PSYC 2317 - Lecture 7

In past lectures, we have learned how to estimate unknown parameters and test hypotheses with a single sample

In this lecture, we will learn how to test hypotheses about two independent samples

Illustrative example: Suppose we want to test the efficacy of a new memory treatment. Eight participants are randomly assigned to one of two groups (treatment or control) and subsequently given a memory test.

Treatment	Control
45	43
55	49
40	35
60	51
$\bar{X}_1 = 50$	$\bar{X}_2 = 45$
$SS_1 = 250$	$SS_2 = 184$

Does the treatment group score significantly higher than the control group?

To work this out, let's measure the "effect" of the treatment.

Effect size

A common measure of effect size is **Cohen's d**

$$\boxed{d = \frac{\text{difference between means}}{\text{standard deviation}}}$$

* difference between means = $\overline{X}_1 - \overline{X}_2$

* but there are TWO standard deviations!

  ↳ solution: "pool" them

Defn: pooled standard deviation

* weighted average of standard deviations from the two groups, with weights determined by relative sample size.

$$\boxed{\hat{\sigma}_p = \sqrt{\frac{SS_1 + SS_2}{df_1 + df_2}}}$$

Note: this is very similar to $\hat{\sigma}$ from previous lecture:

$$\hat{\sigma} = \sqrt{\frac{SS}{N-1}} = \sqrt{\frac{SS}{df}}$$

Let's compute Cohen's d for our example:

$$d = \frac{\bar{X}_1 - \bar{X}_2}{\hat{\sigma}_p} \qquad \hat{\sigma}_p = \sqrt{\frac{SS_1 + SS_2}{df_1 + df_2}} = \sqrt{\frac{250 + 184}{3 + 3}}$$

$$= \sqrt{\frac{434}{6}}$$

$$= \sqrt{72.33}$$

$$= 8.50$$

Thus, $d = \dfrac{\bar{X}_1 - \bar{X}_2}{\hat{\sigma}_p} = \dfrac{50 - 45}{8.50}$

$$= \frac{5}{8.5} = 0.59$$

Note: Cohen recommended the following guidelines for interpretation:

d	size of effect
< 0.2	small
0.5	medium
> 0.8	large

So we have a "medium" effect. Is it statistically significant?

↳ independent samples t-test.

Recall: hypothesis test works as follows:

1. define two competing hypotheses ($\mathcal{H}_0, \mathcal{H}_1$)
2. assume $\mathcal{H}_0$ is true
3. compute probability of observing our data if $\mathcal{H}_0$ is true

↳ convert data to a t-score

↳ find probability of obtaining that observed t-score (or more extreme).

Let μ_1 = population mean of treatment group

μ_2 = population mean of control group.

Define $\mathcal{H}_0: \mu_1 = \mu_2$ Assume $\mathcal{H}_0$ is true.

$\mathcal{H}_1: \mu_1 > \mu_2$ Compute t-score for observed data.

General form of t-score:

$$t = \frac{\text{sample mean} - \text{population mean}}{\text{est. SD} / \sqrt{\text{sample size}}}$$

Specific formula:

$$\boxed{t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\hat{\sigma}_p \sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}}$$

So, if H_0 is true (i.e, $\mu_1 = \mu_2$), we have:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\hat{\sigma}_p \sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

$$= \frac{(50 - 45) - (0)}{8.50 \cdot \sqrt{\frac{1}{4} + \frac{1}{4}}}$$

$$= \frac{5}{6.01} = 0.83$$

Now; compute p-value:

$$P(t > 0.83) = 0.219$$

$$\begin{aligned} df &= df_1 + df_2 \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

The observed data is <u>plausible</u> if H_0 is true.

So we fail to reject H_0.

Conclude: the treatment group does <u>NOT</u> score significantly higher than the control group.