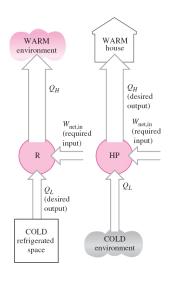
## FIS1523 - Termodinámica

#### Griselda Garcia

Miércoles 15 de Junio

### Ciclos de refrigeración

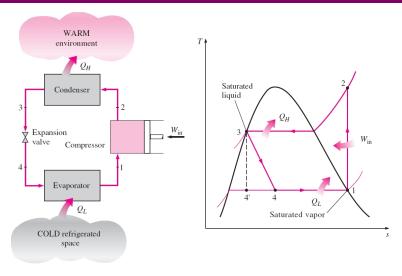


$$Q_{T_{baja} \rightarrow T_{alta}} \iff \text{refrigeradores}$$

Refrigerador y bomba de calor son esencialmente iguales pero que difieren en sus objetivos.

Coeficiente de funcionamiento (CDF)

$$CDF_{refrigerador} = CDF_R = rac{Q_L}{W_{entra}}$$
 $CDF_{bomba\,de\,calor} = CDF_{BC} = rac{Q_H}{W_{entra}}$ 
 $CDF_R > 1$   $CDF_{BC} > 1$ 
 $CDF_{BC} = CDF_R + 1$ 



refrigeradores, sistemas de aire acondicionado y bombas de calor

El ciclo está formado por cuatro procesos reversibles,

- $1 \rightarrow 2$ : compresión isentrópica en un compresor
- $2 \rightarrow 3$ : liberación de calor a P constante en un condensador
- $3 \rightarrow 4$ : estrangulamiento en un dispositivo de expansión
- $4 \rightarrow 1$ : absorción de calor a *P* constante en un evaporador.

#### El ciclo no es reversible debido al proceso de estrangulamiento

Los cuatro componentes asociados al ciclo son dispositivos de flujo estacionario, en los cuales los cambios de energías cinética y potencial pueden despreciarse.

El balance de energía en cada uno se reduce a

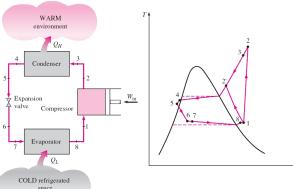
$$(q_{entra} - q_{sale}) + (w_{entra} - w_{sale}) = h_{entrada} - h_{salida}$$

El condensador y el evaporador no involucran trabajo.

El condensador se considera adiabático.

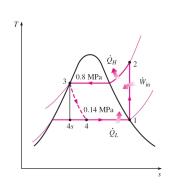
$$CDF_{R} = rac{q_{L}}{w_{entra}} = rac{h_{1} - h_{4}}{h_{2} - h_{1}}$$
  $CDF_{BC} = rac{q_{H}}{w_{entra}} = rac{h_{2} - h_{3}}{h_{2} - h_{1}}$ 

 $h_1 = h_{g@P_1}$  y  $h_3 = h_{f@P_3}$  para el caso ideal



El ciclo real difiere del ideal debido a la fricción del fluido de trabajo y las transf. de calor hacia o desde el entorno

En un refrigerador se usa R-134a como fluido de trabajo y opera en un ciclo ideal de refrigeración por compresión de vapor entre 0.14 MPa y 0.8 MPa. Si el flujo de masa del refrigerante es 0.05 kg/s, determinar la tasa de eliminación de calor del espacio refrigerado, la entrada de potencia al compresor, la tasa de eliminación de calor al ambiente y el coeficiente de funcionamiento del refrigerador.



 $P_1$  = 0.14 MPa  $h_1 = h_{g@0,14MPa}$  = 239.16 kJ/kg  $s_1 = s_{g@0,14MPa}$  = 0.94456 kJ/kg K  $P_2$  = 0.8 MPa y  $s_2 = s_1$   $h_2$  = 275.39 kJ/kg  $P_3$  = 0.8 MPa  $h_3 = h_{f@0,8MPa}$  = 95.47 kJ/kg

Estrangulamiento,  $h_4 \approx h_3 = 95.47 \text{ kJ/kg}$ 

Tasa de remoción de calor del espacio refrigerado

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.05 \times (239.16 - 95.47) = 7.18 \text{ kW}$$

Potencia entregada al compresor

$$\dot{W}_{compresor} = \dot{m}(h_2 - h_1) = 0.05 \times (275.39 - 239.16) = 1.81 \text{ kW}$$

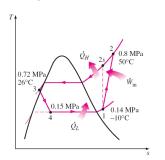
Tasa de eliminación de calor del refrigerante

$$\dot{Q}_{H} = \dot{m}(h_{2} - h_{3}) = 0.05 \times (275.39 - 95.47) = 9.0 \text{ kW} = \dot{Q}_{L} + \dot{W}_{compresor}$$

Coeficiente de funcionamiento del refrigerador

$$CDF_R = \frac{Q_L}{\dot{W}_{compresor}} = \frac{7,18}{1,81} = 3,97$$

Al compresor de un refrigerador entra R-134a como vapor sobrecalentado a 0.14 MPa y -10°C a una tasa de 0.05 kg/s y sale a 0.8 MPa y 50°C. El R-134a se enfría en el condensador a 26°C y 0.72 MPa y se estrangula a 0.15 MPa. Descartar toda transferencia de calor y caída de presión en las líneas de conexión entre componentes. Determinar la tasa de remoción de calor del espacio refrigerado, la entrada de potencia al compresor, la eficiencia isentrópica del compresor y el coeficiente de funcionamiento del refrigerador.



$$P_1 = 0.14 \text{ MPa y } T_1 = -10^{\circ}\text{C}$$
  
 $h_1 = 246.36 \text{ kJ/kg}$ 

$$P_2 = 0.8 \text{ MPa y } T_2 = 50^{\circ}\text{C}$$
  
 $h_2 = 286.69 \text{ kJ/kg}$ 

$$P_3 = 0.72 \text{ MPa y } T_3 = 26^{\circ}\text{C}$$
  
 $h_3 \approx h_{f@26^{\circ}C} = 87.83 \text{ kJ/kg}$ 

Estrangulamiento,  $h_4 \approx h_3 = 87.83 \text{ kJ/kg}$ 

Tasa de remoción de calor

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.05 \times (246.36 - 87.83) = 7.93 \text{ kW}$$

Potencia entregada al compresor

$$\dot{W}_{compresor} = \dot{m}(h_2 - h_1) = 0.05 \times (286.69 - 246.36) = 2.02 \text{ kW}$$

Eficiencia isentrópica del compresor

$$\eta_{compresor} = \frac{h_{2s} - h_1}{h_2 - h_1} = 0.939$$

 $h_{2s}$  = 284.21 kJ/kg para  $P_{2s}$  = 0.8 MPa,  $s_{2s}$  =  $s_1$  = 0.9724 kJ/kg K.

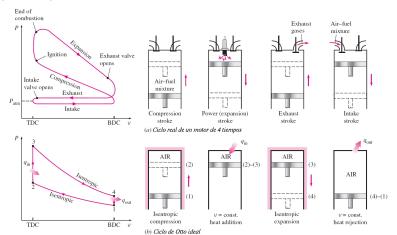
Coeficiente de funcionamiento del refrigerador

$$CDF_{R} = \frac{\dot{Q}_{L}}{\dot{W}_{compresor}} = \frac{7.93}{2.02} = 3.93$$

#### Ciclo de Otto

Es el ciclo de gas ideal correspondiente a un motor a gasolina o de combustión interna

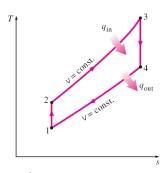
El primer motor fué construído por N. Otto en 1876 usando el ciclo propuesto por Beau de Rochas en 1862.



