Lesson 7-4 – Geometric Series

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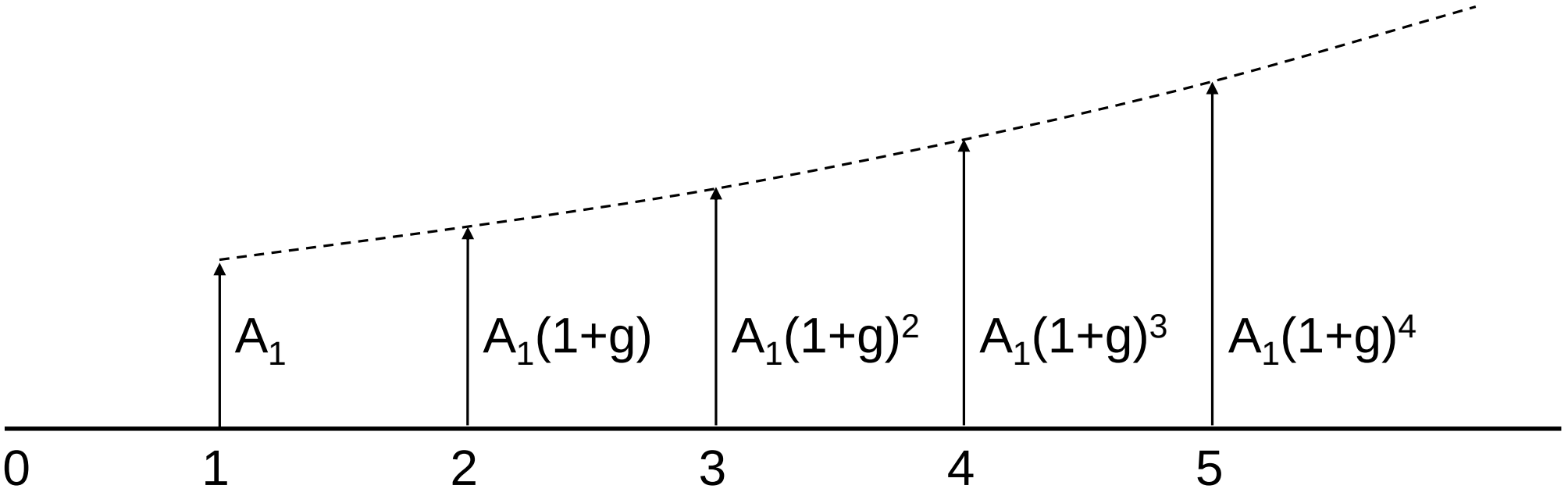
Geometric Gradient Series

- Period-by-period change is at a uniform rate; grows by a constant rate, g
- Expressed as a percentage



Geometric Gradient Series Continued

- Two possible cases:
  - Where: i ≠ g

    GEOMETRIC SERIES PRESENT WORTH FACTOR $$P = A_1\left[\frac{1-(1+g)^n(1+i)^{-n}}{i-g}\right]$$

    GEOMETRIC SERIES COMPOUND AMOUNT FACTOR $$F = A_1\left[\frac{(1+i)^n-(1+g)^n}{i-g}\right]$$

  - Where: i = g

$$P = A_1 n(1+i)^{-1}$$

Geometric Gradient Series: Example 1

- A company has maintenance costs that will be $1500 six months from today and will grow by 4% every six months after that. Find the value today of the maintenance costs over a ten-year period if the rate of interest is 11¼% compounded semi-annually. g = 0.04
  i = 0.1125/2 = 0.05625 n = 10(2) = 20

$$PV = \$1500\left[\frac{1-(1.04)^{20}(1.05625)^{-20}}{0.05625-0.04}\right]$$

$$= \$24{,}610.56$$

Geometric Gradient Series: Example 2

- You will save for a vacation by depositing $200 in one month then 3% less each month for two years. Determine the amount you will have saved after two years if the nominal interest rate is 4.5% compounded monthly. g = -0.03 i = 0.045/12 = 0.00375 n = 2(12) = 24

$$FV = \$200\left[\frac{(1.00375)^{24} - (1-0.03)^{24}}{0.00375-(-0.03)}\right]$$

$$= \$3630.06$$

Reality and the Assumed Uniformity of *A*, *G*, and *g*

- Although the future reality may not exactly follow the uniformity of our series and gradient equations, we may use these formulas because:
  1. It is easier to start with simple models
  2. Convenient for modelling the future with approximate constraints and bounds
  3. Not enough is known about the future and so it is
  4. approximated through uniform series and gradients

