MECH 420
Sensors and Actuators

Presentation Part 5

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Part 5: Performance Specification

- Performance Specification
- Instrument Rating Parameters
- Bandwidth Issues
- Error Propagation

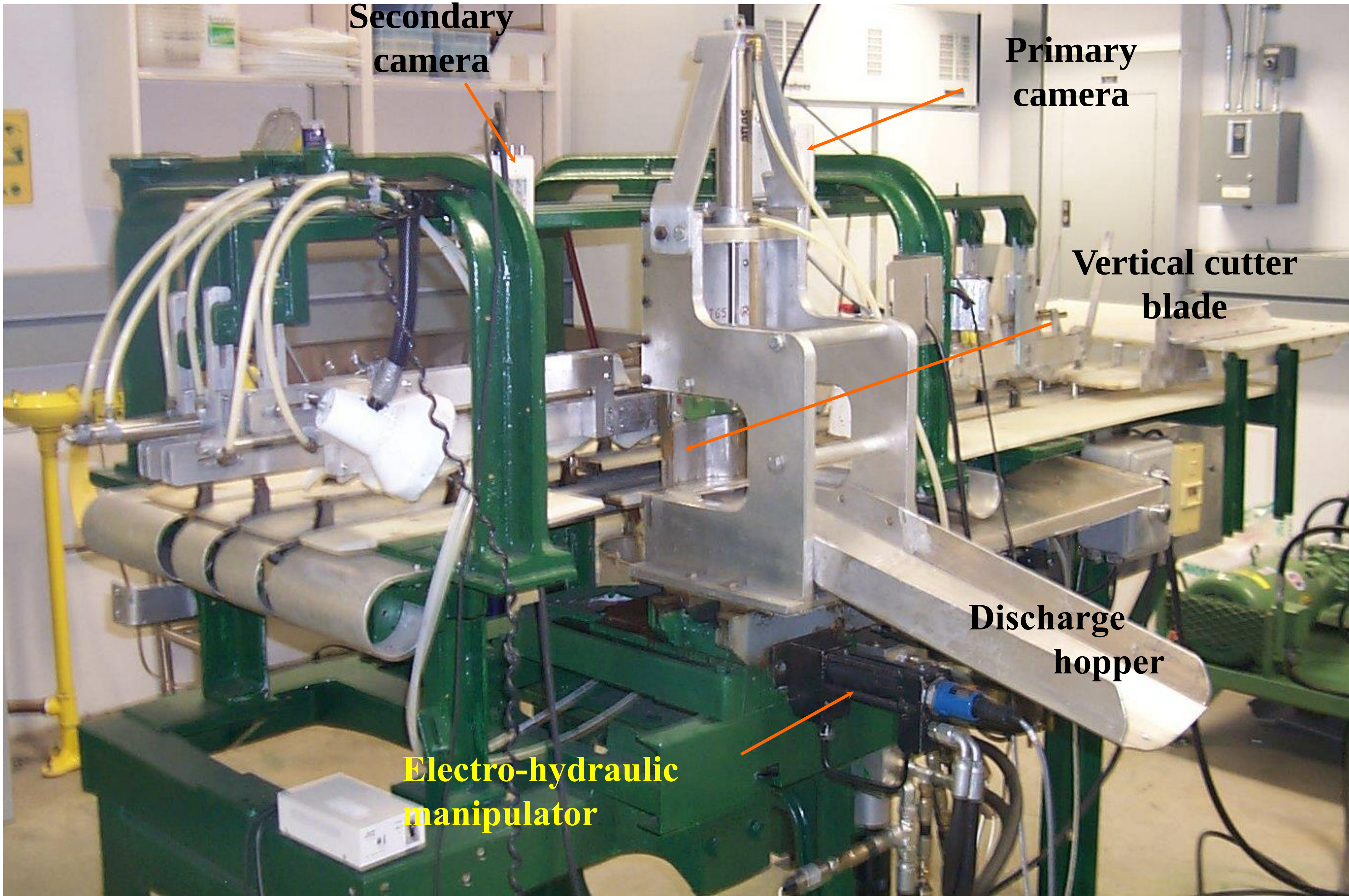
Plan

- **Performance Specification**
- **Categories of Performance Specification**
- **Reference Models for Performance Specification (Time Domain and Frequency Domain)**
- **Analytical and Commercial Parameters of Performance Specification**
- **Sensitivity and Error (Similar Analysis for Both)**
- **Bandwidth Interpretation and Application**
- **Error Analysis**

Performance Specification

Intelligent Iron Butcher

Performance of the overall system depends on that of the individual components, how they are interconnected and interacting, etc.



Performance Specification**

Important for both "System" and "Components"

- **An engineering system consists of an integration of several components**
- **Performance (Intended purpose) of the system depends on the performance of the individual components and how they are interconnected**
- **Performance requirements are specified/ established, and component performance is given using rating parameters (performance parameters)**
- **Instrumentation—Prescription, selection, design, integration of components/instruments having rating parameters that match the required performance of the system**

Parameter Categories of Performance Specification

One Classification: 1. Speed of performance; 2. Stability; 3. Accuracy (for control systems too)

Another Classification:

- **Parameters used in engineering practice (e.g., listed in commercial component data sheets)**
- **Parameters defined using engineering theoretical considerations (1. Time domain, 2. Frequency domain; both use reference models)**

Further Classifications (may fall within previous ones): Static parameters; Dynamic parameters; Model (reference)-based; Signal/Data-based

Perfect Measurement Device

1. **Output instantly reaches the measured value (*fast response*)**
2. **Output is sufficiently large (*high gain, low output impedance, high "direct sensitivity"*)**
3. **Output remains at measured value (without *drifting* or getting affected by environmental effects, disturbances, noise, etc.) when "measurand" is steady (*stability and robustness*)** What is a "measurand"?
4. **Output signal is proportional to the measurand (*static linearity*)**
5. **Does not distort the measurand (*no loading; matched impedances*)**
6. **Efficient, Low power consumption (*high input impedance*)** Why?

Reference Models

(Models of "required performance" <u>Not</u> device models)

Dynamic Reference Model

- **Not a model of the device, but a reference model to define "required performance"**
- **Specifies the required performance/ratings**

Model Categories:

- **Differential-equation models (Time domain)**
- **Transfer-function models (Frequency domain)**

Assumptions: Linear, time-invariant

***Note*: Two model types are equivalent (through Laplace transform or Fourier transform) ➔ Two types are equivalent**

Time-domain Specifications

Time-domain Specification (1st Order Model)

Time-domain Model: $\tau \dot{y} + y = ku$

Saw this under "filters"

Transfer-function Model: $\frac{Y(s)}{U(s)} = H(s) = \frac{k}{\tau s + 1}$

Saw this under "filters"

u **= input,** ***y*** **= output,** τ **= time constant,** ***k*** **= (dc) gain**

Response to step input of magnitude ***A*****:** $y_{\text{step}} = y_0 e^{-t/\tau} + Ak(1 - e^{-t/\tau})$

Derive this. Total response = IC response + Input response

Note **1: Initial tangent line (slope =** $(Ak - y_0)/\tau$ **) will meet final (steady-state) value (*****Ak*****) at time** ***t*** **=** τ **(another interpretation of time constant)**

Note **2: Half-power bandwidth = 1/**τ **(saw in Part 4 of notes)**

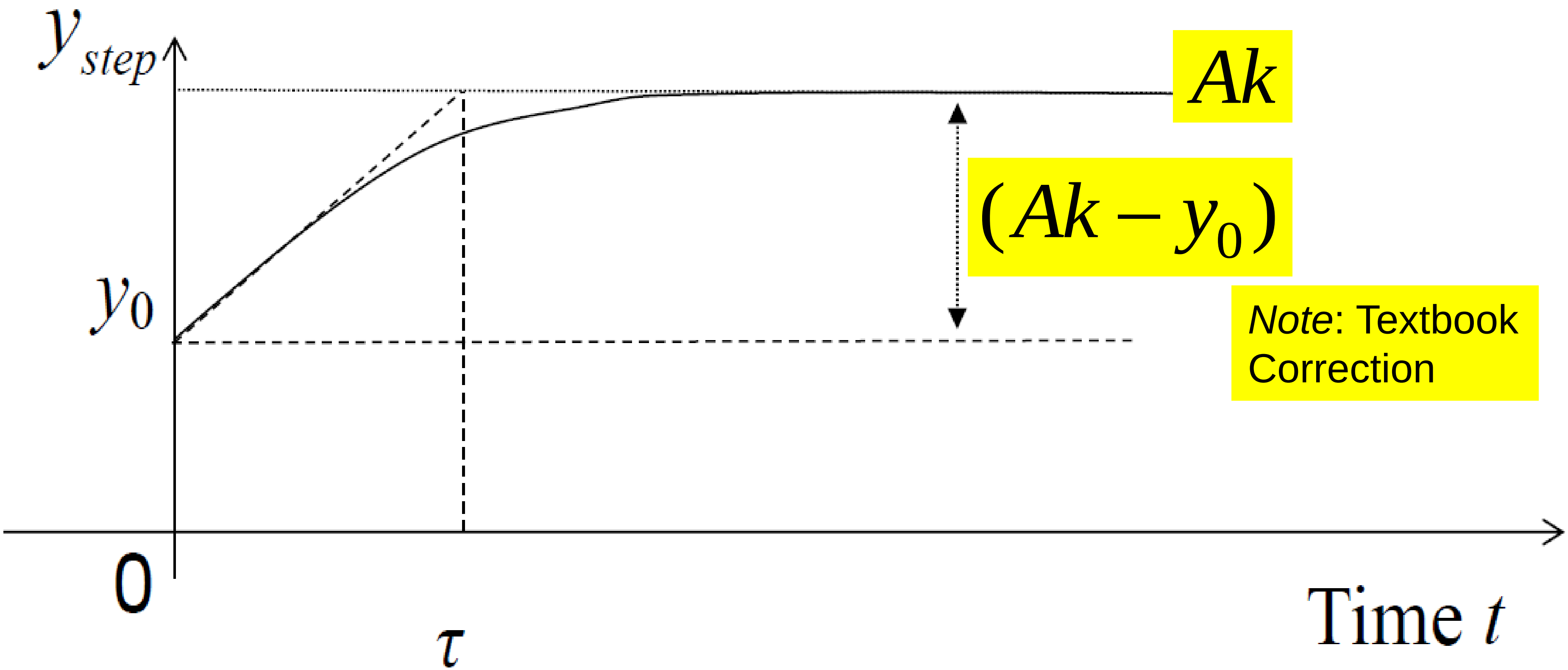
(Power reduction by 2→ amplitude reduction by $\sqrt{2}$ **→ 3dB)**

dB value = 20 log$_{10}$ **()**

First Order Model: Advantages?
Disadvantages?

Performance Parameters Using 1st Order Response

Performance Parameters: Time constant τ and dc gain k

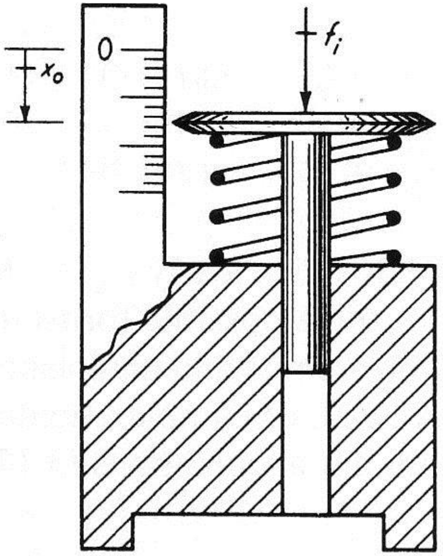


***Notes*: 1.** 1st order reference model has only one performance parameter (time constant τ, which is the "dynamic parameter"); **2.** Gain k is normalized to 1 (and adjusted physically by an amplifier or computationally by simple multiplication); **3.** Cascading two 1st order models cannot represent oscillations (unlike a simple oscillator).

Simple Oscillator Model (2nd Order)

Time Domain: $\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \omega_n^2 u(t)$

Transfer Function: $\dfrac{Y(s)}{U(s)} = H(s) = \left[\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right]$



Step (Unit) Response:

$$y_{step} = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \phi) \quad ; \quad \cos\phi = \zeta$$

ω_n **= undamped natural frequency,** ζ **= damping ratio**

Damped Natural Frequency $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$

Question: Give a performance parameter that this model cannot represent. Why?

