MECH 421

Mechatronics System Instrumentation

Lecture Notes

Jian Gao

2019

Preface

It has been a long term for me to teach MECH 421: mechatronics system instrumentation at UBC. When I was an undergraduate student about ten years ago, I had no chance to study instrumentations systematically, and almost everything I know about instrumentation is from unsuccessful experiments and infinite trials. I hope this course provides a more accessible and more systematic way to guide the students having fundamental knowledge and useful engineering skills as Dr. Xiaodong Lu did before.

I want to thank Prof. Yusuf Altintas's recommendation for me to teach this course and UBC MECH department offering this opportunity. Besides the recommendation, Prof. Altintas gave me endless supports both financially and mentally for my research project about mechatronics system design. I want to acknowledge Dr. Xiaodong Lu, who established this course at UBC. He brought me from China five years ago to let me realize the world-class research and gave me solid fundamental knowledge about instrumentations, controls, and machine design. I want to thank the experience at Shanghai Jiao Tong University, where I stayed more than six years for fundamental knowledge and basic engineering techniques related to mechanical engineering.

Finally, I would like to thank the students in my class, who are in the group of mechatronics option at UBC MECH generally with strong motivations to dig the unknown world.

MECH 421. Lecture Notes

Jian Gao

Lecture #1 : Introduction.

1. Course schedule & information.

① TA office hour & tutorial.

- There is no mandatory turial.
- Check with Scott with his UBC email address & Office hour time. Mon / Wed 2-3 pm

② Homework policies.

10 Homworks, Friday handin.

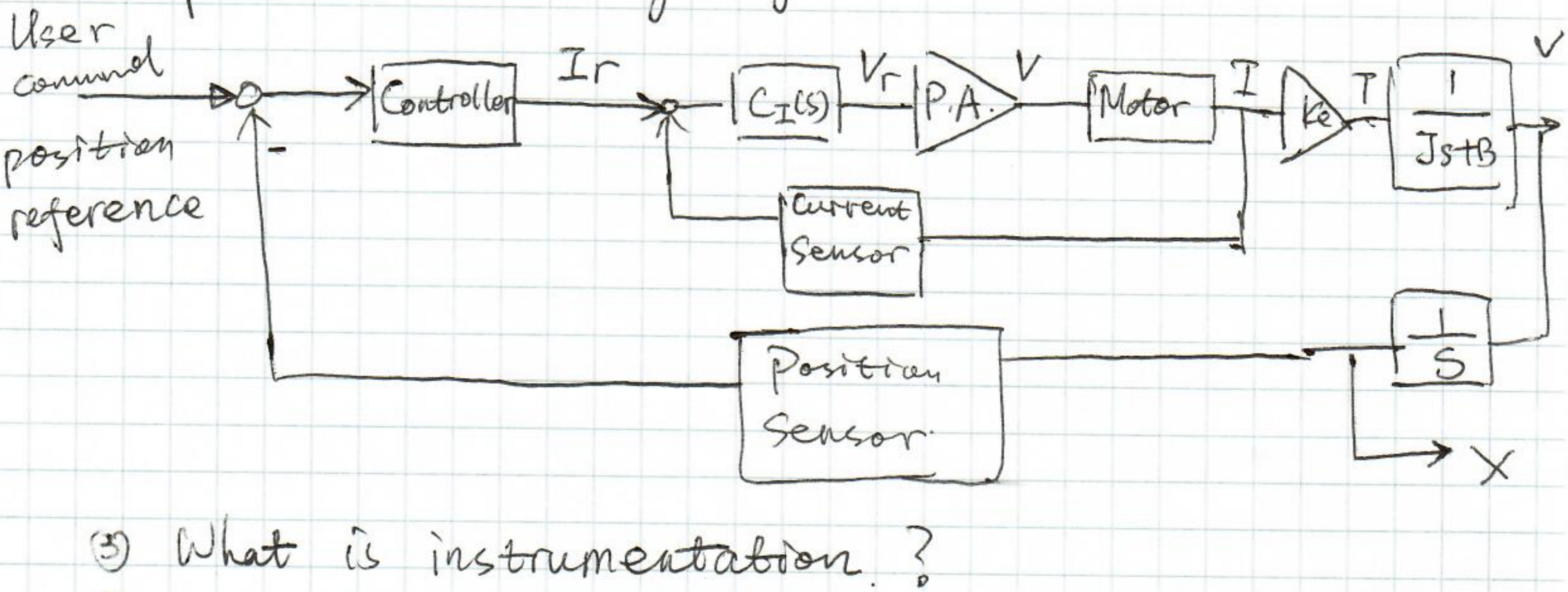
Late policy 10% off for each late day.

2. Mechatronics system introduction.

① Use slides to show the concepts.

② What is mechatronics system ?

Example: Positioning system.



③ What is instrumentation ?

Instrumentation means connection of mechatronics system.

④ How to do instrumentation?

- Select hardware.
- Design controller (softwoare)
- Design circuit.

system identify
Controller design
Performance validation.

3. Outline of the course:

①. Analog instrumentation:

- Design an electronic circuit to control current.

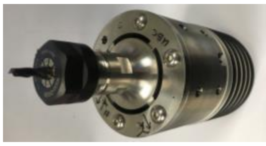
②. Digital instrumentation:

- Design digital control systems to control conveyor belt

③. Vibration of machanical systems.

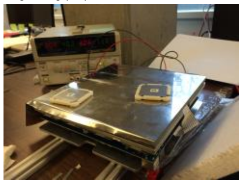
④. Power electronics.

Mechatronics System Instrumentation



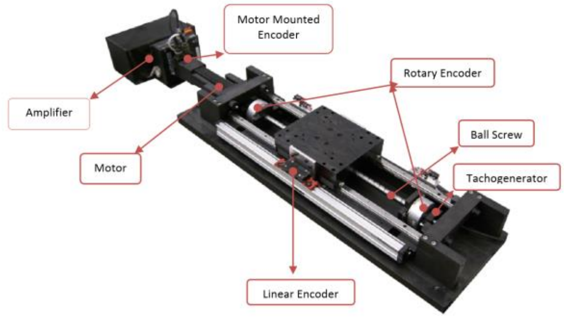
3DOF ultrasonic vibration tool holder

High throughput planar motor

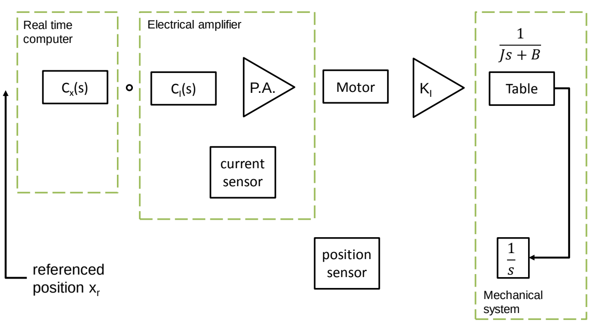


MECH 421 2019 Spring
Instructor: Jian Gao

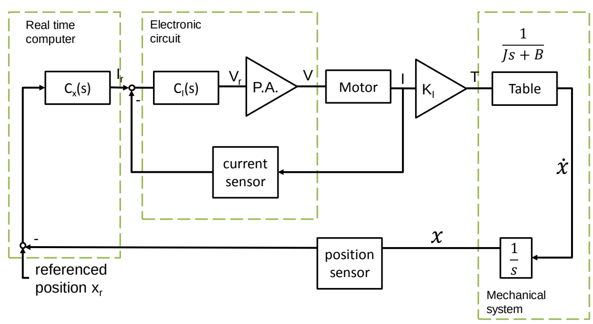
What is mechatronics system?



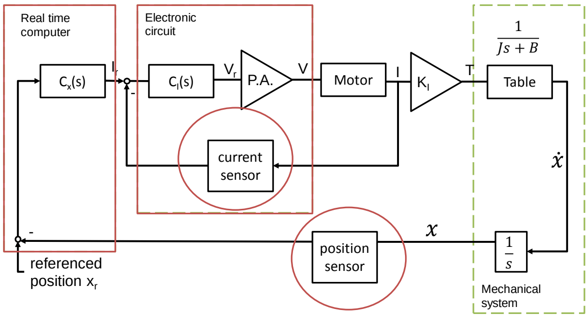
What is mechatronics system?



What is instrumentation?



How to do instrumentation?

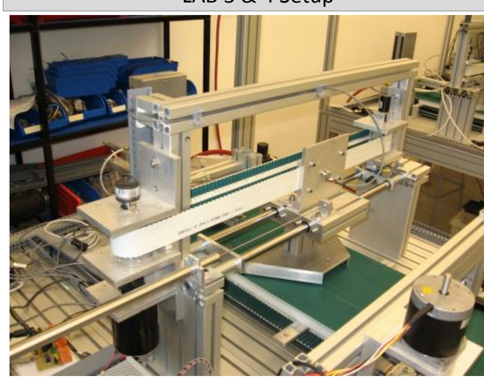


Outline

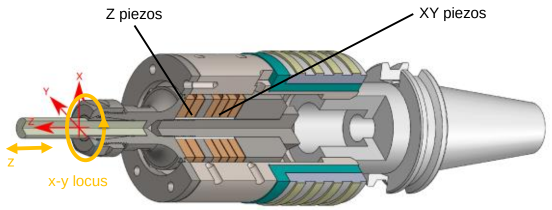
- Review
  - Electronic circuit
  - Frequency response
- Analog instrumentation
  - Design a circuit to control motor current (LAB 1 and LAB 2)
- Digital instrumentation
  - Design digital control systems for belt conveyor (LAB3 and LAB4)
- Vibration of mechanical systems
- Introduction to power electronics

LAB 1 & 2 Setup

LAB 3 & 4 Setup



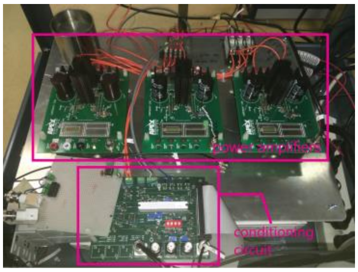
3DOF ultrasonic vibration tool holder



- Z vibration to help drilling
- X and Y vibrations to finish elliptical locus for milling

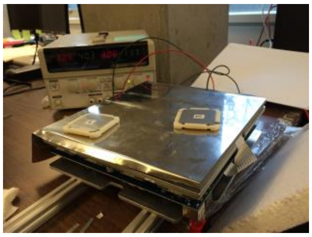
3DOF ultrasonic vibration tool holder

- Purposes of instrumentation
  - Driving piezo actuators
  - Sensorless track the resonance frequency



High throughput planar motor

- Multiple movers capability
- Electromagnetic motor design
  - PCB coil
  - Novel magnet array
- Position sensor design
  - Hall effect sensor array
- Customized multiple PWM power amplifier



Lecture #2 : Frequency response & Bode plots.
step response.

Jian Guo
2018.12

1. Laplace's transform basics.

$$\text{Time Domain} \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\Longleftrightarrow}} s \text{ domain}.$$

Fourier transform. $\searrow$ $\qquad$ $\nearrow\!\!\swarrow$ by letting $s = j\omega$

Frequency Domain

Mathematical calculation:

$$F(s) = \int_0^\infty f(t) e^{-st}\, dt$$

2. Transfer function:

A simple system: $\quad U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s).$

$$T.F. = \frac{Y(s)}{U(s)} = G(s). \qquad (\text{in } s\text{-domain}).$$

3. Frequency response

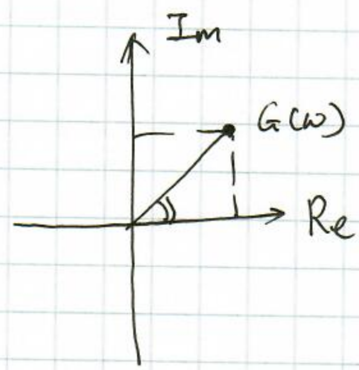
Use a special case as the input: $\quad u(t) = U\cos(\omega t)$

let $s = j\omega$ $\qquad \hookrightarrow y(t) = Y \cdot \cos(\omega t + \phi)$

$$G(j\omega) = |G(j\omega)| \measuredangle G(j\omega).$$

$$\uparrow$$

complex number $\quad = Re[G(j\omega)] + j\, Im[G(j\omega)].$



4. Bode plots.

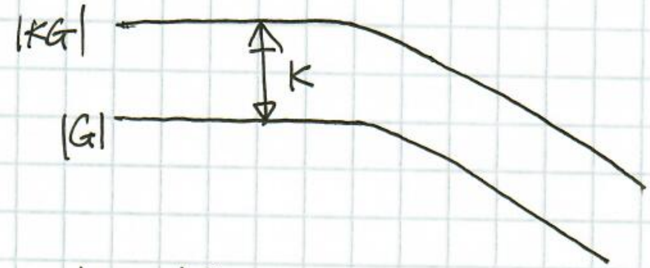
①. Plot G as a function of ω, then we get the frequency response.

$$\begin{cases} \text{Magnitude} : \log|G| \text{ v.s. } \underline{\underline{\log \omega}} \\ \text{Phase} : \angle G \text{ v.s. } \underline{\underline{\log \omega}} \end{cases}$$

Horizontal Axis in log scale. So no real $f=0Hz$ on bodes.

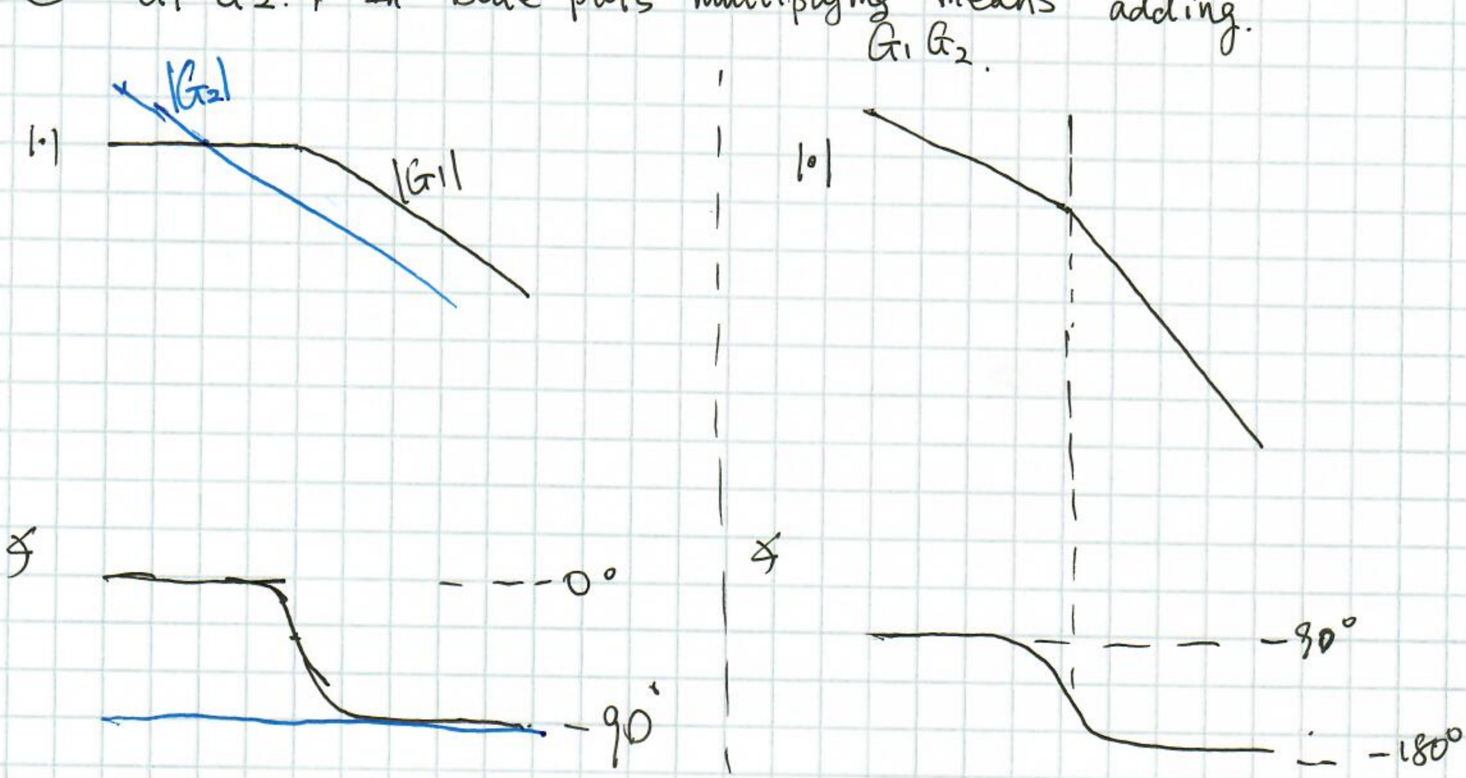
②. Bode plots properties.

(2.1) $K \cdot G$

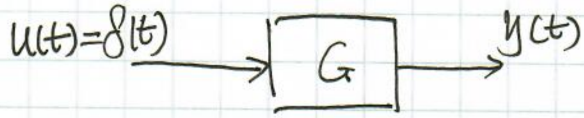


G is shifted up by K times.

(2.2) $G_1 \cdot G_2$. In Bode plots multiplying means adding.



5. Impulse response



Impulse function:

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & t \neq 0. \end{cases}$$

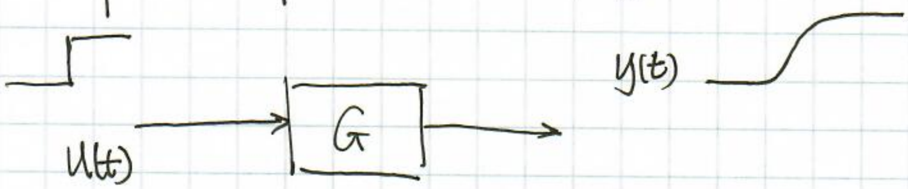
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

In s-domain, $U(s) = 1$

$$Y(s) = G(s) \cdot U(s) = G(s).$$

This is also used for FRF identification.

6. Step response: (Turning on).



$$u(t) = \begin{cases} U, & t>0 \\ 0, & t \leq 0. \end{cases}$$

In s-domain:

$$U(s) = \frac{1}{s}, \quad Y(s) = G(s)\,U(s) = \frac{G(s)}{s}.$$

7. Short summary.

Use different inputs to check system behaviours.

Time domain ~~r~~ response
- Impulse response
- Step response
- Ramp response.

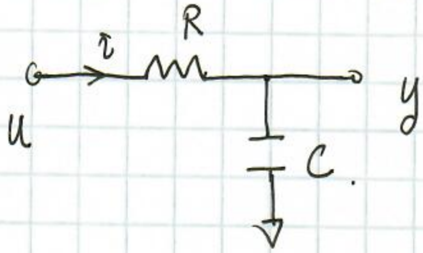
In time domain plots. More about long-term behavior

Frequency response — Bode plots — More about dynamics of the system.

Lecture #3: First order systems.

1. R-C circuit (Example).

① System:



u: input voltage

y: output voltage.

ODE: $i = C\dfrac{dy}{dt} = \dfrac{u-y}{R} \Rightarrow u = y + RC\dfrac{dy}{dt}$

Laplace transform:

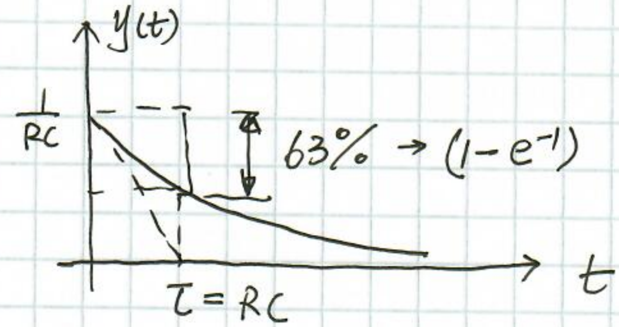
$$U = Y + RCs\,Y$$

$$\Rightarrow \frac{Y}{U} = \frac{1}{RCs+1}.$$

Block diagram: $U \rightarrow \boxed{\dfrac{1}{RCs+1}} \rightarrow Y$

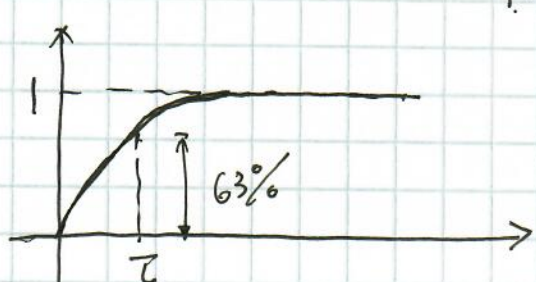
② Impulse response: $u = \delta(t)$ $\quad U(s) = 1.$

$$y(t) = \frac{1}{RC}e^{-\frac{t}{RC}} \qquad Y(s) = \frac{1}{RCs+1}$$



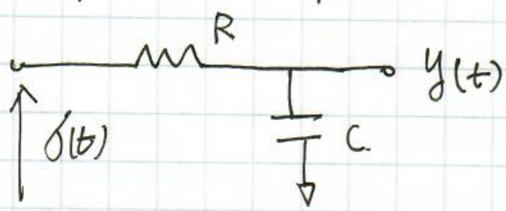
(Time constant).

③ Step response: $u(t) = \begin{cases} 1, & t>0 \\ 0, & t\le 0 \end{cases}$, $\quad y(t) = 1 - e^{-\frac{t}{RC}}$.



2. Intuition of RC circuit

① Impulse response.



From $t=0^-$ to $t=0^+$

$$i(t) = \frac{\delta(t)}{R}$$

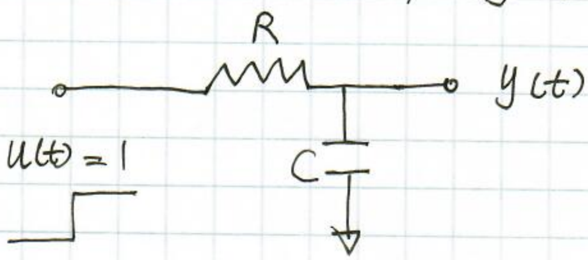
$$q(0^+) = \int_{0^-}^{0^+} i(t)\,dt = \int_{0^-}^{0^+} \frac{\delta(t)}{R}\,dt$$

$$= \frac{1}{R}, \quad \left(\text{Note } \int_{0^-}^{0^+} \delta(t)dt = 1\right)$$

$$y(0^+) = \frac{q(0^+)}{C} = \frac{1}{RC}$$

② Step response :

If $t \ll \tau = RC$, $y(t) \approx 0$



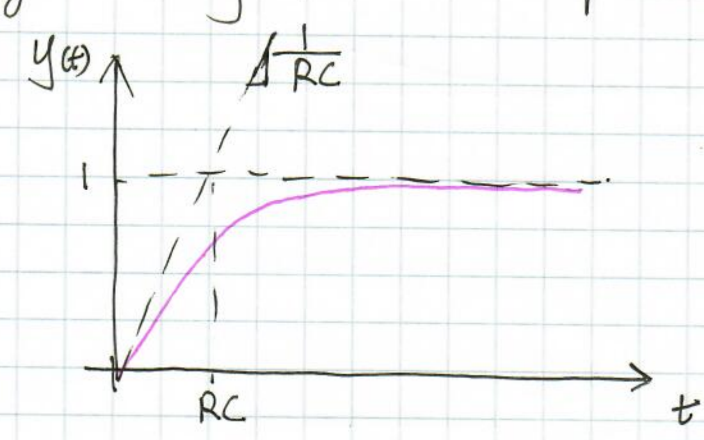
$$i(t) \approx \frac{1 - y(t)}{R} = \frac{1}{R}$$

$$q(t) = \int_0^t i(t)\,dt = \frac{t}{R}$$

$$y(t) = \frac{q(t)}{C} = \frac{t}{RC}$$

If $t \gg RC \Rightarrow i(t) = 0$, $V(t) = 1$.

At beginning Resistor is dominant in the circuit, during steady state Capacitor is dominant.

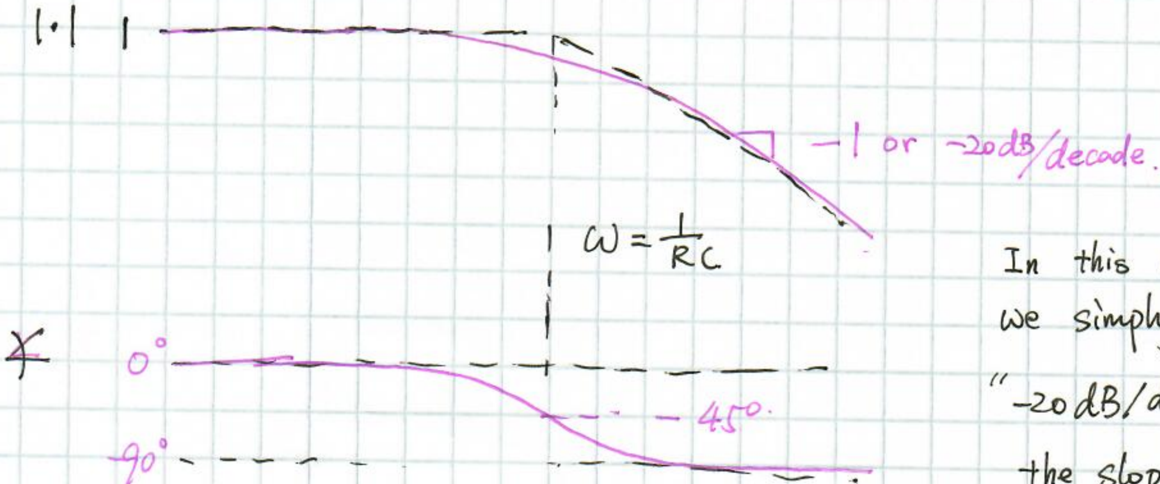


3. Frequency Response.

$$G = \frac{Y}{U} = \frac{1}{RCs+1}, \quad G(j\omega) = \frac{1}{j\omega RC+1}.$$

$$|G(j\omega)| = \begin{cases} 1, & \omega \ll \frac{1}{RC}, \text{ at very low frequency} \\ \frac{1}{RC\omega}, & \omega \gg \frac{1}{RC}, \text{ high frequency.} \\ \frac{1}{\sqrt{2}}. & \omega = \frac{1}{RC}, \text{ corner frequency} \end{cases}$$

$$\measuredangle G(j\omega) = \begin{cases} 0^\circ, & \omega \ll \frac{1}{RC}. \\ -90^\circ, & \omega \gg \frac{1}{RC}. \\ -45^\circ, & \omega = \frac{1}{RC} \end{cases}$$

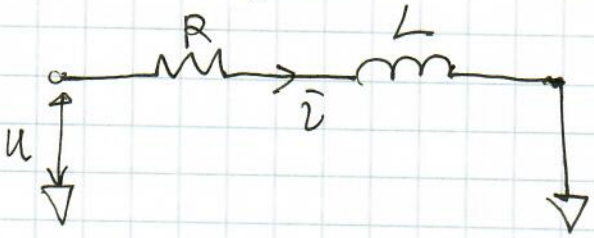


In this course, we simply use "−1" as "−20dB/decade" for the slope of Bode

The standard format of first order system.

$$G = \frac{1}{\tau s+1}, \quad \tau \text{ is the time constant.}$$

4. Similarly, R-L circuit.



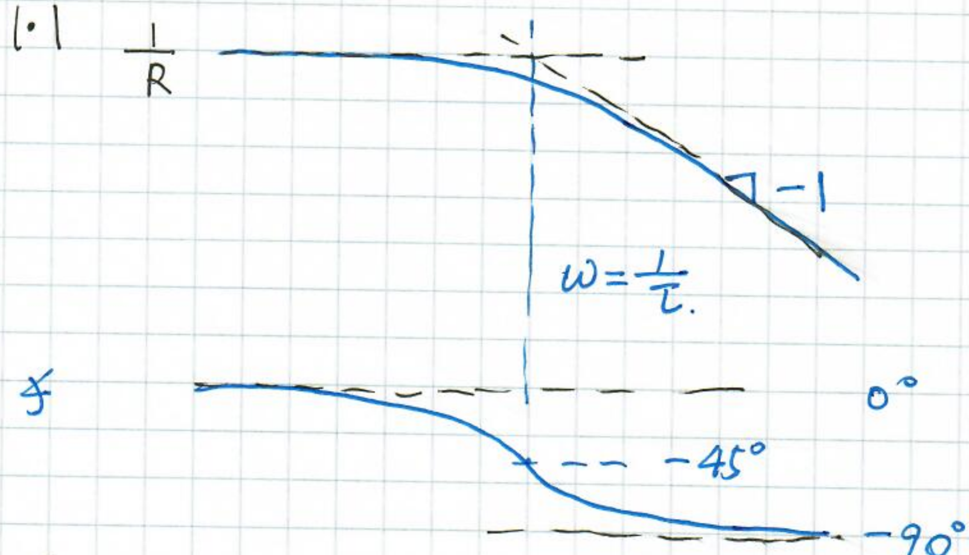
$$G = \frac{I}{U} = \frac{1}{R + Ls} = \frac{\frac{1}{R}}{\frac{L}{R}s + 1}$$

$$\tau = \frac{L}{R}.$$

This is the simpliest electrical model of motor, and it will be used to control DC motor's current.

Frequency response:

$$|G(j\omega)| = \begin{cases} \frac{1}{R}, & \omega \ll \frac{R}{L}, \\ \frac{1}{\omega L} & \omega \gg \frac{R}{L} \\ \frac{1}{R} \cdot \frac{1}{\sqrt{2}} & \omega = \frac{R}{L} \end{cases}$$



5. Impedance of R, L, C,

in s-domain, R, L, C can be treated impedance, which is similar to Resistor

$$Z_R = R$$

$$Z_L = L \cdot s$$

$$Z_C = \frac{1}{Cs}$$

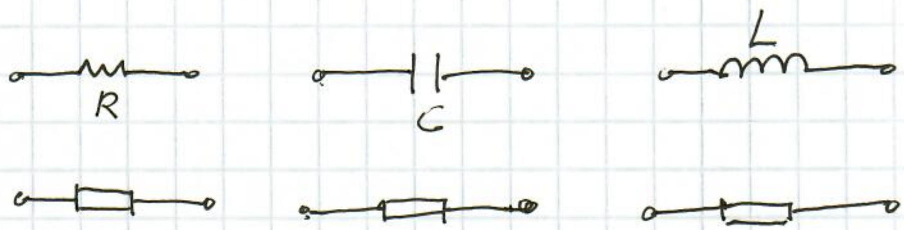
Lecture #4. Fundamentals of analog circuit.

1. Impedance modeling.

- Impedance is the transfer function between current and voltage for any 2-terminal electrical network.

$$Z(s) = \frac{V(s)}{I(s)}.$$

R, L, C models :



$Z = R$ $\quad$ $Z = \frac{1}{Cs}$ $\quad$ $Z = Ls.$

$$i = C\frac{dv}{dt}$$

$$I(s) = Cs\,V(s).$$

So, in s-domain, all resistance, capacitance, inductance can be treated as impedance with a complex number.

Ohm's Law is valid in s-domain as well.

2. Op-amp (operational amplifier)

①. Why do we need op-amps ?

1). Cheap for simple control problem.

2). Calculation for signals: add, subtract, multiply, integration, differentiation

3). High frequency applications.

In this digital age, analog circuits are still widely used in high frequency & high power systems.

② Drawbacks of analog instrumentation,

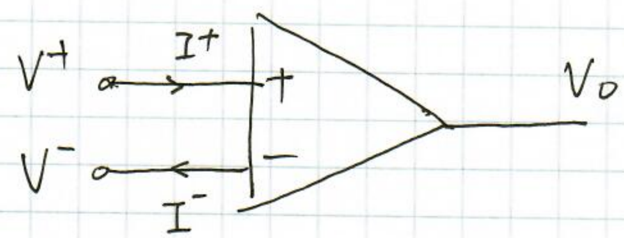
1). Performance related to the exact value of R, C, L.

C and L are not easy to gurantee high resolution.

So tuning procers is always necessary to remove uncertainty.

2). Not flexible. Once design is finished, the system cannot be upgraded except changing hardware.

③. Ideal op amps



Assumptions:

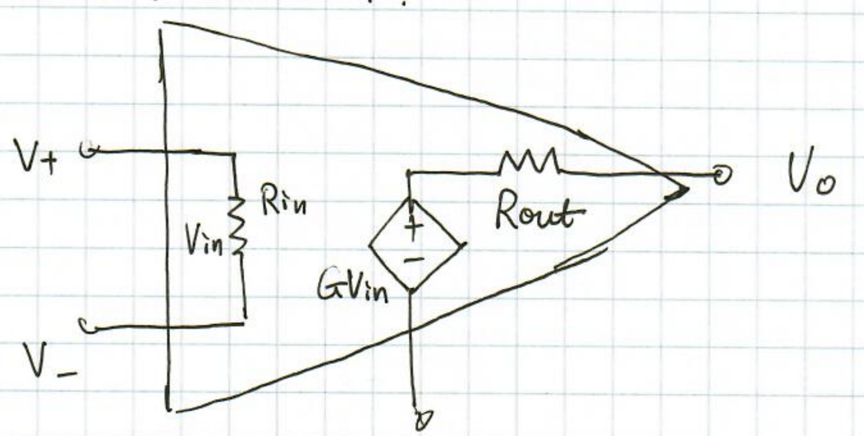
1) Open loop gain is infinity.

$V_+ = V_-$

2) No current flow through

$I_+ = I_- = 0$

Circuit model:



$V_{in} = V_+ - V_-$

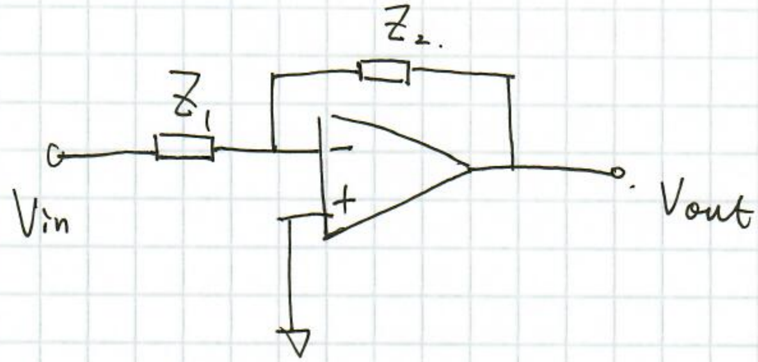
$R_{in} = \infty$

$G = \infty$

$R_{out} = 0$.

3. Common used op-amp circuits

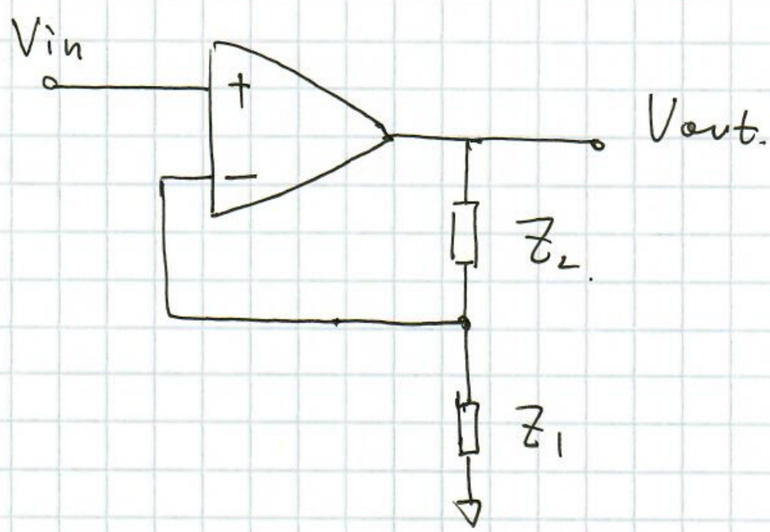
①. Inverting amplifier.



$$\frac{V_{in}-0}{Z_1} = \frac{0-V_{out}}{Z_2}.$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1}$$, Z_1, Z_2 can be combinations of R, L, C.

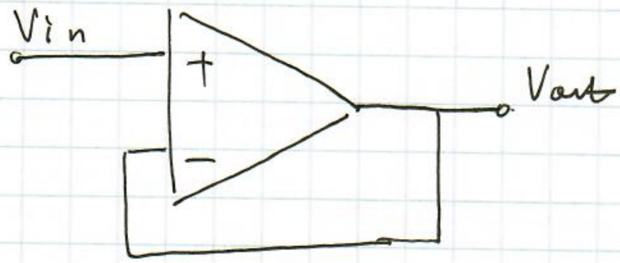
②. Noninverting amplifier.



$$\frac{V_{in}}{Z_1} = \frac{V_{out}}{Z_1+Z_2}.$$

$$\frac{V_{out}}{V_{in}} = 1 + \frac{Z_2}{Z_1}$$

③ Follower



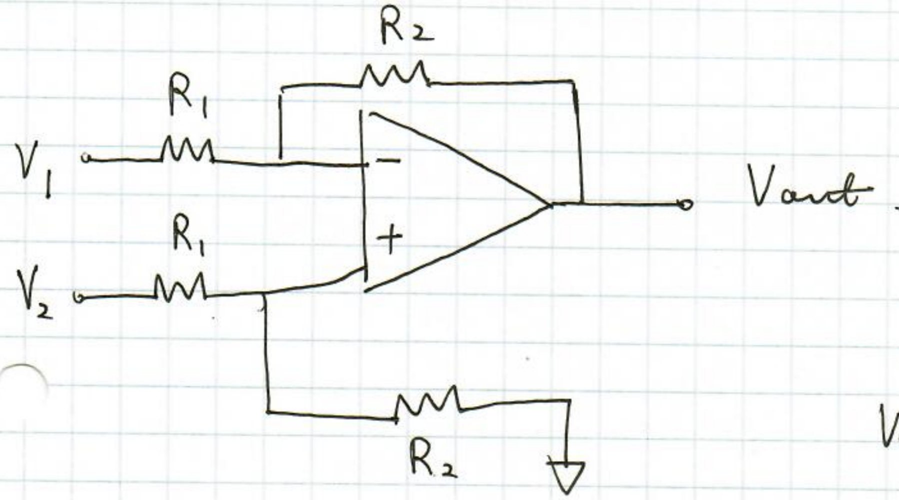
$V_{in} = V_{out}$

$Z_{out} = 0$

$Z_{in} = \infty$

Followers are used for isolation and buffering.

④. Difference amplifier



$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

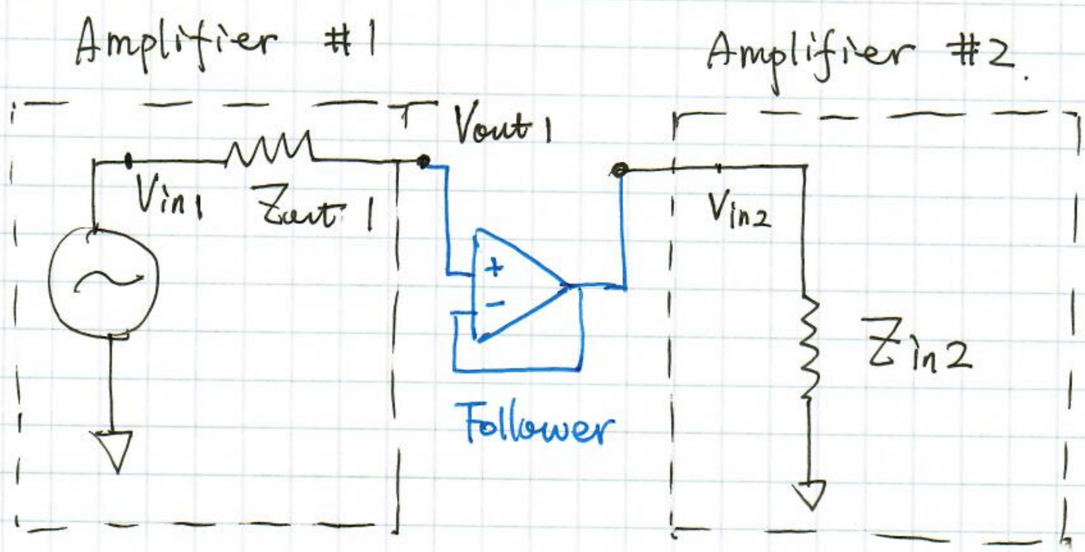
This circuit requires high accuracy resistors, and many models are available in the market.

Analog circuit hand book:

"Art of Electronics"

Paul Horowitz & Winfield Hill.

· Impedance Matching of opamp circuit.



Ideal case $Z_{out\,1} = 0$, $Z_{in\,2} = \infty$

In engineering design view ,

① Make Z_{out} 1 as low as possible

Make Z_{in} 2 as low as possible

②. Insert a voltage follower

Recall the definitions of input & output impedance.

$$Z_{in} = \frac{V_{in}}{I_{in}}$$

$$Z_{out} = \frac{V_{out} - V_{in}}{I_{out}}$$

· Lecture #5: Non-ideal op-amps & closed loop system.

1. Realistic op-amps.

① In the reality, the open-loop gain of op-amp is not infinity, which is

$$G(s) \neq \infty$$

G(s) is the frequency dependent transfer function.

②. G(s) is available in device datasheet, as the plot of "Open-Loop Gain" V.s. Frequency.

Different op-amps for different purposes have different G(s).

