Question 1 - Distance-Redshift relation

In this problem, you will compute distances as a function of redshift **numerically**. For the comoving radial distance $D(z)$ you will need to compute numerically the integral

$$D(z) = \int_0^z \frac{\mathrm{d}z}{H(z)} \tag{1}$$

$$H(z) = H_0\sqrt{\Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{DE}(1+z)^{3(1+w)}} \tag{2}$$

$$\Omega_k = 1 - (\Omega_m + \Omega_r + \Omega_{DE}) \tag{3}$$

From $D(z)$ you can obtain other distance definitions. I **highly** suggest you write a program in C/C++ or Fortran or Python so you can easily combine with other cosmological codes later. You can then find a free numerical integrator (e.g. Simpson, Romberg, etc) to incorporate to your program. Plot the 3 distances (radial, angular-diameter and luminosity) as a function of redshift z for the fiducial case and cosmology variations indicated in problem set 3).

Item a)

The single-component analytical solutions for the $H(z)$ can be obtained by zeroing every unwanted component, leaving only the desired one.

All numerical solutions will be obtained with `Python` package `scypy` (`quad`) and all analytical solutions are selected from the lecture notes.

Also, we can always compute the Angular Diameter Distance $D_A(z)$ and the Luminosity Distance $D_L(z)$ from the following equations

$$D_A(z) = \frac{D(z)}{1+z}, \tag{4}$$

$$D_L(z) = D(z) \cdot (1+z). \tag{5}$$

Matter

For the matter dominated Universe, we have

$$D(z) = \frac{2c}{H_0\sqrt{\Omega_m}}\left(1 - \frac{1}{\sqrt{1+z}}\right). \tag{6}$$

The comparison with the numerical solution can be observed in Figure 1

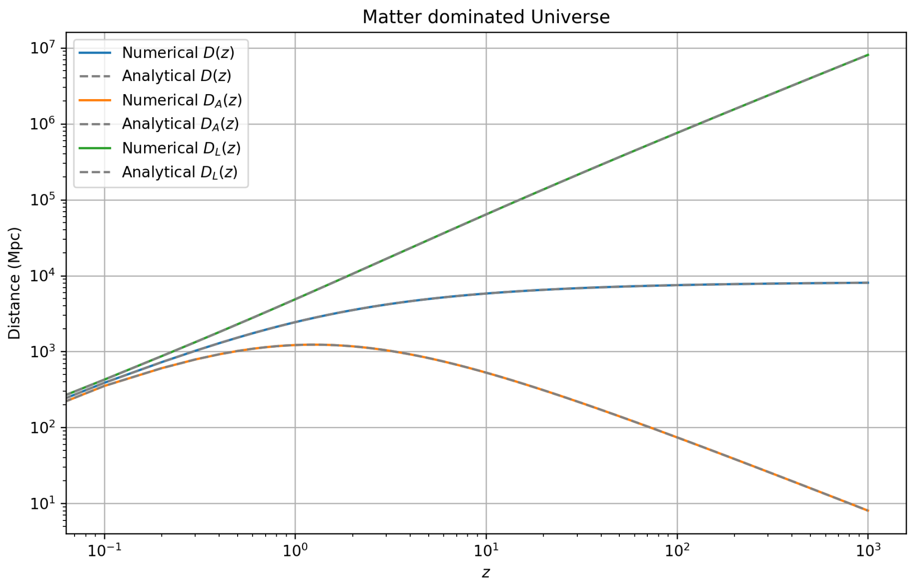

Figure 1: Matter dominated Universe, comparison between numerical and analytical solutions.

Radiation

For a Radiation dominated Universe, we have

$$D(z) = \frac{c}{H_0\sqrt{\Omega_r}}\frac{z}{1+z}. \tag{7}$$

Then, comparing with the numerical solution (Figure 2).

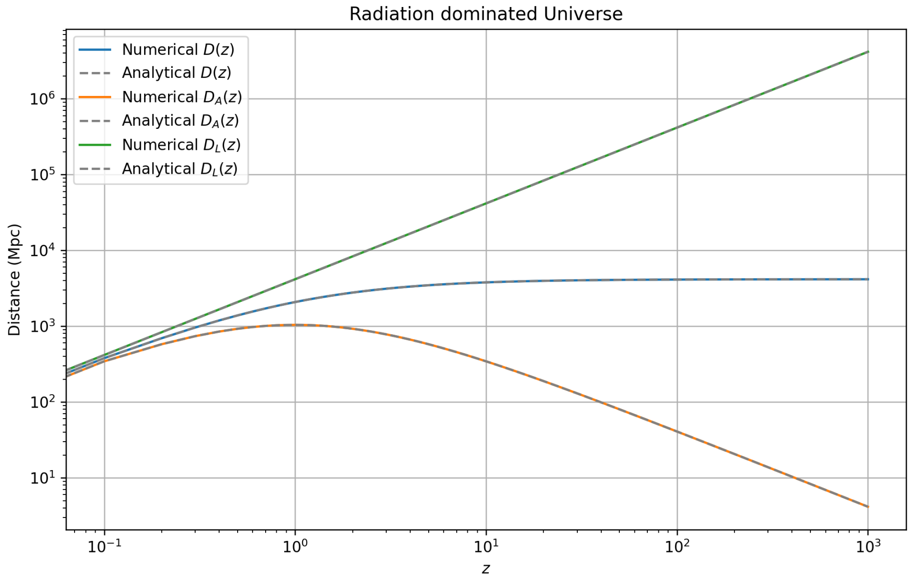

Figure 2: Radiation dominated Universe, comparison between numerical and analytical solutions.