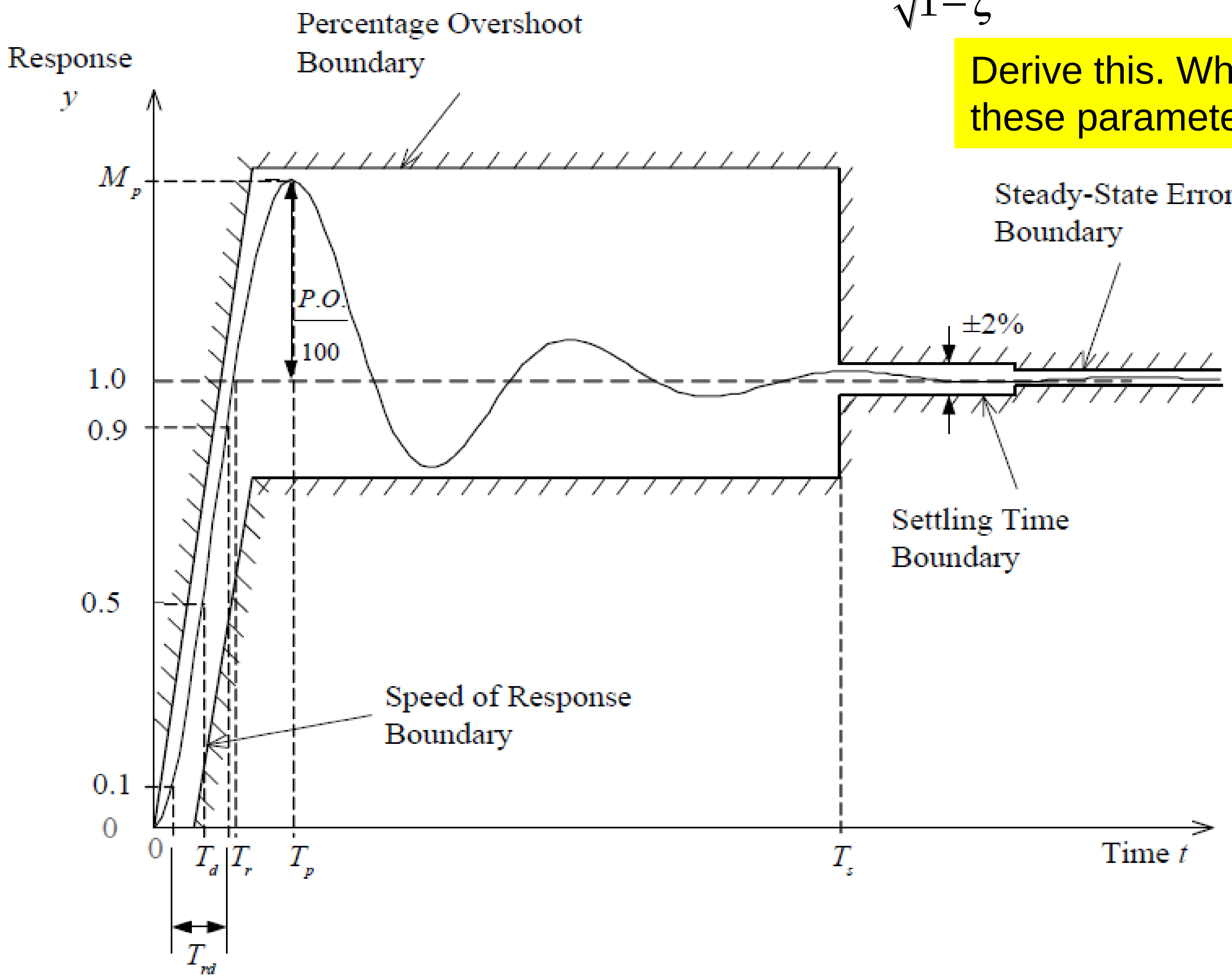
Advantages?
Disadvantages?

Time-domain Specifications (2^{nd} Order)

Based on "unit step" response:

$$y_{step} = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \phi)$$

Derive this. What are these parameters?



Time-domain Specifications

Time-Domain performance parameters using the simple oscillator model.	
Performance Parameter	**Expression**
Rise Time	$T_r = \dfrac{\pi - \phi}{\omega_d}$ with $\cos\phi = \zeta$
Peak Time	$T_p = \dfrac{\pi}{\omega_d}$
Peak Value	$M_p = 1 - e^{-\pi \zeta / \sqrt{1-\zeta^2}}$
Percentage Overshoot (PO)	$\mathrm{PO} = 100\, e^{-\pi \zeta / \sqrt{1-\zeta^2}}$
Time Constant	$\tau = \dfrac{1}{\zeta\omega_n}$
Settling Time (2%)	$T_s = -\dfrac{\ln\left[0.02\sqrt{1-\zeta^2}\right]}{\zeta\omega_n} \approx 4\tau = \dfrac{4}{\zeta\omega_n}$

Of speed, stability, and error, what do these parameters represent?

What performance attribute is not represented in this table?

Example 1

For an application:

Fastest signal component that needs to be accurately measured = 100 Hz

Estimate an upper limit for sensor time constant

Why “upper” (not “lower”) limit?

Solution:

Fastest signal component = $(100 \times 2\pi)$ rad/s

Make the sensor 10 times faster (a rule of thumb)

$\rightarrow$ $\frac{1}{\tau} = 10 \times (100 \times 2\pi)$ rad/s $\rightarrow$ $\tau = \frac{1}{10 \times (100 \times 2\pi)}$ s = 159 μs

τ **= sensor time constant**

***Note*: This is a very conservative estimate. For typical applications, it is adequate for the sensor to be twice as fast** $\rightarrow$ $\tau = 0.8$ ms

Example 2

Automobile:

Weight = 1000 kg.

Equivalent stiffness at each wheel (including suspension system) ≈ 60.0 × 10^3 N/m

Suspension is designed for a PO = 1%

Estimate damping constant needed at each wheel.

Solution:

simple oscillator model (quarter model): $m\ddot{y} + b\dot{y} + ky = ku(t)$

***m* = 250 kg; *k* = 60.0 × 10^3 N/m**

PO: $1.0 = 100 \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$ → $\zeta = 0.83$

$\zeta = \frac{b}{2\sqrt{km}}$ → $0.83 = \frac{b}{2\sqrt{60\times10^3\times250.0}}$ → ***b* = 6.43 × 10^3 N/m/s**

Prove

Frequency-domain Specifications

Simple Oscillator Model (2nd Order)

Transfer Function: $$\frac{Y(s)}{U(s)} = H(s) = \left[\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right]$$

(normalized)

ω_n **= undamped natural frequency,** ζ **= damping ratio**

Frequency Transfer Function (FTF) or Frequency Response Function (FRF)—set $s = j\omega$:

$$H(j\omega) = \left[\frac{\omega_n^2}{\omega_n^2 - \omega^2 + 2j\zeta\omega_n\omega} \right]$$

$$\text{At } \omega = \omega_n\text{: Gain} = \frac{1}{2\zeta}; \text{ Phase lead} = -\frac{\pi}{2}$$

Prove

Resonant Frequency $\omega_r = \sqrt{1-2\zeta^2}\,\omega_n$

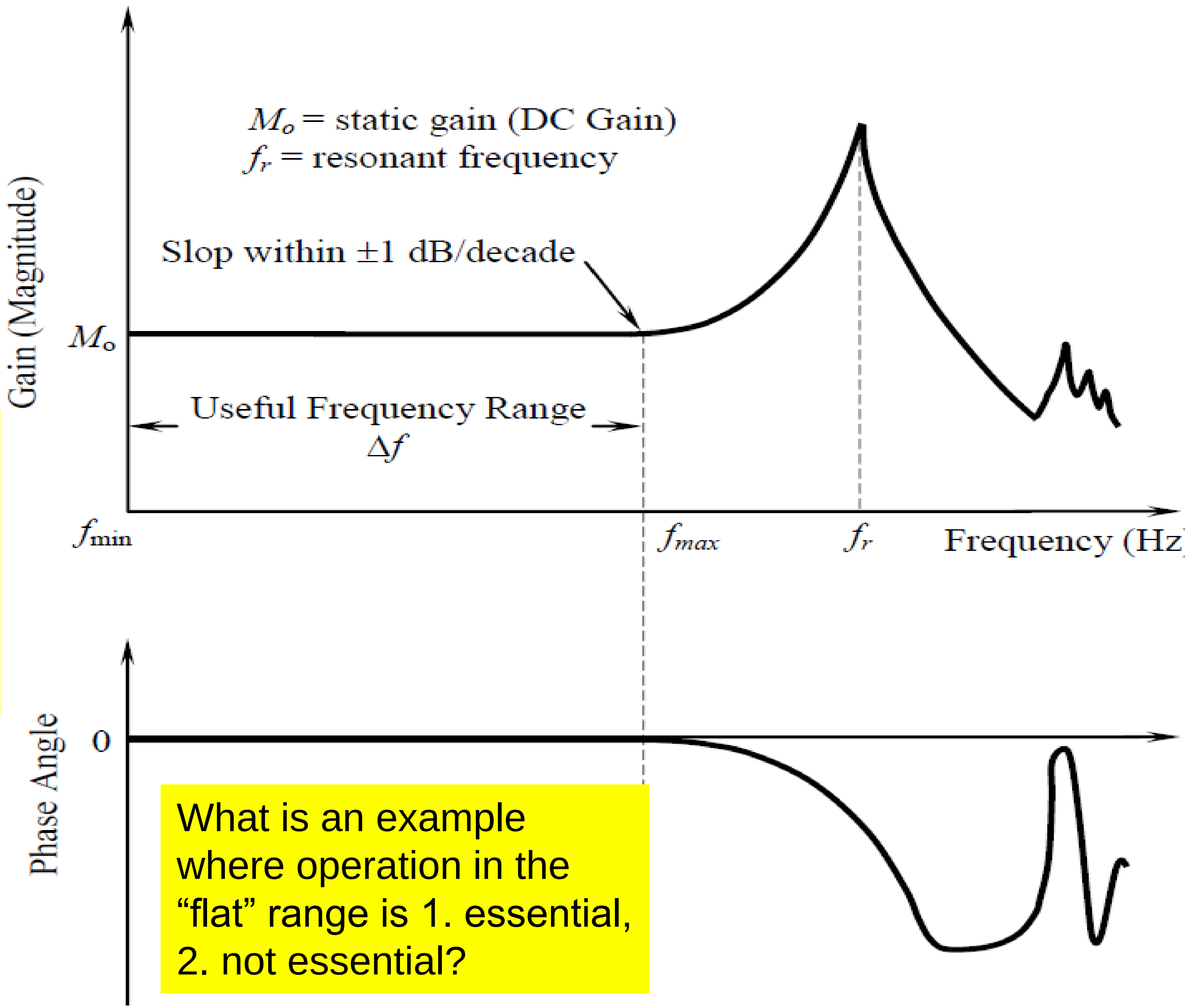
(valid for $\zeta \le 1/\sqrt{2}$ **)**

What is resonant frequency?

Compare: Undamped natural frequency, damped natural frequency, resonant frequency

Frequency-domain Specifications

Response parameters for frequency-domain specification of performance



Can we determine such "Bode plots" for nonlinear systems?

What is an example where operation in the "flat" range is 1. essential, 2. not essential?

Frequency-domain Specifications

Performance parameters for frequency-domain specification:

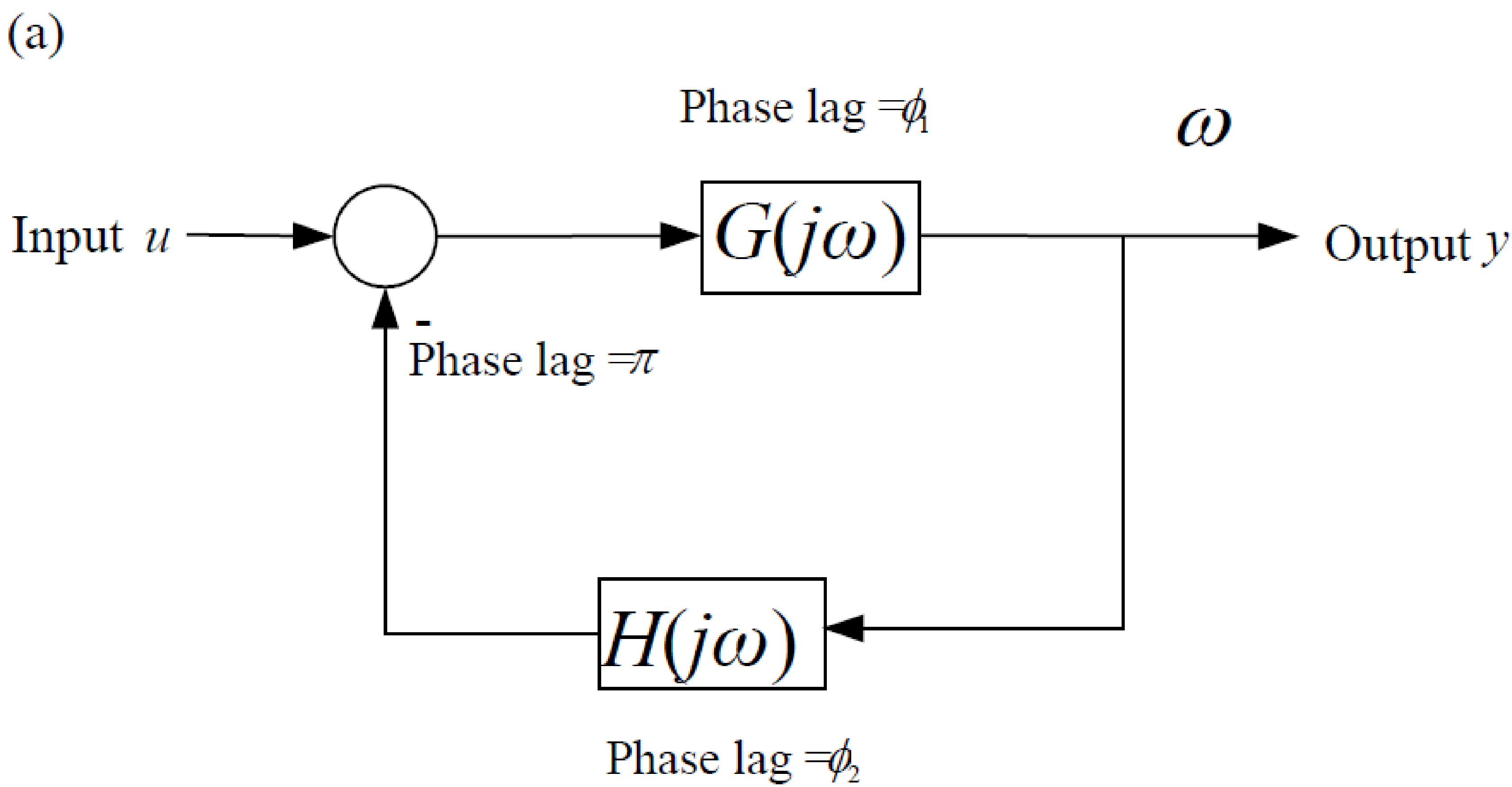
- **Useful frequency range (*operating interval*)**
- **Bandwidth (*speed of response, operating range, etc.*)**
- **Static gain—dc gain (*steady-state performance*)**
- **Resonant frequency (*speed* and *critical frequency region*)**
- **Magnitude at resonance (*stability*)**
- **Input impedance (*loading*, *interconnectability*, *efficiency*, *maximum power transfer*, *signal reflection*)**
- **Output impedance (*loading*, *interconnectability*, *efficiency*, *maximum power transfer*, *signal level*)**
- **Gain margin (*stability*)**
- **Phase margin (*stability*)**

Why does BW represent speed of response?

Stability: Gain Margin and Phase Margin

Qualitative Explanation of the Stability Margins.

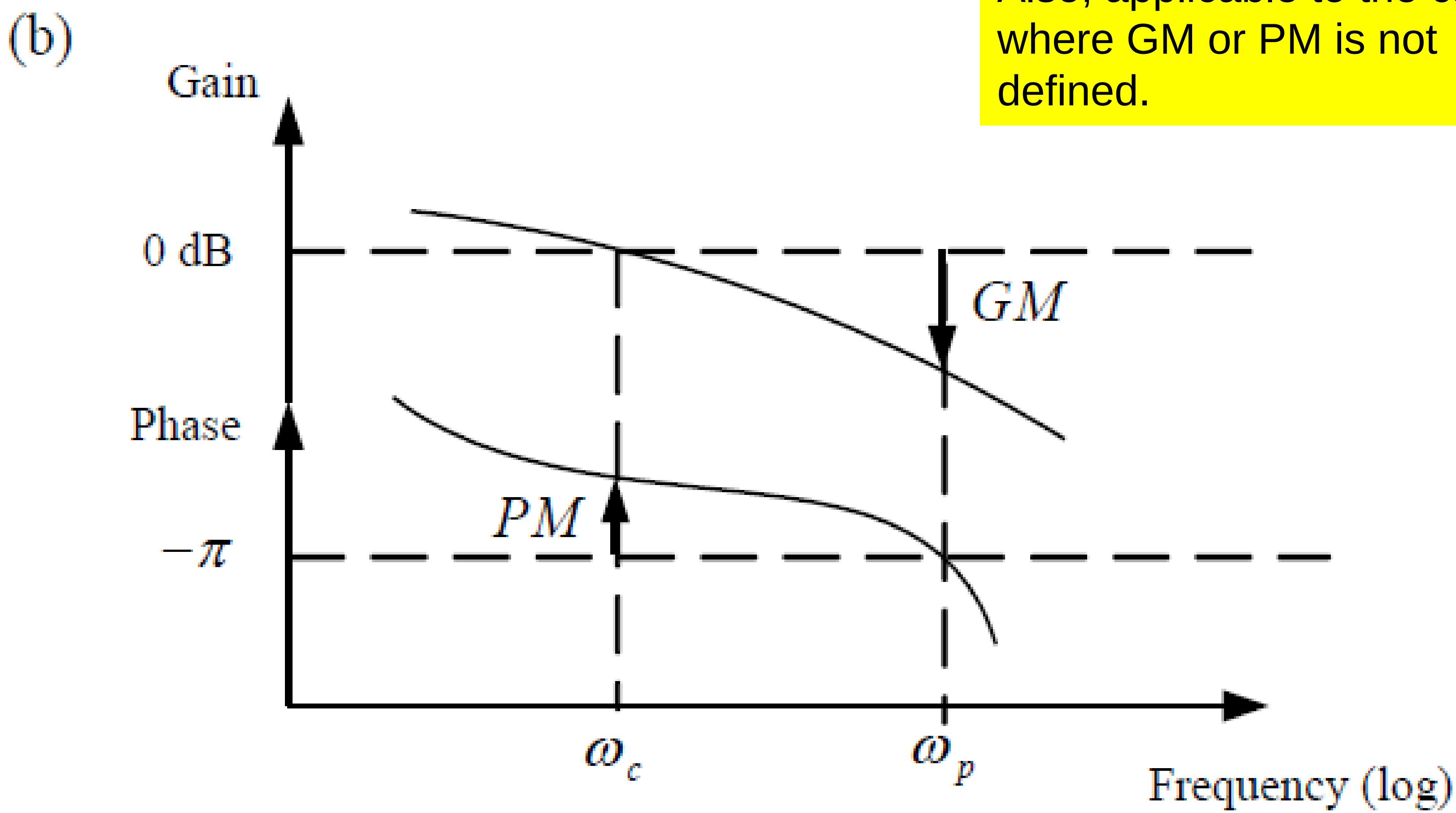
Consider the feedback system:



Gain Margin and Phase Margin

Bode diagram:

Also, applicable to the case where GM or PM is not defined.



Questions

What quantities can you use to specify the following characteristic of a device (in time domain and frequency domain)?

(a) Speed of Response

Time domain:

Frequency-domain:

(b) Stability

Time domain:

Frequency-domain:

Yes or No?

Transfer function of a device assumes linearity:

Linearity

Linearity

- **Linear device➔ 1. Principle of superposition (PoS) applies; 2. Can be modeled by linear differential equations (LDEs) of time *t* or transfer functions of frequency ω**

PoS: If input $\boldsymbol{u}_1$ gives an output $\boldsymbol{y}_1$, and if input $\boldsymbol{u}_2$ gives an output $\boldsymbol{y}_2$, then, input $a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2$ gives an output $a_1\boldsymbol{y}_1 + a_2\boldsymbol{y}_2$ for any a_1 and a_2

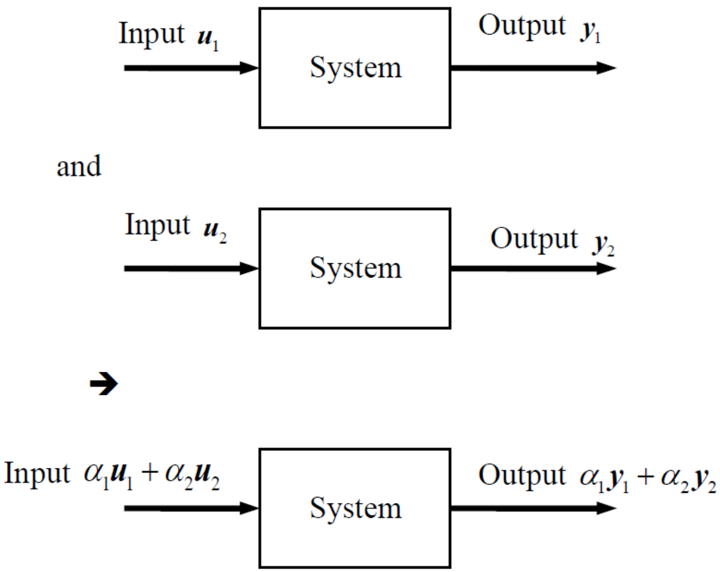
(*Note*: Vector—multiple inputs and outputs)

- **Laplace transform cannot be used to solve nonlinear differential equations**

***Note*: Transfer function representation (analytical, polynomial ratio) assumes linearity**

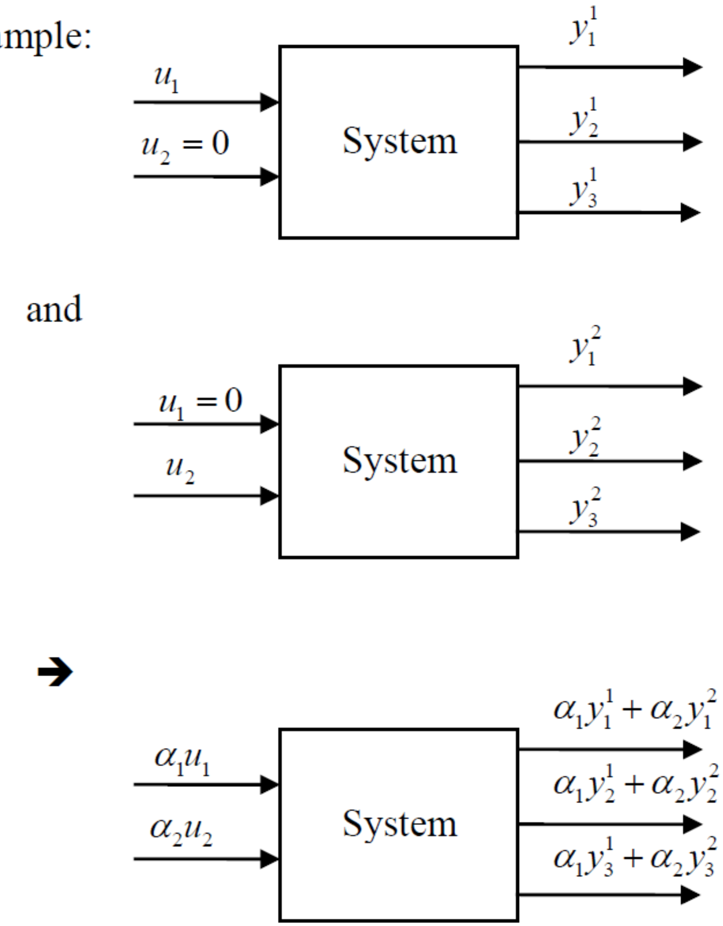
- **An important property of a nonlinear system: Stability may depend on the system inputs and/or initial conditions**
- **All physical devices are nonlinear to some degree (due to saturation, hysteresis, dead zones, friction, creep, backlash, etc.)**
- **Nonlinear devices are often analyzed using linear techniques by considering small excursions about an operating point (local linearization)**

Principle of Superposition (PoS)



General, Vector Case

(b) Two-input Example:



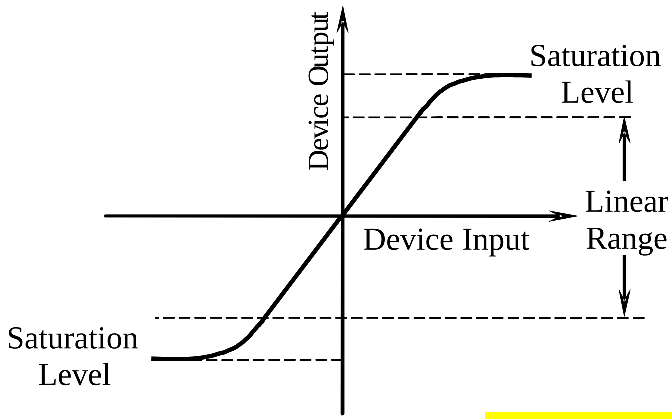
Two General Categories of Practical Nonlinearities:

- Physical nonlinearities
- Geometric nonlinearities

What are these?

