20/20 LM

MECH 463: Mechanical Vibrations
Midterm Examination 2

Time: 45 minutes   31st October 2013   Maximum Available Mark: 20

Student Name: Student Number:

Write your answers on this sheet (4 pages in total). Do not remove pages.

Q1. Consider a motorcycle shown in Fig.(1). The totals mass, including the rider, is 250 kg and the stiffness of the suspension is 70 kN/m. The motorcycle travels over a terrain **with constant velocity** v, approximately sinusoidal with a distance between peaks of 10 m and the distance from peak to valley is 10 cm.

$m = 250\,kg$
$k = 70\,000\ N/m$

Terrain

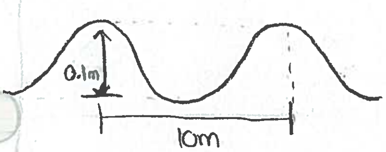

Figure 1: Figure for midterm question. **Ignore damping and fee vibration response.**

a) Write a SDOF model for this problem along with the equation of motion. **4 marks are for FBD and 2 marks for co-ordinate. Accuracy of FBD is marked not just the equation of motion.** **(10 marks)**

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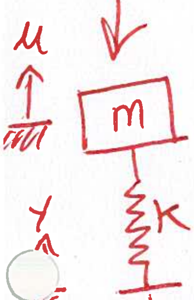
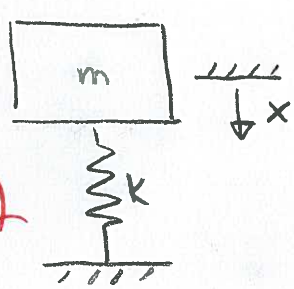
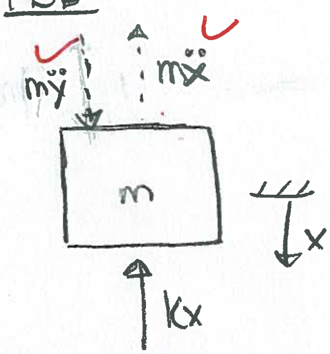
Answer (15 minutes or less): FBD

Never mind

SDOF



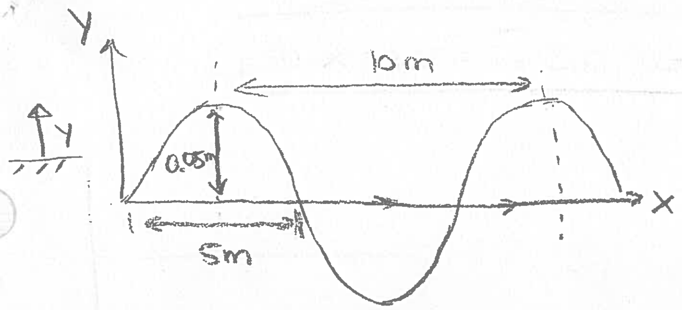




Assume:
- no damping
- Front and back suspension = k
- Constant horizontal velocity $\vec{V}$

→ relative displacement here.
$F_k = k(y-u)$

$$\Sigma F_x = 0 : -m\ddot{x} - kx + F = 0$$
$$m\ddot{x} + kx = F$$



$$y = b\sin\left(\frac{\pi x}{c}\right) \qquad y = 0.05\sin\left(\frac{\pi x}{5}\right)$$

$b = 0.05m$

$C = 5m$

$$V = \frac{x}{t} \Rightarrow x = Vt$$

$$\boxed{y = 0.05\sin\left(\frac{\pi V t}{5}\right)}$$

The acceleration due to the car moving over the hills causes a force. This is in the y coordinate frame. The x coordinate frame is the displacement from static equilibrium.

$$\dot{y} = (0.05)\frac{\pi V}{5}\cos\left(\frac{\pi V t}{5}\right)$$

$$\ddot{y} = -(0.05)\left(\frac{\pi V}{5}\right)^2\sin\left(\frac{\pi V t}{5}\right)$$

$$\ddot{y} = -0.002(\pi V)^2\sin\left(\frac{\pi V t}{5}\right)$$

$$m\ddot{x} + kx = m\ddot{y}$$

$$m\ddot{x} + kx = -m\,0.002(\pi v)^2\sin\left(\frac{\pi v t}{5}\right)$$

b) Deduce the expression for the maximum acceleration felt by the rider due to the **forced vibration** from the rough terrain. **Ignore free vibration (homogeneous solution).** Hence find the maximum **acceleration** experienced by the rider while travelling at $v = 30$ m/s, $v = 60$ m/s and $v = 120$ m/s. **(6 marks)** 5/6

