Design and Analysis of Algorithms

L42: Backtracking Algorithms Approach

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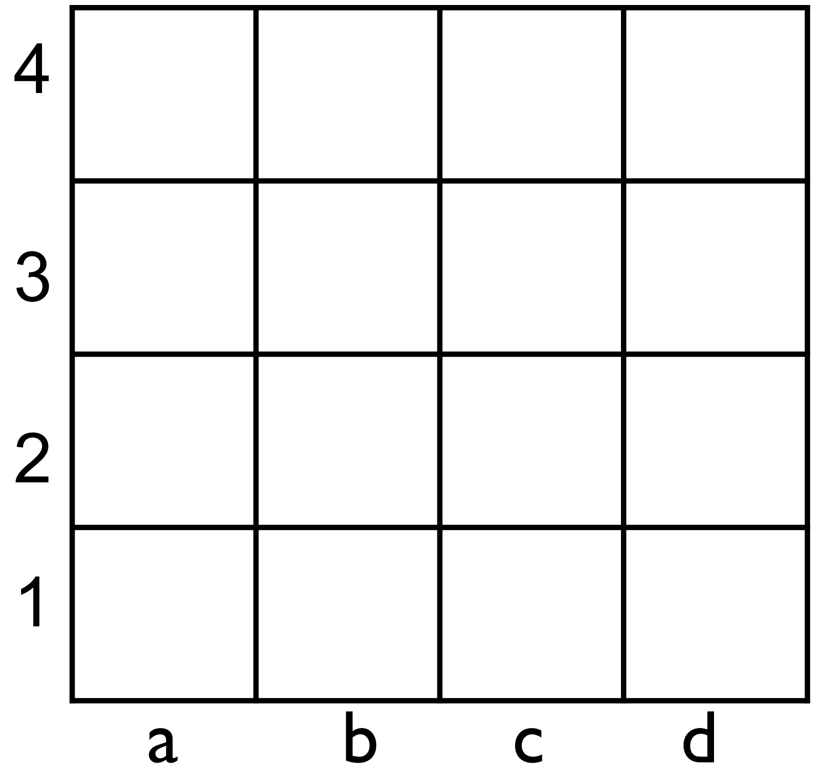
Resources

- Text book 2: Horowitz
  - Sec **7.**1,7.2,7.3,7.4,7.5,8.2,11.1
- Text book 1: Levitin
  - Sec 12.1,12.2
- R1: Introduction to Algorithms
  - Cormen et al.

Overview of Backtracking

- Basic approach of backtracking
  - Determine problem solutions by systematically searching the solution space
- Approach to search the solution space
  - Construct a tree of solution space
  - Node of this tree corresponds to a tuple variable assigned a possible feasible value
  - Edge of tree corresponds to tuple variables x_i where x_i is assigned a possible value
  - Leaf node satisfying the constraints (criterion function) represents a solution
- Two kind of trees
  - Static trees, Dynamic trees

