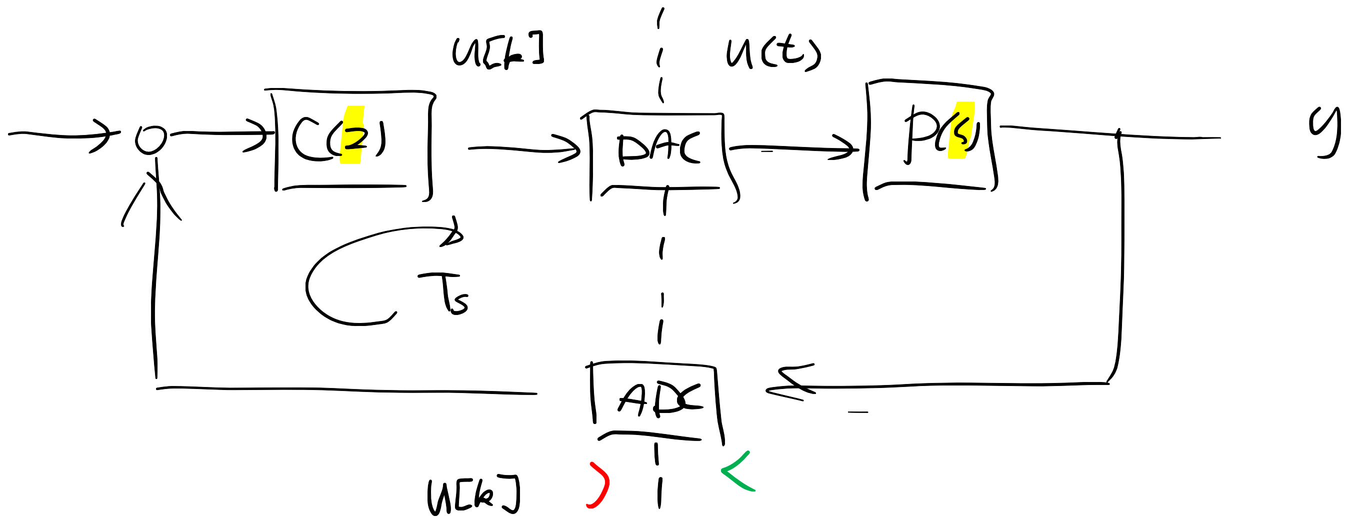
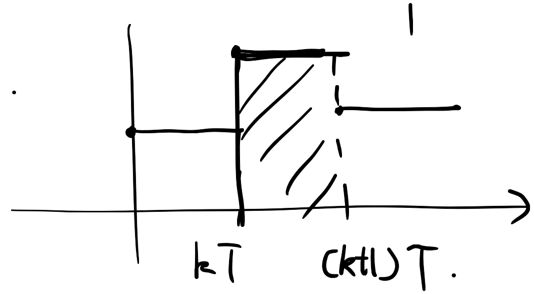
L19 – Digital Control Design

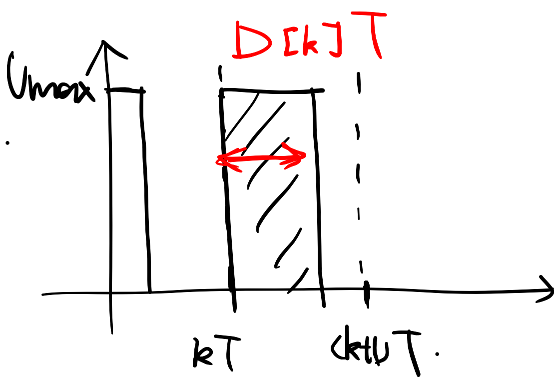


① PAM.



$$\int_{kT}^{(k+1)T} u(t)\,dt = u[k]\cdot T$$

② PWM.



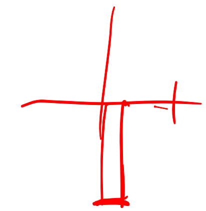
$$\int_{kT}^{(k+1)T} u(t)\,dt = u[k]\,T$$

$$= U_{max}\cdot D[k]\cdot T$$

- If $u[k]\cancel{T} = U_{max} D[k]\cancel{T}$

$$D[k] = \frac{u[k]}{U_{max}}$$

$u[k] < 0$?