Reality and the Assumed Uniformity of *A*, *G*, and *g*

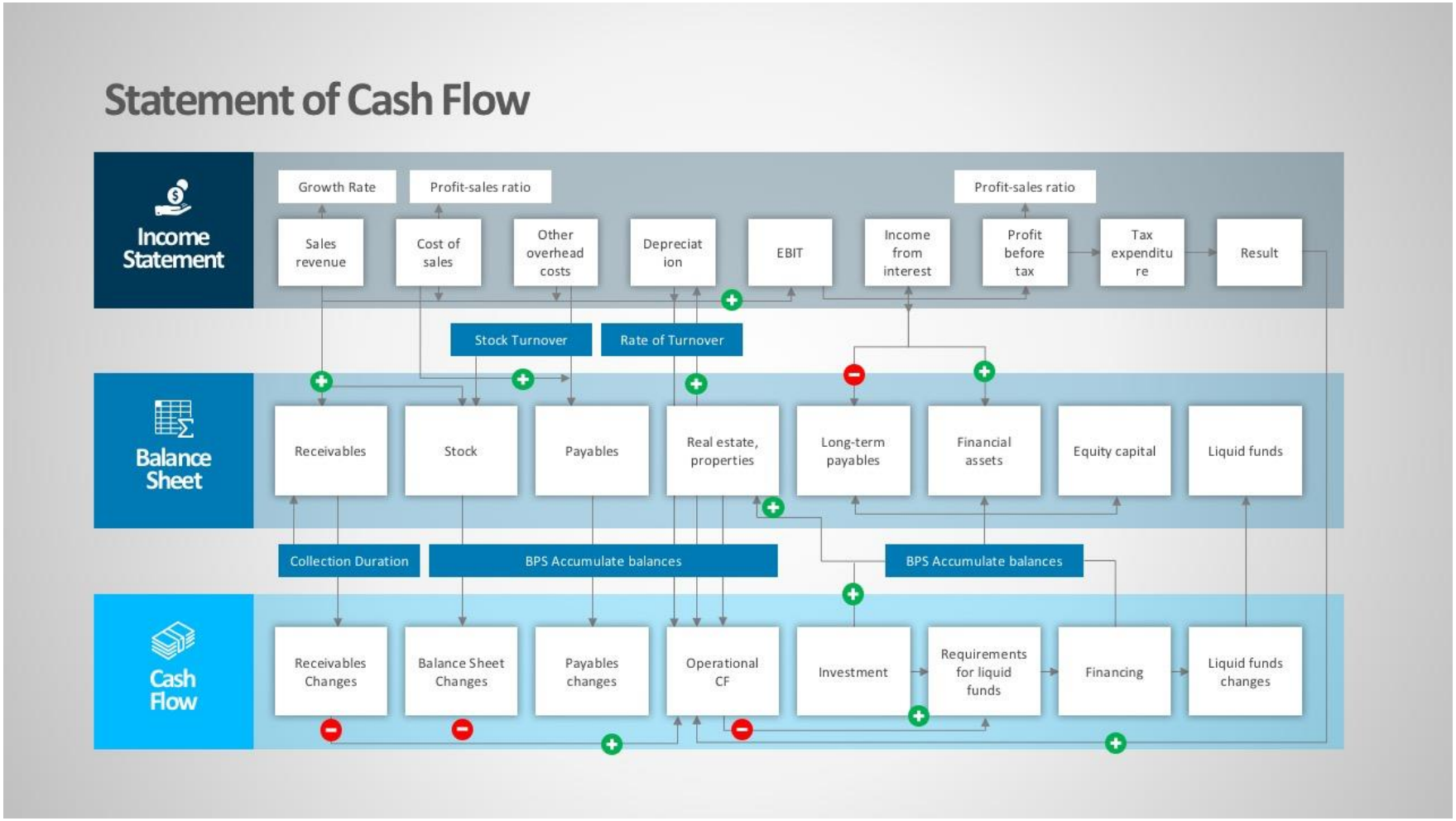


Image source: https://slidemodel.com/templates/statement-cashflows-powerpoint-diagrams/