Lesson 10-1 – Internal Rate of Return

Introduction

- Building on the analysis of different cash flows, we can combine them to do worth or value analysis

- Net Present Value/Net Present Worth
- Equivalent Annual Cash Flow
  - What do they represent?
  - When can we use one vs. the other?

The Challenge of NPV

- Two main issues with NPV
  - Need to assume or get an interest rate
  - Only concerned with 'absolute value,' doesn't consider how much 'bang for your buck' you're getting.

- Consider two alternatives at 10% interest.
  - Alternative A: Invest $1,000,000, receive $1,190,000 at the end of year 1.
  - Alternative B: Invest $100,000, receive $200,000 at the end of year 1.
  - Both have the same NPV: $81,818 and only take one year
  - Which is the better alternative?

Overview

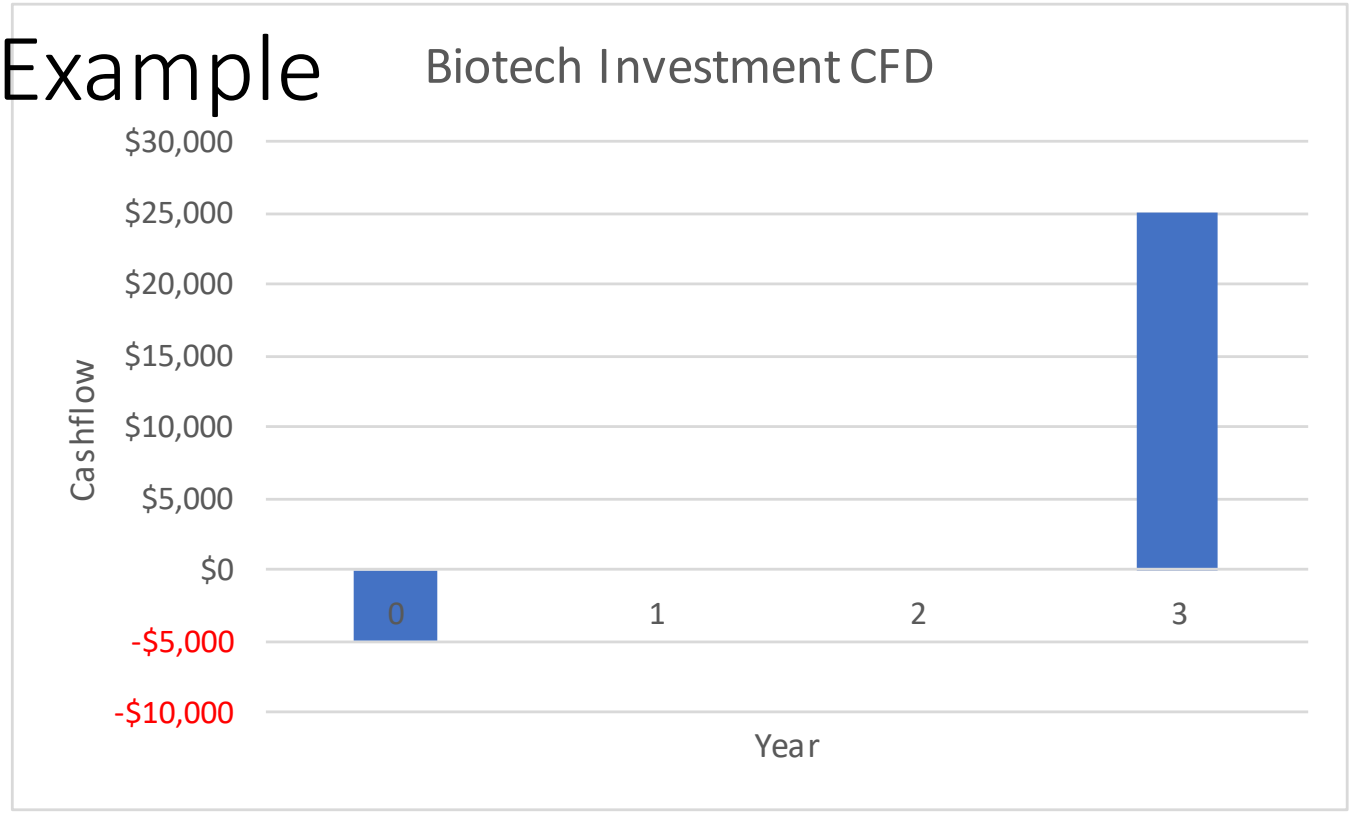
- Introduce Internal Rate of Return (IRR) method of evaluating cashflows
  - Widely used in industry
  - Provides a measure of a project's desirability in terms that are easily understood.
- Relate IRR to Net Present Value
- Make decisions to proceed based on comparison to a
  - Minimum Attractive Rate of Return (MARR)

