# PSYC 2317 - Lecture 5

Learning goal: to answer research questions by translating to statistical questions.

Example: Suppose we are testing a treatment that has been proposed to increase intelligence (as measured by IQ)

A sample of $N = 25$ people is given the treatment, and the average IQ for the sample is $\bar{X} = 107$.

Did the treatment work?

We can answer by translating this research question to a statistical question.

Let $\mu$ = mean of the population who receive treatment

Is $\mu > 100$, the average IQ for the general population?

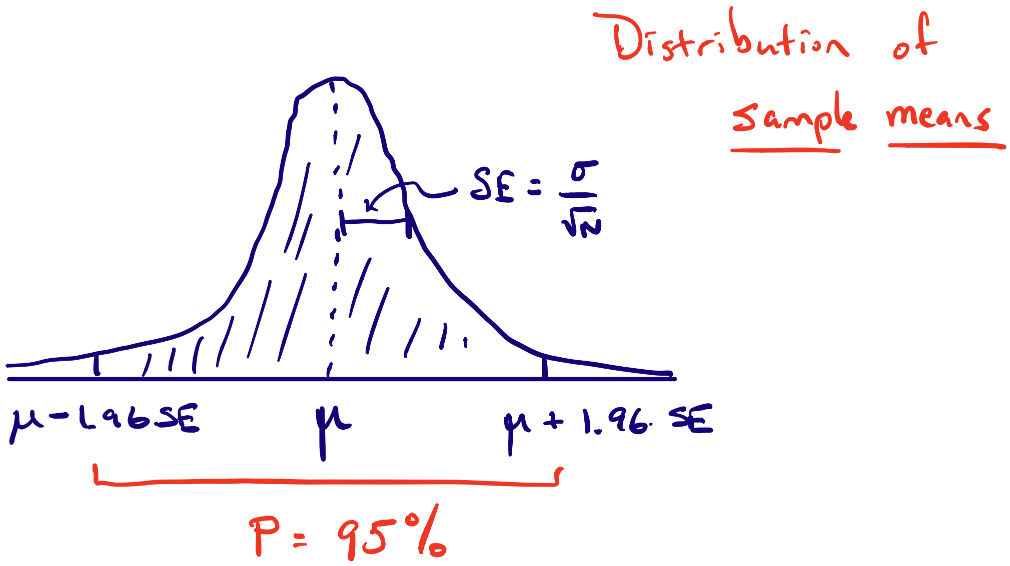
Two methods:

(1) estimate $\mu$ from $\bar{x}$

(2) test competing hypotheses about $\mu$.

# Method 1 - Estimation

From lecture 4, we know that 95% of sample means are within (almost) two standard deviations of the population mean $\mu$.




So, there is a 95% probability that any given sample mean is between $\mu - 1.96 \cdot SE$ and $\mu + 1.96 \cdot SE$

$$\hookrightarrow \quad \mu - 1.96 \cdot \frac{\sigma}{\sqrt{N}} \leq \bar{X} \leq \mu + 1.96 \cdot \frac{\sigma}{\sqrt{N}}$$

A little algebra converts this to:

$$\bar{X} - 1.96 \cdot \frac{\sigma}{\sqrt{N}} \leq \mu \leq \bar{X} + 1.96 \cdot \frac{\sigma}{\sqrt{N}}$$

**Definition:** A 95% confidence interval for $\mu$ is given by the interval

$$\left(\bar{x} - 1.96 \cdot \frac{\sigma}{\sqrt{N}} \ , \ \bar{x} + 1.96 \cdot \frac{\sigma}{\sqrt{N}}\right)$$

Back to our example: recall that our treatment sample $(N = 25)$ had a mean of $\bar{X} = 107$. Let's compute a 95% confidence interval (CI) for $\mu$.

Recall: for distribution of IQ scores, we know $\sigma = 15$.

$$\begin{aligned}
\text{So} \quad 95\%\,\text{CI} &= \left(\bar{x} - 1.96 \cdot \frac{\sigma}{\sqrt{N}} \ , \ \bar{x} + 1.96 \cdot \frac{\sigma}{\sqrt{N}}\right) \\
&= \left(107 - 1.96 \cdot \frac{15}{\sqrt{25}} \ , \ 107 + 1.96 \cdot \frac{15}{\sqrt{25}}\right) \\
&= \left(107 - 5.88 \ , \ 107 + 5.88\right) \quad \text{"Margin of error"} \\
&= (101.12 \ , \ 112.88).
\end{aligned}$$

So, we are 95% confident that $\mu$ is between 101.12 and 112.88

Since our estimate for $\mu$ is greater than 100, we conclude that the treatment worked.

## Method 2 - Hypothesis testing

We define two competing hypotheses: ← about $\mu$.

$H_0: \mu = 100$ ("null hypothesis" / no tmt effect)

$H_1: \mu > 100$ ("alternative hypothesis" / positive tmt effect)

Let us assume that $H_0$ is true (that is, $\mu = 100$)

What is the probability of observing our sample mean $\bar{x} = 107$ (or more extreme) if $H_0$ is true?

$$p(\bar{x} \geq 107)$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{N}} = \frac{107 - 100}{15/\sqrt{25}} = \frac{7}{15/5} = \frac{7}{3} = 2.33.$$

$$\hookrightarrow P(z \geq 2.33) = 0.0099$$

Conclusion: our data is rare under $H_0$.

So, we reject $H_0$ as a plausible hypothesis.

This gives support for $H_1: \mu > 100$.

## Take home:

* translate research questions to statistical questions about some population parameter (e.g., $\mu$)

* Estimation – compute 95% confidence interval for $\mu$

* Hypothesis testing – define competing hypotheses about $\mu$ ($\mathcal{H}_0, \mathcal{H}_1$)
  - assume $\mathcal{H}_0$ is true
  - if observed data is rare under $\mathcal{H}_0$, we reject $\mathcal{H}_0$ and conclude support for $\mathcal{H}_1$.