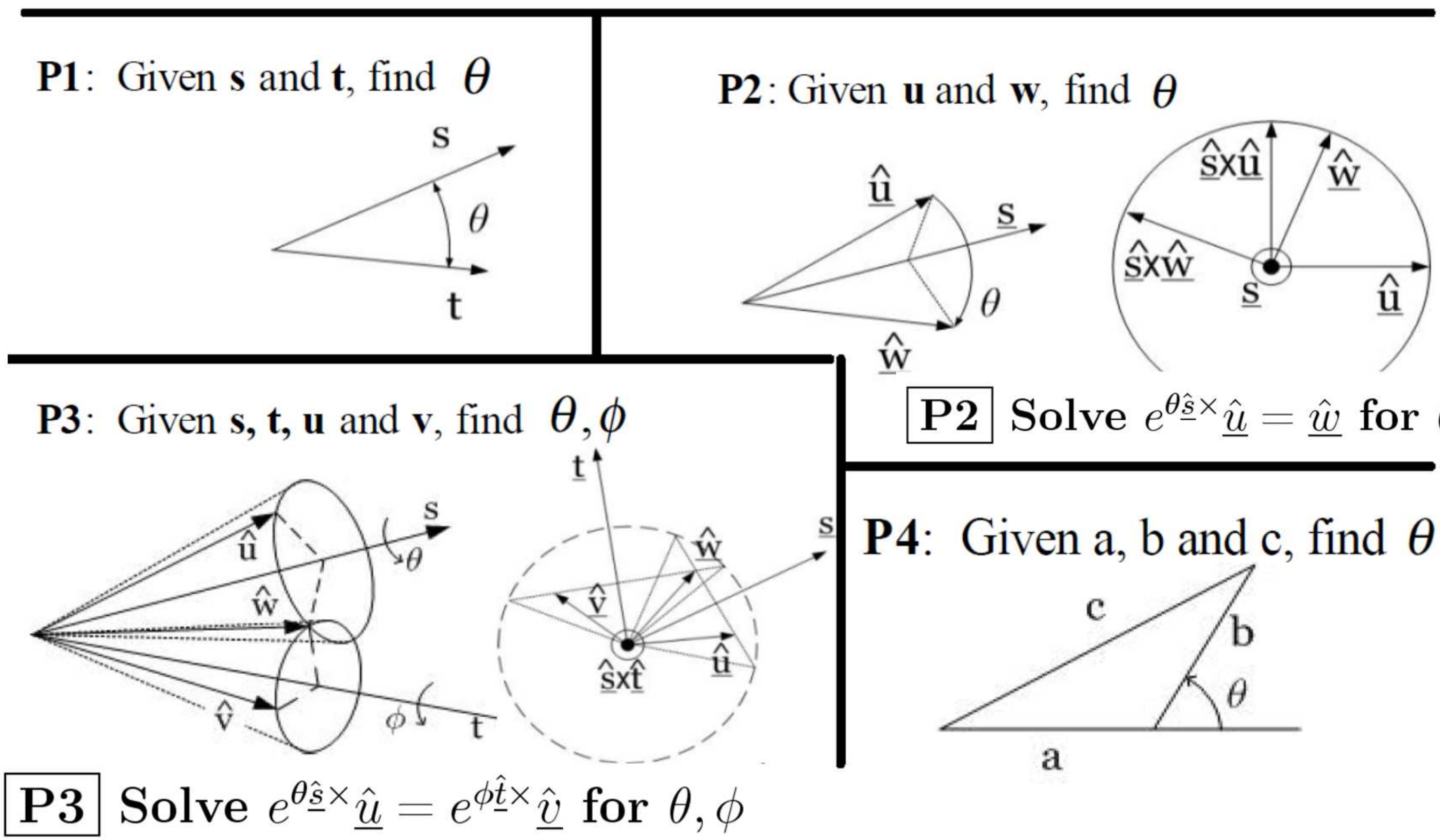


Figure P1.a: Platform shown in a different orientation

Figure P2





P1: Given **s** and **t**, find θ
s
θ
t
P2: Given **u** and **w**, find θ
$\underline{\hat{u}}$
s
θ
$\underline{\hat{w}}$
$\underline{\hat{s}}\times\underline{\hat{u}}$
$\underline{\hat{w}}$
$\underline{\hat{s}}\times\underline{\hat{w}}$
s
$\underline{\hat{u}}$
P2 Solve $e^{\theta\hat{\underline{s}}\times}\underline{\hat{u}} = \underline{\hat{w}}$ for θ
P3: Given **s, t, u** and **v**, find θ, ϕ
s
$\hat{u}$
θ
$\hat{w}$
$\hat{v}$
ϕ
t
t
$\underline{\hat{w}}$
$\underline{\hat{v}}$
s
$\underline{\hat{u}}$
$\hat{s}\times\hat{t}$
P4: Given a, b and c, find θ
c
b
θ
a
P3 Solve $e^{\theta\hat{\underline{s}}\times}\underline{\hat{u}} = e^{\phi\hat{\underline{t}}\times}\underline{\hat{v}}$ for θ, ϕ

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Solutions to Midterm Examination #2

Problem 1.

Consider the three degree of freedom rotational motion Stewart platform shown in Figure 1, which has a coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$ attached to a base (base hinge points shown in red), and a coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$ (not shown) attached to the platform shown in green. The platform is pyramidal in shape. It has a ball joint such that $\underset{\sim}{o}_1$ and $\underset{\sim}{o}_0$ always coincide ($\underset{\sim}{o}_1$ is fixed), and three mounting hinges (also ball joints) $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$, $\underset{\sim}{p}_3$ to which actuating cylinders are mounted. When the platform is at its nominal position, $\underline{C}_0$ and $\underline{C}_1$ coincide.
By extending the leg lengths q_1, q_2, q_3, an object attached to the platform, such as a camera attached to a plate formed by $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$, $\underset{\sim}{p}_3$, can be oriented with high speed. The base hinge points $\underset{\sim}{b}_1$, $\underset{\sim}{b}_2$, $\underset{\sim}{b}_3$ lie at the vertices of a cube with side length of l attached to $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and have fixed coordinates b_1, b_2, b_3 with respect to $\{\underset{\sim}{o}_0, \underline{C}_0\}$. The platform hinge points $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$, $\underset{\sim}{p}_3$ lie at the vertices of the pyramid. These coincide with the vertices of the cube when the platform is in its nominal position. These vertices have fixed coordinates p_1, p_2, p_3 with respect to $\{\underset{\sim}{o}_1, \underline{C}_1\} = \{\underset{\sim}{o}_0, \underline{C}_1\}$.
(a) *(10 marks)* Find p_1, p_2, p_3 and b_1, b_2, b_3 and solve the platform inverse kinematics, i.e., find the leg lengths q_1, q_2 and q_3 (with $q_1 = \|\underset{\sim}{p}_1 - \underset{\sim}{b}_1\|$, etc) given the coordinates Q of a platform-attached frame $\underline{C}_1$ in base frame $\underline{C}_0$, i.e. $\underline{C}_1 = \underline{C}_0 Q$.
(b) *(10 marks)* Find the 3×3 platform Jacobian giving the leg extension rates as a function of platform angular velocity in base frame ω (recall $\dot{Q} = \omega \times Q$). When will the platform be in a singular configuration? Explain.

The base hinge points are given as a function of base coordinates by

$$\underset{\sim}{b}_1 = \underset{\sim}{o}_0 + l(\underline{i}_0 + \underline{j}_0) = \underset{\sim}{o}_0 + \underline{C}_0 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0b_1 = \underset{\sim}{o}_0 + \underline{C}_0 b_1 \tag{1}$$

$$\underset{\sim}{b}_2 = \underset{\sim}{o}_0 + l(\underline{j}_0 + \underline{k}_0) = \underset{\sim}{o}_0 + \underline{C}_0 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0b_2 = \underset{\sim}{o}_0 + \underline{C}_0 b_2 \tag{2}$$

$$\underset{\sim}{b}_3 = \underset{\sim}{o}_0 + l(\underline{k}_0 + \underline{i}_0) = \underset{\sim}{o}_0 + \underline{C}_0 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0b_3 = \underset{\sim}{o}_0 + \underline{C}_0 b_3 \tag{3}$$

$$\underset{\sim}{p}_1 = \underset{\sim}{o}_0 + l\underline{i}_1 = \underset{\sim}{o}_0 + \underline{C}_1 l \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \underset{\sim}{o}_0 + \underline{C}_0 Q\, {}^1p_1 = \underset{\sim}{o}_0 + \underline{C}_0 Q p_1 \tag{4}$$

$$\underset{\sim}{p}_2 = \underset{\sim}{o}_0 + l\underline{j}_1 = \underset{\sim}{o}_0 + \underline{C}_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \underset{\sim}{o}_0 + \underline{C}_0 Q\, {}^1p_2 = \underset{\sim}{o}_0 + \underline{C}_0 Q p_2 \tag{5}$$

$$\underset{\sim}{p}_3 = \underset{\sim}{o}_0 + l\underline{k}_1 = \underset{\sim}{o}_0 + \underline{C}_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \underset{\sim}{o}_0 + \underline{C}_0 Q\, {}^1p_3 = \underset{\sim}{o}_0 + \underline{C}_0 Q p_3 \tag{6}$$

Therefore, for $i = 1, 2, 3$:

$$q_i = \|\underset{\sim}{p}_i - \underset{\sim}{b}_i\| = \|\underset{\sim}{o}_0 + \underline{C}_0 Q p_i - \underset{\sim}{o}_0 - \underline{C}_0 b_i\| = \|\underline{C}_0 Q p_i - \underline{C}_0 b_i\| = \|Q p_i - b_i\| \tag{7}$$

$$= \sqrt{(Qp_i - b_i)^T (Qp_i - b_i)} = \sqrt{p_i^T p_i + b_i^T b_i + 2p_i^T Q^T b_i} \tag{8}$$

This completes the inverse kinematics, giving the leg lengths as a function of platform orientation.

From

$$q_i^2 = p_i^T p_i + b_i^T b_i + 2p_i^T Q^T b_i \tag{9}$$

$$\tag{10}$$

we obtain that

$$2q_i\dot{q}_i = 2p_i^T \dot{Q}^T b_i = 2p_i^T ((\omega\times)Q)^T b_i = -2p_i^T Q^T (\omega\times) b_i = 2p_i^T Q^T (b_i\times)\omega \tag{11}$$

Thus

$$J = \left[\frac{1}{q_1} p_1^T Q^T (b_1\times) \quad \frac{1}{q_2} p_2^T Q^T (b_2\times) \quad \frac{1}{q_3} p_3^T Q^T (b_3\times)\right] \tag{12}$$

The platform is singular when the columns of the Jacobian become co-planar. From Figure P.1, when the platform legs are each of length $l\sqrt{2}$, $\underset{\sim}{p}_1$ coincides with $\underset{\sim}{b}_3$, etc. Alternatively, when the leg lengths are shortened so the platform is rotated 60 degrees around the cube diagonal from $\underset{\sim}{o}_0$ to the opposite vertex, the platform is also singular.

Problem 2.

Consider the 6-DOF manipulator shown in Figure 2.
(a) *(10 marks)* Label the axes and the origins of the coordinate systems attached to each link. Find the manipulator Jacobian and use it to find the manipulator singular configurations.

(b) *(10 marks)* Solve the manipulator inverse kinematics. You may specify all solutions in terms of Kahan's problems (see attached page).

(a) The Jacobian for is the same as the one of the Stanford manipulator (the manipulator has the Stanford arm without the shoulder offset). Follow the Stanford arm notes, the arm singularity is when the arm is straight up or down, wrist as usual.
(b) The inverse kinematics is also identical to the inverse kinematics of the Stanford arm, but the arm is in a different nominal position.

NAME:
Student #:

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Department of Electrical and Computer Engineering
EECE 487 (Winter 2010): Introduction to Robotics
Miderm Examination # 2, April 1, 2010
Closed Book - 80 Minutes
Maximum - 40 marks

Problem 1.

Consider the manipulator shown in Figure P1. All joint parameters are zero in the nominal configuration shown in the figure.
(a) *(10 marks)* Find the manipulator Jacobian and use it to find and discuss the manipulator singular configurations.
(b) *(15 marks)* Solve the inverse kinematics for this manipulator. Carefully discuss the case of multiple solutions or lack of solutions. If you use Kahan's problems P1-P4 (see attached sheets), you must clearly specify the input data.

Problem 2.

(15 marks) Consider the RP/pendulum with running bead planar manipulator shown in Figure P2. Assume the only non-zero mass is m.
Derive the equations of motion of this manipulator. What is the manipulator mass matrix $D(q)$? What is the gravitational vector $G(q)$?

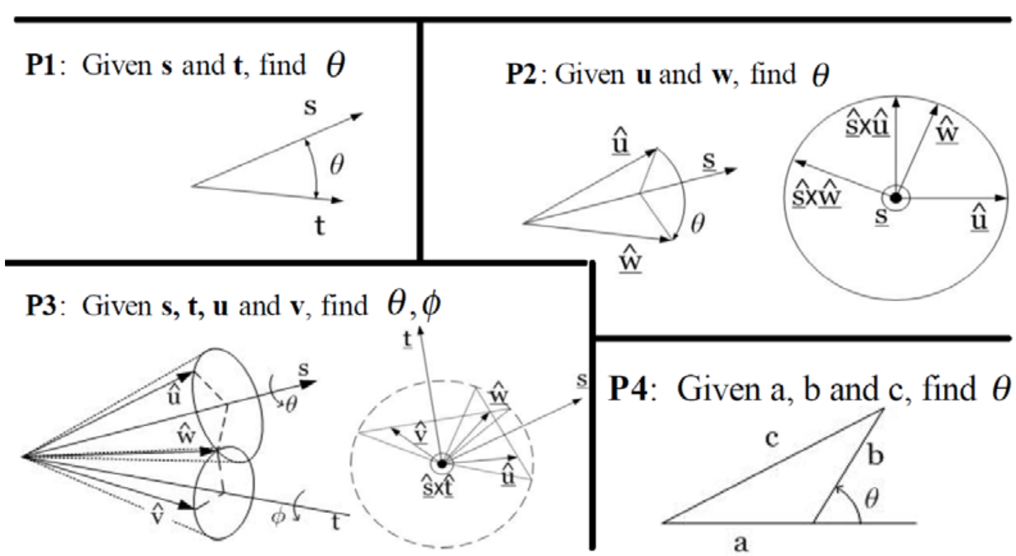


Figure P1

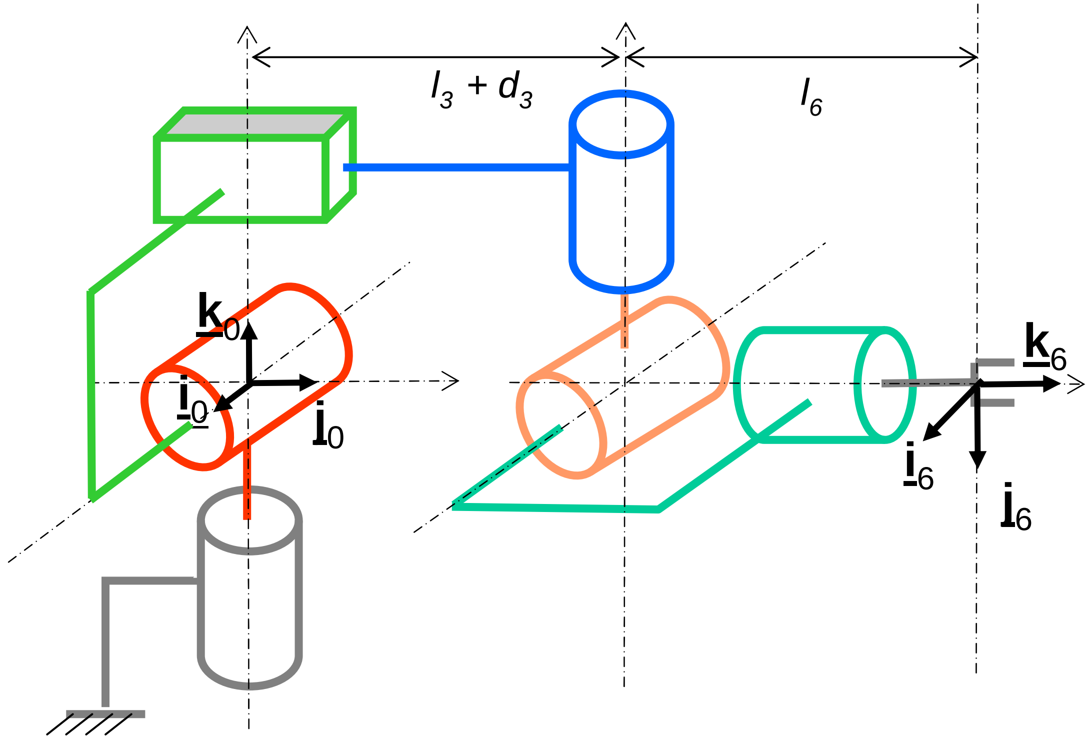
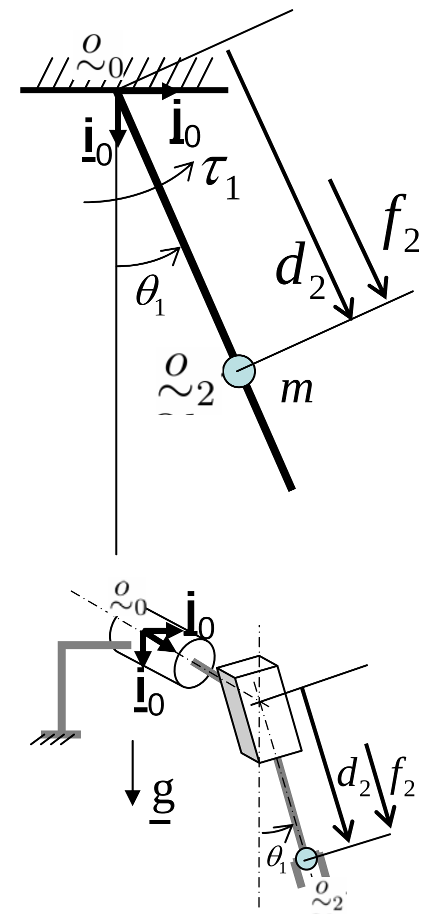