#### Ciclo de Otto

El ciclo ideal Otto consiste de 4 procesos reversibles:

- compresión isoentrópica
- adición de calor a volumen constante
- expansión isoentrópica
- liberación de calor a volumen constante



El ciclo Otto se ejecuta en un sistema cerrado, prácticamente sin cambios de energía cinética y potencial.

$$q_{entra} = u_3 - u_2 = c_v (T_3 - T_2)$$
  $q_{sale} = u_4 - u_1 = c_v (T_4 - T_1)$ 

$$\eta_{Otto} = \frac{w_{neto}}{q_{entra}} = 1 - \frac{q_{sale}}{q_{entra}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1 \left( T_4 / T_1 - 1 \right)}{T_2 \left( T_3 / T_2 - 1 \right)}$$

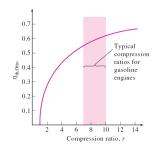
#### Ciclo de Otto

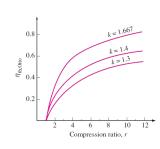
Considerando que los procesos 1  $\rightarrow$  2 y 3  $\rightarrow$  4 son isoentrópicos y que  $v_2=v_3$  y  $v_4=v_1$  resulta que

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$

$$\eta_{Otto} = 1 - \frac{T_1 (T_4/T_1 - 1)}{T_2 (T_3/T_2 - 1)} = 1 - \frac{1}{r^{k-1}}$$
  $r = \frac{v_{max}}{v_{min}} = \frac{v_1}{v_2}$ 

r es la razón de compresión y  $k=c_p/c_v$  es el índice adiabático.





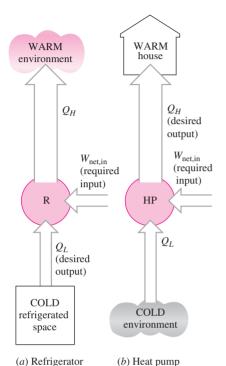
 $\eta_{\mathit{Otto}} pprox 25 - 30\,\%$ 

### Ciclo Otto - Ejemplos

- La razón de compresión en un ciclo de Otto estándar es 10. Al comienzo del proceso de compresión la presión es 0.1 MPa y la temperatura 15°C. El calor transferido al aire en cada ciclo es1800 kJ/kg. Determine la presión y temperatura al final de cada proceso en el ciclo, la eficiencia térmica, la presión media efectiva y la potencia desarrollada para 400 rpm.
- Un ciclo Otto ideal tiene una relación de compresión de 9.2 y usa opera con aire como fluido de trabajo. Al comiendo de la compresión, el aire está a 98 kPa y 27°. La presión es duplicada durante el proceso de agregar calor a volumen constante. Considerando la variación con la temperatura de los calores específicos, determinar la cantidad de calor transferido al aire, el trabajo neto generado, la eficiencia térmica y la presión media efectiva para el ciclo.



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#### FIGURE 11-1

The objective of a refrigerator is to remove heat  $(Q_L)$  from the cold medium; the objective of a heat pump is to supply heat  $(Q_H)$  to a warm medium.

#### 11-1 - REFRIGERATORS AND HEAT PUMPS

We all know from experience that heat flows in the direction of decreasing temperature, that is, from high-temperature regions to low-temperature ones. This heat-transfer process occurs in nature without requiring any devices. The reverse process, however, cannot occur by itself. The transfer of heat from a low-temperature region to a high-temperature one requires special devices called **refrigerators**.

Refrigerators are cyclic devices, and the working fluids used in the refrigeration cycles are called **refrigerants.** A refrigerator is shown schematically in Fig. 11–1a. Here  $Q_L$  is the magnitude of the heat removed from the refrigerated space at temperature  $T_L, Q_H$  is the magnitude of the heat rejected to the warm space at temperature  $T_H$ , and  $W_{\rm net,in}$  is the net work input to the refrigerator. As discussed in Chap. 6,  $Q_L$  and  $Q_H$  represent magnitudes and thus are positive quantities.

Another device that transfers heat from a low-temperature medium to a high-temperature one is the **heat pump.** Refrigerators and heat pumps are essentially the same devices; they differ in their objectives only. The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. Discharging this heat to a higher-temperature medium is merely a necessary part of the operation, not the purpose. The objective of a heat pump, however, is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low-temperature source, such as well water or cold outside air in winter, and supplying this heat to a warmer medium such as a house (Fig. 11-1b).

The performance of refrigerators and heat pumps is expressed in terms of the **coefficient of performance** (COP), defined as

$$COP_{R} = \frac{Desired output}{Required input} = \frac{Cooling effect}{Work input} = \frac{Q_{L}}{W_{net,in}}$$
(11–1)

$$COP_{HP} = \frac{Desired output}{Required input} = \frac{Heating effect}{Work input} = \frac{Q_H}{W_{net,in}}$$
(11-2)

These relations can also be expressed in the rate form by replacing the quantities  $Q_L$ ,  $Q_H$ , and  $W_{\text{net,in}}$  by  $\dot{Q}_L$ ,  $\dot{Q}_H$ , and  $\dot{W}_{\text{net,in}}$ , respectively. Notice that both  $\text{COP}_R$  and  $\text{COP}_{HP}$  can be greater than 1. A comparison of Eqs. 11–1 and 11–2 reveals that

$$COP_{HP} = COP_{R} + 1 \tag{11-3}$$

for fixed values of  $Q_L$  and  $Q_H$ . This relation implies that  $COP_{HP} > 1$  since  $COP_R$  is a positive quantity. That is, a heat pump functions, at worst, as a resistance heater, supplying as much energy to the house as it consumes. In reality, however, part of  $Q_H$  is lost to the outside air through piping and other devices, and  $COP_{HP}$  may drop below unity when the outside air temperature is too low. When this happens, the system normally switches to the fuel (natural gas, propane, oil, etc.) or resistance-heating mode.

The *cooling capacity* of a refrigeration system—that is, the rate of heat removal from the refrigerated space—is often expressed in terms of **tons of refrigeration.** The capacity of a refrigeration system that can freeze 1 ton (2000 lbm) of liquid water at 0°C (32°F) into ice at 0°C in 24 h is said to be

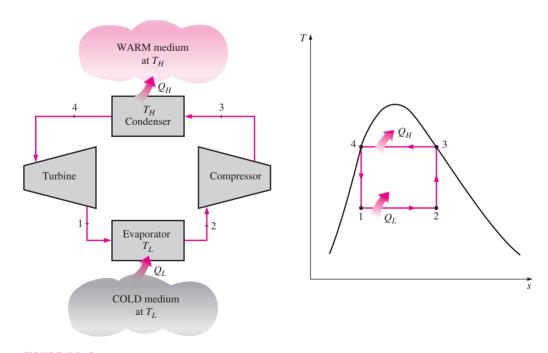
1 ton. One ton of refrigeration is equivalent to 211 kJ/min or 200 Btu/min. The cooling load of a typical 200-m² residence is in the 3-ton (10-kW) range.

#### 11-2 - THE REVERSED CARNOT CYCLE

Recall from Chap. 6 that the Carnot cycle is a totally reversible cycle that consists of two reversible isothermal and two isentropic processes. It has the maximum thermal efficiency for given temperature limits, and it serves as a standard against which actual power cycles can be compared.

