MECH 467 - Tutorial 9 – Trajectory Generation

Solutions

1) Based on the G-code given, the trajectory can be plotted as below.

N010 G01 X4.0 Y3.0 F12900
N020 G03 X44.0 I20.0 J0.

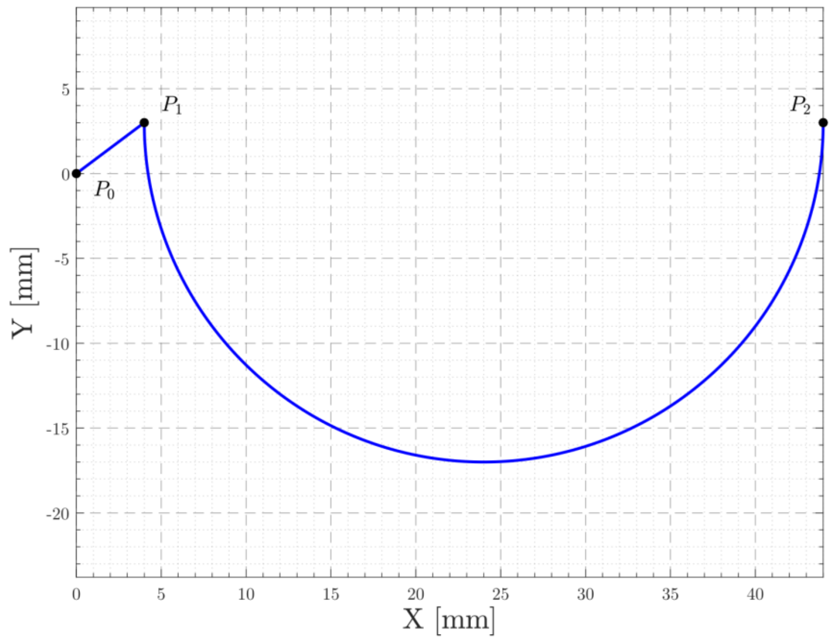


Fig. 1. The trajectory based on the given G-code

The trajectory starts at P_0 = (0,0) and linearly goes to P_1 = (4,3) with a constant feed-rate of 12900 [mm/min]. Then, the trajectory circulates around a circle with a center located with a positive 20 [mm] offset in x-direction, and 0 [mm] offset in y-direction. Finally, the trajectory reaches P_2 = (44,3).

Based on the offsets, the coordinates of the center of the circle are (x_c, y_c) = (24,3). The radius of the circle is R = 20 [mm].

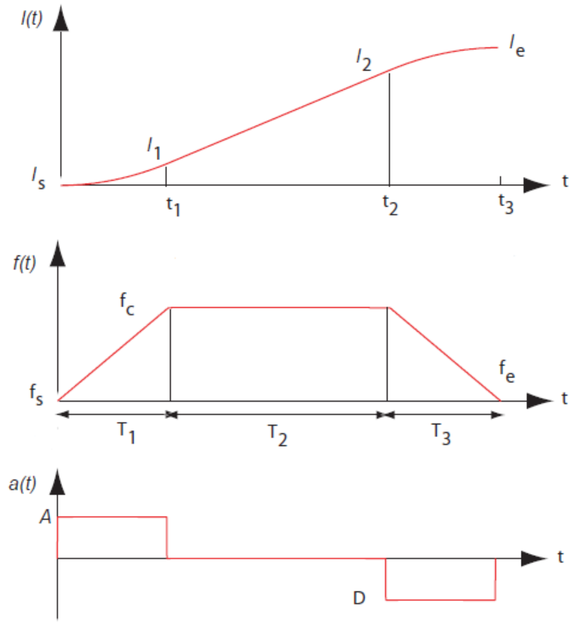
2) The length of the linear segment is calculated as follows.

$$L = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} = \sqrt{(4-0)^2 + (3-0)^2} = 5\, mm$$

The length of the circular segment would be calculated as the half of circle perimeter.

