Problem 2 (35 points)

Let us consider an op-amp circuit in Figure 2. We assume that the op-amp has infinite input impedance, zero output impedance, and open-loop transfer function $A(s)$.

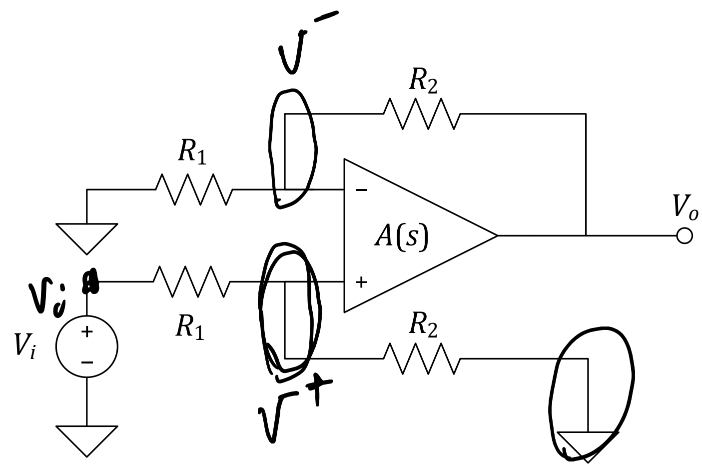


Figure 2: Op-amp circuit for the problem.

(a) (5 pt.) Draw a block diagram that shows the relation between the input voltages V_i and the the output voltage V_o. The block diagram should show a feedback loop around $A(s)$.

$$v^-: \quad \frac{v^- - 0}{R_1} + \frac{V^- - V_o}{R_2} = 0$$

$$v^+: \quad \frac{v^+ - V_i}{R_1} + \frac{v^+ - 0}{R_2} = 0$$

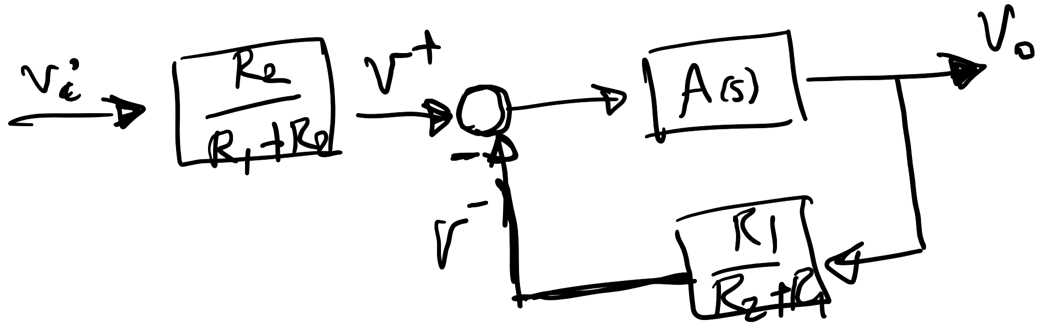
$$\times R_1R_2 \Rightarrow \quad R_2 v^- + R_1(v^- - V_o) = 0$$

$$\Rightarrow \quad R_2(v^+ - v_i) + R_1 v^+ = 0$$

$$\Rightarrow \quad (R_2 + R_1)\, v^- = R_1 V_o \Rightarrow V^- = \frac{R_1}{R_2 + R_1} V_o \quad (1)$$

$$(R_2 + R_1)\, v^+ = R_2 V_i \Rightarrow v^+ = \frac{R_2}{R_1 + R_2} V_i \quad (2)$$

$$\rightarrow (v^+ - v^-)\, A(s) = V_o$$

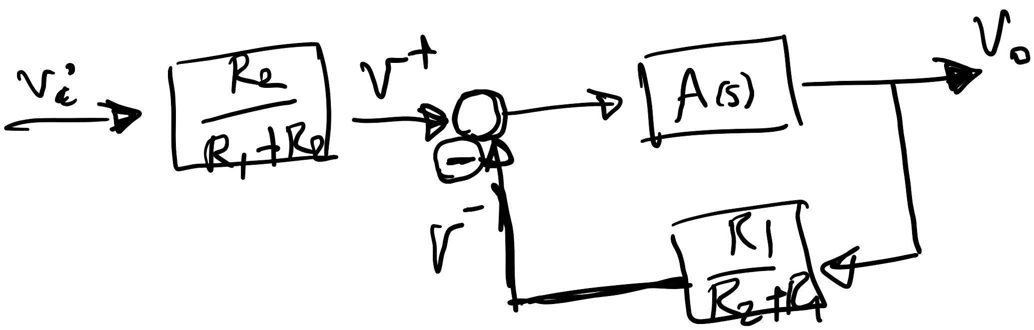


v_i
$\frac{R_2}{R_1+R_2}$
V^+
$A(s)$
V_o
V^-
$\frac{R_1}{R_2+R_1}$

(b) (5 pt.) Express the loop return ratio $L(s)$ in terms of R_1, R_2, and $A(s)$.

