MECH 364: Mechanical Vibrations
Midterm Examination 3

Time: 45 minutes   23rd November 2011   Maximum Available Mark: 20

Please read the question carefully.

Q1.

a) Explain in two sentences or less the working principle involved in the design of an isolation system? **(3 marks)**

b) An electronic control unit, of mass $m = 2$ kg located in the stores pod of a space mission shown below in Fig.(1), needs to be isolated from vibration inputs originating at the base in the form of acceleration $\ddot{x}$ as shown. The acceleration input $\ddot{x}$ is a *broad band* random vibration within the range 10 Hz and 1 kHz. A damping ratio $\zeta = 0.1$ has been chosen to limit the maximum relative displacement (with respect to the stores pod) amplitude around *resonance* within reasonable limits. Design the isolator stiffness such that the maximum transmitted displacement ratio never exceeds 0.6 in the *entire* input frequency range. *Why is displacement transmission ratio preferred here?* You may use the formula $TR_d = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$, $r = \frac{\omega}{\omega_n}$ **(14 marks)**

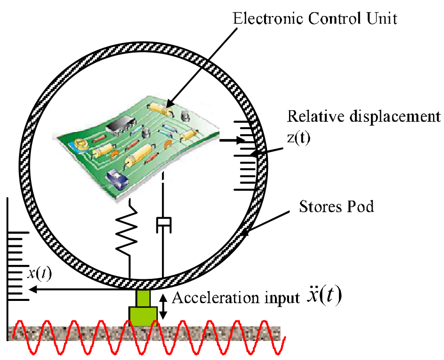

Figure 1: Figure for part b) .

b) Is the acceleration transmission ratio (defined as the ratio of transmitted acceleration amplitude to the applied acceleration amplitude) same as displacement transmission ratio at any *steady* operating frequency ω? Explain your answer in less than 5 sentences. **(3 marks)**

All the Best!

SOLUTION

Q1)

a) THE MAIN WORKING PRINCIPLE INVOLVED IN THE DESIGN OF AN ISOLATION SYSTEM IS TO SLOW DOWN THE DYNAMICS OF COMBINED SYSTEM (ISOLATOR + ORIGINAL SYSTEM) BY REDUCING ITS NATURAL FREQUENCY ω_n RELATIVE TO FORCING FREQUENCY ω SUCH THAT $\frac{\omega}{\omega_n} = r > \sqrt{2}$ AND $TR < 1$.

b) GIVEN : $10 \times 2\pi \leq \omega \leq 1000 \times 2\pi \Rightarrow 20\pi \leq \omega \leq 2000\pi$

$$\zeta = 0.1 \; ; \; m = 2\,kg$$

$$TR_d = 0.6 = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

SQUANIG BOTH SIDES

$$(0.6)^2 = \frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}$$

$$(0.6)^2\left[(1-r^2)^2+(2\zeta r)^2\right] = 1+(2\zeta r)^2$$

CALL $r^2 \equiv x$

$$(0.6)^2\left[(1-x)^2+4\zeta^2 x\right] = 1+4\zeta^2 x$$

$$\Rightarrow (0.6)^2 x^2 + x\left[-2(0.6)^2+(0.6)^2 4\zeta^2 - 4\zeta^2\right] + (0.6)^2 - 1 = 0$$

$$\Rightarrow 0.36x^2 - 0.7456\,x - 0.64 = 0$$

$$\Rightarrow x = \frac{0.7456 \pm \sqrt{(0.7456)^2+4 \times 0.64 \times 0.36}}{2 \times 0.36}$$

SOLUTION TO A QUADRATIC EQUATION

$\Rightarrow x = 2.7238$ or $x = -0.6527$ (IGNORE)

$\Rightarrow x = r^2 = 2.7338 \Rightarrow r = \frac{\omega}{\omega_n} = \sqrt{2.7338} = 1.6504$

$\therefore K = m\omega_n^2 = m\left(\frac{\omega}{r}\right)^2 = 2\left(\frac{20\pi}{1.6504}\right)^2 = 2.9 \times 10^3 \text{ N/m}$

FOR ω = LOWER END OF FREQUENCY

$= 20\pi$

CHECK TR_d FOR $\omega = \omega_{upper} = 2000\pi$

$r = \frac{\omega}{\omega_n} = \frac{2000\pi}{\sqrt{\frac{2.9 \times 10^3}{2}}} = 165.0392 \quad \because \omega_n = \sqrt{\frac{K}{m}}$

$$[TR_d]_{\omega = 2000\pi} = \frac{\sqrt{1 + (2 \times 0.1 \times 165.039)^2}}{\sqrt{(1 - 165.039^2)^2 + (2 \times 0.1 \times 165.039)^2}}$$

$= 0.0012 < 0.6$ OK ✓

$\boxed{\text{So CHOOSE } K = 2.9 \text{ kN/m}}$

c) TR_d IS THE SAME AS TR ACCELERATION SINCE IN THE STEADY STATE: $x = X \cos\omega t$, $y = Y \cos\omega t$ $\Rightarrow TR_d = \left|\frac{Y}{X}\right|$

$\ddot{x} = -X\omega^2 \cos\omega t$, $\ddot{y} = -Y\omega^2 \cos\omega t$ $\Rightarrow TR_a = \left|\frac{-Y\omega^2}{-X\omega^2}\right| = TR_d$

—— THE END ——