Design and Analysis of Algorithms

L41: Intro to
Backtracking

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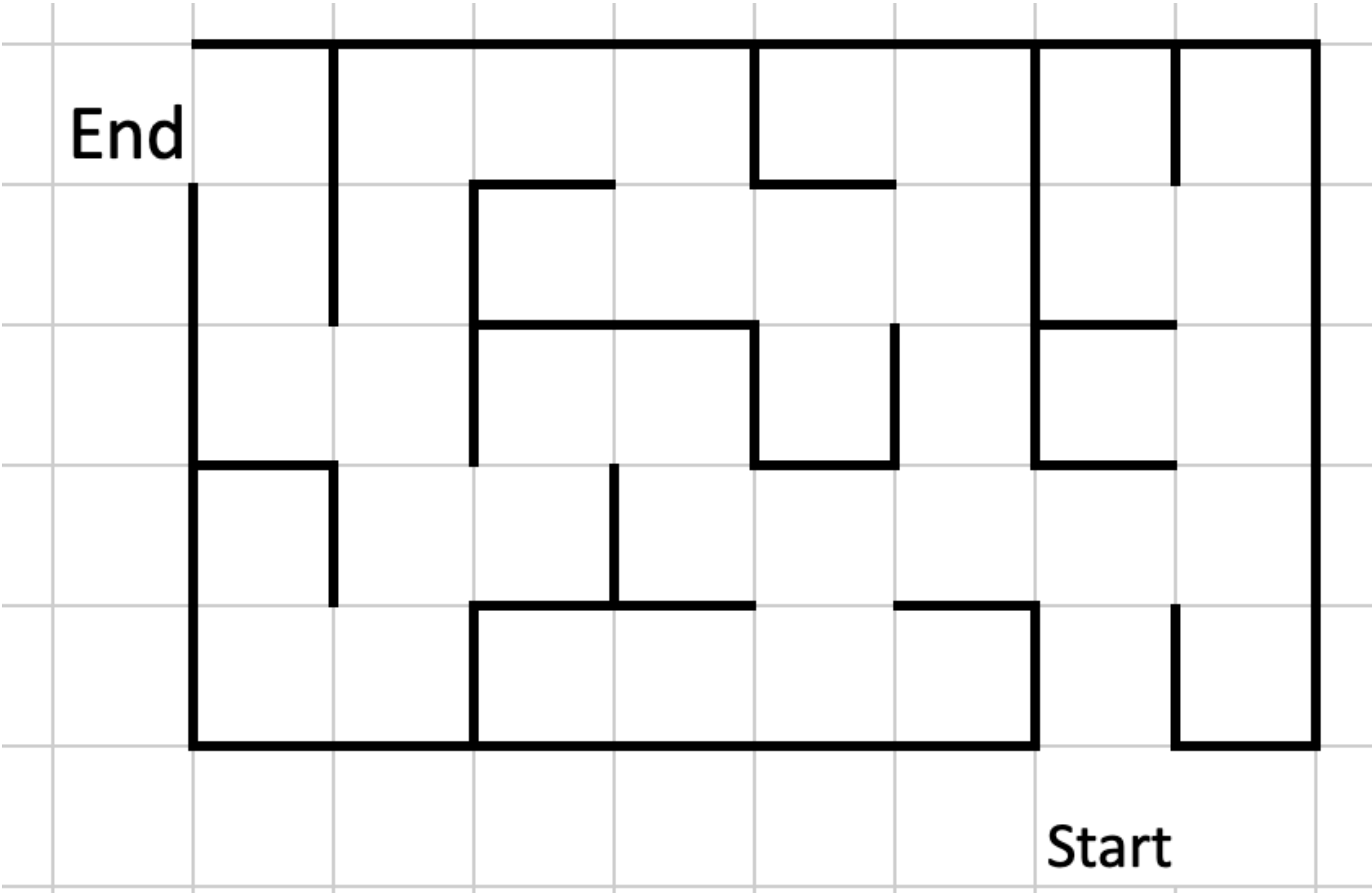
Resources

