you do if you weren't afraid?" I try to look at this daily, and I aim to do something fearless every day. What are you afraid of? Don't let it stop you!

- *Redos come with the territory:* If you don't like the way it turned out—do it again!
- *Criticism makes us better:* By exposing our work to others, and by externalizing it so we can inspect it ourselves, we gain unique perspective and insight and develop new and improved plans for the next version.
- *Be willing to be disagreeable.* There is a negative correlation between the level of creativity and "agreeableness," so those who are the most disagreeable tend to be most creative. Looking back at the few times when I found something novel, it was because I challenged the existing answers. So I believe the creative way is advanced whenever we strip a problem back to its roots and question our own assumptions (along with assumptions suggested by others); then repeat!

{ 4 }

chunking and avoiding illusions of competence:

The Keys to Becoming an "Equation Whisperer"

Solomon Shereshevsky first came to his boss's attention because he was lazy. Or so his boss thought.

Solomon was a journalist. At that time, in the mid-1920s in the Soviet Union, being a journalist meant reporting what you were told, no more, no less. Daily assignments were given out—detailing whom to see, at what address, and to obtain what information. The editor in charge began to notice that everyone took notes. Everyone, that is, except Solomon Shereshevsky. Curious, the editor asked Solomon what was going on.

Solomon was surprised—why should he take notes, he asked, when he could remember whatever he heard? With that, Solomon repeated part of the morning's lecture, word for word. What Solomon found surprising was that he thought *everyone* had a memory like his. Perfect. Indelible.[1]

Wouldn't you love to have the gift of such a memory?

Actually, you probably wouldn't. Because hand-in-hand with his extraordinary memory, Solomon had a problem. In this chapter, we'll be talking about precisely what that problem is—involving how *focus* links to both *understanding* and *memory*.

What Happens When You Focus Your Attention?

We learned in the last chapter about that irritating situation when you become stuck in one way of looking at a problem and can't step back to see easier, better ways—*Einstellung*. Focused attention, in other words, can often help solve problems, but it can also create problems by blocking our ability to see new solutions.

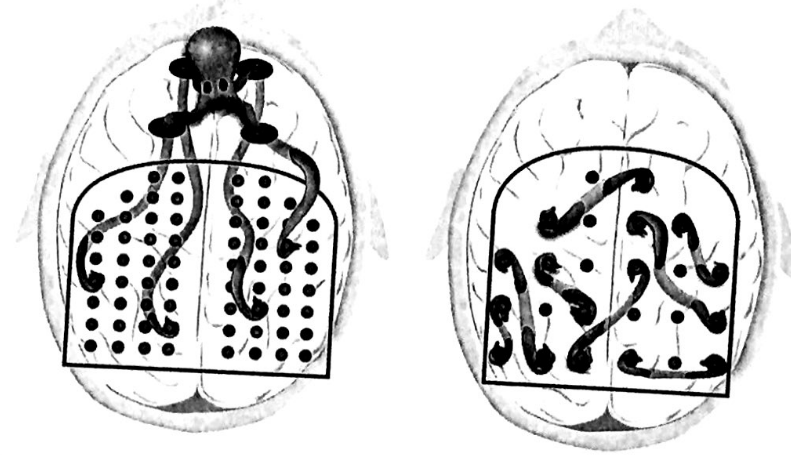
When you turn your attention to something, your attentional octopus stretches its neural tentacles to connect different parts of the brain. Are you focusing on a shape? If so, one tentacle of consciousness reaches from the thalamus back toward the occipital lobe, even as another tentacle reaches toward the wrinkled surface of the cortex. The result? A whispered sense of *roundness*.

Are you focusing instead on color? The attentional tentacle in the occipital lobe shifts slightly and a sense of *green* arises.

More tentacle connections. You conclude that you are looking at a particular type of apple—a Granny Smith. Yum!

Focusing your attention to connect parts of the brain is an important part of the focused mode of learning. Interestingly, when you are stressed, your attentional octopus begins to lose the ability to make some of those connections. This is why your brain doesn't seem to work right when you're angry, stressed, or afraid.[2]

Let's say you want to learn how to speak Spanish. If you're a child hanging around a Spanish-speaking household, learning Spanish is as natural as breathing. Your mother says "mama," and you parrot "mama" back to her. Your neurons fire and wire together in a shimmering mental loop, cementing the relationship in your

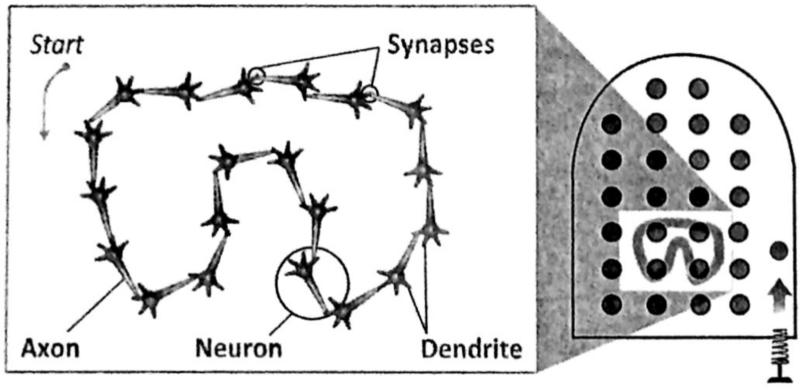


The octopus of your focused attention *(left)* reaches out through the four slots of your working memory to deliberately connect the neural bumpers of your tightly focused brain. The diffuse mode (right) has its bumpers spread farther apart. This mode consists of a wild and crazy hodgepodge of potential connections.

mind between the sound *mama* and your mother's smiling face. That scintillating neural loop is one memory trace—connected, of course, to many other related memory traces.

