K.S. INSTITUTE OF TECHNOLOGY, BANGALORE - 560109

III SESSIONAL TEST QUESTION PAPER 2018 – 19 EVEN SEMESTER

Scheme and Answers

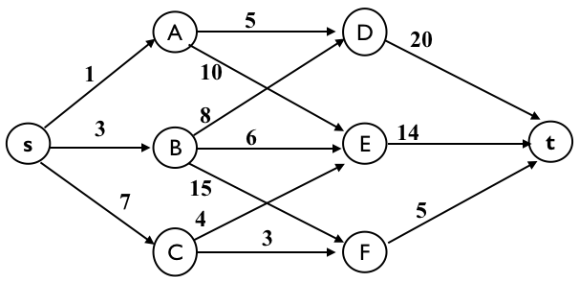
SET – A/~~B~~

USN										

Degree	:	**B.E**	**Semester**	:	**IV**
Branch	:	**Computer Science and Engineering**	**Course Code**	:	**17CS43**
Course Title	:	**Design and Analysis of Algorithms**	**Date**	:	**21-May-2019**
Duration	:	**90 Minutes**	**Max Marks**	:	**30**

Note: Answer ONE full question from each part.

<table>
<tr><th>Q No.</th><th>Question</th><th>Marks</th></tr>
<tr><td>1(a)</td><td>Compare the principle of optimality with Greedy approach. Consider the problem of optimal merge pattern to merge N files, and determine which pair of files should be merged at each. Analyze if decision sequences follows the principle of optimality.</td><td>5</td></tr>
<tr><td>Sch & Ans</td><td>Sch: 3 marks for comparison, 2 marks for merging N files

Ans:
With principle of optimality, the next decision in sequence of decisions is always optimal, whereas in Greedy approach, the next decision may not be optimal. For example, using Dijkstra's algo with negative weights, using greedy approach it may not give optimal results. The optimal merge pattern is always pick two smallest size files and merge them. Put the merge file back and this become another file. Thus each step, number of files decreases by 1, and merging will be complete after N-1 merges.</td><td></td></tr>
<tr><td>(b)</td><td>Calculate the shortest path from source s to destination t for the following multi-stage graph using backward approach (forward reasoning).

</td><td>5</td></tr>
<tr><td>Sch & Ans</td><td>Sch: 3 marks for defining the right steps, and 2 marks for computation.

Ans:
d(s,t)=min{d(s,D)+20, d(s,E)+14, d(s,F)+5}
d(s,D)=min{d(s,A)+5, d(s,B)+8}
d(s,E)=min{d(s,A)+10, d(s,B)+6, d(s,C)+4}
d(s,F)=min{d(s,B)+15, d(s,C)+3}

Computation:
d(s,A)=1, d(s,B)=3, d(s,C)=7
d(s,D)=min{1+5,3+8}=6
d(s,E)=min{1+10,3+6,7+4}=9
d(s,F)=min{3+15,7+3}=10
d(s,t)=min{6+20,9+14,10+5}=15</td><td></td></tr>
</table>

(c)	**Identify** the reasons that are used to terminate a search path in a state-space tree of a branch-and-bound algorithm.	**5**
Sch & Ans	Sch: 2 marks each for first 2 reason, 1 mark for remaining Ans: A search in state space tree using Branch and Bound terminates, when any of the following conditions are met i. The value of a node's bound value is worse than the best solution seen so far ii. The node represents a situation which is not feasible due to constraints imposed on the problem. iii. The node has become a leaf node and reached the end of solution. At this point, if the node's objective function is better than the best solution so far, update the value of best solution.	
2(a)	**List** the OptimalBST algorithm and **analyze** its space and time complexity.	**5**

Sch & Ans	Sch: 3 marks for algo, and 2 marks for time complexity.
	
	Ans:
	Algo optimalBST(P[1:n])
	# P[1:n] represents the probabilities of occurrence of n keys.
	For i=1 to n do
	   C[i,i-1]=0
	   C[i,i]=P[i]
	   R[i,i]=i
	C[n+1,n]=0
	For d=1 to n-1 do # diagonal count
	   For i=1 to n-d to
	      j=i+d
	      minval = inf
	      for k=i to j do
	         if [Ci,k-1]+C[k+1,j] <minval then
	            minval = C[I,k-1]+C[k+1,j]
	            kmin = k
	      R[i,j] = kmin
	      Sum = P[i]
	      For s=i+1 to j do
	         Sum = sum+P[s]
	      C[i,j]=minval + sum
	Return C[1,n],R	