Since it is a reversible cycle, all four processes that comprise the Carnot cycle can be reversed. Reversing the cycle does also reverse the directions of any heat and work interactions. The result is a cycle that operates in the counterclockwise direction on a *T-s* diagram, which is called the **reversed Carnot cycle.** A refrigerator or heat pump that operates on the reversed Carnot cycle is called a **Carnot refrigerator** or a **Carnot heat pump.** 

Consider a reversed Carnot cycle executed within the saturation dome of a refrigerant, as shown in Fig. 11–2. The refrigerant absorbs heat isothermally from a low-temperature source at  $T_L$  in the amount of  $Q_L$  (process 1-2), is compressed isentropically to state 3 (temperature rises to  $T_H$ ), rejects heat isothermally to a high-temperature sink at  $T_H$  in the amount of  $Q_H$  (process 3-4), and expands isentropically to state 1 (temperature drops to  $T_L$ ). The refrigerant changes from a saturated vapor state to a saturated liquid state in the condenser during process 3-4.



**FIGURE 11–2** Schematic of a Carnot refrigerator and *T-s* diagram of the reversed Carnot cycle.

The coefficients of performance of Carnot refrigerators and heat pumps are expressed in terms of temperatures as

$$COP_{R,Carnot} = \frac{1}{T_H/T_L - 1}$$
 (11-4)

and

$$COP_{HP,Carnot} = \frac{1}{1 - T_L/T_H}$$
 (11-5)

Notice that both COPs increase as the difference between the two temperatures decreases, that is, as  $T_L$  rises or  $T_H$  falls.

The reversed Carnot cycle is the *most efficient* refrigeration cycle operating between two specified temperature levels. Therefore, it is natural to look at it first as a prospective ideal cycle for refrigerators and heat pumps. If we could, we certainly would adapt it as the ideal cycle. As explained below, however, the reversed Carnot cycle is not a suitable model for refrigeration cycles.

The two isothermal heat transfer processes are not difficult to achieve in practice since maintaining a constant pressure automatically fixes the temperature of a two-phase mixture at the saturation value. Therefore, processes 1-2 and 3-4 can be approached closely in actual evaporators and condensers. However, processes 2-3 and 4-1 cannot be approximated closely in practice. This is because process 2-3 involves the compression of a liquid–vapor mixture, which requires a compressor that will handle two phases, and process 4-1 involves the expansion of high-moisture-content refrigerant in a turbine.

It seems as if these problems could be eliminated by executing the reversed Carnot cycle outside the saturation region. But in this case we have difficulty in maintaining isothermal conditions during the heat-absorption and heat-rejection processes. Therefore, we conclude that the reversed Carnot cycle cannot be approximated in actual devices and is not a realistic model for refrigeration cycles. However, the reversed Carnot cycle can serve as a standard against which actual refrigeration cycles are compared.



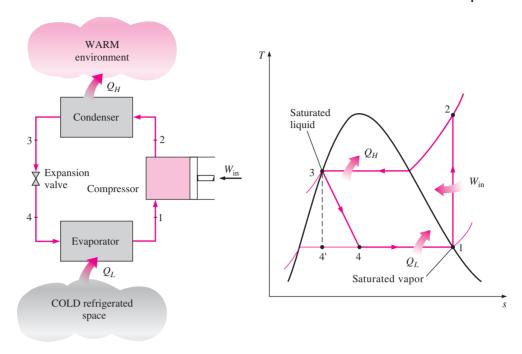
SEE TUTORIAL CH. 11, SEC. 2 ON THE DVD.

# 11–3 • THE IDEAL VAPOR-COMPRESSION REFRIGERATION CYCLE

Many of the impracticalities associated with the reversed Carnot cycle can be eliminated by vaporizing the refrigerant completely before it is compressed and by replacing the turbine with a throttling device, such as an expansion valve or capillary tube. The cycle that results is called the **ideal vapor-compression refrigeration cycle**, and it is shown schematically and on a *T-s* diagram in Fig. 11–3. The vapor-compression refrigeration cycle is the most widely used cycle for refrigerators, air-conditioning systems, and heat pumps. It consists of four processes:

- 1-2 Isentropic compression in a compressor
- 2-3 Constant-pressure heat rejection in a condenser
- 3-4 Throttling in an expansion device
- 4-1 Constant-pressure heat absorption in an evaporator

In an ideal vapor-compression refrigeration cycle, the refrigerant enters the compressor at state 1 as saturated vapor and is compressed isentropically to the condenser pressure. The temperature of the refrigerant increases during



**FIGURE 11–3** Schematic and *T-s* diagram for the ideal vapor-compression refrigeration cycle.

this isentropic compression process to well above the temperature of the surrounding medium. The refrigerant then enters the condenser as superheated vapor at state 2 and leaves as saturated liquid at state 3 as a result of heat rejection to the surroundings. The temperature of the refrigerant at this state is still above the temperature of the surroundings.

The saturated liquid refrigerant at state 3 is throttled to the evaporator pressure by passing it through an expansion valve or capillary tube. The temperature of the refrigerant drops below the temperature of the refrigerated space during this process. The refrigerant enters the evaporator at state 4 as a low-quality saturated mixture, and it completely evaporates by absorbing heat from the refrigerated space. The refrigerant leaves the evaporator as saturated vapor and reenters the compressor, completing the cycle.

In a household refrigerator, the tubes in the freezer compartment where heat is absorbed by the refrigerant serves as the evaporator. The coils behind the refrigerator, where heat is dissipated to the kitchen air, serve as the condenser (Fig. 11–4).

Remember that the area under the process curve on a *T-s* diagram represents the heat transfer for internally reversible processes. The area under the process curve 4-1 represents the heat absorbed by the refrigerant in the evaporator, and the area under the process curve 2-3 represents the heat rejected in the condenser. A rule of thumb is that the *COP improves by 2 to 4 percent for each* °*C the evaporating temperature is raised or the condensing temperature is lowered*.

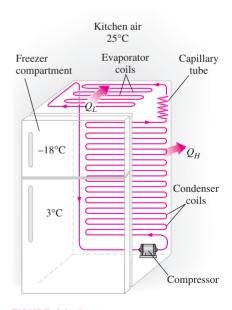


FIGURE 11–4
An ordinary household refrigerator.

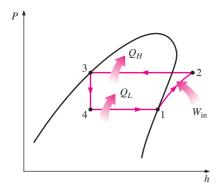


FIGURE 11–5
The *P-h* diagram of an ideal vapor-compression refrigeration cycle.

Another diagram frequently used in the analysis of vapor-compression refrigeration cycles is the *P-h* diagram, as shown in Fig. 11–5. On this diagram, three of the four processes appear as straight lines, and the heat transfer in the condenser and the evaporator is proportional to the lengths of the corresponding process curves.

Notice that unlike the ideal cycles discussed before, the ideal vapor-compression refrigeration cycle is not an internally reversible cycle since it involves an irreversible (throttling) process. This process is maintained in the cycle to make it a more realistic model for the actual vapor-compression refrigeration cycle. If the throttling device were replaced by an isentropic turbine, the refrigerant would enter the evaporator at state 4' instead of state 4. As a result, the refrigeration capacity would increase (by the area under process curve 4'-4 in Fig. 11–3) and the net work input would decrease (by the amount of work output of the turbine). Replacing the expansion valve by a turbine is not practical, however, since the added benefits cannot justify the added cost and complexity.

