MECH 420 SENSORS AND ACTUATORS
Solutions to Assignment 3

Sol-Problem 1 (Problem 3.3 from Textbook)-rev

(a) Spatial resolution = 2 mm

Element area (average) covered by each sensor $= 2 \times 2 \text{ mm}^2 = 4 \times 10^{-6} \text{ m}^2$

Note: Since the spacing of the sensor elements is given (2 mm), we assume that each sensor element covers half this distance on either side (approximately; strictly a circle, not a square, is a better approximation). Anyhow, we assume that any part of the area that is under the influence of the adjoining sensor element is not counted under the sensing area for a specific sensor element.

(b) Force resolution in each element = 0.01 N

$$\text{Load (pressure) resolution} = \frac{0.01}{4 \times 10^{-6}} \text{ N/m}^2 = 2.5 \times 10^3 \text{ N/m}^2$$

$$= 2500 \text{ N/m}^2$$

(c) Dynamic range $= 20 \log_{10} \dfrac{50}{0.01}$ dB = 74.0 dB

Sol-Problem 2 (Problem 3.6 from Textbook)

Consider the simple oscillator model with the usual notation.

Rise time = $\dfrac{\pi - \phi}{\omega d}$, where $\phi = \cos^{-1} \zeta$

For low ζ we have $\phi \simeq \dfrac{\pi}{2}$

Also $1/10$ of the damped natural frequency may be used to represent the device bandwidth. Then, with the given units, we have $T_r \times 10^{-9} = \dfrac{\pi/2}{10 \times f_b \times 2\pi \times 10^6}$

or, $T_r f_b = 25 \cdot 0$

Sol-Problem 3 (Problem 3.7 from Textbook)

Limit cycles, saturation, deadbands, frequency creation, and jump phenomenon.

$$\frac{dy}{dt} = u^3(t) = \left(a_1 \sin \omega_1 t + a_2 \sin \omega_2 t\right)^3 \quad \rightarrow \quad y = \int \left(a_1 \sin \omega_1 t + a_2 \sin \omega_2 t\right)^3 dt$$

Integrate using $\int \sin^3 t \; dt = -\frac{1}{3}\cos t\left(\sin^2 t + 2\right) + C$ and apply:

$\sin A \cos B = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$ and $\sin^2 A = \frac{1}{2}(1-\cos 2A)$

In the result, notice the frequency components other than ω_1 and ω_2 that are present. They are the new frequencies that are created.

Sol-Problem 4 (Problem 3.8 from Textbook)

The input-output behavior of the device is sketched in Figure S3.8(a), using the following MATLAB script:

```
% static nonlinear device
u=[]; y=[];  % declare vectors
p=1.0; c=0.2; u=0; y=0.2; %initialize variables
for i=1:100
a=0.01*i; % input
u(end+1)=a; % store input
y(end+1)=p*a^2+c; % output
end
plot(u,y,'-')
```

The sine response of the device is sketched in Figure S3.8(b), using the following MATLAB script:

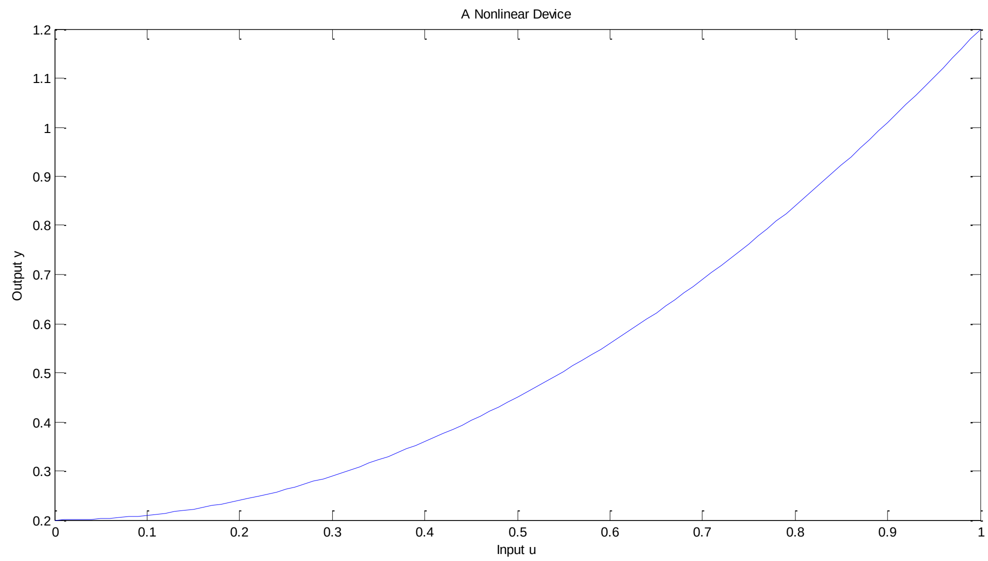
```
% Sine response of nonlinear device
t=[]; u=[]; y=[];  % declare vectors
p=1.0; c=0.2; t=0; u=0; y=c; %initialize variables
for i=1:100
a=0.1*i; % time
t(end+1)=a; % store time
u(end+1)=sin(a); % input
y(end+1)=p*(sin(a))^2+c; % output
end
plot(t,u,'--',t,y,'-')
```

The device can be linearized simply by changing the scale (i.e., coordinate transformation) as follows:

$$y' = y - c \quad \leftarrow \text{ offsetting}$$
$$u' = u^2$$

This gives the linear model $y' = pu'$ without sacrificing accuracy or operating range.

(a)



(b)

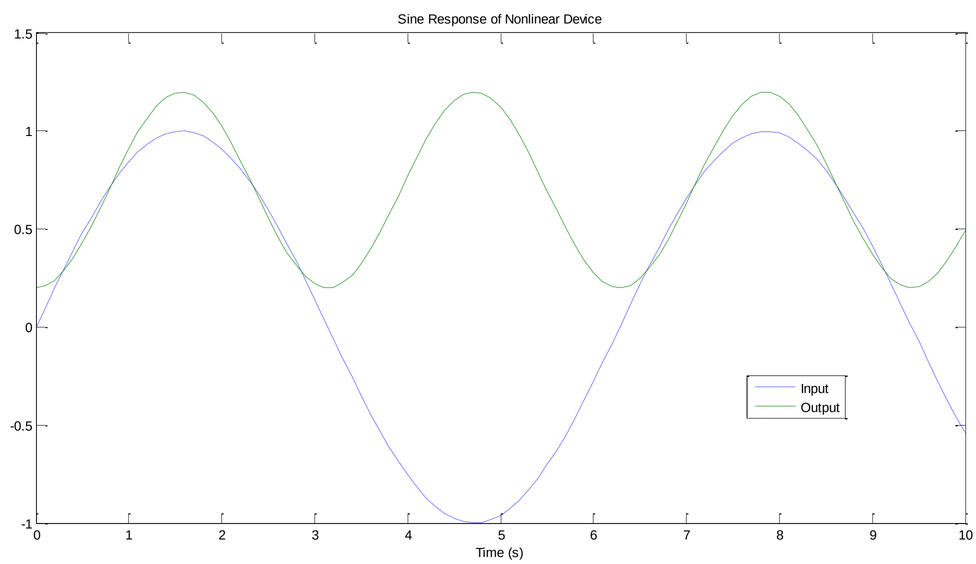


Figure S3.8: (a) Input-output behavior of a static nonlinear device; (b) Sine response.

<u>Sol-Problem 5 (Problem 3.17 from Textbook)</u>

(a) Operating bandwidth f_b = significant frequency content (or maximum frequency) of the operation.
Control bandwidth f_c should be $\geq f_b$.