Rate of Return Example

Problem:

Your investment of $5,000 into a Biotech startup's common stock proved to be very profitable. At the end of three years you sold the stock for $25,000. What was the rate of return?

Rate of Return Example



Solution:

$F = P(1 + i)^n$

$\$25,000 = \$5,000(1 + i)^3$

$(1 + i) = (\$25,000/\$5,000)^{1/3} = 1.71$

i = 0.71 and thus the rate of return was 71%

Internal Rate of Return (IRR)

- This measure of a project's return is called the IRR.
  - Internal Rate of Return (IRR): the interest rate at which the PV and EACF are equal to zero.
  - In other words: the interest rate that causes the unpaid balance on a loan to equal zero when the final payment has been made (from the borrower's perspective)
  - From the investor's perspective: the IRR is the interest rate that causes the unrecovered investment to equal zero at the end of the life of the investment.
  - **The IRR is the interest rate at which the present worth of the benefits are equivalent to the present worth of the costs.**

Calculating Internal Rate of Return

- Given a cash flow, there are five forms of equations that can be used to solve for the unknown interest rate:
  - PV of benefits – PV of costs = 0
  - PV of benefits/PV of costs = 1
  - PV of costs = PV of benefits
  - Present worth = Net present worth = 0
  - EUCF = EUAB – EUAC = 0

Solving for i

- The base formula to solve for i is very simple: $P_{benefits} = P_{costs}$
- Actually solving it analytically is challenging with all the exponential terms
- How can we solve for i?
  - Analytically (for very simple cases, typically single payments)
  - Trial and error: guess an i, solve for NPV, and if not zero, iterate
  - Graphical: Plot NPV vs. i for various values, read i intercept
  - Spreadsheets can speed up the trial and error and graphical methods

Internal Rate of Return Example

- Problem:

You spend $1000 and in return receive two payments of $1,094.60; one at the end of three years and the other at the end of six years. Calculate the resulting rate of return.

- Solution:

PW of costs = PW of benefits

$1,000 = $1094.60 ((P/F, i%, 3) + (P/F, i%, 6))

Using the single payment formula: $P = F(1+i)^{-n}$

Solve for i use trial and error to pick i

Guess i = 10%:

