A Causal Inference Perspective on Therapist Effects—Online Supplement

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Introduction

This online supplement for *A Causal Inference Perspective on Therapist Effects* (Magnusson, 2023, in preparation) contains the code for all calculations, simulations, and figures presented in the article.

- Preprint
- GitHub repository
- OSF repository
- Interactive visualization

An HTML version of this document can also be viewed at https://rpsychologist.github.io/causal-therapist-effects-paper

```r
knitr::opts_chunk$set(
    message = FALSE,
    warning = FALSE,
    cache = TRUE
)
```

```r
library(ggplot2)
library(readr)
library(dplyr)
library(tidyr)
library(purrr)
library(powerlmm)
library(knitr)
library(parallel)
library(lme4)
library(lmerTest)
library(svglite)
```

Therapist-Outcome Confounding Simulation

The functions below were used to simulate a data set with therapist effects and a therapist-outcome confounder.

```r
#' Get the mean difference between prognostic groups
#'
#' @param sd the SD added to the therapist level due to the confounder
#' @param n2 the number of patients per therapist
#'
#' @returns the mean difference between the two binary groups
#' @examples
#' n2 <- 50
#' M <- solve_for_mean(0.075, n2)
#' # we solve for M in this
#' sd(c(rep(0, n2), rep(M, n2)))
solve_for_mean <- function(sd, n2) {
    2 * sqrt(sd^2 * (n2 * 2 - 1) / (n2 * 2))
}

#' Get the therapist SD from thee ICC
#'
#' @param icc the ICC
#' @param sd_error the error SD
#'
#' @returns the random therapist SD
get_therapist_sd_from_icc <- function(icc, sd_error) {
    (sqrt(icc) * sd_error) / (sqrt(1 - icc))
```

```r
}

#' Simulate confounded therapist effects
#'
#' @param n1
#' @param n2
#' @param sd_therapist
#' @param sd_therapist_confounding
#' @param sd_error
#' @param ATE
#'
#' @returns a data.frame
simulate_therapist_effect_confounding <- function(
    n1,
    n2,
    sd_therapist,
    sd_therapist_confounding,
    sd_error,
    ATE,
    ...) {
    tot_n <- 2 * n1 * n2
    b_pre_prognosis <- solve_for_mean(
        sd = sd_therapist_confounding,
        n2 = n2
    )
    Z <- rep(c(0, 1), each = n1 * n2)
    therapist_cc <- 1:n2
    therapist_tx <- (n2 + 1):(n2 * 2)
    therapist_effect_cc <- rnorm(n2, 0, sd_therapist)
    therapist_effect_tx <- rnorm(n2, 0, sd_therapist)
    therapist_effects <- c(therapist_effect_cc, therapist_effect_tx)
    error <- rnorm(tot_n, 0, sd_error)
    d <- data.frame(
        Z,
        pre_prognosis = rbinom(tot_n, 1, 0.5),
        therapist = NA,
        therapist_random = NA
    )
    d$therapist <- ifelse(
        Z == 0,
        # control
```

```r
    ifelse(
        d$pre_prognosis == 0,
        sample(
            x = 1:(n2 / 2),
            size = n1 * n2 / 2,
            replace = TRUE
        ),
        sample(
            x = (n2 / 2 + 1):(n2),
            size = n1 * n2 / 2,
            replace = TRUE
        )
    ),
    # treatment
    ifelse(
        d$pre_prognosis == 0,
        sample(
            x = 1:(n2 / 2),
            size = n1 * n2 / 2,
            replace = TRUE
        ),
        sample(
            x = (n2 / 2 + 1):(n2),
            size = n1 * n2 / 2,
            replace = TRUE
        )
    ) + n2
)
d$therapist_random[d$Z == 0] <- sample(
    x = therapist_cc,
    size = n1 * n2,
    replace = TRUE
)
d$therapist_random[d$Z == 1] <- sample(
    x = therapist_tx,
    size = n1 * n2,
    replace = TRUE
)
# non-random allocation of therapists
d$therapist_effect <- therapist_effects[d$therapist]
d$y <- 10 +
```

```
            d$Z * ATE +
            d$therapist_effect +
            error +
            d$pre_prognosis * b_pre_prognosis
    # random allocation of therapists
    d$therapist_effect_random <- therapist_effects[d$therapist_random]
    d$y_random <- 10 +
            d$Z * ATE +
            d$therapist_effect_random +
            error +
            d$pre_prognosis * b_pre_prognosis
    # required by powerlmm
    d$time <- 0


    d
}
```