See two examples: OP 27 & PA13.

(2.1) op 27.

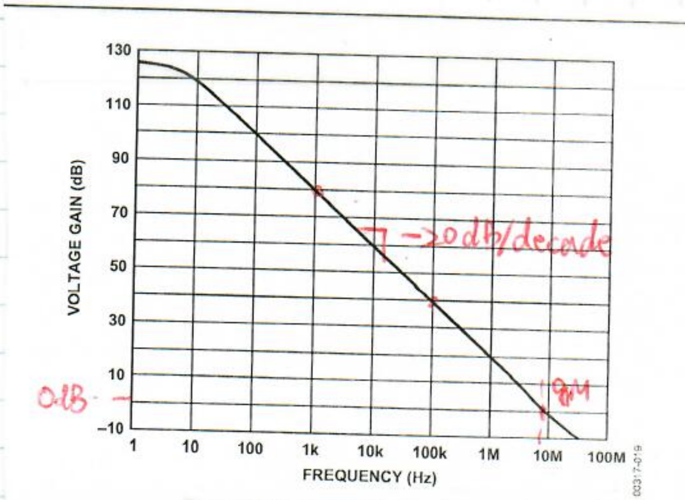


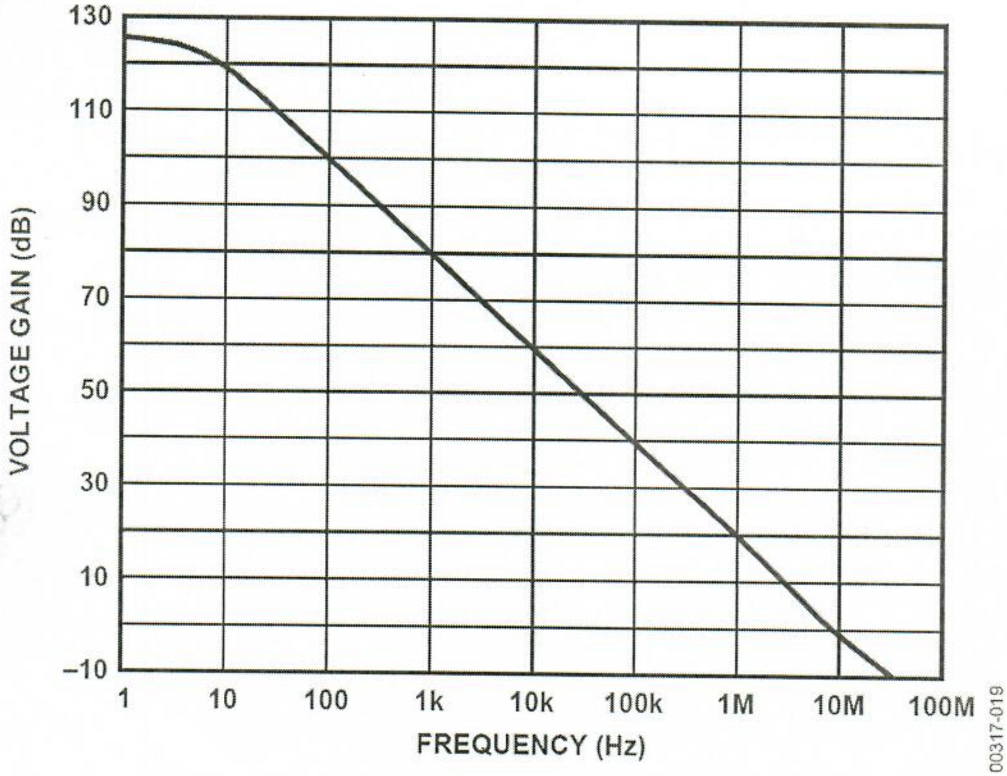
Figure 19. Open-Loop Gain vs. Frequency

From Analog Device datasheet.

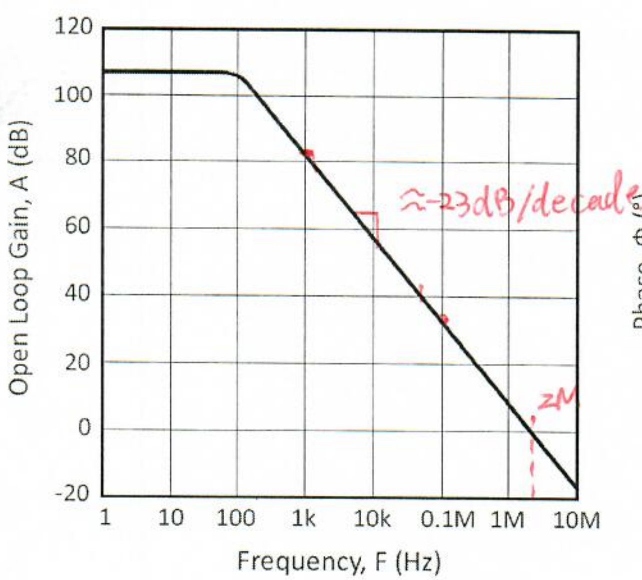
Below 10MHz, it is similar to a first order system.

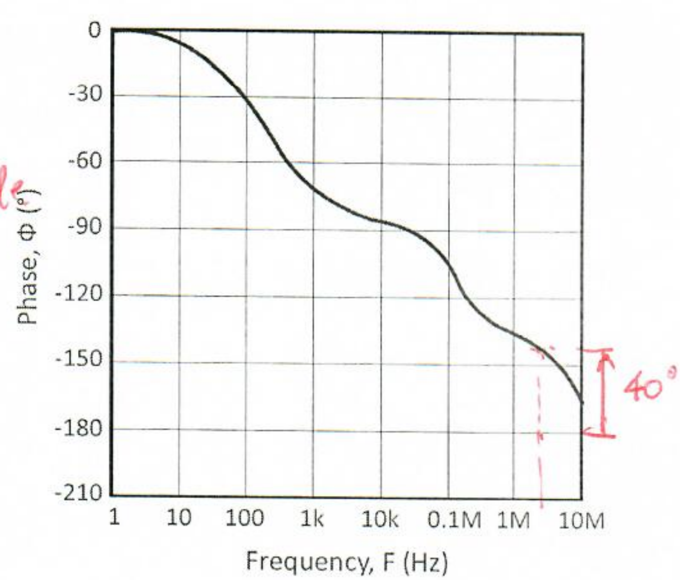
Gain Crossover frequency $\omega_{cr} \doteq 8MHz$

OP 27 open loop frequency response



PA 13 open loop frequency response

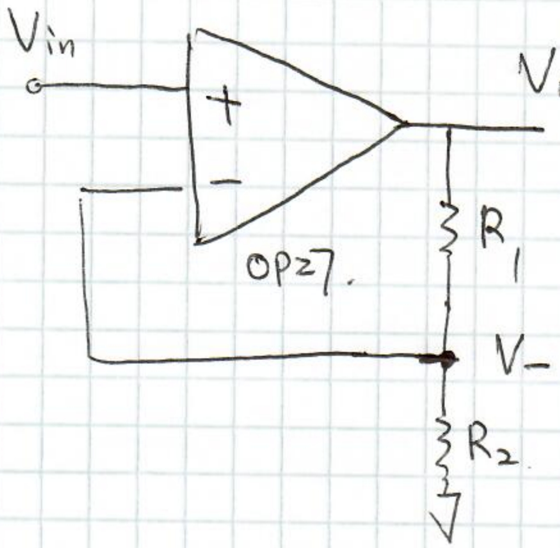




(2.2) PA 13 is a high power, high current op-amp.
From the gain Bode plot, it cannot be treated as first-order system, and the phase is dropping down.

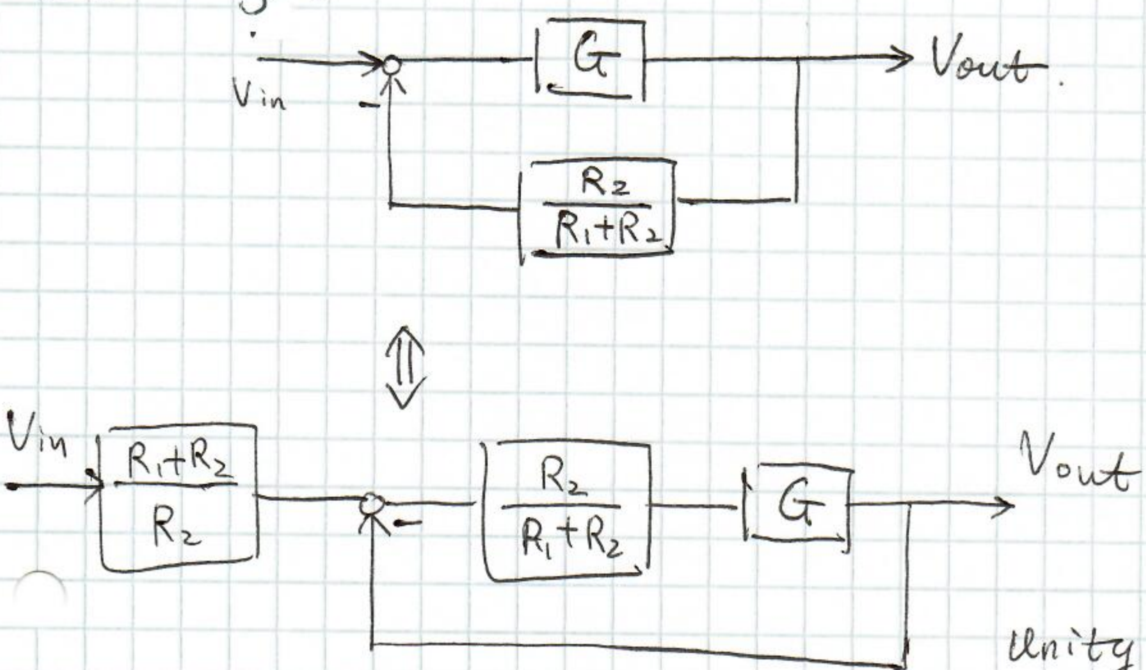
2. Closed loop system & bandwidth.

Use OP27 and consider a noninverting circuit.



$$V_- = \frac{R_2}{R_1+R_2} \cdot V_{out}$$
$$V_+ = V_{in}$$

Block diagram:



Unity feedback layout

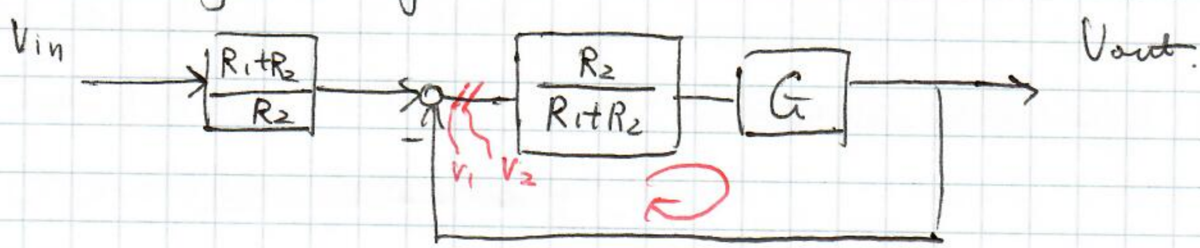
Closed loop transfer function:

$$G_{cl} = \left(\frac{R_1+R_2}{R_2}\right) \frac{\frac{R_2}{R_1+R_2}\cdot G}{1+\frac{R_2}{R_1+R_2}\cdot G}$$

- Recall the key point from control courses:

When analysing the behaviour of closed loop system the open loop transfer function need to be designed well.

Block diagram again:



Cut the loop, so we have

$$V_2 \cdot \frac{R_2}{R_1+R_2}\cdot G \cdot (-1) = V_1$$

$$TF = \frac{V_1}{V_2} = -\frac{R_2}{R_1+R_2}G$$

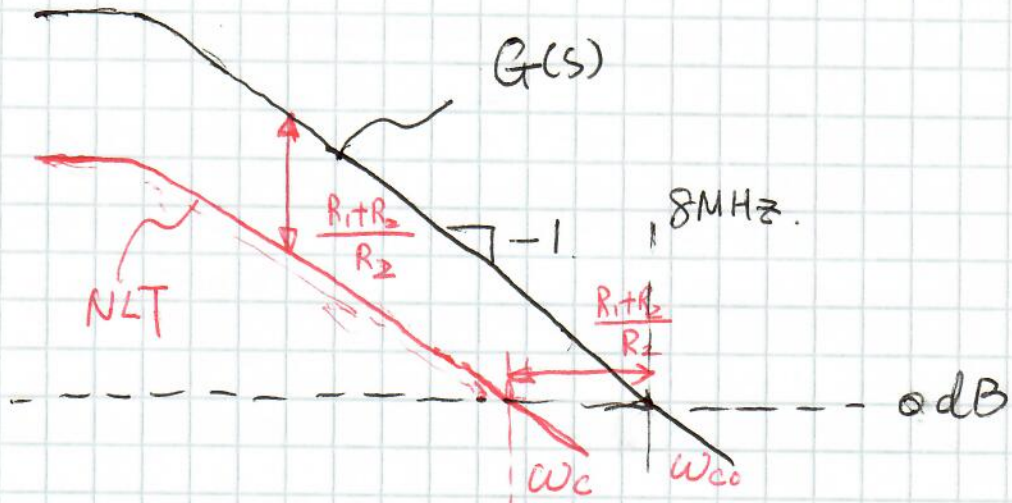
In the negative feedback system,

the negative loop transmission is more convenient

$$\downarrow \qquad \downarrow \qquad \downarrow$$
$$N \qquad L \qquad T$$

$$NLT = -TF = \frac{R_2}{R_1+R_2}G.$$

Transfer function of OP27 ($G(s)$) is known.



So the bode of NLT can be drawn.
and the NLT has lower open loop gain and lower crossover frequency.

Consider OP27 as a first order system.

$$\omega_c = \frac{R_2}{R_1+R_2} \cdot \omega_{co}$$

The closed loop transfer function is:

$$G_{cl} = \frac{R_1+R_2}{R_2} \cdot \frac{NLT}{1+NLT}$$

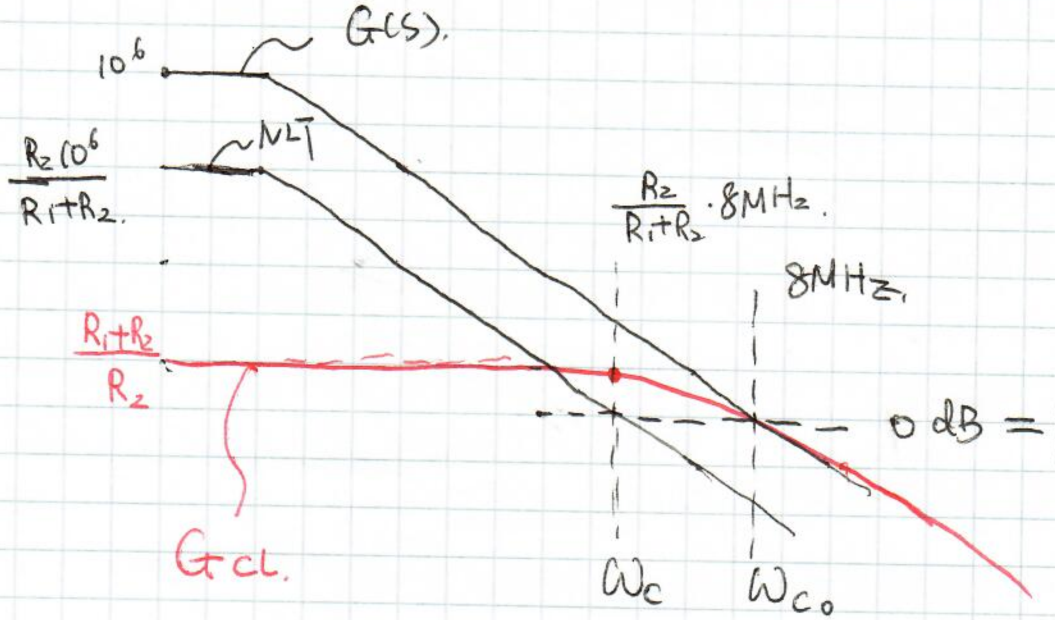
At low frequency $(\omega \ll \omega_c)$, NLT is large, then $\frac{NLT}{1+NLT} \approx 1$

At high frequency $(\omega \gg \omega_c)$, NLT is comparable to one.

$$\text{So, } G_{cl} = \frac{R_1+R_2}{R_2} \cdot NLT = \frac{R_1+R_2}{R_2} \cdot \frac{R_2 G}{R_1+R_2} = G.$$

In sum, $G_{cl} = \begin{cases} \frac{R_1+R_2}{R_2}, & \omega \ll \omega_c \\ G(s), & \omega \gg \omega_c \end{cases}$

Bode plots again:



Key features of Closed loop system.

1). DC gain: $\omega \to 0$, $|G| =$ DC gain.

$$|G_{DC}| = \frac{R_1+R_2}{R_2}$$

2) Band width, (ω_{BW}), $|G(\omega_{BW})| = 0.707\,|G_{DC}|$

Theoretically, ω_{BW} is not equal to crossover frequency (ω_{cr}) for most cases, but they are close to each other.

Therefore, in engineering practice,

we use $\omega_{BW} \approx \omega_{cr}$

3). Gain, bandwidth product.

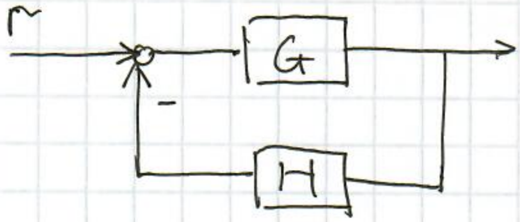
$GBP = G_{DC} \cdot \omega_{BW}$

(key feature of op-amp device).

Lecture Supplementary Notes:

1. What is NLT?

Consider a general system,



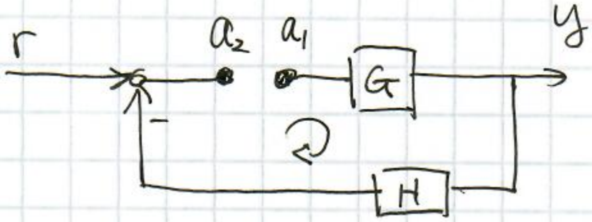
Closed loop transfer function

$$G_{CL} = \frac{G}{1+GH} = \frac{y}{r}$$

$\boxed{1+GH}$ is the characteristic eq, which shows the sys. behaviour such as stability.

So "GH" is important.

If we open the loop, Loop transmission is:



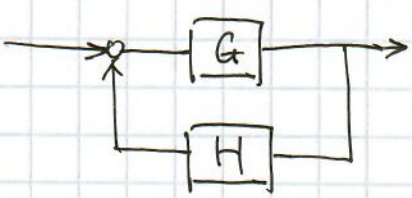
$$a_2 = GH\, a_1 (-1)$$

$$\frac{a_2}{a_1} = -GH.$$

Negative transmission is

$$NLT = (-1)\cdot\frac{a_2}{a_1} = GH$$

2. Why we need NLT? Why not use GH for all case?



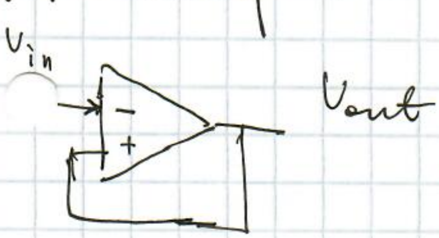
Consider a positive feedback.

$$NLT = -GH,$$ the system is not stable

But the forward transfer function / open loop is the same as negative feed back sys.

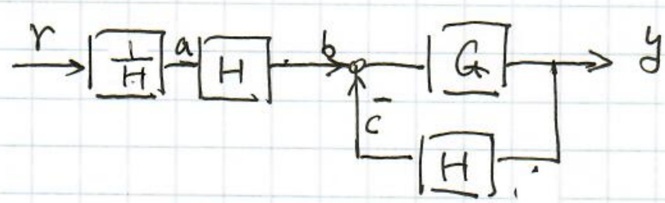
it can not show the correct behaviour of the sys.

For example

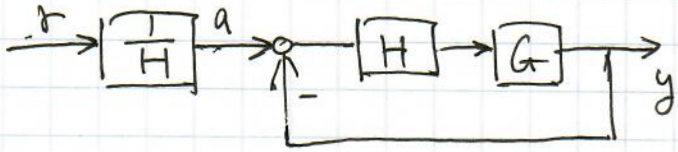


3. Unity feedback system. algebra tricks:

$\Rightarrow$



$\Rightarrow$



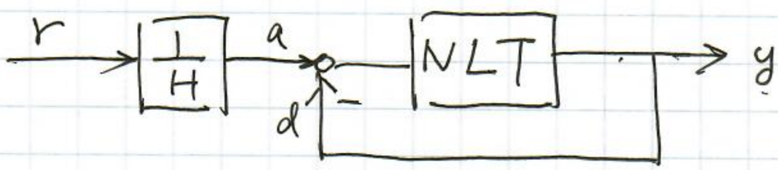
$$\begin{cases} aH = b \\ (b-c)G = y \\ c = Hy \end{cases}$$

$$\Rightarrow (aH - yH)\cdot G = d$$

$$(a-y)GH = d.$$

NLT = GH.

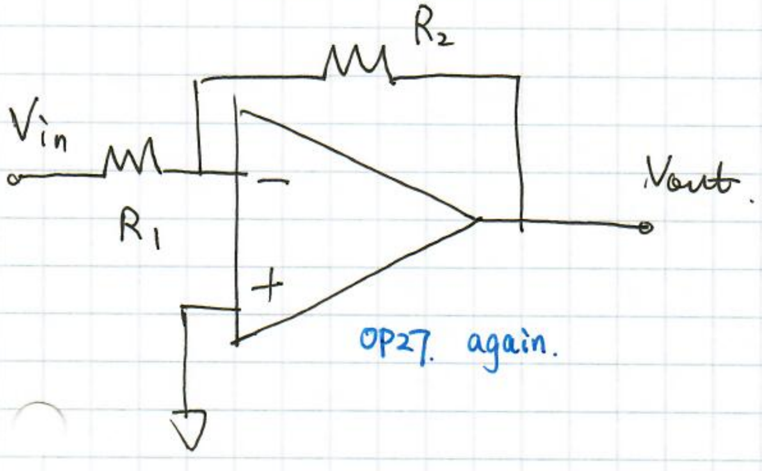
$\Rightarrow$



Unity feedback loop shows NLT clearly.

Again, NLT is the Key of closed loop system.

4. Another example of opamp circuit.



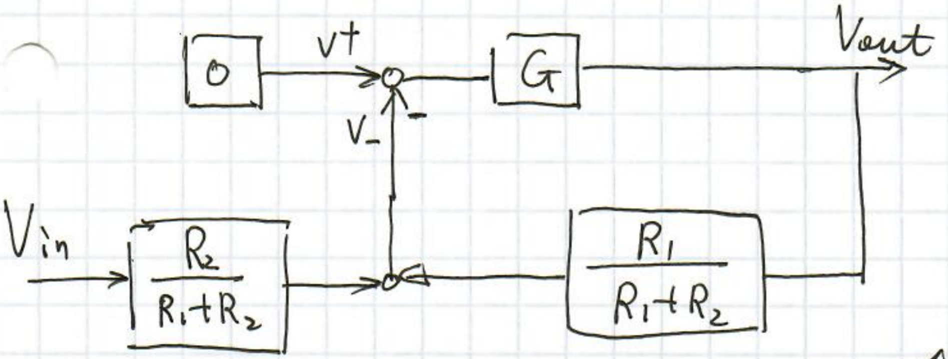
ideal case: $\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$

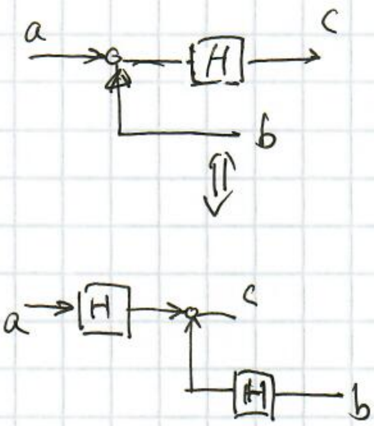
$$\frac{V_{in} - V_-}{R_1} = \frac{V_- - V_{out}}{R_2}$$

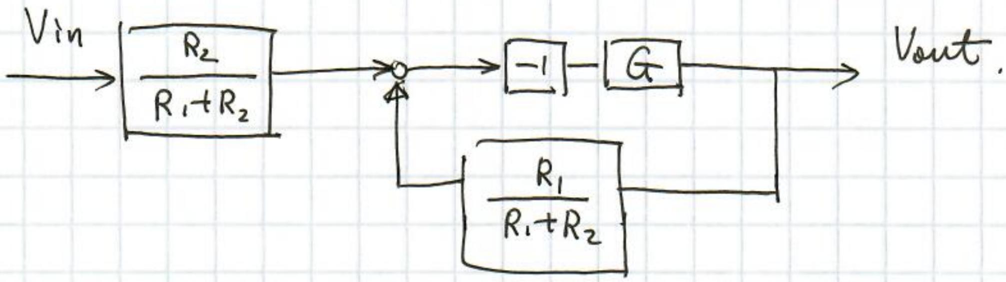
$$(R_1 + R_2)V_- = R_2 V_{in} + R_1 V_{out}$$

$$V_- = \frac{R_2}{R_1+R_2} V_{in} + \frac{R_1}{R_1+R_2} V_{out}$$

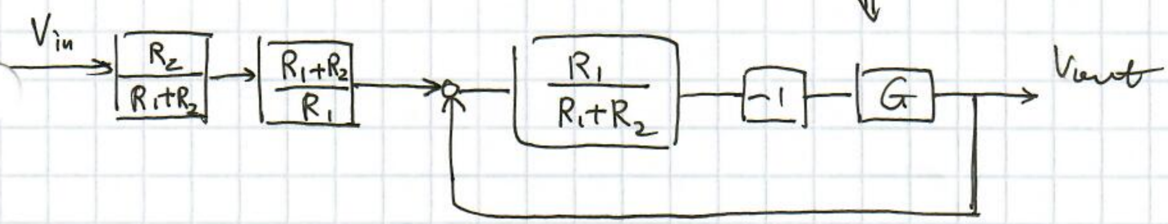
Block diagram:

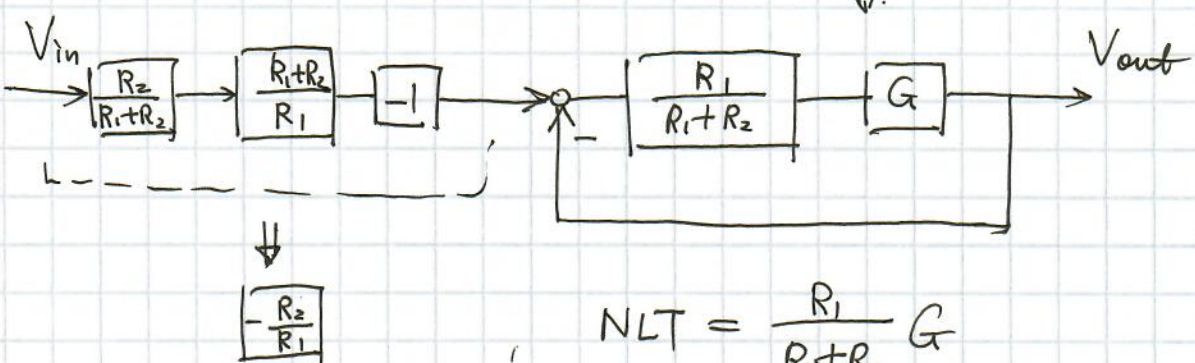






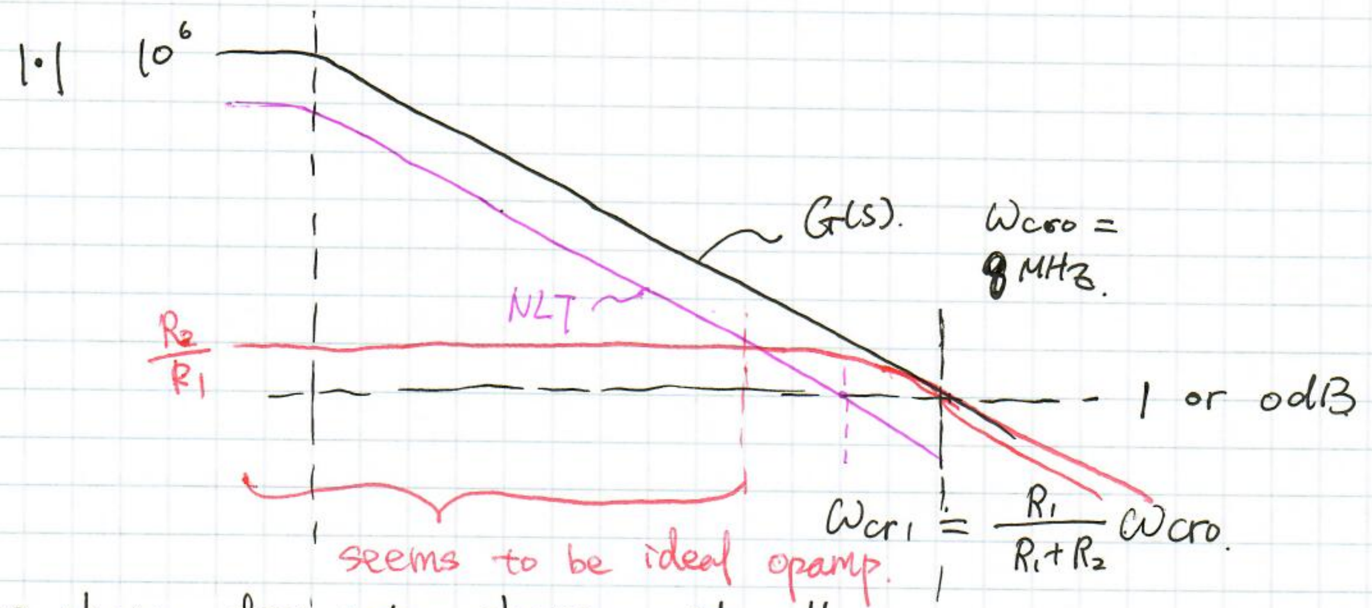
same tech as before



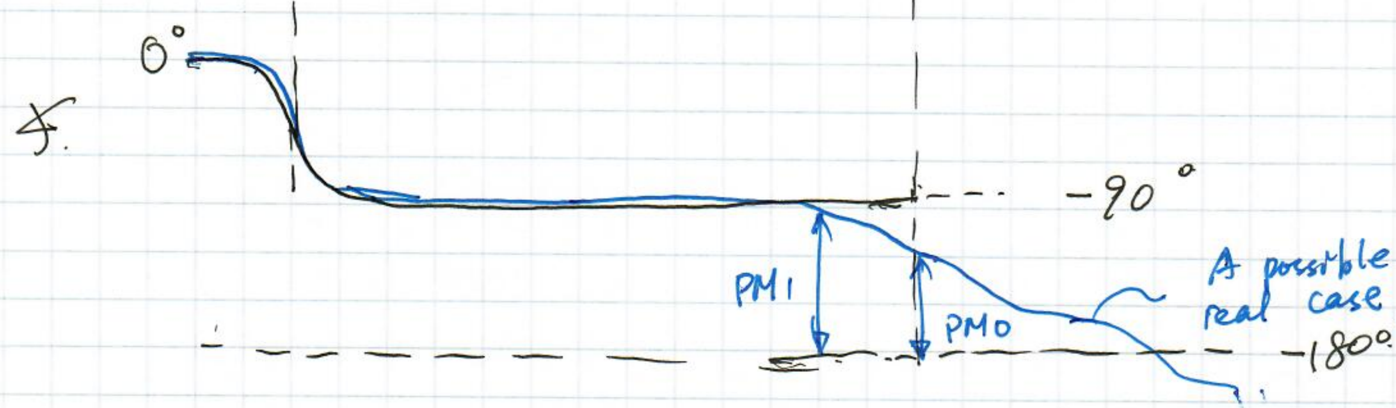


$$NLT = \frac{R_1}{R_1+R_2} G$$

Bode plots:



The phase does not change at all.



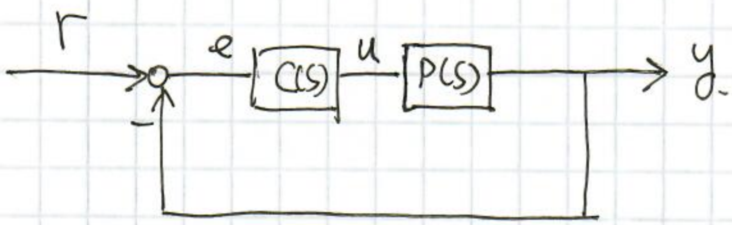
PM is a key concept when we analyze stability.

Lecture 6: Stability & Nyquist criteria.

From last lecture, we connected a non-inverting circuit of op-amps, which is a closed loop system.

Now we consider a general case for the feedback system.

1. Feedback control system.



r: reference input
e: error
u: controller output
y: system output
C(s): controller
P(s): plant.

$NLT = CP$

Reference transfer function:

$$\frac{y}{r} = \frac{NLT}{1+NLT}$$

Error transfer function:

$$r - e \cdot NLT = e \quad \Rightarrow \quad \frac{e}{r} = \frac{1}{1+NLT}$$

2. Stability: the <u>No. 1</u> property of closed loop system.

Quick review: How to check the stability

①. Locations of poles. All poles of the closed loop system should be in left half of the complex plane.

②. Nyquist criteria., Let P is the number of unstable poles of NLT.

The closed loop system is stable $\Leftrightarrow$ if & only if the contour of $NLT(j\omega)$ counter clockwise encircle the "-1" P times in the Nyquist plot.

Why do we need Nyquist criteria ?

①. It can present the system is whether stable or not, more importantly, Nyquist can show <u>how stable</u> the system is.

②. It brings a easier way to design the stability of the system in frequency domain.

<u>Tip</u>: Until nowadays, industrial control systems are dominated by classic control methods, which are based on s-domain descriptions.

Laplace transform

Classic control — transfer function — s domain.

↕ ↕

Modern control — state space model — time domain.

Linear algebra.

---

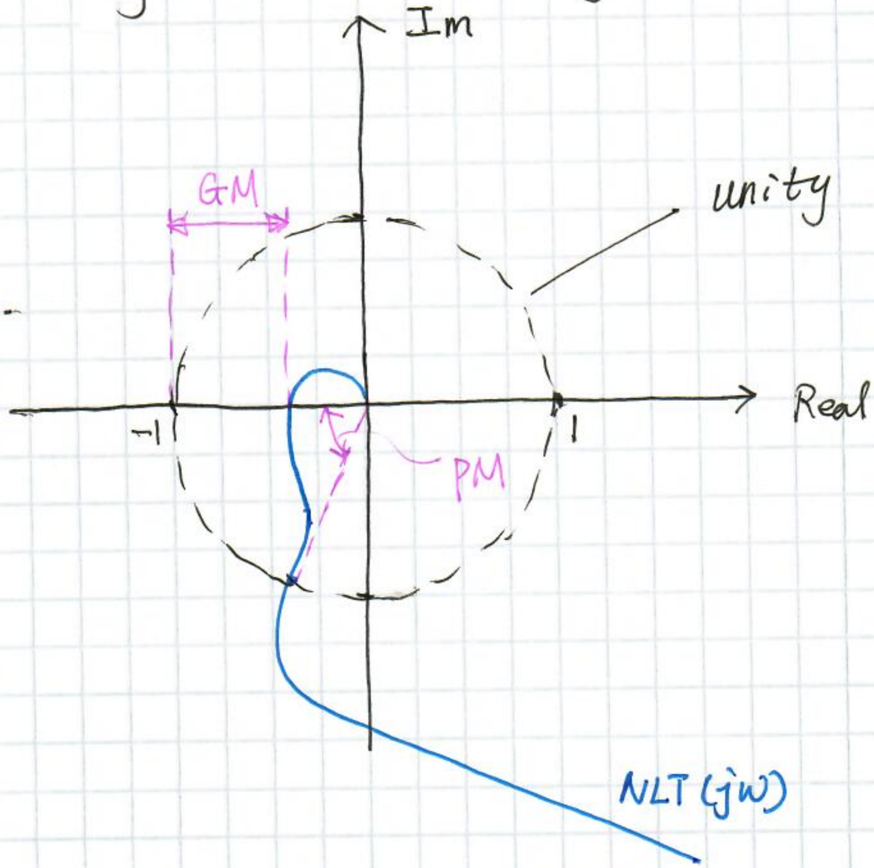
③. It doesn't rely on the accurate transfer function, Bode plots are enough for most cases.

Another tip: For the most of mechatronic systems, the NLT doesn't have unstable pole, which means the NLT(jw) never encircles "−1" point if the system is stable.

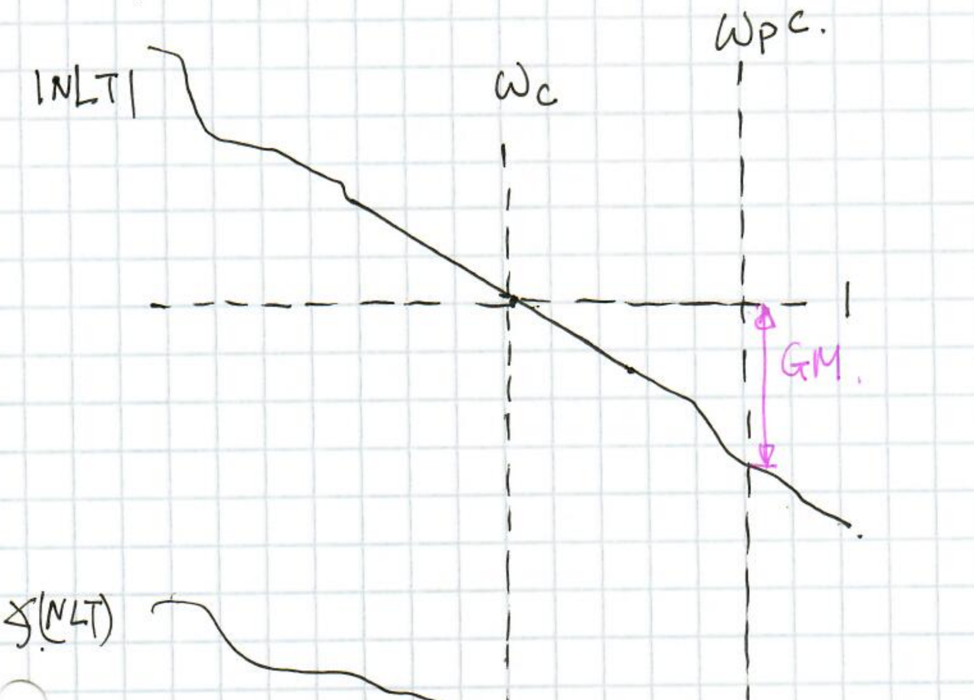
The system is called as minimum phase system, details can be found in control books.

3. Phase margin & Gain margin:

Nyquist Diagram.



Bode plots:



At gain crossover frequency ω_c, $|NLT(j\omega_c)| = 1$

At phase crossover frequency ω_{pc}, $\measuredangle NLT(j\omega_{pc}) = -180^\circ$

Phase margin:

$$PM = 180^\circ + \measuredangle NLT(j\omega_c)$$

Gain margin:

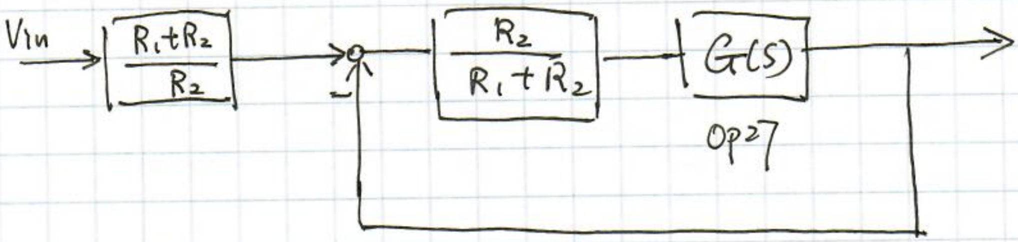
$$GM = \frac{1}{|NLT(j\omega_{pc})|}$$

In short, if NLT has no unstable pole,

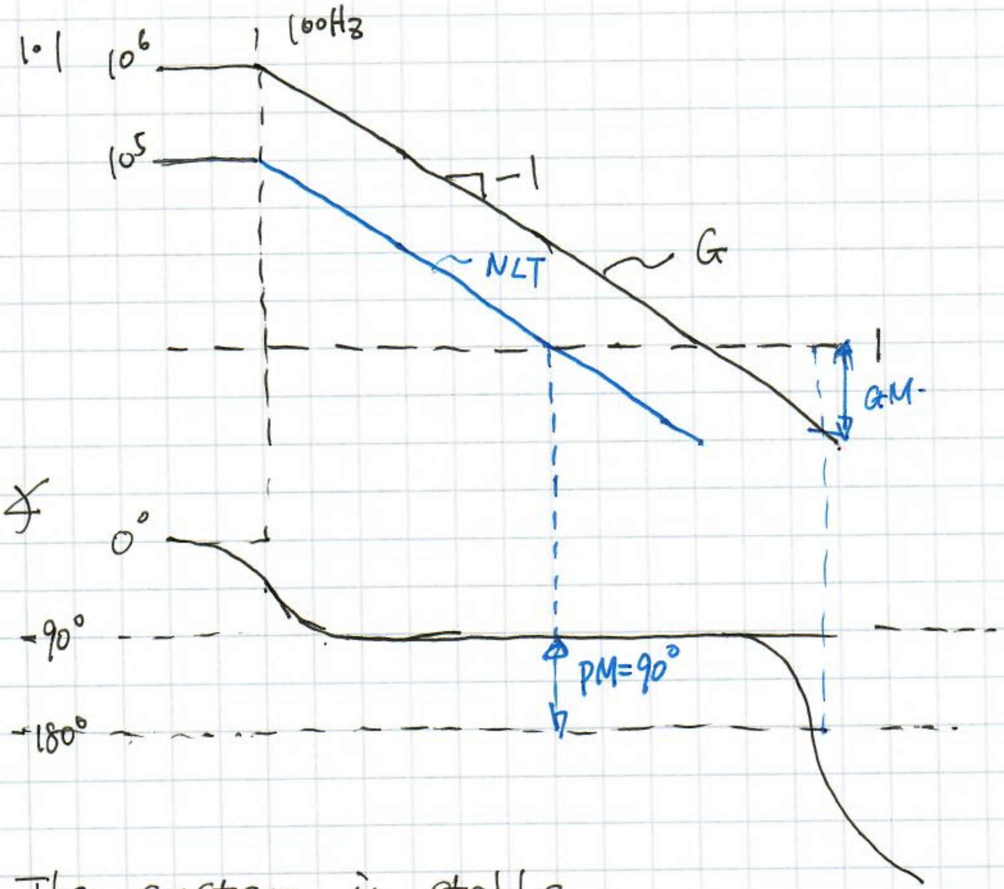
closed loop system is stable $\Longleftrightarrow \begin{cases} PM > 0 \\ GM > 1 \end{cases}$

In engineering applications, we normally require $PM \geq 60°$ & $GM \geq 2$ to gurantee the stability when design control system.