Dark Energy

For a Dark Energy dominated Universe, we have

$$D(z) = \frac{c}{H_0\sqrt{\Omega_{DE}}}z \tag{8}$$

Then, comparing with the numerical solution (Figure 3).

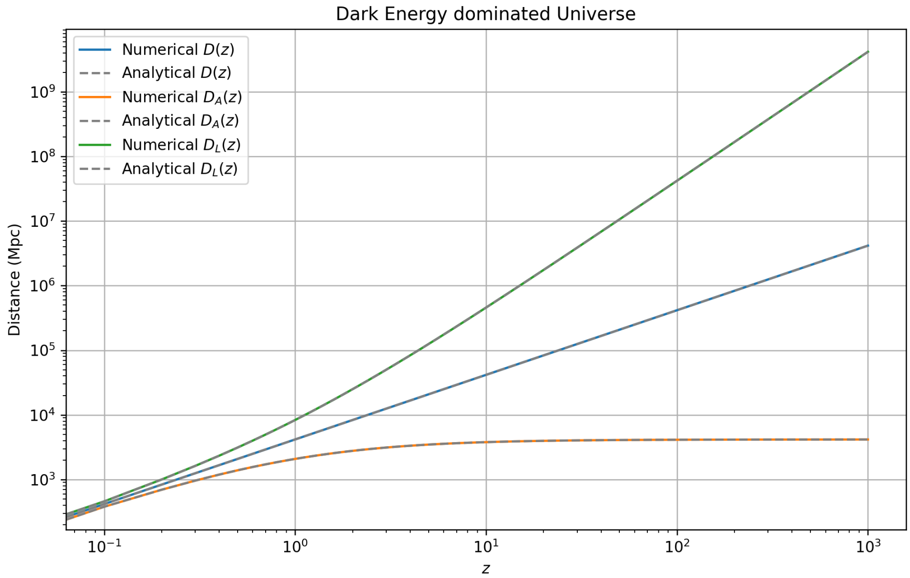

Figure 3: Dark Energy dominated Universe, comparison between numerical and analytical solutions.

Curvature

For a Curvature dominated Universe, we have

$$D(z) = \frac{c}{H_0\sqrt{\Omega_k}} \ln(1+z). \tag{9}$$

Then, comparing with the numerical solution (Figure 4).

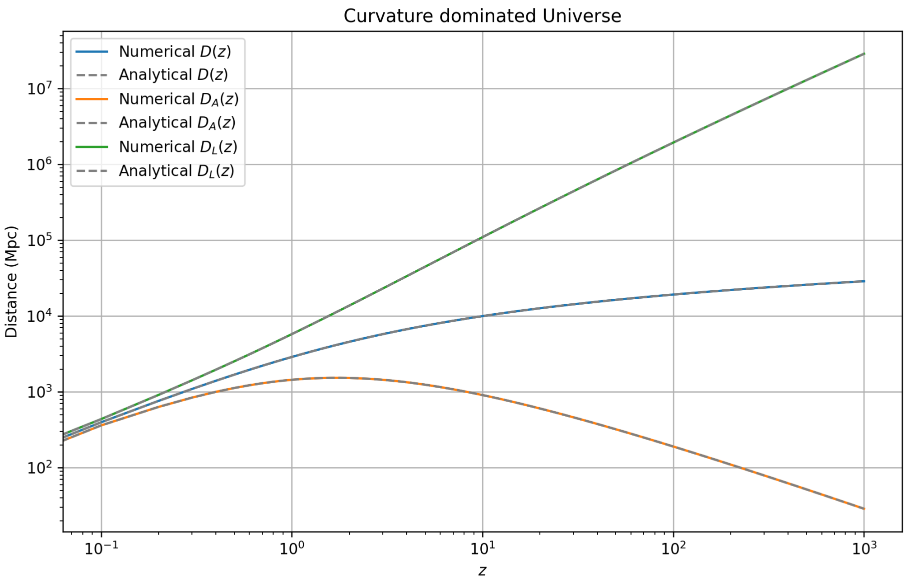

Figure 4: Curvature dominated Universe, comparison between numerical and analytical solutions.

Item b)

Here, we are computing the Matter + Curvature and Matter + Dark Energy solutions. We can see the results in the following images.