$P_{benefits}$ = $1440.26 =/= P_{costs}

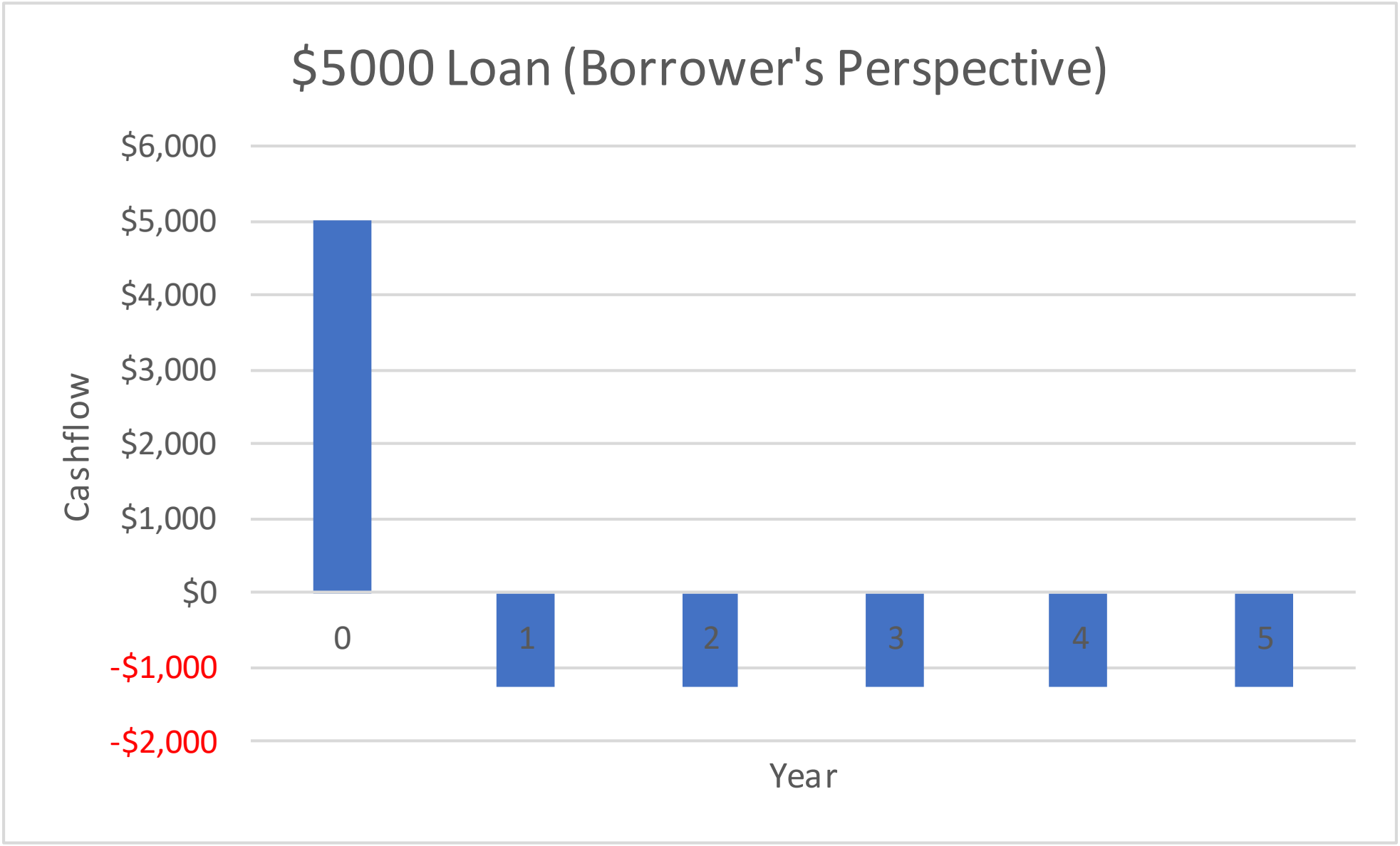
Guess i = 20%

$P_{benefits}$ = $1094.60(0.5787 + 0.3349) = $1000 = P_{costs}

Rate of Return is 20%

Calculating Internal Rate of Return

	Plan			
Year	***A***	***B***	***C***	***D***
0	$5,000.00	$5,000.00	$5,000.00	$5,000.00
1	-$1,400.00	-$400.00	-$1,252.28	$0.00
2	-$1,320.00	-$400.00	-$1,252.28	$0.00
3	-$1,240.00	-$400.00	-$1,252.28	$0.00
4	-$1,160.00	-$400.00	-$1,252.28	$0.00
5	-$1,080.00	-$5,400.00	-$1,252.28	-$7,346.64
IRR=	8.00%	8.00%	8.00%	8.00%


