Question 1 - Correlation Function in Redshift Space

In class, we saw that the correlation function in redshift space is anisotropic in space and given by

$$\xi_g^s(\mathbf{s}) = \xi_g^s(s_\parallel, s_\perp) = \xi_g^s(s, \mu_s) = b^2 \sum_{l=0,2,4} c_l(\beta) L_l(\mu_s) \xi_g^s(s) \tag{1}$$

where $L_l(\mu_s)$ is the Legendre Polinomial of order l, $\mu_s = \cos(\theta_s)$ is the cosine of the angle between the vector $\mathbf{s}$ and the line-of-sight $\hat{\mathbf{z}}$, the coefficients

$$c_l(\beta) = \frac{2l+1}{2} \int_{-1}^{1} (1+\beta x^2)^2 L_l(x) \mathrm{d}x = \begin{cases} 1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2, & l = 0 \\ \frac{4}{3}\beta + \frac{4}{7}\beta^2, & l = 2 \\ \frac{8}{35}\beta^2, & l = 4 \end{cases} \tag{2}$$

where $\beta = f/b$, b is the galaxy bias and $f = \frac{\mathrm{d}\ln D}{\mathrm{d}\ln a}$, and the multipoles

$$\xi_l^s(s) = i^l \int \frac{k^2 \mathrm{d}k}{2\pi^2} j_l(ks) P^r(k) \tag{3}$$

Assume the fiducial cosmology from previous problem sets in the calculations below.
a) From the real-space matter power spectrum $P^r(k)$ (e.g. from CAMB), use Eq. 3 to compute the multipoles $\xi_l^s(s)$ for $l = 0, 2, 4$. Plot each multipole as a function of separation s in log-scale and appropriate ranges. Notice that if the spectrum has k in units of h/Mpc, the separation s will naturally be in units of Mpc/h. Similarly $P^r(k)$ is in units of [Mpc/h]^3, and therefore the multipoles are unitless. Note that $j_0(x) = \sin(x)/x$ and other values of l can be obtained by recurrence relations.
b) Assuming $b = 1$ compute explicitly the analytical integral in Eq. 2 to derive the coefficients $c_l(\beta)$. If you have a numerical growth function $D(z)$ from the previous problem set 8, use it to compute f; otherwise use a fitting function [e.g. $f = \Omega_m^\gamma(z)$ where $\Omega_m(z) = \Omega_m(1+z)^3/E^2(z)$]. Then plot $f(z)$ and $c_l[\beta(z)]$ (for $l = 0, 2, 4$) as a function of redshift z.
c) Finally, use Eq. 1 to obtain the redshift-space correlation function as a function of parallel and perpendicular directions at $z = 0$. Make a 2D plot of your results, with $s_\perp = s\cos\theta_s$ in the y-axis, and a color coding for the value of $\xi^s(s_\parallel, s_\perp)$. Compare these results to a similar 2D plot for the isotropic real-space correlation $\xi^r(s) = \xi_{l=0}^s(s)$. Repeat the same plots at $z = 1.0$.

Table 1: Standard cosmology values (without units of measure).

Parameter	Value
$\Omega_b h^2$	0.022
$\Omega_c h^2$	0.12
h	0.675
n_s	0.965
A_s	$2.1 \cdot 10^{-9}$
τ	0.06
Ω_k	0.0
w	-1
N_{eff}	3.046
T_{CMB}	2.7255

Here, we are considering $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$.

Item a)

We can compute the multipoles $\xi_l^s(s)$ from the matter power spectrum.

