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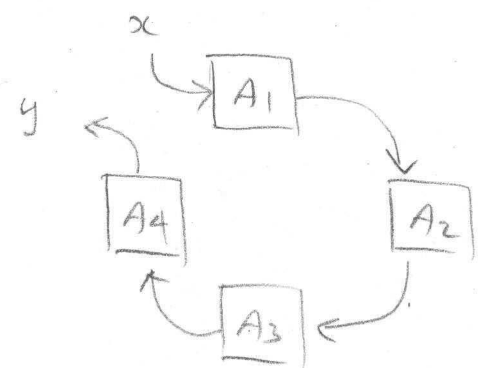
< Feedback & Stability >

◦ Objective

· Understand the condition for marginal stability

· Understand the effect of a time delay to the closed-loop system stability.

◦ Nyquist's original idea.



$L.T. \triangleq A_1 A_2 A_3 A_4$

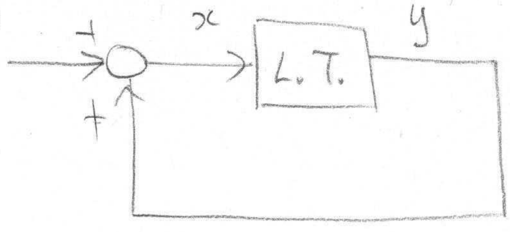
- Let $x = \cos(\omega_0 t)$

If $L.T.(j\omega_0) = 1$, then $y = \cos(\omega_0 t)$.

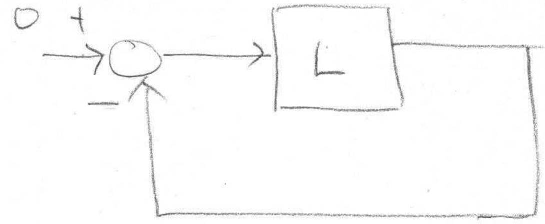
- Now, if we connect the two arrows together.

$x = y = \cos(\omega_0 t)$ "keep oscillating"

- If $L.T.(j\omega_0) = 1$. the loop can maintain a persistant sinusoid of frequency ω_0.



$\Longleftrightarrow$

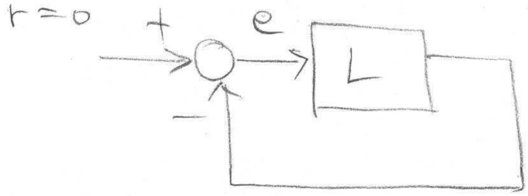


$L.T.(j\omega_0) = 1$ for marginal stability.

$L(j\omega_0) = -1$

$$\boxed{\begin{aligned} |L| &= 1 \\ \angle L &= -180^\circ \end{aligned}}$$

- Sensitivity function

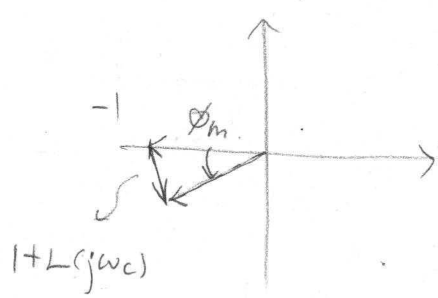


$$\frac{E}{R} = \frac{1}{1+L} \triangleq S$$

- The system would be on the verge of instability if $r = 0 \rightarrow e \neq 0$.

- If $L(j\omega_0) = -1$. $S(j\omega_0) = \dfrac{1}{1+L(j\omega_0)} \rightarrow \infty$.

This is the case when $\phi_m = 0$. deg

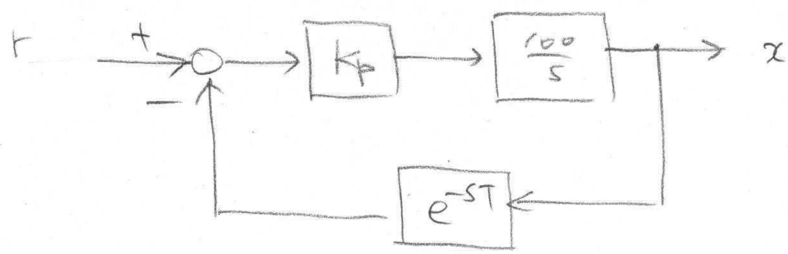


$\underline{Q}$. What if $\phi < 0$?

then $\dfrac{1}{|1+L(j\omega_c)|} < \infty$?

$\underline{A}$. We need to assess the stability via Nyquist test.

- Example : p-control with measurement delay.