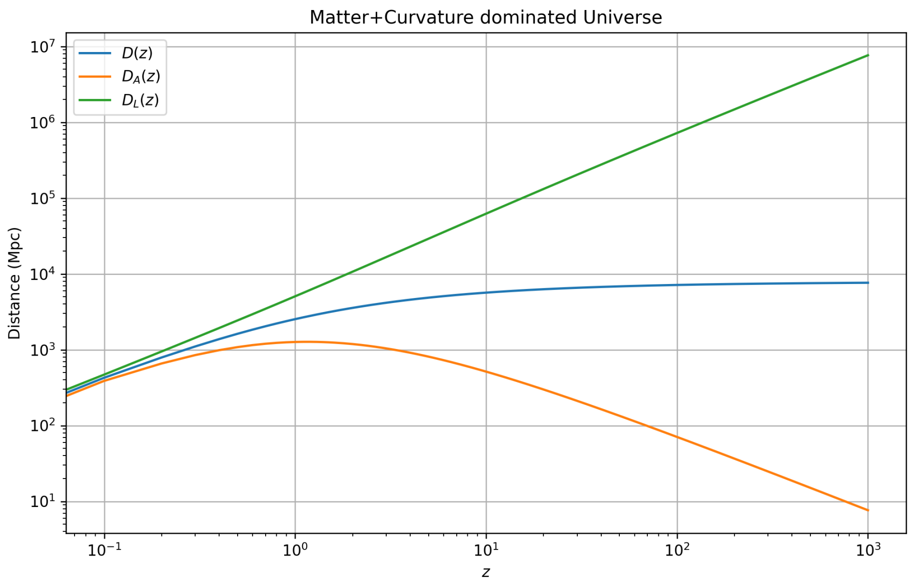

Figure 5: Matter + Curvature dominated Universe. We have $\Omega_k = -0.5$ and $\Omega_m = 1.3$ (all else is zero).

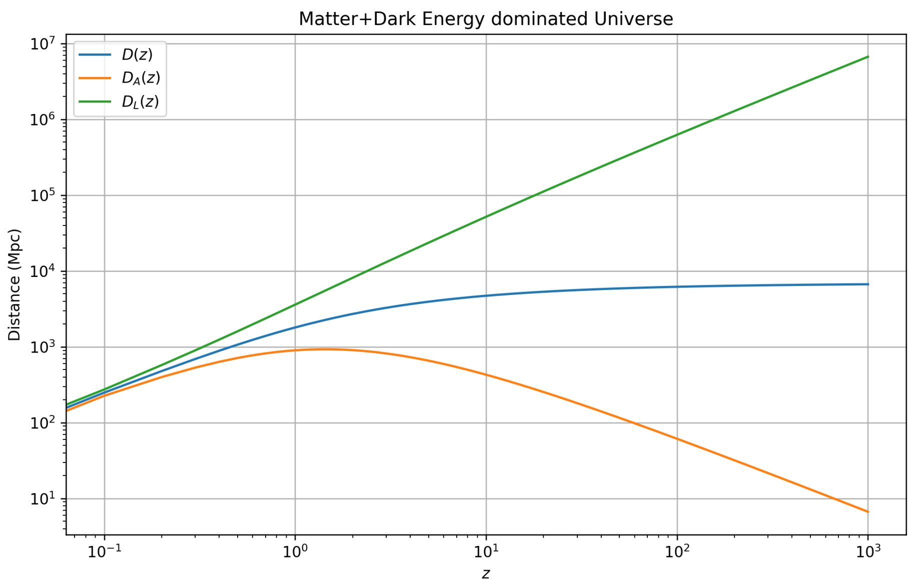

Figure 6: Matter + Dark Energy dominated Universe. We have $\Omega_m = 1.3$, $\Omega_{DE} = 1.3$ and $w = -1$ (all else is zero).

Item c)

Now we can produce the numerical solutions with all components (with $\Omega_k = 0$ for the Flat Universe cases).

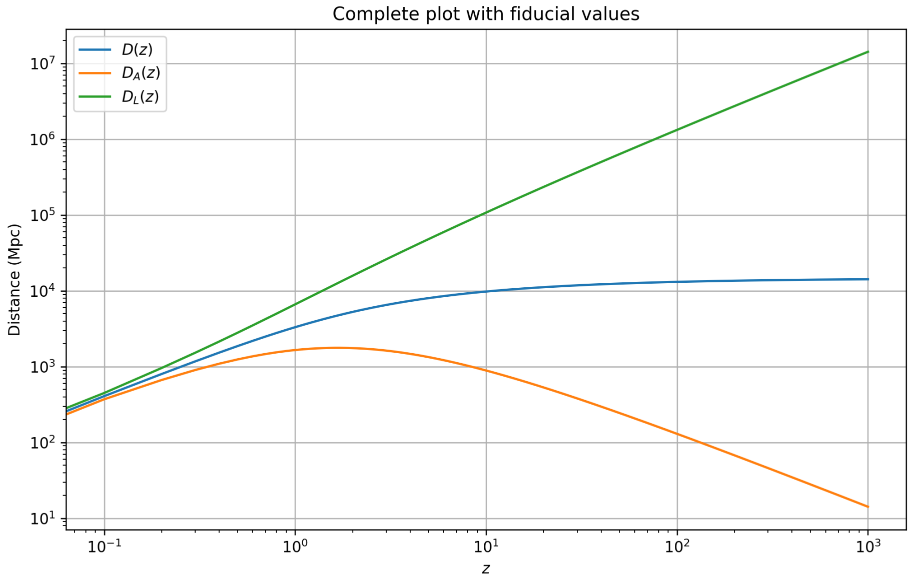

Figure 7: Plot with all fiducial values.

