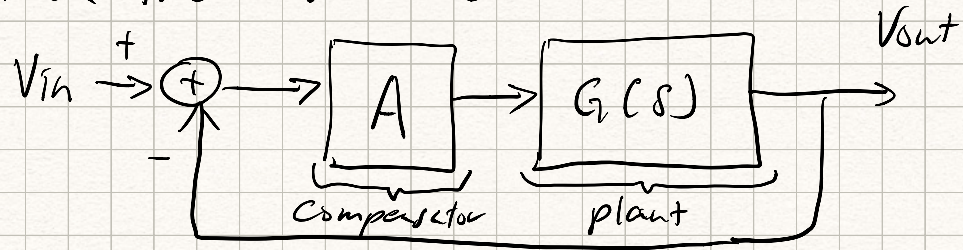
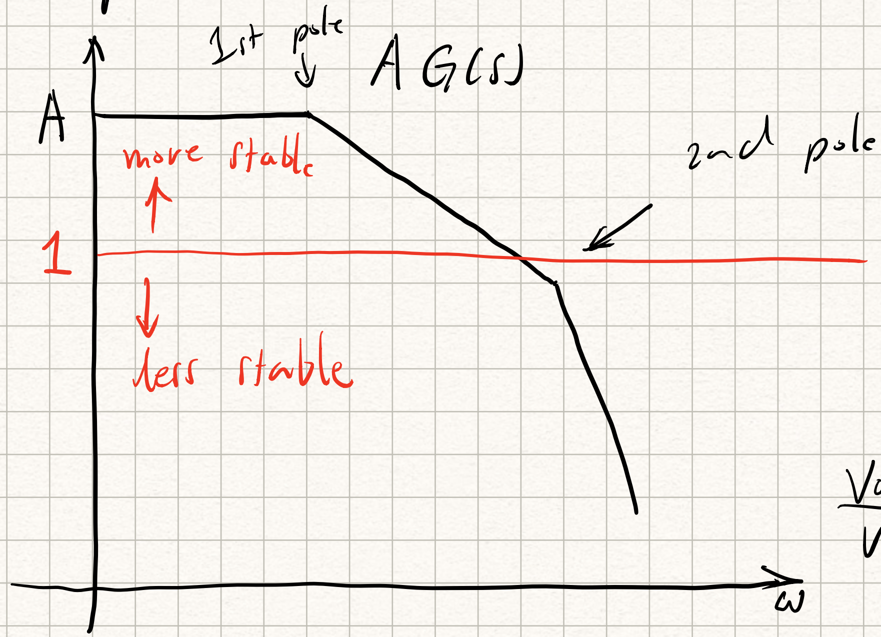
MECH 423 Lecture 16

From the last time:



$$G(s) = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$$

Proportional Control



$$\frac{V_{out}}{V_{in}} = \frac{A\,G(s)}{1 + A\,G(s)}$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{(\tau_1 s+1)(\tau_2 s+1)+A}$$

$$= \frac{A}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + A + 1}$$

since $A \gg 1$

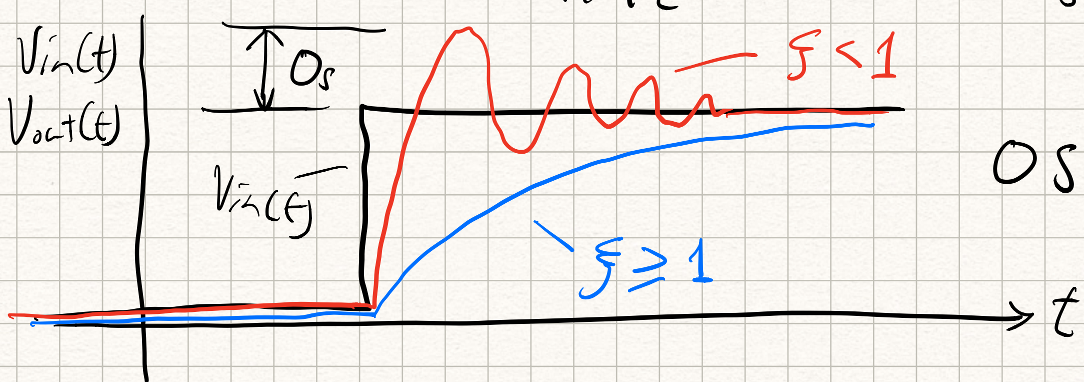
$$\frac{V_{out}}{V_{in}} = \frac{A/\tau_1\tau_2}{s^2 + \left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}\right)s + \frac{A}{\tau_1\tau_2}}$$