All four components associated with the vapor-compression refrigeration cycle are steady-flow devices, and thus all four processes that make up the cycle can be analyzed as steady-flow processes. The kinetic and potential energy changes of the refrigerant are usually small relative to the work and heat transfer terms, and therefore they can be neglected. Then the steady-flow energy equation on a unit–mass basis reduces to

$$(q_{\rm in} - q_{\rm out}) + (w_{\rm in} - w_{\rm out}) = h_e - h_i$$
 (11-6)

The condenser and the evaporator do not involve any work, and the compressor can be approximated as adiabatic. Then the COPs of refrigerators and heat pumps operating on the vapor-compression refrigeration cycle can be expressed as

$$COP_{R} = \frac{q_{L}}{w_{\text{net,in}}} = \frac{h_{1} - h_{4}}{h_{2} - h_{1}}$$
 (11-7)

and

$$COP_{HP} = \frac{q_H}{w_{\text{net in}}} = \frac{h_2 - h_3}{h_2 - h_1}$$
 (11-8)

where  $h_1 = h_{g@P_1}$  and  $h_3 = h_{f@P_3}$  for the ideal case.

Vapor-compression refrigeration dates back to 1834 when the Englishman Jacob Perkins received a patent for a closed-cycle ice machine using ether or other volatile fluids as refrigerants. A working model of this machine was built, but it was never produced commercially. In 1850, Alexander Twining began to design and build vapor-compression ice machines using ethyl ether, which is a commercially used refrigerant in vapor-compression systems. Initially, vapor-compression refrigeration systems were large and were mainly used for ice making, brewing, and cold storage. They lacked automatic controls and were steam-engine driven. In the 1890s, electric motor-driven smaller machines equipped with automatic controls started to replace the older units, and refrigeration systems began to appear in butcher shops and households. By 1930, the continued improvements made it possible to have vapor-compression refrigeration systems that were relatively efficient, reliable, small, and inexpensive.

## EXAMPLE 11-1 The Ideal Vapor-Compression Refrigeration Cycle

A refrigerator uses refrigerant-134a as the working fluid and operates on an ideal vapor-compression refrigeration cycle between 0.14 and 0.8 MPa. If the mass flow rate of the refrigerant is 0.05 kg/s, determine (a) the rate of heat removal from the refrigerated space and the power input to the compressor, (b) the rate of heat rejection to the environment, and (c) the COP of the refrigerator.

**Solution** A refrigerator operates on an ideal vapor-compression refrigeration cycle between two specified pressure limits. The rate of refrigeration, the power input, the rate of heat rejection, and the COP are to be determined. **Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

**Analysis** The *T-s* diagram of the refrigeration cycle is shown in Fig. 11–6. We note that this is an ideal vapor-compression refrigeration cycle, and thus the compressor is isentropic and the refrigerant leaves the condenser as a saturated liquid and enters the compressor as saturated vapor. From the refrigerant-134a tables, the enthalpies of the refrigerant at all four states are determined as follows:

$$P_1 = 0.14 \text{ MPa} \longrightarrow h_1 = h_{g \oplus 0.14 \text{ MPa}} = 239.16 \text{ kJ/kg}$$
 
$$s_1 = s_{g \oplus 0.14 \text{ MPa}} = 0.94456 \text{ kJ/kg} \cdot \text{K}$$

$$P_2 = 0.8 \text{ MPa}$$

$$s_2 = s_1$$

$$h_2 = 275.39 \text{ kJ/kg}$$

$$P_3 = 0.8 \text{ MPa} \longrightarrow h_3 = h_{f@0.8 \text{ MPa}} = 95.47 \text{ kJ/kg}$$

$$h_4 \cong h_3 \text{ (throttling)} \longrightarrow h_4 = 95.47 \text{ kJ/kg}$$

(a) The rate of heat removal from the refrigerated space and the power input to the compressor are determined from their definitions:

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05 \text{ kg/s})[(239.16 - 95.47) \text{ kJ/kg}] = 7.18 \text{ kW}$$

and

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.05 \text{ kg/s})[(275.39 - 239.16) \text{ kJ/kg}] = 1.81 \text{ kW}$$

(b) The rate of heat rejection from the refrigerant to the environment is

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.05 \text{ kg/s})[(275.39 - 95.47) \text{ kJ/kg}] = 9.0 \text{ kW}$$

It could also be determined from

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 7.18 + 1.81 = 8.99 \text{ kW}$$

(c) The coefficient of performance of the refrigerator is

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.18 \text{ kW}}{1.81 \text{ kW}} = 3.97$$

That is, this refrigerator removes about 4 units of thermal energy from the refrigerated space for each unit of electric energy it consumes.

**Discussion** It would be interesting to see what happens if the throttling valve were replaced by an isentropic turbine. The enthalpy at state 4s (the turbine exit with  $P_{4s}=0.14$  MPa, and  $s_{4s}=s_3=0.35404$  kJ/kg·K) is 88.94 kJ/kg,

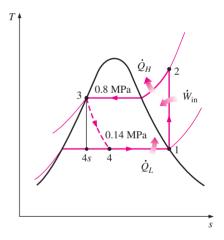


FIGURE 11-6

*T-s* diagram of the ideal vapor-compression refrigeration cycle described in Example 11–1.

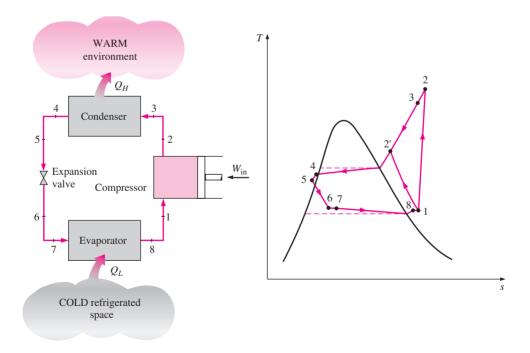
and the turbine would produce 0.33 kW of power. This would decrease the power input to the refrigerator from 1.81 to 1.48 kW and increase the rate of heat removal from the refrigerated space from 7.18 to 7.51 kW. As a result, the COP of the refrigerator would increase from 3.97 to 5.07, an increase of 28 percent.



# 11-4 - ACTUAL VAPOR-COMPRESSION REFRIGERATION CYCLE

An actual vapor-compression refrigeration cycle differs from the ideal one in several ways, owing mostly to the irreversibilities that occur in various components. Two common sources of irreversibilities are fluid friction (causes pressure drops) and heat transfer to or from the surroundings. The *T-s* diagram of an actual vapor-compression refrigeration cycle is shown in Fig. 11–7.

In the ideal cycle, the refrigerant leaves the evaporator and enters the compressor as *saturated vapor*. In practice, however, it may not be possible to control the state of the refrigerant so precisely. Instead, it is easier to design the system so that the refrigerant is slightly superheated at the compressor inlet. This slight overdesign ensures that the refrigerant is completely vaporized when it enters the compressor. Also, the line connecting



**FIGURE 11–7** Schematic and *T-s* diagram for the actual vapor-compression refrigeration cycle.

the evaporator to the compressor is usually very long; thus the pressure drop caused by fluid friction and heat transfer from the surroundings to the refrigerant can be very significant. The result of superheating, heat gain in the connecting line, and pressure drops in the evaporator and the connecting line is an increase in the specific volume, thus an increase in the power input requirements to the compressor since steady-flow work is proportional to the specific volume.

The *compression process* in the ideal cycle is internally reversible and adiabatic, and thus isentropic. The actual compression process, however, involves frictional effects, which increase the entropy, and heat transfer, which may increase or decrease the entropy, depending on the direction. Therefore, the entropy of the refrigerant may increase (process 1-2) or decrease (process 1-2') during an actual compression process, depending on which effects dominate. The compression process 1-2' may be even more desirable than the isentropic compression process since the specific volume of the refrigerant and thus the work input requirement are smaller in this case. Therefore, the refrigerant should be cooled during the compression process whenever it is practical and economical to do so.

