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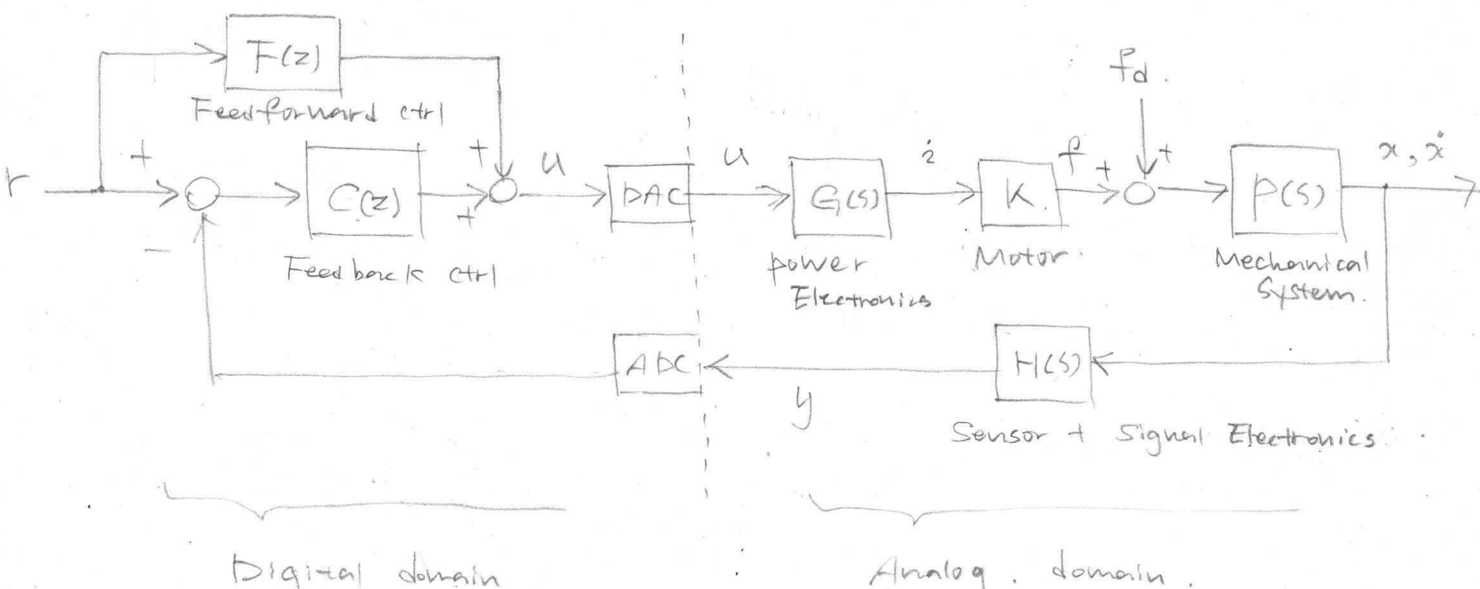
2021 / 03 / 05.

< Motion Control System >

- Objective
  - Servo system architecture
  - 2nd-order system review
  - pz-map / Step resp / Bode plot

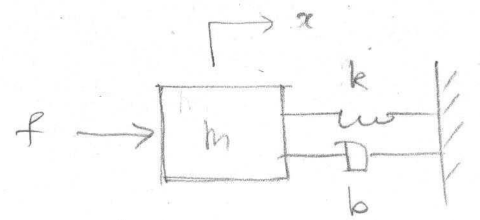
- System architecture (Lab 3 picture)



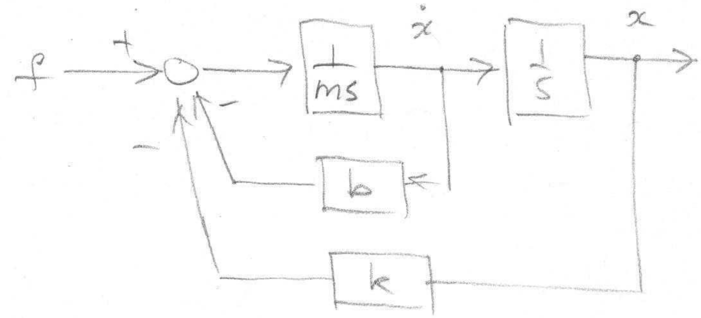
- So far, we studied some controls, motors, and analog circuits
  - power Amplifier : Voltage Amp. Transconductance Amp.
  - Signal Amplifier : Differential Amp. Instrumentation Amp
- Some of their functions can be implemented with digital syst.
  - power : Switching (Class-D) Amplifier.
  - signal : Digital sensors. → will cover briefly later in the course
- From now, we will study the rest modules
  : mechanical syst. ADC/DAC topics / digital control.

