Question 1 - CAMB

Download and install CAMB at `camb.info`. This is a Fortran code which evolves the linear perturbations for the Einstein-Boltzmann equations accounting for various species (dark matter, baryons, photons, neutrinos, scalar fields, etc.). Notice CAMB can also be used within Python. Run the code to generate the matter power spectrum $P(k)$ today, i.e. at $z = 0$.
Plot $P(k)$ versus k in logarithmic scale (indicating units in the axis!), for the values of cosmological parameters from problem 1 in Problem Set 3.

i) $\Omega_m = 0.1, 0.25, 0.5, 1.0$ (with all other parameters equal to fiducial)

ii) $\Omega_{DE} = 0.5, 0.75, 1.0$ (with all other parameters equal to fiducial)

iii) $w = -0.8, -1.0, -1.2$ (with all other parameters equal to fiducial)

iv) $\Omega_r = (6, 8, 10) \times 10^{-5}$ (with all other parameters equal to fiducial)

v) Flat cases: $(\Omega_m, \Omega_{DE}) = (0.0, 1.0), (0.1, 0.9), (0.25, 0.75), (0.75, 0.25), (1.0, 0.0)$.

Describe (using the plots) the effects of changing the various parameters.

The only density parameters that can be set directly in CAMB are $\Omega_b \cdot h^2$ (baryon density times the reduced Hubble constant), $\Omega_c \cdot h^2$ (cold dark matter density times the reduced Hubble constant) and Ω_k. We know that

$$\Omega_b + \Omega_c = \Omega_m. \tag{1}$$

The fiducial values of these densities are $\Omega_b \cdot h^2 = 0.022$ and $\Omega_c \cdot h^2 = 0.122$, and for $h = 0.72$, we find that $\Omega_c \approx 0.84\Omega_m$ and $\Omega_b \approx 0.16\Omega_m$. With these approximations we can find the respective matter density components for an desired Ω_m.

The parameter of Ω_{DE} is automatically calculated by CAMB as

$$\Omega_{DE} = 1 - \Omega_k - \Omega_m - \Omega_r, \tag{2}$$

so we need to manipulate Ω_k, Ω_m and Ω_r in order to find the Ω_{DE} that we desire. Ω_r is dependant on the CMB temperature (T_{CMB}), which can be obtained by

$$\Omega_r = \frac{\pi^2}{15}\left(\frac{T_{CMB}}{a}\right)^4 \frac{1}{8.098 \cdot 10^{-11} h^2 \mathrm{eV}^4}, \tag{3}$$

where $a = 1$ for today time and 1 eV = 11605 K, and then

$$T_{CMB} = \left(\frac{\Omega_r h^2 \cdot 15 \cdot 8.098 \cdot 10^{-11} \cdot 11605^4}{\pi^2}\right)^{1/4}, \tag{4}$$

in this way we can find the correct temperature for a given Ω_r. Now, we have the results

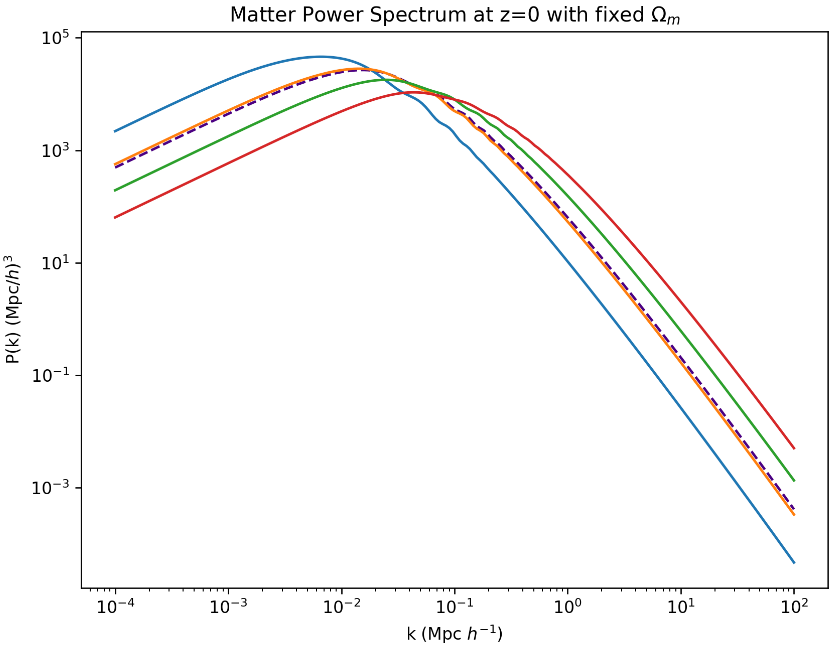

Figure 1: $P(k)$ for fixed Ω_m cases.

