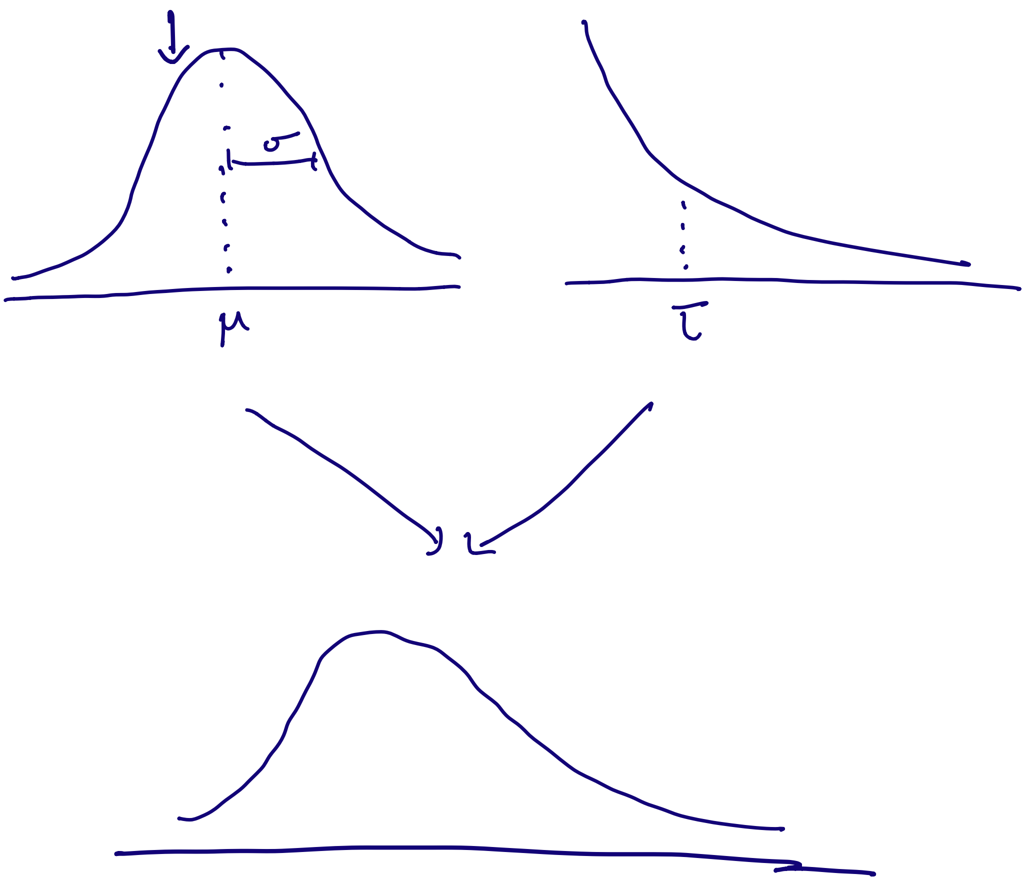
Lecture 8 - the Wald model

Recall: the ex-Gaussian model combines a

normal distribution with exponential tail



Three parameters:

* μ = mean of normal component
* σ = sd of normal component
* τ = mean of "tail" component

Do these parameters have psychological interpretation?

Table 2
Cognitive Interpretations Attributed to the Ex-Gaussian Parameters

Authors	μ	τ
Balota and Spieler (1999)	stimulus driven automatic (nonanalytic) processes	central attention demanding (analytic) processes
Blough (1988, 1989)	component of RT unrelated to stimulus variables (e.g., neural transmission and motor response)	momentary probability of target detection/ search component of RT
Epstein et al. (2006), Leth-Steensen et al. (2000)	—	attentional lapses
Gholson and Hohle (1968a, 1968b)	—	response choice latency/response competition
Gordon and Carson (1990), Hohle (1965), Madden et al. (1999), Possamaï (1991), Rotello and Zeng (2008)	duration of residual processes (e.g., sensory and motor processes)	durations of the decisional phase of RT
Kieffaber et al. (2006)	attentional cognitive processes	intentional cognitive processes
Penner-Wilger, Leth-Steensen, and Lefevre (2002)	retrieval processes	nonretrieval/procedure use
Rohrer (1996, 2002), Rohrer and Wixted (1994), Wixted, Ghadisha, and Vera (1997), Wixted and Rohrer (1993)	initial pause preceding the retrieval of the first response	mean recall latency/ongoing memory search
Schmiedek, Oberauer, Wilhelm, Süß, and Wittmann (2007)	—	higher cognitive functioning (e.g., working memory and reasoning)
Spieler, Balota, and Faust (1996)	—	more central processing component

Note—A dash indicates that the parameter is not given any cognitive interpretation.

↳ from Matzke & Wagenmakers (2009)

↳ showed that ex-Gaussian model parameters do not uniquely reflect cognitive processes

