Marking Scheme for Vibration due to Rotating Unbalance Experiment

Names:
Student Numbers:

Item	Marks Allocated	Marks Awarded
Abstract	**10**	
Precise summary of the overall report	7	
Critical Comparison with theory	2	
Within word limit of 300	1	
Introduction and Methodology	**15**	
Clear motivation for this lab exercise and relevance to industrial practise	5	
Stated phenomena observed and their practical importance	5	
Clear and illustrative diagrams with components indicated	5	
Results	**44**	
<u>In-Phase</u>	<u>24</u>	
(a) Time trace for damping ratio with calculation	2	
(b) Typical pre-and post-filtered signal, tachometer signal	1	
(c) Amplitude vs. rot. speed graph; Phase vs. rot. speed graph	4	
(d) Hand calculation of phase and comparison with MATLAB	5	
(f) Acceleration/Displacement relation for harmonic motion	1	
(g) Theoretical prediction for amplitude vs. rotation speed relation with **damping uncertainty included through bounding curves**	11	
<u>Out-of-phase Results</u>	<u>20</u>	
(a) Damping ratio calculation with time trace	2	
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(d) Moment of inertia & equivalent model	10	
Discussion/Conclusion	**16**	
Concise and specific summary of measurements	2	
Physical meaning of resonant frequencies	2	
Damping ratio explanation	2	
Answered all questions listed at the end of handout	8	
Comment on errors	2	
General	**15**	
Overall presentation, format and tidiness	3	
Labelling of axes & units mentioned	2	
Pre-Lab and Quiz	5	
Critical analysis and discussions	5	
Total	**100**	

Contribution of each group member

Write the contribution of each group member to this report for each section.

DEPARTMENT OF MECHANICAL ENGINEERING–THE UNIVERSITY OF BRITISH COLUMBIA

VIBRATION DUE TO A ROTATING UNBALANCE

MECH 463: Experiment 1

Location: ICICS X039 (Opposite PACE Lab)

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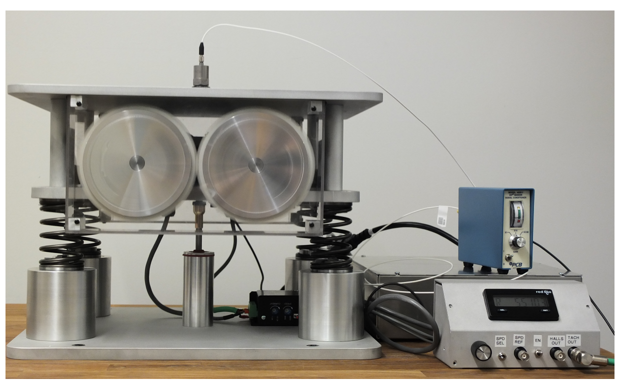

Lab Preparation:

Bring your answers to the pre-lab exercise

Group Report Deadline:

Submit in CEME 1054, within 2 weeks after the completion of the lab (including holidays)

VIBRATION DUE TO A ROTATING UNBALANCE

1.1 Objectives

1. To measure the excitation frequency dependent vibration response of a rotating system with unbalance.
2. To observe the occurrence of multiple resonant/natural frequencies and associated vibration modeshapes.
3. To observe how different unbalanced forces can affect the vibration response.
4. To measure damping ratio of a lightly damped structure.
5. To use typical vibration measuring instrumentation such as a Piezoelectric accelerometer, signal conditioner, optical encoder (motion/vibration sensor), data acquisition system, and MATLAB.
6. To develop SDOF models and predict measured vibration response.

1.2 Introduction

Rotating parts are common components of many mechanical devices. Typical examples are motors, pumps, shafts, pulleys, gears, and wheels. With most of these devices, some care is usually taken to ensure that the centre of mass is located at the axis of rotation. That way, one can avoid the inertial (centrifugal) forces generated by the mass eccentricity—the rotation component is "balanced."

In practice, however, it is not possible to balance a rotor perfectly. Furthermore, many rotating devices have reciprocating devices connected to them (e.g., automotive engine) which are difficult to Balance fully. Harmonic forces are generated due to a rotating mass, situated away from the axis of rotation[1]. The main frequency of such forces equals the rotation speed of the device. The resulting effect depends mainly on the magnitude, frequency, location, and direction of the forces, and the vibration response of the device (which depends on its dynamic characteristics including the flexible mounts). One case of particular concern is when the excitation frequency is close to the resonant frequency of the device. The resulting vibration

[1] Rotating an eccentric mass such as a stone tied to a string is a good example to recall. The force experienced by the person increases as the length of the string is increased, or radius of circular motion is increased

can become quite large, and may cause malfunctions, damage, or even catastrophic failure.
A washing machine is a typical household example where you might have seen this type of vibration. Wet clothes represent the rotating eccentric masses. The resulting vibrations are more complex than the laboratory exercise you will now perform.
This experiment is designed to illustrate salient features of the response of vibrating systems to excitation forces created by the mass eccentricity of a rotating component. This situation is common in an engine with rotating components. In addition to "dynamic balancing," proper engine mounts (i.e., spring-damper devices) may be used to avoid the resulting adverse vibrations (not considered in this experiment, but quite relevant in the study of mechanical vibration). **The underlying theory is provided in Appendix A**.

1.3 Apparatus

The experimental apparatus consists of a box-shaped structure mounted on four spring supports. This structure is an approximate model of an automobile engine secured on engine mounts. Out-of-balance forces are created by eccentric masses attached to two counter-rotating shafts in the structure, in two separate planes. In the experiments you will learn about harmonically excited (forced) vibrations, a damping measurement method, multiple resonances, vibration mode shapes and vibration measurement instrumentation.
The schematic diagram in **Fig.(1)** shows the main components of the experimental set-up.