$$L3 = \pi R = 20\pi\ mm$$

3,4) The general trapezoidal feed-rate trajectory generation is provided below.



$$l(t) = \begin{cases} \frac{1}{2}At^2 + f_s t & T_1 \\ f_c t + l_1 & T_2 \\ \frac{1}{2}Dt^2 + f_c t + l_2 & T_3 \end{cases}$$

$$f(t) = \begin{cases} At + f_s & T_1 \\ f_c & T_2 \\ Dt + f_c & T_3 \end{cases}$$

$$a(t) = \begin{cases} A & T_1 \\ 0 & T_2 \\ D & T_3 \end{cases}$$

Fig. 2. General trapezoidal feed-rate trajectory conventions

Linear Segment

$$f_c = 12900 \left[\frac{mm}{min}\right] = 215 \left[\frac{mm}{s}\right]$$

$$T_1 = \frac{f_c - f_s}{A} = \frac{215 - 0}{1000} = 0.215\ sec$$

$$T_3 = \frac{f_e - f_c}{D} = \frac{0 - 215}{-1000} = 0.215\ sec$$

$$T_2 = \left(\frac{L}{f_c}\right) - \left[\left(\frac{1}{2A} - \frac{1}{2D}\right) f_c + \left(\frac{f_e^2}{2D} - \frac{f_s^2}{2A}\right)\frac{1}{f_c}\right]$$

$$\rightarrow T_2 = \left(\frac{5}{215}\right) - \left[\left(\frac{215}{1000}\right)\right] = -0.192\ \text{sec} \quad \textbf{\textit{Not Acceptable}}$$

If $T_2 \leq 0$, we should modify the feed-rate and the other properties of trajectory.

$$T_{2m} = 0$$

$$f_{cm} = \sqrt{\frac{2ADL - (f_e^2 A - f_s^2 D)}{D - A}} = \sqrt{\frac{2 \times 1000 \times -1000 \times 5}{-1000 - 1000}} = 70.7107\ \frac{mm}{s}$$

$$T_{1m} = \frac{f_{cm} - f_s}{A} = \frac{70.7107 - 0}{1000} = 0.0707\ sec$$

$$T_{3m} = \frac{f_e - f_{cm}}{D} = \frac{0 - 70.7107}{-1000} = 0.0707\ sec$$

The number of interpolation periods are calculated as follows.

$$N_1 = ceil\left(\frac{T_{1m}}{T_i}\right) = ceil\left(\frac{0.0707}{0.001}\right) = 71 \quad \rightarrow \quad T_1' = N_1 T_i = 0.071\ sec$$

$$N_2 = ceil\left(\frac{T_{2m}}{T_i}\right) = ceil\left(\frac{0}{0.001}\right) = 0 \quad \rightarrow \quad T_2' = N_2 T_i = 0\ sec$$

$$N_3 = ceil\left(\frac{T_{3m}}{T_i}\right) = ceil\left(\frac{0.0707}{0.001}\right) = 71 \quad \rightarrow \quad T_3' = N_3 T_i = 0.071\ sec$$

With the new timings, the machine feed-rate and accelerations must be updated.

$$f_c' = \frac{2L - f_s T_1' - f_e T_3'}{T_1' + 2T_2' + T_3'} = \frac{2 \times 5}{0.071 + 2 \times 0 + 0.071} = 70.4225\ \frac{mm}{s}$$

$$A' = \frac{f_c' - f_s}{T_1'} = \frac{70.4225 - 0}{0.071} = 991.866\ \frac{mm}{s^2}$$

$$D' = \frac{f_e - f_c'}{T_3'} = \frac{0 - 70.4225}{0.071} = -991.866\ \frac{mm}{s^2}$$