∘ 2nd-order System Review. (pz-map / Step resp / Bode plot).

< 1 DoF system >



"free-body"



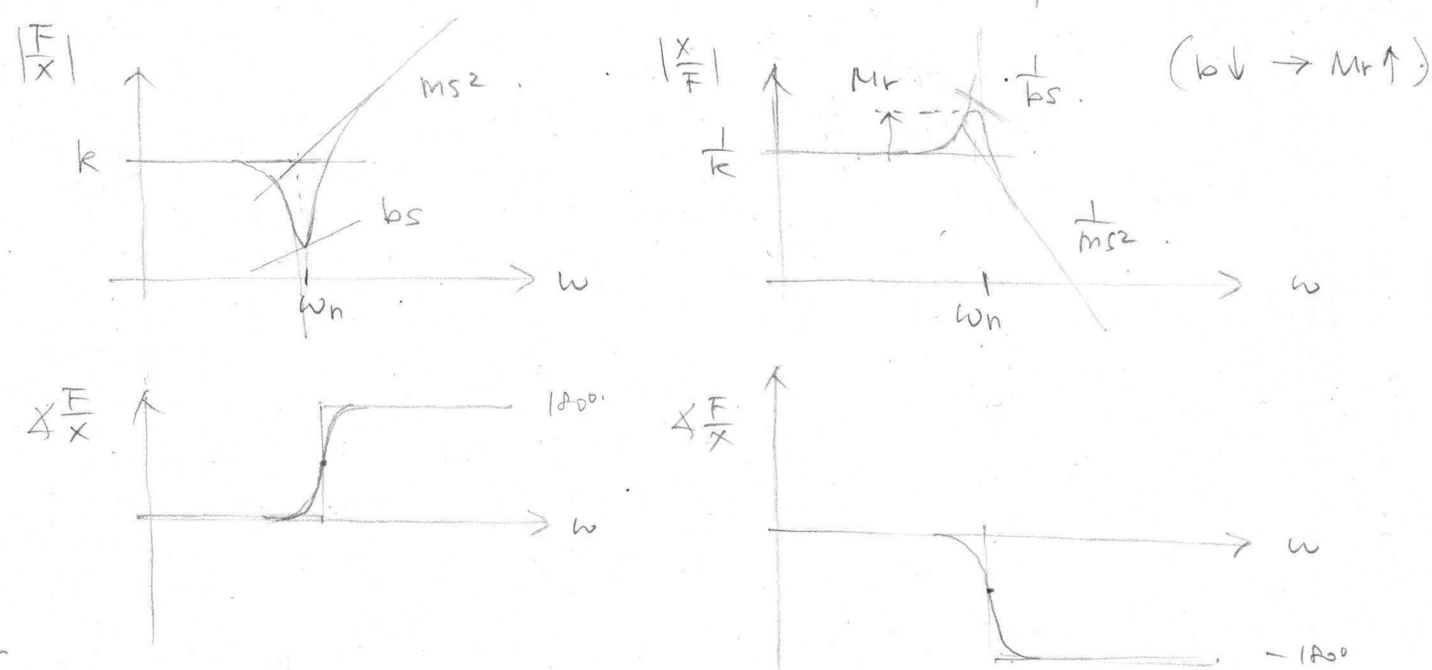
· Momentum principle : $m\ddot{x} = \Sigma f = f - kx - b\dot{x}$.

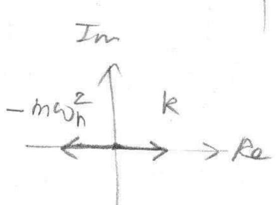
$$\Rightarrow \quad m\ddot{x} + b\dot{x} + kx = f$$

· We will study $\frac{X}{F}$ in class. Check past note for $\frac{\dot{X}}{F}$

· Laplace transform.

$$\underbrace{(ms^2 + bs + k)}_{\text{Stiffness}} X = F \quad \Rightarrow \quad \frac{X}{F} = \underbrace{\frac{1}{ms^2 + bs + k}}_{\text{Compliance}}.$$





$\omega_n \triangleq \sqrt{\frac{k}{m}}$

"Natural freq"

poles $= \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$

$b_c \triangleq 2\sqrt{mk}$: critical damping.

