MECH 421: Mechatronics System Instrumentation
2020/21 Winter Session – Term 2

Homework 4

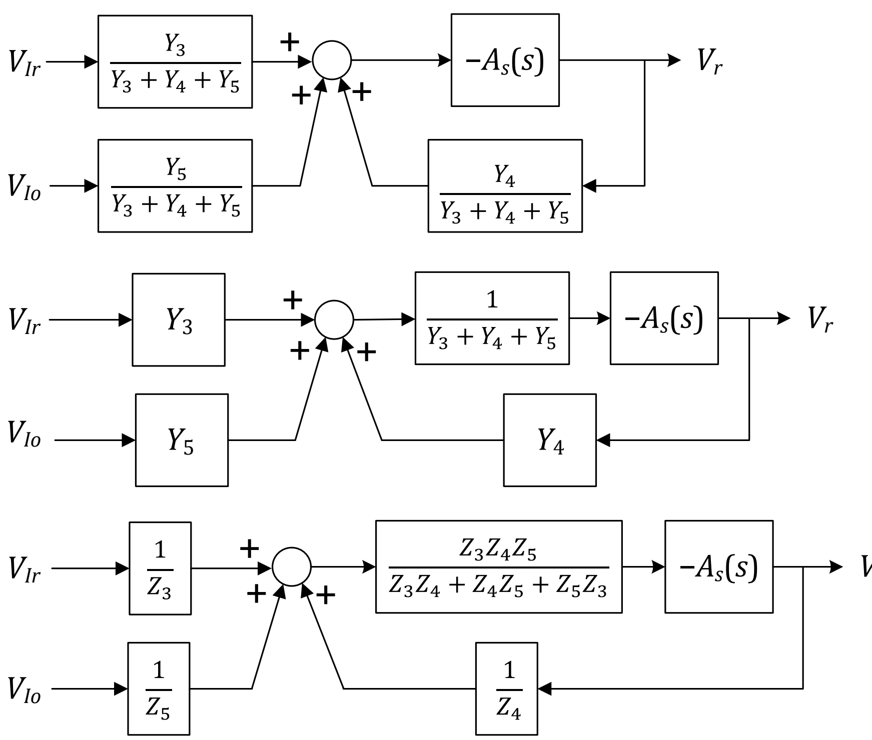
Assigned: Mar 2, 2020
Due: Mar 9, 2020

Problem 1

(a) The voltage at the op-amp inverting terminal is

$$V_- = \frac{Y_3}{Y_3 + Y_4 + Y_5} V_{Ir} + \frac{Y_4}{Y_3 + Y_4 + Y_5} V_r + \frac{Y_5}{Y_3 + Y_4 + Y_5} V_{Io},$$

where $Y_3 = 1/Z_3$, $Y_4 = 1/Z_4$, and $Y_5 = 1/Z_5$. We can construct a block diagram and carry out block-diagram algebra as follows.

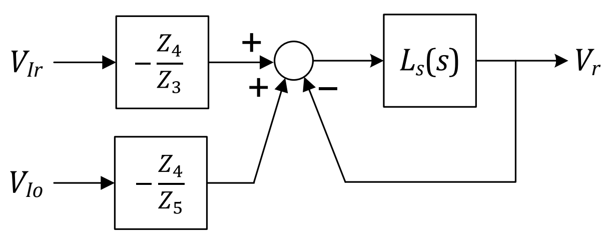


Any block diagram equivalent to the last one is acceptable.

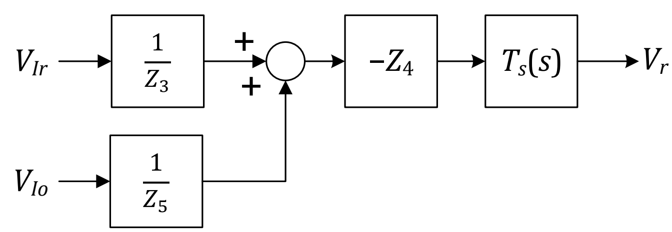
(b)

$$L_s(s) = -\text{L.T.} = \frac{Z_3 Z_5}{Z_3 Z_4 + Z_4 Z_5 + Z_5 Z_3} A_s(s)$$

(c)



V_{Ir}
$-\frac{Z_4}{Z_3}$
V_{Io}
$-\frac{Z_4}{Z_5}$
+
+
−
$L_s(s)$
V_r



V_{Ir}
$\frac{1}{Z_3}$
V_{Io}
$\frac{1}{Z_5}$
+
+
$-Z_4$
$T_s(s)$
V_r

Problem 2

(a) Assuming $L(j\omega)_{\omega=0} \to \infty$,

$$\left.\frac{I_o(j\omega)}{V_{Ir}(j\omega)}\right|_{\omega=0} = -\frac{1}{R_3}\frac{R_5}{R_s} = -\frac{5\times 10^3}{R_3}.$$

To match the DAC range and the motor current range, the dc transconductance magnitude should be

$$\left|\frac{I_o(j\omega)}{V_{Ir}(j\omega)}\right|_{\omega=0} = 0.2\,\text{A/V}.$$

Therefore,

$$R_3 = \frac{5\times 10^3}{0.2\,\text{A/V}} = 25\,\text{k}\Omega.$$

(b) Let us define the plant $P(s)$ and controller $C(s)$ as

$$P(s) = \left(\frac{1}{L_m s + R_m + R_s}\right)\left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_s}{R_5}\right) T_p(s)\cancelto{1}{T_s(s)}$$

$$C(s) = \frac{1}{C_4 s} + R_4.$$

When $C_4 \to \infty$, the controller simplifies to $C(s) = R_4$.

