1

1.1 Problem description

Given the region P in 2D space, defined by

$$\forall i, a_i x + b_i y \leq c_i$$

,

define a linear program whose solution describes the largest axis-aligned square that lies entirely within P.

1.2 Solution

The reduction is based on 3 variables representing the x and y coordinates of a potential square, and the dimension of the square. From those 3 variables, we can compute the x and y coordinates of each corner of the square. Since P is convex, the square is contained within P iff each of the corners is within P. Writing down the linear program is then straightforward:

Maximize:

$$x_{dim}$$

Subject to constraints:

$$\forall i, a_i x_x + b_i x_y \leq c_i$$
$$\forall i, a_i x_x + a_i x_{dim} + b_i x_y \leq c_i$$
$$\forall i, a_i x_x + b_i x_y + b_i x_{dim} \leq c_i$$
$$\forall i, a_i x_x + a_i x_{dim} + b_i x_y + b_i x_{dim} \leq c_i$$

2

2.1 Problem description

Given the region P in 2D space, defined by

$$\forall i, a_i x + b_i y \leq c_i$$

,

define a linear program whose solution describes the maximum-perimeter axis-aligned rectangle that lies entirely within P.

2.2 Solution

The reduction is nearly the same as the first part, with the exception that now we must define the parameters of the rectangle based on 4 variables instead of 3 because the width and height can differ, and additionally we need to maximize the perimeter, which is equivalent to maximizing the sum of the dimensions.

Maximize:

$$x_w + x_h$$

Subject to constraints:

$$\forall i, a_i x_x + b_i x_y \leq c_i$$
$$\forall i, a_i x_x + a_i x_w + b_i x_y \leq c_i$$
$$\forall i, a_i x_x + b_i x_y + b_i x_h \leq c_i$$
$$\forall i, a_i x_x + a_i x_w + b_i x_y + b_i x_h \leq c_i$$