Common Manifestations of Nonlinearity

Saturation

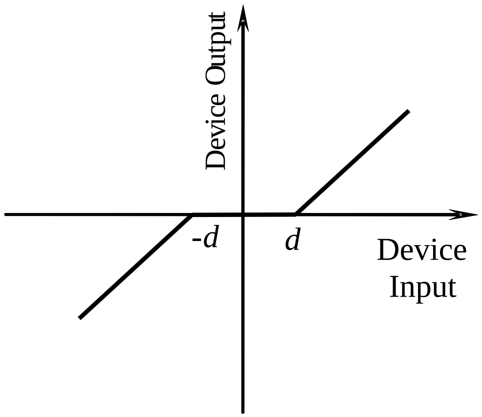


This nonlinearity may result from

- Magnetic saturation – transformers
- Plasticity – mechanical components
- Nonlinear deformation – springs

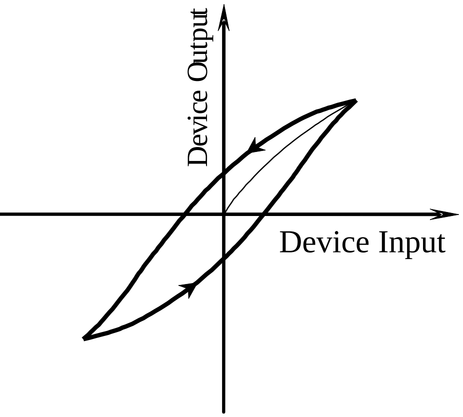
Saturation example in electrical/electronic domain?

Dead Zone:
A region in which a device would not respond to an input, due to:
Mechanical: backlash, stiction, etc.
Electrical: electronic component bias (reverse bias until breakdown, opamps), etc.



Common Manifestations of Nonlinearity

Hysteresis



Hysteresis: Output depends on the direction of the input (whether increasing or decreasing).

Mechanical:

- **Loose components – gears**
- **Nonlinear damping**
- **Nonlinear stress-strain curve (with residual strain)**

Magnetic Devices:

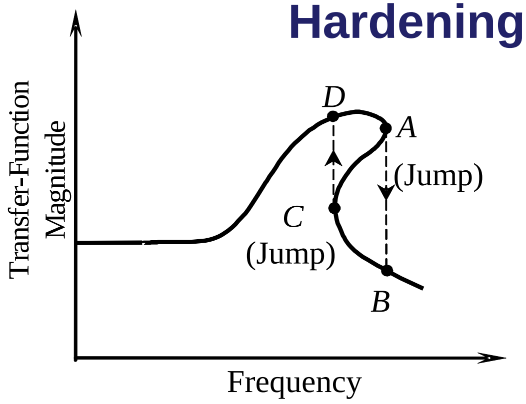
DC current through a coil wrapped around ferromagnetic core versus magnetic field

Area of hysteresis loop = ?; Does a loop always mean nonlinearity?

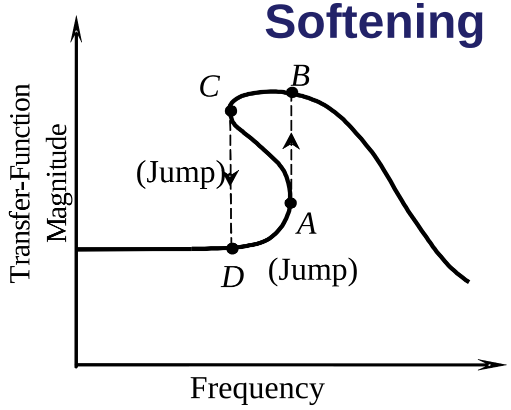
Jump Phenomenon

Why are these called softening/hardening?

Hardening

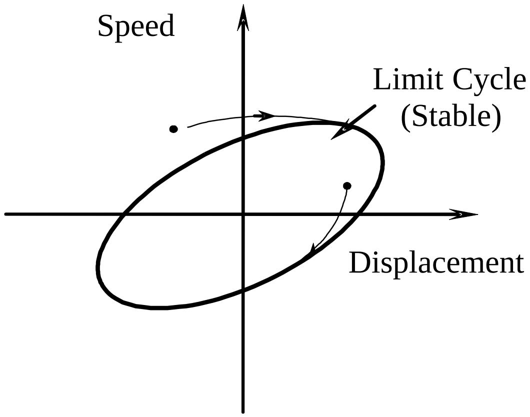


Softening



Common Manifestations of Nonlinearity

Limit Cycles



Nonlinear devices may produce limit cycles:

- **Closed trajectory in the state space**
- **Corresponds to sustained oscillations**
- **Amplitude is independent of the initial condition**

Oscillations in *LC* circuit or mass-spring device: Are these limit cycles?

Frequency Creation:

At steady state nonlinear devices can create frequencies that are not present in the excitation frequency

- **Harmonics (integer multiples of excitation frequency)**
- **Sub-harmonics (integer fractions of excitation frequency)**
- **Non-harmonics (rational fractions of excitation frequency)**

Example of Signal Distortion Due to Nonlinearity

Input (*u*)-output (*y*) behavior of a device: $y = ke^{pu}$

Sinusoidal input: $u = u_0 \sin \omega t$

Output will not be sinusoidal

"Transform" the problem as: $\log(y) = pu + \log(k)$

➔ Input-output (log) relationship is now linear without loss of accuracy

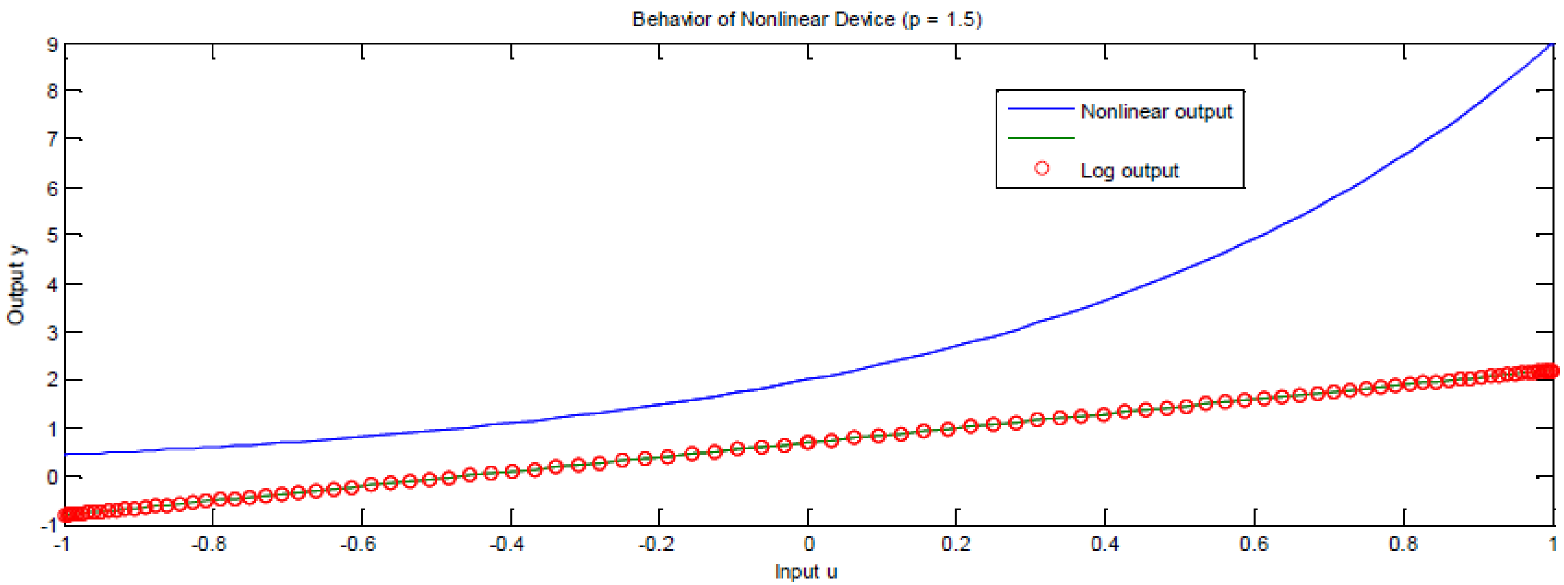
Calibration: Use log scale for output and add a constant offset

***Note*: In the recalibrated form, the output is purely sinusoidal**

From the linearized input-output curve, we can get the parameters *p* and *k* as:

Slope = *p*; *y*-intercept = log *k*

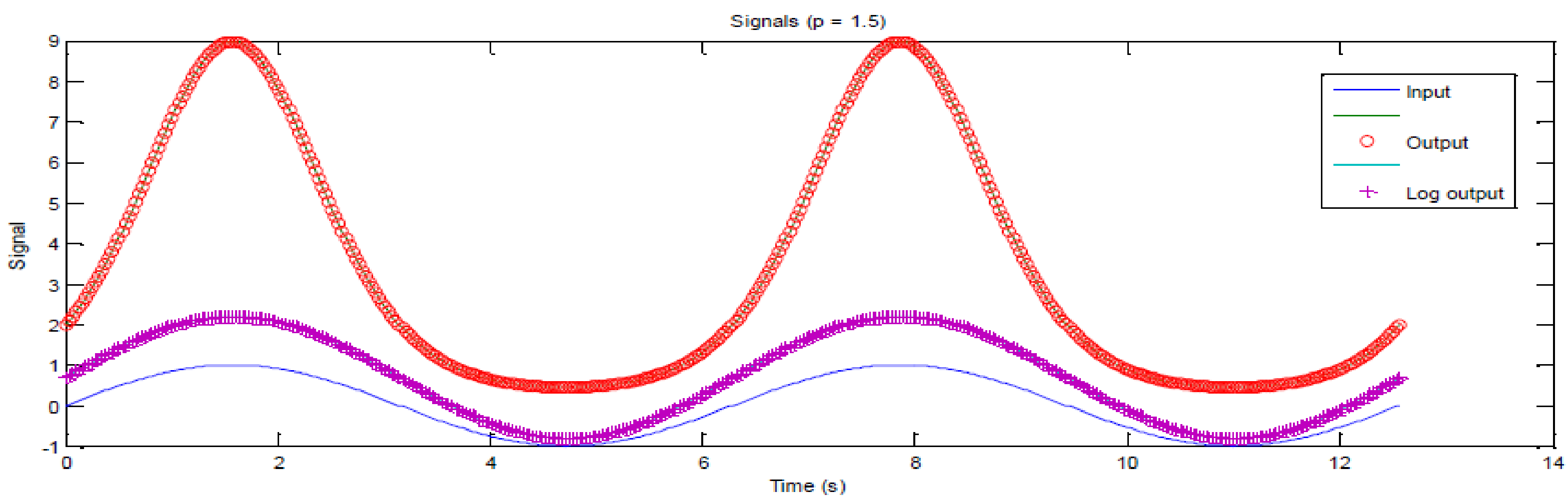
Example of Signal Distortion Due to Nonlinearity



For linear curve:

Slope = p= 3/2 = 1.5

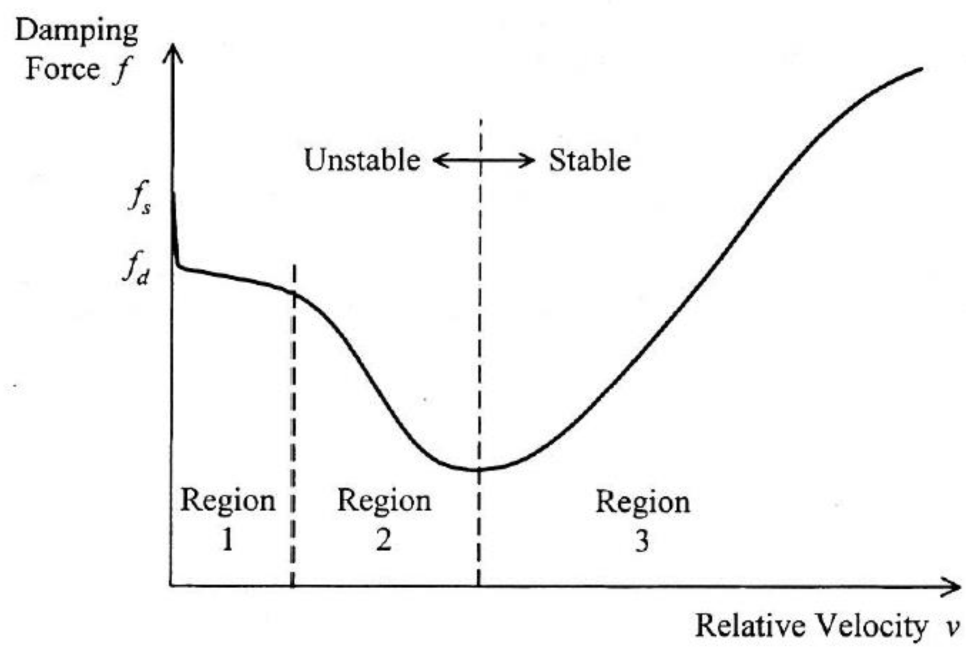
y-intercept = log k = -0.7 → k = 0.2



Problems with Local Linearization

- If operating conditions change considerably, a single local slope will not be valid over the entire range
- Local slope may be zero, multiple, infinite or may not exist (e.g., Coulomb friction)
- In some nonlinear systems, the use of local slopes (e.g., negative damping in a control law) may lead to undesirable consequences (e.g., instability)

Stribeck Friction:



Removing Nonlinear Behavior in Response

Approaches:

1. **Calibration and rescaling (e.g., log) the output (in the static case)** See previous example
2. **Use of linearizing elements (e.g., resistors, amplifiers in bridge circuits) to neutralize the nonlinear effects** See Section 2.83 in textbook (bridge circuits)
3. **Use of nonlinear feedback (feedback linearization): Feed back (negative) the nonlinear term of the system (to cancel that nonlinearity)** Example?