The Bode plot of $P(s)$ is shown in Figure 1. Here, the markers are placed on a candidate

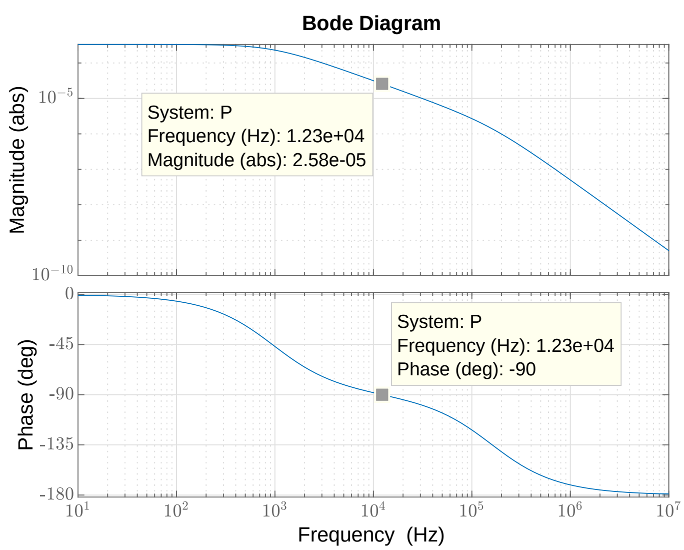


Figure 1: Bode Plot of $P(s)$.

crossover frequency 12.3 kHz for $\phi_m \geq 90°$. Since $|P(j\omega)|_{\omega=12.3\,\text{kHz}} = 2.58\times 10^{-5}\,\Omega^{-1}$, the required proportional gain for the loop to achieve unity-gain crossover at 12.3 kHz is

$$R_4 = \frac{1}{2.58 \times 10^{-5}\,\Omega^{-1}} \approx 39\,\mathrm{k}\Omega.$$

The Bode plot of the loop transfer function with proportional control is shown in Figure 2. Here, the crossover frequency ω_c and phase margin ϕ_m are

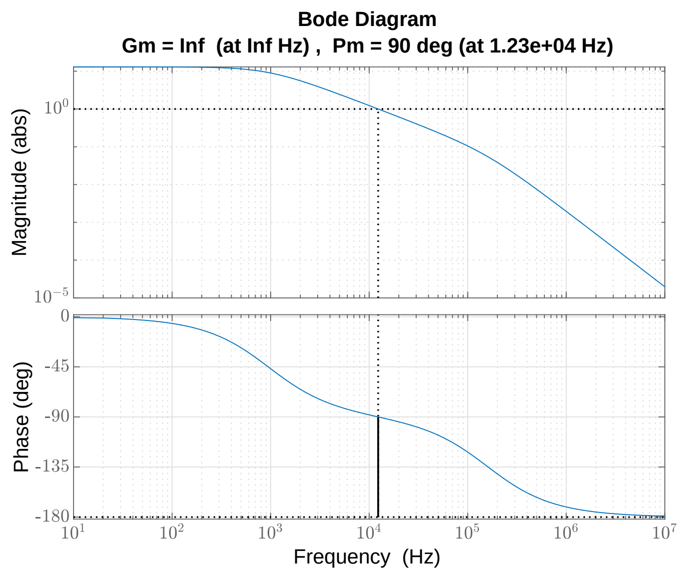


Figure 2: Bode plot of $L(s)$ with P control ($\omega_c = 12.3\,\mathrm{kHz}, \phi_m = 90°$).

(c) Now, we introduce an integral action by assigning a value for C_4. The controller can be re-written as

$$C(s) = R_4 \left(\frac{1}{R_4 C_4 s} + 1 \right)$$

to explicitly show the PI control break frequency $\omega_{\mathrm{PI}} = \frac{1}{R_4 C_4}$. As a initial guess, we can think of placing the break frequency at around 1 kHz to change the loop shape in Figure 2 to a desired one.

$$\frac{1}{R_4 C_4} = 6.28 \times 10^3\,\mathrm{rad/s} \qquad\qquad C_4 = \frac{1}{R_4} \frac{1}{6.28 \times 10^3\,\mathrm{rad/s}} \approx 4\,\mathrm{nF}.$$

Decreasing C_4 will push the break frequency higher, thereby increasing the loop gain at $\omega < \omega_{\mathrm{PI}}$, but at the expense of decreasing phase margin. The capacitance value that satisfies $\phi_m \geq 85°$ turns out to be $C_4 = 4\,\mathrm{nF}$. The Bode plot of $L(s)$ with PI control is shown in Figure 3 Note that the crossover frequency is not changed that much.