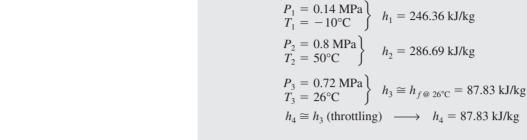
In the ideal case, the refrigerant is assumed to leave the condenser as *saturated liquid* at the compressor exit pressure. In reality, however, it is unavoidable to have some pressure drop in the condenser as well as in the lines connecting the condenser to the compressor and to the throttling valve. Also, it is not easy to execute the condensation process with such precision that the refrigerant is a saturated liquid at the end, and it is undesirable to route the refrigerant to the throttling valve before the refrigerant is completely condensed. Therefore, the refrigerant is subcooled somewhat before it enters the throttling valve. We do not mind this at all, however, since the refrigerant in this case enters the evaporator with a lower enthalpy and thus can absorb more heat from the refrigerated space. The throttling valve and the evaporator are usually located very close to each other, so the pressure drop in the connecting line is small.

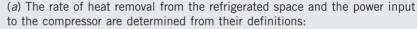
## EXAMPLE 11-2 The Actual Vapor-Compression Refrigeration Cycle

Refrigerant-134a enters the compressor of a refrigerator as superheated vapor at 0.14 MPa and  $-10^{\circ}$ C at a rate of 0.05 kg/s and leaves at 0.8 MPa and 50°C. The refrigerant is cooled in the condenser to 26°C and 0.72 MPa and is throttled to 0.15 MPa. Disregarding any heat transfer and pressure drops in the connecting lines between the components, determine (a) the rate of heat removal from the refrigerated space and the power input to the compressor, (b) the isentropic efficiency of the compressor, and (c) the coefficient of performance of the refrigerator.

**Solution** A refrigerator operating on a vapor-compression cycle is considered. The rate of refrigeration, the power input, the compressor efficiency, and the COP are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.





**Analysis** The *T-s* diagram of the refrigeration cycle is shown in Fig. 11–8. We note that the refrigerant leaves the condenser as a compressed liquid and enters the compressor as superheated vapor. The enthalpies of the refrigerant at various states are determined from the refrigerant tables to be

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.05 \text{ kg/s})[(246.36 - 87.83) \text{ kJ/kg}] =$$
7.93 kW

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.05 \text{ kg/s})[(286.69 - 246.36) \text{ kJ/kg}] = 2.02 \text{ kW}$$

(b) The isentropic efficiency of the compressor is determined from

$$\eta_C \cong \frac{h_{2s} - h_1}{h_2 - h_1}$$

where the enthalpy at state 2s ( $P_{2s}=0.8$  MPa and  $s_{2s}=s_1=0.9724$  kJ/kg · K) is 284.21 kJ/kg. Thus,

$$\eta_C = \frac{284.21 - 246.36}{286.69 - 246.36} = \mathbf{0.939} \text{ or } \mathbf{93.9\%}$$

(c) The coefficient of performance of the refrigerator is

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.93 \text{ kW}}{2.02 \text{ kW}} = 3.93$$

**Discussion** This problem is identical to the one worked out in Example 11–1, except that the refrigerant is slightly superheated at the compressor inlet and subcooled at the condenser exit. Also, the compressor is not isentropic. As a result, the heat removal rate from the refrigerated space increases (by 10.4 percent), but the power input to the compressor increases even more (by 11.6 percent). Consequently, the COP of the refrigerator decreases from 3.97 to 3.93.

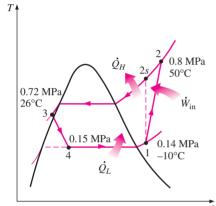


FIGURE 11–8

T-s diagram for Example 11–2.

= 
$$(0.05 \text{ kg/s})[(270.92 - 251.88) \text{ kJ/kg}]$$
  
+  $(0.039 \text{ kg/s})[(255.93 - 239.16) \text{ kJ/kg}]$   
=  $1.61 \text{ kW}$ 

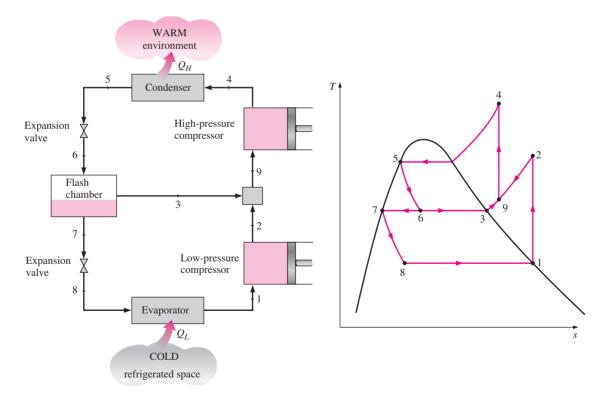
(c) The COP of a refrigeration system is the ratio of the refrigeration rate to the net power input:

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{7.18 \text{ kW}}{1.61 \text{ kW}} = 4.46$$

**Discussion** This problem was worked out in Example 11–1 for a single-stage refrigeration system. Notice that the COP of the refrigeration system increases from 3.97 to 4.46 as a result of cascading. The COP of the system can be increased even more by increasing the number of cascade stages.

### **Multistage Compression Refrigeration Systems**

When the fluid used throughout the cascade refrigeration system is the same, the heat exchanger between the stages can be replaced by a mixing chamber (called a *flash chamber*) since it has better heat transfer characteristics. Such systems are called **multistage compression refrigeration systems**. A two-stage compression refrigeration system is shown in Fig. 11–12.



**FIGURE 11-12** 

A two-stage compression refrigeration system with a flash chamber.

In this system, the liquid refrigerant expands in the first expansion valve to the flash chamber pressure, which is the same as the compressor interstage pressure. Part of the liquid vaporizes during this process. This saturated vapor (state 3) is mixed with the superheated vapor from the low-pressure compressor (state 2), and the mixture enters the high-pressure compressor at state 9. This is, in essence, a regeneration process. The saturated liquid (state 7) expands through the second expansion valve into the evaporator, where it picks up heat from the refrigerated space.

The compression process in this system resembles a two-stage compression with intercooling, and the compressor work decreases. Care should be exercised in the interpretations of the areas on the *T-s* diagram in this case since the mass flow rates are different in different parts of the cycle.

## EXAMPLE 11-4 A Two-Stage Refrigeration Cycle with a Flash Chamber

Consider a two-stage compression refrigeration system operating between the pressure limits of 0.8 and 0.14 MPa. The working fluid is refrigerant-134a. The refrigerant leaves the condenser as a saturated liquid and is throttled to a flash chamber operating at 0.32 MPa. Part of the refrigerant evaporates during this flashing process, and this vapor is mixed with the refrigerant leaving the low-pressure compressor. The mixture is then compressed to the condenser pressure by the high-pressure compressor. The liquid in the flash chamber is throttled to the evaporator pressure and cools the refrigerated space as it vaporizes in the evaporator. Assuming the refrigerant leaves the evaporator as a saturated vapor and both compressors are isentropic, determine (a) the fraction of the refrigerant that evaporates as it is throttled to the flash chamber, (b) the amount of heat removed from the refrigerated space and the compressor work per unit mass of refrigerant flowing through the condenser, and (c) the coefficient of performance.

