thought processes so you can understand how your mind learns—and also how your mind sometimes fools you into believing you're learning, when you're actually not. The book also includes plenty of skill-building exercises that you can apply directly to your current studies. **If you're *already* good at numbers or science, the insights in this book can help make you better.** They will broaden your enjoyment, creativity, and equation-solving elegance.

If you're simply convinced you don't have a knack for numbers or science, this book may change your mind. You may find it hard to believe, but there's hope. When you follow these concrete tips based on how we actually learn, you'll be amazed to see the changes within yourself, changes that can allow new passions to bloom.

What you discover will help you be more effective and creative, not only in math and science, but in almost everything you do.

Let's begin!

{ 2 }

easy does it:

Why Trying Too Hard Can Sometimes Be Part of the Problem

If you want to understand some of the most important secrets to learning math and science, look at the following picture.

The man on the right is legendary chess grand master Garry Kasparov. The boy on the left is thirteen-year-old Magnus Carlsen. Carlsen has just wandered away from the board during the height of a speed chess game, where little time is given to think about moves or strategy. That's a little like casually deciding to do a backflip while walking a tightrope across Niagara Falls.

Yes, Carlsen was psyching out his opponent. Rather than obliterating the upstart youngster, the flustered Kasparov played to a draw. But the brilliant Carlsen, who went on to become the youngest top-rated chess player in history, was doing something far beyond playing mind games with his older opponent. Gaining insight into Carlsen's approach can help us understand how the mind learns

Thirteen-year-old Magnus Carlsen *(left)*, and legendary genius Garry Kasparov playing speed chess at the "Reykjavík Rapid" in 2004. Kasparov's shock is just beginning to become apparent.

math and science. Before we go into how Carlsen psyched out Kasparov, we need to cover a couple of important ideas about how people think. (But I promise, we'll come back to Carlsen.)

We're going to be touching on some of the main themes of the book in this chapter, so don't be surprised if you have to toggle around a bit in your thinking. Being able to toggle your thinking—getting a glimpse of what you are learning before returning later to more fully understand what's going on, is itself one of the main ideas in the book!

NOW YOU TRY!

Prime Your Mental Pump

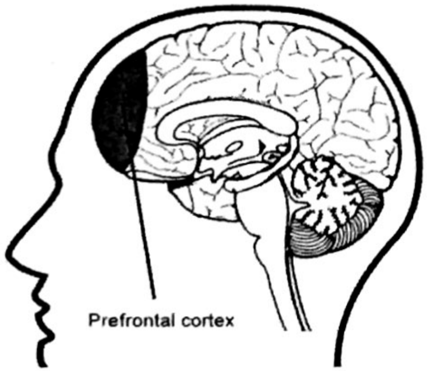
As you first begin looking at a chapter or section of a book that teaches concepts of math or science, it helps to take a "picture walk" through the chapter, glancing not only at the graphics, diagrams, and photos, but also at the section headings, summary, and even questions at the end of the chapter, if the book has them. This seems counterintuitive—you haven't actually read the chapter yet, but it helps prime your mental pump. So go ahead now and glance through this chapter and th e questions at the end of the chapter.

You'll be surprised at how *spending a minute or two glancing ahead before you read in depth will help you organize your thoughts.* You're creating little neural hooks to hang your thinking on, making it easier to grasp the concepts.

Focused versus Diffuse Thinking

Since the very beginning of the twenty-first century, neuroscientists have been making profound advances in understanding the two different types of networks that the brain switches between—*highly attentive states* and more relaxed *resting state networks.*[1] We'll call the thinking processes related to these two different types of networks the **focused mode** and **diffuse mode**, respectively—these modes are highly important for learning.[2] It seems you frequently switch back and forth between these two modes in your day-to-day activities. You're in either one mode or the other—not consciously in both at the same time. The diffuse mode does seem to be able to work quietly in the background on something you are not actively focusing on.[3] Sometimes you may also flicker for a rapid moment to diffuse-mode thinking.

Focused-mode thinking is essential for studying math and science. It involves a direct approach to solving problems using rational, sequential, analytical approaches. The focused mode is associated with the concentrating abilities of the brain's prefrontal cortex, located right behind your forehead.[4] Turn your attention to something and *bam*—the focused mode is *on*, like the tight, penetrating beam of a flashlight.



