

# MECH 463: MECHANICAL VIBRATIONS MIDTERM EXAMINATION 3

Time: 45 minutes

19th November 2013

Maximum Available Mark: 20

READ THE QUESTION BEFORE YOU ANSWER.

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14.5  
20  
RZ

Write your answers on this sheet (4 pages in total). Do not remove pages.

- Q1. Consider a motorcycle shown in Fig.(1). The total mass, including the rider, is 250 kg and the stiffness of the suspension is 70 kN/m. The motorcycle travels with constant horizontal velocity  $v$  over a terrain, approximately sinusoidal with a distance between peaks of 10 m and the distance from peak to valley is 10 cm.

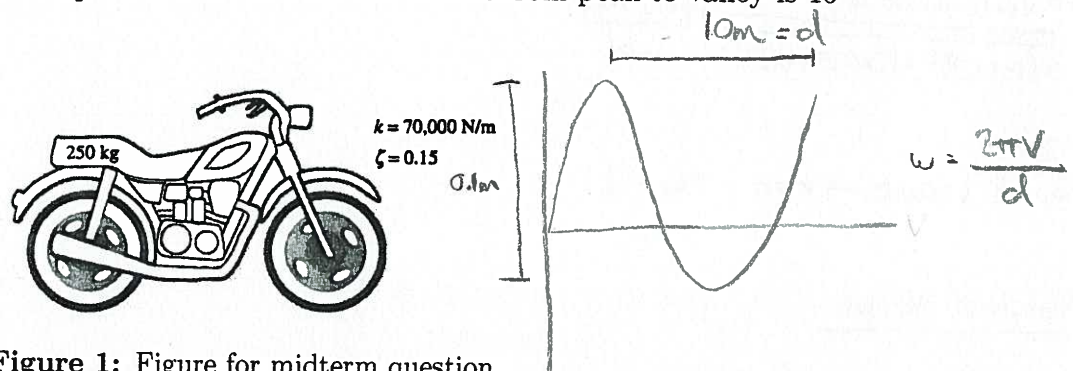


Figure 1: Figure for midterm question.

- a) Calculate the displacement transmissibility, ratio of transmitted to applied displacement in the steady state, for the speeds  $v = 5$  m/s,  $v = 16$  m/s and  $v = 30$  m/s. Sketch the transmissibility for all speeds starting from zero. You can use the transmissibility formula  $[TR]_d = \frac{\sqrt{1+[2\zeta r]^2}}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$  with  $r = \frac{\omega}{\omega_n}$ . No need to show derivation. Numerical accuracy is important in this problem. 3 marks for each speed of which 2 marks are for correct  $r$  value. 3 marks are for the TR plot. (12 marks)

10.5  
12

Answer (20 minutes or less):

$$\omega = \frac{2\pi v}{10} = \frac{v\pi}{5} \text{ [rad/s]}$$

$$\omega_n = \sqrt{k/m} = \sqrt{70,000 \text{ N/m} / 250 \text{ kg}} = 16.73 \text{ rad/s}$$

$$r = \omega/\omega_n = \frac{\pi v}{5(16.73)} = 0.0376 v$$

for  $v = 5 \text{ m/s}$ :

$$r = 0.0376v = 0.188$$

$$R_D = \frac{\sqrt{1 + (2 \times 0.15 \times 0.188)^2}}{\sqrt{(1 - 0.188^2)^2 + (2 \times 0.15 \times 0.188)^2}} = 1.037$$

for  $v = 16 \text{ m/s}$ :

$$r = 0.0376v = 0.601$$

$$R_D = \frac{\sqrt{1 + (2 \times 0.15 \times 0.601)^2}}{\sqrt{(1 - 0.601^2)^2 + (2 \times 0.15 \times 0.601)^2}} = 1.531$$