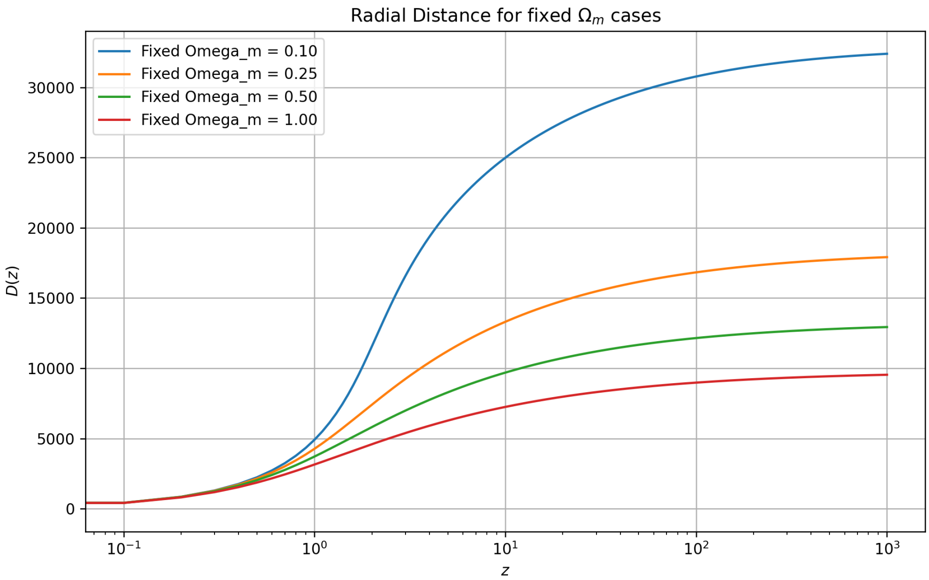

Figure 8: Radial Distance plot with Ω_m fixed.

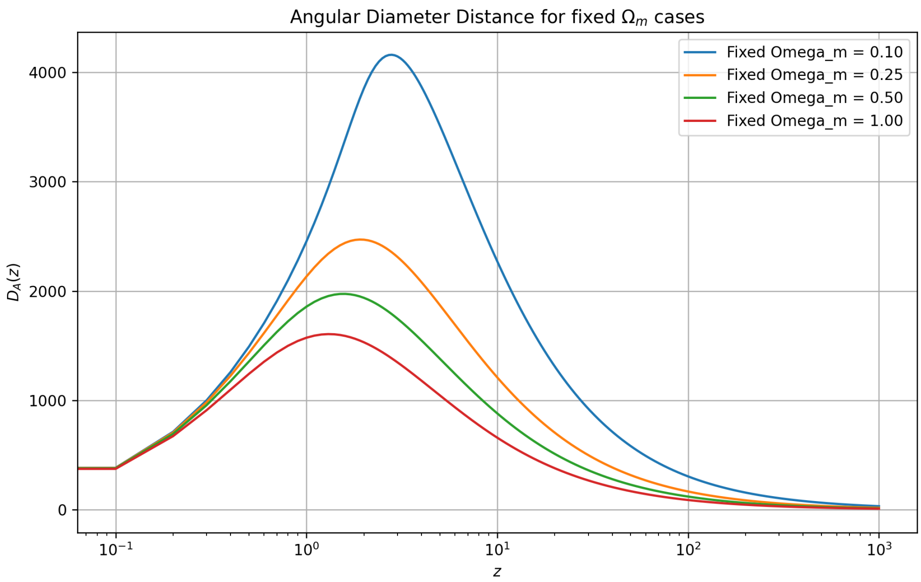

Figure 9: Angular Diameter Distance plot with Ω_m fixed.

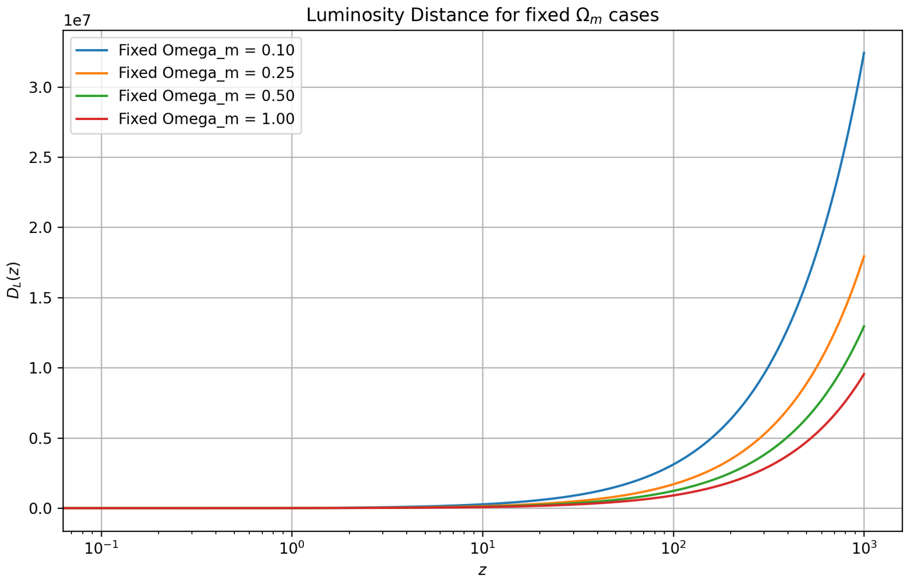

Figure 10: Luminosity Distance plot with Ω_m fixed.