4-Queens Problem



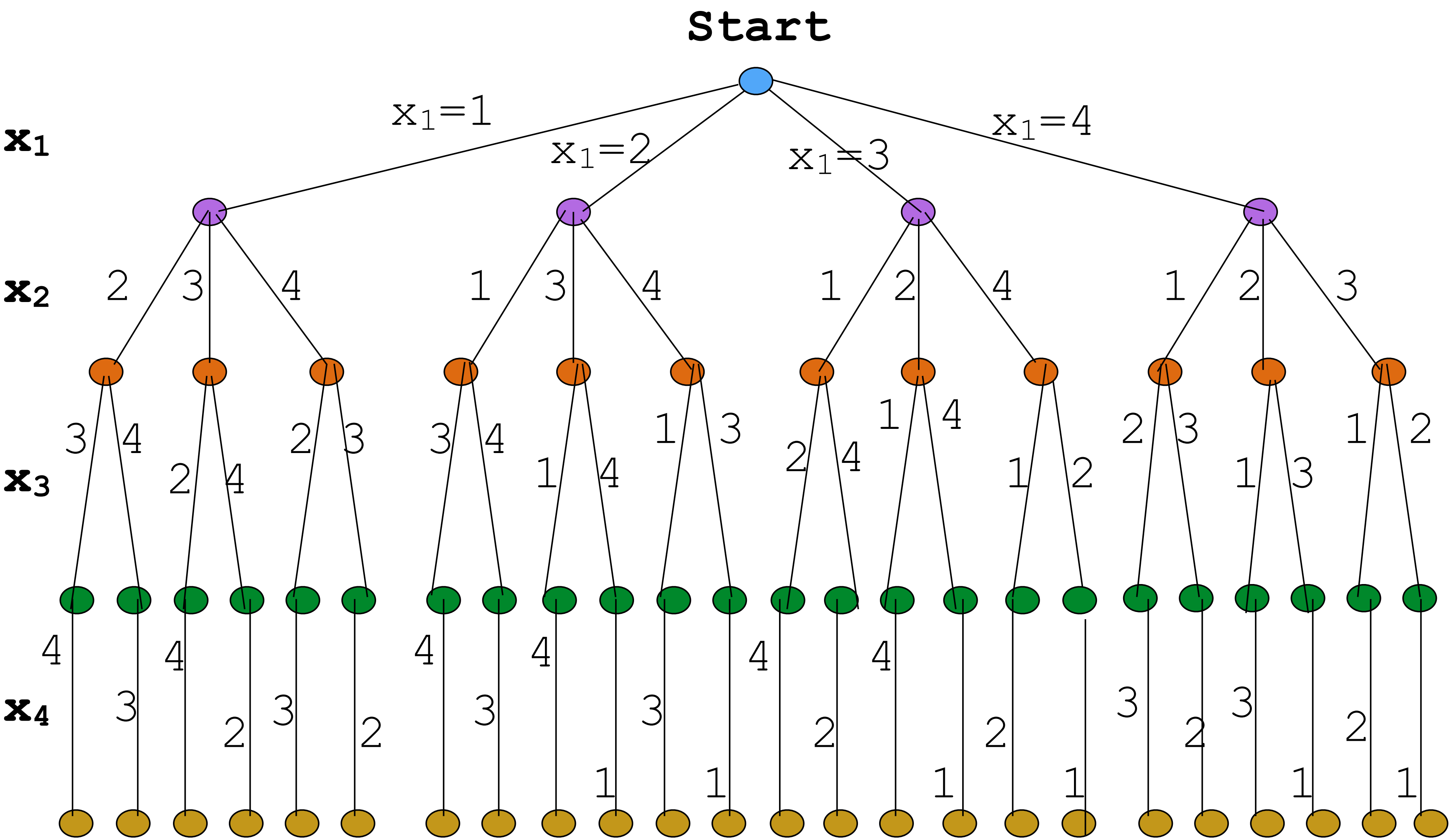
Q: Place 4 queens on the board such that no queen attacks another

Approach: For each column, assign a variable,

e.g. x_1, x_2, x_3, x_4 respectively for columns `a, b, c, d`

Each xi can take values from 1 to 4.