1. **Engine model**: The box-shaped structure supported on four springs is an approximate model of an automobile engine secured on engine mounts. A dashpot attached to the centre of the box provides damping, representing, for example, the damping present in an automobile shock absorber. The box includes two counter rotating shafts with four discs carrying eccentric masses (two separate planes of rotating disks; each plane has two counter-rotating eccentric masses). These eccentric masses supply the excitation force through their inertial forces, as described in Appendix A.
2. **Drive motor**: All four eccentric masses (on the four disks) are driven by a single motor, using proper gearing. The motor is of the stepper category, which is moved in response to a sequence of pulses generated by the drive hardware (indexer) inside the speed control box. As the speed control knob is turned clockwise, the pulse rate increases thereby increasing the motor speed, and *vice versa*.
3. **Motor speed sensor**: The motor shaft carries an optical encoder. It generates one optical pulse per rotation. The pulse count gives the angle of rotation (angular position) since each pulse corresponds to a rotation of 2π radians. The pulse rate gives the speed. Both readings are displayed, as determined by the electronics inside the control box.
4. **Accelerometer & other instrumentation**: An accelerometer device contains a piezo-

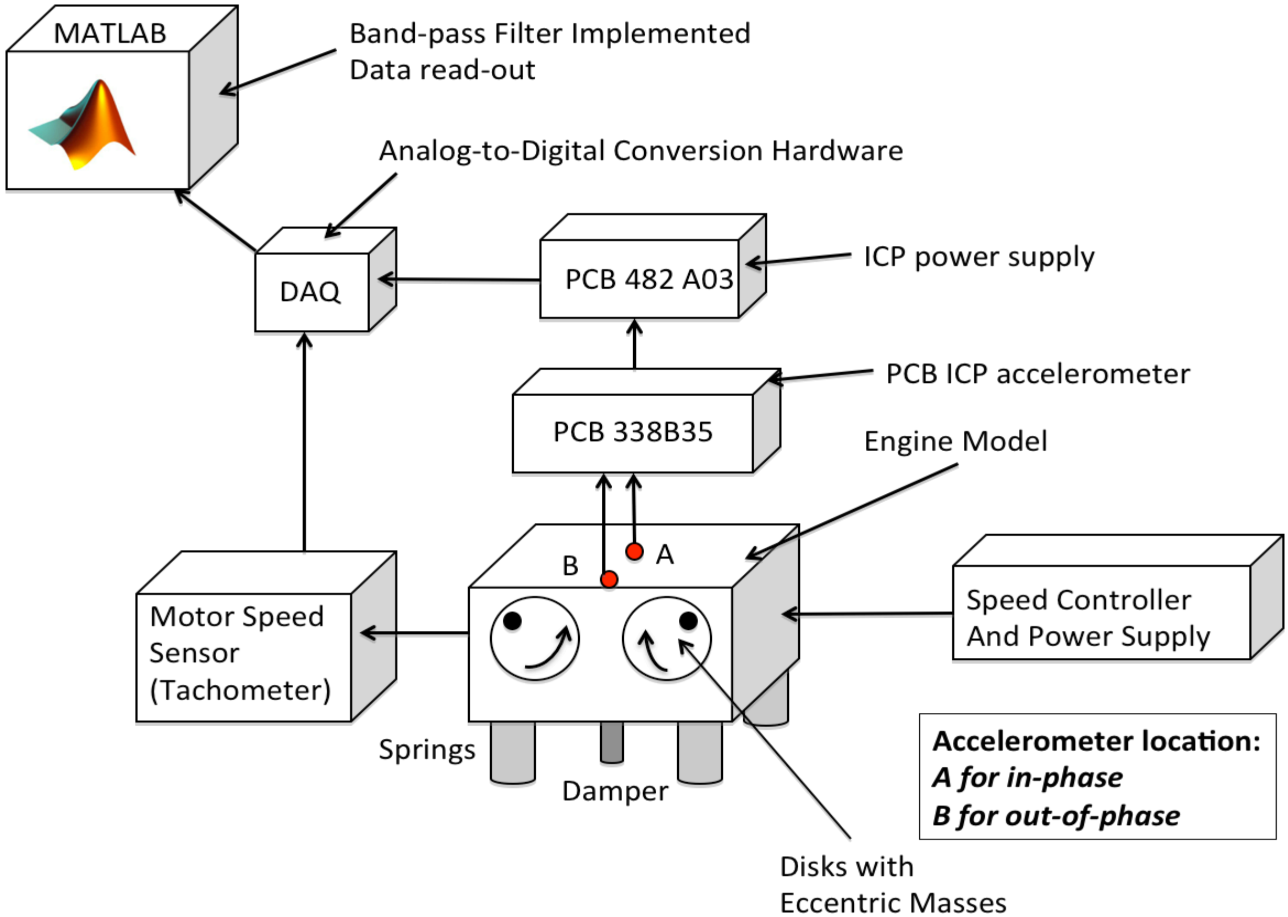

Figure 1: Schematic of the apparatus: note the different accelerometer locations A and B for in-phase and out-of-phase experiments, respectively.

electric element (a crystal) that produces an electric charge when subjected to a load (strain) due to acceleration. When accelerated, an inertia load (proportional to the acceleration) causing it to produce charge under strain. It has a magnetic base to allow easy and secure attachment at various points on the vibrating box. An ICP accelerometer used in this experiment has a built-in quartz sensing element operating in shear mode and a IC circuit gives voltage as the output. The accelerometer requires an external power supply (20-30 VDC). The data acquisition (DAQ) board has analog-to-digital conversion hardware. The software for band-pass filter is implemented in the MATLAB.

PLEASE NOTE ALL DETAILS ABOUT THE SENSORS, MOTORS AND SIGNAL CONDITIONERS AND INCLUDE THEM IN YOUR REPORT.

1.4 Experimental Procedure

The experiment has three overlapping parts. In the first part you will measure the in-phase frequency response. In the second part, you will measure damping. In the third part, the out-of-phase frequency response will be measured. Software written in MATLAB will be used to acquire and process data. So the first step is to setup the software.**You may find it useful to measure damping ratio when you lock-in to resonance in Parts I and III. This ensures that you get accurate data with minimum effort.**

1.4.1 Setting up MATLAB Software and accelerometer gains (TA will help)

1. Turn the computer on and log in to lab account. **Create a folder to store your data.**

2. Start the MATLAB program by clicking the icon on the desktop.

3. In the command window type **logmech463** to start a GUI. This is general purpose data acquisition software. It can be used to perform advanced vibration analyses. We will use it for our tasks: acquire, process, and store data.

4. To operate the MATLAB GUI, please follow the instructions from the TA. **Ask the TA to show how to acquire a typical reading and ask about all features you see in the signals, including phase angle calculation.**

1.4.2 "In-Phase" Vibration Response Measurements (PART I)

1. Make sure that **all eccentric masses are securely fastened and are in phase**; i.e. they all reach their uppermost positions at the same time. Attach the accelerometer to point "A" shown in **Fig.(1)**. Note that there are 4 masses in total, situated in two parallel planes. So we have unbalance forces acting in two parallel planes. In-phase or out-of-phase refers to the phase relation between forces in these two planes of vibration.