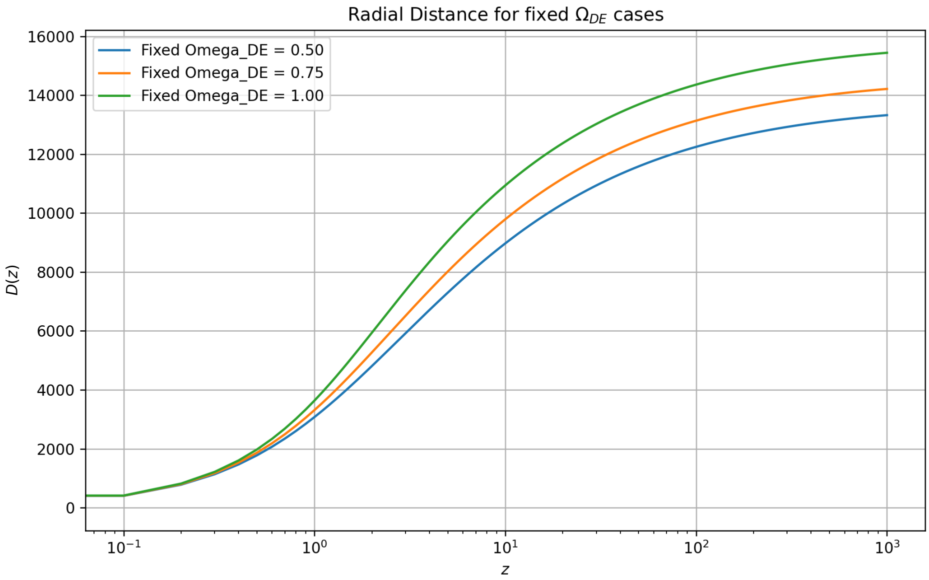

Figure 11: Radial Distance plot with Ω_{DE} fixed.

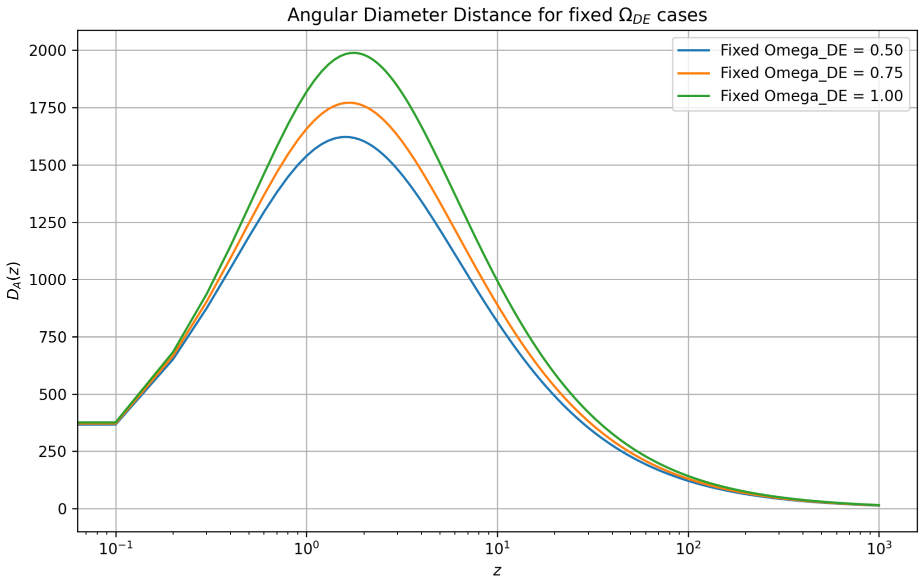

Figure 12: Angular Diameter Distance plot with Ω_{DE} fixed.

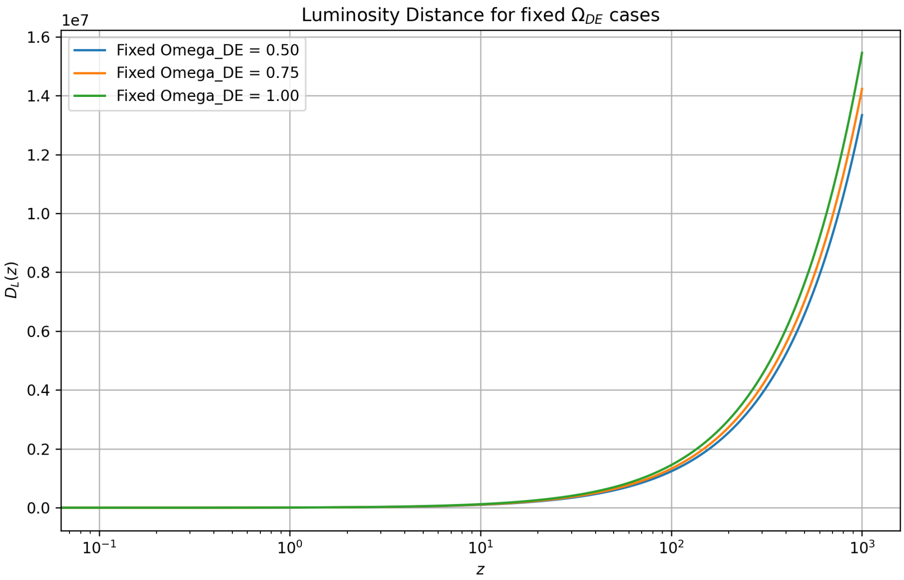

Figure 13: Luminosity Distance plot with Ω_{DE} fixed.