– Standard 2nd order system

Define: $$\omega_n^2 = \frac{A}{\tau_1\tau_2}$$

$$2\zeta\omega_n = \frac{\tau_1+\tau_2}{\tau_1\tau_2}$$

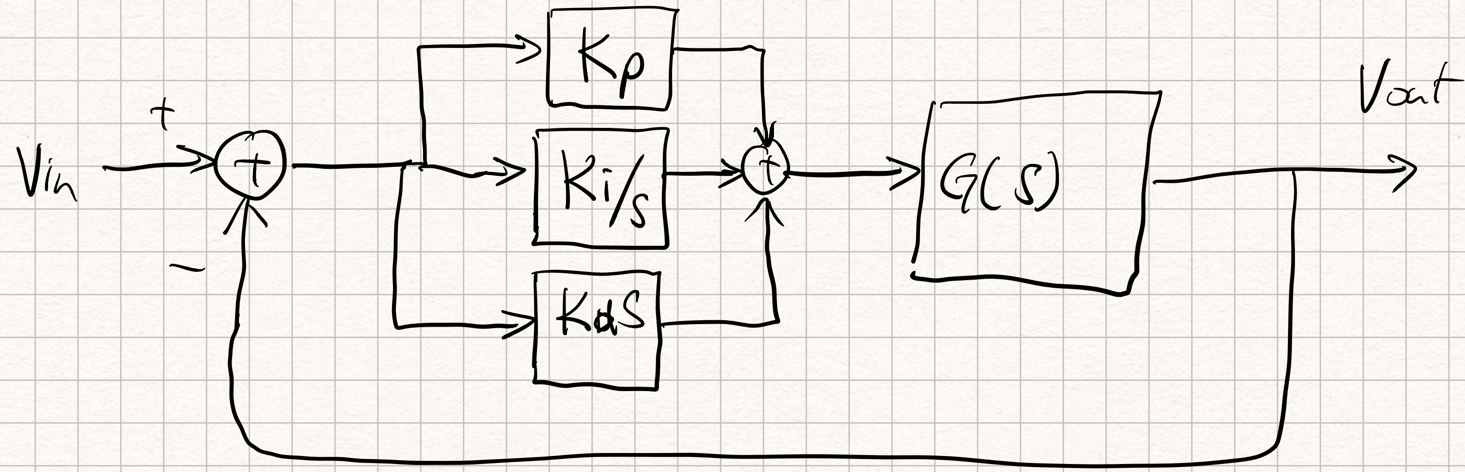
$$\frac{V_{out}}{V_{in}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$