**Solution** A two-stage compression refrigeration system operating between specified pressure limits is considered. The fraction of the refrigerant that evaporates in the flash chamber, the refrigeration and work input per unit mass, and the COP are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible. 3 The flash chamber is adiabatic.

**Properties** The enthalpies of the refrigerant at various states are determined from the refrigerant tables and are indicated on the *T-s* diagram.

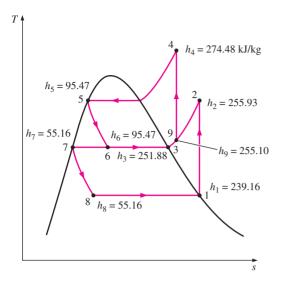
*Analysis* The *T-s* diagram of the refrigeration cycle is shown in Fig. 11–13. We note that the refrigerant leaves the condenser as saturated liquid and enters the low-pressure compressor as saturated vapor.

(a) The fraction of the refrigerant that evaporates as it is throttled to the flash chamber is simply the quality at state 6, which is

$$x_6 = \frac{h_6 - h_f}{h_{fg}} = \frac{95.47 - 55.16}{196.71} =$$
**0.2049**

(b) The amount of heat removed from the refrigerated space and the compressor work input per unit mass of refrigerant flowing through the condenser are

$$q_L = (1 - x_6)(h_1 - h_8)$$
  
=  $(1 - 0.2049)[(239.16 - 55.16) \text{ kJ/kg}] = 146.3 \text{ kJ/kg}$ 



#### **FIGURE 11-13**

*T-s* diagram of the two-stage compression refrigeration cycle described in Example 11–4.

and

$$w_{\text{in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}} = (1 - x_6)(h_2 - h_1) + (1)(h_4 - h_9)$$

The enthalpy at state 9 is determined from an energy balance on the mixing chamber,

$$\dot{E}_{\mathrm{out}} = \dot{E}_{\mathrm{in}}$$

$$(1)h_9 = x_6h_3 + (1 - x_6)h_2$$

$$h_9 = (0.2049)(251.88) + (1 - 0.2049)(255.93) = 255.10 \text{ kJ/kg}$$

Also,  $s_9=0.9416$  kJ/kg · K. Thus the enthalpy at state 4 (0.8 MPa,  $s_4=s_9$ ) is  $h_4=274.48$  kJ/kg. Substituting,

$$w_{\rm in} = (1 - 0.2049)[(255.93 - 239.16) \text{ kJ/kg}] + (274.48 - 255.10) \text{ kJ/kg}$$
  
= 32.71 kJ/kg

(c) The coefficient of performance is

$$COP_R = \frac{q_L}{w_{in}} = \frac{146.3 \text{ kJ/kg}}{32.71 \text{ kJ/kg}} = 4.47$$

**Discussion** This problem was worked out in Example 11-1 for a single-stage refrigeration system (COP = 3.97) and in Example 11-3 for a two-stage cascade refrigeration system (COP = 4.46). Notice that the COP of the refrigeration system increased considerably relative to the single-stage compression but did not change much relative to the two-stage cascade compression.

#### 9-4 - AN OVERVIEW OF RECIPROCATING ENGINES

Despite its simplicity, the reciprocating engine (basically a piston-cylinder device) is one of the rare inventions that has proved to be very versatile and to have a wide range of applications. It is the powerhouse of the vast majority of automobiles, trucks, light aircraft, ships, and electric power generators, as well as many other devices.

The basic components of a reciprocating engine are shown in Fig. 9–10. The piston reciprocates in the cylinder between two fixed positions called the **top dead center** (TDC)—the position of the piston when it forms the smallest volume in the cylinder—and the **bottom dead center** (BDC)—the position of the piston when it forms the largest volume in the cylinder. The distance between the TDC and the BDC is the largest distance that the piston can travel in one direction, and it is called the **stroke** of the engine. The diameter of the piston is called the **bore.** The air or air—fuel mixture is drawn into the cylinder through the **intake valve**, and the combustion products are expelled from the cylinder through the **exhaust valve**.

The minimum volume formed in the cylinder when the piston is at TDC is called the **clearance volume** (Fig. 9–11). The volume displaced by the piston as it moves between TDC and BDC is called the **displacement volume**. The ratio of the maximum volume formed in the cylinder to the minimum (clearance) volume is called the **compression ratio** r of the engine:

$$r = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}}$$
 (9-3)

Notice that the compression ratio is a *volume ratio* and should not be confused with the pressure ratio.

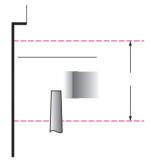
Another term frequently used in conjunction with reciprocating engines is the **mean effective pressure** (MEP). It is a fictitious pressure that, if it acted on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle (Fig. 9–12). That is,

$$W_{\mathrm{net}} = \mathrm{MEP} \times \mathrm{Piston}$$
 area  $\times$  Stroke = MEP  $\times$  Displacement volume

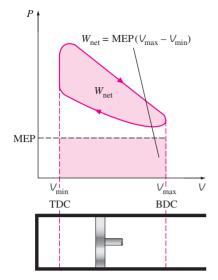
or

$$MEP = \frac{W_{\text{net}}}{V_{\text{max}} - V_{\text{min}}} = \frac{w_{\text{net}}}{v_{\text{max}} - v_{\text{min}}} \qquad \text{(kPa)}$$

The mean effective pressure can be used as a parameter to compare the performances of reciprocating engines of equal size. The engine with a larger value of MEP delivers more net work per cycle and thus performs better.



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#### FIGURE 9-12

The net work output of a cycle is equivalent to the product of the mean effective pressure and the displacement volume. Reciprocating engines are classified as **spark-ignition** (**SI**) **engines** or **compression-ignition** (**CI**) **engines**, depending on how the combustion process in the cylinder is initiated. In SI engines, the combustion of the air–fuel mixture is initiated by a spark plug. In CI engines, the air–fuel mixture is self-ignited as a result of compressing the mixture above its self-ignition temperature. In the next two sections, we discuss the *Otto* and *Diesel cycles*, which are the ideal cycles for the SI and CI reciprocating engines, respectively.

# 9-5 OTTO CYCLE: THE IDEAL CYCLE FOR SPARK-IGNITION ENGINES

The Otto cycle is the ideal cycle for spark-ignition reciprocating engines. It is named after Nikolaus A. Otto, who built a successful four-stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862. In most spark-ignition engines, the piston executes four complete strokes (two mechanical cycles) within the cylinder, and the crankshaft completes two revolutions for each thermodynamic cycle. These engines are called **four-stroke** internal combustion engines. A schematic of each stroke as well as a *P-v* diagram for an actual four-stroke spark-ignition engine is given in Fig. 9–13(*a*).

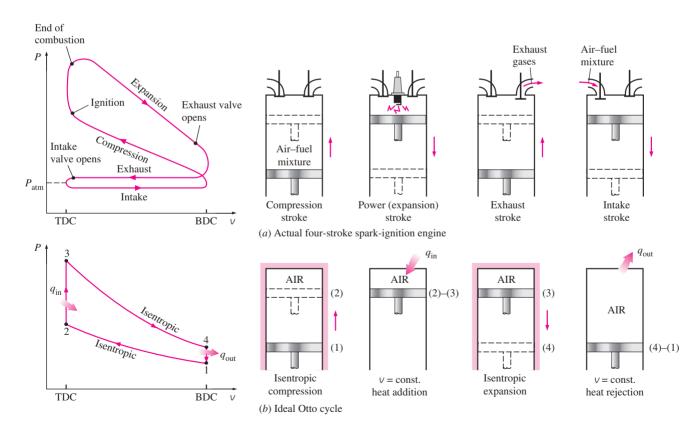


FIGURE 9–13 Actual and ideal cycles in spark-ignition engines and their P-V diagrams.

