Stan

Software Ecosystem for Modern Bayesian Inference

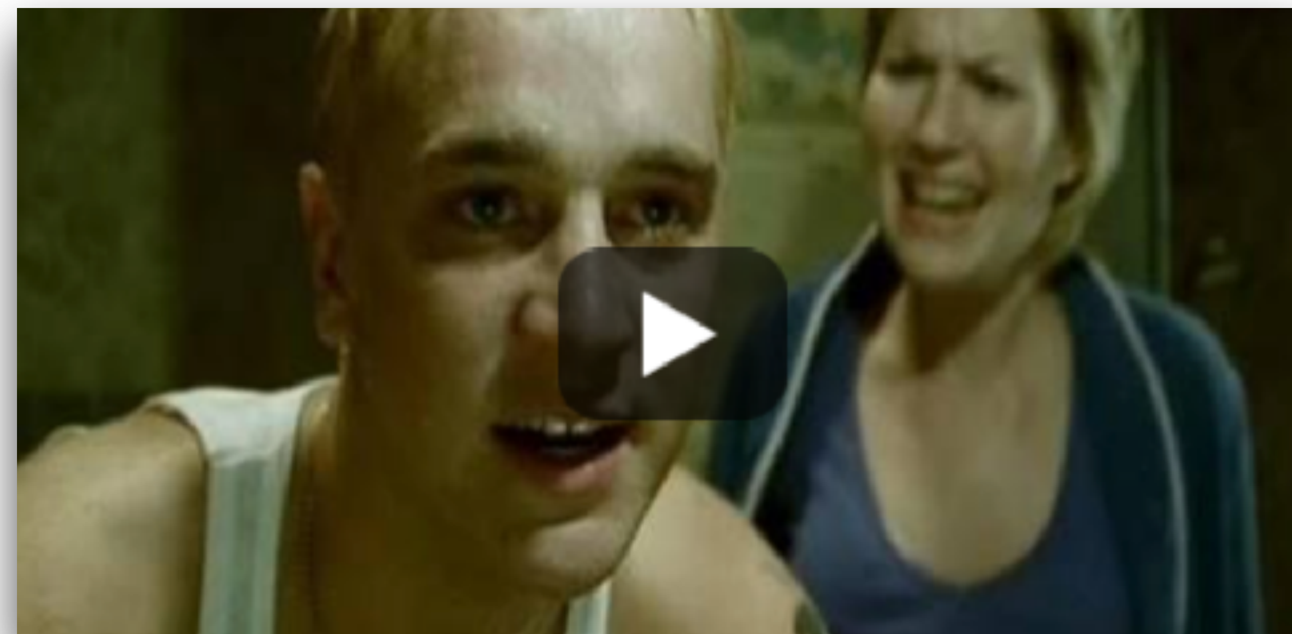
Course materials:
https://github.com/rpruim/StanWorkshop

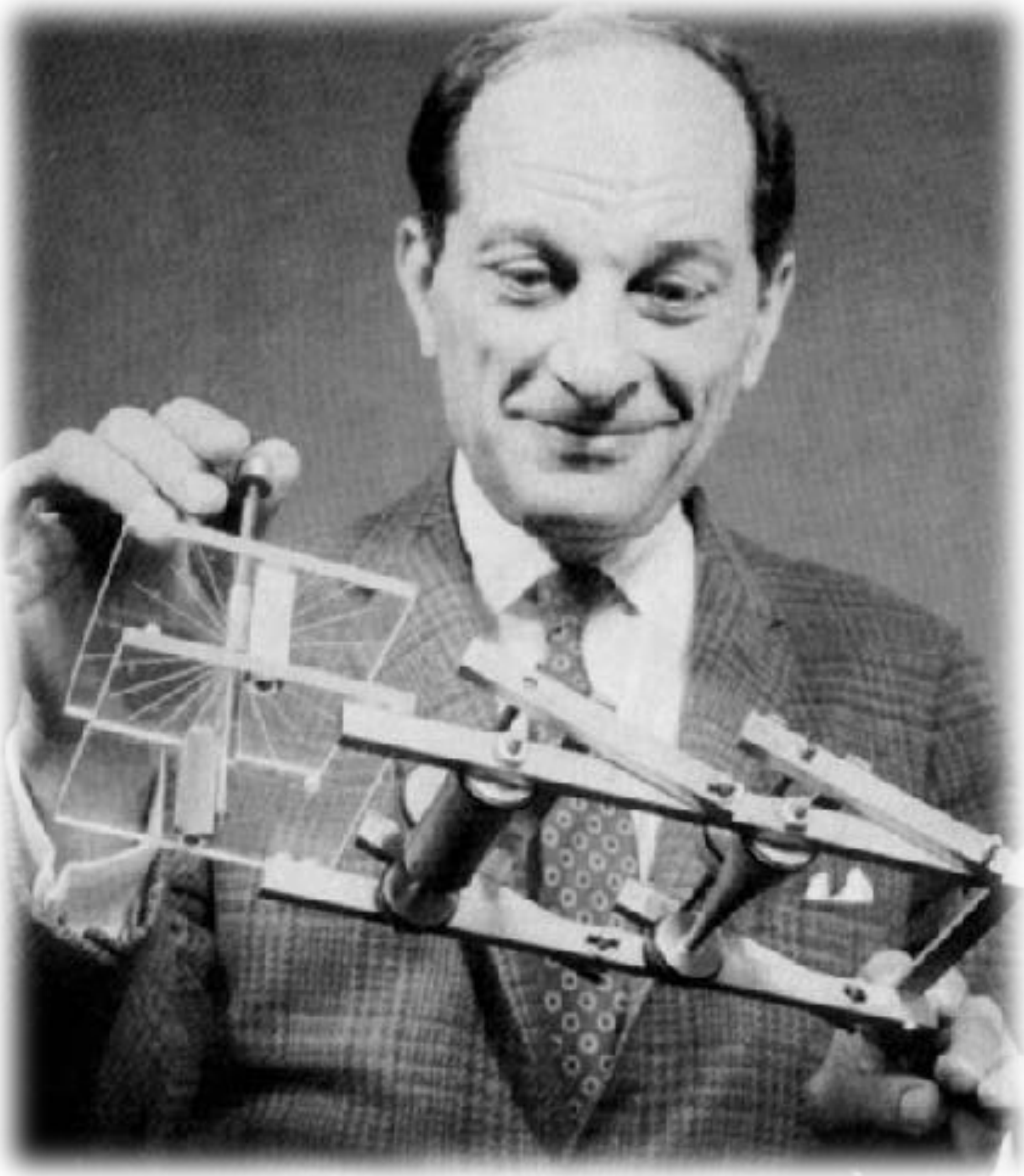
Jonah Gabry
Columbia University

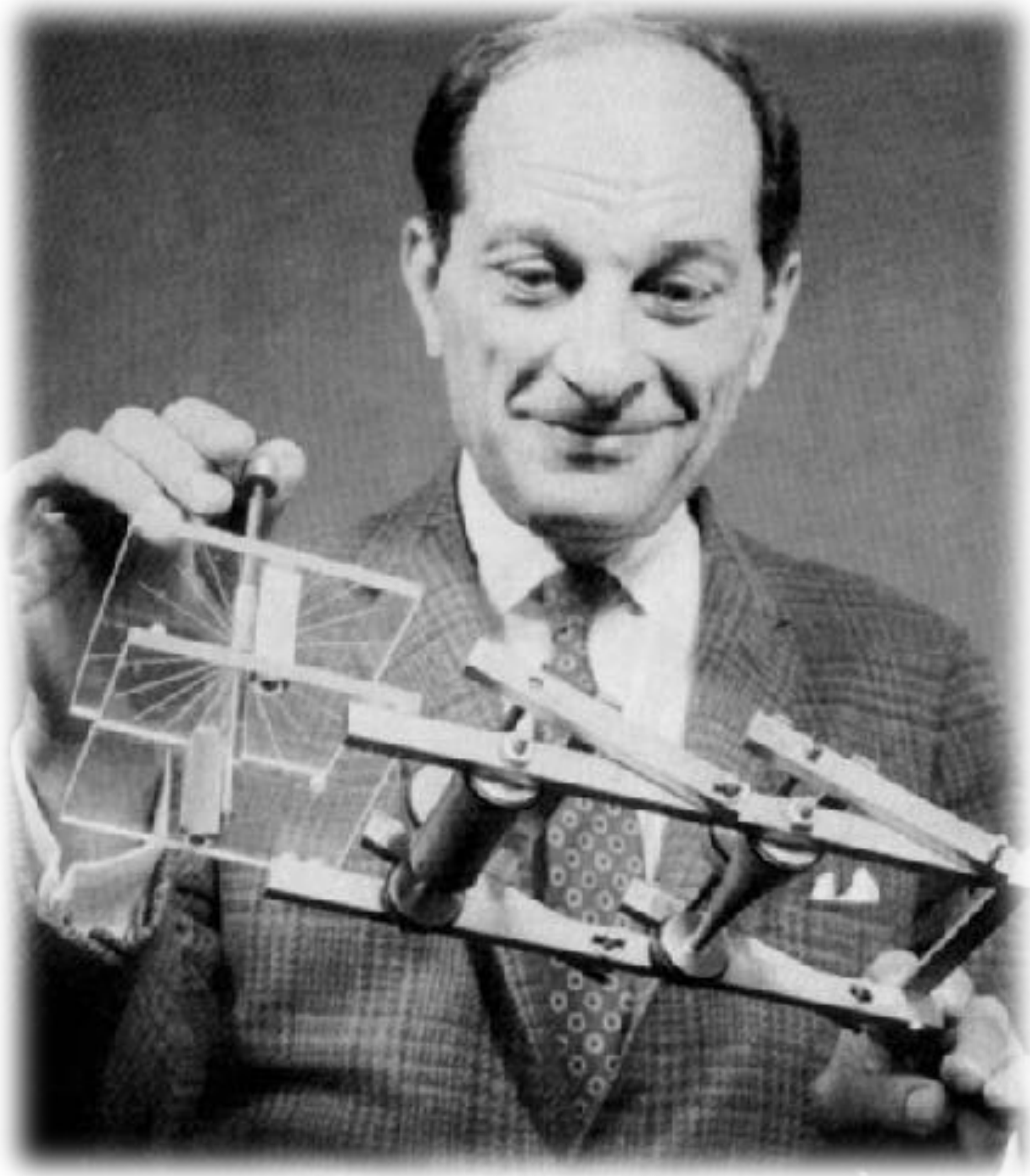
Vianey Leos Barajas
Iowa State University

Why "Stan"?

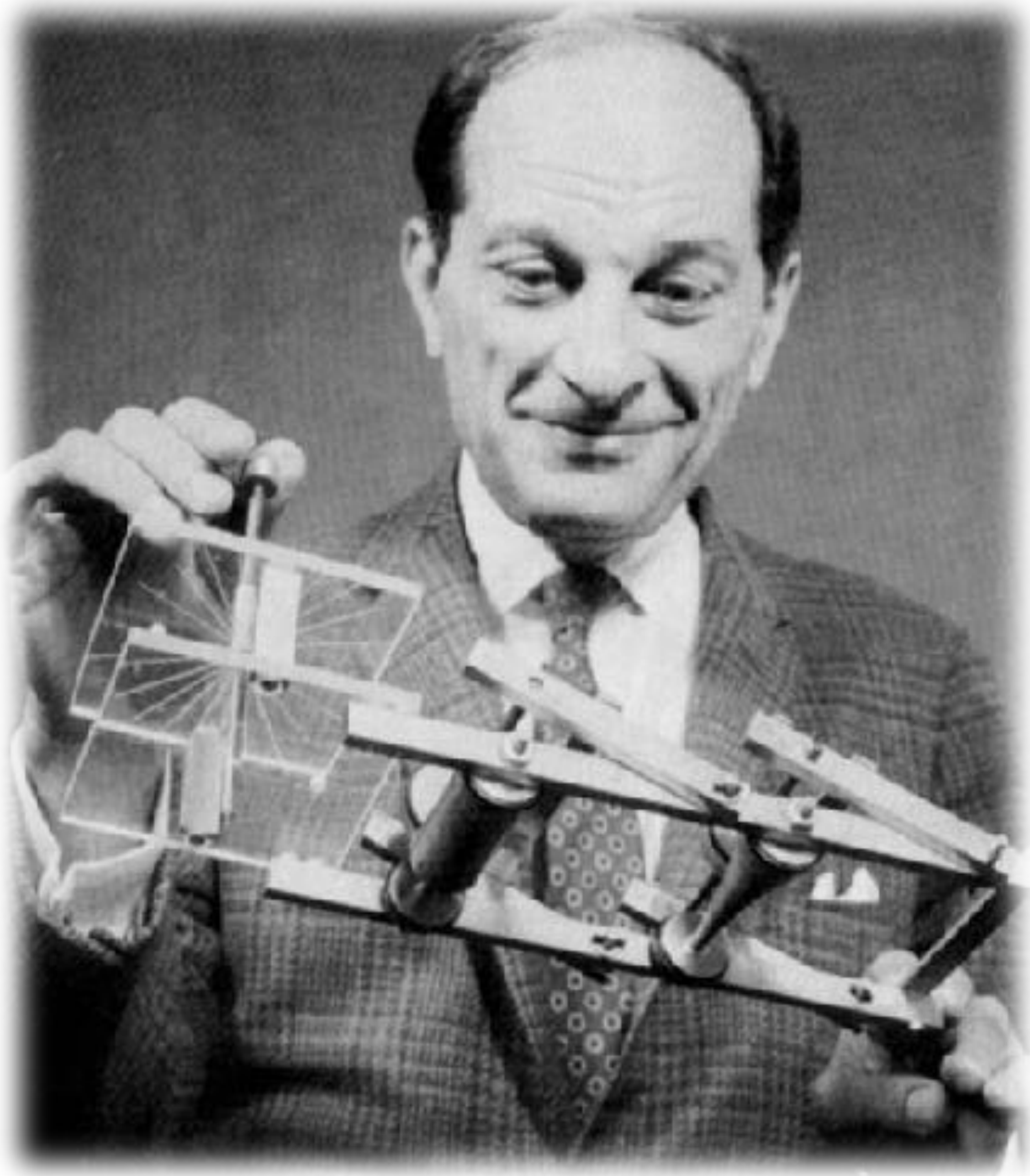
suboptimal SEO







Stanislaw Ulam
(1909–1984)



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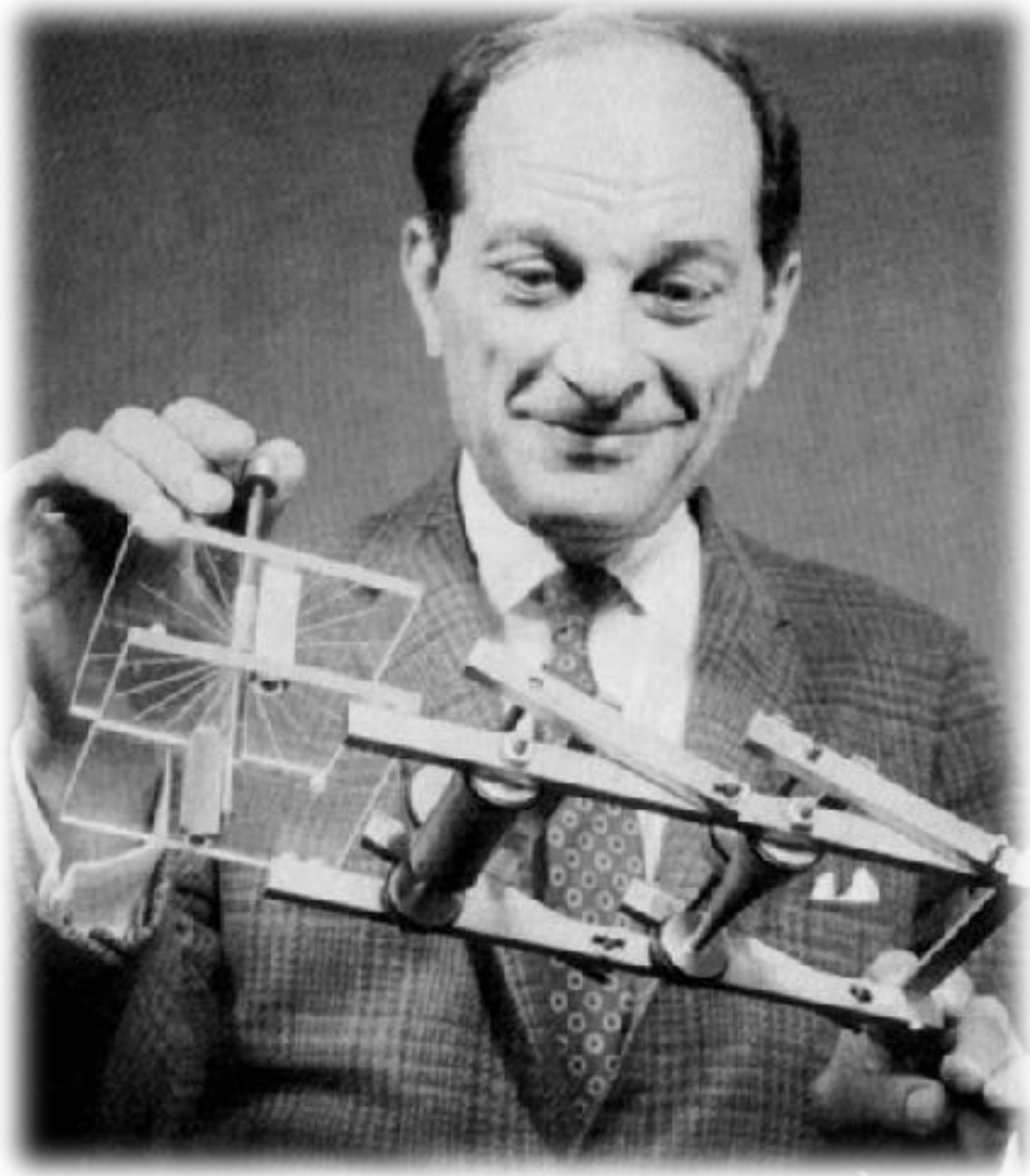
Monte Carlo
Method



