MECH 364: Mechanical Vibrations
Midterm Examination 1

Time: 45 minutes   28th September 2011   Maximum Available Mark: 20

Q1.

a) Consider a rigid disc of radius r rolling without slipping on a horizontal plane as shown in Fig.(1). Choosing an appropriate displacement co-ordinate as a degree of freedom, draw the free body diagram (FBD) for small amplitudes of oscillations about equilibrium position. Draw FBD as appropriate for Newton's second law or D'Alembert's principle. **Indicate all forces. You can ignore gravity.** **(8 marks)**

b) Formulate the equations of motion for the above system. *You may use Newton's second law or D'Alembert's principle depending on the FBD you drew in part a).* **(8 marks)**

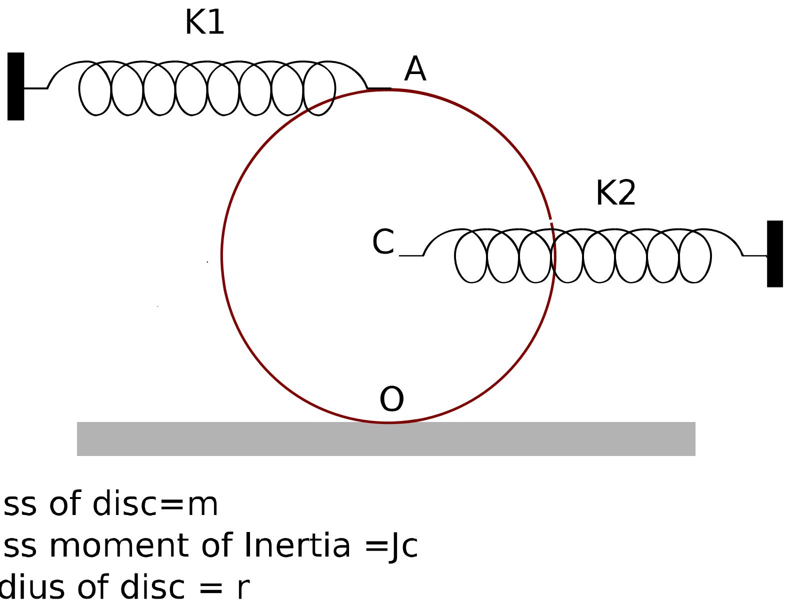


Figure 1: Disc rolling without slipping on a horizontal plane.

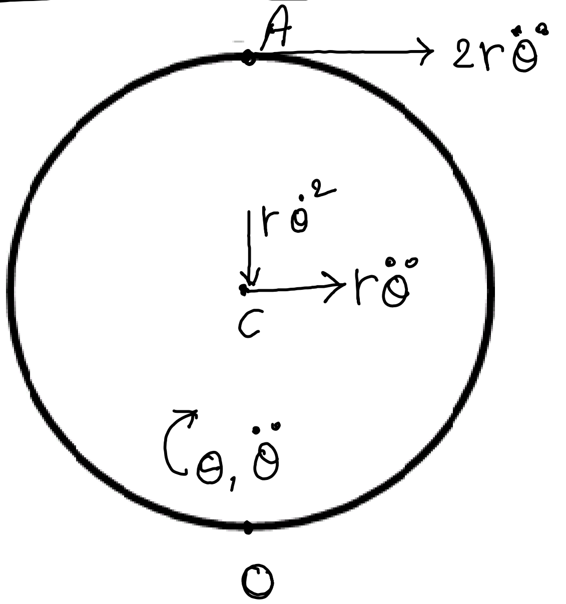
c) Can you suggest at least two alternative choices for co-ordinates in addition to the one you have already used? What is the relationship among these co-ordinates? **(4 marks)**

All the Best!