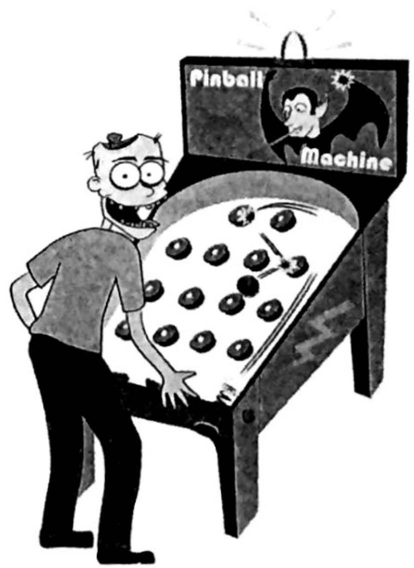
The prefrontal cortex is the area right behind the forehead.

Diffuse-mode thinking is also essential for learning math and science. It allows us to suddenly gain a new insight on a problem we've been struggling with and is associated with "big-picture" perspectives. Diffuse-mode thinking is what happens when you relax your attention and just let your mind wander. This relaxation can allow different areas of the brain to hook up and return valuable insights. Unlike the focused mode, the diffuse mode seems less affiliated with any one area of the brain—you can think of it as being "diffused" throughout the brain.[5] Diffuse-mode insights often flow from preliminary thinking that's been done in the focused mode. (The diffuse mode must have clay to make bricks!)

Learning involves a complex flickering of neural processing among different areas of the brain, as well as back and forth between hemispheres.[6] So this means that thinking and learning is more complicated than simply switching between the focused and diffuse modes. But fortunately, we don't need to go deeper into the physical mechanisms. We're going to take a different approach.

The Focused Mode—A Tight Pinball Machine

To understand focused and diffuse mental processes, we're going to play some pinball. (Metaphors are powerful tools for learning in math and science.) In the old game of pinball, you pull back on a spring-loaded plunger and it whacks a ball, which ends up bouncing randomly around the circular rubber bumpers.



This happy zombie is playing neural pinball.

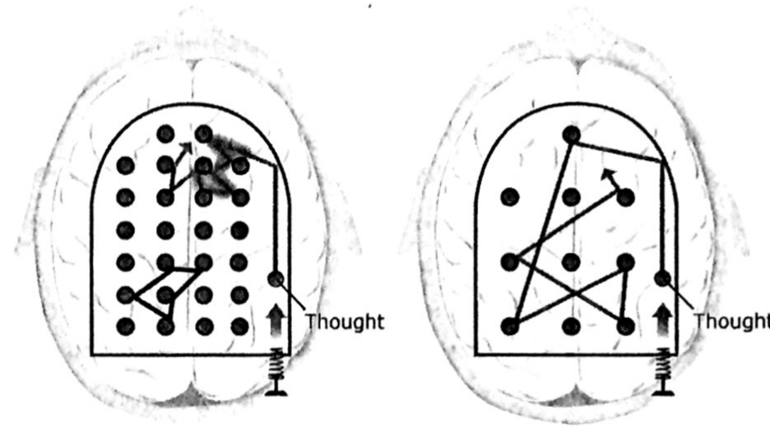
Look at the following illustration. When you focus your attention on a problem, your mind pulls back the mental plunger and releases a thought. Boom—that thought takes off, bumping around like the pinball in the head on the left. This is the *focused mode* of thinking.

Notice how the round bumpers are very close together in the focused mode. In contrast, the diffuse mode on the right has its circular rubber bumpers farther apart. (If you want to pursue the metaphor still further, you can think of each bumper as a cluster of neurons.)

The close bumpers of the focused mode mean that you can more easily think a precise thought. Basically, the focused mode is used to concentrate on something that's already tightly connected in your mind, often because you are familiar and comfortable with the underlying concepts. If you look closely at the upper part of the focused-mode thought pattern, you'll see a wider, "well-trodden" part of the line. That broader path shows how the focused-mode thought is following along a route you've already practiced or experienced.

For example, you can use the focused mode to multiply numbers—if you already know how to multiply, that is. If you're studying a language, you might use the focused mode to become more fluent with the Spanish verb conjugation you learned last week. If you're a swimmer, you might use the focused mode to analyze your breaststroke as you practice staying low to allow more energy to go into your forward motion.

