René Rinfret
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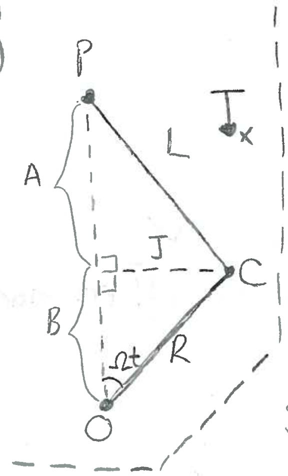
September 18, 201

19.5/20 BL

MECH463 - ASSIGNMENT #1

PART A



Based on diagram (see left):

$x_p = -A - B$ [Sign convention]

$B = R\cos(\Omega t)$

$J = R\sin(\Omega t)$

$A = \sqrt{L^2 - J^2}$ [from Pythagoreas]

$A = \sqrt{L^2 - R^2\sin(\Omega t)^2}$

So: $x_p = -\sqrt{L^2 - R^2\sin(\Omega t)^2} - R\cos(\Omega t)$

For $\sqrt{L^2 - R^2\sin(\Omega t)^2}$, let $x = \left[\frac{\sin(\Omega t)R}{L}\right]^2$

$\therefore L\sqrt{1-x}$ becomes $L(1-\frac{x}{2})$ since $x \ll 1$ [i.e. $R^2/L^2 \ll 1$]

Finally: $x_p = \frac{R^2}{2L}\left[\sin(\Omega t)^2\right] - L - R\cos(\Omega t)$

$V_p = \frac{dx_p}{dt} = \frac{2R^2}{2L}\left[\sin(\Omega t)\right]\left[\Omega\cos(\Omega t)\right] + R\Omega\sin(\Omega t)$

$\hookrightarrow$ since $2\sin\theta\cos\theta = \sin 2\theta \rightarrow V_p = \frac{R^2\Omega}{2L}\left[\sin(2\Omega t)\right] + \Omega R\sin(\Omega t)$

$A_p = \frac{dV_p}{dt} = \frac{dx_p^2}{d^2t} = \frac{2\Omega^2R^2}{2L}\left[\cos(2\Omega t)\right] + \Omega^2R\cos(\Omega t) = R\Omega^2\left[\cos(\Omega t) + \frac{R}{L}\cos(2\Omega t)\right]$

Now, include acceleration of crankshaft axis, $\ddot{x}$, to find final A_p vector:

$$\boxed{\vec{A_p} = \ddot{x}\hat{e}_1 + \left[R\Omega^2\right]\left[\cos(\Omega t) + \frac{R}{L}\cos(2\Omega t)\right]\hat{e}_1}$$ ANSWER ✓

$x_c = -R\cos(\Omega t)\hat{e}_1 - R\sin(\Omega t)\hat{e}_2$

$V_c = \frac{dx_c}{dt} = R\Omega\left[\sin(\Omega t)\hat{e}_1 - \cos(\Omega t)\hat{e}_2\right]$

$A_c = \frac{dx_c^2}{d^2t} = R\Omega^2\left[\cos(\Omega t)\hat{e}_1 + \sin(\Omega t)\hat{e}_2\right]$

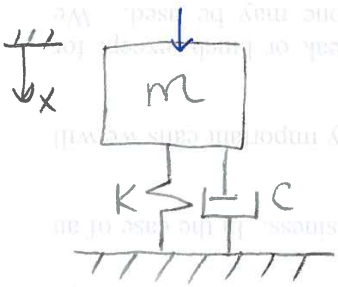
Now, include acceleration of crankshaft axis, $\ddot{x}$, to find final A_c vector:

$$\boxed{\vec{A_c} = \ddot{x}\hat{e}_1 + \Omega^2R\left[\cos(\Omega t)\hat{e}_1 + \sin(\Omega t)\hat{e}_2\right]}$$ ANSWER ✓

Part (B)

Force from unbalanced rotation

System as S.D.O.F



m: lumped masses of all components ✓

C: damping, to represent realistic model? friction

K: spring stiffness

✓

−0.5

END OF SUBMISSION

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Sept. 25, 2013

MECH 463 - HOMEWORK #2

$\frac{19.5}{20}$ BL

PART A

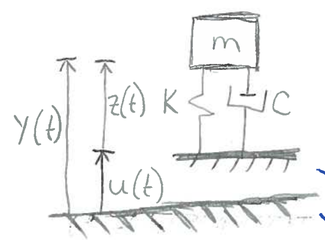
- Generic wave equation: $u(t) = A\sin(\omega x)$
- For this question: $x = vt$ (since $v = \frac{x}{t}$)

