

1. Recall the globe tossing model from the lecture. Compute and plot the grid approximate posterior distribution for each of the following sets of observations. In each case, assume a uniform prior for  $p$ , and use a grid of  $n = 30$  points.
  - (a) W, W, W
  - (b) W, W, W, L
  - (c) L, W, W, L, W, W, W
2. Now assume a prior for  $p$  that is equal to 0 when  $p < 0.5$  and is a positive constant when  $p > 0.5$ . Again, compute and plot the grid approximate posterior distribution for each of the sets of observations in the problem above.
3. Suppose that the globe tossing data turned out to be 6 waters in 9 tosses (as in the lecture). Compute a grid approximate posterior for the binomial parameter  $p$  using  $n = 1000$  points. Use the same flat prior as before. Then, draw 10,000 samples from the posterior and answer the following questions:
  - (a) How much posterior probability lies below  $p = 0.2$ ?
  - (b) How much posterior probability lies below  $p = 0.8$ ?
  - (c) How much posterior probability lies between  $p = 0.2$  and  $p = 0.8$ ?
  - (d) 20% of the posterior probability lies below which value of  $p$ ?
  - (e) 20% of the posterior probability lies above which value of  $p$ ?
  - (f) Which values of  $p$  contain the *narrowest* interval equal to 60% of the posterior probability?
4. Suppose the globe tossing data had turned out to be 8 waters in 15 tosses.
  - (a) Construct a posterior distribution, using grid approximation with  $n = 1000$  points. Use the same flat prior as before. Then, draw 10,000 samples from the posterior distribution, compute the posterior mode, and compute a 90% HPDI for  $p$ . Interpret what these values mean.
  - (b) Construct a posterior predictive check for this model and data. That is, simulate the distribution of samples, averaging over the posterior uncertainty in  $p$ . What is the probability of observing 8 water in 15 tosses?
5. Repeat the previous problem, but now use a prior that is zero below  $p = 0.5$  and a constant above  $p = 0.5$ . This prior corresponds to our *a priori* knowledge that a majority of the Earth's surface is water. Compare the answers to these two problems. How well does each compare to the true value of  $p = 0.7$ ? What difference does a better prior make?