When you focus on something, the consciously attentive prefrontal cortex automatically sends out signals along neural pathways. These signals link different areas of your brain related to what you're thinking about. This process is a little like an octopus that sends its tentacles to different areas of its surroundings to fiddle



In the game "pinball," a ball, which represents a thought, shoots up from the spring-loaded plunger to bounce randomly against rows of rubber bumpers. These two pinball machines represent **focused** *(left)* and **diffuse** *(right)* ways of thinking. The focused approach relates to intense concentration on a specific problem or concept. But while in focused mode, sometimes you inadvertently find yourself focusing intently and trying to solve a problem using erroneous thoughts that are in a different place in the brain from the "solution" thoughts you need to actually need to solve the problem.

As an example of this, note the upper "thought" that your pinball first bounces around in on the left-hand image. It is very far away and completely unconnected from the lower pattern of thought in the same brain. You can see how part of the upper thought seems to have an underlying broad path. This is because you've thought something similar to that thought before. The lower thought is a new thought—it doesn't have that underlying broad pattern.

The diffuse approach on the right often involves a big-picture perspective. This thinking mode is useful when you are learning something new. As you can see, the diffuse mode doesn't allow you to focus tightly and intently to solve a specific problem—but it can allow you to get closer to where that solution lies because you're able to travel much farther before running into another bumper.

with whatever it's working on. The octopus has only so many tentacles to make connections, just as your working memory has only so many things it can hold at once. (We'll talk more about the working memory later.)

You often first funnel a problem into your brain by focusing your attention on words—reading the book or looking at your notes from a lecture. Your attentional octopus activates your focused mode. As you do your initial focused noodling around with the problem, you are thinking tightly, using the pinball bumpers that are close together to follow along familiar neural pathways related to something you already know or are familiar with. Your thoughts rattle easily through the previously ingrained patterns and quickly settle on a solution. In math and science, however, it often doesn't take much of a change for a problem to become quite different. Problem solving then grows more difficult.

Why Math and Science Can Be More Challenging

Focused problem solving in math and science is often more effortful than focused-mode thinking involving language and people.[7] This may be because humans haven't evolved over the millennia to manipulate mathematical ideas, which are frequently more abstractly encrypted than those of conventional language.[8] Obviously, we can still think *about* math and science—it's just that the *abstractness* and *encryptedness* adds a level—sometimes a number of levels—of complexity.

What do I mean by abstractness? You can point to a real live *cow* chewing its cud in a pasture and equate it with the letters *c-o-w* on the page. But you can't point to a real live *plus sign* that the symbol "+" is modeled after—the idea underlying the plus sign is more *abstract*. By *encryptedness*, I mean that one symbol can stand for a number of different operations or ideas, just as the multiplication sign symbolizes repeated addition. In our pinball analogy, it's as if the abstractness and encryptedness of math can make the pinball bumpers a bit spongier—it takes extra practice for the bumpers to harden and the pinball to bounce properly. This is why dealing with procrastination, while important in studying any discipline, is particularly important in math and science. We'll be talking more about this later.

Related to these difficulties in math and science is another challenge. It's called the ***Einstellung* effect** (pronounced *EYE-nshtellung*). In this phenomenon, an idea you already have in mind, or your simple initial thought, prevents a better idea or solution from being found.[9] We saw this in the focused pinball picture, where your initial pinball thought went to the upper part of the brain, but the solution thought pattern was in the lower part of the image. (The German word *Einstellung* means "mindset"—basically you can remember *Einstellung* as installing a roadblock because of the way you are initially looking at something.)

This kind of wrong approach is especially easy to do in science because sometimes your initial intuition about what's happening is misleading. You have to unlearn your erroneous older ideas even while you're learning new ones.[10]

The *Einstellung* effect is a frequent stumbling block for students. It's not just that sometimes your natural intuitions need to be retrained—it's that sometimes it is tough even figuring out where to begin, as when tackling a homework problem. You bumble about—your thoughts far from the actual solution—because the crowded bumpers of the focused mode prevent you from springing to a new place where the solution might be found.

This is precisely why **one significant mistake students sometimes make in learning math and science is *jumping into the water before they learn to swim.*** [11] In other words, they blindly start working on homework without reading the textbook, attending lectures, viewing online lessons, or speaking with someone knowledgeable.

This is a recipe for sinking. It's like randomly allowing a thought to pop off in the focused-mode pinball machine without paying any real attention to where the solution truly lies.

