MECH 467/589 -Final Exam 12:00-2:30pm (December 7, 2019) Prof. Y. Altintas
CLOSED BOOK EXAM: No phone/calculators. PUT A BOX AROUND YOUR ANSWERS.
WRITE YOUR ANSWERS IN A LEGIBLE WAY TO AVOID LOOSING MARKS

1. Open loop dynamics of a process is defined by the following differential equation where $u(t)$ is the input and $y(t)$ is the output to the physical, uncontrolled process:

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 12y = \frac{du}{dt} + 2u$$

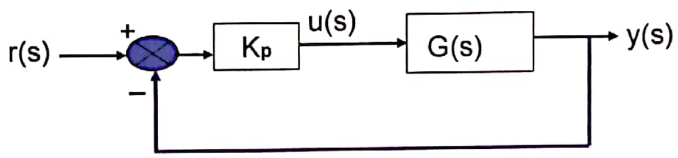

Figure 1: Block diagram of a closed loop system with a proportional controller.

- (5) Obtain the transfer function of the open loop dynamics of the process
- (10) Plot the Bode diagram of the open loop system approximately (i.e.$\omega = 1, 10, 100$)? (You can use logarithmic scale in dB or just a linear scale)
- (10) If a proportional controller is used to control the closed loop system, find the value of proportional gain K_p for the critically stable closed loop system? What is the range of K_p to ensure the stability?

2. A ball screw system is driving a table with a single degree of flexibility between the motor and ball screw assembly system. The position command is x_r, the position of the motor shaft is x_m and the table position is x The block diagram of the system is given in the figure. The equation of the motion for the flexible system is : $m\ddot{x} = -c(\dot{x} - \dot{x}_m) - k(x - x_m)$ which has a natural frequency of $\omega_n = \sqrt{k/m}$. The controller is a low pass filter with a gain $C(s) = K_p\omega_n/(s + \omega_n)$.

$C(s) = K_p \frac{\omega_n}{s+\omega_n}$
Low pass filter

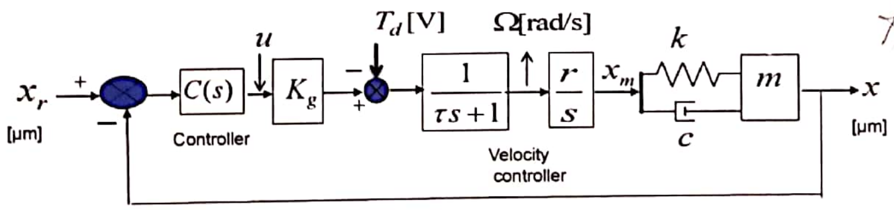

Figure 2: Block diagram of a closed loop system with a flexible table attachment. Note: $C(s) = K_p s/(s + \omega_n)$

- (10) Express the frequency response of the loop transfer function of the system (Magnitude and Phase)
- (10) Express the closed loop transfer functions $(G_x(s), G_d(s))$ of the system including disturbance torque ($x(s) = G_x(s)x_r(s) + G_d(s)T_d(s)$)
- (10) What is the steady error for a ramp position input ($x_r(t) = ft$) and step disturbance input ($T_d(t) = T_0$)

3. A plant's open loop transfer function is given as $G(z) = \frac{z^{-2}B(z^{-1})}{A(z^{-1})} = \frac{z^{-2}b}{1+a_1z^{-1}}$.

- (15)) Design a pole placement controller to achieve a damping ratio of ζ and natural frequency ω_n? $(R(z^{-1}) = 1 + r_1z^{-1} + r_2z^{-2} + .. + r_pz^{-p} \rightarrow \deg(R) = p;\ S(z) = s_0 + s_1z^{-1} + s_2z^{-2} + .. + s_fz^{-f} \rightarrow \deg(S) = f; t_0 = ?)$
- (10) Express the control law at each time interval, i.e. $u(k) = ??x(k) + ??x(k-1) + .. + ??u(k-1) + ??u(k-2) + ...$

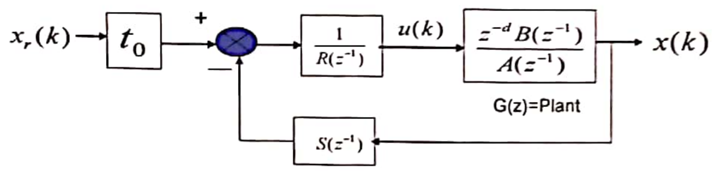

Figure 3: A pole placement controller system.

Notes:

Diophentine casuility equilibrium - $p+f+1=d+n+f$; $p+f+1=m+p$; mapping of a pole from s to z plane: $z^{-1}=e^{-sT}$ where T is the control time interval.

Pole placement principle: $G_{cl}(z)=\frac{x(k)}{x_r(k)}=\frac{z^{-d}Bt_0}{AR+z^{-d}BS}\equiv\frac{z^{-d}B_m}{A_m}$ where A_m is the characteristic equation with the desired damping ratio ζ and natural frequency ω_n.

4. (20) Design a linear interpolator between points $P(0,0)$ and $P(x_2,y_2)$.The cruising tangential velocity is f_c, the acceleration and deceleration are the same $A=A$ and $D=-A$. The initial ($f(0)=0$) and final ($f_e(t_3)=0$) velocities are zero, and the interpolation time interval is T_i. Total travel time is $T=T_1+T_2+T_3$. Express T_1, T_2, T_3: $l(k)=$??, $\Delta l(k)=?$, $\Delta x(k)=?$, $\Delta y(k)=?\rightarrow k=1,2,..N$. for each zone

$$\left\{\begin{array}{lllll} \text{Zone I} & a(t)=A\ ; & f(t)=At; & l(t)=\int f(t)dt & 0\le t\le t_1\rightarrow T_1=t_1 \\ \text{Zone II} & a(t)=0\ ; & f_c; & l(t)=l(t_1)+\int f_c dt & t_1\le t\le t_2\rightarrow T_2=t_2-t_1 \\ \text{Zone III} & a(t)=-A\ ; & f_c-At; & l(t)=l(t_2)+\int f(t)dt & t_2\le t\le t_3\rightarrow T_3=t_3-t_2 \end{array}\right\}$$

Additional **Formulas:**

x(t)	1	e^{-at}	t		Final value theory
X(s)	$\frac{1}{s}$	$\frac{1}{s+a}$	$\frac{1}{s^2}$		Continuous time domain $\lim_{t\to+\infty} f(t)=\lim_{s\to+0} sF(s)$
$X(z)$	$\frac{1}{1-z^{-1}}$	$\frac{1}{1-e^{-aT}z^{-1}}$	$\frac{Tz^{-1}}{(1-z^{-1})^2}$		Discrete time domain $\lim_{t\to+\infty} f(t)=\lim_{z\to1}(1-z^{-1})F(z)$