

Design and Analysis of Algorithms

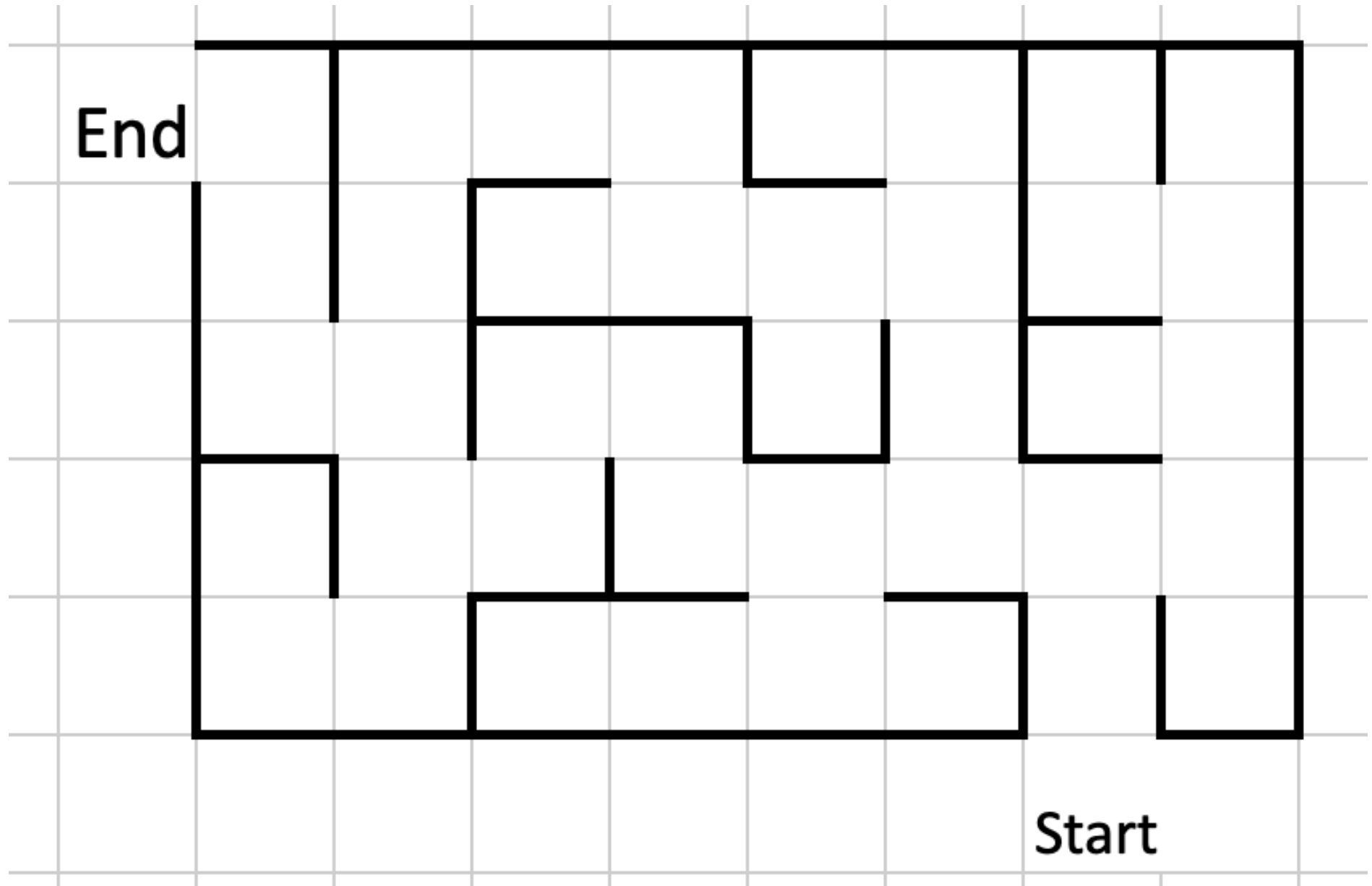
L41: Intro to Backtracking

Dr. Ram P Rustagi
Sem IV (2019-H1)
Dept of CSE, KSIT/KSSEM
rprustagi@ksit.edu.in

Resources

- Text book 2: Horowitz
 - Sec 7.1, 7.2, 7.3, 7.4, 7.5, 8.2, 11.1
- Text book 1: Levitin
 - Sec 12.1, 12.2
- RI: Introduction to Algorithms
 - Cormen et al.
- https://en.wikipedia.org/wiki/Dynamic_programming
- <https://www.codechef.com/wiki/tutorial-dynamic-programming>
- <https://www.hackerearth.com/practice/algorithms/dynamic-programming/introduction-to-dynamic-programming-I/tutorial/>

