MECH 421: Mechatronics System Instrumentation
2019/20 Winter Session – Term 2

Mid-term Exam

Date: Feb 24, 2020
Time: 3:00 – 4:00pm

Problem 1 (130 points)

Let us consider an op-amp circuit in Figure 1. We assume that the op-amp has infinite input impedance, zero output impedance, and open-loop transfer function $A(s)$. Figure 2 shows the Bode plot of $A(s)$.

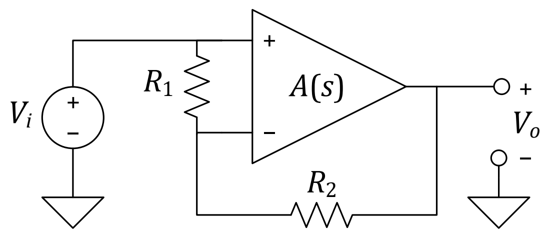

Figure 1: Op-amp circuit for Problem1.

(a) (20 pt.) Draw a block diagram that shows the feedback relation between the input voltage $V_i(s)$ and output voltage $V_o(s)$.

(b) (10 pt.) Find the expression for the loop transfer function $L(s)$ in terms of R_1, R_2, and $A(s)$.

(c) (20 pt.) For $R_1 \to \infty$, $R_2 = 1\,\text{k}\Omega$, and $A(s)$ given in Figure 2, find the gain crossover frequency ω_c and phase margin ϕ_m of $L(s)$.

(d) (30 pt.) For $R_1 = 1\,\text{k}\Omega$ and $A(s)$ given in Figure 2, find the resistance value R_2 that makes the closed-loop transfer function $G(s) = V_o(s)/V_i(s)$ achieve a $-3\,\text{dB}$ bandwidth of $100\,\text{kHz}$.

(e) (20 pt.) What is the dc gain of $G(s)$ designed in part (d)?

(f) (30 pt.) Suppose $G(s)$ designed in part (d) is excited with an input voltage

$$V_i(t) = \cos(2\pi \times 10^5\ t),$$

which is a persistent sinusoid defined for all time including $t < 0$. Find the magnitude M_o and phase ϕ_o of the output voltage