4. An Example: recall the noninverting circuit for OP27.



$$NLT = \frac{R_2}{R_1+R_2} \cdot G(s). \quad , \text{Let } \frac{R_1+R_2}{R_2} = 10$$



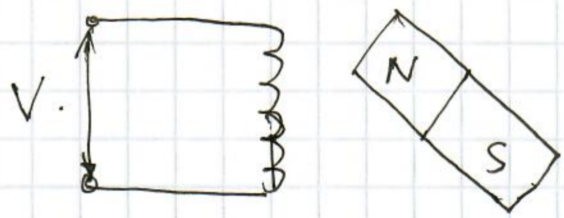
For the real device, phase drops at high freque

The system is stable.

Lecture #7: Voltage stage for motor.

1. A simple model of motor.

Consider a DC motor.

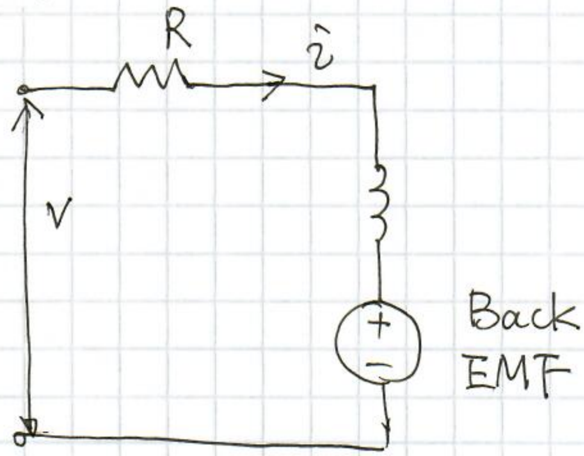


Electric field. ⇒ Magnetic field

↳ Force on permanent magnet.

(Lorentz)

Circuit Model:

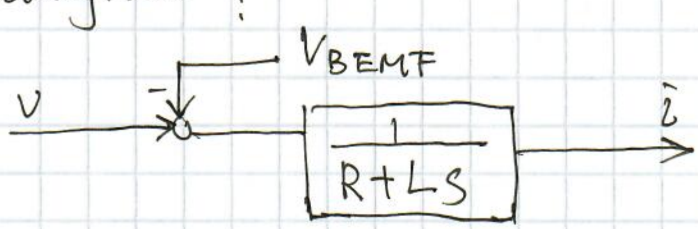


$V_{B\text{-}EMF} = BS\omega$.

B is magnetic field flux density

EMF: Eletromotive force, which is a voltage.

Block diagram:



Neglect disturbance (V_{BEMF}). the transfer function is:

$$\frac{\hat{i}}{V} = \frac{1}{R+LS}.$$

This is a typical first order system.

2. Power amplifier of motor.
(P.A.)

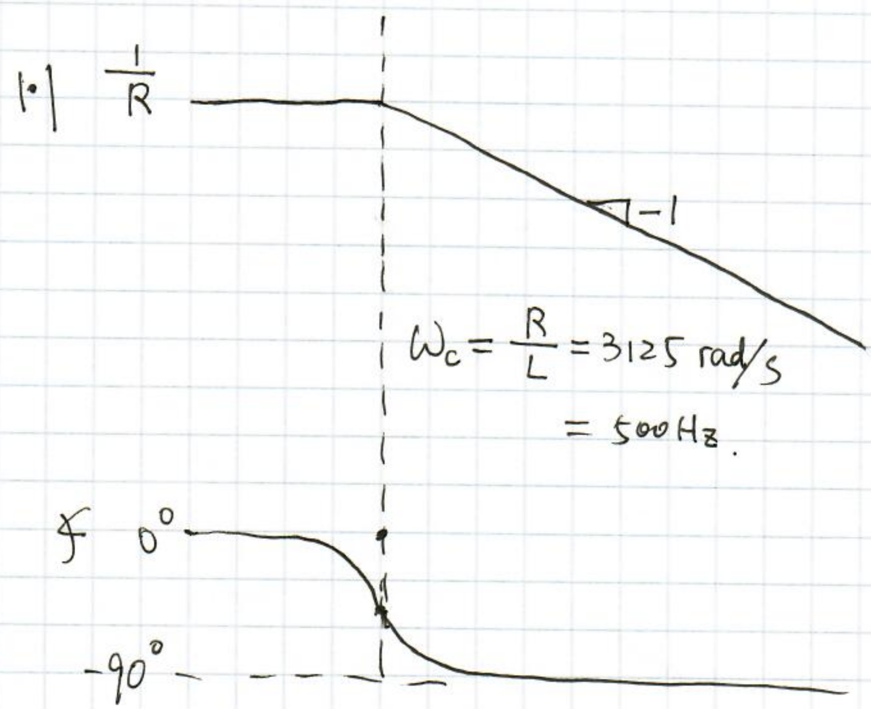
- P.A. provides a current to drive the motor following the referenced command.
- Usually, the driving currents for motors are up to ampere level, so the small signal op-amps are not available for the purpose.
- In this course, we use a high power op-amp (PA13) to drive motor.

3. Frequency response of motor.

An example with parameters: A DC motor

$R = 5\Omega$, $L = 1.6mH$.

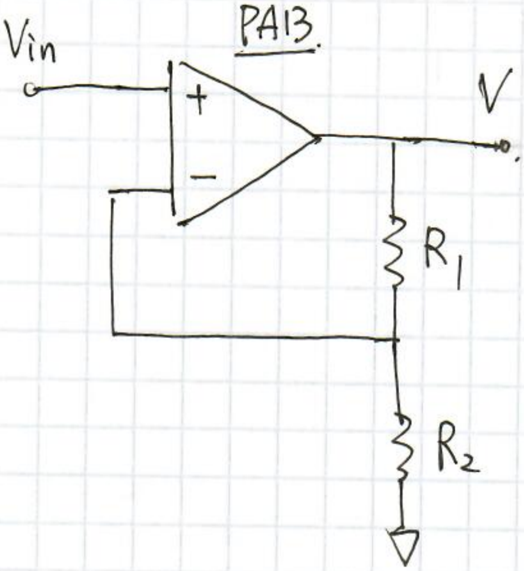
$$G_{motor} = \frac{1}{R+Ls} = \frac{I}{V}$$



4. Design the circuit for PA 13.

Use an noninverting circuit to drive motor.

Again:

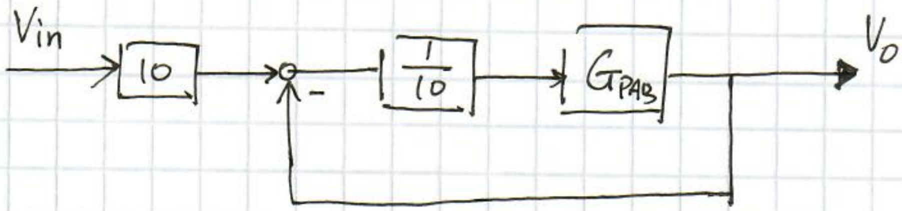


Let the gain of P.A. is 10, $A=10$, so $R_1 = 9k\Omega$, $R_2 = 1k\Omega$.

Tricky questions: ① Why not use $R_1 = 9\Omega$, $R_2 = 1\Omega$?

② Why not use $R_1 = 9M\Omega$, $R_2 = 1M\Omega$?

Block diagram:



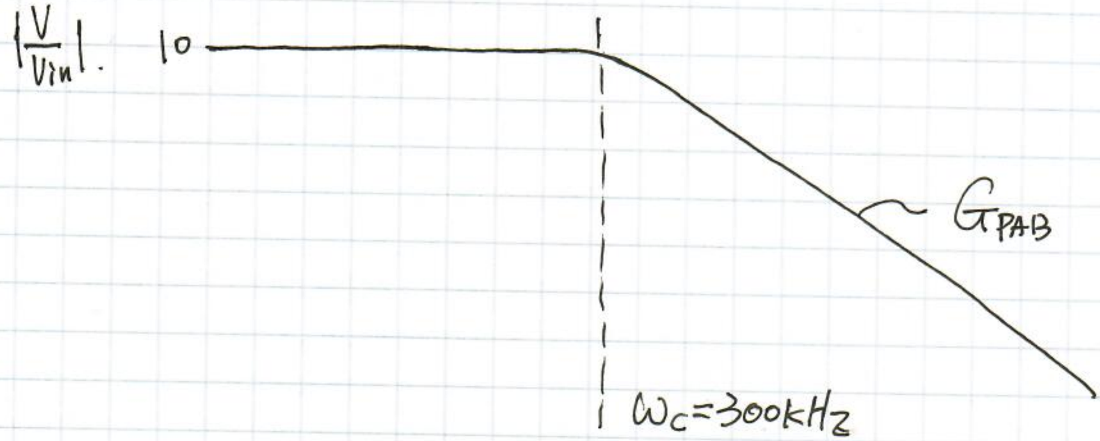
check the band width & stability first for the Bode plots on datasheet.

$$NLT = \frac{1}{10} G_{PA13}.$$

$\omega_c = 300kHz$.

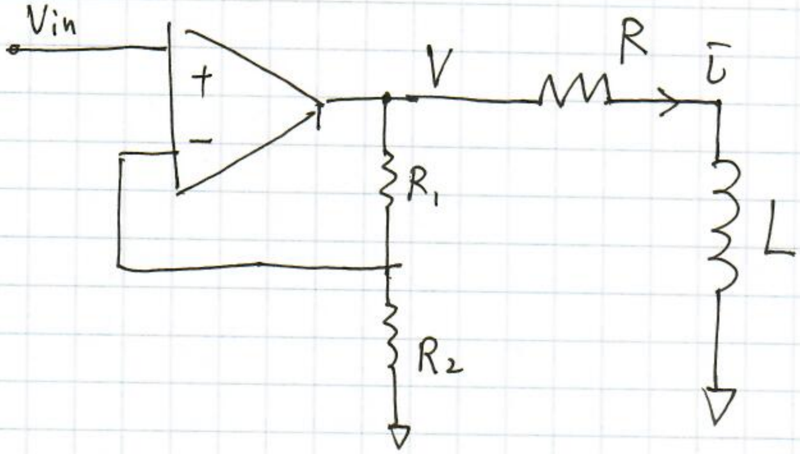
$PM = 55°$

The closed loop Bode plots is:

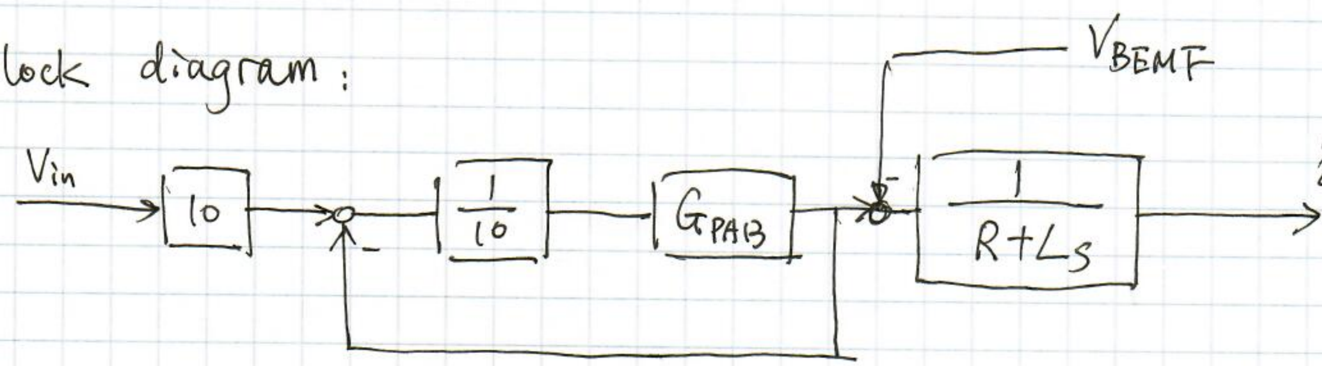


5. Combine motor with P.A.

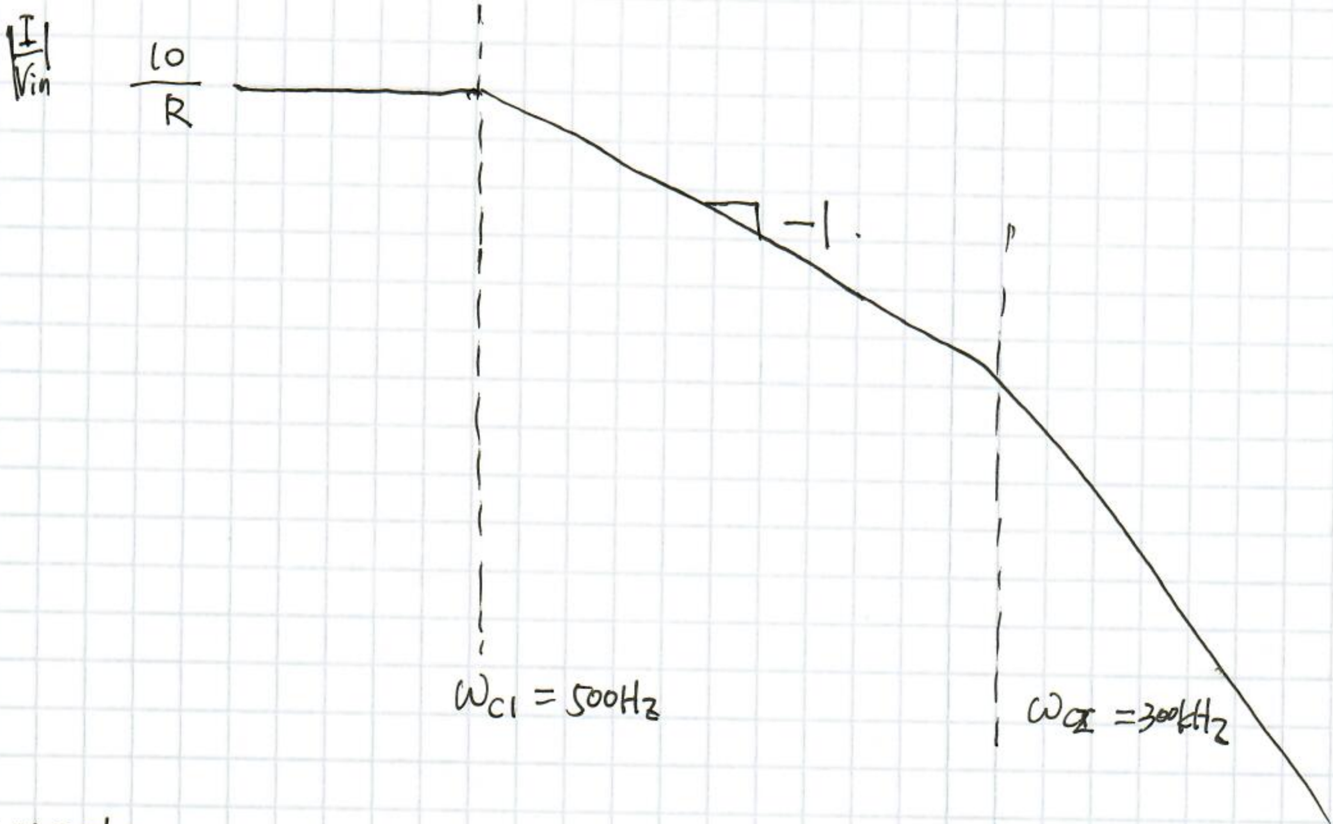
Circuit:



Block diagram:



Bode plots of $\left|\frac{I}{V_{in}}\right|$.



Comments:

① For some simple situations, we can use this system to control the current flowing through motor. by $I = \frac{10}{R} \cdot V_{in}$

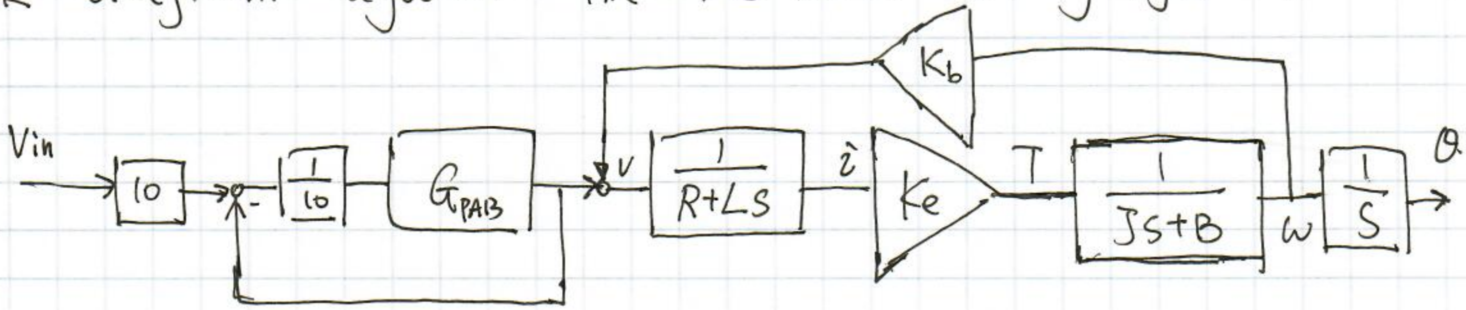
② However, the bandwidth is low, $\omega_c = 500Hz$, Also, it depends on high-accuracy modeling R and L.

③. For better performance, current controller is required.

· Lecture #9,10: Current controller design.

1. Why do we need to control current of motor?

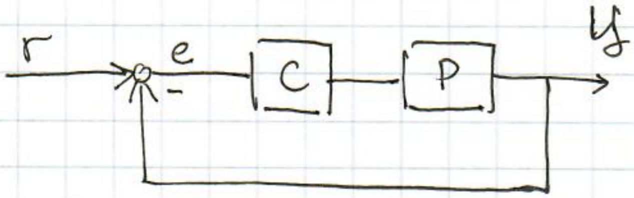
Block diagram again: the DC motor driving system.



The torque from the motor is proportional to current because of Lorentz's Law.

If there is no current controller, the bandwidth is poor, and it is easy to be affected by disturbance such as back EMF.

2. Review control system:



Objectives: ①. The tracking error is as small as possible

②. The response is as fast as possible

③ The closed loop system is as stable as possible.

$\underline{\underline{\text{ASAP}^3}}$.

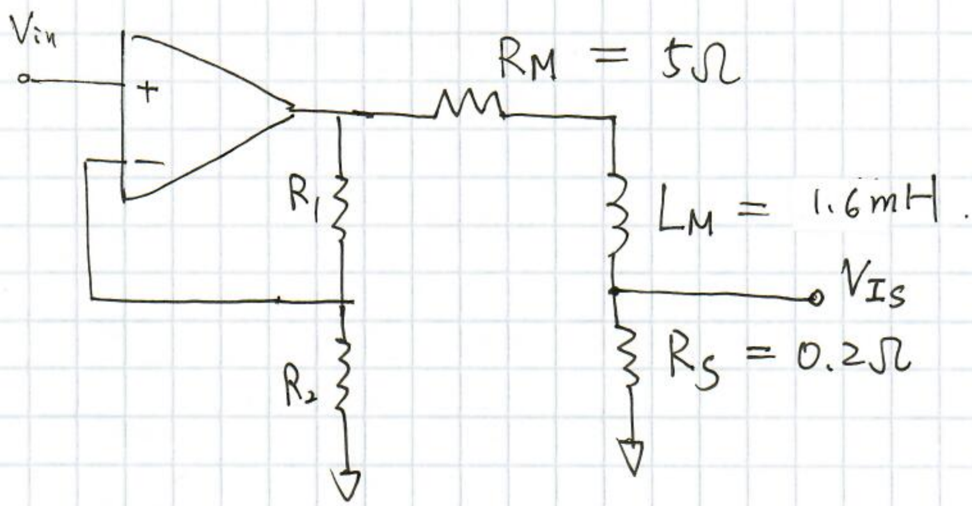
In this course, controller design is in frequency domain by shaping "NLT" in the Bodes.

3. Current controller, PI controller

The actual current flowing through DC motor has to be realized when we control the current,

and a resistor can be used to sense the current.

The circuit of P.A. + Motor is.



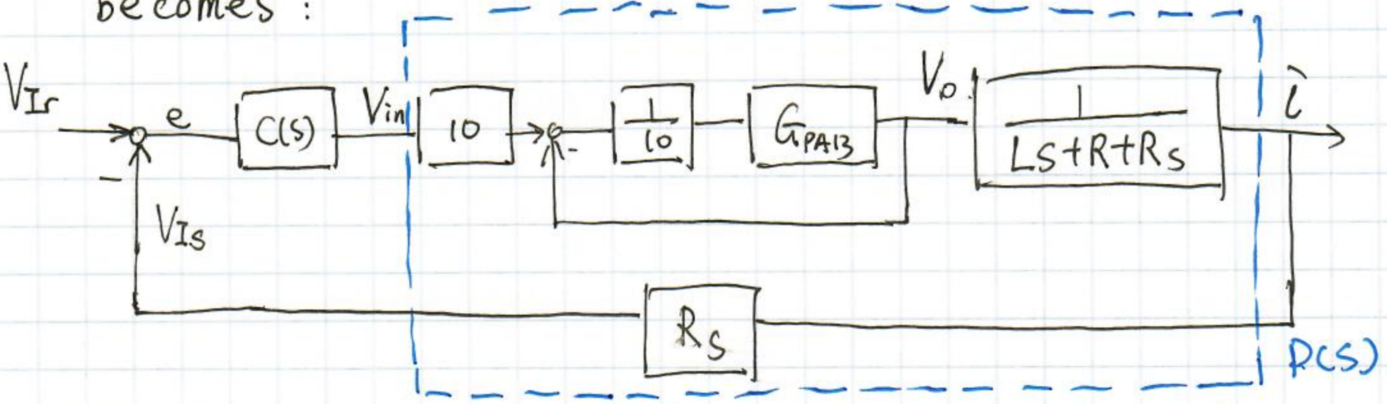
R_S is the sensing resistor, V_{IS} is the voltage signal to represent current.

Tips: The resistance value needs to be well tuned in the real circuit, since too much resistance consumes power but too small resistance will bring noise.

In addition, the type of resistor and power level need to be selected based on the amplitude and frequency of the current.

High accuracy, high bandwidth current sensing is not easy.

So with sensing resistor the block diagram becomes :

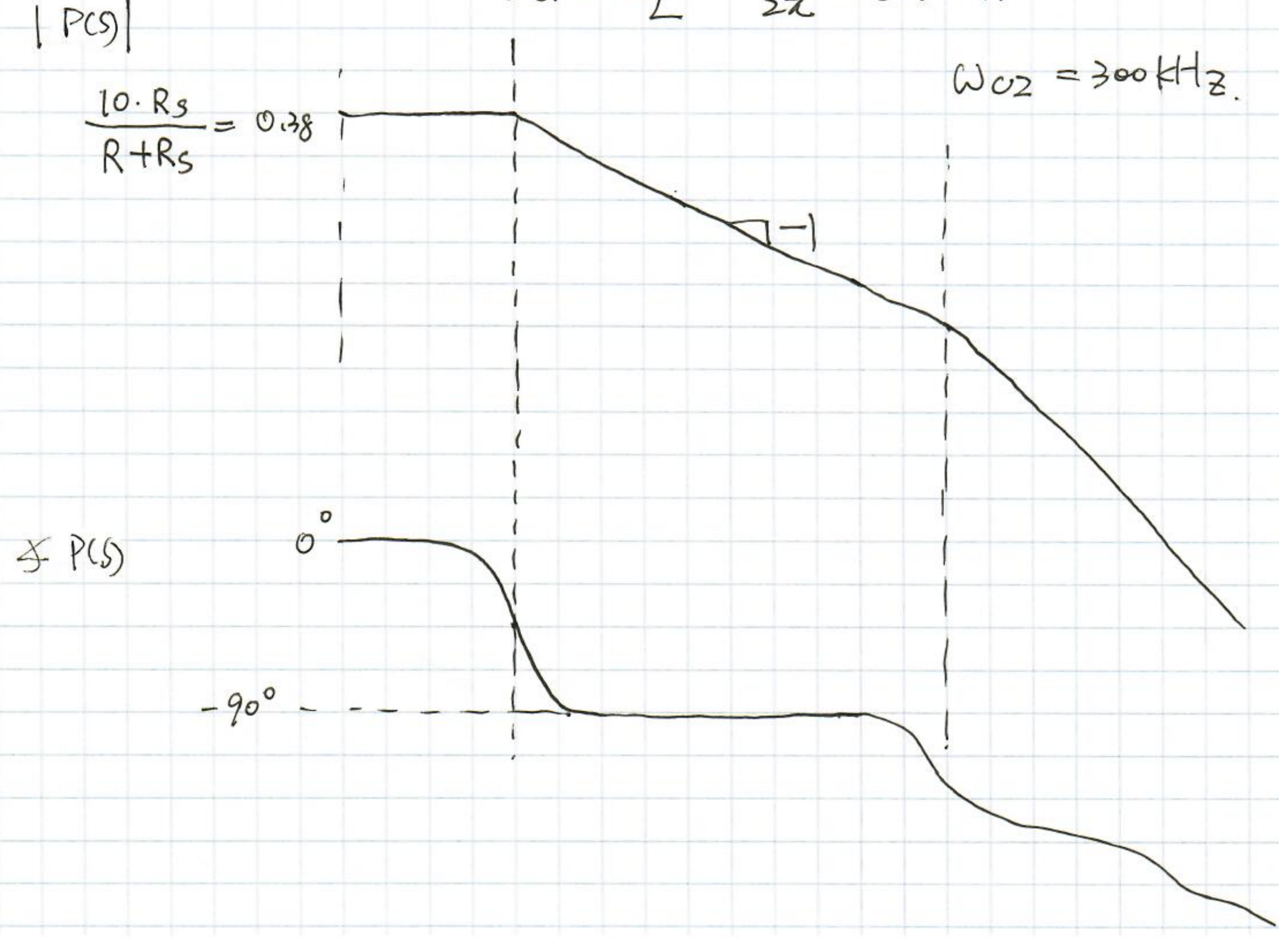


① C(S) is the controller need to be designed.

②. V_{Ir} is the command of referenced current, and this command is a voltage signal.

Bode plots of P(S) ...

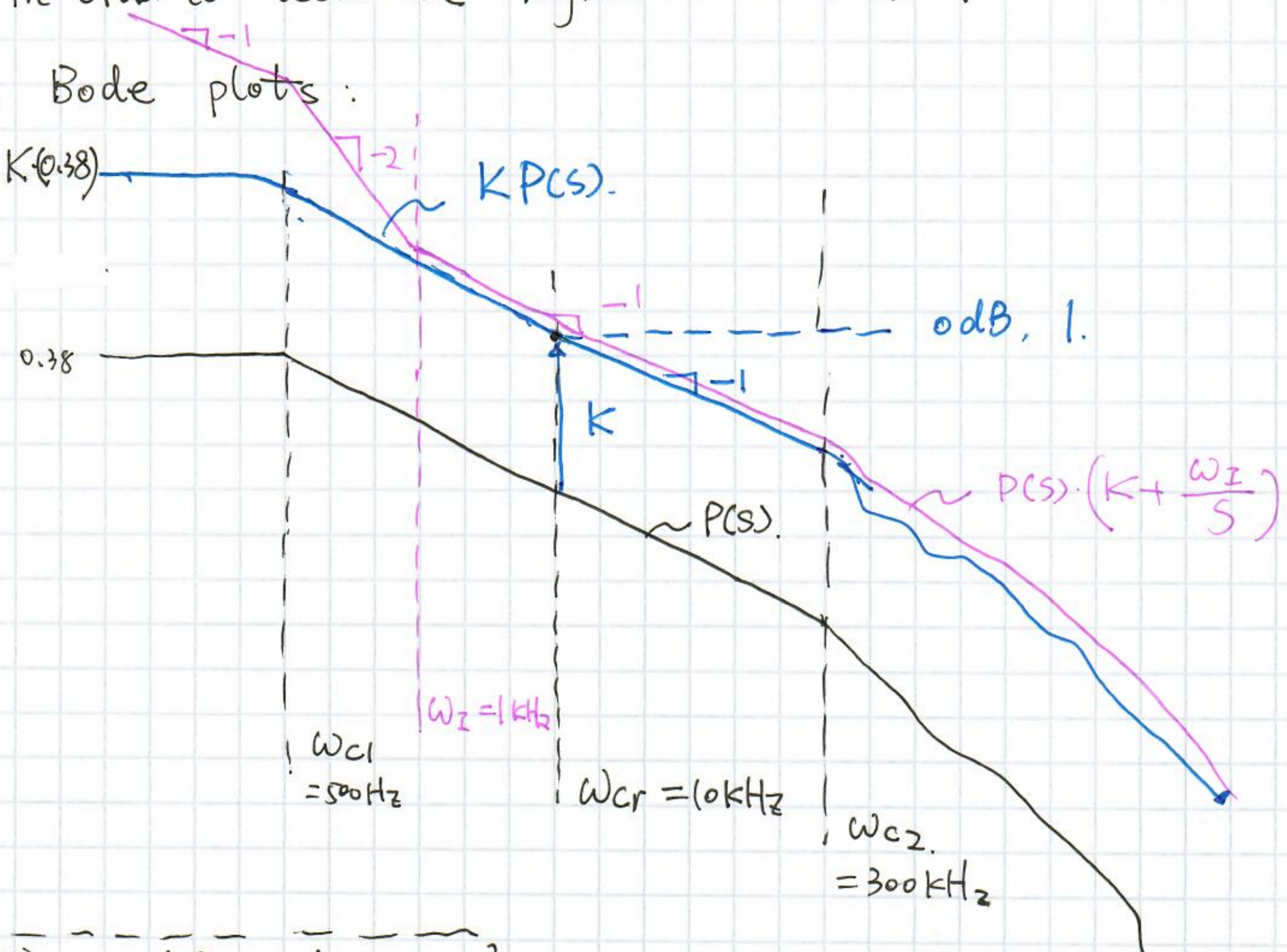
$$\omega_{c1} = \frac{R+R_S}{L} \cdot \frac{1}{2\pi} = 317\,Hz.$$

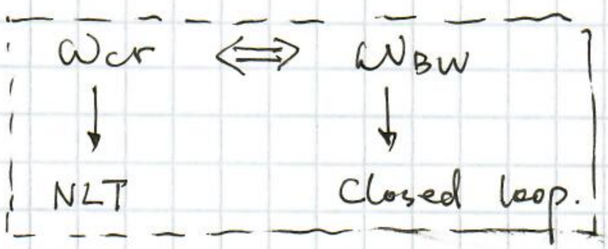


PI controller is used for current control.

①. C(S) changes the crossover frequency of P(S) in order to achieve higher bandwidth.

In Bode plots:





$$K = \frac{10k}{500} / (0.38) = 52.$$

Also, $K \left| \frac{10\,(0.2)}{5.2 + 1.6e^{-3}(2\pi \cdot 10k \cdot j)} \right| = 1$

In Bodes, P(S) is moved up by K to catch the selected ωcr. ωcr is the key to design controller.

In this course, we don't discuss optimization, which is a main topic of control people now, we rely on try & check.

The K value, or ωcr also related the device behaviour, such as slew rate of op-amp.

②. PI controller can remove steady state error as well.

If only Proportional control, $C(s) = K$

at very low frequency, $f \to 0$.

$$G_{DC} = \frac{1}{K \cdot P(0)} = \frac{1}{20} > 0. \quad e_{ss} \text{ exists.}$$

From Bodes, at $f \to 0$, $NLT(0) \to \infty$

so. $e_{ss} = 0$.

Integrator is used to change the low frequency behaviour.

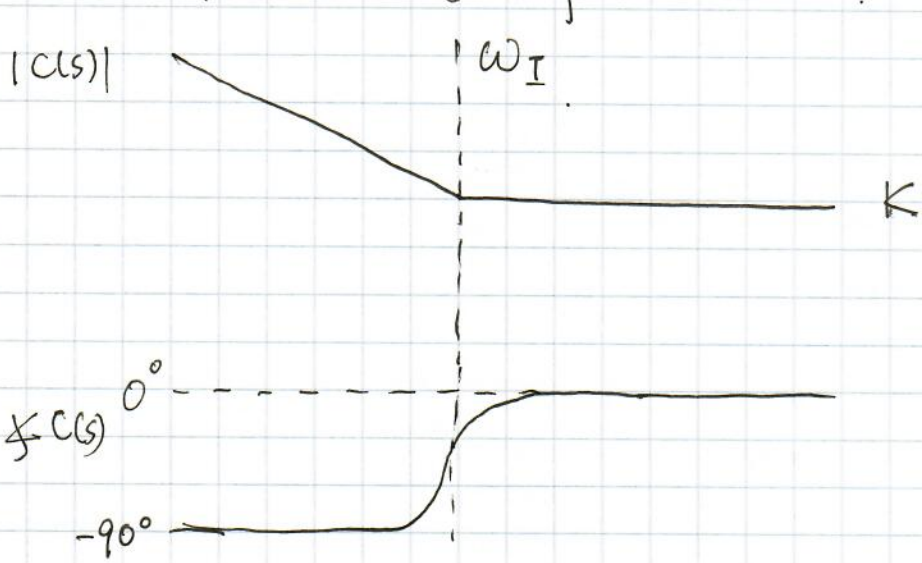
See the Bodes in last page.

The PI controller is defined as follows:

$$C(s) = K\left(1 + \frac{\omega_I}{s}\right).$$

Usually, we choose corner frequency of integrator (ω_I) much lower than crossover frequency (ω_{cr}) such as, $\omega_I = \frac{1}{10}\omega_{cr}$ to gurantee the Phase Margin.

So the Bodes of $C(s)$ is.



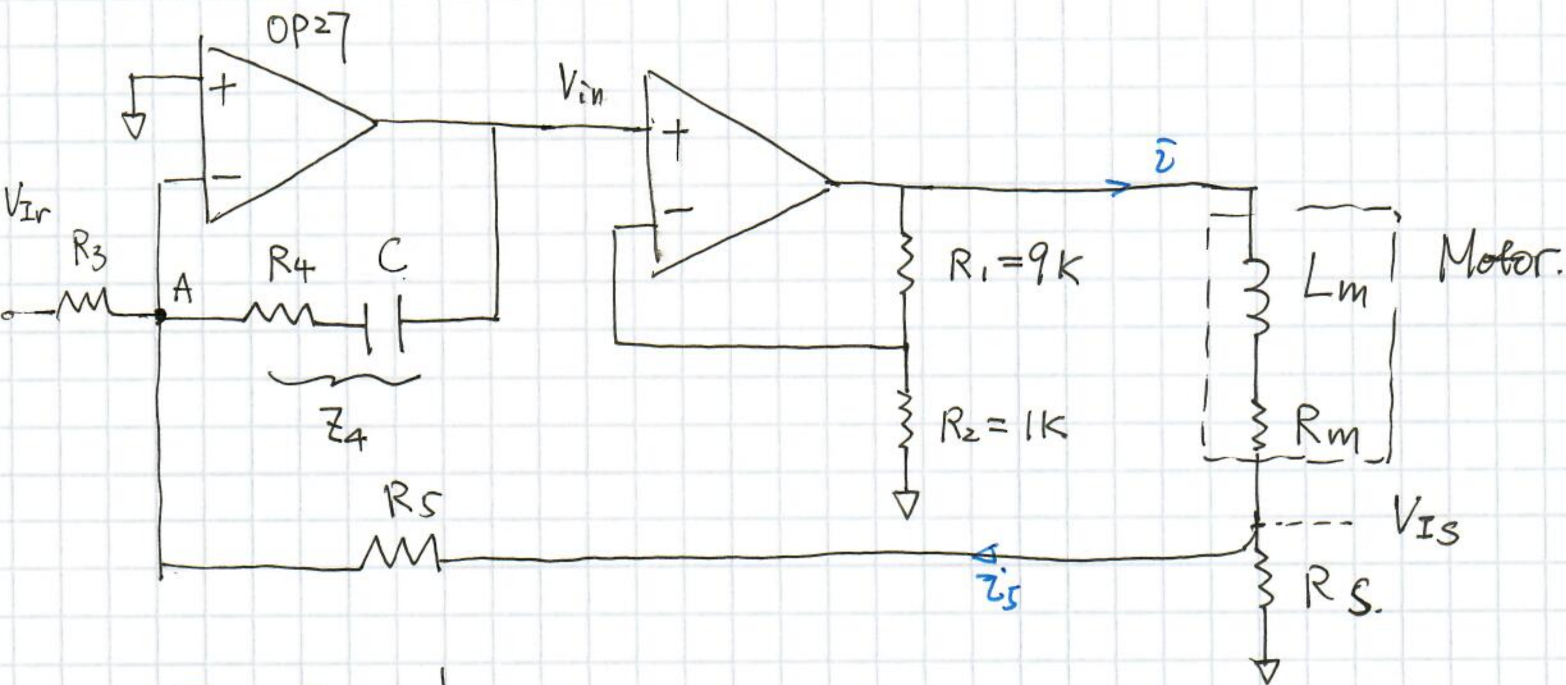
Stability consideration:

the plant around the crossover frequency is a first order system, and the PM = 90° theoretically if only using P item

It is easy to gurantee PM ≥ 60° if the ω_I is much lower than ω_{cr} such as $\omega_I = \frac{1}{10}\omega_{cr}$.