The best language programs—such as those at the Defense Language Institute, where I learned Russian—incorporate structured practice that includes plenty of repetition and rote, focused-mode learning of the language, along with more diffuse-like free speech with native speakers. The goal is to embed the basic words and patterns so you can speak as freely and creatively in your new language as you do in English.[3]

Focused practice and repetition—the creation of memory traces—are also at the heart of an impeccably played golf stroke, a master chef's practiced flip of an omelet, or a basketball free throw. In dance, it's a long way from a toddler's clumsy pirouette to the choreographed grace of a professional dancer. But that path to expertise is built bit by bit. Small memorized free spins, heel turns and kicks become incorporated into larger, more creative interpretations.



The left image symbolizes the compact connections when one chunk of knowledge is formed—neurons that fire together wire together. The image on the right shows the same pattern in your mind's symbolic pinball machine. Such a memory trace is easy to recall when you need it.

What Is a Chunk? Solomon's Chunking Problem

Solomon Shereshevsky's extraordinary memory came with a surprising drawback. His individual memory traces were each so colorful and emotional—so rich with connections—that they interfered with his ability to put those traces together and create conceptual **chunks**. He couldn't see the forest, in other words, because his imagery of each of the individual trees was so vivid.

Chunks are pieces of information that are bound together through meaning. You can take the letters *p*, *o*, and *p* and bind them into one conceptual, easy-to-remember chunk, the word *pop*. It's like converting a cumbersome computer file into a .zip file. Underneath that simple *pop* chunk is a symphony of neurons that have learned to trill in tune with one another. The complex neural activity that ties together our simplifying, abstract chunks of thought—

whether those thoughts pertain to acronyms, ideas, or concepts—are the basis of much of science, literature, and art.

Let's take an example. In the early 1900s, German researcher Alfred Wegener put together his theory of continental drift. As Wegener analyzed maps and thought about the information he'd gleaned from his studies and exploration, he realized that the different land masses fit together like puzzle pieces. The similarity of rocks and fossils between the land masses reinforced the fit. Once Wegener put the clues together, it was clear that all the continents had once, very long ago, been joined together in a single landmass. Over time, the mass had broken up and the pieces had drifted apart to form the continents separated by oceans we see today.

Continental drift! Wow—what a great discovery!

But if Solomon Shereshevsky had read this same story about the discovery of continental drift, he wouldn't have gotten the point. Even though he would have been able to repeat every individual word in the story, the concept of continental drift would have been very difficult for him to grasp, since he was unable to link his individual memory traces together to create conceptual chunks.

As it turns out, **one of the first steps toward gaining expertise in math and science is to create conceptual chunks—mental leaps that unite separate bits of information through meaning.**[4] Chunking the information you deal with helps your brain run more efficiently. Once you chunk an idea or concept, you don't need to remember all the little underlying details; you've got the main idea—the chunk—and that's enough. It's like getting dressed in the morning. Usually you just think one simple thought—*I'll get dressed.* But it's amazing when you realize the complex swirl of underlying activities that take place with that one simple chunk of a thought.

When you are studying math and science, then, how do you form a chunk?

Basic Steps to Forming a Chunk

Chunks related to different concepts and procedures can be molded in many different ways. It's often quite easy. You formed a simple chunk, for example, when you grasped the idea of continental drift. But since this is a book about how to learn math and science in general rather than geology in particular, we're going to take as our initial, illustrative chunk *the ability to understand and work a certain type of math or science problem.*

When you are learning new math and science material, you are almost always given sample problems with worked-out solutions. This is because, when you are first trying to understand how to work a problem, you have a heavy cognitive load—so it helps to start out with a fully worked-through example. It's like using a GPS unit when you are driving on unfamiliar roads in the middle of the night. Most of the details in the worked-out solution are right there, and your task is simply to figure out why the steps are taken the way they are. That can help you see the key features and underlying principles of a problem.

Some instructors do not like to give students extra worked-out problems or old tests, as they think it makes matters too easy. But there is bountiful evidence that having these kinds of resources available helps students learn much more deeply.[5] The one concern about using worked-out examples to form chunks is that it can be all too easy to focus too much on why an individual step works and not on the *connection* between steps—that is, on why this particular step is the next thing you should do. So keep in mind that I'm not talking about a cookie-cutter "just do as you're told" mindless approach when following a worked-out solution. It's more like using a guide to help you when traveling to a new place. Pay attention to

When you first look at a brand-new concept in science or math, it sometimes doesn't make much sense, as shown by the puzzle pieces above on the left. Just memorizing a fact *(center)* without understanding or context doesn't help you understand what's really going on, or how the concept fits together with the other concepts you are learning—notice there are no interlocking puzzle edges on the piece to help you fit into other pieces. **Chunking** *(right)* is the mental leap that helps you unite bits of information together through meaning. The new logical whole makes the chunk easier to remember, and also makes it easier to fit the chunk into the larger picture of what you are learning.

what's going on around you when you're with the guide, and soon you'll find yourself able to get there on your own. You will even begin to figure out new ways of getting there that the guide didn't show you.