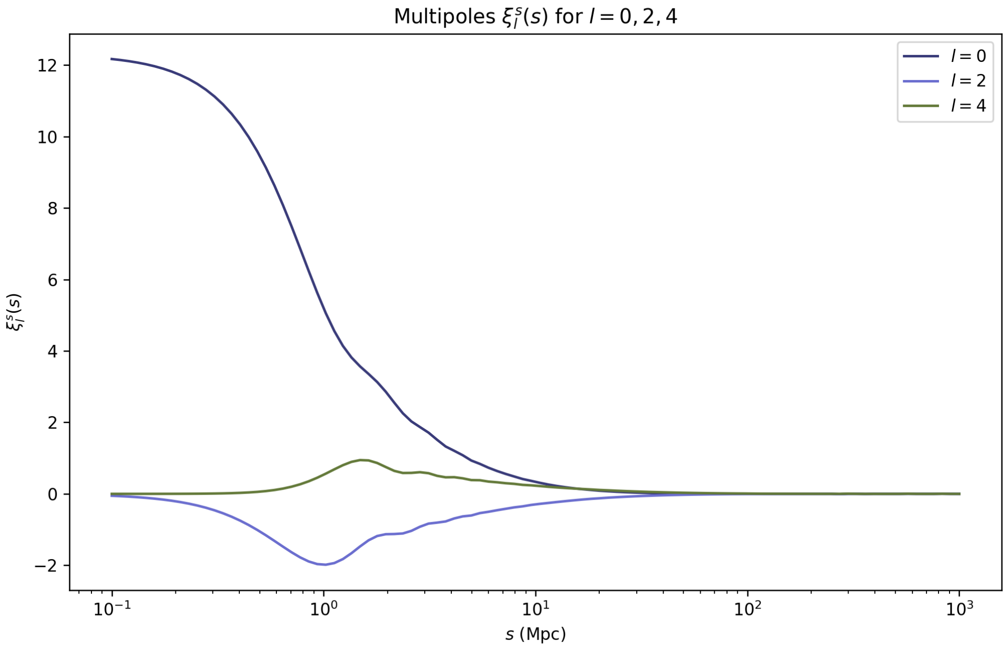

Figure 1: Multipoles $\xi_l^s(s)$ for $l = 0, 2, 4$.

Item b)

From the equations given by the problem, we get Figure 2.

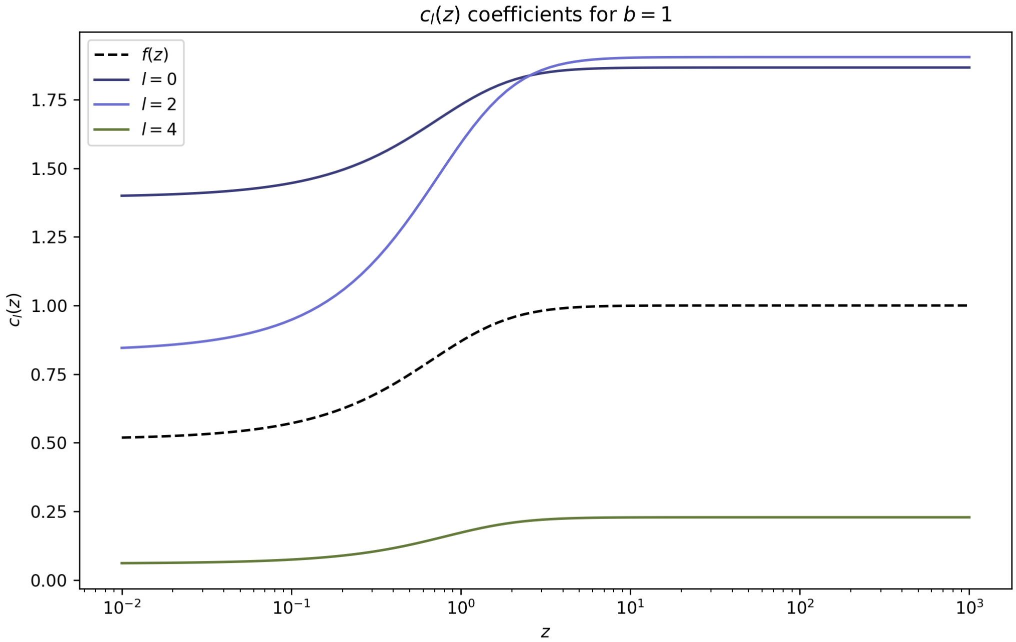

Figure 2: c_l values for $l = 0, 2, 4$ (Compared with $f(z)$).

We can observe that c_l for $l = 4$ grows slower than both $l = 0$ and $l = 2$. We can also see that as $c_{l=2}$ grows, it surpasses $c_{l=0}$, which indicates a significant level of anisotropy at this level.

Item c)

Calculating the Redshif-space correlation function, we can observe that both plots for $l = 0, 2, 4$ (Figure 3) and $l = 0$ (Figure 4) are similar.