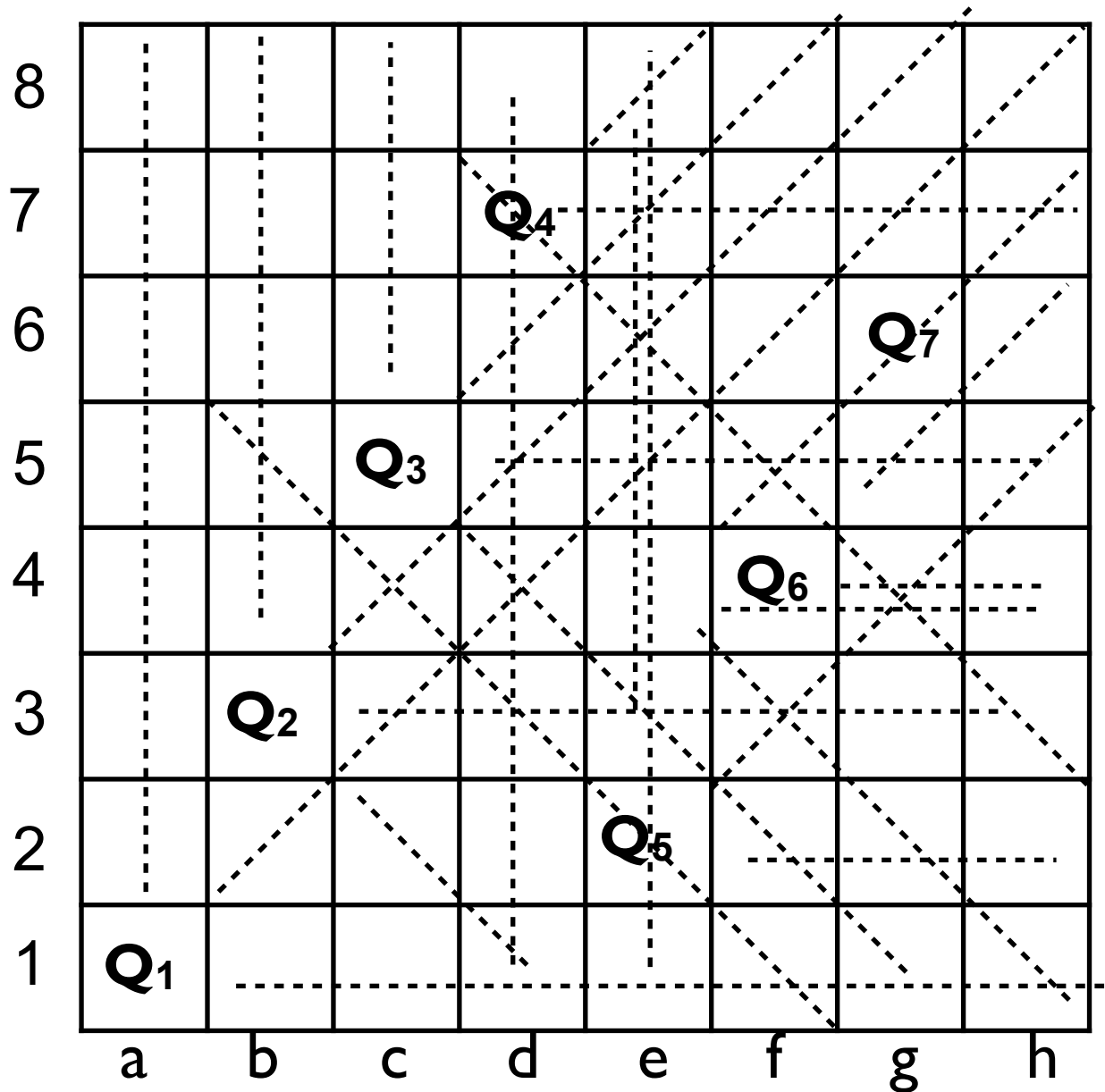
Finding a way in Maze



Overview of Backtracking

- Backtracking
 - Start from some solution.
 - Keep exploring for next part of solution
 - When exploration of solution stops (not possible to proceed further)