- Text book 2: Horowitz
  - Sec **7.1**, 7.2, 7.3, 7.4, 7.5, 8.2, 11.1
- Text book 1: Levitin
  - Sec 12.1, 12.2
- R1: Introduction to Algorithms
  - Cormen et al.
- Youtube link of video lecture
  - https://www.youtube.com/watch?v=Jcnk_hwS08A

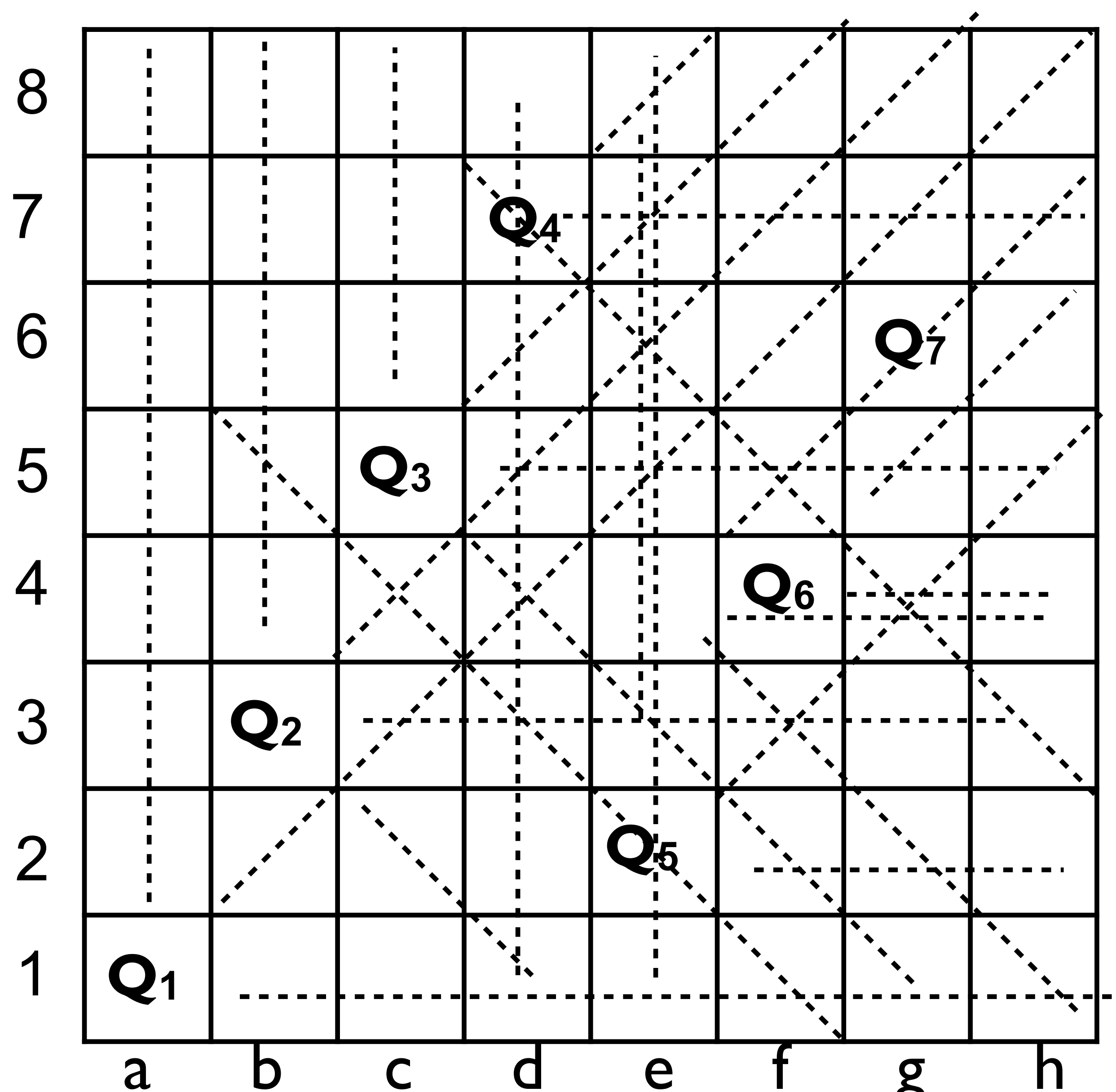
Finding a way in Maze



Overview of Backtracking

- Backtracking