Initially, both the intake and the exhaust valves are closed, and the piston is at its lowest position (BDC). During the *compression stroke*, the piston moves upward, compressing the air–fuel mixture. Shortly before the piston reaches its highest position (TDC), the spark plug fires and the mixture ignites, increasing the pressure and temperature of the system. The high-pressure gases force the piston down, which in turn forces the crankshaft to rotate, producing a useful work output during the *expansion* or *power stroke*. At the end of this stroke, the piston is at its lowest position (the completion of the first mechanical cycle), and the cylinder is filled with combustion products. Now the piston moves upward one more time, purging the exhaust gases through the exhaust valve (the *exhaust stroke*), and down a second time, drawing in fresh air–fuel mixture through the intake valve (the *intake stroke*). Notice that the pressure in the cylinder is slightly above the atmospheric value during the exhaust stroke and slightly below during the intake stroke.

In **two-stroke engines**, all four functions described above are executed in just two strokes: the power stroke and the compression stroke. In these engines, the crankcase is sealed, and the outward motion of the piston is used to slightly pressurize the air–fuel mixture in the crankcase, as shown in Fig. 9–14. Also, the intake and exhaust valves are replaced by openings in the lower portion of the cylinder wall. During the latter part of the power stroke, the piston uncovers first the exhaust port, allowing the exhaust gases to be partially expelled, and then the intake port, allowing the fresh air–fuel mixture to rush in and drive most of the remaining exhaust gases out of the cylinder. This mixture is then compressed as the piston moves upward during the compression stroke and is subsequently ignited by a spark plug.

The two-stroke engines are generally less efficient than their four-stroke counterparts because of the incomplete expulsion of the exhaust gases and the partial expulsion of the fresh air–fuel mixture with the exhaust gases. However, they are relatively simple and inexpensive, and they have high power-to-weight and power-to-volume ratios, which make them suitable for applications requiring small size and weight such as for motorcycles, chain saws, and lawn mowers (Fig. 9–15).

Advances in several technologies—such as direct fuel injection, stratified charge combustion, and electronic controls—brought about a renewed interest in two-stroke engines that can offer high performance and fuel economy while satisfying the stringent emission requirements. For a given weight and displacement, a well-designed two-stroke engine can provide significantly more power than its four-stroke counterpart because two-stroke engines produce power on every engine revolution instead of every other one. In the new two-stroke engines, the highly atomized fuel spray that is injected into the combustion chamber toward the end of the compression stroke burns much more completely. The fuel is sprayed after the exhaust valve is closed, which prevents unburned fuel from being ejected into the atmosphere. With stratified combustion, the flame that is initiated by igniting a small amount of the rich fuel-air mixture near the spark plug propagates through the combustion chamber filled with a much leaner mixture, and this results in much cleaner combustion. Also, the advances in electronics have made it possible to ensure the optimum operation under varying engine load and speed conditions.



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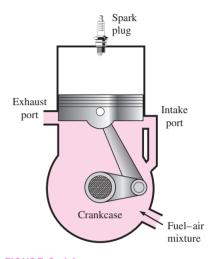


FIGURE 9–14
Schematic of a two-stroke reciprocating engine.



FIGURE 9-15

Two-stroke engines are commonly used in motorcycles and lawn mowers.

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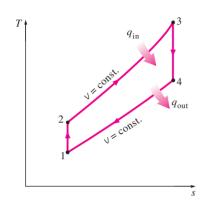


FIGURE 9–16

T-s diagram of the ideal Otto cycle.

Major car companies have research programs underway on two-stroke engines which are expected to make a comeback in the future.

The thermodynamic analysis of the actual four-stroke or two-stroke cycles described is not a simple task. However, the analysis can be simplified significantly if the air-standard assumptions are utilized. The resulting cycle, which closely resembles the actual operating conditions, is the ideal **Otto cycle.** It consists of four internally reversible processes:

- 1-2 Isentropic compression
- 2-3 Constant-volume heat addition
- 3-4 Isentropic expansion
- 4-1 Constant-volume heat rejection

The execution of the Otto cycle in a piston–cylinder device together with a P- $\nu$  diagram is illustrated in Fig. 9–13b. The T-s diagram of the Otto cycle is given in Fig. 9–16.

The Otto cycle is executed in a closed system, and disregarding the changes in kinetic and potential energies, the energy balance for any of the processes is expressed, on a unit-mass basis, as

$$(q_{\rm in} - q_{\rm out}) + (w_{\rm in} - w_{\rm out}) = \Delta u$$
 (kJ/kg) (9-5)

No work is involved during the two heat transfer processes since both take place at constant volume. Therefore, heat transfer to and from the working fluid can be expressed as

$$q_{\rm in} = u_3 - u_2 = c_{\rm v}(T_3 - T_2)$$
 (9-6a)

and

$$q_{\text{out}} = u_4 - u_1 = c_v (T_4 - T_1)$$
 (9-6*b*)

Then the thermal efficiency of the ideal Otto cycle under the cold air standard assumptions becomes

$$\eta_{\text{th,Otto}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

Processes 1-2 and 3-4 are isentropic, and  $v_2 = v_3$  and  $v_4 = v_1$ . Thus,

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{V_3}{V_4}\right)^{k-1} = \frac{T_4}{T_3}$$
 (9-7)

Substituting these equations into the thermal efficiency relation and simplifying give

$$\eta_{\text{th,Otto}} = 1 - \frac{1}{r^{k-1}}$$
(9-8)

where

$$r = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$
 (9-9)

is the **compression ratio** and k is the specific heat ratio  $c_p/c_v$ .

Equation 9–8 shows that under the cold-air-standard assumptions, the thermal efficiency of an ideal Otto cycle depends on the compression ratio of the engine and the specific heat ratio of the working fluid. The thermal efficiency of the ideal Otto cycle increases with both the compression ratio

and the specific heat ratio. This is also true for actual spark-ignition internal combustion engines. A plot of thermal efficiency versus the compression ratio is given in Fig. 9–17 for k = 1.4, which is the specific heat ratio value of air at room temperature. For a given compression ratio, the thermal efficiency of an actual spark-ignition engine is less than that of an ideal Otto cycle because of the irreversibilities, such as friction, and other factors such as incomplete combustion.

We can observe from Fig. 9–17 that the thermal efficiency curve is rather steep at low compression ratios but flattens out starting with a compression ratio value of about 8. Therefore, the increase in thermal efficiency with the compression ratio is not as pronounced at high compression ratios. Also, when high compression ratios are used, the temperature of the air–fuel mixture rises above the autoignition temperature of the fuel (the temperature at which the fuel ignites without the help of a spark) during the combustion process, causing an early and rapid burn of the fuel at some point or points ahead of the flame front, followed by almost instantaneous inflammation of the end gas. This premature ignition of the fuel, called **autoignition**, produces an audible noise, which is called **engine knock**. Autoignition in spark-ignition engines cannot be tolerated because it hurts performance and can cause engine damage. The requirement that autoignition not be allowed places an upper limit on the compression ratios that can be used in sparkignition internal combustion engines.

