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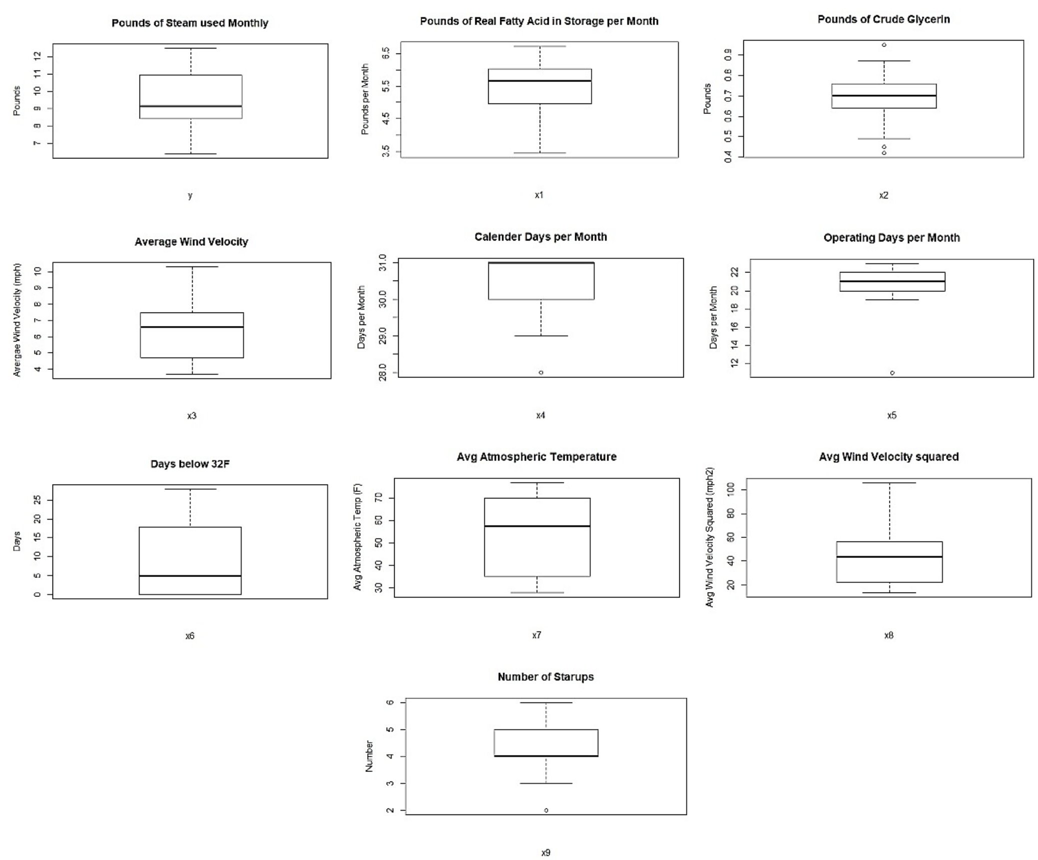
Regression Analysis

Final Exam

21 April 2020

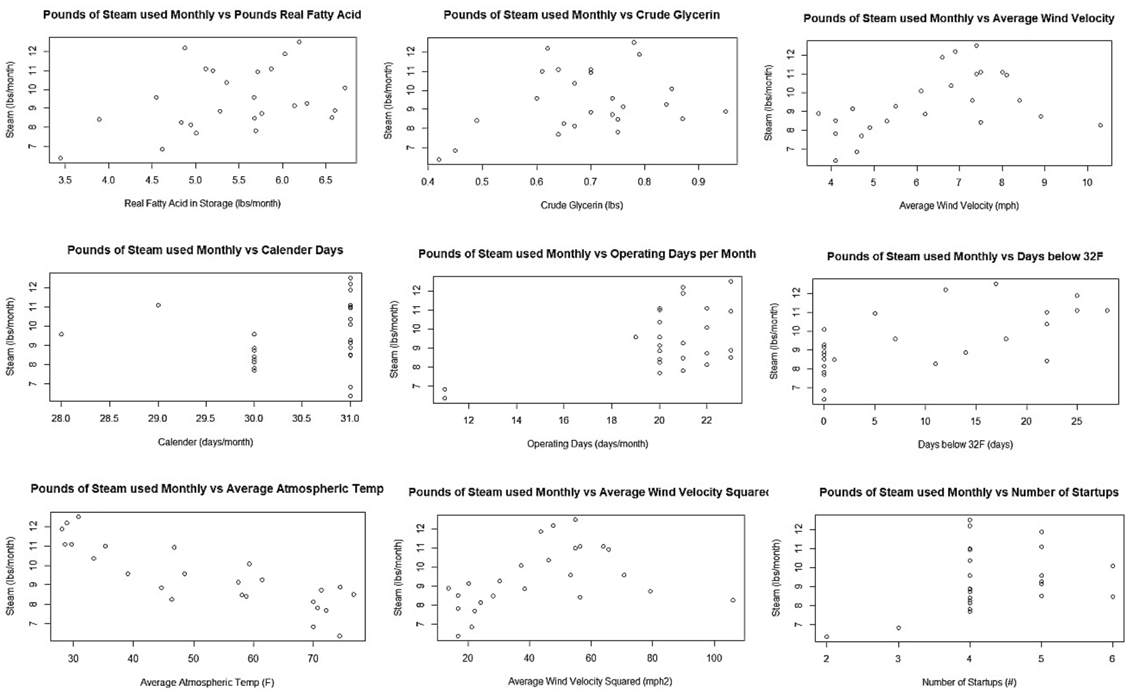
1) The following are box plots of the y variable and x variables for observations at a steam plant. Each boxplot is separate because each variable has different units.

   Figure 1: Boxplots depicting the spread of the y variable and the regressor variables of observations at a steam plant.



One of the requirements for a data set so that statistical analyses are valid is normal distribution, which means equal distribution of data points around the mean that have roughly 95% of the data points within two standard deviations of the mean and no outliers. Based on the box plots from this data set, certain variables appear to fulfill this requirement and others appear not to. The pounds of steam used monthly (y), pounds of real fatty acid in storage per month (x1), average wind velocity (x3), average atmospheric temperature (x7) and average wind velocity squared (x8) appear to have a normal distribution and no outliers. On the other hand, pounds of crude glycerin (x2) has outliers at both extremes, calendar days per month (x4), operating days per month (x5) and number of startups (x9) have high extreme outliers, and calendar days per month (x4) and days below 32F (x6) are missing extremes for the boxplot in the data set. Therefore, certain variables appear to fulfill one of the requirements for statistical analysis while others do not.

2) **Figure 2**: The following are scatterplots relating steam (lbs/month) to each regressor variable accounted for in observations from a steam plant.



Based on the scatter plots, certain regressor variables seem to have a linear relationship with steam (lbs/month) while others do not. Real fatty acid (lbs/month) and average atmospheric temperature (F) seem to have a fairly strong linear relationship with steam (lbs/month). Crude glycerin (lbs), average wind velocity (mph) and average wind velocity squared (mph2) seem to have more of a parabolic relationship with steam (lbs/month). Days below 32F may have a linear relationship with steam (lbs/month) but there are quite a few data points where there is a decent spread of steam (lbs/month) with zero days below 32F, so this regressor does not seem particularly linear. The same could be said for operating days per month for slightly different reasons; though fewer operating days seem to correlate with lower steam in lbs/month, more operating days correlates with a wide range of steam in lbs. Finally, calendar days and number of startups do not really seem to have a linear relationship with steam in lbs/month, but it is difficult to tell because, like operating days and number of days below 32F, it is a discrete numerical variable compared to a continuous numerical variable. Further analysis for these variables are warranted.

3) The full model for relating the y variable and regressors for the steam model is as follows (**note: coefficients that correspond to regressors that are measured in the same units**

as the y variable (lbs/month) remain unitless in order to correctly relate the regressor to the y variable):

Steam (lbs/month) = 0.700(x1) -1.868[month^{-1}](x2) +1.140[lbs*hr*month^{-1}*miles^{-1}](x3) + 0.123[lbs*days^{-1}](x4) +0.180[lbs*days^{-1}](x5) -0.018[lbs*days^{-1}*month^{-1}](x6) -0.077[lbs*°F^{-1}*month^{-1}](x7) -0.086[lbs*hr^{2}*miles^{-2}*month^{-1}](x8) -0.346[lbs*number of startups^{-1}*month^{-1}](x9) +1.761 (lbs/month)

The summary statistics for the full model area as follows:

```
        Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.76116    6.96637   0.253 0.803847
x1           0.70084    0.56248   1.246 0.231880
x2          -1.86794    4.12852  -0.452 0.657421
x3           1.14038    0.74289   1.535 0.145591
x4           0.12253    0.20374   0.601 0.556546
x5           0.17957    0.08060   2.228 0.041619 *
x6          -0.01831    0.02440  -0.751 0.464557
x7          -0.07734    0.01652  -4.681 0.000295 ***
x8          -0.08626    0.05178  -1.666 0.116445
x9          -0.34610    0.20979  -1.650 0.119777


Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5673 on 15 degrees of freedom
Multiple R-squared:  0.9242,   Adjusted R-squared:  0.8788
F-statistic: 20.33 on 9 and 15 DF,  p-value: 7.576e-07
```

Though the adjusted R^2 value is fairly high (0.8788), only two of the regressors (operating days per month and average atmospheric temperature) are significant, so the model likely could use some alteration.

Figure 3.1: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model and a plot of the residuals from the full model versus the y-values the constructed model predicts.

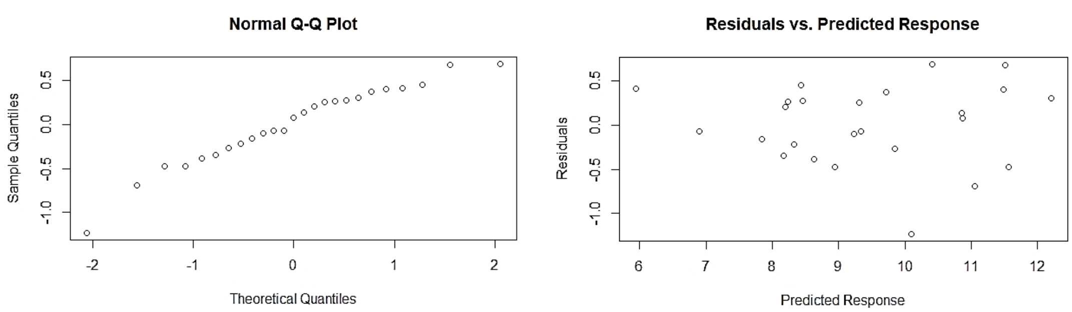


Table 3.1: The following is a table of the standardized residuals for the full model relating steam (lbs/month) to all of the provided regressor variables.

1	2	3	4	5	6
0.2542561	1.4748225	0.6363896	0.6141057	-0.1488749	0.6723499
7	**8**	**9**	**10**	**11**	**12**
1.1986003	0.5678766	-0.7316112	-0.2035156	-1.7552012	1.6696907
13	**14**	**15**	**16**	**17**	**18**
0.8503303	0.6887397	0.1596926	-0.5628649	0.8309716	-0.4543152
19	**20**	**21**	**22**	**23**	**24**
-0.2101913	1.1567576	-0.3102859	-1.1720115	-2.4195457	-1.3487168
25					
-1.0444930					

Table 3.2: The following is a table of the studentized residuals for the full model relating steam (lbs/month) to all of the provided regressor variables.

1	2	3	4	5	6
0.2461658	1.5409084	0.6232825	0.6008842	-0.1439332	0.6595664
7	**8**	**9**	**10**	**11**	**12**
1.2177407	0.5546151	-0.7197621	-0.1968867	-1.9022436	1.7877397
13	**14**	**15**	**16**	**17**	**18**
0.8420427	0.6761632	0.1544091	-0.5496144	0.8219363	-0.4419615
19	**20**	**21**	**22**	**23**	**24**
-0.2033638	1.1709832	-0.3007313	-1.1879701	-2.9935535	-1.3899885
25					
-1.0479030					

Table 3.3: The following is a table of the VIF values for the full model relating steam (lbs/month) to all of the provided regressor variables.

x1	x2	x3	x4	x5	x6
15.746595	20.137114	126.625618	1.836626	4.411920	4.695013
x7	**x8**	**x9**			
6.067426	107.590891	2.385046			

Figure 3.2: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except pounds of real fatty acid per month (x1), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x1 versus the residuals for the x1 explained by the other regressor variables.

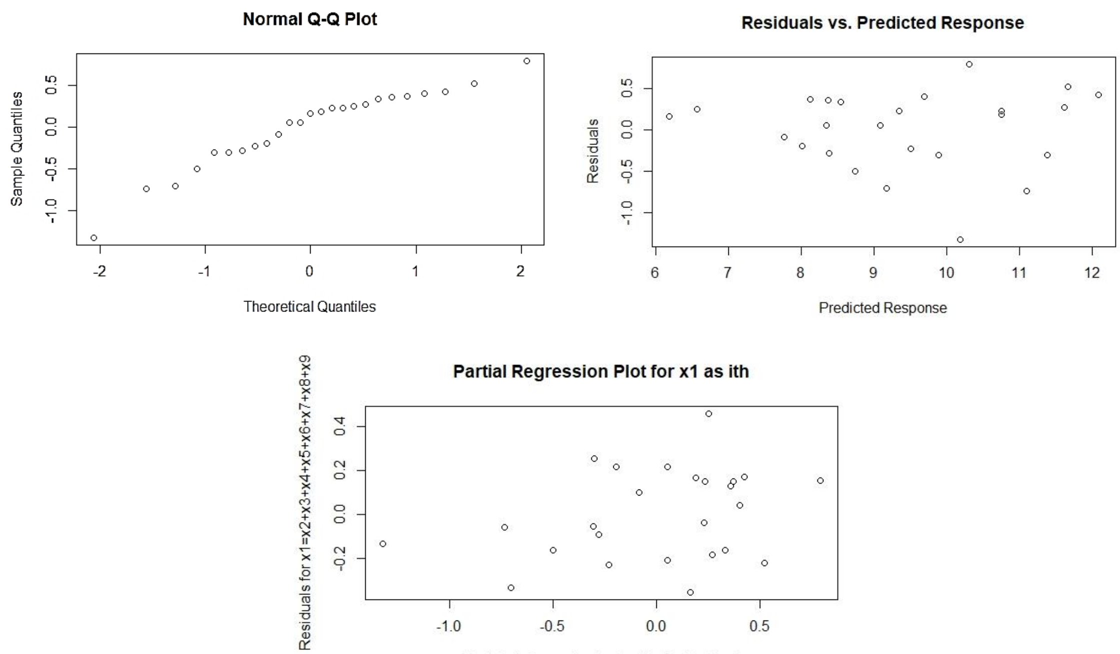


Table 3.4: The following is a table of the standardized residuals for the partial model without x1 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.4441977	1.6510694	0.8543183	0.1531863	-0.4543402	0.8720314
7	8	9	10	11	12
0.4103772	0.7668158	-0.3919954	0.1078524	-2.0623675	1.2142951
13	14	15	16	17	18
0.5545728	0.6053296	0.4001498	-0.6318953	0.8814951	-0.5757276
19	20	21	22	23	24
0.6058804	0.8247769	-0.1650782	-1.5598131	-2.5312539	-1.4006357
25					
-0.6138719					

Table 3.5: The following is a table of the studentized residuals for the partial model without x1 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.4327693	1.7551351	0.8467278	0.1484309	-0.4427785	0.8651502
7	8	9	10	11	12
0.3994538	0.7564971	-0.3813837	0.1044656	-2.3305308	1.2339697
13	14	15	16	17	18
0.5421991	0.5929368	0.3893967	-0.6196102	0.8750158	-0.5633113
19	20	21	22	23	24
0.5934890	0.8161244	-0.1599726	-1.6401235	-3.1652623	-1.4478207
25					
-0.6015045					

Table 3.6: The following is a table of the VIF values for the partial model without x1 relating steam (lbs/month) to all of the remaining regressor variables.

x2	x3	x4	x5	x6	x7
5.180428	117.669441	1.370684	4.024388	4.667488	6.064969
x8	**x9**				
100.986695	2.232906				

Figure 3.3: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except pounds of crude glycerin (x2), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x2 versus the residuals for the x2 explained by the other regressor variables.

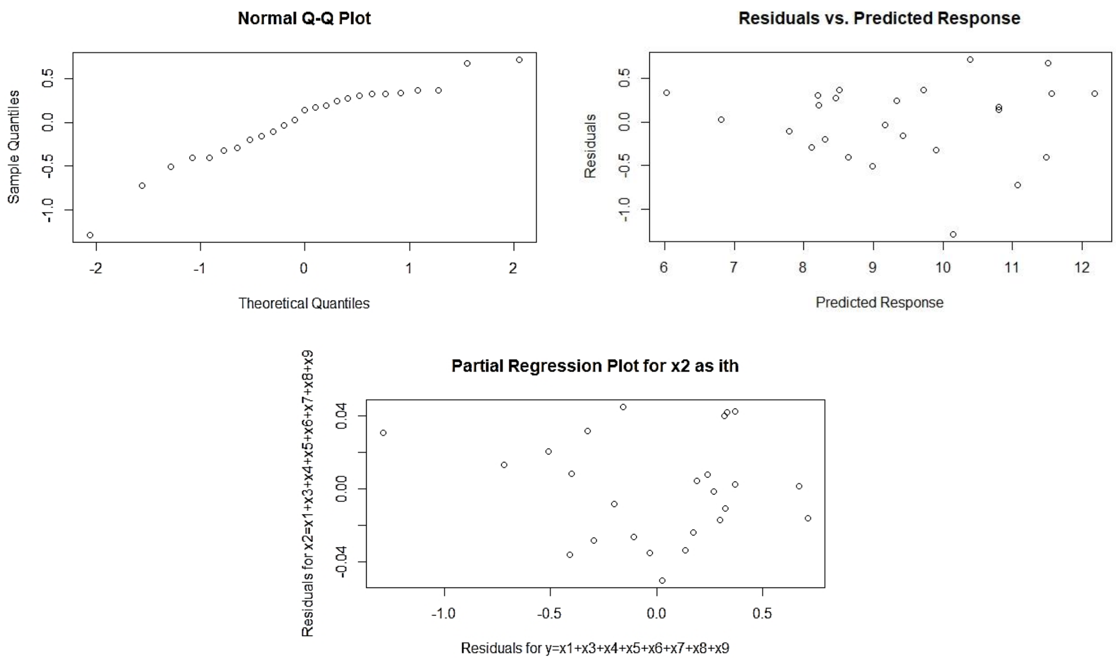


Table 3.7: The following is a table of the standardized residuals for the partial model without x2 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.34458364	1.56370197	0.69349876	0.60394268	-0.30824780	0.69710608
7	**8**	**9**	**10**	**11**	**12**
0.89015998	0.64587856	-0.61682186	-0.06514238	-1.85042861	1.70507243
13	**14**	**15**	**16**	**17**	**18**
0.66716441	0.66500086	0.29151391	-0.67997466	0.84178824	-0.43011399
19	**20**	**21**	**22**	**23**	**24**
0.07162692	0.89155551	-0.21381134	-1.27219570	-2.52041442	-1.42484640
25					
-0.87446700					

Table 3.8: The following is a table of the studentized residuals for the partial model without x2 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.33488660	1.64494915	0.68180255	0.59154650	-0.29934982	0.68545982
7	8	9	10	11	12
0.88406284	0.63368463	-0.60446533	-0.06308221	-2.02091565	1.82504421
13	14	15	16	17	18
0.65515642	0.65297132	0.28300970	-0.66810683	0.83372911	-0.41888477
19	20	21	22	23	24
0.06936359	0.88552121	-0.20731833	-1.29926334	-3.14275077	-1.47644878
25					
-0.86768771					

Table 3.9: The following is a table of the VIF for the partial model without x2 relating steam (lbs/month) to all of the remaining regressor variables.

x1	x3	x4	x5	x6	x7
4.050933	103.260596	1.650794	2.704032	4.665612	5.999323
x8	x9				
90.287806	2.384648				

Figure 3.4: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except average wind velocity in miles per hour (x3), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x3 versus the residuals for the x3 explained by the other regressor variables.