$A = b$ (amplitude)

$\omega = \frac{\pi}{c}$ (half period of length c)

- So, $\boxed{u(t) = b\sin\left(\frac{\pi v t}{c}\right)}$

- SDOF:



m: mass of all vehicle components

K: suspension — tire stiffness can be added as well

C: friction

what do these refer to? $\cancel{-1}$ -0.5

- $y(t) = u(t) + z(t)$

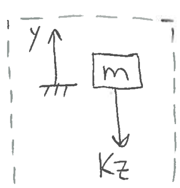
↳ this accounts for C.O.M. changing based on motion, due primarily to suspension

u is just ground displacement

- Forces due to wind/unbalanced engine neglected, as instructed

PART B

Newtons FBD



$\therefore \Sigma \uparrow F_y = ma_y$

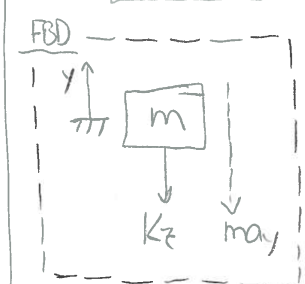
$-Kz = m\frac{d^2y}{dt^2}$

$-Kz = m\left[\frac{d^2u}{dt^2} + \frac{d^2z}{dt^2}\right]$

$-Kz = m\ddot{u} + m\ddot{z}$

$\therefore \boxed{m\ddot{z} + Kz = -m\ddot{u}}$

D'Alembert



$\therefore \uparrow \Sigma F_y = 0$

$-Kz - ma_y = 0$

$-Kz = m\left[\frac{d^2u}{dt^2} + \frac{d^2z}{dt^2}\right]$

$-Kz = m\ddot{u} + m\ddot{z}$

$\therefore \boxed{m\ddot{z} + Kz = -m\ddot{u}}$

20 / BC

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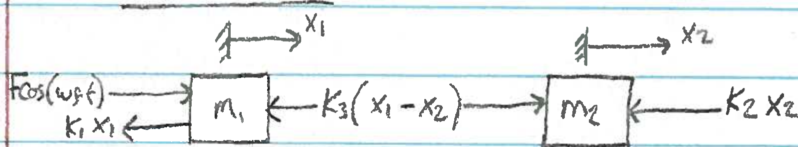
OCT. 2, 2013

MECH 463: HOMEWORK 3

Q1

1a: FBD (Newton)



eqns. of motion:

$\xrightarrow{+} \sum F_x = m_1\ddot{x}_1 = F\cos(\omega_f t) - k_1 x_1 - k_3(x_1 - x_2)$

$\therefore \boxed{F\cos(\omega_f t) = m_1\ddot{x}_1 + k_1 x_1 + k_3(x_1 - x_2)}$ Eqn. 1 ✓

$\xrightarrow{+} F_x = m_2\ddot{x}_2 = k_3(x_1 - x_2) - k_2 x_2$

$\therefore \boxed{m_2\ddot{x}_2 + k_2 x_2 + k_3(x_2 - x_1) = 0}$ Eqn. 2 ✓

1b:

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}}_{\ddot{x}} + \underbrace{\begin{bmatrix} k_1 + k_3 & -k_3 \\ -k_3 & k_2 + k_3 \end{bmatrix}}_{K} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} F\cos\omega_f t \\ 0 \end{bmatrix}}_{f}$$ ✓

Output: 2 equations

$\rightarrow m_1\ddot{x}_1 + 0(\cancel{\ddot{x}_2}) + (k_1 + k_3)x_1 - k_3 x_2 = F\cos\omega_f t$

$\therefore \boxed{m_1\ddot{x}_1 + k_1 x_1 + k_3(x_1 - x_2) = F\cos(\omega_f t)}$; same as Eqn. 1 from Part 1a

$\rightarrow 0(\cancel{\ddot{x}_1}) + m_2\ddot{x}_2 - k_3 x_1 + (k_2 + k_3)x_2 = 0$