Understanding how to obtain *real* solutions is important, not only in math and science problem solving, but for life in general. For example, a little research, self-awareness, and even self-experimentation can prevent you from being parted with your money—or even your good health—on products that come with bogus "scientific" claims.[12] And just having a little knowledge of the relevant math can help prevent you from defaulting on your mortgage—a situation that can have a major negative impact on your life.[13]

The Diffuse Mode—A Spread-Out Pinball Machine

Think back several pages to the illustration of the diffuse-mode pinball machine brain, where the bumpers were spread far apart. This mode of thinking allows the brain to look at the world from a much broader perspective. Can you see how a thought can travel much further before it runs into a bumper? The connections are further apart—you can quickly zoom from one clump of thought to another that's quite far away. (Of course, it's hard to think precise, complex thoughts while in this mode.)

If you are grappling with a new concept or trying to solve a new problem, you don't have preexisting neural patterns to help guide your thoughts—there's no fuzzy underlying pathway to help guide you. You may need to range widely to encounter a potential solution. For this, diffuse mode is just the ticket!

Another way to think of the difference between focused and diffuse modes is to think of a flashlight. You can set a flashlight so it has a tightly focused beam that can penetrate deeply into a small

area. Or you can set the flashlight onto a more diffuse setting where it casts its light broadly, but not very strongly in any one area.

If you are trying to understand or figure out something new, your best bet is to turn off your precision-focused thinking and turn on your "big picture" diffuse mode, long enough to be able to latch on to a new, more fruitful approach. As we'll see, the diffuse mode has a mind of its own—you can't simply command it to turn on. But we'll soon get to some tricks that can help you transition between modes.

COUNTERINTUITIVE CREATIVITY

"When I was learning about the diffuse mode, I began to notice it in my daily life. For instance, I realized my best guitar riffs always came to me when I was 'just messing around' as opposed to when I sat down intent on creating a musical masterpiece (in which case my songs were often clichéd and uninspiring). Similar things happened when I was writing a school paper, trying to come up with an idea for a school project, or trying to solve a difficult math problem. I now follow the rule of thumb that is basically: The harder you push your brain to come up with something creative, the less creative your ideas will be. So far, I have not found a single situation where this does not apply. Ultimately, this means that relaxation is an important part of hard work—and good work, for that matter."

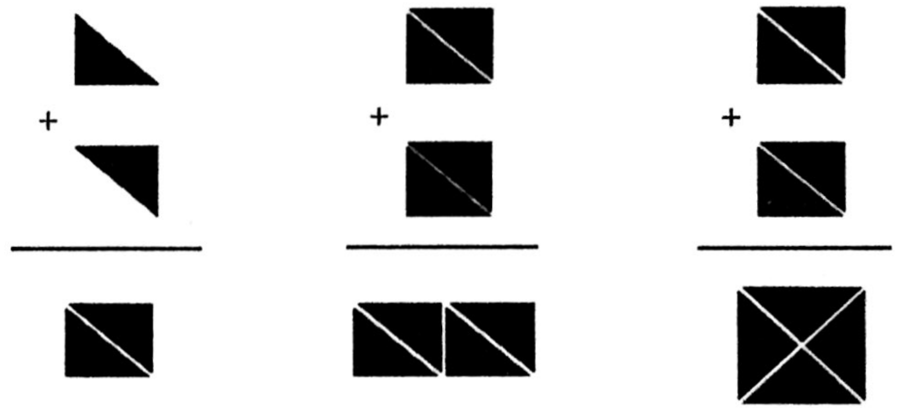
—Shaun Wassell, freshman, computer engineering

Why Are There Two Modes of Thinking?

Why do we have these two different thinking modes? The answer may be related to two major problems that vertebrates have had in staying alive and passing their genes on to their offspring. A bird, for example, needs to focus carefully so it can pick up tiny pieces of grain as it pecks the ground for food, and at the same time, it must scan the horizon for predators such as hawks. What's the best way to carry out those two very different tasks? Split things up, of course. You can have one hemisphere of the brain more oriented toward the focused attention needed to peck at food and the other oriented toward scanning the horizon for danger. When each hemisphere tends toward a particular type of perception, it may increase the chance of survival.[14] If you watch birds, they'll first peck, and then pause to scan the horizon—almost as if they are alternating between focused and diffuse modes.