Control cycle rate (rate at which the control action is generated) $f_a = 2\, f_c \geq 2\, f_b$ assuming that the analog circuitry can accommodate the digital control action generated by the digital controller.
Response sampling rate f_s should be $\geq 2\, f_a \geq 4\, f_b$.
Now refer to Figure S3.17.

(i) $\underline{f_b = f_n / 10}$

- Good, trouble-free operation is possible and the link will move in phase with the drive torque (link will behave like a rigid arm)
- A sensor at the motor (joint) alone would be adequate for control
- A simple controller (e.g., proportional-integral-derivative or PID servo) would be adequate
- Low sampling rate (~ 0.4 f_n) and control cycle (action) rate (~ 0.2 f_n)

(ii) $\underline{f_b \approx f_n}$

- Resonant conditions cause large-amplitude oscillations or vibrations (instability problems) that have a phase difference (about 90°) with respect to the drive torque
- In addition to a joint sensor (at the motor), a motion sensor at the end effector would be needed to accurately control the payload in the presence of oscillations (*Note*: Since only the first flexible mode of the arm is considered, there will not be a "node point" along the arm, except at the joint. Hence, a sensor may be mounted anywhere along the arm, in addition to the one at the joint, in order to know the dynamics of the arm)
- A sophisticated controller that guarantees stability (e.g., linear quadratic Gaussian or LQG control, modal control) would be needed
- High sampling rate for sensor signals (> 4 f_n) and control cycle rate (> 2 f_n)

(iii) $\underline{f_b = 2\, f_n}$

- The robot arm will behave like a high-impedance (mechanical) device and will respond with a phase difference (about 180°) with respect to the drive torque (Relatively high drive torque would be needed for a given response amplitude)
- A motion sensor at the payload (end effector) would be needed (in addition to a sensor at the joint motor), particularly to accommodate the phase difference between the drive torque and the response
- A powerful control action would be needed, with a fast and effective controller
- High sampling rate (~ 8 f_n) and control action cycle rate (~ 4 f_n)

(b) Mass per unit length $m = \dfrac{400}{10} = 40$ kg/m

$\lambda_1 l = 1.875$ with $l = 10$ m → $\lambda_1 = 0.1875$. Also, $EI = 8.25 \times 10^9$ N.m^2

Substitute values: $\omega_1 = (0.1875)^2 \sqrt{\dfrac{8.25\times10^9}{40}}$ rad/s $= 505.0$ rad/s

→ $f_n = 80$ Hz
We should operate at significantly below the resonance.
Suggested operating bandwidth $f_b = f_n / 5 = 16$ Hz.
Sampling rate f_s should be $> 4\, f_b = 64$ Hz.
A higher sampling rate would be preferable. We pick $f_s = 200$ Hz.

Then, control cycle (action) rate of f_a = 100 Hz would be possible.
Control bandwidth f_c = 50 Hz.
This assumes that the operating frequency range (flat region of the spectrum) of the analog hardware of the controller is at least 50 Hz.