2. Adjust the rotation speed to be at 20 Hz (1200 rpm). The rotation speed in the front panel is in rpm. The indicator on the front panel of the apparatus reads two values: a count of the number of rotations (letter C shows on the display) and the rate of rotation (letter R shows on the display).

3. Acquire the data using the MATLAB GUI. Choose the following menu options: **Options → Log data→ Log time series data**. You will now see another window asking for data acquisition parameters. Assign the following values: number of channels =2; sampling

frequency = 5 kHz; sampling time =20 s; leave the pre-trigger samples as negative (you do not need to change this[2]).

4. Once you press start the system will acquire data for 20s which will be displayed in the main GUI window. You can see unfiltered "raw data". The first channel displays pulses and the second channel shows the vibration measured by the accelerometer. **Can you see a clear harmonic acceleration response in Channel 2? Why not? Hint: The accelerometer reports what it measures. So listen to the "sounds" generated by the apparatus, are they harmonic? Can you hear rattling? What could be the source of this?** All realistic vibration measurements contain noise and any mathematical theory should be robust to explain phenomena in a noisy environment.

5. Filtering is required to distinguish the signal[3] (vibration due to rotating eccentric mass) from noise (all other vibrations). A zero-phase **digital filter** is implemented inside the MATLAB software. It calculates the forcing frequency from the channel 1 data (pulses generated by the optical encoder). It uses this frequency to set the limits for a zero-phase, band-pass, and Butterworth filter. The filtered data is used to estimate the acceleration phase lead ahead of the force. To invoke the filter follow the options **UBC→ Calculate amplitude and phase** in the main GUI. Now, a portion of the filtered acceleration signal and the force signal are displayed in a separate window.

6. Use the filtered accelerometer data to estimate **phase lead of acceleration response** and steady state **acceleration amplitude** estimation (from the time traces) by **hand calculation for 600rpm. You may do this after the lab. But you MUST include the hand calculation in your final report. Ask TA if you need help with this during the lab session.** Compare your answer with the median value of phase lead given by the program in the MATLAB command window for 600 rpm. Comment on sources of errors. From now on for other values of rpm, you need not perform the hand calculation. The computer will do it for you!

7. Record the vibration amplitude and phase indicated in the MATLAB command window (you can copy and paste them in a text file or spreadsheet). The phase here represents the **phase lead of the acceleration response**, measured from the distance along the time axis between the up-ward zero-crossing of the filtered acceleration signal and the front edge of the tachometer signal (beginning of the pulse) in the response graph. **TA**

[2] The sampling frequency is chosen based on Nyquist criterion and the sampling time to suit the needs of the *digital filter*. MECH 506 and courses on digital signal processing will help you understand this. Ask the TA if you want to know more. This topic is outside the scope of this course.

[3] Here you will apply a filter. The actual **filter design** is a separate topic in itself. Such filter design is unavoidable in industrial practice as noise is omnipresent!

will show you how to identify the relationship between the signal from the tachometer and the position of the discs. The **phase lead** in degrees is given by $\phi_a = 360^0 - \frac{\omega\Delta\tau\times180}{\pi}$. Remember that Figure A3 in the Appendix shows the phase lag between displacement and the force. **In this experiment we are measuring acceleration phase lead, whereas the theory described in Appendix pertains to displacement phase lag.**

8. Save the time series by following **Options→ Save time series data** in the main GUI. You WILL need this data during the preparation of the report. Store it in a flash drive. A MATLAB script **read_mech463.m is posted on vista that reads the saved .mat files (consuming 100 MB space in total).**

9. Repeat steps 2-6 for rotation speeds at 1 Hz intervals in the range from 20 Hz down to 2 Hz (4 Hz for in-phase). Record your readings in a table. You may notice several resonances. **Do some extra measurements around the resonances to reveal the details of the vibration response near these important frequencies.**

10. On separate graphs, plot vibration amplitude and phase measured at point A vs. rotation speed in Hz. Compare your plots with Figure A3. You have to convert acceleration amplitude to displacement amplitude. Recall that the acceleration amplitude (A) is related to displacement amplitude (X) via $A = \omega^2 X$ for harmonic motion of frequency ω. **Comment on the phase changes near resonance and why it occurs?**

1.4.3 Damping Ratio Estimation (PART II)

The damping ratio ζ is a measure of how quickly energy is dissipated from a vibration system. In reality, all physical systems possess some amount of damping. If one looks at a time domain trace of a simple spring-mass-damper oscillator displaced by an initial distance y_0 and then released, the "free vibration" decay would look similar to **Fig.(2)**. (Note: We are now dealing with a homogeneous version of the general equation of motion (A5). The right hand side of (A5) would be zero for this situation since there is no driving force). The decay is governed by an equation of the form (see equation (A6)):

$$y(t) = e^{-\zeta\omega_n t}\cos?(\omega_d t) \tag{1}$$

Where the "damped natural frequency" ω_d is is given by:

$$y(t) = \omega_n\sqrt{1-\zeta^2} \tag{2}$$

As the damping ratio increases the decay also increases. For example, in an automobile shock absorber the damping ratio ranges between 0.1 (old shock absorber which is relatively inef-

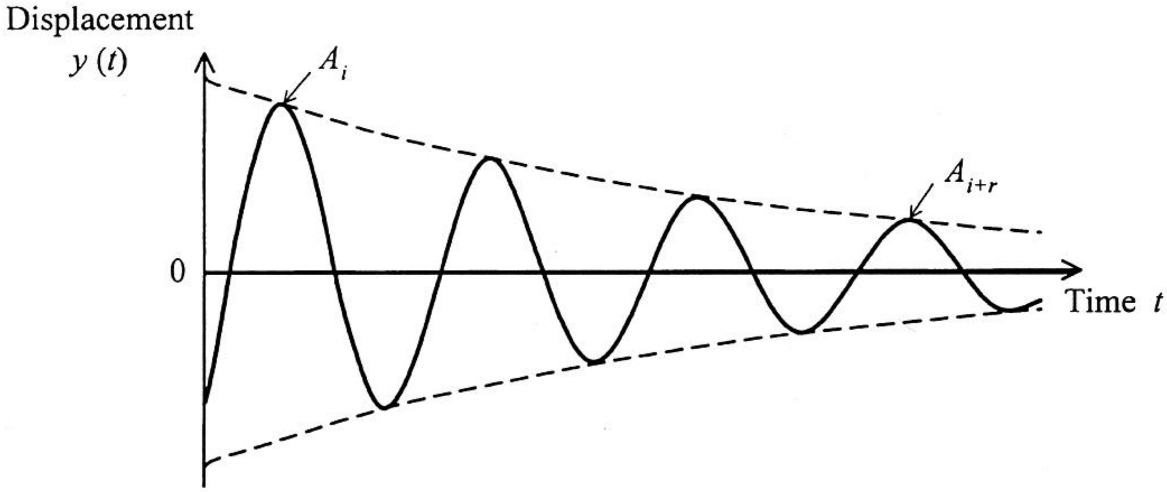

Figure 2: Free vibration response of a damped single degree of freedom system (from reference [1]). Note the amplitudes at two time intervals indicated.