Mitigation of Nonlinear Effects

Precautions:

- Avoid operation over a wide range of inputs
- Avoid operation over a wide frequency band
- Avoid devices that generate large deformations (deviation from Hooke's law—physical nonlinearity)
- Avoid devices that generate large mechanical motions (trigonometric terms: geometric or kinematic nonlinearity)
- Minimize physical nonlinearities like Coulomb friction and stiction (e.g., using proper lubrication)
- Avoid loose joints and gear coupling (e.g., use direct-drive mechanisms, harmonic drives, etc.)
- Minimize environmental influences
- Minimize sensitivity to undesirable influences
- Minimize wear and tear

1. Reduce the causes (e.g., by lubrication);
2. Reduce the effects (e.g., by calibration);
3. Avoid nonlinear regions.

Rating Parameters: (Commercial)

E.g., Sensitivity

Rating Parameters

Typical rating parameters provided by instrument manufacturers and vendors (in the product data sheets):

- **Sensitivity and sensitivity error**
- **Signal-to-noise ratio**
- **Dynamic range**
- **Resolution**
- **Offset or bias**
- **Linearity**
- **Zero drift, full scale drift, and calibration drift (Stability)**
- **Useful frequency range**
- **Bandwidth**
- **Input and output impedances**

What is important is to understand the meanings of these terms

Sensitivity

Input Sensitivity: **Output (peak, rms value, etc.) corresponding to a unit input**

Parameter Sensitivity: **Sensitivity to parameter changes**

***Note*:** **A parameter change may result from an input signal (unwanted/noise/disturbance/environmental) ➔**
Parameter sensitivity and input sensitivity are related

That means only one type of sensitivity needs to be considered?

Sensitivity

Input Sensitivity ➔ Local slope of input-output curve; partial derivative of input-output relationship; gain; etc.

- **Can be defined for many inputs and many outputs (vector-matrix case)**
- **Many factors affect the device output ➔**

Many sensitivities have to be considered

Cross-sensitivity: Input sensitivity along directions orthogonal to primary direction of sensitivity (expressed as a % of direct sensitivity)

Example of cross-sensitivity?

High direct sensitivity and low cross-sensitivity are desirable

Sensitivity to parameter changes, disturbances, and noise should be small ➔ *robustness*

Adaptive control and self-tuning control: Sensitivity to control parameters has to be high

Often, sensitivity and robustness are conflicting requirements

Why do we non-demensionalize sensitivity?

Why?

Examples of Sensor Sensitivity (Input Sensitivity)

Sensor	Sensitivity
Blood Pressure Sensor	10 mV/V/mm Hg
Capacitive Displacement Sensor	10.0 V/mm
Charge sensitivity of Piezoelectric (PZT) Accelerometer	110 pC/N (pico-coulombs per newton)
Current Sensor	2.0 V/A
DC Tachometer	5 V± 0.1% error for 1000 rpm
Fluid Pressure Sensor	80 mV/kPa What is a "Pa"?
Light Sensor (digital output with ADC)	50 counts/lux
Strain gauge (gauge factor)	150 ΔR/R/strain (nondimensional)
Temperature Sensor (Thermistor)	5 mV/K

***Note*: Sensitivity may be expressed with respect to more than one input variable.**
E.g., Potentiometric displacement sensor with power supply (or, reference voltage) is 10 V. It produces an output of 1.5 V for a displacement of 5 cm.
Sensitivity = 1.5 (V)/5.0 (cm)/10.0 (V) = 30.0 mV/cm/V

What is pico?; why **± in a parameter?; maximum count an *n*-bit device can record?**

Sensitivity in Digital Devices
(Input Sensitivity)

Digital devices generate digital outputs (e.g., devices that generate pulses or counts or those with built-in ADCs)

Sensitivity = Digital Output/Corresponding Input

An n-bit device: Represents 2^n values or counts (including 0)

Maximum value = 2^n-1; minimum value = 0

Leaving for a sign bit, n bits can represent 2^{n-1} values (including 0)

Digital sensitivity of a device may be expressed as "counts per unit input"

Why are "0" and sign irrelevant for a counting device?

Digital Sensitivity Example

Photovoltaic light sensor: can detect a maximum of 20 lux of light and generates a corresponding voltage **(full-scale)** of 5.0 V.

The device has an **8-bit ADC**, which gives its maximum count for the full-scale input of 5.0 V.

What is the overall sensitivity of the device?

Max count of ADC = 2^8 = 256 counts

This corresponds to: 5.0 V input to ADC = sensor output for max light level = 20 lux

➔ Overall sensitivity of the device =

256 (counts) / 20.0 (lux) = 12.8 counts/lux

Why is the sensitivity not 256 (counts) / 5.0 (V)/20 (lux) = 2.56 counts/V/lux ?

***Note*:** Sensitivity of the ADC alone = 256 (counts) / 5.0 (V) = 51.2 counts/V

Sensitivity Error

= Rated sensitivity - Actual sensitivity

Reasons for Error:

- **Effect of cross-sensitivities of undesirable inputs**
- **Drifting due to wear, environmental effects, instability, etc.**
- **Dependence on the input value (slope changes with input value) ← indicates device *nonlinearity***
- **Local slope of the input-output curve (*local sensitivity*) may not be defined or may be insignificant**

Sensitivity Error (Cont'd)

Local slope of input-output curve (*local sensitivity*) may not be defined or may be insignificant.

Reasons:

- **Local slope (derivative) may be Zero (as in saturation or deadzone) or Infinity (as in Coulomb friction)**
- **Higher derivatives in nonlinear model (i.e., O(2) terms of Taylor series expansion) cannot be neglected**

Global sensitivity = Full-scale output/Corresponding input

Another way to indicate Sensitivity Error:

Average sensitivity ±range of variation

= Max sensitivity – Min sensitivity

← Measure of the static nonlinearity of the device

See my examples

Other Rating Parameters (Commercial)

Signal-to-Noise Ratio

Signal-to-Noise Ratio: [Signal magnitude (rms)]/[Noise magnitude (rms)] in dB ➔

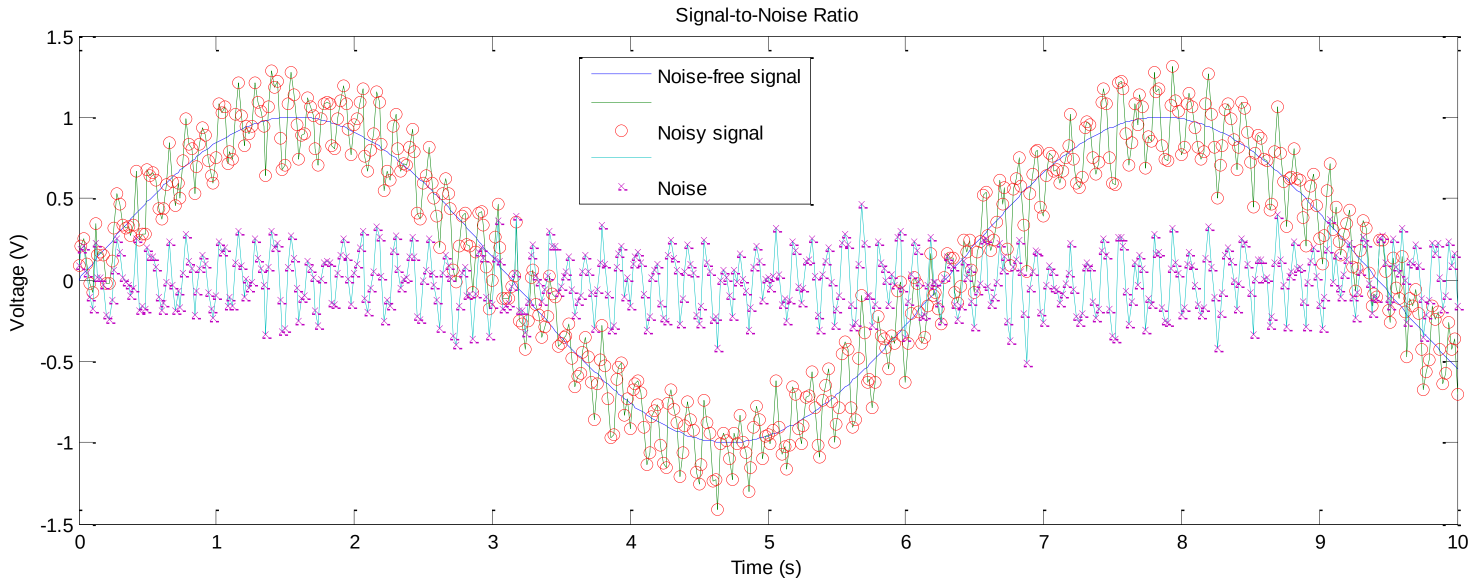
Why dB?

$$SNR = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) = 20\log_{10}\left(\frac{M_{signal}}{M_{noise}}\right)$$

P **= signal power;** ***M*** **=signal magnitude;** $P \propto M^2$

Rule of Thumb: SNR ≥ 10 dB is good; SNR ≤ 3 dB (half-power for noise) is bad

Example:



Signal_rms = 0.6663; Noise_rms = 0.1706; SNR = 11.8330 dB

Dynamic Range

Dynamic Range (DR): Or “range” of an instrument:

Allowed lower to upper range of output, while maintaining a required level of output accuracy; i.e., useful output range (Expressed as a ratio in dB)

Why dB?

Typically, lower limit = Instrument resolution (using zero as the lower limit is not meaningful)

➔ **DR = (Range of operation)/(Resolution)**

Justify the denominator term

Resolution

Resolution: Smallest change in a signal (input) that can be accurately detected and presented (in output) by the instrument.

E.g., Sensor, transducer, signal conversion hardware (ADC, etc.)

Usually expressed as a % of max range of instrument

Or, Inverse of Dynamic Range ratio

➔ DR and resolution are closely related

***n*-bit Digital Instrument (e.g., ADC at output):**

Resolution = 1 bit = δy **; Range =** $y_{\max} - y_{\min} = (2^n - 1)\delta y$

➔ DR = $\dfrac{y_{\max} - y_{\min}}{\delta y} = 20\log_{10}\left[\dfrac{2^n - 1}{1}\right]$

***Example*: For a 12-bit device,** $DR = 20\log_{10}\left[\dfrac{2^{12} - 1}{1}\right] = 72\text{ dB}$

Does this mean DR depends only on the digital device (*n*) if one is present?

How would you measure: 1. Force resolution; 2. Spatial resolution of your finger or palm?

Rating Parameters (Commercial)

Offset (Bias): *Zero Offset* = device output when input = 0

(e.g., output of imperfect bridge under balanced conditions; output of imperfect difference amp when the two input signals are equal).

Any others?

Methods of Correction (when offset is known):

1. Recalibrate; 2. Program digital output (i.e., subtract offset); 3. Use analog offsetting hardware at device output

Linearity Discussed before.

