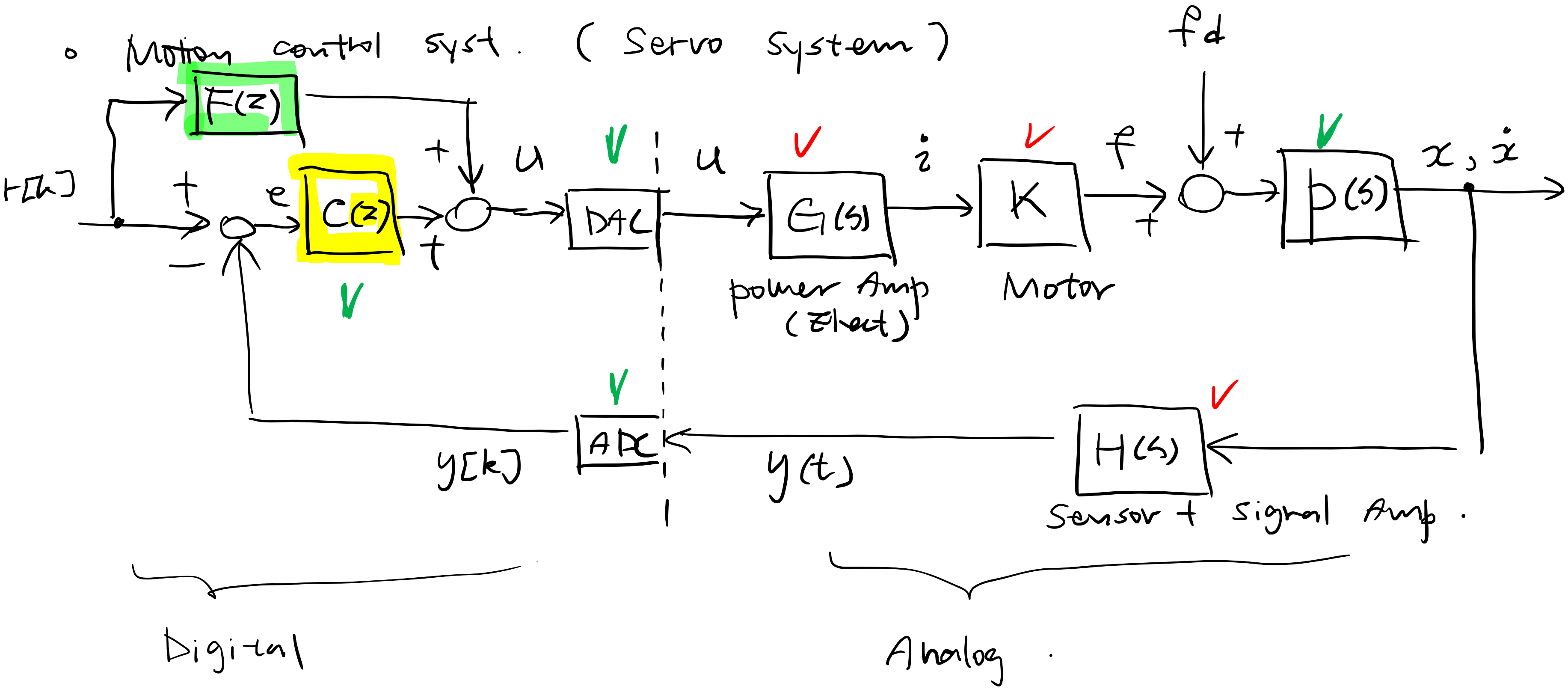
L13 – Second-order System

- Motion control syst. (Servo System)



- Covered: control, motor, analog circuits.

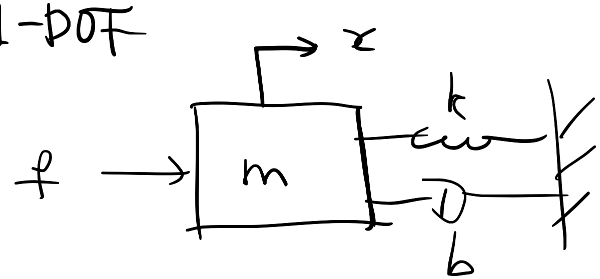
- <u>Remark</u>
  - power Amp : voltage. , Transcond.
  - Signal Amp : Diff Amp , In-Amp

" Digital Alternatives".