Now, for the position, feed-rate, and acceleration in discrete time domain, we have:

$$l(k) = \begin{cases} \frac{1}{2}A'(kT_i)^2 + f_s(T_i) & N_1 \\ f_c'(kT_i) + l_1 & N_2 \\ \frac{1}{2}D'(kT_i)^2 + f_c'(kT_i) + l_2 & N_3 \end{cases}$$

$$x(k) = \frac{x_1 - x_0}{L}\, l(k)$$

$$y(k) = \frac{y_1 - y_0}{L}\, l(k)$$

$$f(k) = \begin{cases} A'(kT_i) + f_s & N_1 \\ f_c' & N_2 \\ D'(kT_i) + f_c' & N_3 \end{cases}$$

$$\dot{x}(k) = \frac{x_1 - x_0}{L}\, f(k)$$

$$\dot{y}(k) = \frac{y_1 - y_0}{L}\, f(k)$$

$$a(t) = \begin{cases} A' & N_1 \\ 0 & N_2 \\ D' & N_3 \end{cases}$$

$$\ddot{x}(k) = \frac{x_1 - x_0}{L}\, a(k)$$

$$\ddot{y}(k) = \frac{y_1 - y_0}{L}\, a(k)$$

The results for the initial steps are tabulated below.

Table 1. Results for initial interpolation steps in the linear segment

k	t [s]	l [mm]	f [mm/s]	a [mm/s^2]	x [mm]	y [mm]	$\dot{x}$ [mm/s]	$\dot{y}$ [mm/s]	$\ddot{x}$ [mm/s^2]	$\ddot{y}$ [mm/s^2]
0	0	0	0	0	0	0	0	0	0	0
1	0.001	0.0005	0.9919	991.8667	0.0004	0.0003	0.7935	0.5951	793.4934	595.1200
2	0.002	0.0020	1.9837	991.8667	0.0016	0.0012	1.5870	1.1902	793.4934	595.1200
3	0.003	0.0045	2.9756	991.8667	0.0036	0.0027	2.3805	1.7854	793.4934	595.1200

Also, the displacement, feed-rate, and acceleration of the machine along the linear segment are presented below.

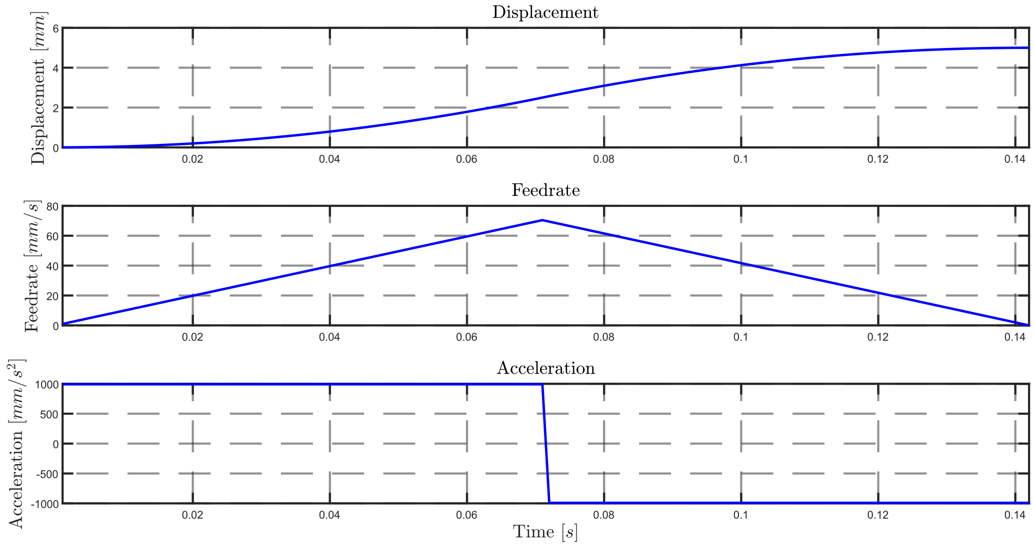


Fig. 3. Displacement, feed-rate, and acceleration along the linear segment

The position, velocity, and acceleration of the machine in x- and y-directions are presented below.