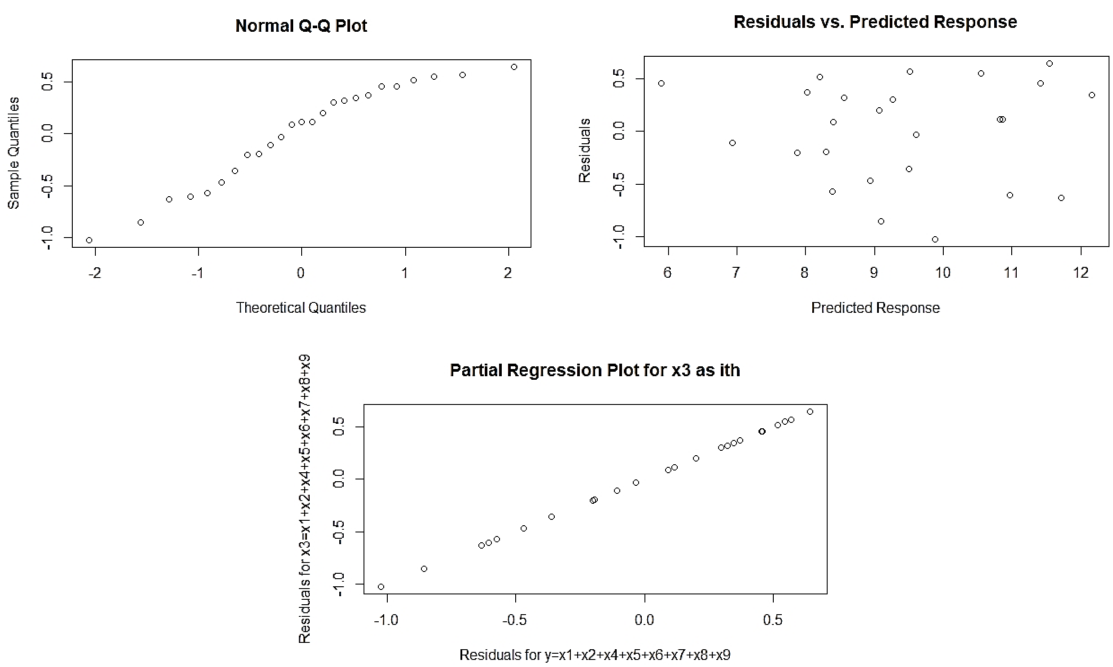


Table 3.10: The following is a table of the standardized residuals for the partial model without x3 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.21978832	1.11306319	0.69462526	1.04461250	0.37127291	1.15551286
7	**8**	**9**	**10**	**11**	**12**
1.26741150	0.18096145	-1.10497637	-0.67945367	-2.17941076	1.53195823
13	**14**	**15**	**16**	**17**	**18**
0.94586367	0.77027276	0.24323331	-0.06193097	1.17340759	-0.39424347
19	**20**	**21**	**22**	**23**	**24**
-0.32270450	0.78449261	-0.38018695	-1.12298900	-1.87259191	-1.12532471
25					
-1.29792337					

Table 3.11: The following is a table of the studentized residuals for the partial model without x3 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.2131311	1.1220345	0.6829445	1.0478032	0.3610420	1.1686447
7	**8**	**9**	**10**	**11**	**12**
1.2938309	0.1753948	-1.1132064	-0.6675797	-2.5165479	1.6057460
13	**14**	**15**	**16**	**17**	**18**
0.9425598	0.7600386	0.2359463	-0.0599716	1.1884327	-0.3835923
19	**20**	**21**	**22**	**23**	**24**
-0.3134791	0.7746255	-0.3697885	-1.1328924	-2.0518635	-1.1354510
25					
-1.3285962					

Table 3.12: The following is a table of the VIF values for the partial model without x3 relating steam (lbs/month) to all of the remaining regressor variables.

x1	x2	x4	x5	x6	x7	x8
14.632845	16.421404	1.824200	3.856707	4.694497	5.056370	1.867641
x9						
2.256168						

Figure 3.5: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except calendar days per month (x4), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x4 versus the residuals for the x4 explained by the other regressor variables.

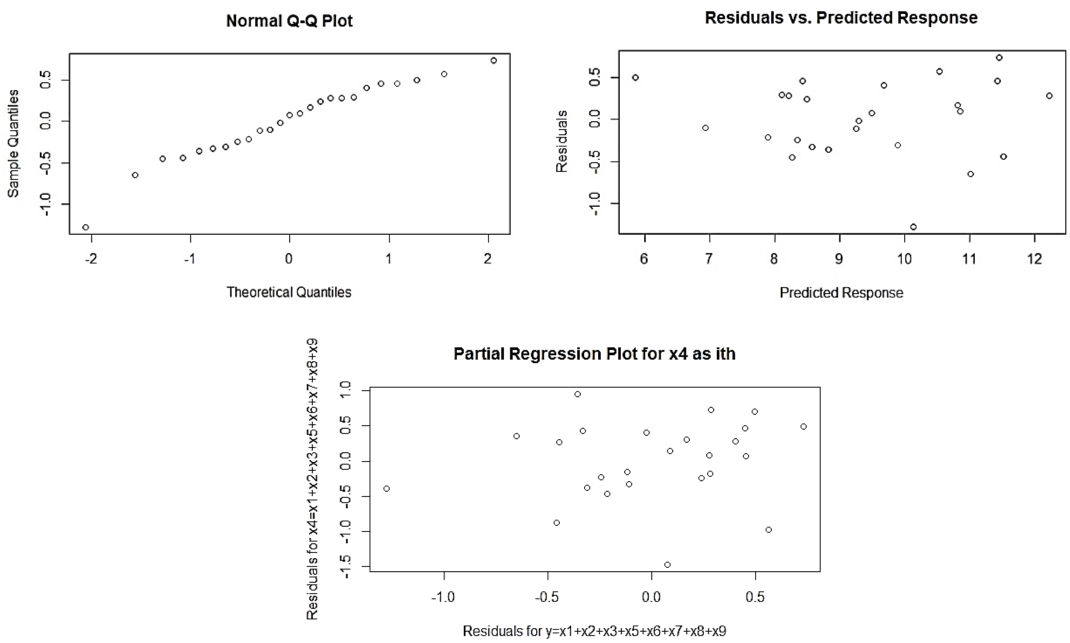


Table 3.13: The following is a table of the standardized residuals for the partial model without x4 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.33290107	1.14265177	0.59932418	0.82460700	-0.04766458	0.60577340
7	8	9	10	11	12
1.36514236	0.60142111	-0.91375395	-0.24626395	-1.43178824	1.80107286
13	14	15	16	17	18
0.97282224	0.16106991	0.20167227	-0.66457396	0.91734269	-0.52126578
19	20	21	22	23	24
-0.33453219	1.20181466	-0.42317941	-0.81231332	-2.53402695	-1.27840162
25					
-0.98515588					

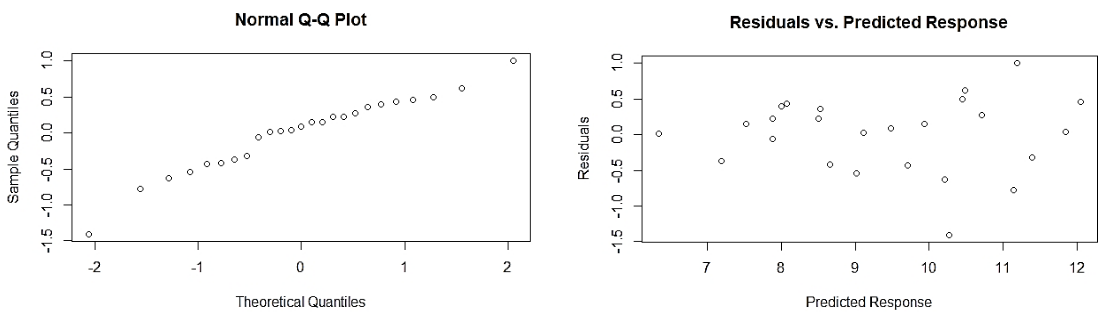
Table 3.14: The following is a table of the studentized residuals for the partial model without x4 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.32345220	1.15447460	0.58691850	0.81594885	-0.04615431	0.59338168
7	8	9	10	11	12
1.40622356	0.58901944	-0.90876778	-0.23889723	-1.48469617	1.95306785
13	14	15	16	17	18
0.97108823	0.15608186	0.19551699	-0.65254024	0.91253467	-0.50905441
19	20	21	22	23	24
-0.32504817	1.22002141	-0.41205416	-0.80325686	-3.17105230	-1.30632039
25					
-0.98418962					

Table 3.15: The following is a table of the VIF values for the partial model without x4 relating steam (lbs/month) to all of the remaining regressor variables.

x1	x2	x3	x5	x6	x7	x8	x9
11.751771	18.099620	125.768897	4.392236	4.634851	5.862202	107.419983	2.080472

Figure 3.6: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except operating days per month (x5), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x5 versus the residuals for the x5 explained by the other regressor variables.



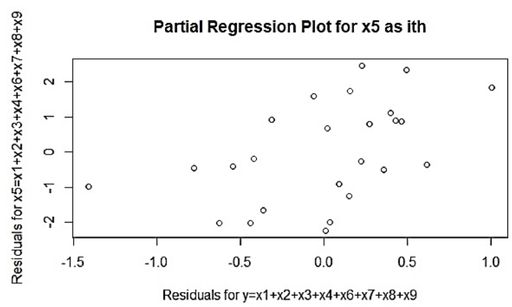


Table 3.16: The following is a table of the standardized residuals for the partial model without x5 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.47361134	1.18993889	0.85685321	1.06250175	-0.76465028	0.49306366
7	**8**	**9**	**10**	**11**	**12**
0.02267385	0.80864384	-0.10868656	0.04194676	-1.70069400	2.09083363
13	**14**	**15**	**16**	**17**	**18**
0.06206378	0.21576074	0.89235610	-1.12637653	0.29228189	0.39875392
19	**20**	**21**	**22**	**23**	**24**
-0.94037856	0.81973694	0.26525590	-1.20661706	-2.44826380	-1.34668784
25					
-0.60626942					

Table 3.17: The following is a table of the studentized residuals for the partial model without x5 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.46182082	1.20678884	0.84936084	1.06709696	-0.75427958	0.48107568
7	8	9	10	11	12
0.02195422	0.79947330	-0.10527418	0.04061701	-1.81932195	2.37467866
13	14	15	16	17	18
0.06010023	0.20921402	0.88635800	-1.13660364	0.28375928	0.38802469
19	20	21	22	23	24
-0.93677305	0.81091801	0.25739951	-1.22538351	-2.99759874	-1.38476469
25					
-0.59387897					

Table 3.18: The following is a table of the VIF residuals for the partial model without x5 relating steam (lbs/month) to all of the remaining regressor variables.

x1	x2	x3	x4	x6	x7
14.363452	12.341883	110.690568	1.828431	4.530146	5.840032
x8	x9				
97.334598	2.354898				

Figure 3.7: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except days below 32F (x6), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x6 versus the residuals for the x6 explained by the other regressor variables.

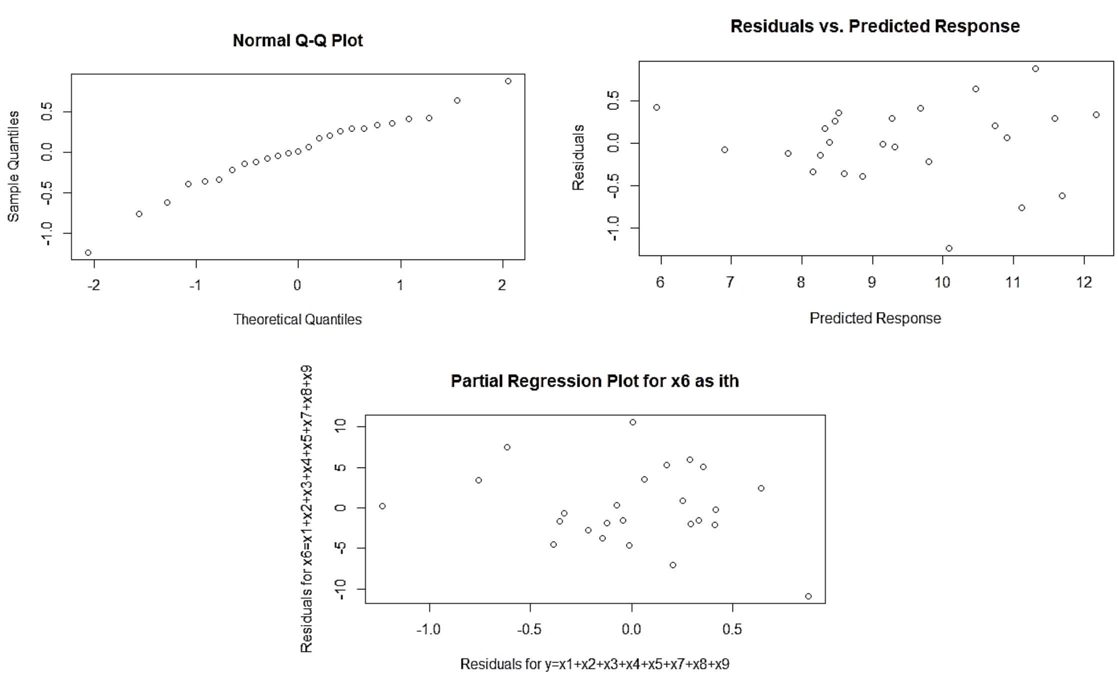


Table 3.19: The following is a table of the standardized residuals for the partial model without x6 relating steam (lbs/month) to all of the remaining regressor variables.

1	**2**	**3**	**4**	**5**	**6**
0.12579090	1.38574126	0.70218304	0.01215781	-0.08966443	0.63946578
7	**8**	**9**	**10**	**11**	**12**
1.22778901	0.35485753	-0.71384817	-0.02629473	-1.61570740	1.83234109
13	**14**	**15**	**16**	**17**	**18**
0.59561943	0.79049787	0.42062397	-0.45870909	0.92144628	-0.30746771
19	**20**	**21**	**22**	**23**	**24**
-0.22933923	0.88660864	-0.24655628	-0.94537103	-2.45962806	-1.47255796
25					
-1.26552346					

Table 3.20: The following is a table of the studentized residuals for the partial model without x6 relating steam (lbs/month) to all of the remaining regressor variables.

1	**2**	**3**	**4**	**5**	**6**
0.12185679	1.43031190	0.69061010	0.01177180	-0.08683903	0.62722706
7	8	9	10	11	12
1.24909999	0.34494943	-0.70245721	-0.02546032	-1.71011900	1.99588273
13	**14**	**15**	**16**	**17**	**18**
0.58320791	0.78079527	0.40953799	-0.44709273	0.91684497	-0.29858774
19	**20**	**21**	**22**	**23**	**24**
-0.22242264	0.88035321	-0.23918189	-0.94203979	-3.01994138	-1.53349456
25					
-1.29168898					

Table 3.21: The following is a table of the VIF values for the partial model without x6 relating steam (lbs/month) to all of the remaining regressor variables.

x1	**x2**	**x3**	**x4**	**x5**	**x7**
15.654279	20.011010	126.611716	1.813091	4.256994	2.707750
x8	**x9**				
107.589948	2.384853				

Figure 3.8: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except average atmospheric temperature (x7), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x7 versus the residuals for the x7 explained by the other regressor variables.