$\therefore \boxed{m_2\ddot{x}_2 + k_2 x_2 + k_3(x_2 - x_1) = 0}$; same as Eqn. 2 from Part 1a

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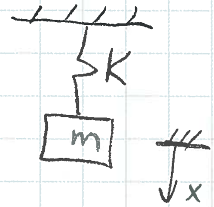
18/20 BL

Homework # 5

given: $m = 72 Kg$
$L = 61 m$
$K = 1751.3 \ N/m = 1751.3 \ Kg/s^2$



• spring is massless

a) $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1751.3 \ Kg/s^2}{72 Kg}} = \boxed{4.932 \ s^{-1}}$ ANS ✓

b) maximum amplitude of displacement:

$\dot{x}$ = velocity → to find velocity of jumper after he drops 61m:

$$PE_{TOTAL} = KE_{TOTAL} \rightarrow \cancel{m}gL = \frac{1}{2}\cancel{m}v^2 \therefore v = \sqrt{2gL} = \sqrt{2(9.81)(61)} = 34.60 m/s$$ ✓

→ for free harmonic responses:

$$X_h(t) = A_1 \cos(\omega_n t + \phi) + A_2 \sin(\omega_n t + \phi)$$

→ to find A_1, A_2:

$x(0) = 0$, since that is how I decided to set my $t = 0$
$\dot{x}(0) = V = 34.60 m/s$

if you decided that then $V_0 = \sqrt{2g(\ell + \delta_{static})}$

$x(0) = 0 = A_1 + 0 \quad \therefore \boxed{A_1 = 0}$

−1

$$\dot{x}(t) = -\omega_n A_1 \sin(\omega_n t + \phi) + \omega_n A_2 \cos(\omega_n t + \phi)$$
$$\dot{x}(0) = 34.60 = (4.932) A_2 \cos(\phi)$$

But, $\phi = \tan^{-1}\left(\frac{x(0)\,\omega_n}{\dot{x}(0)}\right) = \tan^{-1}(0) = 0$

$\therefore\ 34.60 = 4.932\,A_2$

$A_2 = 7.015$

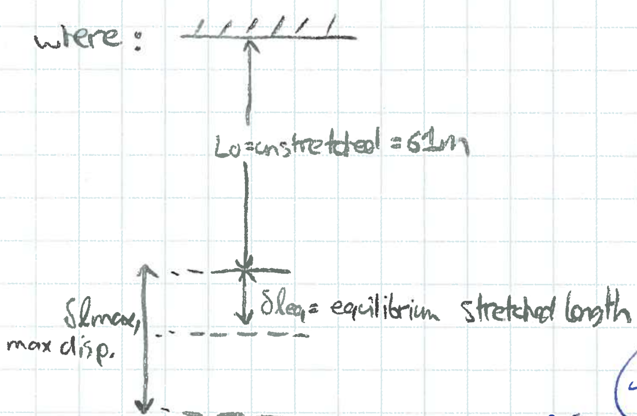
$x(t) = 7.015\sin(4.932t)$

So, maximum amplitude of displacement is about $\boxed{7.02\,m,\ \text{or} \approx 280\,in}$

ANS

(c) $K_{AMP} = \frac{\delta_{max\ disp}}{\delta_{EQ}} = \frac{E\varepsilon_{max\ disp}}{E\varepsilon_{equil.}} = \frac{\delta \ell_{max}}{\delta \ell_{eq}}$

where:



$\therefore K_{AMP} = \frac{7.02m}{\frac{mg}{k}} = \frac{7.02m + \frac{mg}{k}}{\frac{(72kg)(9.81\,m/s^2)}{1751.3\,kg/s^2}} = \boxed{17.41}$ ANS

-1

remember that static displacement also contribute to spring force / energy in dynamic condition

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Oct. 30, 2013

20/20

MECH463 - HOMEWORK 6

(1) from Homework 2: $u = b\sin\left(\frac{\pi v t}{c}\right) \therefore \ddot{u} = -b\left[\frac{\pi v}{c}\right]^2 \sin\left(\frac{\pi v t}{c}\right)$