(b)	**Construct** an OptimalBST for following 3 keys. Pa=0.2, Pb=0.5, Pc=0.3	**5**
Sch & Ans	Sch: 0.5 mark each for diagonal computations C[1,1], C[2,2],C[3,3]: total 1.5 marks 1 mark each for computing C[1,2], and C[2,3], total : 2 marks 1.5 marks for computing C[1,3] Thus, total = 1.5+2+1.5=5 marks Ans: C(1,0)=0, C(2,1)=0, C(3,2)=0 C(1,1)=p_1=0.2, R(1,1)=1	2

C(2,2)=p_2=0.5, R(2,2)=2
C(3,3)=p_1=0.3, R(3,3)=3
C(1,2)=$\Sigma_{1\le s\le 2}p_s$+min{C(1,0)+C(2,2),C(1,1)+C(3,2)}
     =0.7+min{0+0.5,0.2+0}=0.9
R(1,2)=2
C(2,3)=$\Sigma_{2\le s\le 3}p_s$+min{C(2,1)+C(3,3),C(2,2)+C(4,3)}
     =0.8+min{0+0.3,0.5+0}=1.1
R(2,3)=2
C(1,3)=$\Sigma_{1\le s\le 3}p_s$+min{C(1,0)+C(2,3),C(1,1)+C(3,3),C(1,2)+C(4,3)
     =1.0+min{0+1.1,0.2+0.3,0.9+0}=1.0+0.5=1.5
R(1,3)=2

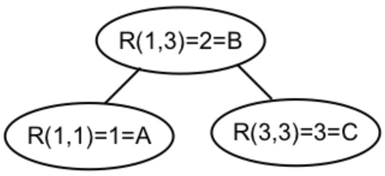
Comparison table and Root table

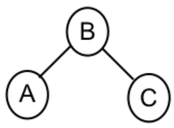
C(i,j)

	0	1	2	3
1	0	0.2	0.9	1.5
2		0	0.5	1.1
3			0	0.3
4				0

Root table

	0	1	2	3
1		1	2	2
2			2	2
3				3
4				





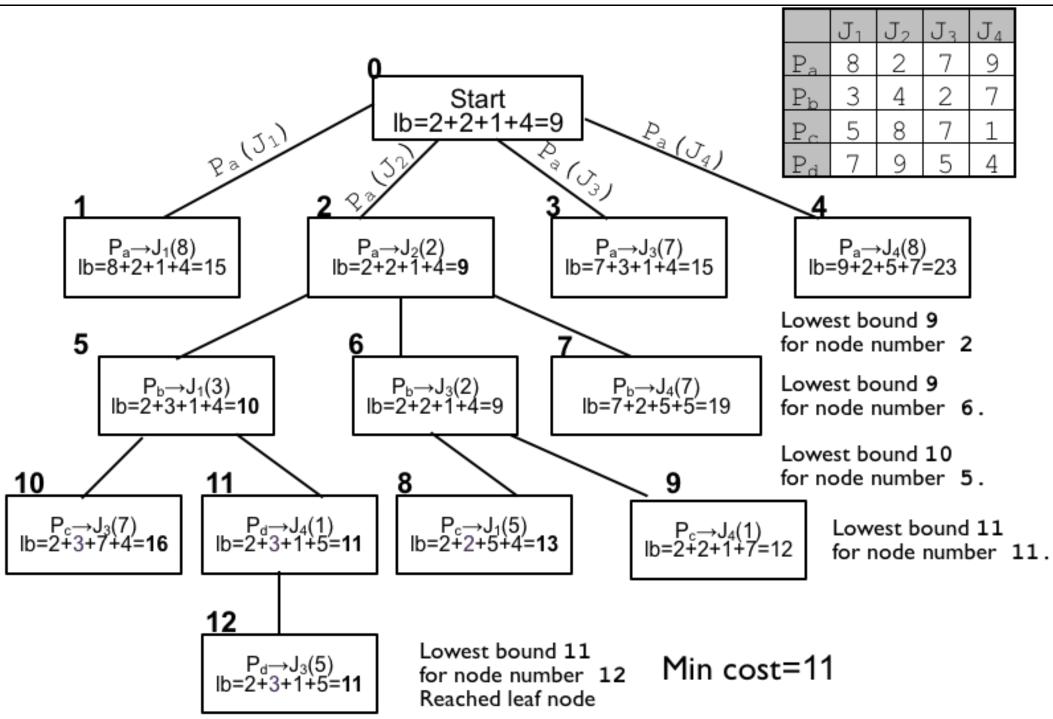
(c) **Apply** the branch and bound approach to **solve** the instance of following assignment problem among 4 persons for 4 jobs. **5**

