Calculating Probability Theoretically

Stat 241

Theoretical Probability calculations combine

1.

2.

3.

Probability by Axioms and Rules

Three Axioms

Let S be a sample space for a random experiment and A and B be events (sets of outcomes).

- The events A and B are **mutually exclusive** (ME) if A and B have no outcomes in common; i.e., A and B cannot occur simultaneously.

- The event "A or B" is the event consisting of all outcomes that are either in A or in B (possibly both).
- The event "A and B" is the event that consists of all outcomes that are in both A and B.
- The event "not A" consists of all outcomes not in the event A.

1. $0 \leq P(A) \leq 1$.
2. $P(S) = 1$.
3. (Sum Rule) If A and B are mutually exclusive then $P(A \text{ or } B) = P(A) + P(B)$.

   In fact, this works for multple events: $P(A_1 \text{ or } A_2 \text{ or } A_3 \cdots A_k) = P(A_1) + P(A_2) + P(A_3) + \cdots P(A_k)$ as long as all the events are mutuall exclusive (at most one can happen on any given trial).

Derived Rules

4. (Complement Rule) $P(\text{not } A) = 1 - P(A)$.
5. (General Sum Rule) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
6. (Equally Likely Rule) If the sample space consists of n equally likely outcomes, then

$$P(A) = \frac{|A|}{|S|} = \frac{\text{number of outcomes in } A}{\text{number of outcomes in } S}$$

Examples

1. Carol has applied for admission to Harvard and to Princeton. The probability that Harvard accepts her is .3, the probability that Princeton accepts her is .4, and the probability that both accept her is .2. What is the probability that neither accept her?

2. Toss a fair die.

- $S =$

- P(die comes up prime) =

- P(die comes up odd) =

3.. Toss a fair coin 3 times.

- $S =$

- If $X =$ number of heads, then the probability table for X is

4. Toss a pair of dice. Let X = the sum of the two numbers rolled, then

- $S =$

- The probability table for X is

- Compare with the simulated result

2	3	4	5	6	7	8	9	10	11	12
0.03	0.053	0.084	0.108	0.134	0.163	0.146	0.112	0.088	0.056	0.027

- $P(X \geq 10) =$

- $P(X < 10) =$

- $P(6 \text{ or doubles}) =$