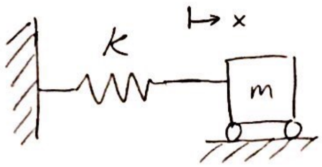
Lecture 2

1-DOF vibration



FBD $\Longrightarrow$

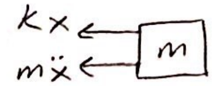
$F = ma$

$-kx = m\ddot{x}$

$m\ddot{x} + kx = 0$

<u>Newton</u>

or



$F = ma$

$F - ma = 0 \quad (\Sigma F = 0)$

$-m\ddot{x} - kx = 0$

$m\ddot{x} + kx = 0$

<u>D'Alembert</u>

$m\ddot{x} + kx = 0$

See Canvas announcements for Office Hours

Sign up for labs today, first on Friday.

Solutions to $m\ddot{x} + kx = 0$

#1) $x = A\cos(\omega t) - B\sin(\omega t)$

$\ddot{x} = -\omega^2 A\cos(\omega t) + \omega^2 B\sin(\omega t)$

$\Rightarrow m\ddot{x} + kx = (-m\omega^2 + k)(A\cos(\omega t) - B\sin(\omega t)) = 0$

↳ one, or both of products is zero.

$A\cos(\omega t) - B\sin(\omega t)$ only = zero if

A & B are zero (no motion)

$\Rightarrow -m\omega^2 + k = 0$ is the <u>Characteristic Equation</u>

$\Rightarrow \omega = \sqrt{\frac{k}{m}}$ is the <u>Natural Frequency</u> $\left[\frac{\text{rad}}{\text{sec}}\right]$

increase ω, increase k.

decrease ω, increase m.

Solve A, B with Initial Conditions:

x_0 - initial position

$\dot{x}_0$ - initial velocity

$$x = A\cos(\omega t) - B\sin(\omega t)$$

$$\dot{x} = -\omega A\sin(\omega t) - \omega B\cos(\omega t)$$

When $t \to 0$ $\quad A = x_0$

$\quad B = -\dot{x}_0/\omega$

$$\Rightarrow \boxed{x = x_0\cos(\omega t) + \frac{\dot{x}_0}{\omega}\sin(\omega t)}$$

<u>Period of vibration</u> $\tau = \frac{2\pi}{\omega}$

<u>Vibration frequency</u> $f = \frac{1}{\tau} = \frac{\omega}{2\pi}$ [Hz]

Solution 1 is convenient because we have solution from initial conditions. A, B have no physical meaning.

$\boxed{\text{Solution 2}}$

Let $A = C\cos\phi$ and $B = C\sin\phi$

Sub in solution 1:

$$\Rightarrow x = A\cos(\omega t) - B\sin(\omega t)$$

$$x = C\cos(\phi)\cos(\omega t) - \sin(\phi)\sin(\omega t)$$

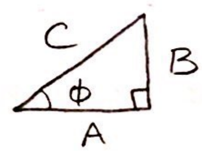
w/ trig identity: $\boxed{x = C\cos(\omega t + \phi)}$

Phase shift ϕ (angular position @ $t_0 = 0$)

Amplitude C

The benefit here is physical meaning of constants.

$$\boxed{\frac{B}{A} = \frac{C\sin\phi}{C\cos\phi} = \tan\phi}$$



for inversion

$\boxed{\text{Solution 3}}$

$x = Ge^{\lambda t}$, differentiate twice $\ddot{x} = \lambda^2 Ge^{\lambda t}$

Euler: $e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

Sub into equation of motion $m\ddot{x} + kx = 0$

$$\Rightarrow (m\lambda^2 + k)(Ge^{\lambda t}) = 0$$

One product must be zero, and $Ge^{\lambda t} \neq 0$ generally

$\Rightarrow m\lambda^2 + k = 0$

$\Rightarrow \lambda^2 = -\frac{k}{m} = -\omega^2$

$\Rightarrow \lambda = \pm i\omega$

original equation $\boxed{x = Ge^{i\omega t} + He^{-i\omega t}}$ G,H are complex

Use solutions 1 or 2 for undamped systems.

