MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

Homework 4

Assigned: Mar 2, 2021
Due: Mar 9, 2021

Problem 1

Figure 1 shows a transconductance amplifier driving a brushed dc motor. Here, $A_p(s)$ is the open-loop gain of a power op-amp, $A_s(s)$ is the open-loop gain of a signal op-amp, and Z_3, Z_4, and Z_5 are impedances to be designed. We assume infinite input impedance and zero output impedance for the op-amps.

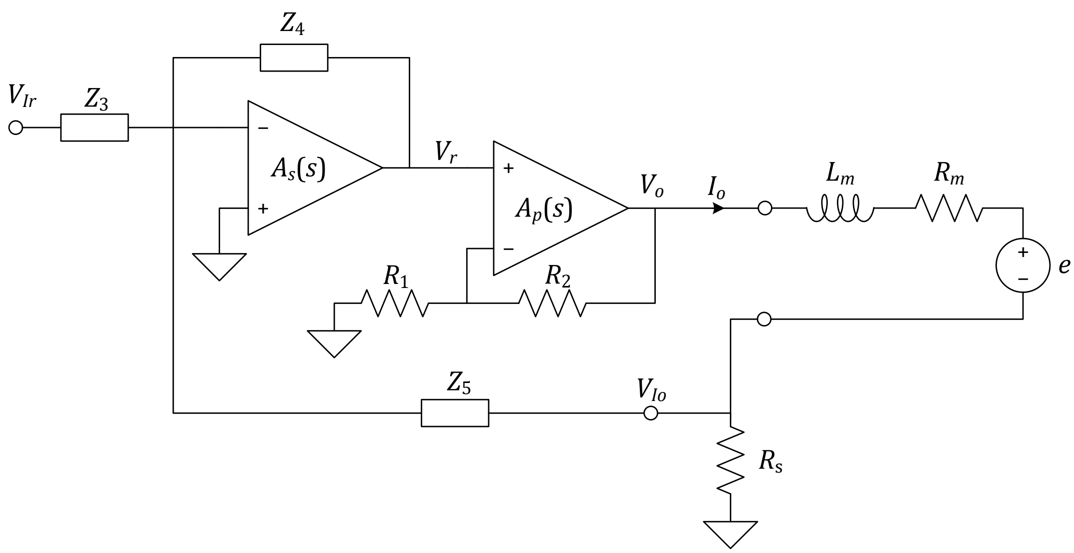

Figure 1: Schematic of a transconductance amplifier.

(a) For the signal op-amp stage, draw a block diagram that shows the relation between the V_{Ir}, V_{Io}, and V_r. The block diagram should show a feedback loop around $A_s(s)$ (Tip: do not use the virtual short approximation for V_-. Instead, use the superposition method to find the inverting terminal voltage V_-).

(b) Find the expression for the loop transfer function $L_s(s)$ of the signal op-amp stage in terms of Z_3, Z_4, Z_5, and $A_s(s)$.

(c) Carry out block diagram algebra on the result of part (a) to complete the equivalent block diagrams in Figure 2.

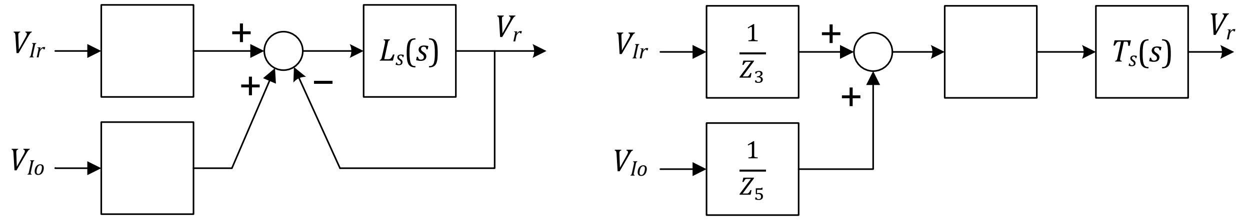

Figure 2: Equivalent block diagram, where $T_s(s) = \frac{L_s(s)}{1+L_s(s)}$

Problem 2

Figure 3 shows the full block diagram of the transconductance amplifier. Here, the motor back EMF e is modeled as a disturbance signal, and the load impedance and feedback gain are approximated for $R_s \ll R_5$. The op-amp open-loop transfer functions and circuit parameters are given as follows

$$A_s(s) = \frac{5 \times 10^7}{s} \qquad A_p(s) = \frac{10^7}{s} \qquad R_1 = 1\,\mathrm{k\Omega} \qquad R_2 = 9\,\mathrm{k\Omega}$$
$$L_m = 1\,\mathrm{mH} \qquad R_m = 5.8\,\Omega \qquad R_s = 0.2\,\Omega \qquad R_5 = 1\,\mathrm{k\Omega}.$$

We will select values for R_3, R_4, and C_4 to complete the amplifier design. Use MATLAB for the design process.

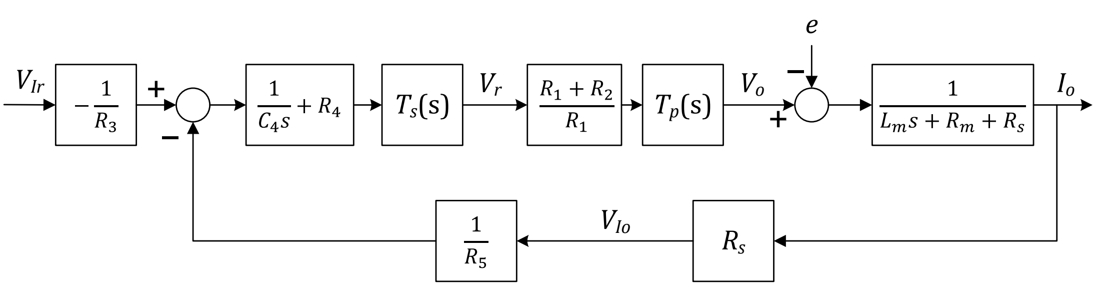

Figure 3: Transconductance amplifier block diagram.

(a) Selecting R_3: Suppose we use a digital-to-analog converter (DAC) whose output range is $\pm 10\,\mathrm{V}$ to drive V_{Ir}. Also, suppose the motor is designed for dc current range of $\pm 2\,\mathrm{A}$. Find the value R_3 that allows maximum utilization of the DAC dynamic range. Assume that the loop transfer function has infinite dc gain.

(b) Selecting R_4: This step corresponds to the design of proportional control. For now, let us assume that the bandwidth of $T_s(s)$ is infinitely high (i.e., $T_s = 1$) and capacitance C_4 is infinitely large. Find the value R_4 rounded to the nearest kΩ that makes the loop transfer function achieve the highest crossover frequency ω_c while guaranteeing phase margin $\phi_m \geq 90°$. Note that this step should account for the finite bandwidth of $T_p(s)$. Draw the Bode plot of the loop transfer function and find ω_c and ϕ_m.

(c) Selecting C_4: This step corresponds to the design of integral control. A capacitor C_4 in series with R_4 makes the loop transfer function achieve higher dc gain, thereby improving the closed-loop tracking and disturbance rejection performance. However, C_4 can decrease the phase margin. Find the value C_4 rounded to the nearest nF that achieves the highest loop gain while guaranteeing phase margin $\phi_m \geq 85°$. Draw the Bode plot of the loop transfer function and find ω_c and ϕ_m.

(d) Including $T_s(s)$: Using the R_3, R_4, and C_4 values obtained above, draw the Bode plot of the final loop transfer function including $T_s(s)$ and find ω_c and ϕ_m.