State Space: 4 Queens



Size of 4-queens state space: $4! = 24$

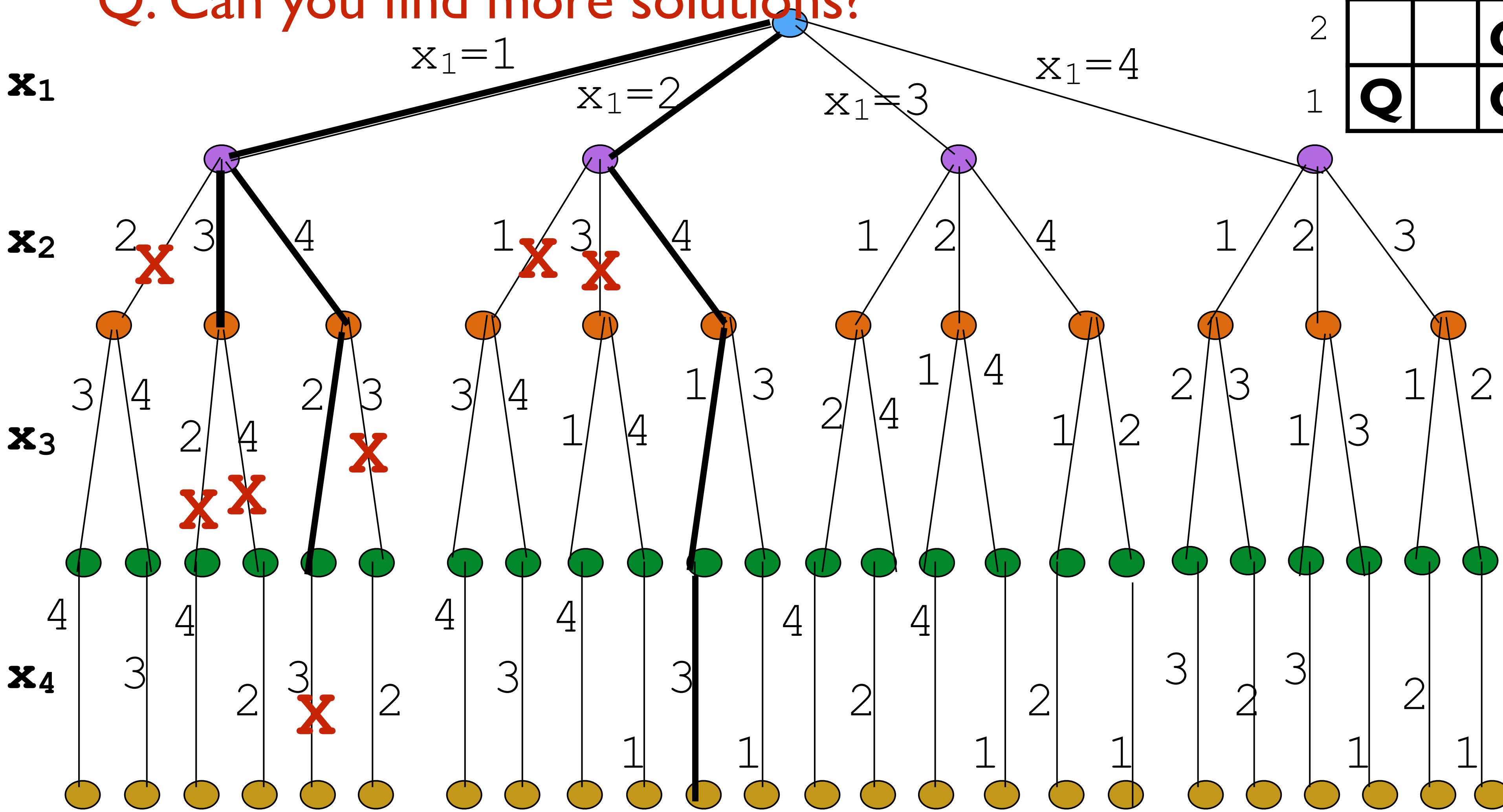
Solution: 4-Queens

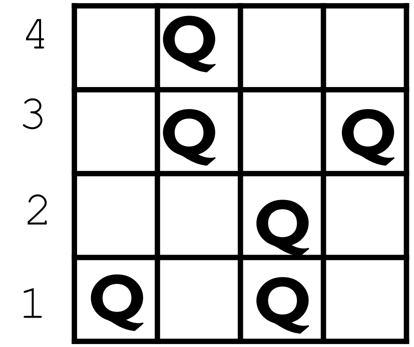
	a	b	c	d
4		Q_2		
3				Q_4
2	Q_1			
1			Q_3	

State Space: 4 Queens

Solution: (2,4,1,3)

Q: Can you find more solutions?





