MECH 364: Mechanical Vibrations
Midterm Examination 2

Time: 45 minutes 26th October 2011 Maximum Available Mark: 20

Q1)

a) Determine the natural frequency of free vibrations for the system shown in Figure 1 (left). **State your assumptions. 2 marks are reserved for this.** **(8 marks)**

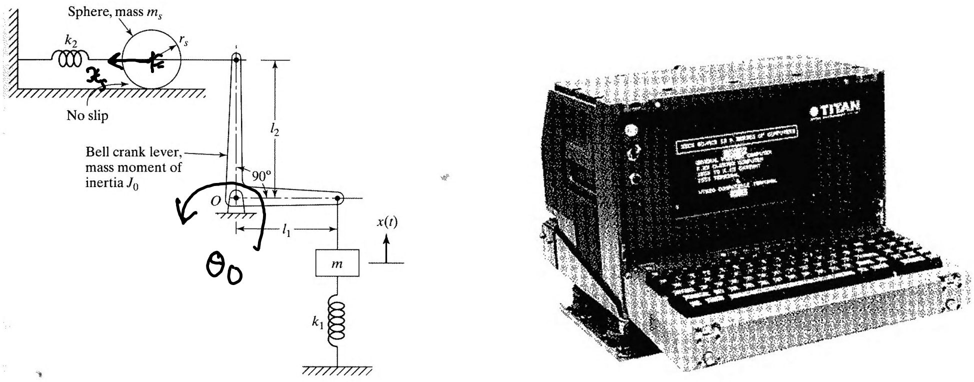

Figure 1: Figure for part a) (left) and part b) (right).

b) An electronic chassis weighing 500 N is isolated by supporting it on four identical helical springs, as shown in Figure 1 (right). Design the springs such that the unit can be used in an environment in which the vibratory frequency of external forces ranges from 0 to 5Hz by placing the natural frequencies outside this range. **(8 marks)**

c) A weight is being lowered from the helicopter shown below at a constant downward speed of V. To avoid a mishap the weight is suddenly stopped. What are the initial conditions for the ensuing free vibration, counting time from the instant the weight is stopped? **(4 marks)**

Figure 2: Figure for part c).

All the Best!

SOLUTION : MIDTERM EXAM 2

a) ASSUMING (i) ALL LINKS ARE RIGID
(ii) NO SLIP BETWEEN DISC & GROUND

KINETIC ENERGY $= T = \frac{1}{2} m_{eq} \dot{x}^2$

$$= \frac{1}{2} m \dot{x}^2 + \underbrace{\frac{1}{2} J_0 \dot{\theta}_0^2}_{\text{CRANK}} + \underbrace{\frac{1}{2} m_s (\dot{x}_s)^2 + \frac{1}{2} J_s \dot{\theta}_s^2}_{\text{SPHERE}} \quad -(1)$$

USING KINEMATICS TO EXPRESS VELOCITIES INTERMS OF $\dot{x}$

$$\left.\begin{array}{l} \dot{\theta}_0 = \dfrac{\dot{x}}{L_1} \;;\quad \dot{x}_s = L_2 \dot{\theta}_0 = \dfrac{L_2 \dot{x}}{L_1} \\ \dot{\theta}_s = \dfrac{\dot{x}_s}{r_s} = \dfrac{L_2 \dot{x}}{r_s L_1} \end{array}\right\} - (2)$$

(2) IN (1) GIVES

$$T = \frac{1}{2} m_{eq} \dot{x}^2 = \frac{1}{2}\left[m \dot{x}^2 + J_0 \left(\frac{\dot{x}}{L_1}\right)^2 + m_s \left(\frac{L_2 \dot{x}}{L_1}\right)^2 + J_s \left(\frac{L_2 \dot{x}}{r_s L_1}\right)^2\right]$$

$$= \frac{1}{2} \dot{x}^2 \left[m + \frac{J_0}{L_1^2} + m_s \left(\frac{L_2}{L_1}\right)^2 + J_s \left(\frac{L_2}{r_s L_1}\right)^2\right]$$

$$\Rightarrow \boxed{m_{eq} = m + \frac{J_0}{L_1^2} + m_s \left(\frac{L_2}{L_1}\right)^2 + J_s \left(\frac{L_2}{L_1 r_s}\right)^2} \quad -(3)$$

POTENTIAL ENERGY $= PE = \frac{1}{2} K_{eq} x^2 = \frac{1}{2} K_1 x^2 + \frac{1}{2} K_2 x_s^2$

$$= \frac{1}{2} \left[K_1 + K_2 \left(\frac{L_2}{L_1}\right)^2\right] x^2$$

$$\Rightarrow \boxed{K_{eq} = K_1 + K_2 \left(\frac{L_2}{L_1}\right)^2} \quad -(4)$$

NATURAL FREQUENCY $= \omega_n = \sqrt{\dfrac{K_{eq}}{M_{eq}}} = \sqrt{\dfrac{K_1 + K_2(L_2/L_1)^2}{m + \dfrac{J_0}{L_1^2} + m_s\left(\dfrac{L_2}{L_1}\right)^2 + J_s\left(\dfrac{L_2}{L_1 r_s}\right)^2}}$

b) MASS $= M = \dfrac{500}{9.81}$ kg

FOR 4 SUPPORT SRINGS OF K EACH IN PARALLEL $= K_{eq} = 4K$

WE WANT : $\omega_n =$ NATURAL FREQUENCY $= \sqrt{\dfrac{K_{eq}}{M}} = \sqrt{\dfrac{4K}{M}} > 5 \times 2\pi$

$\Rightarrow \quad 4K > 25 \times 4\pi^2 \times M \quad \Rightarrow \quad K > \dfrac{25 \times 4 \times \left(\frac{22}{7}\right)^2 \times 500}{9.81 \times 4}$

$\Rightarrow \quad K > 1.26 \times 10^4$ N/m

SOFT SPRINGS REDUCE VIBRATION TRANSMITTED BUT INCREASE DISPLACEMENTS.

c) SINCE GRAVITATIONAL WEIGHT IS ACTING EVEN WHEN THE WEIGHT IS BEING LOWERED, NO ADDITIONAL SHIFT IN EQUILIBRIUM TAKES PLACE WHEN THE MASS IS STOPPED.

$t = 0$ WHEN MASS IS STOPPED

$x = 0$ ABOUT THE EQUILIBRIUM CONFIGURATION AT $t = 0$

INITIAL CONDITIONS: $x(0) = 0$

$\dot{x}(0) = V$ = INITIAL VELOCITY

—— THE END ——