Figure P2

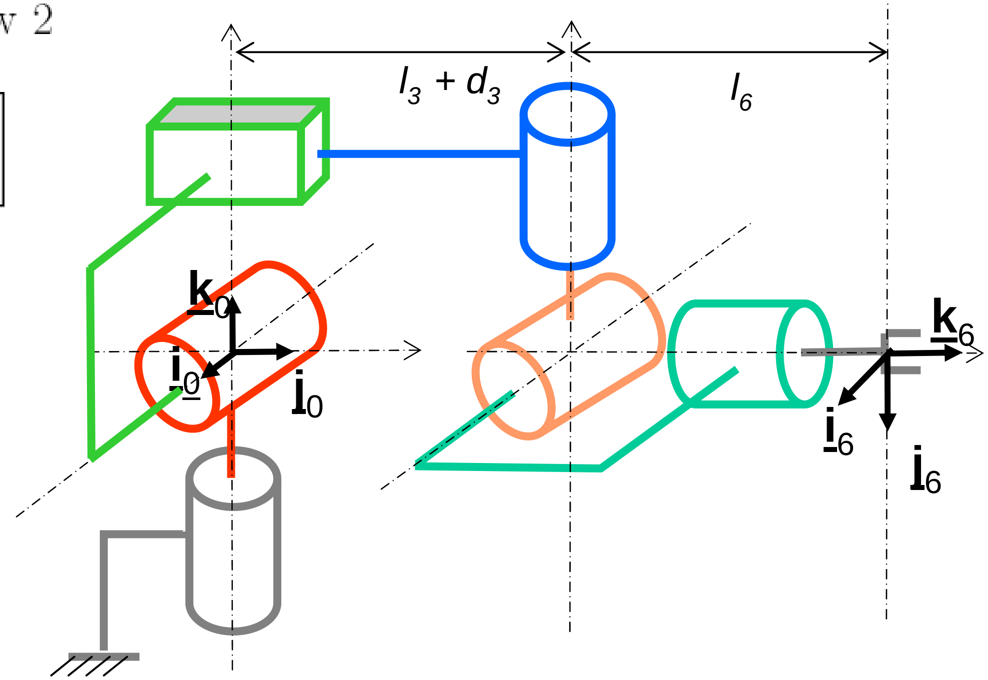


Problem 1.

$$\underline{J} = \begin{bmatrix} \underline{k}_0 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_0) & \underline{k}_1 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_1) & \underline{k}_2 & \underline{k}_3 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) & \underline{k}_4 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) & \underline{k}_5 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) \\ \underline{k}_0 & \underline{k}_1 & 0 & \underline{k}_3 & \underline{k}_4 & \underline{k}_5 \end{bmatrix}$$

$$\text{row } 1 \longleftarrow \text{row } 1 + (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) \times \text{row } 2$$

(pre-multiplying by) $\begin{bmatrix} I & (\underset{\sim}{o}_6 - \underset{\sim}{o}_3)\times \\ 0 & I \end{bmatrix}$



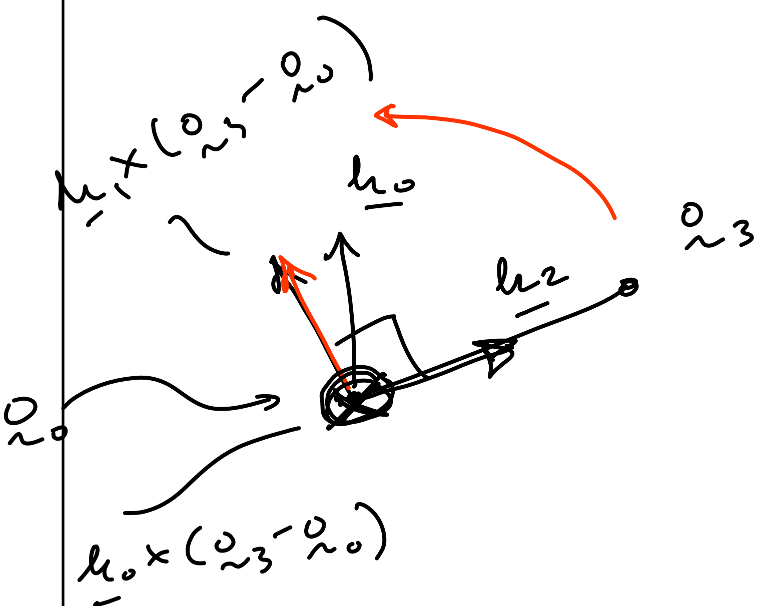
$$\underline{J} \sim \underbrace{\begin{bmatrix} \underline{k}_0 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_0) & \underline{k}_1 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_0) & \underline{k}_2 \\ \underline{k}_0 & \underline{k}_1 & 0 \end{bmatrix}}_{\text{arm singularities}} \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ \underline{k}_3 & \underline{k}_4 & \underline{k}_5 \end{bmatrix}}_{\text{wrist singularities}} = \begin{bmatrix} \underline{J}_{11} & 0 \\ \underline{J}_{21} & \underline{J}_{22} \end{bmatrix}$$

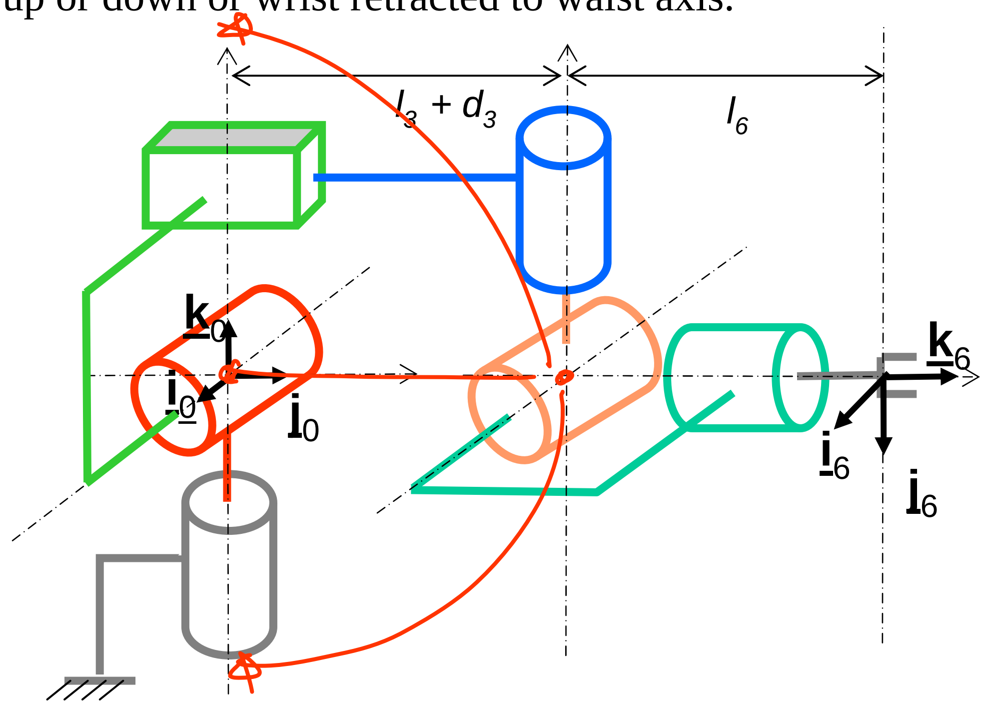
Singularities

$\underline{J}_{11}$ is singular when $\underline{k}_0 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_0)$, $\underline{k}_1 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_0)$, and $\underline{k}_2$ are *coplanar.*

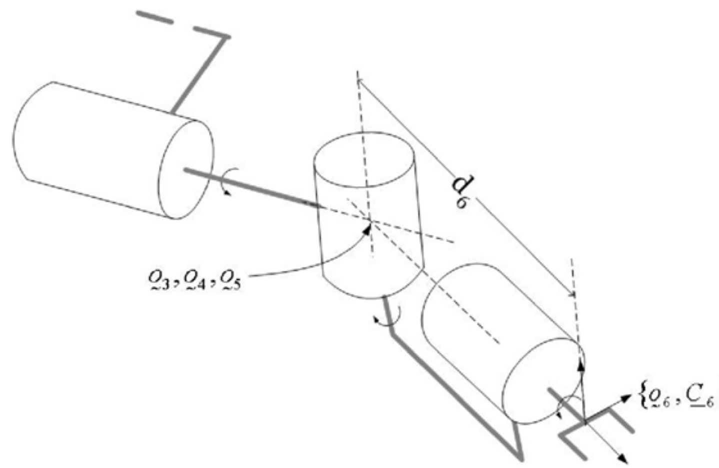
$\underline{J}_{22}$ is singular when $\underline{k}_3$, $\underline{k}_4$ and $\underline{k}_5$ are *coplanar.*

- Wrist singular when gripper is pointing down, and first and last wrist axes are aligned
- Arm is singular when wrist center aligned with waist axis, in which case no outward radial motion is possible. Arm up or down or wrist retracted to waist axis.





1. **Find spherical wrist center**
2. Solve inverse arm kinematics
3. Find wrist base by direct kinematics
4. Solve inverse wrist kinematics



$$\underset{\sim}{o}_6 = \underset{\sim}{o}_d \quad ; \quad \underline{C}_6 = \underline{C}_d$$

$$\underset{\sim}{o}_3 = \underset{\sim}{o}_d - \underline{C}_d l_6 k$$

1. Find spherical wrist center
2. **Solve inverse arm kinematics (Kahan's problem P3)**
3. Find wrist base frame by direct kinematics
4. Solve inverse wrist kinematics

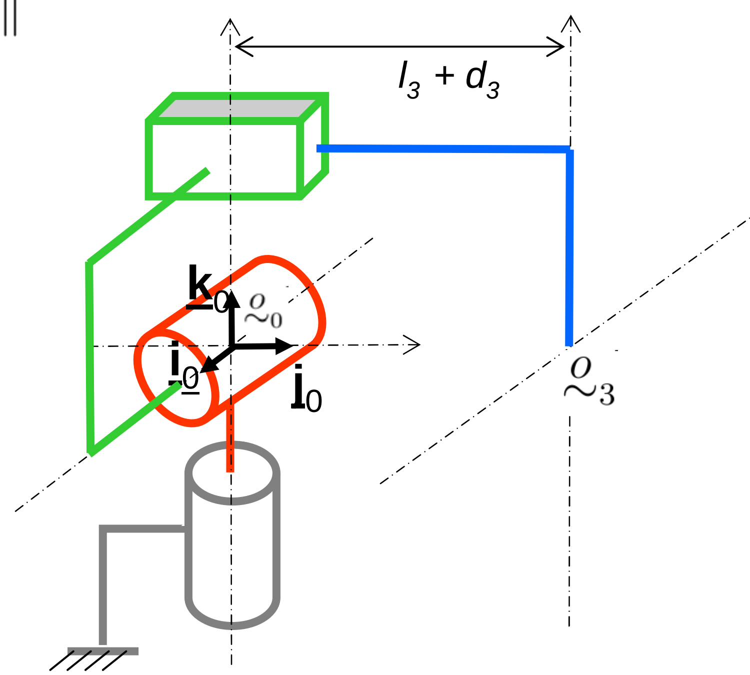
$$\underset{\sim}{o}_3 - \underset{\sim}{o}_0 = e^{\theta_1 \underline{k}_0 \times} e^{\theta_2 \underline{i}_0 \times} [(l_3 + d_3) \underline{j}_0]$$

$$d_3 = -l_3 \pm \|\underset{\sim}{o}_3 - \underset{\sim}{o}_0\|$$

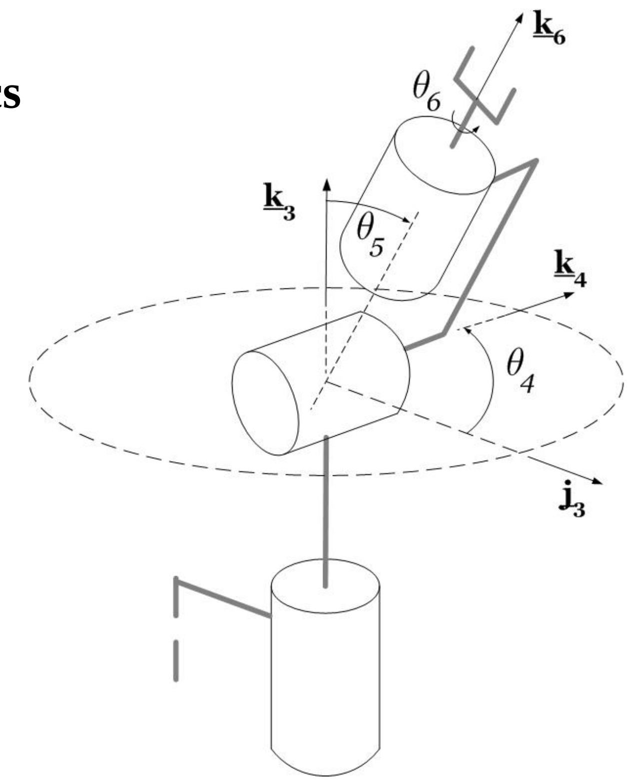
$$\underline{u} \triangleq \underset{\sim}{o}_3 - \underset{\sim}{o}_0$$

$$\underline{v} \triangleq \|\underset{\sim}{o}_3 - \underset{\sim}{o}_0\| \, \underline{j}_0$$

$$e^{-\theta_1 \underline{k}_0 \times} \underline{u} = e^{\theta_2 \underline{j}_0 \times} \underline{v}$$



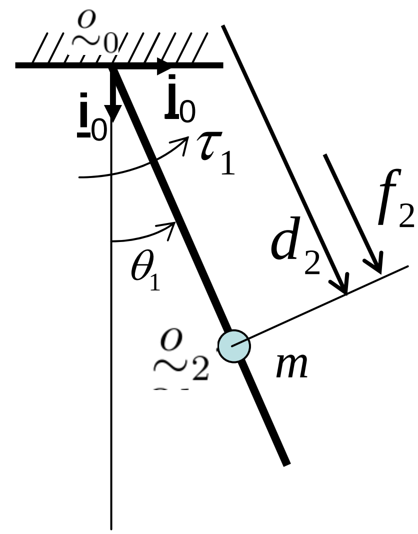
1. Find spherical wrist center
2. Solve inverse arm kinematics
3. **Find wrist base frame by direct kinematics**
4. **Solve inverse wrist kinematics**



$$\underline{k}_4 = \pm \frac{\underline{k}_3 \times \underline{k}_6}{\|\underline{k}_3 \times \underline{k}_6\|}$$

$$e^{-\theta_6 \underline{k}_6 \times} \underline{j}_6 = e^{-\theta_6 \underline{k}_6 \times} \underline{C}_6 j = \underline{k}_4 = e^{\theta_4 \underline{k}_3 \times} \underline{C}_3 j = e^{\theta_4 \underline{k}_3 \times} \underline{j}_3$$

$$T = \frac{1}{2} m \left(\dot d_2^2 + \dot\theta_1^2 d_2^2 \right) \quad ; \quad V = -mg d_2 \cos\theta_1$$



$$L = T - V = \frac{1}{2} m \left(\dot d_2^2 + \dot\theta_1^2 d_2^2 \right) + mg d_2 \cos\theta_1$$

$$\frac{\partial L}{\partial \dot\theta_1} = m \dot\theta_1 d_2^2 \qquad \frac{\partial L}{\partial \theta_1} = -mg d_2 \sin\theta_1$$

$$\frac{\partial L}{\partial \dot d_2} = m \dot d_2 \qquad \frac{\partial L}{\partial d_2} = m \dot\theta_1^2 d_2 + mg \cos\theta_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1} \right) = m \ddot\theta_1 d_2^2 + 2m \dot\theta_1 d_2 \dot d_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot d_2} \right) = m \ddot d_2$$

$$\begin{cases} m d_2^2 \ddot\theta_1 + 2m \dot\theta_1 \dot d_2 d_2 + mg d_2 \sin\theta_1 = \tau_1 \\ m \ddot d_2 - m \dot\theta_1^2 d_2 - mg \cos\theta_1 = f_2 \end{cases} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1} \right) - \frac{\partial L}{\partial \theta_1} = \tau_1$$

$$D(\theta_1, d_2) = \begin{bmatrix} m d_2^2 & 0 \\ 0 & m \end{bmatrix} \quad ; \quad G(\theta_1, d_2) = \begin{bmatrix} m g d_2 \sin\theta_1 \\ -mg \cos\theta_1 \end{bmatrix}$$

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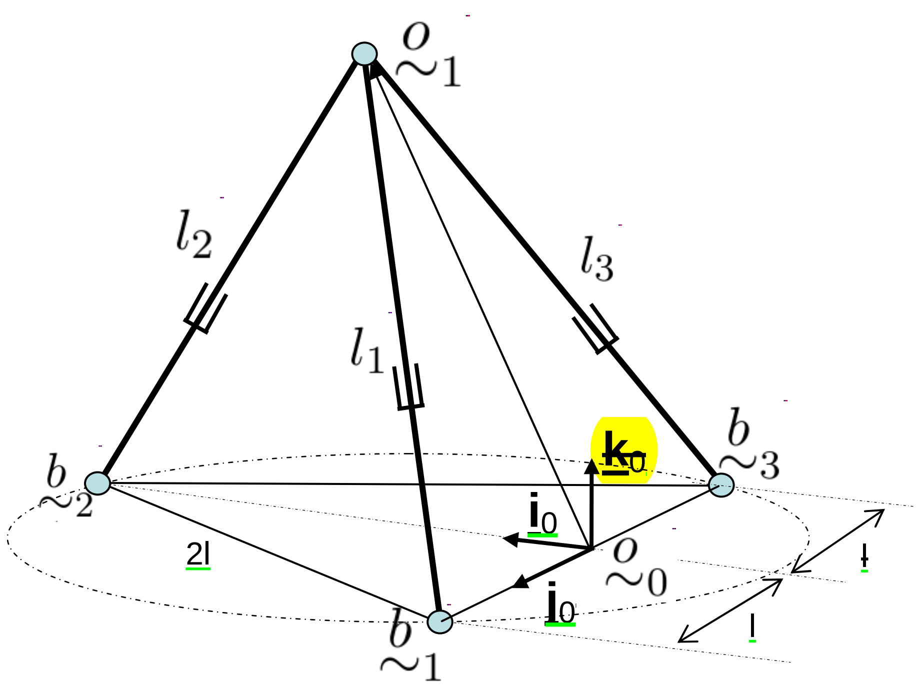
Problem 1.