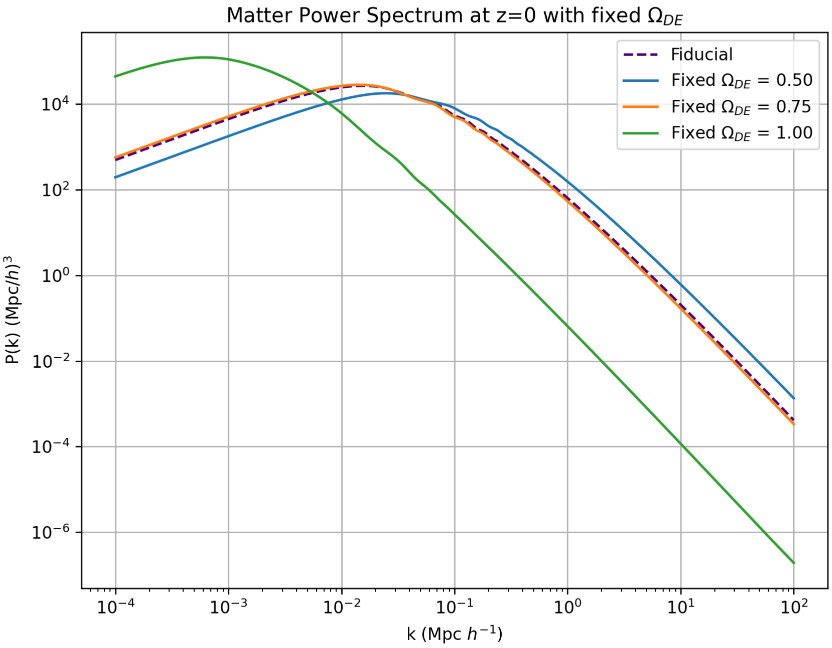

Figure 2: $P(k)$ for fixed Ω_{DE} cases.

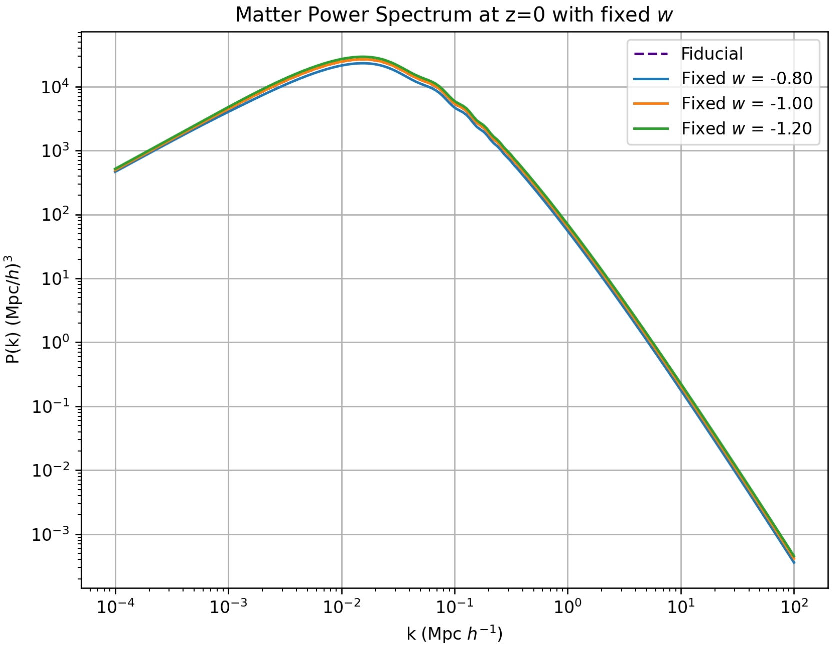

Figure 3: $P(k)$ for fixed w cases.