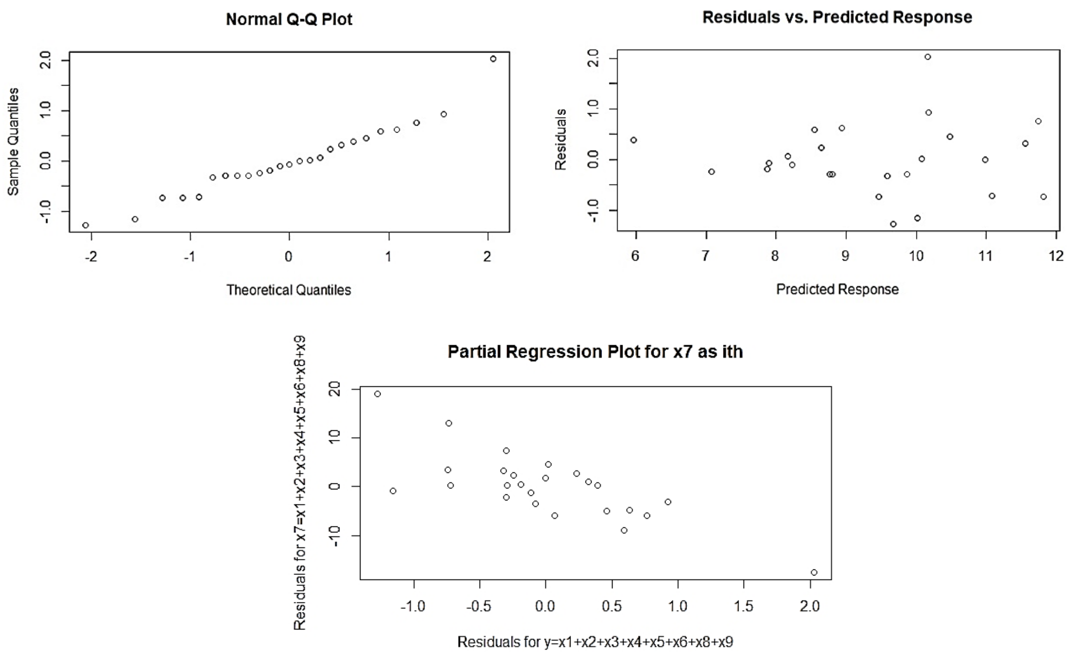


Table 3.22: The following is a table of the standardized residuals for the partial model without x7 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5
-0.004280291	1.308122679	1.027464977	-1.854430998	-0.435022623
6	7	8	9	10
-1.064714863	0.757797127	-0.404796745	-0.102563672	0.775142622
11	12	13	14	15
0.192660293	2.690707046	0.453713494	1.098732540	0.652335753
16	17	18	19	20
-0.402602580	0.024830698	-0.156057688	-0.495848853	0.398504864
21	22	23	24	25
-0.244553018	-0.486886978	-1.495983602	-0.917094485	-1.057002079

Table 3.23: The following is a table of the studentized residuals for the partial model without x7 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5
-0.004144376	1.340281420	1.029377425	-2.026481852	-0.423722146
6	7	8	9	10
-1.069488942	0.747266530	-0.393965308	-0.099339509	0.765030619
11	12	13	14	15
0.186759282	3.520931882	0.442159823	1.106401147	0.640192143
16	17	18	19	20
-0.391807939	0.024042683	-0.151217336	-0.483835440	0.387779906
21	22	23	24	25
-0.237231229	-0.474957952	-1.561820663	-0.912274062	-1.061157694

Table 3.24: The following is a table of the VIF values for the partial model without x7 relating steam (lbs/month) to all of the remaining regressor variables.

x1	x2	x3	x4	x5	x6
15.740218	19.911086	105.525135	1.774504	4.246571	2.095274

x8	x9
93.393373	2.377952

Figure 3.9: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except average wind velocity squared (x8), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x8 versus the residuals for the x8 explained by the other regressor variables.

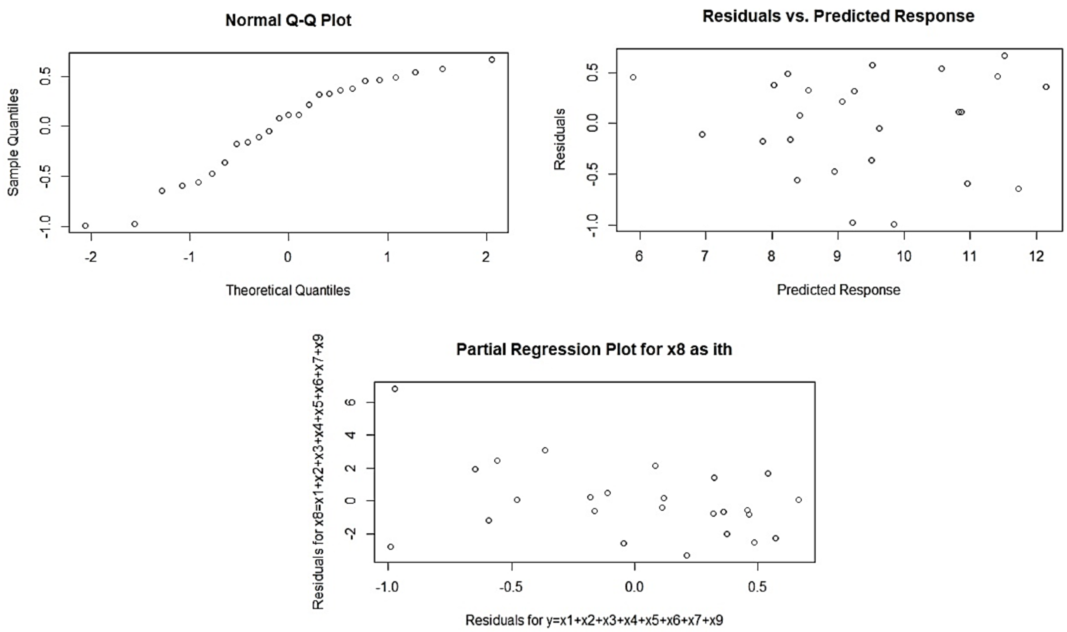


Table 3.25: The following is a table of the standardized residuals for the partial model without x8 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.21729176	1.08816727	0.71558471	1.04173314	0.39239001	1.09680110
7	**8**	**9**	**10**	**11**	**12**
1.26167965	0.16465417	-1.07432000	-0.68097205	-2.22223837	1.56615747
13	**14**	**15**	**16**	**17**	**18**
0.94789723	0.81749603	0.22764192	-0.08670867	1.16616172	-0.32307847
19	**20**	**21**	**22**	**23**	**24**
-0.32605162	0.77911432	-0.33327258	-1.12071787	-1.77808752	-1.08372175
25					
-1.30830038					

Table 3.26: The following is a table of the studentized residuals for the partial model without x8 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.21070296	1.09490739	0.70422246	1.04470461	0.38177135	1.10429795
7	8	9	10	11	12
1.28733150	0.15956096	-1.07988362	-0.66911603	-2.58777304	1.64799950
13	14	15	16	17	18
0.94470665	0.80860444	0.22077115	-0.08397504	1.18041000	-0.31384478
19	20	21	22	23	24
-0.31675218	0.76910468	-0.32381570	-1.13040583	-1.92195507	-1.09007920
25					
-1.34048530					

Table 3.27: The following is a table of the VIF values for the partial model without x8 relating steam (lbs/month) to all of the remaining regressor variables.

x1	x2	x3	x4	x5	x6	x7
14.780030	16.898604	2.198060	1.833708	3.991346	4.694972	5.266779
x9						
2.224572						

Figure 3.10: The following is a normal probability plot relating steam (lbs/month) to the regressor variables in the full model except number of starups (x9), a plot of the residuals from this model versus the y-values the constructed model predicts, and a partial regression plot of residuals for steam (lbs/month) explained by the all regressors except x9 versus the residuals for the x9 explained by the other regressor variables.

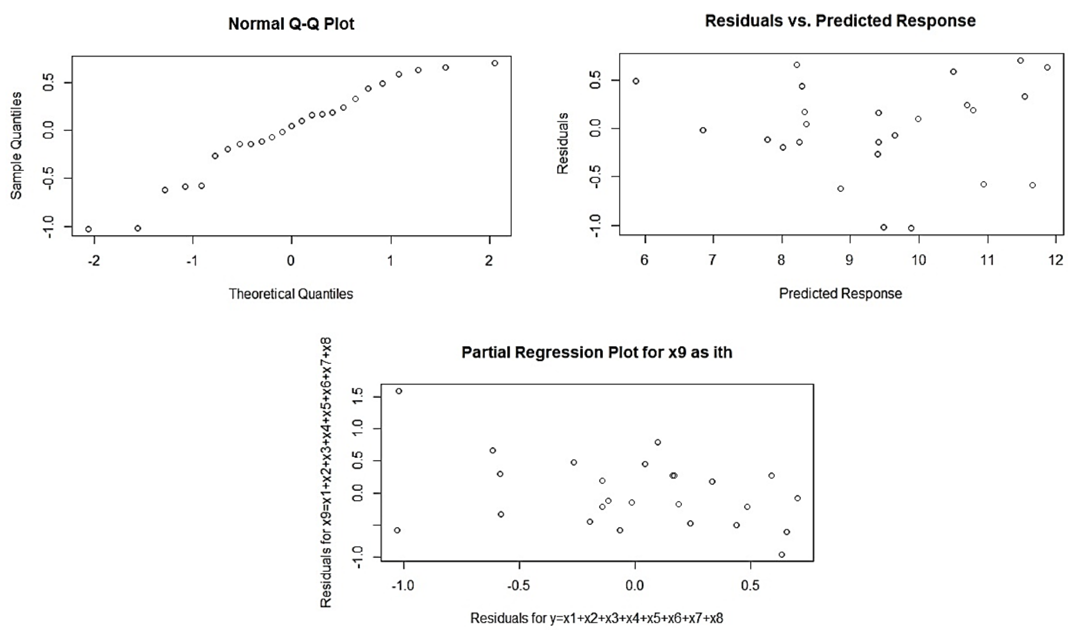


Table 3.28: The following is a table of the standardized residuals for the partial model without x9 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.35337483	1.20330596	1.16282131	0.12237207	-0.27203442	1.01269469
7	**8**	**9**	**10**	**11**	**12**
1.33820146	0.34627126	-0.37873464	-0.50954470	-2.24821617	1.65498827
13	**14**	**15**	**16**	**17**	**18**
0.67871459	0.41607185	0.48517966	-0.12808026	0.19531462	-0.28329663
19	**20**	**21**	**22**	**23**	**24**
-0.04110146	1.53347520	-0.21573584	-1.85710423	-1.86770605	-1.05948298
25					
-1.19638859					

Table 3.29: The following is a table of the studentized residuals for the partial model without x9 relating steam (lbs/month) to all of the remaining regressor variables.

1	2	3	4	5	6
0.34349676	1.22168583	1.17671617	0.11854173	-0.26400744	1.01355829
7	8	9	10	11	12
1.37493465	0.33653909	-0.36836314	-0.49741690	-2.63187596	1.76016016
13	14	15	16	17	18
0.66683201	0.40505710	0.47326755	-0.12407680	0.18933841	-0.27499134
19	20	21	22	23	24
-0.03979841	1.60760985	-0.20918981	-2.03020507	-2.04501474	-1.06383604
25					
-1.21397061					

Table 3.30: The following is a table of the VIF values for the partial model without x9 relating steam (lbs/month) to all of the remaining regressor variables.

x1	x2	x3	x4	x5	x6
14.742133	20.133751	119.783271	1.602086	4.356152	4.694632
x7	**x8**				
6.049379	100.351820				

Table 3.31: The following is a table of the PRESS statistics for all of the models represented above.

Model	**PRESS Statistic**
Full model	18.785
x1 ith	18.605
x2 ith	17.347
x3 ith	16.504
x4 ith	14.913
x5 ith	22.269
x6 ith	15.784
x7 ith	25.748
x8 ith	16.202
x9 ith	20.962

4) For analyzing model adequacy, figures have been provided with side-by-side comparison plots.

Figure 4.1: The following is a side-by-side comparison of all of the normality plots created in Question 3:

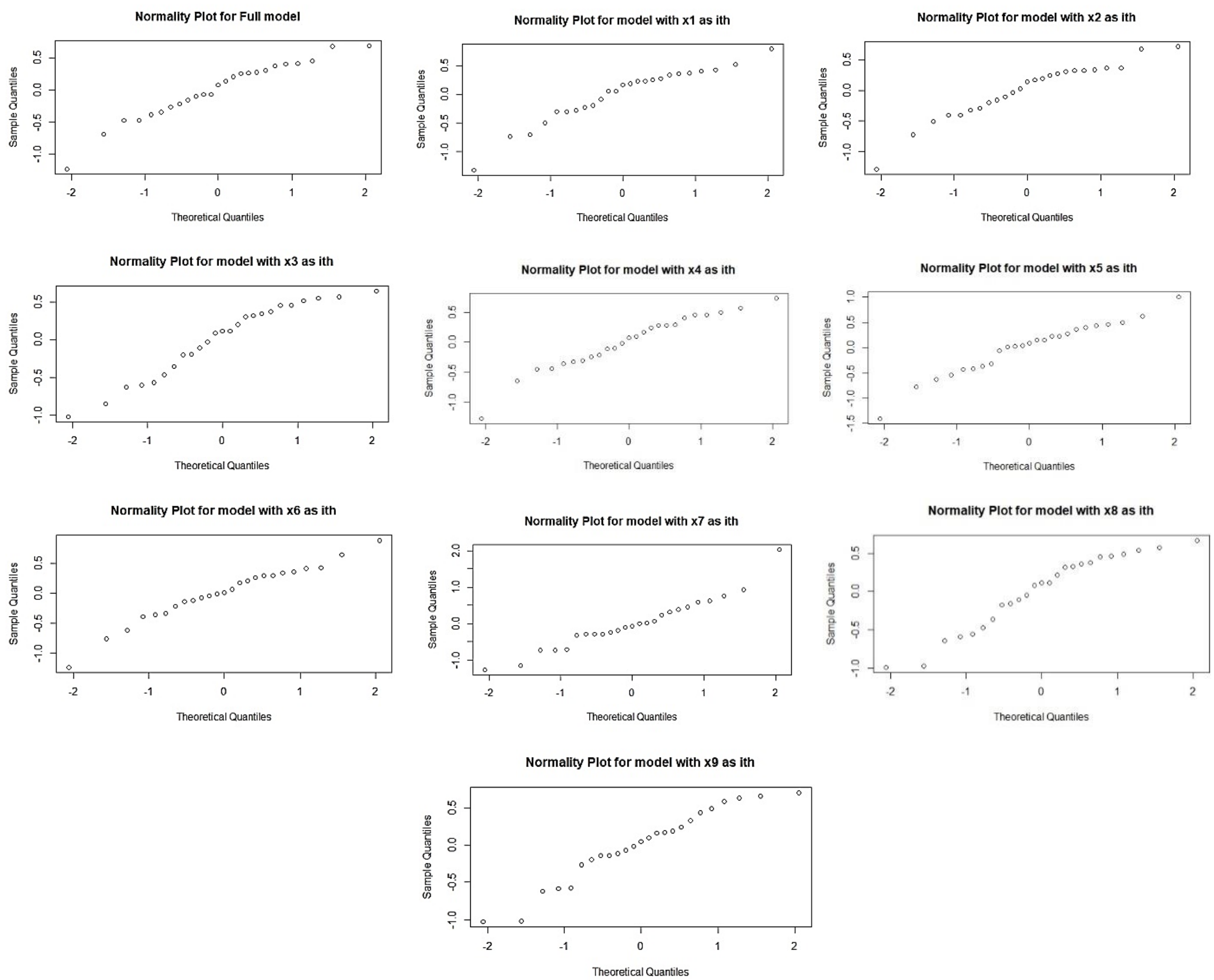


Figure 4.2: The following is a side-by-side comparison of all of the residual plots created in Question 3:

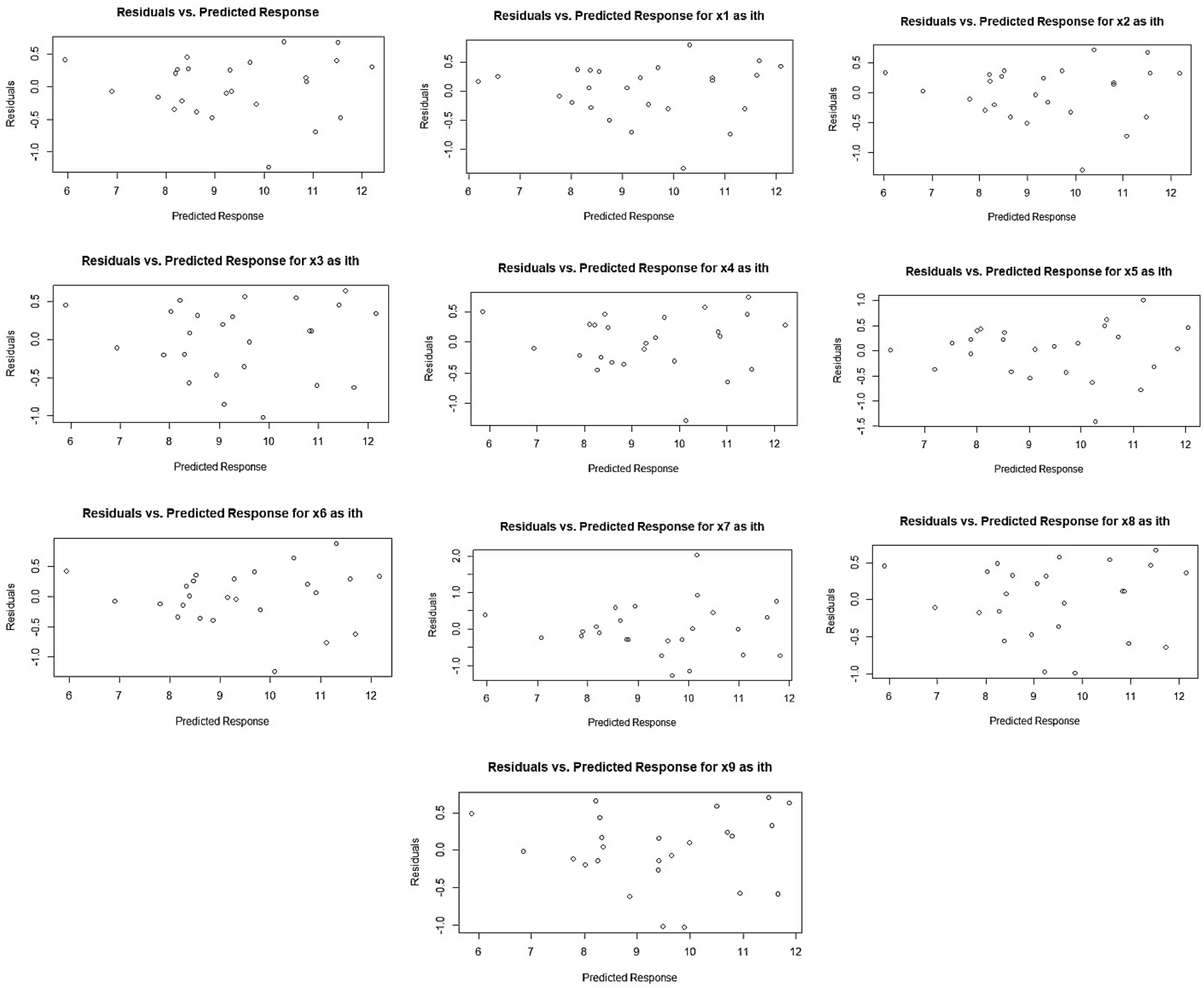
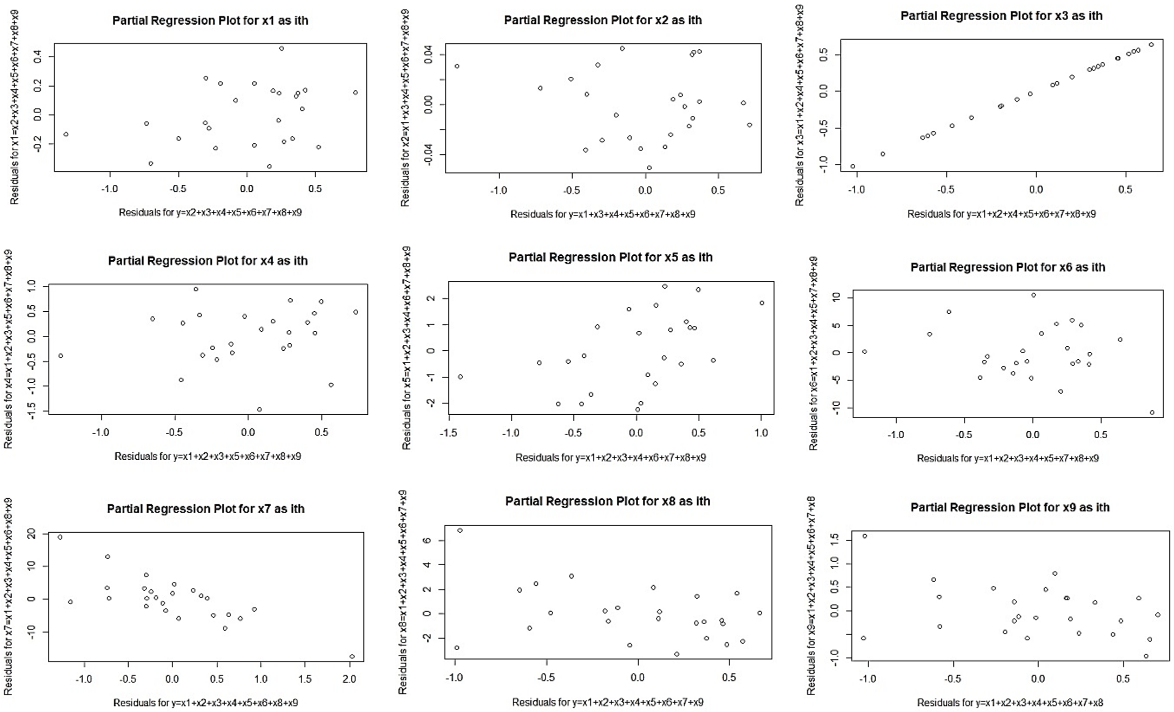


Figure 4.3: The following is a side-by-side comparison of all of the partial plots created in Question 3:



The normal probability plot of residuals, when the model follows the rules of normal distribution, independent distribution and equal variance, should have all the residuals fall on a straight line. When comparing the normality plots for the full model to the plots with each regressor removed individually, it appears that the full model normality plot (upper left corner) deviates more from a straight line when average wind velocity in miles per hour (x3), average wind velocity squared (x8) and number of startups (x9) are removed (Figure 4.1). This suggests that the model has a more-normal distribution when these regressors are present if they are taken out individually.

If the data set is normally distributed, independently distributed and has a constant variance, there will be no pattern in the data points in the constructed residuals vs. predicted response plot. The original model has a fairly randomly-distributed residual plot, suggesting little if any transformation to the model must be done (Figure 4.2, upper left corner). Removing regressors for partial regressions also does not really alter the residual plot, suggesting removal of regressors will not yield a model with a more-constant variance.

The standardized and studentized residuals are scaled residuals that can indicate if the data set is normally distributed and has a constant variance. If the absolute value of any of these residuals is greater than 3, it could indicate an outlier in the data set. An absolute value of a studentized residual or a R-student residual that is greater than 3 indicates the associated data point's y-coordinate is likely an outlier. None of the standardized and studentized residuals from the full model have an absolute value greater than 3 (though the studentized residual for the 23rd y value is very close, at 2.994) (Table 3.2). It is likely that there are no glaring outliers created by this model. The partial plots do not drastically reduce the standardized and studentized residuals either, though the removal of certain regressors individually are accompanied by a spike in certain residuals over 3 when the absolute value is taken and suggest the regressor should remain in the model to control for outliers (Tables 3.5, 3.8, 3.14, 3.20). However, these spikes are seen in the studentized residuals, which are much more conservative than the standardized residuals and are therefore more likely to consider a residual an outlier than the corresponding standardized residuals.

When considering PRESS residuals, the model that has the lowest PRESS residual when one regressor is removed is when the average wind velocity in miles per hour (x3) regressor is removed. However, the partial plot for removing this regressor shows that it is the regressor that, when removed individually, explains a great amount of the variance of the model after the other regressor variables are accounted for (Figures 3.4 and 4.3). Therefore, it should remain in the model. Other regressors, when removed individually, increase the PRESS statistic compared to the original model, such as calendar days per month (x4) and average atmospheric temperature in degrees F (x7) (Table 3.31). The higher PRESS statistic means that, when the individual regressor is removed, influential points in the data set affect the model more than when the regressor was present, and suggests that the model would be benefited by the regressor in question remaining in the model.

Finally, removal of regressor variables alters the VIF values differently depending on the regressor variable that is removed. All regressors have moderate to low collinearity in the full model except pounds of real fatty acid in storage per month (x1), pounds of crude glycerin made per month (x2), average wind velocity (x4) and average wind velocity squared (x8) (Table 3.3). These collinearites make sense when considering the regressors; if more real fatty acid is stored per month, then more glycerin can be made because glycerin is comprised primarily of fatty acids. When x1 is removed from the model, x2's VIF value drops from 20.137 to 5.180 (Tables 3.3 and 3.6). Similarly, when x2 is removed from the model, x1's VIF value drops from 15.747 to 4.051 (Tables 3.3 and 3.9). The collinearity between average wind velocity and average wind velocity squared is obvious; average wind velocity squared is simply average wind velocity multiplied by itself. When x8 is removed, x3's VIF value drops from 126.626 to 2.198 (Tables 3.3 and 3.27). Similarly, when x3 is removed, x8's VIF value drops from 107.591 to 1.868 (Tables 3.3 and 3.12). However, multicollinearity is not a good enough reason to remove a regressor from a model.

Based on these analyses, there is no definitive indication that any particular regressor should be removed from the model for the model to perform better. It could be argued that certain regressors must stay in the model for it to perform well, such as average wind velocity (x3) based on its partial regression plot, but there is no evidence that any regressor should absolutely be removed from a model (Figures 3.4 and 4.3).

Based on the original scatter plots, the regressors that show a pattern with steam (lbs/month) that is nonlinear are average wind velocity (x3) and average wind velocity squared (x8). Both plots show a slight parabolic curve, suggesting that squaring these regressors may be warranted. There is a slight parabolic curve to the pounds of crude glycerin made per month (x2) as well, so this may also warrant squaring as well. Finally, because of the large number of data points at zero days for the number of days below 32F (x6), there is a slight square-root-like curve to the scatterplot and could perhaps benefit from square-rooting that regressor. The transformed model that is a better fit for the data set is as follows: (**note: coefficients that correspond to regressors that are measured in the same units as the y variable (lbs/month) remain unitless in order to correctly relate the regressor to the y variable**):

Steam (lbs/month) = 0.022(x1) +1.724[lbs^{-1}*month](x2^2) +0.070[lbs*hr^2*$month^{-1}$*$miles^{-2}$](x3^2) + 0.206[lbs*$days^{-1}$](x4) +0.147[lbs*$days^{-1}$](x5) -0.246[lbs*$days^{-0.5}$*$month^{-1}$](sqrt[x6]) -0.091[lbs*°F^{-1}*$month^{-1}$](x7) -0.00063[lbs*hr^2*$miles^{-2}$*$month^{-1}$](x8^2) -0.291[lbs*number of $startups^{-1}$*$month^{-1}$](x9) + 4.232 (lbs/month)

The summary statistics for the new model are as follows:

```
Residuals:
     Min       1Q   Median       3Q      Max 
-1.07899 -0.20368  0.05293  0.26547  0.60527 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.2319065  5.5446609   0.763   0.4572    
x1           0.0221917  0.4810566   0.046   0.9638    
sqx2         1.7240965  2.2457111   0.768   0.4546    
sqx3         0.0698773  0.0269608   2.592   0.0204 *  
x4           0.2060570  0.1731591   1.190   0.2525    
x5           0.1473051  0.0631305   2.333   0.0340 *  
sqrtx6      -0.2455904  0.1387612  -1.770   0.0971 .  
x7          -0.0911187  0.0170326  -5.350  8.1e-05 ***
sqx8        -0.0006255  0.0002186  -2.861   0.0119 *  
x9          -0.2908944  0.1795418  -1.620   0.1260    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4971 on 15 degrees of freedom
Multiple R-squared:  0.9418,	Adjusted R-squared:  0.9069 
F-statistic: 26.98 on 9 and 15 DF,  p-value: 1.106e-07
```

Compared to the statistics for the original model, the adjusted R^2 value for the transformed model has increased, and more regressor coefficients have become significant for the model (x3 and x8 have become significant to an alpha level of 0.05, and x6 has become significant to an alpha value of 0.1).

5) Based on the results from the stepwise regression in R, the model of best-fit is as follows: (**note: coefficients that correspond to regressors that are measured in the same units as the y variable (lbs/month) remain unitless in order to correctly relate the regressor to the y variable**):

Steam (lbs/month) = 0.488(x1) +0.108[lbs*$days^{-1}$](x5) -0.076[lbs*°F^{-1}*$month^{-1}$](x7) + 8.556 (lbs/month)

The output and summary statistics for this model are as follows:

```
                                 Stepwise Selection Summary
-----------------------------------------------------------------------------
-------
                        Added/                            Adj.
Step    Variable        Removed      R-Square           R-Square      C(p)          AIC
RMSE
-----------------------------------------------------------------------------
-------
   1       x7         addition        0.714           0.701      52.7850     69.0538
0.8903
   2       x1         addition        0.860           0.847      17.1190     53.1954
0.6369
   3       x5         addition        0.880           0.862      14.0670     51.4289
0.6046
```

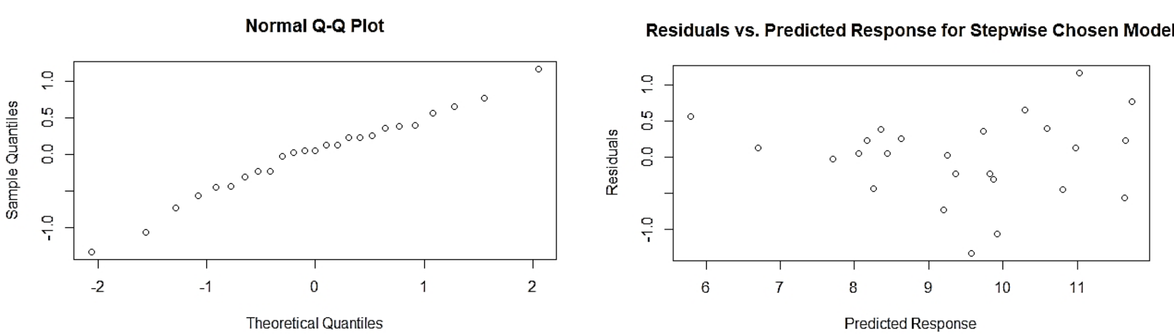
```
Call:
lm(formula = y ~ x1 + x5 + x7)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.33205 -0.30490  0.05466  0.35996  1.16505 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  8.55609    1.03675   8.253 4.98e-08 ***
x1           0.48842    0.21162   2.308   0.0313 *  
x5           0.10827    0.05859   1.848   0.0788 .  
x7          -0.07572    0.00746 -10.150 1.49e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6046 on 21 degrees of freedom
Multiple R-squared:  0.8795,	Adjusted R-squared:  0.8623 
F-statistic:  51.1 on 3 and 21 DF,  p-value: 7.988e-10
```

Figure 5.1: The following is a normal probability plot relating steam (lbs/month) to the pounds of real fatty acid per month (x1), operating days per month (x5) and average atmospheric temperature (F) in the stepwise model and a plot of the residuals from the full model versus the y-values the constructed model predicts.



Though the adjusted R^2 value is lower than the original model and the transformed model, it is still quite high and only uses three of the nine regressors of the model, which suggests it is a much more efficient model and uses the regressors that best explain the y variable. The normality plot is more linear than the original normality plot, and the residual plot is still fairly randomly distributed, suggesting a normally-distributed data set with independence and constant variance (Figure 5.1). To summarize, the computer is much better at predicting regressors to use in a model than I am.

Rcode with output:

library(car)

library(qpcR)