- power : Switching Amp (Class-D). → will cover
- signal : Digital sensors. Inductance-to-Digital conv LDC from TI.

- 2nd-order System. (pz map vs. step vs. Bode)

1-DOF



$$m\ddot{x} = \Sigma f = f - kx - b\dot{x}$$

$$\Rightarrow m\ddot{x} + b\dot{x} + kx = f$$

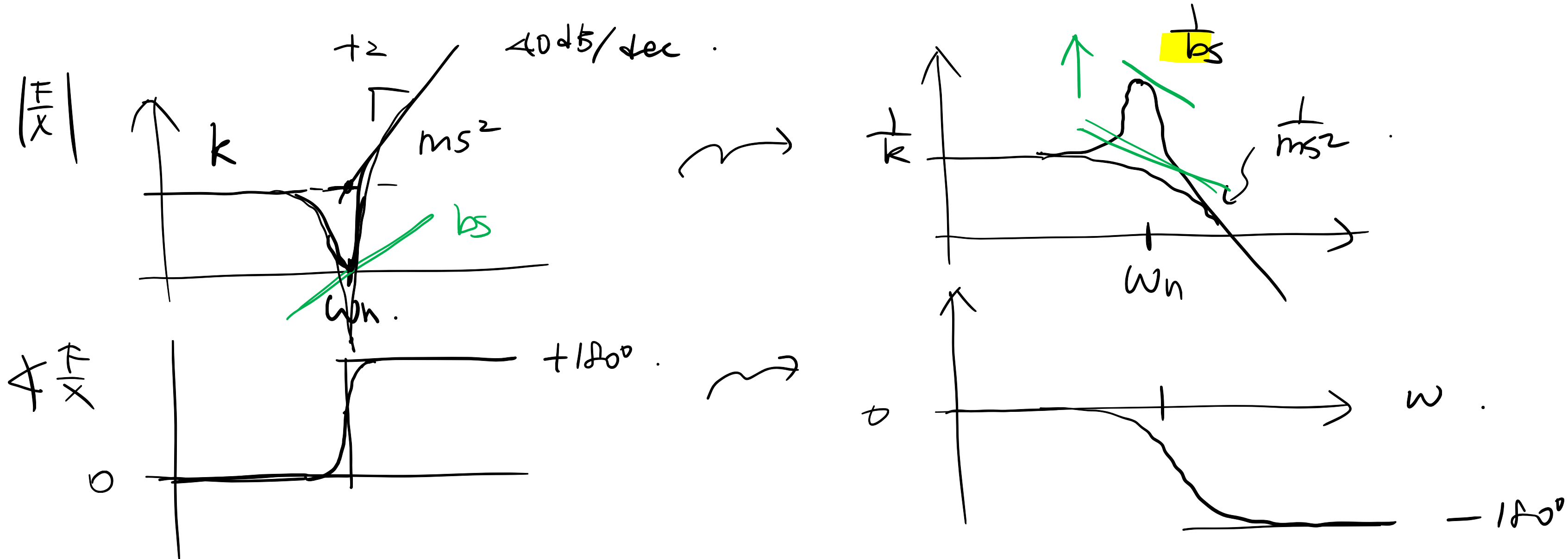
$$\Rightarrow \underbrace{(ms^2+bs+k)}_{\text{"Dynamic stiffness"}} X = F.$$

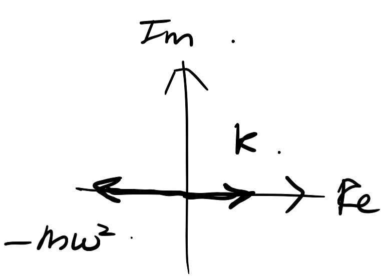
$$\frac{F}{X} = ms^2 + bs + k \quad \text{"Stiffness"}$$

$$\frac{X}{F} = \frac{1}{ms^2 + bs + k} \quad \text{"Compliance"}$$

$\frac{F}{\dot{X}}$: Impedance.