- PWM works well for "Inductive plant"

  (e.g. power electronic conv. Motors)

- $\to \left[\boxed{\text{DAC}} \to \boxed{\text{Amp}} \right] \to$

- Sampling rate: $f_s = \frac{1}{T_s}$

$$f_s > 10 f_c$$

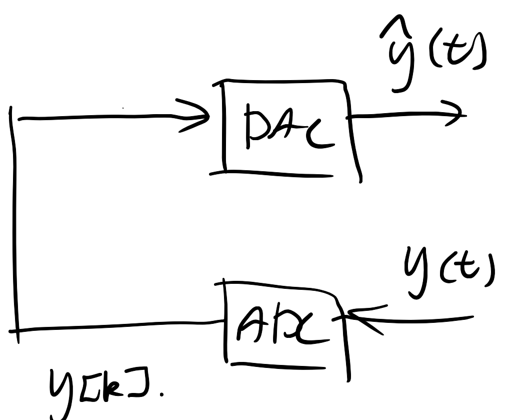
target bandwidth (crossover)

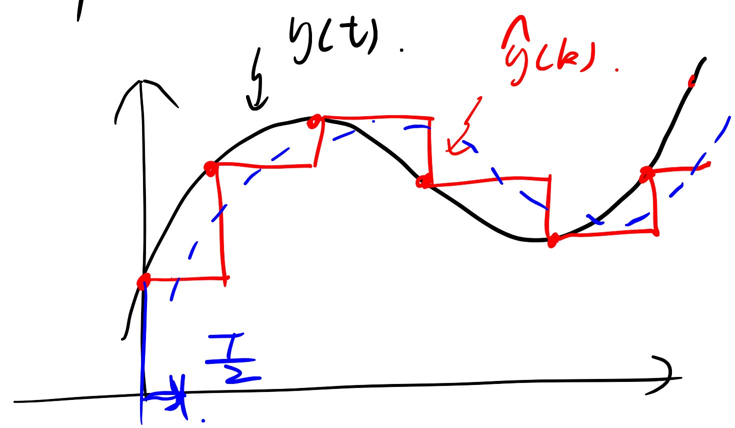
○ Two Method. "ZOH"

① . $P(s) \longrightarrow P_{eq}(z) \longrightarrow C(z)$. MECH 467.

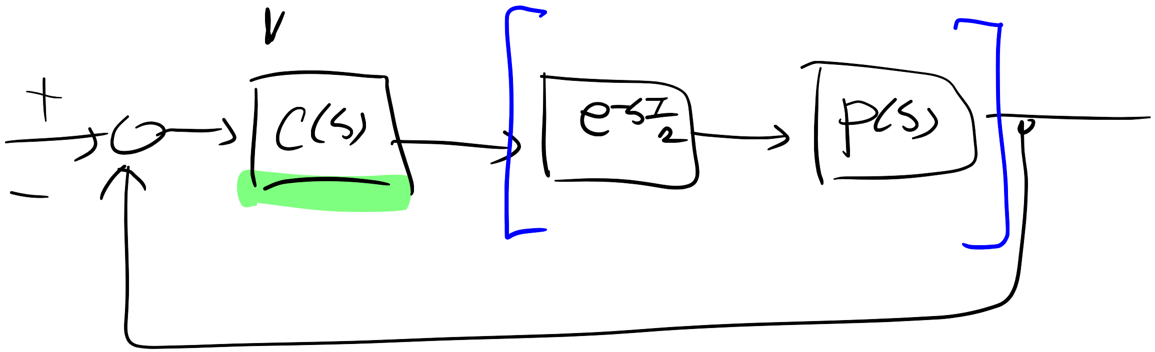
② Indirect design via DT approx.

- Design $C(s) \longrightarrow$ Implement $C(z)$.
- "Emulation"
- Good result if delay is counted.





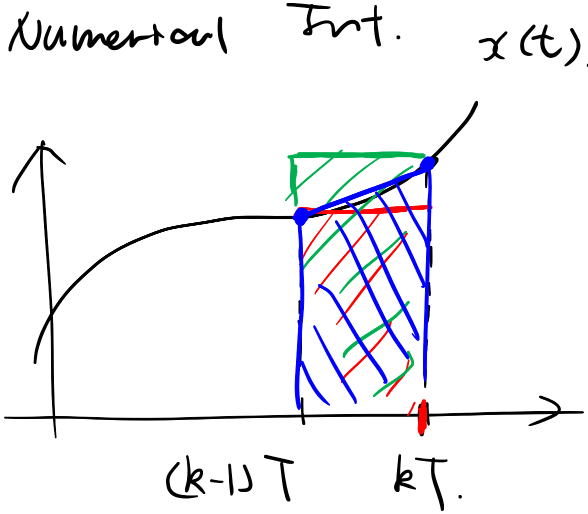
$$\simeq \quad y(t) \longrightarrow \boxed{e^{-s\frac{T}{2}}} \longrightarrow \hat{y}(t)$$



- Now $\frac{T}{2}$ is the minimum possible. (e.g. $\frac{T}{2} + T$)

- DT Approximations.

① Numerical Int.