#Add Data

```
#Subset
> y=Data[,1]
> x1=Data[,2]
> x2=Data[,3]
> x3=Data[,4]
> x4=Data[,5]
> x5=Data[,6]
> x6=Data[,7]
> x7=Data[,8]
> x8=Data[,9]
> x9=Data[,10]
>
> #Question 1: Box Plots
> boxplot(y, main = "Pounds of Steam used Monthly", xlab = "y", ylab = "Pound
s")
> boxplot(x1, main = "Pounds of Real Fatty Acid in Storage per Month", xlab=
"x1", ylab = "Pounds per Month")
> boxplot(x2, main = "Pounds of Crude Glycerin", xlab = "x2", ylab = "Pounds"
)
> boxplot(x3, main = "Average Wind Velocity", xlab = "x3", ylab = "Avergae Wi
nd Velocity (mph)")
> boxplot(x4, main = "Calender Days per Month", xlab = "x4", ylab = "Days per
Month")
> boxplot(x5, main = "Operating Days per Month", xlab = "x5", ylab= "Days per
Month")
> boxplot(x6, main = "Days below 32F", xlab = "x6", ylab = "Days")
> boxplot(x7, main = "Avg Atmospheric Temperature", xlab = "x7", ylab = "Avg
Atmospheric Temp (F)")
> boxplot(x8, main = "Avg Wind Velocity squared", xlab = "x8", ylab = "Avg Wi
nd Velocity Squared (mph2)")
> boxplot(x9, main = "Number of Starups", xlab = "x9", ylab = "Number")


> #Question 2: Scatter Plots
> plot(x1, y, main = "Pounds of Steam used Monthly vs Pounds Real Fatty Acid"
, xlab= "Real Fatty Acid in Storage (lbs/month)", ylab = "Steam (lbs/month)")
> plot(x2, y, main = "Pounds of Steam used Monthly vs Crude Glycerin", xlab=
"Crude Glycerin (lbs)", ylab = "Steam (lbs/month)")
> plot(x3, y, main = "Pounds of Steam used Monthly vs Average Wind Velocity",
xlab= "Average Wind Velocity (mph)", ylab = "Steam (lbs/month)")
```

```
> plot(x4, y, main = "Pounds of Steam used Monthly vs Calender Days", xlab= "
Calender (days/month)", ylab = "Steam (lbs/month)")
> plot(x5, y, main = "Pounds of Steam used Monthly vs Operating Days per Mont
h", xlab= "Operating Days (days/month)", ylab = "Steam (lbs/month)")
> plot(x6, y, main = "Pounds of Steam used Monthly vs Days below 32F", xlab=
"Days below 32F (days)", ylab = "Steam (lbs/month)")
> plot(x7, y, main = "Pounds of Steam used Monthly vs Average Atmospheric Tem
p", xlab= "Average Atmospheric Temp (F)", ylab = "Steam (lbs/month)")
> plot(x8, y, main = "Pounds of Steam used Monthly vs Average Wind Velocity S
quared", xlab= "Average Wind Velocity Squared (mph2)", ylab = "Steam (lbs/mon
th)")
> plot(x9, y, main = "Pounds of Steam used Monthly vs Number of Startups", xl
ab= "Number of Startups (#)", ylab = "Steam (lbs/month)")


#Question 3: Linear Model
> lm.full=lm(y~x1+x2+x3+x4+x5+x6+x7+x8+x9)
> lm.full

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
    1.76116      0.70084     -1.86794      1.14038      0.12253
         x5           x6           x7           x8           x9
    0.17957     -0.01831     -0.07734     -0.08626     -0.34610

> summary(lm.full)

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.22921 -0.26565  0.07307  0.30513  0.68420

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.76116    6.96637   0.253 0.803847
x1           0.70084    0.56248   1.246 0.231880
x2          -1.86794    4.12852  -0.452 0.657421
x3           1.14038    0.74289   1.535 0.145591
x4           0.12253    0.20374   0.601 0.556546
x5           0.17957    0.08060   2.228 0.041619 *
x6          -0.01831    0.02440  -0.751 0.464557
x7          -0.07734    0.01652  -4.681 0.000295 ***
x8          -0.08626    0.05178  -1.666 0.116445
x9          -0.34610    0.20979  -1.650 0.119777
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5673 on 15 degrees of freedom
Multiple R-squared:  0.9242,   Adjusted R-squared:  0.8788
F-statistic: 20.33 on 9 and 15 DF,  p-value: 7.576e-07

>
> residfull=resid(lm.full)
> residfull
          1           2           3           4           5           6
 0.12971046  0.68420417  0.30512860  0.19897269 -0.07212726  0.26888236
          7           8           9          10          11          12
 0.41150181  0.26680258 -0.34906745 -0.09772008 -0.38567331  0.67409485
         13          14          15          16          17          18
```

```
 0.39507038  0.25510806  0.07307058 -0.26564789  0.37327741 -0.21481118
         19          20          21          22          23          24
-0.06727046  0.44771277 -0.15717316 -0.47223264 -1.22921459 -0.69490449
         25
-0.47769422
>
> predictlmfull=predict(lm.full)
> predictlmfull
        1         2         3         4         5         6         7
10.850290 10.415796 12.204871  8.201027  9.342127  8.461118  5.948498
        8         9        10        11        12        13        14
 8.233197  8.169067  9.237720  8.625673 11.515905 11.484930  9.314892
       15        16        17        18        19        20        21
10.866929  9.845648  9.716723  8.324811  6.897270  8.432287  7.837173
       22        23        24        25
 8.942233 10.089215 11.054904 11.557694
> plot(predictlmfull,residfull, main= "Residuals vs. Predicted Response", xla
b ="Predicted Response", ylab="Residuals")
> qqnorm(residfull, main = "Normality Plot for Full model")
>
> standfull=rstandard(lm.full)
> standfull
         1          2          3          4          5          6
 0.2542561  1.4748225  0.6363896  0.6141057 -0.1488749  0.6723499
         7          8          9         10         11         12
 1.1986003  0.5678766 -0.7316112 -0.2035156 -1.7552012  1.6696907
        13         14         15         16         17         18
 0.8503303  0.6887397  0.1596926 -0.5628649  0.8309716 -0.4543152
        19         20         21         22         23         24
-0.2101913  1.1567576 -0.3102859 -1.1720115 -2.4195457 -1.3487168
        25
-1.0444930
> studentfull=rstudent(lm.full)
> studentfull
         1          2          3          4          5          6
 0.2461658  1.5409084  0.6232825  0.6008842 -0.1439332  0.6595664
         7          8          9         10         11         12
 1.2177407  0.5546151 -0.7197621 -0.1968867 -1.9022436  1.7877397
        13         14         15         16         17         18
 0.8420427  0.6761632  0.1544091 -0.5496144  0.8219363 -0.4419615
        19         20         21         22         23         24
-0.2033638  1.1709832 -0.3007313 -1.1879701 -2.9935535 -1.3899885
        25
-1.0479030
> vif(lm.full)
        x1         x2         x3         x4         x5         x6
 15.746595  20.137114 126.625618   1.836626   4.411920   4.695013
        x7         x8         x9
  6.067426 107.590891   2.385046
> PRESS(lm.full)
.........10.........20.....
$stat
[1] 18.78545

$residuals
 [1]  0.16039785  1.02311518  0.42716331  0.60999105 -0.09889503
 [6]  0.54107699  1.12358967  0.38899908 -0.49349421 -0.13640873
[11] -2.57078019  1.33101141  0.58902199  0.59843466  0.11232023
[16] -0.38382449  0.59534866 -0.30923483 -0.21136629  0.96186910
[21] -0.19714075 -0.93613495 -1.53274768 -0.84245581 -0.73500760

$P.square
[1] 0.7051611
```

```
>
> #Partial x1 ith
> lm.x1=lm(y~x2+x3+x4+x5+x6+x7+x8+x9)
> lm.x1

Call:
lm(formula = y ~ x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)          x2           x3           x4           x5
   -1.91505      2.56535      1.38655      0.25039      0.14980
         x6           x7           x8           x9
   -0.02064     -0.07775     -0.10224     -0.28008

> lm.x1ith=lm(x1~x2+x3+x4+x5+x6+x7+x8+x9)
> lm.x1ith

Call:
lm(formula = x1 ~ x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)          x2           x3           x4           x5
 -5.2454368    6.3256737    0.3512527    0.1824399   -0.0424695
         x6           x7           x8           x9
 -0.0033219   -0.0005911   -0.0228058    0.0942014

> resid.x1=resid(lm.x1)
> resid.x1
          1           2           3           4           5           6
 0.23354461  0.79268140  0.42492442  0.05365364 -0.23156537  0.36067629
          7           8           9          10          11          12
 0.16533581  0.37234436 -0.19640345  0.05434779 -0.49858432  0.52116620
         13          14          15          16          17          18
 0.26828176  0.22843196  0.19011488 -0.30397607  0.40333156 -0.27849061
         19          20          21          22          23          24
 0.25294612  0.33364620 -0.08559584 -0.70416688 -1.32193831 -0.73547262
         25
-0.29923352
> resid.x1ith=resid(lm.x1ith)
> resid.x1ith
          1           2           3           4           5           6
 0.14815643  0.15478143  0.17093144 -0.20734944 -0.22749529  0.13097676
          7           8           9          10          11          12
-0.35124356  0.15059297  0.21782961  0.21697903 -0.16110780 -0.21820724
         13          14          15          16          17          18
-0.18090915 -0.03806297  0.16700542 -0.05468881  0.04288295 -0.09086140
         19          20          21          22          23          24
 0.45690310 -0.16275662  0.10213056 -0.33093687 -0.13230344 -0.05788491
         25
 0.25463780
> plot(resid.x1,resid.x1ith,main="Partial Regression Plot for x1 as ith", xla
b= "Residuals for y=x2+x3+x4+x5+x6+x7+x8+x9", ylab= "Residuals for x1=x2+x3+x
4+x5+x6+x7+x8+x9")
> qqnorm(resid.x1, main = "Normality Plot for model with x1 as ith")
>
> summary(lm.x1)

Call:
lm(formula = y ~ x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3219 -0.2785  0.1653  0.3337  0.7927
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.91505    6.41875  -0.298 0.769273    
x2           2.56535    2.12985   1.204 0.245922    
x3           1.38655    0.72840   1.904 0.075113 .  
x4           0.25039    0.17902   1.399 0.180993    
x5           0.14980    0.07830   1.913 0.073782 .  
x6          -0.02064    0.02475  -0.834 0.416490    
x7          -0.07775    0.01680  -4.628 0.000279 ***
x8          -0.10224    0.05102  -2.004 0.062304 .  
x9          -0.28008    0.20647  -1.357 0.193759    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.577 on 16 degrees of freedom
Multiple R-squared:  0.9164,	Adjusted R-squared:  0.8746 
F-statistic: 21.92 on 8 and 16 DF,  p-value: 3.126e-07

> predictlm.x1=predict(lm.x1)
> predictlm.x1
        1         2         3         4         5         6         7 
10.746455 10.307319 12.085076  8.346346  9.501565  8.369324  6.194664 
        8         9        10        11        12        13        14 
 8.127656  8.016403  9.085652  8.738584 11.668834 11.611718  9.341568 
       15        16        17        18        19        20        21 
10.749885  9.883976  9.686668  8.388491  6.577054  8.546354  7.765596 
       22        23        24        25 
 9.174167 10.181938 11.095473 11.379234 
> plot(predictlm.x1,resid.x1, main= "Residuals vs. Predicted Response for x1 
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx1=rstandard(lm.x1)
> standx1
         1          2          3          4          5          6 
 0.4441977  1.6510694  0.8543183  0.1531863 -0.4543402  0.8720314 
         7          8          9         10         11         12 
 0.4103772  0.7668158 -0.3919954  0.1078524 -2.0623675  1.2142951 
        13         14         15         16         17         18 
 0.5545728  0.6053296  0.4001498 -0.6318953  0.8814951 -0.5757276 
        19         20         21         22         23         24 
 0.6058804  0.8247769 -0.1650782 -1.5598131 -2.5312539 -1.4006357 
        25 
-0.6138719 
> studentx1=rstudent(lm.x1)
> studentx1
         1          2          3          4          5          6 
 0.4327693  1.7551351  0.8467278  0.1484309 -0.4427785  0.8651502 
         7          8          9         10         11         12 
 0.3994538  0.7564971 -0.3813837  0.1044656 -2.3305308  1.2339697 
        13         14         15         16         17         18 
 0.5421991  0.5929368  0.3893967 -0.6196102  0.8750158 -0.5633113 
        19         20         21         22         23         24 
 0.5934890  0.8161244 -0.1599726 -1.6401235 -3.1652623 -1.4478207 
        25 
-0.6015045 
> vif(lm.x1)
        x2         x3         x4         x5         x6         x7 
  5.180428 117.669441   1.370684   4.024388   4.667488   6.064969 
        x8         x9 
100.986695   2.232906 
> PRESS(lm.x1)
.........10.........20.....
$stat
[1] 18.60529
```

```
$residuals
 [1]  0.28129156  1.14500099  0.57187560  0.14561780 -0.29679908
 [6]  0.70197309  0.33913559  0.52578859 -0.26048733  0.07126088
[11] -2.84032033  0.94198950  0.38168032  0.53407329  0.28041606
[16] -0.43734548  0.64143325 -0.39627567  0.48319193  0.67882932
[21] -0.10599872 -1.15038550 -1.61374196 -0.88809147 -0.41929439

$P.square
[1] 0.7079888

> 
> #Partial x2 ith
> lm.x2=lm(y~x1+x3+x4+x5+x6+x7+x8+x9)
> lm.x2

Call:
lm(formula = y ~ x1 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x3           x4           x5  
    0.65648      0.48151      1.28476      0.15185      0.15688  
         x6           x7           x8           x9  
   -0.01744     -0.07655     -0.09566     -0.34487  

> lm.x2ith=lm(x2~x1+x3+x4+x5+x6+x7+x8+x9)
> lm.x2ith

Call:
lm(formula = x2 ~ x1 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x3           x4           x5  
  0.5913909    0.1174171   -0.0772954   -0.0156973    0.0121469  
         x6           x7           x8           x9  
 -0.0004677   -0.0004240    0.0050293   -0.0006567  

> resid.x2=resid(lm.x2)
> resid.x2
          1           2           3           4           5           6 
 0.17457402  0.71451258  0.32557851  0.19103415 -0.15583568  0.27180123 
          7           8           9          10          11          12 
 0.33342933  0.29923543 -0.29556999 -0.03186179 -0.40111895  0.67114083 
         13          14          15          16          17          18 
 0.32057940  0.24096869  0.13588405 -0.32450889  0.36871177 -0.19880378 
         19          20          21          22          23          24 
 0.02663444  0.36888969 -0.10798310 -0.51070158 -1.28678190 -0.71968254 
         25 
-0.41012592 
> resid.x2ith=resid(lm.x2ith)
> resid.x2ith
           1            2            3            4            5 
-0.024017670 -0.016225579 -0.010947847  0.004249893  0.044813234 
           6            7            8            9           10 
-0.001562614  0.041796044 -0.017362898 -0.028639822 -0.035257186 
          11           12           13           14           15 
 0.008268808  0.001581436  0.039878686  0.007569498 -0.033627143 
          16           17           18           19           20 
 0.031511193  0.002444214 -0.008569550 -0.050271917  0.042197878 
          21           22           23           24           25 
-0.026333860  0.020594313  0.030818615  0.013264913 -0.036172639 
> plot(resid.x2,resid.x2ith,main="Partial Regression Plot for x2 as ith", xla
b= "Residuals for y=x1+x3+x4+x5+x6+x7+x8+x9", ylab= "Residuals for x2=x1+x3+x
4+x5+x6+x7+x8+x9")
> qqnorm(resid.x2, main = "Normality Plot for model with x2 as ith")
```

```
>
> summary(lm.x2)

Call:
lm(formula = y ~ x1 + x3 + x4 + x5 + x6 + x7 + x8 + x9)

Residuals:
    Min      1Q  Median      3Q     Max
-1.2868 -0.2956  0.1359  0.3206  0.7145

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.65648    6.36028   0.103 0.919074
x1           0.48151    0.27811   1.731 0.102620
x3           1.28476    0.65398   1.965 0.067079 .
x4           0.15185    0.18829   0.806 0.431801
x5           0.15688    0.06151   2.550 0.021388 *
x6          -0.01744    0.02371  -0.735 0.472698
x7          -0.07655    0.01601  -4.780 0.000205 ***
x8          -0.09566    0.04624  -2.069 0.055118 .
x9          -0.34487    0.20450  -1.686 0.111103
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.553 on 16 degrees of freedom
Multiple R-squared:  0.9232,   Adjusted R-squared:  0.8848
F-statistic: 24.04 on 8 and 16 DF,  p-value: 1.616e-07

> predictlm.x2=predict(lm.x2)
> predictlm.x2
        1         2         3         4         5         6         7
10.805426 10.385487 12.184421  8.208966  9.425836  8.458199  6.026571
        8         9        10        11        12        13        14
 8.200765  8.115570  9.171862  8.641119 11.518859 11.559421  9.329031
       15        16        17        18        19        20        21
10.804116  9.904509  9.721288  8.308804  6.803366  8.511110  7.787983
       22        23        24        25
 8.980702 10.146782 11.079683 11.490126
> plot(predictlm.x2,resid.x2, main= "Residuals vs. Predicted Response for x2
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx2=rstandard(lm.x2)
> standx2
          1           2           3           4           5           6
 0.34458364  1.56370197  0.69349876  0.60394268 -0.30824780  0.69710608
          7           8           9          10          11          12
 0.89015998  0.64587856 -0.61682186 -0.06514238 -1.85042861  1.70507243
         13          14          15          16          17          18
 0.66716441  0.66500086  0.29151391 -0.67997466  0.84178824 -0.43011399
         19          20          21          22          23          24
 0.07162692  0.89155551 -0.21381134 -1.27219570 -2.52041442 -1.42484640
         25
-0.87446700
> studentx2=rstudent(lm.x2)
> studentx2
          1           2           3           4           5           6
 0.33488660  1.64494915  0.68180255  0.59154650 -0.29934982  0.68545982
          7           8           9          10          11          12
 0.88406284  0.63368463 -0.60446533 -0.06308221 -2.02091565  1.82504421
         13          14          15          16          17          18
 0.65515642  0.65297132  0.28300970 -0.66810683  0.83372911 -0.41888477
         19          20          21          22          23          24
 0.06936359  0.88552121 -0.20731833 -1.29926334 -3.14275077 -1.47644878
         25
-0.86768771
```

```
> vif(lm.x2)
        x1         x3         x4         x5         x6         x7 
  4.050933 103.260596   1.650794   2.704032   4.665612   5.999323 
        x8         x9 
 90.287806   2.384648 
> PRESS(lm.x2)
.........10.........20.....
$stat
[1] 17.34658

$residuals
 [1]  0.20801686  1.04661496  0.45177739  0.58394141 -0.18647556
 [6]  0.54680839  0.72681049  0.42636110 -0.39368424 -0.04073296
[11] -2.61072029  1.32483215  0.42463811  0.56127105  0.19126651
[16] -0.43576034  0.58777020 -0.28459768  0.05891118  0.65900561
[21] -0.12947757 -0.96923591 -1.50982920 -0.86274804 -0.57024107

$P.square
[1] 0.7277444

> 
> #Partial x3 ith
> lm.x3=lm(y~x1+x2+x4+x5+x6+x7+x8+x9)
> lm.x3

Call:
lm(formula = y ~ x1 + x2 + x4 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x4           x5  
   6.356212     0.930473    -4.590272     0.096805     0.223461  