 - Resume back from the last point where decision was made to explore the current path.
 - Explore with the next path.

8-Queens Problem



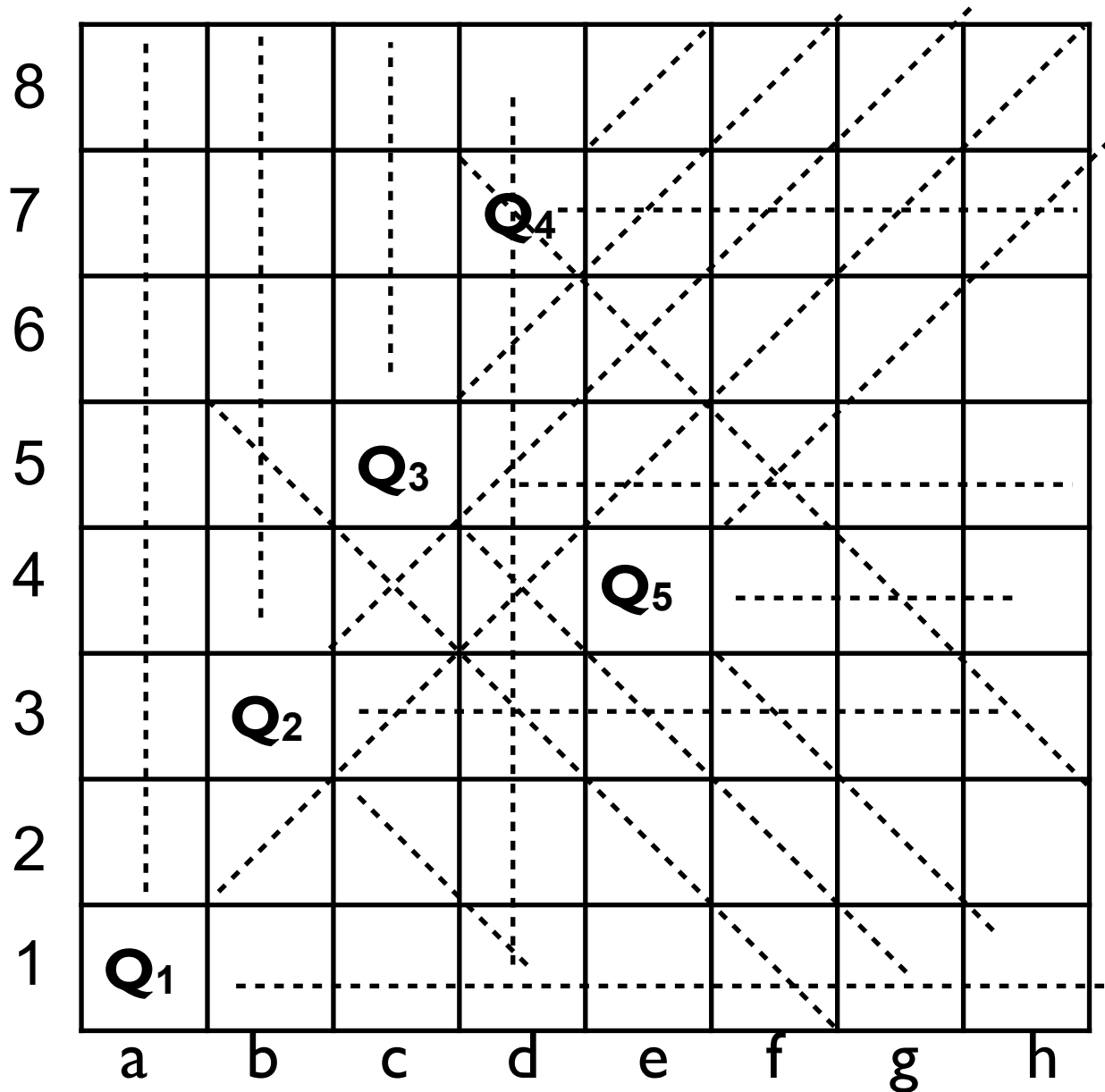
Q₈ can't be placed.