	J_1	J_2	J_3	J_4
P_a	8	2	7	9
P_b	3	4	2	7
P_c	5	8	7	1
P_d	7	9	5	4

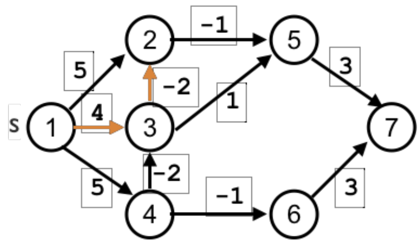
Sch & Ans

Sch: 1 mark for each level in state space tree. Total 5 levels→5 marks

Ans:

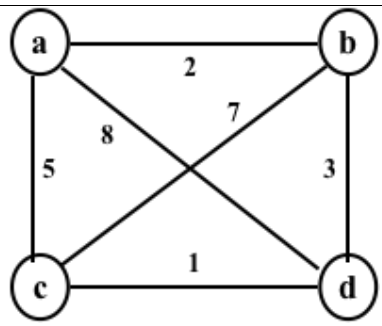
<table>
<tr><td></td><td>


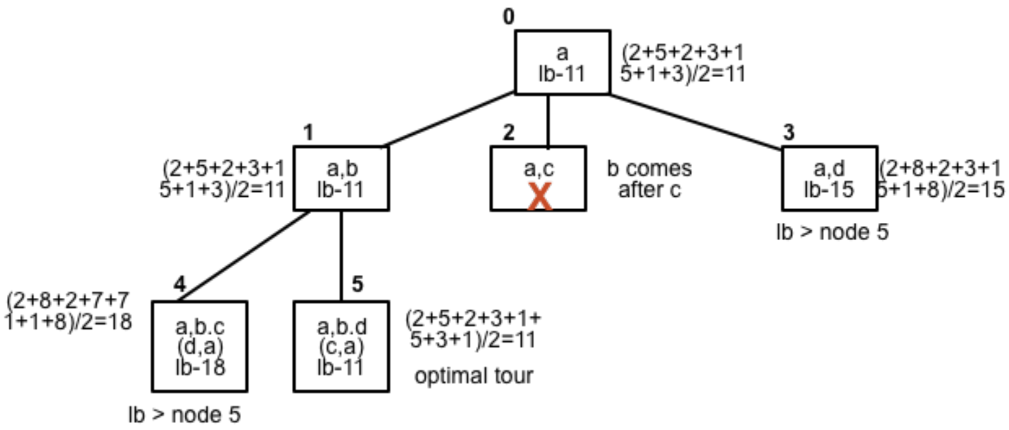
</td><td></td></tr>
<tr><td>3(a)</td><td>
Calculate the shortest path from node 2 to all the nodes for the graph below using Bellman Ford algorithm and list output after each of the iteration step.



</td><td>5</td></tr>
<tr><td>Sch & Ans</td><td>
Sch: 1 mark each for each iteration step

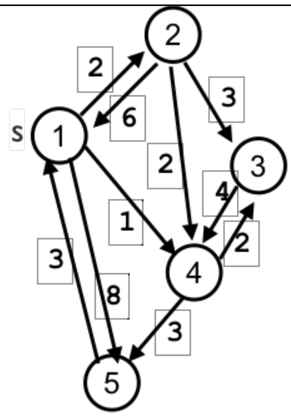
Ans:
<table>
<tr><td>k↓</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>1</td><td>0</td><td>6</td><td>5</td><td>5</td><td>∞</td><td>∞</td><td>∞</td></tr>
<tr><td>2</td><td>0</td><td>3</td><td>3</td><td>5</td><td>5</td><td>4</td><td>∞</td></tr>
<tr><td>3</td><td>0</td><td>1</td><td>3</td><td>5</td><td>2</td><td>4</td><td>7</td></tr>
<tr><td>4</td><td>0</td><td>1</td><td>3</td><td>5</td><td>0</td><td>4</td><td>5</td></tr>
<tr><td>5</td><td>0</td><td>1</td><td>3</td><td>5</td><td>0</td><td>4</td><td>3</td></tr>
<tr><td>6</td><td>0</td><td>1</td><td>3</td><td>5</td><td>0</td><td>4</td><td>3</td></tr>
</table>
</td><td></td></tr>
<tr><td>(b)</td><td>
Establish the shortest tour for the Traveling Salesperson Problem using Dynamic Programming for the following graph. Discover all the applicable <code>g(i,S)</code> values in your analysis of TSP steps.
</td><td>5</td></tr>
</table>