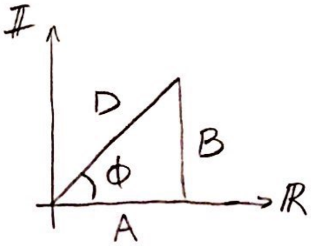
To relate A,B,G,H : $\boxed{\begin{array}{l} G = \frac{1}{2}(A + iB) \\ H = \frac{1}{2}(A - iB) \end{array}}$

$\boxed{\text{Solution 4}}$

$x = Re\left[De^{i\omega t}\right]$ Re - "real part of"

D is a complex number, a <u>phasor</u>.

<u>Phasors</u>:



"Argand Diagram"

$D = A + iB$

$D = C\cos\phi + iC\sin\phi$ (Sol. #2)

$D = C(\cos\phi + i\sin\phi)$

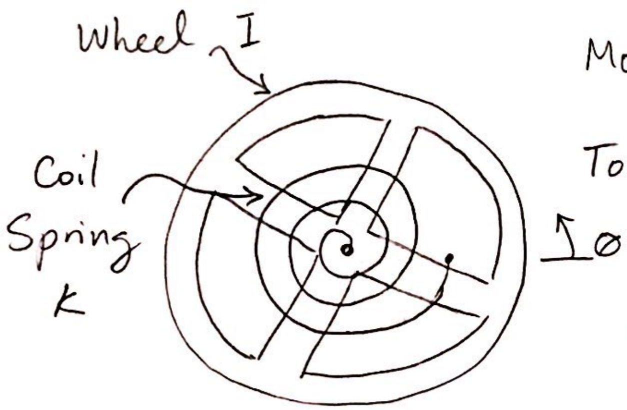
$D = Ce^{i\phi}$

$\Rightarrow x = Re\left[(A + iB)(\cos(\omega t) + i\sin(\omega t))\right]$

$= Re\left[A\cos(\omega t) - B\sin(\omega t) + iA\sin(\omega t) + iB\cos(\omega t)\right]$

$\boxed{x = A\cos(\omega t) - B\sin(\omega t)}$

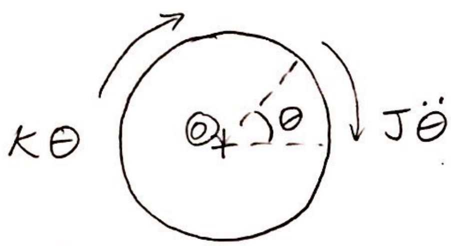
Ex: Torsion Wheel



Moment of Inertia J

Torsional Stiffness k $\left[\frac{\text{Torque}}{\text{rad}}\right]$

FBD:

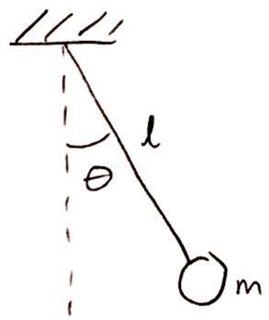


$\circlearrowright^{+} \Sigma M = 0$ (D'Alembert)

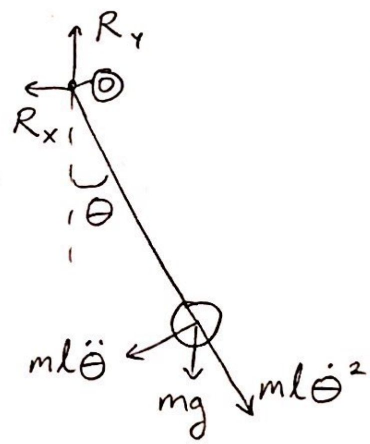
$$J\ddot{\theta} + k\theta = 0$$

$$\boxed{\omega = \sqrt{\frac{k}{J}}}$$

Ex: Pendulum



FBD



$\circlearrowright^{+} \Sigma_0 = 0$

$$ml\ddot{\theta} + mgl\sin\theta = 0$$

$$ml\ddot{\theta} + mgl\theta = 0$$

(small angle)

$$\Rightarrow \boxed{\omega = \sqrt{\frac{g}{l}}}$$

Draw diagrams in displaced states