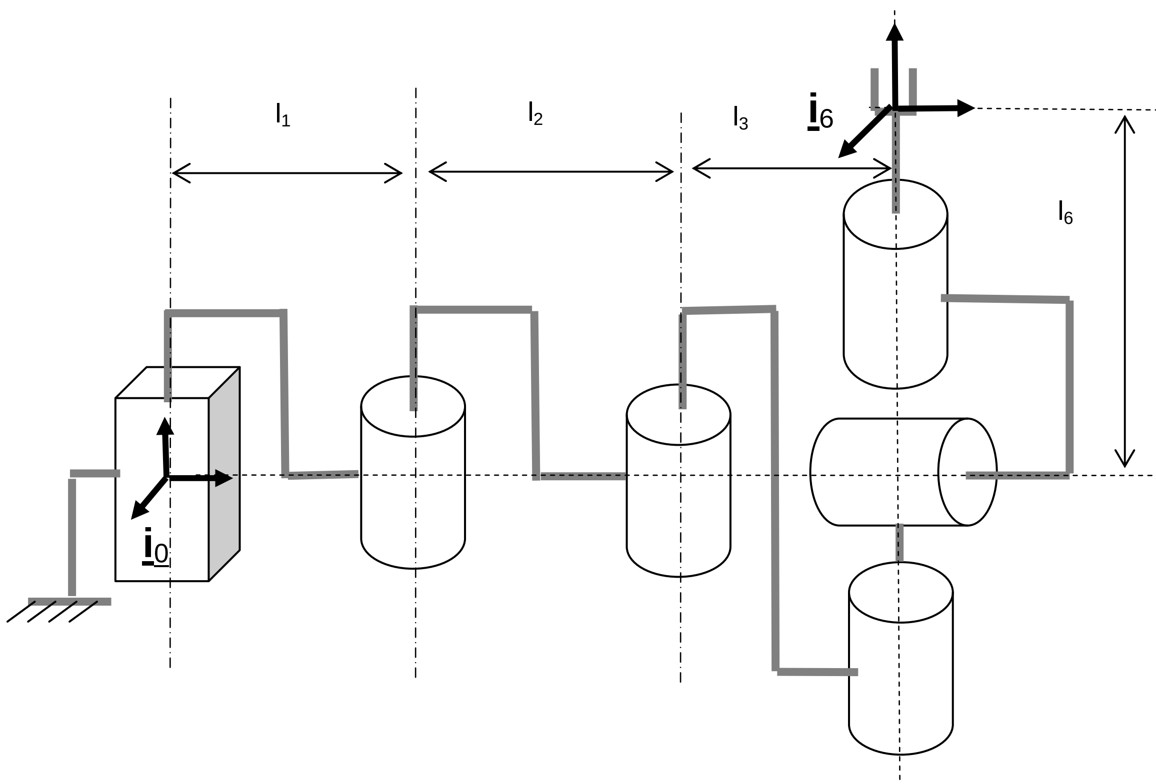
Consider the three degree of freedom Stewart platform shown below, where we assume that all joints are spherical with unlimited angular motion range. The base hinge points $\underset{\sim}{b}_1$, $\underset{\sim}{b}_2$, $\underset{\sim}{b}_3$ lie at the corners of an equilateral triangle with side $2l$.
(a) *(5 marks)* Solve the platform inverse kinematics, i.e., find l_1, l_2 and l_3 given the coordinates ${}^0d_1 = d$ of the platform center $\underset{\sim}{o}_1$ with respect to the base $\underset{\sim}{o}_0$.
(b) *(10 marks)* Find the 3×3 platform Jacobian giving the leg extension rates as a function of platform linear velocity in base frame $\dot{d}$. When will the platform be in a singular configuration? Explain.



Problem 2.

Consider the 6-DOF manipulator shown below.
(a) *(5 marks)* Label the axes and the origins of the coordinate systems attached to each link. Find the manipulator Jacobian and use it to find the manipulator singular configurations.

(b) *(20 marks)* Solve the manipulator inverse kinematics. Specify all solutions in terms of Kahan's problems (see attached page).



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Problem 1.

Consider the three degree of freedom Stewart platform shown below, where we assume that all joints are spherical with unlimited angular motion range. The base hinge points $\underset{\sim}{b}_1$, $\underset{\sim}{b}_2$, $\underset{\sim}{b}_3$ lie at the corners of an equilateral triangle with side $2l$.
(a) *(5 marks)* Solve the platform inverse kinematics, i.e., find l_1, l_2 and l_3 given the coordinates ${}^0d_1 = d$ of the platform center $\underset{\sim}{o}_1$ with respect to the base $\underset{\sim}{o}_0$.

For $i = 1, 2, 3$:

$$l_i = \|\underset{\sim}{o}_1 - \underset{\sim}{b}_i\| = \|\underline{C}_0\,{}^0d_1 - \underline{C}_0\,{}^0b_i\| = \|\,{}^0d_1 - {}^0b_i\| \tag{1}$$

$${}^0b_1 = \begin{bmatrix} 0 \\ l \\ 0 \end{bmatrix} \quad {}^0b_2 = \begin{bmatrix} \sqrt{3}l \\ 0 \\ 0 \end{bmatrix} \quad {}^0b_3 = \begin{bmatrix} 0 \\ -l \\ 0 \end{bmatrix} \tag{2}$$

(b) *(10 marks)* Find the 3×3 platform Jacobian giving the leg extension rates as a function of platform linear velocity in base frame $\dot{d}$. When will the platform be in a singular configuration? Explain.

$$\begin{aligned} l_i^2 &= (\,d - {}^0b_i)^T(d - {}^0b_i) && (3) \\ 2l_i\dot{l}_i &= 2(\,d - {}^0b_i)^T\dot{d} && (4) \\ \dot{l}_i &= \frac{1}{l_i}(\,d - {}^0b_i)^T\dot{d} && (5) \end{aligned}$$

$${}^0J = \begin{bmatrix} \frac{1}{l_1}(\,d - {}^0b_1)^T \\ \\ \frac{1}{l_2}(\,d - {}^0b_2)^T \\ \\ \frac{1}{l_3}(\,d - {}^0b_3)^T \end{bmatrix} \tag{6}$$

Therefore ${}^0J$ is singular when $\underset{\sim}{o}_1 - \underset{\sim}{b}_1$, $\underset{\sim}{o}_1 - \underset{\sim}{b}_2$, $\underset{\sim}{o}_1 - \underset{\sim}{b}_3$ lie in the same plane i.e. when $\underset{\sim}{o}_1$ is in the plane of $\underset{\sim}{b}_1$, $\underset{\sim}{b}_2$, $\underset{\sim}{b}_3$.

Problem 2.

Consider the 6-DOF manipulator shown below.
(a) *(5 marks)* Label the axes and the origins of the coordinate systems attached to each link. Find the manipulator Jacobian and use it to find the manipulator singular configurations.

$$\underline{J} = \begin{bmatrix} \underline{k}_0 & \underline{k}_1 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_1) & \underline{k}_2 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_2) & \underline{k}_3 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) & \underline{k}_4 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) & \underline{k}_5 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) \\ 0 & \underline{k}_1 & 0 & \underline{k}_3 & \underline{k}_4 & \underline{k}_5 \end{bmatrix} \tag{7}$$

$$\underline{J} \sim \left[\underbrace{\begin{matrix} \underline{k}_0 & \underline{k}_1 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_0) & \underline{k}_2 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_2) \\ 0 & \underline{k}_1 & \underline{k}_2 \end{matrix}}_{\text{arm singularities}} \quad \underbrace{\begin{matrix} 0 & 0 & 0 \\ \underline{k}_3 & \underline{k}_4 & \underline{k}_5 \end{matrix}}_{\text{wrist singularities}} \right] \tag{8}$$

Wrist singular when $\underline{k}_3$ and $\underline{k}_5$ parallel.
Arm singular when fully extended or retracted.

(b) *(20 marks)* Solve the manipulator inverse kinematics. Specify all solutions in terms of Kahan's problems (see attached page).
Find the wrist center $\underset{\sim}{o}_3 = \underset{\sim}{o}_6 - l_6\underline{k}_6$.

$$\underset{\sim}{o}_3 = \underset{\sim}{o}_0 + d_1\underline{k}_0 + l_1\underline{j}_0 + e^{\theta_2\underline{k}_0\times}(l_2\underline{j}_0 + e^{\theta_3\underline{k}_0\times} l_3\underline{j}_0) \tag{9}$$

Find $d_1 = \underline{k}_0^T(\underset{\sim}{o}_3 - \underset{\sim}{o}_0)$.
Find θ_3 from Kahan's Problem 4:

$$\|\underset{\sim}{o}_3 - \underset{\sim}{o}_0 - d_1\underline{k}_0 - l_1\underline{j}_0\| = \|l_2\underline{j}_0 + e^{\theta_3\underline{k}_0\times} l_3\underline{j}_0\| \tag{10}$$

with $a = l_2$, $b = l_3$ and $c = \|\underset{\sim}{o}_3 - \underset{\sim}{o}_0 - d_1\underline{k}_0 - l_1\underline{j}_0\|$.
Let $\underline{v}','' = e^{\theta_2\underline{k}_0\times}(l_2\underline{j}_0 + e^{\theta_3',''\underline{k}_0\times} l_3\underline{j}_0)$ and $\underline{u} = \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - d_1\underline{k}_0 - l_1\underline{j}_0$. Find θ_2 from Kahan's Problem 2 that solves for θ_2 given $\underline{u} = e^{\theta_2\underline{k}_0\times}\underline{v}$.
Find wrist base $\underline{C}_3(d_1, \theta_2, \theta_3)$. Solve inverse wrist as in class notes.

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Maximum - 30 marks

Reason your solutions. Pay attention to your notation; partial credit will not be given to you if you do not follow the notation convention we use in this course.

Problem 1.
Consider two coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and $\{\underset{\sim}{o}_1, \underline{C}_1\}$, with $\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\,{}^0d_1$ and $\underline{C}_1 = \underline{C}_0\,{}^0C_1(t)$.

(4 marks)
(a) If ${}^0C_1(t) = e^{\theta(t)s\times}R$, where the rotation matrix R and the axis s are constant, what is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$ in terms of R, s and $\dot{\theta}(t)$?

What are the coordinates of this angular velocity vector in the frame $\underline{C}_1$?

(2 marks)

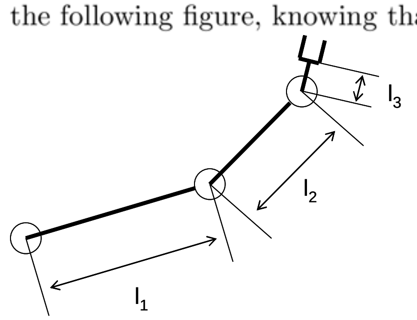
(b) If a point $\underset{\sim}{x}$ has coordinates ${}^{0}x$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and a point $\underset{\sim}{y}$ has coordinates ${}^{1}y$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$, what is the distance between $\underset{\sim}{x}$ and $\underset{\sim}{y}$ in terms of ${}^{0}x$, ${}^{1}y$, ${}^{0}d_1$, R, s and θ?

(3 marks)

(c) Suppose $\{\underset{\sim}{o}_0, \underline{C}_0\}$ is the base of a robot and $\{\underset{\sim}{o}_1, \underline{C}_1\}$ is attached to link 1. If the Denavit-Hartenberg parameters of link 1 are θ, d, a, α, what is the homogeneous transformation ${}^{0}T_1$ that expresses the relationship between the coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and $\{\underset{\sim}{o}_1, \underline{C}_1\}$?

(2 marks)

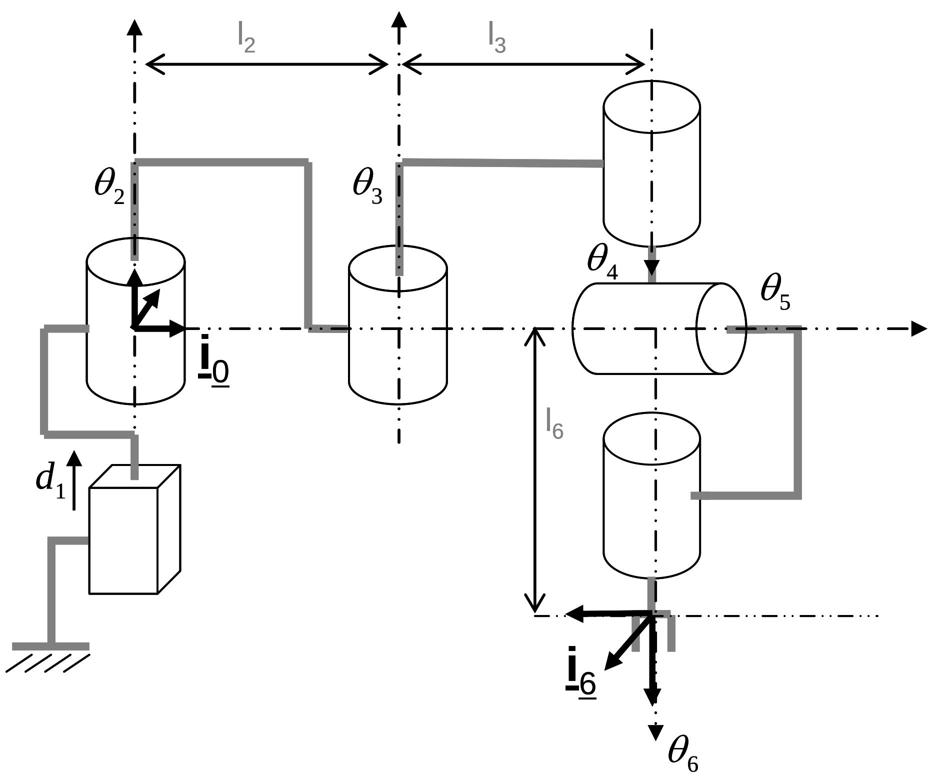
What is the linear velocity of the gripper center of the 3-dof planar manipulator shown in the following figure, knowing that the joint rates are $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$?



Problem 2.
Consider the manipulator shown in the figure below.

(*8 marks*)
Find the manipulator Jacobian and use elementary row operations to discuss its singularities.

(*12 marks*)
Give a detailed solution to the inverse kinematics of this manipulator, i.e., if $\{\underset{\sim}{o}_6, \underline{C}_6\}$ $= \{\underset{\sim}{o}_d, \underline{C}_d\}$, find $d_1, \theta_i, i = 2, ..., 6$. Discuss multiple solutions or lack of solutions of this problem. All angles are at zero degrees in the nominal configuration shown in the figure and oriented to be positive (right hand rule) for the axes shown. If you use Kahan's problems P1-P4 (see attached page), you must clearly specify their inputs and outputs.



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Reason your solutions. Pay attention to your notation; partial credit will not be given to you if you do not follow the notation convention we use in this course.

Problem 1.
Consider two coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and $\{\underset{\sim}{o}_1, \underline{C}_1\}$, with $\underset{\sim}{o}_1 = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0d_1$ and $\underline{C}_1 = \underline{C}_0\, {}^0C_1(t)$.