4. Current controller instrumentation.

Here we use analog circuit to implement circuit controller. See the below circuit:

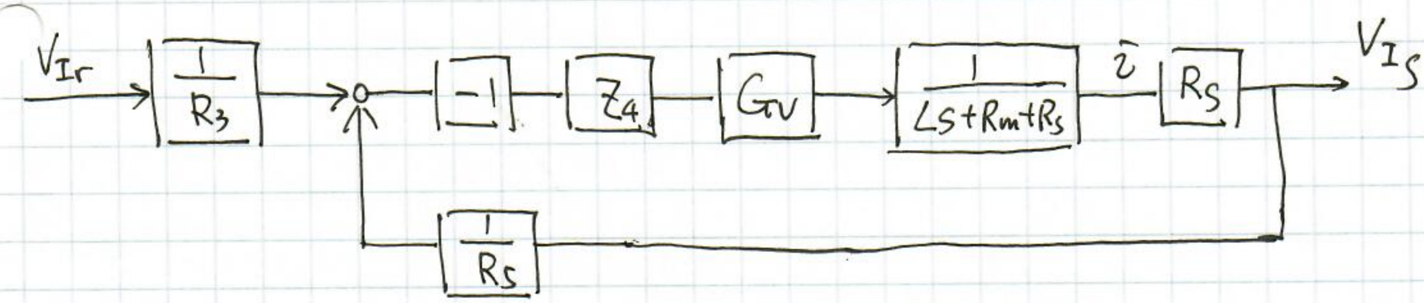


$$Z_4 = R_4 + \frac{1}{CS}$$

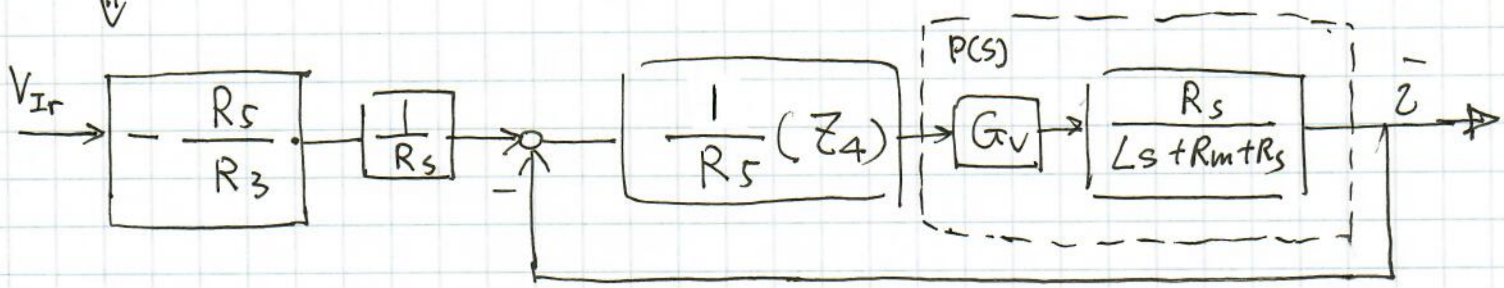
$$\frac{V_{Ir} - 0}{R_3} + \frac{V_{Is} - 0}{R_5} + \frac{V_{in} - 0}{Z_4} = 0, \quad \text{at point A.}$$

So $$V_{in} = -Z_4\left(\frac{V_{Is}}{R_5} + \frac{V_{Ir}}{R_3}\right).$$

The block diagram is :



$\Updownarrow$

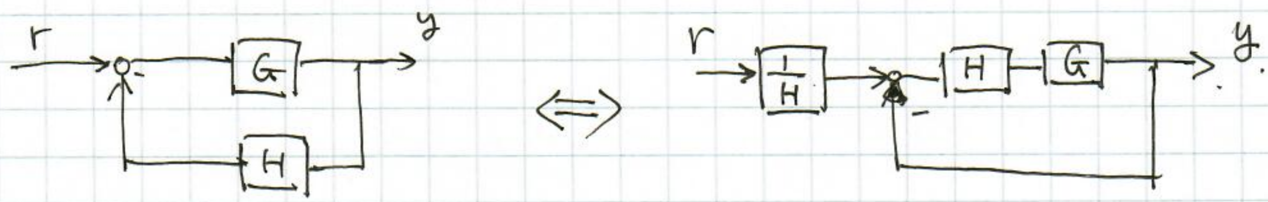


So, $C(s) = \frac{1}{R_5}\left(R_4 + \frac{1}{Cs}\right) = K\left(1+\frac{\omega_I}{s}\right)$.

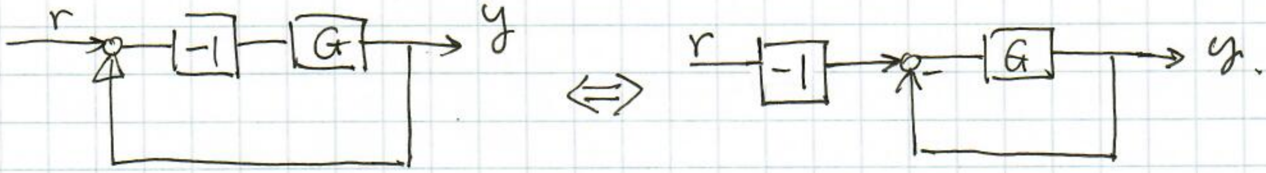
$K = \frac{R_4}{R_5}$, $\omega_I = \frac{1}{R_4 C}$.

Tips: for the Block diagram transformation.

①.



②.



$$\frac{y}{r} = -\frac{G}{1+G}$$

Comments of this current controller:

①. This is one way to implement, and other circuits can also do the same thing.

②. The output has inversed polarity to input.

③. The DC gain is modulated by $\frac{R_5}{R_3} \cdot \frac{1}{R_S}$

For convenience, we prefer $\left| \frac{i}{V_{I_r}} \right| = 1$ at DC

so, $\frac{R_5}{R_3} \cdot \frac{1}{R_S} = 1$.

④. According to the circuit, $R_5 \gg R_S \Rightarrow i_5 \ll i$

⑤. For a common DC motor, $i \approx (\cdot)A$, so the reasonable sensing resistance, $0.1 \leq R_S \leq 1$

Then $R_5 \doteq 1K\Omega$ is reasonable.

⑥. Parameters realization:

- select Capacitance first because of less options.
  CBB (plastic) cap is prefered for small charge leakage.
- OP27 is considered as an ideal op-amp because the crossover frequency of NLT is much lower than OP27 bandwidth.

Just a quick calculation: Let $R_5 = 1k$, $R_3 = 5K$, $R_S = 0.2\Omega$

$$R_4 = K \cdot R_5 = 50 \cdot (1k) = 50k$$

$$C = \frac{1}{R_4 \omega_I} = \frac{1}{50K \cdot (6280)} = 3.2nF$$

ω_I in "rad/s"

Summary of Current controller design:

1. Objectives:
- Improve dynamic response → 10kHz BW.
- Accurately track reference → $e_{ss} = 0$
- Ensure stability → $PM = 60°$

2. Design procedure

①. Understand plant of the system ($P(s)$).

- Draw Bode plots to show frequency response.

②. Select a controller according to $P(s)$ and specs.
Roughly speaking, designing a controller is reshape the Bode plots of NLT in frequency domain.

• Tips: 1). We always rely on NLT to design a controller,
$NLT = C \cdot P$ in this general system.
NLT is not a closed loop transfer function,
The closed loop TF is: $G_{cl} = \dfrac{CP}{1+CP} = \dfrac{y}{r}$

2). Two techniques can be used to reshape NLT in Bodes
- a. move magnitude up and down → K
- b. Change the slope of magnitude: $\left(1+\dfrac{\omega_I}{s}\right)$, $\left(\dfrac{\frac{s}{\omega_c/\alpha}+1}{\frac{s}{\alpha\omega_c}+1}\right)$

↓ phase is also changed.

Integrator

Lead-Lag Compensator.

③. Details about PI controller design:

Step #1: select NLT crossover frequency, ω_{cr}

#2: calculate $|P(\omega_{cr})|$

#3: calculate $K = \frac{1}{|P(\omega_{cr})|}$

#4: select integrator corner frequency, $\omega_I = \frac{\omega_{cr}}{10}$.

#5: check the phase margin by realize $NLT(j\omega_{cr})$, $PM = \angle NLT(j\omega_{cr}) + 180°$

Outline of analog instrumentation:

One double-side crib sheet for midterm exam.

1. Mathematical tools: block diagram, transfer function, Laplace transform.
2. 1^{st} order system: RC, RL circuit.
3. Op-amps' circuits: consider a nonideal op-amp, with frequency response of G.

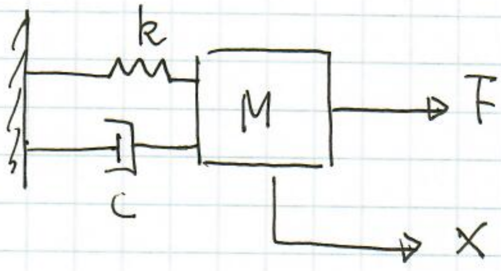
   Block diagrams of inverting opamp circuit & noninverting amp ckt.
4. control properties:

   NLT, crossover frequency, closed loop T.F., B.W,
5. Stability: Nyquist criteria, Phase margin, gain margin.
6. PI controller design just discussed.

Lecture #11 : 2^{nd} order system of 1 DoF mechanical system

Jian Gao
2019.1.

1. 1 DOF mechanical system.



1 DOF means one mass moving along one axis.

Transfer function : $G = \frac{X}{F} = \frac{1}{Ms^2+cs+k}$

Also, $G = \frac{X}{F} = \frac{1}{k} \cdot \frac{1}{\frac{s^2}{\omega_n^2} + 2\frac{\zeta s}{\omega_n} + 1} = \frac{1}{M} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

where natural frequency, $\omega_n = \sqrt{\frac{k}{M}}$,

damping ratio, $\zeta = \frac{c}{2\sqrt{kM}}$

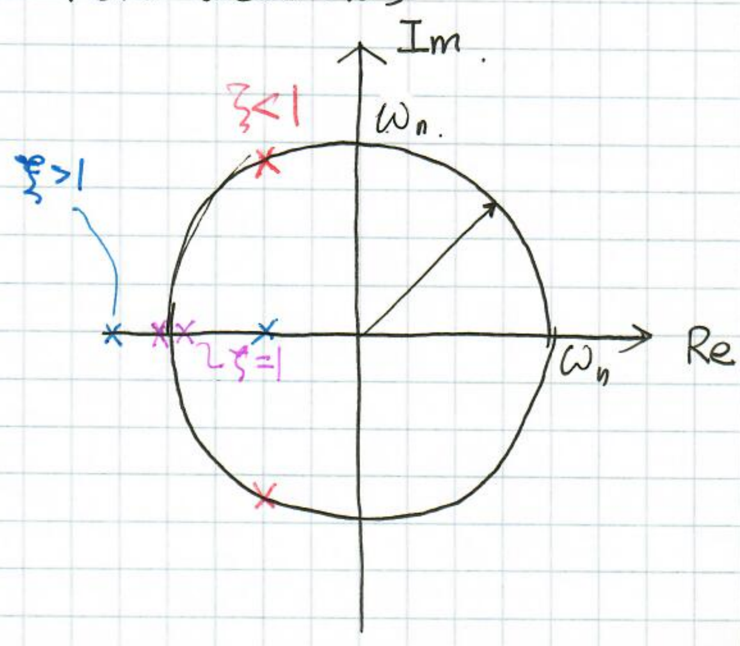
Poles of transfer function. $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$.

Under damped, $\zeta < 1$

critical damping $\zeta = 1$

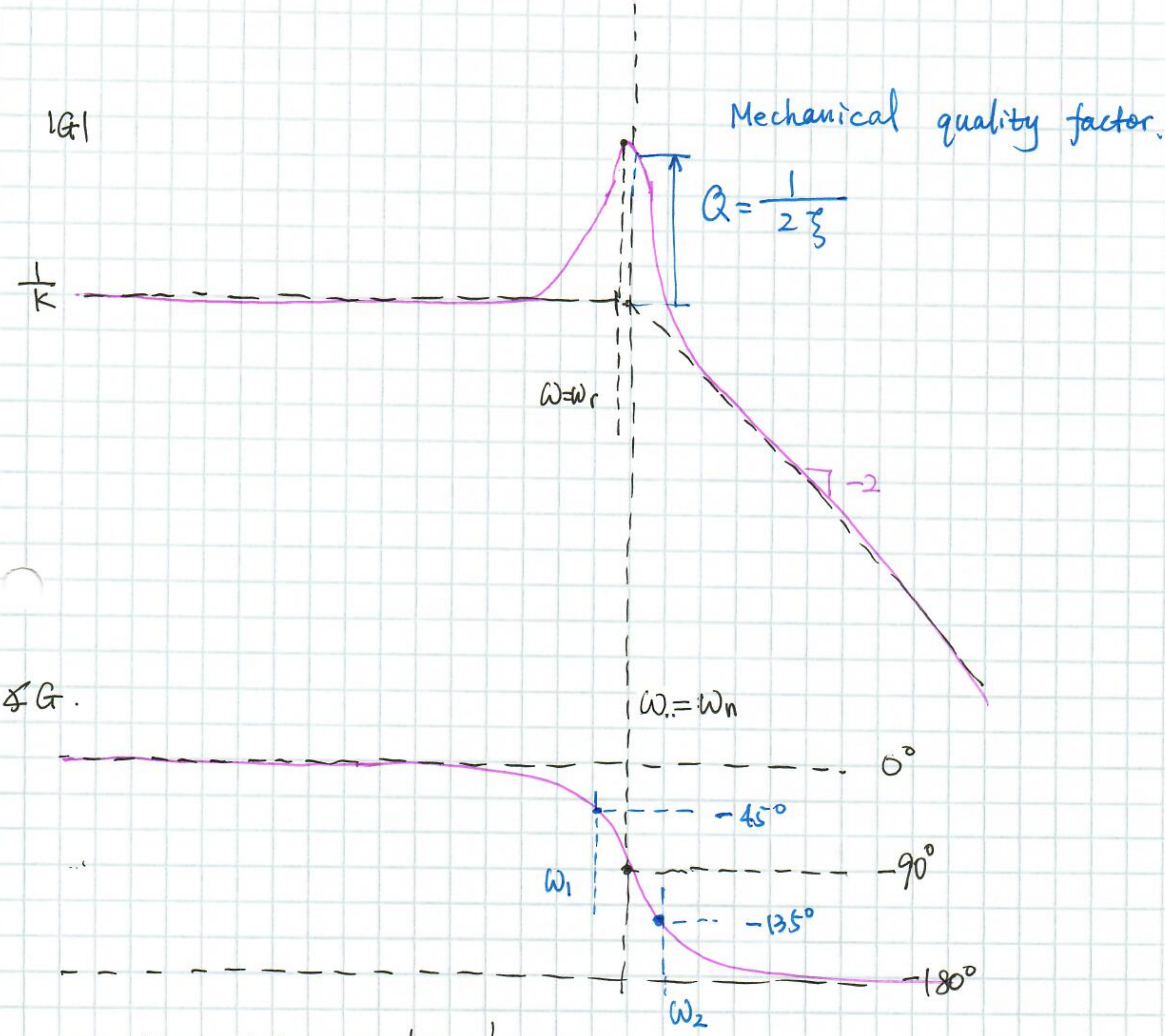
Over damped $\zeta > 1$

Pole locations. $\omega_d = \omega_n\sqrt{1-\zeta^2}$



2. Frequency response of 2^{nd} system.

For $\zeta < 1$, under damped



At $\omega = \omega_n$, $G(j\omega_n) = \frac{1}{K} \cdot \frac{1}{2\xi j} =$

Maximum point, $\omega = \omega_r = \omega_n \sqrt{1 - 2\xi^2}$, $\quad 0 < \zeta < 0.7$

$$|G(j\omega_r)| = \frac{1}{2\xi} \cdot \frac{1}{\sqrt{1-\xi^2}} \cdot \frac{1}{K}$$

Peak point is not at natural frequency, but they are close.

Reconstruct the system from phase:

At $\omega=\omega_1$, $\measuredangle G(j\omega_1) = -45°$, $\omega=\omega_n \Rightarrow \measuredangle G(j\omega_n) = -90°$

At $\omega=\omega_2$, $\measuredangle G(j\omega_2) = -135°$.

$$G(j\omega) = \frac{1}{k}\cdot\frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right] + 2\zeta\left(\frac{\omega}{\omega_n}\right)j}$$

If imaginary part equal to real part, respect phase of $-45°$ and $-135°$

$$\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 = 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2$$

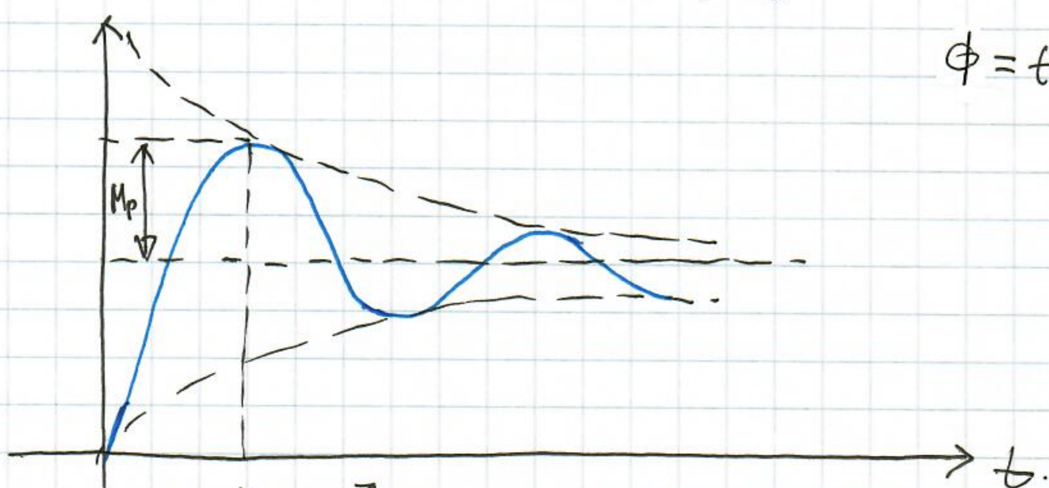
Let $r=\left(\frac{\omega}{\omega_n}\right)^2$, then $r^2-2(1+2\zeta^2)r+1=0$

$$\begin{cases} r_1+r_2 = 2(1+2\zeta^2) \\ r_1\cdot r_2 = 1 \end{cases} \Rightarrow \begin{cases} (\sqrt{r_1})^2+(\sqrt{r_2})^2-2\sqrt{r_1r_2} = 4\zeta^2 \\ \sqrt{r_1r_2} = 1 \end{cases}$$

$$\Rightarrow \begin{cases} \omega_1\omega_2 = \omega_n^2 \\ \omega_2-\omega_1 = 2\zeta\omega_n \end{cases}$$

3. Step response: $x(t) = \frac{1}{k}\left[\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)+1\right]$.

$$\phi = \tan^{-1}\sqrt{\frac{1-\zeta^2}{\zeta^2}} + \pi.$$



$t_p = \frac{\pi}{\omega_d}$ Overshoot: $M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

4. Practice Draw bode plots of 2^{nd} system.

$$G = \frac{1}{5s^2 + 10s + 500}$$

Step 1. convert to standard form.

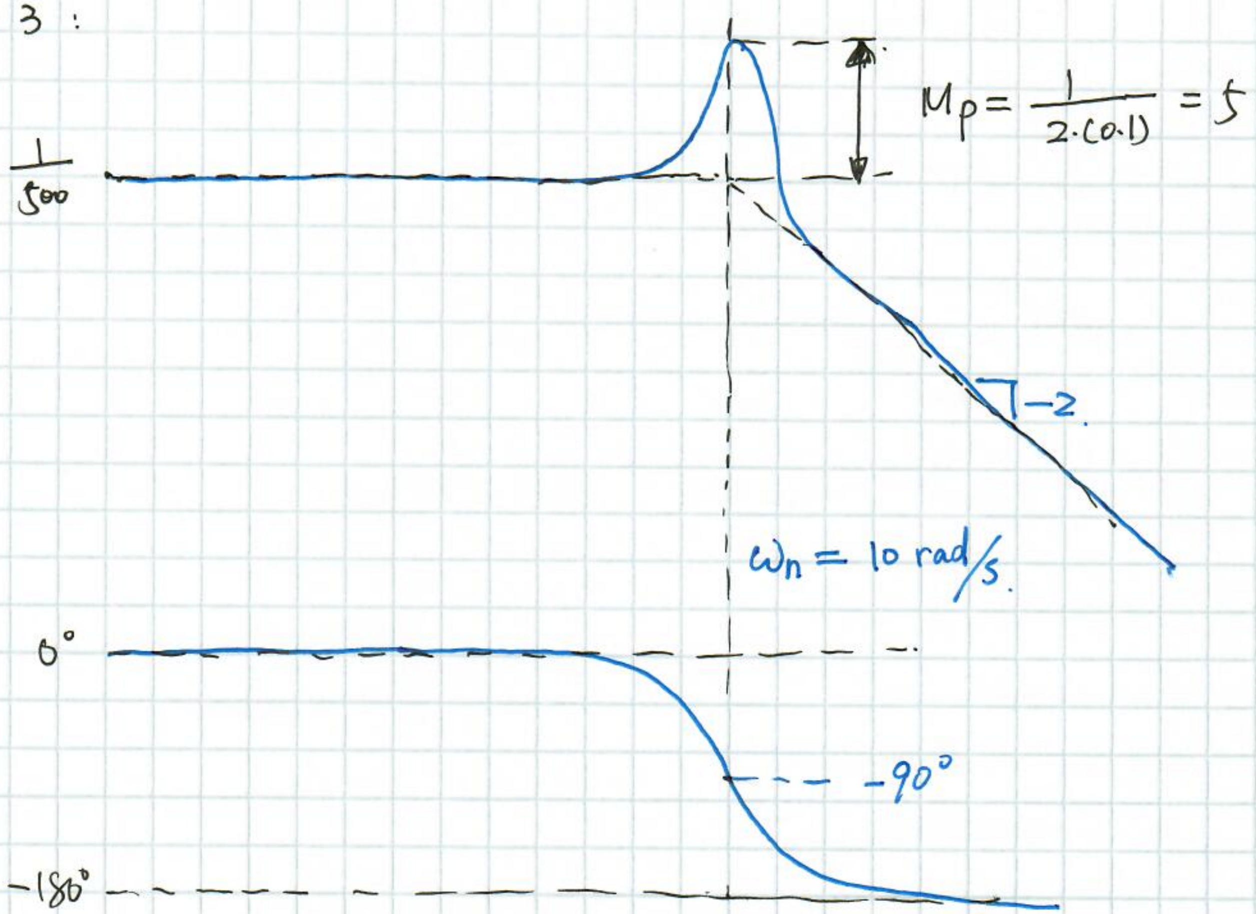
$$G = \frac{1}{500} \cdot \frac{1}{\frac{s^2}{100} + 2 \cdot \left(\frac{0.1}{10}\right) s + 1}$$

Step 2.

$\zeta = 0.1, < 1$, under damped system.

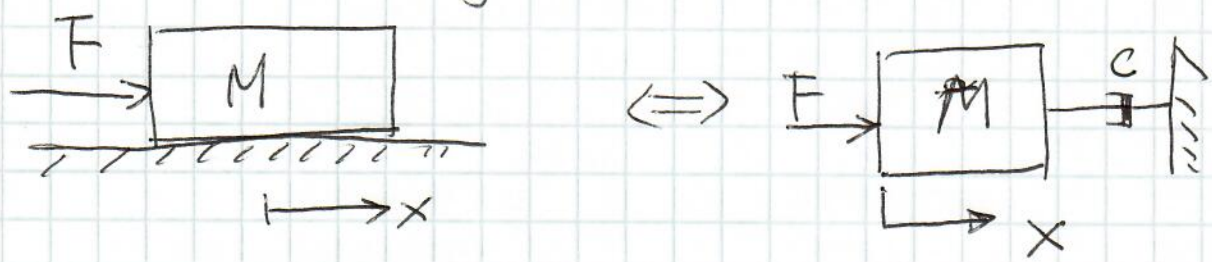
$\omega_n = 10$ rad/s ≈ 1.6 Hz.

Step 3:



5. An example of 2^{nd} order sys.

Consider a motion stage: such as ball-scraw drive

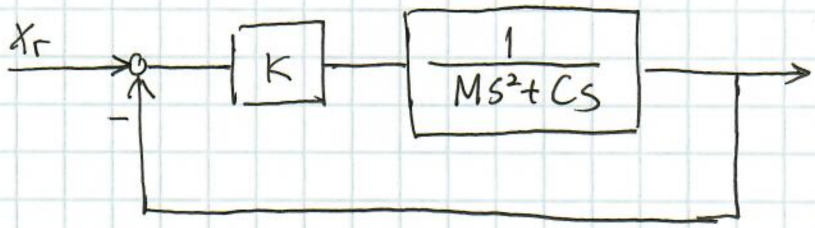


Transfer function of the open loop system:

$$\frac{X}{F} = \frac{1}{Ms^2+Cs}.$$

- If a proportional controller is used to close the loop,

the block diagram will be:



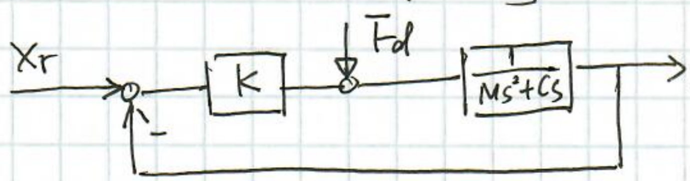
$$\omega_n = \sqrt{\frac{K}{m}}$$

$$\zeta = \frac{C}{2\sqrt{Km}}$$

$$NLT = \frac{K}{Ms^2+CS}, \quad G_{cl} = \frac{K}{Ms^2+Cs+K}.$$

The controller define the natural frequency & damping ratio.

- A disturbance input



Stiffness of the system is: (Dynamic)

$$\frac{F}{X} = \frac{1+\frac{K}{Ms^2+CS}}{\frac{1}{Ms^2+Cs}} = Ms^2+Cs+k$$

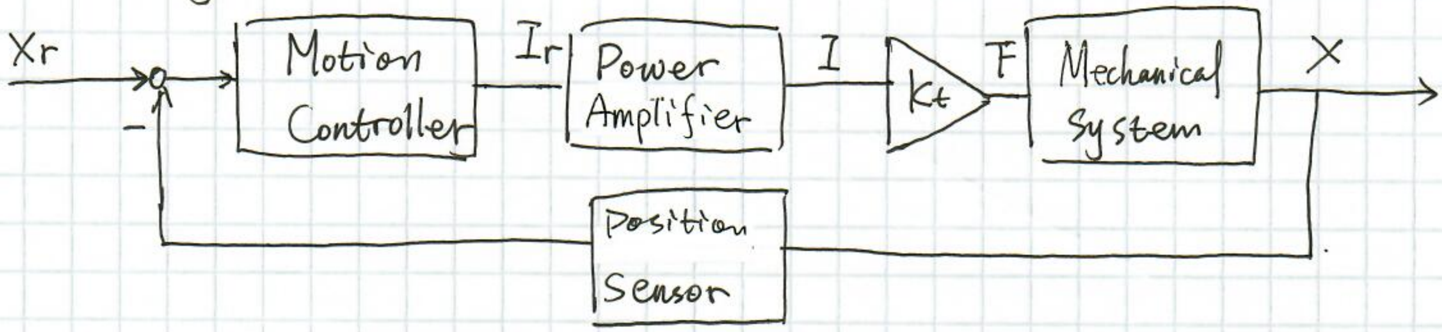
"K" also changes dynamic stiffness.

How about replace K by C(s).

Lecture 12. Loop shaping techniques for motion control sys.

1. A typical motion control system:

Block diagram:

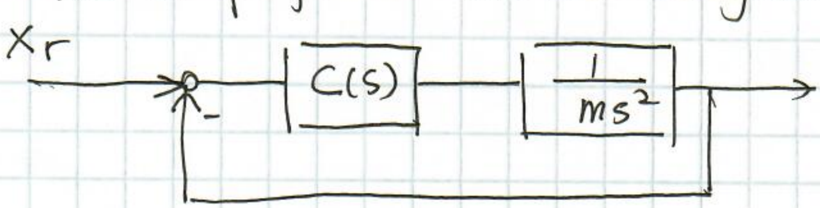


- Current controller is embedded in P.A., which is discussed in the previous lectures, and its bandwidth is up to kHz.
- K_t is the motor constant to show the linear relationship between F and I : $F = K_t \cdot I$
- Mechanical system is a 2^{nd} order system, when using transfer function between X and F

For a simpliest case : $G_m(s) = \frac{X}{F} = \frac{1}{ms^2}$

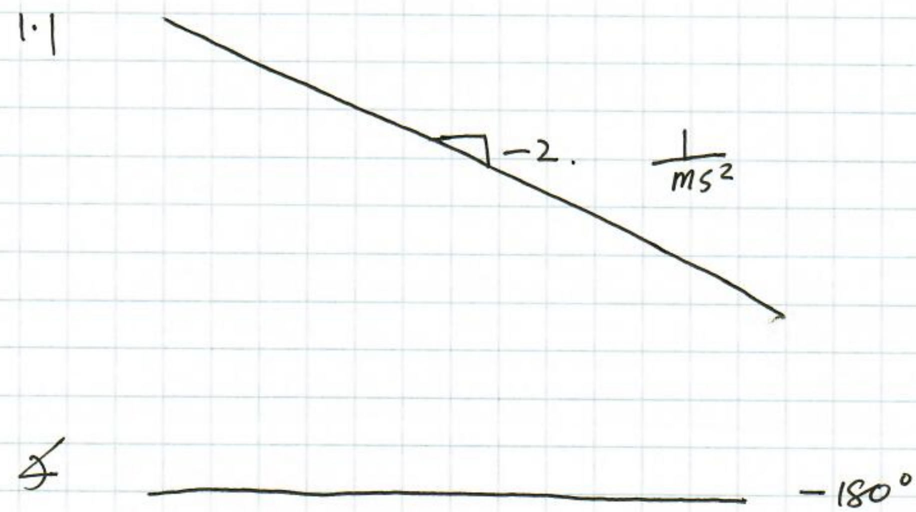
Regardless damping c and stiffness K.

So the simplified block diagram becomes:



$$NLT = C(s) \cdot \frac{1}{ms^2}$$

Bode plots of this simpliest mechanical system.



Controller design concerns:

If we use PI controller as we did for current control, the. PM will never touch a reasonable value i.e. $PM \geq 60^\circ$

Therefore, the desired controller should reshape the phase of NLT bode plots to obtain enough PM at gain crossover frequency.

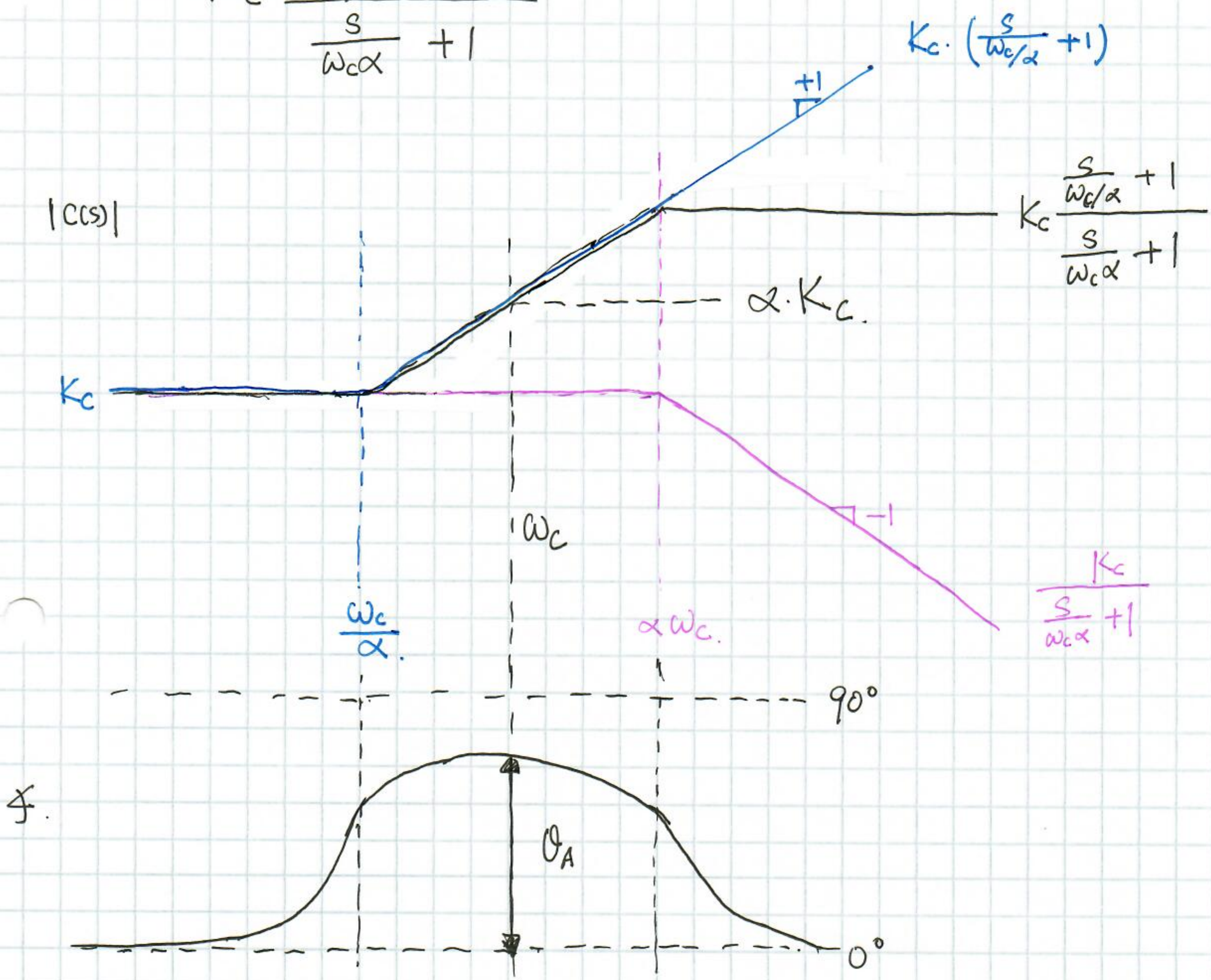
2. Lead-lag compensator.

$$C(s) = K \cdot \frac{\frac{s}{\omega_c/\alpha} + 1}{\frac{s}{\alpha\omega_c} + 1}$$, ω_c is the gain crossover frequency.

In engineering practice, the Key to design a controller is to select a crossover frequency from the measured Bode plots.

Bode plots of Lead-Lag compensator:

$$C(S) = K_c \cdot \frac{\frac{s}{\omega_c/\alpha} + 1}{\frac{s}{\omega_c\alpha} + 1}$$



θ_A is the advanced angle at ω_c

$$\theta_A = \measuredangle \frac{1 + \frac{j\omega_c}{\omega_c/\alpha}}{1 + \frac{j\omega_c}{\omega_c \cdot \alpha}} = \measuredangle(1+\alpha j) - \measuredangle\left(1+\frac{1}{\alpha}j\right)$$

θ_A depends on "α" not "ω_c"

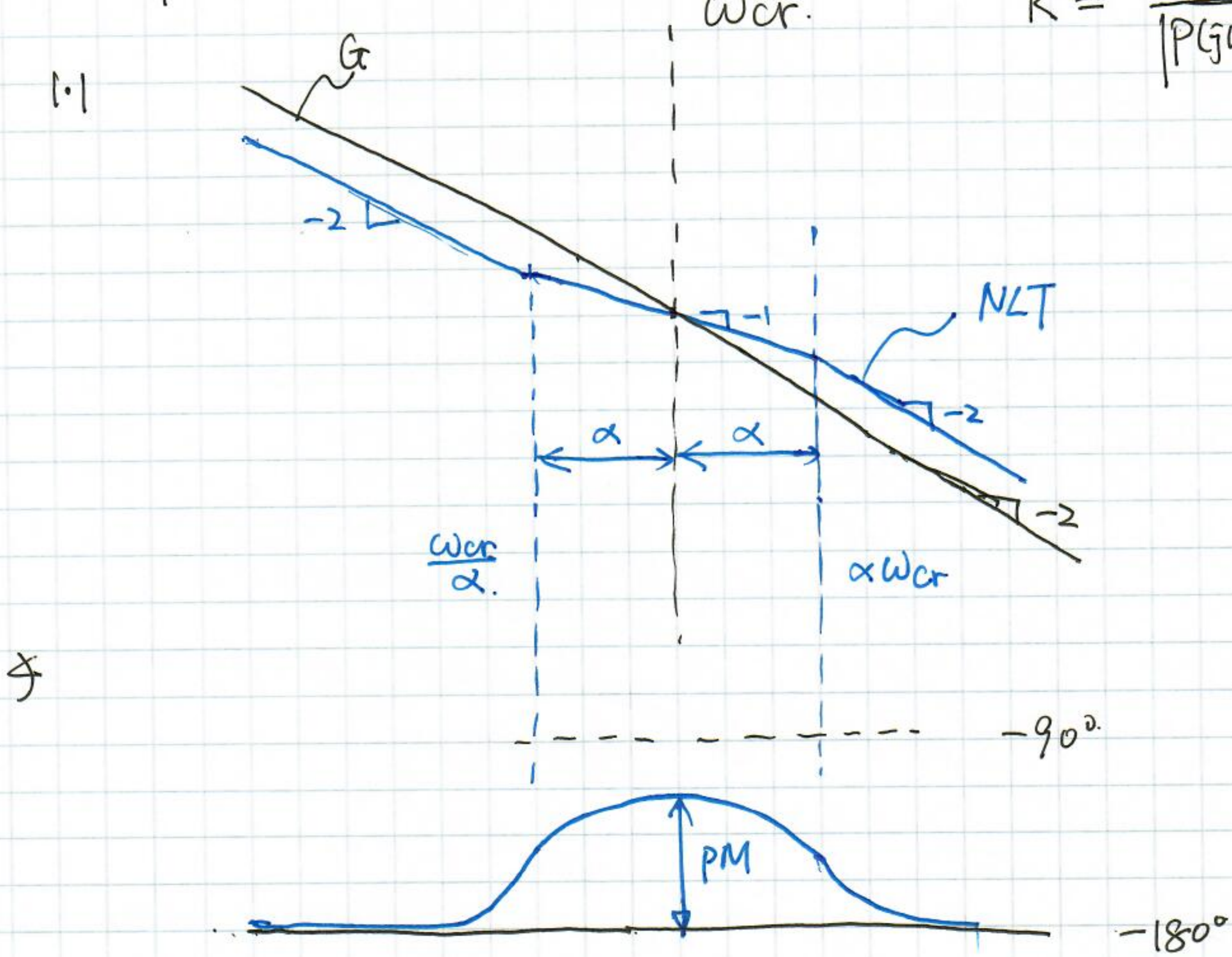
Usually we select $\alpha = 3, 4, 5$.

α	θ_A
3	$53°$
4	$62°$
5	$67°$

$|C(j\omega_{cr})| = \alpha$.

3. NLT with $C(s)$:

$K = \frac{1}{|P(j\omega_{cr})|} \cdot \frac{1}{\alpha}$



Let $\alpha = 4$, $PM = 62°$

With lead-lag compensator, the shape of NLT Bodes has smaller slope around crossover frequency to get enough phase margin.

But too large "α" will pick noise at high frequency.

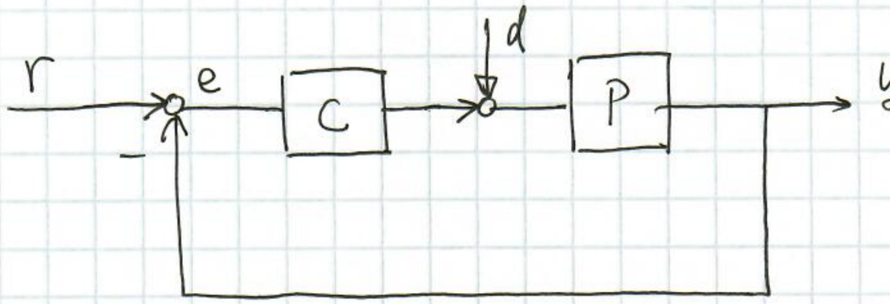
4. Zero steady-state error. ($e_{ss}=0$)

For the mechanical system $G(s)=\frac{1}{ms^2}$, there is no e_{ss} for a reference step input

$NLT = C(s)\,G(s)$,

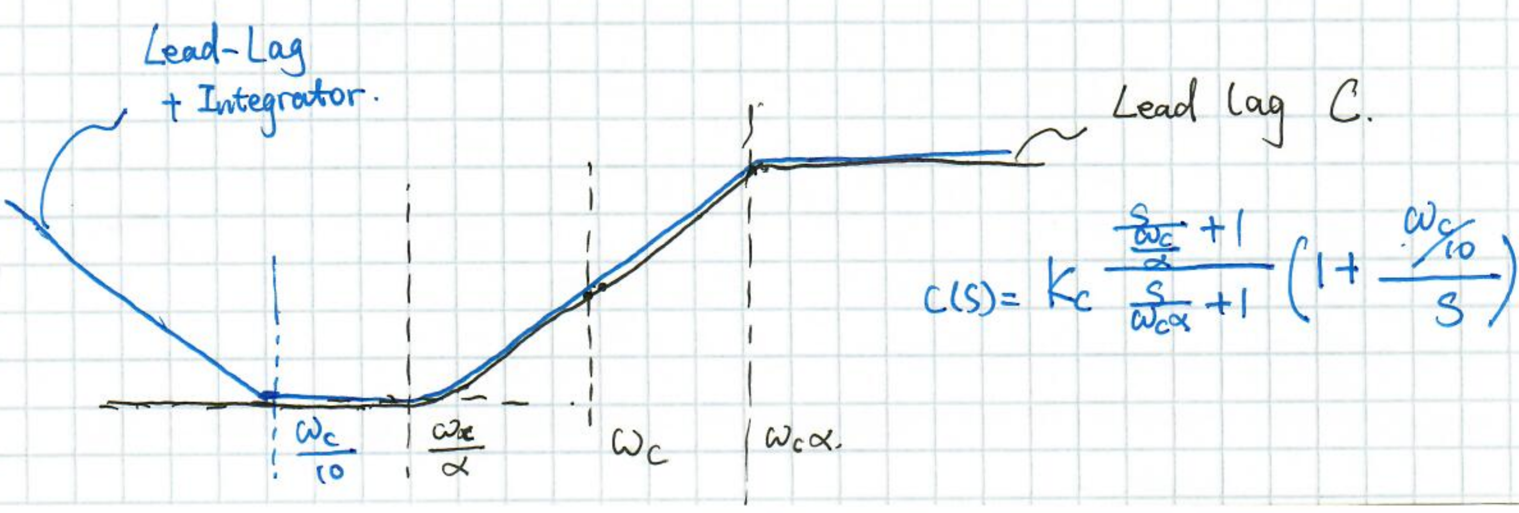
$s\to 0$, $NLT(0)\to\infty$, $\frac{y}{r}=\frac{NLT}{1+NLT}$

However, the steady-state error happens if there is a disturbance input.