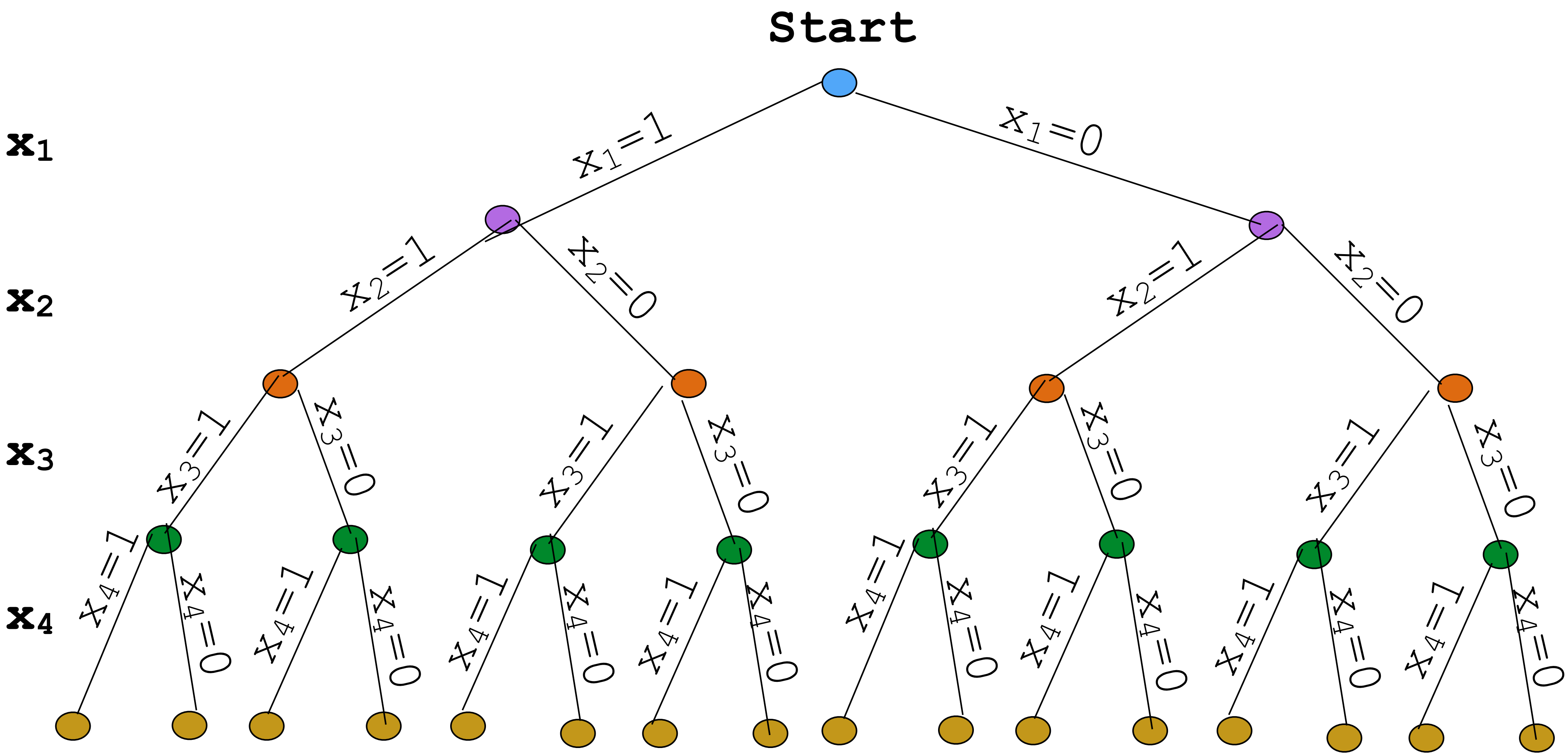
Size of 4-queens state space: 4!=24

State Space: 4 Queens

- Solving 4-Queens problems
  - Build a complete possible tree
    - Called a static approach
  - Explore (traverse) the tree for possible solutions.
  - Prune the tree when come to a node where one can not traverse further
  - Backtrack to explore next path.
  - When leaf node is reached, a solution is found.
- Note: Building a static tree is independent of problem instance.
  - Tree is built for all possible solutions.