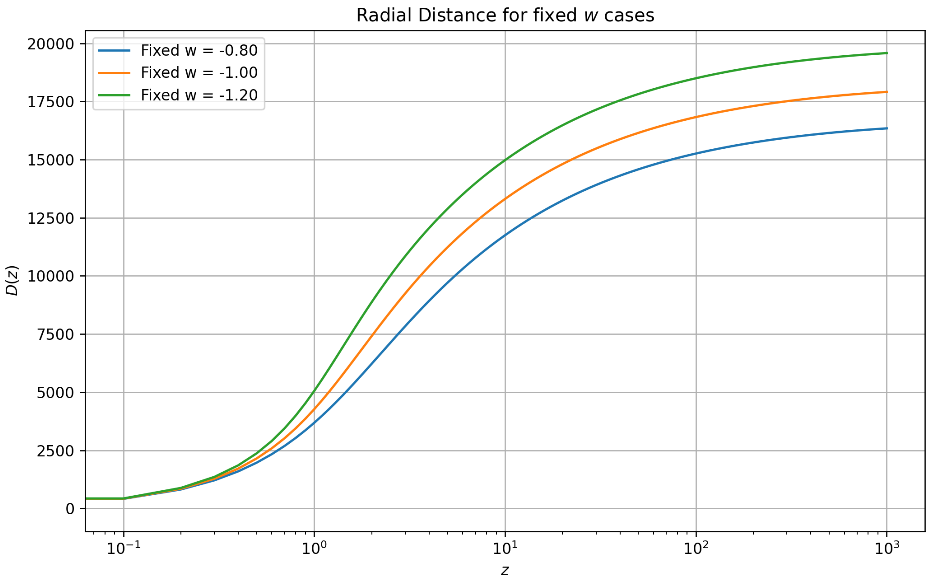

Figure 14: Radial Distance plot with w fixed.

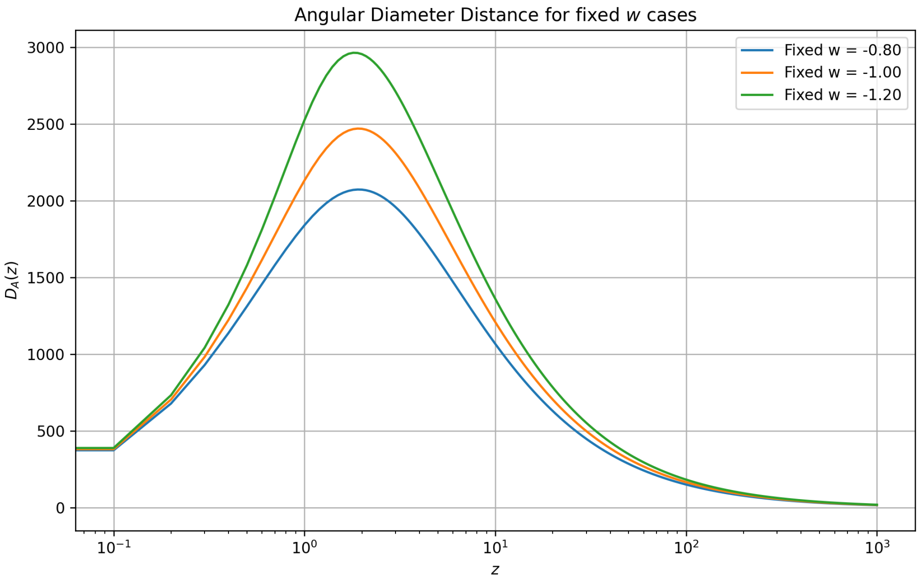

Figure 15: Angular Diameter Distance plot with w fixed.

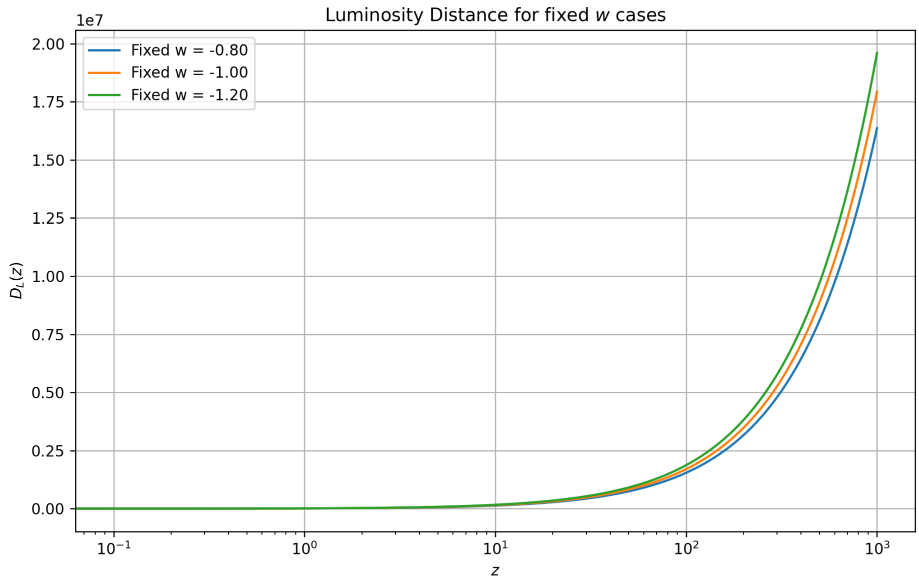

Figure 16: Luminosity Distance plot with w fixed.