1. **The first step in chunking, then, is to simply *focus your attention* on the information you want to chunk.**[6] If you have the television going in the background, or you're looking up every few minutes to check or answer your phone or computer messages, it means that you're going to have difficulty making a chunk, because your brain is not really focusing on the chunking. When you first begin to learn something, you are making new neural patterns and connecting them with preexisting patterns that are spread through many areas of the brain.[7] Your octopus tentacles can't make connections very well if some of them are off on other thoughts.

2. **The second step in chunking is to *understand* the basic idea you are trying to chunk,** whether it is understanding a concept such as continental drift, the idea that force is proportional to mass, the economic principle of supply and demand, or a particular type of math problem. Although this step of basic understanding—synthesizing the gist of what's important—was difficult for Solomon Shereshevsky, most students figure out these main ideas naturally. Or at least, they can grasp those ideas if they allow the focused and diffuse modes of thinking to take turns in helping them figure out what's going on.

   Understanding is like a superglue that helps hold the underlying memory traces together. It creates broad, encompassing traces that link to many memory traces.[8] Can you create a chunk if you don't understand? Yes, but it's a useless chunk that won't fit in with other material you are learning.

   That said, it's important to realize that *just understanding how a problem was solved does not necessarily create a chunk that you can easily call to mind later.* Do *not* confuse the "aha!" of a breakthrough in understanding with solid expertise! (That's part of why you can grasp an idea when a teacher presents it in class, but if you don't review it fairly soon after you've first learned it, it can seem incomprehensible when it comes time to prepare for a test.) Closing the book and testing yourself on how to solve the problems will also speed up your learning at this stage.

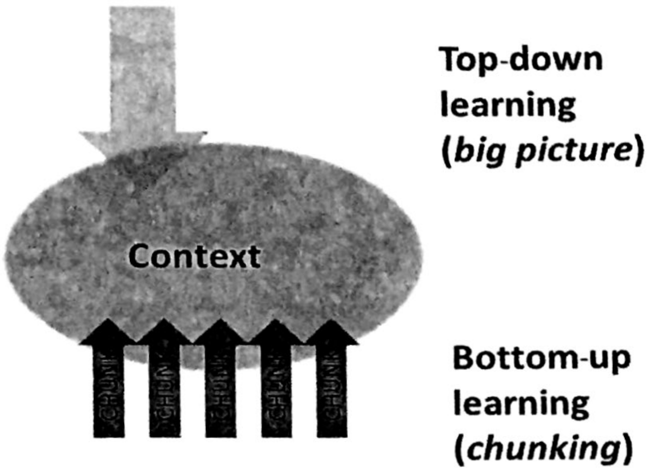
3. **The third step to chunking is gaining context so you see not just how, but also *when* to use this chunk.** Context means going beyond the initial problem and seeing more broadly, repeating and practicing with both related and unrelated problems so you see not only when to use the chunk, but when *not* to use it. This helps you see how your newly formed chunk fits into the bigger picture. In other words, you may have a tool in your strategy or problem-solving toolbox, but if you don't know when to use that tool, it's not going to do you a lot of good. Ultimately, practice helps you broaden the networks of neurons connected to your chunk, ensuring that it is not only firm, but also accessible from many different paths.

   There are chunks related to both concepts and procedures that reinforce one another. Solving a lot of math problems provides an opportunity to learn why the procedure works the way it does or why it works at all. Understanding the underlying concept makes it easier to detect errors when you make them. (Trust me, you *will* make errors, and that's a good thing.) It also makes it much easier to apply your knowledge to novel problems, a phenomenon called *transfer.* We'll talk more about transfer later.

As you can see from the following "top-down, bottom-up" illustration, learning takes place in two ways. There is a **bottom-up chunking process** where practice and repetition can help you both build and strengthen each chunk, so you can easily gain access to it when needed. And there is a **top-down "big picture" process** that allows you to see where what you are learning fits in.[9] *Both processes are vital in gaining mastery over the material.* Context is where bottom-up and top-down learning meet. To clarify here—chunking may involve your learning *how* to use a certain problem-solving technique. Context means learning *when* to use that technique instead of some other technique.

Those are the essential steps to making a chunk and fitting that chunk into a greater conceptual overview of what you are learning.

But there's more.

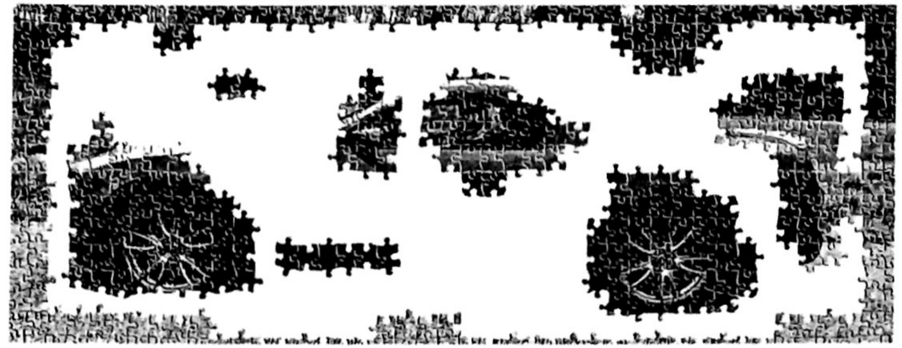


Both top-down, big-picture learning, and bottom-up chunking are important in becoming an expert in math and science.