  - Start from some solution.
  - Keep exploring for next part of solution
  - When exploration of solution stops (not possible to proceed further)
    - Resume back from the last point where decision was made to explore the current path.
    - Explore with the next path.

8-Queens Problem



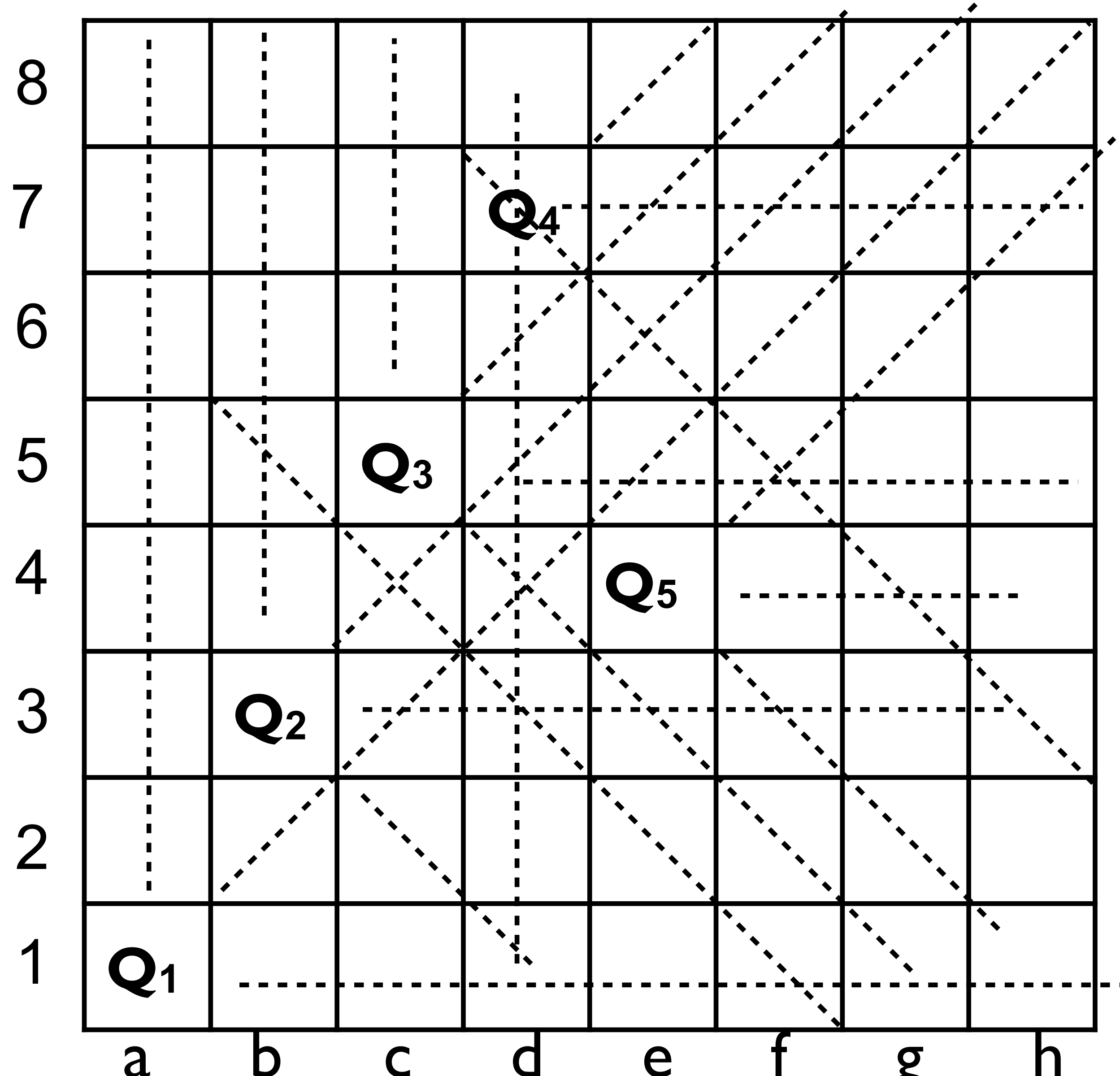
Q_8 can't be placed.