(4 marks)
(a) If ${}^0C_1(t) = e^{\theta(t)s\times}R$, where the rotation matrix R and the axis s are constant, what is the angular velocity of $\underline{C}_1$ with respect to $\underline{C}_0$ in terms of R, s and $\dot\theta(t)$?

$$ {}^0\omega_{1,0}\times = \left[\frac{d}{dt}\left(e^{\theta s\times}R\right)\right]\left(e^{\theta s\times}R\right)^T = \dot\theta s\times e^{\theta s\times}RR^T\left(e^{\theta s\times}\right)^T $$

$$ = \dot\theta s\times \quad \Rightarrow \quad {}^0\omega_{1,0} = \dot\theta s $$

$$ \underline{\omega}_{1,0} = \underline{C}_0\, {}^0\omega_{1,0} = \dot\theta\, \underline{C}_0 s $$

What are the coordinates of this angular velocity vector in the frame $\underline{C}_1$?

$$ {}^1\omega_{1,0} = {}^1C_0\, {}^0\omega_{1,0} = {}^0C_1^T\, {}^0\omega_{1,0} = R^T\left(e^{\theta s\times}\right)^T \dot\theta s $$

$$ = R^T\dot\theta s = \dot\theta R^T s $$

(2 marks)
(b) If a point $\underset{\sim}{x}$ has coordinates ${}^0x$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and a point $\underset{\sim}{y}$ has coordinates ${}^1y$ in $\{\underset{\sim}{o}_1, \underline{C}_1\}$, what is the distance between $\underset{\sim}{x}$ and $\underset{\sim}{y}$ in terms of ${}^0x$, ${}^1y$, ${}^0d_1$, R, s and θ?

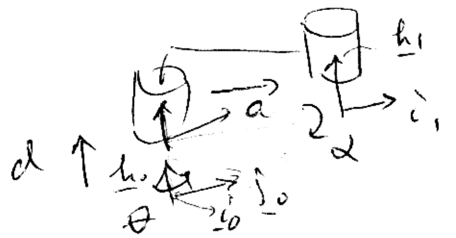
$$\underset{\sim}{x} = \underset{\sim}{o}_0 + \underline{C}_0\, {}^0x \quad ; \quad \underset{\sim}{y} = \underset{\sim}{o}_1 + \underline{C}_1\, {}^1y$$

$$\| \underset{\sim}{x} - \underset{\sim}{y} \| = \| \underset{\sim}{o}_0 + \underline{C}_0\, {}^0x - \underset{\sim}{o}_1 - \underline{C}_0 \underbrace{\left(e^{\theta_0 \times} R\right)}_{{}^0C_1} {}^1y \|$$

$$= \| \cancel{\underset{\sim}{o}_0} + \underline{C}_0\, {}^0x - \cancel{\underset{\sim}{o}_0} - \underline{C}_0\, {}^0d_1 - \underline{C}_0 \left(\quad \right) {}^1y \|$$

$$= \| {}^0x - {}^0d_1 - e^{\theta_0 \times} R\, {}^1y \|$$

(4 marks)
(c) Suppose $\{\underset{\sim}{o}_0, \underline{C}_0\}$ is the base of a robot and $\{\underset{\sim}{o}_1, \underline{C}_1\}$ is attached to link 1. If the Denavit-Hartenberg parameters of link 1 are θ, d, a, α, what is the homogeneous transformation ${}^0T_1$ that expresses the relationship between the coordinate systems $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and $\{\underset{\sim}{o}_1, \underline{C}_1\}$?



$${}^0T_1 = \begin{bmatrix} e^{\theta k \times} & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} I & dk \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} I & ai \\ 0^T & 0 \end{bmatrix} \begin{bmatrix} e^{\alpha i \times} & 0 \\ 0^T & 1 \end{bmatrix}$$

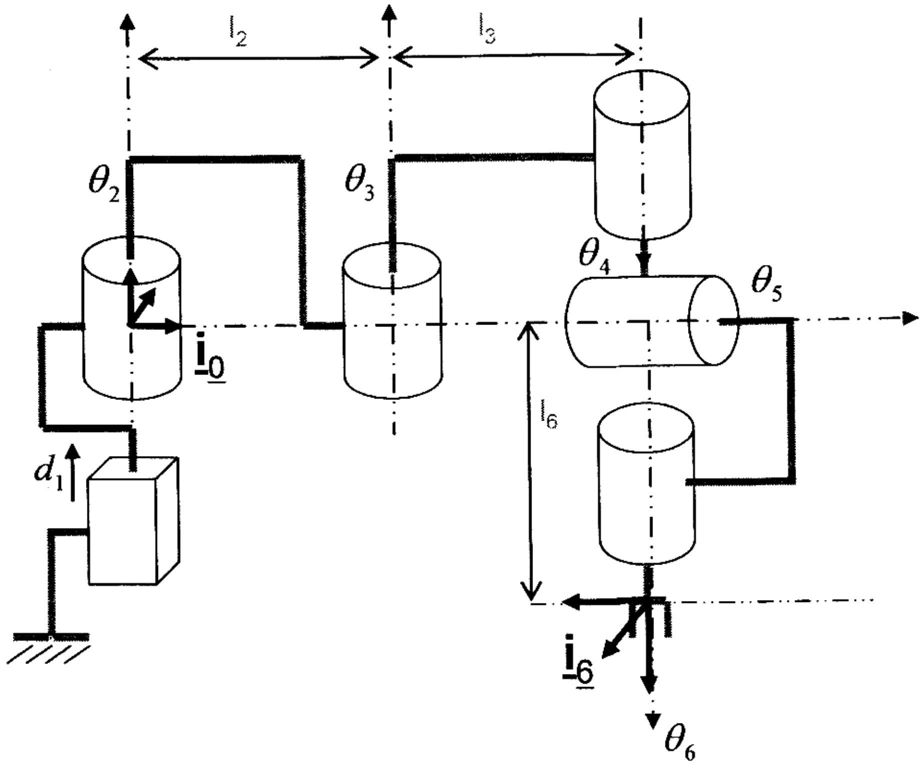
$$= \begin{bmatrix} e^{\theta k \times} & e^{\theta k \times} dk \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} e^{\alpha i \times} & ai \\ 0^T & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{\theta k \times} e^{\alpha i \times} & e^{\theta k \times} ai + dk \\ 0^T & 1 \end{bmatrix}$$

Problem 2.
Consider the manipulator shown in the figure below.

(*8 marks*)
Find the manipulator Jacobian and use elementary row operations to discuss its singularities.

(*12 marks*)
Give a detailed solution to the inverse kinematics of this manipulator, i.e., if $\{\underset{\sim}{o}_6, \underline{C}_6\} = \{\underset{\sim}{o}_d, \underline{C}_d\}$, find $d_1, \theta_i, i = 2, \ldots, 6$. Discuss multiple solutions or lack of solutions of this problem. All angles are at zero degrees in the nominal configuration shown in the figure and oriented to be positive (right hand rule) for the axes shown. If you use Kahan's problems P1-P4 (see attached page), you must clearly specify their inputs and outputs.



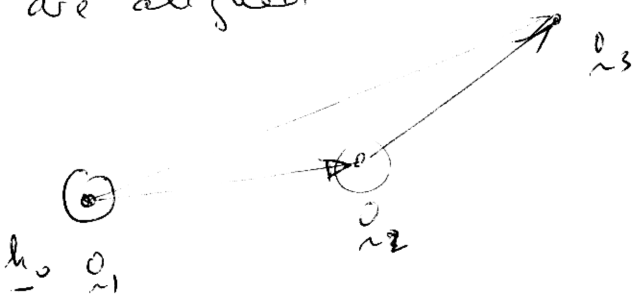
(a)

$$\underline{J} = \begin{bmatrix} \underline{h}_0 & \underline{h}_1 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_1) & \underline{h}_2 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_2) & \underline{h}_3 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) & \underline{h}_4 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) & \underline{h}_5 \times (\underset{\sim}{o}_6 - \underset{\sim}{o}_3) \\ 0 & \underline{h}_1 & \underline{h}_2 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|ccc} \underline{h}_0 & \underline{h}_1 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_1) & \underline{h}_2 \times (\underset{\sim}{o}_3 - \underset{\sim}{o}_2) & \underline{0} & \underline{0} & \underline{0} \\ \hline 0 & \underline{h}_1 & \underline{h}_2 & \underline{h}_3 & \underline{h}_4 & \underline{h}_5 \end{array}\right]$$

Arm singularity when $\underline{h}_0$, $\underline{h}_1 \times (\underset{\sim}{O}_3 - \underset{\sim}{O}_2)$, $\underline{h}_2 \times (\underset{\sim}{O}_3 - \underset{\sim}{O}_1)$ are aligned

$\underline{h}_1 = \underline{h}_0$, $\underline{h}_2 = \underline{h}_0$



$\underline{h}_0$, $\underline{h}_0 \times (\underset{\sim}{O}_3 - \underset{\sim}{O}_2)$, $\underline{h}_0 \times (\underset{\sim}{O}_3 - \underset{\sim}{O}_1)$ aligned when arm fully extended or fully folded. $(\underset{\sim}{O}_3 - \underset{\sim}{O}_1) \parallel (\underset{\sim}{O}_3 - \underset{\sim}{O}_2)$

Wrist singularity when $\underline{h}_3 \parallel \underline{h}_5$.

For inverse kinematics, see PUMA inverse kinematics, set offsets to zero.

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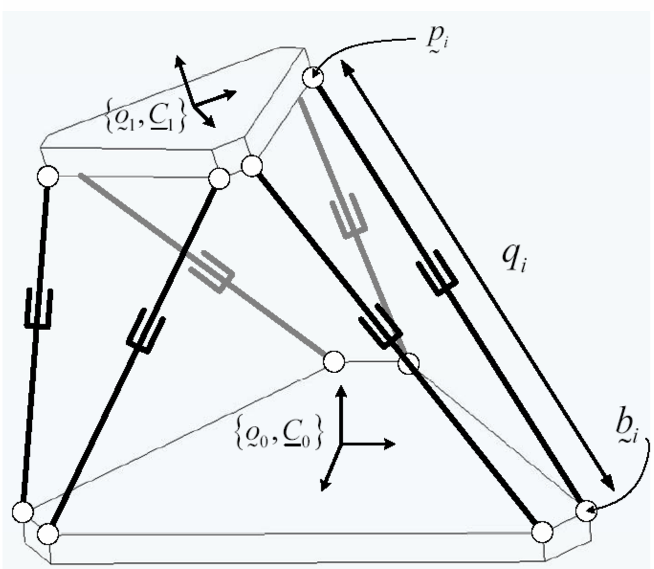
Maximum - 30 marks

Solve all the problems. Reason your solutions. Pay attention to your notation; partial credit will not be given to you if you do not follow the notation convention.

Problem 1.
Consider a parallel manipulator, where the leg hinge points $\underset{\sim}{b}_i$, $i = 1, ..., 6$ on the base have coordinates b_i, $i = 1, ..., 6$ in a base-attached coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$. The leg hinge points $\underset{\sim}{p}_i$, $i = 1, ..., 6$ on the platform have coordinates p_i, $i = 1, ..., 6$ in a platform-attached coordinate system $\{\underset{\sim}{o}_1, \underline{C}_1\}$.
(5 marks)
(a) Solve the inverse kinematics problem, i.e. find the platform leg lengths q_i given that you know the coordinates d of $\underset{\sim}{o}_1$ in $\{\underset{\sim}{o}_0, \underline{C}_0\}$ and the coordinates Q of $\underline{C}_1$ in $\underline{C}_0$.

(5 marks)

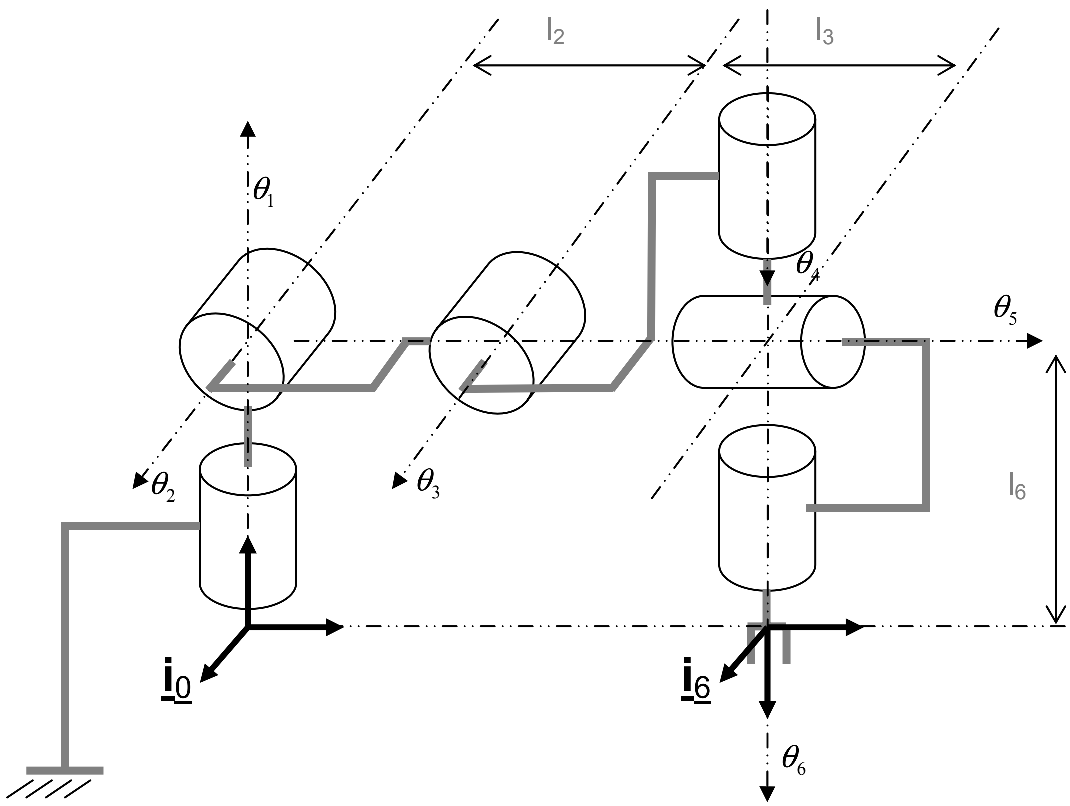
(b) Find the Jacobian $J(d, Q)$ that relates leg extension rates $\dot{q}$ to platform velocity $\begin{bmatrix} \dot{d} \\ \omega \end{bmatrix}$.



Problem 2.

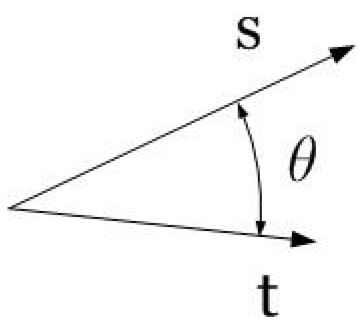
(*20 marks*)

Give a detailed solution (discuss multiple solutions or lack of solutions) to the inverse kinematics of the following manipulator. All angles are at zero degrees in the nominal configuration shown in the figure and oriented to be positive (right hand rule) for the axes shown. If you use Kahan's problems P1-P3, you must clearly specify the given (input) data and provide a solution for that particular case.

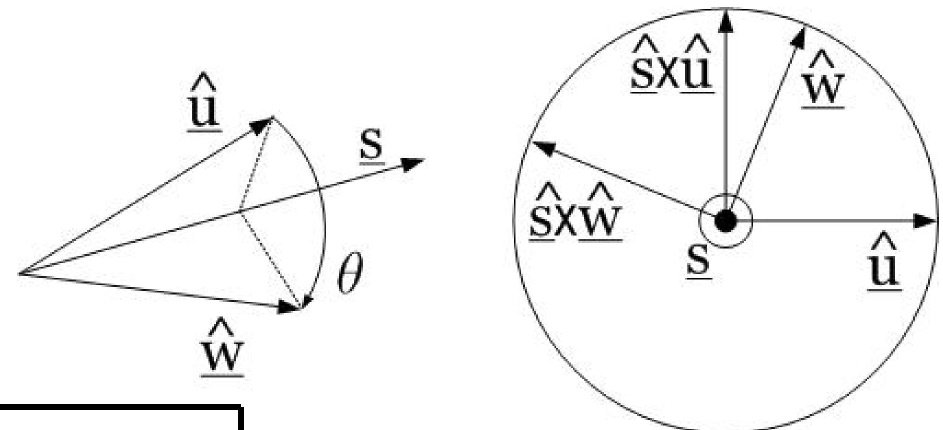


Kahan's Problems

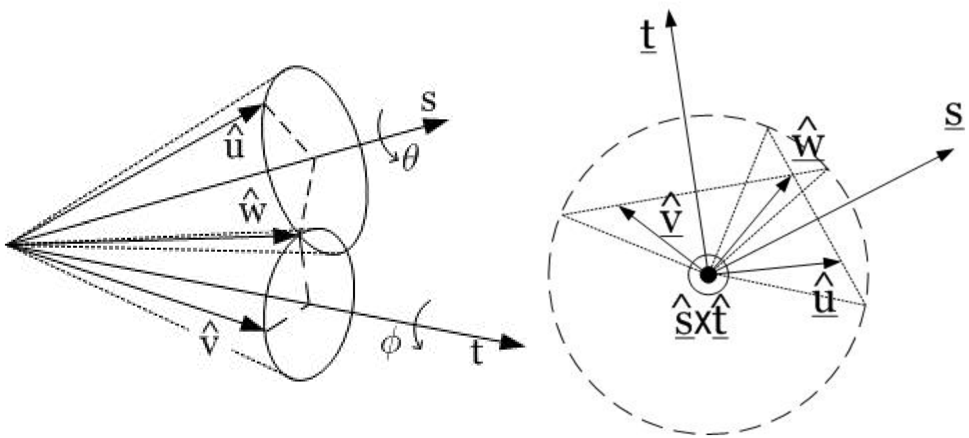
Kahan P1: Given **s** and **t**, find θ.