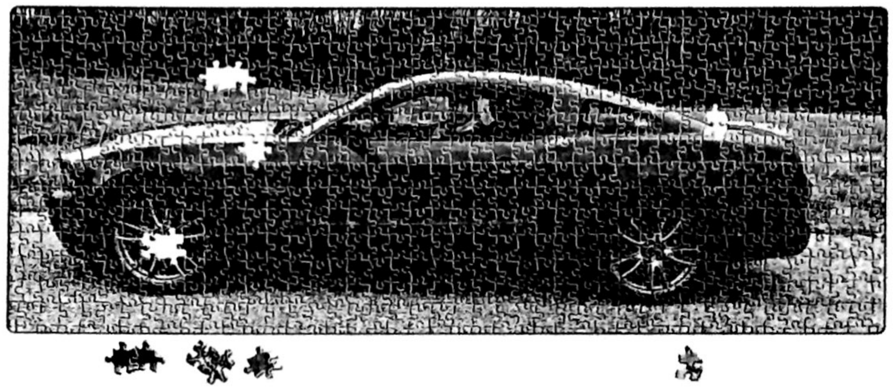
NOW I LAY ME DOWN TO SLEEP

"I tell my students that internalizing the accounting fundamentals is like internalizing how to type on a keyboard. In fact, as I write this myself, I'm not thinking of the act of typing, but of formulating my thoughts—the typing comes naturally. My mantra at the end of each class is to tell students to look at the Debit and Credit Rules as well as the Accounting Equation just before they tuck themselves in at night. Let those be the last things they repeat to themselves before falling asleep. Well, except meditation or prayers, of course!"

—*Debra Gassner Dragone, Accounting Instructor, University of Delaware*



Skimming through a chapter or listening to a very well-organized lecture can allow you to gain a sense of the big picture. This can help you know where to put the chunks you are constructing. Learn the major concepts or points first—these are often the key parts of a good instructor or book chapter's outline, flow charts, tables, or concept maps. Once you have this done, fill in the details. Even if a few of the puzzle pieces are missing at the end of your studies, you can still see the big picture.



Illusions of Competence and the Importance of Recall

Attempting to *recall* the material you are trying to learn—retrieval practice—is far more effective than simply rereading the material.[10] Psychologist Jeffrey Karpicke and his colleagues have shown that many students experience *illusions of competence* when they are studying. Most students, Karpicke found, "repeatedly read their notes or textbook (despite the limited benefits of this strategy), but relatively few engage in self-testing or retrieval practice while studying."[11] When you have the book (or Google!) open right in front of you, it provides the illusion that the material is also in your brain. *But it's not.* Because it can be easier to look at the book instead of recalling, students persist in their illusion—studying in a far less productive way.

This, indeed, is why just *wanting* to learn the material, and spending a lot of time with it, doesn't guarantee you'll actually learn it. As Alan Baddeley, a renowned psychologist and expert on memory, notes: **"Intention to learn is helpful only if it leads to the use of good learning strategies."**[12]

You may be surprised to learn that highlighting and underlining must be done carefully—otherwise they can be not only ineffective but also misleading. It's as if the motion of your hand can fool you into thinking you've placed the concept in your brain. When marking up the text, train yourself to look for main ideas before making any marks, and keep your text markings to a minimum—one sentence or less per paragraph.[13] Words or notes in a margin that synthesize key concepts are a good idea.[14]

Using recall—mental retrieval of the key ideas—rather than passive rereading will make your study time more focused and effective. The only time rereading text seems to be effective is if you let time pass between rereadings so that it becomes more of an exercise in spaced repetition.[15]

Along these same lines, always work through homework problems in math and science on your own. Some textbooks include solutions at the back of the book, but you should look at these only to check your answer. This will help ensure that the material is more deeply rooted in your mind and make it much more accessible when you really need it. This is why instructors place so much emphasis

on showing your work and giving your reasoning on tests and homework problems. Doing so forces you to think your way through a problem and provides a self-test of your understanding. This additional information about your thinking also gives graders a better opportunity to provide useful feedback.

You don't want to wait too long for the recall practice, so that you have to start the reinforcement of the concept from scratch every time. Try to touch again on something you're learning within a day, especially if it's new and rather challenging. This is why many professors recommend that, if at all possible, you rewrite your notes during the evening after a lecture. This helps to solidify newly forming chunks and reveals the holes in your understanding that professors just *love* to target on tests. Knowing where the holes are, of course, is the first step toward getting them filled in.

Once you've got something down, you can expand the time between "upkeep" repetitions to weeks or months—and eventually it can become close to permanent. (Returning to Russia on a visit, for example, I found myself annoyed by an unscrupulous taxi driver. To my amazement, words I hadn't thought or used for twenty-five years popped from my mouth—I hadn't even been consciously aware I *knew* those words!)

MAKE YOUR KNOWLEDGE SECOND NATURE

"Getting a concept in class versus being able to apply it to a genuine physical problem is the difference between a simple student and a full-blown scientist or engineer. The only way I know of to make that jump is to work with the concept until it becomes second nature, so you can begin to use it like a tool."

—Thomas Day, Professor of Audio Engineering, McNally Smith College of Music

Later, we'll discuss useful apps and programs that can help with learning. But for now, it's worth knowing that well-designed electronic flash card systems, such as Anki, have built into them the appropriate spaced repetition time to optimize the rate of learning new material.

One way to think about this type of learning and recall is shown in the following working-memory illustration. As we mentioned earlier, there are four or so spots in working memory.