Improvement of the thermal efficiency of gasoline engines by utilizing higher compression ratios (up to about 12) without facing the autoignition problem has been made possible by using gasoline blends that have good antiknock characteristics, such as gasoline mixed with tetraethyl lead. Tetraethyl lead had been added to gasoline since the 1920s because it is an inexpensive method of raising the octane rating, which is a measure of the engine knock resistance of a fuel. Leaded gasoline, however, has a very undesirable side effect: it forms compounds during the combustion process that are hazardous to health and pollute the environment. In an effort to combat air pollution, the government adopted a policy in the mid-1970s that resulted in the eventual phase-out of leaded gasoline. Unable to use lead, the refiners developed other techniques to improve the antiknock characteristics of gasoline. Most cars made since 1975 have been designed to use unleaded gasoline, and the compression ratios had to be lowered to avoid engine knock. The ready availability of high octane fuels made it possible to raise the compression ratios again in recent years. Also, owing to the improvements in other areas (reduction in overall automobile weight, improved aerodynamic design, etc.), today's cars have better fuel economy and consequently get more miles per gallon of fuel. This is an example of how engineering decisions involve compromises, and efficiency is only one of the considerations in final design.

The second parameter affecting the thermal efficiency of an ideal Otto cycle is the specific heat ratio k. For a given compression ratio, an ideal Otto cycle using a monatomic gas (such as argon or helium, k=1.667) as the working fluid will have the highest thermal efficiency. The specific heat ratio k, and thus the thermal efficiency of the ideal Otto cycle, decreases as the molecules of the working fluid get larger (Fig. 9–18). At room temperature it is 1.4 for air, 1.3 for carbon dioxide, and 1.2 for ethane. The working

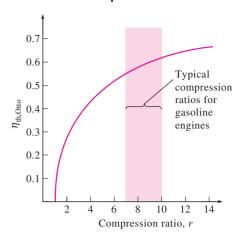


FIGURE 9–17
Thermal efficiency of the ideal Otto cycle as a function of compression ratio (k = 1.4).

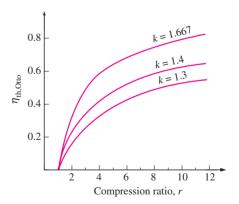


FIGURE 9–18

The thermal efficiency of the Otto cycle increases with the specific heat ratio k of the working fluid.

fluid in actual engines contains larger molecules such as carbon dioxide, and the specific heat ratio decreases with temperature, which is one of the reasons that the actual cycles have lower thermal efficiencies than the ideal Otto cycle. The thermal efficiencies of actual spark-ignition engines range from about 25 to 30 percent.

#### EXAMPLE 9-2 The Ideal Otto Cycle

An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and  $17^{\circ}\text{C}$ , and 800 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Accounting for the variation of specific heats of air with temperature, determine (a) the maximum temperature and pressure that occur during the cycle, (b) the net work output, (c) the thermal efficiency, and (d) the mean effective pressure for the cycle.

**Solution** An ideal Otto cycle is considered. The maximum temperature and pressure, the net work output, the thermal efficiency, and the mean effective pressure are to be determined.

**Assumptions** 1 The air-standard assumptions are applicable. 2 Kinetic and potential energy changes are negligible. 3 The variation of specific heats with temperature is to be accounted for.

**Analysis** The P- $\nu$  diagram of the ideal Otto cycle described is shown in Fig. 9–19. We note that the air contained in the cylinder forms a closed system.

(a) The maximum temperature and pressure in an Otto cycle occur at the end of the constant-volume heat-addition process (state 3). But first we need to determine the temperature and pressure of air at the end of the isentropic compression process (state 2), using data from Table A–17:

$$T_1 = 290 \text{ K} \rightarrow u_1 = 206.91 \text{ kJ/kg}$$
  
 $v_{r1} = 676.1$ 

Process 1-2 (isentropic compression of an ideal gas):

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} = \frac{1}{r} \rightarrow v_{r2} = \frac{v_{r1}}{r} = \frac{676.1}{8} = 84.51 \rightarrow T_2 = 652.4 \text{ K}$$

$$u_2 = 475.11 \text{ kJ/kg}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \rightarrow P_2 = P_1 \left(\frac{T_2}{T_1}\right) \left(\frac{v_1}{v_2}\right)$$
$$= (100 \text{ kPa}) \left(\frac{652.4 \text{ K}}{290 \text{ K}}\right) (8) = 1799.7 \text{ kPa}$$

Process 2-3 (constant-volume heat addition):

$$q_{\text{in}} = u_3 - u_2$$
  
 $800 \text{ kJ/kg} = u_3 - 475.11 \text{ kJ/kg}$   
 $u_3 = 1275.11 \text{ kJ/kg} \rightarrow T_3 = 1575.1 \text{ K}$   
 $v_{r3} = 6.108$ 

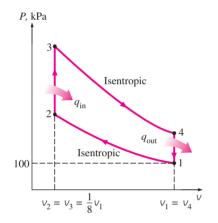


FIGURE 9–19

P-v diagram for the Otto cycle discussed in Example 9–2.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \rightarrow P_3 = P_2 \left(\frac{T_3}{T_2}\right) \left(\frac{v_2}{v_3}\right)$$
$$= (1.7997 \text{ MPa}) \left(\frac{1575.1 \text{ K}}{652.4 \text{ K}}\right) (1) = 4.345 \text{ MPa}$$

(b) The net work output for the cycle is determined either by finding the boundary ( $P \, dV$ ) work involved in each process by integration and adding them or by finding the net heat transfer that is equivalent to the net work done during the cycle. We take the latter approach. However, first we need to find the internal energy of the air at state 4:

Process 3-4 (isentropic expansion of an ideal gas):

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = r \rightarrow v_{r4} = r v_{r3} = (8)(6.108) = 48.864 \rightarrow T_4 = 795.6 \text{ K}$$

$$u_4 = 588.74 \text{ kJ/kg}$$

Process 4-1 (constant-volume heat rejection):

$$-q_{\text{out}} = u_1 - u_4 \rightarrow q_{\text{out}} = u_4 - u_1$$
  
 $q_{\text{out}} = 588.74 - 206.91 = 381.83 \text{ kJ/kg}$ 

Thus,

$$w_{\text{net}} = q_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 800 - 381.83 = 418.17 \text{ kJ/kg}$$

(c) The thermal efficiency of the cycle is determined from its definition:

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{418.17 \text{ kJ/kg}}{800 \text{ kJ/kg}} = 0.523 \text{ or } 52.3\%$$

Under the cold-air-standard assumptions (constant specific heat values at room temperature), the thermal efficiency would be (Eq. 9–8)

$$\eta_{\text{th,Otto}} = 1 - \frac{1}{r^{k-1}} = 1 - r^{1-k} = 1 - (8)^{1-1.4} = 0.565 \text{ or } 56.5\%$$

which is considerably different from the value obtained above. Therefore, care should be exercised in utilizing the cold-air-standard assumptions.

(d) The mean effective pressure is determined from its definition, Eq. 9-4:

MEP = 
$$\frac{w_{\text{net}}}{v_1 - v_2} = \frac{w_{\text{net}}}{v_1 - v_1/r} = \frac{w_{\text{net}}}{v_1(1 - 1/r)}$$

where

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(290 \text{ K})}{100 \text{ kPa}} = 0.832 \text{ m}^3/\text{kg}$$

Thus,

MEP = 
$$\frac{418.17 \text{ kJ/kg}}{(0.832 \text{ m}^3/\text{kg})(1 - \frac{1}{8})} \left(\frac{1 \text{ kPa} \cdot \text{m}^3}{1 \text{ kJ}}\right) = 574 \text{ kPa}$$

**Discussion** Note that a constant pressure of 574 kPa during the power stroke would produce the same net work output as the entire cycle.