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- Stan **inference**
  - MCMC for full Bayes
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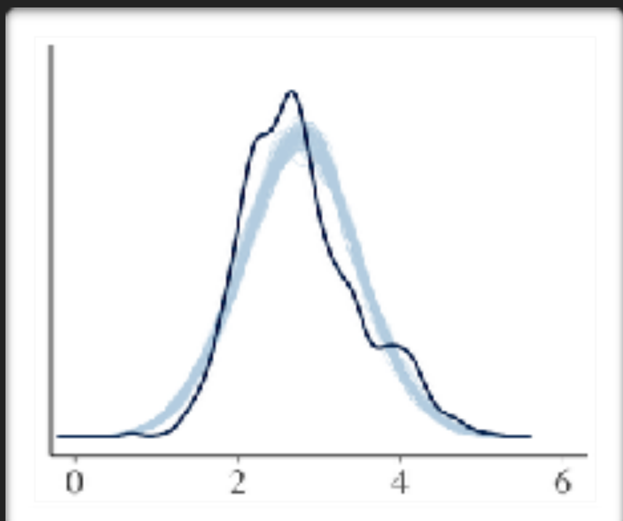
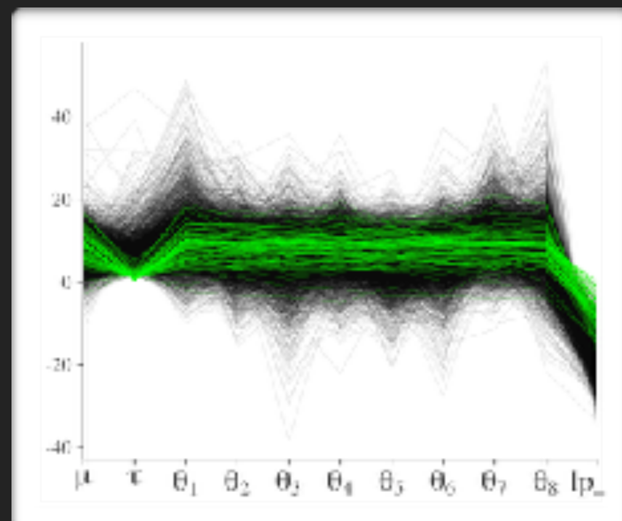
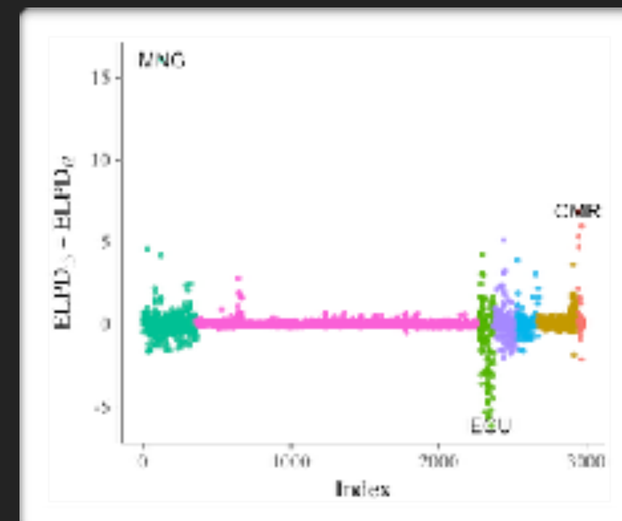
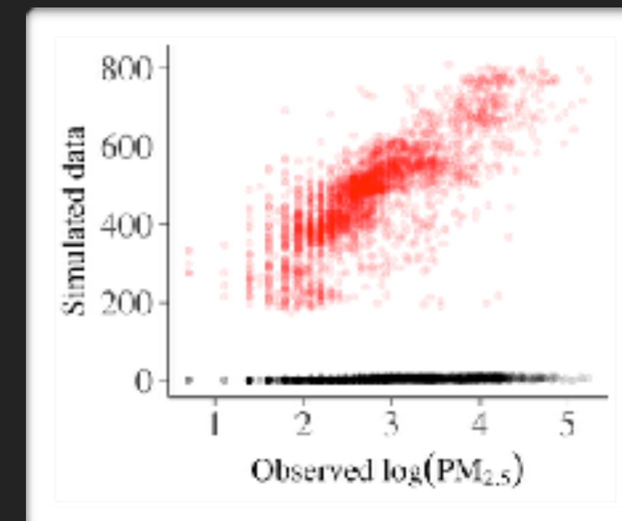
- Stan **ecosystem**
  - lang, math library (C++)
  - interfaces and tools (R, Python, many more)
  - documentation (example model repo, user guide & reference manual, case studies, R package vignettes)
  - online community (Stan Forums on Discourse)

Visualization in Bayesian workflow









Jonah Gabry

Columbia University
Stan Development Team

Workflow

Bayesian data analysis

Workflow
Bayesian data analysis

- Exploratory data analysis

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Bayesian data analysis

- Exploratory data analysis
- *Prior* predictive checking

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- Model comparison (e.g., via cross-validation)

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Gabry, J., Simpson, D., Vehtari, A., Betancourt, M., and Gelman, A. (2019).
Visualization in Bayesian workflow.
Journal of the Royal Statistical Society Series A

Journal version: rss.onlinelibrary.wiley.com/doi/full/10.1111/rssa.12378
arXiv preprint: arxiv.org/abs/1709.01449
Code: github.com/jgabry/bayes-vis-paper

Example

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Goal Estimate global PM2.5 concentration

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Problem Most data from noisy satellite measurements (ground monitor network provides sparse, heterogeneous coverage)

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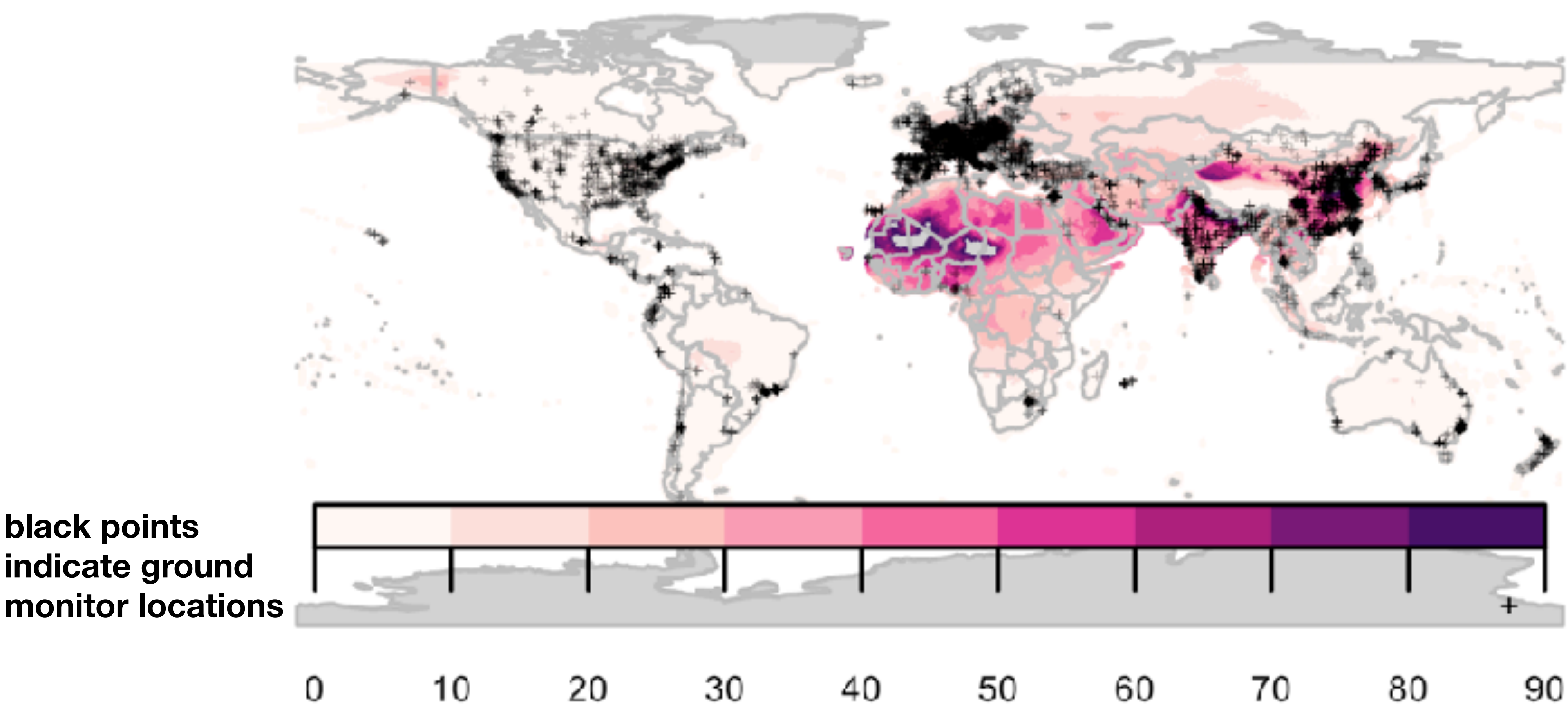


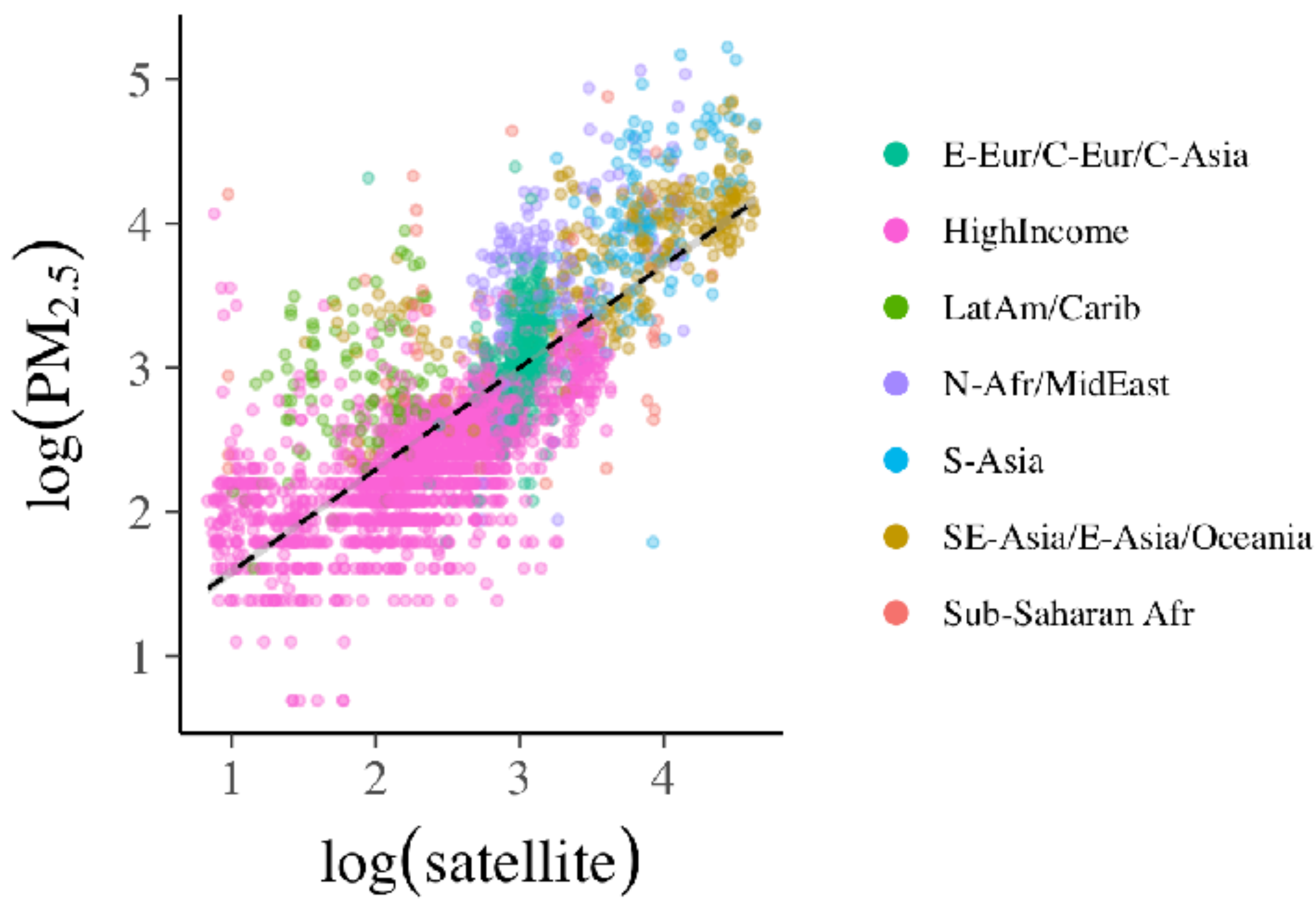
Satellite estimates of PM2.5 and ground monitor locations

Exploratory Data Analysis

Building a network of models

Exploratory data analysis

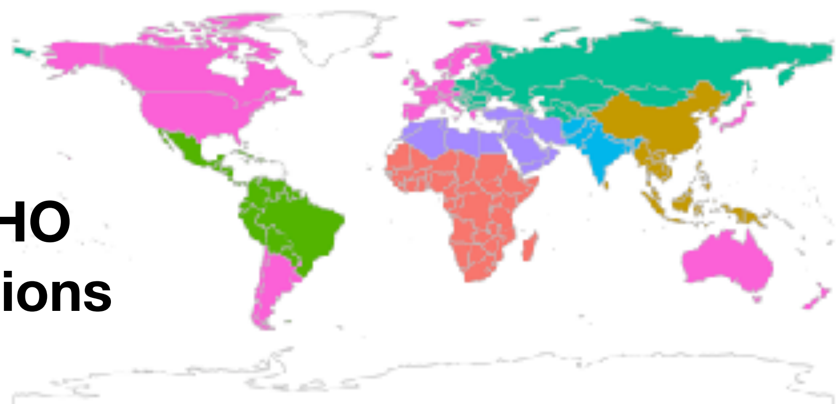
building a network of models



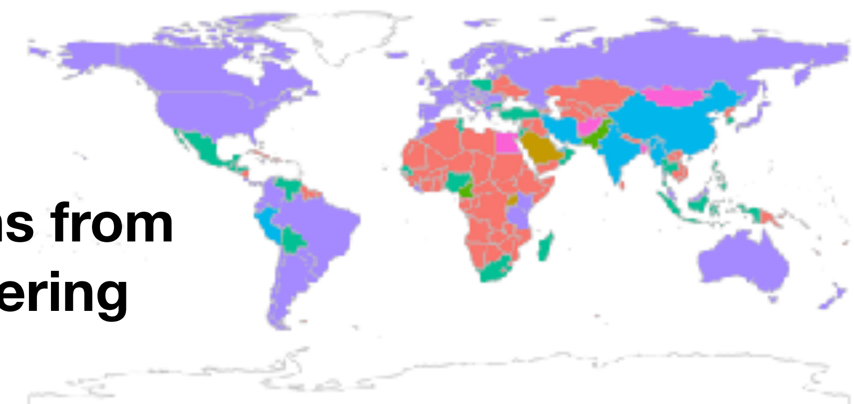
Exploratory data analysis

building a network of models

WHO
Regions



Regions from
clustering



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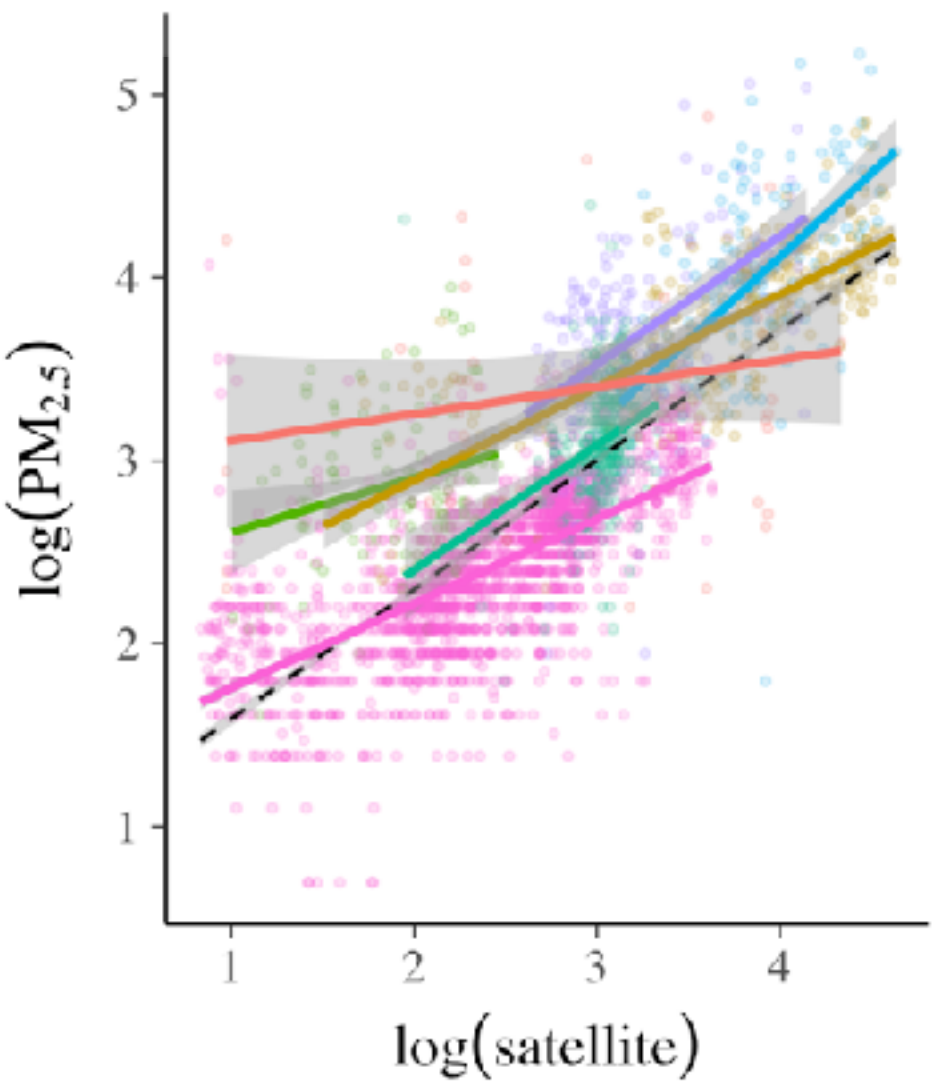