$$L(s) = -L.T. = -\left(-\frac{R_1}{R_1+R_2}A(s)\right) = \frac{R_1}{R_1+R_2}A(s)$$

$$= f\,A(s)$$



$$\boxed{L(s) = f\,A(s)}$$

(c) (5 pt.) For $R_1 = 1\ \text{k}\Omega$, $R_2 = 1\ \text{k}\Omega$, and $A(s)$ given in Figure 3, find the unity-gain crossover frequency ω_c and phase margin ϕ_m of $L(s)$.

$$|L(j\omega)|_{\omega=\omega_c} = 1 \qquad f = \frac{R_1}{R_1+R_2} = \frac{1}{2}$$

$$f\,|A(j\omega)|_{\omega=\omega_c} = 1$$

$$\Rightarrow \frac{1}{2}|A(j\omega)|_{\omega=\omega_c} = 1 \Rightarrow |A(j\omega)|_{\omega=\underline{\omega_c}} = 2$$

$$f_c = 5\,\text{MHz}, \quad \omega_c = 2\pi f_c = \pi \times 10^7\ \text{rad/s}$$

$$\phi_m = \angle L(j\omega)\big|_{\omega=\omega_c} - (-180) = \angle A(j\omega)\big|_{\omega=5\text{MHz}} + 180$$

$$= -105 + 180 = \underline{75^\circ}$$

$$L(s) = f A(s) \Rightarrow \angle L(j\omega) = \angle A(j\omega)$$

(d) (5 pt.) For $R_1 = 1\ \mathrm{k}\Omega$ and $A(s)$ given in Figure 3, find the resistance value R_2 that makes the closed-loop transfer function $G(s) = V_o/V_i$ achieve a -3 dB bandwidth of 10 kHz.

at 10kHz

$$\angle A(j\omega) = \angle L(j\omega) = -90^\circ \Rightarrow \boxed{\phi_m = 90^\circ}$$

-3dB bandwidth = crossover frequency

$$|G(j\omega)|_{\omega = 10\text{kHz}} = -3\text{dB} = \frac{1}{\sqrt{2}}$$

$$|L(j\omega)|_{\omega = 10\text{kHz}} = 1$$

$$f\,|A(j\omega)|_{\omega = 10\text{kHz}} = 1 \Rightarrow |A(j\omega)|_{\omega = 10\text{kHz}} = \frac{1}{f}$$

$$10^3 = \frac{R_1 + R_2}{R_1} = \frac{1 + R_2}{1}$$

$$\boxed{R_2 = 10^3 - 1 = 999\ \mathrm{k}\Omega}$$

(e) (5 pt.) For the circuit designed in part (d), determine the dc gain of $G(s) = V_o/V_i$.

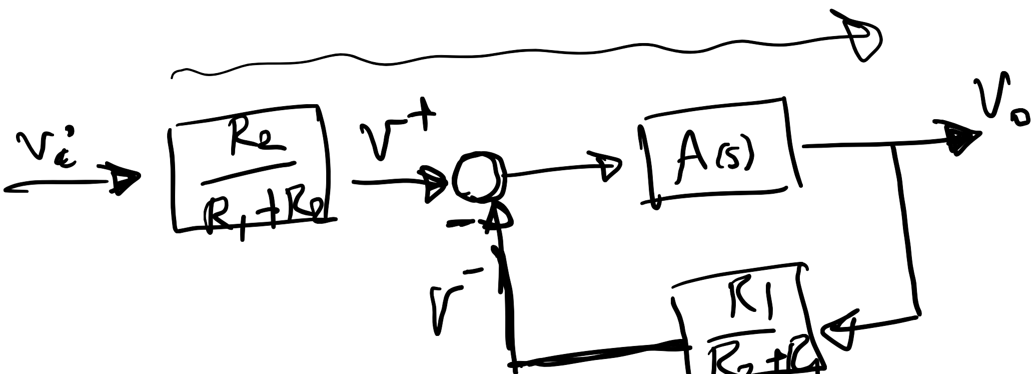
$$|G(j\omega)|_{\omega=0}$$

$R_2 = 999\,k\Omega$

$R_1 = 1k\Omega$

$$G(s) = \frac{V_o}{V_i} = \frac{\frac{R_2}{R_1+R_2} A(s)}{1 + \frac{R_1}{R_1+R_2} A(s)}$$

$$G(s) = \frac{R_2 A(s)}{R_1+R_2+R_1 A(s)} = \frac{999\,A(s)}{1000 + A(s)}$$



$$|G(j\omega)|_{\omega=0} = \frac{999\,A(j\omega)|_{\omega=0}}{1000 + A(j\omega)|_{\omega=0}} = \frac{999(10^6)}{1000+10^6}$$

$$\approx \underline{998}$$

(f) (10 pt.) Suppose the circuit designed in part (d) is excited with an input voltage

