University of British Columbia
Department of Mechanical Engineering

MECH468 Modern Control Engineering
MECH522 Foundations in Control Engineering
Final exam

Examiner: Dr. Ryozo Nagamune

December 11 (Friday), 2015, 8:30-11am

Last name, First name

Name: Student #:

Signature:

Exam policies

- Allowed: Lecture note print-outs, hand-written notes, homework, books.
- Not-allowed: PC, calculators, mobile phones.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 100 points in total. Mark will be scaled later.

Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

If you finish early ...

- If you would like to leave the room **before 10:50am**, **raise your hand with this booklet**, and **wait at your seat** until the invigilator comes to you and collects your exam booklet.

To be filled in by the instructor/marker

Problem #	Mark	Full mark
1		20
2		30
3		25
4		25
Total		100

Extra page. Write the problem number before writing your answer.

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1. Answer the following true-or-false questions. Write (T) (meaning *true*) or (F) (meaning *false*). **No need to motivate your answers.** (2pt each)

   Below, consider the continuous-time linear time-invariant system

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{cases} \tag{1}$$

   where x, u and y denote respectively state, input and output vectors. By applying the state coordinate transformation $z = Tx$ with a nonsingular matrix T, we can obtain a system:

$$\begin{cases} \dot{z}(t) &= TAT^{-1}z(t) + TBu(t), \\ y(t) &= CT^{-1}z(t) + Du(t). \end{cases} \tag{2}$$

   (a) If the system (1) is asymptotically stable, then it is always observable.

   (b) If the system(1) is observable, then it is always asymptotically stable.

   (c) If the system (1) is observable, then it is always detectable.

   (d) If the system (1) is detectable, then it is always observable.

   (e) If the system (1) is detectable, then it is always asymptotically stable.

   (f) If the system (1) is asymptotically stable, then it is always detectable.

   (g) If the system (1) is stabilizable, then it is always detectable.

   (h) If the system (1) is detectable, then it is always stabilizable.

   (i) If the system (1) is observable, then the system (2) is always observable.

   (j) If the system (1) is detectable, then the system (2) is always detectable.

Question	**Write your answer here**
(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	
(i)	
(j)	

2. Select **only one** correct statement, **by circling one of the numbers i, ii, iii or iv**, for the following sentences. **No need to motivate your answers.** (3pt each)

   (a) If we linearize the state equation $\dot{x}(t) = -\sin x(t) + \cos u(t)$ around an input $u_0 = \frac{\pi}{2}$, then the corresponding equilibrium input x_0 and the linearized state equation will be ($\delta x(t) := x(t) - x_0$, $\delta u(t) := u(t) - u_0$):

      i. $x_0 = \frac{\pi}{2}$ and $\delta\dot{x}(t) = -\delta x(t) + \delta u(t)$.
      ii. $x_0 = -\frac{\pi}{2}$ and $\delta\dot{x}(t) = -\delta x(t) + \delta u(t)$.
      iii. $x_0 = \frac{\pi}{2}$ and $\delta\dot{x}(t) = \delta x(t) - \delta u(t)$.
      iv. None of i, ii, iii is correct.

   (b) If we discretize (with the zero-order-hold method) a continuous-time linear time-invariant system which is controllable, observable and asymptotically stable, then the discretized system for any sampling time is:

      i. always controllable, observable and asymptotically stable.
      ii. always controllable and observable, but not necessarily asymptotically stable.
      iii. always observable and asymptotically stable, but not necessarily controllable.
      iv. None of i, ii, iii is correct.

   (c) For a state equation $x[k+1] = -x[k] + 2w[k]$ where the expected value and variance of w and given by $E\{w\} = 1$ and $R_w = 1$, respectively, the prediction step of the Kalman filter will be:

      i. $\hat{x}[k+1|k] = -\hat{x}[k|k] + 2$ and $P[k+1|k] = P[k|k] + 2$.
      ii. $\hat{x}[k+1|k] = -\hat{x}[k|k] + 2$ and $P[k+1|k] = P[k|k] + 4$.
      iii. $\hat{x}[k+1|k] = -\hat{x}[k|k]$ and $P[k+1|k] = P[k|k] + 2$.
      iv. $\hat{x}[k+1|k] = -\hat{x}[k|k]$ and $P[k+1|k] = P[k|k] + 4$.

