

# Lesson 5-4 – Cashflow Equivalence

# Take a step back

- The last few videos we've looked at why we value money differently over time, and some simple tools for calculating the value of different cashflows at different times. Why do we care?
- Go back to our earlier question – would you rather have \$10,000 now, or \$10,000 five years from now?
- Comparing the values of these cashflows over time allows us to determine when they have the same value, or are equivalent to us.
  - Would I rather have \$10,000 today, or \$14,000 five years from now? Honestly, I could go either way – they're both about the same to me
- If we can determine when two cashflows are equivalent, we can also determine when they are not – i.e. one cashflow is more valuable than the other

# Equivalence

- Equivalence with respect to the “time value of money” implies that a sum of money in one time period may have the same “value” to a different sum in another time period with respect to an interest rate.
- Example: \$1000 now is equivalent to:
  - \$1100 one year from now at 10% per year
  - \$1050 one year from now at 5% per year
  - \$1210 two years from now at 10% per year
  - \$1102 two years from now at 5% per year

## Equivalence Continued...

- **Equivalence is dependent on interest rate**
- Equivalence is useful when:
  - There are cash flows (positive and/or negative) over “ $n$ ” time periods that need to be compared
  - There are alternative comparisons of multiple cash flows
  - Equivalent cash flows are considered to have the same value.

The 4 plans discussed on the next slides are considered equivalent because the principle amount and interest rates are identical, as each plan repays the same amount of money (that has the same value today), at the same interest rate.

# Repaying a Debt

- Four plans for Loan Repayment of the same principle amount (See pgs. 78-80):

| <b><i>Plan</i></b> | <b><i>Repay Principal</i></b> | <b><i>Repay Interest</i></b> | <b><i>Interest Amount Paid</i></b> |
|--------------------|-------------------------------|------------------------------|------------------------------------|
| 1                  | Equal annual amounts          | On the unpaid balance        | Declines                           |
| 2                  | At the end of the loan        | On the unpaid balance        | Constant                           |
| 3                  | Equal annual amounts          |                              | Declines at an increasing rate     |
| 4                  | At the end of the loan        | At the end of the loan       | Grows at an increasing rate        |

# Repay a Debt – Plan 4

- Borrow \$5,000 at time zero
- 8% interest, five year term
- No payments until end of term

- How much do we pay at the end of the term?

- $F = P(F/P, i, n) = \$5000(F/P, 8\%, 5), = \$5,000 * 1.469 = \$7,345$
- $F = P(1+i)^n = \$5000(1+0.08)^5 = \$7,347$

8%

| <i>n</i> | Single Payment  |   |
|----------|---|---|
|          | Compound<br>Amount<br>Factor<br>Find <i>F</i><br>Given <i>P</i><br><i>F/P</i> | Present<br>Worth<br>Factor<br>Find <i>P</i><br>Given <i>F</i><br><i>P/F</i> |
| 1        | 1.080   | .9259   |
| 2        | 1.166   | .8573   |
| 3        | 1.260   | .7938   |
| 4        | 1.360   | .7350   |
| 5        | 1.469   | .6806   |

TABLE 3-1

## Four Plans for Repayment of \$5,000 in Five Years with Interest at 8%

| (a)  | (b)                                    | (c)   | (d)  | (e)                  | (f)                           |
|--|--|---|--|----------------------|-------------------------------|
| Year   | Amount Owed<br>at Beginning of<br>Year | Interest Owed for<br>That Year,<br>$8\% \times (b)$ | Total Owed at<br>End of Year,<br>$(b) + (c)$ | Principal<br>Payment | Total End-of-<br>Year Payment |
| <b>Plan 1:</b> Constant \$1,000 principal payment plus interest due.                   |  |   |  |                      |                               |
| 1  | \$5,000                                | \$ 400  | \$5,400                                      | \$1,000              | \$1,400                       |
| 2  | 4,000                                  | 320   | 4,320  | 1,000                | 1,320                         |
| 3  | 3,000                                  | 240   | 3,240  | 1,000                | 1,240                         |
| 4  | 2,000                                  | 160   | 2,160  | 1,000                | 1,160                         |
| 5  | 1,000                                  | 80  | 1,080  | 1,000                | 1,080                         |
|  |  | <u>\$1,200</u>                                      |  | <u>\$5,000</u>       | <u>\$6,200</u>                |
| <b>Plan 2:</b> Annual interest payment due and principal payment at end of five years. |  |   |  |                      |                               |
| 1  | \$5,000                                | \$ 400  | \$5,400                                      | \$ 0                 | \$ 400                        |
| 2  | 5,000                                  | 400   | 5,400  | 0                    | 400                           |
| 3  | 5,000                                  | 400   | 5,400  | 0                    | 400                           |
| 4  | 5,000                                  | 400   | 5,400  | 0                    | 400                           |
| 5  | 5,000                                  | 400   | 5,400  | 5,000                | 5,400                         |
|  |  | <u>\$2,000</u>                                      |  | <u>\$5,000</u>       | <u>\$7,000</u>                |
| <b>Plan 3:</b> Constant annual payments.   |  |   |  |                      |                               |
| 1  | \$5,000                                | \$ 400  | \$5,400                                      | \$ 852               | \$1,252*                      |
| 2  | 4,148                                  | 331   | 4,479  | 921                  | 1,252                         |
| 3  | 3,227                                  | 258   | 3,485  | 994                  | 1,252                         |
| 4  | 2,233                                  | 178   | 2,411  | 1,074                | 1,252                         |
| 5  | 1,159                                  | 93  | 1,252  | 1,159                | 1,252                         |
|  |  | <u>\$1,260</u>                                      |  | <u>\$5,000</u>       | <u>\$6,260</u>                |
| <b>Plan 4:</b> All payments at end of five years.                                      |  |   |  |                      |                               |
| 1  | \$5,000                                | \$ 400  | \$5,400                                      | \$ 0                 | \$ 0                          |
| 2  | 5,400                                  | 432   | 5,832  | 0                    | 0                             |
| 3  | 5,832                                  | 467   | 6,299  | 0                    | 0                             |
| 4  | 6,299                                  | 504   | 6,803  | 0                    | 0                             |
| 5  | 6,803                                  | 544   | 7,347  | 5,000                | 7,347                         |
|  |  | <u>\$2,347</u>                                      |  | <u>\$5,000</u>       | <u>\$7,347</u>                |

# Repay a Debt – Present Values

|              |                    |                                 |                      |                      |
|--------------|--------------------|---------------------------------|----------------------|----------------------|
|              |                    | <b>Borrow \$5,000 at time 0</b> |                      |                      |
|              |                    | <b>Interest Rate:</b>           | 8%                   |                      |
|              | Plan 1             |                                 | Plan 4               |                      |
| <b>Year</b>  | <b>EoY Payment</b> | <b>PV of Payment</b>            | <b>EoY Payment 4</b> | <b>PV of Payment</b> |
| 1            | \$1,400            | \$1,296                         | \$0                  | \$0                  |
| 2            | \$1,320            | \$1,132                         | \$0                  | \$0                  |
| 3            | \$1,240            | \$984                           | \$0                  | \$0                  |
| 4            | \$1,160            | \$853                           | \$0                  | \$0                  |
| 5            | \$1,080            | \$735                           | \$7,347              | \$5,000              |
|              |                    |                                 |                      |                      |
| <b>Total</b> | <b>\$6,200</b>     | <b>\$5,000</b>                  | <b>\$7,347</b>       | <b>\$5,000</b>       |



# Spreadsheet Example