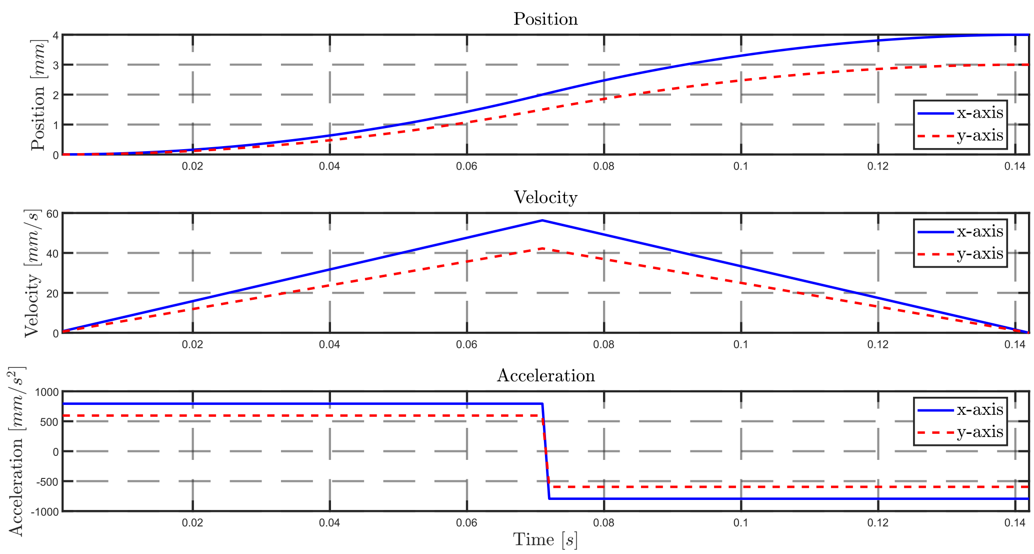


Fig. 4. Position, velocity, and acceleration in the x- and y-directions

Circular Segment

$$T_1 = \frac{f_c - f_s}{A} = \frac{215 - 0}{1000} = 0.215\ sec$$

$$T_3 = \frac{f_e - f_c}{D} = \frac{0 - 215}{-1000} = 0.215\ sec$$

$$T_2 = \left(\frac{L}{f_c}\right) - \left[\left(\frac{1}{2A} - \frac{1}{2D}\right) f_c + \left(\frac{f_e^2}{2D} - \frac{f_s^2}{2A}\right) \frac{1}{f_c}\right]$$

$$\rightarrow T_2 = \left(\frac{20\pi}{215}\right) - \left[\left(\frac{215}{1000}\right)\right] = 0.07724\ sec$$

The number of interpolation periods are calculated as follows.

$$N_1 = ceil\left(\frac{T_1}{T_i}\right) = ceil\left(\frac{0.215}{0.001}\right) = 215 \quad \rightarrow \quad T_1' = N_1 T_i = 0.215\ sec$$

$$N_2 = ceil\left(\frac{T_2}{T_i}\right) = ceil\left(\frac{0.07724}{0.001}\right) = 78 \quad \rightarrow \quad T_2' = N_2 T_i = 0.078\ sec$$

$$N_3 = ceil\left(\frac{T_3}{T_i}\right) = ceil\left(\frac{0.215}{0.001}\right) = 71 \quad \rightarrow \quad T_3' = N_3 T_i = 0.215\ sec$$

With the new timings, the machine feed-rate and accelerations must be updated.

$$f_c' = \frac{2L - f_s T_1' - f_e T_3'}{T_1' + 2T_2' + T_3'} = \frac{2 \times 20\pi}{0.215 + 2 \times 0.078 + 0.215} = 214.4432\ \frac{mm}{s}$$

$$A' = \frac{f_c' - f_s}{T_1'} = \frac{214.4432 - 0}{0.215} = 997.41\ \frac{mm}{s^2}$$

$$D' = \frac{f_e - f_c'}{T_3'} = \frac{0 - 70.4225}{0.071} = -997.41\ \frac{mm}{s^2}$$

To calculate the position, velocity, and acceleration along the circular path, the following conventions are used.