- Delay by T : $\delta(t) \longrightarrow \boxed{\text{Delay}} \longrightarrow \delta(t-T) = h(t)$

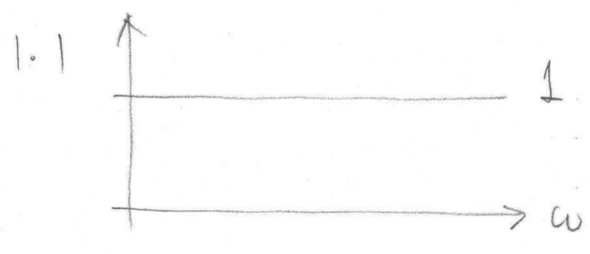
$$\therefore H(s) = \int_{-\infty}^{\infty} \delta(t-T)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t-T)\, e^{-sT}\, dt \qquad \text{"Sampling"}$$

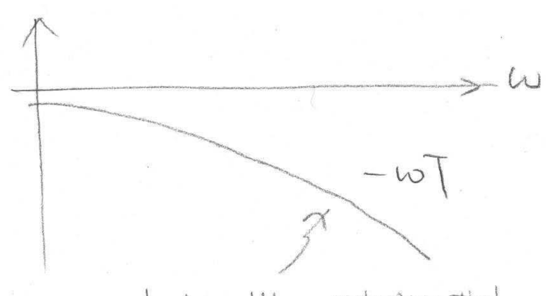
$$= e^{-sT} \underbrace{\int_{-\infty}^{\infty} \delta(t-T)\, dt}_{1} \qquad \text{"Unit area"}$$

$$= e^{-sT} \qquad \begin{cases} |e^{-j\omega T}| = 1 \\ \measuredangle e^{-j\omega T} = -\omega T \end{cases}$$

- Bode plot.



"All-pass" System



" Linear-phase" System.

" Non-minimum phase (Lag) System.

- looks like exponential because horizontal axis is logarithmic.

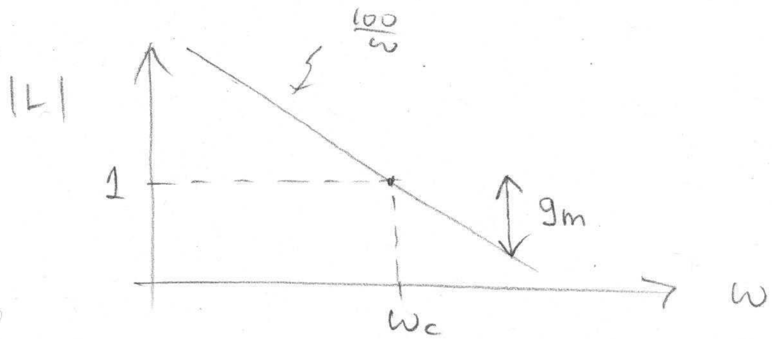
- Loop Return Ratio

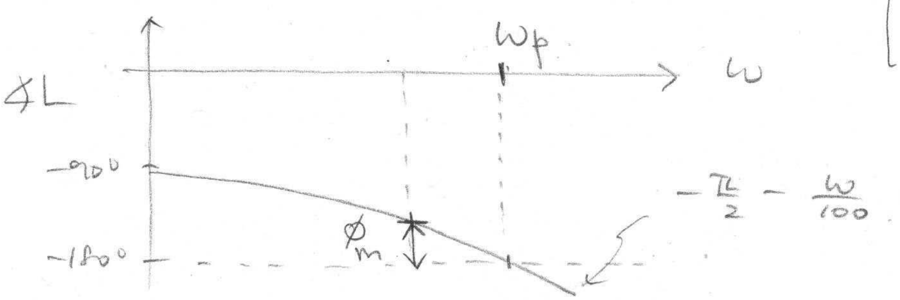
$$L(s) = K_p \cdot \frac{100}{s} e^{-sT}$$

For $K_p = 1$, $T = 10\ ms$

$$L(s) = \frac{100}{s} e^{-0.01s}$$

$$\begin{cases} |L| = \frac{100}{\omega} \\ \measuredangle L = -\frac{\pi}{2} - \frac{\omega}{100} \end{cases}$$





$\begin{cases} \omega_c : \text{gain crossover freq} \\ \phi_m : \text{phase margin.} \end{cases}$

$\begin{cases} \omega_p : \text{phase crossover freq} \\ g_m : \text{gain margin.} \end{cases}$

Two "suspicious frequencies" for marginal stability

① ω_c : $|L(j\omega_c)| = 1$. $\frac{100}{\omega_c} = 1$ $\therefore \omega_c = 100\ rad/s$

$\measuredangle L(j\omega_c) \neq -180°$. $\simeq$ 16 Hz.