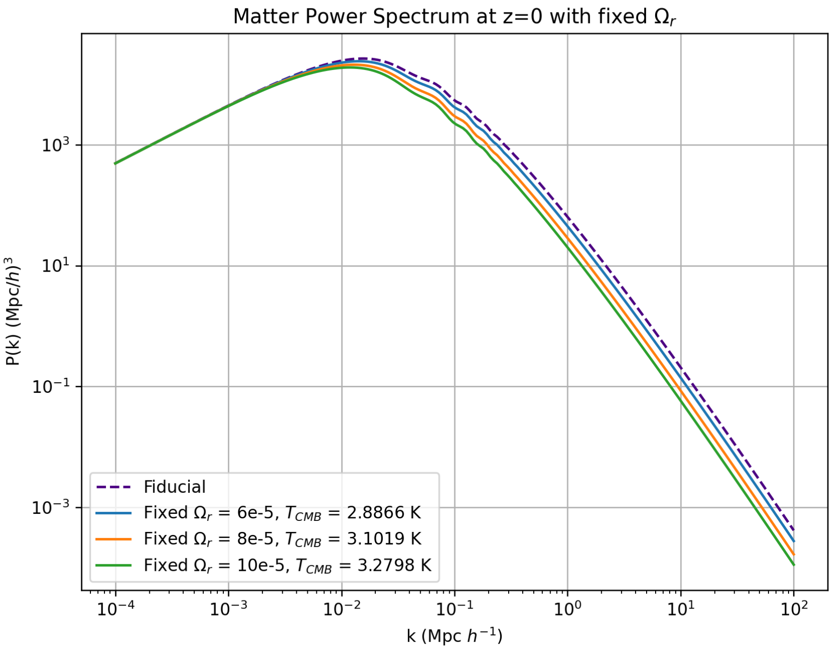

Figure 4: $P(k)$ for fixed Ω_r cases.

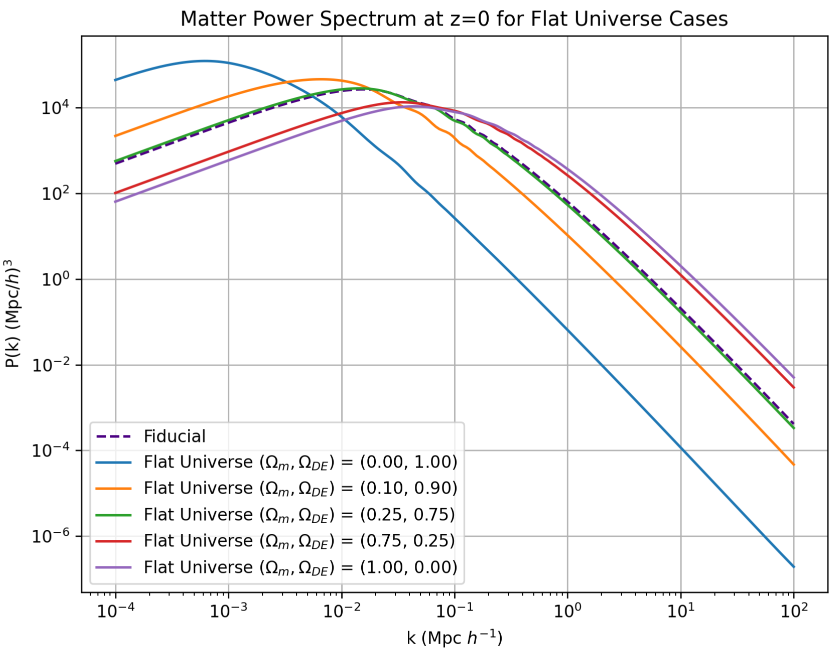

Figure 5: $P(k)$ for flat cases.

Question 2 - Top-Hat Window

Define the radial top-hat radial window funcion $W(\mathbf{x}, R)$ on a scale R as

$$W(\mathbf{x}, R) = \begin{cases} 3/(4\pi R^3) & \text{if } r < R, \\ 0 & \text{if } r > R, \end{cases} \tag{5}$$

where $r = |\mathbf{x}|$. Show that the Fourier Transform of $W(\mathbf{x}, R)$ is given by

$$\tilde{W}(kR) = \frac{3}{k^2R^2}\left[\frac{\sin(kR)}{kR} - \cos(kR)\right] \tag{6}$$