In humans, we see a similar splitting of brain functions. The left side of the brain is somewhat more associated with careful, focused attention. It also seems more specialized for handling sequential information and logical thinking—the first step leads to the second step, and so on. The right seems more tied to diffuse scanning of the environment and interacting with other people, and seems more associated with processing emotions.[15] It also is linked with handling simultaneous, big-picture processing.[16]

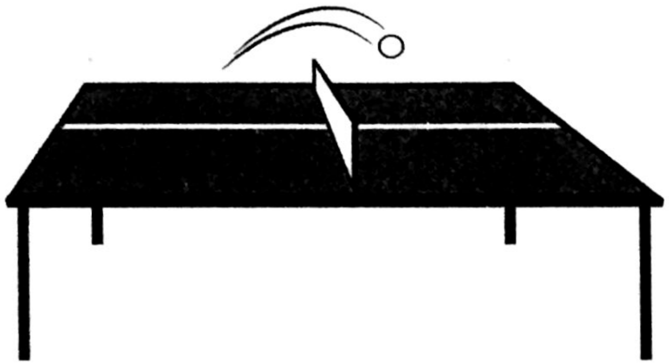
The slight differences in the hemispheres give us a sense of why two different processing modes may have arisen. But be wary of the idea that some people are "left-brain" or "right-brain" dominant—research indicates that is simply not true.[17] Instead it is clear that *both* hemispheres are involved in focused as well as diffuse modes of thinking. **To learn about and be creative in math and science,**



Here's a quick example that gives a sense of the difference between focused and diffuse thinking. If you are given two triangles to put together into a square shape, it's easy to do, as shown on the left. If you are given two more triangles and told to form a square, your first tendency is to erroneously put them together to form a rectangle, as shown in the middle. This is because you've already laid down a focused-mode pattern that you have a tendency to follow. It takes an intuitive, diffuse leap to realize that you need to completely rearrange the pieces if you want to form another square, as shown on the right.[18]

we need to strengthen and use both the focused and diffuse modes.[19]

Evidence suggests that to grapple with a difficult problem, we must first put hard, focused-mode effort into it. (We learned that in grade school!) Here's the interesting part: The diffuse mode is *also* often an important part of problem solving, especially when the problem is difficult. *But as long as we are consciously focusing on a problem, we are blocking the diffuse mode.*



There's a winner at Ping-Pong only if the ball is able to go back and forth.

EMBRACE BEFUDDLEMENT!

"Befuddlement is a healthy part of the learning process. When students approach a problem and don't know how to do it, they'll often decide they're no good at the subject. Brighter students, in particular, can have difficulty in this way—their breezing through high school leaves them no reason to think that being confused is normal and necessary. But the learning process is all about working your way out of confusion. Articulating your question is 80 percent of the battle. By the time you've figured out what's confusing, you're likely to have answered the question yourself!"

—Kenneth R. Leopold, Distinguished Teaching Professor, Department of Chemistry, University of Minnesota

The bottom line is that problem solving in any discipline often involves an exchange between the two fundamentally different modes. One mode will process the information it receives and then send the result back to the other mode. This volleying of information back and forth as the brain works its way toward a conscious solution appears essential for understanding and solving all but trivial problems and concepts.[20] The ideas presented here are extremely helpful for understanding learning in math and science. But as you are probably beginning to see, they can be just as helpful for many other subject areas, such as language, music, and creative writing.

NOW YOU TRY!

Shifting Modes

Here's a cognitive exercise that can help you feel the shift from focused to diffuse mode. See whether you can form a new triangle that points down by moving only three coins.

When you relax your mind, releasing your attention and focusing on nothing in particular, the solution can most easily come to you.

You should know that some children get this exercise instantly,

while some highly intelligent professors finally just give up. To answer this question, it helps to summon your inner child. The solutions for this challenge and for all the "Now You Try!" challenges in the book can be found in the endnotes.[21]

Procrastination Prelude

Many people struggle with procrastination. We'll have a lot to say later in this book about how to deal effectively with procrastination. For now, keep in mind that **when you procrastinate, you are leaving yourself only enough time to do superficial focused-mode learning.** You are also increasing your stress level because you know you have to complete what feels like an unpleasant task. The resulting neural patterns will be faint and fragmented and will quickly disappear—you'll be left with a shaky foundation. In math and science in particular, this can create severe problems. If you cram for a test at the last minute or quickly breeze through your homework, you won't have time for either learning mode to help you tackle the tougher concepts and problems or to help you synthesize the connections in what you are learning.