Backtrack to Q_7 which can't be placed too

Backtrack to Q_6 which can't be placed too

Backtrack to Q_5 which can be placed at e4

8-Queens Problem



Q_5 is placed at e4

Q_6 can't be placed

Backtrack to Q_5

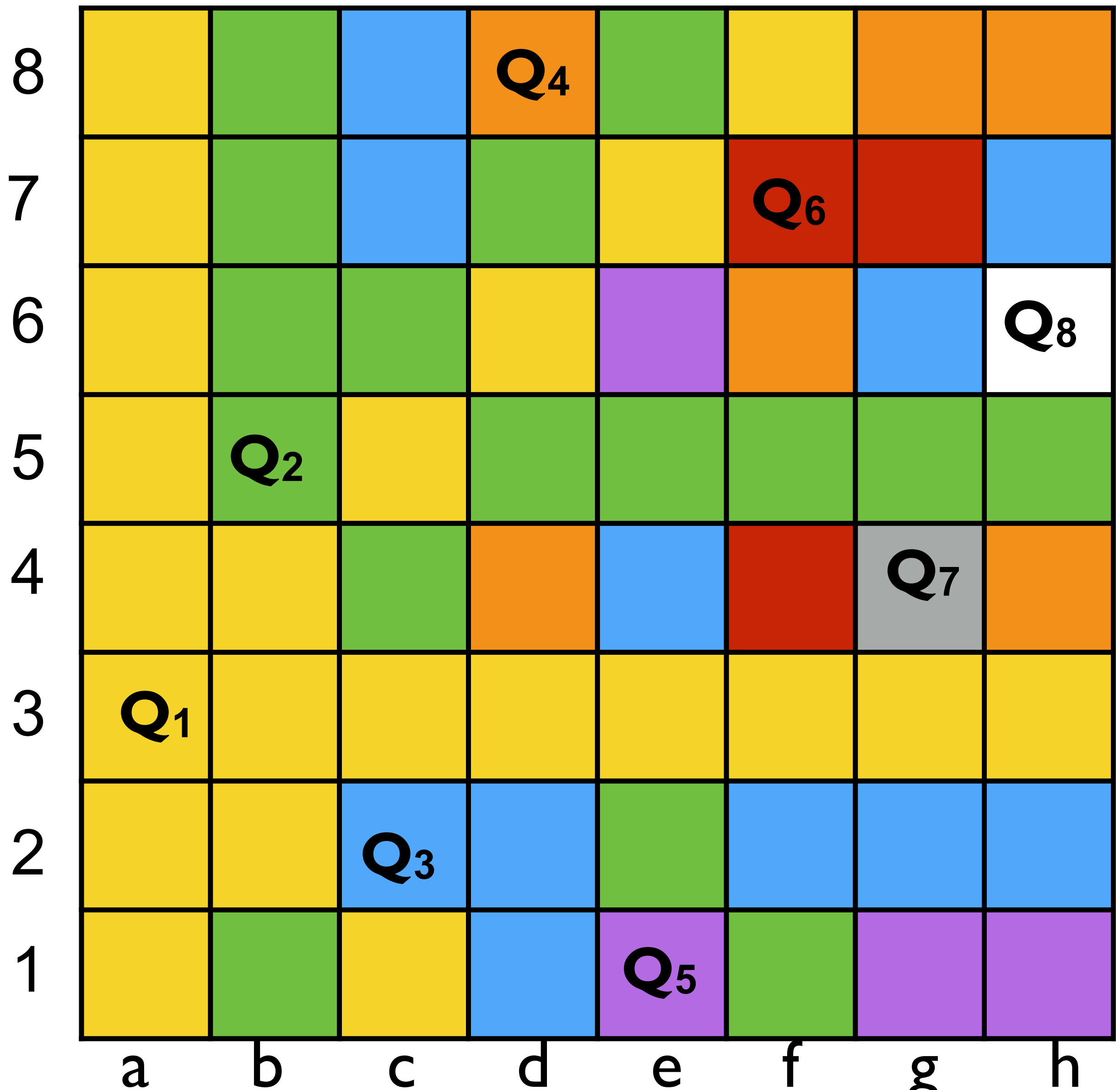
Q_5 can't be placed

Backtrack to Q_4

Q_4 is moved to d8

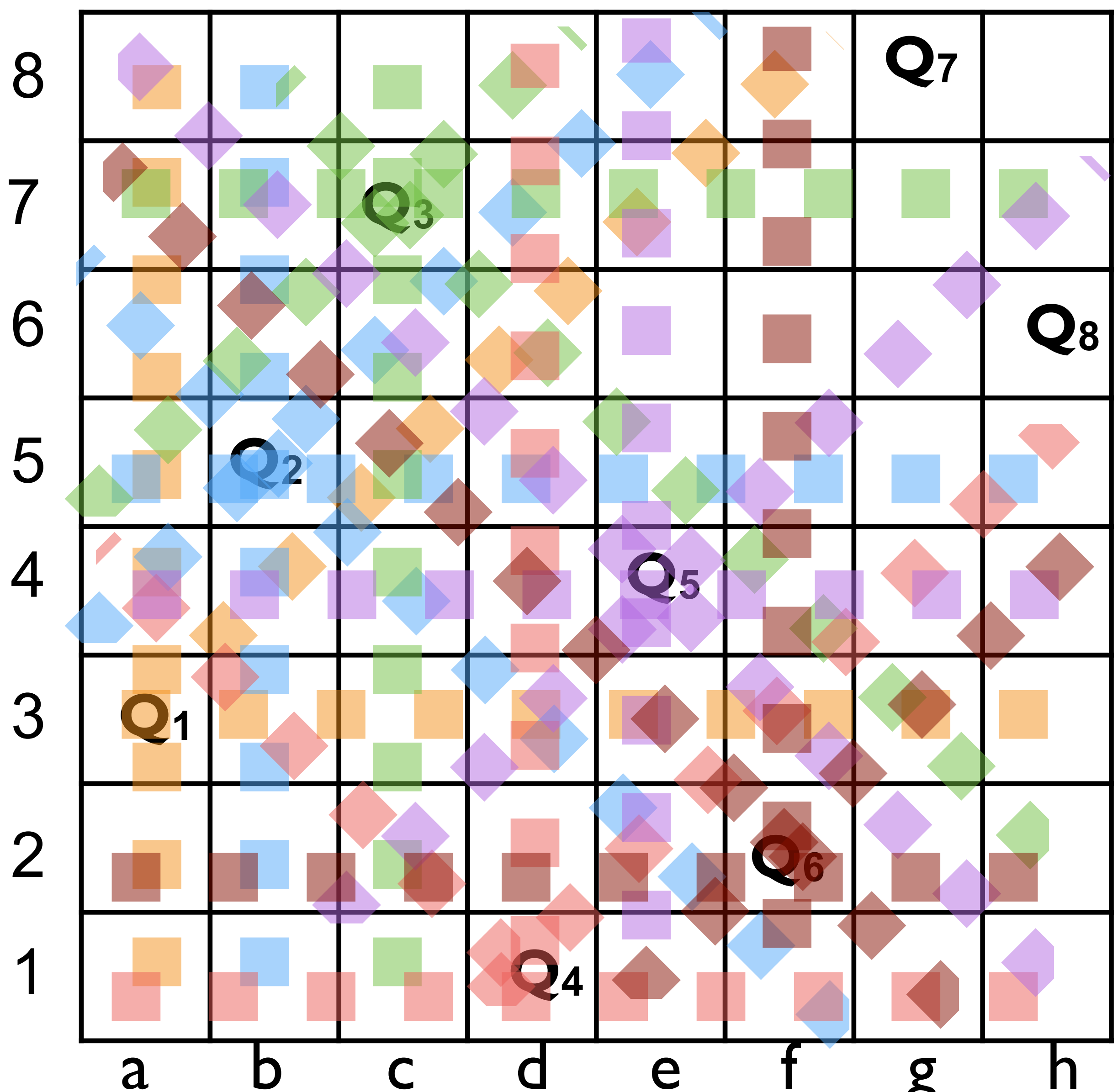
Continue in this way. Find the positions where 8 queens can be placed.