$$V_o(t) = M_o \cos(2\pi \times 10^5\ t + \phi_o).$$

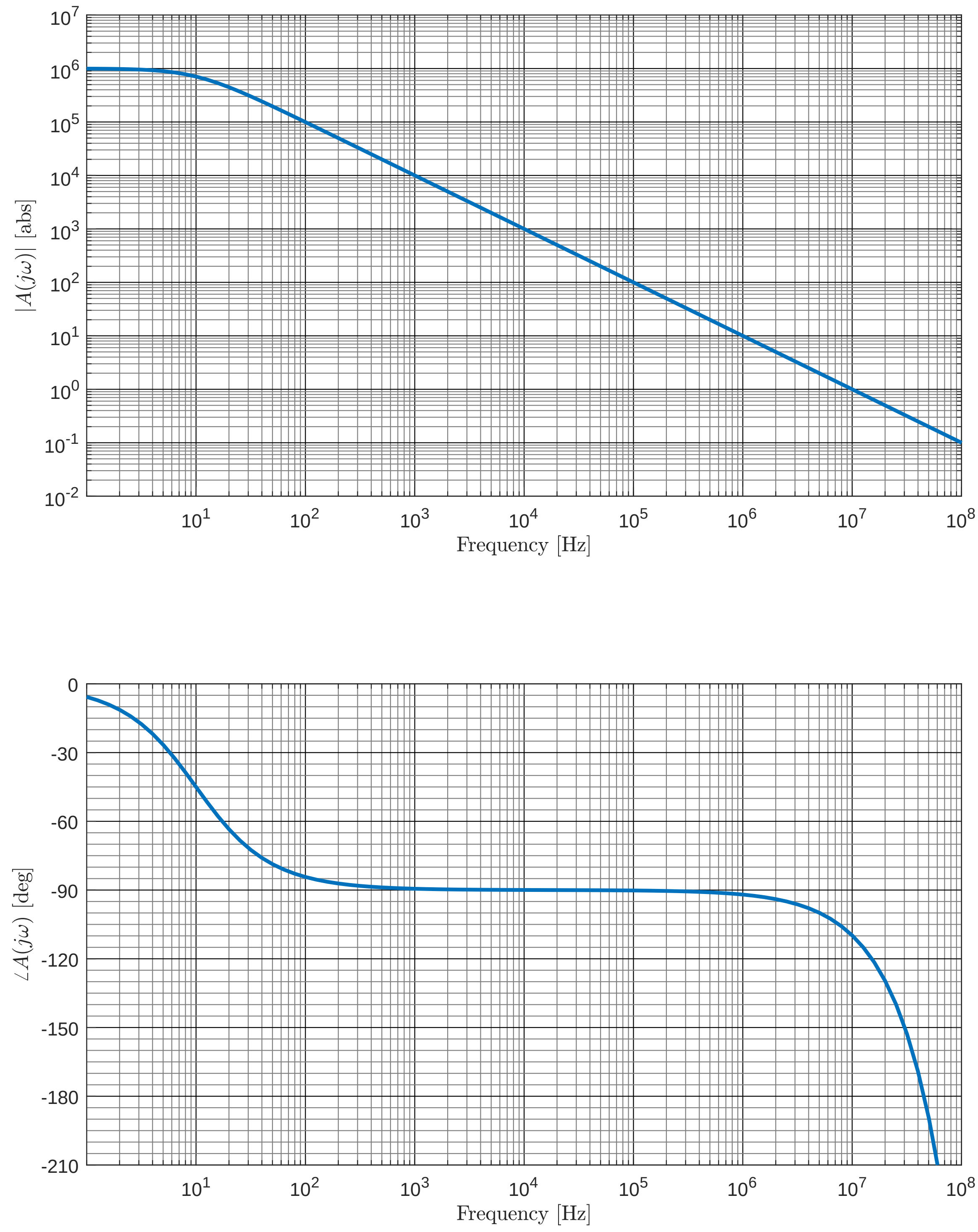

Figure 2: Bode plot of $A(s)$.

Problem 2 (70 points)

Let us consider a full-bridge strain gauge circuit in Figure 3. Here, $V_s = 5\,\text{V}$ is the supply voltage, R is the nominal resistance, and r is the resistance change due to the strain.

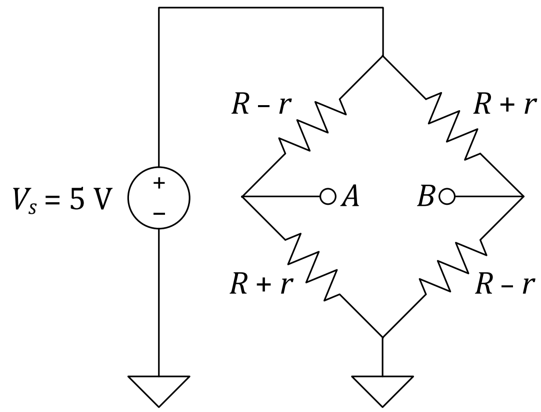

Figure 3: Full-bridge strain gauge circuit.

(a) (10 pt.) Find the expression for the voltage between the output terminals A and B in terms of R and r.

(b) (30 pt.) The output terminals A and B of the bridge circuit are connected to the input terminals A and B of the op-amp circuit in Figure 4. The grounds of the two systems are also connected together. Suppose the op-amps are ideal (i.e., infinite input impedance, zero output impedance, and infinite open-loop gain) and do not saturate, and r varies such that $-0.01R < r < 0.01R$. Find the range of the output voltage V_o.

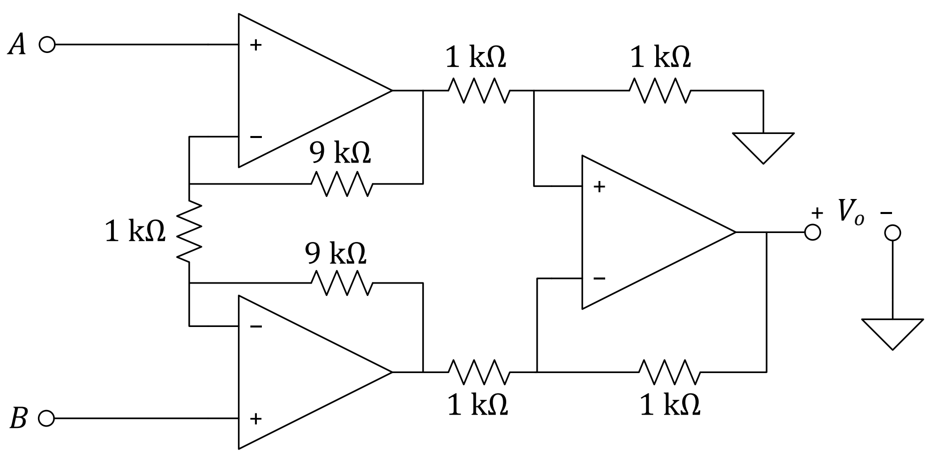

Figure 4: Instrumentation amplifier.

(c) (30 pt.) The output terminals A and B of the bridge circuit are connected to the input terminals A and B of the op-amp circuit in Figure 5. The grounds of the two systems are also connected together. Suppose the op-amp is ideal (i.e., infinite input impedance, zero output impedance, and infinite open-loop gain) and does not saturate, and r varies such that $-0.01R < r < 0.01R$. Find the range of the output voltage V_o.

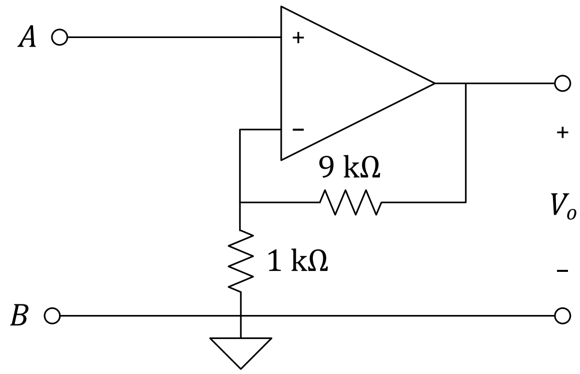

Figure 5: Non-inverting amplifier.