$z_p(t)$ = particular solution = $A\cos\left(\frac{\pi v t}{c}\right) + B\sin\left(\frac{\pi v t}{c}\right)$

$$\ddot{z}_p(t) = -A\left(\frac{\pi v}{c}\right)^2 \cos\left(\frac{\pi v t}{c}\right) - B\left(\frac{\pi v}{c}\right)^2 \sin\left(\frac{\pi v t}{c}\right)$$

for forced vibration: $m\ddot{z} + Kz = -m\ddot{u}$

↳ since RHS only has "sin" terms, all "cos" terms are cancelled ou

$$\therefore m\left(-B\left[\frac{\pi v}{c}\right]^2 \sin\left(\frac{\pi v t}{c}\right)\right) + K\left(B\sin\left(\frac{\pi v t}{c}\right)\right) = -m\left(-b\left[\frac{\pi v}{c}\right]^2 \sin\left(\frac{\pi v t}{c}\right)\right)$$

$$m\left(-B\left[\frac{\pi v}{c}\right]^2\right) + KB = mb\left[\frac{\pi v}{c}\right]^2$$

$$B\left(K - m\left[\frac{\pi v}{c}\right]^2\right) = mb\left[\frac{\pi v}{c}\right]^2$$

$$\therefore B = \left[mb\left[\pi v/c\right]^2\right] \Big/ \left[K - m\left[\frac{\pi v}{c}\right]^2\right]$$

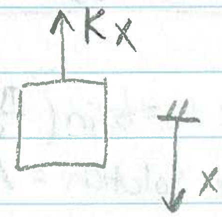
$$\therefore Z_p(t) = -B\left(\frac{\pi v}{c}\right)^2 \sin\left(\frac{\pi v t}{c}\right)$$

$$\boxed{= \frac{mb\left[\frac{\pi v}{c}\right]^2}{K - m\left[\frac{\pi v}{c}\right]^2} \sin\left(\frac{\pi v t}{c}\right)}$$ ANS

for maximum $z_p(t)$, set $K - m\left[\frac{\pi v}{c}\right]^2 = 0$

$$\therefore \boxed{V = \sqrt{\frac{K}{m}}\,\frac{c}{\pi}}$$ ANS

(2) a) FBD



$$\downarrow \Sigma F_x = m\ddot{x} = -Kx \quad \therefore \; m\ddot{x} + Kx = 0$$

so, $x(t) = A\cos(\omega t) + B\sin(\omega t)$

I.C. $\rightarrow x(0) = 0 = A \quad \therefore \boxed{A = 0}$

I.C. $\rightarrow \dot{x}(0) = V_0$

$$\dot{x}(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t) = B\omega\cos(\omega t)$$

$$\dot{x}(0) = V_0 = B\omega \rightarrow B = \frac{V_0}{\omega}$$

$$\therefore \; x(t) = \frac{V_0 \sin(\omega t)}{\omega} = \boxed{V_0\sqrt{\frac{WL}{AE}}\,\sin\left(\sqrt{\frac{AE}{LW}}\,t\right)} \quad \text{ANS}$$

b) $\sigma_{Dyn} = E\epsilon = E\frac{\delta}{L}$

$$\delta = \delta_{st} + \delta_{dyn,max} = \frac{WgL}{AE} + V_0\sqrt{\frac{WL}{AE}}$$

$$\therefore \boxed{\sigma_{Dyn} = \frac{E\left[\frac{WgL}{AE} + V_0\sqrt{\frac{WL}{AE}}\right]}{L}} \quad \text{ANS}$$

c) $\delta = \frac{Wg L}{AE} + V_0\sqrt{\frac{WL}{AE}} = \frac{4536(9.81)(18.3)}{(1.61\times10^{-3})(103.5\times10^{9})} + 0.91\sqrt{\frac{(4536)(18.3)}{(1.61\times10^{-3})(103.5\times10^{9})}}$

$= \boxed{0.0252\text{m}}$

$\sigma_{Dyn} = \frac{E\delta}{L} = \frac{103.5\times10^{9}(0.0252)}{18.3} = \boxed{142.5\text{MPa}}$