Exploratory data analysis
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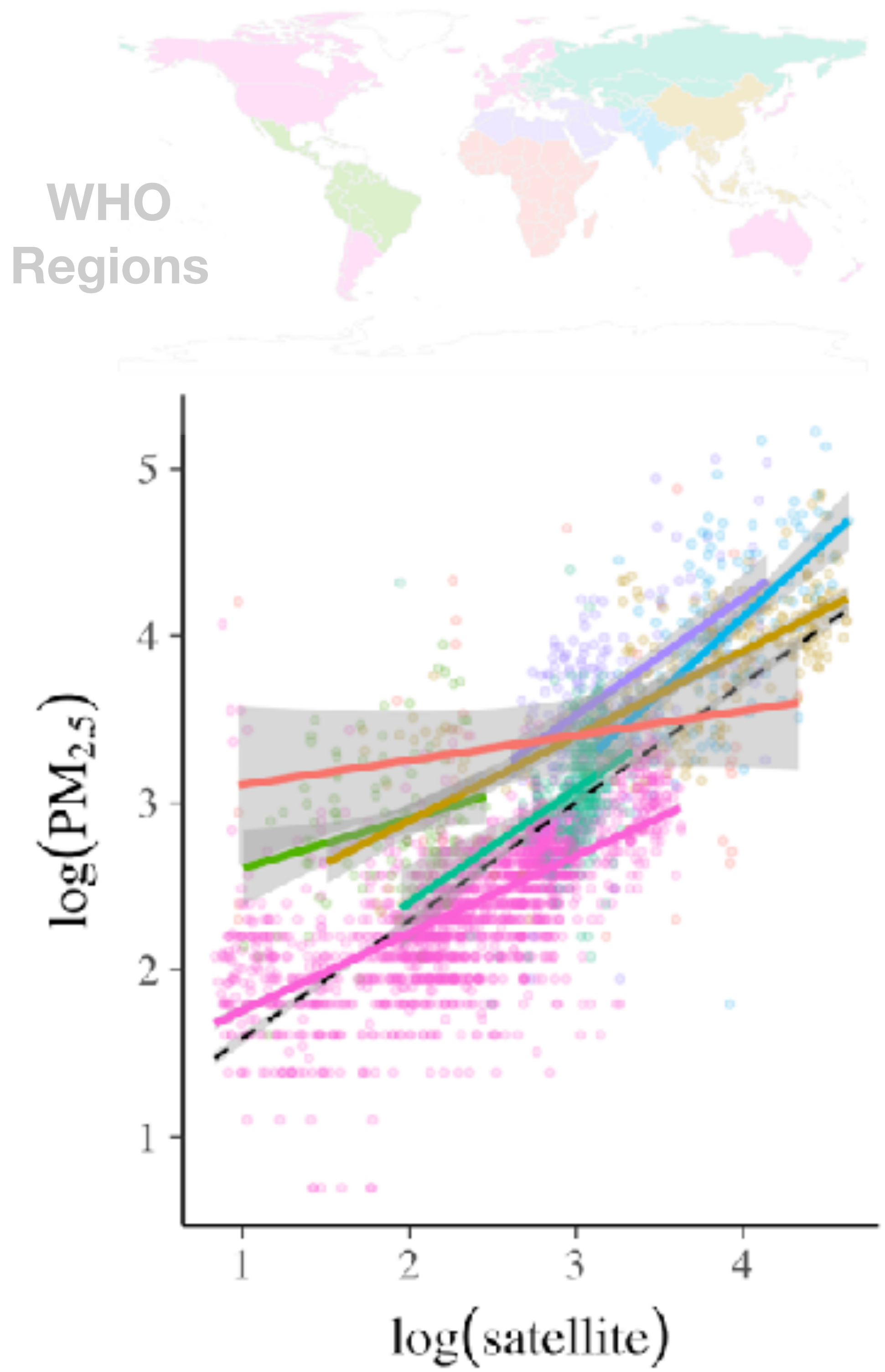

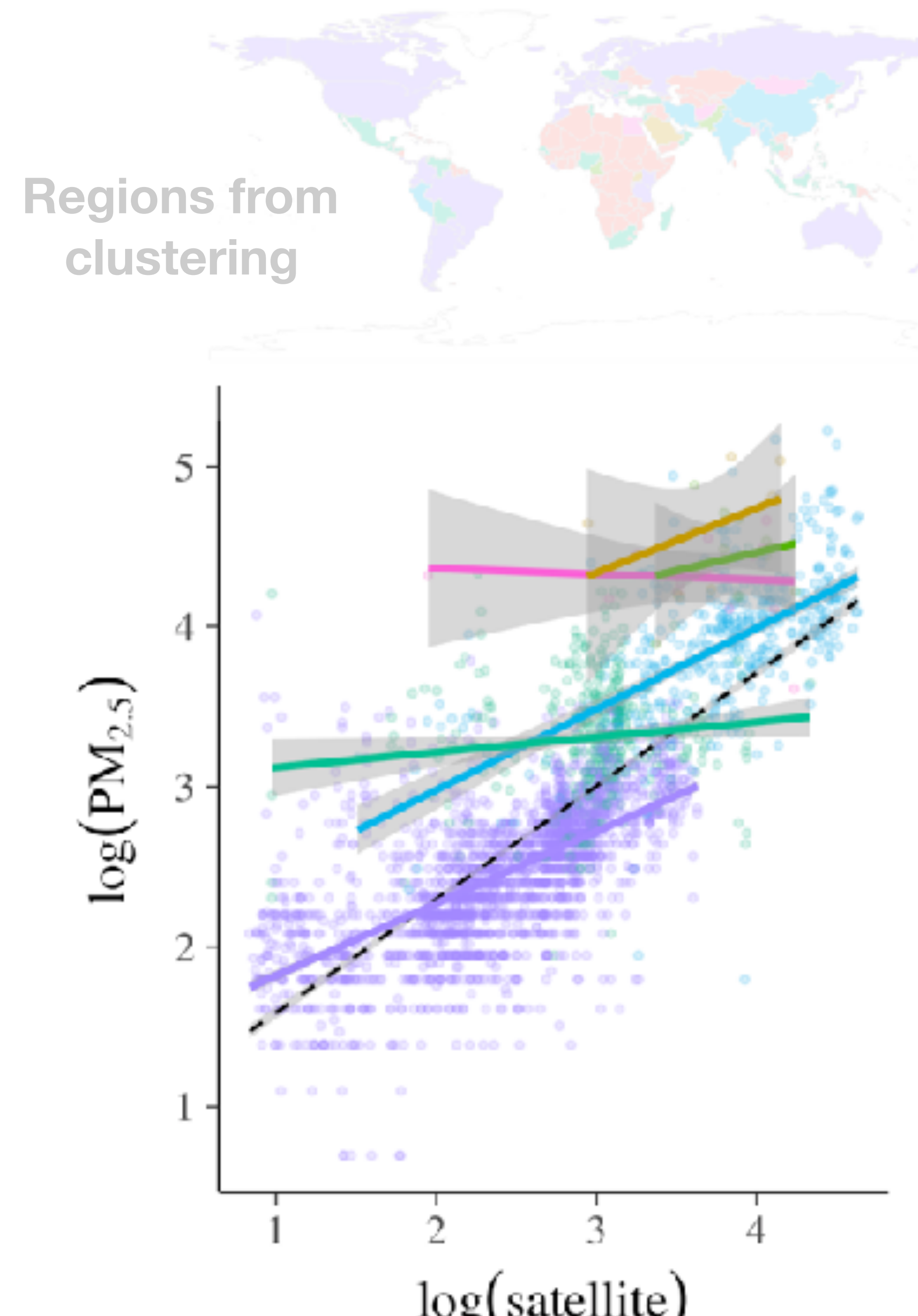

Exploratory data analysis
building a network of models

For measurements $n = 1, \ldots, N$

and regions $j = 1, \ldots, J$

Model 1

Exploratory data analysis
building a network of models

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Model 1

$$\log\left(\mathrm{PM}_{2.5,nj}\right) \sim N(\alpha + \beta \log\left(\mathrm{sat}_{nj}\right), \sigma)$$

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Models 2 and 3

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$$\alpha_j \sim N(0, \tau_\alpha) \quad \beta_j \sim N(0, \tau_\beta)$$

Prior predictive checks

Fake data can be almost as valuable as real data

A Bayesian modeler commits to an a priori *joint distribution*

Likelihood x Prior

$$p(\mathbf{y}, \boldsymbol{\theta}) = p(\mathbf{y} \mid \boldsymbol{\theta})p(\boldsymbol{\theta}) = p(\boldsymbol{\theta} \mid \mathbf{y})p(\mathbf{y})$$

Posterior x
Marginal Likelihood

Data
(observed)

Parameters
(unobserved)

Generative models

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$$

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$$\tau_\alpha^2 \sim \mathrm{InvGamma}(1, 100)$$
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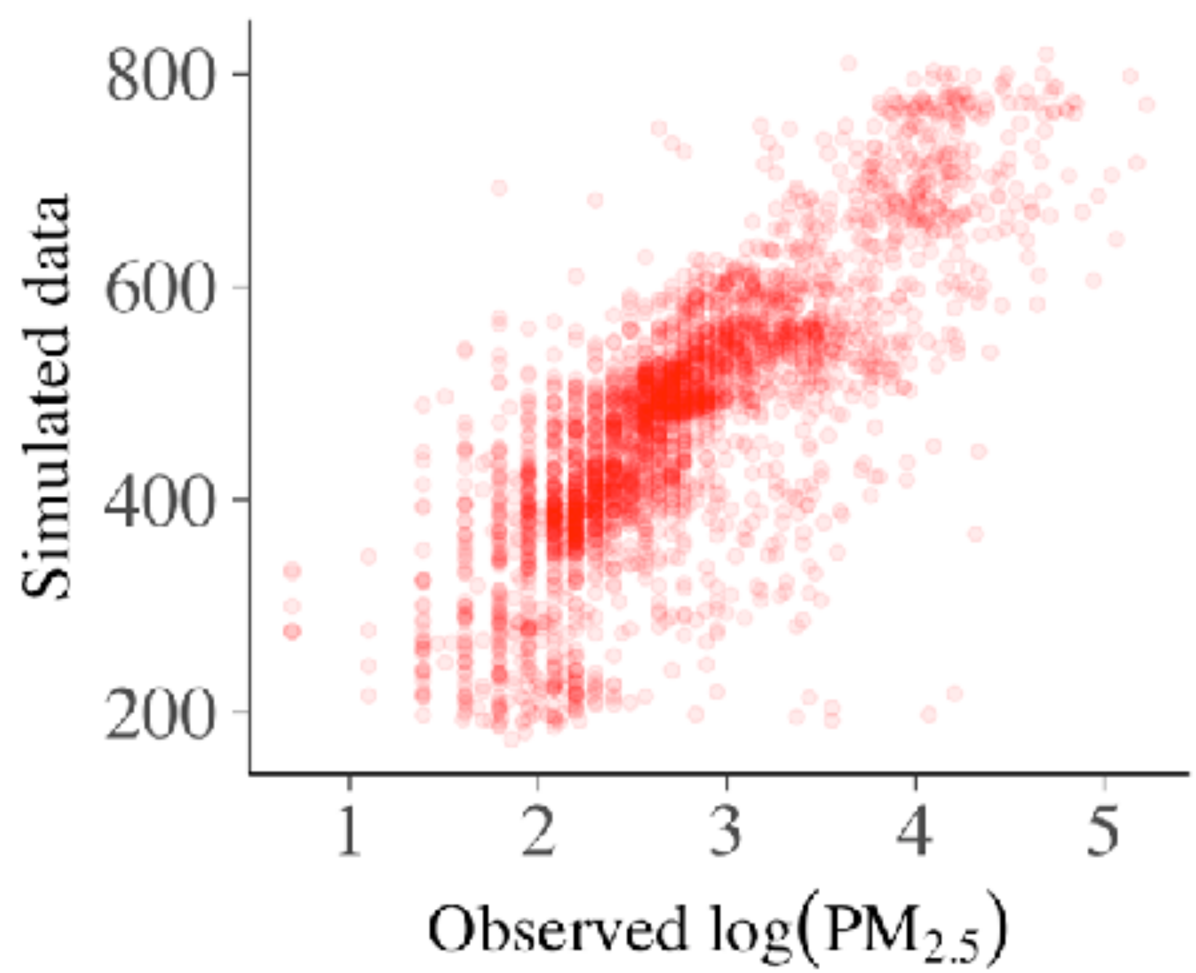

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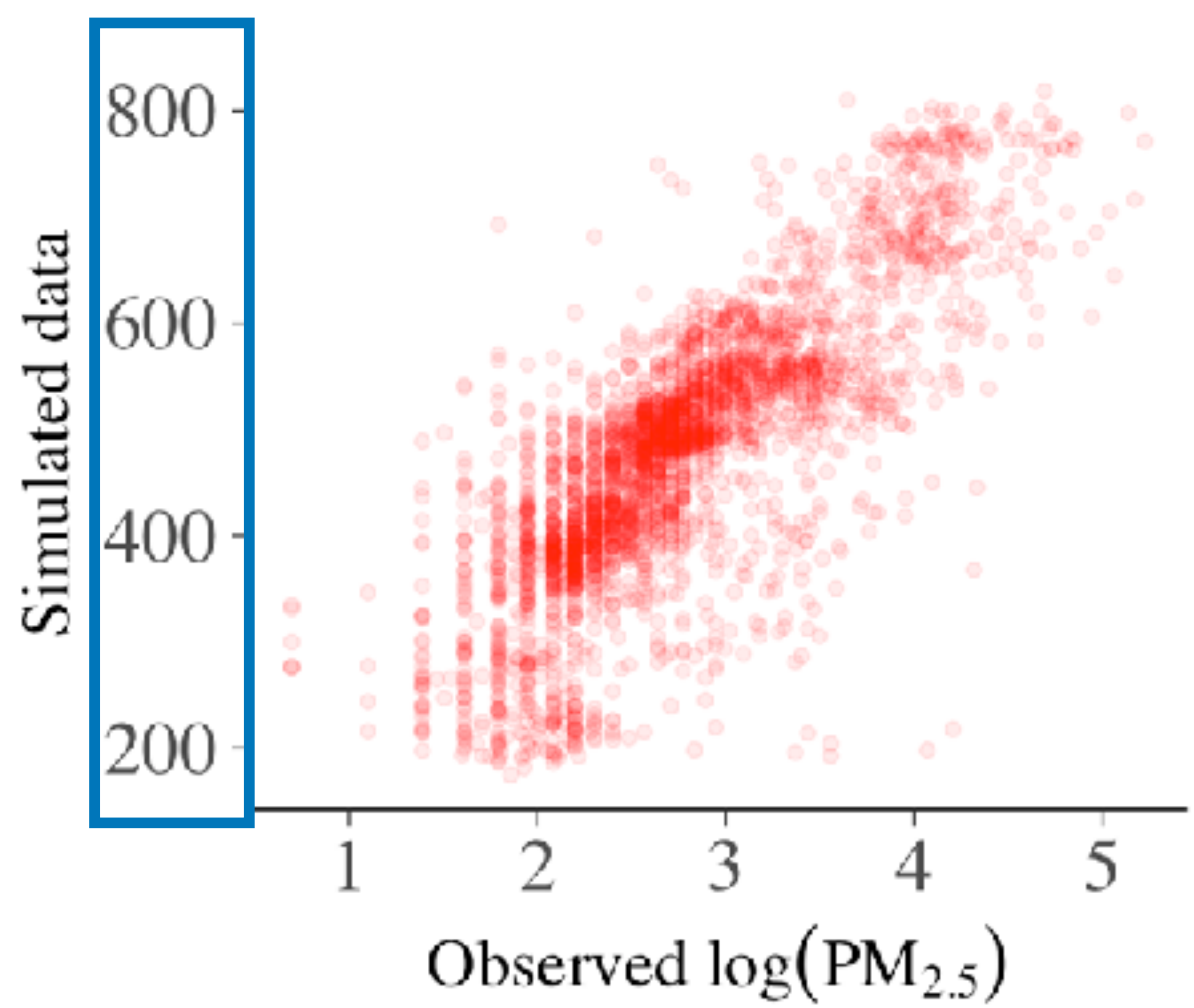