When you are first chunking a concept, its pre-chunked parts take up all your working memory, as shown on the left. As you begin to chunk the concept, you will feel it connecting more easily and smoothly in your mind, as shown in the center. Once the concept is chunked, as shown at the right, it takes up only one slot in working memory. It simultaneously becomes one smooth strand that is easy to follow and use to make new connections. The rest of your working memory is left clear. That dangling strand of chunked material has, in some sense, increased the amount of information available to your working memory, as if the slot in working memory is a hyperlink that has been connected to a big webpage.[16]

When you are first learning how to solve a problem, your entire working memory is involved in the process, as shown by the mad tangle of connections between the four slots of working memory on the left. But once you become smoothly familiar with the concept or method you are learning and have it encapsulated as a single chunk, it's like having one smooth ribbon of thought, as shown on the right. The chunking, which enlists long-term memory, frees the rest of your working memory to process other information. Whenever you want, you can slip that ribbon (chunk) from long-term memory into your working memory and follow along the strand, smoothly making new connections.

Now you understand why it is key that you are the one doing the problem solving, not whoever wrote the solution manual. If you work a problem by just looking at the solution, and then tell yourself, "Oh yeah, I see why they did that," then the solution is not really yours—you've done almost nothing to knit the concepts into your underlying neurocircuitry. Merely glancing at the solution to a problem and thinking you truly know it yourself is one of the most common illusions of competence in learning.

NOW YOU TRY!

Understanding Illusions of Competence

Anagrams are rearrangements of letters so that one word or phrase can spell something different. Let's say you have the phrase "Me, radium ace." Can you rearrange it to spell the last name of a honorific famous physicist?[17] It may take you a bit of thought to do it. But if you saw the solution here on the page, your subsequent "aha!" feeling would make you think that your anagram-solving skills are better than they actually are.

Similarly, students often erroneously believe that they are learning by simply rereading material that is on the page in front of them. They have an illusion of competence *because the solution is already there.*[18]

Pick a mathematical or scientific concept from your notes or from a page in the book. Read it over, then look away and see what you can recall—working toward understanding what you are recalling at the same time. Then glance back, reread the concept, and try it again.

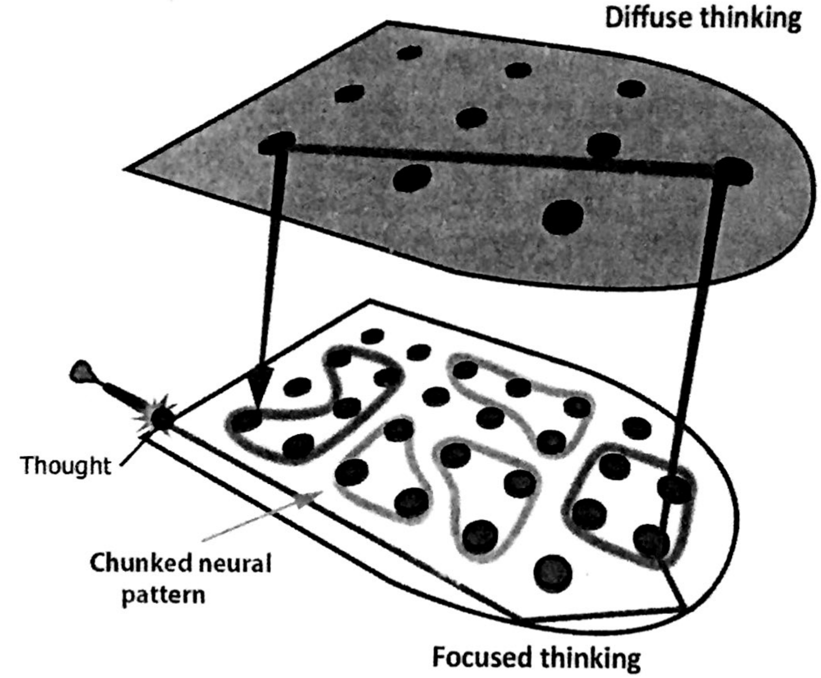
At the end of this exercise, you will probably be surprised to see how much this simple recall exercise helped improve your *understanding* of the concept.

You must have information persisting in your memory if you are to master the material well enough to do well on tests and think creatively with it.[19] The ability to combine chunks in novel ways underlies much of historical innovation. Steven Johnson, in his brilliant book *Where Good Ideas Come From*, describes the "slow hunch"—the gentle, years-long simmering of focused and diffuse processes that has resulted in creative breakthroughs ranging from Darwin's evolutionary theory to the creation of the World Wide Web.[20] Key to the slow hunch is simply having mental access to aspects of an idea. That way, some aspects can tentatively and randomly combine with others until eventually, beautiful novelty can emerge.[21] Bill Gates and other industry leaders, Johnson notes, set aside extended, week-long reading periods so that they can hold many and varied ideas in mind during one time. This fosters their own innovative thinking by allowing fresh-in-mind, not-yet-forgotten ideas to network among themselves. (An important side note here is that a key difference between creative scientists and technically competent but nonimaginative ones is their breadth of interest.[22])

The bigger your chunked mental library, the more easily you will be able to solve problems. Also, as you gain more experience in chunking, you will see that the chunks you are able to create are bigger—the ribbons are longer.

You may think there are so many problems and concepts just in a single chapter of the science or math subject you are studying that there's no way to do them all! This is where the **Law of Serendipity** comes to play: **Lady Luck favors the one who tries.**[23]

Just focus on whatever section you are studying. You'll find that once you put the first problem or concept in your library, *whatever it is*, then the second concept will go in a bit more easily. And the third more easily still. Not that all of this is a snap, but it does get easier.



If you have a library of concepts and solutions internalized as chunked patterns, you can more easily skip to the right solution to a problem by listening to the whispers from your diffuse mode. Your diffuse mode can also help you connect two or more chunks together in new ways to solve unusual problems.