$5000 Loan (Borrower's Perspective)
$6,000
$5,000
$4,000
$3,000
$2,000
$1,000
$0
-$1,000
-$2,000
Cashflow
0
1
2
3
4
5
Year

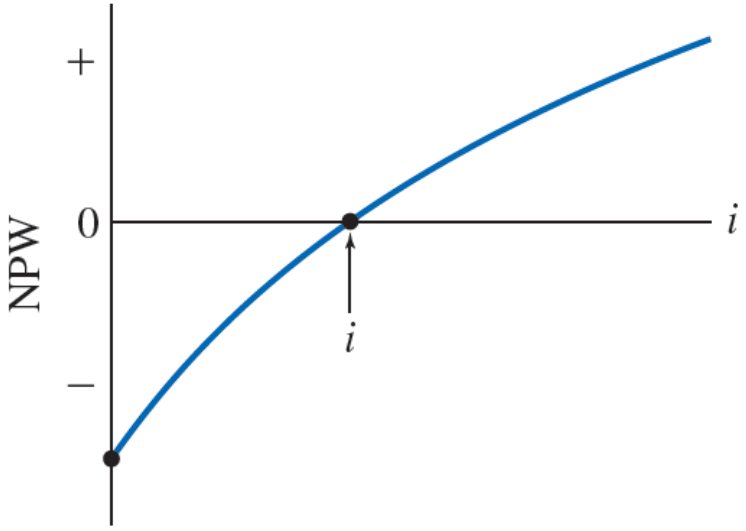
Calculating Internal Rate of Return

Plan C		Discounted Cashflows		
Year	**Cashflow**	**4%**	**8%**	**12%**
0	$ 5,000.00	$5,000.00	$5,000.00	$5,000.00
1	$(1,252.28)	($1,204.12)	($1,159.52)	($1,118.11)
2	$(1,252.28)	($1,157.80)	($1,073.63)	($998.31)
3	$(1,252.28)	($1,113.27)	($994.10)	($891.35)
4	$(1,252.28)	($1,070.45)	($920.46)	($795.85)
5	$(1,252.28)	($1,029.28)	($852.28)	($710.58)
	NPV	**($574.93)**	**$0.01**	**$485.81**

Plot of NPW versus Interest Rate *i*

- A typical plot for borrowed money:
  - Viewpoint of the borrower:

Year	Cash Flow
0	$+P$
1	$-A$
2	$-A$
3	$-A$
4	$-A$
.	.
.	.
.	.

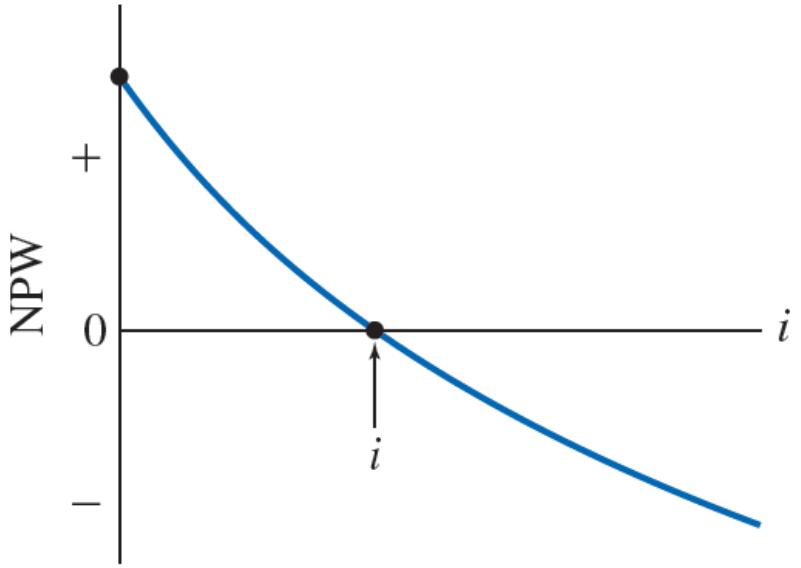


- The IRR is located where the plot crosses the NPW = 0 point.