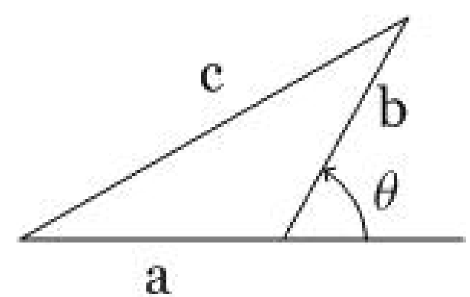
Kahan P2: Given the vectors shown, find θ.



Kahan P3: Given the **s, t, u** and **v**, find θ and ϕ.



Kahan P4: Given a, b and c, find θ.



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Solutions to Midterm #2

Problem # 2.

Because the robot has a spherical wrist, the inverse kinematics problem decouples into inverse position and inverse orientation problems. So, we first set

$$\begin{aligned} \underline{k}_6 &= \underline{k}_d && (1) \\ \underset{\sim}{o}_3 &= \underset{\sim}{o}_d - l_6 \underline{k}_6 && (2) \end{aligned}$$

to obtain the center of the spherical wrist.
The inverse arm problem can be solved by noting that

$$\underset{\sim}{o}_3 - \underset{\sim}{o}_0 = l_6 \underline{k}_0 + e^{\theta_1 \underline{k}_0 \times} e^{\theta_2 \underline{i}_0 \times} [l_2 \underline{j}_0 + e^{\theta_3 \underline{i}_0 \times} l_3 \underline{j}_0] \ . \quad (3)$$

and therefore

$$\| \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - l_6 \underline{k}_0 \| = \| l_2 \underline{j}_0 + e^{\theta_3 \underline{i}_0 \times} l_3 \underline{j}_0 \| \ . \quad (4)$$

This is Kahan's problem P4. Obtain two solutions - elbow down θ_3' and elbow up θ_3'', unless $\underset{\sim}{o}_3$ happens to be such that $\| \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - l_6 \underline{k}_0 \| = l_2 + l_3 =$ length of the arm, in which case we have one solution. We have no solutions if $\| \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - l_6 \underline{k}_0 \| > l_2 + l_3$, or if $\| \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - l_6 \underline{k}_0 \| < |l_2 - l_3|$.
Let

$$\begin{aligned} \underline{u} &= \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - l_6 \underline{k}_0 && (5) \\ \underline{v}' &= l_2 \underline{j}_0 + e^{\theta_3' \underline{i}_0 \times} l_3 \underline{j}_0 && (6) \\ \underline{v}'' &= l_2 \underline{j}_0 + e^{\theta_3'' \underline{i}_0 \times} l_3 \underline{j}_0 \ . && (7) \end{aligned}$$

Now solve for θ_1 and θ_2 as done for the Stanford manipulator:

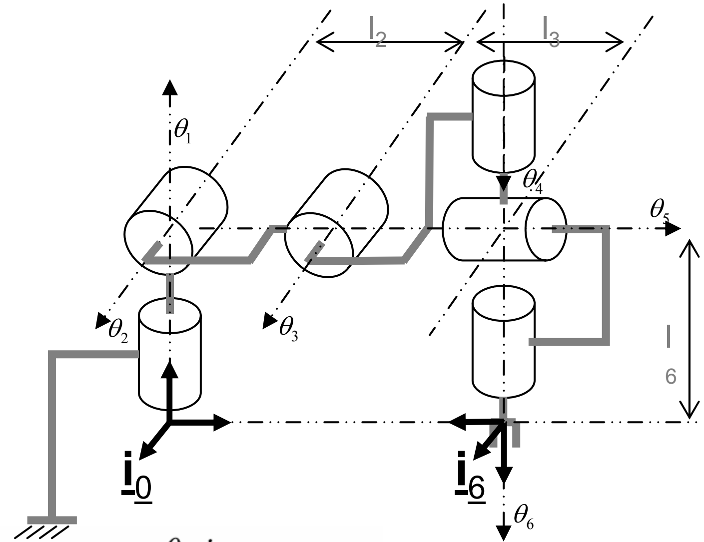
$$\begin{aligned} e^{-\theta_1 \underline{k}_0 \times} \hat{\underline{u}} &= e^{\theta_2 \underline{i}_0 \times} \hat{\underline{v}'} && (8) \\ e^{-\theta_1 \underline{k}_0 \times} \hat{\underline{u}} &= e^{\theta_2 \underline{i}_0 \times} \hat{\underline{v}''} \ . && (9) \end{aligned}$$

There are two solutions for each elbow up and down configurations, corresponding to the arm swinging over the top or not. Joint limits may remove some of these.
Compute $\underline{C}_3(\theta_1, \theta_2, \theta_3)$ for the four combinations of arm angles, according to

$$\underline{C}_3 = e^{\theta_1 \underline{k}_0 \times} e^{\theta_2 \underline{j}_0 \times} e^{\theta_3 \underline{j}_0 \times} \underline{C}_0 e^{\pi i \times} \quad (10)$$

and solve the inverse wrist problem as done in class.

Problem 2.



- Find wrist center:

$$\begin{aligned} \underline{k}_6 &= \underline{k}_d \\ \underset{\sim}{o}_3 &= \underset{\sim}{o}_d - l_6 \underline{k}_6 \end{aligned}$$

- Solve inverse arm problem:

$$\underset{\sim}{o}_3 - \underset{\sim}{o}_0 = l_6 \underline{k}_0 + e^{\theta_1 \underline{k}_0 \times} e^{\theta_2 \underline{i}_0 \times} [l_2 \underline{j}_0 + e^{\theta_3 \underline{i}_0 \times} l_3 \underline{j}_0]$$

- First, elbow angle by Kahan's P4:

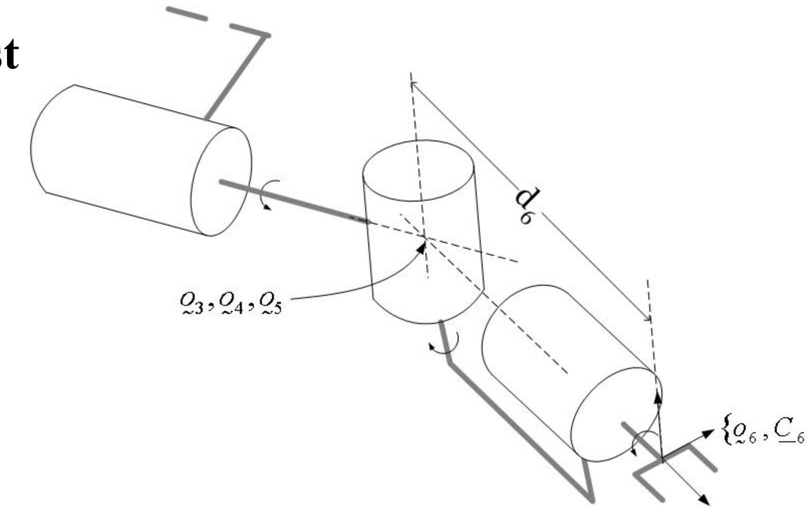
$$\| \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - l_6 \underline{k}_0 \| = \| l_2 \underline{j}_0 + e^{\theta_3 \underline{i}_0 \times} l_3 \underline{j}_0 \|$$

- Then, waist and shoulder angle by Kahan's P3:

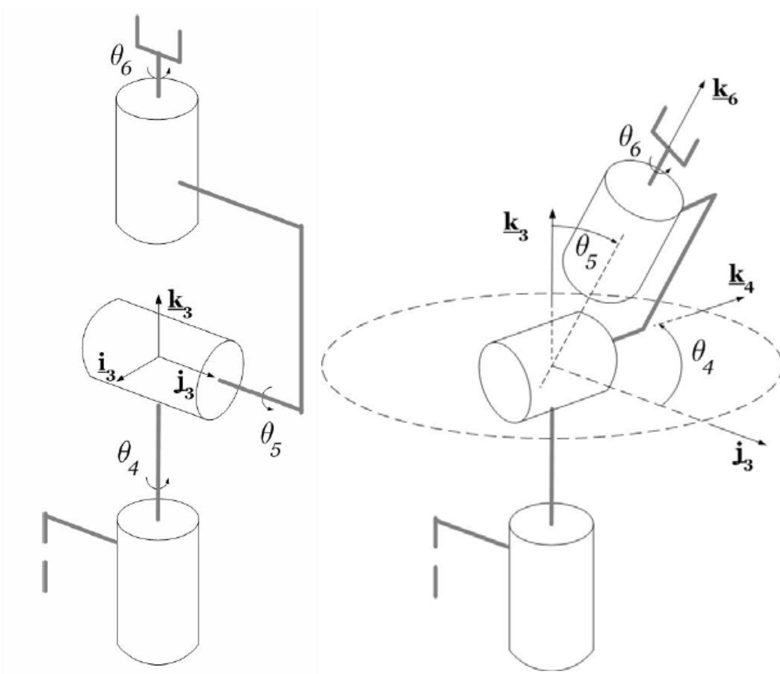
$$\begin{aligned} e^{-\theta_1 \underline{k}_0 \times} \hat{\underline{u}} &= e^{\theta_2 \underline{i}_0 \times} \hat{\underline{v}'} \\ e^{-\theta_1 \underline{k}_0 \times} \hat{\underline{u}} &= e^{\theta_2 \underline{i}_0 \times} \hat{\underline{v}''} \end{aligned}$$ ith

$$\begin{aligned} \underline{u} &= \underset{\sim}{o}_3 - \underset{\sim}{o}_0 - l_6 \underline{k}_0 \\ \underline{v}' &= l_2 \underline{j}_0 + e^{\theta_3' \underline{i}_0 \times} l_3 \underline{j}_0 \\ \underline{v}'' &= l_2 \underline{j}_0 + e^{\theta_3'' \underline{i}_0 \times} l_3 \underline{j}_0 \end{aligned}$$

Inverse Kinematics for Spherical Wrist



Easy to find wrist center



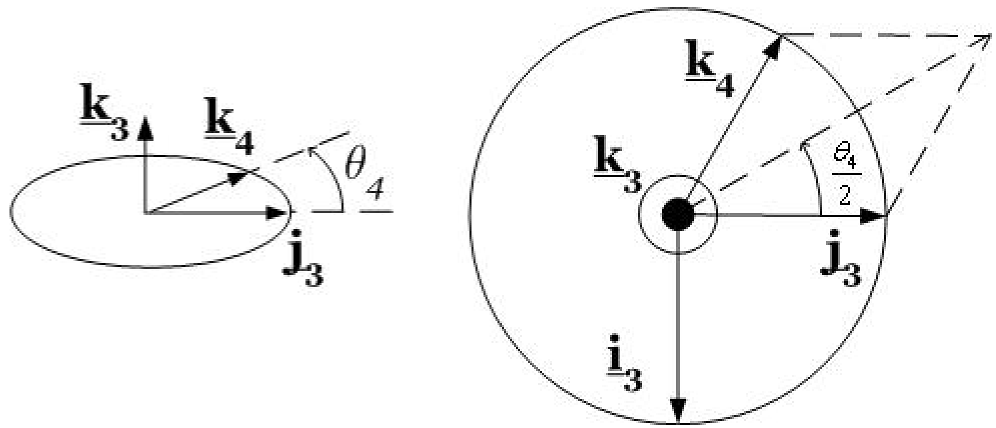
Suppose we know $\underline{C}_3$. Can we find θ_4, θ_5 and θ_6 to get $\underline{C}_6 = \underline{C}_d$?

$$\underline{k}_4 = \pm \frac{\underline{k}_3 \times \underline{k}_6}{\|\underline{k}_3 \times \underline{k}_6\|}$$

$$e^{-\theta_6 \underline{k}_6 \times} \underline{j}_6 = e^{-\theta_6 \underline{k}_6 \times} \underline{C}_6 j = \underline{k}_4 = e^{\theta_4 \underline{k}_3 \times} \underline{C}_3 j = e^{\theta_4 \underline{k}_3 \times} \underline{j}_3$$

Inverse Kinematics for Spherical Wrist

Finding θ_4 and θ_6:



$$e^{\theta_4 \underline{k}_3 \times} \underline{j}_3 = \underline{k}_4$$

$$|\theta_4| \quad = \quad 2 \arctan \frac{\|\underline{k}_4 - \underline{j}_3\|}{\|\underline{k}_4 + \underline{j}_3\|}$$

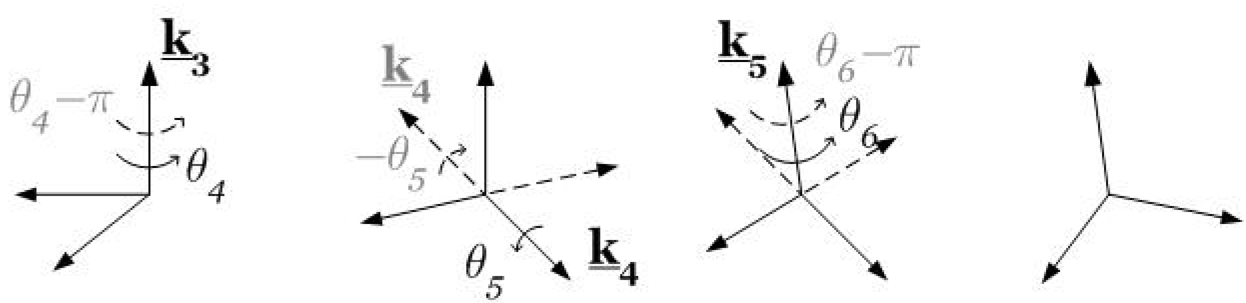
$$\operatorname{sign}(\theta_4) \quad = \quad -\operatorname{sign}[\underline{k}_4^T \underline{i}_3]$$

Inverse Kinematics for Spherical Wrist

Finding θ_5: $e^{\theta_5 \underline{k}_4 \times} \underline{k}_3 = \underline{k}_6$

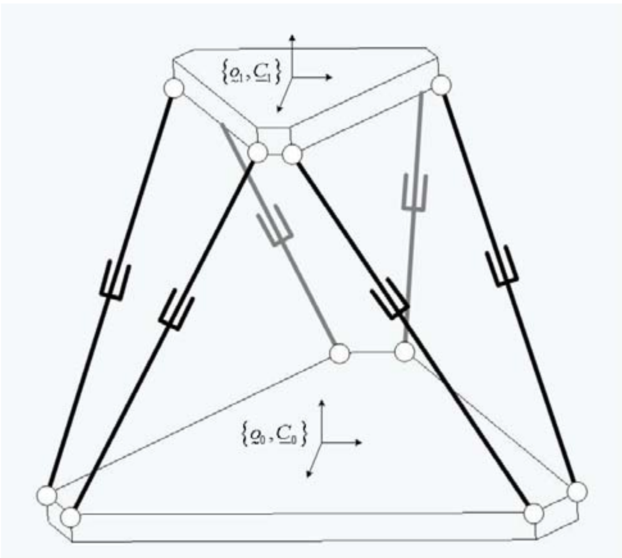
$$
\begin{aligned}
|\theta_5| &= 2\arctan \frac{\|\underline{k}_6 - \underline{k}_3\|}{\|\underline{k}_6 + \underline{k}_3\|} \\
\mathrm{sign}(\theta_5) &= \mathrm{sign}[\underline{k}_6^T (\underline{k}_4 \times \underline{k}_3)]
\end{aligned}
$$

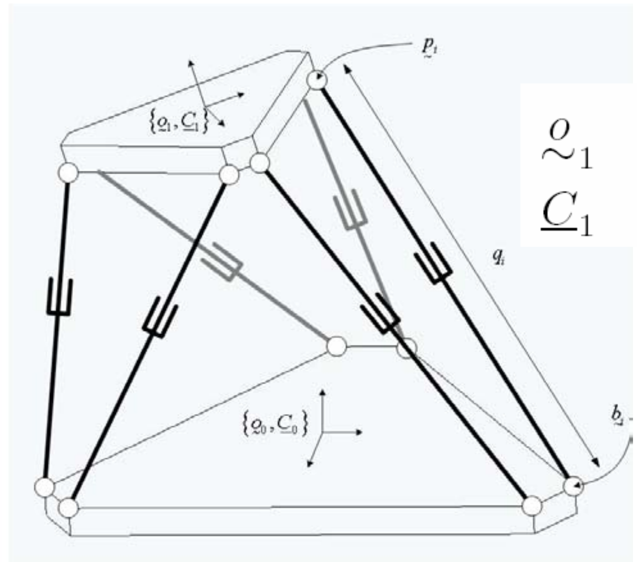
Multiple solutions:



$$
\begin{aligned}
\underline{C}_6 &= \underline{C}_3 e^{\theta_4 k \times} e^{\theta_5 j \times} e^{\theta_6 k \times} \\
\underline{C}_6 &= \underline{C}_3 e^{-(\pi-\theta_4) k \times} e^{-\theta_5 j \times} e^{-(\pi-\theta_6) k \times}
\end{aligned}
$$