NOW YOU TRY!

Focusing Intently but Briefly

If you often find yourself procrastinating, as many of us do, here's a tip. Turn off your phone and any sounds or sights (or websites) that might signal an interruption. Then set a timer for twenty-five minutes and put yourself toward doing a twenty-five-minute interlude of work focused on a task—any task. Don't worry about finishing the task—just worry about working on it. Once the twenty-five minutes is up, reward yourself with web surfing, checking your phone, or whatever you like to do. *This reward is as important as the work itself.* You'll be amazed at how productive a focused twenty-five-minute stint can be—especially when you're just focusing on the work itself, *not* on finishing. (This method, known as the Pomodoro technique, will be discussed in more detail in chapter 6.)

If you want to apply a more advanced version of this approach, imagine that at the end of the day, you are reflecting on the *one* most important task that you accomplished that day. What would that task be? Write it down. Then work on it. Try to complete at least three of these twenty-five-minute sessions that day, on whatever task or tasks you think are most important.

At the end of your workday, look at what you crossed off your list and savor the feeling of accomplishment. Then write a few key things that you would like to work on the next day. This early preparation will help your diffuse mode begin to think about how you will get those tasks done the next day.

SUMMING IT UP

- Our brain uses two very different processes for thinking—the focused and diffuse modes. It seems you toggle back and forth between these modes, using one or the other.
- It is typical to be stumped by new concepts and problems when we first focus on them.
- To figure out new ideas and solve problems, it's important not only to focus initially, but also to subsequently turn our focus *away* from what we want to learn.
- The *Einstellung* effect refers to getting stuck in solving a problem or understanding a concept as a result of becoming fixated on a flawed approach. Switching modes from focused to diffuse can help free you from this effect. Keep in mind, then, that sometimes you will need to be flexible in your thinking. You may need to switch modes to solve a problem or understand a concept. Your initial ideas about problem solving can sometimes be very misleading.

PAUSE AND RECALL

Close the book and look away. What were the main ideas of this chapter? Don't worry if you can't recall very much when you first begin trying this. As you continue practicing this technique, you'll begin noticing changes in how you read and how much you recall.

ENHANCE YOUR LEARNING

1. How would you recognize when you are in the diffuse mode? How does it feel to be in the diffuse mode?
2. When you are consciously thinking of a problem, which mode is active and which is blocked? What can you do to escape this blocking?
3. Recall an episode where you experienced the *Einstellung* effect. How were you able to change your thinking to get past the preconceived, but erroneous, notion?
4. Explain how the focused and diffuse modes might be equated to an adjustable beam on a flashlight. When can you see farther? When can you see more broadly, but less far?
5. Why is procrastination sometimes a special challenge for those who are studying math and science?

SHIFTING OUT OF BEING STUCK: INSIGHT FROM NADIA NOUI-MEHIDI, A SENIOR STUDYING ECONOMICS

"I took Calculus I in eleventh grade and it was a nightmare. It was so profoundly different from anything I had learned before that I didn't even know how to learn it. I studied longer and harder than I ever had before, yet no matter how many problems I did or how long I stayed in the library I was learning nothing. I ultimately just stuck to what I could get by with through memorizing. Needless to say, I did not do well on the AP [advanced placement] exam.

"I avoided math for the next two years, and then as a sophomore in college, I took Calculus I and got a 4.0. I don't think I was any smarter two years later, but there was a complete shift in the way I was approaching the subject.

"I think in high school I was stuck in the focused mode of thinking (*Einstellung!*) and felt that if I kept trying to approach problems in the same way it would eventually click.

"I now tutor students in math and economics and the issues are almost always that they are fixated on looking at the details of the problem for clues on how to solve it, and not on understanding the problem itself. I don't think you can tutor someone on how to think—it's kind of a personal journey. But here are some things that have helped me understand a concept that at first seems complicated or confusing.

1. I understand better when I read the book rather than listen to someone speak, so I always read the book. I skim first so I know basically what the chapter is trying to get at and then I read it in detail. I read the chapter more than once (but not in a row).
2. If after reading the book, I still don't fully understand what's going on, I Google or look at YouTube videos on the subject. This isn't because the book or professor isn't thorough, but rather because sometimes hearing a slightly different way of