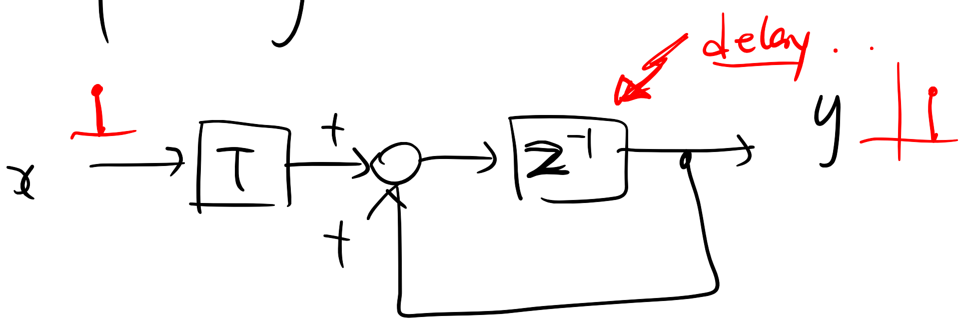
- $y(t) = \int_{-\infty}^{t} x(t)\, dt$.

- Find $y[k]$ that approx $y(t)$.

- Forward Rect. (Euler Method).

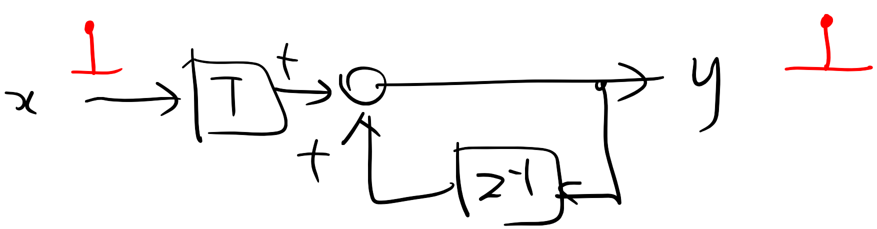
$$y[k] = y[k-1] + T\,x[k-1].$$

$$Y(z)\left(1 - z^{-1}\right) = T\,z^{-1}X(z). \qquad \therefore \frac{Y}{X} = T\left(\frac{z^{-1}}{1-z^{-1}}\right).$$



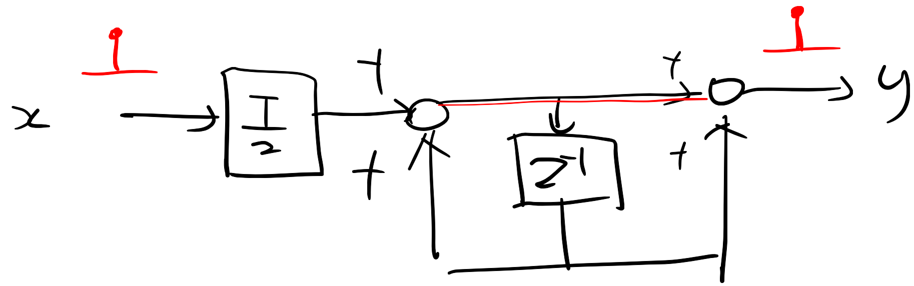
- Backward.

$$y[k] = y[k-1] + T \cdot x[k]. \qquad \frac{Y}{X} = T\left(\frac{1}{1-z^{-1}}\right).$$



- Tustin.

$$y[k] = y[k-1] + \frac{T}{2}\left(x[k] + x[k-1]\right). \quad \frac{Y}{X} = \frac{T}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)$$



- Summary

	Substitution Rule.	Mapping Rule.
① Forward.		
$\frac{1}{s} \leftrightarrow T\frac{z^{-1}}{1-z^{-1}}$	$s \leftrightarrow \frac{z-1}{T}$	$z \leftrightarrow 1+Ts$
② Back		
$\frac{1}{s} \leftrightarrow T\frac{1}{1-z^{-1}}$	$s \leftrightarrow \frac{z-1}{Tz}$.	$z \leftrightarrow \frac{1}{1-Ts}$
③ Tustin		
$\frac{1}{s} \leftrightarrow \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}}$	$s \leftrightarrow \frac{2}{T}\frac{z-1}{z+1}$.	$z \leftrightarrow \frac{1+2Ts}{1-2Ts}$

I Use sub rule. $C(s) \longrightarrow C(z)$.

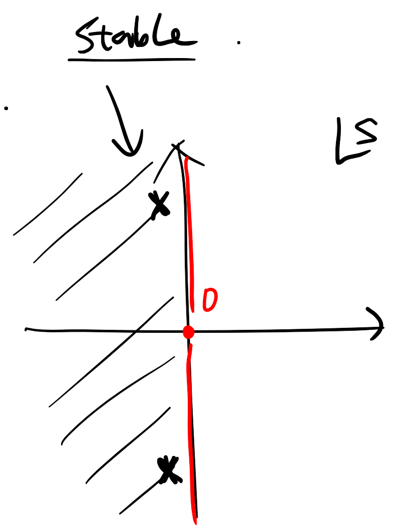
- Simulink uses "Euler" by default.