8-Queens Problem



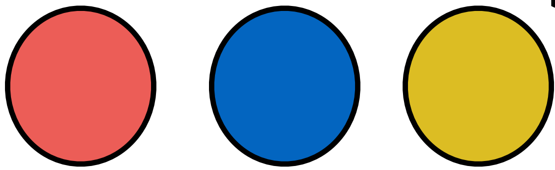
Continuing in this way. positions of 8 queens.

8-Queens Problem: Soln 2

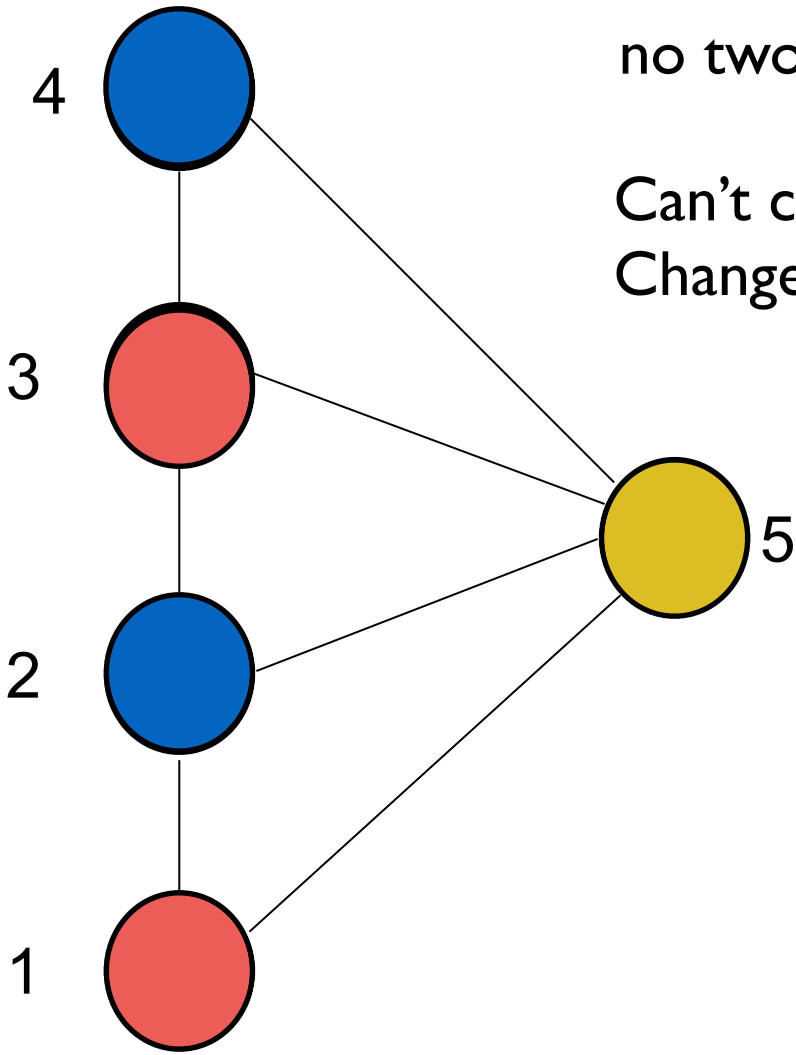


Continuing further another solution for 8 queens problem

3-Color Problem



Consider three colors are : Red, Blue, Yellow
Color the nodes with these 3 colors such that no two adjacent nodes have same color.



Can't color node 5, so backtrack
Change 4 to Blue

Can't color node 5, so backtrack
Can't change 4 to Yellow
Backtrack to 3

Solution: 1-R, 2-B, 3-R, 4-B, 5-Y

Are there other solutions?

Sum of Subset Problem

- Given a set S of numbers and a value m,
  - Find all subsets $S_i \subseteq S$ so that their sum of elements in S_i equals m.