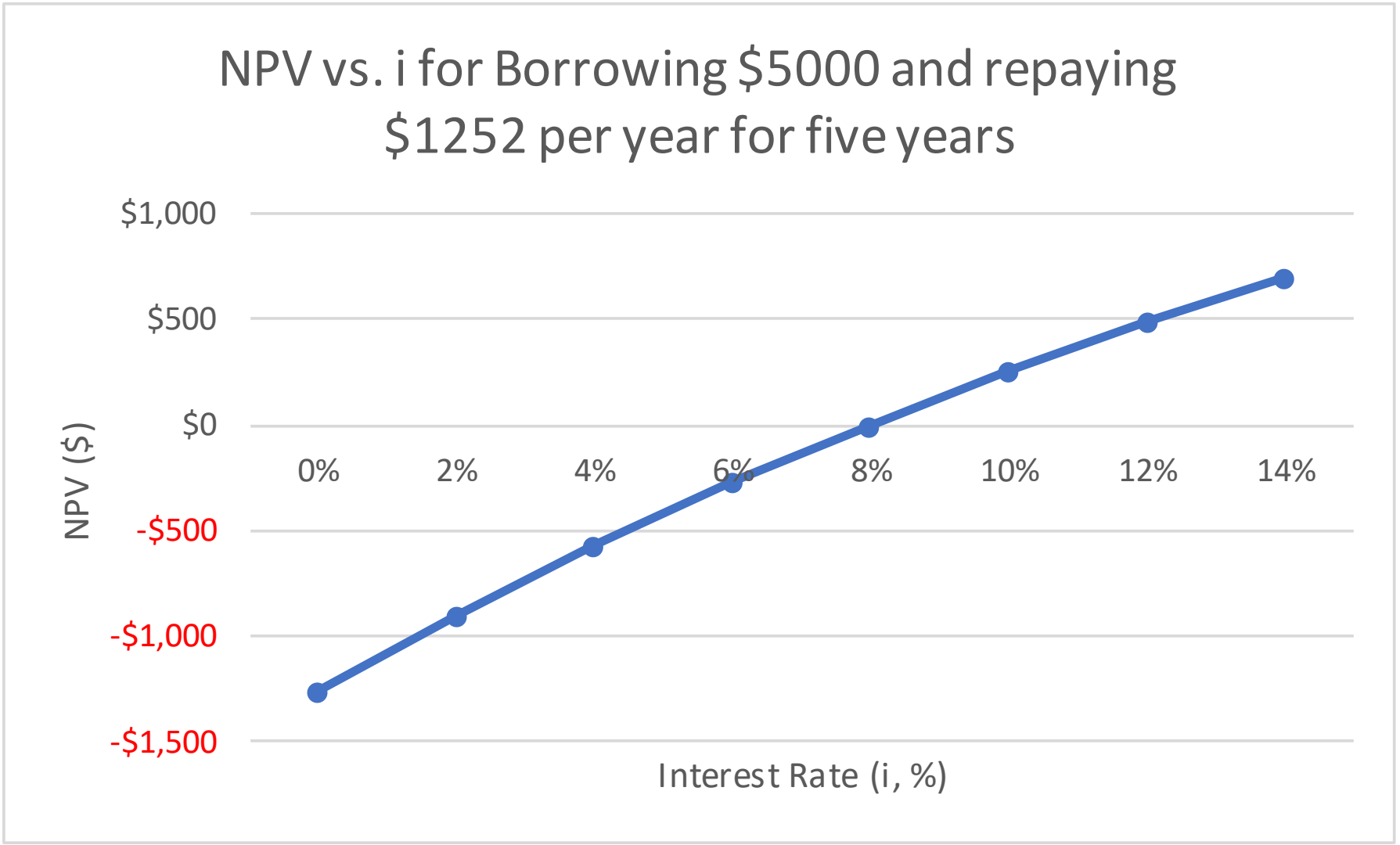
Plot of NPW versus Interest Rate *i*, Continued…

- A typical plot for invested money:
  - Viewpoint of the investor:

Year	Cash Flow
0	$-P$
1	+Benefit A
2	$+A$
3	$+A$
4	$+A$
.	.
.	.
.	.