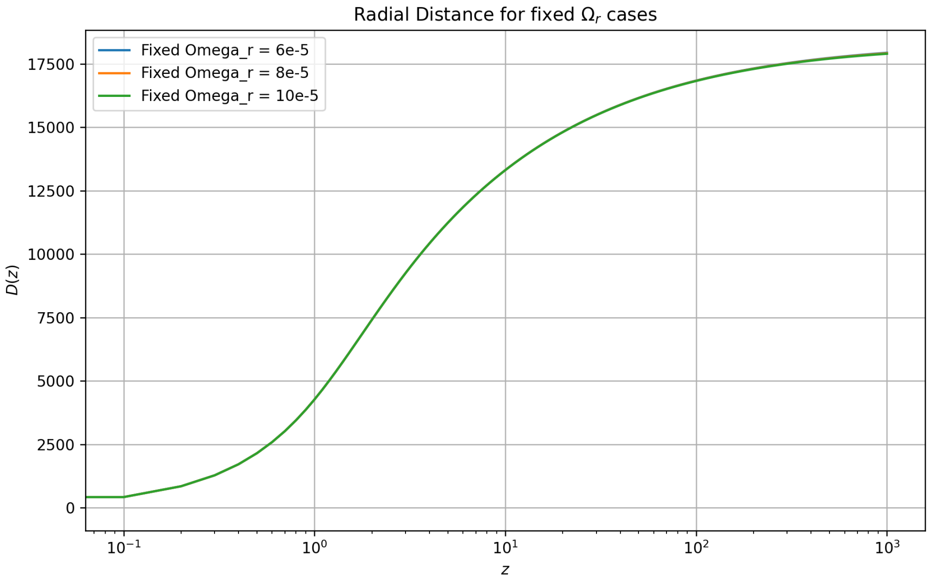

Figure 17: Radial Distance plot with Ω_r fixed.

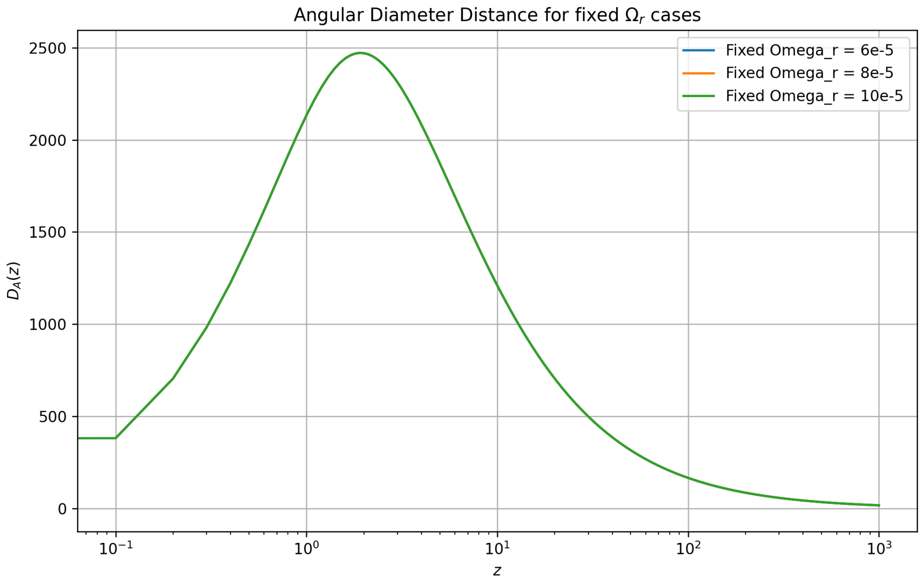

Figure 18: Angular Diameter Distance plot with Ω_r fixed.

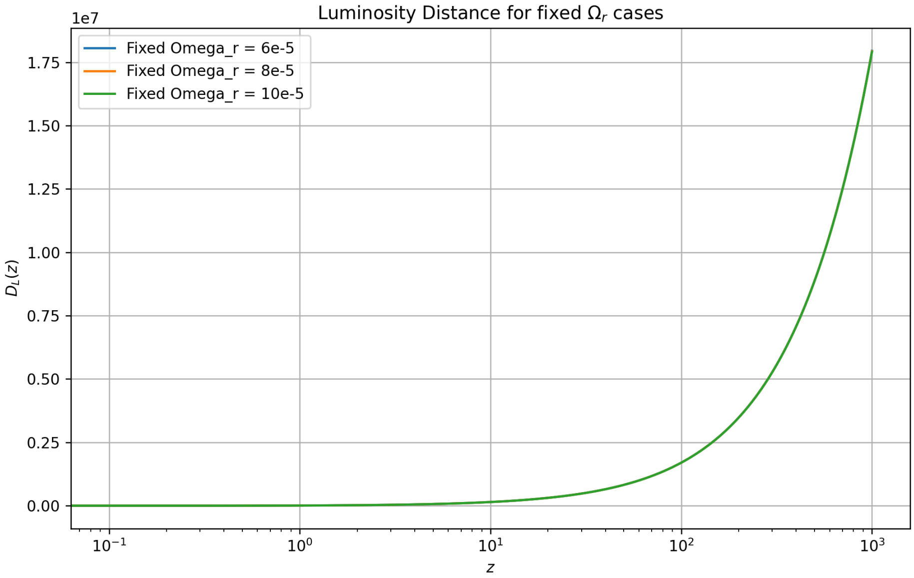

Figure 19: Luminosity Distance plot with Ω_r fixed.