         x6           x7           x8           x9  
  -0.018707    -0.087690    -0.007475    -0.271240  

> lm.x3ith=lm(x3~x1+x2+x4+x5+x6+x7+x8+x9)
> lm.x3ith

Call:
lm(formula = x3 ~ x1 + x2 + x4 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x4           x5  
  4.0293949    0.2013638   -2.3872116   -0.0225582    0.0384888  
         x6           x7           x8           x9  
 -0.0003442   -0.0090780    0.0690877    0.0656458  

> resid.x3=resid(lm.x3)
> resid.x3
          1           2           3           4           5           6 
 0.11679845  0.54762053  0.34745389  0.37361302  0.19944411  0.52000836 
          7           8           9          10          11          12 
 0.45471574  0.09113951 -0.57452882 -0.36078770 -0.85678713  0.64488688 
         13          14          15          16          17          18 
 0.45957264  0.29799958  0.11613558 -0.03198664  0.57114445 -0.19422652 
         19          20          21          22          23          24 
-0.10793624  0.32311824 -0.20089488 -0.47127023 -1.02443718 -0.60755376 
         25 
-0.63324186 
> resid.x3ith=resid(lm.x3ith)
> resid.x3
          1           2           3           4           5           6 
 0.11679845  0.54762053  0.34745389  0.37361302  0.19944411  0.52000836 
          7           8           9          10          11          12 
 0.45471574  0.09113951 -0.57452882 -0.36078770 -0.85678713  0.64488688 
```

```
          13          14          15          16          17          18
 0.45957264  0.29799958  0.11613558 -0.03198664  0.57114445 -0.19422652
          19          20          21          22          23          24
-0.10793624  0.32311824 -0.20089488 -0.47127023 -1.02443718 -0.60755376
          25
-0.63324186
> plot(resid.x3,resid.x3,main="Partial Regression Plot for x3 as ith", xlab=
"Residuals for y=x1+x2+x4+x5+x6+x7+x8+x9", ylab= "Residuals for x3=x1+x2+x4+x
5+x6+x7+x8+x9")
> qqnorm(resid.x3, main = "Normality Plot for model with x3 as ith")
>
> summary(lm.x3)

Call:
lm(formula = y ~ x1 + x2 + x4 + x5 + x6 + x7 + x8 + x9)

Residuals:
    Min      1Q  Median      3Q     Max
-1.0244 -0.3608  0.1161  0.3736  0.6449

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.356212   6.551668   0.970   0.3464
x1           0.930473   0.564739   1.648   0.1189
x2          -4.590272   3.883029  -1.182   0.2544
x4           0.096805   0.211479   0.458   0.6533
x5           0.223461   0.078489   2.847   0.0117 *
x6          -0.018707   0.025415  -0.736   0.4723
x7          -0.087690   0.015708  -5.583 4.12e-05 ***
x8          -0.007475   0.007105  -1.052   0.3084
x9          -0.271240   0.212519  -1.276   0.2201
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5909 on 16 degrees of freedom
Multiple R-squared:  0.9123,   Adjusted R-squared:  0.8685
F-statistic: 20.81 on 8 and 16 DF,  p-value: 4.516e-07

> predictlm.x3=predict(lm.x3)
> predictlm.x3
        1         2         3         4         5         6         7
10.863202 10.552379 12.162546  8.026387  9.070556  8.209992  5.905284
        8         9        10        11        12        13        14
 8.408860  8.394529  9.500788  9.096787 11.545113 11.420427  9.272000
       15        16        17        18        19        20        21
10.823864  9.611987  9.518856  8.304227  6.937936  8.556882  7.880895
       22        23        24        25
 8.941270  9.884437 10.967554 11.713242
> plot(predictlm.x3,resid.x3, main= "Residuals vs. Predicted Response for x3
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx3=rstandard(lm.x3)
> standx3
          1           2           3           4           5           6
 0.21978832  1.11306319  0.69462526  1.04461250  0.37127291  1.15551286
          7           8           9          10          11          12
 1.26741150  0.18096145 -1.10497637 -0.67945367 -2.17941076  1.53195823
          13          14          15          16          17          18
 0.94586367  0.77027276  0.24323331 -0.06193097  1.17340759 -0.39424347
          19          20          21          22          23          24
-0.32270450  0.78449261 -0.38018695 -1.12298900 -1.87259191 -1.12532471
          25
-1.29792337
> studentx3=rstudent(lm.x3)
> studentx3
```

```
        1         2         3         4         5         6 
0.2131311  1.1220345  0.6829445  1.0478032  0.3610420  1.1686447 
        7         8         9        10        11        12 
1.2938309  0.1753948 -1.1132064 -0.6675797 -2.5165479  1.6057460 
       13        14        15        16        17        18 
0.9425598  0.7600386  0.2359463 -0.0599716  1.1884327 -0.3835923 
       19        20        21        22        23        24 
-0.3134791  0.7746255 -0.3697885 -1.1328924 -2.0518635 -1.1354510 
       25 
-1.3285962 
> vif(lm.x3)
       x1        x2        x4        x5        x6        x7        x8 
14.632845 16.421404  1.824200  3.856707  4.694497  5.056370  1.867641 
       x9 
 2.256168 
> PRESS(lm.x3)
.........10.........20.....
$stat
[1] 16.50452

$residuals
 [1]  0.14439181  0.78982402  0.48481314  1.01966816  0.24128799
 [6]  0.89641612  1.23329143  0.12543995 -0.74193281 -0.44672309
[11] -1.93542198  1.27051773  0.67963126  0.69509449  0.17784892
[16] -0.04186178  0.84163201 -0.27937715 -0.33683186  0.66494643
[21] -0.25118631 -0.93422484 -1.19500953 -0.72768164 -0.92875018

$P.square
[1] 0.7409605

> 
> #Partial x4 ith
> lm.x4=lm(y~x1+x2+x3+x5+x6+x7+x8+x9)
> lm.x4

Call:
lm(formula = y ~ x1 + x2 + x3 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x5  
    5.53799      0.87123     -2.65774      1.10363      0.18281  
         x6           x7           x8           x9  
   -0.01998     -0.07916     -0.08502     -0.39119  

> lm.x4ith=lm(x4~x1+x2+x3+x5+x6+x7+x8+x9)
> lm.x4ith

Call:
lm(formula = x4 ~ x1 + x2 + x3 + x5 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x5  
   30.82361      1.39056     -6.44574     -0.29993      0.02643  
         x6           x7           x8           x9  
   -0.01356     -0.01491      0.01013     -0.36797  

> resid.x4=resid(lm.x4)
> resid.x4
          1           2           3           4           5           6 
 0.16767898  0.56490838  0.28243712  0.28759656 -0.02294737  0.23917935 
          7           8           9          10          11          12 
 0.49733412  0.27705107 -0.45576115 -0.11609789 -0.33238735  0.73349946 
         13          14          15          16          17          18 
 0.45195940  0.07515008  0.09060310 -0.31127565  0.40688469 -0.24265573 
```

```
         19          20          21          22          23          24
-0.10713394  0.45607275 -0.21361358 -0.35580336 -1.27636840 -0.65165858
         25
-0.44465205
> resid.x4ith=resid(lm.x4ith)
> resid.x4ith
          1           2           3           4           5           6
 0.30987078 -0.97360354 -0.18519099  0.72328205  0.40136968 -0.24241391
          7           8           9          10          11          12
 0.70049944  0.08364055 -0.87075463 -0.14998599  0.43488037  0.48481625
         13          14          15          16          17          18
 0.46428591 -1.46868320  0.14308741 -0.37237985  0.27427764 -0.22724648
         19          20          21          22          23          24
-0.32533609  0.06822794 -0.46062476  0.95020923 -0.38483431  0.35294090
         25
 0.26966562
> plot(resid.x4,resid.x4ith,main="Partial Regression Plot for x4 as ith", xla
b= "Residuals for y=x1+x2+x3+x5+x6+x7+x8+x9", ylab= "Residuals for x4=x1+x2+x
3+x5+x6+x7+x8+x9")
> qqnorm(resid.x4, main = "Normality Plot for model with x4 as ith")
>
> summary(lm.x4)

Call:
lm(formula = y ~ x1 + x2 + x3 + x5 + x6 + x7 + x8 + x9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.27637 -0.31128  0.07515  0.28760  0.73350

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.53799    2.95468   1.874 0.079266 .
x1           0.87123    0.47613   1.830 0.085973 .
x2          -2.65774    3.83522  -0.693 0.498257
x3           1.10363    0.72546   1.521 0.147702
x5           0.18281    0.07880   2.320 0.033895 *
x6          -0.01998    0.02376  -0.841 0.412833
x7          -0.07916    0.01591  -4.975 0.000138 ***
x8          -0.08502    0.05069  -1.677 0.112933
x9          -0.39119    0.19199  -2.038 0.058487 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5559 on 16 degrees of freedom
Multiple R-squared:  0.9224,	Adjusted R-squared:  0.8836
F-statistic: 23.77 on 8 and 16 DF,  p-value: 1.75e-07

> predictlm.x4=predict(lm.x4)
> predictlm.x4
        1         2         3         4         5         6         7
10.812321 10.535092 12.227563  8.112403  9.292947  8.490821  5.862666
        8         9        10        11        12        13        14
 8.222949  8.275761  9.256098  8.572387 11.456501 11.428041  9.494850
       15        16        17        18        19        20        21
10.849397  9.891276  9.683115  8.352656  6.937134  8.423927  7.893614
       22        23        24        25
 8.825803 10.136368 11.011659 11.524652
> plot(predictlm.x4,resid.x4, main= "Residuals vs. Predicted Response for x4
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx4=rstandard(lm.x4)
> standx4
          1           2           3           4           5           6
 0.33290107  1.14265177  0.59932418  0.82460700 -0.04766458  0.60577340
```

```
          7           8           9          10          11          12
 1.36514236  0.60142111 -0.91375395 -0.24626395 -1.43178824  1.80107286
         13          14          15          16          17          18
 0.97282224  0.16106991  0.20167227 -0.66457396  0.91734269 -0.52126578
         19          20          21          22          23          24
-0.33453219  1.20181466 -0.42317941 -0.81231332 -2.53402695 -1.27840162
         25
-0.98515588
> studentx4=rstudent(lm.x4)
> studentx4
          1           2           3           4           5           6
 0.32345220  1.15447460  0.58691850  0.81594885 -0.04615431  0.59338168
          7           8           9          10          11          12
 1.40622356  0.58901944 -0.90876778 -0.23889723 -1.48469617  1.95306785
         13          14          15          16          17          18
 0.97108823  0.15608186  0.19551699 -0.65254024  0.91253467 -0.50905441
         19          20          21          22          23          24
-0.32504817  1.22002141 -0.41205416 -0.80325686 -3.17105230 -1.30632039
         25
-0.98418962
> vif(lm.x4)
        x1         x2         x3         x5         x6         x7
 11.751771  18.099620 125.768897   4.392236   4.634851   5.862202
        x8         x9
107.419983   2.080472
> PRESS(lm.x4)
.........10.........20.....
$stat
[1] 14.91339

$residuals
 [1]  0.20422160  0.71416687  0.39296308  0.73056738 -0.03059204
 [6]  0.47407455  1.15786400  0.40341071 -0.56607133 -0.16140882
[11] -1.90573718  1.36650958  0.64701966  0.10667173  0.13870727
[16] -0.43842106  0.63906021 -0.34600136 -0.32277408  0.97856750
[21] -0.25904195 -0.57304276 -1.55452008 -0.77493348 -0.67443410

$P.square
[1] 0.7659334


#Partial x5 ith
> lm.x5=lm(y~x1+x2+x3+x4+x6+x7+x8+x9)
> lm.x5

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
  -0.368080     0.329448     3.854711     1.727504     0.152849
         x6           x7           x8           x9
  -0.008127    -0.070212    -0.121875    -0.293553

> lm.x5ith=lm(x5~x1+x2+x3+x4+x6+x7+x8+x9)
> lm.x5ith

Call:
lm(formula = x5 ~ x1 + x2 + x3 + x4 + x6 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
  -11.85754     -2.06825     31.86884      3.26962      0.16884
         x6           x7           x8           x9
```

```
    0.05673      0.03968     -0.19833      0.29264

> resid.x5=resid(lm.x5)
> resid.x5
           1            2            3            4            5 
 0.271993846  0.617901205  0.463949832  0.399031314 -0.436784619 
           6            7            8            9           10 
 0.220578892  0.009821334  0.429469371 -0.060027689  0.022639821 
          11           12           13           14           15 
-0.418351147  1.004493185  0.034111634  0.091016307  0.493330485 
          16           17           18           19           20 
-0.628092036  0.150255019  0.228490638 -0.364155846  0.356378118 
          21           22           23           24           25 
 0.155748369 -0.544836425 -1.406229673 -0.777027849 -0.313704086 
> resid.x5ith=resid(lm.x5ith)
> resid.x5ith
         1          2          3          4          5          6 
 0.7923612 -0.3692343  0.8844588  1.1141054 -2.0307385 -0.2689970 
         7          8          9         10         11         12 
-2.2369164  0.9058743  1.6096321  0.6702717 -0.1819794  1.8399536 
        13         14         15         16         17         18 
-2.0101413 -0.9138097  2.3403833 -2.0184134 -1.2419883  2.4687013 
        19         20         21         22         23         24 
-1.6533235 -0.5086331  1.7426271 -0.4043228 -0.9857784 -0.4573364 
        25 
 0.9132438 
> plot(resid.x5,resid.x5ith,main="Partial Regression Plot for x5 as ith", xla
b= "Residuals for y=x1+x2+x3+x4+x6+x7+x8+x9", ylab= "Residuals for x5=x1+x2+x
3+x4+x6+x7+x8+x9")
> qqnorm(resid.x5, main = "Normality Plot for model with x5 as ith")
> 
> summary(lm.x5)

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x6 + x7 + x8 + x9)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.40623 -0.36416  0.09102  0.35638  1.00449 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)   
(Intercept) -0.368080   7.707917  -0.048  0.96250   
x1           0.329448   0.600066   0.549  0.59057   
x2           3.854711   3.610297   1.068  0.30150   
x3           1.727504   0.775848   2.227  0.04069 * 
x4           0.152849   0.227068   0.673  0.51047   
x6          -0.008127   0.026775  -0.304  0.76540   
x7          -0.070212   0.018105  -3.878  0.00133 **
x8          -0.121875   0.055009  -2.216  0.04157 * 
x9          -0.293553   0.232855  -1.261  0.22550   
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1 

Residual standard error: 0.6337 on 16 degrees of freedom
Multiple R-squared:  0.8992,	Adjusted R-squared:  0.8487 
F-statistic: 17.83 on 8 and 16 DF,  p-value: 1.331e-06 

> predictlm.x5=predict(lm.x5)
> predictlm.x5
        1         2         3         4         5         6         7 
10.708006 10.482099 12.046050  8.000969  9.706785  8.509421  6.350179 
        8         9        10        11        12        13        14 
 8.070531  7.880028  9.117360  8.658351 11.185507 11.845888  9.478984 
```

```
       15        16        17        18        19        20        21
10.446670 10.208092  9.939745  7.881509  7.194156  8.523622  7.524252
       22        23        24        25
 9.014836 10.266230 11.137028 11.393704
> plot(predictlm.x5,resid.x5, main= "Residuals vs. Predicted Response for x5
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx5=rstandard(lm.x5)
> standx5
          1           2           3           4           5           6
 0.47361134  1.18993889  0.85685321  1.06250175 -0.76465028  0.49306366
          7           8           9          10          11          12
 0.02267385  0.80864384 -0.10868656  0.04194676 -1.70069400  2.09083363
         13          14          15          16          17          18
 0.06206378  0.21576074  0.89235610 -1.12637653  0.29228189  0.39875392
         19          20          21          22          23          24
-0.94037856  0.81973694  0.26525590 -1.20661706 -2.44826380 -1.34668784
         25
-0.60626942
> studentx5=rstudent(lm.x5)
> studentx5
          1           2           3           4           5           6
 0.46182082  1.20678884  0.84936084  1.06709696 -0.75427958  0.48107568
          7           8           9          10          11          12
 0.02195422  0.79947330 -0.10527418  0.04061701 -1.81932195  2.37467866
         13          14          15          16          17          18
 0.06010023  0.20921402  0.88635800 -1.13660364  0.28375928  0.38802469
         19          20          21          22          23          24
-0.93677305  0.81091801  0.25739951 -1.22538351 -2.99759874 -1.38476469
         25
-0.59387897
> vif(lm.x5)
        x1         x2         x3         x4         x6         x7
 14.363452  12.341883 110.690568   1.828431   4.530146   5.840032
        x8         x9
 97.334598   2.354898
> PRESS(lm.x5)
.........10.........20.....
$stat
[1] 22.26887

$residuals
 [1]  0.33115331  0.92018314  0.63545650  1.13604707 -0.53752957
 [6]  0.44257415  0.02101959  0.61140130 -0.07902128  0.03120814
[11] -2.77622969  1.74757554  0.04534380  0.20538543  0.64816031
[16] -0.81112521  0.22830679  0.27943804 -0.97512917  0.75715009
[21]  0.18140557 -1.07304215 -1.71160743 -0.93721945 -0.47049515

$P.square
[1] 0.6504887

>
> #Partial x6 ith
> lm.x6=lm(y~x1+x2+x3+x4+x5+x7+x8+x9)
> lm.x6

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
    0.41533      0.73316     -1.62274      1.14622      0.13984
         x5           x7           x8           x9
    0.16823     -0.06811     -0.08615     -0.34468
```

```
> lm.x6ith=lm(x6~x1+x2+x3+x4+x5+x7+x8+x9)
> lm.x6ith

Call:
lm(formula = x6 ~ x1 + x2 + x3 + x4 + x5 + x7 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
  73.483795    -1.764852   -13.388023    -0.318968    -0.945086
         x5           x7           x8           x9
   0.618941    -0.503772    -0.006281    -0.077402

> resid.x6=resid(lm.x6)
> resid.x6
           1            2            3            4            5
 0.064210634  0.639295090  0.333045549  0.004969973 -0.042981268
           6            7            8            9           10
 0.252588594  0.415787678  0.170457276 -0.336130958 -0.012793301
          11           12           13           14           15
-0.355830336  0.873768227  0.285973270  0.291198968  0.202940240
          16           17           18           19           20
-0.215633735  0.410744196 -0.146047434 -0.072406531  0.355191105
          21           22           23           24           25
-0.123652956 -0.389488667 -1.232450231 -0.758263112 -0.614492270
> resid.x6ith=resid(lm.x6ith)
> resid.x6ith
          1           2           3           4           5           6
  3.5763422   2.4520714  -1.5242876  10.5927016  -1.5913944   0.8896527
          7           8           9          10          11          12
 -0.2340117   5.2605297  -0.7063425  -4.6370693  -1.6294501 -10.9023238
         13          14          15          16          17          18