$\zeta \triangleq \frac{b}{b_c}$: damping ratio. [unitless].

- Let $p(s) = \dfrac{1}{ms^2+bs+k} = \dfrac{1}{m}\,\dfrac{1}{s^2+\frac{b}{m}s+\frac{k}{m}}$

$$= \frac{1}{m}\,\frac{1}{\underbrace{s^2+2\zeta\omega_n s+\omega_n^2}}$$ "Evans Form"

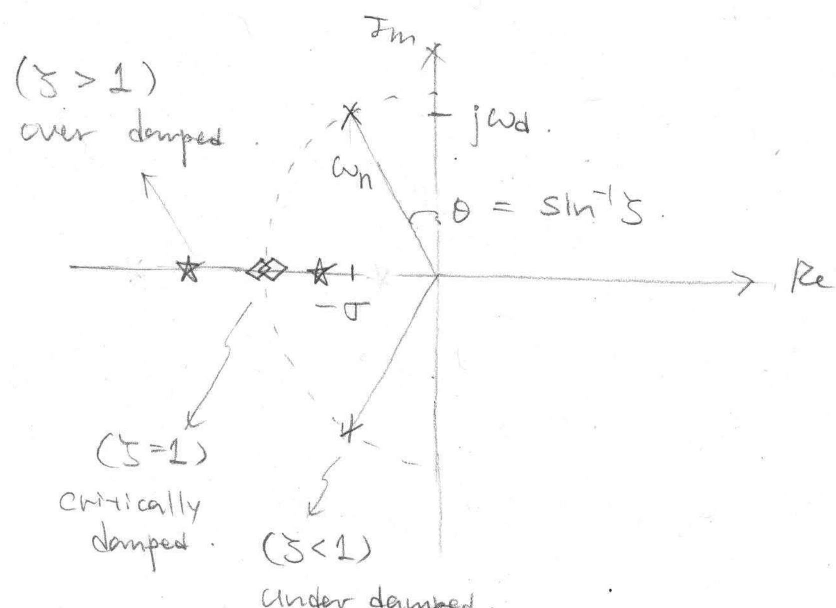
- If $\zeta = 0$ → Resonance
- If $\zeta = 1$ → Critical damping

OR, $p(s) = \dfrac{1}{ms^2+bs+k} = \underbrace{\dfrac{1}{k}}_{\text{DC gain}}\;\underbrace{\dfrac{1}{\left(\frac{s}{\omega_n}\right)^2+2\zeta\frac{s}{\omega_n}+1}}_{\text{Dynamics}}$ "Bode Form"

- pole-zero map. (ω_n, ζ)

$f(s) = s^2+2\zeta\omega_n s+\omega_n^2$ "Evans Form"

roots $= -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2-\omega_n^2} = \underbrace{-\zeta\omega_n}_{\sigma} \pm j\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}$.



- $\omega_n = \sqrt{\frac{k}{m}}$: natural freq.
- $\zeta = \frac{b}{2\sqrt{mk}}$: damping ratio
- $\sigma = \zeta\omega_n$: decay rate
- $\omega_d = \omega_n\sqrt{1-\zeta^2}$: damped natural freq.

