1. Let X denote the random variable that counts the number of times we observe "heads" out of 15 coin flips. Let θ denote the probability of landing heads on any one of those coin flips.

   (a) Plot the probability function $p(x)$, given $\theta = 0.3$.

   (b) Plot the likelihood function $f(\theta)$, given $x = 7$.

   (c) Explain the differences between these two plots.

   (d) Find the maximum likelihood estimate for θ given that we've observed 7 successes.

2. Like the binomial distribution, the *Poisson* distribution can be used to describe probabilities of certain events. As a random variable, the Poisson distribution describes the number of rare events that occur within a certain timeframe. For example, it can be used to model the number of car accidents during rush hour, the number of earthquakes in a year in a certain region, or the number of deer in an area of land. The probability function for the Poisson random variable is given by

$$p(x \mid \lambda) = \frac{\lambda^x e^{-\lambda}}{\lambda!}$$

   where $x = 0, 1, 2, \ldots$, and λ is the "rate" parameter (that is, the expected number of occurrences for a given timeframe.

   (a) Plot the probability function for $\lambda = 0.5$ with $x = 0, 1, \ldots, 10$. You can use the `dpois` function in R to do this. Just type `?dpois` in the console to see the help page.

   (b) Plot the probability function for $\lambda = 10$ with $x = 0, 1, \ldots, 30$.

   (c) Plot the likelihood for $x = 4$. Hint: you'll need to make sure you have a suitable range for values of λ. Use your previous two plots to get a feel for what λ might be in this case.

   (d) Find the maximum likelihood estimate for λ, given $x = 4$. What do you notice?

3. The command `data=read.csv("https://git.io/v5RO6")` will load a set of 1000 observations into R. Your task is to fit a normal model to this data. Using the techniques demonstrated in the lecture notes, compute maximum likelihood estimates for μ and σ. Then, plot the density curves for both the raw data (solid line) and the normal model (dashed line). Does the model fit the data well? Explain.

   - Note: the initial parameter values we did in the lecture will NOT work with this data. You'll have to play around with this a bit to make it work.

4. The command `data=read.csv("https://git.io/v5ROF")` will load a set of 1000 response times into R. Your task in this problem is to fit the data two ways: first, with a normal model, then second, with an ex-Gaussian model.

   (a) Assume $RT \sim \text{Normal}(\mu, \sigma)$. Compute maximum likelihood estimates for μ and σ. Hint: use starting values of $\mu = 2$ and $\sigma = 0.1$.

   (b) Assume $RT \sim \text{ExGaussian}(\mu, \sigma, \tau)$. Compute maximum likelihood estimates for μ, σ, and τ. Hint: use starting values of $\mu = 2$, $\sigma = 0.1$, and $\tau = 0.1$.

   (c) Plot both models along with the density curve of the RT data. Which is the better fit?