Lecture 3 - Statistical Properties of Composite Tests

Guiding example: GRE scores are composed of two subtests:

- GRE Verbal: $\mu_1 = 150$, $\sigma_1 = 10$
- GRE Quantitative: $\mu_2 = 150$, $\sigma_2 = 10$

Suppose you score a 160 on each subtest. What is the resulting percentile rank?

To answer this, we need to know how to compute descriptives (i.e., mean & variance) of a composite test score.

This requires talking about three things:

(1) mean of a composite

(2) correlation / covariance between subtests

(3) variance of a composite.

What exactly do we mean by composite?

* Definition: a composite test score is the sum of two (or more) subtest scores.

For example, we can write $C = X_1 + X_2$

How to compute mean of a composite

Recall that mean is given by $\mu_x = \dfrac{\sum x}{N}$

So $\mu_c = \dfrac{\sum c}{N} = \dfrac{\sum (x_1 + x_2)}{N}$

$$= \frac{\sum x_1 + \sum x_2}{N}$$

$$= \frac{\sum x_1}{N} + \frac{\sum x_2}{N}$$

$$= \mu_1 + \mu_2$$

Thus, the mean of a composite is equal to the <u>sum of the subtest means.</u>

<u>Example:</u> mean composite GRE $= \mu_V + \mu_Q$

$$= 150 + 150$$

$$= 300$$

Next question: how does our observed composite score of 320 compare to this mean?

↳ need to know standard deviation / variance

How to compute variance of a composite

Recall that variance is given by $\sigma_x^2 = \dfrac{\sum (x-\mu_x)^2}{N}$

So $$\sigma_c^2 = \frac{\sum (c-\mu_c)^2}{N}$$

$$= \frac{\sum \left[(x_1+x_2)-(\mu_1+\mu_2)\right]^2}{N}$$

$$= \frac{\sum \left[(x_1-\mu_1)+(x_2-\mu_2)\right]^2}{N}$$

$$= \frac{\sum \left[(x_1-\mu_1)^2 + 2(x_1-\mu_1)(x_2-\mu_2) + (x_2-\mu_2)^2\right]}{N}$$

$$= \frac{\sum (x_1-\mu_1)^2 + \sum 2(x_1-\mu_1)(x_2-\mu_2) + \sum (x_2-\mu_2)^2}{N}$$

$$= \underbrace{\frac{\sum (x_1-\mu_1)^2}{N}}_{\text{variance of } x_1} + \underbrace{\frac{\sum (x_2-\mu_2)^2}{N}}_{\text{variance of } x_2} + 2\underbrace{\frac{\sum (x_1-\mu_1)(x_2-\mu_2)}{N}}_{\text{covariance of } x_1, x_2}$$

$$= \sigma_{x_1}^2 + \sigma_{x_2}^2 + 2\sigma_{x_1 x_2}$$

In words, the variance of a composite is equal to the sum of the variances plus twice the covariance of the subtests.

Back to our example: $\sigma^2_{GRE} = \sigma^2_V + \sigma^2_Q + 2\sigma_{VQ}$

* we know the variances:

$$\sigma^2_V = 10^2 = 100$$

$$\sigma^2_Q = 10^2 = 100.$$

* what about the covariance?

Recall from last time – covariance is related to correlation:

$$r = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

to find covariance, it suffices to know the correlation of the two subtests!

Suppose $r_{VQ} = 0.35$. Then

$$0.35 = \frac{\sigma_{XY}}{(10)(10)} \longrightarrow \sigma_{XY} = 0.35 \times 100 = 35.$$

Thus, we have

$$\sigma^2_{GRE} = \sigma^2_V + \sigma^2_Q + 2\sigma_{VQ}$$

$$= 100 + 100 + 2(35)$$

$$= 270.$$

This implies $\sigma_{GRE} = \sqrt{270} = 16.43$

Now we can find the percentile rank of our score of 320.

Just use z-scores and a normal distribution calculator:

$$z = \frac{x - \mu}{\sigma} = \frac{320 - 300}{16.43} = 1.22$$

Percentile rank $\longrightarrow P(z < 1.22) = 0.889$

$\longrightarrow 88.9\%$

thus, we scored higher than 88.9 percent of the population of GRE test takers.

Quick note: we will often talk about the

Variance - covariance matrix (vcov)

of a composite test:

	GRE-V	GRE-Q
GRE-V	100	35
GRE-Q	35	100

These are helpful for computing variance of composites

because Variance = sum of the VCOV matrix

$$= 100 + 100 + 35 + 35$$

$$= 270$$