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- The prior model is **two orders of magnitude** off the real data
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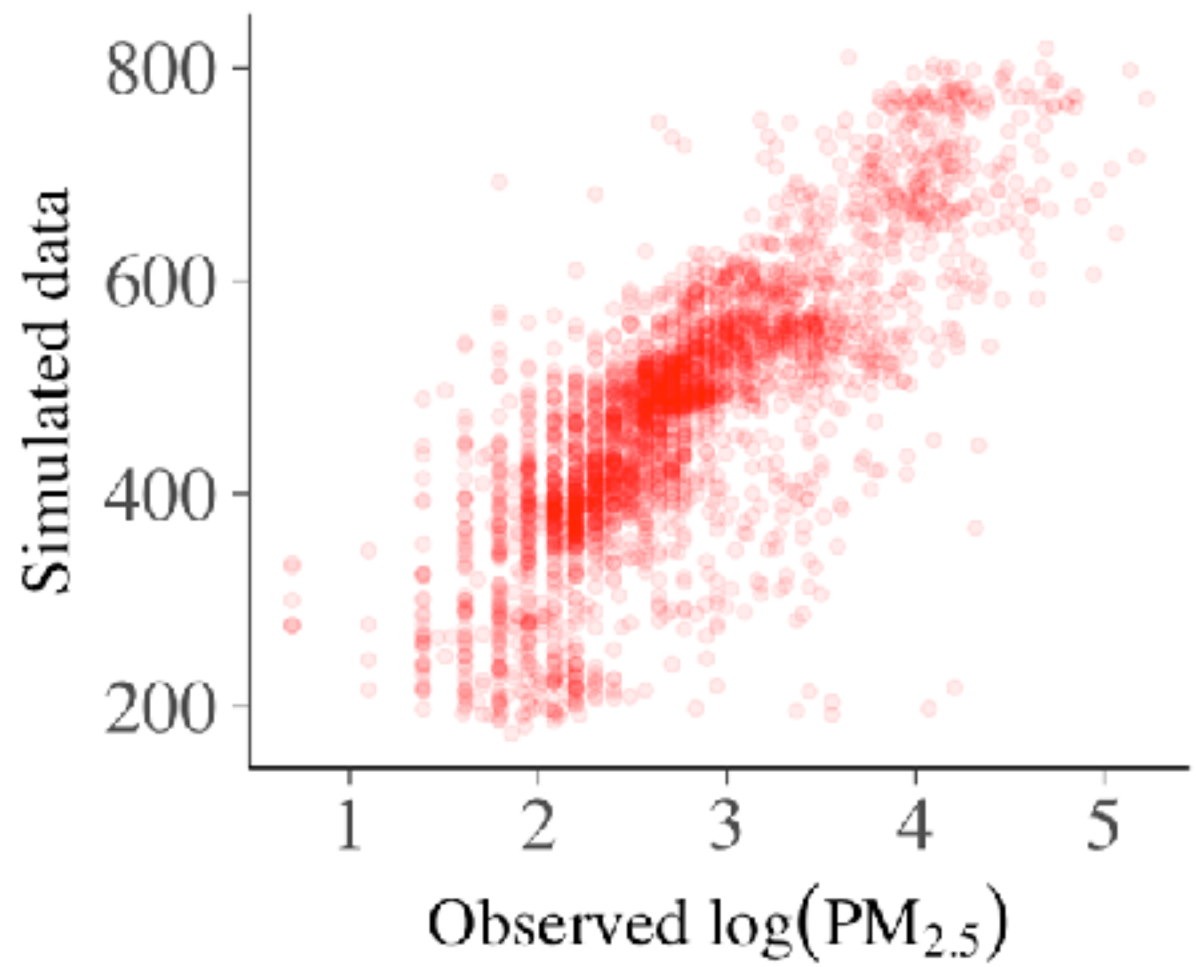

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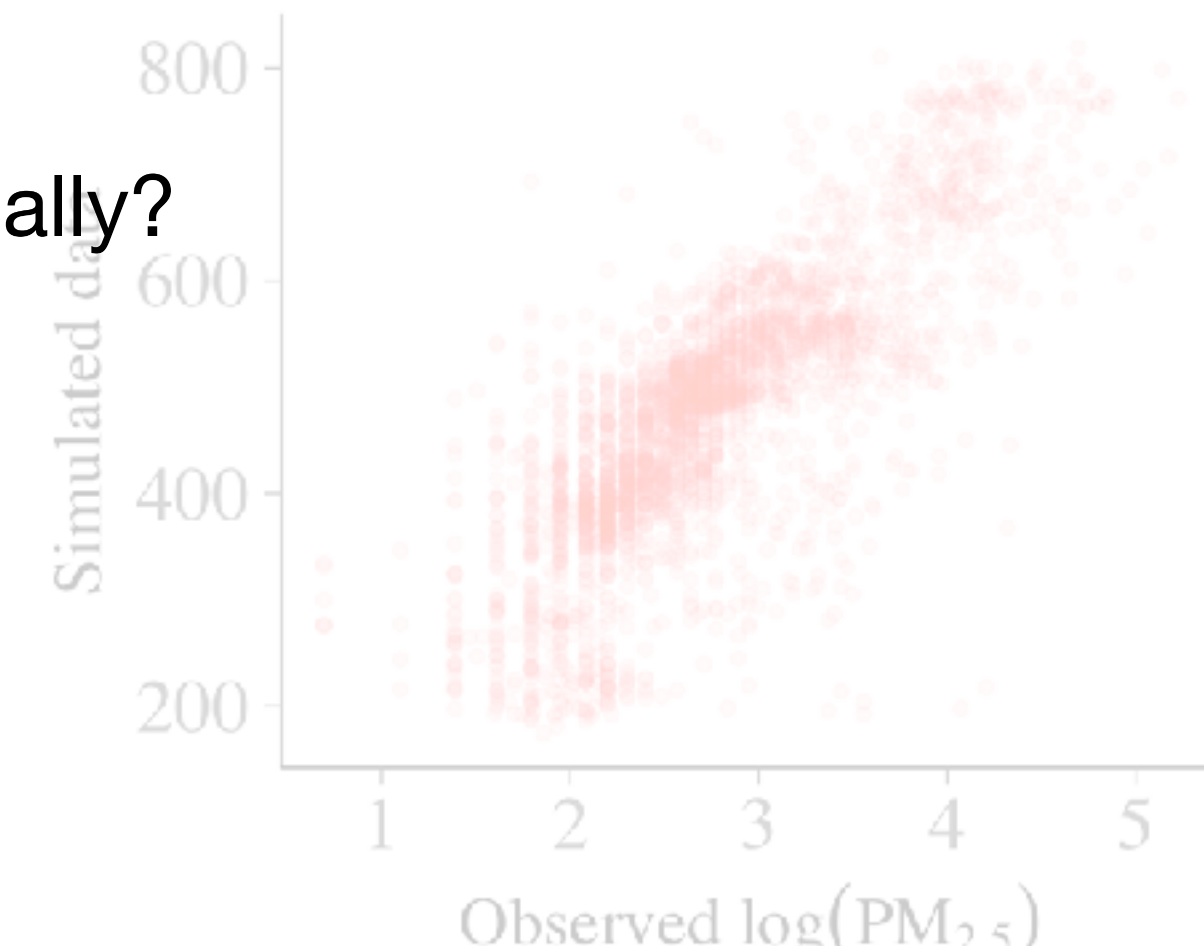

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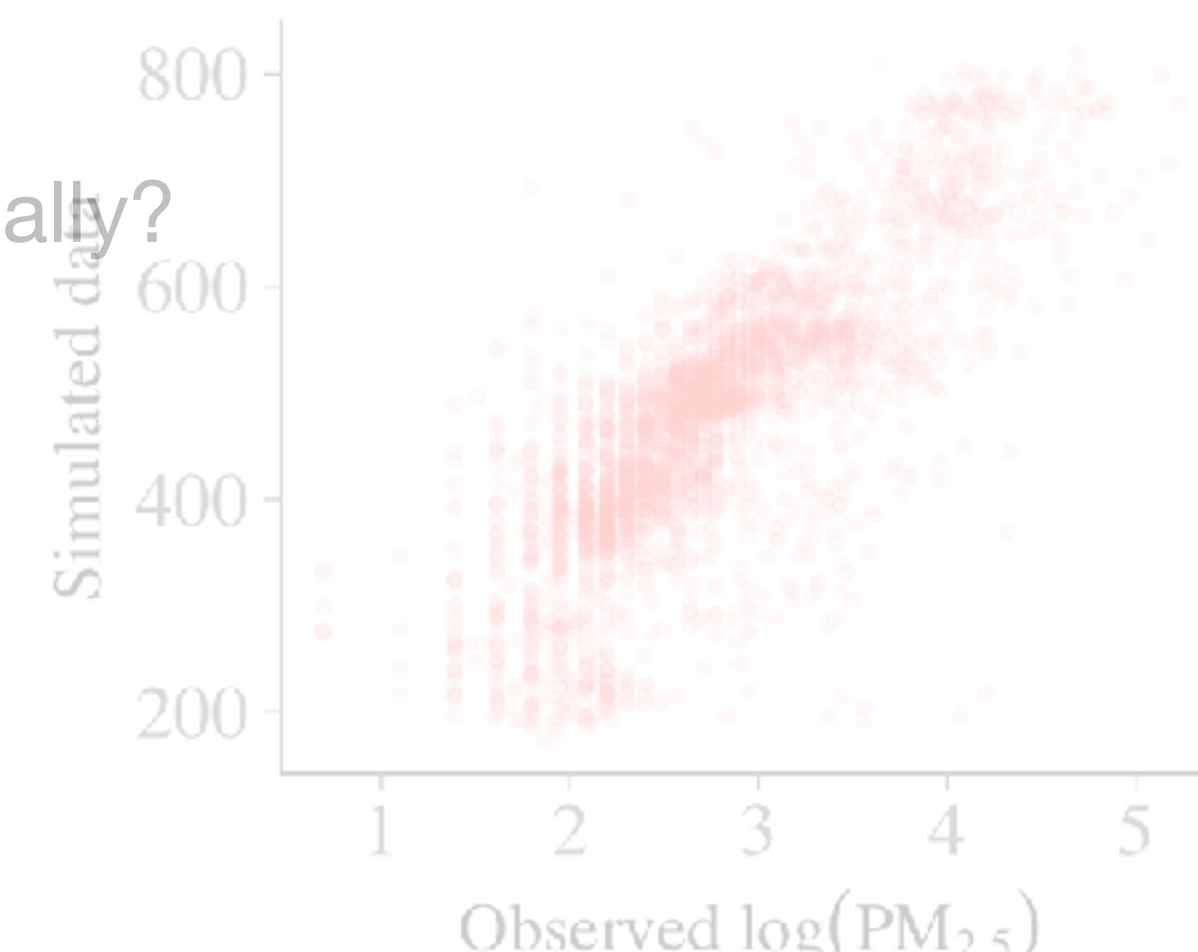
- What does this mean practically?
- The data will have to overcome the prior…



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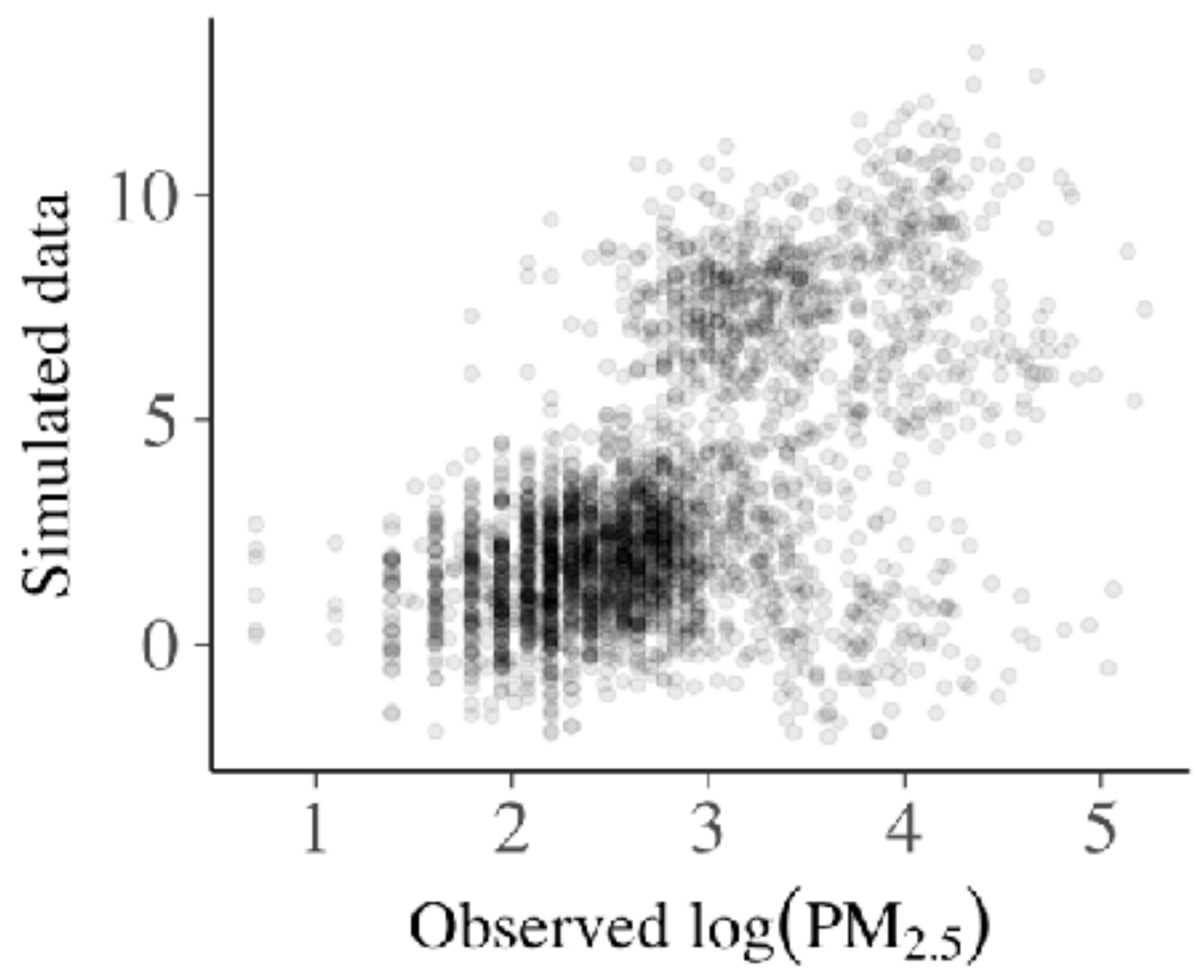

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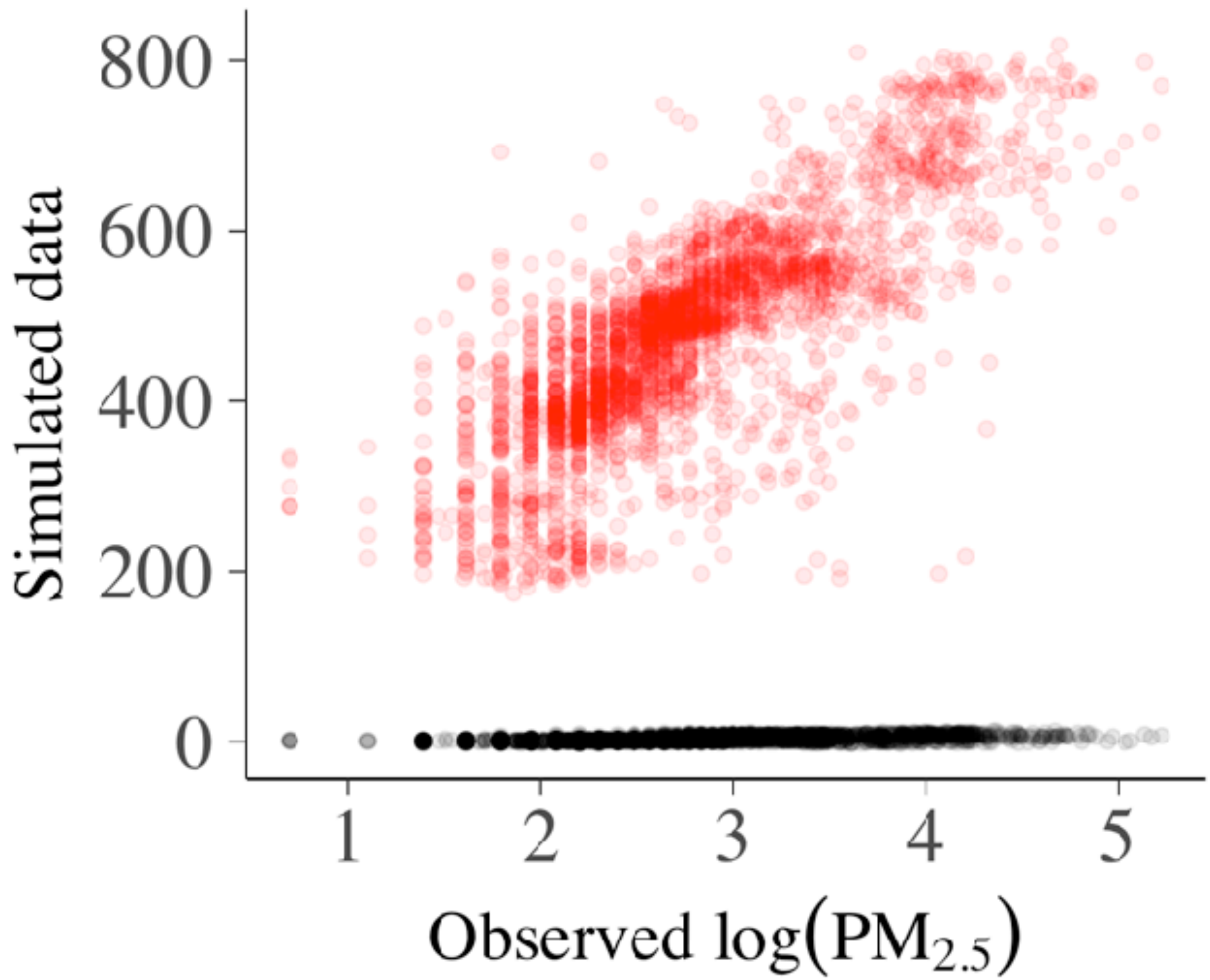