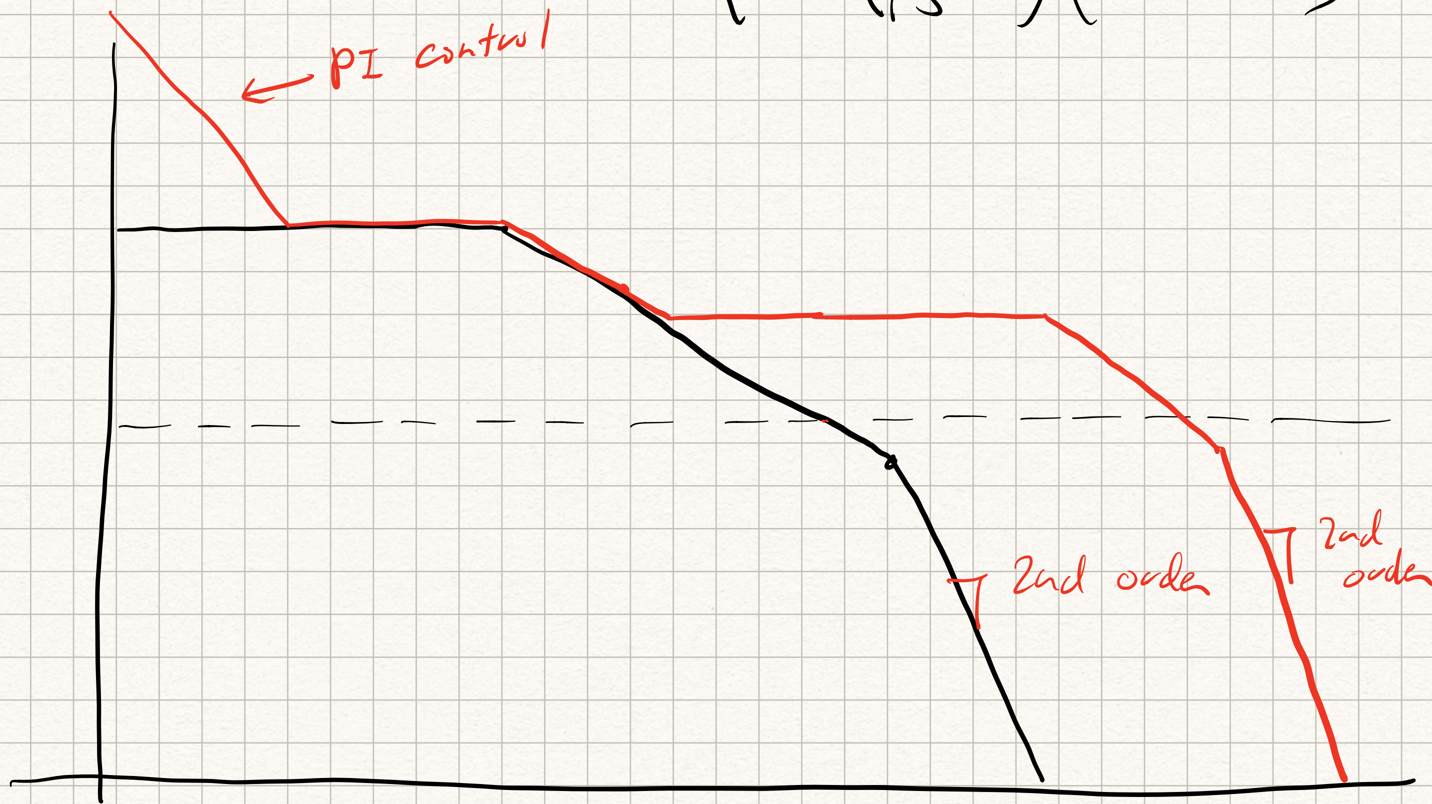
$$OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

PID Control

– Easy to implement on a digital system



$$K_p + \frac{K_i}{s} + K_d s = K_c\left(\frac{\tau_i s + 1}{\tau_i s}\right)\left(\tau_d s + 1\right)$$