$\frac{\dot{X}}{F}$: Admittance.





$$\frac{1}{ms^2 + bs + k}$$

$$\text{poles} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$$b_c \triangleq 2\sqrt{mk} \quad \text{"Critical damping"}$$

$$\omega_n \triangleq \sqrt{\frac{k}{m}}$$

* "Natural freq"

$$\zeta \triangleq \frac{b}{b_c}$$

"*Damping Ratio" [unitless].

$$P(s) = \frac{1}{ms^2+bs+k} = \frac{1}{m}\frac{1}{s^2+\frac{b}{m}s+\frac{k}{m}}.$$

"Evans Form"

$$= \frac{1}{m}\left[\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}\right] \rightarrow s^2+2\omega_n s+\omega_n^2.$$

if $\zeta = 0.$ $\rightarrow$ Resonance. V

if $\zeta = 1$ $\rightarrow$ critical damping. V

$$P(s) = \frac{1}{ms^2+bs+k} = \frac{1}{k}\left[\frac{1}{\frac{m}{k}s^2+\frac{b}{k}s+1}\right]$$

$$= \underbrace{\frac{1}{k}}_{\text{DC gain}}\underbrace{\left(\frac{1}{\left(\frac{s}{\omega_n}\right)^2+2\zeta\left(\frac{s}{\omega_n}\right)+1}\right)}_{\text{Dynamics.}}.$$

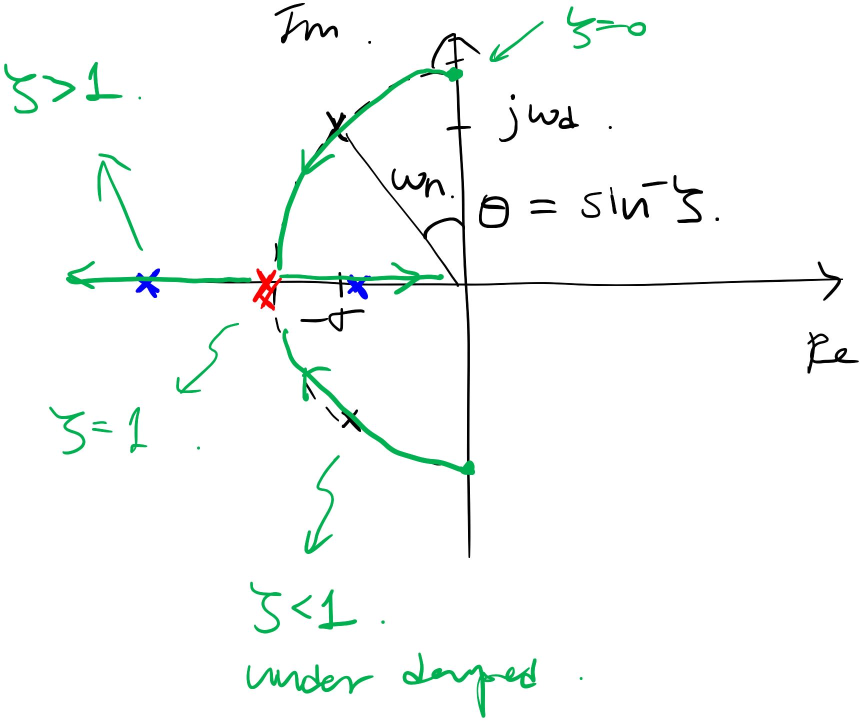
"Bode form"

- pole-zero map. (ω_n, ζ)

$$f(s) = s^2+2\zeta\omega_n \cdot s+\omega_n^2.$$

$$\text{Roots} = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2-\omega_n^2}$$

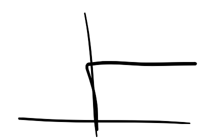
$$= -\underbrace{\zeta\omega_n}_{\sigma} \pm j\,\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}.$$



- $\omega_n = \sqrt{\frac{k}{m}}$. "Nat. freq"
- $\zeta = \frac{b}{2\sqrt{mk}}$.
- $\sigma = \zeta\omega_n$ "decay rate"
- $\omega_d = \omega_n\underbrace{\sqrt{1-\zeta^2}}$ "Damped Nat. freq"

$\zeta^2 \ll 1$

$\omega_n \simeq \omega_d$.