END OF SUBMISSION

MECH463 - ASSIGNMENT 7

21/20 LM

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November 13th, 2013

Q1

$f = C_c \frac{\dot{x}}{|\dot{x}|} = \pm C_c$

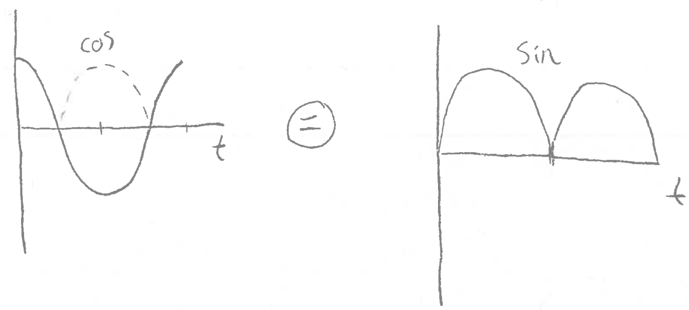
$x(t) = X\sin(\omega t)$ ✓

$\dot{x}(t) = X\omega\cos(\omega t)$

So, $W = f(t)\dot{x}\,dt = C_c X\omega\cos(\omega t)\,dt$

↳ when $\dot{x}(t)<0$, $C_c<0$ $\therefore W>0$

when $\dot{x}(t)>0$, $C_c>0$ $\therefore W>0$



So, ΔW can be rewritten as:

$$\Delta W = 2XC_c\omega\int_0^{\pi/\omega}\sin(\omega t)\,dt = \frac{2XC_c\cancel{\omega}}{\cancel{\omega}}\Big[-\cos(\omega t)\Big]_0^{\pi/\omega} = 2XC_c\big(-\cos(\pi)+\cos(0)\big) = 4XC_c$$

Now, $\Delta W = W_{eq\text{-}viscous} = \pi C_{eq}\omega X^2$

So, $4XC_c = \pi C_{eq}\omega X^2$

$\therefore \boxed{C_{eq} = \frac{4C_c}{\pi\omega X}}$ ANS. ✓

Q2

- Solve ω_n for each case:

$\omega_{n,A} = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{80{,}000\ N/m}{2000\ kg}} = 6.325\ rad/s$

$\omega_{n,B} = \sqrt{\frac{800{,}000\ N/m}{2000\ kg}} = 20\ rad/s$

$\omega_{n,C} = \sqrt{\frac{80{,}000\ N/m}{2000\ kg}} = 6.325\ rad/s$

- Solve ζ for each case: ✓

$zeta,a = \frac{c}{2m\omega_n} = \frac{20{,}000\ Ns/m}{2(2000)(6.325)} = 0.791 \rightarrow$ underdamped

$zeta,b = \frac{20{,}000\ Ns/m}{2(2000)(20)} = 0.25 \rightarrow$ underdamped

$zeta,c = \frac{40{,}000}{\quad} = 1.58 \rightarrow$ overdamped

these can be somewhat interpreted graphically, but determining them numerically gives a better understanding.

• Solve ω_d for each case:

$\omega_{d,A} = \omega_{n,A}\sqrt{1-\zeta^2} = 6.325\sqrt{1-0.791^2} = 3.87 \text{ rad/s}$

$\omega_{d,B} = 20\sqrt{1-0.25^2} = 19.36 \text{ rad/s}$

$\omega_{d,C} = \omega_n\sqrt{\zeta^2-1} = 6.325\sqrt{1.58^2-1} = 7.737 \text{ rad/s}$

Solve for times taken to reach maxima:

→ for case A, B:

$$x_h(t) = e^{-\zeta\omega_n t} A\cos(\omega_d t - \phi)$$

$$\phi = \frac{\pi}{2}$$

$$x_h(t) = e^{-\zeta\omega_n t} A\sin(\omega_d t)$$

$$\dot{x}_h(t) = -\zeta\omega_n e^{-\zeta\omega_n t} A\sin(\omega_d t) + e^{-\zeta\omega_n t} A\omega_d\cos(\omega_d t)$$

at x_{hmax}, $\dot{x}_h = 0$

$$\therefore\ 0 = -\zeta\omega_n \sin(\omega_d t) + \omega_d\cos(\omega_d t)$$

$$\therefore\ \frac{\sin(\omega_d t)}{\cos(\omega_d t)} = \tan(\omega_d t) = \frac{\omega_d}{\zeta\omega_n}$$

$$\therefore\ t = \tan^{-1}\left[\omega_d/\zeta\omega_n\right]/\omega_d$$

$t_{case\,A} = \tan^{-1}\left(3.87/0.791\times 6.325\right)/3.87 = \boxed{0.17 \text{ sec}}$ } both agree with graphs!