↳ descriptive model only

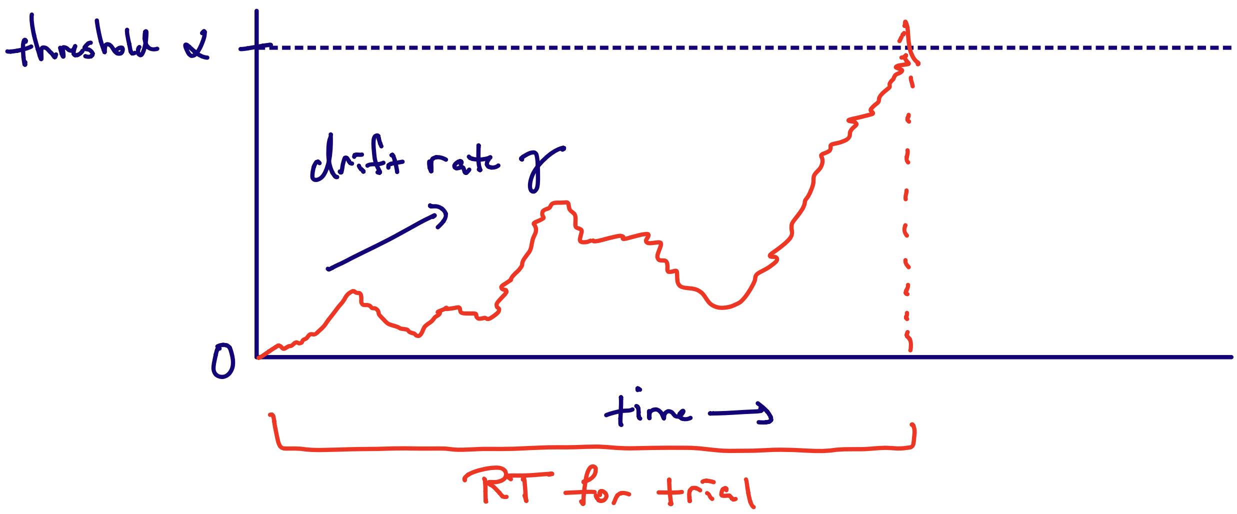
What about the sequential sampling models we touched on in Lecture 6?

↳ drift rate

↳ response threshold

$\left.\begin{array}{l}\text{drift rate}\\\text{response threshold}\end{array}\right\} \rightarrow$ cognitive interpretations?

Wald model: sequential sampling model w/ one boundary



Density function:

$$f(x \mid \alpha, \gamma) = \frac{\alpha}{\sqrt{2\pi x^3}} \exp\left[-\frac{(\alpha - \gamma x)^2}{2x}\right]$$

Parameters and their cognitive interpretations

* Drift rate γ = task difficulty
  ↳ influenced by participant ability or task demands

* Response threshold α = response caution
  ↳ influenced by task instructions

Fitting the Wald model in R

Let's fit the Schwarz (2001) data from Lecture 7.

```r
# density for shifted Wald
dwald = function(x, alpha, gamma){
  return((alpha/(sqrt(2*pi*x^3)))*exp(-(alpha-gamma*x)^2/(2*x)))
}

# Step 1: define NLL for shifted wald
nll.wald = function(data, pars){
  alpha = pars[1] # response threshold
  gamma = pars[2] # drift rate
  return(-sum(log(dwald(data,alpha,gamma))))
}

# Step 2: create function to give initial parameter estimate
# from Heathcote (2004)
waldInit = function(x, p = 0.9) {
  theta = p*min(x)
  gamma = sqrt(mean(x)/var(x))
  alpha = gamma*mean(x)
  return(c(alpha,gamma))
}

initPar = waldInit(X$RT)

# Step 3: perform optimization
model = optim(par = initPar,
              fn = nll.wald,
              data = X$RT)
```

Just like ex-Gaussian, ← hand-code the Wald density

← objective function

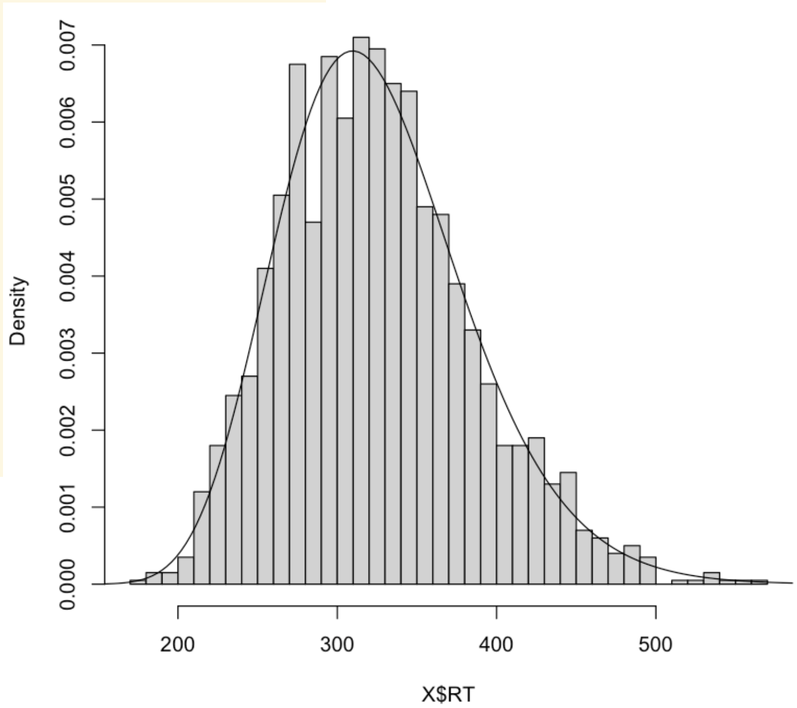
← function to estimate initial parameters

← optimization.

```r
# extract parameters
alpha = model$par[1]
gamma = model$par[2]

# plot model against raw data
hist(X$RT, breaks = 30, probability = T)
x = seq(from = 0, to = 600, length.out=200)
lines(x, dwald(x, alpha, gamma))

# compute BIC
k = 3 # two parameters
N = length(X$RT)
BIC1 = k*log(N) + 2*model$value
```



How does it compare to ex-Gaussian?

↳ from Lecture 7, BIC_{EG} : 21,989.92

↳ here, we have BIC_{Wald} : 21,967.62

(better fit!)