  - An element in a subset is to be considered only once.
- Example

  $S=\{11,13,24,7\}$, and $m=31$
  - Possible subsets are

    $S_1 = \{11,13,7\}$

    $S_2 = \{24,7\}$

Backtracking: General Method

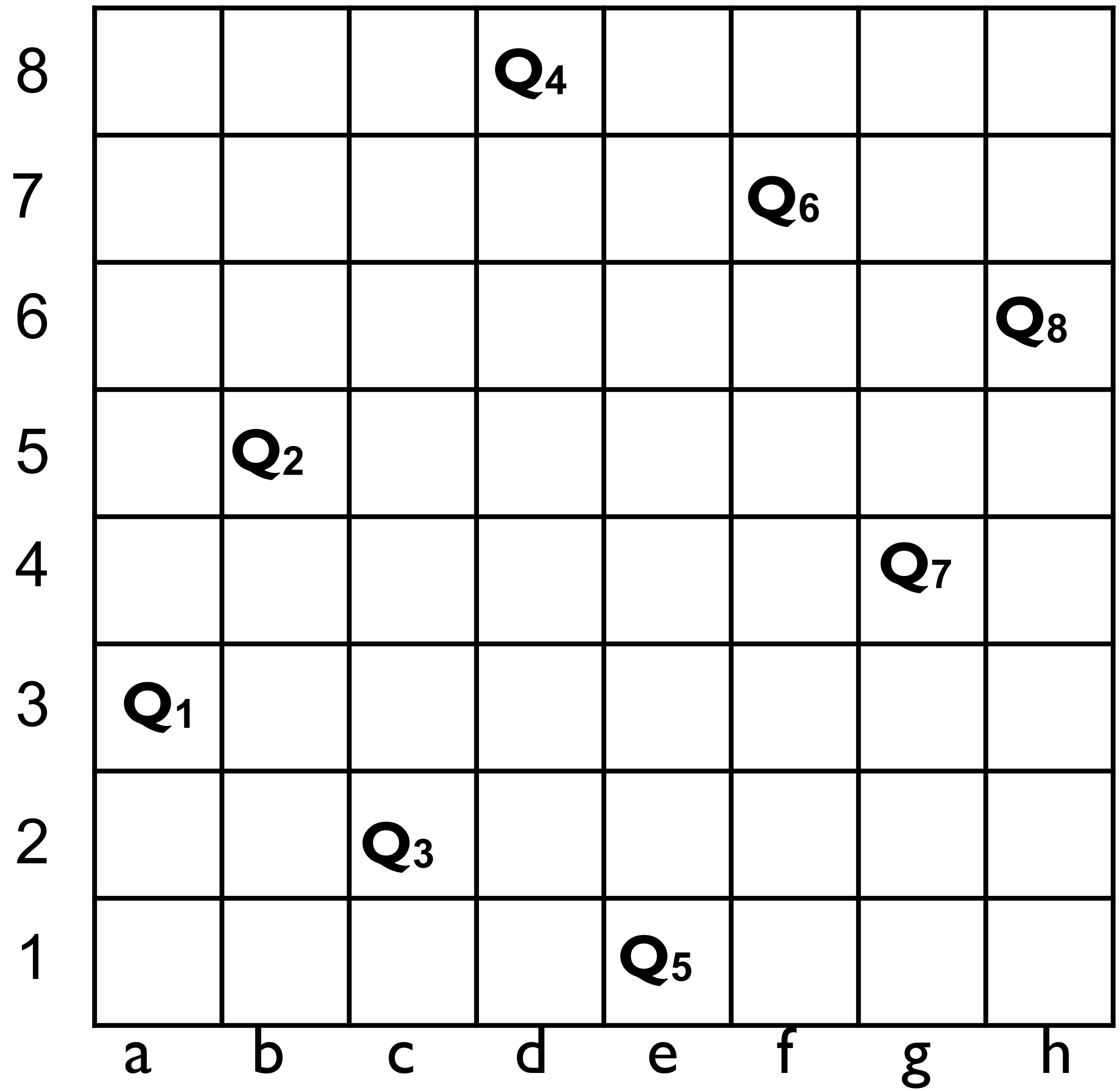
- General solution is an n-tuple (x_1, ..., x_n), where
  - x_i is chosen from some finite set S_i.
  - While choosing x_i, it has to follow some constraints
    - or meet a criterion function P(x_1, ..., x_n)
- Suppose, the size of each set S_i is m_i
- Then, total number of possible tuples are $M=m_1*m_2*...*m_n$
- Identify those tuples that satisfies the contraints i.e. Criterion function.
- Backtracking approach provides the answer in far fewer trials than M.