fective at damping out vehicle motion) and 0.4 (new shock absorber which is relatively effective at damping out suspension motion). In many applications it is desirable to have a large damping ratio to quickly eliminate undesirable motions (dampers for doors). In other applications (music), high damping ratios translate into higher energy dissipation (losses) and hence less desired. Also, high damping can create thermal problems due to heat generated by the damper. Therefore measuring the damping ratio is critical to understand the behaviour of any mechanical system subjected to dynamic loads such as shock, impact, or blast.
In this experiment we use the logarithmic decrement method to estimate the damping ratio. The procedure is as follows.

1. Adjust the rotation speed to read the maximum amplitude near the in-phase resonance. It is crucial that you excite the system as close to resonance as you can. Why? Acquire the data using the MATLAB GUI. Choose the following menu options: **Options** ⇒ **Log data** ⇒ **Log time series data**. You will now see another window asking for data acquisition parameters. Assign the following values: number of channels =2; sampling frequency = 5 kHz; sampling time = 20 s.

2. After pressing start, wait for a second or so and then switch off the motor power supply. The motor will stop rapidly, and the vibration amplitude will diminish exponentially. You will see a time trace that shows transients as well as an exponentially decaying part that resembles **Fig.(2)**. Save the time series. The first part of the trace is the response when the motor is still running, although slowing. Focus on the later part of the trace which should correspond to the acceleration history when the motor has completely stopped. Identify a reasonably clean integer number (r) of cycles of decay from the

time response trace (**Fig.(2)**) and measure the start and end amplitudes (call them A_i and A_{i+r}) in this time interval.

From equation Eq.(1) we have:

$$\frac{A_i}{A_{i+r}} = e^{-\zeta \frac{\omega_n}{\omega_d} 2\pi r} = e^{-\zeta \frac{1}{\sqrt{1-\zeta^2}} 2\pi r} \tag{3}$$

By definition, the logarithmic decrement ? is given by (per unit cycle):

$$\delta = \frac{1}{r} \ln\left(\frac{A_i}{A_{i+r}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \tag{4}$$

Then using equation (4) we can estimate the damping ratio from the logarithmic decrement as:

$$\zeta = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta}\right)^2}} \approx \frac{\delta}{2\pi} \tag{5}$$

1.4.4 "Out-of-Phase" Vibration Response Measurements (PART III)

1. Unscrew the two eccentric masses on the **front side** of the vibrating box. Insert and securely fasten them so that they are out of phase relative to the masses on the other side; i.e., the two masses on the front are at their lowest positions when the other two on the back are at their highest positions. Move the accelerometer to the front, position B in **Fig.(1)**.
2. Repeat steps 2-3-4-5-6 describe earlier for in-phase measurements of PART I.
3. Repeat steps 7-8, but adjust the motor speed to achieve the maximum response around all resonances. Remember the accelerometer should be at position B. Save the time traces. **Measure the damping ratios for at least one resonance: the large amplitude rocking mode for which you will be developing an equivalent model**. Without this measurement you will not be able to develop equivalent SDOF model for the out-of-phase case.

Pre-Lab Exercise (Attach this to the main report)

1. Draw the free-body diagram of the shaky table for both "in-phase" and "out-of-phase" vibrations. Use the isometric view of the two parallel shafts, or the shaky table, to represent the forces exerted by the eccentric masses on the two parallel shafts.
2. What types of motion do you expect to see in the two cases? Hint: Use the forces in the free-body diagram.
3. Derive the relation between the phase lag of acceleration and phase lag of displacement.
4. Determine the displacement amplitude of a harmonic motion of frequency ω rad/s whose acceleration amplitude is A m/s^2.

Before you leave the lab ...

1. **Save data** in .mat format for each rpm for in-phase, out-of-phase, and damping estimation.
2. Take the data with you and use the program **read_mech463.m** to read the time series data and generate the plots for the report.
3. Take ALL geometric measurements of the apparatus: a) eccentricity of each mass; b) distance between shafts, discs in all planes; c) distances between all four springs; d) length and width of the shaky table.

Shaky Table Data Useful in Calculations

Quantity	Old (with metal guards)	New (with perspex glass)
Mass	33.054 lbs or 14.993 Kg (Table+Accelerometer spring mounts+Upper Spring attachment+Center damper piston)	34.554 lbs or 15.673 Kg (Table+Accelerometer spring mounts+Upper Spring attachment+Center damper piston)
Mass of Accelerometer	0.222 lbs or 100 g	0.222 lbs or 100 g
Mass of damper piston		0.250 lbs or 113.4 g
Spring constant (each)	82 lb/in=14360.4 N/m	82 lb/in=14360.4 N/m
Eccentric mass (each)	15.2 g	15.2 g
Calibration constant for accelerometer	99 mV/g g=9.81 m/s2	98 mV/g g=9.81 m/s2

Things You Should Bring to the Lab

You should prepare your own

1. **Pre-lab assignment**: Do the pre-lab assignment and submit the solutions to the TA before you begin the experiment. The pre-lab assignment must be submitted individually (by each member of the group).
2. **Ruler or tape measure**: To measure the distance between the front and the back disks, distance between front and back springs, and the eccentric radius e.
3. **Protractor**: The technician has adjusted the rotating shaft such that the phase is measured between the upward zero crossing of the acceleration signal and the front edge of the signal from the tachometer. The TA will show you how to relate the tachometer signal to the angular position of the discs.
4. **Flash disk**: To store a data file of your experimental result.
5. **Digital camera or camera-enabled phone**: To take pictures of the experimental set-up from different views. You should include some photographs to help you describe the experimental set-up in your report.
6. **Calculator**: Bring a calculator to laboratories in case you need to do some simple calculations.