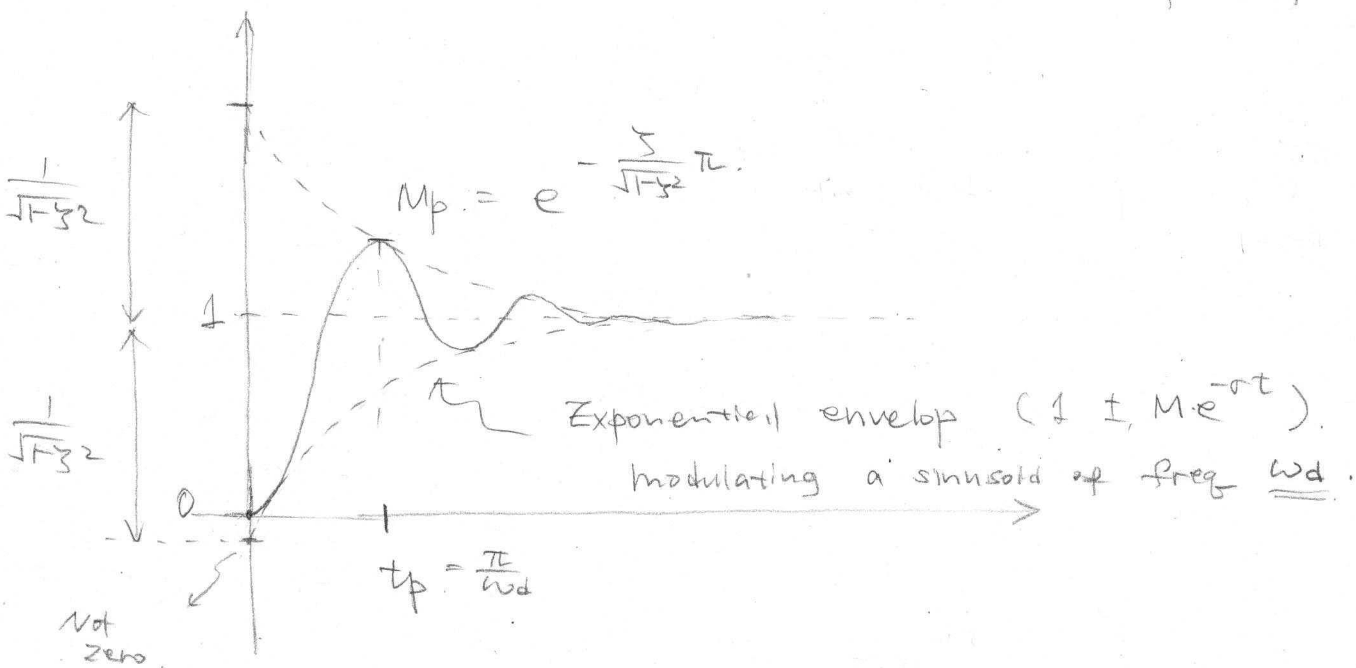
- Step Response. (ω_n, ζ)

$$x(t) = \frac{1}{k}\left[1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right)\right] u(t)$$

$$= \frac{1}{k}\left[1 - e^{-\sigma t} M \cdot \cos(\omega_d t + \phi)\right] u(t).$$

$$\begin{cases} M = \sqrt{1+\frac{\sigma^2}{\omega_d^2}} = \sqrt{1+\frac{\zeta^2}{1-\zeta^2}} = \frac{1}{\sqrt{1-\zeta^2}} \\ \phi = \tan^{-1}\left(\frac{\sigma}{\omega_d}\right) = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) \end{cases}$$

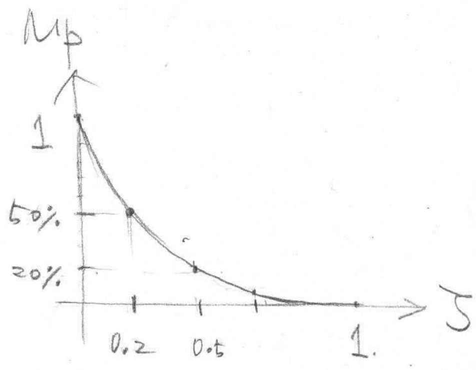
Let $\hat{x}(t) = k\,x(t)$: "Normalized" step response.



