MECH 463: Mechanical Vibrations
Midterm Examination 1

SOLUTION WITH MARKING SCHEME

Time: 45 minutes 26th September 2013 Maximum Available Mark: 20

Student Name: SRIKANTH Student Number: #0

Write your answers on this sheet (4 pages in total). Do not remove pages.

Q1. Unbalanced masses are a common source of vibrations in rotating systems. Consider an idealized Single Degree of Freedom (SDOF) model shown in Fig.(1). The rotating mass m_2 is unbalanced, since it lies at a distance r from the centre of rotation, which moves with the mass m_1. m_1 rests on guided supports and is free to move in the vertical direction. Because of forces exerted by rotating unbalanced mass m_2, m_1 oscillates in the vertical direction. Take y the vertical displacement of mass m_1, positive downwards, as the displacement co-ordinate from the stretched spring (static equilibrium) position p. θ is the **given** angular displacement of the rotating unbalance mass m_2, measured from the vertical, positive counter clockwise. $\dot{\theta}$ is **given** angular velocity of the rotating mass.

NOTE: $\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$, $\underline{e}_4$ ARE UNIT VECTORS

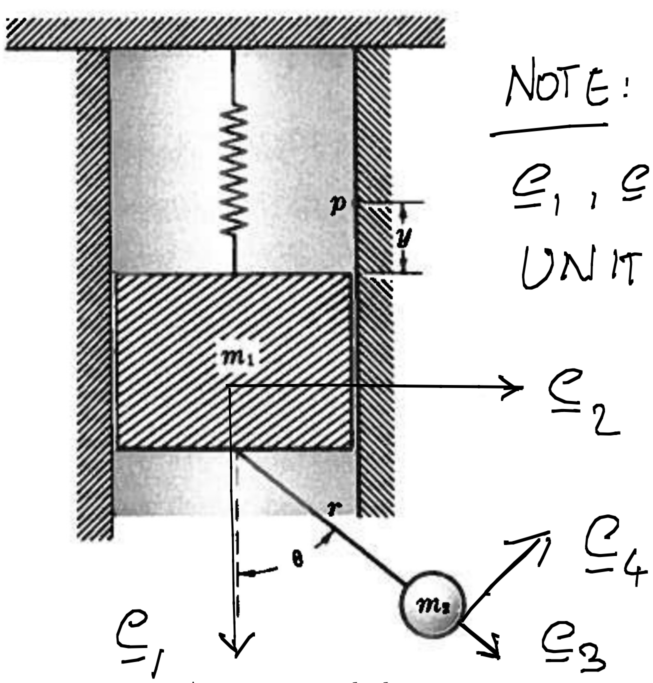


Figure 1: Figure for midterm question. A rotating unbalance m_2 causes vertical vibrations of m_1. p is the static equilibrium position and y measured with respect to p, positive downwards, is the dynamic displacement. m_1 is supported on guides and can move in the vertical direction.

a) Determine the acceleration of m_1 and m_2 with respect to a fixed observer in terms of y, r, and θ. **(4 marks)**

Answer: ACCELERATION OF m_1 REFER TO FIGURE 1 ON PAGE 1.

1 MARK ← DISPLACEMENT VECTOR $= \underline{r}_{m1/O} = y\,\underline{e}_1$ $\quad y$: ABSOLUTE DISPLACEMENT OF m_1 w.r.t. A FIXED POINT O.

1 MARK

VELOCITY OF $m_1 = \dot{\underline{r}}_{m1/O} = \frac{d}{dt}(y\,\underline{e}_1) = \dot{y}\,\underline{e}_1 + y\,\cancelto{0}{\dot{\underline{e}}_1}$ $\quad \underline{e}_1$ DOES NOT ROTATE

ACCELERATION OF $m_1 = \ddot{\underline{r}}_{m1/O} = \frac{d}{dt}(\dot{\underline{r}}_{m1/O}) = \frac{d}{dt}(\dot{y}\,\underline{e}_1) = \ddot{y}\,\underline{e}_1 + \dot{y}\,\cancelto{0}{\dot{\underline{e}}_1}$

$\therefore$ ABSOLUTE ACCELERATION OF $m_1 = \ddot{\underline{r}}_{m1/O} = \ddot{y}\,\underline{e}_1$

ACCELERATION OF m_2

1 MARK ← DISPLACEMENT VECTOR : $\underline{r}_{m2/O} = \underline{r}_{m2/m1} + \underline{r}_{m1/O} = r\,\underline{e}_3 + y\,\underline{e}_1$