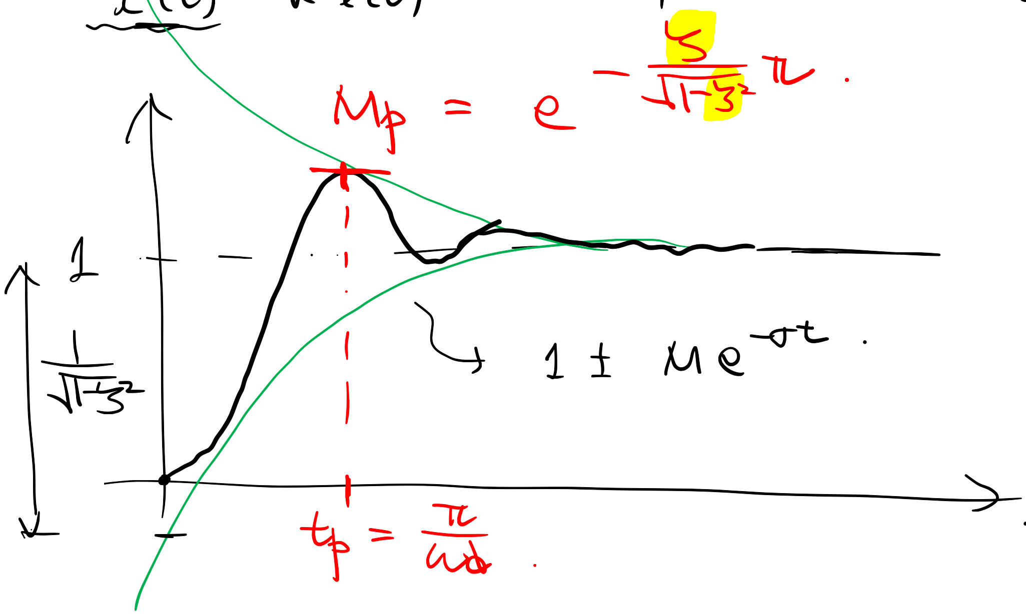
- Step Resp. $(\omega_n, \zeta) \longrightarrow (\omega_d, \sigma)$.

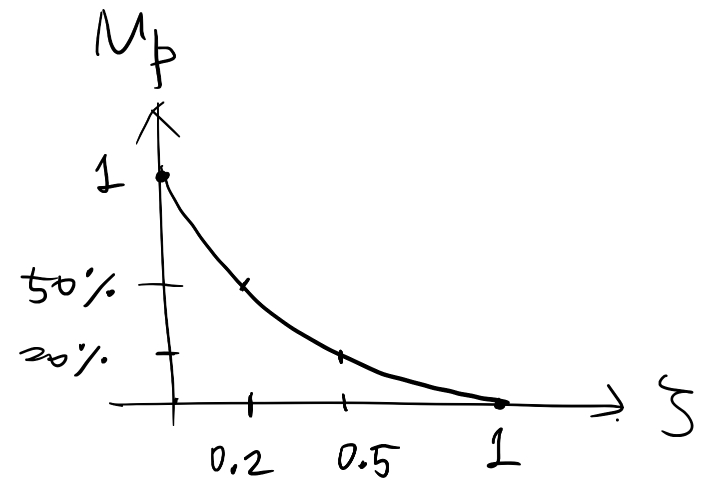
$$x(t) = \frac{1}{k}\left[1 - \underbrace{e^{-\sigma t}}_{\text{Envelope}}\underbrace{\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right)}_{\text{Sinusoid}}\right]u(t)$$

$$= \frac{1}{k}\left[1 - e^{-\sigma t} M \cos(\omega_d t + \phi)\right] u(t)$$

$$\begin{cases} M = \sqrt{1+\frac{\sigma^2}{\omega_d^2}} = \frac{1}{\sqrt{1-\zeta^2}} \\ \phi = \tan^{-1}\left(\frac{\sigma}{\omega_d}\right) = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right). \end{cases}$$