There are two ways to solve problems—first, through sequential, step-by-step reasoning, and second, through more holistic intuition. Sequential thinking, where each small step leads deliberately toward the solution, involves the focused mode. Intuition, on the other hand, often seems to require a creative, diffuse mode linking of several seemingly different focused mode thoughts.

Most difficult problems are solved through intuition, because they make a leap away from what you are familiar with.[24] Keep in mind that the diffuse mode's semi-random way of making connections means that the solutions it provides with should be carefully verified using the focused mode. Intuitive insights aren't always correct![25]

In building a chunked library, you are training your brain to recognize not only a specific problem, but different *types* and *classes* of problems so that you can automatically know how to quickly solve whatever you encounter. You'll start to see patterns that simplify problem solving for you and will soon find that different solution techniques are lurking at the edge of your memory. Before midterms or finals, it is easy to brush up and have these solutions at the mental ready.

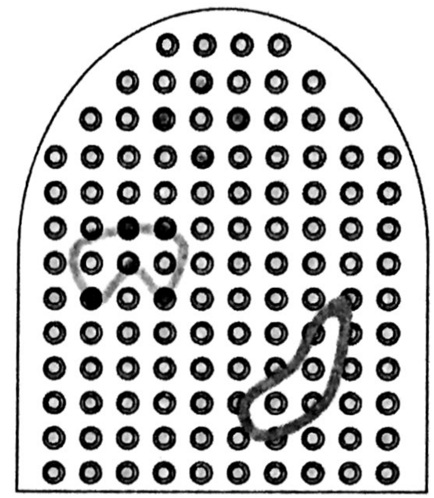
NOW YOU TRY!

What to Do If You Can't Grasp It

If you don't understand a method presented in a course you are taking, stop and work backward. Go to the Internet and discover who first figured out the method or some of the earliest people to use it. Try to understand how the creative inventor arrived at the idea and why the idea is used—you can often find a simple explanation that gives a basic sense of why a method is being taught and why you would want to use it.

Practice Makes Permanent

I've already mentioned that just *understanding* what's going on is *not* usually enough to create a chunk. You can get a sense of what I mean by looking at the "brain" picture shown on p. 69. The chunks (loops) shown are really just extended memory traces that have arisen because you have knit together an understanding. A chunk, in other words, is simply a more complex memory trace. At the top is a faint chunk. That chunk is what begins to form after you've understood a concept or problem and practiced just a time or two. In the middle, the pattern is darker. This is the stronger neural pattern that results after you've practiced a little more and seen the chunk in more contexts. At the bottom, the chunk is very dark.



Solving problems in math and science is like playing a piece on the piano. The more you practice, the firmer, darker, and stronger your mental patterns become.

You've now got a solid chunk that's firmly embedded in long-term memory.

Incidentally, strengthening an initial learning pattern within a day after you first begin forming it is important. Without the strengthening, the pattern can quickly fade away. Later, we'll talk more about the importance of spaced repetition in learning. Also, you can reinforce a "wrong" process by doing the same problems over and over the wrong way. This is why checking things is so important. Even getting the right answer can occasionally mislead you if you get it by using an incorrect procedure.

THE IMPORTANCE OF CHUNKING

"Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through the same process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics."[26]

—William Thurston, winner of the Fields Medal, the top award in mathematics

The challenge with repetition and practice, which lie behind the mind's creation of solid chunks, is that it can be boring. Worse yet, in the hands of a poor instructor, like my old math teacher, Mr. Crotchety, practice can become an unrelenting instrument of torture. Despite its occasional misuse, however, it's critical. Everybody knows you can't effectively learn the chunked patterns of chess, language, music, dance—just about anything worthwhile—without repetition. Good instructors can explain why the practice and repetition is worth the trouble.

Ultimately, both bottom-up chunking and top-down big-picture approaches are vital if you are to become an expert with the material. We love creativity and the idea of being able to learn by seeing the big picture. **But you can't learn mathematics or science without also including a healthy dose of practice and repetition to help you build the chunks that will underpin your expertise.**[27]

Research published in the journal *Science* provided solid evidence along these lines.[28] Students studied a scientific text and then

practiced it by recalling as much of the information as they could. Then they restudied the text and recalled it (that is, tried to remember the key ideas) once more.

The results?

In the same amount of time, *by simply practicing and recalling the material,* students learned far more and at a much deeper level than they did using any other approach, including simply rereading the text a number of times or drawing concept maps that supposedly enriched the relationships in the materials under study. This improved learning comes whether students take a formal test or just informally test themselves.

This reinforces an idea we've alluded to already. When we retrieve knowledge, we're not being mindless robots—*the retrieval process itself enhances deep learning and helps us begin forming chunks.*[29] Even more of a surprise to researchers was that the students themselves predicted that simply reading and recalling the materials wasn't the best way to learn. They thought concept mapping (drawing diagrams that show the relationship between concepts) would be best. But if you try to build connections between chunks *before the basic chunks are embedded in the brain*, it doesn't work as well. It's like trying to learn advanced strategy in chess before you even understand the basic concepts of how the pieces move.[30]

Practicing math and science problems and concepts in a variety of situations helps you build chunks—solid neural patterns with deep, contextual richness.[31] The fact is, when learning *any* new skill or discipline, you need plenty of varied practice with different contexts. This helps build the neural patterns you need to make the new skill a comfortable part of your way of thinking.