$\frac{e}{d}=\frac{P}{1+PC}$, If $s\to 0$, $\frac{e}{d}=\frac{1}{C(0)}=\frac{1}{K_c}$.

Therefore, an integrator should be introduced to improve the low frequency behaviour.



Lecture #14: Digital controller implementation.

1. • Motion control systems are usually implemented by digital controller

• Benefits of digital system:

① More flexible: changing parameters fast compared to replace resistors & capacitors.

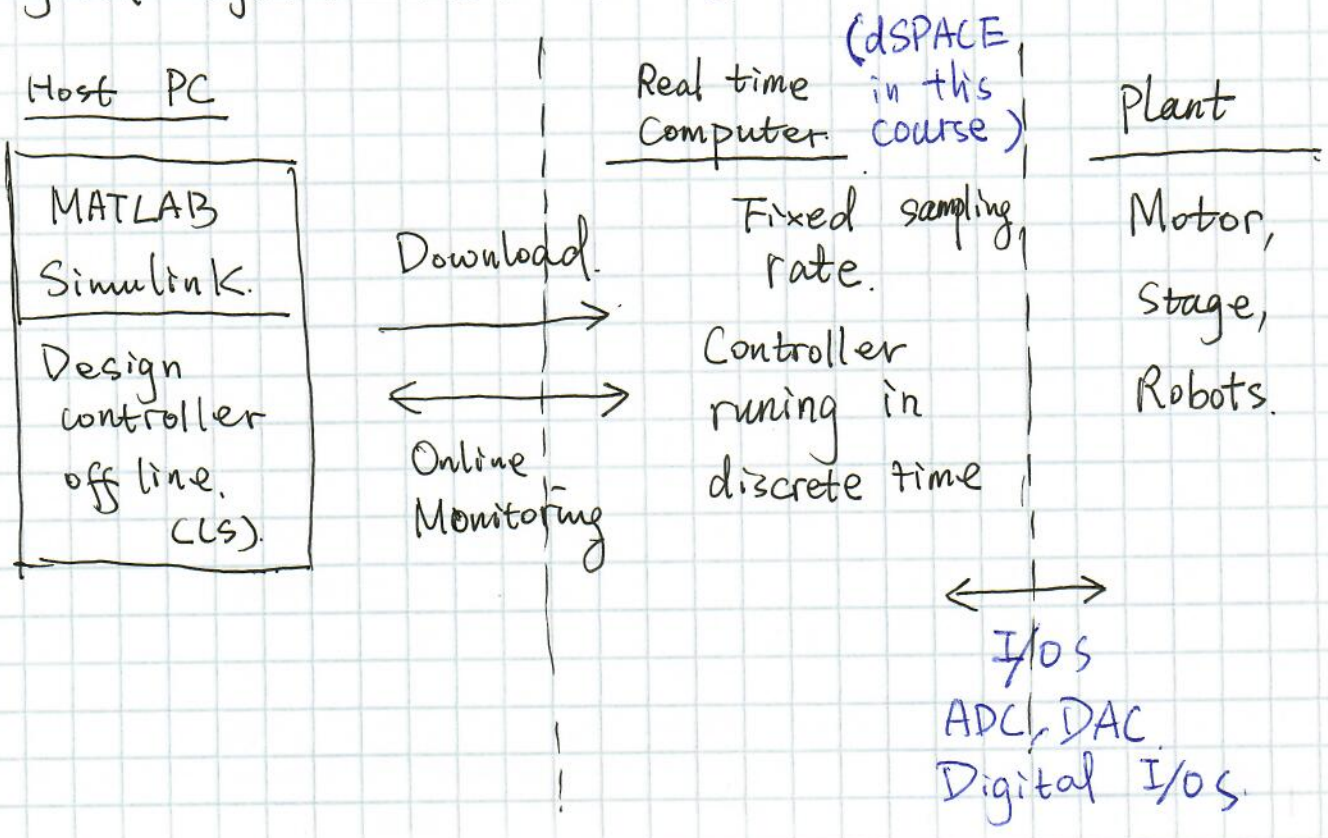
② Independence of components quality. Not a lot of temperature related properties.

③ Easy to design complex systems

④ Fast to prototype.

• Drawbacks: expansive, performance relying on sampling rate, requirements of I/Os.

2. Digital system architecture



3. Digitalization:

— The system is understood in S-domain, and the controller is designed in S-domain, but it has to be implemented in Z-domain, which corresponds to discrete time.

— Different ODE Numerical methods can be used in the digital control system such as Euler's method and tustin's method., etc.

In simulink, the ODEs methods can be selected.

— ZOH with Z transform is another usual way to digitalize the system, this has been introduced in previous course.

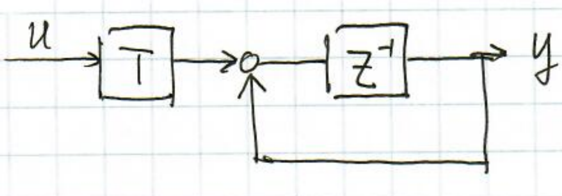
Tips: Since we rely on commercial tool chain (Simulink + dSPACE) to fast prototype mechatronic systems, using existing ODE numerical methods is easier with more intuition in S domain.

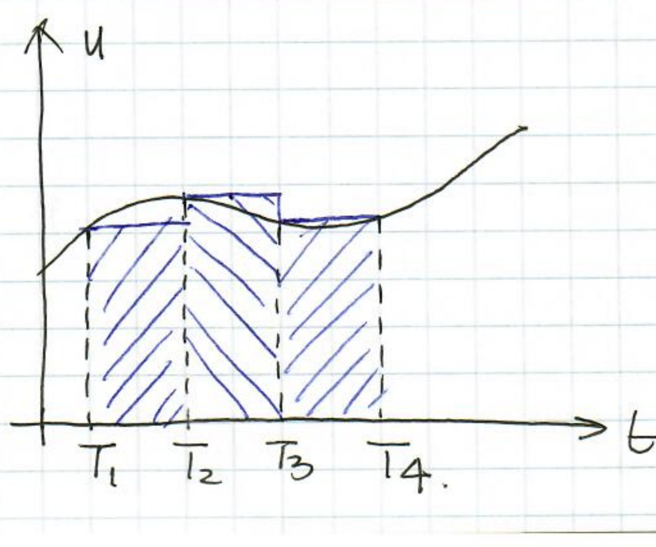
- Example ①. Forward Rectangular Rule, Euler Method (ODE1).

$$u \rightarrow \boxed{\frac{1}{s}} \rightarrow y.$$

In discrete time domain:

$$y(n+1) = y(n) + T \cdot u(n).$$





Then $$\frac{y(n)}{u(n)} = \frac{T \cdot z^{-1}}{1 - z^{-1}}$$

With ODE 1

s domain	z domain
$\frac{1}{s}$	$\frac{T z^{-1}}{1 - z^{-1}}$
s	$\frac{1}{T}(z-1)$

Example ②: $G = \frac{1}{s+100}$

· CT:

Frequency response: $$u = u_0 \sin\omega t \longrightarrow \boxed{G} \longrightarrow y = y_0 \sin(\omega t + \phi)$$

Let $s = j\omega$, $G(j\omega) = \frac{1}{j\omega + 100}$

Discrete Time:

$$u(k) = u_0 \sin(\omega \cdot T \cdot k)$$

$$y(k) = Y_0 \sin(\omega T \cdot k + \phi)$$

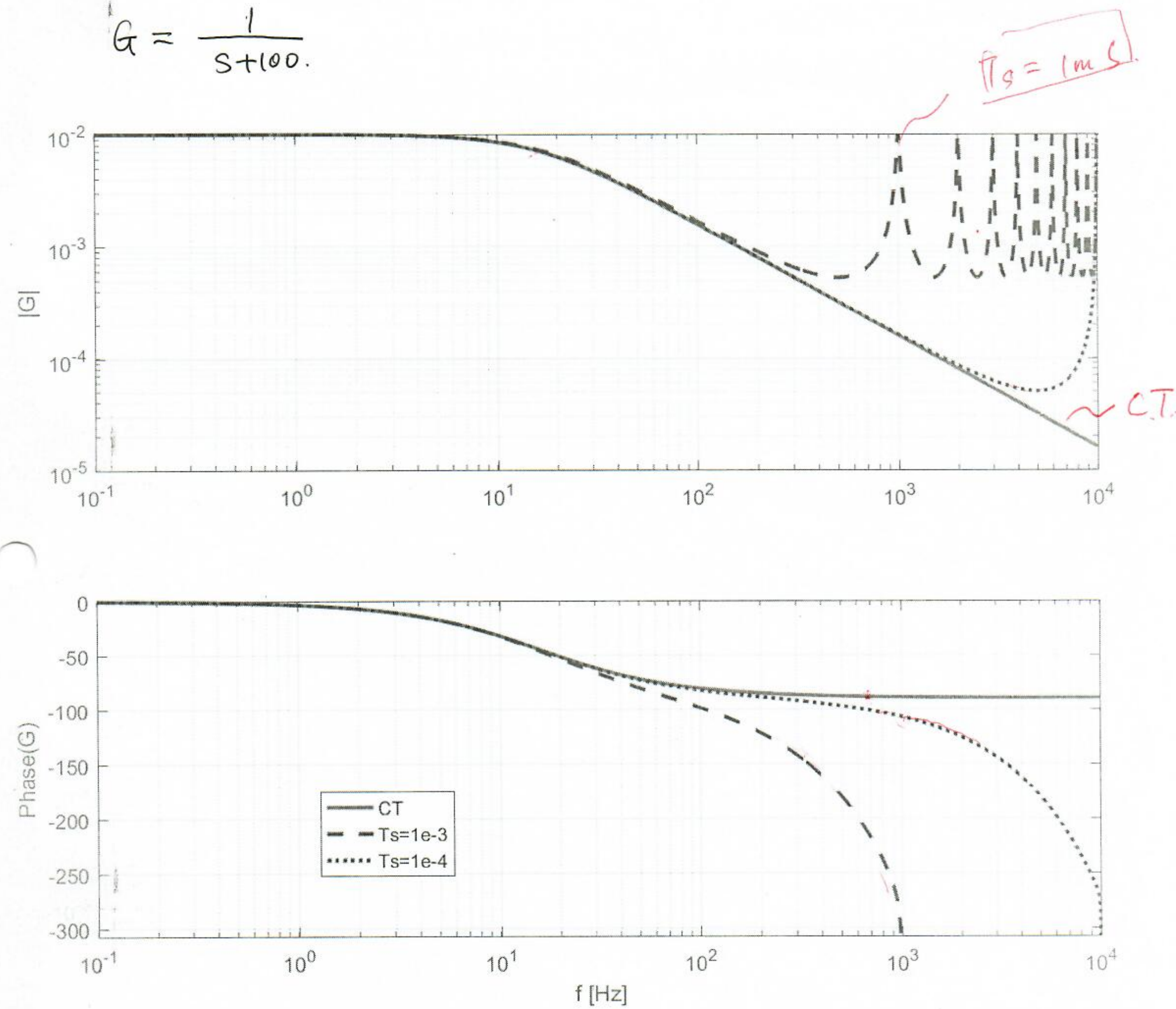
Let $z = e^{j\omega T}$, T is the sampling time

$$G_d(j\omega) = \frac{1}{\frac{1}{T}(e^{j\omega T} - 1) + 100}$$

e^{jT} is a phase delay caused by sampling cycle.

This will affect the stability by reducing PM.

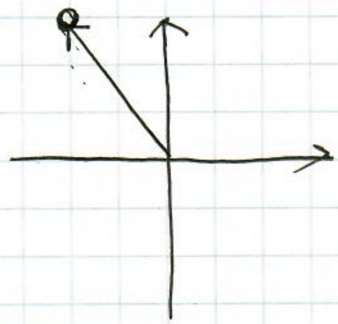
$G = \frac{1}{s+100}$



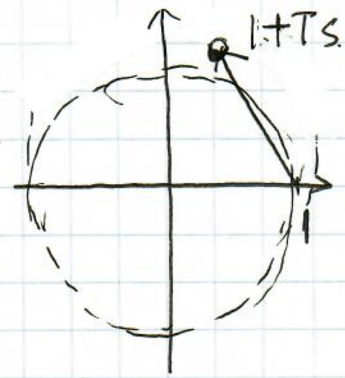
4. Stability of digital system:

If using Euler Method : $z := 1 + Ts$

s domain: $\Longleftrightarrow$ z domain.



Stable.



unstable.

A stable pole in s-domain is mapped to a unstable pole in z-domain.

Reducing sampling time can make the system stable again!

An Example: Use ODE 1, $T_s = \frac{1}{1000}$ Sec.

$$G(s) = \frac{1}{0.0016s+5}, \quad \text{pole: } s = -3125$$

Let $s = \frac{z-1}{T_s}$

$$\Rightarrow G(z) = \frac{1}{0.0016\left(\frac{z-1}{T_s}\right)+5} = \frac{1}{1.6z+3.4}$$

pole: $z = -2.125$, unstable !

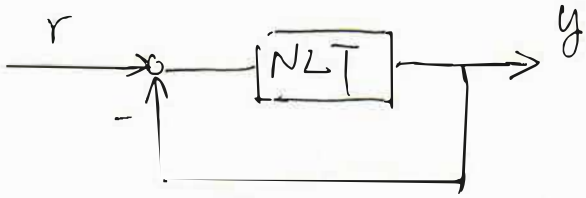
Lecture #15: System identification & Real-time Simulation.

Jian Guo

2019.2.

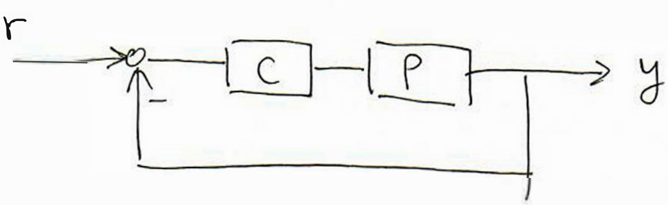
1. Frequency response measurement.

A simple model:



NLT is the key to design a feedback control system, since it can show the bandwidth of the closed loop system by gain crossover frequency and the stability by phase margin.

Normally, to design the controller $C(s)$, we need the frequency response of $P(s)$.



Methods of frequency response measurement:

① Sinusoidal sweeping:

$$u = U_0 \sin\omega t \longrightarrow \boxed{NLT} \longrightarrow y = Y_0 \sin(\omega t + \phi)$$

$$\left|\frac{Y}{U}\right| = \frac{Y_0}{U_0}$$

$$\angle\left(\frac{Y}{U}\right) = -\phi$$

Advantages:
- Better Signal-noise ratio
- Easy to control input signal amplitude.

Disadvantage: Slow, need to measure point by point.

② White Noise

$$W(t) \longrightarrow \boxed{NLT} \xrightarrow{y(t)} \qquad E[W(t)] = 0, \quad E[W(t)W^T(t)] = Q.$$

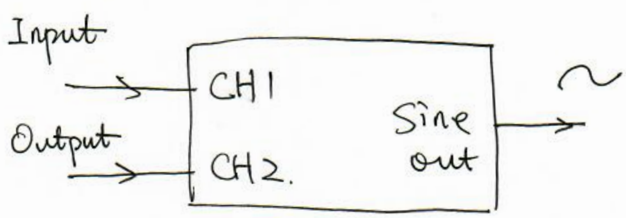
W(t) contains multiple frequency components.
and W(t) is zero mean and finite variance.

Advantages: Fast, high resolution of frequency

Disadvantages: Low SNR, and coherence of signals need to be check.

In this course, Sine sweeping will be implemented to identify the system.

. Dynamic signal analyzer.

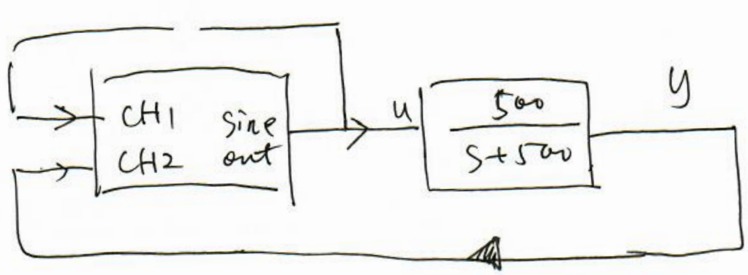


DSA

The DSA is embedded in realtime computer, and it can produce sine functions with different frequencies & amplitudes.

Multiple channels can be used to measure signals.

Example:



$$\frac{y}{u} = \frac{CH2}{CH1}$$

3. Simulink setup real time computer:

Fast prototyping relies on commercial real-time computer such as dSPACE, xPC ~~target~~, TwinCAT, NI DAQ.

They usually have Simulink interface in order to reduce debug time.

Setup procedure:

① Select target file for code generation.

② Select ODE solver

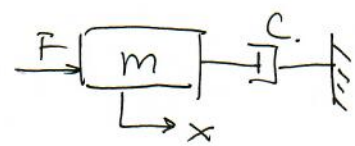
③ Set sampling time

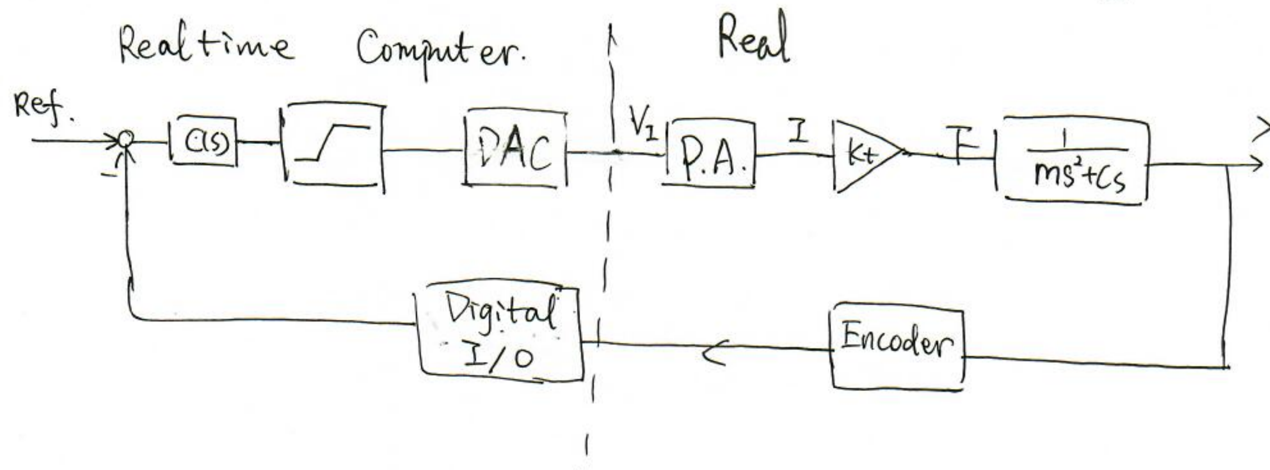
④ Develop model in Simulink.

⑤ Compile and download

4. Example:

Consider a moving mass system.





Without using real experimental setup, the system can simulated in dSPACE. and the frequency response can be measured by DSA.

An Example of real-time simulation:

Consider a moving mass with friction driven by a motor. The transfer function of motor is: $G_a = \frac{500}{s+500}$. The mass and damping are 3.5 kg and 10 N/(m/s).

In Simulink, the system is modeled as following figure. The DSA is used to measure frequency responses.

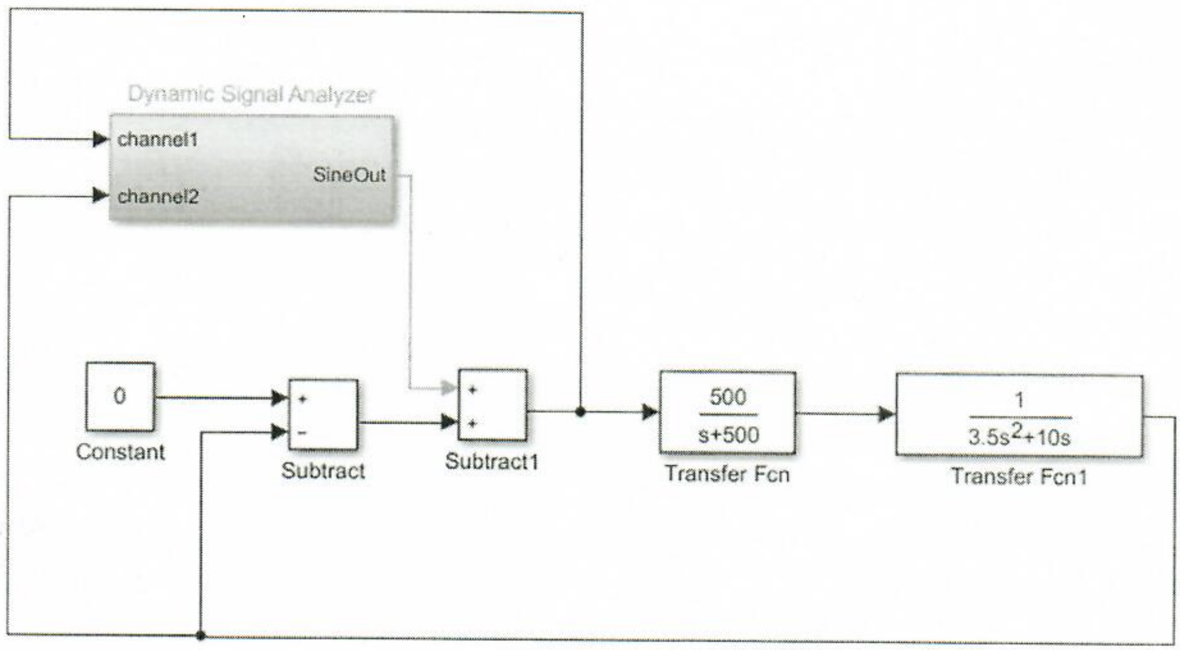
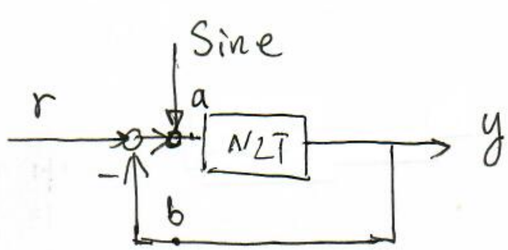


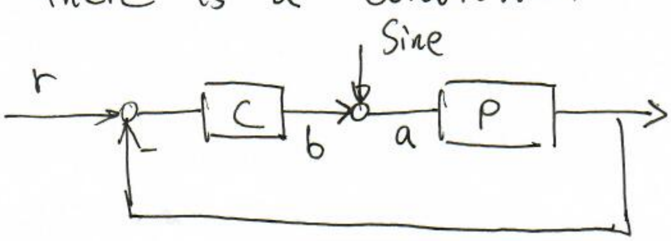
Figure 1: Simulink model

Usually, to avoid stability issues, we close the loop first and input sine waves as disturbances.



$$NLT = \frac{b}{a}.$$

If there is a controller.



$$\frac{b}{a} = P(-1)C = -PC.$$

Figure 2: Closed loop dSA measurement.

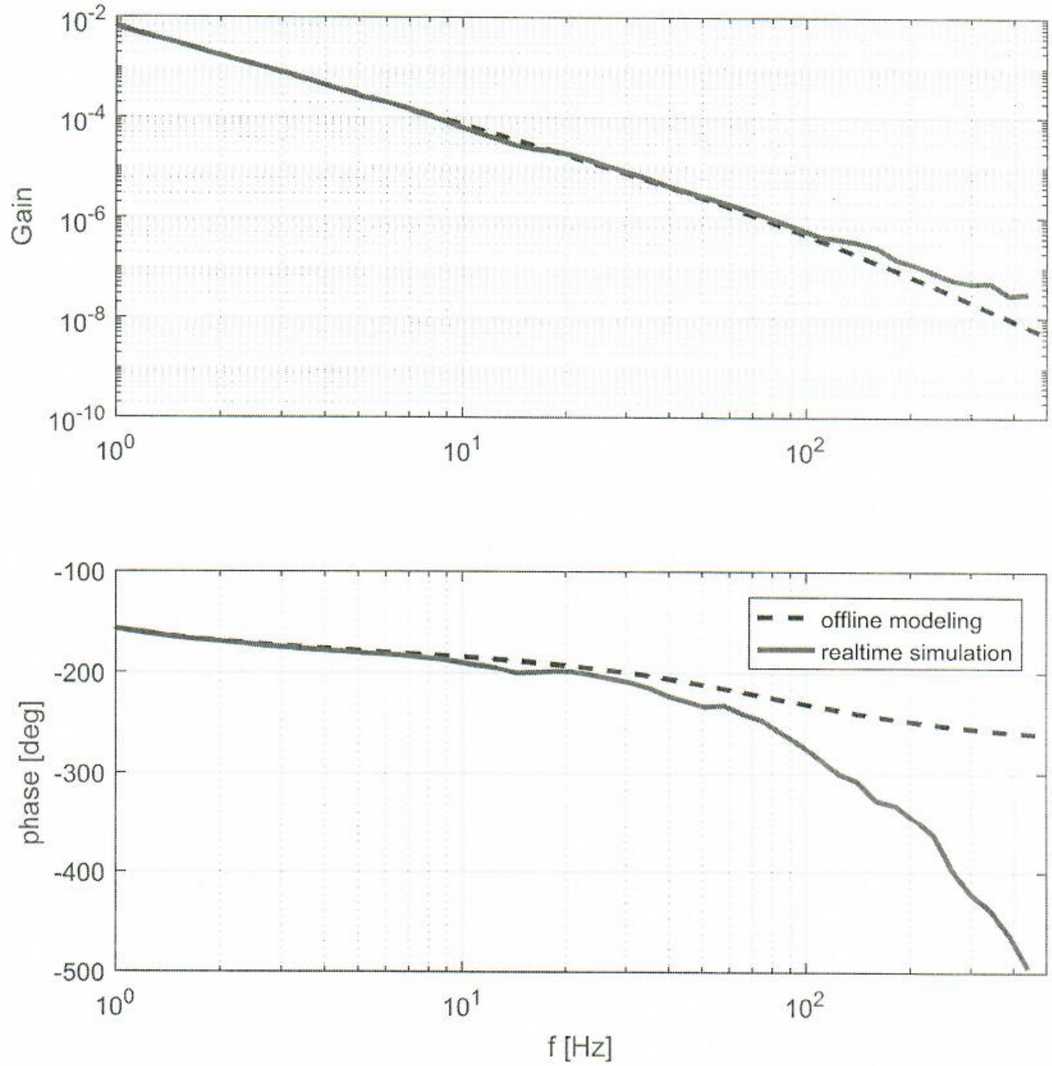
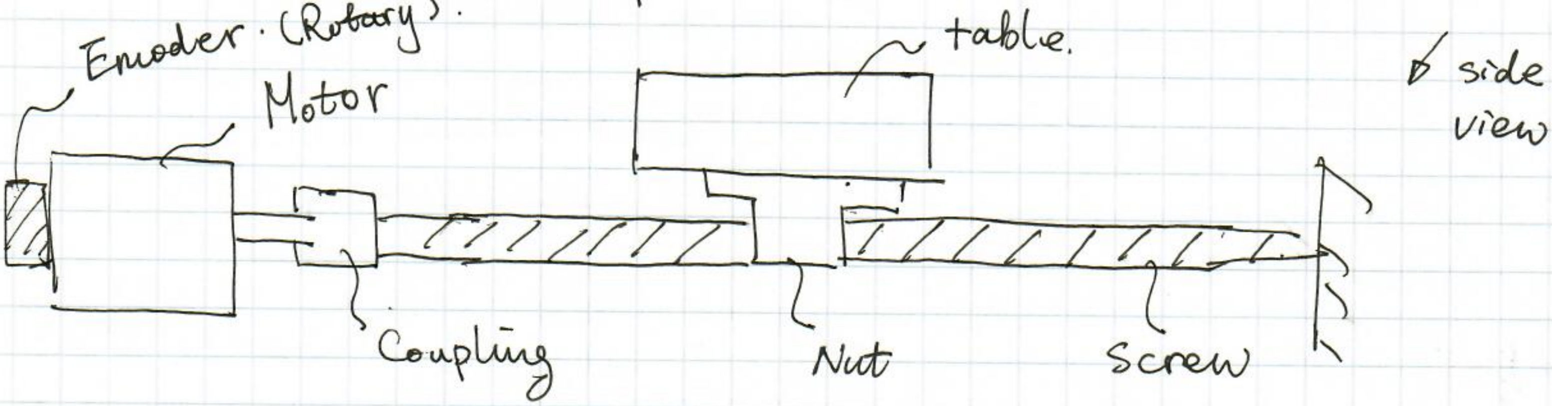
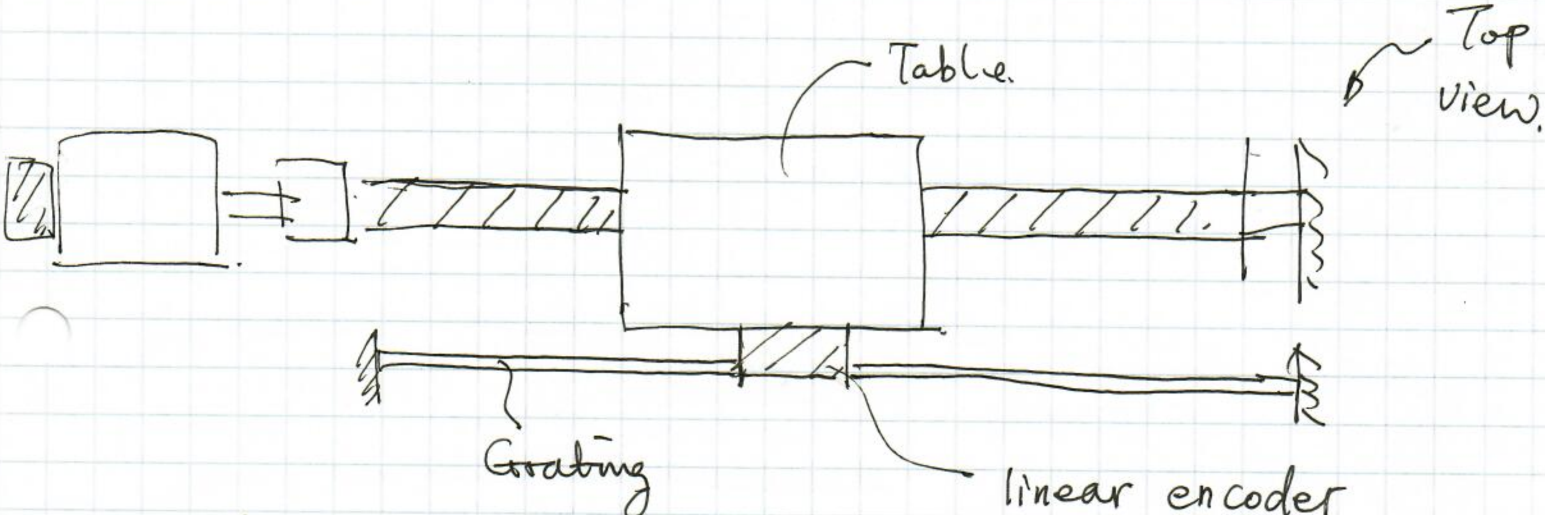


Figure 3: DSA measurement results of the moving mass system.

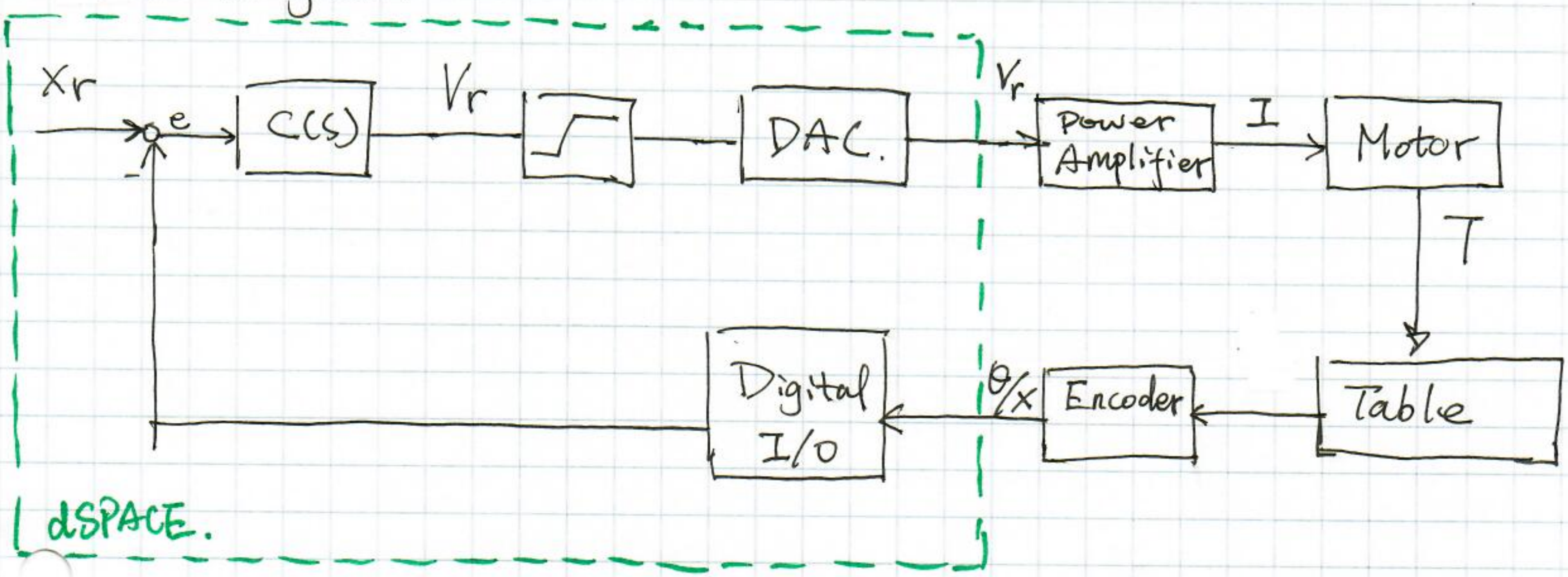
Lecture #16 Case study: Ball-screw stage.

1. Experimental setup:



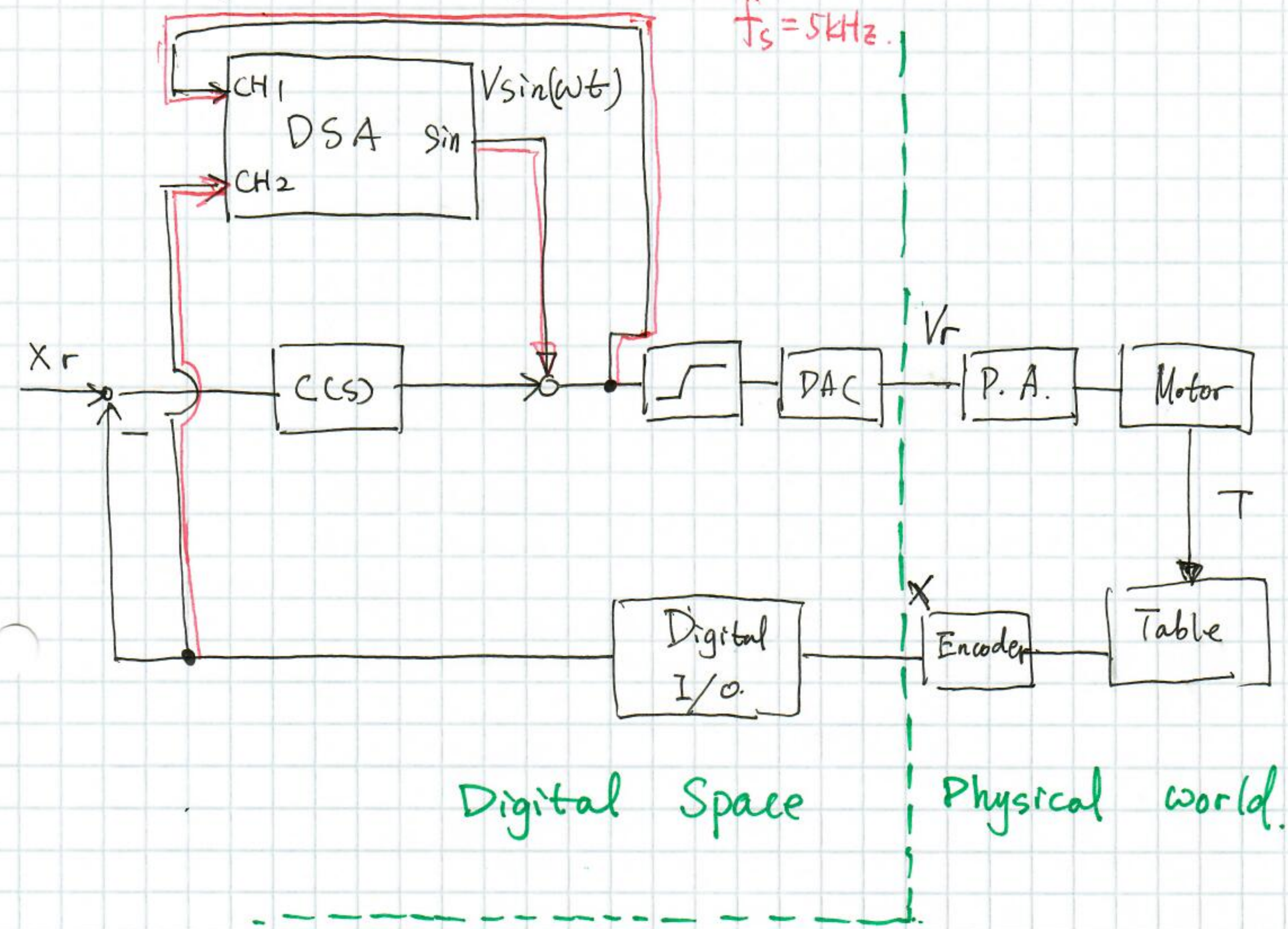


Block diagram:

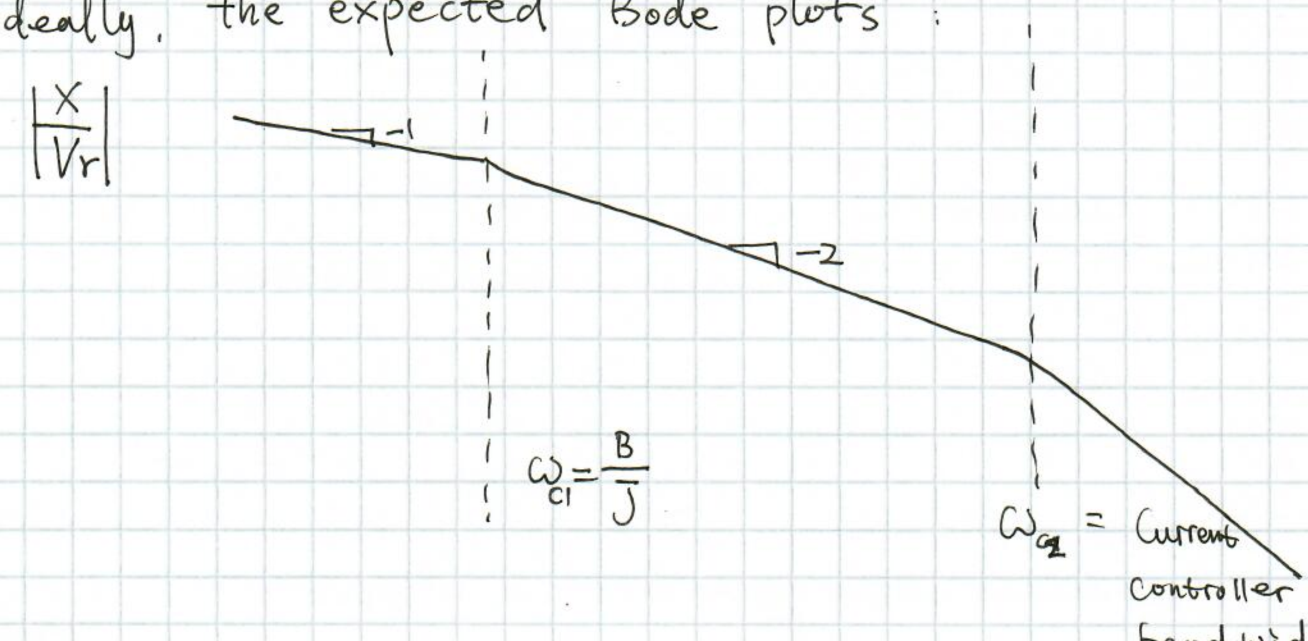


2. Frequency response measurement.

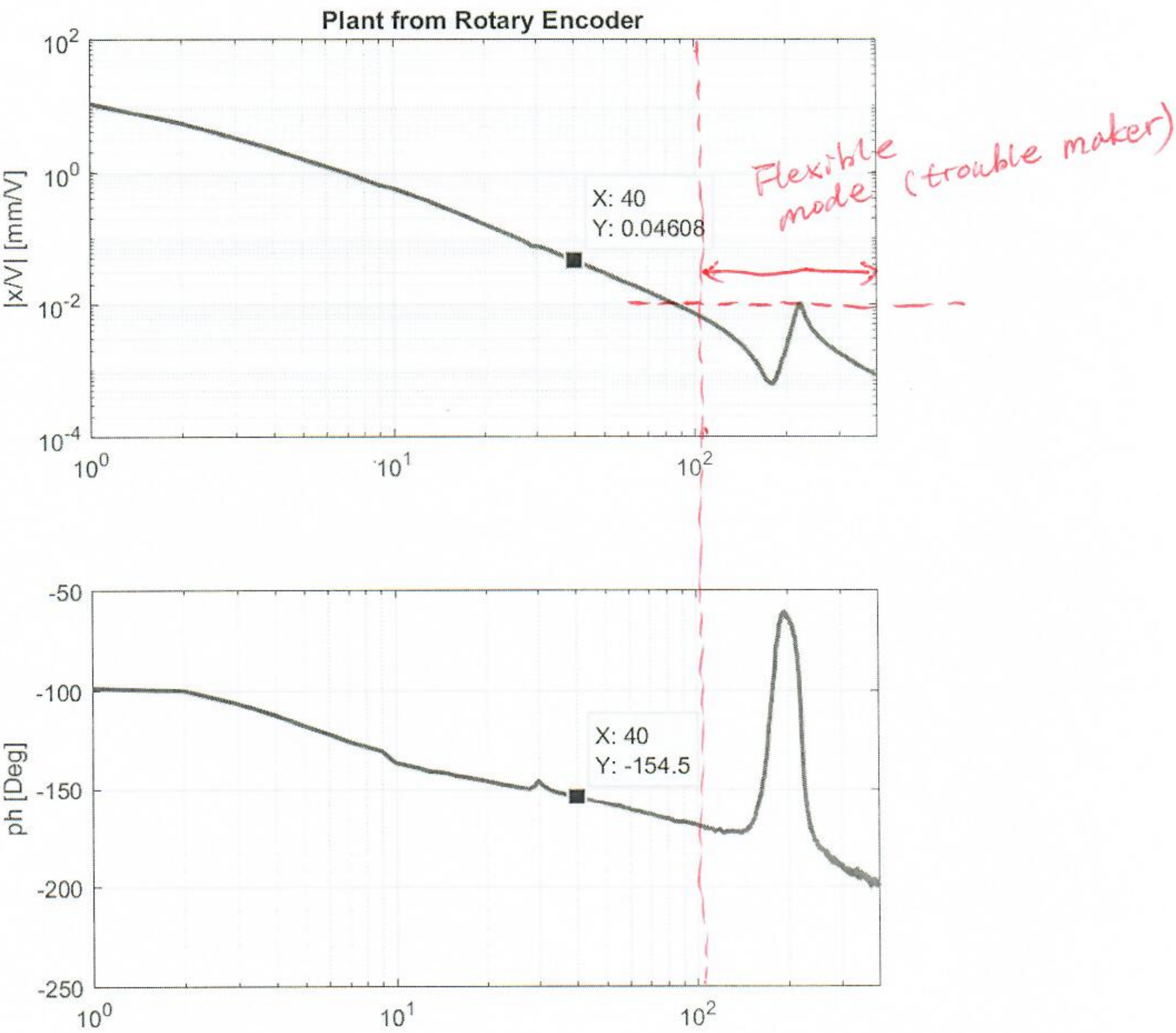
Use dynamic signal analyzer (DSA) to sweep the plant by sinusoidal functions.