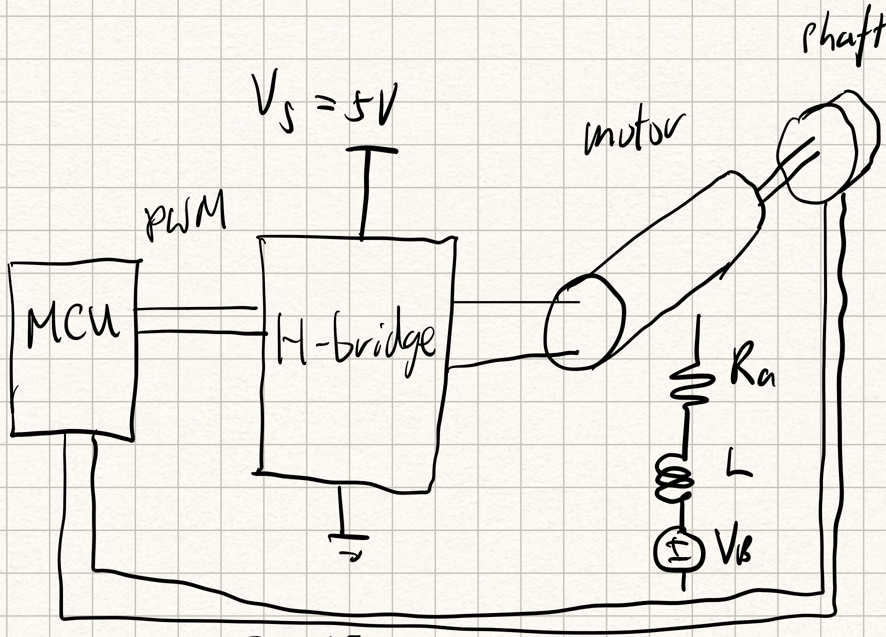


PID tuning

	Speed (rise time)	SS error	Instability (overshoot)
K_p	↑	↓	↑
K_i	↑	↓ eliminate -wind up	↑
K_d	~	~	↓

* Open-loop TF usually cannot be predicted

* Approach: Measure & model as a 2nd order system.
- all systems can be modeled as 2nd order.



K_M — Motor (torque) constant.

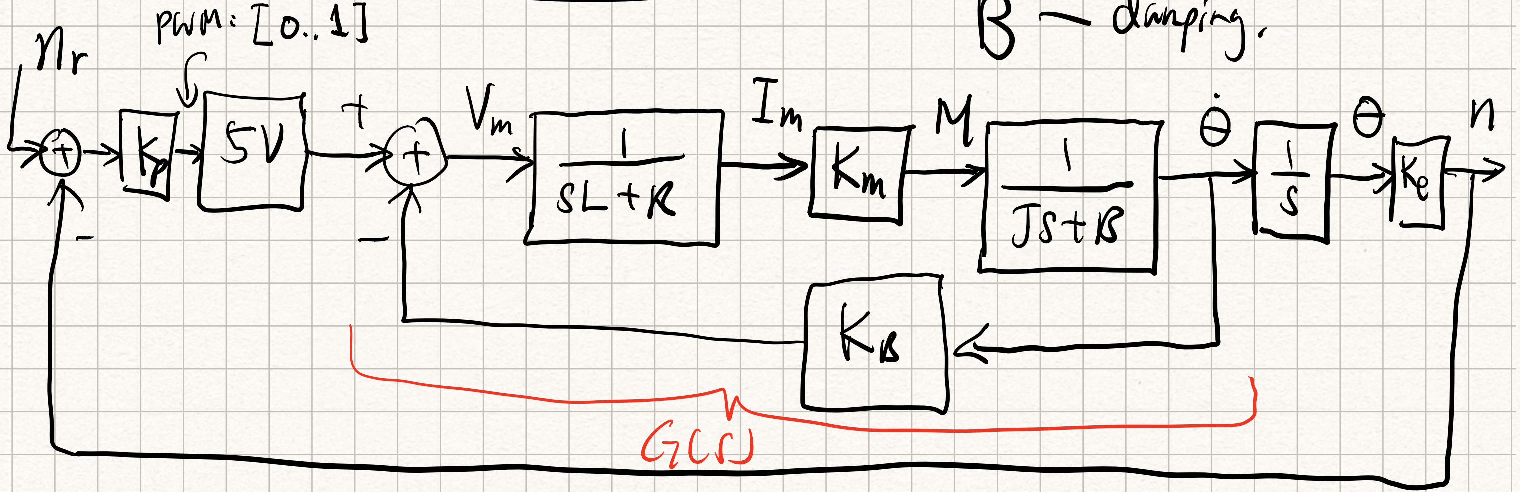
$M = K_m I$

K_B — speed constant

$V_B = K_B \dot{\theta}$

J — inertia
- shaft, gearbox, load

B — damping.



Typically, mech response is slower than the elec. response.