Background Theory and Report Requirements

You need to submit your report in a "ready-to-publish" professional format. Each student must submit group lab report of 2000 words maximum for the main report. Attach the pre-lab exercise with initials from TA. The report should address all points mentioned in the marking scheme posted on VISTA and the Theory and report requirements document posted on VISTA.
Please ensure that your report satisfies all the items in the **checklist** provided.

All the Best!

Engineering problems are under-defined, there are many solutions, good, bad and indifferent. The art is to arrive at a good solution. This is a creative activity, involving imagination, intuition and deliberate choice.

–Ove Arup

VIBRATION DUE TO A ROTATING UNBALANCE
THEORY & REPORT REQUIREMENTS

MECH 463: Experiment 1

Location: ICICS X039 (Opposite PACE Lab)

Dr. A. Srikantha Phani

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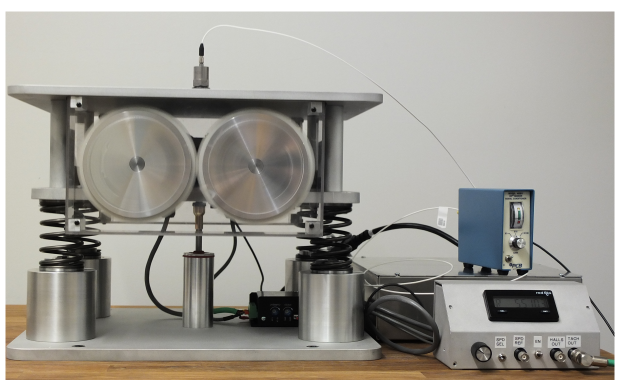

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Appendix A: Theory and Report Requirements

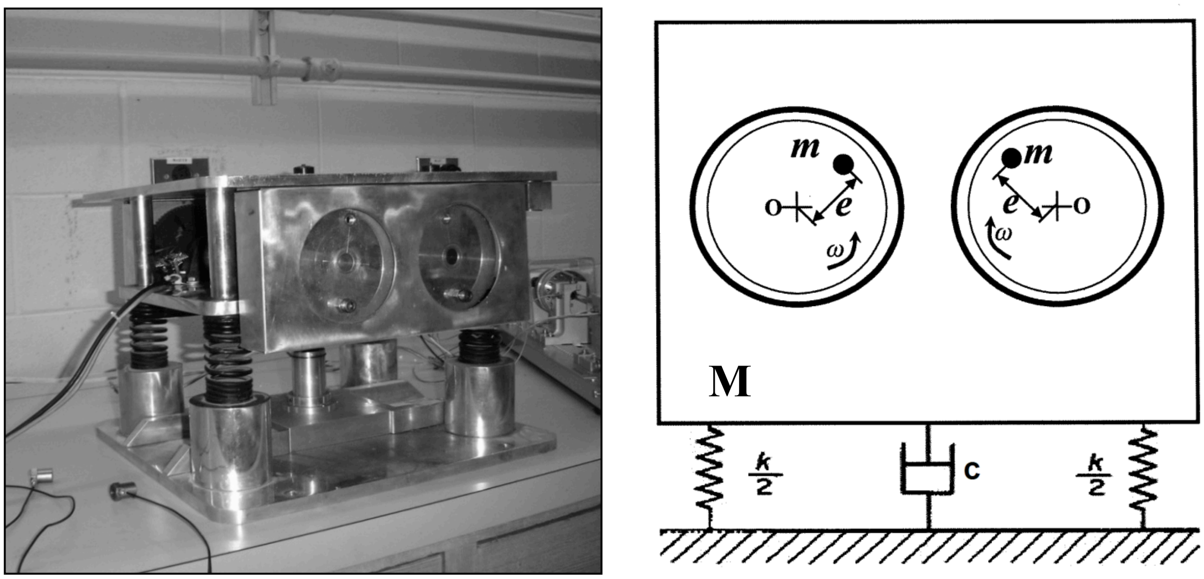

Figure A1: Experimental apparatus (left) and a schematic diagram (right). The counter rotating- discs are turning at a ***constant*** angular speed ω. The eccentric masses m generate a harmonic vertical force whose magnitude varies as the square of the angular speed ω. Note that the mass of the entire system is M. The vertical displacement measured with reference to the static equilibrium configuration is denoted by $y(t)$.

Figure A1 depicts the experimental apparatus in the left half and a simplified model in the right half. The model on the right shows the front view of the apparatus. It can be seen that the four springs in the original apparatus are represented by two equivalent springs of constant $\frac{k}{2}$ each, in the simplified model, so that the total stiffness of the system is k. **Find k for your apparatus using the spring constant data provided. Can you use the same value of k in the case of out-of-phase vibration response? Explain why not?**

We require an equation that describes the motion of the apparatus. We consider vertical motion only and pretend that this motion is not influenced by other motions of the apparatus. The **absolute** displacement of the eccentric mass, assumed to be positive upwards, is equal to the sum of the displacements of the centre of the shaft O and the displacement of the eccentric mass relative to O.

$$y_m(t) = y(t) + e\sin\omega t \tag{A1}$$

The unbalanced force exerted on the non-rotating system (without eccentric masses of mass

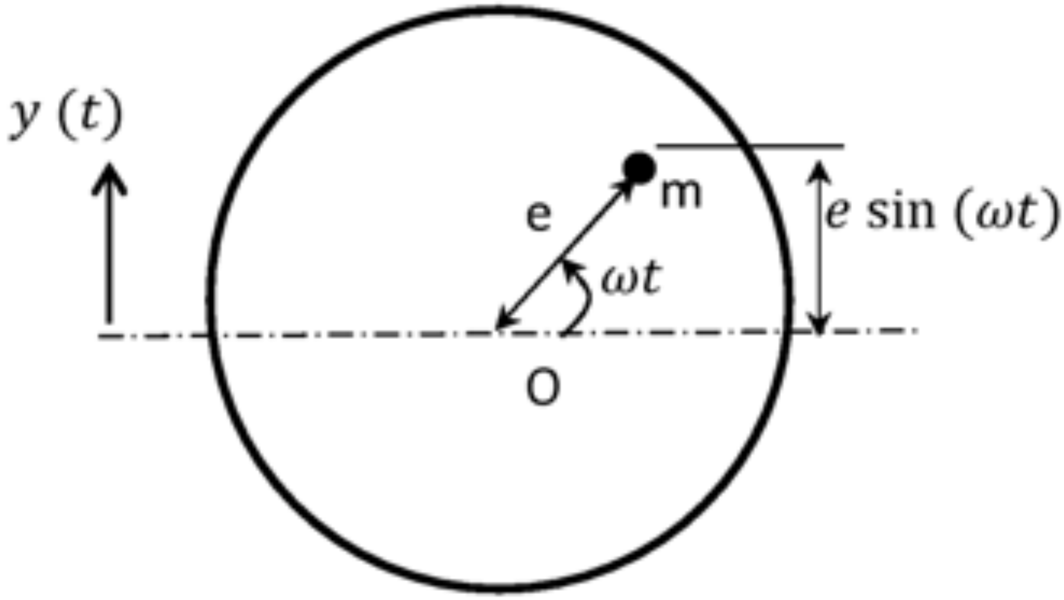

Figure A2: Diagram indicating vertical component of displacement of the rotating unbalance and the centre of the shaft. Note that this represents a particular instant of time t during the motion.