Problem 1:
- Inverse kinematics





$$
\begin{aligned}
\underset{\sim}{o}_1 &= \underset{\sim}{o}_0 + \underline{C}_0 {}^0 d_1 \\
\underline{C}_1 &= \underline{C}_0 {}^0 C_1
\end{aligned}
$$

$$
\begin{aligned}
q_i &= \|\underset{\sim}{p}_i - \underset{\sim}{b}_i\| = \\
&= \|\underset{\sim}{o}_1 + \underline{C}_1 {}^1 p_i - \underset{\sim}{o}_0 - \underline{C}_0 {}^0 b_i\| \\
&= \|\underset{\sim}{o}_1 - \underset{\sim}{o}_0 + \underline{C}_0 {}^0 C_1 {}^1 p_i - \underline{C}_0 {}^0 b_i\| \\
&= \|\underline{C}_0 ({}^0 d_1 + {}^0 C_1 {}^1 p_i - {}^0 b_i)\| \\
&= \|{}^0 d_1 - {}^0 b_i + {}^0 C_1 {}^1 p_i\| \\
&= \sqrt{\|{}^0 d_1 - {}^0 b_i\|^2 + 2({}^0 d_1 - {}^0 b_i)^T \; {}^0 C_1 {}^1 p_i + \|{}^0 C_1 \; {}^1 p_i\|^2} \\
&= \sqrt{\|{}^0 d_1 - {}^0 b_i\|^2 + 2({}^0 d_1 - {}^0 b_i)^T \; {}^0 C_1 {}^1 p_i + \|{}^1 p_i\|^2} \;.
\end{aligned}
$$

Problem 1:
- Inverse kinematics

$$^{0}C_1 \triangleq Q,\ ^{0}d_1 \triangleq d,\ ^{1}p_i \triangleq p_i,\ ^{0}b_i \triangleq b_i,\ i = 1, 2, \cdots 6$$

$$q_i^2 = \|d - b_i\|^2 + 2(d - b_i)^T Q p_i + \|p_i\|^2$$

$$\begin{aligned}
\frac{dq_i^2}{dt} = 2q_i\,\dot{q}_i &= 2(d - b_i)^T \dot{d} + 2\dot{d}^T Q p_i + 2(d - b_i)^T \dot{Q} p_i \\
&= 2(d - b_i)^T \dot{d} + 2p_i^T Q^T \dot{d} + 2(d - b_i)^T (\omega\times) Q p_i \\
&= 2(d - b_i + Qp_i)^T \dot{d} + 2(d - b_i)^T (\omega\times) Q p_i \\
&= 2(d - b_i + Qp_i)^T \dot{d} - 2p_i^T Q^T (\omega\times)(d - b_i) \\
&= 2(d - b_i + Qp_i)^T \dot{d} + 2p_i^T Q^T [(d - b_i)\times]\omega \\
&= 2(d - b_i + Qp_i)^T \dot{d} - 2[(d - b_i) \times Qp_i]^T \omega
\end{aligned}$$

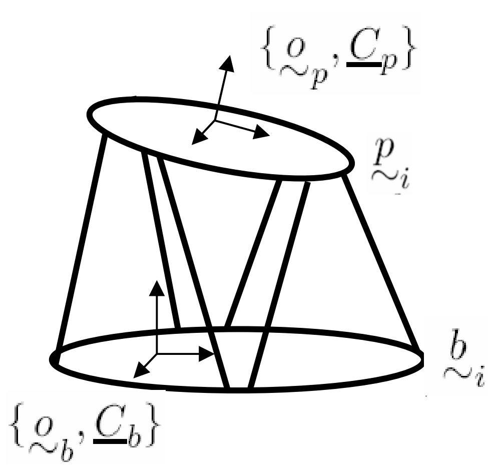
$$\dot{q}_i = \frac{1}{q_i}\left[(d + Qp_i - b_i)^T \qquad -[(d - b_i) \times Qp_i]^T\right] \begin{bmatrix} \dot{d} \\ \omega \end{bmatrix}$$

NAME:

University of British Columbia
Department of Electrical and Computer Engineering
EECE 487 (Winter 2006): Introduction to Robotics
Midterm Examination #2, Thursday March 16, 2006
Closed Book - 80 Minutes
Maximum - 30 marks

Solve all the problems. Reason your solutions. Pay attention to your notation; partial credit will not be given to you if you do not follow the notation convention.

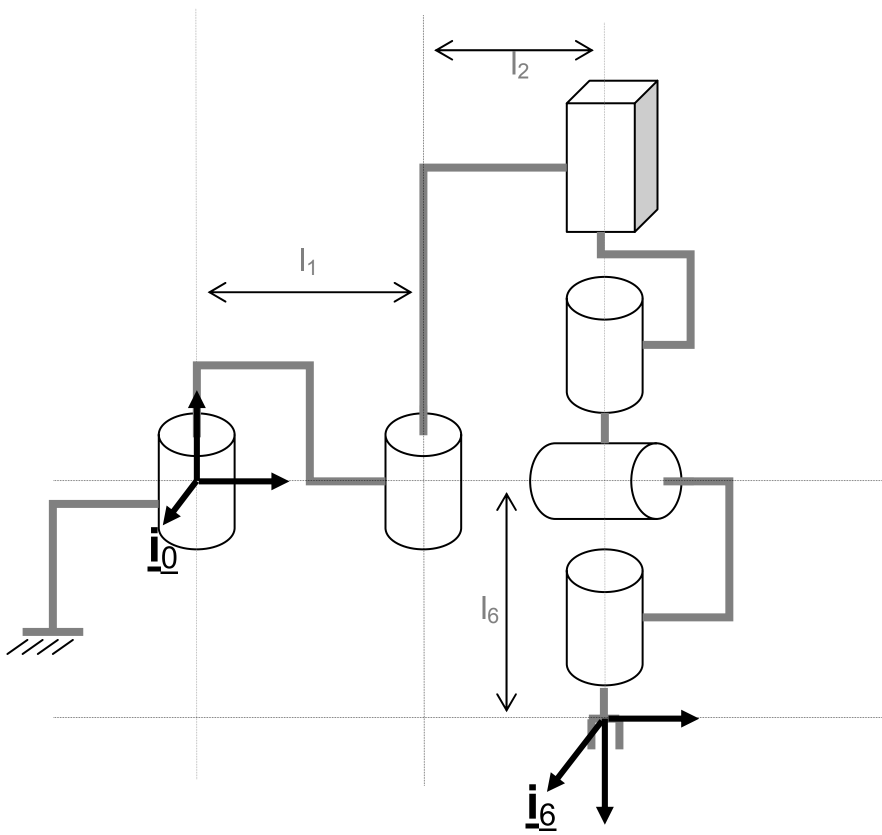
Problem 1.
(10 marks)
Consider a parallel manipulator, where the leg hinge points $\underset{\sim}{b}_i$, $i = 1, ..., 6$ on the base have coordinates b_i, $i = 1, ..., 6$ in a base-attached coordinate system $\{\underset{\sim}{o}_b, \underline{C}_b\}$. The leg hinge points $\underset{\sim}{p}_i$, $i = 1, ..., 6$ on the platform have coordinates p_i, $i = 1, ..., 6$ in a platform-attached coordinate system $\{\underset{\sim}{o}_p, \underline{C}_p\}$. Solve the inverse kinematics problem (leg lengths l_i from platform-base offsets) for this platform.



Problem 2.

(*15 marks*)

Give a detailed solution (discuss multiple solutions or lack of solutions) to the inverse kinematics of the following SCARA manipulator with spherical wrist. Assume that θ_3 and θ_5 have joint range $-\theta_{3max}$, θ_{3max}, and $-\theta_{5max}$, θ_{5max}. All angles are at zero degrees in the nominal configuration shown in the figure. If you use Kahan's problems P1-P3, you must describe their solution. The solution to P4 is given on the last page of the exam.

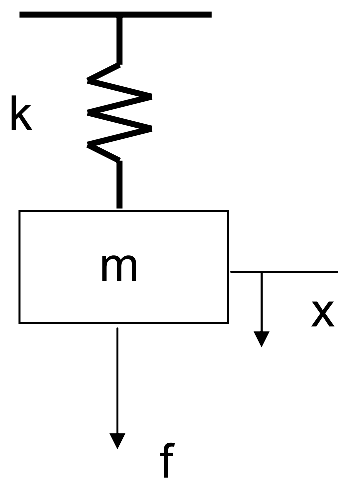


Problem 3.

(*5 marks*)

(a) Define the kinetic energy of a rigid body with mass M, center of mass velocity $\underline{v}$, angular velocity $\underline{\omega}$ and inertia tensor $\underline{\underline{J}}_{\underset{\sim}{c}}$ with respect to the center of mass $\underset{\sim}{c}$.

(b) Use Lagrange's equations to find the equation of motion of the mass-spring system shown below.



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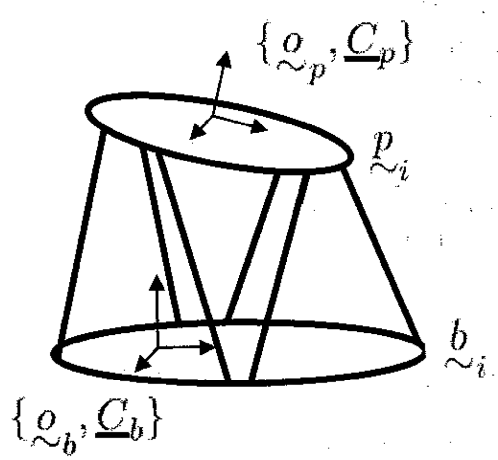


$$\underset{\sim}{p}_i = \underset{\sim}{o}_p + \underline{C}_p p_i$$

$$\underset{\sim}{b}_i = \underset{\sim}{o}_b + \underline{C}_b b_i$$

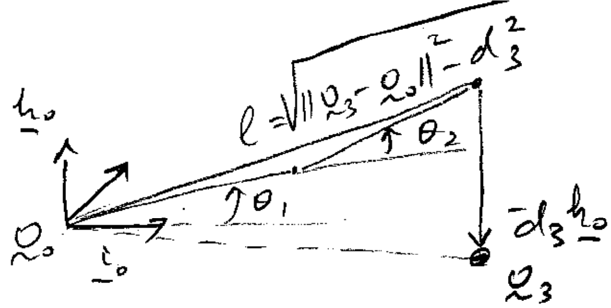
$$\underset{\sim}{p}_i - \underset{\sim}{b}_i = \underset{\sim}{o}_p - \underset{\sim}{o}_b + \underline{C}_p p_i - \underline{C}_b b_i$$

$$= \underline{C}_b\, {}^b d_p + \underline{C}_b\, {}^b C_p p_i - \underline{C}_b b_i$$

$$= \underline{C}_b ({}^b d_p + {}^b C_p p_i - b_i)$$

$$\|\underset{\sim}{p}_i - \underset{\sim}{b}_i\|_2 = l_i = \sqrt{({}^b d_p - b_i + {}^b C_p p_i)^T ({}^b d_p - b_i + {}^b C_p p_i)}$$

$$= \sqrt{\|{}^b d_p - b_i\|_2^2 + \|p_i\|_2^2 + 2 p_i^T\, {}^b C_p^T ({}^b d_p - b_i)}$$



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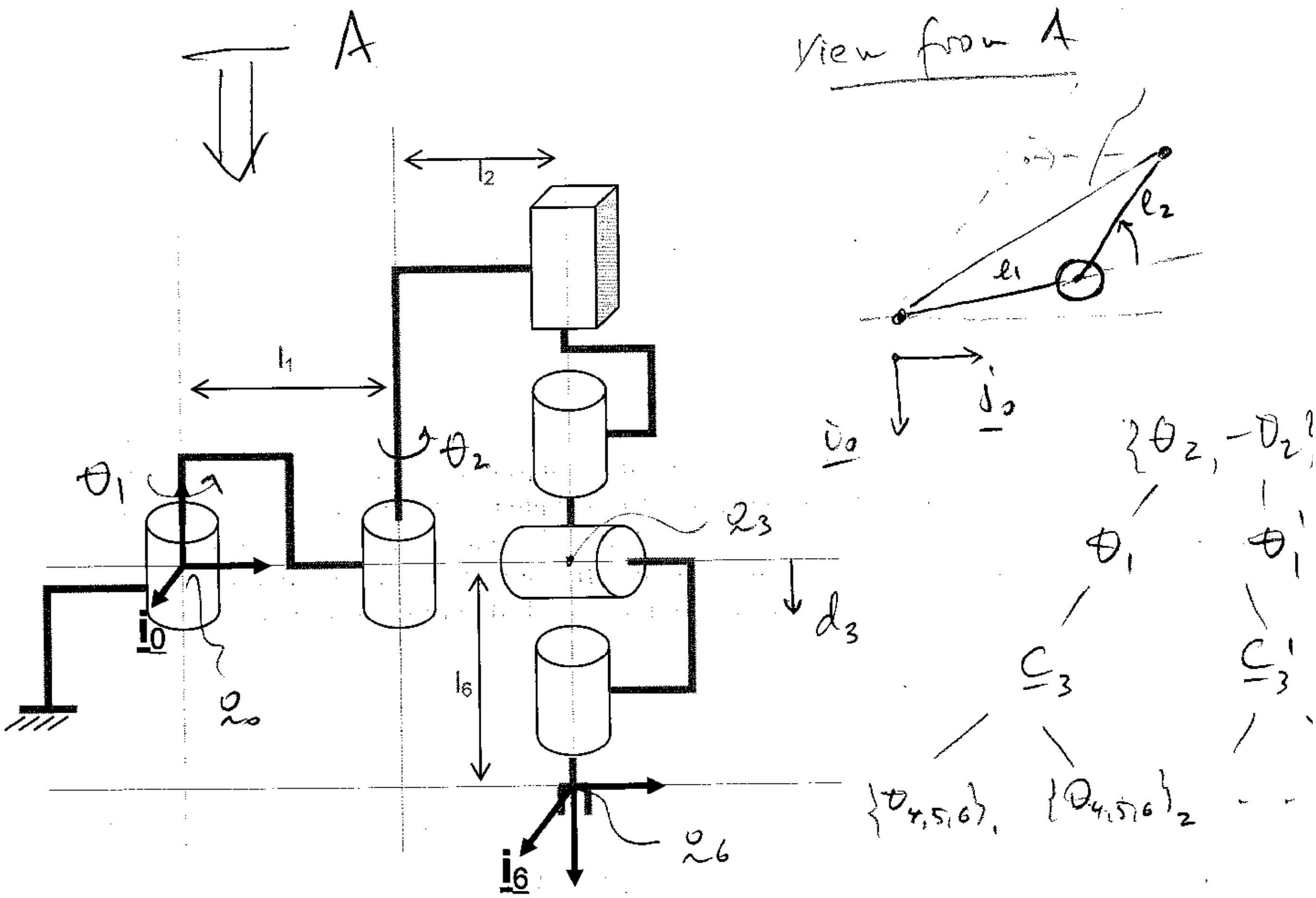


$$o_3 = o_0 + e^{\theta_1 h_0 \times}\left[l_1 j_0 + e^{\theta_2 h_0 \times}(l_2 j_0 - d_3 h_0)\right]$$

$$o_3 = o_6 - l_6 h_6 = \text{known}$$

$$o_3 - o_0 = -d_3 h_0 + e^{\theta_1 h_0 \times}\left[l_1 j_0 + e^{\theta_2 h_0 \times} l_2 j_0\right]$$

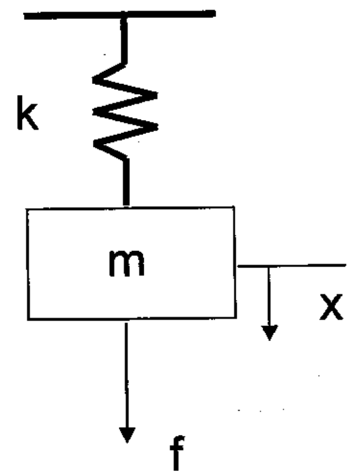
$$d_3 = -h_0^T(o_3 - o_0)$$

θ_2 from KP4, θ_1 from KP2 $\Rightarrow C_3$; solve wrist

Problem 3.
(5 marks)
(a) Define the kinetic energy of a rigid body with mass M, center of mass velocity $\underline{v}$, angular velocity $\underline{\omega}$ and inertia tensor $\underline{\underline{J}}_{\underset{\sim}{c}}$ with respect to the center of mass $\underset{\sim}{c}$.