KEEP YOUR LEARNING AT THE TIP OF YOUR TONGUE

"By chance, I have used many of the learning techniques described in this book. As an undergraduate I took physical chemistry and became fascinated with the derivations. I got into a habit of doing every problem in the book. As a result, I hardwired my brain to solve problems. By the end of the semester I could look at a problem and know almost immediately how to solve it. I suggest this strategy to my science majors in particular, but also to the nonscientists. I also talk about the need to study every day, not necessarily for long periods of time but just enough to keep what you are learning at the tip of your tongue. I use the example of being bilingual. When I go to France to work, my French takes a few days to kick in, but then it is fine. When I return to the States and a student or colleague asks me something on my first or second day back, I have to search for the English words! When you practice every day the information is just there—you do not have to search for it."

—Robert R. Gamache, Associate Vice President, Academic Affairs, Student Affairs, and International Relations, University of Massachusetts, Lowell

Recall Material While Outside Your Usual Place of Study: The Value of Walking

Doing something physically active is especially helpful when you have trouble grasping a key idea. As mentioned earlier, stories abound of innovative scientific breakthroughs that occurred when the people who made them were out walking.[32]

In addition, **recalling material when you are outside your usual place of study helps you strengthen your grasp of the material by viewing it from a different perspective.** People sometimes lose subconscious cues when they take a test in a room that looks different

from where they studied. By thinking about the material while you are in various physical environments, you become independent of cues from any one location, which helps you avoid the problem of the test room being different from where you originally learned the material.[33]

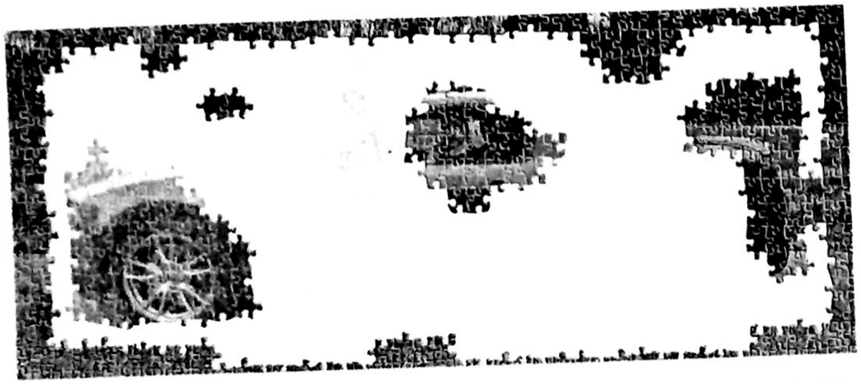
Internalizing math and science concepts can be *easier* than memorizing a list of Chinese vocabulary words or guitar chords. After all, you've got the problem there to speak to you, telling you what you need to do next. In that sense, problem solving in math and science is like dance. In dance, you can *feel* your body hinting at the next move.

Different types of problems have different review time frames that are specific to your own learning speed and style.[34] And of course, you have other obligations in your life besides learning one particular topic. You have to prioritize how much you're able to do, also keeping in mind that you *must* schedule some time off to keep your diffuse mode in play. How much internalizing can you do at a stretch? It depends—everyone is different. But, here's the real beauty of internalizing problem solutions in math and science. The more you do it, the easier it becomes, and the more useful it is.

ORGANIZE, CHUNK—AND SUCCEED

"The first thing I always do with students who are struggling is ask to see how they are organizing their notes from class and reading. We often spend most of the first meeting going over ways they can organize or chunk their information rather than with my explaining concepts. I have them come back the next week with their material already organized, and they are amazed at how much more they retain."

—Jason Dechant, Ph.D., Course Director, Health Promotion and Development, School of Nursing, University of Pittsburgh



If you don't practice with your growing chunks, it is harder to put together the big picture—the pieces are simply too faint.

Interleaving—Doing a Mixture of Different Kinds of Problems—versus Overlearning

One last important tip in becoming an equation whisperer is interleaving.[35] **Interleaving means practice by doing a mixture of different kinds of problems requiring different strategies.**

When you are learning a new problem-solving approach, either from your teacher or from a book, you tend to learn the new technique and then practice it over and over again during the same study session. Continuing the study or practice after it is well understood is called *overlearning*. Overlearning can have its place—it can help produce an automaticity that is important when you are executing a serve in tennis or playing a perfect piano concerto. But be wary of repetitive overlearning during a single session in math and science learning—research has shown it can be a waste of valuable learning time.[36] (Revisiting the approach mixed with other approaches during a subsequent study session, however, is just fine.)

In summary, then, once you've got the basic idea down during a session, continuing to hammer away at it during the same session ... strengthen the kinds of long-term memory con-

nections you want to have strengthened. Worse yet, focusing on one technique is a little like learning carpentry by only practicing with a hammer. After a while, you think you can fix anything by just bashing it.[37]

The reality is, mastering a new subject means learning to select and use the proper technique for a problem. The only way to learn that is by practicing with problems that require *different* techniques. Once you have the basic idea of a technique down during your study session (sort of like learning to ride a bike with training wheels), start interleaving your practice with problems of different types.[38] Sometimes this can be a little tough to do. A given section in a book, for example, is often devoted to a specific technique, so when you flip to that section, you already know which technique you're going to use.[39] Still, do what you can to mix up your learning. It can help to look ahead at the more varied problem sets that are sometimes found at the end of chapters. Or you can deliberately try to make yourself occasionally pick out why some problems call for one technique as opposed to another. **You want your brain to become used to the idea that just knowing *how* to use a particular problem-solving technique isn't enough—you also need to know *when* to use it.**

Consider creating index cards with the problem question on one side, and the question and solution steps on the other. That way you can easily shuffle the cards and be faced with a random variety of techniques you must call to mind. When you first review the cards, you can sit at a desk or table and see how much of the solution you can write on a blank sheet of paper without peeking at the back of the card. Later, when mastery is more certain, you can review your cards anywhere, even while out for a walk. Use the initial question as a cue to bring to mind the steps of the response, and flip the card over if necessary to verify that you've got the procedural steps all in mind. You are basically strengthening a new chunk. Another idea is to open the book to a randomly chosen page and work a problem while, as much as possible, hiding from view everything but the problem.