$(M-4m)$ by each of the four rotating masses (m), due to their inertia[1], is

$$F_e = -m\ddot{y}_m = -m\ddot{y} + me\omega^2 \sin\omega t. \tag{A2}$$

Now the equation of motion of the shaky table model in Figure A1, without the four eccentric masses, is given by the NewtonÕs second law applied to the non-rotating mass $(M-4m)$:

$$\uparrow_{+ve} \sum F = (M-4m)\,\ddot{y}. \tag{A3}$$

The forces on the left hand side of the above equation are due to: the springs and dashpot (both downwards), and the force exerted by the eccentric masses on the shaky table (upwards)

$$-ky - c\dot{y} + 4F_e = (M-4m)\,\ddot{y}. \tag{A4}$$

Using (A.2) the above simplifies to

$$\mathbf{M\ddot{y} + c\dot{y} + ky = 4me\boldsymbol{\omega}^2 \sin\boldsymbol{\omega} t}. \tag{A5}$$

[1]The inertial force here arises due to the reluctance of the eccentric masses to rotate. You can also think in terms of Newton's third law. The rotating masses exert a reaction (centrifugal force) on the non-rotating mass on account of their reluctance to rotate.

Note that the above equation of motion is that of a single degree of freedom system subjected to a harmonic force whose amplitude varies as the square of rotational speed. **Hence, doubling the rpm of the shaft quadruples the excitation force. This feature suggests that eccentric masses—however small their magnitude may be—can cause large inertial forces at high rotational speeds.** The displacement response y(t) is obtained by solving the above linear, second-order, ordinary differential equation. Recall that the solution of equation (A5) is

$$y(t) = \underbrace{e^{-\zeta\omega_n t}\left[A\cos\omega_d t + B\sin\omega_d t\right]}_{\text{homogeneous solution}} + \underbrace{Y\sin\left(\omega t - \phi\right)}_{\text{particular solution}}. \tag{A6}$$

where the new parameters introduced are defined as follows:

$\omega_n = \sqrt{\dfrac{k}{M}}$ Undamped natural frequency

$\zeta = \dfrac{c}{2m\omega_n}$ Damping ratio

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ Damped natural frequency

A and B Unknown constants in the transient solution

Y Displacement amplitude of particular solution

ϕ Phase lag of displacement in relation to force

The solution in equation (A6) comprises two parts. The first part of the total solution—called homogeneous solution, or transient solution —represents the natural, unforced response obtained by setting the forcing term to zero in equation (A5).

$$M\ddot{y} + c\dot{y} + ky = 0. \tag{A7}$$

The second part of the solution —called particular or steady-state solution—represents the forced response obtained by solving equation (A5). The sum of these two solutions determines the total response of the system at any time. It is important to realise that the arbitrary constants A and B in the homogeneous solution are to be determined from the specified initial conditions on the total solution, not the homogeneous solution! For damped systems the homogeneous solution decays exponentially with time. Thus, given sufficient time after the vibration is started at a particular rpm of the shaft, it is the steady state solution that decides the amplitude of the response. Henceforth, we **assume that this condition is ensured in the experiment**. The two unknowns in the steady-state or particular solution can be solved and they are given by

$$Y = \frac{4me}{M}\frac{r^2}{\sqrt{\left[1-r^2\right]^2 + [2\zeta r]^2}}, \quad \tan\phi = \frac{2\zeta r}{1-r^2}, \quad r = \frac{\omega}{\omega_n} \tag{A8}$$

By varying the rpm of the shaft one is changing the parameter ω, or equivalently r, in equation (A8). The equation in (A8) relates the steady state vibration displacement amplitude Y and phase lag ϕ to non-dimensional excitation frequency r. It is convenient to non-dimensionalise Y by dividing with another quantify of same dimension (length) which in our case is $u = \frac{4me}{M}$. Note that this is a measure of the amplitude of input excitation to the system (why?[2]) and the ratio of the output to input expressed as a function of excitation frequency is called **frequency response function**. It is obtained from equation (A8) as follows:

$$\frac{Y}{u} = \frac{r^2}{\sqrt{\left[1-r^2\right]^2 + [2\zeta r]^2}}, \quad \tan\phi = \frac{2\zeta r}{1-r^2}, \quad r = \frac{\omega}{\omega_n}, \quad u = \frac{4me}{M} \tag{A9}$$

As can be verified from equation (A9), the **frequency response function** of a linear system depends only upon a system's properties such as inertia, stiffness, and damping. These curves are also known as **Bode plots** when logarithmic scales are used. Such plots are frequently used by control engineers. In vibrations, however, it is a common practice to employ linear scale for frequency and a logarithmic (decibel) scale[3] for the normalized displacement amplitude, or magnitude of frequency response function.

The steady-state displacement amplitude Y and its phase lag ϕ behind the force are **measured** in this experiment as function of r for two different arrangements of the eccentric masses: "in-phase" and "out-of-phase". The damping ratio ζ is **also measured** from the decay of transient or free vibration.

Typical frequency response curves are displayed in a graphical form in Figure A3 for different values of the damping ratio ζ. Notice that the frequency associated with maximum displacement response or resonant frequency is slightly above the value $r = 1$. Why?

Another feature worth noting in Figure A3 is the phase lag of displacement behind the excitation force. At zero frequency $r = 0$ the displacement is in-phase with the force. As the shaft rpm is increased the displacement lags behind the force. At $r = 1$ the displacement is 90^0 behind the force and for large values of r (or rotational speeds) the displacement is 180^0 out-of-phase with respect to the force. This means that when the force pushes the shaky table downwards the displacement is upwards! This is opposite to the static case associated with $r = 0$. This is the phase relation between the displacement and force. But, **we measure acceleration in this experiment**.