(b) Use Lagrange's equations to find the equation of motion of the mass-spring system shown below.

a) $$T = \frac{1}{2} M \dot{\underset{\sim}{c}}^T \dot{\underset{\sim}{c}} + \frac{1}{2} \underline{\omega}^T \underline{\underline{J}}_{\underset{\sim}{c}} \underline{\omega}$$



b) $$T = \frac{1}{2} m \dot{x}^2$$

$$V = \frac{1}{2} k x^2$$

$$L = T - V = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{1}{2} m \, 2\dot{x} = m\dot{x}$$

$$\frac{\partial L}{\partial x} = -\frac{1}{2} k \, 2x = -kx$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} \quad ; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = f$$

$$\therefore \quad m\ddot{x} - (-kx) = f$$

$$m\ddot{x} + kx = f.$$

If you considered gravity as well, $V = \frac{1}{2} k x^2 - mgx$

$$\therefore \quad \frac{\partial L}{\partial x} = -kx + mg.$$

$$\therefore \quad m\ddot{x} + kx - mg = f$$

$$m\ddot{x} + kx = f + mg.$$

THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering
EECE 487 – Introduction to Robotics
Instructor: Dr. Joseph Yan
Spring 2005 Midterm 2 Examination

One double-sided hand-written 8.5" x 11" sheet is permitted. Calculators are not permitted.

Time: 75 minutes.

This examination consists of 5 pages. Please check that you have a complete copy. You may use both sides of each sheet if needed.

READ THIS

Surname First name

Student Number

#	MAX	GRADE
1	11	
2	9	
3	20	
4	20	
TOTAL	60	

IMPORTANT NOTE: The announcement "stop writing" will be made at the end of the examination. Anyone writing after this announcement will receive a score of 0. No exceptions, no excuses.

All writings must be on this booklet. The blank sides on the reverse of each page may also be used.

Each candidate should be prepared to produce, upon request, his/her Library/AMS card.

Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one-half hour, or to leave during the first half-hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination-questions.

***Caution** - Candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:*

Making use of any books, papers or memoranda, calculators, audio or visual cassette players or other memory aid devices, other than as authorized by the examiners.

Speaking or communicating with other candidates.

Purposely exposing written papers to the view of other candidates.

The plea of accident or forgetfulness shall not be received.

Problem #1 (11pts): Provide definitions for the terms below. Observe point values for an idea of how involved your definitions should be.

a) **Inverse Kinematics Problem** (3pts):

b) **Direct (Forward) Dynamics Problem** (3pts):

c) **Newton-Euler Formulation** (5pts):

Problem #2 (9pts): **Circle either T or F.** Explain your answers to receive any points.

a) (4pts) When the manipulator is in a static (stationary) configuration, the required motor generalized forces are all zero. **T / F**

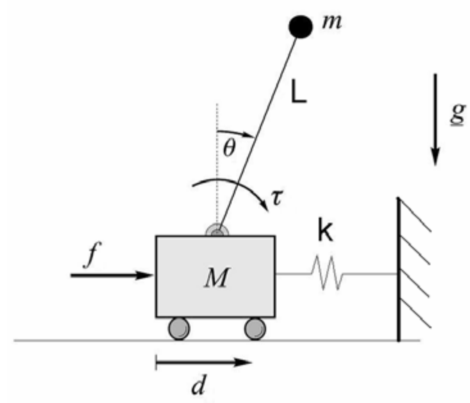
b) (5pts) It is possible for the manipulator inertia matrix to take on the value $D = \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}$

(assume units are compatible with joint variables). **T / F**
(Hint: I only want you to determine if D is positive definite)

Problem #3 (20 pts): <u>Inverted Pendulum on a Cart</u>

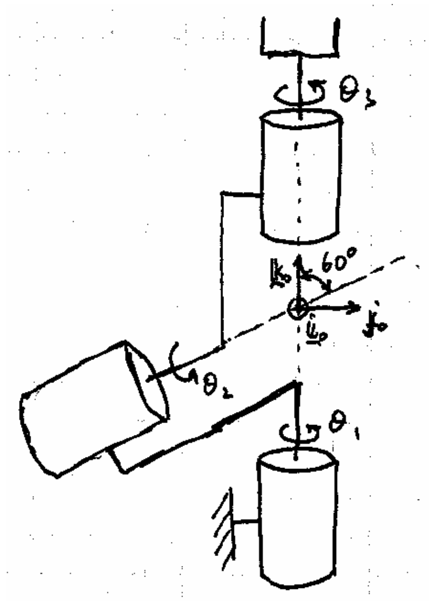
a) For the inverted pendulum on a cart, attached to a wall by a Hookean spring of stiffness *k*, derive the Lagrangian function. Assume the spring is relaxed when *d*=0. (7pts).

b) Now determine the equations of motion in standard form for robot manipulator dynamics (identify your expressions for $D(q), C(q,\dot{q}),$ and $G(q)$) (13pts).



Problem #4 (20pts): Inverse Kinematics for the Oblique Spherical Wrist

a) (6pts) For the oblique spherical wrist shown, assign and label D-H coordinate systems (preferably with different colours; only need to label $\underline{i}_i$ and $\underline{k}_i$ vectors), starting from the base coordinate system $\{\underset{\sim}{o}_0, \underline{C}_0\}$ already given. Generate the D-H table. Assume the configuration shown is the home position and enclose variable joint parameters in parentheses. Add appropriate labels on the figure for any additional dimensions you need to complete the table.



b) (4pts) Given $\underline{C}_0$ and a desired end-effector orientation, $\underline{C}_d$, establish a necessary and sufficient condition for the inverse kinematics solution to exist (assume no joint limits).

c) (10pts) Assuming the condition in (b) is satisfied, solve the inverse kinematics problem for θ_1, θ_2, and θ_3. Express your solutions as Kahan problem solutions (after adequately defining the needed ones). Discuss solution uniqueness.

Problem #1 (11pts): Provide definitions for the terms below. Observe point values for an idea of how involved your definitions should be.

a) **Inverse Kinematics Problem** (3pts): THE PROBLEM OF DETERMINING THE SET(S) OF JOINT DISPLACEMENTS ($\underset{\sim}{q}$) THAT PLACE THE END-EFFECTOR AT THE SPECIFIED DESIRED LOCATION ($\underset{\sim}{P}_d$) AND ORIENTATION ($\underline{C}_d$)

b) **Direct (Forward) Dynamics Problem** (3pts): THE PROBLEM OF DETERMINING JOINT ACCELERATIONS (AND HENCE, END-EFFECTOR MOTION) FROM ACTUATOR GENERALIZED FORCES.

c) **Newton-Euler Formulation** (5pts): A METHODOLOGY OF DETERMINING MANIPULATOR DYNAMICS (BETTER SUITED FOR INVERSE DYNAMICS) USING

A : FORWARD PROPAGATION OF VELOCITIES AND ACCELERATIONS

B : REVERSE PROPAGATION OF FORCES AND TORQUES

C : PROJECTION OF GENERALIZED FORCES ON MOTOR AXES.

Problem #2 (9pts): **Circle either T or F.** Explain your answers to receive any points.

a) (4pts) When the manipulator is in a static (stationary) configuration, the required motor generalized forces are all zero. **T** / **F**

$$D(q)\cancel{\ddot{q}}^{0} + C(q,\dot{q})\cancel{\dot{q}}^{0} + G(q) = u$$

⇒ WHEN STATIC, THE MOTOR GENERALIZED FORCES NEED TO TAKE CARE OF GRAVITATIONAL FORCES (AND POSSIBLY ENVIRONMENT FORCES)

SO $u = G(q)$

b) (5pts) It is possible for the manipulator inertia matrix to take on the value $D = \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}$

(assume units are compatible with joint variables) **T** / **F**

(Hint: I only want you to determine if D is positive definite)

$$x^T D x = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2x_1 - x_2 \\ -x_1 + 4x_2 \end{bmatrix}$$

$$= 2x_1^2 - x_1 x_2 - x_2 x_1 + 4x_2^2$$

$$= x_1^2 + (x_1^2 - 2x_1 x_2 + x_2^2) + 3x_2^2$$

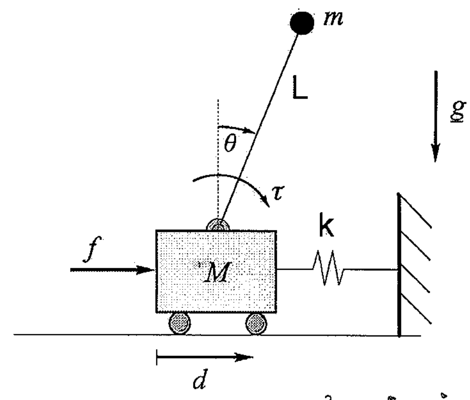
$$= x_1^2 + (x_1 - x_2)^2 + 3x_2^2 \geq 0$$

WITH EQUALITY ONLY FOR $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Problem #3 (2.0 pts): Inverted Pendulum on a Cart

a) For the inverted pendulum on a cart, attached to a wall by a Hookean spring of stiffness k, derive the Lagrangian function. Assume the spring is relaxed when d=0. (7pts).

b) Now determine the equations of motion in standard form for robot manipulator dynamics (identify your expressions for $D(q), C(q,\dot{q})$, and $G(q)$) (13pts).



a) $T = \frac{1}{2} M \dot{d}^2 + \frac{1}{2} m v_m^2$

WHERE $x_m = d + L s_\theta \rightarrow \dot{x}_m = \dot{d} + L c_\theta \dot{\theta}$

$y_m = L c_\theta \rightarrow \dot{y}_m = -L s_\theta \dot{\theta}$

$\Big\} \; v_m^2 = \dot{x}_m^2 + \dot{y}_m^2 = \dot{d}^2 + 2\dot{d} L \dot{\theta} c_\theta + L^2 \dot{\theta}^2$

$\Rightarrow T = \frac{1}{2}\left\{ (M+m)\dot{d}^2 + m L^2 \dot{\theta}^2 + 2 m L \dot{d} \dot{\theta} c_\theta \right\}$

$V = mgLc_\theta + \frac{1}{2} k d^2$ WHERE $g = 9.81 \, m/s^2$

$$\Rightarrow \boxed{L = T - V = \frac{1}{2}\left\{ (M+m)\dot{d}^2 + mL^2\dot{\theta}^2 + 2mL\dot{d}\dot{\theta}c_\theta \right\} - mgLc_\theta - \frac{1}{2}kd^2}$$

b) $f = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{d}}\right) - \frac{\partial L}{\partial d} = \frac{d}{dt}\left((M+m)\dot{d} + mL\dot{\theta}c_\theta \right) - (-kd)$

$= (M+m)\ddot{d} + mL(\ddot{\theta}c_\theta + \dot{\theta}^2(-s_\theta)) + kd$

$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(mL^2\dot{\theta} + mL\dot{d}c_\theta \right) - \left(mL\dot{d}\dot{\theta}(-s_\theta) - mgL(-s_\theta) \right)$

$= mL\left(L\ddot{\theta} + \ddot{d}c_\theta + \cancel{\dot{d}\dot{\theta}(-s_\theta)} + s_\theta\cancel{(\dot{d}\dot{\theta})} - g s_\theta\right)$

$$\boxed{\begin{bmatrix} f \\ \tau \end{bmatrix} = \underbrace{\begin{bmatrix} M+m & mLc_\theta \\ mLc_\theta & mL^2 \end{bmatrix}}_{D(q)} \begin{bmatrix} \ddot{d} \\ \ddot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -mL\dot{\theta}s_\theta \\ 0 & 0 \end{bmatrix}}_{C(q,\dot{q})} \begin{bmatrix} \dot{d} \\ \dot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} kd \\ -mgLs_\theta \end{bmatrix}}_{G(q)}}$$

Problem #4 (20pts): <u>Inverse Kinematics for the Oblique Spherical Wrist</u>

a) (6pts) For the oblique spherical wrist shown, assign and label D-H coordinate systems (preferably with different colours), starting from the base coordinate system $\{\underline{O}_0, \underline{C}_0\}$ already given. Generate the D-H table. Assume the configuration shown is the home position and enclose variable joint parameters in parentheses. Add appropriate labels on the figure for any additional dimensions you need to complete the table.

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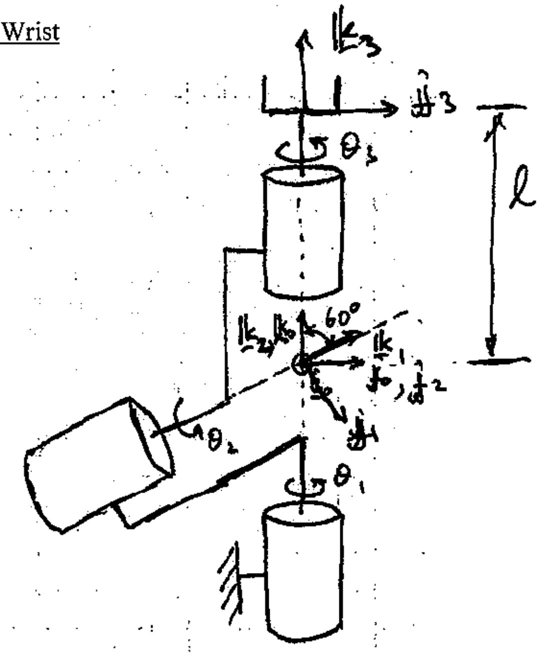


a)

	θ_i	d_i	a_i	α_i
1	(θ_1)	0	0	$-\frac{\pi}{3}$
2	(θ_2)	0	0	$\frac{\pi}{3}$
3	(θ_3)	l	0	0

$$\Rightarrow \underline{C}_1 = \underline{C}_0 e^{\theta_1 \underline{k}\times} e^{-\frac{\pi}{3}\underline{i}\times}$$

$$\Rightarrow \underline{C}_2 = \underline{C}_1 e^{\theta_2 \underline{k}\times} e^{\frac{\pi}{3}\underline{i}\times}$$

$$\Rightarrow \underline{C}_3 = \underline{C}_2 e^{\theta_3 \underline{k}\times}$$

$$\underline{k}_3 = \underline{k}_2 = e^{\theta_2 \underline{k}_1\times}\left(e^{\frac{\pi}{3}\underline{i}_1\times}\underline{k}_1\right)$$

b) NEED $\boxed{\underline{k}_d^T \underline{k}_0 \geq \cos\left(2\times\frac{\pi}{3}\right) = -\frac{1}{2}}$ $\Rightarrow$ For $\underline{k}_d^T\underline{k}_0$:
- $< -\frac{1}{2}$ SOLUTION DOESN'T EXIST
- $= -\frac{1}{2}$ " UNIQUE
- $= 1$ INFINITE SOL'NS (SINGULARITY)
- OTHERWISE 2 SOLUTIONS

c) FROM (a) $\Rightarrow$ $\underline{C}_3 e^{-\theta_3 \underline{k}\times} = \underline{C}_2 = \underline{C}_0 e^{\theta_1 \underline{k}\times} e^{-\frac{\pi}{3}\underline{i}\times} e^{\theta_2 \underline{k}\times} e^{\frac{\pi}{3}\underline{i}\times}$

$$\Rightarrow e^{-\theta_3 \underline{k}_3\times}\underline{C}_3 = \underline{C}_0 e^{\theta_1 \underline{k}\times} e^{\theta_2\left(e^{-\frac{\pi}{3}\underline{i}\times}\underline{k}\right)\times}$$

OPERATE ON $e^{-\frac{\pi}{3}\underline{i}\times}\underline{k}$ $\Rightarrow$ $e^{-\theta_3 \underline{k}_3\times}\left(e^{-\frac{\pi}{3}\underline{i}_3\times}\underline{k}_3\right) = \overbrace{\underline{C}_0 e^{\theta_1 \underline{k}\times} e^{-\frac{\pi}{3}\underline{i}\times}}^{\underline{C}_1}\underline{k} = e^{\theta_1 \underline{k}_0\times}\left(e^{-\frac{\pi}{3}\underline{i}_0\times}\underline{k}_0\right) = \underline{k}_1$

ASSUME HAVE FOLLOWING FUNCTIONS TO SOLVE KAHAN PROBLEMS:

a) $e^{\theta \hat{s}\times} u = w \rightarrow \theta = \text{kahanP2}(\hat{s}, u, w)$

b) $e^{\theta \hat{s}\times} u = e^{\phi \hat{t}\times} v \rightarrow [\theta, \phi] = \text{kahanP3}(\hat{s}, \hat{t}, u, v)$ [BOTH SETS FOUND]

$$\Rightarrow \boxed{\begin{array}{l}[\theta_1, \theta_3] = \text{kahanP3}\left(\underline{k}_0, -\underline{k}_3, e^{-\frac{\pi}{3}\underline{i}_0\times}\underline{k}_0, e^{-\frac{\pi}{3}\underline{i}_3\times}\underline{k}_3\right) \\ \theta_2 = \text{kahanP2}\left(\underline{k}_1, e^{\frac{\pi}{3}\underline{i}_1\times}\underline{k}_1, \underline{k}_2\right)\end{array}}$$

THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering
EECE 487 – Introduction to Robotics
Instructor: Dr. Joseph Yan
Fall 2003 Midterm

One single-sided hand-written 8.5” x 11” sheet is permitted. Calculators are not permitted.

Time: 50 minutes.

This examination consists of 7 pages. Please check that you have a complete copy. You may use both sides of each sheet if needed.

READ THIS

Surname First name

Student Number

#	MAX	GRADE
1	12	
2	20	
3	20	
4	18	
Bonus	15	
TOTAL	85	

IMPORTANT NOTE: The announcement “stop writing” will be made at the end of the examination. Anyone writing after this announcement will receive a score of 0. No exceptions, no excuses.

All writings must be on this booklet. The blank sides on the reverse of each page may also be used.

Each candidate should be prepared to produce, upon request, his/her Library/AMS card.

Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one-half hour, or to leave during the first half-hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination-questions.