Ideally, the expected Bode plots:



However, real world always give us surprise.



However, real world always give us surprise.

3. Controller design procedure.

① Pick a reasonable crossover frequency.

$\boxed{\text{Tips}}$: 1) If the frequency is too low, the bandwidth of the closed loop system will be low. Therefore, we want "ω_{cr}" as high as possible.

2). If the frequency is too high, the flexible mode will have effects to generate vibration.

So the stability will be a problem.

Stay away from the flexible mode, and let $\omega_{cr} = 40\,Hz$

② Select "α"

$\angle P(\omega_c) = -154.5°$, $\alpha = 4$ should be enough for $PM = 60°$.

③ Calculate "K_c":

$$K_c = \frac{1}{\alpha} \cdot \frac{1}{0.04608} = 5.425.$$

④ Set ω_I, $\omega_I = \frac{\omega_c}{10} = 4\,Hz$.

⑤. Implement in Simulink.

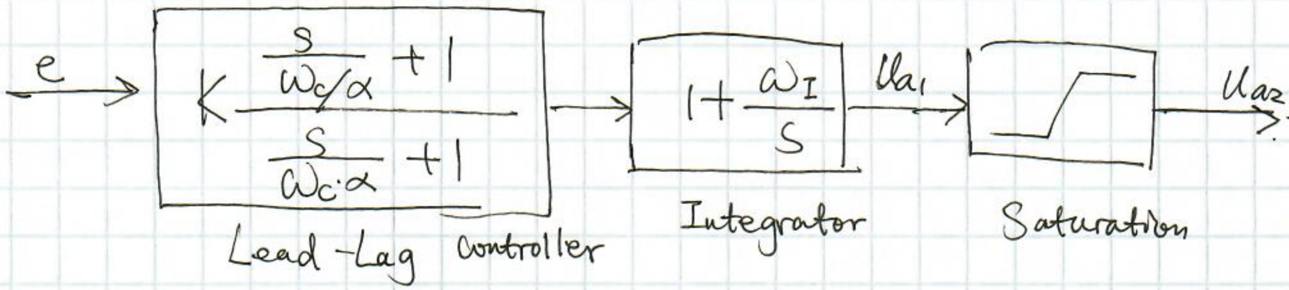
Practical concerns: ①. Anti-Windup of integrator.

②. Sampling rate of computer. (Aliasing).

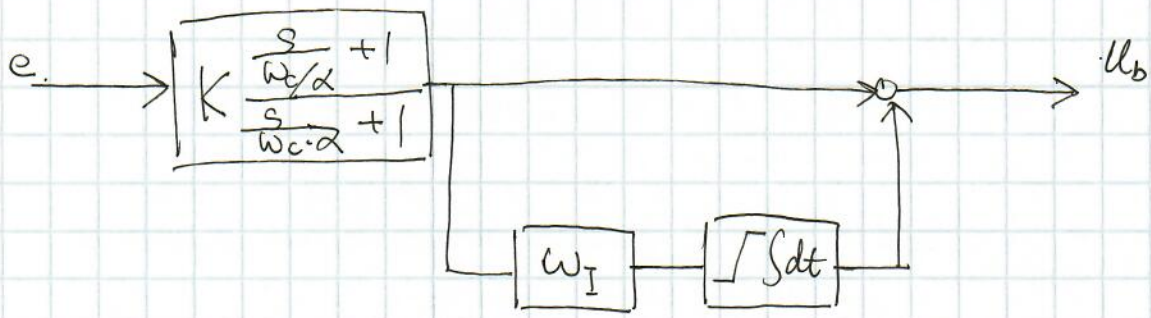
4. Integral control implementation:

Compare a) and b). for saturation

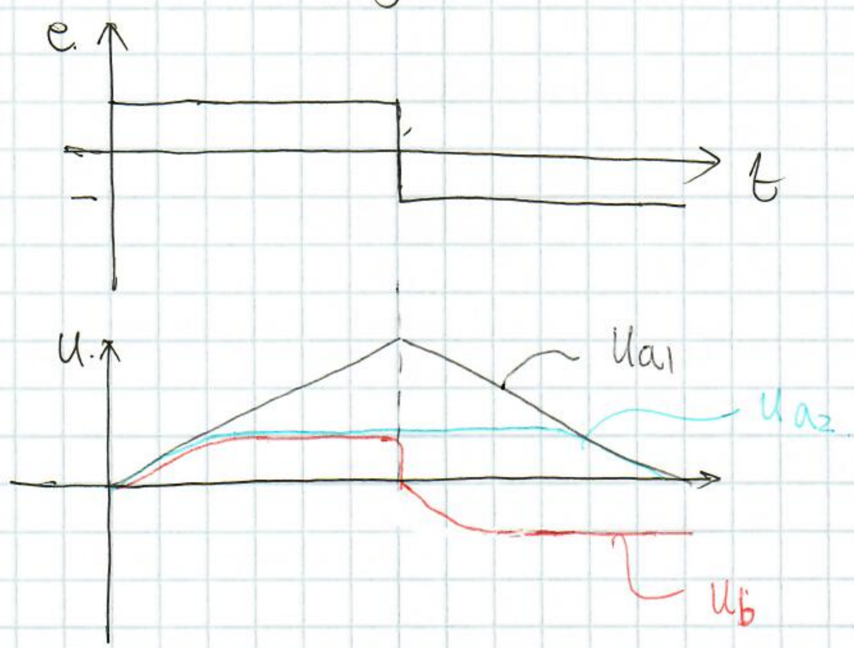
a) : Use transfer function of controller



b). Anti-windup configuration.



There is a difference between a) & b) when the error is inversed suddenly.



u_a cannot be inversed follow the error because of integration.

5. Experimental results :

①. Check the NLT first to verify the crossover frequency.

For the ball screw stage, at low frequency, there might not be a steeper slope corresponding to the integrator because of static friction (see Figure in following page)

However, the integral effect can be shown in the time domain

From the measured NLT,

Crossover frequency, $\omega_{cr} = 41.32 Hz$.

phase margin, $PM' = -100.4 + 180 = 79.6°$

②. Check transient response in time domain.

The step response can show the steady-state error of the system.

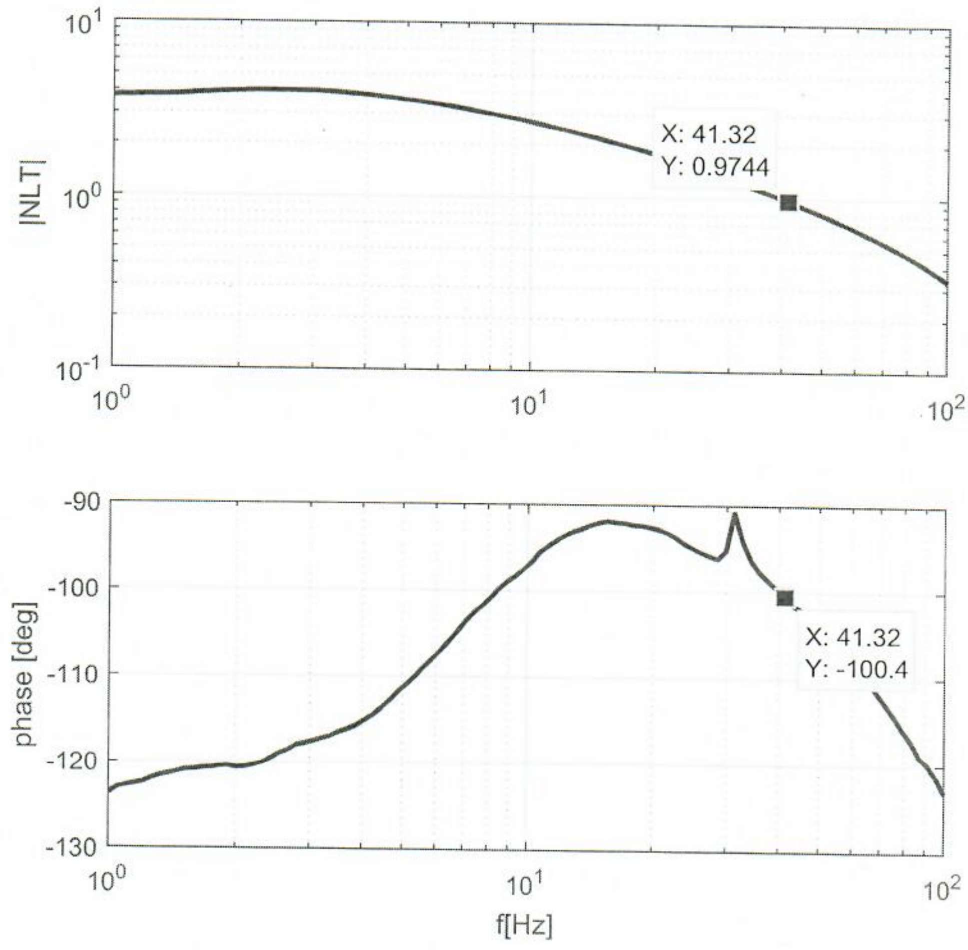
6. Why there is a resonance at around 100 Hz ?

Because of coupling's stiffness, and it can be modeled as a two-mass damping system.

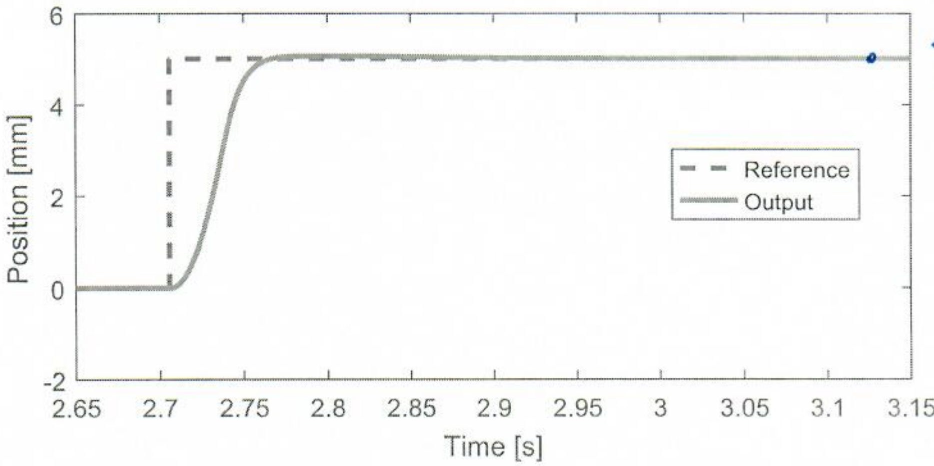
This will be covered later in this course.

Bode plots of NLT after implementing controller

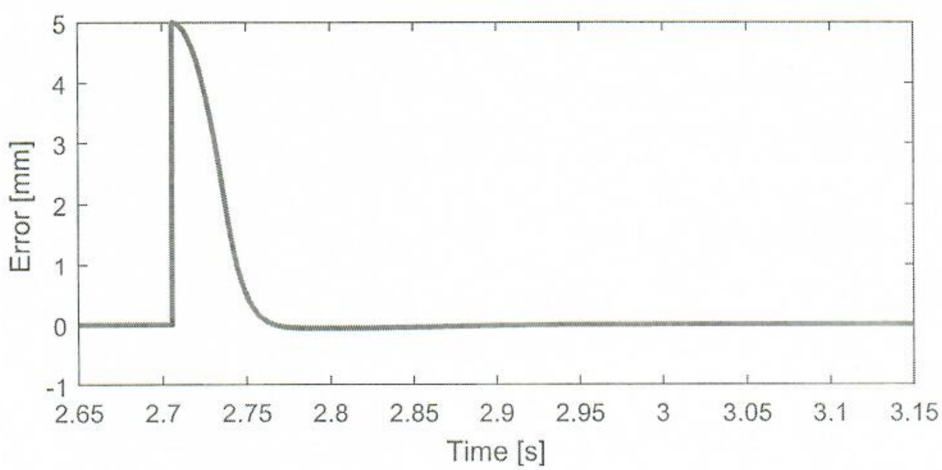
$NLT = C \cdot P.$



Step response of position reference.



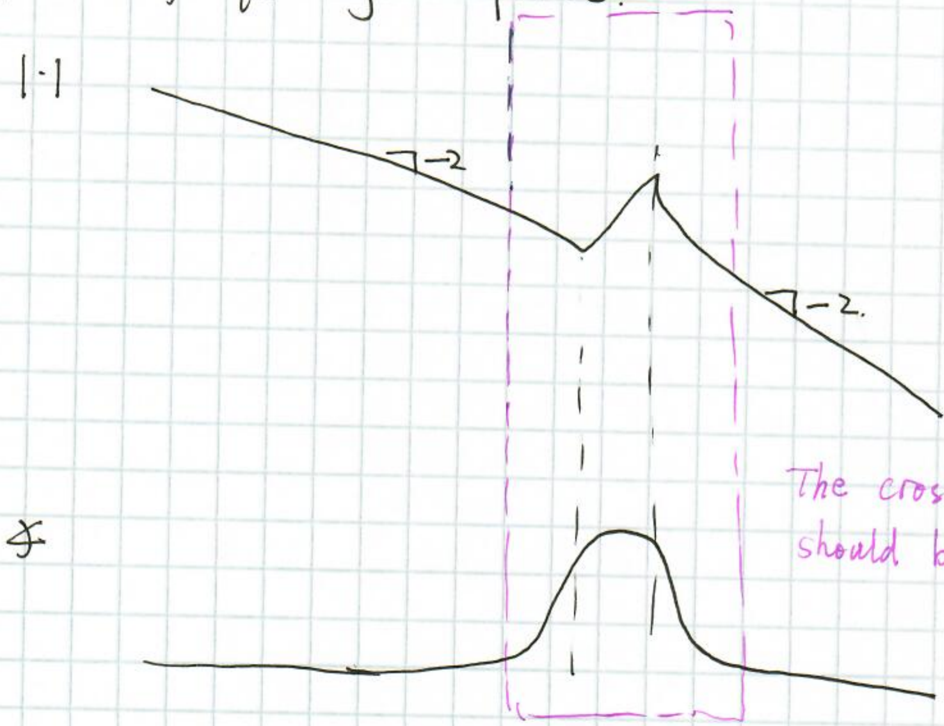
$E_{ss} = 0$.



Lecture #17: Mechanical stiffness modeling.

1. Motivation: Why and how does the vibration showing up in the frequency response of the ball-screw stage?

Typical frequency response:



The crossover frequency should be away from this part.

This is because of the resonance related to the flexible mode.

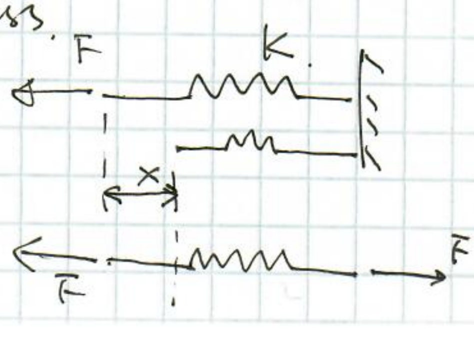
So the mechanical system should be modeled to check the resonance of the vibrations.

2. Mechanical impedance modeling.

Basic idea: Force is "voltage", and Displacement is "current".

Three types of components:

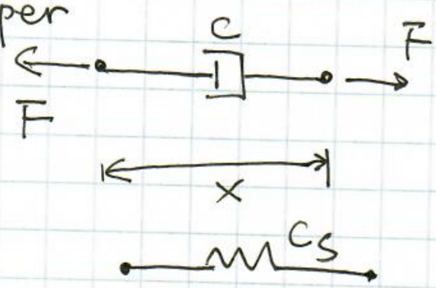
① Stiffness.



$$F = k \cdot x$$

$$Z_k = \frac{F(s)}{X(s)} = K.$$

② Damper

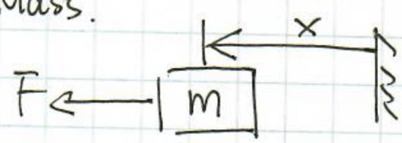


$$F = c\frac{dx}{dt}$$

$\Rightarrow$ s domain:

$$F = cs \cdot X, \quad Z = \frac{F}{X} = cs.$$

③. Mass.



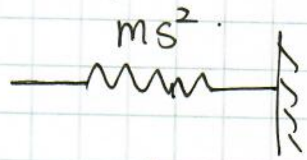
$$F = m\frac{d^2x}{dt^2}$$

Mass only has one terminal and the other end must contact inertia ground.

$\Rightarrow$ s domain:

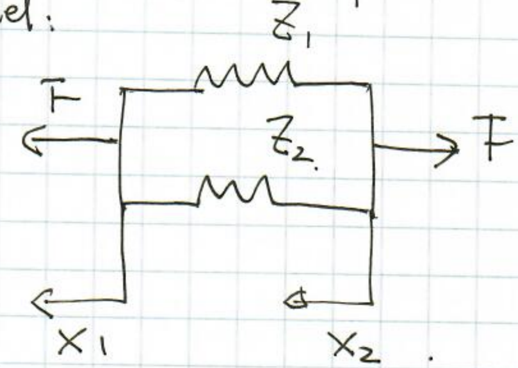
$$F = ms^2 \cdot X$$

$$Z = \frac{F}{X} = ms^2.$$



3. Serial & parallel connection:

①. parallel:

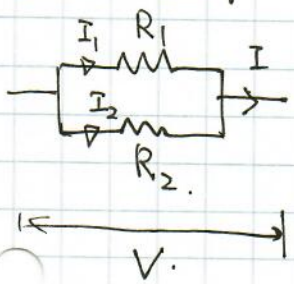


Z_1. Z_2 are mechanical impedance, which might be stiffness, damper or mass.

$$Z = Z_1 + Z_2 = \frac{F(s)}{X_1 - X_2}$$

This is different from electrical impedance:

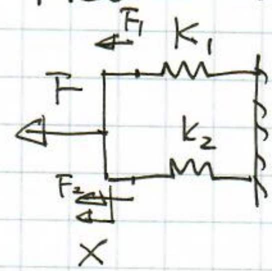
Electrical:



$$R_{//} = R_1 // R_2$$

$$R_{//} = \frac{R_1 R_2}{R_1 + R_2}.$$

Mechanical:



$$F_1 = K_1 \cdot X$$

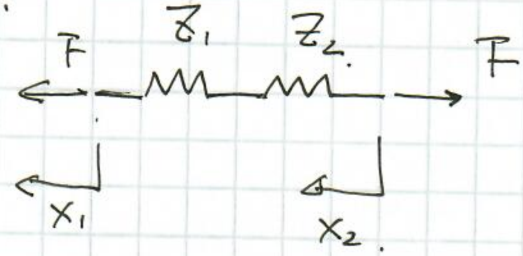
$$F_2 = K_2 \cdot X$$

$$F = F_1 + F_2 = (K_1 + K_2)X$$

$$K_{//} = K_1 + K_2.$$

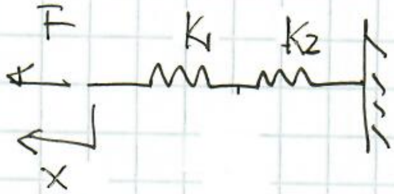
$$\Rightarrow Z_{//} = Z_1 + Z_2.$$

② Serial:



$$Z = Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Still use stiffness as an example:



$$X = X_1 + X_2$$

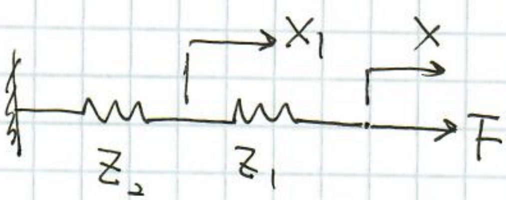
$$F = K_{eq}(X_1 + X_2)$$

$$F_1 = K_1 X_1, \quad F_2 = K_2 X_2$$

$$\frac{F}{K_{eq}} = \frac{F_1}{K_1} + \frac{F_2}{K_2}, \quad F = F_1 = F_2$$

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}, \quad K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$$

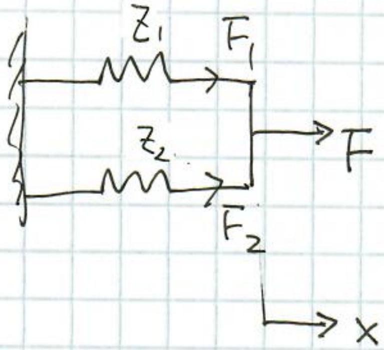
③. Serial connection displacement distribution:



$$\frac{X_1}{X} = \frac{X_1}{F} \cdot \frac{F}{X} = \frac{1}{Z_2} \cdot \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{Z_1}{Z_1 + Z_2}$$

④ Force distribution in parallel connection:



$$\frac{F_1}{F} = \frac{F_1}{X} \cdot \frac{X}{F} = Z_1 \cdot \frac{1}{Z_1 + Z_2}$$

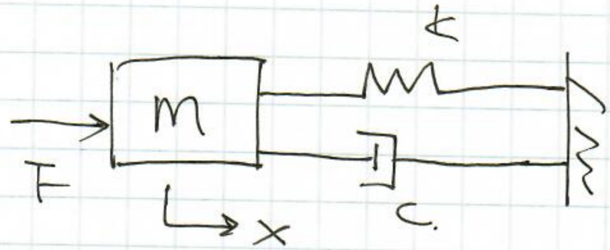
$$\frac{F_2}{F} = \frac{Z_2}{Z_1 + Z_2}$$

4. Why do we need impedance modeling?

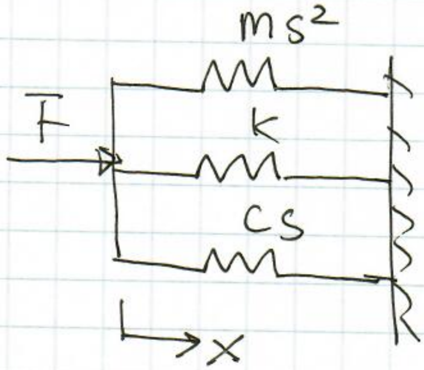
For complex mechanical system, we don't need to draw free body diagrams.

Example:

①



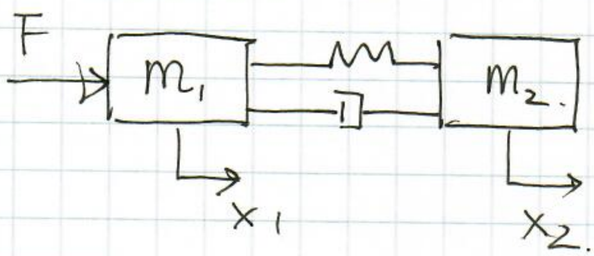
Impedance modeling:



Transfer function:

$$\frac{X}{F} = \frac{1}{Z} = \frac{1}{ms^2+cs+k}$$

② Two-mass damping system. (Important)

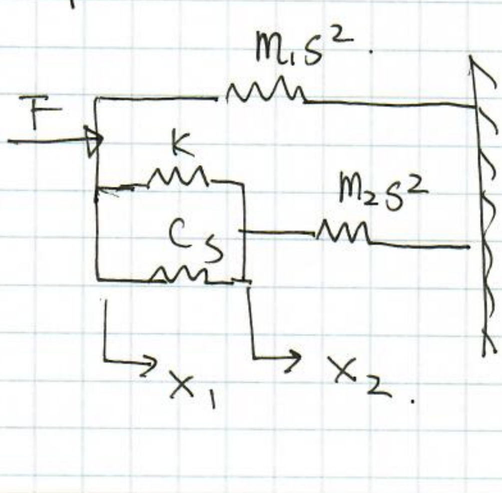


$$Z = m_1 s^2 + \frac{(k+cs)\cdot m_2 s^2}{m_2 s^2 + k + cs}$$

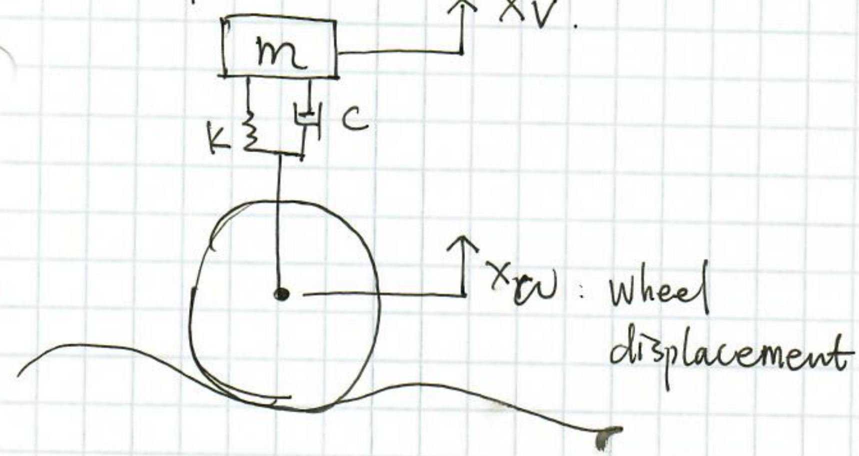
$$\frac{X_1}{F} = \frac{1}{Z}$$

$$\frac{X_2}{F} = \frac{X_2}{X_1}\cdot\frac{X_1}{F} = \frac{k+cs}{k+cs+ms^2}\cdot\frac{1}{Z}$$

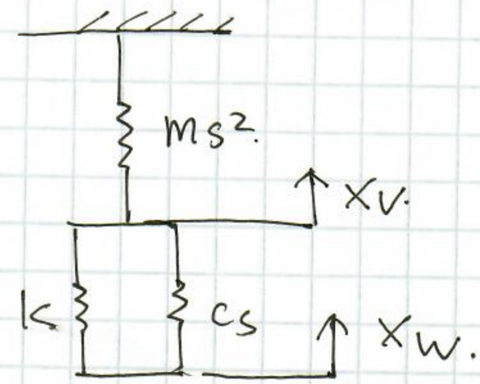
Impedance model:



③. Suspension:



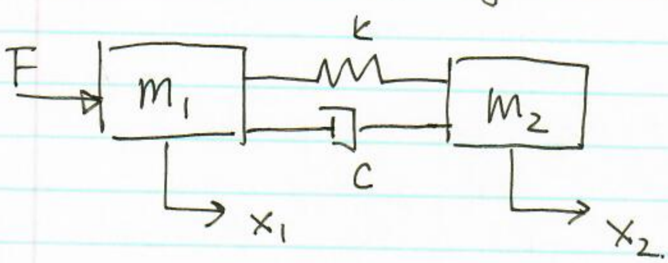
Impedance model:



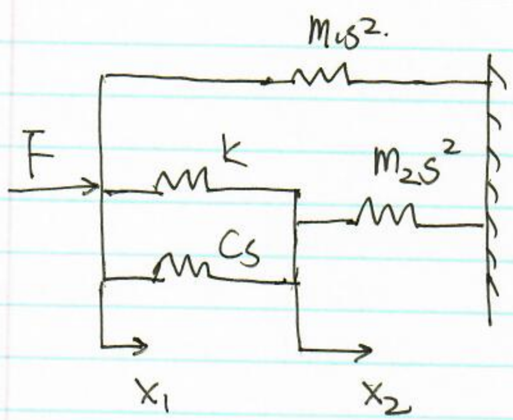
$$\frac{X_V}{X_W} = \frac{k + Cs}{ms^2 + cs + k}.$$

Lecture #18 Two-mass damping system.

1. A mechanical system with 2 degrees of freedom,



2. Mechanical impedance modeling:



3. Transfer functions

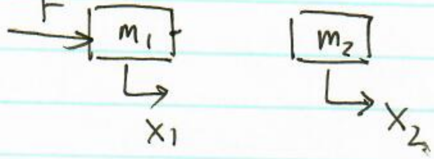
T.F.s can be derived quickly with the impedance model.

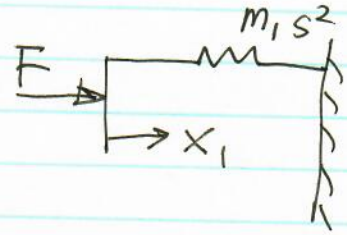
$$\frac{X_1}{F} = \frac{1}{m_1s^2 + m_2s^2 \,//\, (Cs+k)}$$

$$= \frac{1}{m_1s^2 + \dfrac{m_2s^2(Cs+k)}{m_2s^2+Cs+k}}$$

$$= \frac{m_2s^2+Cs+k}{s^2\left[m_1m_2s^2 + (m_1+m_2)Cs + k(m_2+m_1)\right]}$$

2). At very high frequency, $m_1 s^2 >> k$
$m_2 s^2 >> k$

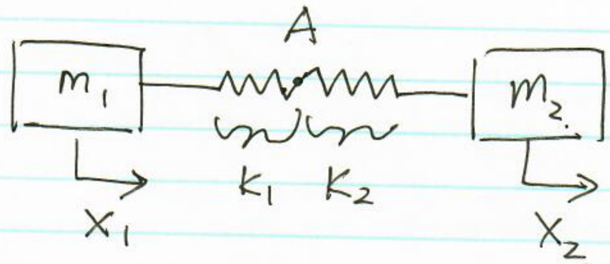




$$\frac{X_1}{F} = \frac{1}{m_1 s^2}$$

3). Two mass resonance.

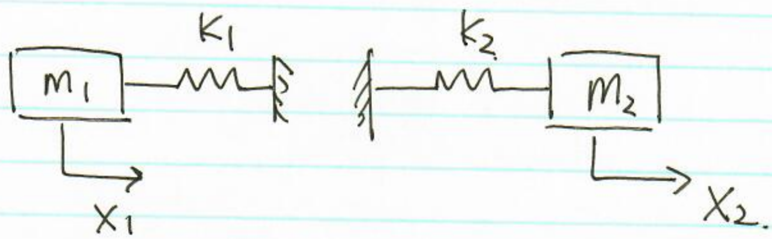
Critical frequenies should be analyzed when drawing Bode plots:



$K = k_1 // k_2$.

Two-mass resonance means two objects vibrate at the same natural frequency.

Exist fixed A to split the spring into two stiffnesses. (k_1, k_2)



Two single mass systems have the same natural frequency.

$$\omega_n = \omega_{n1} = \omega_{n2} \iff \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{k_2}{m_2}}$$

$$\frac{X_2}{F} = \frac{X_1}{F} \cdot \frac{X_2}{X_1}$$

$$= \frac{cs+k}{m_2 s^2 + cs + k} \cdot \frac{m_2 s^2 + cs + k}{s^2 (m_1+m_2)\left[(m_1 // m_2) s^2 + cs + k)\right]}$$

$$= \frac{cs+k}{(m_1+m_2) s^2 \left[(m_1 // m_2) s^2 + cs + k\right]}$$

4. Frequency response of two-mass system.

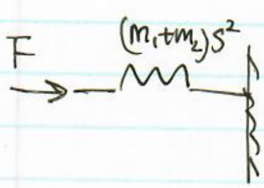
Tips: The Bode plots can be drawn with known transfer functions, however, the physical intepretion is more important to bring intuitive considerations.

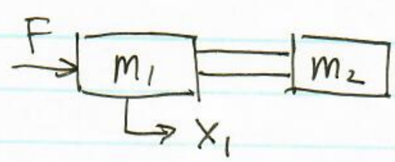
(4.1). $\frac{X_1}{F}$.

Neglect damping (C) first, since it can be converted to stiffness in s-domain.

1). At very low frequency, $m_1 s^2 = m_2 s^2 \ll k$,

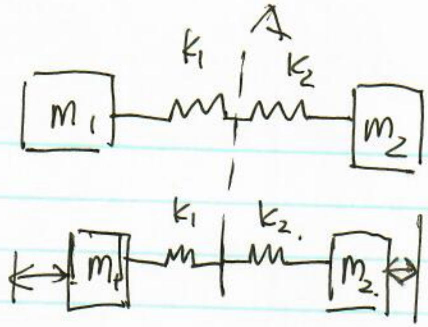
the stiffness model is:





$$\frac{X_1}{F} \approx \frac{1}{(m_1+m_2) s^2}$$

Also, $$k = \frac{k_1 k_2}{k_1 + k_2}$$



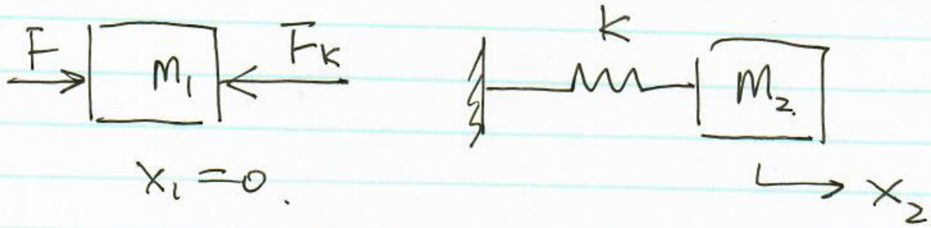
$$\Rightarrow \begin{cases} k_1 = \dfrac{m_1 + m_2}{m_2} k \\ k_2 = \dfrac{m_1 + m_2}{m_1} k \end{cases}$$

$$\omega_n = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{m_1 + m_2}{m_1 m_2} k} = \sqrt{\frac{k}{m_1 // m_2}}$$

4). Zero dynamics.

At ω_z, m_1 has no displacement, $x_1 = 0$.

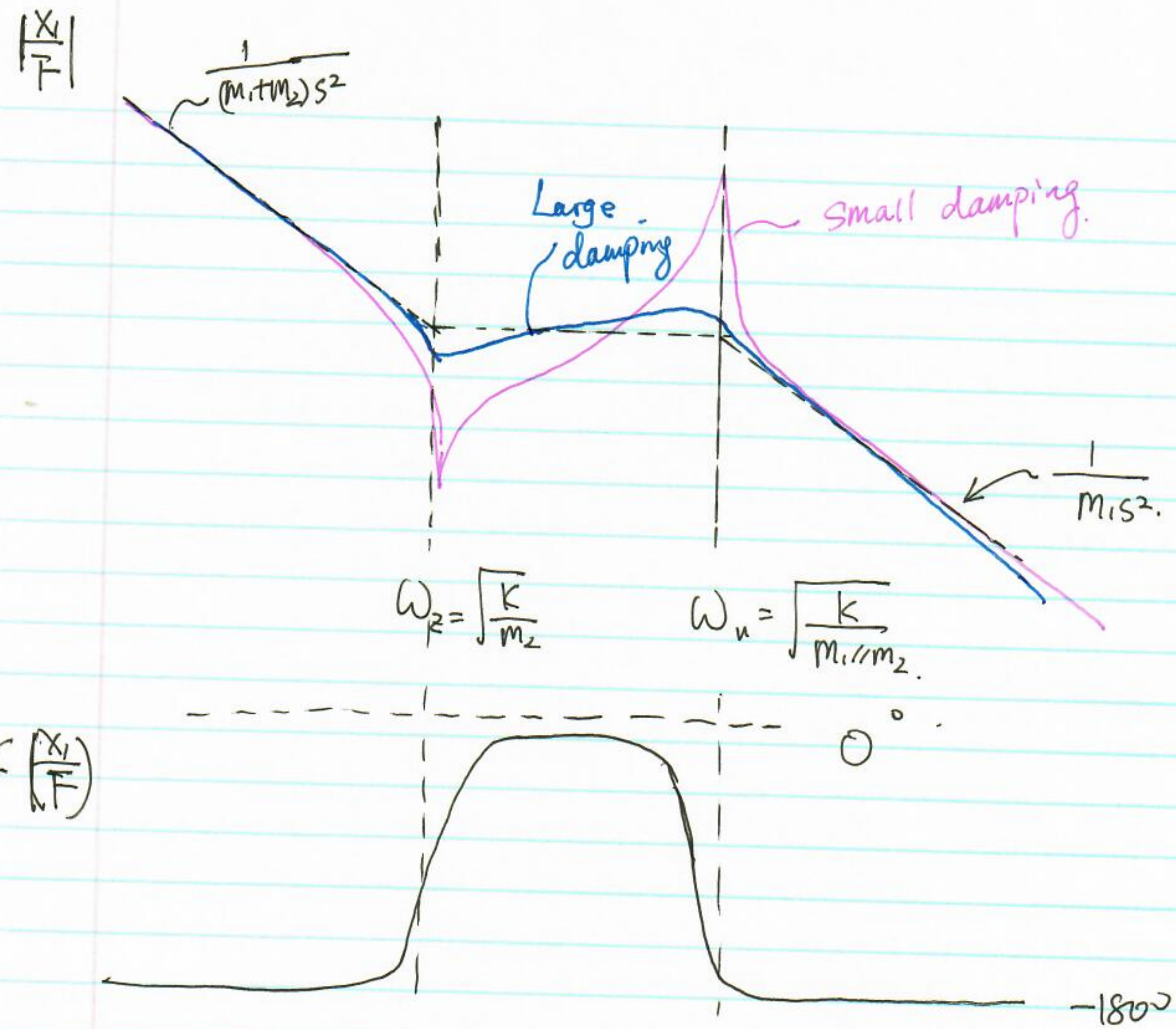
The externally applied force "F" is balanced by the force in spring.



$$-k X_2 = m_2 s^2 \quad , \quad s = j\omega.$$

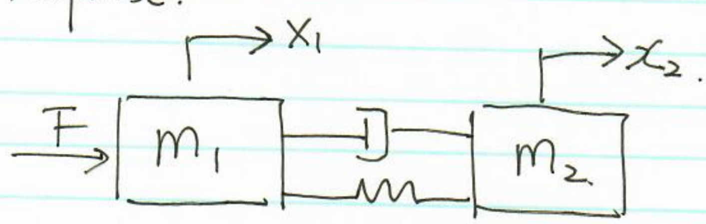
$$\omega = \sqrt{\frac{k}{m_2}}.$$

If combine these four situations together, the frequency response can be drawn.



5. Recall the ball-screw sample.

The ball-screw stage can be modeled as a two-mass damping system, with a similar frequency response.