Static Calibration Curve: Curve of output (peak or rms) vs. input value under static (or steady-state) conditions within DR of instrument. *Note*: Slope of this curve = sensitivity

Its closeness to a "fitted" straight line → linearity

If least squares fit is used → **independent linearity**

Nonlinearity: Expressed as: 1. % Variation/(Actual reading at operating point or full scale reading); 2. Max variation of sensitivity/ reference sensitivity (%) ← Sensitivity error

Rating Parameters (Commercial)

Zero Drift: **Drift from null reading of instrument while input is maintained steady for a long period**

Full Scale Drift: **Drift from full scale reading while input is maintained at full scale value**

Parametric Drift: **Drift in device parameter values (due to environmental effects, etc.)**

Sensitivity Drift: **Drift in the sensitivity of a parameter**

Scale-factor Drift: **Drift in the scaling factor of output**

(Above three are closely related)

Calibration Drift: **Drift in calibration curve of device (due to changes in the device and the operating conditions)**

Rating Parameters (Commercial)

Useful Frequency Range: Corresponds to flat *gain curve* and a zero *phase curve* in frequency response (*frequency transfer function* FTF—*frequency response function FRF*) of "instrument" (e.g., sensor, not plant)

Upper frequency in this range < (rule of thumb: half, one-fifth or one-tenth) × dominant resonant frequency

🡐 measure of instrument bandwidth

Five Interpretations of Bandwidth: Discussed before

1. **Speed of response of device**
2. **Pass band of filter**
3. **Operating frequency range of device**
4. **Uncertainty in frequency content of signal**
5. **Information capacity of communication network**

Static Gain (DC Gain)

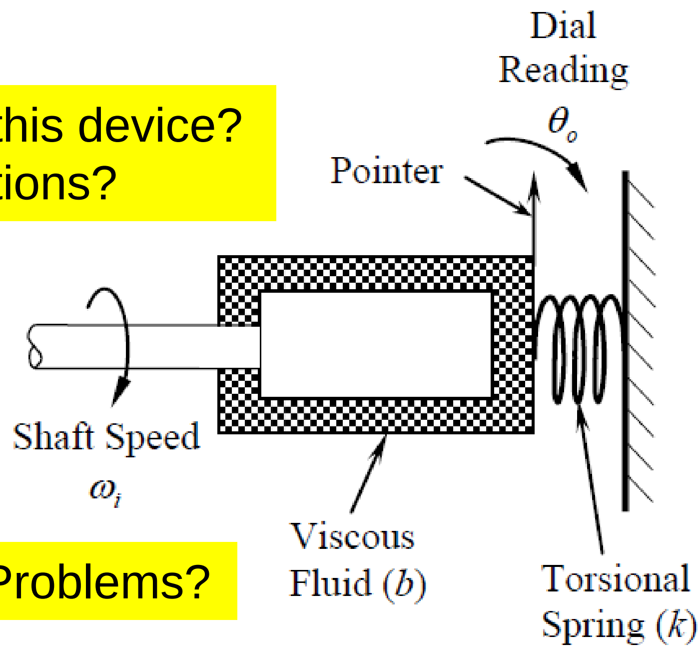
Gain (transfer function magnitude) within the useful (flat) frequency range (or at very low frequencies) of a device

High static gain ➔ High sensitivity ➔ Increases output level, Increases speed of response, Reduces steady-state error of a device (e.g., in a feedback control system)

But, typically, makes it less stab'-

Example: $b\dot{\theta}_o + k\theta_o = b\omega_i$

What is this device?
Assumptions?



➔ $$\frac{\theta_o}{\omega_i} = \frac{b}{[bs+k]} = \frac{b/k}{[(b/k)s+1]} = \frac{k_g}{[\tau s+1]}$$

Static gain: $k_g = \dfrac{b}{k}$

Time constant: $\tau = \dfrac{b}{k}$

FRF: $G(j\omega) = \dfrac{k_g}{\tau j\omega + 1}$

What happens to τ when DC gain increases'

Problems?

Half-power bandwidth: $$\frac{k_g}{|\tau j\omega_b + 1|} = \frac{k_g}{\sqrt{2}} \quad ➔ \quad \omega_b = \frac{1}{\tau}$$

- Data Sampling
- System Design Using Bandwidth Considerations

Aliasing Distortion due to Signal Sampling

Shannon's Sampling Theorem: Sampled data of a signal sampled at equal steps of ΔT (sampling frequency $f_s = 1/(\Delta T)$) has no information regarding signal spectrum beyond frequency $f_c = 1/(2\Delta T)$

This is called *Nyquist frequency*

Aliasing Error (Distortion): Folding of high-frequency spectrum beyond Nyquist frequency onto on to low-frequency side, due to sampling

➔ Spectrum at frequency f_2 appears as spectrum at f_1

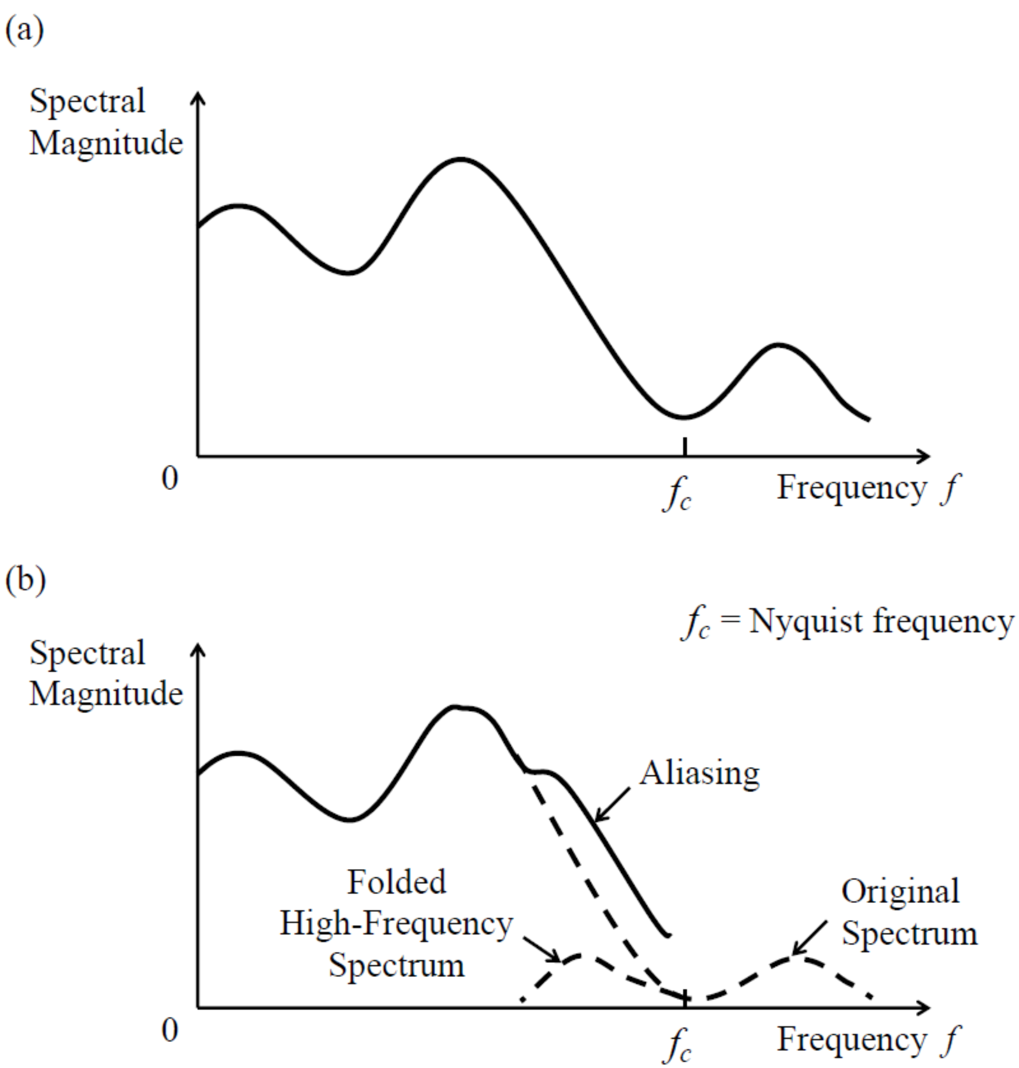
These two frequencies are related Through: $f_2 - f_c = f_c - f_1$

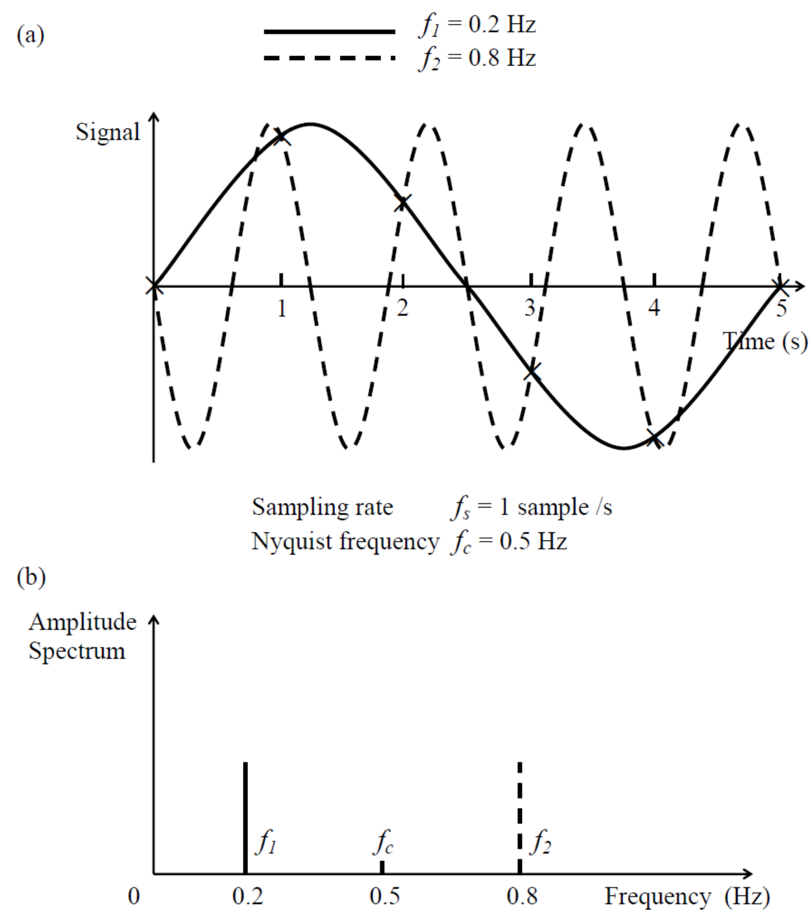
➔ $\dfrac{f_2 + f_1}{2} = f_c = \dfrac{1}{2} f_s$

Note: Increasing ΔT reduces aliasing

Antialiasing Filter: To remove aliasing, low-pass filter with cutoff at Nyquist frequency; Better cutoff: $f_c / 1.28\ (\cong 0.8 f_c)$

Aliasing Distortion due to Signal Sampling





"Bandwidth Design" of a Control System

Example: Digital control of mechanical positioning system;

TF of drive amplifier and electromagnetic circuit of motor:

$\dfrac{k_e}{(s^2 + 2\zeta_e\omega_e s + \omega_e^2)}$; TF of mechanical system **(plant)**: $\dfrac{k_m}{(s^2 + 2\zeta_m\omega_m s + \omega_m^2)}$

k = equivalent gain, ζ = damping ratio, ω = natural frequency

$(\ )_e$; $(\ )_m$: electrical ; mechanical components

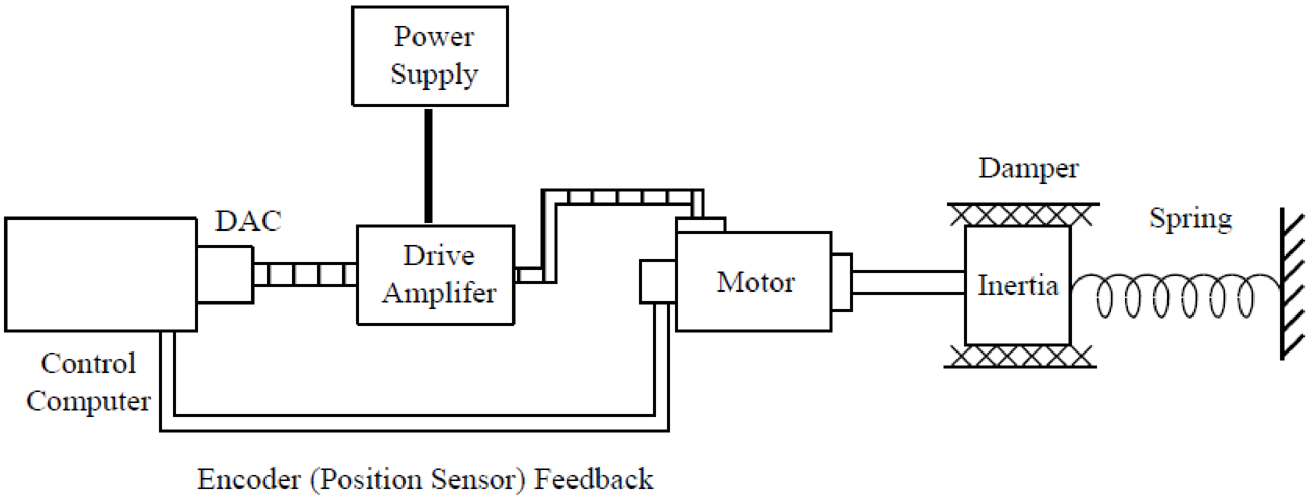
ΔT_c = time taken to compute each control action

ΔT_p = pulse period of position sensor (encoder)

Given values: $\omega_e = 1000\pi$ rad/s, $\zeta_e = 0.5$, $\omega_m = 100\pi$ rad/s, $\zeta_m = 0.3$

Neglect loading effects and coupling effects

Justify this



Design Through "Bandwidth Considerations" of a Control System

Questions:

(a) Explain why control bandwidth of system cannot be (and need not be) >> 50 Hz

(b) If ΔT_c = 0.02 s, estimate the control bandwidth

(c) Explain significance of ΔT_p in this application. Why, typically, ΔT_p should not be > 0.5 ΔT_c?

(d) Estimate the operating bandwidth of system

when significant plant dynamics should be avoided.

(e) If $\omega_m = 500\pi$ rad/s and ΔT_c = 0.02 s (remaining parameters kept same), estimate the operating bandwidth (so as not to excite significant plant dynamics)

Meaning of, 1. Control bandwidth; 2. Operating bandwidth?