		
Sch & Ans	Sch: 1 mark each computing g(i,S) for |S|-0, 1, 2. 1 mark for computing corresponding J values, and 1 marks for working out the min cost tour. Ans: Goal:g(1,V-{1}) Power set of {2,3,4} Ø,{2},{3},{4},{2,3},{2,4},{3,4},{2,3,4} g(1,Ø)=c_{11}=0 g(2,Ø)=c_{21}=2 g(3,Ø)=c_{31}=5 g(4,Ø)=c_{41}=8 Compute g(i,S), for |S|=1 g(2,{3})=c_{23}+g(3,Ø)=7+5=12 g(2,{4})=c_{24}+g(4,Ø)=3+8=11 g(3,{2})=c_{32}+g(2,Ø)=7+2=9 g(3,{4})=c_{34}+g(4,Ø)=1+8=9 g(4,{2})=c_{42}+g(2,Ø)=3+2=5 g(4,{3})=c_{43}+g(3,Ø)=1+5=6 J(2,{3})=3,J(2,{4})=4,J{3,{2}=2 J(3,{4})=4,J(4,{2})=2,J(4,{3}=3 Compute g(i,S), for |S|=2 g(2,{3,4})=min{c_{23}+g(3,{4}),c_{24}+g(4,{3})=min{7+9,3+6}=9, J(2,{3,4})=4 g(3,{2,4})=min{c_{32}+g(2,{4}),c_{34}+g(4,{2})=min{7+11,1+5}=6,J(3,{2,4})=4 g(4,{2,3})=min{c_{42}+g(2,{3}),c_{43}+g(3,{2})=min{3+12,1+9}=10,J(2,{3,4})=3 g(1,{2,3,4})= min{c_{12}+g(2,{3,4}),c_{13}+g(3,{2,4}),c_{14}+g(4,{2,3})} =min{2+9, 5+6, 8+10} = 11 J(1,{2,3,4})=2 (can be 3 as well) The optimal tour length is 11, path is 1,2,4,3 J(1,{2,3,4})=2, J(2,{3,4})=4, J(4,{3})=3 If J(1,{2,3,4})=3, then J(3,{2,4})=4, J(4,{2})=2 And thus path is 1,3,4,2.	
(c)	**Apply** the branch and bound algorithm to solve Traveling Salesperson Problem for the graph Q3b.	**5**
Sch & Ans	Sch: 1 mark each for each node of state space tree. Ans:	

<table>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>4(a)</td><td>Analyze the problem of designing a system that is composed of n devices connected in series and maximizing its reliability using device duplication. Examine the maximization expression along with applicable constraints to be used for solving this problem using dynamic programming.</td><td>5</td></tr>
<tr><td>Sch & Ans</td><td>Sch: 2 marks for problem analysis, 3 marks for maximization expression

Ans:
Designing a system that is composed of n devices connected in series so as to maximize the reliability of the system with a upper limit c of cost.
r_i: reliability of device D_i functioning properly
c_i: cost of device D_i
m_i: number devices D_i that are arranged in duplicate
Thus, the probability that all devices D_i with duplicate setting will malfunction is $(1-r_i)^{mi}$
Let reliability at i^{th} stage with m_i duplicate devices is given by $\varnothing_i(m_i)$
Then the problem of reliability is given is specified as
Maximize $\Pi_{1\le i\le n}\ \varnothing_i(m_i)$
Subject to $\Sigma_{1\le i\le n}\ c_i m_i \le C$
Where $m_i >= 1$ is an integer, and $1 \le i \le n$</td><td></td></tr>
<tr><td>(b)</td><td>Calculate the shortest distance between all pairs for the diagraph with the below weight matrix.
$$\begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$</td><td>5</td></tr>
<tr><td>Sch & Ans</td><td>Sch: 1 mark each for each of first 3 iteration (total 4 iterations), 2 marks for last iteration

Ans:</td><td></td></tr>
</table>



k↓	1	2	3	4	5
1	0	2	∞	1	8
2	6	0	3	2	∞
3	∞	∞	0	4	∞
4	∞	∞	2	0	3
5	3	∞	∞	∞	0

D(0)

k↓	1	2	3	4	5
1	0	2	∞	1	8
2	6	0	3	2	**14**
3	∞	∞	0	4	∞
4	∞	∞	2	0	3
5	3	**5**	∞	**4**	0