To run the simulation we pass the simulation function `simulate_therapist_effect_confounding` and use a custom `powerlmm` model to setup all the parameters.

```
ds <- study_design(custom = TRUE)
ICC <- 0.05
sd_error <- 1.5
sd_therapist <- sqrt((ICC * sd_error^2) / (1 - ICC))
sd_therapist^2 / (sd_therapist^2 + sd_error^2)
```

```
[1] 0.05
```

```
p <- powerlmm:::study_parameters.plcp_design_custom(
    design = ds,
    n1 = 20,
    n2 = 10,
    sd_therapist = sd_therapist,
    sd_error = sd_error,
    ATE = 0,
    data_gen = simulate_therapist_effect_confounding
)
p$sd_therapist_confounding <- get_therapist_sd_from_icc(
    icc = 0.075,
    sd_error = p$sd_error
```

```
)
tot_sd <- sqrt(
    p$sd_error^2 + p$sd_therapist^2 + p$sd_therapist_confounding^2
)
p$ATE <- 0.5 * tot_sd
p$tot_n <- 2 * p$n1 * p$n2
p$df <- 2 * p$n2 - 2
```

True parameter values

```
p$thetas_FE <- list(
    "(Intercept)" = 10,
    "Z" = p$ATE
)
p$thetas_RE <- list(
    "therapist_(Intercept)" = p$sd_therapist^2,
    "therapist_random_(Intercept)" = p$sd_therapist^2,
    "error" = p$sd_error^2
)
```

We then specify the statistical models

```
f0 <- sim_formula(
    "y_random ~ Z + (1 | therapist_random)",
    test = "Z"
)
f1 <- sim_formula(
    "y_random ~ Z + pre_prognosis + (1 | therapist_random)",
    test = "Z"
)
f2 <- sim_formula(
    "y ~ Z + (1 | therapist)",
    test = "Z"
)
f3 <- sim_formula(
    "y ~ Z + pre_prognosis + (1 | therapist)",
    test = "Z"
)
f4 <- sim_formula(
    "y ~ Z",
    test = "Z"
```

```
    )
    f5 <- sim_formula(
        "y ~ Z + pre_prognosis",
        test = "Z"
    )
    f <- sim_formula_compare(
        "rand" = f0,
        "rand_adj" = f1,
        "confounding" = f2,
        "adjusted" = f3,
        "ignored" = f4,
        "ignored_adjusted" = f5
    )
```

Finally, we run the simulation and save the results.

```
    # Load cache if it exists
    file_path <- "tmp/simulation.rds"
    if (file.exists(file_path)) {
        res <- read_rds(file = file_path)
    } else {
        MAX_CORES <- as.numeric(Sys.getenv("MAX_CORES"))
        N_SIM <- as.numeric(Sys.getenv("N_SIM"))
        if (is.na(MAX_CORES)) MAX_CORES <- parallel::detectCores(logical = FALSE) - 1
        cl <- makeCluster(MAX_CORES)
        clusterExport(
            cl,
            c(
                "solve_for_mean",
                "get_therapist_sd_from_icc"
            )
        )
        res <- simulate(
            p,
            nsim = ifelse(is.na(N_SIM), 10000, N_SIM),
            cores = MAX_CORES,
            formula = f,
            cl = cl,
            satterthwaite = TRUE
        )
        stopCluster(cl)
```

```
    write_rds(
        res,
        file = file_path,
        compress = "gz"
    )
}
```

