MECH468 Modern Control Engineering
MECH509 Controls

Homework 2. Due: February 15 (Monday), 11:59 pm, 2021.

1 Theoretical (hand-calculation) questions

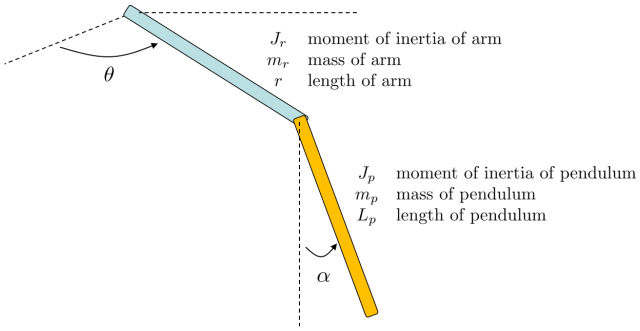
Let us consider the following continuous-time system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u, \\ y &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x. \end{aligned}$$

1. Check if the system is BIBO stable.
   Hint: For a block-diagonal matrix $M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}$, $M^{-1} = \begin{bmatrix} M_1^{-1} & 0 \\ 0 & M_2^{-1} \end{bmatrix}$.
2. Check if the system is asymptotically stable, marginally stable, or unstable.
   Hint: For a block-diagonal matrix $M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}$, the set of eigenvalues of M consists of the set of eigenvalues of M_1 and the set of eigenvalues of M_2.
3. Check the controllability.
4. Check the observability.
5. Obtain the Kalman decomposition.

2 Matlab question

Consider a rotary pendulum shown below. This system has been taken from https://www.quanser.com/products/qube-servo-2/. All the equations and parameter values were given in HW1.



In HW1, we derived the linearized model for the pendulum system, i.e., around

$$\theta = 0,\ \dot{\theta} = 0,\ \alpha = 0,\ \dot{\alpha} = 0.$$

Task:

1. By hand-calculation, derive the linearized model for the inverted pendulum system, i.e., around

   $$\theta = 0,\ \dot{\theta} = 0,\ \alpha = \pi,\ \dot{\alpha} = 0.$$

   Hint: See HW1 for the derivation of the linearized model for the pendulum system, and think how to modify it.

2. Using Simulink and the linearized model, simulate for the case when all the initial states are zero except $\alpha(0) = \pi + 0.1$ [rad], and with no input. Plot the outputs $\theta(t)$ and $\alpha(t)$.

3. For both (pendulum and inverted pendulum) linearized systems, compute the eigenvalues of A-matrices and determine the internal stability.

Attach your Matlab code(s) (m-file and Simulink block) in your report.

Note: The requirement to attach Matlab codes to your homework assignments is:

- for making sure that each student did the homework independently, and
- for pointing out possible errors if the marker feels something is wrong.

We will not aim at checking your Matlab codes in detail.