Prior predictive checking:

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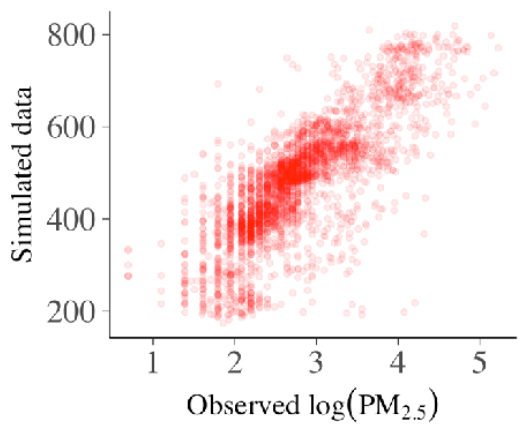
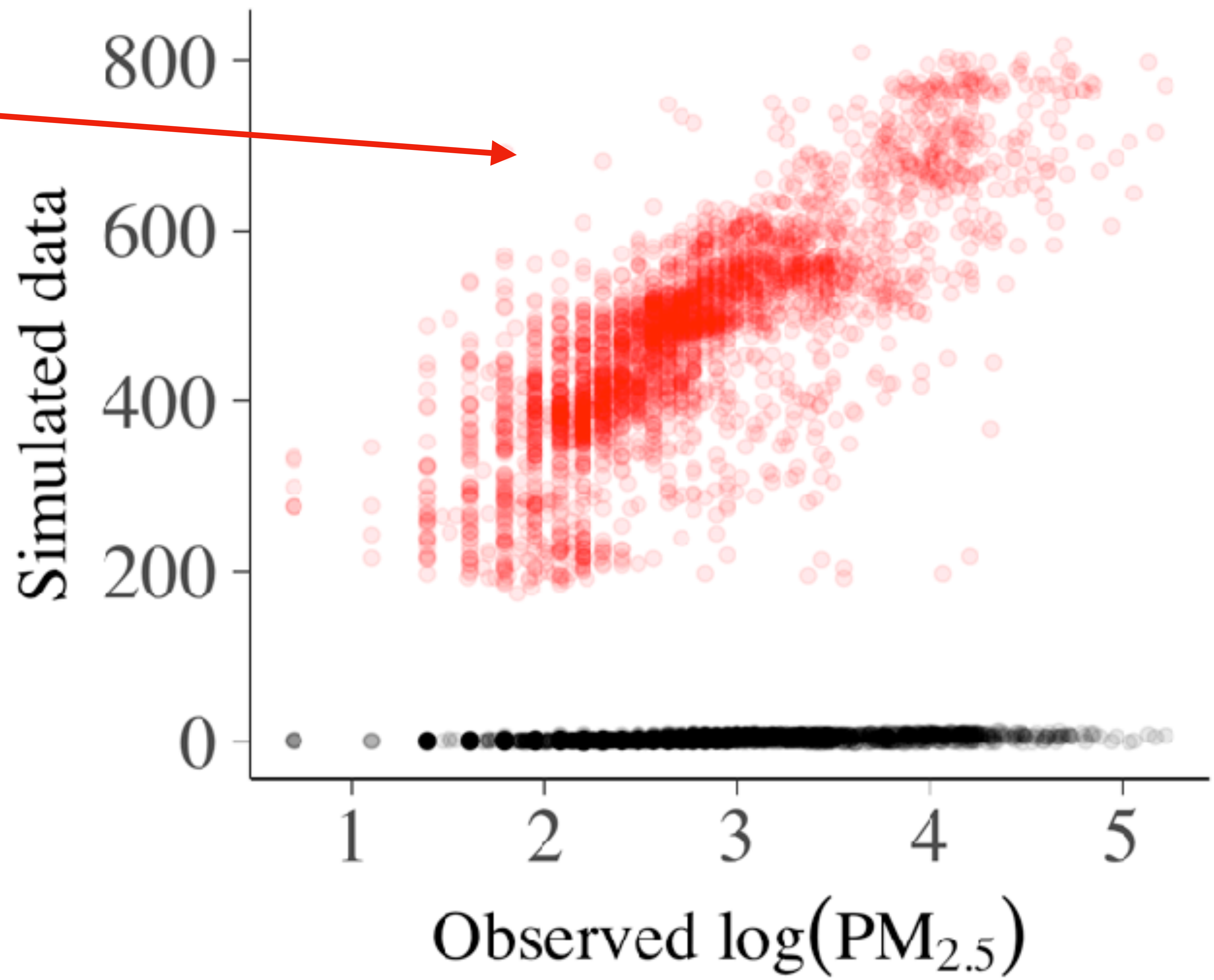
Non-informative





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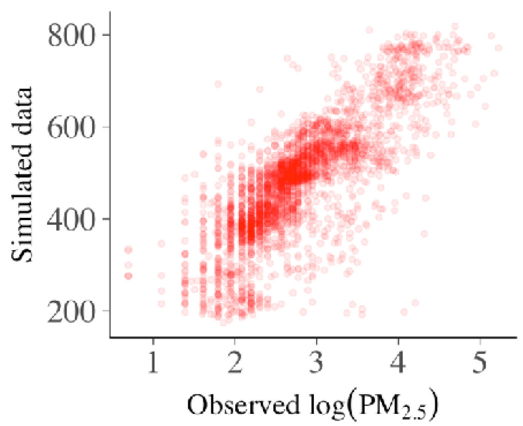
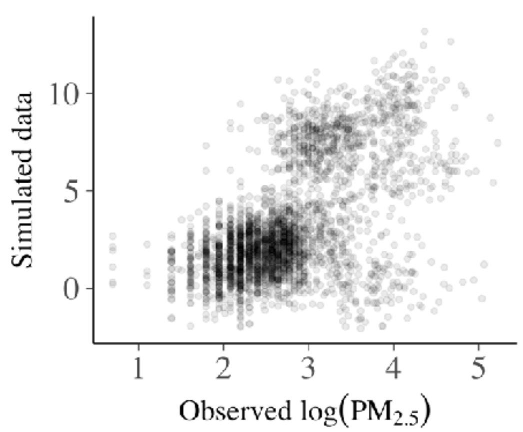
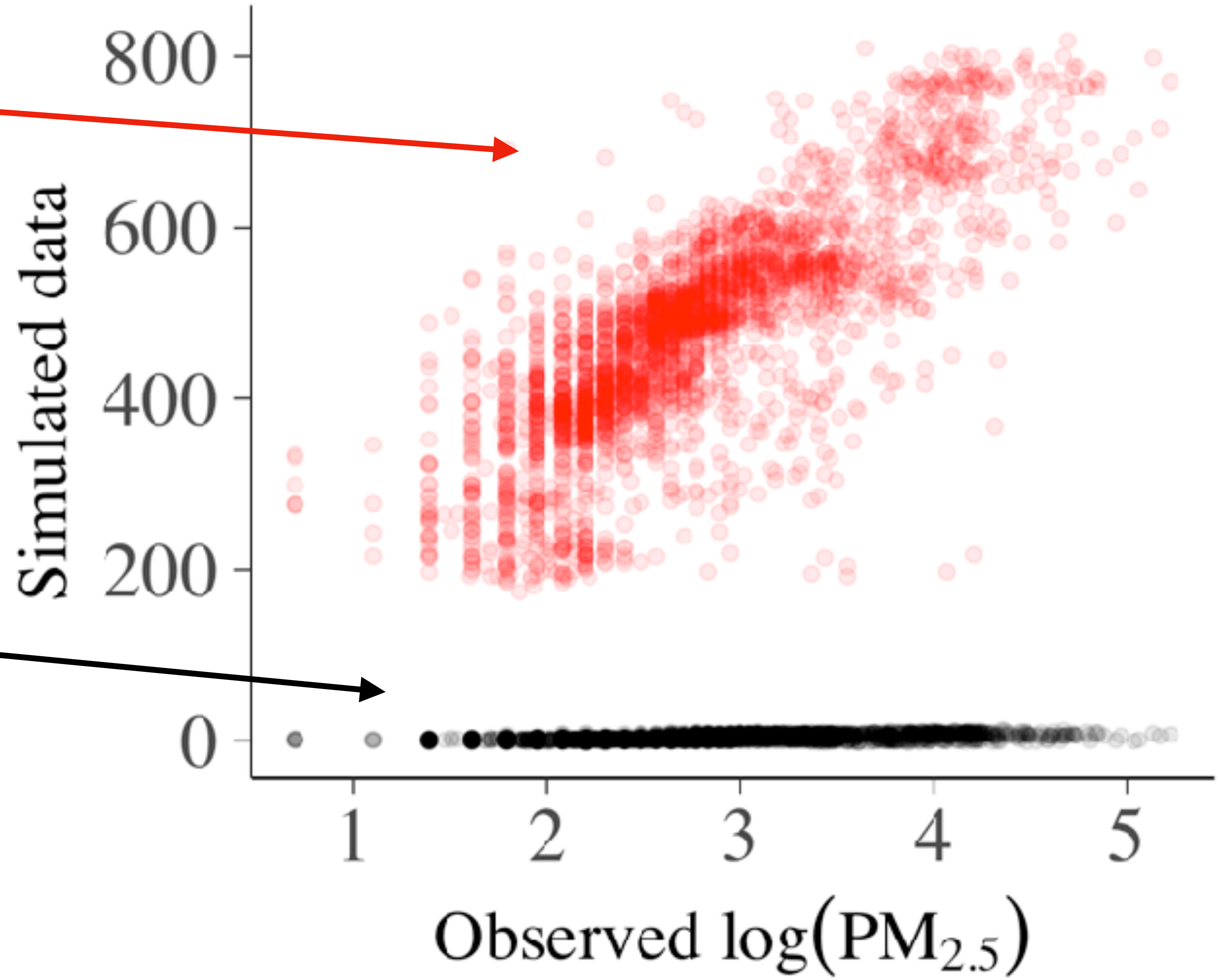


Weakly informative

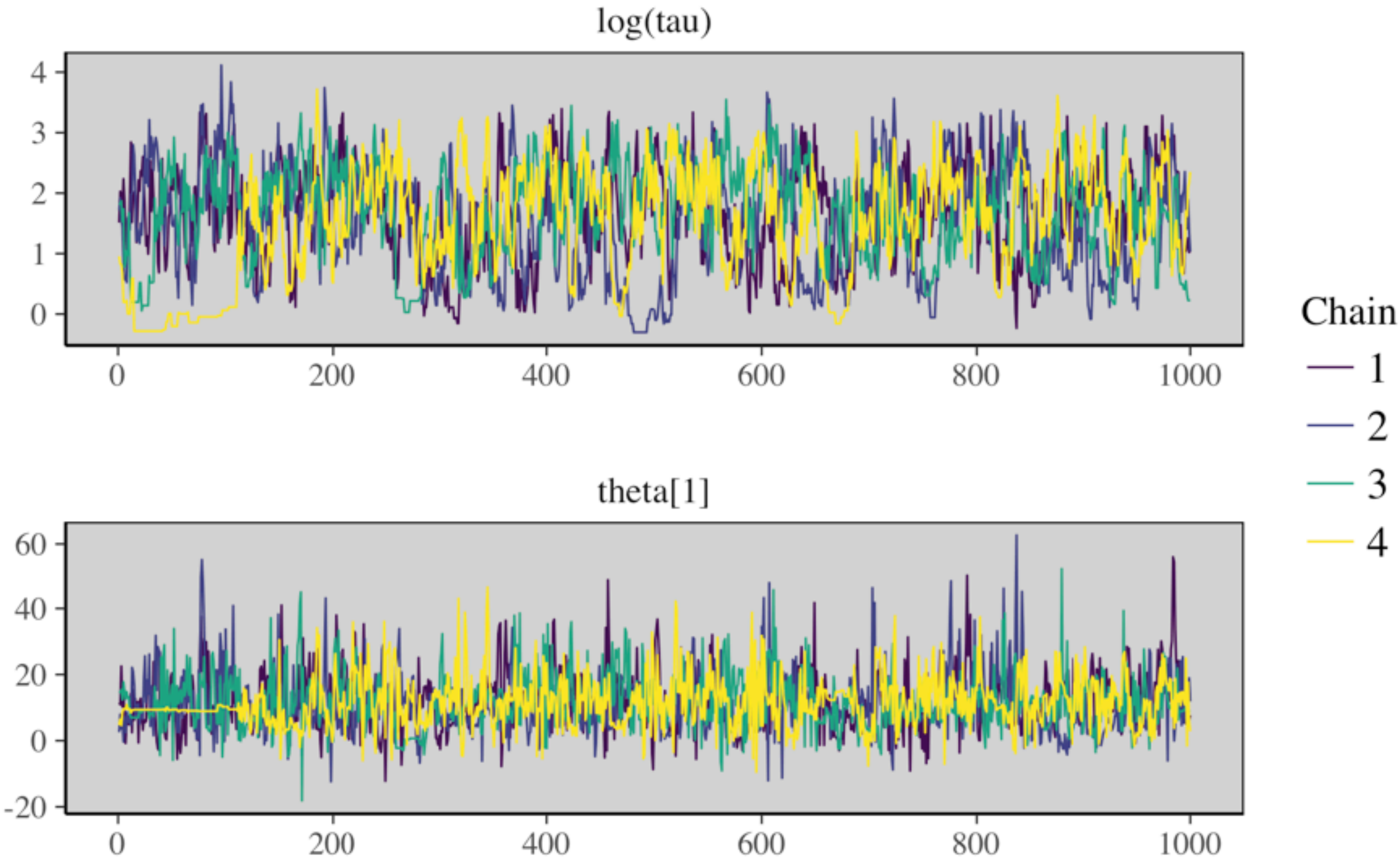


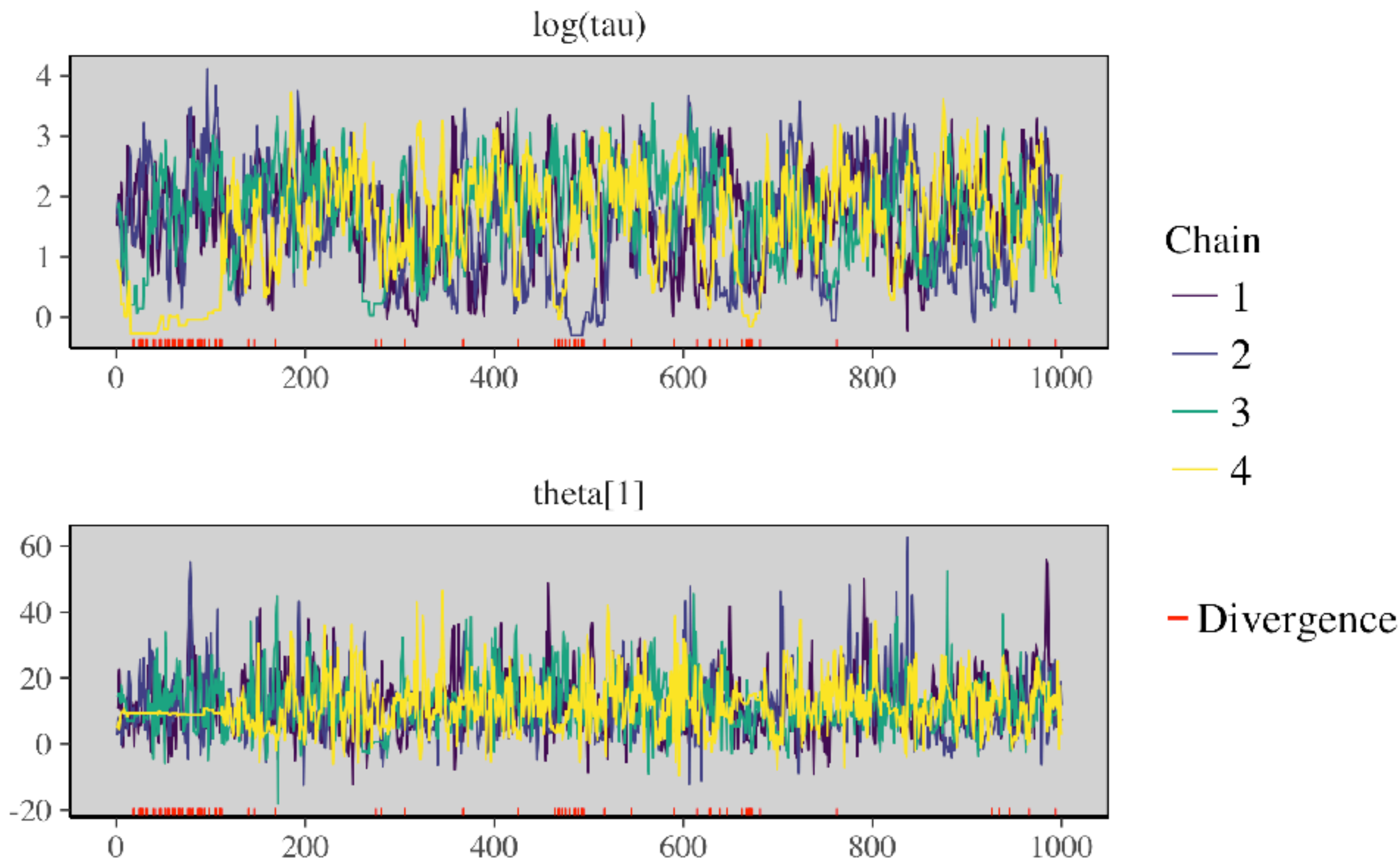


MCMC diagnostics

Beyond trace plots

https://chi-feng.github.io/mcmc-demo/


log(tau)
4
3
2
1
0
0
200
400
600
800
1000
theta[1]
60
40
20
0
-20
0
200
400
600
800
1000
Chain
1
2
3
4



