Slide 18:

Normalized relation of
$$v_o = \frac{R}{(R+R_c)}v_{ref}$$
: $\frac{v_o}{v_{ref}} = \frac{R/R_c}{(R/R_c+1)}$

This is nonlinear. Also, see the characteristic curve.

For small changes:

$$v_o = \frac{R}{(R+R_c)}v_{ref}$$
 \rightarrow $\delta v_o = \frac{\partial v_o}{\partial R}\delta R + \frac{\partial v_o}{\partial R_c}\delta R_c$ (the familiar differential relation)

Differentiate the original relation wrt the two parameters:

$$\frac{\partial v_o}{\partial R} = \frac{(R + R_c) - R}{(R + R_c)^2} v_{ref} = \frac{R_c}{(R + R_c)^2} v_{ref}; \quad \frac{\partial v_o}{\partial R_c} = -\frac{R}{(R + R_c)^2} v_{ref}$$

Substitute:
$$\delta v_o = \frac{R_c}{(R+R_c)^2} v_{ref} \delta R - \frac{R}{(R+R_c)^2} v_{ref} \delta R_c = \frac{v_{ref}}{(R+R_c)^2} [R_c \delta R - R \delta R_c]$$

Note: Now, it is linear.

Problem: In the beginning, the circuit output is $v_o = \frac{1}{2}v_{ref}$ if $R_0 = R_{0c}$

→Any associated change (e.g., noise) can completely mask the needed output change.

Temperature Compensation:

For a temperature change of ΔT the corresponding change in the output is

$$\delta v_o = \frac{v_{ref}}{\left(R_0 + R_{0c}\right)^2} \left[R_{0c}R_0(1 + \alpha \Delta T) - R_0R_{0c}(1 + \alpha \Delta T)\right] = 0 \text{ where } ()_0 \text{ denotes initial values, and } \alpha$$

denotes the coefficient of thermal resistance of the material

Note: Since the material is assumed to be the same for the two resistors, α is the same as well Other assumption: The changes are small (O(2) terms in the Taylor series expansion are neglected.