We can write the Fourier Transform in spherical coordinates of $W(r, R)$ as it follows

$$\begin{aligned}
\tilde{W}(kR) &= \frac{3}{4\pi R^3}\int_0^R r^2 \,\mathrm{d}r \int_0^\pi \sin\theta e^{-ikr\cos\theta}\,\mathrm{d}\theta \int_0^{2\pi}\mathrm{d}\varphi \\
&= \frac{3}{2R^3}\int_0^R r^2 \,\mathrm{d}r \int_0^\pi \sin\theta e^{-ikr\cos\theta}\,\mathrm{d}\theta \\
&= \frac{3}{2R^3}\int_0^R r^2 \left(\frac{e^{ikr\cos\theta}}{ikr}\right)\Bigg|_\pi^0 \mathrm{d}r \\
&= \frac{3}{2R^3}\int_0^R r \left(\frac{e^{ikr} - e^{-ikr}}{ik}\right) \mathrm{d}r \\
&= \frac{3}{2R^3}\int_0^R r\frac{2\sin(kr)}{k}\,\mathrm{d}r,
\end{aligned}$$

and then

$$\boxed{\tilde{W}(kR) = \frac{3}{k^2R^2}\left[\frac{\sin(kR)}{kR} - \cos(kR)\right]}$$

Question 3 - Filtered Variance

Use the CAMB power spectrum $P(k)$ for the fiducial cosmology in problem 1) to compute (numerically) the variance of the linear density field filtered on a scale R:

$$\sigma_R^2 = \int \frac{d^3k}{(2\pi)^3} P(k) |\tilde{W}(kR)|^2 = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2 P(k) |\tilde{W}(kR)|^2 \tag{7}$$

where $\tilde{W}(kR)$ is given in problem 2). Obviously the numerical integral cannot be done in the formal limits above, so make some convergence test to assure your answer does not depend on the specific range used.
Plot σ_R versus R in logarithmic scale from $R = 10^{-2}\, h^{-1}\,\text{Mpc}$ to $R = 10^{2}\, h^{-1}\,\text{Mpc}$. What is the value you obtain for σ_R when $R = 8\, h^{-1}\,\text{Mpc}$? This is known as σ_8 and is typically quoted as a useful cosmological parameter related to the amplitude of the spectrum at $z = 0$. How does the value you obtain compare to the value outputted by CAMB? How does it compare to the value obtained by recent experiments (e.g. from the Planck satellite)?

The convergence test can be seen in the attached script, but the function created doesn't deppend on an specific range as the values of σ_R don't change too much when using different k_{min} and k_{max} values. The plot of σ_R can be seen below.

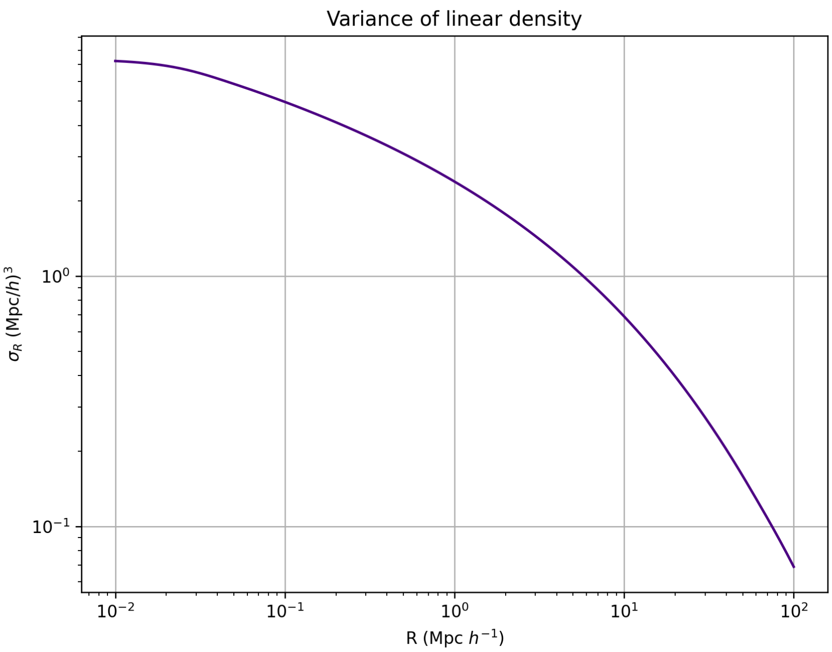

Figure 6: Variance of linear density.

The obtained value of $\sigma_R(8)$ is $\sigma_8 = 0.8064$, which is close to the Planck Collaboration result of $\sigma_{8,\text{Planck}} = 0.8111 \pm 0.0060$.