   (d) For an output equation $y[k] = x[k] + v[k]$ where the expected value and variance of v and given by $E\{v\} = 1$ and $R_v = 1$, respectively, the correction step of the Kalman filter will be:

      i. $$\begin{cases} \hat{x}[k|k] = \hat{x}[k|k-1] + P[k|k](y[k] - \hat{x}[k|k-1] - 1), \\ P[k|k] = \frac{P[k|k-1]+1}{P[k|k-1]}. \end{cases}$$

      ii. $$\begin{cases} \hat{x}[k|k] = \hat{x}[k|k-1] + P[k|k](y[k] - \hat{x}[k|k-1] - 1), \\ P[k|k] = \frac{P[k|k-1]}{P[k|k-1]+1}. \end{cases}$$

      iii. $$\begin{cases} \hat{x}[k|k] = \hat{x}[k|k-1] + P[k|k](y[k] - \hat{x}[k|k-1]), \\ P[k|k] = \frac{P[k|k-1]+1}{P[k|k-1]}. \end{cases}$$

      iv. $$\begin{cases} \hat{x}[k|k] = \hat{x}[k|k-1] + P[k|k](y[k] - \hat{x}[k|k-1]), \\ P[k|k] = \frac{P[k|k-1]}{P[k|k-1]+1}. \end{cases}$$

(e) By infinite-horizon LQR optimal control with weighting matrices $Q \geq 0$ and $R > 0$ and controllable (A, B) and observable (A, Q), the closed-loop system becomes:

i. always asymptotically stable.
ii. always marginally stable.
iii. always unstable.
iv. None of i, ii, iii is correct.

(f) For a system $\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$, by selecting an appropriate control input $u(t)$, it is possible to transfer state:

i. from $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T$ to $x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$.
ii. from $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T$ to $x(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T$.
iii. from $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T$ to $x(1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T$.
iv. All of i, ii, iii are correct.

(g) In the infinite-horizon LQR problem with a cost function

$$\min_{u(\cdot)} \int_0^\infty \left(Qx(t)^2 + Ru(t)^2\right) dt, \quad Q > 0,\ R > 0,$$

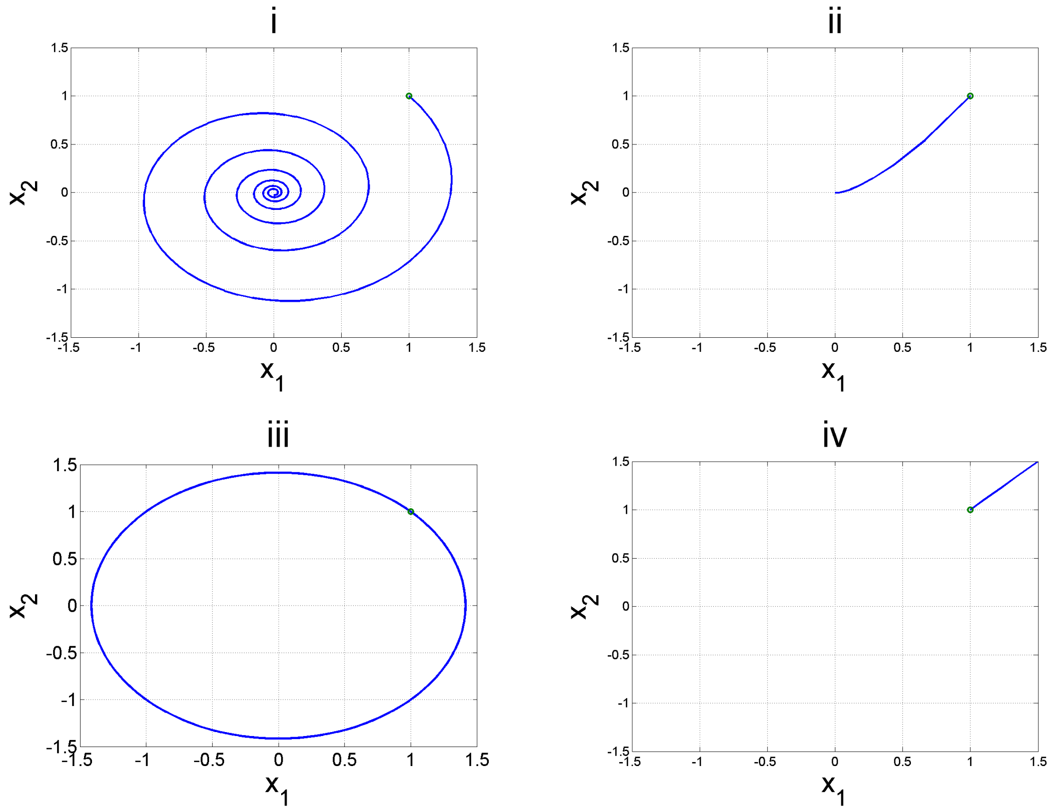
with a state equation (for example, $\dot{x}(t) = -x(t) + u(t)$), during the design iteration while searching for appropriate Q and R, if we would like to reduce the input amplitude, then we should:

i. Increase Q without changing R.
ii. Increase R without changing Q.
iii. Increase Q and R by the same multiple (for example, $2Q$ and $2R$).
iv. None of i, ii, iii is correct.

(h) In the observer-based state-feedback controller design using pole-placement technique, there are two types of poles, that is, the eigenvalues of $A - BK$, denoted by $\sigma(A - BK)$ and the eigenvalues of $A - LC$, denoted by $\sigma(A - LC)$. As a rule of thumb, we should place the poles so that:

i. $\sigma(A - BK)$ and $\sigma(A - LC)$ are located in similar distances from the origin.
ii. $\sigma(A - BK)$ are located far left, compared to $\sigma(A - LC)$.
iii. $\sigma(A - LC)$ are located far left, compared to $\sigma(A - BK)$.
iv. None of i, ii, iii is correct.

(i) A state equation $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ with an initial condition $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ has the following phase plot (small 'o'-mark at $(x_1, x_2) = (1, 1)$ indicates the initial condition):



(j) A continuous-time linear state-space model

$$\begin{cases} \dot{x}(t) &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t), \end{cases}$$

is:

i. stabilizable and detectable.
ii. stabilizable but not detectable.
iii. detectable but not stabilizable.
iv. neither stabilizable nor detectable.

3. Consider a system expressed by a transfer function:

$$G(s) = \frac{1}{0.5s + \alpha}.$$

(a) Obtain one minimal realization of $G(s)$. (5pt)

(b) For the obtained realization of $G(s)$, with $\alpha = 1$, design a state feedback controller with an integrator (i.e., state feedback gain K and integrator gain K_a) such that the closed-loop poles are $s = -1, -2$. (10pt)

(c) Suppose that our modelling is inaccurate and that the actual α is not 1. For the designed controller in (b), what is the range of the parameter α that results in zero steady-state tracking error for any step reference input? (In this question, we are checking how robust the controller designed in (b) is against the uncertainty of α-value.) (10pt)

Write your answer here for Problem 3.

Write your answer here for Problem 3.

4. Consider the following continuous-time infinite-horizon optimal control problem:

$$\min_{u(\cdot)} \int_0^\infty \left\{2y(t)^2 + u(t)^2\right\} dt$$

subject to

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t). \end{cases}$$

(a) Is the system above controllable? (3pt)

(b) Is the system above observable? (3pt)

(c) Obtain the Algebraic Riccati Equation (ARE) associated with the infinite-horizon optimal control problem above, by explicitly showing what the matrices A, B, Q and R are. (3pt)

(d) Find the positive definite solution to the ARE obtained in (c). (10pt)

(e) Determine the optimal control law $u(t)$. (3pt)

(f) Verify that the closed-loop system with the optimal control law obtained in (e) is asymptotically stable. (3pt)

<u>**Write your answer here for Problem 4.**</u>

Write your answer here for Problem 4.

Extra page. Write the problem number before writing your answer.