② ω_p : $|L(j\omega_p)| \neq 1$ $-\frac{\pi}{2} - \frac{\omega_p}{100} = -\pi$

$\measuredangle L(j\omega_p) = -180°$ $\therefore \omega_p = \frac{\pi}{2} \times 100$

$= 50\pi\ rad/s$

$\simeq$ 25 Hz.

- phase margin

$$\phi_m = \measuredangle L(j\omega_c) - (-180°)$$
$$= \left(-\frac{\pi}{2} - 1\right) + \pi =$$
$$= \frac{\pi}{2} - 1 = 0.57 \text{ rad} \qquad \therefore \underline{\phi_m \doteqdot 33°}$$

Meaning : ϕ_m tells us how much delay the loop can tolerate before loosing stability

- Gain margin.

$$g_m = \frac{1}{|L(j\omega_p)|}$$
$$= \frac{1}{100/50\pi} = \frac{\pi}{2} \qquad \therefore \underline{g_m \simeq 1.57}$$

Meaning : g_m tells us how much gain increas the loop can tolerate before losing stability.

- Now, we can think of two ways to destabilize the system.

① Increase time delay T such that $\measuredangle L(j\omega_c) = -180°$

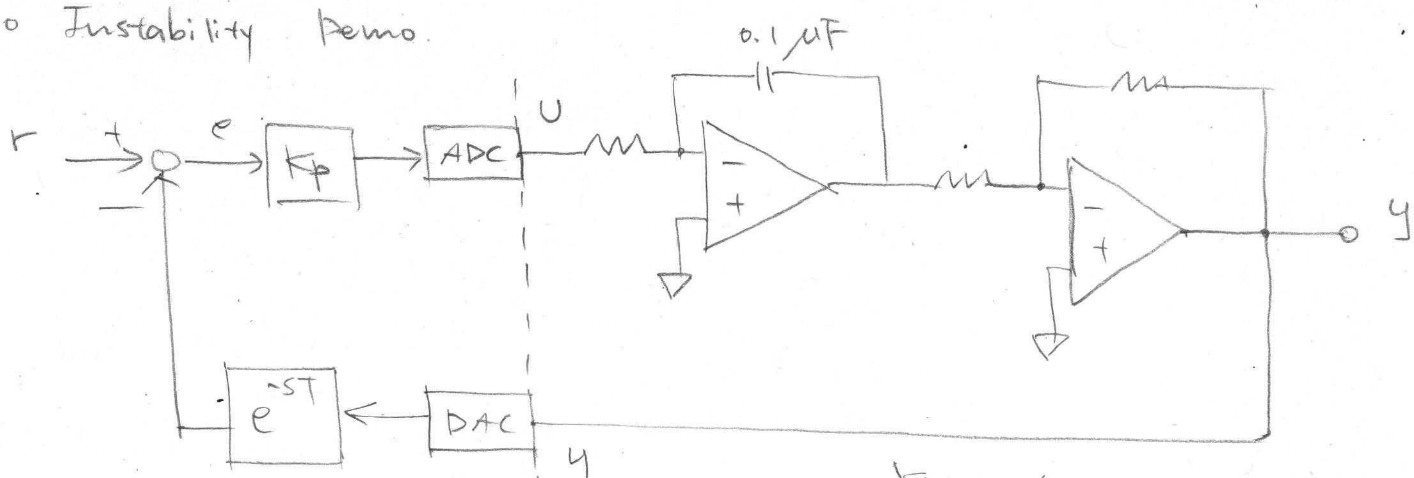
Since $\phi_m = 0.57$ rad, additional time delay of

$\Delta T = \frac{0.57 \text{ rad}}{100 \text{ rad/s}} \simeq 5.7$ ms will make the system marginally stable.

② Increase k_p such that $|L(j\omega_p)| = 1$

Since $g_m = 1.57$, increasing the gain to $k_p = 1.57$ will make it marginally stable.

∘ Instability Demo.



$$\frac{y}{u} = \frac{100}{s}.$$

① For $K_p = 1$, $T = 10$ ms

- Check the step resp. r = step
- See the overshoot.

② For $K_p = 1$. $T = 10$ ms

- Turn off the ref. $r = 0$.
- Gradually increase T untill seeing a sinusoid.
- Measure the frequency ! $\sim \omega_c$

③ For $K_p = 1$. $T = 10$ ms

- Turn off the ref. $r = 0$
- Gradually increase K_p untill seeing a sinusoid.
- Measure the frequency.

④ For $K_p = 1$, $T = 10$ ms.

- Set the ref. to step
- Vary the delay T and observe the overshoot