

$$1.9a \quad G(s) = \frac{1}{s^2(s+1)} \begin{bmatrix} s & s+1 \end{bmatrix}$$

$$= \frac{1}{s^3+s^2} \begin{bmatrix} [1 \ 1]s & [0 \ 1] \end{bmatrix} \quad N \rightarrow 1 \times 2$$

$$\begin{array}{ll} d_1 = 1 & N_3 = (0 \ 1) \\ d_2 = 0 & N_2 = (1 \ 1) \\ d_3 = 0 & N_1 = (0 \ 0) \end{array}$$

$r=3$

$$\dot{x} = \left(\begin{array}{cc|cc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array} \right) x + \left(\begin{array}{c} 0 \ 0 \\ 0 \ 0 \\ \hline 0 \ 0 \\ 0 \ 0 \\ \hline 1 \ 0 \\ 0 \ 1 \end{array} \right) u$$

$$y = (0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0) x$$

$$1.16 \quad G(s) = \begin{pmatrix} \frac{1}{s^2+s} \\ \frac{1}{s^2} \end{pmatrix} = \frac{1}{s^3+s^2} \begin{bmatrix} (1 \ 1) s & (0 \ 1) \end{bmatrix} \quad N = 2 \times 1$$

$$\begin{array}{ll} d_1 = 1 & N_3 = (0 \ 1) \\ d_2 = 0 & N_2 = (1 \ 1) \\ d_3 = 0 & N_1 = (0 \ 0) \end{array}$$

$r=3$

$$\dot{x} = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & -1 \end{array} \right) x + \left(\begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right) u$$

$$y = (0 \ 1 \ 0) x$$

$$HC \quad G(s) = \frac{1}{s^2(s+1)} \begin{pmatrix} s^2 & s \\ s & s+1 \end{pmatrix} = \frac{1}{s^3+s^2} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} s^2 + \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} s + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right)$$

$$d_1 = 1 \quad N_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$d_2 = 0 \quad N_2 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$d_3 = 0 \quad N_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$N \rightarrow 2 \times 2 \quad r=3$$

$\begin{matrix} q & \\ p & \end{matrix}$

$$\dot{x} = \left[\begin{array}{cc|cc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array} \right] x + \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \right] u$$

$$y = \left[\begin{array}{cc|cc} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array} \right] x$$

1.2a $\dot{x} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}x + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}u$

$y = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}x$

1.2b $\dot{x} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}x + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}u$

$y = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}x$

1.2c $\dot{x} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}x + \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}u$

$y = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}x$

$$1.3a. \quad A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad C = (0 \quad 0 \quad 1)$$

$$C = (B \quad AB \quad A^2B) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{pmatrix}$$

$$\text{Im}(C) = \left\{ x_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + x_5 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \mid x_i \in \mathbb{R} \right\}$$

$$\Rightarrow \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$0 = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\ker(0) = \left\{ x \in \mathbb{R}^3 \mid \begin{array}{l} x_3 = 0 \\ x_2 - x_3 = 0 \\ x_1 - x_2 + x_3 = 0 \end{array} \right\} \sim \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$T_{C0} = \text{Im}(C) \cap \ker(0) = 0$$

$$T_{C0} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$T_{C0}^- = 0$$

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$$T^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$TAT^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \quad TB = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & 0 \end{pmatrix} \quad CT^4 = (0 \quad 0 \quad 1)$$

$$136 \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$C = (B \ AB \ A^2B) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\text{Im}(C) = \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$0 = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\ker(0) = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$T_{\bar{C}0} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$T_{\bar{C}0} - T_{\bar{C}0} = T_{\bar{C}0} = 0$$

$$T^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$TAT^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad TB = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad CT^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$13c \quad A^2 = \begin{pmatrix} 0 & 0 & 10 & 00 \\ 0 & 0 & 01 & 00 \\ 0 & 0 & 00 & 10 \\ 0 & 0 & 00 & 01 \\ 0 & 0 & 00 & -10 \\ 0 & 0 & 00 & 0-1 \end{pmatrix} \quad B = \begin{pmatrix} 00 \\ 00 \\ 00 \\ 01 \\ 10 \\ 00 \end{pmatrix} \quad C = \begin{pmatrix} 000110 \\ 011100 \end{pmatrix}$$

$$C = [B \ AB \ A^2B \ A^3B \ A^4B \ A^5B]$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\text{Im}(C) = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\} \quad (\text{6 unit vectors}) \quad \text{colspace}$$

$$0 = \begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 11 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ \vdots & \vdots & \vdots & \vdots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & -1 & 1 \\ \vdots & \vdots & \vdots & \vdots & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{pmatrix}$$

$$\ker(0) = \left\{ \begin{pmatrix} 0 \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\} \quad \text{null}$$

$$T_{\bar{C}0} = \text{im}(c) \cap \ker(M) = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$T_{C0} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \dots \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$T_{\bar{C}0} = 0$$

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$$T^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/\sqrt{2} \\ 1 & 0 & 0 & 0 & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{2} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$TAT^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$TB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$CT = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$