Backtrack to Q₇ which
can't be placed too

Backtrack to Q₆ which
can't be placed too

Backtrack to Q₅ which
can be placed at e4

8-Queens Problem



Q₅ is placed at e4

Q₆ can't be placed

Backtrack to Q₅

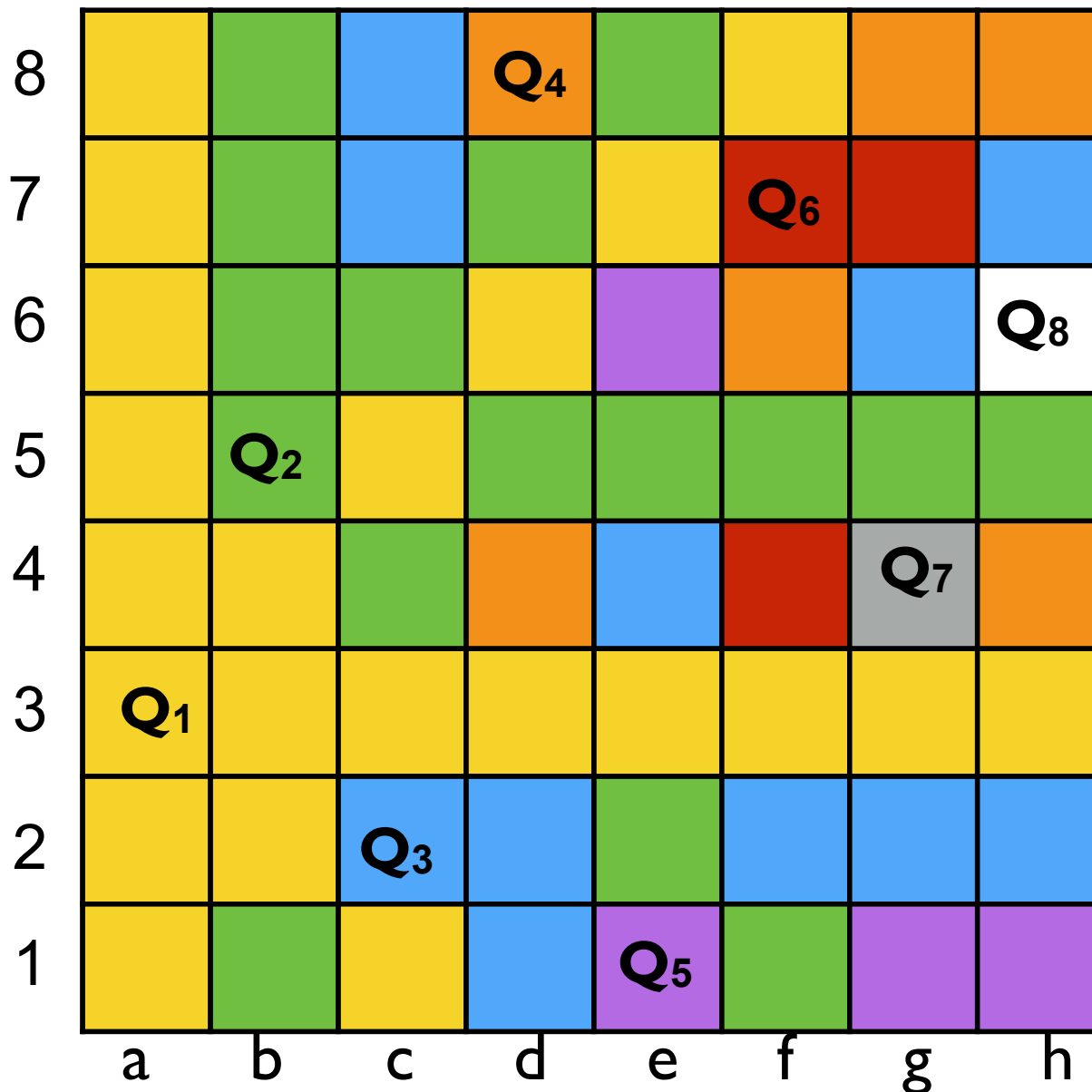
Q₅ can't be placed

Backtrack to Q₄

Q₄ is moved to d8

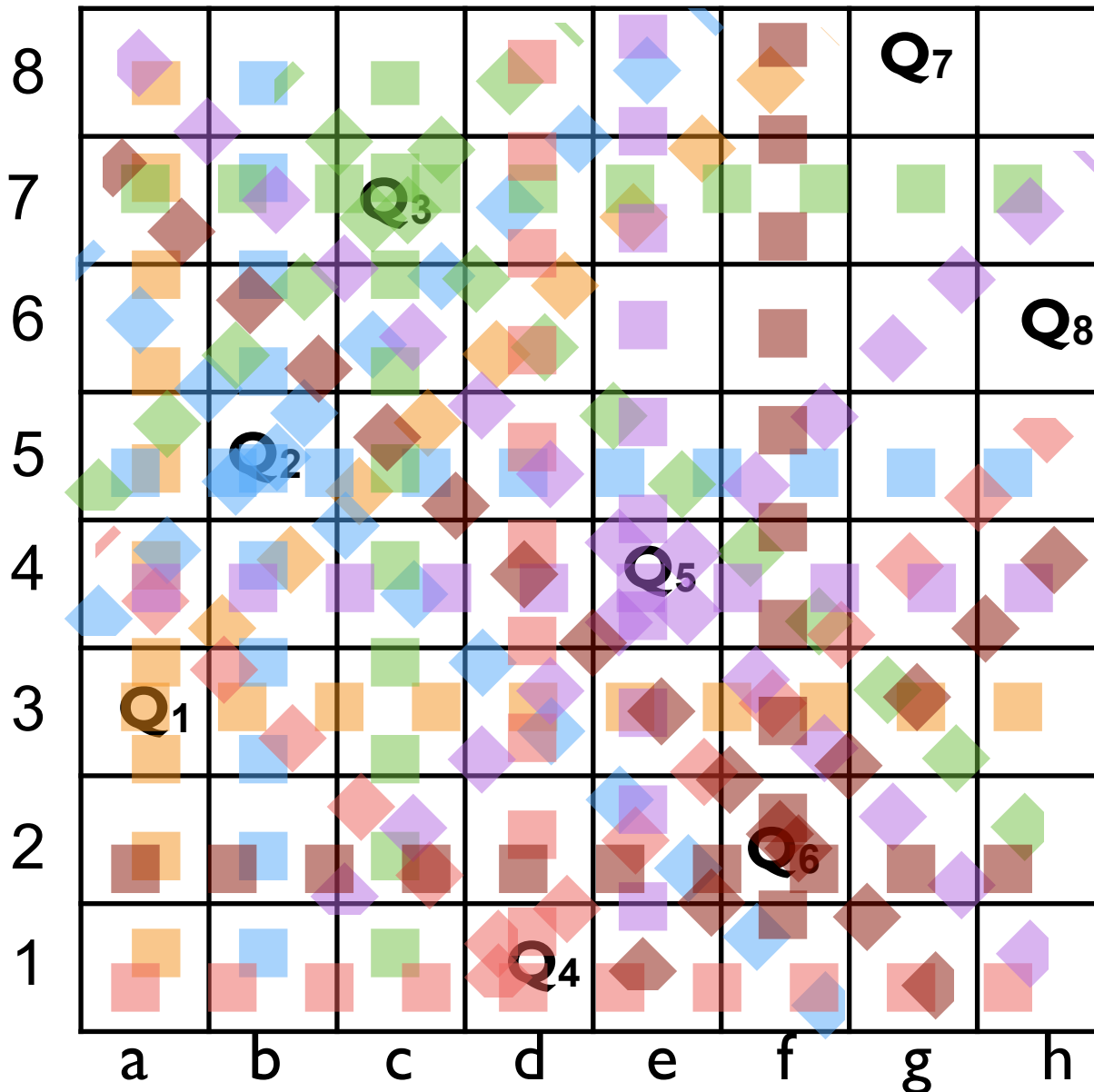
Continue in this way.
Find the positions
where 8 queens can
be placed.