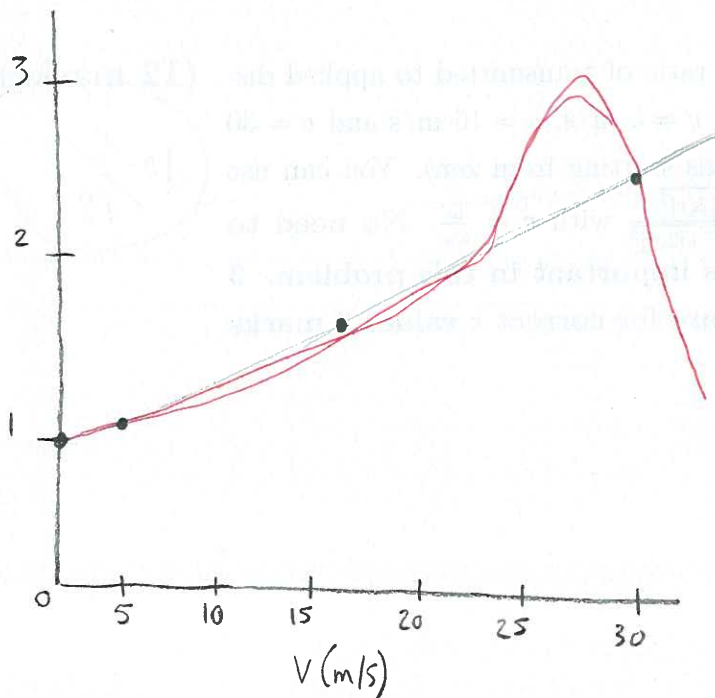
for  $v = 30 \text{ m/s}$ :

$$r = 0.0376v = 1.127$$

$$R_D = \frac{\sqrt{1 + (2 \times 0.15 \times 1.127)^2}}{\sqrt{(1 - 1.127^2)^2 + (2 \times 0.15 \times 1.127)^2}} = 2.44$$

Also, at  $v = 0 \text{ m/s} \rightarrow r = 0 \therefore TR_D = 1$

Sketch of  $TR$  plot



\* Here is a plot of my obtained  $TR$  values for each given velocity.

Not a complete diagram and also not showing the points in the right position.

b) List at least three design criteria for selecting the suspension stiffness in a).

(3 marks)

Answer (10 minutes or less):

Design criteria:

2/3

1. Firstly, ensure  $r = \frac{w}{w_n} > \sqrt{2}$ ; so when designing, you need to make sure that  $K < \frac{mw^2}{2}$
2. Account for displacements  $\rightarrow$  reducing  $w_n$  by decreasing  $K$  ( $w_n = \sqrt{K/m}$ ) would increase displacements, which is not desirable typically on a motorcycle.
3. Ensure  $TR < 1$  for a chosen range of velocities (and therefore,  $w$ ) of the motorcycle!  
This form is more useful:  $TR = \frac{\sqrt{K^2 + (cw)^2}}{\sqrt{(K-mw^2)^2 + (cw)^2}}$  where  $w = \frac{2\pi v}{10}$ , and  $c = 52mw_n$

Summary: In terms of designing  $K$ ,  $K$  has an upper bound from Criteria 1 ( $K < \frac{mw^2}{2}$ ), a lower bound from Criteria 2 (displacements), and a final benchmark parameter for  $TR$  from Criteria 3.

- c) What is the working principle of vibration isolation systems? Explain how you will apply it to design suspension stiffness in a). What is the relation between displacement and velocity transmissibilities? (5 marks)

Answer (5 minutes or less):

The main principle in isolation systems is to slow down dynamics of combine system (i.e. isolator + original system) by reducing its  $\omega_n$  relative to  $\omega$  such that  $r = \omega/\omega_n > \sqrt{2}$  and  $TR < 1$ .  $\omega_n < \omega$

Based on my part a), all TR values are greater than or equal to 1. The way I would apply it to design suspension stiffness would be this way:

choose  $k$  such that  $\omega_n = \sqrt{\frac{k}{m}} < \omega$

→ Goal: decrease  $r$ , so that  $TR < 1$

→  $r = \omega/\omega_n = \frac{\omega}{\sqrt{k/m}}$  → if  $m$  is fixed, we should increase spring stiffness

Assuming  $\zeta$ , sinusoidal path, and motorcycle mass remain constant:  $k$  must increase from 70 kN/m (current value), but remain true for  $k < \frac{m\omega^2}{2}$  ~~if  $k < 5000$  in this case.~~

Ultimately, the current configuration does need to change because from part a),  $TR > 1$  for all velocities.

$$[TR]_d = [TR]_v$$

ALL THE BEST!