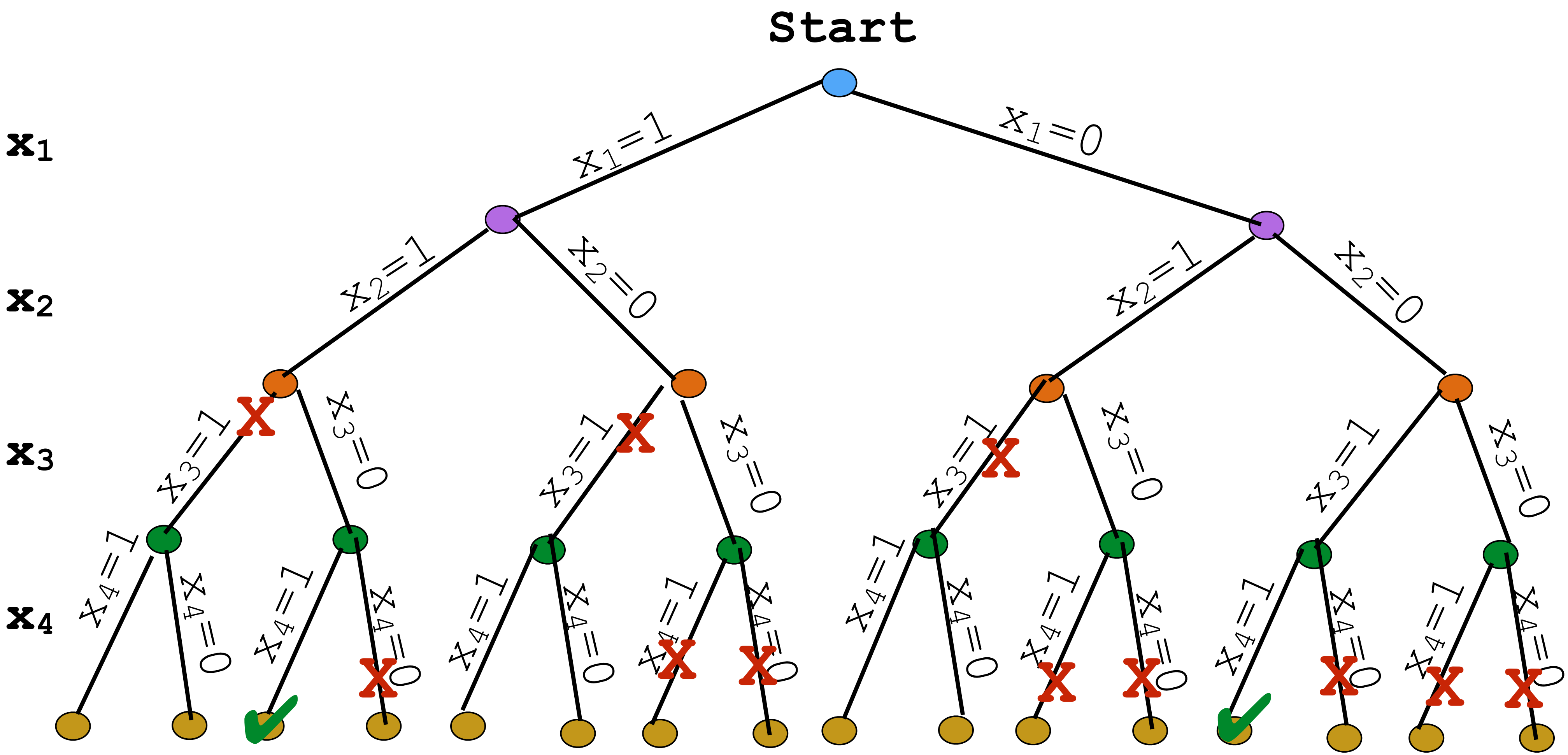
State Space: Sum of Subset problem

- **Ex:** `S={11,13,24,7}`, **and** `m=31`



Solution Space: Subset sum problem

- **Ex:** `S={11,13,24,7}`, **and** `m=31`



Sol^n 1: `{1,1,0,1}` Sol^n 2=`{0,0,1,1}`

State Space: Subset Sum

- Solving sum of subset problems
  - Build a complete possible tree
    - Again a static approach
  - Explore (traverse) the tree for possible solutions.
  - Prune the tree on reaching a node where can not traverse further
  - Backtrack to explore next path.
  - When reach the leaf node, it is not necessary that a solution is found.
- Note: Building a static tree is independent of problem instance. e.g. even if value of elements of set or sum total changes, tree remains the same.
  - Tree is built for all possible solutions.