8-Queens Problem



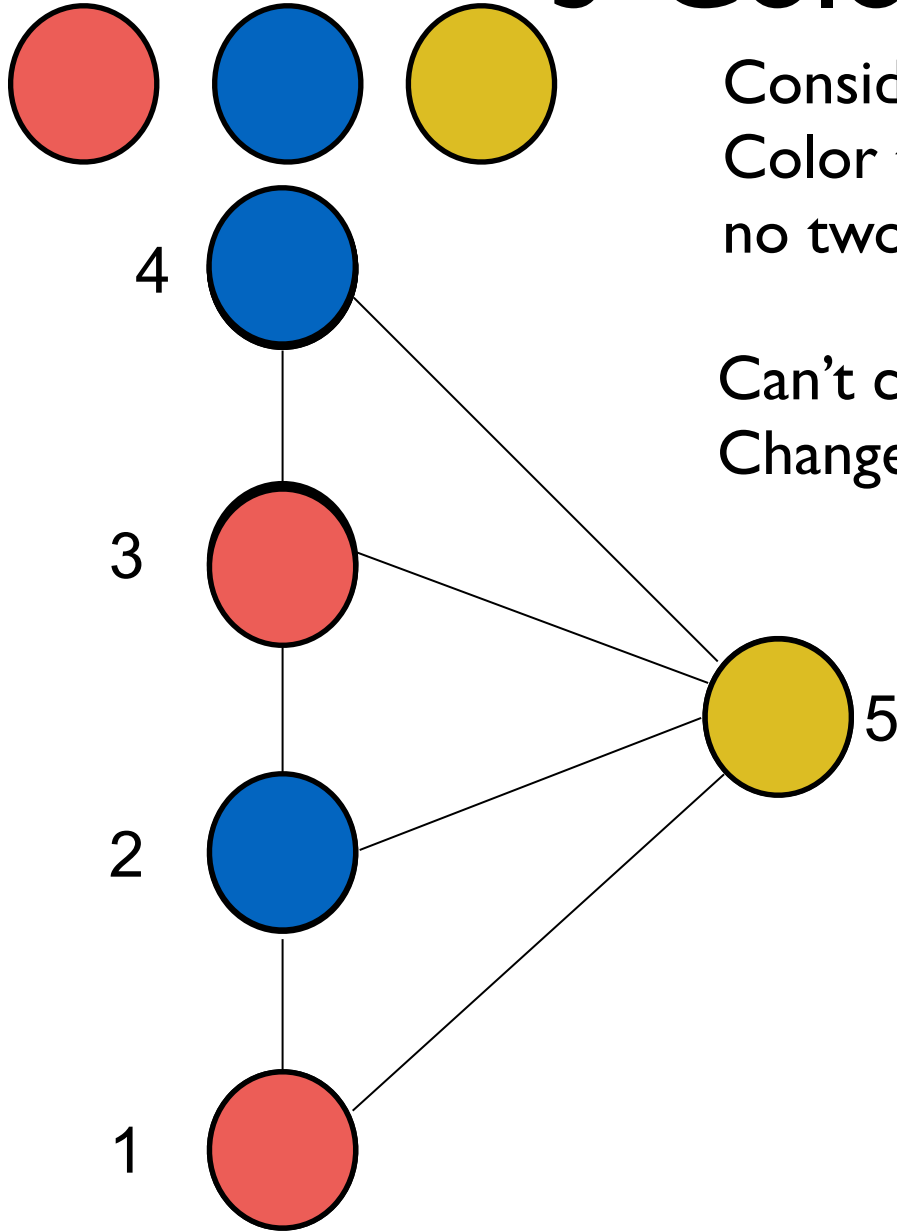
Continuing in this way.
positions of 8 queens.