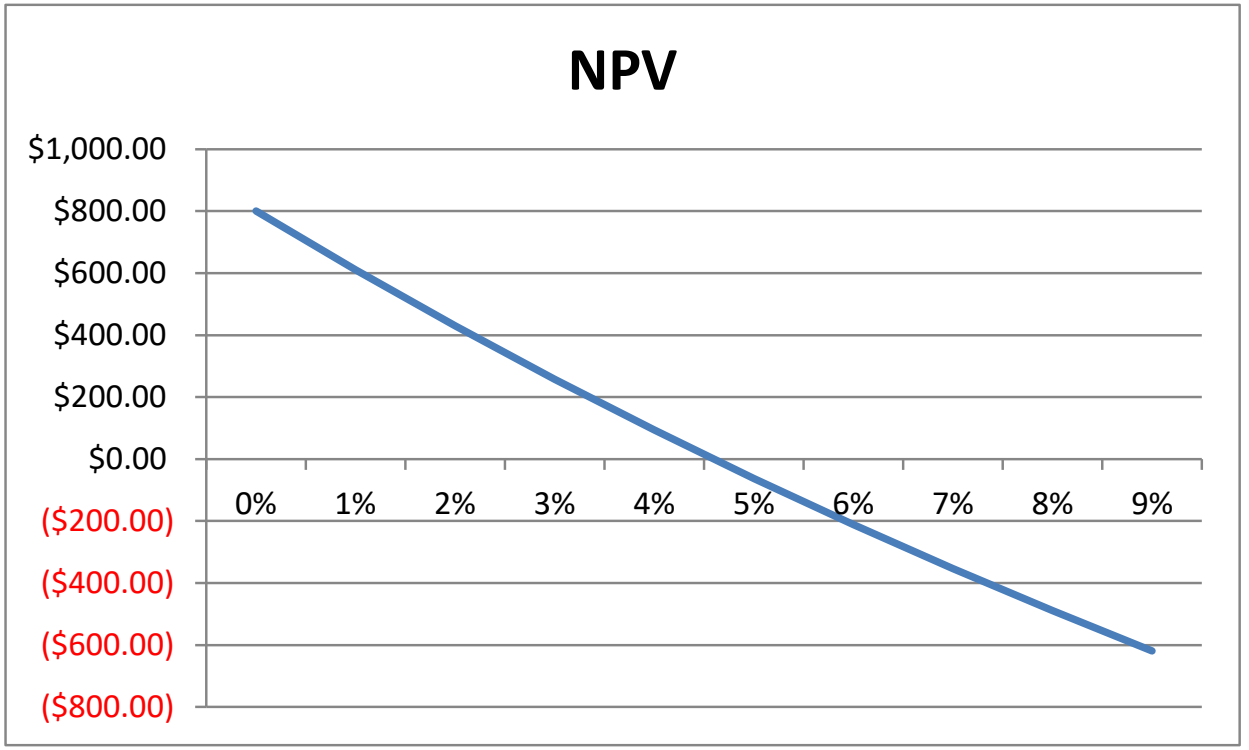
Borrow $5000 Example



Plot of NPW versus Interest Rate *i*, Continued…

- A possible plot for a plant investing in machinery:
  - Viewpoint of the owners:

Year	Cashflow
-	$ (5,000)
1	$ 1,000
2	$ 1,500
3	$ (500)
4	$ 2,000
5	$ 2,000



- The IRR is where the plot crosses the NPW = 0 point.