1 MARK ←

VELOCITY OF $m_2 = \dot{\underline{r}}_{m2/O} = \frac{d}{dt}\left[r\,\underline{e}_3 + y\,\underline{e}_1\right] = \cancelto{0}{\dot{r}}\,\underline{e}_3 + r\,\dot{\underline{e}}_3 + \dot{y}\,\underline{e}_1 + y\,\cancelto{0}{\dot{\underline{e}}_1}$ $\quad r =$ CONSTANT

$= r\dot{\theta}\,\underline{e}_4 + \dot{y}\,\underline{e}_1$ $\quad \because \dot{\underline{e}}_3 = \dot{\theta}\,\underline{e}_4$

ACCELERATION OF $m_2 = \ddot{\underline{r}}_{m2/O} = \frac{d}{dt}\left[r\dot{\theta}\,\underline{e}_4 + \dot{y}\,\underline{e}_1\right]$

$= \cancelto{0}{\dot{r}}\,\dot{\theta}\,\underline{e}_4 + r\ddot{\theta}\,\underline{e}_4 + r\dot{\theta}\,\dot{\underline{e}}_4 + \ddot{y}\,\underline{e}_1 + \dot{y}\,\cancelto{0}{\dot{\underline{e}}_1}$

$= r\ddot{\theta}\,\underline{e}_4 + r\dot{\theta}(-\dot{\theta}\,\underline{e}_3) + \ddot{y}\,\underline{e}_1$

$= r\ddot{\theta}\,\underline{e}_4 - r\dot{\theta}^2\,\underline{e}_3 + \ddot{y}\,\underline{e}_1$ $\quad \because \dot{\underline{e}}_4 = -\dot{\theta}\,\underline{e}_3$

$\therefore$ ABSOLUTE ACCELERATION OF $m_2 = \ddot{\underline{r}}_{m2/O} = r\ddot{\theta}\,\underline{e}_4 - r\dot{\theta}^2\,\underline{e}_3 + \ddot{y}\,\underline{e}_1$

ADDITIONAL POINTS:

(1) KINEMATICS IS AN ESSENTIAL STEP IN SOLVING ANY PRACTICAL VIBRATION PROBLEM.

(2) VECTORS ARE USEFUL.

(3) ABSOLUTE ACCELERATIONS SINCE NEWTON/D'ALEMBERT REQUIRE THEM.

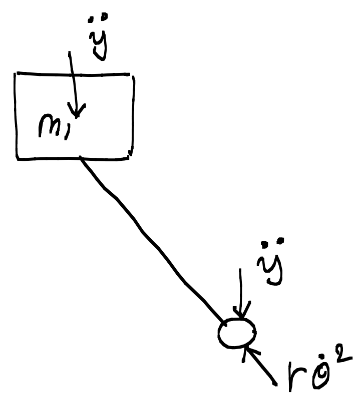
b) Consider the case when $\dot{\theta}$, the given angular velocity is constant ($\dot{\theta}$ = constant). **(8 marks)** Sketch an appropriate free body diagram and formulate the equations of motion in terms of y, θ, and r. **5 marks are for FBD indicating all forces with correct location of their points of action.** You can use Newton's method or D'Alembert's principle. Ignore gravity.

Answer: IF $\dot{\theta}$ = CONST THEN $\ddot{\theta} = 0$

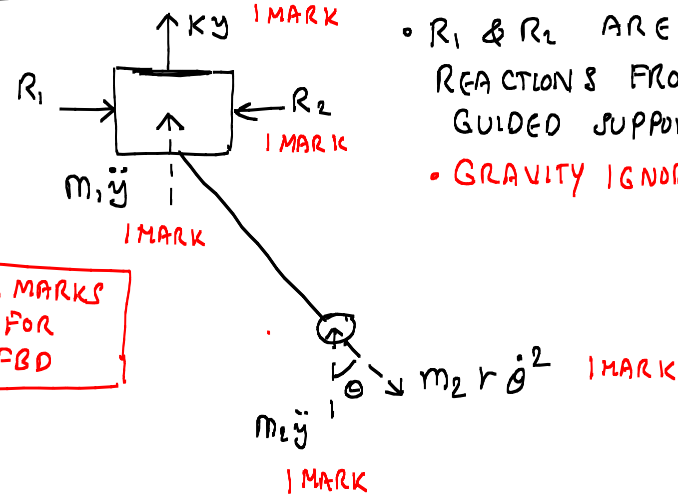
ABSOLUTE ACCELERATION OF $m_1 = \ddot{y}\, \underline{e}_1$

ABSOLUTE ACCELERATION OF $m_2 = \ddot{y}\, \underline{e}_1 + r\cancel{\ddot{\theta}}^{\,0}\, \underline{e}_4 - r\dot{\theta}^2\, \underline{e}_3 = \ddot{y}\, \underline{e}_1 - r\dot{\theta}^2\, \underline{e}_3$

