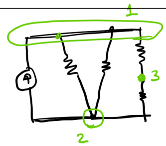
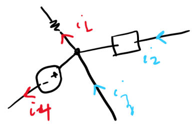
- Node Method
- superposition
- Thevenin

- Node:

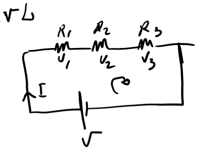


KCL



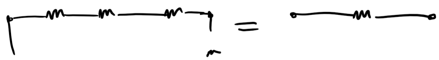
$i_1+i_4=i_2+i_3$

KVL



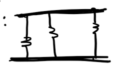
$-V+V_1+V_2+V_3=0$

Series:



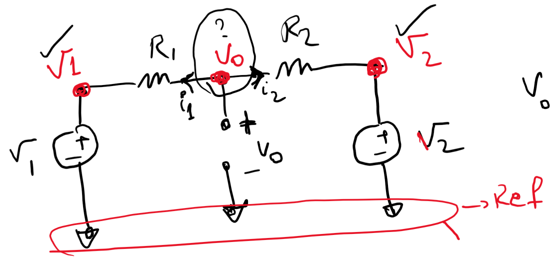
$R_{eq} = \sum_{k=1}^{m} R_k$

Parallel:



$\frac{1}{R_{eq}} = \sum_{k=1}^{m} \frac{1}{R_k} \longrightarrow R_{eq} = \frac{R_1R_2}{R_1+R_2}$

<Node Method>



$V_0 = ?$

- Identify all the nodes.
- Pick one of the nodes as the reference node.
- Assign a voltage to each node except the reference node.
- Write the KCL equation for all nodes except the reference node.

$$V_0: \underbrace{\frac{V_0-V_1}{R_1}+\frac{V_0-V_2}{R_2}}_{\text{outgoing}} = \underbrace{0}_{\text{incoming}}$$

NOTES:

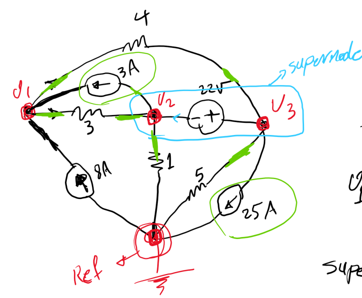
- Each node's voltage is measured w.r.t the reference node.
- Usually the node with the highest number of branches is picked as the reference node.
- If voltage sources are present in the circuit, and if they are not connected to the reference node (ground) on neither side, consider the voltage source and the two nodes around it as a supernode and write KCL equation for the whole supernode.
- As a convention, when writing KCL for a node, we assume all the currents are outgoing except the ones determined by current sources.

$$\xrightarrow{\times R_1R_2} R_2(V_0-V_1)+R_1(V_0-V_2)=0$$

$$(R_2+R_1)V_0 - R_2V_1 - R_1V_2 = 0$$

$$V_0 = \frac{R_1V_2 + R_2V_1}{R_1+R_2}$$

$$\boxed{= \frac{R_1}{R_1+R_2}V_2 + \frac{R_2}{R_1+R_2}V_1}$$



$$\boxed{v_3 - v_2 = 22} \quad ①$$

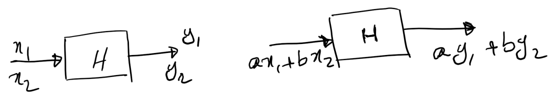
$v_1, v_2, v_3 = ?$

$$v_1 : ② \quad \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{3} = 3 + 8$$

$$\text{supernode: } ③ \quad \frac{v_2 - v_1}{3} + \frac{v_3 - v_1}{4} + \frac{v_3 - 0}{5} + \frac{v_2 - 0}{1} + 25 + 3 = 0$$

<superposition>

- Linear system:



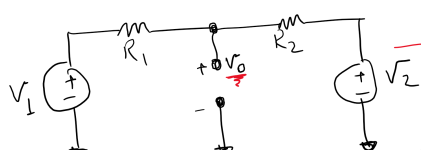
$\underline{1}\checkmark \quad \underline{2} \quad \underline{3} \quad \underline{4} \to y_1$

$\underline{1} \quad \underline{2}\checkmark \quad \underline{3} \quad \underline{4} \to y_2$

$\underline{1} \quad \underline{2} \quad \underline{3}\checkmark \quad \underline{4} \to y_3$