[2] Think of the case $e = 0$. What is the amplitude of the excitation force for any non-zero r?

[3] Decibel (dB) is a logarithmic ratio (base 10) of two quantities: power, pressure, sound pressure, voltage etc. It is a useful scale to display very large and very small numbers. In the original definition of dB $10log_{10}$? is used, but in our case (and control engineers) use $20log_{10}$. This is to reflect the fact that power is proportional to the square of the displacement amplitude.

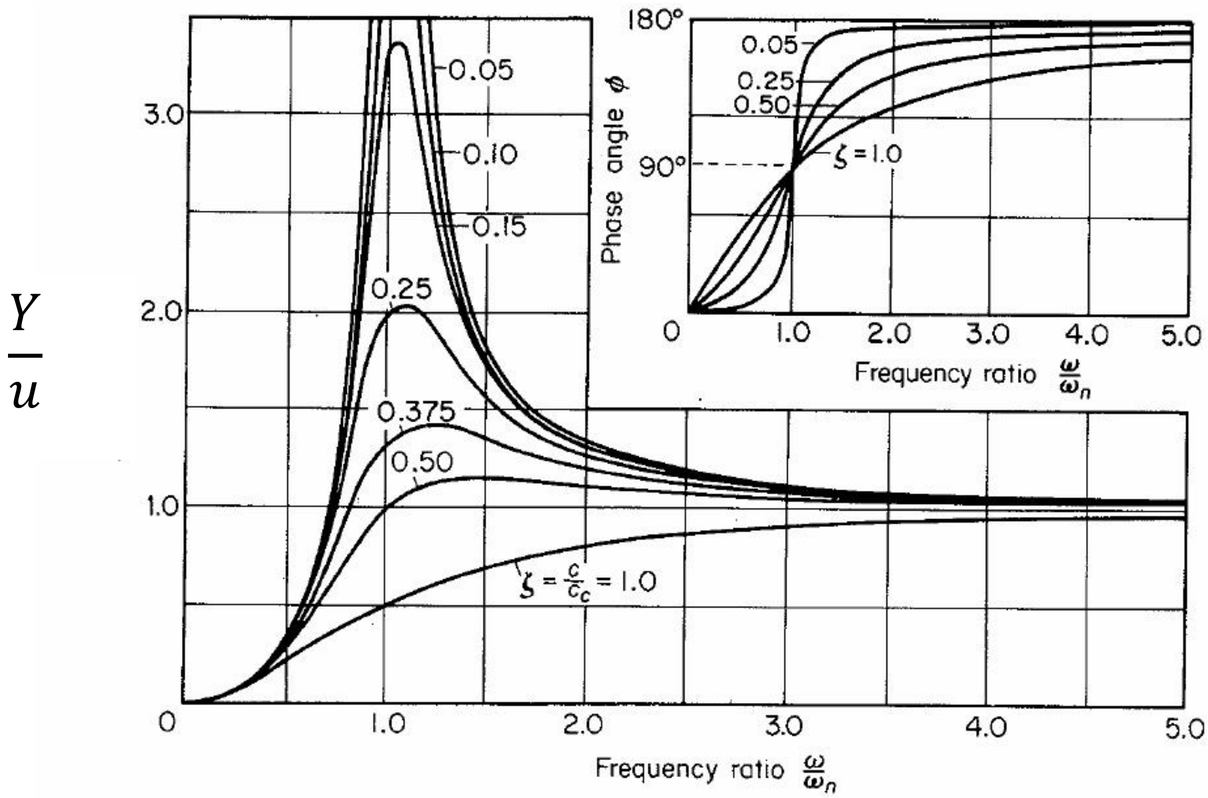

Figure A3: Frequency response curves illustrating equation (A9). The figure is adopted from reference [2].) .

Damping Measures

Damping is one of the least known aspects of vibration theory. Damping is always measured! In the present experiment we use logarithmic decrement method as described earlier. This is a time-domain technique based on the decay of free vibration of a system excited at a single resonant frequency. **The success of this technique depends crucially upon exciting only one mode of vibration which may not always be possible. Did you observe an exponentially decaying sinusoid of one time period in the experiment? Why not?**

An alternative approach is a frequency-domain measure of damping using the half-power-bandwidth method. In this method, we pick two frequencies on either side of a selected resonance peak. These two frequencies correspond to the amplitude which is 3 dB (or half power) below the peak amplitude at resonance. Specifically, consider Figure A4, which gives the Displacement/Force frequency response function of a mass-spring-damper. The Q-factor, which measures the sharpness of resonant peak, is defined by:

$$Q = \frac{\omega_n}{\Delta\omega_n} \approx 2\zeta \tag{A10}$$

where ω_n is undamped natural frequency, ζ is damping ratio and $\Delta\omega_n$ is Half-Power-Bandwidth

(HPBW)