Bandwidth Design of a Control System

Solution:

(a) Drive system (hardware) resonant frequency < 500 Hz

➔ 1. Flat region of drive spectrum (operating region) ≈ 50 Hz

2. The plant need not be operated beyond its resonant frequency (critical frequency) ➔ Required max spectral component of drive signal ≈ 50 Hz

➔ Control (analog) bandwidth cannot be (from 1.) and need not be (from 2.) > 50 Hz

Note: Plant resonance = 50 Hz ➔ The plant will behave like a "rigid wall" much beyond this frequency (gain close to zero)

(b) Rate of digital control signal generation = 1/0.02 Hz = 50 Hz

Shannon's sampling theorem ➔ Effective (useful) spectrum of control signal is limited to $\frac{1}{2} \times 50 \text{ Hz} = 25 \text{ Hz}$

Drive system (analog) can accommodate a bandwidth of 50 Hz

But, control bandwidth would be limited to 25 Hz (due to the limit of the digital control bandwidth)

Design Through "Bandwidth Considerations" of a Control System

ΔT_p

Solution (Cont'd):

(c) = sampling period of sensor signal (for feedback) ➔ Useful spectrum ≤ 1/2 ΔT_p (sampling theorem)

➔Feedback signal will be unable to provide useful information of plant beyond frequency $1/2\Delta T_p$

➔ To generate control signal at $1/\Delta T_c$ samples/s,

process information (from sensor) has to be provided at least up to $1/\Delta T_c$ Hz ➔ We need: $\frac{1}{2\Delta T_p} \geq \frac{1}{\Delta T_c} \quad \rightarrow \quad \Delta T_p \leq 0.5\,\Delta T_c$

➔ At least 2 sampled data points from sensor have to be used for computing a control action

Bandwidth Design of a Control System

Solution (Cont'd):

(d) Plant resonant frequency ~ $\frac{100\pi}{2\pi}$ Hz = 50 Hz.

Near resonance plant dynamics will interfere with control (avoid this, unless plant resonances--modes need to be controlled)

For frequencies >> resonance, plant will not significantly respond to control action (like a rigid wall)

➔ To avoid plant dynamics, operating bandwidth has to be sufficiently smaller than* 50 Hz (Control BW is 25 Hz ➔ it limits the operating BW to 25 Hz)

**Note*: This is a matter of design judgment, based on the application (e.g., excavator, disk drive). If plant dynamics have to be controlled, use the entire control bandwidth (available max control speed) as the operating bandwidth.

In present example, the entire available control BW (25 Hz) avoids plant resonance

Design Through "Bandwidth Considerations" of a Control System

Solution (Cont'd):

(e) Now, plant resonance ≈ $500\pi/(2\pi)$ Hz = 250 Hz

➔ Operating bandwidth limited to (say) ≈ ½×250 Hz = 125 Hz

(to avoid plant resonance)

But, still, control bandwidth ~ 25 Hz because ΔT_c = 0.02 s

➔Operating bandwidth cannot be > 25 Hz

➔Operating bandwidth is still $\simeq$25 Hz

Bandwidth Design of a Control System

Steps:

1: Decide on max frequency of operation (BW_o) based on the plant and application requirements

2: Select the drive components (electro-mechanical, analog hardware ➔ interface analog hardware, filters, amplifiers, actuators, etc.) that have capacity to operate (flat frequency spectrum) at least up to BW_o

3: Select feedback sensors with flat frequency spectrum (sensor operating frequency range) > 4 × BW_o

4: Develop digital controller with: (a) Feedback sensor signal sampling rate > 4 × BW_o (i.e., within flat spectrum of sensors) and digital control cycle time (period) < 1/(2 × BW_o).

Note: Generate digital control actions at rate > 2 × BW_o

5: Integrate the system and test the performance. If the performance specs are not satisfied, make necessary adjustments and test again.

Questions: Give Yes or No Answers

Bandwidth represents,

(a) Speed of Response of a Device?

(b) Operating Frequency Range?

(c) The Range of Allowed Frequencies of a Band-pass Filter?

(d) Frequency range in which the “Power” of a Signal is Half?

(e) Frequency (or Noise) Uncertainty of a Signal?

(f) Data/Information Carried by a Communication Channel?

(g) Highest Control Speed of a Control Device (Analog or Digital)?

Error Considerations

Error Considerations

Error = (Measured value) – (True value)

Correction = (True value) – (Measured value)

Causes for Error in an Engineering System (having interconnected and interacting multiple components):

- **Instrument instability**
- **Noise and external disturbances (undesirable inputs)**
- **Poor calibration**
- **System errors (due to inaccurate analytical models, control laws, etc.)**
- **Parameter changes (e.g., from environmental changes, aging, and wear)**
- **Unknown nonlinearities**
- **Sensor errors**
- **Improper use of the instruments (measurement setup, operating conditions, human error, etc.)**

***Note*: 1. Signal errors (at source); 2. Measurement errors**

Classify above into these two; Give other examples.

Instrument (Measurement System) Accuracy

Depends On:

- Calibration
- Physical hardware
- Actual operating conditions (power, signal levels, load, speed, environmental factors, etc.)
- Design operating conditions (operating conditions for which the instrument is designed for: normal, steady operating conditions;)
- Extreme Operating Conditions: (extreme transient conditions, emergency start-up and shutdown conditions) Examples?
- Instrument setup shortcomings
- Other components and systems to which the instrument is connected (e.g., dynamic coupling, loading, or noise from other connected devices)

Error Considerations

Classification of Error: Deterministic (systematic) and Random (stochastic)

Deterministic Errors:

- Caused by well-defined factors (e.g., Nonlinearities, offsets in readings)
- Correctable by calibration and analytical/ computational means (test data, calibration charts, error ratings, etc.)

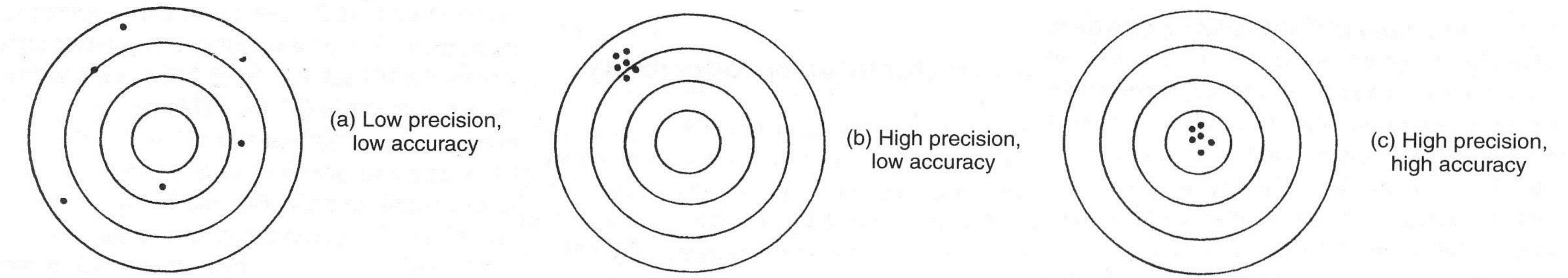
Random Errors:

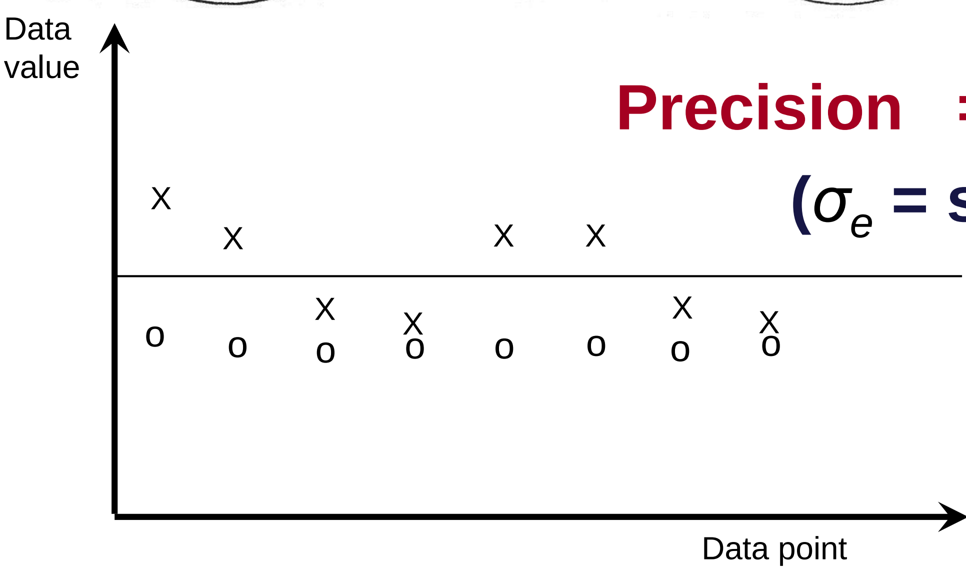
- Caused by uncertain factors (e.g., Noise, Unknown random variations in operating environment, random inputs)
- May be compensated for (not completely) through statistical analysis (using a sufficiently large number of data to "estimate" random errors)
- Results are expressed as *mean error* μ_e (systematic part) + *standard deviation* σ_e or *confidence interval* (random)

Precision

Precision is determined by reproducibility (repeatability) of instrument reading (e.g. accurate clock with wrong time setting ➔ precise, not accurate)

Precise ➔ Low random error





Precision = (Measurement range)/σ_e

(σ_e = standard deviation of error)

X: Accurate ? Precise?

o: Accurate ? Precise?

Difficulties in Error Analysis

1. The true value is unknown (chicken or egg came first?)
2. Instrument reading may contain random error ((a) Error of the measuring system, including sensor error; (b) Other random errors that enter into the engineering system; e.g., noise, disturbance inputs, human error) which cannot be determined exactly
3. Error may be a complex (i.e., not simple) function of many quantities (physical parameters, input variables and state variables or response variables); may not be additive (e.g., multiplicative)
4. Monitored system may be multi-component, having complex interrelations (dynamic coupling, multiple degree-of-freedom responses, nonlinearities, etc.), and each component may contribute to the overall error

Error Propagation and Combination

Error Propagation and Combination

(a) How component errors are reflected in the final output—error propagation; (b) combining component errors—error combination.

Both Depend on Errors in:

1. **Components (their variables and parameters), their errors, and how they interact**
2. **Measured variables or parameters (of individual components, etc.) that are used to compute (estimate) the required quantity (variable or parameter value)**
3. **Relation among components (model)**

It is important to know:

1. **How component errors propagate within a multicomponent system (Error propagation)**
2. **How individual errors in variables/parameters contribute toward overall error (Error combination)**

Error Propagation and Combination

Examples:

- **Variable Error:** **Power output of a gas turbine, computed by measuring torque and speed at output shaft (errors in two measured variables directly contribute to error in power computation)**
- **Model/Parameter Error:** **Natural frequency of vehicle suspension system is computed by measured parameters (mass and spring stiffness) ➔ Estimate is directly affected by errors in mass and stiffness measurements (and also by nonlinearities—model errors)**
- **Component Error:** **In a robotic manipulator, accuracy of actual end effector trajectory will depend on the accuracy of: (a) joint sensors; (b) joint actuators; (c) robot controller (may be based on robot model)**

Analytical Basis for Error Combination

Component contribution to output: $y = f(x_1, x_2, \ldots, x_r)$

x_i = independent system variable/parameter values whose errors propagate/combine into error in output y (or required parameter value), represented by increment of y

→ $$\delta y = \frac{\partial f}{\partial x_1}\delta x_1 + \frac{\partial f}{\partial x_2}\delta x_2 + \ldots\ldots + \frac{\partial f}{\partial x_r}\delta x_r$$

What assumptions?

Note the “sensitivity” terms

Fractional Error: $$\frac{\delta y}{y} = \sum_{i=1}^{r}\left[\frac{x_i}{y}\frac{\partial f}{\partial x_i}\frac{\delta x_i}{x_i}\right] \Rightarrow e_y = \sum_{i=1}^{r}\left[\frac{x_i}{y}\frac{\partial f}{\partial x_i}e_i\right]$$

Non-dimensional; Significance of the signs of the summed terms?

$\delta y/y = e_y$ = overall (propagated) error (fractional—nondimensional)

$\delta x_i/x_i = e_i$ = component error (fractional—nondimensional)

$\frac{x_i}{y}\frac{\partial f}{\partial x_i}$ **= sensitivity of error in x_i on combined (propagated) error in y (non-dimensional)**

Note the similarity of “error combination” and “sensitivity combination”

Does *f* represent a system model?

Error Propagation/Combination Estimation

Absolute Error: $$e_{\text{ABS}} = \sum_{i=1}^{r} \left| \frac{x_i}{y} \frac{\partial f}{\partial x_i} \right| e_i$$

When is this method appropriate?