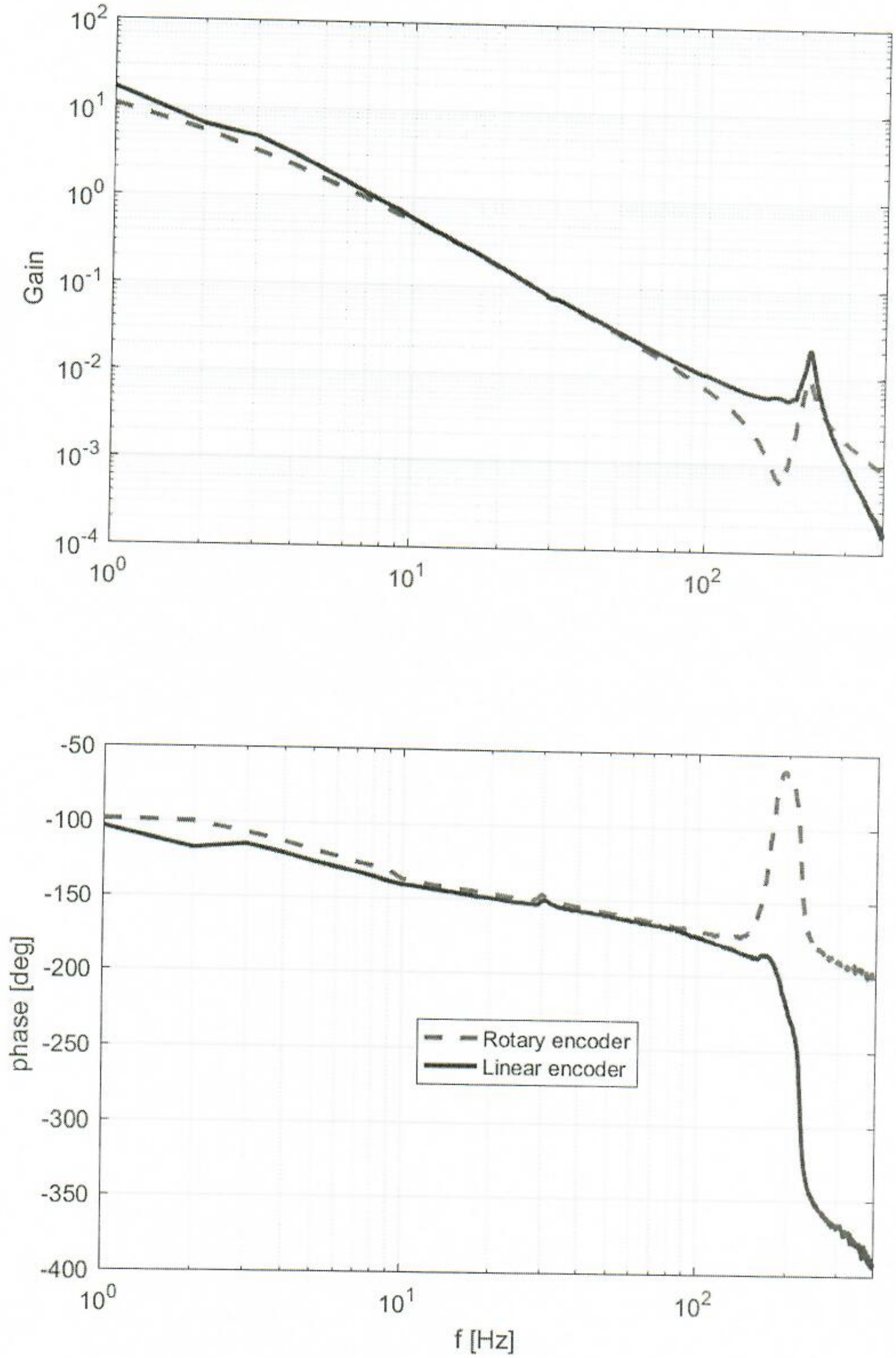


m_1: Motor + coupling

m_2: table.

x_1: the Rotary encoder output

x_2: the linear encoder output.

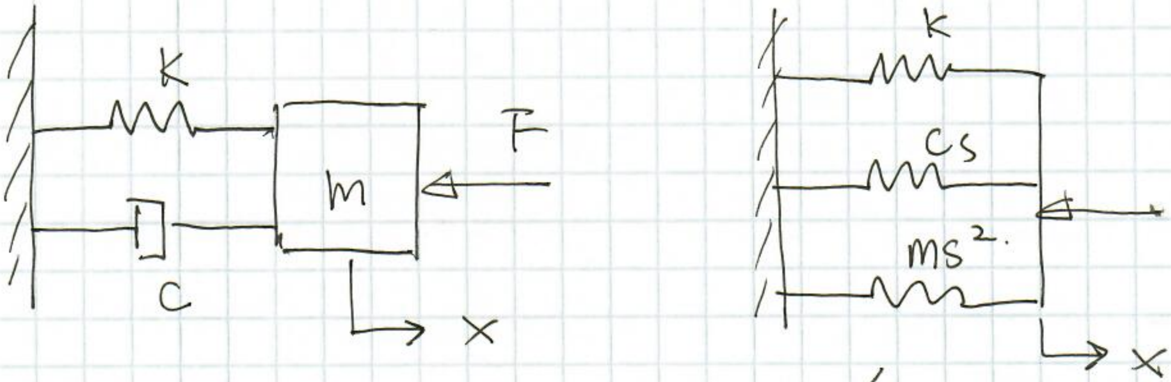


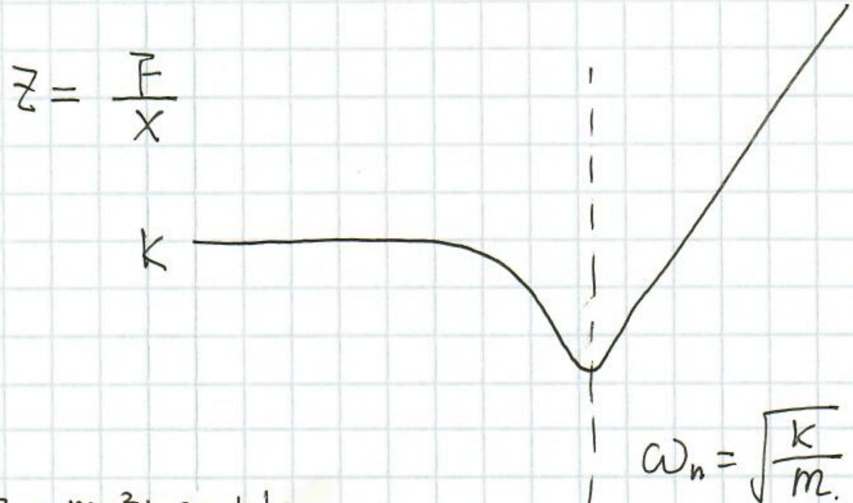
If Loop Shaping Controller is applied
Which encoder is better to use ?

Lecture #19 : Tuned-mass damper. (A way of vibration suppression)

— Jian Gao 2019.3.

1. A normal second order system, the stiffness around natural frequency is low.



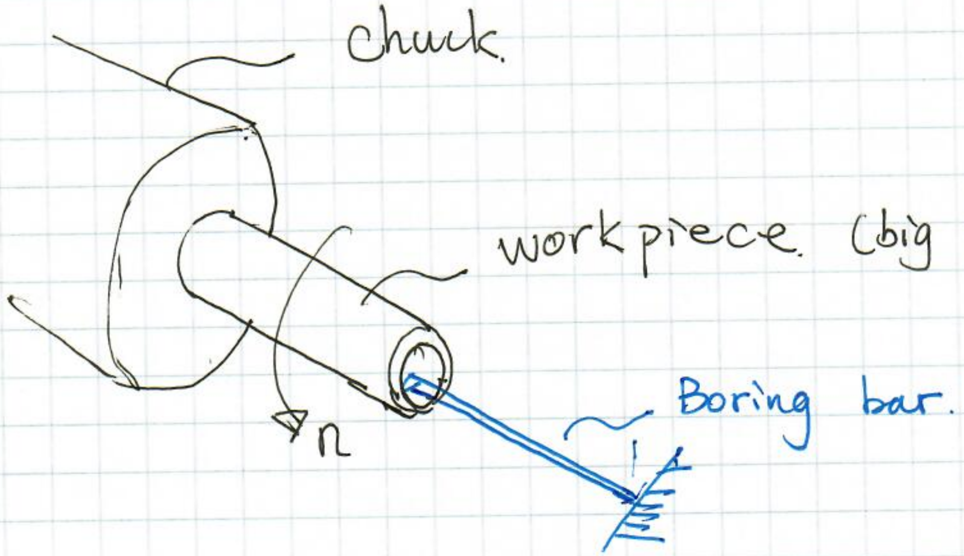


$Z = ms^2 + cs + k$.

If the input is around the natural frequency, the overall mechanical system will experience large vibration amplitude. which is bad for most of systems.

A real example in cutting process.

Consider a boring process for a long hollow bar.



The boring bar is a mass with stiffness & damping.

the input force is periodical with spindle speed.

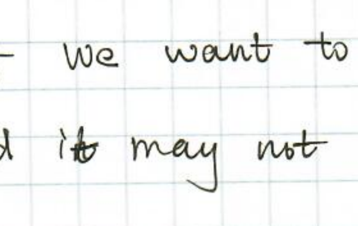
$F_c = F_0 \cdot \sin(\omega t)$.

$\omega = 2\pi \cdot \frac{n}{60}$, n is in RPM.

The stiffness of the boring bar is depend on length & crosssection area.

$r = 5mm$, $l = 200mm$, steel

$K = \frac{3EI}{l^3} = 11718.5\ N/m$, $I = \frac{\pi}{2} r^4$, $E = 200GPa$.



$m = \rho \cdot V = 0.1225\ kg$.

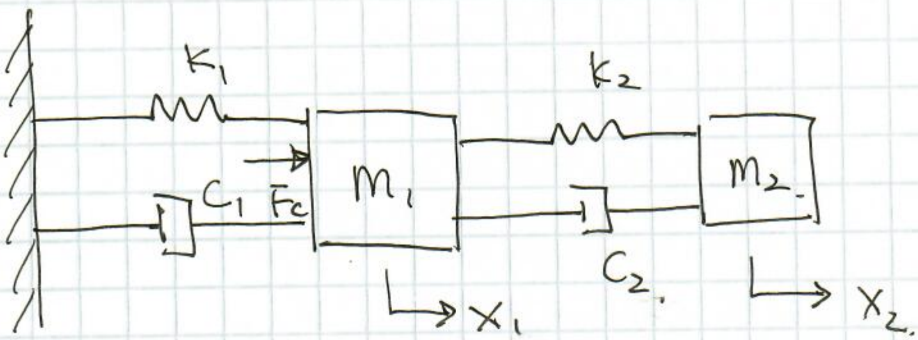
$\omega_n = \sqrt{\frac{k}{m}} = 309\ rad/s = 50\ Hz$, $n = 3000$

This ω_n is easy to be exited by F_c.

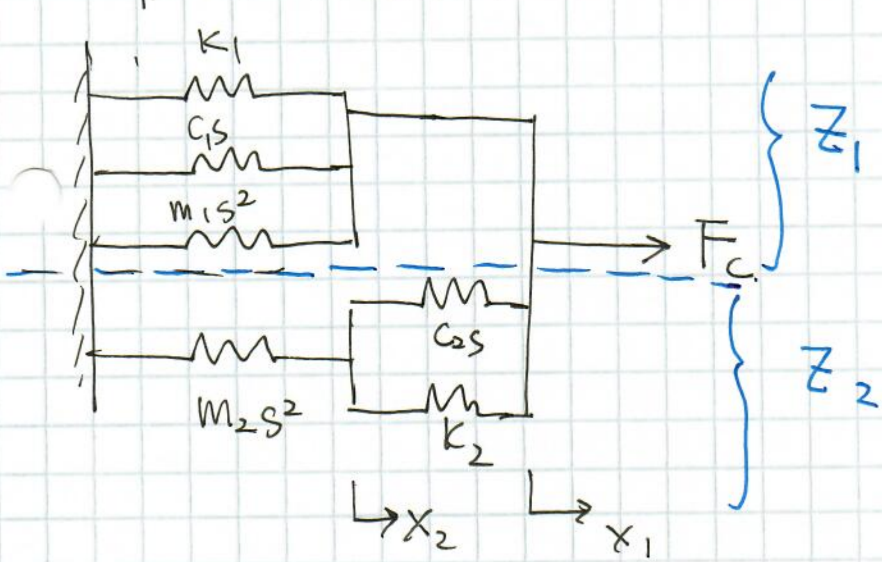
If we want to push ω_n high, larger cross-section is necessary and it may not go into the tube for boring.

2. Is there any other ways to improve the stiffness at certain frequency?

Tuned - mass damper basics:



Stiffness model:



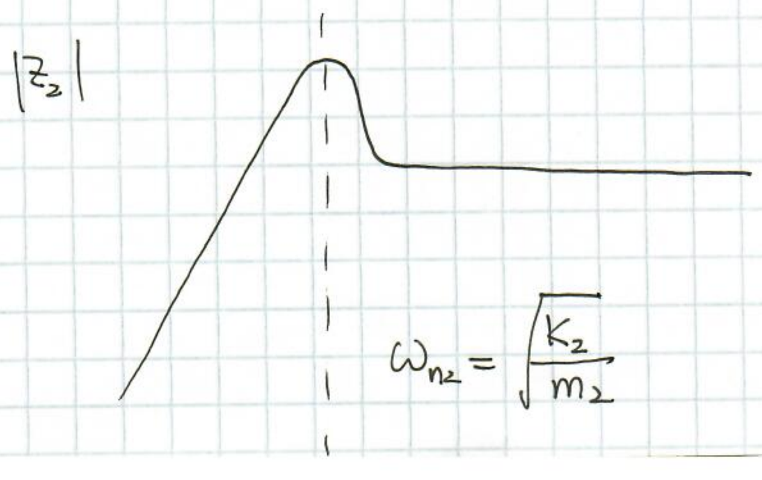
$Z_1 = m_1 s^2 + C_1 s + k_1$

the Bodes plots we know

$$Z_2 = (k_2 + C_2 s) \,//\, m_2 s^2 = \frac{m_2 s^2 (k_2 + C_2 s)}{m_2 s^2 + C_2 s + k_2}$$

For Z_2: at low frequency, $m_2 s^2$ is dominant.

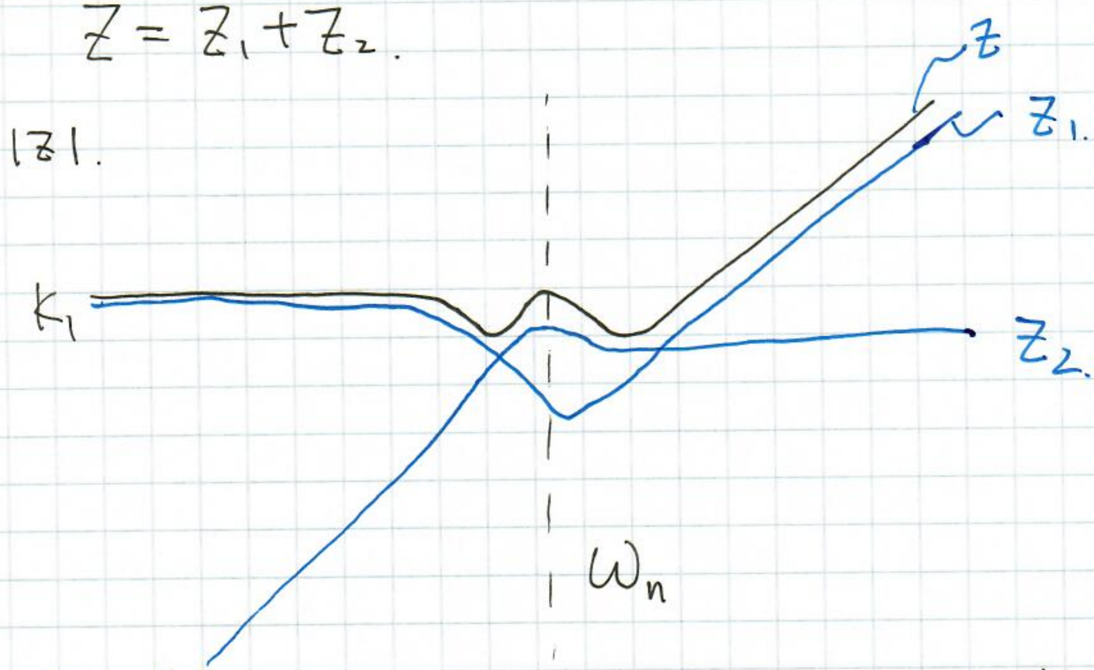
at high frequency, k is dominant.



If we select m_2, K_2 to achieve $\omega_{n_1} = \omega_{n_2}$.

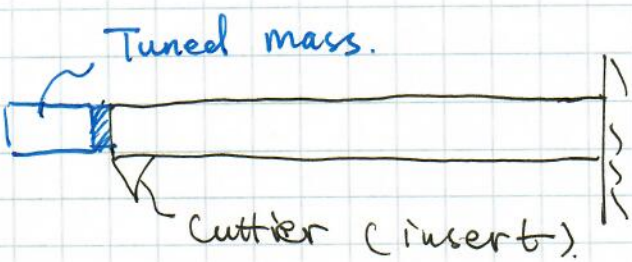
then the overall stiffness in frequency domain.

$Z = Z_1 + Z_2$.



So the stiffness at ω_n is much higher

Return to the example of boring bar.

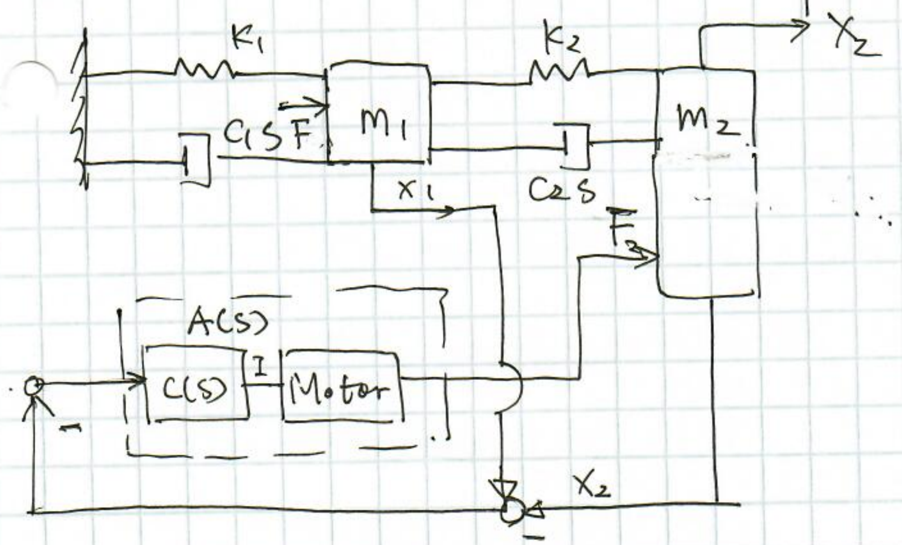


Another concern, if the boring bar clamped at different position, is the tuned mass damper working?

- No. Because the length is different, hence the natural frequency is different. $\omega_{n1} \neq \omega_{n2}$.

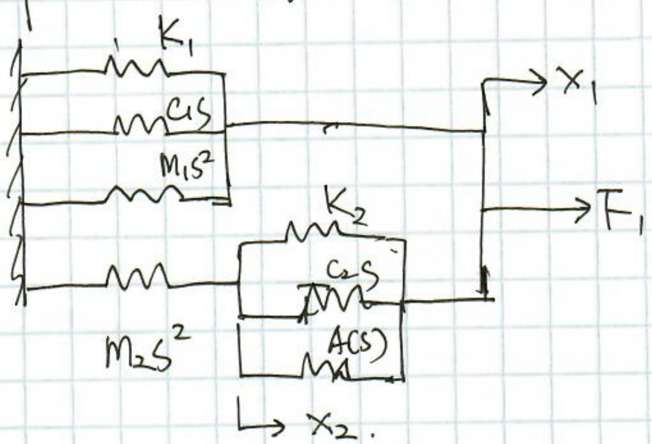
Then what will be the solution?

3. Active tuned mass damper:



Assume there is a actuator to produce force.

Impedance model:

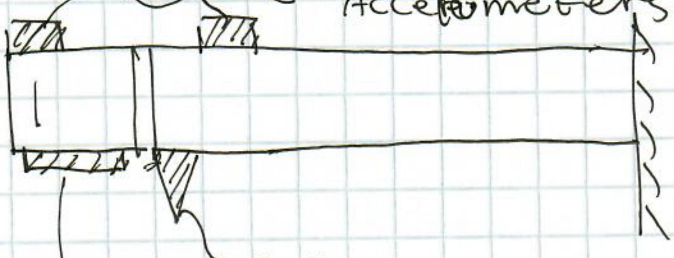


The actuator with the controller can change the stiffness of the system.

In this case, X_1 & X_2 should be as feedbacks in the system.

If we reconsider the boring bar,

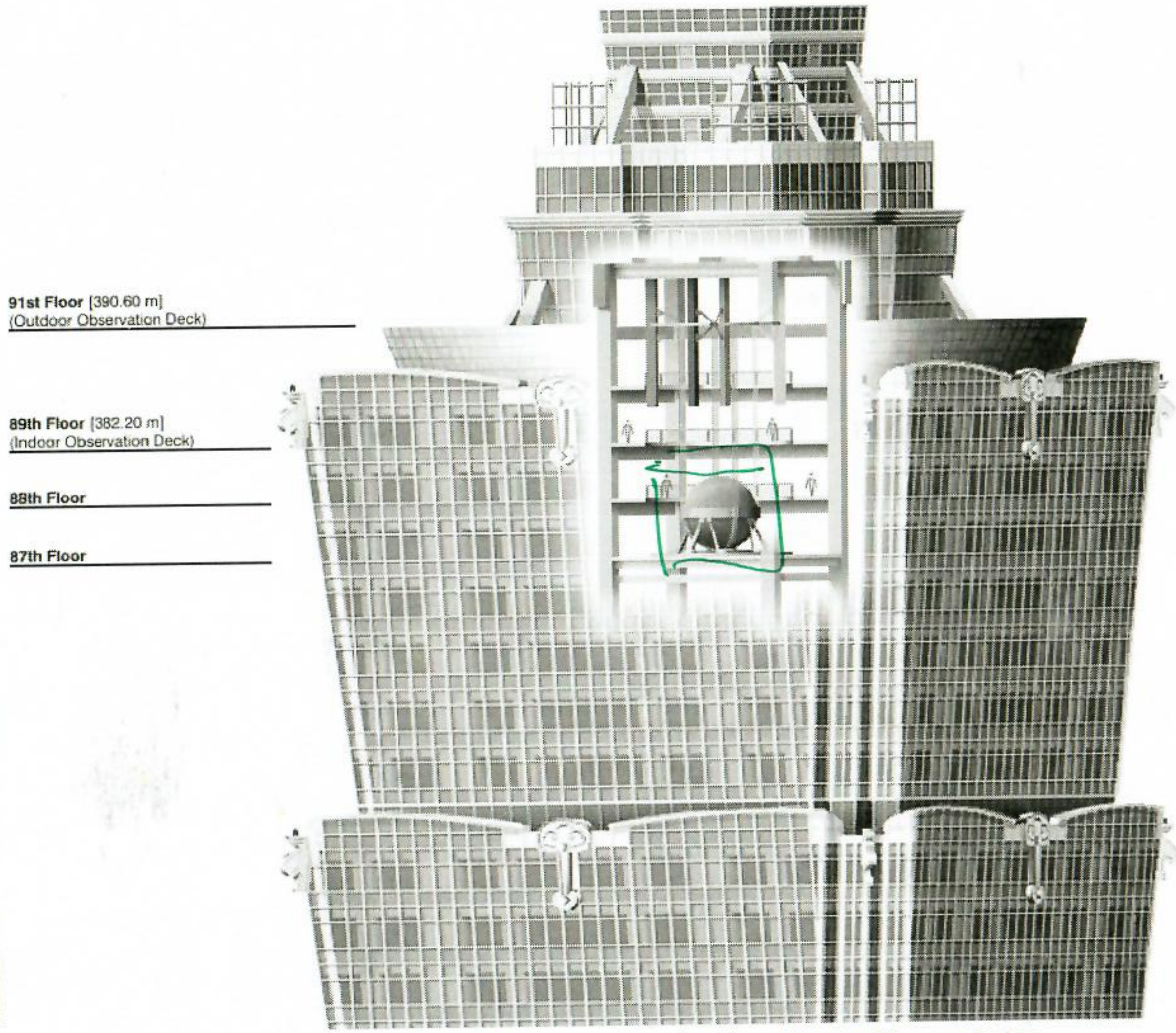
Accelerometers : feed back sensors



Cutter.

Piezo actuator. To produce bending moment.

Taipei 101 (http://www.taipei-101.com.tw)

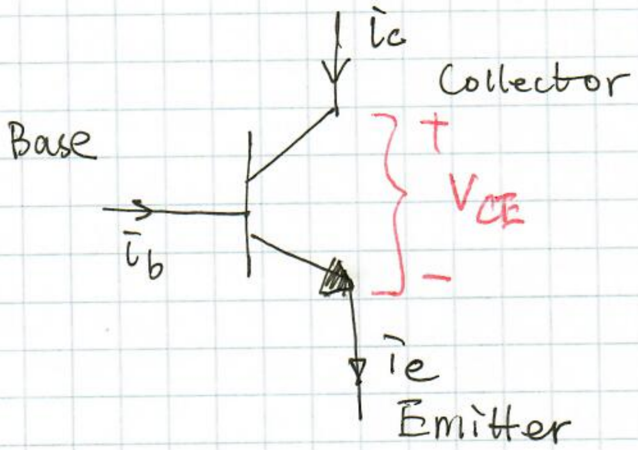


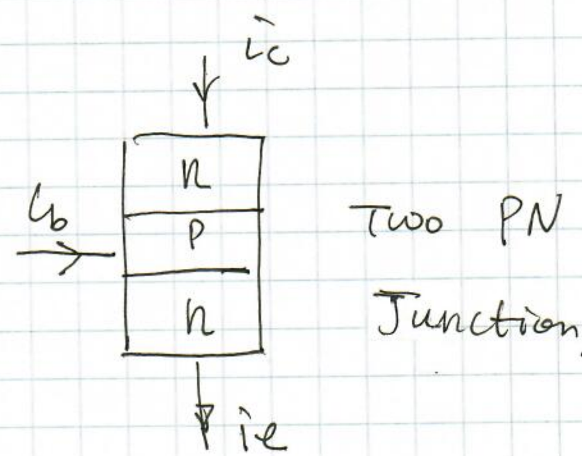
Lecture #20 . Power electronics : Part I

1. Motivations: To select appropriate power amplifier modules in the market, not to design the circuit from zero.

2. Fundamental components of power electronics.

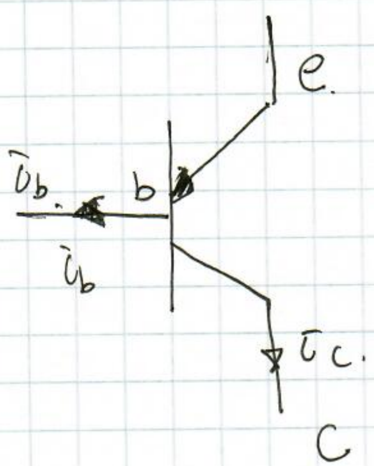
①. Bipolar Junction transistor

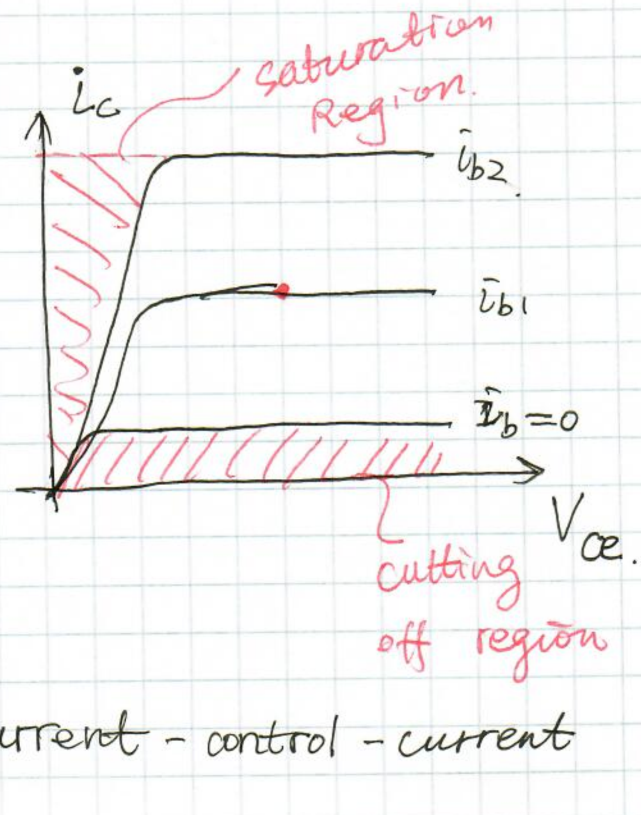




$V_{BE} = 0.7V$.

NPN . BJT. , $\beta = \frac{i_c}{i_b}$

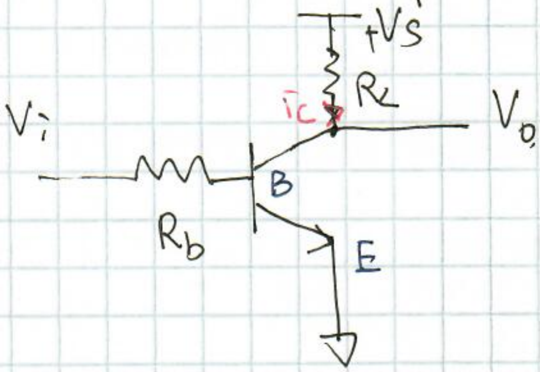


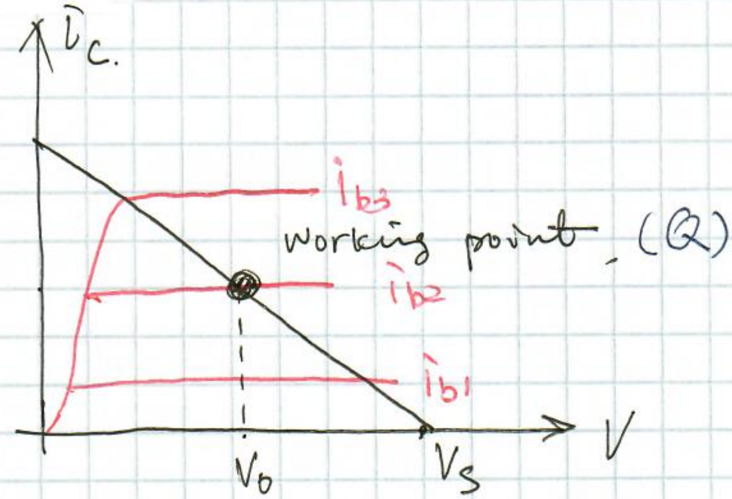


PNP . BJT.

BJT is a current - control - current device.

A circuit example:



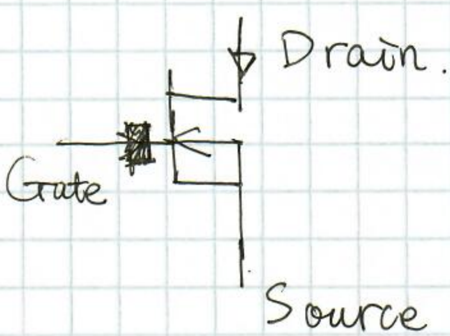


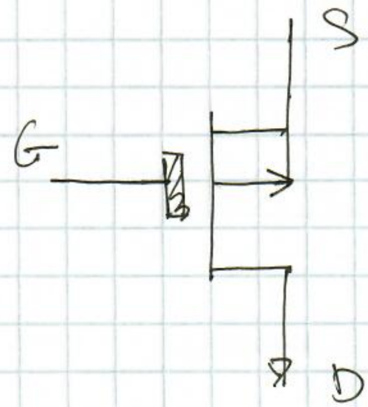
$$i_b = \frac{V_i - 0.7}{R_b}$$

$$V_o = V_s - i_c R_L$$

Some times transistors are cascaded together
It's called Darlington pair

② MOSFET : More common in high-power applications
(Metal-Oxide semiconductor fieled effect transistor)



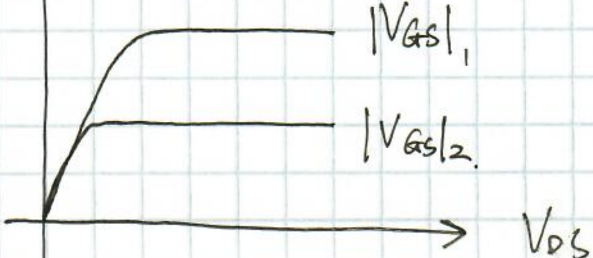


$V_{GS} \geq 0$ $V_{GS} < 0$

N - channel. P-channel

MOSFET is voltage controlled device.

It usually works in saturation - cut off mode for switching.

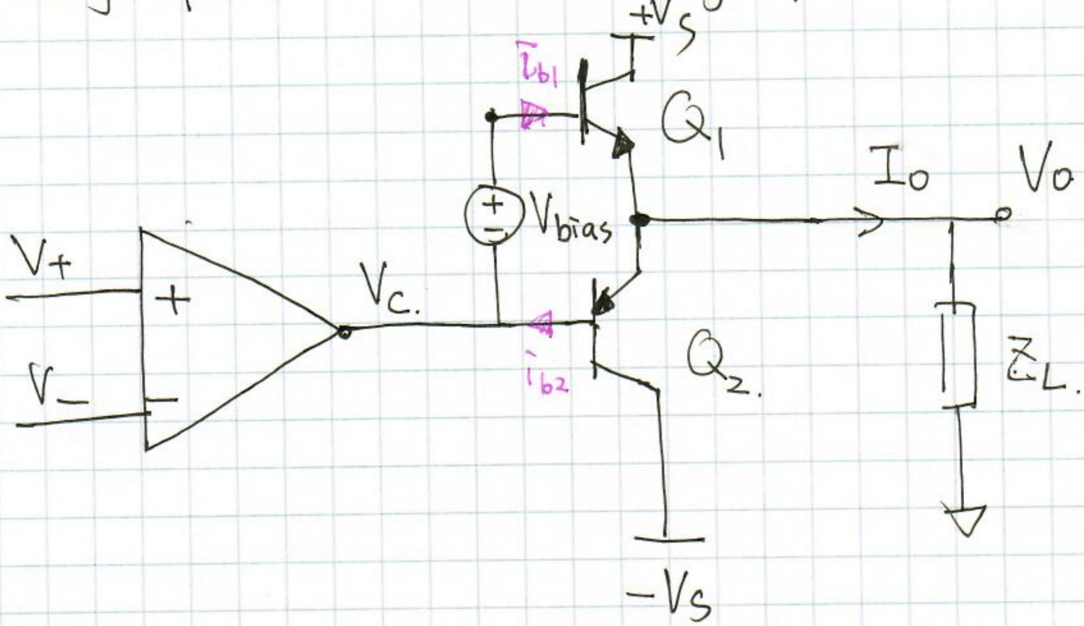


Remarks:

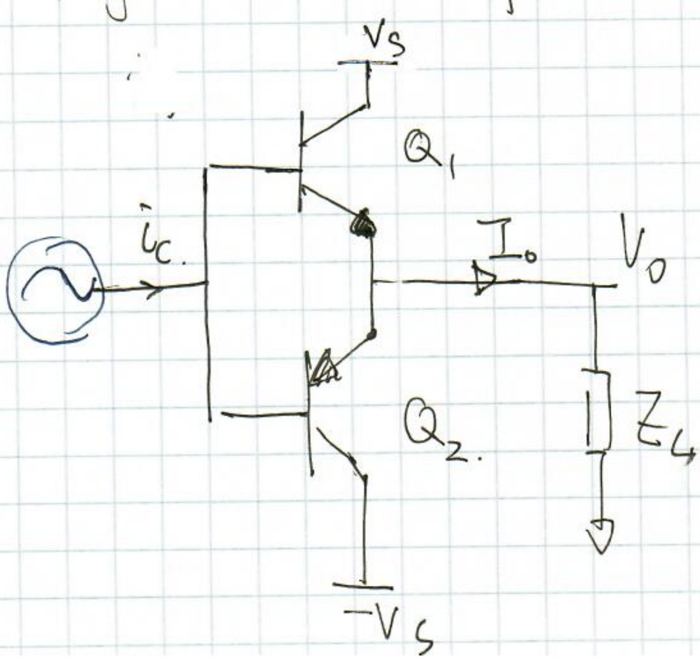
Power Amplifier
- Linear type: BJT, low noise, low efficiency
- Switching type: MOSFET, high noise, high efficiency.

3. Linear power amplifer.

Normally, linear P.A. is based on push-pull circuit by input a continuous signal.



Neglect V_{bias} first.



If $I_o > 0$, Q_1 working
$i_c > 0$

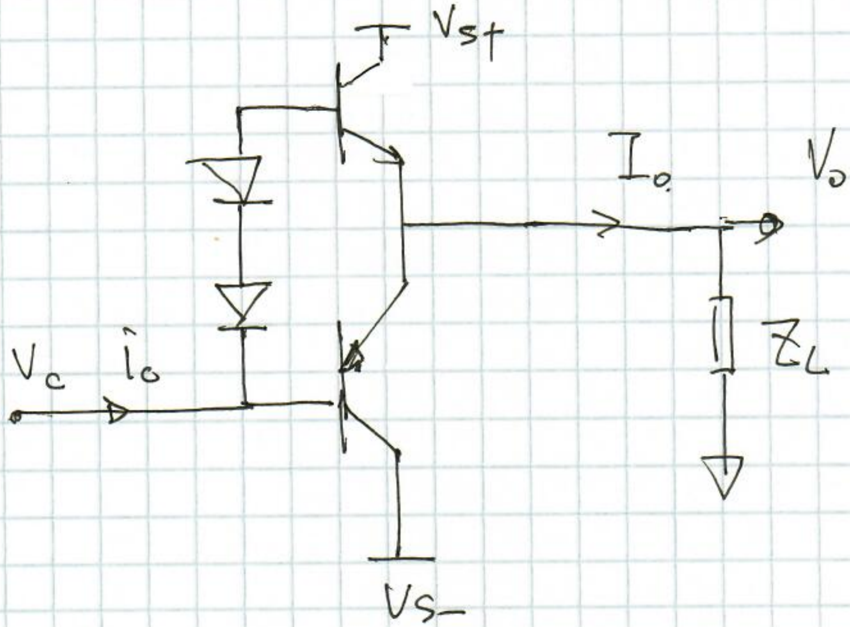
If $I_o < 0$, Q_2 working.
$i_c < 0$.

Typically, the transistor has a threshold voltage between emitter and base for BJT or between gate and source for MOSFET.

Just like a backlash, so a bias voltage is used to compensate it.

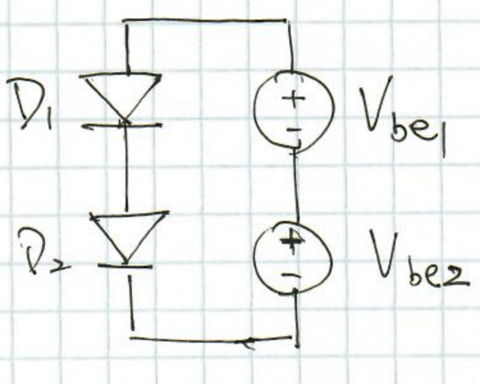
$V_{bias} = 2V_{th}$ in push pull amplifier.

Two diodes in serial is used to generate V_{bias}.



What is the drawbacks of this design?

Temperature will be an issue.

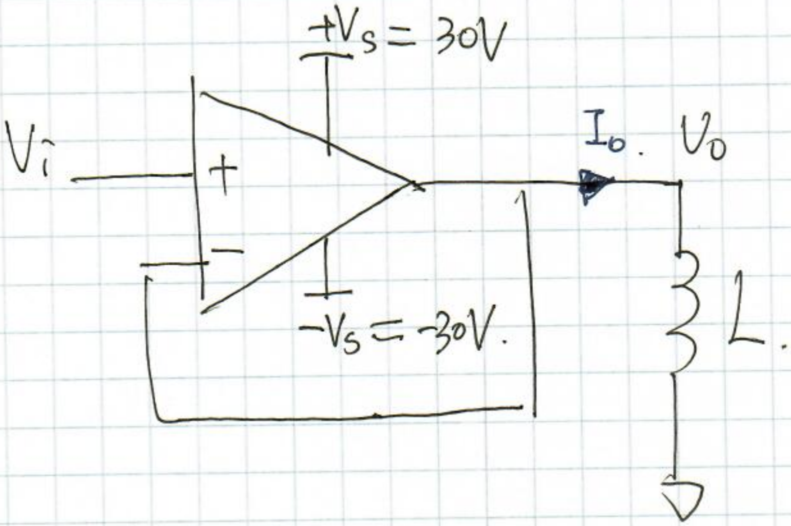


As temperature increasing V_{be1} and V_{be2} will change so D_1 and D_2 cannot balance it.