Illustrating the relationship between value and interest– GICs

- Pretend you just got your first job and your first paycheck

- Go the bank and buy a $1000 five year GIC at 3% per year.
  - Pays $1,159 at term.
- As you walk out the door, the sign changes – drat!
  - Five year GIC's now 5%! (pays $1276 at term)

- Just now your friend walks up to the bank also wants to buy a $1000 GIC - should she buy yours for $1,000?

- How much is your GIC worth now?

Rate of Return: Problem 1

With an initial investment of $6549.32 in a new machine, you will provide your company with $4000 more incoming dollars per year over the next four years. However, over those four years, the maintenance of the machine will cost $800 per year. Also, in Year 2 a refit of the machine will cost $5,100. What is the rate of return?

How would we set this up analytically?

Rate of Return: Problem 1

$NPV = PV_{cost} + PV_{benefits} + PV_{maint} + PV_{overhaul}$

$= -\$6549.32 + \$4000(P/A, i, 4) - \$800(P/A, i, 4) - \$5100(P/F, i, 2)$

Take a guess
at i and solve.

10%	Compound Interest Factors					
	Single Payment		Uniform Payment Series			
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*
1	1.100	.9091	1.0000	1.1000	1.000	0.909
2	1.210	.8264	.4762	.5762	2.100	1.736
3	1.331	.7513	.3021	.4021	3.310	2.487
4	1.464	.6830	.2155	.3155	4.641	3.170
5	1.611	.6209	.1638	.2638	6.105	3.791

- $=-\$6549.32 + \$4000(P/A, 10\%, 4) - \$800(P/A, 10\%, 4) - \$5100(P/F, 10\%, 2)$
- $-\$6549.32 + \$4000*3.17 - \$800*3.17 - \$5100*.8264 = -\$619.96$

Rate of Return: Problem 1

Negative NPV. Investor problem, so try a lower interest rate.

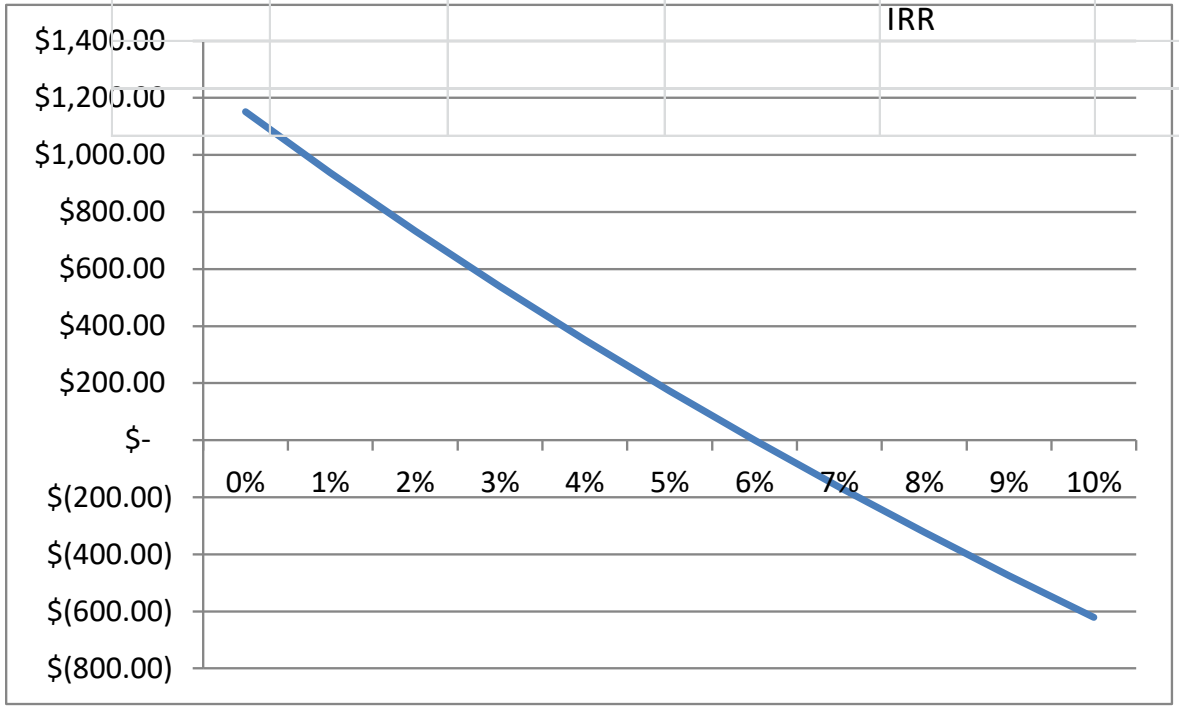
6%	Compound Interest Factors					
	Single Payment		Uniform Payment Series			
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*
1	1.060	.9434	1.0000	1.0600	1.000	0.943
2	1.124	.8900	.4854	.5454	2.060	1.833
3	1.191	.8396	.3141	.3741	3.184	2.673
4	1.262	.7921	.2286	.2886	4.375	3.465
5	1.338	.7473	.1774	.2374	5.637	4.212