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Divergence

MCMC diagnostics
beyond trace plots

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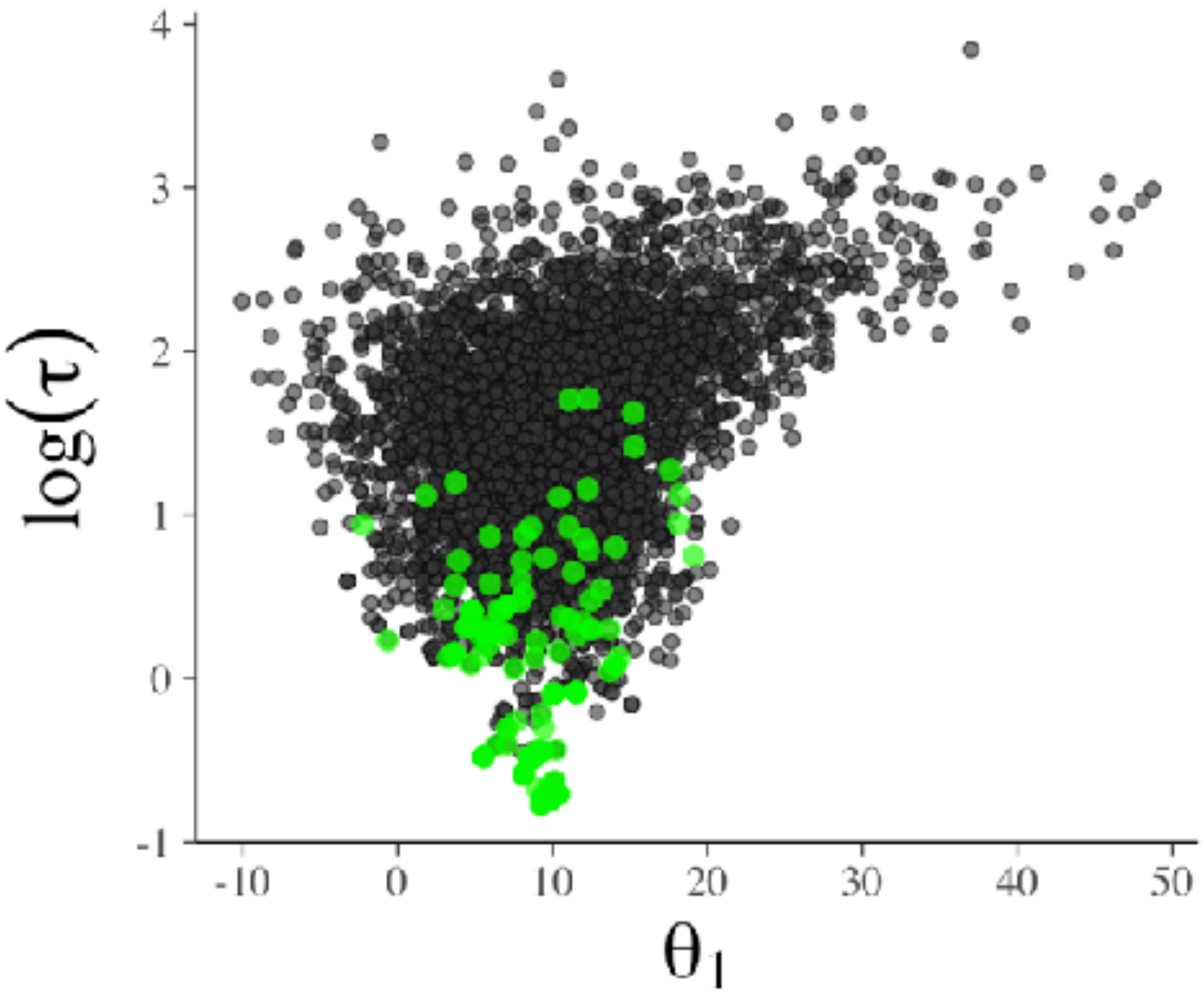

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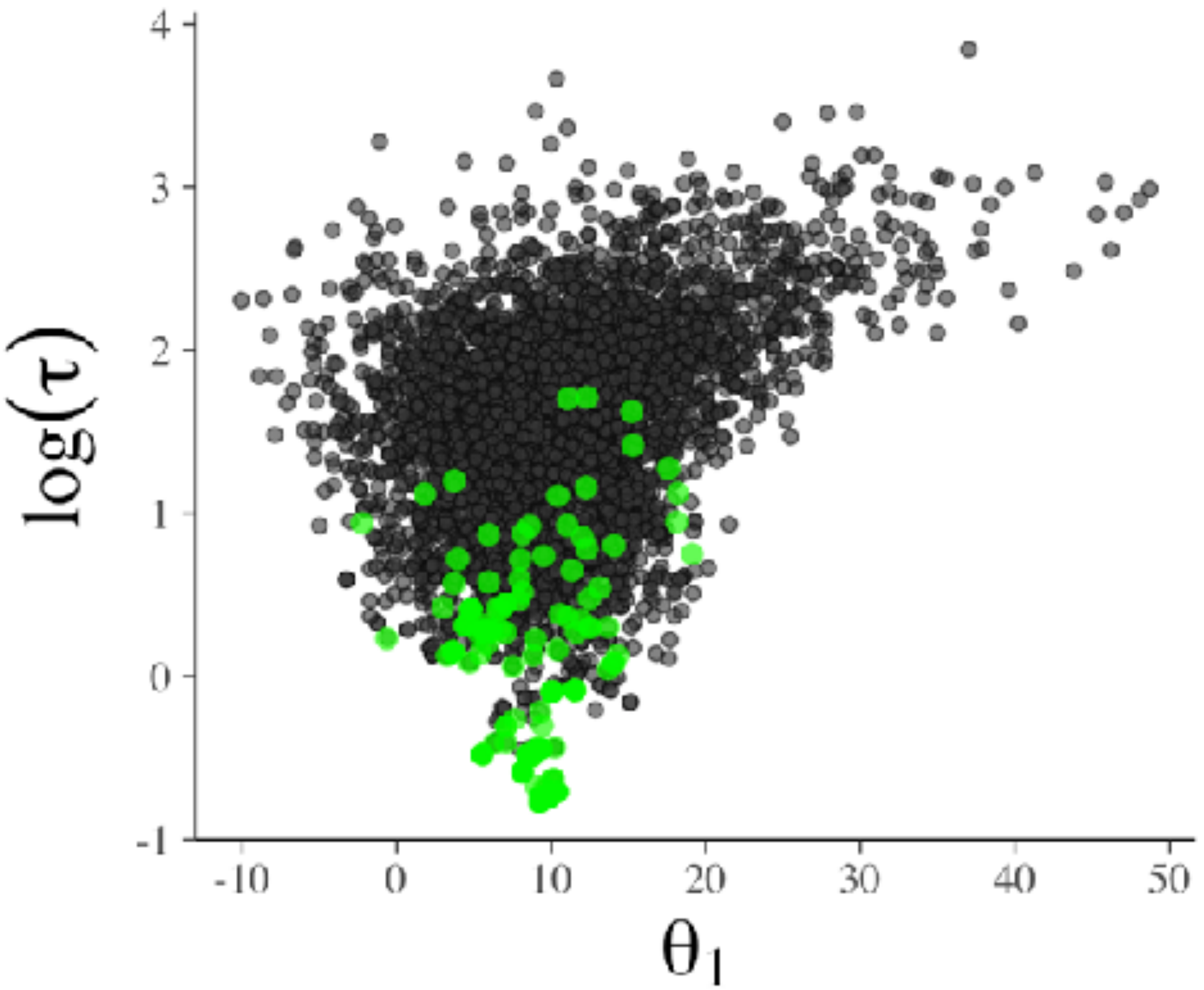

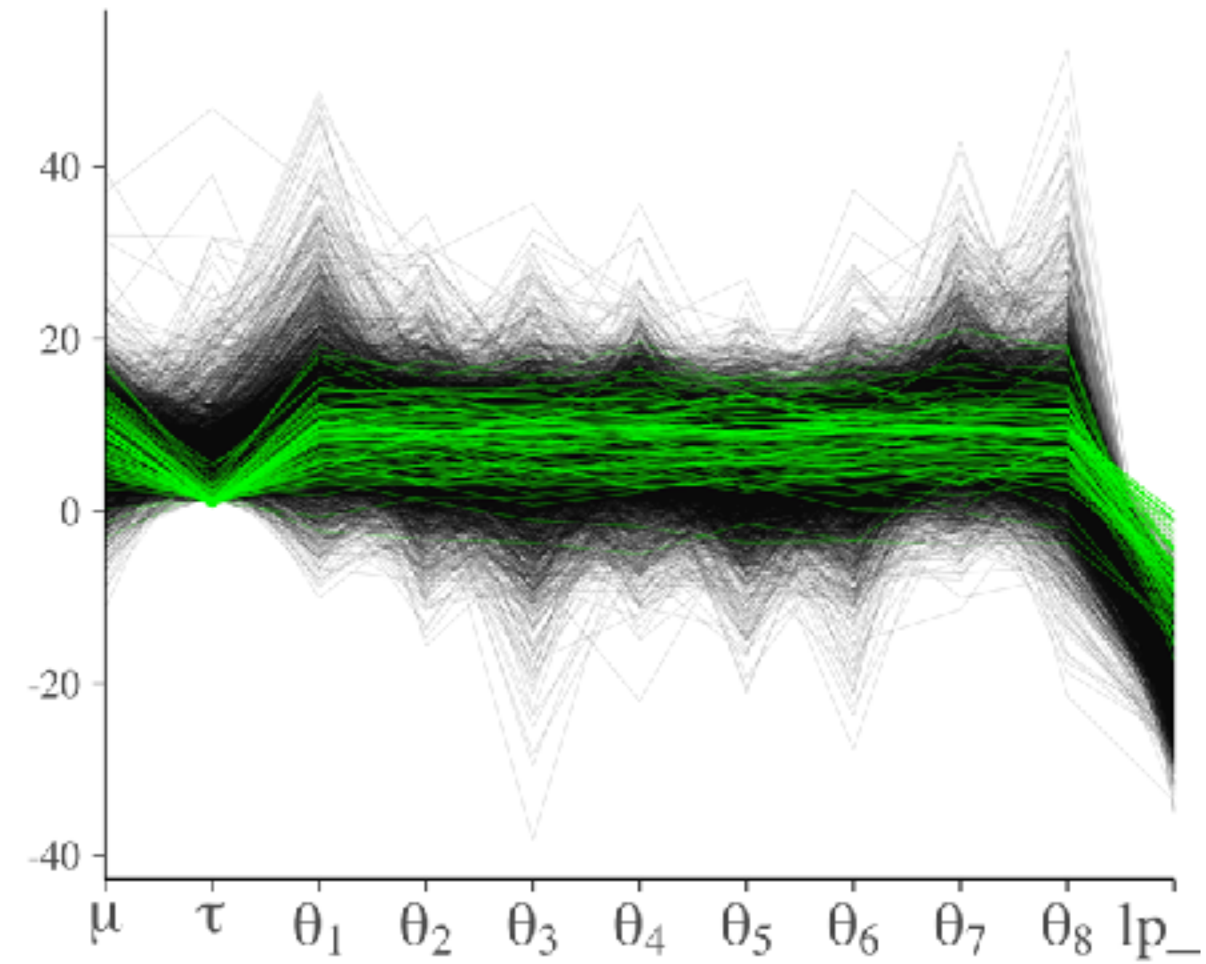

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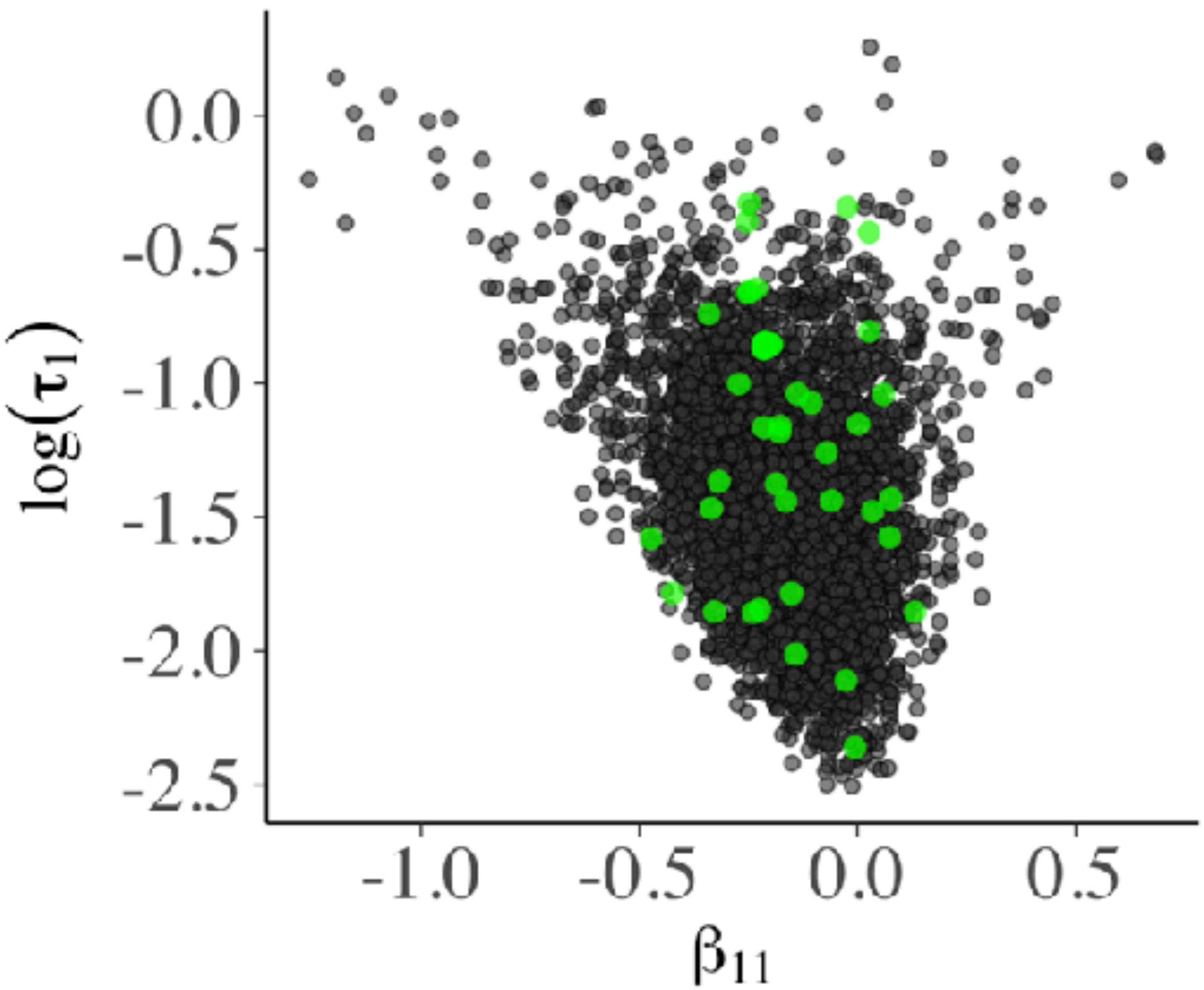