D(1)

k↓	1	2	3	4	5
1	0	2	5	1	8
2	6	0	3	2	**14**
3	∞	∞	0	4	∞
4	∞	∞	2	0	3
5	3	**5**	8	4	0

D(2)

k↓	1	2	3	4	5
1	0	2	5	1	8
2	6	0	3	2	14
3	∞	∞	0	4	∞
4	∞	∞	2	0	3
5	3	5	8	4	0

D(3)

k↓	1	2	3	4	5
1	0	2	3	1	4
2	6	0	3	2	5
3	∞	∞	0	4	7
4	∞	∞	2	0	3
5	3	5	6	4	0

D(4)

k↓	1	2	3	4	5
1	0	2	3	1	4
2	6	0	3	2	5
3	10	12	0	4	7
4	6	8	2	0	3
5	3	5	6	4	0

D(5)

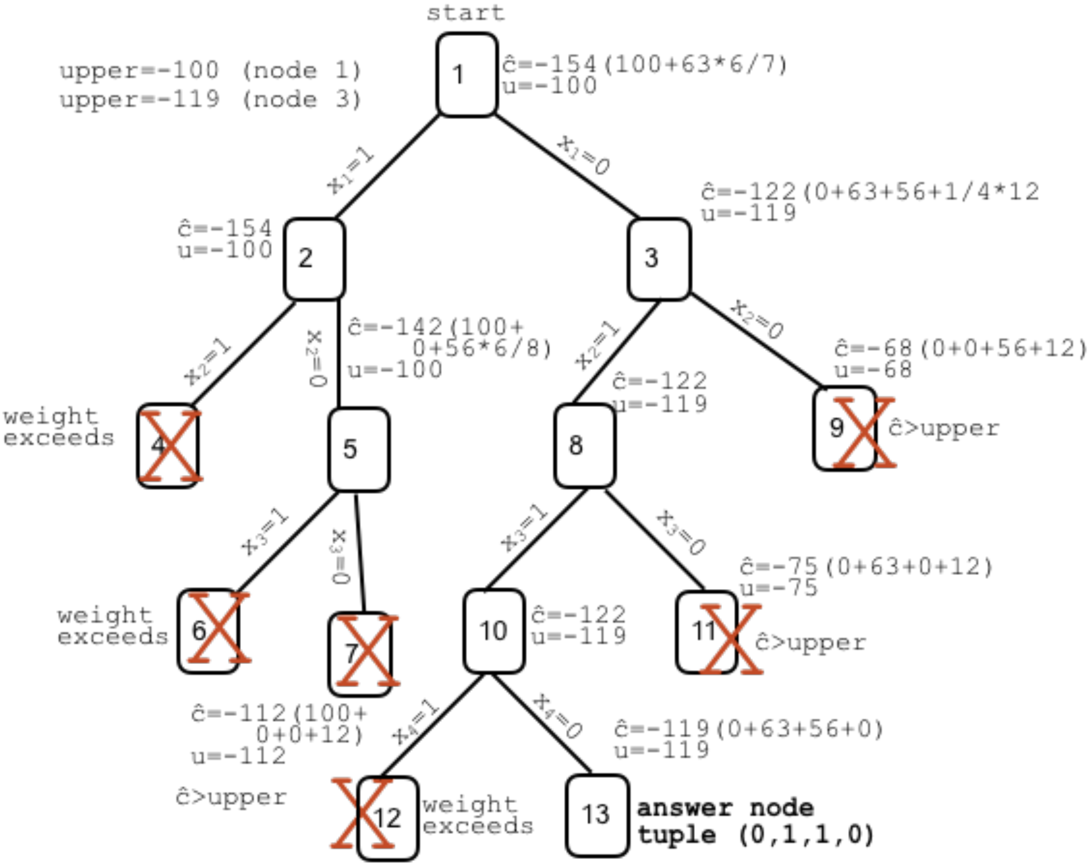
(c) **Apply** the branch and bound technique to solve the following knapsack problem with knapsack weight of 16, with weights and values for 4 items are given below. **5**

Item	Weight	Value
1	10	Rs 100
2	7	Rs 63
3	8	Rs 56
4	4	Rs 12

Sch & Ans

Sch: 1 mark each or each level of state space tree

Ans:



Signature of Course in charge **Signature of HOD-CSE**