- =-$6549.32 + $4000(P/A, 6%, 4) - $800(P/A, 6%, 4) - $5100(P/F, 6%, 2)
- -$6549.32 + $4000*3.465 - $800*3.465 - $5100*.890 = -$0.32

- How can we set this up in a spreadsheet?

Rate of Return: Problem 1

Interest rate		0.06		**Time**		4			
Initial Cost	$	(6,549.20)		**Year**	**Net Cashflow**			**Discount Rate**	**NPV**
Annual Benefits		$13,860.42		0	$(6,549.20)			0%	$1,150.80
Maintenance costs		($2,772.08)		1	$ 3,200.00			1%	$ 937.58
Overhaul		($4,538.98)		2	$(1,900.00)			2%	$ 733.57
				3	$ 3,200.00			3%	$ 538.28
NPV	$	0.16		4	$ 3,200.00			4%	$ 351.23
								5%	$ 171.99
				NPV	$0.16			6%	$ 0.16
								7%	$ (164.66)
				IRR	6.0%			8%	$ (322.82)
								9%	$ (474.66)
								10%	$ (620.51)



Evaluating Projects based on IRR

- Once we calculate a projects IRR, what does that tell us?
- Generally, higher is better, but what is the minimum acceptable?
- "Minimum Acceptable Rate of Return" or MARR
  - Often comes from our Weighted Average Cost of Capital or other sources of interest rates
- If IRR > MARR, project is worth doing
- How is this relate to an NPV analysis? If IRR = MARR, what is the NPV of our project?

Evaluation

- You're bootstrapping a small tech startup out of your garage, and have developed potential sales to three different clients.
- You are financing this off your credit card at 24.99% interest, and only have enough credit limit to cover the costs of developing for one client.

Client A	Client B	Client C
IRR: 28.5%	IRR: 23.2%	IRR: 32.3%

- Which is your preferred client?
- Is there any client you would not work with?

Interest Rates When There Are Fees or Discounts

- The internal rate of return is affected by fees or discounts.
  - An electronics retailer offers a 2% interest rate on purchases. However, they charge a financing fee of $200.00 to provide the service.
    - The fee increases the true internal interest rate of the purchase.
    - Example 7-6 demonstrates a good example (Pg. 235)

IRR Limitations

- Comparing projects strictly on IRR can disagree with value or worth assessments
- The IRR technique does not distinguish between investing and borrowing; the criterion for acceptance depends on which it is.
- Uses the same rate for financing and reinvesting – not usually realistic

- We have other methods to address these limitations
  - ΔIRR or incremental IRR allows us to account for values
  - Modified Internal Rate of Return, or MIRR, addresses different financing rates