  5.9567881  -1.9705922  -7.0909861  -2.7308123  -2.0457159  -3.7545547
         19          20          21          22          23          24
  0.2804335   5.0517561  -1.8302296  -4.5178863   0.1766686   3.4594309
         25
  7.4692814
> plot(resid.x6,resid.x6ith,main="Partial Regression Plot for x6 as ith", xla
b= "Residuals for y=x1+x2+x3+x4+x5+x7+x8+x9", ylab= "Residuals for x6=x1+x2+x
3+x4+x5+x7+x8+x9")
> qqnorm(resid.x6, main = "Normality Plot for model with x6 as ith")
>
> summary(lm.x6)

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x7 + x8 + x9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.23245 -0.21563  0.00497  0.29120  0.87377

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.41533    6.63911   0.063   0.9509
x1           0.73316    0.55312   1.326   0.2036
x2          -1.62274    4.05901  -0.400   0.6946
x3           1.14622    0.73264   1.565   0.1373
x4           0.13984    0.19965   0.700   0.4937
x5           0.16823    0.07809   2.154   0.0468 *
x7          -0.06811    0.01088  -6.257 1.14e-05 ***
x8          -0.08615    0.05106  -1.687   0.1110
x9          -0.34468    0.20690  -1.666   0.1152
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Residual standard error: 0.5595 on 16 degrees of freedom
Multiple R-squared:  0.9214,   Adjusted R-squared:  0.8821
F-statistic: 23.44 on 8 and 16 DF,  p-value: 1.937e-07

> predictlm.x6=predict(lm.x6)
> predictlm.x6
        1         2         3         4         5         6         7
10.915789 10.460705 12.176954  8.395030  9.312981  8.477411  5.944212
        8         9        10        11        12        13        14
 8.329543  8.156131  9.152793  8.595830 11.316232 11.594027  9.278801
       15        16        17        18        19        20        21
10.737060  9.795634  9.679256  8.256047  6.902407  8.524809  7.803653
       22        23        24        25
 8.859489 10.092450 11.118263 11.694492
> plot(predictlm.x6,resid.x6, main= "Residuals vs. Predicted Response for x6
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx6=rstandard(lm.x6)
> standx6
          1           2           3           4           5           6
 0.12579090  1.38574126  0.70218304  0.01215781 -0.08966443  0.63946578
          7           8           9          10          11          12
 1.22778901  0.35485753 -0.71384817 -0.02629473 -1.61570740  1.83234109
         13          14          15          16          17          18
 0.59561943  0.79049787  0.42062397 -0.45870909  0.92144628 -0.30746771
         19          20          21          22          23          24
-0.22933923  0.88660864 -0.24655628 -0.94537103 -2.45962806 -1.47255796
         25
-1.26552346
> studentx6=rstudent(lm.x6)
> studentx6
          1           2           3           4           5           6
 0.12185679  1.43031190  0.69061010  0.01177180 -0.08683903  0.62722706
          7           8           9          10          11          12
 1.24909999  0.34494943 -0.70245721 -0.02546032 -1.71011900  1.99588273
         13          14          15          16          17          18
 0.58320791  0.78079527  0.40953799 -0.44709273  0.91684497 -0.29858774
         19          20          21          22          23          24
-0.22242264  0.88035321 -0.23918189 -0.94203979 -3.01994138 -1.53349456
         25
-1.29168898
> vif(lm.x6)
        x1         x2         x3         x4         x5         x7
 15.654279  20.011010 126.611716   1.813091   4.256994   2.707750
        x8         x9
107.589948   2.384853
> PRESS(lm.x6)
.........10.........20.....
$stat
[1] 15.78359

$residuals
 [1]  0.077144164  0.940317496  0.463456124  0.009310396 -0.058556181
 [6]  0.506795168  1.134978047  0.231261904 -0.474585846 -0.016918683
[11] -2.296645938  1.202895727  0.388350491  0.671773968  0.272917547
[16] -0.305470714  0.647113105 -0.202636032 -0.227400044  0.692808845
[21] -0.153900214 -0.718325600 -1.536671644 -0.895233773 -0.815897434

$P.square
[1] 0.7522756

>
> #Partial x7 ith
> lm.x7=lm(y~x1+x2+x3+x4+x5+x6+x8+x9)
> lm.x7
```

```
Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
  -14.10165      0.75383      0.17962      2.56000      0.29794
         x5           x6           x8           x9
    0.10652      0.06669     -0.17431     -0.29254

> lm.x7ith=lm(x7~x1+x2+x3+x4+x5+x6+x8+x9)
> lm.x7ith

Call:
lm(formula = x7 ~ x1 + x2 + x3 + x4 + x5 + x6 + x8 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
   205.1118      -0.6852     -26.4756     -18.3561      -2.2680
         x5           x6           x8           x9
     0.9445      -1.0992       1.1385      -0.6926

> resid.x7=resid(lm.x7)
> resid.x7
           1            2            3            4            5
-0.003321882  0.927552635  0.763735527 -1.270856030 -0.322066598
           6            7            8            9           10
-0.733645272  0.395191250 -0.298255126 -0.074886766  0.591294949
          11           12           13           14           15
 0.070336809  2.031485858  0.320375384  0.632635230  0.460747028
          16           17           18           19           20
-0.288626950  0.017183373 -0.112198126 -0.242689145  0.235870612
          21           22           23           24           25
-0.188174654 -0.299230653 -1.154959970 -0.717748923 -0.739748559
> resid.x7ith=resid(lm.x7ith)
> resid.x7ith
          1           2           3           4           5           6
  1.7201549  -3.1465811  -5.9299488  19.0054019   3.2318034  12.9630346
          7           8           9          10          11          12
  0.2109013   7.3063947  -3.5452525  -8.9092065  -5.8963711 -17.5515427
         13          14          15          16          17          18
  0.9658325  -4.8815590  -5.0127927   0.2971273   4.6044211  -1.3268228
         19          20          21          22          23          24
  2.2682252   2.7391936   0.4008602  -2.2369765  -0.9601383   0.2953866
         25
  3.3884547
> plot(resid.x7,resid.x7ith,main="Partial Regression Plot for x7 as ith", xla
b= "Residuals for y=x1+x2+x3+x4+x5+x6+x8+x9", ylab= "Residuals for x7=x1+x2+x
3+x4+x5+x6+x8+x9")
> qqnorm(resid.x7, main = "Normality Plot for model with x7 as ith")
>
> summary(lm.x7)

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x8 + x9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.27086 -0.29923 -0.07489  0.39519  2.03149

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -14.10165    9.24517  -1.525   0.1467
x1            0.75383    0.85419   0.883   0.3906
```

```
x2            0.17962    6.23559   0.029   0.9774  
x3            2.56000    1.03010   2.485   0.0244 *
x4            0.29794    0.30418   0.979   0.3419  
x5            0.10652    0.12011   0.887   0.3883  
x6            0.06669    0.02476   2.693   0.0160 *
x8           -0.17431    0.07327  -2.379   0.0302 *
x9           -0.29254    0.31818  -0.919   0.3715  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8617 on 16 degrees of freedom
Multiple R-squared:  0.8135,   Adjusted R-squared:  0.7203 
F-statistic: 8.726 on 8 and 16 DF,  p-value: 0.0001402

> predictlm.x7=predict(lm.x7)
> predictlm.x7
        1         2         3         4         5         6         7 
10.983322 10.172447 11.746264  9.670856  9.592067  9.463645  5.964809 
        8         9        10        11        12        13        14 
 8.798255  7.894887  8.548705  8.169663 10.158514 11.559625  8.937365 
       15        16        17        18        19        20        21 
10.479253  9.868627 10.072817  8.222198  7.072689  8.644129  7.868175 
       22        23        24        25 
 8.769231 10.014960 11.077749 11.819749 
> plot(predictlm.x7,resid.x7, main= "Residuals vs. Predicted Response for x7 
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx7=rstandard(lm.x7)
> standx7
           1            2            3            4            5 
-0.004280291  1.308122679  1.027464977 -1.854430998 -0.435022623 
           6            7            8            9           10 
-1.064714863  0.757797127 -0.404796745 -0.102563672  0.775142622 
          11           12           13           14           15 
 0.192660293  2.690707046  0.453713494  1.098732540  0.652335753 
          16           17           18           19           20 
-0.402602580  0.024830698 -0.156057688 -0.495848853  0.398504864 
          21           22           23           24           25 
-0.244553018 -0.486886978 -1.495983602 -0.917094485 -1.057002079 
> studentx7=rstudent(lm.x7)
> studentx7
           1            2            3            4            5 
-0.004144376  1.340281420  1.029377425 -2.026481852 -0.423722146 
           6            7            8            9           10 
-1.069488942  0.747266530 -0.393965308 -0.099339509  0.765030619 
          11           12           13           14           15 
 0.186759282  3.520931882  0.442159823  1.106401147  0.640192143 
          16           17           18           19           20 
-0.391807939  0.024042683 -0.151217336 -0.483835440  0.387779906 
          21           22           23           24           25 
-0.237231229 -0.474957952 -1.561820663 -0.912274062 -1.061157694 
> vif(lm.x7)
        x1         x2         x3         x4         x5         x6 
 15.740218  19.911086 105.525135   1.774504   4.246571   2.095274 
        x8         x9 
 93.393373   2.377952 
> PRESS(lm.x7)
.........10.........20.....
$stat
[1] 25.74788

$residuals
 [1] -0.004095079  1.369803834  1.026339519 -2.009208436 -0.436292743
 [6] -1.147309267  1.078943193 -0.407930424 -0.104299454  0.754499674
[11]  0.391833680  2.646177786  0.477094316  1.416871913  0.685772100
```

```
[16] -0.416980954  0.026642160 -0.161170192 -0.752225871  0.499911826
[21] -0.235985283 -0.588234144 -1.438754598 -0.870072825 -1.121418061

$P.square
[1] 0.5958855

>
> #Partial x8 ith
> lm.x8=lm(y~x1+x2+x3+x4+x5+x6+x7+x9)
> lm.x8

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
    6.17909      0.93302     -4.62632     -0.08652      0.10900
         x5           x6           x7           x9
    0.22103     -0.01819     -0.08734     -0.25544

> lm.x8ith=lm(x8~x1+x2+x3+x4+x5+x6+x7+x9)
> lm.x8ith

Call:
lm(formula = x8 ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x9)

Coefficients:
(Intercept)           x1           x2           x3           x4
 -51.215802    -2.691535    31.977132    14.223104     0.156832
         x5           x6           x7           x9
  -0.480643    -0.001395     0.115910    -1.051031

> resid.x8=resid(lm.x8)
> resid.x8
          1           2           3           4           5           6
 0.11685567  0.54114034  0.36257195  0.37399549  0.21246790  0.48632345
          7           8           9          10          11          12
 0.45807282  0.08373865 -0.55875840 -0.36311343 -0.97301299  0.66650186
         13          14          15          16          17          18
 0.46614894  0.32097243  0.10991827 -0.04480286  0.57111765 -0.16138290
         19          20          21          22          23          24
-0.11034738  0.32366967 -0.17799235 -0.47597095 -0.98956739 -0.59269648
         25
-0.64584993
> resid.x8ith=resid(lm.x8ith)
> resid.x8ith
          1           2           3           4           5           6
 0.14902204  1.65849950 -0.66592491 -2.02899093 -3.29923318 -2.52073448
          7           8           9          10          11          12
-0.53988489  2.12220971  2.43088939  3.07663190  6.80886683  0.08802353
         13          14          15          16          17          18
-0.82399412 -0.76354745 -0.42716518 -2.56019546 -2.29350726 -0.61937926
         19          20          21          22          23          24
 0.49937879  1.43799751  0.24135113  0.04333723 -2.77816387 -1.18486926
         25
 1.94938271
> plot(resid.x8,resid.x8ith,main="Partial Regression Plot for x8 as ith", xla
b= "Residuals for y=x1+x2+x3+x4+x5+x6+x7+x9", ylab= "Residuals for x8=x1+x2+x
3+x4+x5+x6+x7+x9")
> qqnorm(resid.x8, main = "Normality Plot for model with x8 as ith")
>
> summary(lm.x8)

Call:
```

```
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x9)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.9896 -0.3631  0.1099  0.3740  0.6665 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  6.17909    6.79000   0.910   0.3763    
x1           0.93302    0.57439   1.624   0.1238    
x2          -4.62632    3.98634  -1.161   0.2629    
x3          -0.08652    0.10317  -0.839   0.4140    
x4           0.10900    0.21457   0.508   0.6184    
x5           0.22103    0.08081   2.735   0.0147 *  
x6          -0.01819    0.02572  -0.707   0.4895    
x7          -0.08734    0.01622  -5.383 6.09e-05 ***
x9          -0.25544    0.21356  -1.196   0.2491    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1 

Residual standard error: 0.598 on 16 degrees of freedom
Multiple R-squared:  0.9102,	Adjusted R-squared:  0.8653 
F-statistic: 20.27 on 8 and 16 DF,  p-value: 5.432e-07 

> predictlm.x8=predict(lm.x8)
> predictlm.x8
        1         2         3         4         5         6         7 
10.863144 10.558860 12.147428  8.026005  9.057532  8.243677  5.901927 
        8         9        10        11        12        13        14 
 8.416261  8.378758  9.503113  9.213013 11.523498 11.413851  9.249028 
       15        16        17        18        19        20        21 
10.830082  9.624803  9.518882  8.271383  6.940347  8.556330  7.857992 
       22        23        24        25 
 8.945971  9.849567 10.952696 11.725850 
> plot(predictlm.x8,resid.x8, main= "Residuals vs. Predicted Response for x8 as ith", xlab ="Predicted Response", ylab="Residuals")
> standx8=rstandard(lm.x8)
> standx8
          1           2           3           4           5           6 
 0.21729176  1.08816727  0.71558471  1.04173314  0.39239001  1.09680110 
          7           8           9          10          11          12 
 1.26167965  0.16465417 -1.07432000 -0.68097205 -2.22223837  1.56615747 
         13          14          15          16          17          18 
 0.94789723  0.81749603  0.22764192 -0.08670867  1.16616172 -0.32307847 
         19          20          21          22          23          24 
-0.32605162  0.77911432 -0.33327258 -1.12071787 -1.77808752 -1.08372175 
         25 
-1.30830038 
> studentx8=rstudent(lm.x8)
> studentx8
          1           2           3           4           5           6 
 0.21070296  1.09490739  0.70422246  1.04470461  0.38177135  1.10429795 
          7           8           9          10          11          12 
 1.28733150  0.15956096 -1.07988362 -0.66911603 -2.58777304  1.64799950 
         13          14          15          16          17          18 
 0.94470665  0.80860444  0.22077115 -0.08397504  1.18041000 -0.31384478 
         19          20          21          22          23          24 
-0.31675218  0.76910468 -0.32381570 -1.13040583 -1.92195507 -1.09007920 
         25 
-1.34048530 
> vif(lm.x8)
       x1        x2        x3        x4        x5        x6        x7 
14.780030 16.898604  2.198060  1.833708  3.991346  4.694972  5.266779 
       x9 
```

```
 2.224572
> PRESS(lm.x8)
.........10.........20.....
$stat
[1] 16.20219

$residuals
 [1]  0.14446878  0.78238173  0.50496957  1.03748996  0.25910742
 [6]  0.88443891  1.24251307  0.11575956 -0.73855111 -0.45661855
[11] -1.81467954  1.31585123  0.68918388  0.74445915  0.16856674
[16] -0.06000074  0.85139145 -0.23125749 -0.34446730  0.67056005
[21] -0.22311827 -0.94351637 -1.14234738 -0.70850110 -0.94759032

$P.square
[1] 0.7457056

>
> #Partial x9 ith
> lm.x9=lm(y~x1+x2+x3+x4+x5+x6+x7+x8)
> lm.x9

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8)

Coefficients:
(Intercept)           x1           x2           x3           x4
   -1.10958      0.46648     -1.77992      0.85549      0.24264
         x5           x6           x7           x8
    0.16462     -0.01795     -0.07585     -0.06410

> lm.x9ith=lm(x9~x1+x2+x3+x4+x5+x6+x7+x8)
> lm.x9ith

Call:
lm(formula = x9 ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8)

Coefficients:
(Intercept)           x1           x2           x3           x4
   8.294526     0.677157    -0.254322     0.823146    -0.347039
         x5           x6           x7           x8
   0.043195    -0.001047    -0.004295    -0.064016

> resid.x9=resid(lm.x9)
> resid.x9
          1           2           3           4           5           6
 0.19021737  0.59177636  0.63604388  0.04345789 -0.13919279  0.44038936
          7           8           9          10          11          12
 0.48789363  0.17247904 -0.19383881 -0.26303752 -0.61588656  0.70389124
         13          14          15          16          17          18
 0.33295606  0.16398528  0.23923609 -0.06568201  0.09847997 -0.14159158
         19          20          21          22          23          24
-0.01391441  0.65747717 -0.11515386 -1.02178799 -1.02720299 -0.57974810
         25
-0.58124672
> resid.x9ith=resid(lm.x9ith)
> resid.x9ith
          1           2           3           4           5           6
-0.17482443  0.26705447 -0.95612353  0.44933362  0.19377444 -0.49554033
          7           8           9          10          11          12
-0.22072119  0.27253186 -0.44850677  0.47765666  0.66516209 -0.08609159
         13          14          15          16          17          18
 0.17946877  0.26328379 -0.48010703 -0.57776746  0.79398055 -0.21155559
         19          20          21          22          23          24
-0.15416325 -0.60607861 -0.12140764  1.58784685 -0.58367821 -0.33272483
```

```
         25
 0.29919738
> plot(resid.x9,resid.x9ith,main="Partial Regression Plot for x9 as ith", xla
b= "Residuals for y=x1+x2+x3+x4+x5+x6+x7+x8", ylab= "Residuals for x9=x1+x2+x
3+x4+x5+x6+x7+x8")
> qqnorm(resid.x9, main = "Normality Plot for model with x9 as ith")
>
> summary(lm.x9)

Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.02720 -0.19384  0.04346  0.33296  0.70389 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.10958    7.09917  -0.156 0.877754    
x1           0.46648    0.57278   0.814 0.427364    
x2          -1.77992    4.34460  -0.410 0.687469    
x3           0.85549    0.76042   1.125 0.277174    
x4           0.24264    0.20026   1.212 0.243246    
x5           0.16462    0.08429   1.953 0.068536 .  
x6          -0.01795    0.02568  -0.699 0.494563    