Caution *- Candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:*

Making use of any books, papers or memoranda, calculators, audio or visual cassette players or other memory aid devices, other than as authorized by the examiners.

Speaking or communicating with other candidates.

Purposely exposing written papers to the view of other candidates.

The plea of accident or forgetfulness shall not be received.

Problem #1 (12pts): Provide definitions for the terms below. Observe point values for an idea of how involved your definitions should be.

a) **Robot**. Also, name 2 reasons to employ a robot instead of a human for a task. (5pts):

b) **Euler angles** (3pts):

c) **Manipulator Jacobian** (4pts):

Problem #2 (20pts): True/False questions. Explain your answers to receive any points.

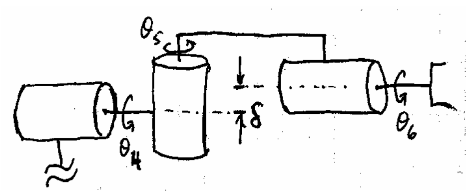
a) (4pts) For a homogenous transformation matrix ${}^{1}T_0$, the transpose and inverse are identical (i.e., $({}^{1}T_0)^T = ({}^{1}T_0)^{-1}$). **T / F**

b) (6pts) (For any true answers here, indicate the required D-H parameters.)

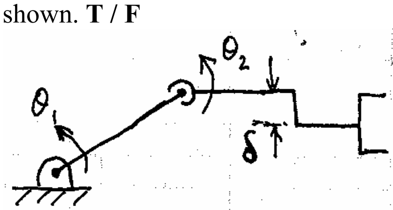
In the D-H convention, it is possible to have $\boldsymbol{i_{i+1}}=\boldsymbol{j_i}$. **T / F**

In the D-H convention, it is possible to have $\boldsymbol{i_{i+1}}=\boldsymbol{k_i}$. **T / F**

c) (5pts) For the orthogonal wrist, we can avoid wrist singularities by incorporating a nonzero offset, δ, as shown. **T / F**

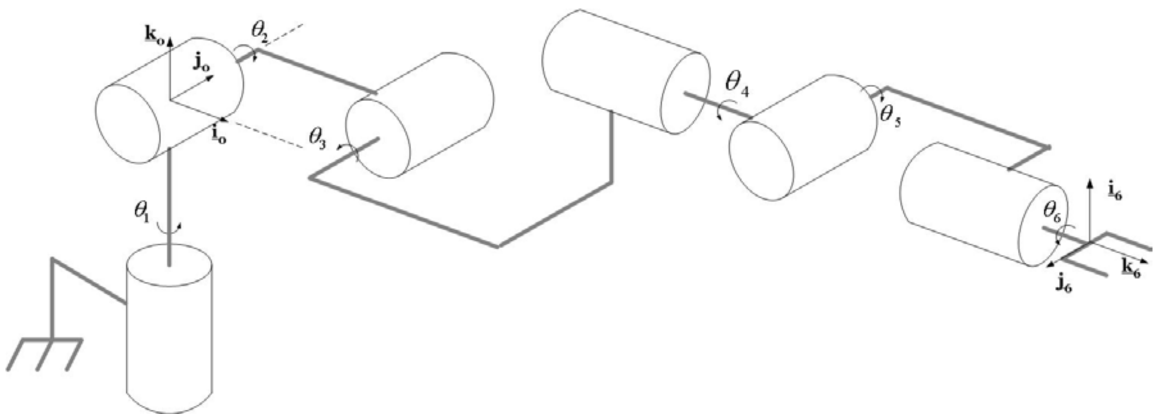


d) (5pts): For the planar 2-link RR arm where we're only concerned with the positioning of the end-effector (not the orientation), we can avoid singularities by incorporating a nonzero offset as shown. **T / F**



Problem #3 (20pts): Puma560

For the Puma560 shown, assign and label D-H frames (only need to label **i** and **k** vectors), starting from the base frame already assigned. Generate the D-H table, adding appropriate labels on the figure for any dimensions you need to complete the table. Assume the configuration shown is the home position and enclose variable joint parameters in parentheses.

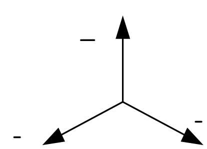


Problem #4 (18ts): Manipulator from D-H Table

a) Sketch the manipulator described by the table of D-H parameters below, starting from the base coordinate system shown. Label all coordinate systems and dimensions. In the table, joint variables are enclosed in parentheses (12 pts).

	θ_i	d_i	a_i	α_i
Link 1	(θ_1)	d_1	a_1	$\pi/2$
Link 2	(θ_2)	d_2	0	$-\pi/2$
Link 3	$\pi/2$	(d_3)	0	0

b) Find the abstract expression for the geometric Jacobian and discuss the existence of singularities (6pts).



Bonus Problem (15 pts): Homogenous Transformation Matrices

Write down the general form of a homogenous transformation matrix, ${}^{1}T_{0}$, and find its eigenvalues. Consider $v = \begin{bmatrix} p_x & p_y & p_z & \xi \end{bmatrix}^T$ to be an eigenvector. For what values of ξ does $\boldsymbol{v}$ have any physical meaning and what is the interpretation?

Problem #1 (11pts): Provide definitions for the terms below. Observe point values for an idea of how involved your definitions should be.

a) **Inverse Kinematics Problem** (3pts): THE PROBLEM OF DETERMINING THE SET(S) OF JOINT DISPLACEMENTS ($\underset{\sim}{q}$) THAT PLACE THE END-EFFECTOR AT THE SPECIFIED DESIRED LOCATION ($\underset{\sim}{p}_d$) AND ORIENTATION ($\underline{C}_d$)

b) **Direct (Forward) Dynamics Problem** (3pts): THE PROBLEM OF DETERMINING JOINT ACCELERATIONS (AND HENCE, END-EFFECTOR MOTION) FROM ACTUATOR GENERALIZED FORCES.

c) **Newton-Euler Formulation** (5pts): A METHODOLOGY OF DETERMINING MANIPULATOR DYNAMICS (BETTER SUITED FOR INVERSE DYNAMICS) USING

A : FORWARD PROPAGATION OF VELOCITIES AND ACCELERATIONS

B : REVERSE PROPAGATION OF FORCES AND TORQUES

C : PROJECTION OF GENERALIZED FORCES ON MOTOR AXES.

Problem #2 (9pts): **Circle either T or F.** Explain your answers to receive any points.

a) (4pts) When the manipulator is in a static (stationary) configuration, the required motor generalized forces are all zero. **T** / **F**

$$D(q)\cancel{\ddot{q}}^{0} + C(q,\dot{q})\cancel{\dot{q}}^{0} + G(q) = u$$

⇒ WHEN STATIC, THE MOTOR GENERALIZED FORCES NEED TO TAKE CARE OF GRAVITATIONAL FORCES (AND POSSIBLY ENVIRONMENT FORCES)

SO $u = G(q)$

b) (5pts) It is possible for the manipulator inertia matrix to take on the value $D = \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}$

(assume units are compatible with joint variables) **T** / **F**

(Hint: I only want you to determine if D is positive definite)

$$x^T D x = [x_1 \quad x_2] \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x_1 \quad x_2] \begin{bmatrix} 2x_1 - x_2 \\ -x_1 + 4x_2 \end{bmatrix}$$

$$= 2x_1^2 - x_1x_2 - x_2x_1 + 4x_2^2$$

$$= x_1^2 + (x_1^2 - 2x_1x_2 + x_2^2) + 3x_2^2$$

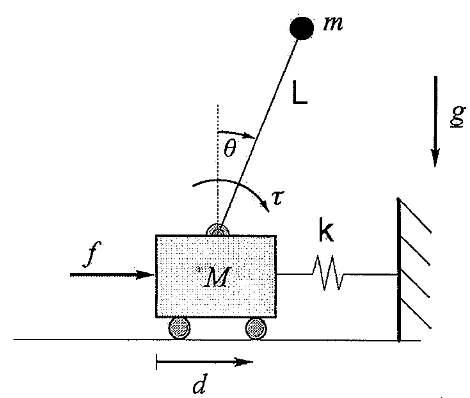
$$= x_1^2 + (x_1 - x_2)^2 + 3x_2^2 \geq 0$$

WITH EQUALITY ONLY FOR $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Problem #3 (2.0 pts): Inverted Pendulum on a Cart

a) For the inverted pendulum on a cart, attached to a wall by a Hookean spring of stiffness k, derive the Lagrangian function. Assume the spring is relaxed when d=0. (7pts).

b) Now determine the equations of motion in standard form for robot manipulator dynamics (identify your expressions for $D(q), C(q,\dot{q})$, and $G(q)$) (13pts).



a) $T = \frac{1}{2}M\dot{d}^2 + \frac{1}{2}m v_m^2$

WHERE $x_m = d + L s_\theta \rightarrow \dot{x}_m = \dot{d} + L c_\theta \dot{\theta}$

$y_m = L c_\theta \rightarrow \dot{y}_m = -L s_\theta \dot{\theta}$

$\Big\} \; v_m^2 = \dot{x}_m^2 + \dot{y}_m^2 = \dot{d}^2 + 2\dot{d}L\dot{\theta}c_\theta + L^2\dot{\theta}^2$

$\Rightarrow T = \frac{1}{2}\{(M+m)\dot{d}^2 + mL^2\dot{\theta}^2 + 2mL\dot{d}\dot{\theta}c_\theta\}$

$V = mgLc_\theta + \frac{1}{2}kd^2$ WHERE $g = 9.81 m/s^2$

$\Rightarrow \boxed{L = T - V = \frac{1}{2}\{(M+m)\dot{d}^2 + mL^2\dot{\theta}^2 + 2mL\dot{d}\dot{\theta}c_\theta\} - mgLc_\theta - \frac{1}{2}kd^2}$

b) $f = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{d}}\right) - \frac{\partial L}{\partial d} = \frac{d}{dt}\left((M+m)\dot{d} + mL\dot{\theta}c_\theta\right) - (-kd)$

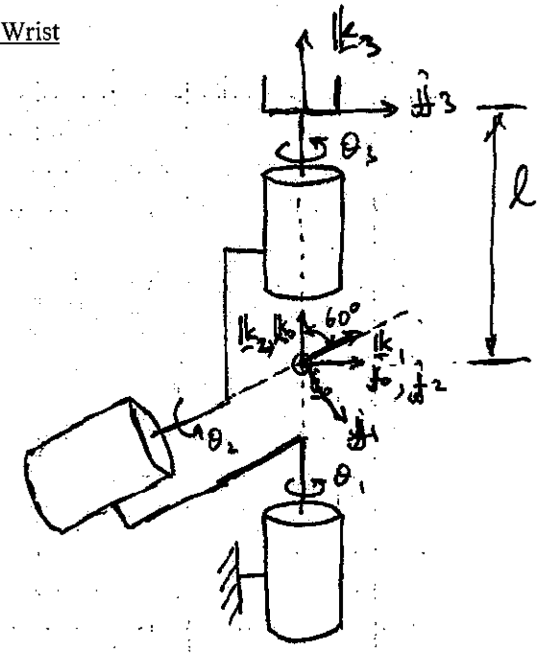
$= (M+m)\ddot{d} + mL(\ddot{\theta}c_\theta + \dot{\theta}^2(-s_\theta)) + kd$

$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(mL^2\dot{\theta} + mL\dot{d}c_\theta\right) - \left(mL\dot{d}\dot{\theta}(-s_\theta) - mgL(-s_\theta)\right)$

$= mL(L\ddot{\theta} + \ddot{d}c_\theta + \cancel{\dot{d}\dot{\theta}(-s_\theta)} + \cancel{s_\theta(\dot{d}\dot{\theta})} - gs_\theta)$

$$\boxed{\begin{bmatrix} f \\ \tau \end{bmatrix} = \underbrace{\begin{bmatrix} M+m & mLc_\theta \\ mLc_\theta & mL^2 \end{bmatrix}}_{D(q)} \begin{bmatrix} \ddot{d} \\ \ddot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -mL\dot{\theta}s_\theta \\ 0 & 0 \end{bmatrix}}_{C(q,\dot{q})} \begin{bmatrix} \dot{d} \\ \dot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} kd \\ -mgLs_\theta \end{bmatrix}}_{G(q)}}$$

Problem #4 (20pts): Inverse Kinematics for the Oblique Spherical Wrist



a) (6pts) For the oblique spherical wrist shown, assign and label D-H coordinate systems (preferably with different colours), starting from the base coordinate system $\{\underline{O}_0, \underline{C}_0\}$ already given. Generate the D-H table. Assume the configuration shown is the home position and enclose variable joint parameters in parentheses. Add appropriate labels on the figure for any additional dimensions you need to complete the table.

b) (4pts) Given $\underline{C}_0$ and a desired end-effector orientation, $\underline{C}_d$, establish a necessary and sufficient condition for the inverse kinematics solution to exist (assume no joint limits).

c) (10pts) Assuming the condition in (b) is satisfied, solve the inverse kinematics problem for θ_1, θ_2, and θ_3. Express your solutions as Kahan problem solutions (after adequately defining the needed ones). Discuss solution uniqueness.

a)

	θ_i	d_i	a_i	α_i
1	(θ_1)	0	0	$-\frac{\pi}{3}$
2	(θ_2)	0	0	$\frac{\pi}{3}$
3	(θ_3)	l	0	0

$$\Rightarrow \underline{C}_1 = \underline{C}_0 e^{\theta_1 \mathbb{k}\times} e^{-\frac{\pi}{3}\mathbb{i}\times}$$

$$\Rightarrow \underline{C}_2 = \underline{C}_1 e^{\theta_2 \mathbb{k}\times} e^{\frac{\pi}{3}\mathbb{i}\times}$$

$$\Rightarrow \underline{C}_3 = \underline{C}_2 e^{\theta_3 \mathbb{k}\times}$$

$$\underline{\mathbb{k}}_3 = \underline{\mathbb{k}}_2 = e^{\theta_2 \underline{\mathbb{k}}_1\times}\left(e^{\frac{\pi}{3}\underline{\mathbb{i}}_1\times}\underline{\mathbb{k}}_1\right)$$

b) NEED $\boxed{\underline{\mathbb{k}}_d^T \underline{\mathbb{k}}_0 \geq \cos\left(2\times\frac{\pi}{3}\right) = -\frac{1}{2}}$ $\Rightarrow$ For $\underline{\mathbb{k}}_d^T \underline{\mathbb{k}}_0$:

- $< -\frac{1}{2}$ SOLUTION DOESN'T EXIST
- $= -\frac{1}{2}$ " UNIQUE
- $= 1$ INFINITE SOL'NS (SINGULARITY)
- OTHERWISE 2 SOLUTIONS

c) FROM (a) $\Rightarrow$ $\underline{C}_3 e^{-\theta_3 \mathbb{k}\times} = \underline{C}_2 = \underline{C}_0 e^{\theta_1 \mathbb{k}\times} e^{-\frac{\pi}{3}\mathbb{i}\times} e^{\theta_2 \mathbb{k}\times} e^{\frac{\pi}{3}\mathbb{i}\times}$

$$\Rightarrow e^{-\theta_3 \underline{\mathbb{k}}_3\times}\underline{C}_3 = \underline{C}_0 e^{\theta_1 \mathbb{k}\times} e^{\theta_2\left(e^{-\frac{\pi}{3}\mathbb{i}\times}\mathbb{k}\right)\times}$$

OPERATE ON $e^{-\frac{\pi}{3}\mathbb{i}\times}\mathbb{k}$ $\Rightarrow$ $e^{-\theta_3 \underline{\mathbb{k}}_3\times}\left(e^{-\frac{\pi}{3}\underline{\mathbb{i}}_3\times}\underline{\mathbb{k}}_3\right) = \underbrace{\underline{C}_0 e^{\theta_1 \mathbb{k}\times}}_{\underline{C}_1} e^{-\frac{\pi}{3}\mathbb{i}\times}\mathbb{k} = e^{\theta_1 \underline{\mathbb{k}}_0\times}\left(e^{-\frac{\pi}{3}\underline{\mathbb{i}}_0\times}\underline{\mathbb{k}}_0\right) = \underline{\mathbb{k}}_1$

ASSUME HAVE FOLLOWING FUNCTIONS TO SOLVE KAHAN PROBLEMS:

a) $e^{\theta\hat{s}\times}u = w \rightarrow \theta = \text{kahanP2}(\hat{s}, u, w)$

b) $e^{\theta\hat{s}\times}u = e^{\phi\hat{t}\times}v \rightarrow [\theta, \phi] = \text{kahanP3}(\hat{s}, \hat{t}, u, v)$ [BOTH SETS FOUND]

$$\Rightarrow \boxed{\begin{array}{l}[\theta_1, \theta_3] = \text{kahanP3}\left(\underline{\mathbb{k}}_0, -\underline{\mathbb{k}}_3, e^{-\frac{\pi}{3}\underline{\mathbb{i}}_0\times}\underline{\mathbb{k}}_0, e^{-\frac{\pi}{3}\underline{\mathbb{i}}_3\times}\underline{\mathbb{k}}_3\right) \\ \theta_2 = \text{kahanP2}\left(\underline{\mathbb{k}}_1, e^{\frac{\pi}{3}\underline{\mathbb{i}}_1\times}\underline{\mathbb{k}}_1, \underline{\mathbb{k}}_2\right)\end{array}}$$