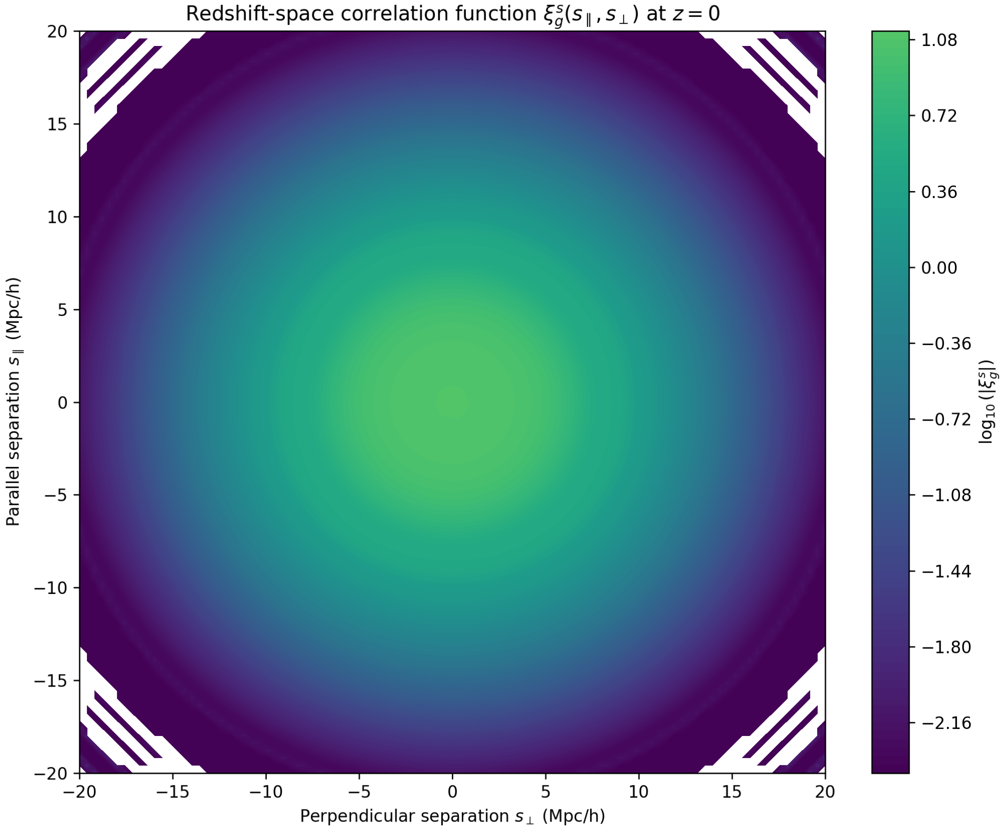

Figure 3: Redshift-space correlation function values for $l = 0, 2, 4$ at $z = 0$.

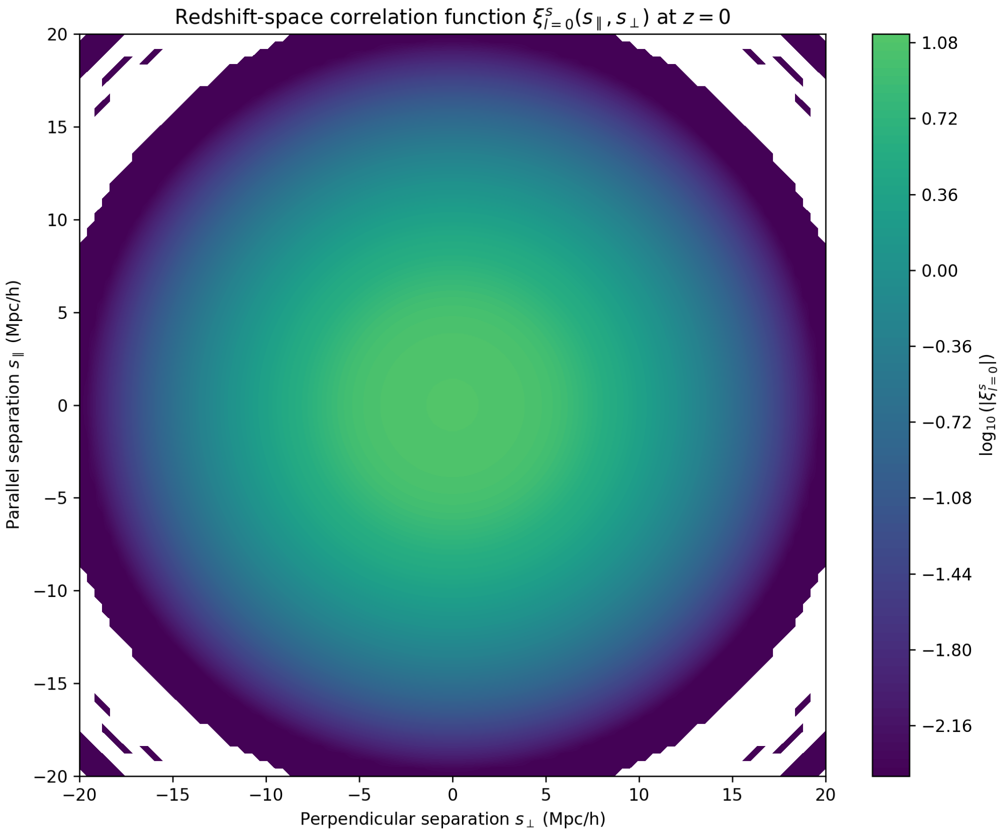

Figure 4: Redshift-space correlation function values for $l = 0$ at $z = 0$.