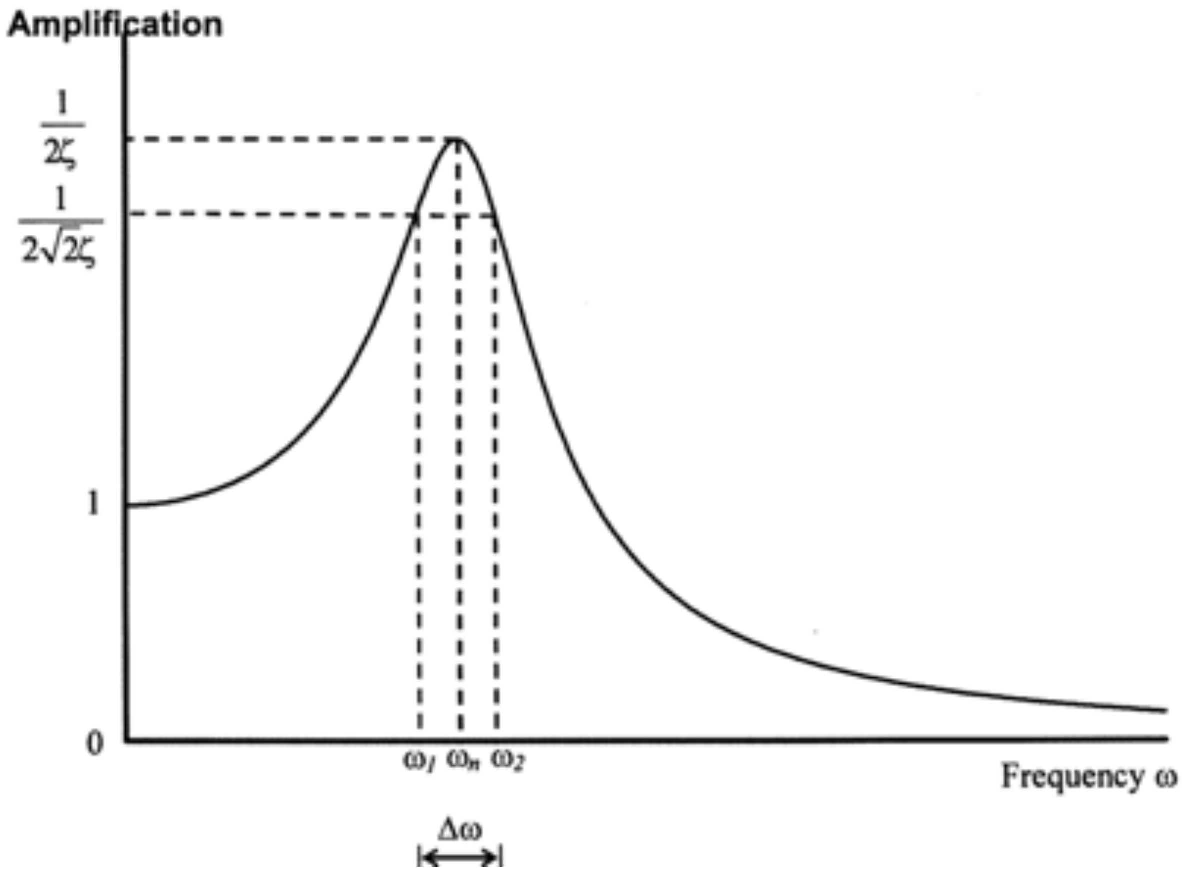

Figure A4: Half-Power-Bandwidth method to determine Q factor (from reference [2]) .

REFERENCES

1. Rao, S.S., Mechanical vibrations, 4th Edition, Pearson, Prentice-Hall, new Jersey, USA. Read Section 3.7 for an alternate derivation of equations of motion given in A1.
2. de Silva, C.W., Vibration —Fundamentals and Practice, 2nd Edition, CRC Press/Taylor&Francis, Boca Raton, FL, 2007.

REPORT GUIDELINES

1. **Abstract**: Keep this to around 300 words or less. Be precise. Highlight the comparison between the measured and the calculated natural frequencies. Mention your conclusions.
2. **Introduction**: Write your own introduction to this lab justifying the importance of vibrations due to rotating eccentric masses. State the phenomena you observed and why are they important.
3. **Methods**: Describe in detail what you see in the experimental set-up, the parameters you measured, or the information that you obtained with the help of the TA. Identify the important components in the pictures you have taken. Provide a schematic diagram of your own, similar to the Fig. 1 of this hand-out but with additional detail. Describe the experimental procedure for the in- phase, out-of-phase, and damping ratio experiments.
4. **Results**

   In-Phase vibration response measurements: This section and the next section are the essential parts of the report. Describe this in as much detail as possible. They should include (a) the time trace used to estimate the damping ratio, (b) a typical pre-filtered signal, post-filtered signal, and a tachometer signal, (c) the amplitude versus rotation speed graph and phase versus rotation speed graph. (d) Compare these experimental results with the theoretical results based on equations (A9). Try to reproduce one curve of Figure A3 (for your damping value) for both experimental and theoretical approaches. How does the accuracy of the theoretical theoretical curves depend upon the damping estimates? Note that in Eq. (A9) we have used the displacement amplitude instead of acceleration.

   Out-of-phase vibration response measurement: We are unable to compare the experimental result with the theoretical one because we do not know the moment of inertia of the apparatus. However, can you use the dominant resonant frequency of this experiment to estimate the moment of inertia of the block? Based on this derive the theoretical prediction on the amplitude versus rotation speed relation as we have for in-phase excitation in Appendix A for the selected mode.
5. **Discussion and conclusion**: Summarize the measurements that you made. Describe the physical meaning of the resonant frequencies. Explain why the damping ratio for the out-of-phase resonance is much smaller than that for the in-phase resonance. Try to identify the source of error causing the discrepancy between the experiment and the theory. Comment on any anomalous results that you observe. Answer ALL questions asked on Page 9 of this document.
6. **Learning outcomes**: Describe briefly what you learned from this lab.

QUESTIONS TO BE ANSWERED IN THE REPORT

1. Can you list at least two differences between this experiment and the vibrations in a washing machine?
2. How long must you wait between recording the data for two different RPMs?
3. Is the system lightly damped? Justify your answer.
4. Explain why r=1 in Fig. A3 is not the location of maximum response.
5. Why does the width of pulse (channel 1 data in your measurements) change with rpm in readings from optical encoder (channel 1)?
6. You can obtain **different estimates** for logarithmic decedent δ. Use these estimates to general the upper and lower bounding curves for the theoretical frequency response curve.
7. **Equivalent models**[4] : For each typical resonance that you observed, deduce an equivalent spring-mass-dashpot model (from your measured damping) according to the following table given for in-phase case. The equation of motion for in-phase case can be expressed as $m\ddot{y} + c\dot{y} + ky = F\sin\omega t$, $F = 4me\omega^2$. Develop an equivalent model for the **dominant mode** in the out-of-phase case. Indicate the appropriate units in the table.

Quantity	Formula for **in-phase**	**Out-of-phase**	Comment
Resonant frequency, ω_n	Use the measured value		
Maximum Acceleration, $\ddot{Y}_{max}$	Use the calibration constant		
Maximum Displacement ,Y_{max}	Can you find this from maximum acceleration?		
Modal stiffness, k	$k = \frac{F}{2\zeta Y_{max}}$		Using $\zeta = \frac{c}{2m\omega_n}$ & $Y_{max} \approx \frac{F}{2\zeta}$
Modal Mass, m	Use $\omega_n = \sqrt{\frac{k}{m}}$		
Modal damping, c	$c = \frac{F}{\omega_n Y_{max}}$ F=Maximum force		

You need to develop equivalent model for the dominant mode in out-of-phase case, by choosing an appropriate displacement co-ordinate.

[4]This table requires you to measure damping in out-of-phase case as well. You need to supplement appropriate expressions in column 2 for out-of-phase configuration. F is not measured in this experiment. Hence, use the estimate from theory.