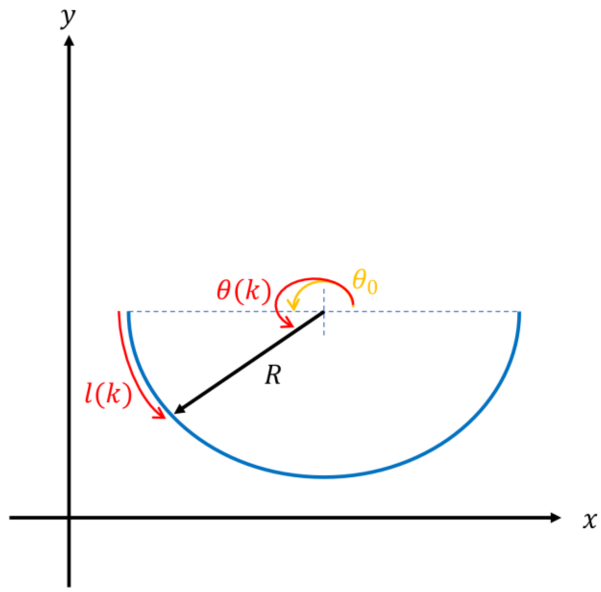


Fig. 5. Schematic of a circular path

$$\theta_0 = \pi, \quad \theta(k) = \theta_0 + \frac{l(k)}{R}$$

Now, for the position, feed-rate, and acceleration in discrete time domain, we have:

$$l(k) = \begin{cases} \frac{1}{2}A'(kT_i)^2 + f_s(T_i) & N_1 \\ f_c'(kT_i) + l_1 & N_2 \\ \frac{1}{2}D'(kT_i)^2 + f_c'(kT_i) + l_2 & N_3 \end{cases}$$

$$x(k) = x_c + R\cos\big(\theta(k)\big)$$
$$y(k) = y_c + R\sin\big(\theta(k)\big)$$

$$f(k) = \begin{cases} A'(kT_i) + f_s & N_1 \\ f_c' & N_2 \\ D'(kT_i) + f_c' & N_3 \end{cases}$$

$$\dot{x}(k) = -f(k)\sin\big(\theta(k)\big)$$
$$\dot{y}(k) = f(k)\cos\big(\theta(k)\big)$$

$$a(t) = \begin{cases} A' & N_1 \\ 0 & N_2 \\ D' & N_3 \end{cases}$$

$$\ddot{x}(k) = -a(k)\sin\big(\theta(k)\big) - \left(\frac{1}{R}\right)f^2(k)\cos\big(\theta(k)\big)$$

$$\ddot{y}(k) = a(k)\cos\big(\theta(k)\big) - \left(\frac{1}{R}\right)f^2(k)\sin\big(\theta(k)\big)$$

The results for the initial steps are tabulated below.

Table 2. Results for initial interpolation steps in the circular segment

k	t [s]	l [mm]	f [mm/s]	a [mm/s^2]	x [mm]	y [mm]	$\dot{x}$ [mm/s]	$\dot{y}$ [mm/s]	$\ddot{x}$ [mm/s^2]	$\ddot{y}$ [mm/s^2]
0	0	0	0	0	0	0	0	0	0	0
1	0.001	0.0005	0.9974	997.4102	4	2.9995	0.024e-3	-0.9974	0.0746	-997.4102
2	0.002	0.0020	1.9948	997.4102	4	2.9980	0.199e-3	1.9948	0.2984	-997.4101
3	0.003	0.0045	2.9922	997.4102	4	2.9955	0.6715e-3	-2.9922	0.6715	-997.4100

Also, the displacement, feed-rate, and acceleration of the machine along the linear segment are presented below.