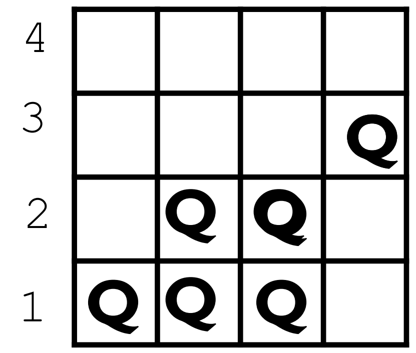
State Space Trees: Terminology

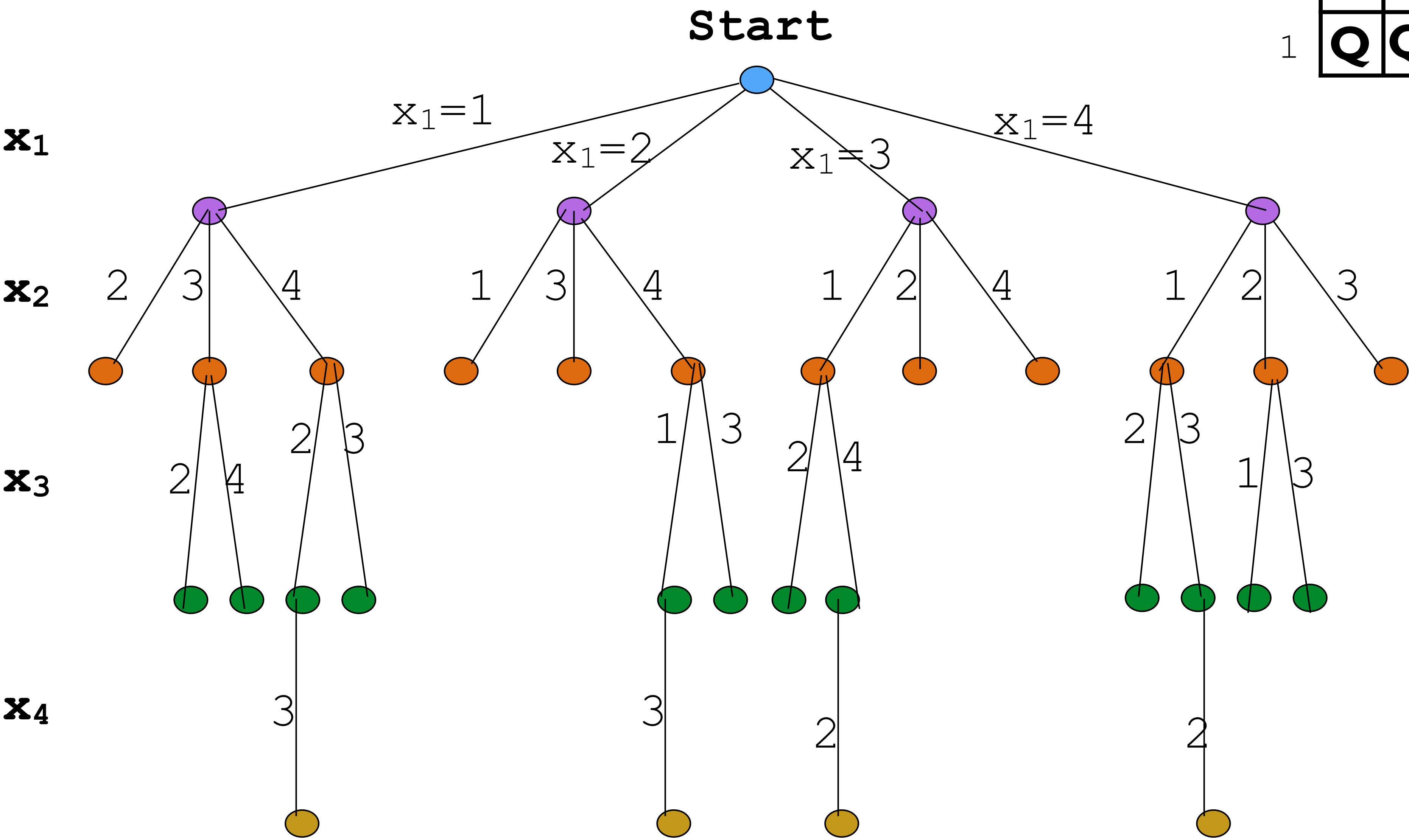
- Terminology
  - Each node in the tree defines a problem state
  - All paths from root to other nodes define *state space* of problem
  - *Solution states* are those problem states `s` for which the path from root to `s` defines a tuple in the solution space.
  - *Answer states* are are those solution states s for which the path from root to s defines a tuple that is a member of the set of solutions.
  - Tree organization is referred to *Space Tree*.

State Space Trees

- Dynamic Trees:
  - Tree organization is determined dynamically as the state space is being searched.
  - Tree organizations that are dependent on problem instance are called dynamic trees.
- Two ways to generate problem states
  - Backtracking, and Branch-n-Bound
  - Both begin with the root and generate other nodes
  - A node whose all children have not been generated is called a *live* node
    - *Live* node whose children are currently being generated is called E−*node*.
  - A *dead* node is a generated node which is not to be expanded further, or whose all children are generated.