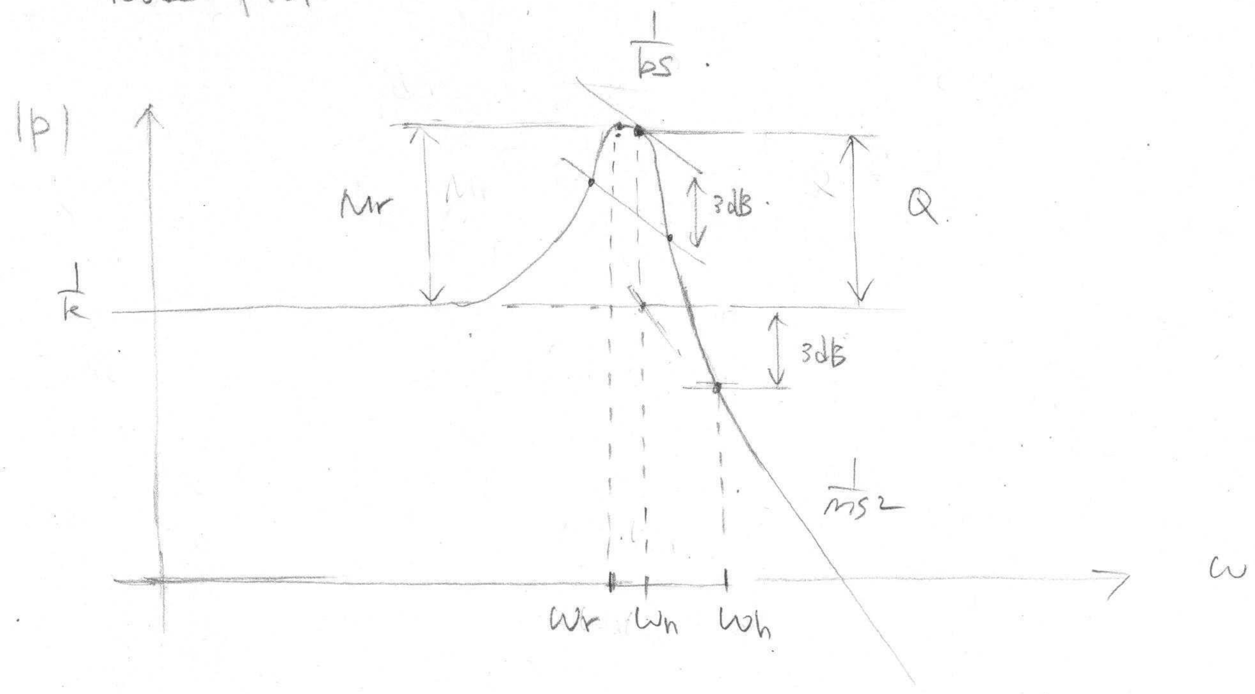
$$\dot{x}(t)\Big|_{t=t_p} = 0 \Rightarrow t_p = \frac{\pi}{\omega_d}$$

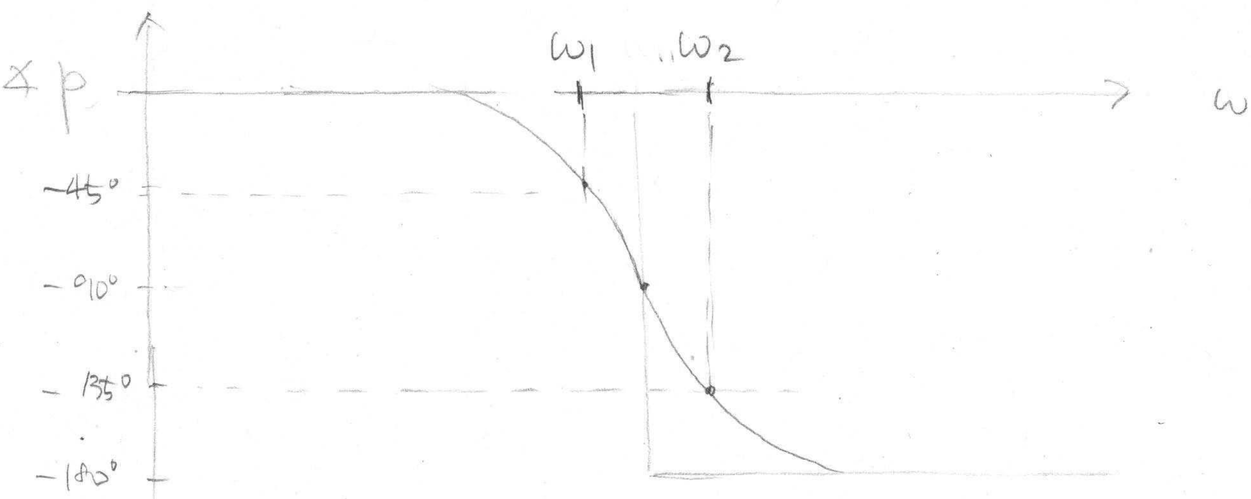
$$\hat{x}(t_p) = 1 + \underbrace{e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}}_{M_p}$$

"Overshoot" [unitless]



∘ Bode plot. (ω_n, ζ)





· $\omega_n = \sqrt{\frac{k}{m}}$: natural frequency

· $\omega_r = \omega_n \sqrt{1-2\zeta^2}$: resonance frequency $(0 < \zeta < \frac{1}{\sqrt{2}})$

· $M_r = \frac{1}{2\zeta} \frac{1}{\sqrt{1-\zeta^2}}$: resonance peak

· $Q = \frac{1}{2\zeta}$: quality factor.

when $\zeta^2 \ll 1$, $M_r \simeq Q = \frac{1}{2\zeta}$

$\omega_r \simeq \omega_n = \sqrt{\frac{k}{m}}$.

Check the Bode plot of

$\frac{\dot{X}}{F} = s\frac{X}{F}$

from the past lecture note.