Backtracking: 8-Queens Method

- Let queens are numbered 1 thru 8, i.e. $Q_1, \ldots, Q_8$
- Each queen must be on a separate column (and row)
  – For simpliciy, let's say Q_i is placed on i^{th} column.
- Thus, solution can be represented by an 8-tuple
  $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$
  – where $x_1 \approx a, x_2 \approx b, x_3 \approx c, x_8 \approx h$
- Each queen must be on a separate row.
- Thus, each x_i can have a value from 1 to 8.
  – Thus, constraint is $x_i \in S_i = \{1,2,3,4,5,6,7,8\}$
- Solution space size before and after the constraint
  - before: 8^8, after: $8!$
- Representation for solution-1
  - $\{3,5,2,8,1,7,4,6\}$

8-Queens Problem

{3, 5, 2, 8, 1, 7, 4, 6}



Backtracking: Sum of Subsets

- Problem: $S=\{11,13,24,7\}$, and $m=31$
- Solution approach 1 :
  - Consider 4-tuple $\{x_1, x_2, x_3, x_4\}$
    - where, $x_i \in S_i=\{0,1\}$
    - Size of solution space: 2^n
  - possible solutions
    - $\{1,1,0,1\}$
    - $\{0,0,1,1\}$
- Solution approach 2
  - Solution contains the index values of elements.
  - Solution is a vector of varying dimensions
  - Possible solutions
    - $(1,2,4)$
    - $(3,4)$

Backtracking: 3-Color problem

- Problem: $G=\{V,E\}$, and 3 colors to color the graph
- Solution vector: n-tuple $(x_1, \ldots, x_n)$
  - where $x_i \in S_i=\{R,B,Y\}$
- Size of total solution space: 3^n
  - An edge reduces solution space from 3^2 to $3*2=6$
  - Any path of length k reduces the solution space from 3^{k+1} to $3*2^k$

Summary

- Overview of backtracking
- Problem examples for backtracking
  - 8-queens problems
  - Sum of subsets
  - 3-color problem
- Solution space
- Possible solution space.