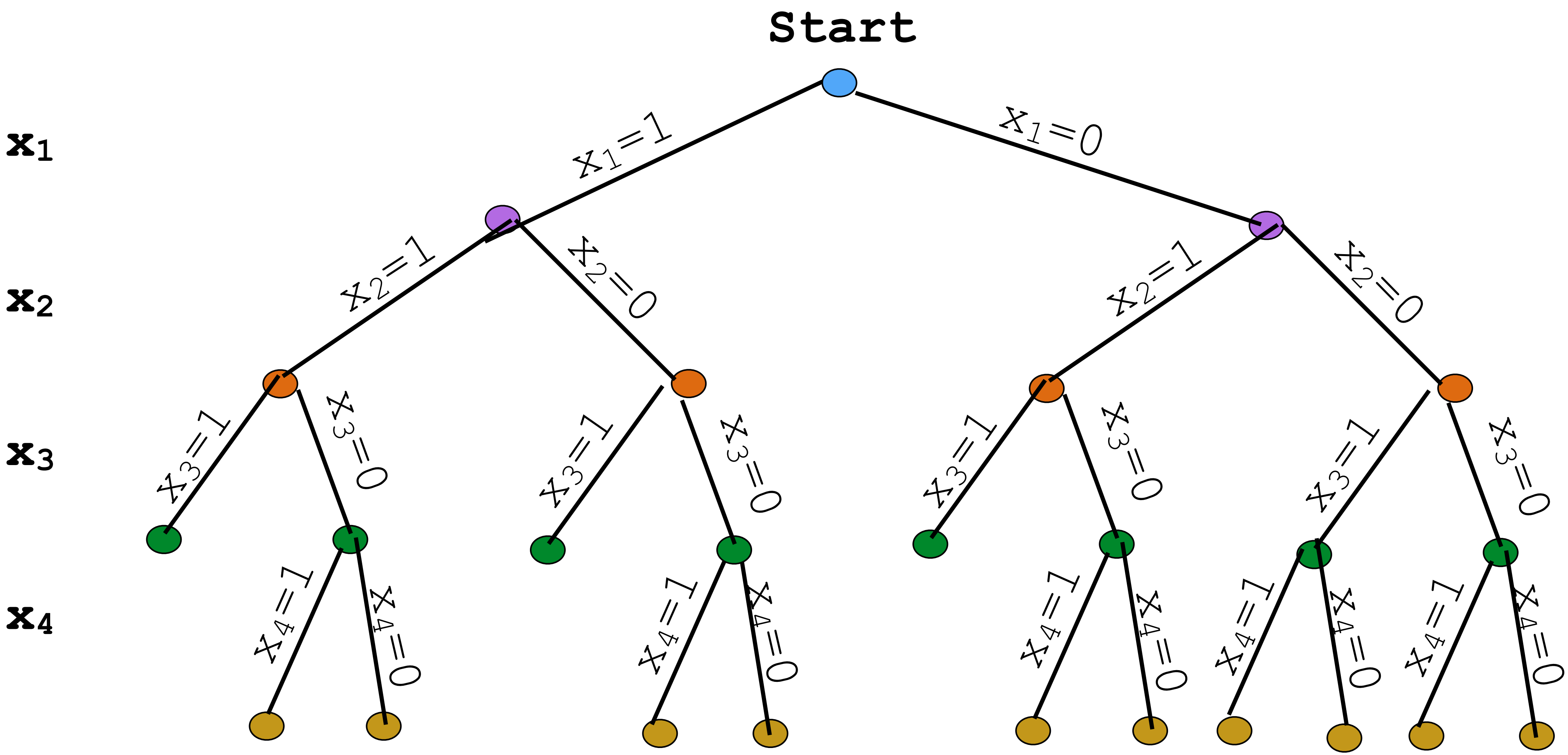
Backtracking Tree: 4 Queens



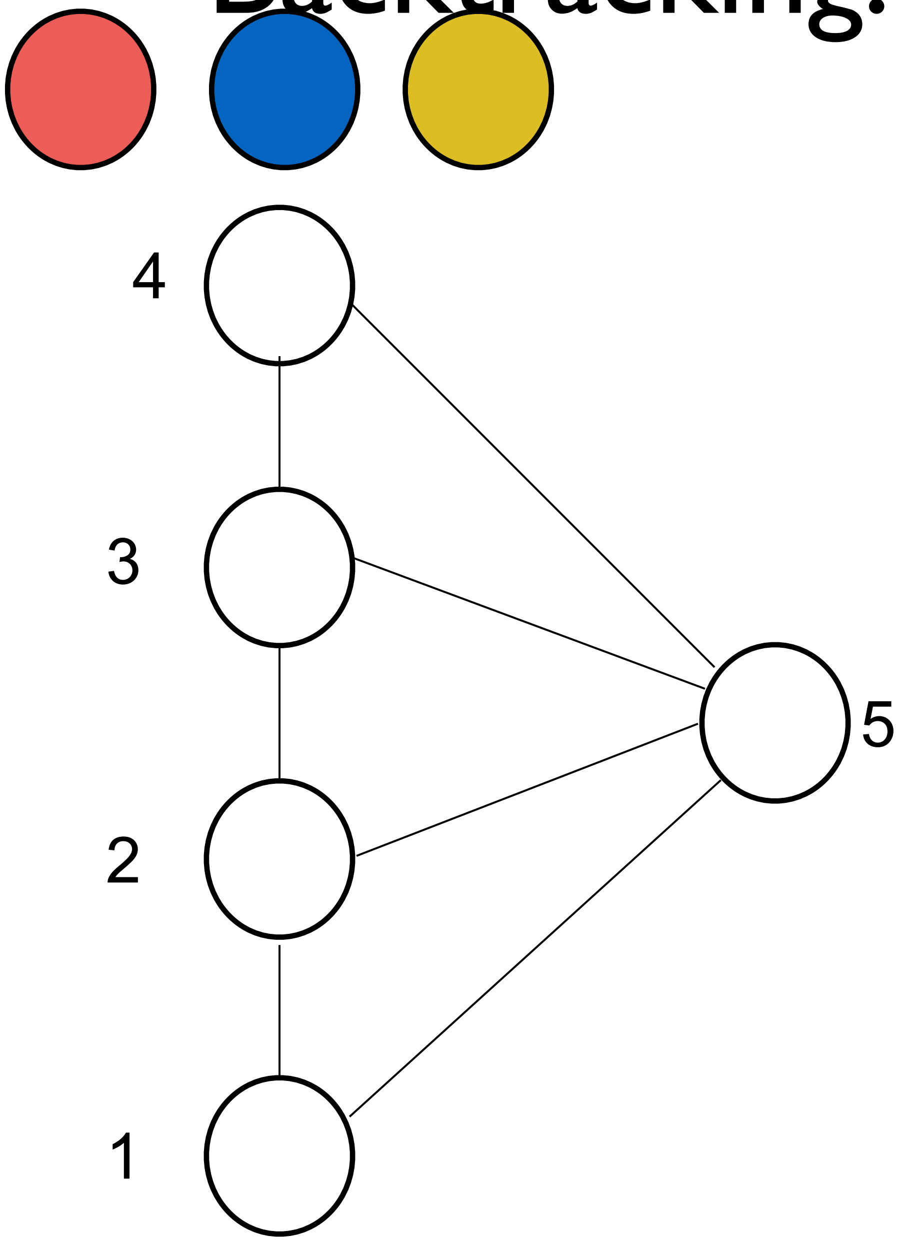


Backtracking Tree: Subset Sum

- **Ex:** `S={11,13,24,7}`, **and** `m=31`



Backtracking: 3-color problem



- Exercise:
  – Construct the state space tree for the graph shown for 3-color problem.

Algo: Backtracking

- Notations:
  - $(x_1, x_2, \ldots, x_i)$ be the path from root to a node in state space tree.
  - $T(x_1, x_2, \ldots, x_i)$ be the set of all possible values for x_{i+1} such that $(x_1, x_2, \ldots, x_{i+1})$ is also a path to problem state
  - B_{i+1} is boundary function predicate i.e. if $B_{i+1}(x_1, x_2, \ldots, x_{i+1})$ is `false` for path $(x_1, \ldots, x_{i+1})$, then path can't be extended to reach an answer state
  - $T(x_1, x_2, \ldots, x_n) = \emptyset$ (no more nodes to be explored)
  - Candidates for position $i+1$ of solution vector $(x_1, \ldots, x_n)$
    - Those values generated by T and satisfies B_{i+1}

Algo: Backtracking

Algo `Backtrack(`***int*** `n)`

    int `k=1`

    while `(k) {`

        if there remains an untried `x[k]` such that `x[k]` is in `T(x[1],…,x[k-1])`, and `B(x[1],…,x[k])` is ***true***, ***then*** `{`

            if `(x[1],…[x[k])` *is a path to an answer node*,

                output `x[1:k]`

            `k++`

        else

            `k--` //backtrack to previous set

    `}`

Summary

- Basic approach of backtracking
- Static tree
- Dynamic tree