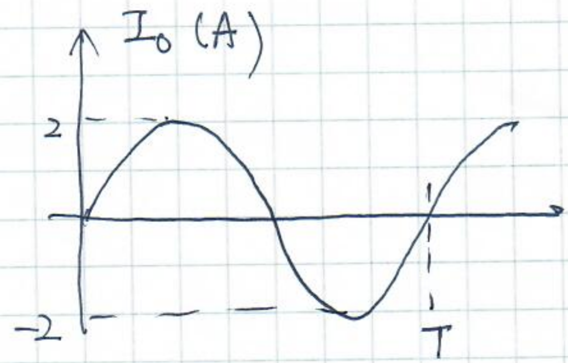
Example : PA13. A familiar name.

4. Power consumption of power amplifier.

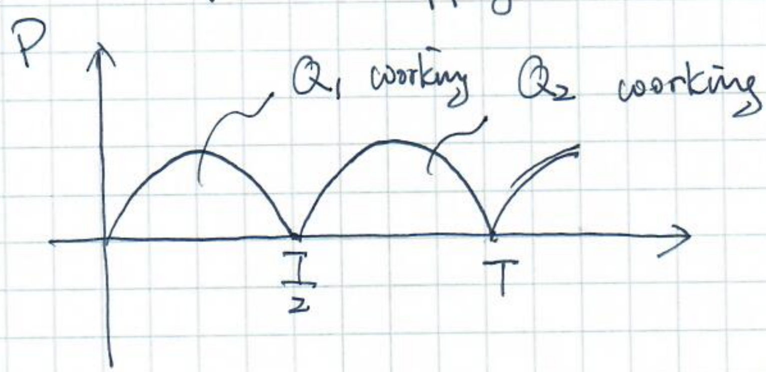
Consider a linear P.A. with unit gain.



Io is a sine wave with 2A amplitude.



The power consumption of P.A. always is calculated from DC power supply.



Average power :

$$\bar{P} = \frac{1}{T}\int_0^T P(t)\,dt$$

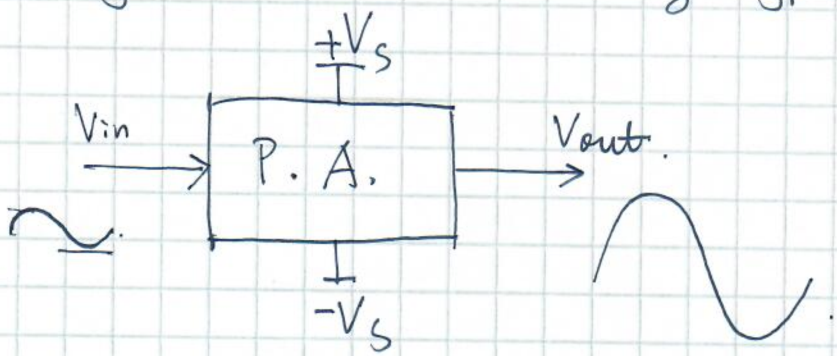
$$= 2\,\frac{1}{\left(\frac{T}{2}\right)} V_s \int_0^{\frac{T}{2}} I_o \sin\left(\frac{2\pi}{T}\cdot t\right) dt$$

$$= \frac{2}{\pi}\cdot 30 \cdot 2 = 38.2\,W.$$

Lecture #21 : Power electronics : switching power, amplifier.

1. Why do we need switching-type power amplifiers ?

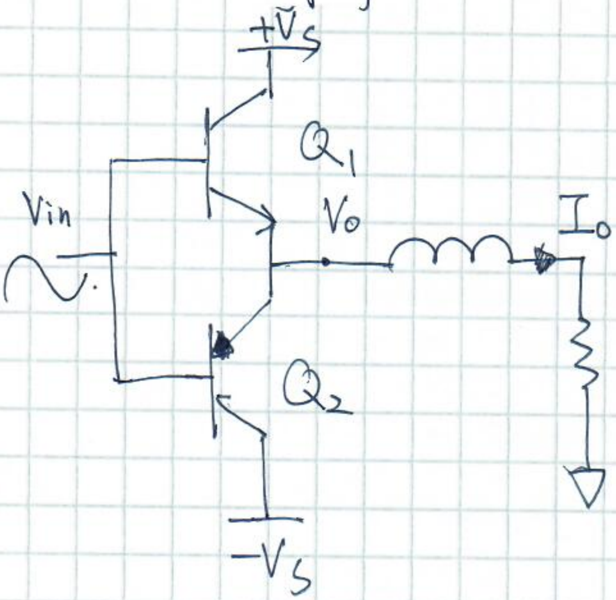
(Low power consumption)



$V_{out} \rightarrow \boxed{1/Z_L} \rightarrow I$

Power amplifers are devices to supply power to drive loads with low impedance. So the power consumption of the power device is very important.

Linear amplifier :



Power on transistors:

When $I_o > 0$, Q_1 working

$$P_D = I_o (V_S - V_o).$$

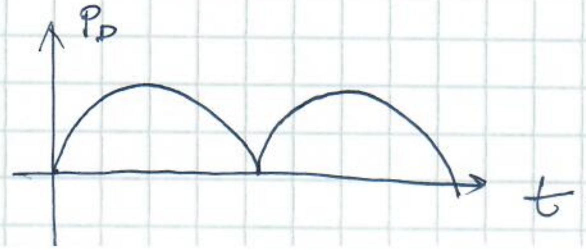
When $I_o < 0$, Q_2 working

$$P_D = -I_o (V_o + V_S).$$

If the input V_{in} is sinusoidal, the average of output power

$$\overline{P_o} = \frac{1}{T}\int_0^T V_o I_o \, dt = 0.$$

But $\overline{P_D} \neq 0$,



For each transistor, $P_D = I_d \cdot V_d$

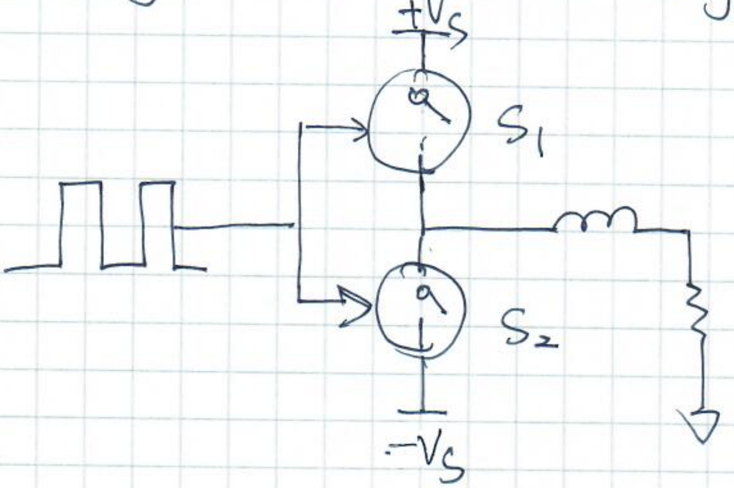
Voltage drop

Current flowing through.

To minimize the power consumption P_D,

$I_d = 0$ or $V_d = 0$.

Then the transistor must work within either cutoff region or saturation region. then Q_1. Q_2 become switch



So the input has to be square wave to saturate the transistors.

2. Bridge circuit and PWM wave.

To drive a motor, two functions have to be achieved.

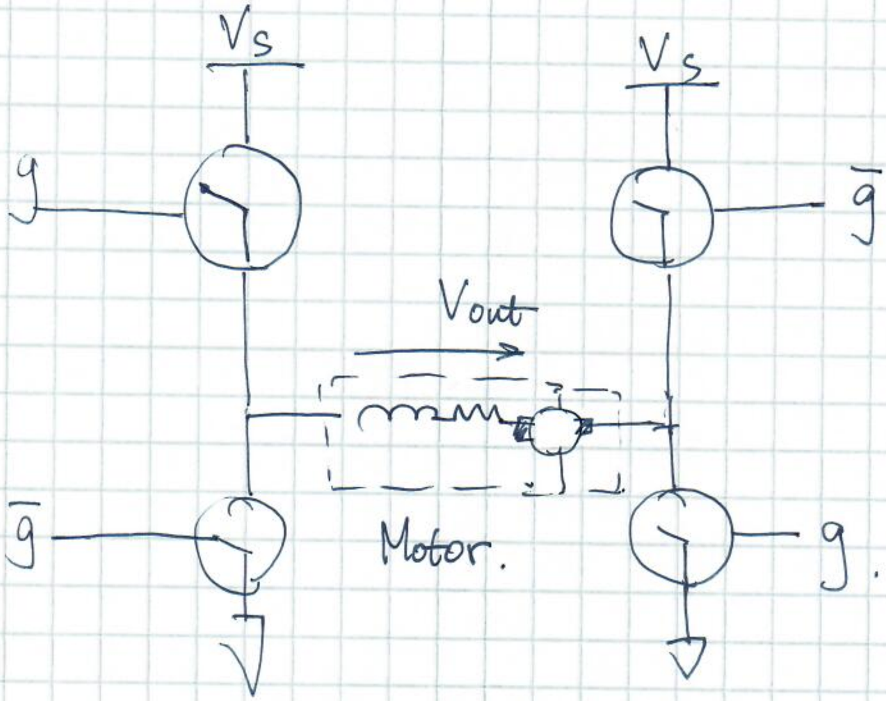
1). Rotate back & forth. (Polarity of voltage)

2). Control the speed (Voltage variation).

Also, single voltage power supply is preferred.

+12V, +24V, +48V. ...

H bridges are widely used in industry for driving DC motor.

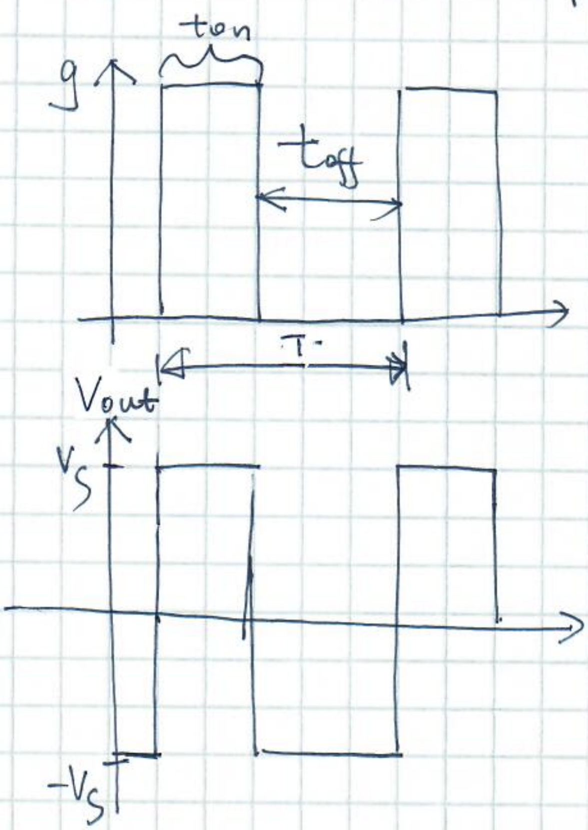


g is the control voltage

$\bar{g}$ = Not g.

$g = 1$, switch close

$g = 0$, switch open.



Average output voltage:

$$\bar{V}_o = \frac{t_{on} V_s + t_{off}(-V_s)}{T}$$

Duty cycle:

$$d = \frac{t_{on}}{T}.$$

g is a PWM wave.

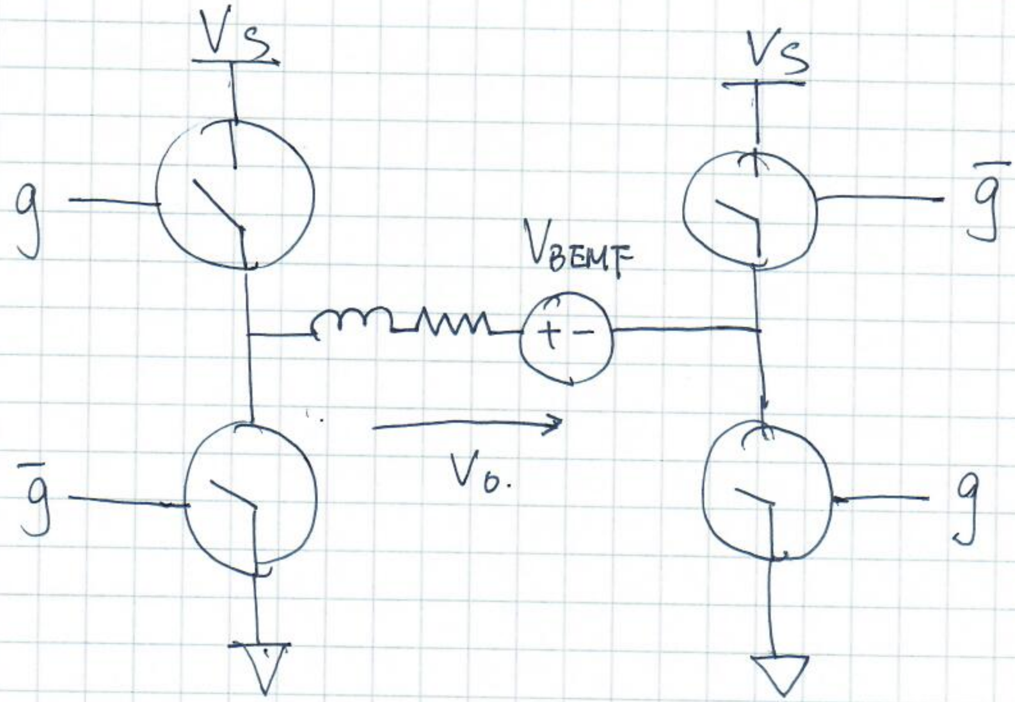
PWM: pulse-width modulation.

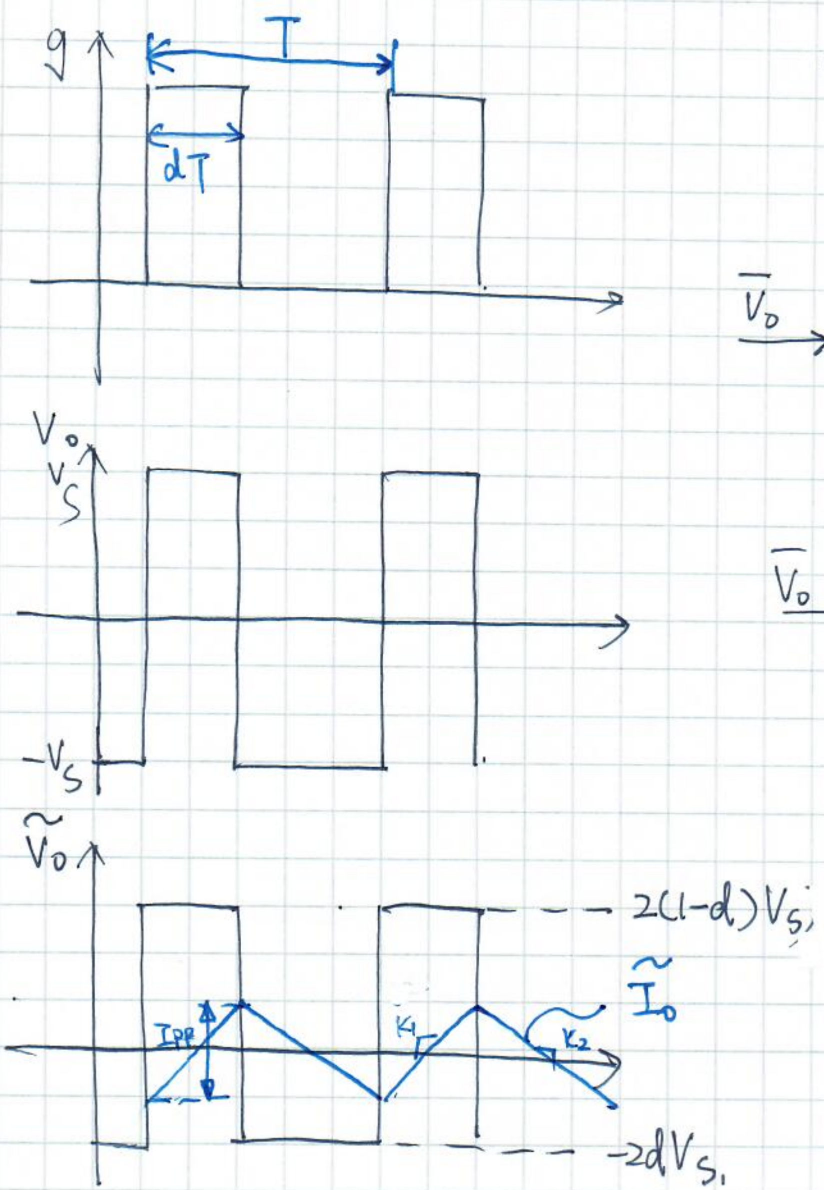
$$\bar{V}_o = dV_S - (1-d)V_S = (2d-1)V_s.$$

AC part of V_{out}

$$\tilde{V}_o = V_o - \bar{V}_o$$

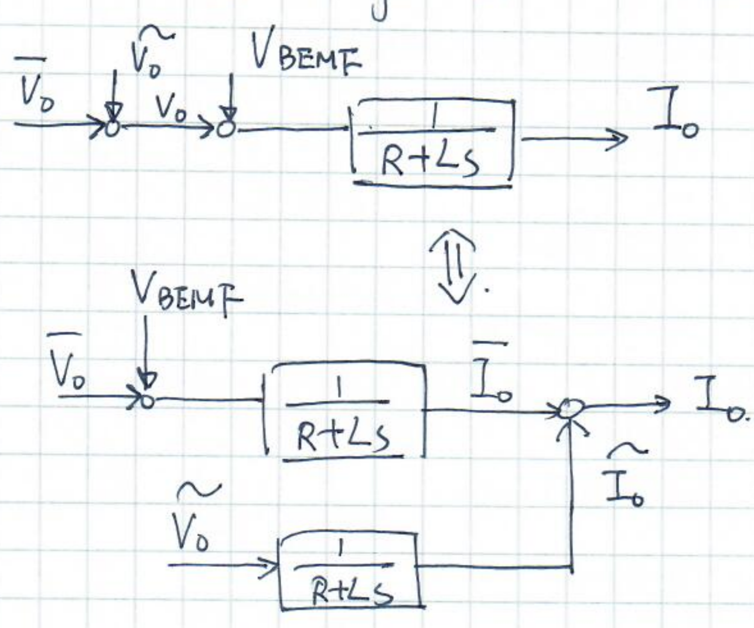
If a DC motor is connected:





$$\bar{V}_o = (2d-1)V_s$$

Block diagram:



$\tilde{V}_o$ is AC part of PWM, and it generates current ripples.

I_{pp} is peak-to-peak amplitude of the current ripples

Only inductance in motor affects the current variation.

$$\Delta I = \frac{V}{L} \cdot \Delta t.$$

If $g=1$, $\quad I_{pp} = \Delta I = \frac{2(1-d)V_s}{L} \cdot d \cdot T$

$$k_1 = \frac{\Delta I}{\Delta t} = \frac{2(1-d)V_s}{L}.$$

If $g=0$, $\quad k_2 = \frac{\Delta I}{\Delta t} = -2d\frac{V_s}{L}.$

Example: $V_s = 30V$, $L = 1.6mH$, $T = 20\mu s$ (50kHz), $d=0.5$.

$$I_{pp} = \frac{2(0.5)(0.5)(30)(20e^{-6})}{1.6e^{-3}} = 0.1875A$$

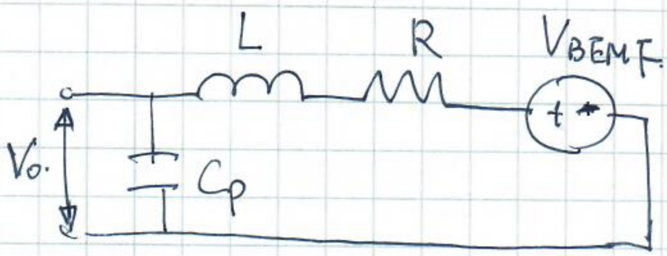
The current ripple is a main drawback of the switching type power amplifier, and it will generate torque/force ripple, so it limits the usage of PWM P.A. in high presision motion applications.

Tips: ① To reduce the ripple (I_{pp}), a compensation inductance can be placed in serial with the motor coil. (larger L)

② The frequency of PWM should be high enough. (smaller T).

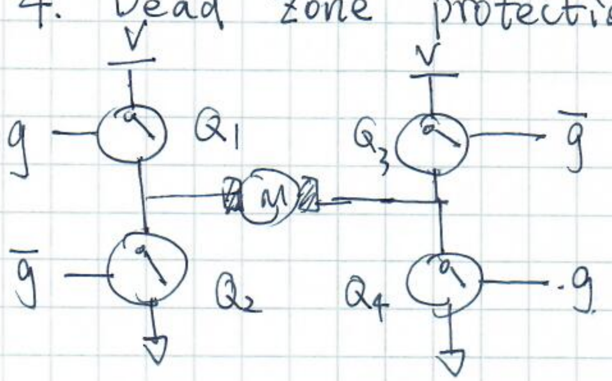
3. Paracitic capacitance.

Small capacitance from actuator's wires and cables brings trouble to the PWM P.A.



At the moment of direction changing, the output Voltage V_o experiences a large variation. (from $+V_s$ to $-V_s$).
So there is a large current flowing through Cp, and it is harmful for the transistors.

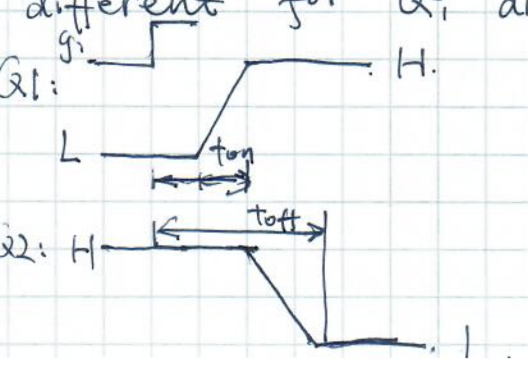
4. Dead zone protection.



Q_1 and Q_2 (Q_3 and Q_4) can not be close at the same time.

Otherwise, MOSFET will burn up. because of extremely low impedance

In pratice, the turning on time and turning off time are different for Q_1 and Q2.



Usually, $t_{on} < t_{off}$, so there is a dead zone for the bridge circuit.
But it can be avoided by adjusting the switching logic (g, $\bar{g}$).

Lecture #22: Power amplifiers for different actuators.

1. Classification of actuators:

P.A.s are selected based on the actuator's properties. According to the P.A. selection, the actuators are classified as:

$$V, I \longrightarrow \boxed{\text{Actuator}} \longrightarrow T, F.$$

Electric field | Mechanical field.

- Static electric field machines: (High voltage) piezo actuators
  - Static Voltage input: stacking type
  - Dynamic voltage input: Ultrasonic motor.
- Dynamic electric field. $\Downarrow$ Magnetic field.: (High current). electromagnetic motors
  - Induction motors. (Asynchronous)
  - Synchronous. AC motor.
  - DC motor. (Brush DC).
  - Solenoid (moving iron motor).

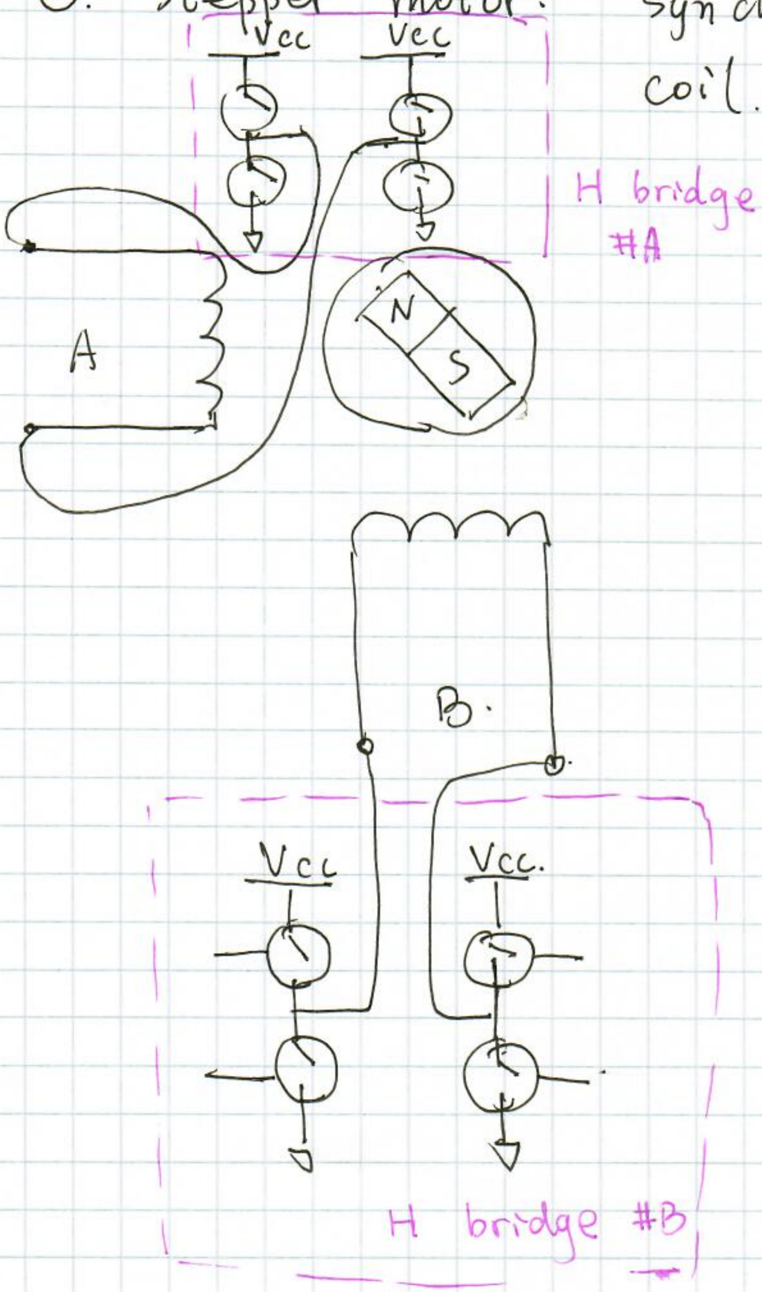
2. Examples :

①. Precision stage : 20μm stroke., high resolution expected

A piezo stacking actuator is selected.

Linear power amplifiers with high supply voltage are prefered.

②. Stepper motor: synchronous motor. with two-phase coil.

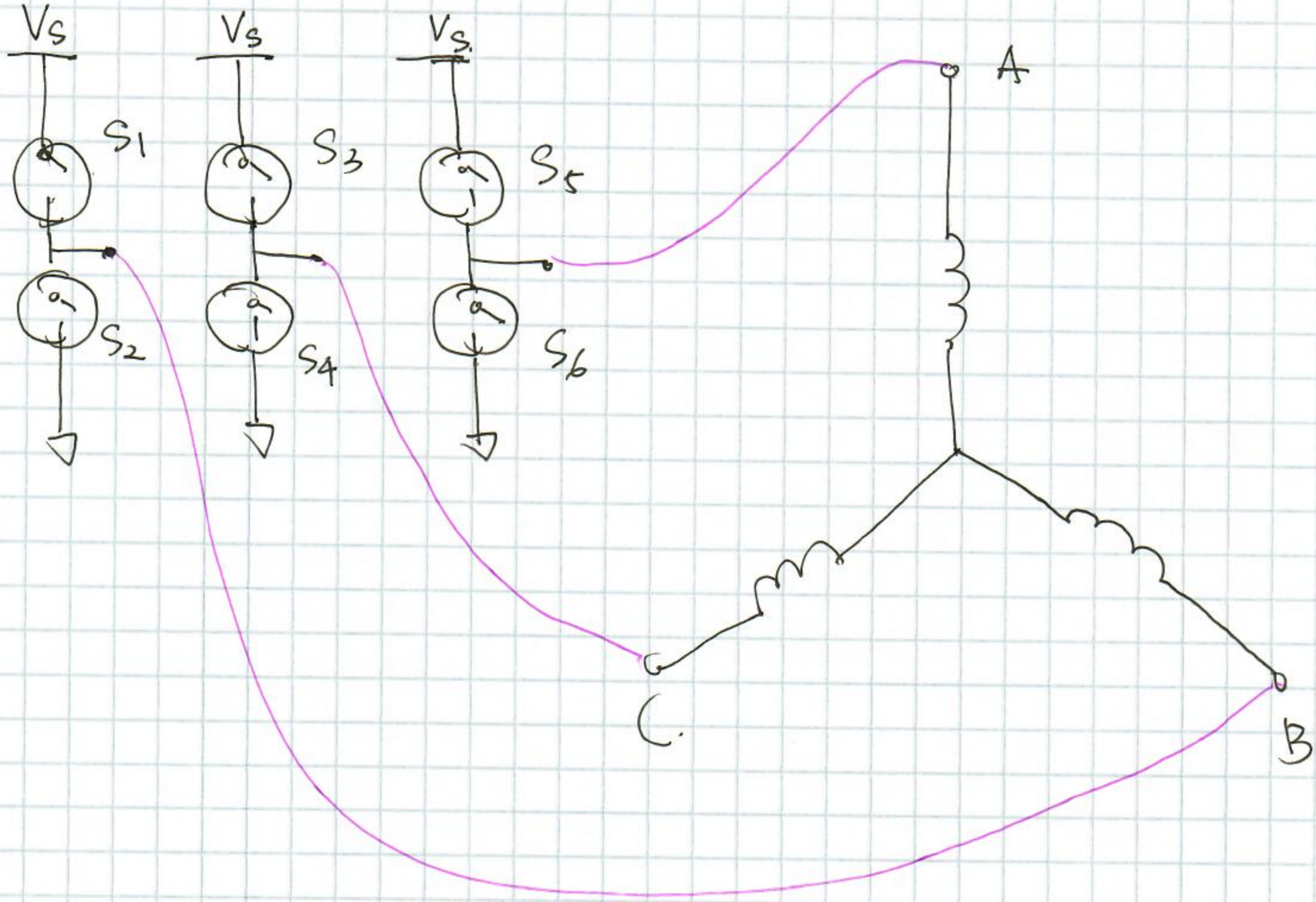


For two phase motor two H-bridges are required. 8 power devices in total.

The motor speed is adjusted by the input frequency of the current.

③. Three-phase motors: induction motors, permanent-magnet synchronous motors (PMSM) can be designed as 3-phase motors.

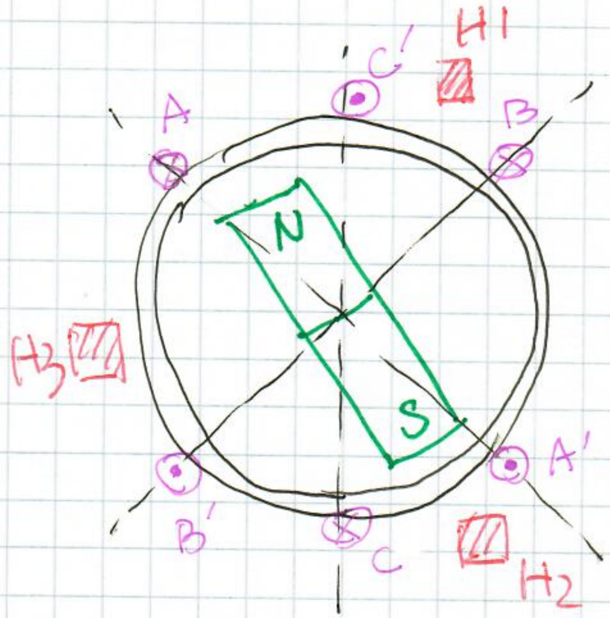
3 phase bridge circuit is used as P.A.



3-phase motor only need 6 power transistors.
in the real application, 3-phase motors are preferred for saving cost and volume regarding P.A.s.

④. Sensors in power amplifier,

Normally for driving PMSM, or called brushless DC (BLDC) motor, 3 Hall effect sensors are placed aside coil to check the position of the rotating permanent magnet.



H1, 2, 3. are Hall effect sensors.

⑤. Dynamic type of piezo actuator, ultrasonic motor.

Normally ultrasonic motors work at up to 30kHz ~ 40kHz.

1). Switching type P.A. can be used, but it cannot adjust the supply ~~on~~ voltage on the motor.

2). linear P.A. can be used as well, and it can adjust supply voltage, but the efficiency is low.

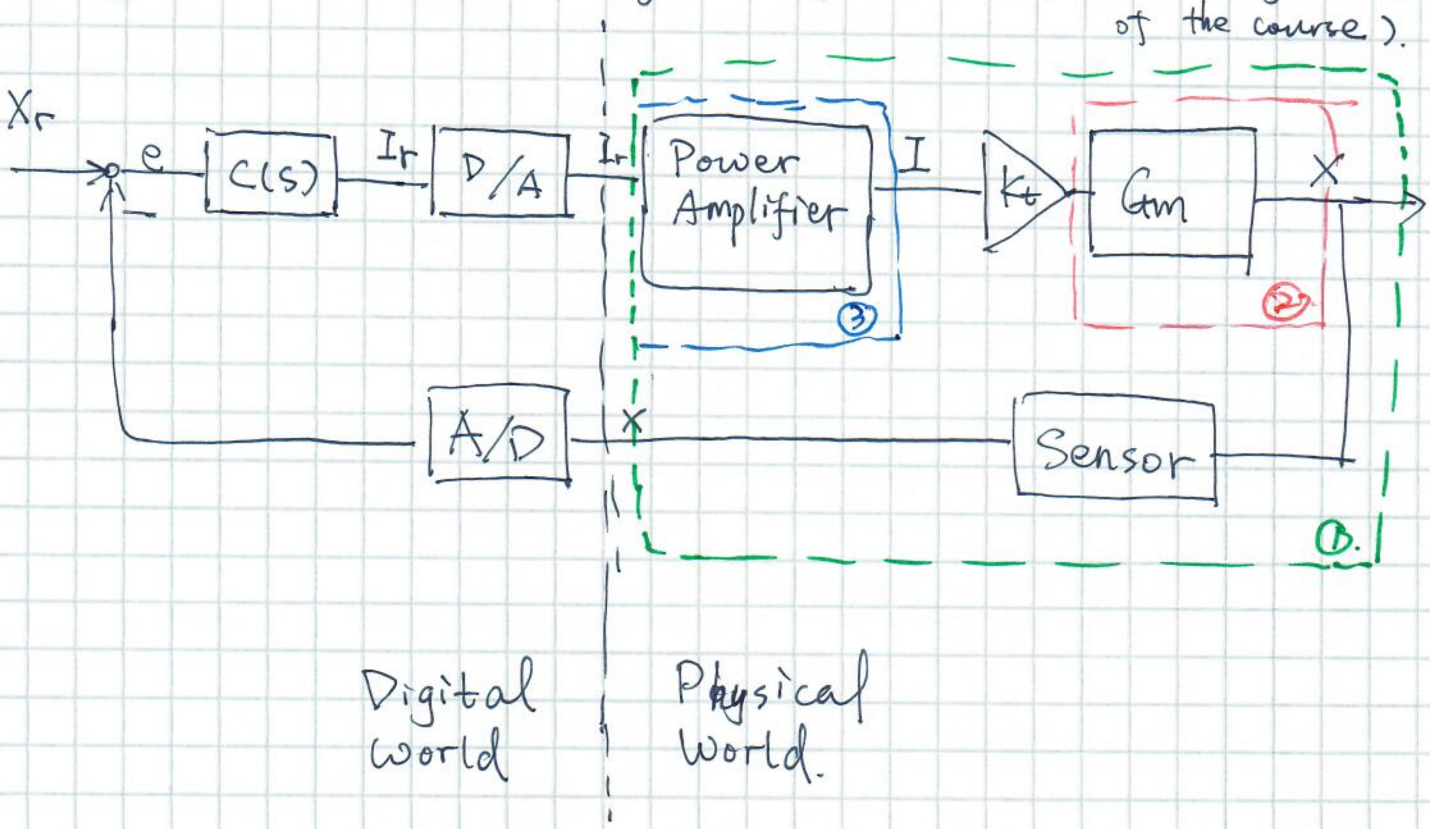
Summary of MECH 421, 2019:

1. About Final Exam:

- April 12^{th}, 12pm – 2pm.
- Two double sided crib sheets
- Calculator is allowed.
- No Matlab.

2. Summary:

A general mechanical system for motion: (We drew a similar block diagram at begining of the course).

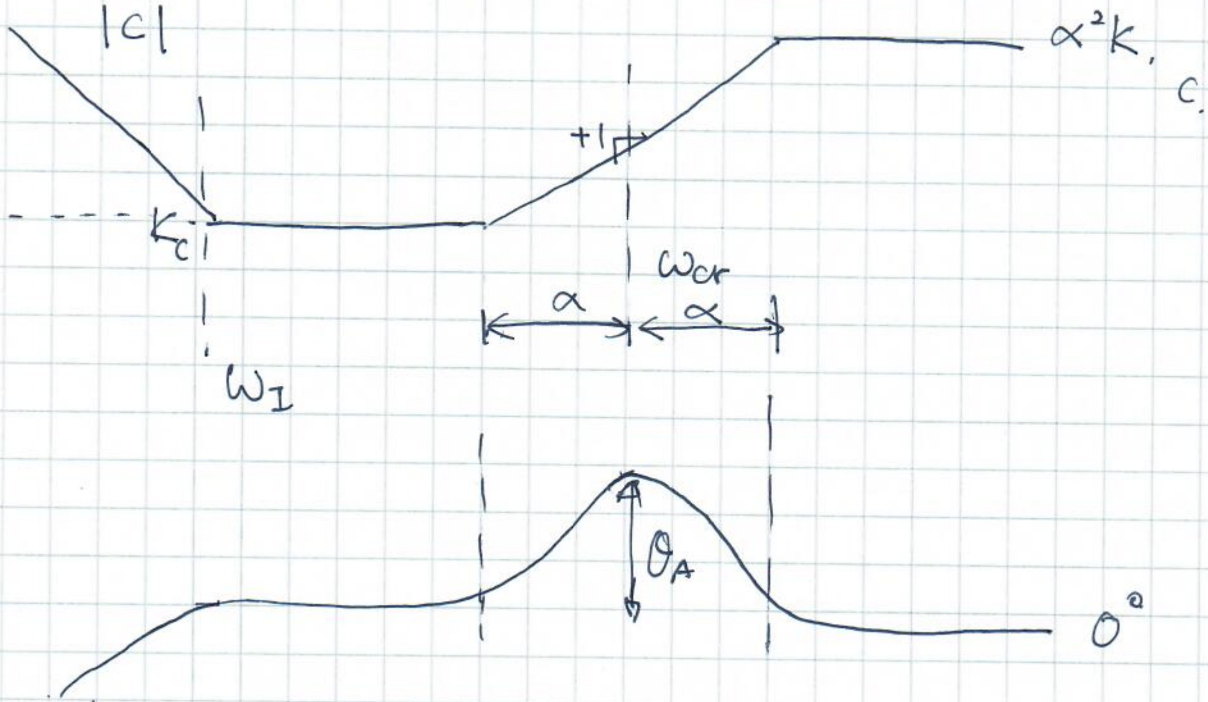


①. Measuring $\frac{X}{I_r}$ frequency response digitally to design C(S). (Techniques to design controller without known the analytical T.F.)

Here, for C(S) we use loop shaping controller.

Specs: fast; stable; accurate.

what does loop shaping controller look like ?



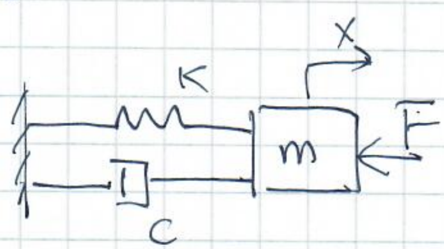
The key to design a controller is to select a proper crossover frequency in NLT.

②. To understand the performance of mechanical system and to improve band width, we model $G_m(s)$.

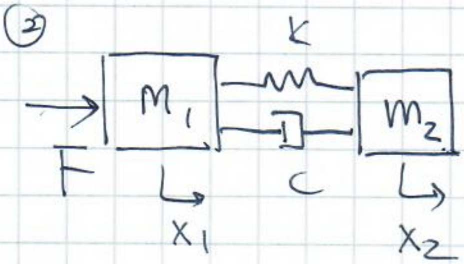
Method: Mechanical impedance with stiffness network.

$$\begin{matrix} K \\ C \\ m \end{matrix} \implies Z_m(s).$$

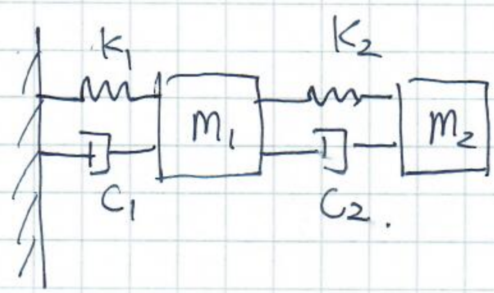
Objects: ①



②



③



③. Power Amplifier: { Switching / linear.

key questions:

1). How to select P.A. types for different applications?

Linear: low power, high accuracy

Switching: high power, low accuracy.

2). Calculation of duty ~~cl~~ cycle, current ripple and average voltage for PWM power amplifier.

3). Issues of PWM amplifier.