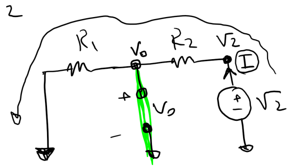
$\underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4}\checkmark \to y_4$

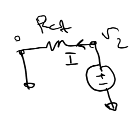
$$y = y_1 + y_2 + y_3 + y_4$$



$\to V_1 = 0 \quad V_2 \checkmark$

- Turn off voltage source: short circuit
- Turn off a current source: open circuit





$$V_2 = (R_1 + R_2) I$$

$$\boxed{I = \frac{V_2}{R_1 + R_2}}$$

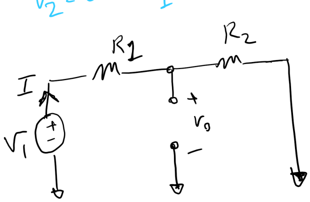
ohm's Law R_2: $v_2 - v_0 = R_2 I$

$$v_0 = v_2 - R_2 I$$

$$v_0 = v_2 - R_2 \left(\frac{v_2}{R_1 + R_2} \right)$$

$$v_{01} = \frac{R_1}{R_1 + R_2} v_2$$

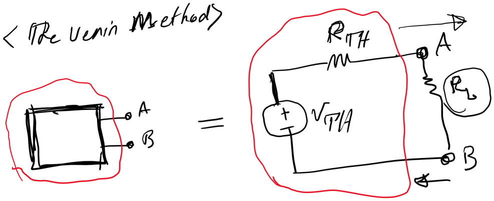
$V_2 = 0 \quad V_1 \checkmark$



$$v_{02} = \frac{R_2}{R_1 + R_2} v_1$$

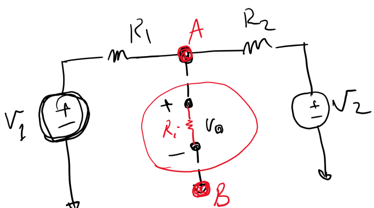
$$\text{Superposition: } v_0 = v_{01} + v_{02} = \frac{R_1}{R_1 + R_2} v_2 + \frac{R_2}{R_1 + R_2} v_1$$

< Thevenin Method >



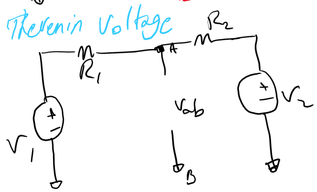
- Thevenin Voltage: The voltage between A and B, when no load is connected to A and B (V_{OC}) (open circuit voltage)

- Thevenin Resistor: The equivalent resistor between A and B when all sources are turned off (killed).



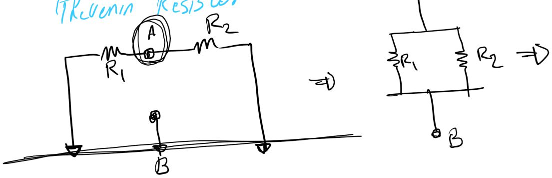
$\underline{v_o = ?}$

Thevenin Voltage



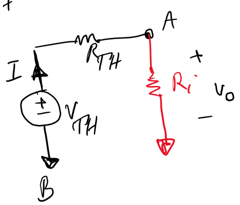
$$\boxed{V_{ab} = V_{OC} = V_{TH} = \frac{R_1}{R_1+R_2} v_2 + \frac{R_2}{R_1+R_2} v_1}$$

Thevenin Resistor



$$\boxed{R_{TH} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}}$$

Equivalent circuit



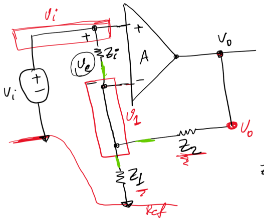
voltage divider

$$\boxed{V_0 = \frac{R_i}{R_i + R_{TH}} V_{TH}}$$

$$I = \frac{V_{TH}}{R_i + R_{TH}}$$

$$V_0 = R_i \times I = \frac{R_i}{R_i + R_{TH}} V_{TH}$$

∘ Non-inverting Op-Amp with Z_i



v_i, v_o

$v_1: \frac{v_1 - 0}{Z_1} + \frac{v_1 - v_o}{Z_2} + \frac{v_1 - v_i}{Z_i} = 0$

$v_o = A v_e = A(v_i - v_1)$

$\xrightarrow{Z_1 Z_2 Z_i}$ $Z_1 Z_2 (v_1 - v_i) + Z_i Z_2 v_1 + Z_i Z_1 (v_1 - v_o) = 0$