ACCELERATION DIAGRAM



FREEBODY DIAGRAM (D'ALEMBERT)



- R_1 & R_2 ARE REACTIONS FROM GUIDED SUPPORTS
- GRAVITY IGNORED

$\boxed{\text{5 MARKS FOR FBD}}$

1 MARK $\downarrow \sum_{+ve} F_y = 0$ IN FBD GIVES

1 MARK $-m_1\ddot{y} - m_2\ddot{y} + m_2 r\dot{\theta}^2 \cos\theta - ky = 0$

1 MARK $\Rightarrow$ $\boxed{(m_1+m_2)\,\ddot{y} + ky = m_2 r\dot{\theta}^2 \cos\theta}$ EQUATION OF MOTION

ADDITIONAL NOTES:

(1) NOTE THE APPEARANCE OF TOTAL MASS m_1+m_2 AS A COEFFICIENT OF $\ddot{y}$

(2) $m_2 r\dot{\theta}^2 \cos\theta = m_2 r\dot{\theta}^2 \cos(\dot{\theta} t)$ = FORCING FUNCTION

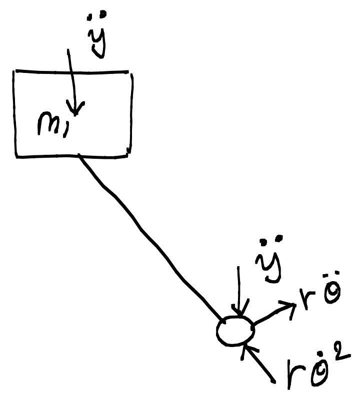
(3) YOU WILL LEARN MORE ABOUT THE ABOVE SYSTEM IN THE LAB.

BONUS POINTS FOR TIDINESS : 2

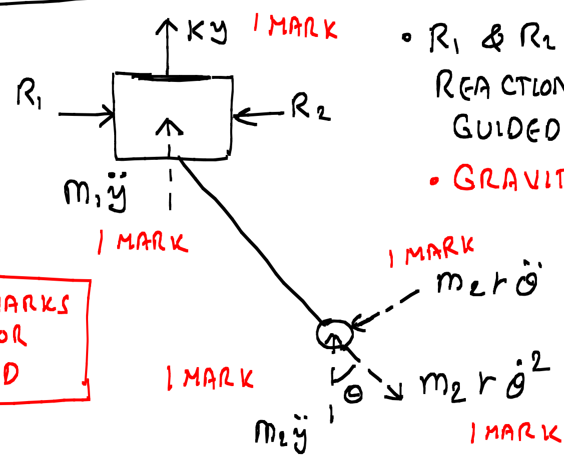
c) Treating $\dot{\theta}$, the given angular velocity not as a constant ($\dot{\theta} \neq$ constant). Sketch an appropriate free body diagram and formulate the equations of motion in terms of y, θ and r. **5 marks are for FBD indicating all forces with correct location of their points of action.** You can use Newton's method or D'Alembert's principle. Ignore gravity. **(8 marks)**

Answer: IN THIS CASE $\ddot{\theta} \neq 0$ SO WE HAVE TO USE ALL TERMS IN THE ACCELERATIONS DERIVED IN PART a)

ACCELERATION DIAGRAM



FREEBODY DIAGRAM (D'ALEMBERT)



5 MARKS FOR FBD

- R_1 & R_2 ARE REACTIONS FROM GUIDED SUPPORTS
- GRAVITY IGNORED

1 MARK NOW $\downarrow_{+ve} \sum F_y = 0$ IN FBD GIVES

1 MARK $-ky - m_1\ddot{y} - m_2\ddot{y} + m_2 r\dot{\theta}^2\cos\theta + m_2 r\ddot{\theta}\sin\theta = 0$

1 MARK $\Rightarrow$ $$\boxed{(m_1+m_2)\ddot{y} + ky = m_2 r\dot{\theta}^2\cos\theta + m_2 r\ddot{\theta}\sin\theta}$$ EQUATION OF MOTION

ADDITIONAL NOTES:

(1) NOTE THE APPEARANCE OF ADDITIONAL INERTIAL FORCE $m_2 r\ddot{\theta}\sin\theta$

(2) INERTIAL FORCES, THE SO CALLED 'FICTITIOUS' FORCES, CAUSE REAL VIBRATIONS!

THIS PROBLEM IS INSPIRED BY TUTORIAL # 2 ON KINEMATICS & HW # 1 ON ENGINE VIBRATIONS.

All the Best! ANY TYPOS etc. TO SRIKANTH@MECH.UBC.CA