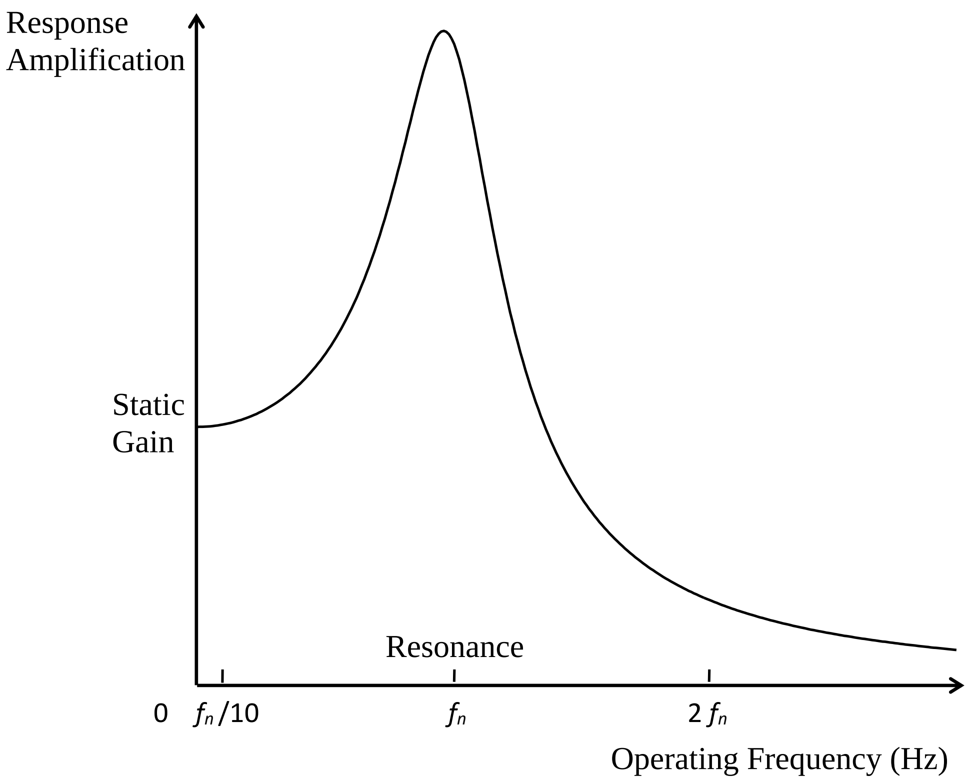


Figure S3.17: A typical frequency response curve.

Sol-Problem 6 (Problem 3.21 from Textbook)

(a) Loading distorts the acquired signal and changes its true value, due to dynamic (electrical., mechanical., etc.) effects of the acquiring device. At steady speed there is no inertia torque and so the torque exerted on the rotating (sensed) mechanical system by the tachometer is just its damping (electrical and mechanical) torque, at steady state. If the speed is transient, the inertia load of the tachometer will be present as well. This can be significant at high acceleration levels, particularly for heavy tachometers.

(b) Consider the rotor circuit shown in Figure S3.21. At steady state, L and C effects do not enter into the circuit. Under open circuit conditions, tachometer output signal = v_g. In the presence of an electrical load, the tachometer output signal is:

$$v_o = \frac{v_g}{(R_a + R_L)} \cdot R_L = \frac{2000}{(20+2000)}\, v_g \;\Rightarrow\; \text{Error} = v_g - \frac{2000}{(20+2000)}\, v_g$$

$\rightarrow \qquad \%\text{Error} = \left(1 - \frac{2000}{20+2000}\right) \times 100\% = \frac{20}{2020} \times 100\% = 0.99\%$

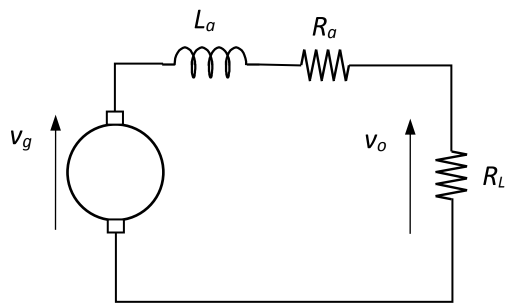


Figure S3.21: Tachometer electrical circuit.

(c) A higher load will be present under accelerating conditions, and a lower load under decelerating conditions, when passing through the same speed value.

Sol-Problem 7 (Problem 3.30 from Textbook)

(a) In the absolute error method, the absolute values of error in various subsystems are added to get an estimate of the overall error in the system. Since there are interactions between subsystems and since some errors can have opposite signs, this estimate is usually a conservative upperbound for the actual error.

In the SRSS method, an estimate for the system error is obtained as the square root of sum of squares of the subsystem error. This is not a conservative upper bound for the system error, and in extreme cases can underestimate the error.

The absolute error method is preferred when it is known that error contributions are of the same sign (and additive) and when underestimation of the system error can lead to very detrimental consequences. The SRSS error method is preferred when the subsystem error sources are known to be random and independent, and when a standard deviation type estimate is all that is needed to represent the error.

