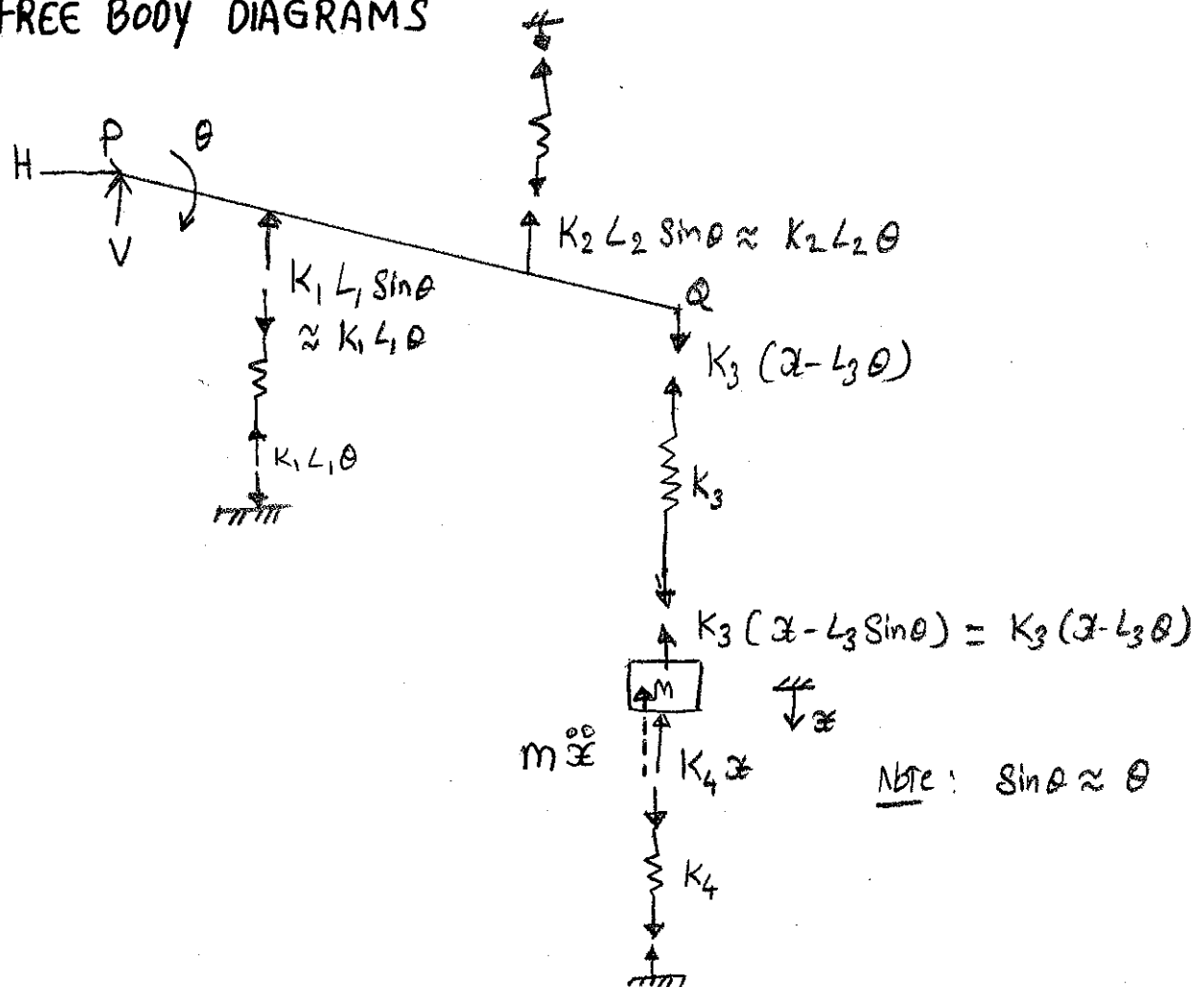


MIDTERM EXAMINATION I : SOLUTIONS

FREE BODY DIAGRAMS



NOTE: (1) BAR 'PQ' HAS NO MASS

(2) GRAVITATIONAL WEIGHT 'mg' AND STATIC FORCES IN SPRINGS CANCEL AT STATIC EQUILIBRIUM, AND HENCE ARE NOT SHOWN.

(3) NO NEED TO SHOW ' $m\ddot{x}$ ' IN THE FBD OF MASS IF WE USE NEWTON'S SECOND LAW.

(4) LINEARISATION (SMALL θ) $\sin \theta \approx \theta$

USE D'ALEMBERT IN FBD OF 'PQ'

$$\sum_{\text{tvc}} M_P = 0 \Rightarrow -(K_1 L_1 \theta) L_1 - (K_2 L_2 \theta) L_2 + K_3 (x - L_3 \theta) L_3 = 0$$

$$\Rightarrow \theta = \left(\frac{K_3 L_3}{K_1 L_1^2 + K_2 L_2^2 + K_3 L_3^2} \right) x \quad \text{--- ①}$$

APPLY D'ALEMBERT'S PRINCIPLE TO FBD OF MASS

(2)

$$\downarrow \sum_{+ve} F_x = 0 \Rightarrow -m\ddot{x} - k_3(x - L_3\theta) - k_4x = 0$$

$$\Rightarrow m\ddot{x} + (k_3 + k_4)x - k_3L_3\theta = 0 \quad \text{--- (2)}$$

USING (1) IN THE ABOVE

$$\Rightarrow m\ddot{x} + (k_3 + k_4)x - k_3L_3 \left(\frac{k_3L_3}{k_1L_1^2 + k_2L_2^2 + k_3L_3^2} \right) x = 0 \quad \text{--- (2)}$$

IF WE WISH TO USE NEWTON'S SECOND LAW

IN FBD OF PQ : $\uparrow \sum_{+ve} M_O = J_O \ddot{\theta}$ BUT $J_O = 0$ \because BAR HAS NO MASS

$$= 0$$

$$\Rightarrow \theta = \left(\frac{k_3L_3}{k_1L_1^2 + k_2L_2^2 + k_3L_3^2} \right) x \quad \text{AS ALREADY SEEN IN (1)}$$

APPLY NEWTON'S 2nd LAW TO FBD OF 'm' : $\downarrow \sum_{+ve} F_x = m\ddot{x}$

$$\Rightarrow -k_3(x - L_3\theta) - k_4x = m\ddot{x}$$

$$\Rightarrow m\ddot{x} + (k_3 + k_4)x - k_3L_3\theta = 0$$

USING $\theta = \left(\frac{k_3L_3}{k_1L_1^2 + k_2L_2^2 + k_3L_3^2} \right) x$ IN THE ABOVE

WE OBTAIN THE SAME SODE EQUATION OF MOTION AS GIVEN IN (2)

———— THE END ————