8-Queens Problem: Soln 2



Continuing further
another solution for
8 queens problem

3-Color Problem



Consider three colors are : Red, Blue, Yellow
Color the nodes with these 3 colors such that
no two adjacent nodes have same color.

Can't color node 5, so backtrack
Change 4 to Blue

Can't color node 5, so backtrack
Can't change 4 to Yellow
Backtrack to 3

Solution: 1-R, 2-B, 3-R, 4-B, 5-Y

Are there other solutions?

Sum of Subset Problem

- Given a set S of numbers and a value m ,
 - Find all subsets $S_i \subseteq S$ so that their sum of elements in S_i equals m .
 - An element in a subset is to be considered only once.
- Example
 - $S = \{11, 13, 24, 7\}$, and $m = 31$
 - Possible subsets are
 - $S_1 = \{11, 23, 7\}$
 - $S_2 = \{24, 7\}$

Backtracking: General Method

- General solution is an n -tuple (x_1, \dots, x_n) , where
 - x_i is chosen from some finite set S_i .
 - While choosing x_i , it has to follow some constraints
 - or meet a criterion function $P(x_1, \dots, x_n)$
- Suppose, the size of each set S_i is m_i
- Then, total number of possible tuples are
$$M = m_1 * m_2 * \dots * m_n$$
- Identify those tuples that satisfies the constraints i.e. Criterion function.
- Backtracking approach provides the answer in far fewer trials than M .

Backtracking: 8-Queens Method

- Let queens are numbered 1 thru 8, i.e. Q_1, \dots, Q_8
- Each queen must be on a separate column (and row)
 - For simplicity, let's say Q_i is placed on i^{th} column.
- Thus, solution can be represented by an 8-tuple
$$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$
 - where $x_1 \approx a, x_2 \approx b, x_3 \approx c, x_8 \approx h$
- Each queen must be on a separate row.
- Thus, each x_i can have a value from 1 to 8.
 - Thus, constraint is $x_i \in S_i = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- Solution space size before and after the constraint
 - before: 8^8 , after: $8!$
- Representation for solution-1
 - $\{3, 5, 2, 8, 1, 7, 4, 6\}$

Backtracking: Sum of Subsets

- Problem: $S = \{11, 13, 24, 7\}$, and $m=31$
- Solution approach 1 :
 - Consider 4-tuple $\{x_1, x_2, x_3, x_4\}$
 - where, $x_i \in S_i = \{0, 1\}$
 - Size of solution space: 2^n
 - possible solutions
 - $\{1, 1, 0, 1\}$
 - $\{0, 0, 1, 1\}$
- Solution approach 2
 - Solution contains the index values of elements.
 - Solution is a vector of varying dimensions
 - Possible solutions
 - $(1, 2, 4)$
 - $(3, 4)$

Backtracking: 3-Color problem

- Problem: $G = \{V, E\}$, and 3 colors to color the graph
- Solution vector: n -tuple (x_1, \dots, x_n)
 - where $x_i \in S_i = \{R, B, Y\}$
- Size of total solution space: 3^n
 - An edge reduces solution space from 3^2 to $3 * 2 = 6$
 - Any path of length k reduces the solution space from 3^{k+1} to $3 * 2^k$

Summary

- Overview of backtracking
- Problem examples for backtracking
 - 8-queens problems
 - Sum of subsets
 - 3-color problem
- Solution space
- Possible solution space.