(d) Finally, we account for $T_s(s)$

$$T_s(s) = \frac{L_s(s)}{1 + L_s(s)} \qquad L_s(s) = \frac{R_3 R_5}{R_3 Z_4 + Z_4 R_5 + R_5 R_3} A_s(s) \qquad Z_4 = \frac{1}{C_4 s} + R_4.$$

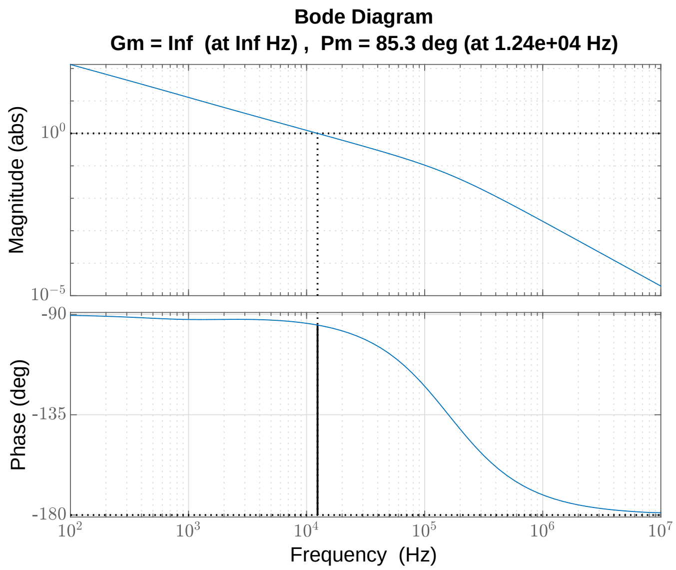


Figure 3: Bode plot of $L(s)$ with PI control ($\omega_c = 12.4\,\text{kHz}, \phi_m = 85.3°$).

The Bode plot of $T_s(s)$ is shown in Figure 4. Here, the -3 dB bandwidth turns out to be 190 kHz, which is about a decade above $\omega_c = 12.4\,\text{kHz}$. Therefore, we can expect that it would not change our designed crossover frequency that much but decrease the phase margin by some degrees.

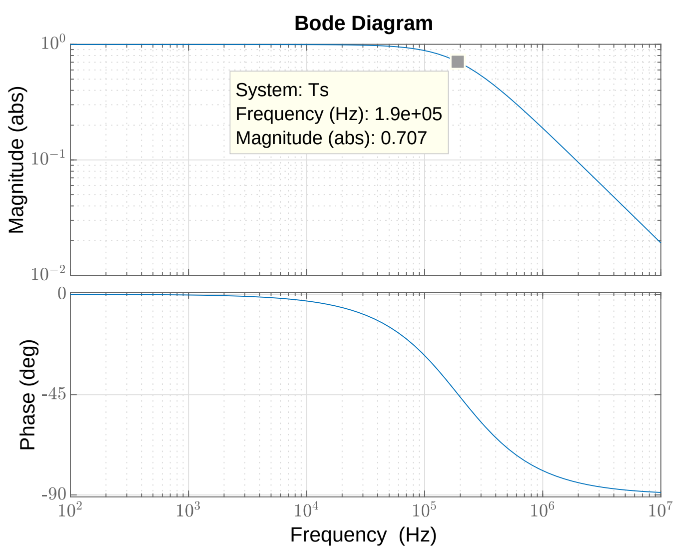


Figure 4: Bode plot of $T_s(s)$.

The Bode plot of $L(s)$ accounting for $T_s(s)$ is shown in Figure 5.

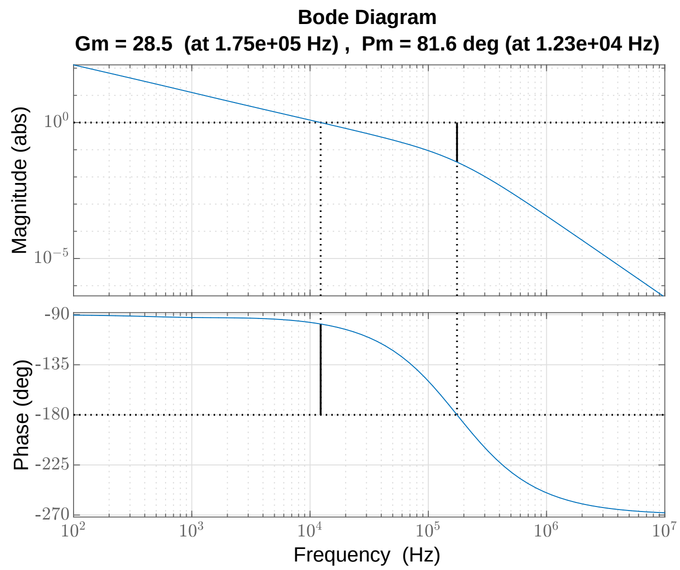


Figure 5: Bode plot of $L(s)$ with $T_s(s)$ accounted for ($\omega_c = 12.3\,\mathrm{kHz}, \phi_m = 81.6°$).