This is an upper bound (conservative estimate) for overall error (individual terms in the previous sum may be –ve)

SRSS (Square Root of Sum of Squares) Error:

$$e_{\text{SRSS}} = \left[\sum_{i=1}^{r} \left(\frac{x_i}{y} \frac{\partial f}{\partial x_i} e_i \right)^2 \right]^{1/2}$$

***Note*: $e_{SRSS} < e_{ABS}$ when two or more nonzero error contributions are present**

SRSS is particularly suitable when the component error is represented by the *standard deviation* of the associated variable/parameter value and when the corresponding error sources are independent

Why?

Error Combination/Propagation Applications

Degree of Importance of Component Error: Measured by the non-dimensional sensitivity: $\frac{x_i}{y}\frac{\partial f}{\partial x_i}$

Condition for Equal Contributions from Individual Errors:

$$\left|\frac{x_1}{y}\frac{\partial f}{\partial x_1}\right|e_1 = \left|\frac{x_2}{y}\frac{\partial f}{\partial x_2}\right|e_2 = \ldots = \left|\frac{x_r}{y}\frac{\partial f}{\partial x_r}\right|e_r = \frac{e_{ABS}}{r}$$

→ **We need** $e_i = e_{ABS} \Big/ \left(r \left| \frac{x_i}{y}\frac{\partial f}{\partial x_i} \right| \right)$ ← **Error inverse to sensitivity**

***Note* 1: These results are useful in: 1. Design of multicomponent systems; 2. Cost effective selection of instrumentation**

***Note* 2: Error values are given with "±" because the sign of error is unknown in general**

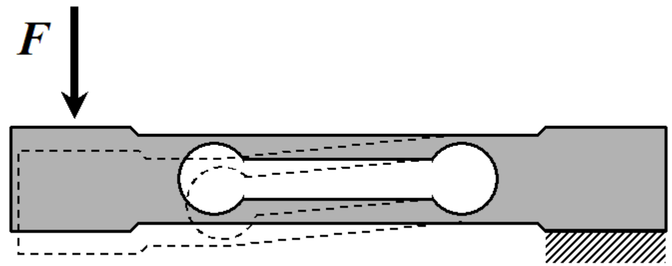
Example: Data Sheet of a Load Cell

Load Cell Used in Lab 2

Specific

Note: All specifications are a maximum, as a % *% of full load*

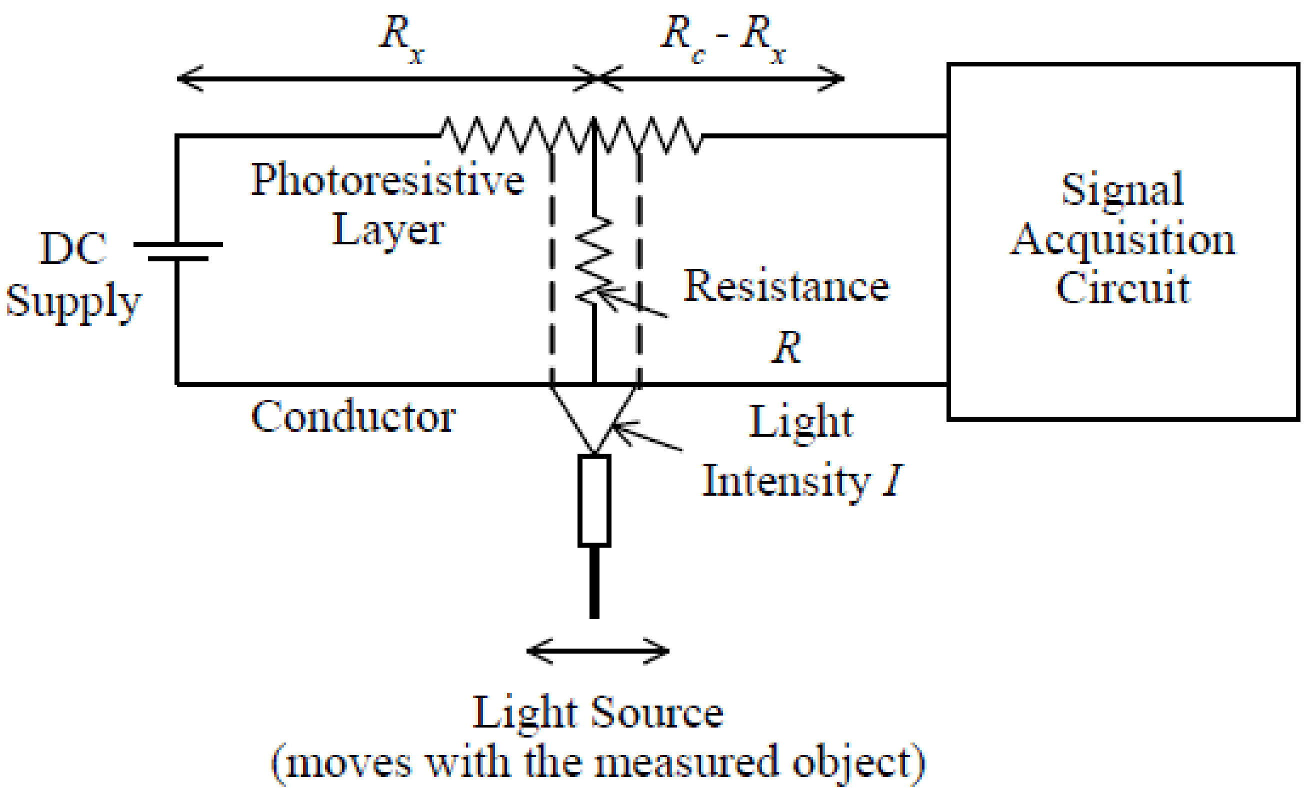
Nominal Capacity	3kg ~ 250kg
Signal Output at Capacity	2mV/V ± 10%
Linearity Error	< 0.020% FSO
Non-Repeatability	< 0.010% FSO
Combined Error	< 0.025% FSO
Hysteresis	< 0.015% FSO
Creep/Zero Return (30 mins)	< 0.030% / 0.020% FSO
Zero Balanc	< 3.000% Capacity
Temperature Effect on Span/10°C	< 0.010% FSO
Temperature Effect on Zero/10°C	< 0.015% Capacity



Error Combination/Propagation Example

Optical Potentiometer

What does this sensor measure?
What other sensor can this device represent?



Note: **Device output (volts) directly depends on the bridging resistance *R* ➔ It may be considered as the parameter of interest in a light intensity sensor**

Error Combination/Propagation Example

Optical device for measuring displacement (optical potentiometer)

Potentiometer element resistance R_c is uniform
Photoresistive layer is sandwiched between it and a conductor

Light source moves with measured object

Light intensity = I (falls on narrow photoresistive layer)

Corresponding resistance = R

(bridges potentiometer element and conductor element)

Empirical relation between R (kΩ) and I (W/m^2):

$$\ln\left(\frac{R}{R_o}\right) = \left(\frac{I_o}{I}\right)^{1/4}$$

R_o and I_o : Empirical constants (have same units as R, I)
They have experimental error

Error Combination/Propagation Example

(a) Sketch *R* vs *I*. Explain the significance of R_o and I_o

(b) Using absolute error method, show that the combined fractional error e_R in *R* can be expressed as:

$$e_R = e_{R_o} + \frac{1}{4}\left(\frac{I_o}{I}\right)^{1/4} [e_I + e_{I_o}]$$

$e_{R_o}, e_I,$ and e_{I_o} : fractional errors in R_o, I, and I_o

(c) Empirical error in sensor model: $e_{R_o} = \pm 0.01$ and $e_{I_o} = \pm 0.01$

Fractional error in $I = \pm 0.01$ (due to effects of power supply variations on light source; variations in ambient lighting)

e_R should be maintained within ±0.02.

At what intensity (I) should the light source operate? Given: $I_o = 2.0$ W/m^2

(d) Discuss advantages and disadvantages of this device as a dynamic displacement sensor

Error Combination/Propagation Example

Solution: (a) $\ln \frac{R}{R_o} = \left(\frac{I_o}{I}\right)^{1/4}$ **: As** $I \to \infty,\ \ln(R/R_o) \to 0$ or $R/R_o \to 1$

➔ R_o = minimum R (i.e., at very high light intensity levels)

When $I = I_o$, $R = 2.7\ R_o$ ➔ I_o represents a lower bound for I for satisfactory operation of sensor

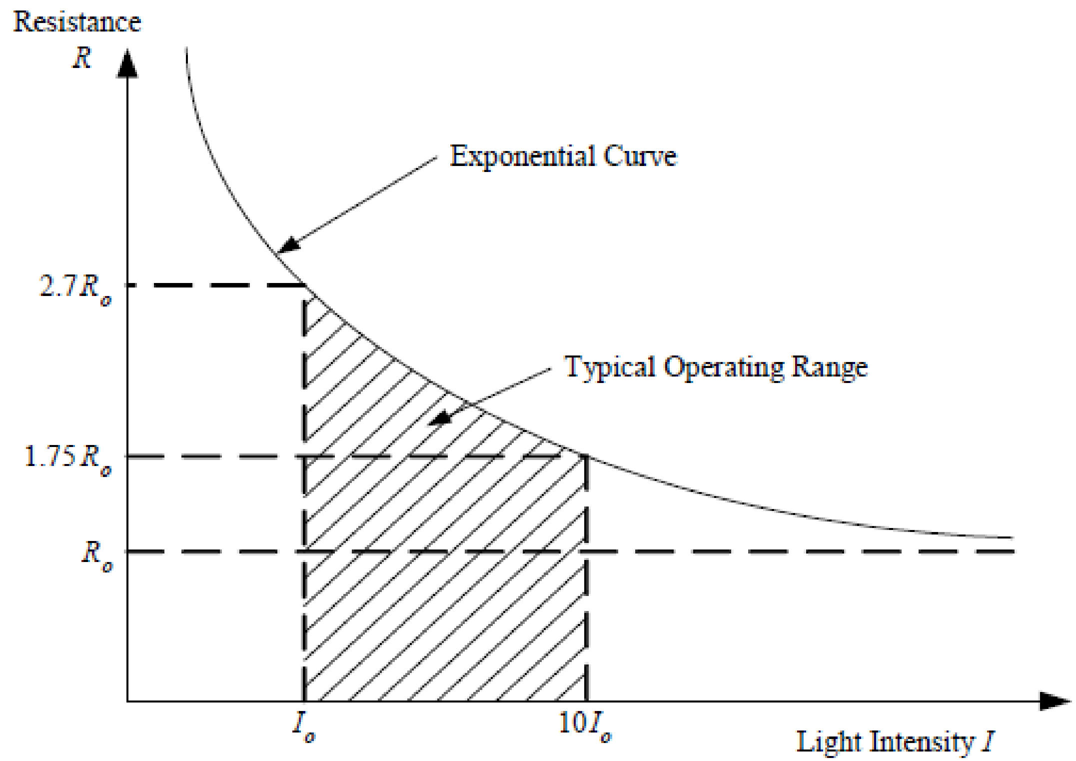
A suitable upper bound:

$I = 10\ I_o$, for satisfactory operation.

➔ $R \approx 1.75 R_o$

Why these values for bounds?

See figure for these characteristics



Error Combination/Propagation Example

Solution: (b) $\ln R - \ln R_o = \left(\frac{I_o}{I}\right)^{1/4}$

Take differentials: $\frac{\delta R}{R} - \frac{\delta R_o}{R_o} = \frac{1}{4}\left(\frac{I_o}{I}\right)^{-3/4}\left[\frac{\delta I_o}{I} - \frac{I_o}{I^2}\delta I\right] = \frac{1}{4}\left(\frac{I_o}{I}\right)^{1/4}\left[\frac{\delta I_o}{I_o} - \frac{\delta I}{I}\right]$

Absolute error combination: $e_R = e_{Ro} + \frac{1}{4}\left(\frac{I_o}{I}\right)^{1/4}\left[e_{I_o} + e_I\right]$

(c) Substitute numerical values: $0.02 = 0.01 + \frac{1}{4}\left(\frac{I_o}{I}\right)^{1/4}[0.01+0.01] \Rightarrow \left(\frac{I_o}{I}\right)^{1/4} = 2$

➔ $I = \frac{1}{16}I_o = \frac{2.0}{16}\,\mathrm{W/m^2} = 0.125\,\mathrm{W/m^2}$

***Note*: For larger *I*, absolute error in *R* would be smaller** Why?

E.g., for $I = 10\,I_o$: $e_R = 0.01 + \frac{1}{4}\left(\frac{1}{10}\right)^{1/4}[0.01+0.01] \approx 0.013.$

(d) Advantages as a Displacement Sensor: Noncontacting; Small moving mass (low inertial loading); All advantages of a potentiometer

Disadvantages: Nonlinear and exponential variation of *R;* Effect of ambient lighting; Possible nonlinear behavior of device (nonlinear input–output relation); Effect of variations in power supply on light source; Effect of aging of the light source

Note: The example shows how conditions can been chosen for getting the desired level of accuracy in the output (*R*)