$t_{case\,B} = \tan^{-1}\left(19.36/0.25\times 20\right)/19.36 = \boxed{0.06 \text{ sec}}$

→ for case C:

Graphically, we can see that it reaches x_{hmax} in less time than Case A but more than Case B.

$$\therefore\ \boxed{0.06\text{s} < t_{,case\,C} < 0.17\text{s}}$$

So to reach maximum displacement: $\boxed{t_{case\,A} > t_{case\,C} > t_{case\,B}}$

Also, $\boxed{\text{max disp.}_{,A} > \text{max disp}_{,C} > \text{max disp}_{,B}}$

.Finally, velocity peaks <u>after</u> $V_0 = 10 m/s$, and the time to get there:

$$|V_{MAX_B}| > |V_{MAX\,A}| > |V_{MAX\,C}|$$

$$t_a > t_b \approx t_c$$

→ Oscillations in case b were quite noticeable relative to the rest of the cases. The system in case b oscillated because the damping was so little ($\zeta = 0.25$, as shown earlier), and so the change in amplitudes (i.e. $\frac{A_2}{A_1}$, $\frac{A_3}{A_2}$, etc) were very large relative to the other cases. The other cases were near critical damping (0.8 for Case A), or over-damped (1.58 for Case C)

$\boxed{\text{END OF SUBMISSION}}$

(+1 for completeness)

MECH 463 : HOMEWORK 8

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Nov. 20, 2013

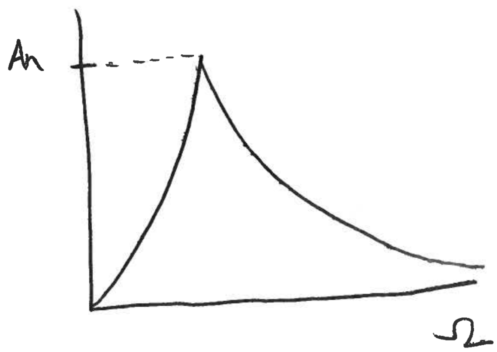
LM

20/20

(1) $\ddot{r} + 2\zeta\omega_n\dot{r} + \omega_n^2 r = 2\Omega^2 e^{i\Omega t}$ ✓

a) Max. Amplitude occurs when $\Omega = 1 \rightarrow A_n = \frac{\varepsilon}{2\zeta}$

b) Minimum amplitude occurs when $\Omega = 0$, or ∞



(2) $\dot{\Omega} = 0$

$0 = T_{motor} - \varepsilon K_x X \sin\Omega t + \varepsilon K_y Y \cos\Omega t$ ✓

$\Rightarrow T_m = \varepsilon K_x X \sin\Omega t - \varepsilon K_y Y \cos\Omega t$

Some parameters include: ✓
- the eccentricity ε
- spring constants K_x and K_y
- angular velocity Ω

3) a) Forward whirl: $\frac{r^+}{r^-} = \frac{(\omega_x^2+\omega_y^2-2\Omega^2)}{(\omega_y^2-\omega_x^2)} = \frac{9+10-2\Omega^2}{10-9} = 19-2\Omega^2$

So, $19-2\Omega^2 > 1$

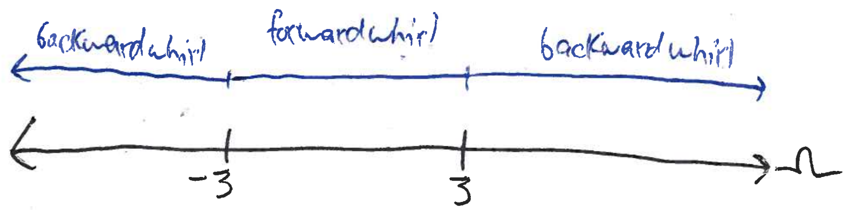
$\Omega^2 < 9 \Rightarrow \boxed{-3 < \Omega < 3}$

Backward whirl:

$19-2\Omega^2 < 1 \Rightarrow \Omega^2 > 9$

$|\Omega| > 3$ ✓

So graphically:



b) You want to avoid cases where $|\Omega| > 3$ because the backward whirl leads to fatigue failures! ✓

END OF SUBMISSION