10MHz

bandwidth 10kHz

$$v_i(t) = \cos(\boxed{2\pi \times 10^7} t)$$

$$v_o(t) = M_o \cos(2\pi \times 10^7 t + \phi_o)$$

$$G(j\omega) = \frac{R_2}{R_1 + R_2} A(j\omega)$$

$$|G(j\omega)|_{\omega = 10MHz} = \frac{999}{1000} |A(j\omega)|_{\omega = 10MHz}$$

$$= 0.999 \times 1 = 0.999$$

$$\angle G(j\omega) = \angle A(j\omega)|_{\omega = 10MHz} = -120^\circ$$

$$v_o(t) = 0.999 \cos(2\pi \times 10^7 t - 120^\circ)$$

$$G(s) = \frac{R_2 A(s)}{R_1 + R_2 + R_1 A(s)}$$

$$A(j\omega)|_{\omega = 10MHz} = 1 \angle -120^\circ$$

$$= e^{-\frac{2\pi}{3} j}$$

$$= \cos\left(-\frac{2\pi}{3}\right) + \sin\left(-\frac{2\pi}{3}\right) j$$

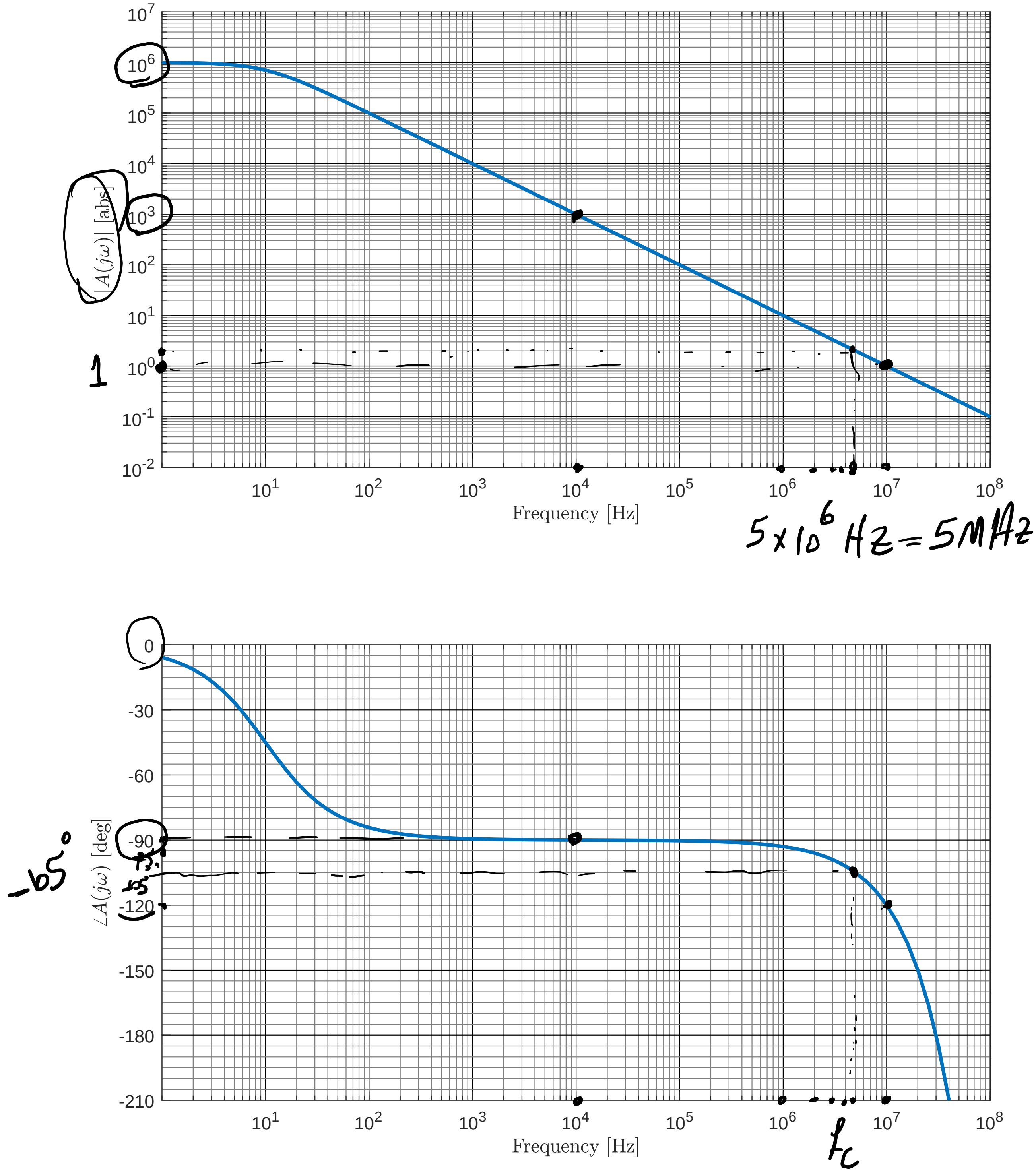


Figure 7: Bode plot of $A(s)$.