EMPHASIZE INTERLEAVING INSTEAD OF OVERLEARNING

Psychologist Doug Rohrer of the University of South Florida has done considerable research on overlearning and interleaving in math and science. He notes:

> "Many people believe overlearning means studying or practicing until mastery is achieved. However, in the research literature, overlearning refers to a learning strategy in which a student continues to study or practice immediately after some criterion has been achieved. An example might be correctly solving a certain kind of math problem and then immediately working several more problems of the same kind. Although working more problems of the same kind (rather than fewer) often boosts scores on a subsequent test, doing too many problems of the same kind in immediate succession provides diminishing returns.
>
> "In the classroom and elsewhere, students should maximize the amount they learn per unit time spent studying or practicing—that is, they should get the most bang for the buck. How can students do this? The scientific literature provides an unequivocal answer: Rather than devote a long session to the study or practice of the same skill or concept so that overlearning occurs, students should divide their effort across several shorter sessions. This doesn't mean that long study sessions are necessarily a bad idea. Long sessions are fine as long as students don't devote too much time to any one skill or concept. Once they understand 'X,' they should move on to something else and return to 'X' on another day."[40]

It's best to write the initial solution, or diagram, or concept, out by hand. There's evidence that writing by hand helps get the ideas into

mind more easily than if you type the answer.[41] More than that, it's often easier to write symbolic material like Σ or Ω by hand than to search out the symbol and type it (unless you use the symbols often enough to memorize the alt codes).[42] But if you then want to photograph or scan the question and your handwritten solution to load it into a flash card program for your smartphone or laptop, that will work just fine. Beware—a common illusion of competence is to continue practicing a technique you know, simply because it's easy and it feels good to successfully solve problems. Interleaving your studies—making a point to review for a test, for example, by skipping around through problems in the different chapters and materials—can sometimes seem to make your learning more difficult. But in reality, it helps you learn more deeply.

AVOID MIMICKING SOLUTIONS—PRACTICE CHANGING MENTAL GEARS

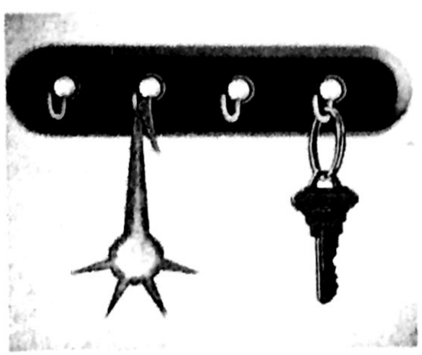
> "When students do homework assignments, they often have ten identical problems in a row. After the second or third problem, they are no longer thinking; they are mimicking what they did on the previous problem. I tell them that, when doing the homework from section 9.4, after doing a few problems, go back and do a problem from section 9.3. Do a couple more 9.4 problems, and then do one from section 9.1. This will give them practice in mentally shifting gears in the same way they'll need to switch gears on the test.
>
> "I also believe too many students do homework just to get it done. They finish a problem, check their answer in the back of the text, smile, and go on to the next problem. I suggest that they insert a step between the smile and going on to the next problem—asking themselves this question: How would I know how to do the problem this way if I saw it on a test mixed together with other problems and I didn't know it was from this

section of the text? Students need to think of every homework problem in terms of test preparation and not as part of a task they are trying to complete."

—*Mike Rosenthal, Senior Instructor of Mathematics, Florida International University*

SUMMING IT UP

- Practice helps build strong neural patterns—that is, conceptual chunks of understanding.
- Practice gives you the mental fluidity and agility you need for tests.
- Chunks are best built with:
  - *Focused attention.*
  - *Understanding* of the basic idea.
  - *Practice* to help you gain big-picture context.
- Simple recall—trying to remember the key points without looking at the page—is one of the best ways to help the chunking process along.



In some sense, recall helps build neural hooks that you can hang your thinking on.

ENHANCE YOUR LEARNING

1. How is a chunk related to a memory trace?
2. Think of a topic you are passionate about. Describe a chunk involving that topic that was at first difficult for you to grasp but now seems easy.
3. What is the difference between top-down and bottom-up approaches to learning? Is one approach preferable to the other?
4. Is *understanding* enough to create a chunk? Explain why or why not.
5. What is your own most common illusion of competence in learning? What strategy can you use to help avoid falling for this illusion in the future?

PAUSE AND RECALL

Next time you are with a family member, friend, or classmate, relate the essence of what you have been learning, either from this book or in regard to a class you are taking. Retelling whatever you are learning about not only helps fuel and share your own enthusiasm, but also clarifies and cements the ideas in your mind, so you'll remember them better in the weeks and months to come. Even if what you are studying is very advanced, simplifying so you can explain to others who do not share your educational background can be surprisingly helpful in building your understanding.