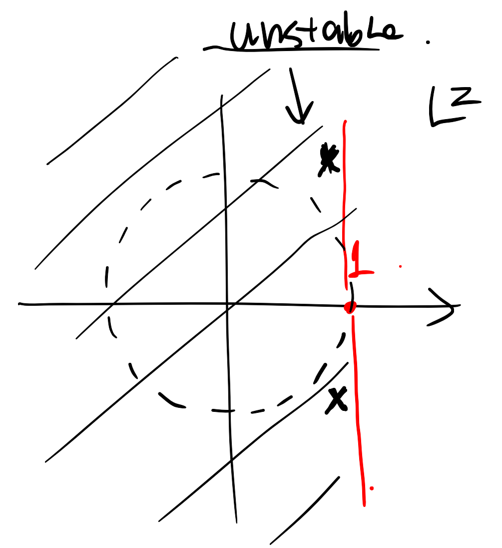
II. $C(z)$ → Simulink or LabVIEW.

→ Convert $C(z)$ to $u[k] = ---$

Implement in text lang.

- Effect on stability.





① Forward.

$$z = 1 + Ts.$$

$$\begin{cases} s = 0 \rightarrow z = 1. \\ s = j\omega \rightarrow z = 1 + jT\omega. \end{cases}$$

② Backward.

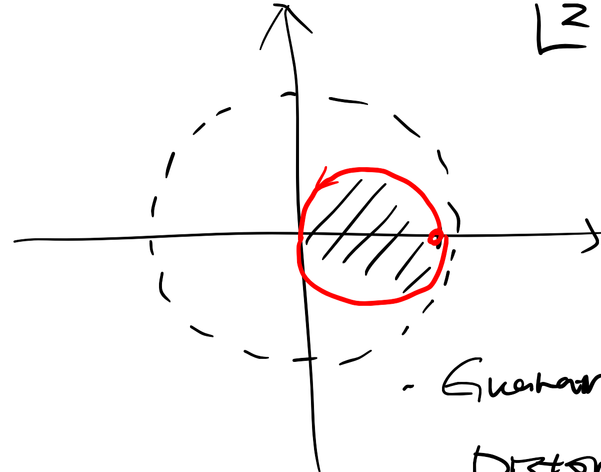
$$Z = \frac{1}{1-Ts}$$

$$\begin{cases} s=0 \rightarrow z=1 \\ s=j\omega \rightarrow z = \dfrac{1}{1-jT\omega} \end{cases}$$

As $\omega \rightarrow \infty$. $z \simeq \dfrac{1}{-jT\omega}$.

$|z| = 0$.

$\measuredangle z = \measuredangle j = 90^\circ$.



- Guarantees stability
- Distorts dynamics.

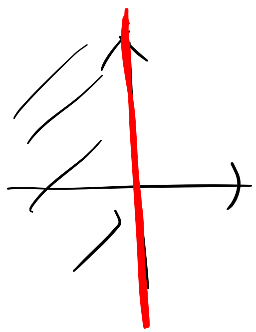
③ Tustin.

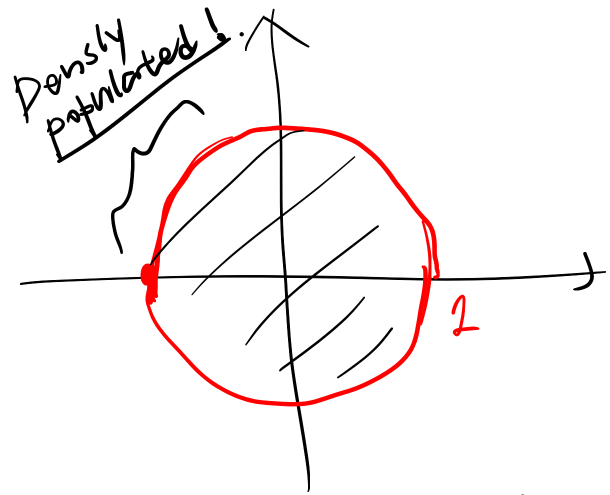
$$z = \frac{1+2Ts}{1-2Ts}.$$

$$\begin{cases} s=0 \rightarrow z=1 \\ s=j\omega \rightarrow z = \dfrac{1+j2T\omega}{1-j2T\omega} \end{cases}$$

As $\omega \rightarrow 0$. $z \simeq \dfrac{j2T\omega}{-j2T\omega}$.

$$\begin{cases} |z| = 1 \\ \measuredangle = \pi = 180^\circ \end{cases}$$





- Stability, exact.
- High-freq distortion