Number of simulations: 10^4

Simulations results

We first summarize the treatment effects, and the result is shown in Table 1.

```
summary(
    res,
    verbose = FALSE,
    para = "Z"
)$summary$summary$FE %>%
    mutate(
        SD_rel_bias = (M_se - SD_est) / SD_est
    ) %>%
    relocate(model, .before = 1) %>%
    select(
        -parameter,
        -Power,
        -Power_bw,
        "Model" = model,
        "Estimate" = M_est,
        "Rel. bias (SE)" = SD_rel_bias,
        "SD(Est.)" = SD_est,
        "Power" = Power_satt,
        "True" = theta
    ) %>%
    kable(digits = 2)
```

Table 1: Simulation results (treatment effects)

	Model	Estimate	True	M_se	SD(Est.)	Power	Rel. bias (SE)
2	rand	0.8	0.8	0.22	0.22	0.93	0.00
21	rand_adj	0.8	0.8	0.21	0.21	0.94	0.00
22	confounding	0.8	0.8	0.29	0.21	0.80	0.35

	Model	Estimate	True	M_se	SD(Est.)	Power	Rel. bias (SE)
23	adjusted	0.8	0.8	0.21	0.21	0.94	0.00
24	ignored	0.8	0.8	0.16	0.22	0.99	-0.28
25	ignored_adjusted	0.8	0.8	0.15	0.22	0.99	-0.29

A summary of the estimated ICCs is shown in Table 2.

```
lapply(
    seq_along(res$res),
    function(i) {
        res$res[[i]]$RE %>%
            group_by(sim) %>%
            summarize(ICC = vcov[1] / sum(vcov)) %>%
            ungroup() %>%
            summarize(
                est_mean = mean(ICC),
                est_sd = sd(ICC),
                est_lwr = quantile(ICC, 0.025),
                est_upr = quantile(ICC, 0.975)
            ) %>%
            ungroup() %>%
            mutate(
                model = names(res$res)[[i]],
                parameter = "ICC",
                .before = 1
            )
    }
) %>%
    bind_rows() %>%
    filter(grepl("ignored", model) == FALSE) %>%
    mutate(
        theta = p$sd_therapist^2 / (p$sd_therapist^2 + p$sd_error^2),
        rel_bias = (est_mean - theta) / theta
    ) %>%
    select(
        "Model" = model,
        "Parameter" = parameter,
        "Estimate" = est_mean,
        "Rel. bias" = rel_bias,
        "SD(Est.)" = est_sd,
        "Est. (2.5%)" = est_lwr,
```

```
        "Est. (97.5%)" = est_upr,
        "True" = theta
    ) %>%
    kable(digits = 2)
```

Table 2: Simulation results (ICCs)

Model	Parameter	Estimate	Rel. bias	SD(Est.)	Est. (2.5%)	Est. (97.5%)	True
rand	ICC	0.05	-0.07	0.03	0.00	0.11	0.05
rand_adj	ICC	0.05	0.00	0.03	0.00	0.12	0.05
confounding	ICC	0.12	1.39	0.04	0.04	0.21	0.05
adjusted	ICC	0.05	-0.01	0.03	0.00	0.12	0.05

Lastly, we summarize the variance components.

```
lapply(
    seq_along(res$res),
    function(i) {
        res$res[[i]]$RE %>%
            group_by(parameter) %>%
            summarize(
                est_mean = mean(vcov),
                est_sd = sd(vcov),
                est_lwr = quantile(vcov, 0.025),
                est_upr = quantile(vcov, 0.975)
            ) %>%
            ungroup() %>%
            mutate(
                model = names(res$res)[[i]],
                .before = 1
            )
    }
) %>%
    bind_rows() %>%
    left_join(
        data.frame(
            parameter = names(p$thetas_RE),
            theta = unlist(p$thetas_RE)
        )
    ) %>%
```

```
    mutate(
        parameter = replace(
            parameter,
            grep("therapist", parameter),
            "therapist"
        ),
        theta = case_when(
            parameter == "therapist" ~ p$sd_therapist^2,
            parameter == "error" & model == "rand" ~ p$sd_error^2 +
                p$sd_therapist_confounding^2,
            model == "ignored" ~ p$sd_error^2 +  p$sd_therapist^2 +
                p$sd_therapist_confounding^2,
            model == "ignored_adjusted" ~ p$sd_error^2 + p$sd_therapist^2,
            parameter == "error" ~ p$sd_error^2
        ),
        rel_bias = (est_mean - theta) / theta
    ) %>%
    select(
        "Model" = model,
        "Parameter" = parameter,
        "Estimate" = est_mean,
        "Rel. bias" = rel_bias,
        "SD(Est.)" = est_sd,
        "Est. (2.5%)" = est_lwr,
        "Est. (97.5%)" = est_upr,
        "True" = theta
    ) %>%
    kable(digits = 2)
```