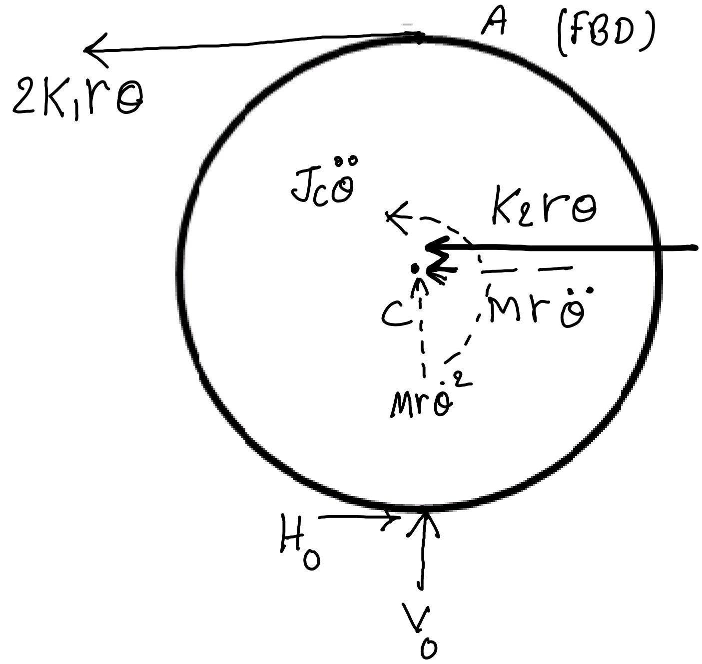
SOLUTION

a) FREE BODY DIAGRAM FOR D'ALEMBERT PRINCIPLE:
ONLY DISC IS SHOWN

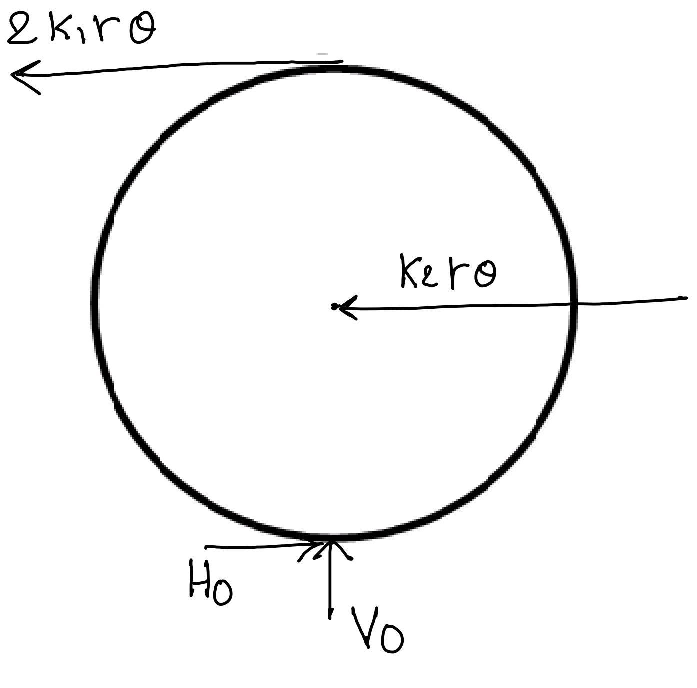
ACCELERATIONS:



FREE BODY DIAGRAM



FBD FOR NEWTON'S SECOND LAW:



b) EQUATION OF MOTION

D'ALEMBERT: $\circlearrowright \sum_{+ve} M_O = 0$

$$\Rightarrow -2k_1 r\theta \times 2r - k_2 r\theta \times r - Mr\ddot{\theta} \times r - J_C\ddot{\theta} = 0$$

$$\Rightarrow \boxed{4k_1 r^2\theta + k_2 r^2\theta + (J_C + Mr^2)\ddot{\theta} = 0 \quad -(1)}$$

NEWTON'S SECOND LAW: $\circlearrowright \sum_{+ve} M_O = J_O \ddot{\theta}$

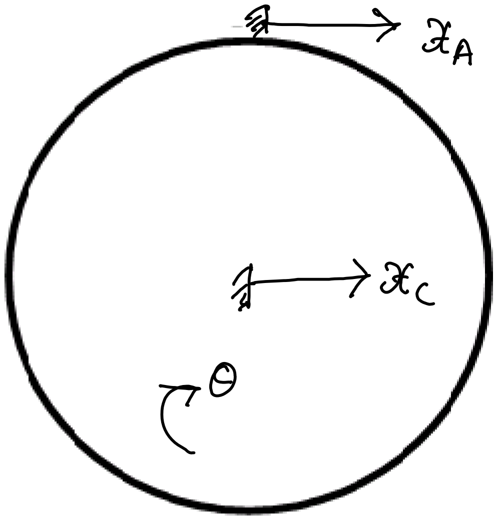
$J_O = J_C + Mr^2$ PARALLEL AXIS THEOREM

$$\Rightarrow -2k_1 r\theta \times 2r - k_2 r\theta \times r = J_O\ddot{\theta}$$

$$\Rightarrow \boxed{J_O\ddot{\theta} + 4k_1 r^2\theta + k_2 r^2\theta = 0 \quad -(2)}$$

NOTING THAT $J_O = J_C + Mr^2$ (1) & (2) ARE IDENTICAL !!

c) WE CAN USE x_A & x_C AS TWO ALTERNATIVE CHOICES FOR CO-ORDINATES HOWEVER, THEY ARE RELATED TO EACH OTHER AND TO θ IN PART a) VIA $\boxed{x_A = \frac{1}{2}x_C = r\theta}$



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