x7          -0.07585    0.01736  -4.369 0.000477 ***
x8          -0.06410    0.05263  -1.218 0.240831    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.597 on 16 degrees of freedom
Multiple R-squared:  0.9105,   Adjusted R-squared:  0.8657 
F-statistic: 20.34 on 8 and 16 DF,  p-value: 5.306e-07

> predictlm.x9=predict(lm.x9)
> predictlm.x9
        1         2         3         4         5         6         7 
10.789783 10.508224 11.873956  8.356542  9.409193  8.289611  5.872106 
        8         9        10        11        12        13        14 
 8.327521  8.013839  9.403038  8.855887 11.486109 11.547044  9.406015 
       15        16        17        18        19        20        21 
10.700764  9.645682  9.991520  8.251592  6.843914  8.222523  7.795154 
       22        23        24        25 
 9.491788  9.887203 10.939748 11.661247 
> plot(predictlm.x9,resid.x9, main= "Residuals vs. Predicted Response for x9
as ith", xlab ="Predicted Response", ylab="Residuals")
> standx9=rstandard(lm.x9)
> standx9
          1           2           3           4           5           6 
 0.35337483  1.20330596  1.16282131  0.12237207 -0.27203442  1.01269469 
          7           8           9          10          11          12 
 1.33820146  0.34627126 -0.37873464 -0.50954470 -2.24821617  1.65498827 
         13          14          15          16          17          18 
 0.67871459  0.41607185  0.48517966 -0.12808026  0.19531462 -0.28329663 
         19          20          21          22          23          24 
-0.04110146  1.53347520 -0.21573584 -1.85710423 -1.86770605 -1.05948298 
         25 
-1.19638859 
> studentx9=rstudent(lm.x9)
> studentx9
          1           2           3           4           5           6 
 0.34349676  1.22168583  1.17671617  0.11854173 -0.26400744  1.01355829 
          7           8           9          10          11          12 
 1.37493465  0.33653909 -0.36836314 -0.49741690 -2.63187596  1.76016016 
```

```
         13          14          15          16          17          18
 0.66683201  0.40505710  0.47326755 -0.12407680  0.18933841 -0.27499134
         19          20          21          22          23          24
-0.03979841  1.60760985 -0.20918981 -2.03020507 -2.04501474 -1.06383604
         25
-1.21397061
> vif(lm.x9)
        x1         x2         x3         x4         x5         x6
 14.742133  20.133751 119.783271   1.602086   4.356152   4.694632
        x7         x8
  6.049379 100.351820
> PRESS(lm.x9)
.........10.........20.....
$stat
[1] 20.96213

$residuals
 [1]  0.23401019  0.87218425  0.75779652  0.12283148 -0.18951548
 [6]  0.83010679  1.30837298  0.24780513 -0.26378071 -0.35185247
[11] -2.92542604  1.38706879  0.49317512  0.37630979  0.35074557
[16] -0.08902904  0.13808144 -0.20205011 -0.04327762  1.27493061
[21] -0.14407208 -1.20316580 -1.21052703 -0.69017998 -0.87780434

$P.square
[1] 0.6709981


#Partial x8 and x1 removed
> lm.nox8x1=lm(y~x2+x3+x4+x5+x6+x7+x9)
> lm.nox8x1

Call:
lm(formula = y ~ x2 + x3 + x4 + x5 + x6 + x7 + x9)

Coefficients:
(Intercept)           x2           x3           x4           x5
    2.12600      0.93669     -0.05979      0.28680      0.18971
         x6           x7           x9
   -0.02146     -0.09055     -0.13797

> summary(lm.nox8x1)

Call:
lm(formula = y ~ x2 + x3 + x4 + x5 + x6 + x7 + x9)

Residuals:
    Min      1Q  Median      3Q     Max
-1.2875 -0.2377  0.1293  0.3050  0.6737

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.12600    6.61225   0.322   0.7517
x2           0.93669    2.13621   0.438   0.6666
x3          -0.05979    0.10664  -0.561   0.5823
x4           0.28680    0.19325   1.484   0.1561
x5           0.18971    0.08217   2.309   0.0338 *
x6          -0.02146    0.02685  -0.799   0.4351
x7          -0.09055    0.01686  -5.370 5.09e-05 ***
x9          -0.13797    0.21040  -0.656   0.5208
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6261 on 17 degrees of freedom
Multiple R-squared:  0.8954,   Adjusted R-squared:  0.8523
```

```
F-statistic: 20.79 on 7 and 17 DF,  p-value: 3.65e-07

> resid.nox8x1=resid(lm.nox8x1)
> resid.nox8x1
          1           2           3           4           5           6 
 0.26074962  0.65740043  0.54757955  0.21388003  0.06112276  0.67366270 
          7           8           9          10          11          12 
 0.12116450  0.18532312 -0.39733621 -0.21717550 -1.28751392  0.44760142 
         13          14          15          16          17          18 
 0.30499911  0.30044597  0.28561069 -0.04112656  0.66573767 -0.23766093 
         19          20          21          22          23          24 
 0.33250817  0.12928548 -0.08194265 -0.80591523 -1.05810124 -0.62337536 
         25 
-0.43692363 
> qqnorm(resid.nox8x1, main = "Normality Plot for model without x1 and x8")
> predictlm.nox8x1=predict(lm.nox8x1)
> plot(predictlm.nox8x1,resid.nox8x1, main= "Residuals vs. Predicted Response 
for x9 as ith", xlab ="Predicted Response", ylab="Residuals")
> PRESS(lm.nox8x1)
.........10.........20.....
$stat
[1] 14.71888

$residuals
 [1]  0.31384925  0.93118232  0.72595561  0.55172715  0.07239647
 [6]  1.14750510  0.24779006  0.25237291 -0.50669009 -0.26556137
[11] -2.00850073  0.80314827  0.43329598  0.69612929  0.41707495
[16] -0.05507631  0.97860166 -0.33757857  0.62948305  0.24732428
[21] -0.10147349 -1.30016411 -1.21448235 -0.74428660 -0.60030286

$P.square
[1] 0.7689863

> standnox8x1=rstandard(lm.nox8x1)
> standnox8x1
          1           2           3           4           5           6 
 0.45690070  1.24963099  1.00699677  0.54865170  0.10624527  1.40426153 
          7           8           9          10          11          12 
 0.27674454  0.34541056 -0.71663780 -0.38356352 -2.56839252  0.95761960 
         13          14          15          16          17          18 
 0.58061874  0.73042828  0.55124379 -0.07601391  1.28915155 -0.45239288 
         19          20          21          22          23          24 
 0.73070590  0.28559966 -0.14564002 -1.63490755 -1.81054219 -1.08791346 
         25 
-0.81797006 
> vif(lm.nox8x1)
      x2       x3       x4       x5       x6       x7       x9 
4.426137 2.142163 1.356569 3.764093 4.666206 5.188386 1.969501 
> 
> lm.nox8x2=lm(y~x1+x3+x4+x5+x6+x7+x9)
> lm.nox8x2

Call:
lm(formula = y ~ x1 + x3 + x4 + x5 + x6 + x7 + x9)

Coefficients:
(Intercept)           x1           x3           x4           x5  
    4.33882      0.36033     -0.05475      0.19119      0.16739  
         x6           x7           x9  
   -0.01558     -0.08821     -0.22268  

> summary(lm.nox8x2)

Call:
```

```
lm(formula = y ~ x1 + x3 + x4 + x5 + x6 + x7 + x9)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.2074 -0.2540  0.1206  0.3071  0.6553 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.33882    6.66926   0.651   0.5240    
x1           0.36033    0.29695   1.213   0.2415    
x3          -0.05475    0.10048  -0.545   0.5929    
x4           0.19119    0.20461   0.934   0.3632    
x5           0.16739    0.06696   2.500   0.0229 *  
x6          -0.01558    0.02588  -0.602   0.5552    
x7          -0.08821    0.01637  -5.388  4.9e-05 ***
x9          -0.22268    0.21383  -1.041   0.3123    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 0.604 on 17 degrees of freedom
Multiple R-squared:  0.9027,	Adjusted R-squared:  0.8626 
F-statistic: 22.52 on 7 and 17 DF,  p-value: 2.018e-07

> resid.nox8x2=resid(lm.nox8x2)
> resid.nox8x2
          1           2           3           4           5           6 
 0.24513144  0.58460682  0.44139017  0.40682238  0.05689118  0.56482843 
          7           8           9          10          11          12 
 0.24262338  0.12061815 -0.46826859 -0.25404659 -1.20738237  0.65534296 
         13          14          15          16          17          18 
 0.26914702  0.30041263  0.30714582 -0.14753715  0.62123314 -0.09696659 
         19          20          21          22          23          24 
 0.15295199  0.05116576 -0.03950754 -0.59070741 -1.08244011 -0.63297300 
         25 
-0.50048192 
> qqnorm(resid.nox8x2, main = "Normality Plot for model without x1 and x8")
> predictlm.nox8x2=predict(lm.nox8x2)
> plot(predictlm.nox8x2,resid.nox8x2, main= "Residuals vs. Predicted Response 
for x9 as ith", xlab ="Predicted Response", ylab="Residuals")
> PRESS(lm.nox8x2)
.........10.........20.....
$stat
[1] 14.88712

$residuals
 [1]  0.29077314  0.84045833  0.60389323  1.12159180  0.06537262
 [6]  1.00384602  0.52170771  0.16609298 -0.60533935 -0.30984152
[11] -1.85679086  1.29316046  0.35555923  0.69535739  0.41911121
[16] -0.19195028  0.91895625 -0.13725586  0.32942392  0.08033740
[21] -0.04716908 -1.11076678 -1.22424611 -0.75361250 -0.68988981

$P.square
[1] 0.7663458

> standnox8x2=rstandard(lm.nox8x2)
> standnox8x2
         1          2          3          4          5          6 
 0.4419997  1.1604740  0.8547454  1.1183177  0.1009638  1.2466289 
         7          8          9         10         11         12 
 0.5890136  0.2343297 -0.8814392 -0.4644852 -2.4788445  1.5240735 
        13         14         15         16         17         18 
 0.5121488  0.7566735  0.5939944 -0.2786060  1.2508928 -0.1909949 
        19         20         21         22         23         24 
 0.3716212  0.1061436 -0.0714684 -1.3410440 -1.9058191 -1.1434364 
```

```
        25 
-0.9728131 
> vif(lm.nox8x2)
      x1       x3       x4       x5       x6       x7       x9 
3.871233 2.043285 1.633958 2.685582 4.658877 5.255365 2.185702 
> #Conclusion: don't remove based on multicollinearity
> 
> #Transformed model attempts
> sqx3=x3^2
> sqx8=x8^2
> lm.sqx3x8=lm(y~x1+x2+sqx3+x4+x5+x6+x7+sqx8+x9)
> summary(lm.sqx3x8)

Call:
lm(formula = y ~ x1 + x2 + sqx3 + x4 + x5 + x6 + x7 + sqx8 + 
    x9)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.16318 -0.13067 -0.00757  0.31610  0.64030 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.0609004  6.0664240   0.505 0.621196    
x1           0.5253783  0.5389080   0.975 0.345074    
x2          -0.8008825  3.8840706  -0.206 0.839410    
sqx3         0.0555070  0.0292446   1.898 0.077107 .  
x4           0.1700484  0.1926556   0.883 0.391349    
x5           0.1607023  0.0759639   2.116 0.051528 .  
x6          -0.0232787  0.0228982  -1.017 0.325454    
x7          -0.0771157  0.0148893  -5.179 0.000112 ***
sqx8        -0.0005262  0.0002384  -2.207 0.043311 *  
x9          -0.2897835  0.1908812  -1.518 0.149773    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5302 on 15 degrees of freedom
Multiple R-squared:  0.9338,	Adjusted R-squared:  0.8941 
F-statistic: 23.52 on 9 and 15 DF,  p-value: 2.831e-07

> plot(sqx3,y)
> plot(sqx8,y)
> residsqx3x8=resid(lm.sqx3x8)
> predictsqx3x8=predict(lm.sqx3x8)
> plot(predictsqx3x8,residsqx3x8, main= "Residuals vs. Predicted Response for x9 as ith", xlab ="Predicted Response", ylab="Residuals")
> 
> sqx2=x2^2
> sqx9=x9^2
> lm.sqx2x3x8=lm(y~x1+sqx2+sqx3+x4+x5+x6+x7+sqx8+x9)
> summary(lm.sqx2x3x8)

Call:
lm(formula = y ~ x1 + sqx2 + sqx3 + x4 + x5 + x6 + x7 + sqx8 + 
    x9)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.20665 -0.17438  0.09663  0.24083  0.66091 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.7667156  5.7807599   0.479 0.639119
x1           0.2371022  0.4821657   0.492 0.630018
```

```
sqx2         1.0970955  2.3331518   0.470 0.644962
sqx3         0.0638924  0.0282053   2.265 0.038734 *
x4           0.2035612  0.1838486   1.107 0.285638
x5           0.1363155  0.0663938   2.053 0.057928 .
x6          -0.0243351  0.0228971  -1.063 0.304682
x7          -0.0777092  0.0148649  -5.228 0.000102 ***
sqx8        -0.0005895  0.0002303  -2.560 0.021758 *
x9          -0.2757266  0.1901496  -1.450 0.167632
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.527 on 15 degrees of freedom
Multiple R-squared:  0.9346,   Adjusted R-squared:  0.8954
F-statistic: 23.82 on 9 and 15 DF,  p-value: 2.598e-07

> plot(sqx2,y)
> lm.sqx2x3x8x9=lm(y~x1+sqx2+sqx3+x4+x5+x6+x7+sqx8+sqx9)
> summary(lm.sqx2x3x8x9)

Call:
lm(formula = y ~ x1 + sqx2 + sqx3 + x4 + x5 + x6 + x7 + sqx8 +
    sqx9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.20393 -0.16998  0.06469  0.24623  0.67286

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.7198552  5.6227538   0.306 0.763905
x1           0.1728404  0.4742948   0.364 0.720634
sqx2         1.3275457  2.3375694   0.568 0.578494
sqx3         0.0654148  0.0284245   2.301 0.036130 *
x4           0.2302134  0.1798594   1.280 0.220000
x5           0.1247320  0.0658305   1.895 0.077569 .
x6          -0.0254494  0.0230791  -1.103 0.287534
x7          -0.0775042  0.0149685  -5.178 0.000112 ***
sqx8        -0.0006003  0.0002325  -2.582 0.020838 *
sqx9        -0.0272822  0.0200343  -1.362 0.193372
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5309 on 15 degrees of freedom
Multiple R-squared:  0.9336,   Adjusted R-squared:  0.8938
F-statistic: 23.45 on 9 and 15 DF,  p-value: 2.89e-07

> sqrtx6=sqrt(x6)
> lm.sqx2x3x8sqrtx6=lm(y~x1+sqx2+sqx3+x4+x5+sqrtx6+x7+sqx8+x9)
> summary(lm.sqx2x3x8sqrtx6)

Call:
lm(formula = y ~ x1 + sqx2 + sqx3 + x4 + x5 + sqrtx6 + x7 + sqx8 +
    x9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.07899 -0.20368  0.05293  0.26547  0.60527

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.2319065  5.5446609   0.763   0.4572
x1           0.0221917  0.4810566   0.046   0.9638
sqx2         1.7240965  2.2457111   0.768   0.4546
sqx3         0.0698773  0.0269608   2.592   0.0204 *
```

```
x4           0.2060570  0.1731591   1.190   0.2525
x5           0.1473051  0.0631305   2.333   0.0340 *
sqrtx6      -0.2455904  0.1387612  -1.770   0.0971 .
x7          -0.0911187  0.0170326  -5.350  8.1e-05 ***
sqx8        -0.0006255  0.0002186  -2.861   0.0119 *
x9          -0.2908944  0.1795418  -1.620   0.1260
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4971 on 15 degrees of freedom
Multiple R-squared:  0.9418,   Adjusted R-squared:  0.9069
F-statistic: 26.98 on 9 and 15 DF,  p-value: 1.106e-07

> plot(sqrtx6,y)
> lm.sqx2x3x8x9sqrtx6=lm(y~x1+sqx2+sqx3+x4+x5+sqrtx6+x7+sqx8+sqx9)
> summary(lm.sqx2x3x8x9sqrtx6)

Call:
lm(formula = y ~ x1 + sqx2 + sqx3 + x4 + x5 + sqrtx6 + x7 + sqx8 +
    sqx9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.07500 -0.20284  0.06192  0.27059  0.61688

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.2166860  5.3840022   0.597   0.5591
x1          -0.0472108  0.4730110  -0.100   0.9218
sqx2         1.9828891  2.2462257   0.883   0.3913
sqx3         0.0716932  0.0271209   2.643   0.0184 *
x4           0.2323285  0.1689150   1.375   0.1892
x5           0.1355870  0.0624261   2.172   0.0463 *
sqrtx6      -0.2547064  0.1398612  -1.821   0.0886 .
x7          -0.0913240  0.0171467  -5.326 8.48e-05 ***
sqx8        -0.0006385  0.0002203  -2.898   0.0110 *
sqx9        -0.0296213  0.0189163  -1.566   0.1382
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4995 on 15 degrees of freedom
Multiple R-squared:  0.9413,   Adjusted R-squared:  0.906
F-statistic:  26.7 on 9 and 15 DF,  p-value: 1.188e-07

> lm.sqx
Error: object 'lm.sqx' not found
>
> #Chosen transformed model
> lm.sqx2x3x8sqrtx6=lm(y~x1+sqx2+sqx3+x4+x5+sqrtx6+x7+sqx8+x9)
> summary(lm.sqx2x3x8sqrtx6)

Call:
lm(formula = y ~ x1 + sqx2 + sqx3 + x4 + x5 + sqrtx6 + x7 + sqx8 +
    x9)

Residuals:
     Min       1Q   Median       3Q      Max
-1.07899 -0.20368  0.05293  0.26547  0.60527

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.2319065  5.5446609   0.763   0.4572
x1           0.0221917  0.4810566   0.046   0.9638
sqx2         1.7240965  2.2457111   0.768   0.4546
```

```
sqx3         0.0698773  0.0269608   2.592   0.0204 *
x4           0.2060570  0.1731591   1.190   0.2525
x5           0.1473051  0.0631305   2.333   0.0340 *
sqrtx6      -0.2455904  0.1387612  -1.770   0.0971 .
x7          -0.0911187  0.0170326  -5.350  8.1e-05 ***
sqx8        -0.0006255  0.0002186  -2.861   0.0119 *
x9          -0.2908944  0.1795418  -1.620   0.1260
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4971 on 15 degrees of freedom
Multiple R-squared:  0.9418,   Adjusted R-squared:  0.9069
F-statistic: 26.98 on 9 and 15 DF,  p-value: 1.106e-07

> lm.sqx2x3x8sqrtx6

Call:
lm(formula = y ~ x1 + sqx2 + sqx3 + x4 + x5 + sqrtx6 + x7 + sqx8 +
    x9)

Coefficients:
(Intercept)           x1         sqx2         sqx3           x4
  4.2319065    0.0221917    1.7240965    0.0698773    0.2060570
         x5       sqrtx6           x7         sqx8           x9
  0.1473051   -0.2455904   -0.0911187   -0.0006255   -0.2908944

>
> #Question 5: Stepwise
> library(olsrr)
> Stepwise=ols_step_both_p(lm.sqx2x3x8sqrtx6)
> Stepwise

                             Stepwise Selection Summary
-----------------------------------------------------------------------------
-------
                     Added/                   Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
-----------------------------------------------------------------------------
-------
   1       x7       addition       0.714       0.701    52.7850    69.0538
0.8903
   2       x1       addition       0.860       0.847    17.1190    53.1954
0.6369
   3       x5       addition       0.880       0.862    14.0670    51.4289
0.6046
-----------------------------------------------------------------------------
-------
>
> lm.stepwise=lm(y~x1+x5+x7)
> lm.stepwise

Call:
lm(formula = y ~ x1 + x5 + x7)

Coefficients:
(Intercept)           x1           x5           x7
    8.55609      0.48842      0.10827     -0.07572

> summary(lm.stepwise)

Call:
lm(formula = y ~ x1 + x5 + x7)
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-1.33205 -0.30490  0.05466  0.35996  1.16505 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  8.55609    1.03675   8.253 4.98e-08 ***
x1           0.48842    0.21162   2.308   0.0313 *  
x5           0.10827    0.05859   1.848   0.0788 .  
x7          -0.07572    0.00746 -10.150 1.49e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1 

Residual standard error: 0.6046 on 21 degrees of freedom
Multiple R-squared:  0.8795,	Adjusted R-squared:  0.8623 
F-statistic:  51.1 on 3 and 21 DF,  p-value: 7.988e-10 

> residste=resid(lm.stepwise)
> qqnorm(residste)
> predictste=predict(lm.stepwise)
> plot(predictste,residste, main= "Residuals vs. Predicted Response for Stepw
ise Chosen Model", xlab ="Predicted Response", ylab="Residuals")
```