(b)

(i) $q = \left[1 + \frac{s}{s_o}\sin\frac{\pi}{2}\left(\frac{f}{f_o + f}\right)\right] / (1 + \beta V / Q)$

Differential:

$$\delta q = \frac{\partial q}{\partial V}\delta V + \frac{\partial q}{\partial Q}\delta Q + \frac{\partial q}{\partial s}\delta s + \frac{\partial q}{\partial f}\delta f = V\frac{\partial q}{\partial V}\frac{\delta V}{V} + Q\frac{\partial q}{\partial Q}\frac{\delta Q}{Q} + s\frac{\partial q}{\partial S}\frac{\delta s}{s} + f\frac{\partial q}{\partial f}\frac{\delta f}{f}$$

Partial differentiation: $\frac{\partial q}{\partial V} = -\frac{\left[1 + \frac{s}{s_o}\sin\frac{\pi}{2}\left(\frac{f}{f_o + f}\right)\right]}{(1 + \beta V / Q)^2}\frac{\beta}{Q}$; $\frac{\partial q}{\partial Q} = \frac{\left[1 + \frac{s}{s_o}\sin\frac{\pi}{2}\left(\frac{f}{f_o + f}\right)\right]}{(1 + \beta V / Q)^2}\frac{\beta V}{Q^2}$;

$$\frac{\partial q}{\partial s} = \frac{1}{s_o}\frac{\sin\frac{\pi}{2}\left(\frac{f}{f_o + f}\right)}{(1+\beta V/Q)};$$

$$\frac{\partial q}{\partial f} = \frac{s}{s_o}\frac{\cos\frac{\pi}{2}\left(\frac{f}{f_o + f}\right)}{(1+\beta V/Q)}\frac{\pi}{2}\frac{(f_o + f - f)}{(f_o + f)^2} = \frac{\pi s f_o}{2s_o(f_o + f)^2}\frac{\cos\frac{\pi}{2}\left(\frac{f}{f_o + f}\right)}{(1+\beta V/Q)}$$

Now, at the operating point $\left(Q = \beta V, \quad f = f_o, \quad s = s_o\right)$ we have:

$$V\frac{\partial q}{\partial V} = -\frac{1}{4}\left[1+\sin\frac{\pi}{4}\right]; \quad Q\frac{\partial q}{\partial Q} = \frac{1}{4}\left[1+\sin\frac{\pi}{4}\right]; \quad s\frac{\partial q}{\partial s} = \frac{1}{2}\sin\frac{\pi}{4}; \quad f\frac{\partial q}{\partial f} = \frac{\pi}{16}\cos\frac{\pi}{4}$$

These are substituted in the error equation to give the absolute error estimate of q:

$$e_q = \frac{1}{4}\left(1+\sin\frac{\pi}{4}\right)e_v + \frac{1}{4}\left(1+\sin\frac{\pi}{4}\right)e_Q + \frac{1}{2}\left(\sin\frac{\pi}{4}\right)e_s + \frac{\pi}{16}\left(\cos\frac{\pi}{4}\right)e_f$$

in which, errors in the measurements of *V*, *Q*, *s*, and *f* are $\pm e_v$, $\pm e_Q$, $\pm e_s$, and $\pm e_f$, respectively.

(ii) For equal contribution of individual errors, we must have

$$\frac{1}{4}\left(1+\sin\frac{\pi}{4}\right)e_v = \frac{1}{4}\left(1+\sin\frac{\pi}{4}\right)e_Q = \frac{1}{2}\left(\sin\frac{\pi}{4}\right)e_s = \frac{\pi}{16}\left(\cos\frac{\pi}{4}\right)e_f$$

Then for $e_v = \pm 1\%$, we get: $e_Q = \pm 1\%$, $e_s = \pm 1.2\%$, $e_f = \pm 3.1\%$