Lecture 18

Approximate Solution Methods

$$[M]\ddot{\vec{x}} + [K]\vec{x} = \vec{0}$$

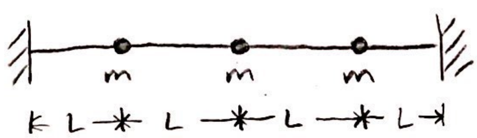
Solution: $\left(-\omega^2[M] + [K]\right)\vec{u} = \vec{0}$

Premultiply $\vec{u}^T$: $-\omega^2 \vec{u}^T[M]\vec{u} + \vec{u}^T[K]\vec{u} = 0$

Rearrange: $\omega^2 = \dfrac{\vec{u}^T[K]\vec{u}}{\vec{u}^T[M]\vec{u}}$ is exact.

Guess mode shape, $\vec{v}$: $\omega_R^2 \approx \dfrac{\vec{v}^T[K]\vec{v}}{\vec{v}^T[M]\vec{v}}$ Rayleigh Quotient

Ex: Stretched string with masses



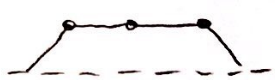
$\Rightarrow$

1^{st} mode shape

$$[M] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \qquad [K] = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix}$$

Tension $= P$

$k = \dfrac{P}{L}$

First guess:  $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$\omega_R^2 = \frac{[1\ 1\ 1][K]\begin{bmatrix}1\\1\\1\end{bmatrix}}{[1\ 1\ 1][M]\begin{bmatrix}1\\1\\1\end{bmatrix}} = \frac{2k}{3m} = 0.667\frac{k}{m} \text{ is } 7\% \text{ high}$$

Second guess:  $\vec{v} = \begin{bmatrix} 0.5 \\ 1 \\ 0.5 \end{bmatrix} \Rightarrow \omega_R^2 = 0.667 \frac{k}{m}$

Third guess:  $\vec{v} = \begin{bmatrix} 2/3 \\ 1 \\ 2/3 \end{bmatrix} \Rightarrow \omega_R^2 = 0.588 \frac{k}{m}$

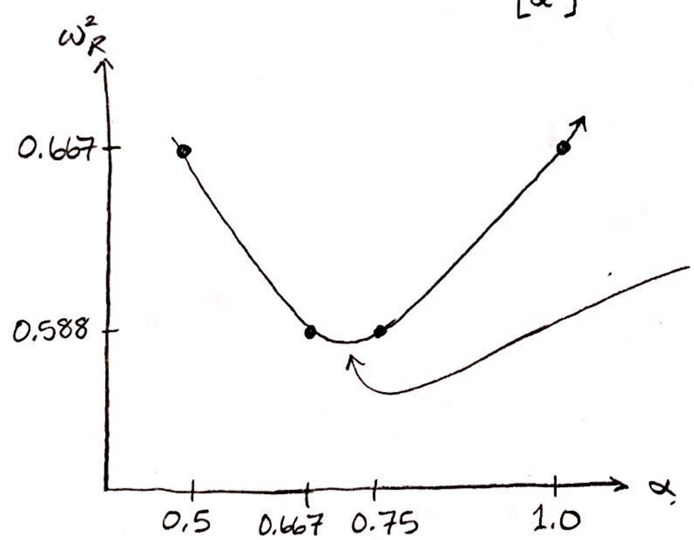
is 0.2% high

Fourth guess:  $\vec{v} = \begin{bmatrix} 3/4 \\ 1 \\ 3/4 \end{bmatrix} \Rightarrow \omega_R^2 = 0.588 \frac{k}{m}$

Guessed mode shape

$$\vec{v} = \begin{bmatrix} \alpha \\ 1 \\ \alpha \end{bmatrix}$$

Plot:



The exact solution is:

$\omega^2 = 0.586 \frac{k}{m}$

$\alpha = 0.707 = \frac{1}{\sqrt{2}}$

Second mode shape:  Guess $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\Rightarrow \omega_R^2 = 2 \frac{k}{m}$ is exact

Third mode shape:  Guess $\vec{v} = \begin{bmatrix} -2/3 \\ 1 \\ -2/3 \end{bmatrix}$

$\Rightarrow \omega_R^2 = 3.412 \frac{k}{m}$

The exact solution is $3.414 \frac{k}{m}$, $\alpha = 0.707$

Plot 1st and 3rd because they have the same form