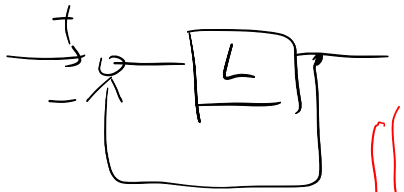
$\hat{x}(t) = k\, x(t)$





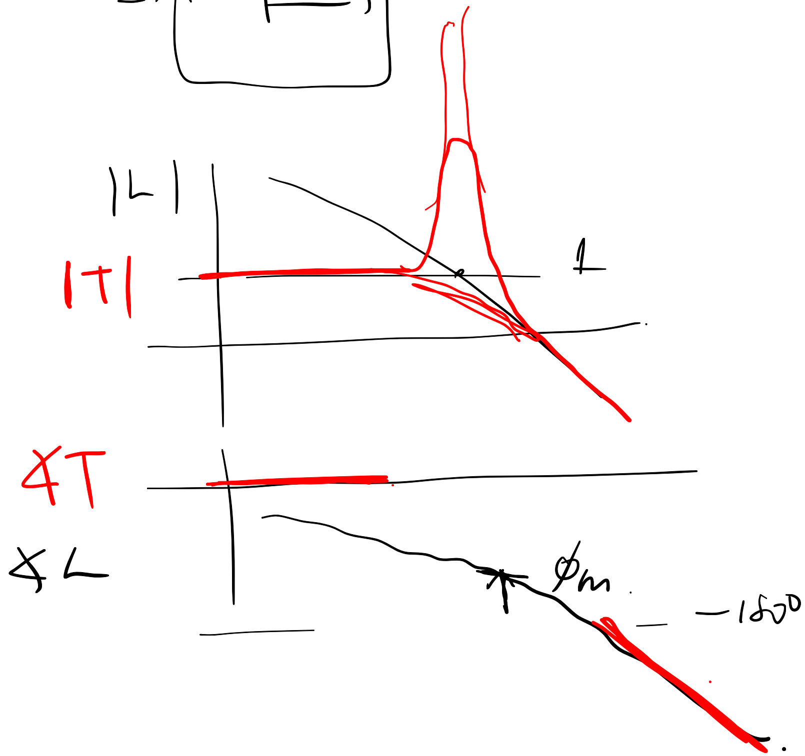
$$\dot{x}(t)\Big|_{t=t_p} = 0. \Rightarrow t_p = \frac{\pi}{\omega_d}.$$

$$\hat{x}(t) = 1 + \underbrace{e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}}_{\text{"Overshoot"}}. \quad [\text{unitless}].$$



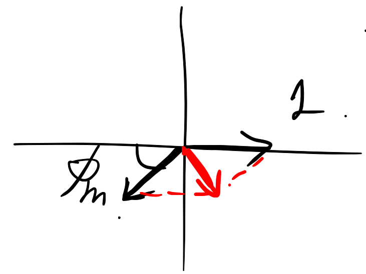
$L(s) \longrightarrow T(s) = \dfrac{L}{1+L}$.

$$T \approx \begin{cases} 1 & |L| \gg 1 \\ L & |L| \ll 1 \end{cases}$$

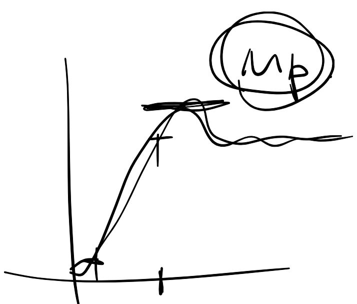


$\longrightarrow$

$|L| \simeq 1$



$$T = \left|\frac{L}{1+L}\right| = \frac{|L|}{|1+L|} = \frac{1}{|1+L|}$$



$\zeta \sim \phi_m$.

$\zeta \simeq \dfrac{\phi_m[\deg]}{100}$. ✓

$t_r \sim \omega_c \simeq \omega_h$.