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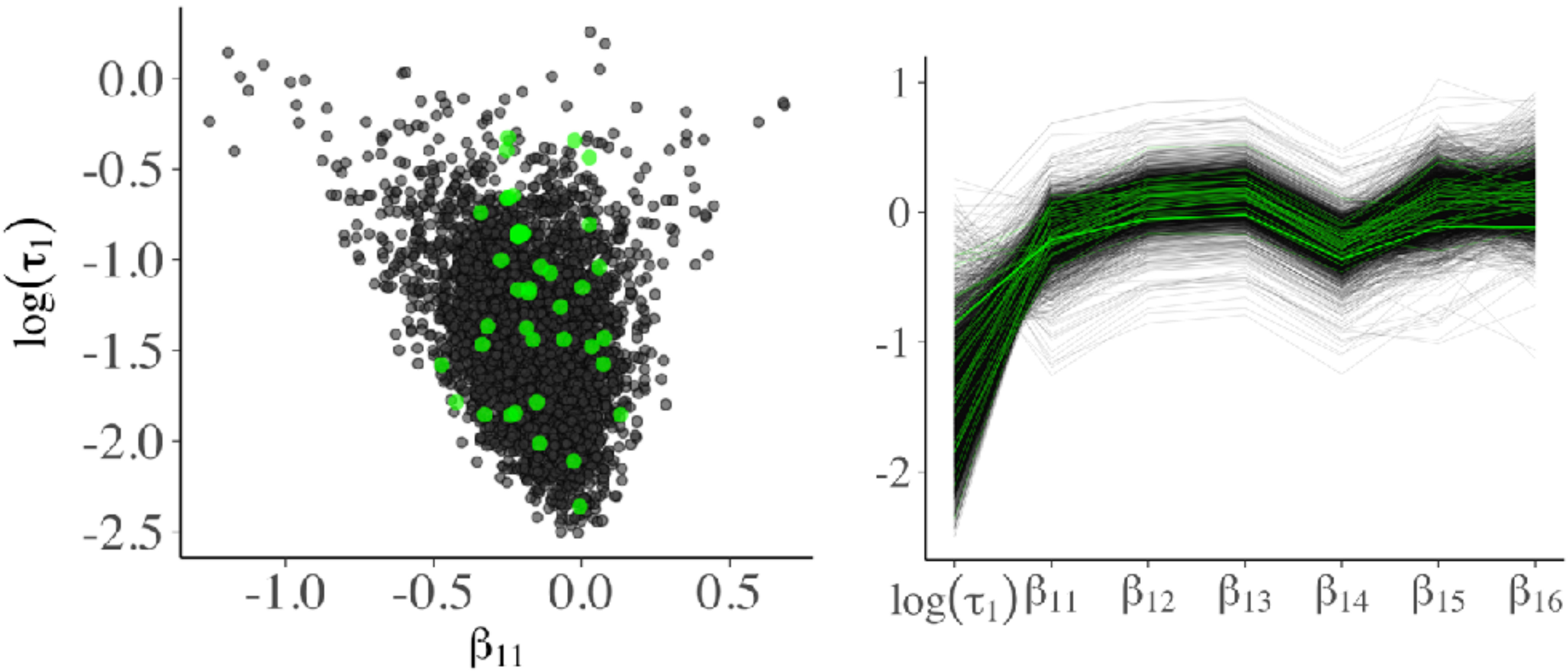
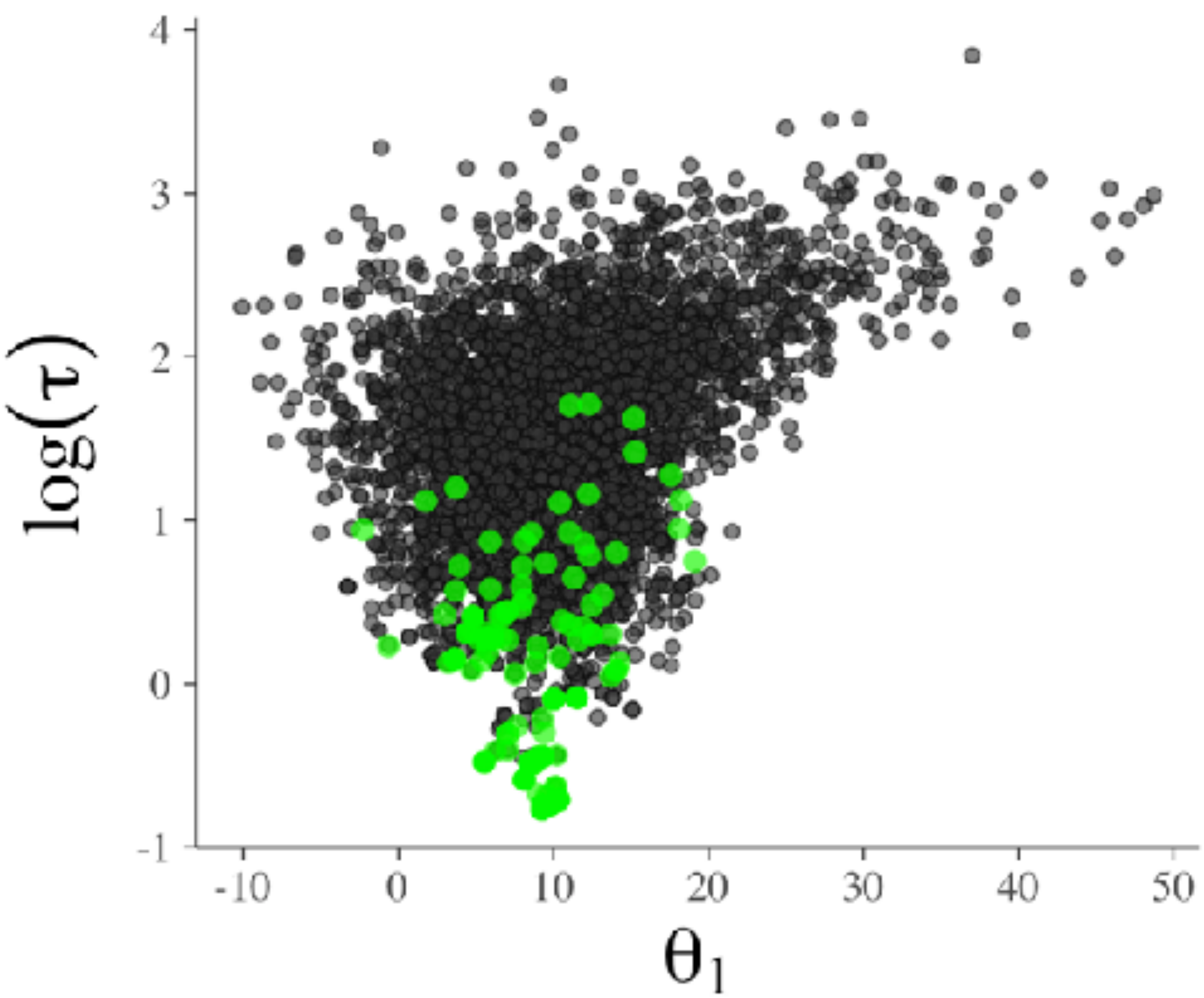
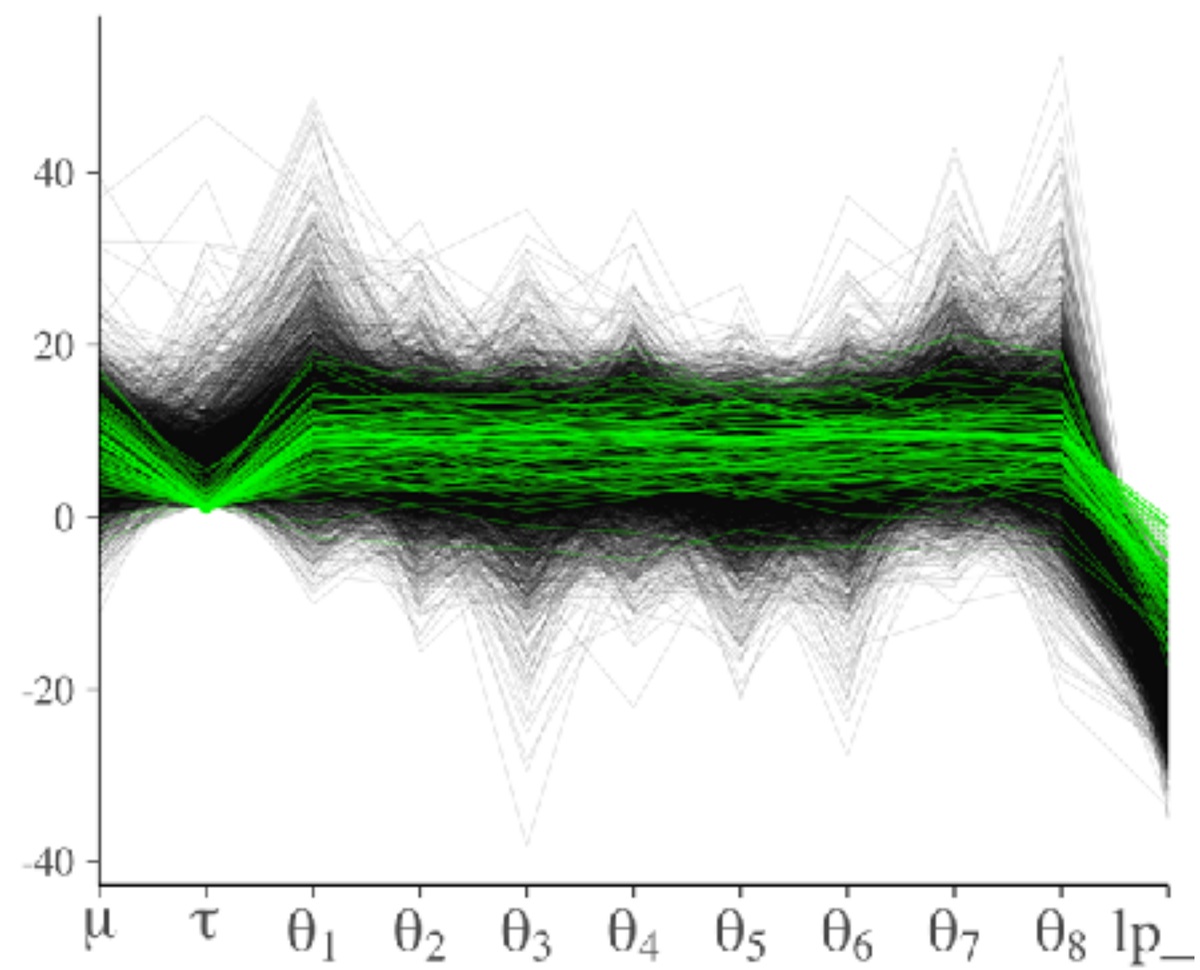
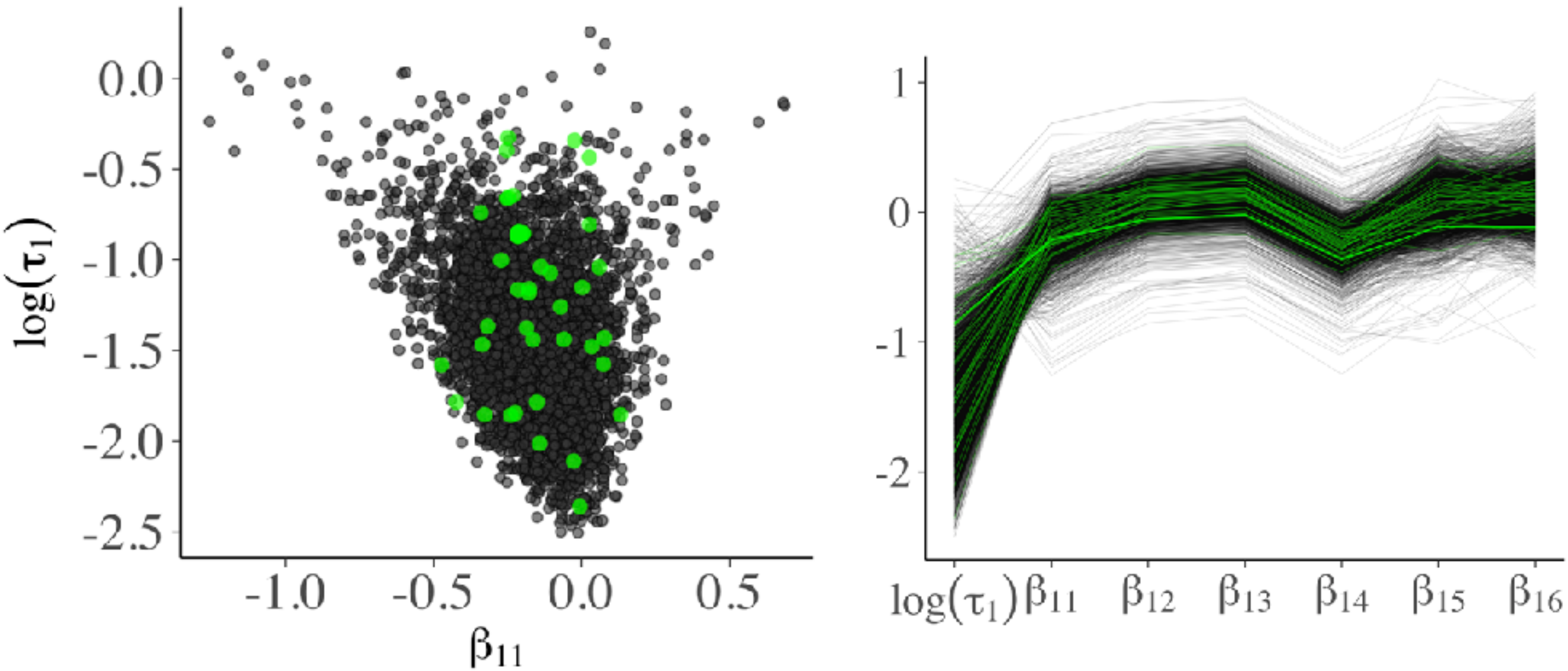


Pathological geometry





“False positives”



Posterior predictive checks

Visual model evaluation

Posterior predictive checking

visual model evaluation

The *posterior predictive distribution* is the average data generation process over the entire model

Posterior predictive checking
visual model evaluation

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$$p(\tilde{y}|y) = \int p(\tilde{y}|\theta)\, p(\theta|y)\, d\theta$$

Posterior predictive checking

visual model evaluation

Posterior predictive checking

visual model evaluation

- Misfitting and overfitting both manifest as tension between measurements and predictive distributions

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visual model evaluation

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- Graphical posterior predictive checks visually compare the observed data to the predictive distribution

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visual model evaluation

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$$\theta^\star \sim p(\theta|y)$$

Posterior predictive checking

visual model evaluation

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$$\theta^\star \sim p(\theta|y)$$

$$\downarrow$$

$$\tilde{y} \sim p(y|\theta^\star)$$

Posterior predictive checking
visual model evaluation

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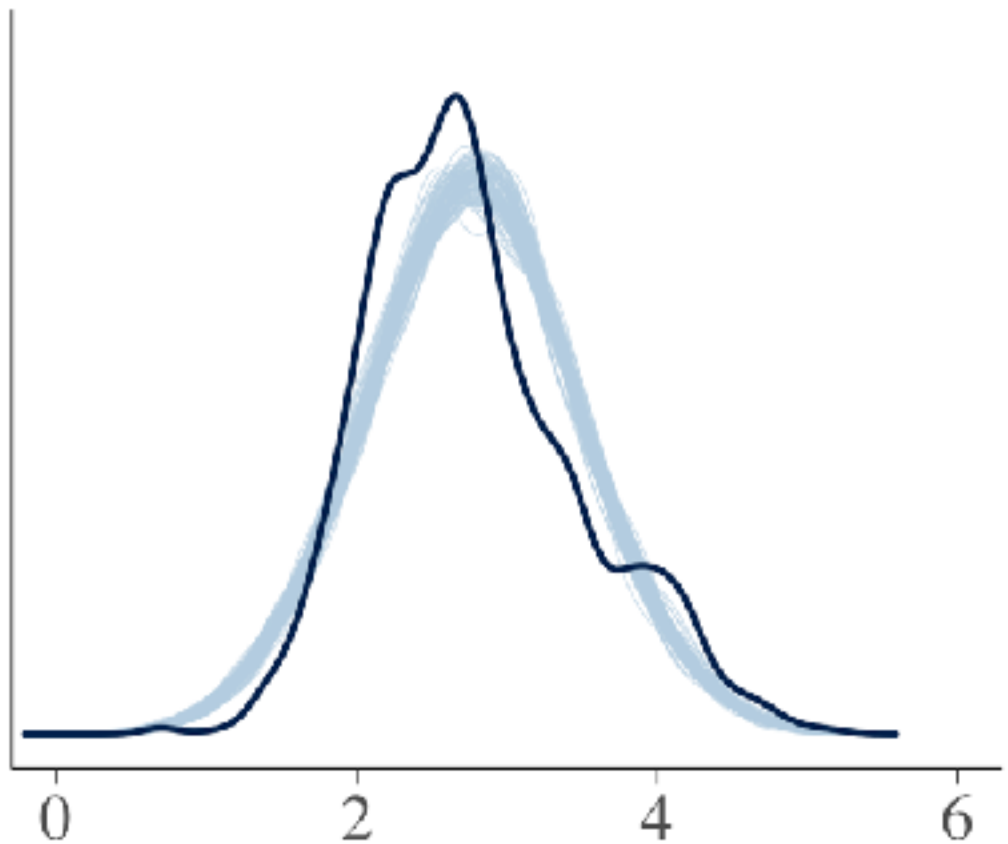
$$\begin{array}{c} \theta^{\star} \sim p(\theta | y) \\ \downarrow \\ \tilde{y} \sim p(y | \theta^{\star}) \end{array} \quad \Longleftrightarrow \quad \tilde{y} \sim p(\tilde{y} | y)$$

Posterior predictive checking

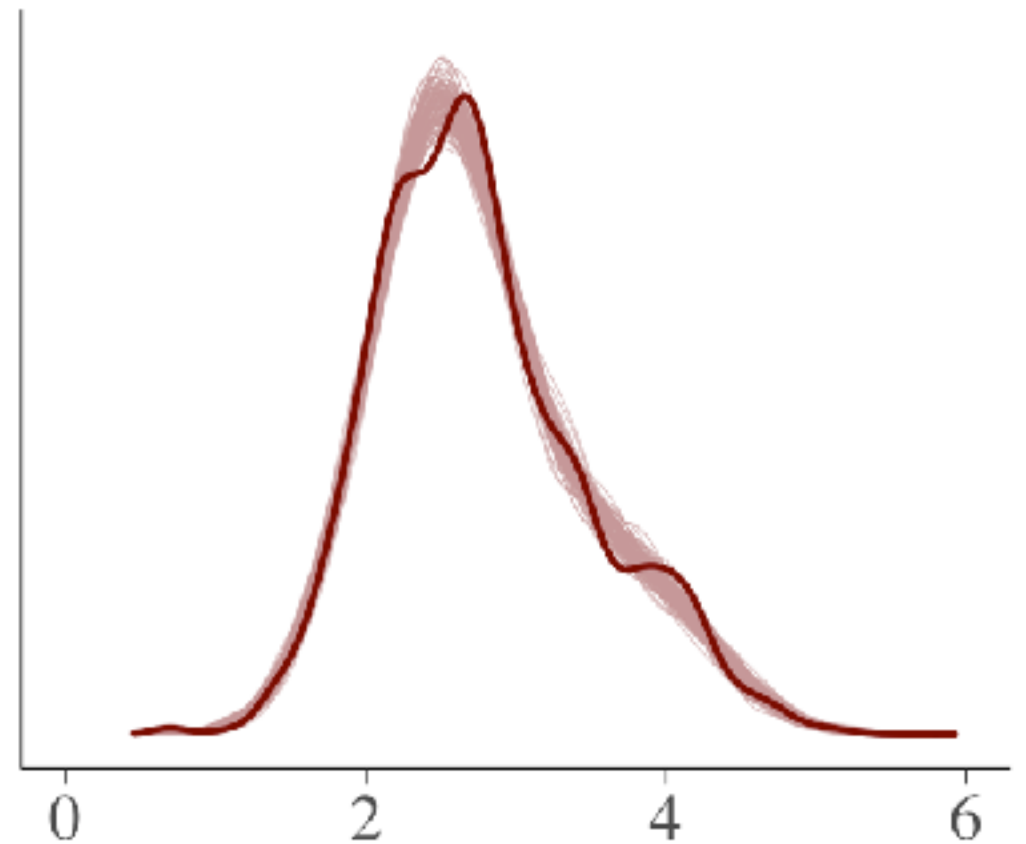
visual model evaluation

Observed data vs posterior predictive simulations

Model 1 (single level)



