Psyc 2317 - Lecture 6

Last time – we translated research questions to **statistical questions** about some population mean μ.

Two approaches:

(1) estimate 95% confidence interval for μ

"We are 95% confident that μ is between $\underline{X}$ and $\underline{Y}$."

(2) define competing models for μ $(\mathcal{H}_0, \mathcal{H}_1)$ and compute probability of observing sample mean $\bar{X}$ if $\mathcal{H}_0$ is true.

Technical note – you may have noticed that every example we've done includes the population standard deviation σ.

What happens if we are not given σ?

Example: A population has a mean of 23. A sample of $N=4$ is given an experimental treatment and had scores of 20, 22, 22, and 20. Does the treatment result in a significantly lower score?

We are not given σ – what can we do?

↳ we need a new technique!

How do we get σ?

- maybe we can estimate it from the observed data

  ↳ $$s = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- Problem: s tends to be too small! It systematically underestimates σ.

- Solution: let's <u>correct</u> the formula to fix the bias

$$\boxed{\begin{aligned}\hat{\sigma} &= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \\ &= \sqrt{\frac{SS}{N-1}}\end{aligned}}$$

← Easier to remember!

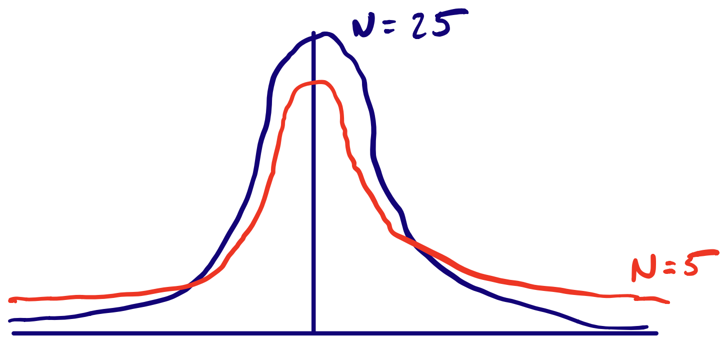
OK, fine ... but the distribution of "z-scores" $\left(\frac{\bar{x}-\mu}{\hat{\sigma}/\sqrt{N}}\right)$ is no longer <u>normal</u>, but something else entirely!

The T-distribution

- details worked out by Gosset (1908)
  - ↳ Biometrika paper written under pseudonym "Student"
  - ↳ nice history of this paper given in Zabell (2008) - Journal of the Amer. Stat. Assoc.

- shape of the distribution depends on sample size
  - ↳ parameter = "degrees of freedom"
  - ↳ $df = N - 1$

- the smaller the sample size, the <u>fatter</u> the tails



- so, sample size (i.e., degrees of freedom) must be specified when we calculate probabilities
  - ↳ T-distribution is built into the probability app we've been using in this course

https://tomfaulkenberry.shinyapps.io/dist_calc

Back to our example:

A population has a mean of 23. A sample of $N = 4$ is given an experimental treatment and had scores of 20, 22, 22, and 20. Does the treatment result in a significantly lower score?

Let μ = mean of the treatment population. Note that $\overline{X} = 21$.

Define: $H_0: \mu = 23$

$H_1: \mu < 23$.

Assume H_0 is true.

Find probability of observing $\overline{X} < 21$ if H_0 is true.

To proceed, we need to compute an estimate $\hat{\sigma}$ of the population standard deviation.

- from above, we have $\hat{\sigma} = \sqrt{\frac{SS}{N-1}}$

X_i	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
20	-1	1
22	1	1
22	1	1
20	-1	1

$\overline{X} = 21$

$\longrightarrow SS = 4$

$\longrightarrow \hat{\sigma} = \sqrt{\frac{SS}{N-1}}$

$= \sqrt{\frac{4}{3}} = 1.15$

Now we can compute a "t - score"

$$t = \frac{\bar{x} - \mu}{\hat{\sigma}/\sqrt{N}} = \frac{21 - 23}{1.15/\sqrt{4}} = \frac{-2}{0.575} = -3.48$$

Finally, we find $P(t < -3.48)$

↳ from app (with $df = 4 - 1 = 3$)

we get $P = 0.02$

Since $P < 0.05$, our data is rare if H_0 is true.

So we reject H_0 in favor of H_1 (i.e, $\mu < 23$)

and conclude that the treatment results in significantly lower scores.

Take home:

- in problems where σ is unknown, we must estimate it from the data.
- when using estimate $\hat{\sigma}$, the distribution of sample means depends on sample size.
- result: t-test.