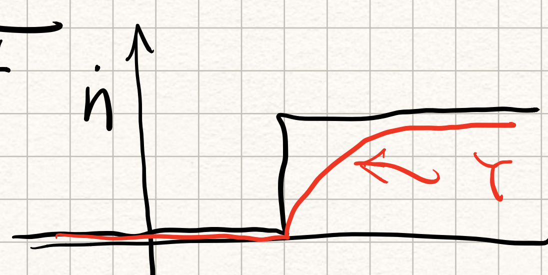
$$\tau_m \gg \tau_e \qquad \frac{J}{B} \gg \frac{L}{R}$$

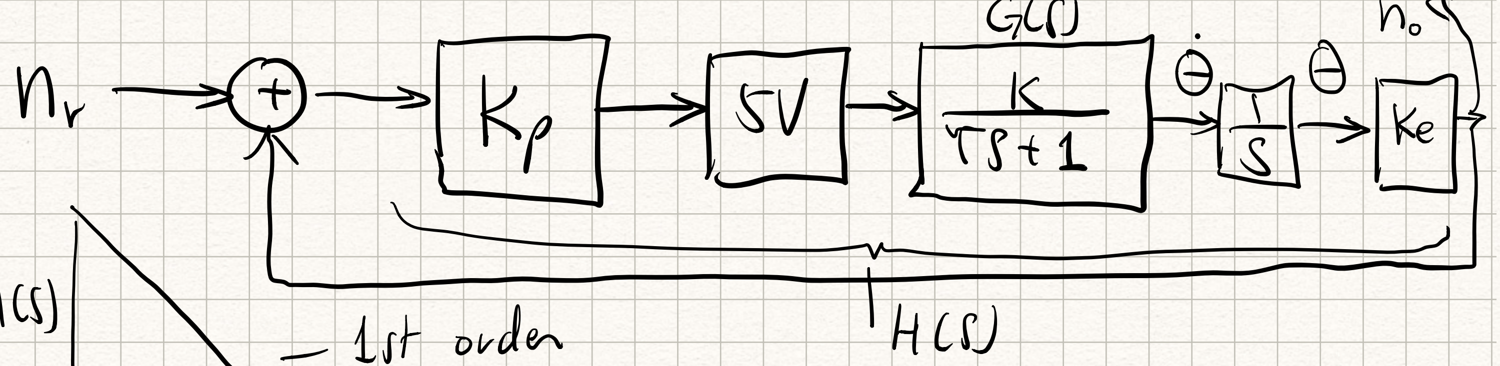
$$\left(\frac{1}{s\frac{L}{R}+1}\right)\left(\frac{1}{s\frac{J}{B}+1}\right)$$

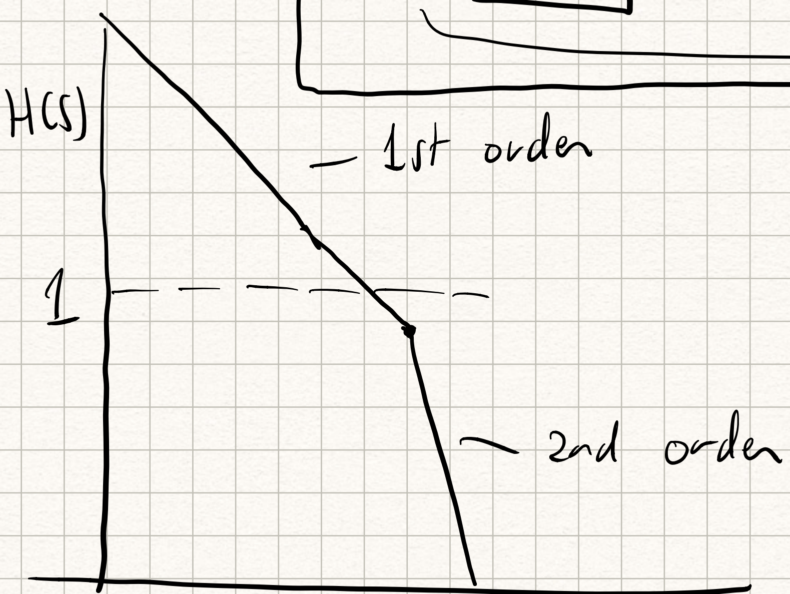
Approximate G(s) as a first order system.

$$G(s) \simeq \frac{K}{\tau s+1}$$

* Measure τ using $\dot{\theta} \rightarrow \dot{n}$







Other issues

* MCU delay.
  — model using e^{st} delay element

* Add saturation to controller.