Model 3 (multilevel)

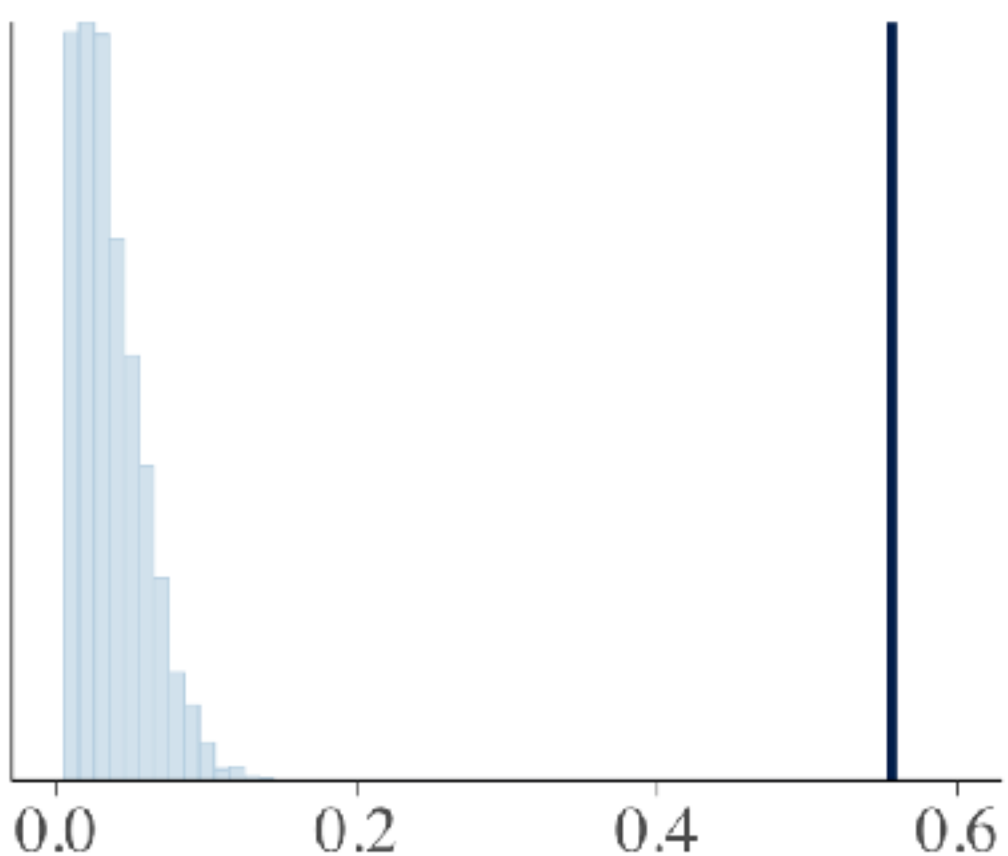


Posterior predictive checking

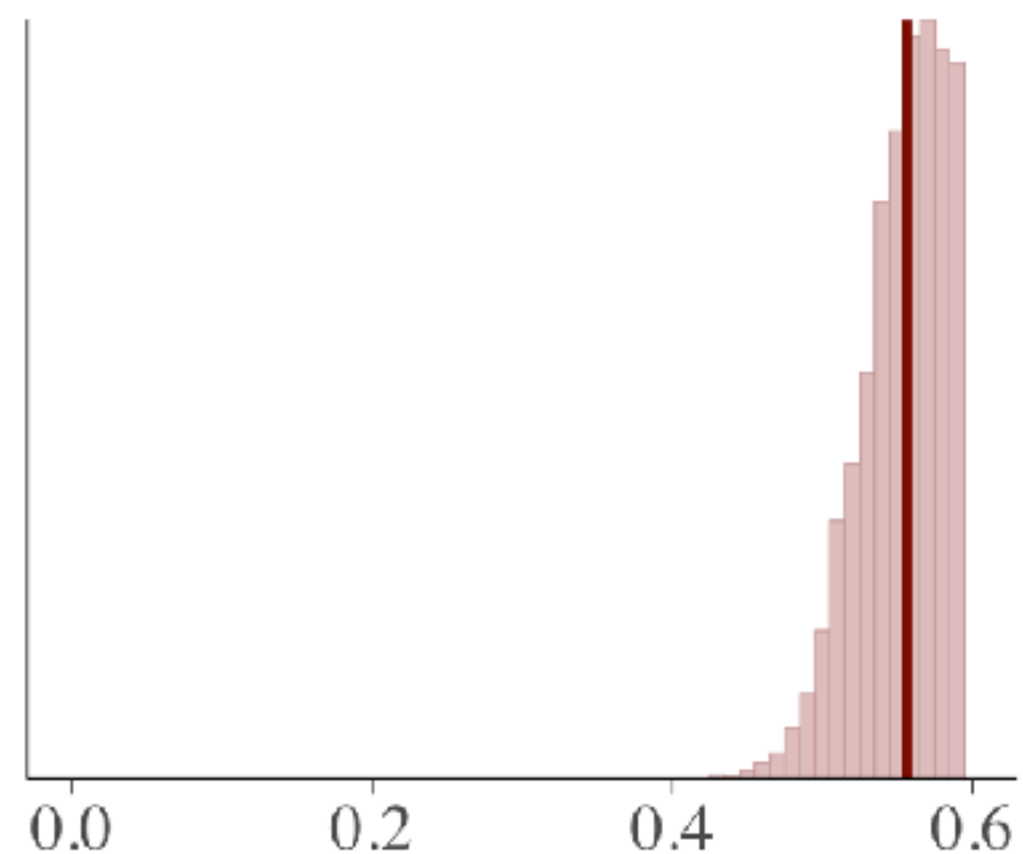
visual model evaluation

Observed statistics vs posterior predictive statistics

Model 1 (single level)

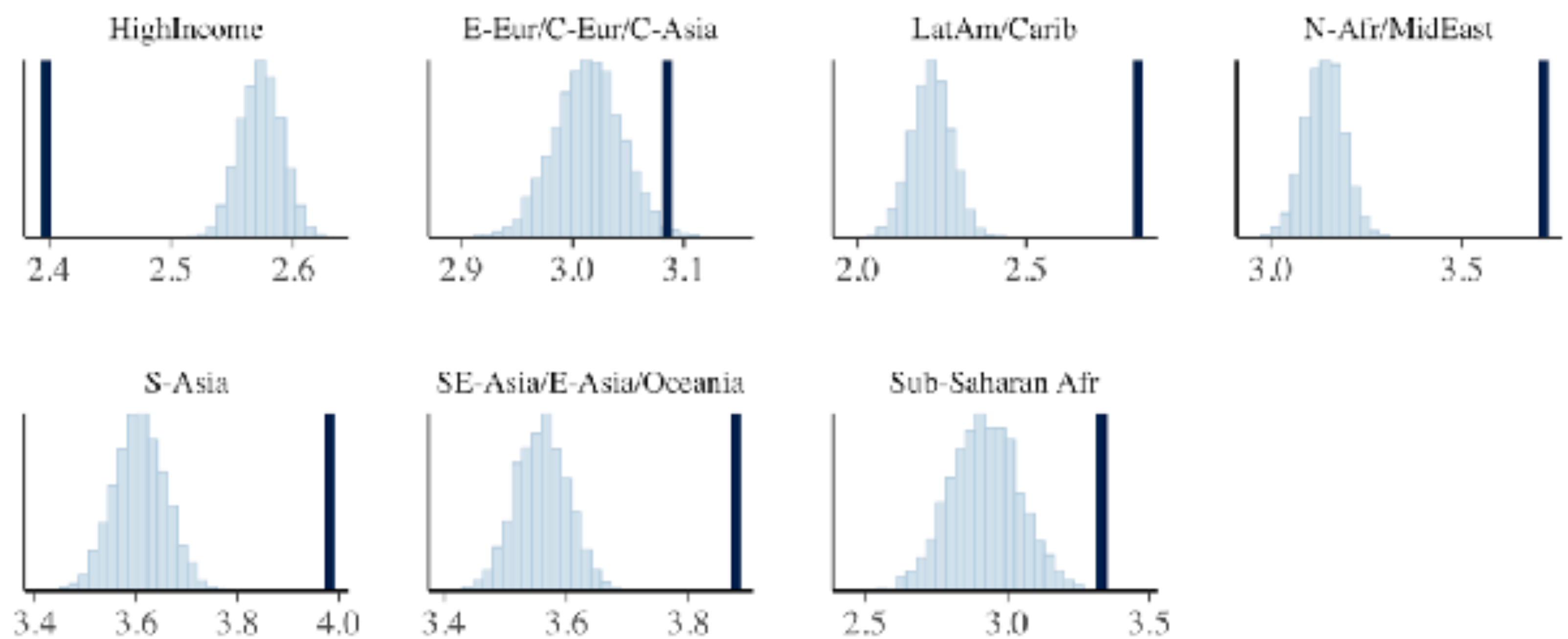


Model 3 (multilevel)



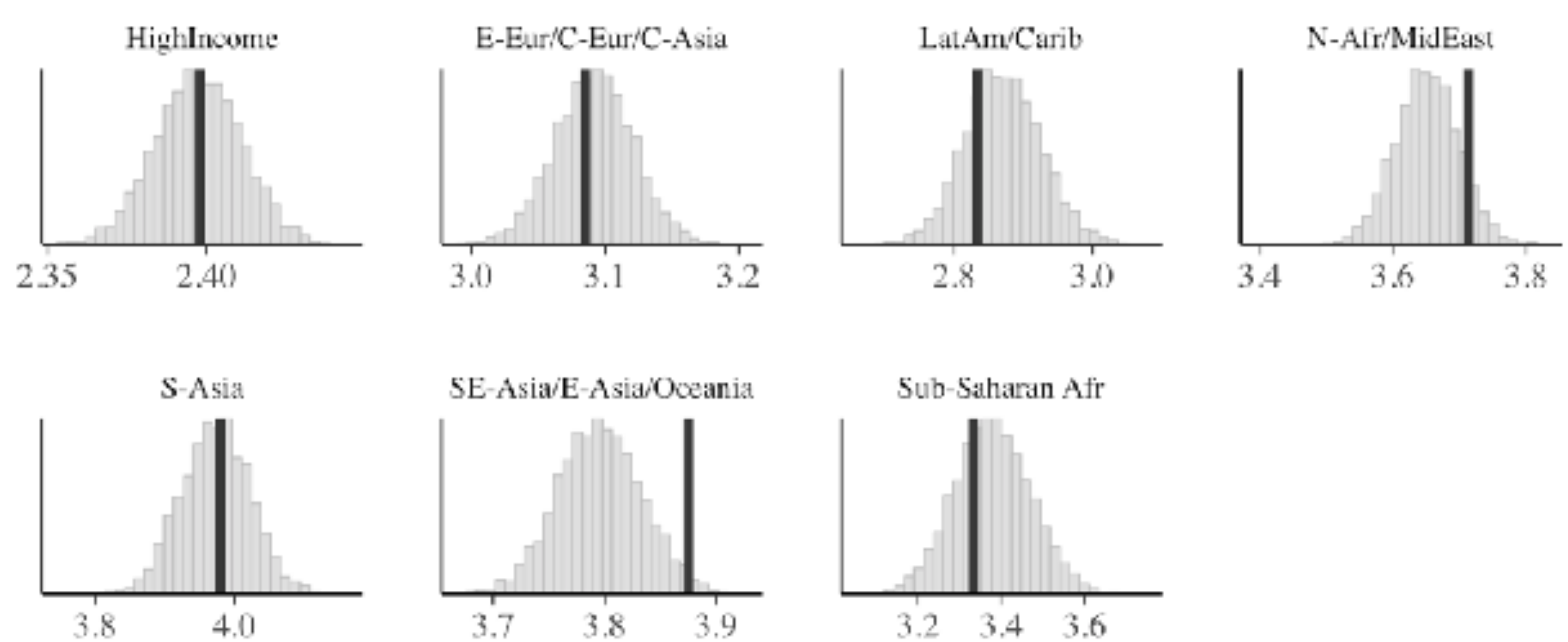
$$T(y) = \mathrm{skew}(y)$$

Posterior predictive checking:
visual model evaluation



Model 1 (single level)

$$T(y) = \mathrm{med}(y|\mathrm{region})$$



Model 2 (multilevel)

Model comparison

Pointwise predictive comparisons & LOO-CV

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- Visual PPCs can also identify unusual/influential (outliers, high leverage) data points

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$$p(y_i | y_{-i})$$

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pointwise predictive comparisons & LOO-CV

- Visual PPCs can also identify unusual/influential (outliers, high leverage) data points
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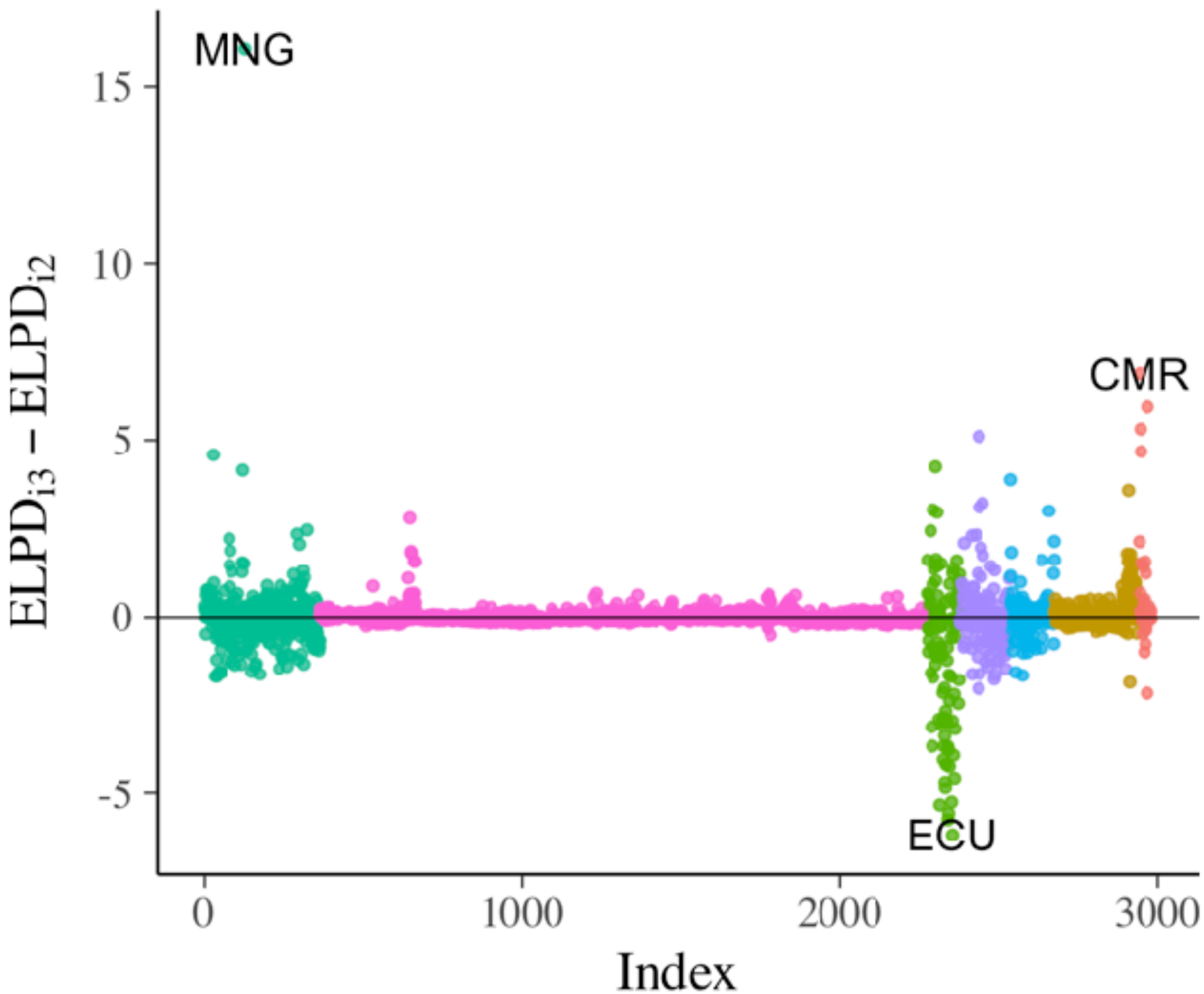
$$p(y_i | y_{-i})$$

- Which model best predicts each of the data points that is left out?

Model comparison

pointwise predictive comparisons & LOO-CV



Model comparison

Efficient approximate LOO-CV

Vehtari, A., Gelman, A., and Gabry, J. (2017).
Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC.
Statistics and Computing. 27(5), 1413–1432.
doi: [10.1007/s11222-016-9696-4](https://doi.org/10.1007/s11222-016-9696-4)

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Pareto smoothed importance sampling.
working paper
arXiv: arxiv.org/abs/1507.02646/

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- Asymptotically equivalent to WAIC
- Advantage: PSIS-LOO CV more robust + has diagnostics (check assumptions)

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Diagnostics

Pareto shape parameter & influential observations