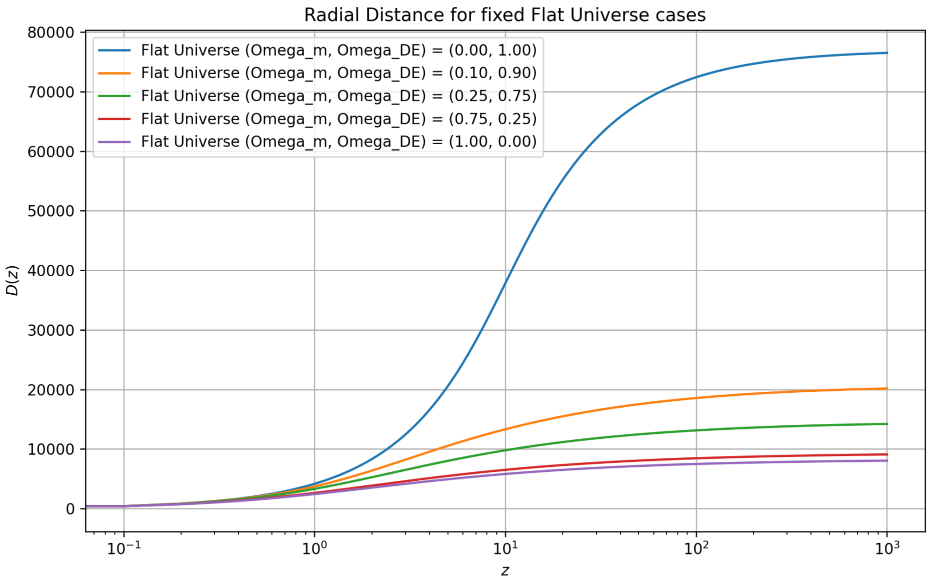

Figure 20: Radial Distance plot with Flat Universe cases.

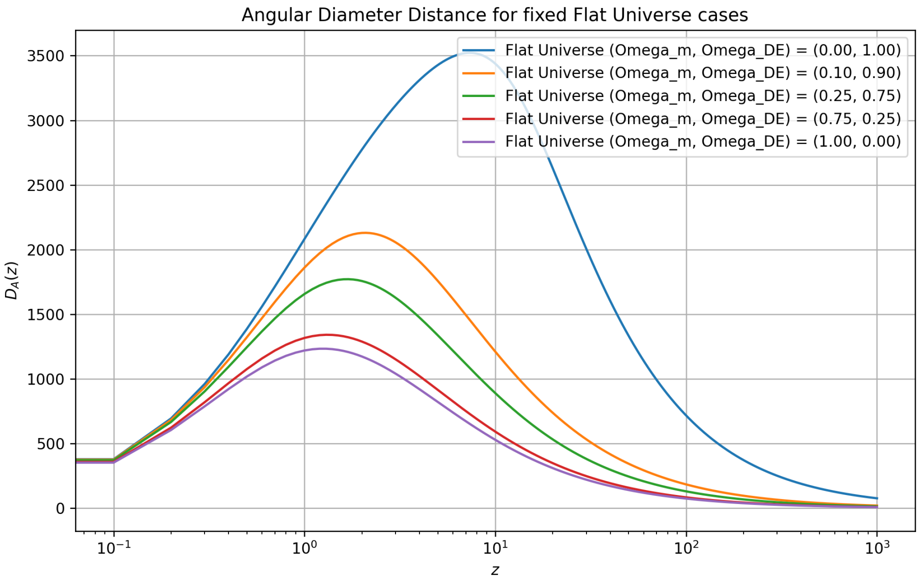

Figure 21: Angular Diameter Distance plot with Flat Universe cases.

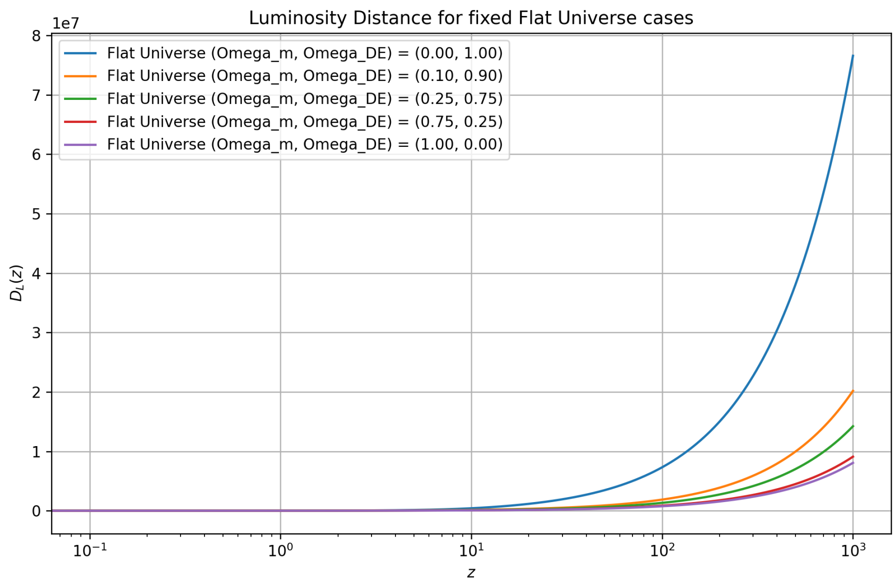

Figure 22: Luminosity Distance plot with Flat Universe cases.