$(Z_1 Z_2 + Z_i Z_2 + Z_i Z_1) v_1 - Z_1 Z_2 v_i - Z_i Z_1 v_o = 0$

$$v_1 = \underbrace{\frac{Z_1 Z_2}{Z_1 Z_2 + Z_i Z_2 + Z_i Z_1}}_{a} v_i + \underbrace{\frac{Z_i Z_1}{Z_1 Z_2 + Z_i Z_2 + Z_i Z_1}}_{b} v_o$$

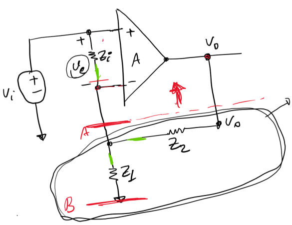
$v_1 = a v_i + b v_o$

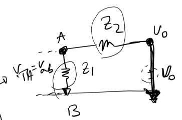
$v_e = v_i - v_1 = v_i - (a v_i + b v_o)$

$v_e = (1-a) v_i - b v_o$

$v_o = A v_e = A(1-a) v_i - Ab v_o$

$$\Rightarrow \boxed{\frac{v_o}{v_i} = \frac{A(1-a)}{1+Ab}}$$





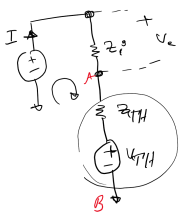
voltage divider

$$\boxed{V_{TH} = \frac{Z_1}{Z_1 + Z_2} V_o}$$

$$\boxed{R_{TH} = Z_1 \| Z_2}$$

KVL:

$-v_i + Z_i I + Z_{TH} I + V_{TH} = 0$



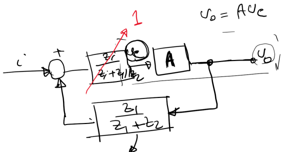
$I = \frac{v_i - V_{TH}}{Z_i + Z_{TH}}$

$v_e = Z_i I = \frac{Z_i}{Z_i + Z_{TH}} (v_i - V_{TH})$

$$\boxed{v_e = \frac{Z_i}{Z_i + Z_1 \| Z_2} \left(v_i - \frac{Z_1}{Z_1 + Z_2} v_o \right)} \checkmark$$

$v_o = A v_e$

$v_o = A v_e$



$Z_i \gg Z_1 \| Z_2$

$\frac{Z_i}{Z_i + Z_1 \| Z_2} \to 1$

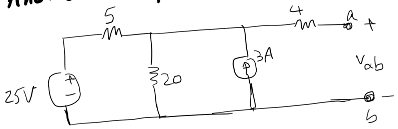
$f = \frac{Z_1}{Z_1 + Z_2}$

$A \to \infty, \; Z_i \gg Z_1 \| Z_2$

$G = \frac{1}{f} = \frac{Z_1 + Z_2}{Z_1}$

closed-loop Gain

Another example for Thevenin method (Not solved in class)

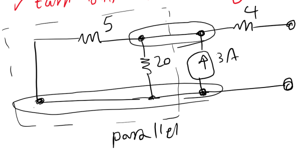


< Thevenin Voltage >

$V_{oc} = V_{ab} = ?$

- Let's use superposition for finding V_{ab}

✓ turn off the voltage source and find V_{ab}



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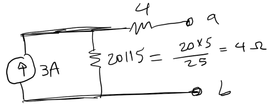

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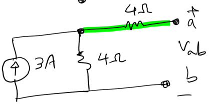
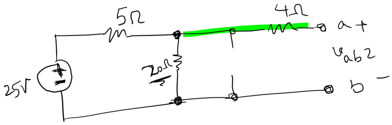


$V_{ab1} = 3 \times 4 = 12\,V$

✓ turn off the current source

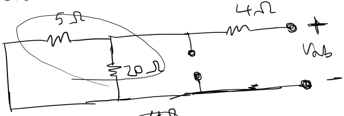


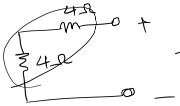
voltage divider → $V_{ab2} = \frac{20}{20+5} \times 25 = \frac{20}{25} \times 25 = 20\,V$

⇒ Superposition: $V_{ab} = V_{ab1} + V_{ab2} = 20\,V + 12\,V = 32\,V$

⇒ $V_{oc} = V_{TH} = 32\,V$

< Thevenin Resistor >





⇒ $R_{TH} = 4 + 4 = 8\,\Omega$

Ans.