Answer (15 minutes or less):

$$m\ddot{x}_p + kx_p = -m(0.002)(\pi v)^2\sin\left(\frac{\pi v t}{5}\right)$$

Where $x_p = X\sin\left(\frac{\pi v t}{5}\right)$

$$\dot{x}_p = X\frac{\pi v}{5}\cos\left(\frac{\pi v t}{5}\right)$$

$$\ddot{x}_p = \frac{X(-\pi^2 v^2)}{25}\sin\left(\frac{\pi v t}{5}\right)$$

$$-m\frac{X\pi^2 v^2}{25}\cancel{\sin\left(\frac{\pi v t}{5}\right)} + kX\cancel{\sin\left(\frac{\pi v t}{5}\right)} = -\frac{m\pi^2 v^2}{500}\sin\left(\frac{\pi v t}{5}\right)$$

$$-\frac{m\pi^2 v^2}{25}X + kX = -\frac{m\pi^2 v^2}{500} \qquad (25k - m\pi^2 v^2)X = -\frac{m\pi^2 v^2}{20}$$

$$X = \frac{-m\pi^2 v^2}{500k - 20m\pi^2 v^2}$$

$$\ddot{x}_p = \frac{-X \pi^2 v^2}{25} \sin\left(\frac{\pi v t}{5}\right) = \frac{-m\pi^2 v^2}{500k - 20m\pi^2 v^2} \left(\frac{-\pi^2 v^2}{25}\right) \underbrace{\sin\left(\frac{\pi v t}{5}\right)}_{\text{max when this} = 1}$$

$$\ddot{x}_{p_{max}} = \frac{m\pi^4 v^4}{12500k - 500m\pi^2 v^2}$$

$$\ddot{x}_{p_{max}} = \frac{(250kg)\pi^4 v^4}{(12500)(70\,000 N/m) - 500(250kg)\pi^2 v^2}$$

$$\ddot{x}_{p_{max}} = \frac{250\pi^4 v^4}{875\,000\,000 - 125000\pi^2 v^2}$$

$m = 250kg$

$K = 70\,000\ N/m$

$V = 30\ m/s,\ 60\ m/s,\ 120\ m/s$

$$\boxed{\ddot{x}_{p_{max}} = \frac{\pi^4 v^4}{3\,500\,000 - 50\pi^2 v^2}}$$

<u>$V = 30\ m/s$</u>

$\ddot{x}_{p_{max}} = 25.8\ m/s^2$

<u>$V = 60\ m/s$</u>

$\ddot{x}_{p_{max}} = 732.5\ m/s^2$

Some of these are very large?

<u>$V = 120\ m/s$</u>

$\ddot{x}_{p_{max}} = -5601.2\ m/s^2$

equations look good, but numerics don't quite work...

(-1)

(+1 for neatness)

c) Based on the form of the solution above, sketch the variation of the displacement felt by the rider as a function of speed and indicate the critical (resonant) speed. **(4 marks)** 4/4

Answer (5 minutes or less):

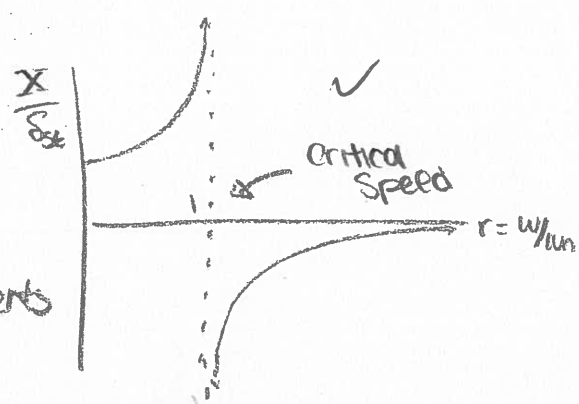
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{70\,000 \text{ N/m}}{250 \text{ kg}}} = 16.7 \text{ rad/s} \checkmark$$

$$\frac{\pi V}{(5m)} = \omega_n$$

$$V = \frac{5}{\pi}\omega_n = \frac{5(16.7 \text{ rad/s})}{\pi} = 26.6 \text{ m/s}$$ is the critical or resonant speed. ✓

$$X_p = X \sin\left(\frac{\pi V t}{5}\right) = \frac{-m\pi^2 V^2}{500k - 20m\pi^2 V^2} \sin\left(\frac{\pi V t}{5}\right) = \frac{-250\pi^2 V^2}{3\,500\,000 - 5000\pi^2 V^2} \sin\left(\frac{\pi V t}{5}\right)$$

$$X_p = \frac{-\pi^2 V^2}{14\,000 - 20\pi^2 V^2} \sin\left(\frac{\pi V t}{5}\right)$$



✓

as the rider gets closer to resonant frequency the displacements get larger.

ALL THE BEST!