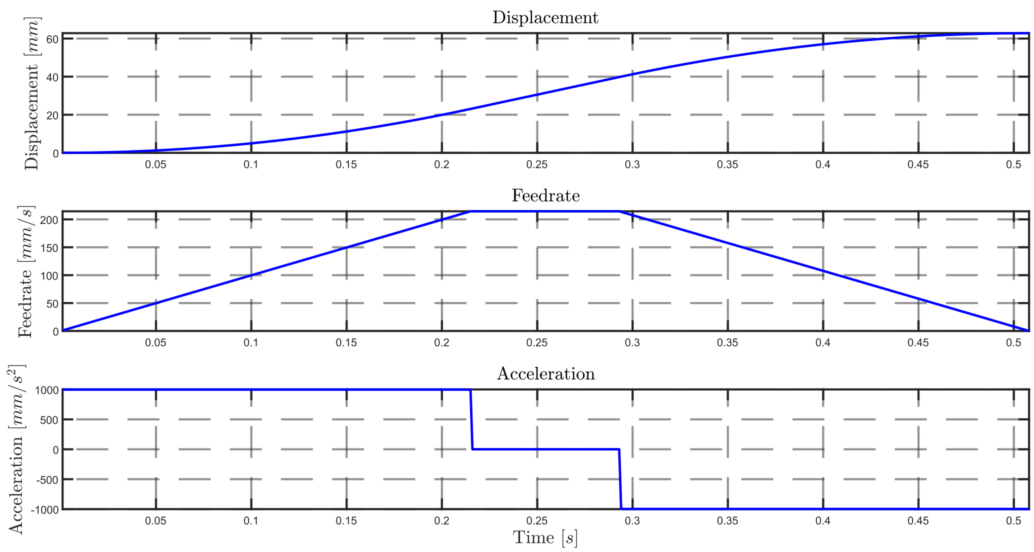


Fig. 6. Displacement, feed-rate, and acceleration along the circular segment

The position, velocity, and acceleration of the machine in x- and y-directions are presented below.

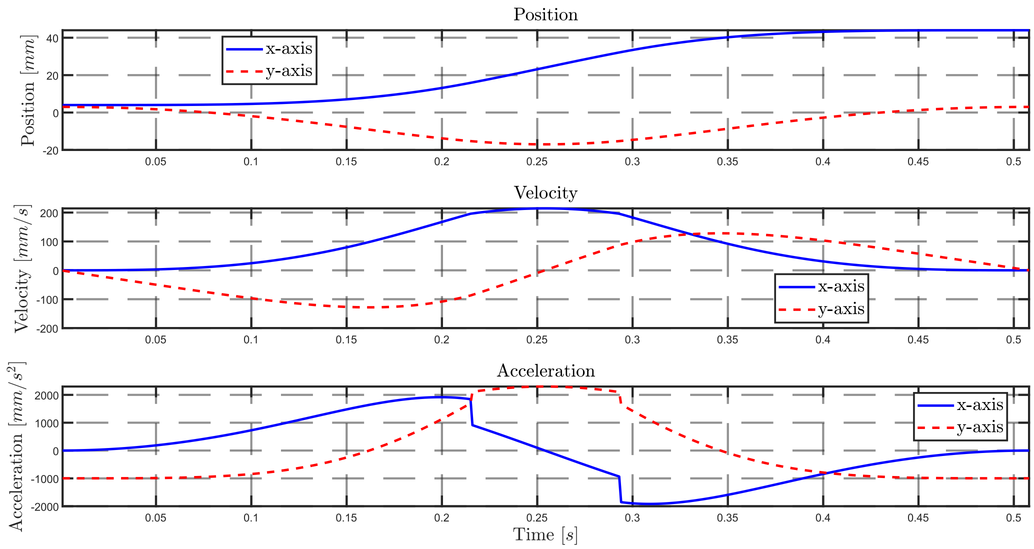


Fig. 7. Position, velocity, and acceleration in the x- and y-directions