Table 3: Simulation results (variance components)

Model	Parameter	Estimate	Rel. bias	SD(Est.)	Est. (2.5%)	Est. (97.5%)	True
rand	error	2.42	0.00	0.18	2.08	2.78	2.43
rand	therapist	0.12	0.01	0.08	0.00	0.30	0.12
rand_adj	error	2.25	0.00	0.16	1.93	2.58	2.25
rand_adj	therapist	0.12	0.01	0.08	0.00	0.30	0.12
confounding	error	2.25	0.00	0.17	1.93	2.58	2.25
confounding	therapist	0.31	1.61	0.13	0.10	0.59	0.12
adjusted	error	2.25	0.00	0.17	1.93	2.58	2.25
adjusted	therapist	0.12	0.00	0.08	0.00	0.30	0.12

Model	Parameter	Estimate	Rel. bias	SD(Est.)	Est. (2.5%)	Est. (97.5%)	True
ignored	error	2.53	-0.01	0.19	2.16	2.93	2.55
ignored_adjusted	error	2.35	-0.01	0.17	2.03	2.70	2.37

A More Intuitive Interpretation of Therapist Effects

This section presents R code for calculating the overlap measures presented in the manuscript.

Calculate the overlap between therapist distributions

Overlap can be calculated using the examples presented below, and several equivalent parameterizations are shown, using both standardized and raw effect sizes.

```r
# Integration
int_f <- function(x, mu1, mu2, sd1, sd2) {
    f1 <- dnorm(x, mean = mu1, sd = sd1)
    f2 <- dnorm(x, mean = mu2, sd = sd2)
    pmin(f1, f2)
}
cohensd <- 0.2
tot_sd <- sqrt(p$sd_therapist^2 + p$sd_error^2)
ATE <- cohensd * tot_sd
# standardize using therapist SD
z <- ATE / p$sd_therapist
2 * pnorm(-abs(z) / 2)
```

```
[1] 0.6547208
```

```r
# unstandardized
2 * pnorm(
    -abs(ATE) / 2,
    sd = p$sd_therapist
)
```

```
[1] 0.6547208
```

```r
# Integrate unstandardized
integrate(
    int_f,
    -Inf,
    Inf,
    mu1 = 0,
    mu2 = ATE,
    sd1 = p$sd_therapist,
    sd2 = p$sd_therapist
)
```

0.6547209 with absolute error < 1.6e-05

```r
# cohen's d
2 * pnorm(
    -abs(cohensd) / 2,
    sd = p$sd_therapist / tot_sd
)
```

[1] 0.6547208

```r
ICC <- p$sd_therapist^2 / (p$sd_therapist^2 + p$sd_error^2)
2 * pnorm(
    -abs(cohensd) / 2,
    sd = sqrt(ICC)
)
```

[1] 0.6547208

Plot Overlap

Figure 1 visualizes the overlapping therapist effect distributions.

```r
SD <- sqrt(0.05)
mean1 <- 0.2
# create x axis
x_min <- 0 - 3 * SD
```

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```r
x_max <- mean1 + 3 * SD
x <- seq(x_min, x_max, length.out = 2e4)
df_control <- rbind(
    data.frame("x" = x_min, "y" = 0),
    data.frame("x" = x, "y" = dnorm(x, 0, SD)),
    data.frame("x" = x_max, "y" = 0)
)
df_tx <- rbind(
    data.frame("x" = x_min, "y" = 0),
    data.frame("x" = x, "y" = dnorm(x, mean1, SD)),
    data.frame("x" = x_max, "y" = 0)
)
poly_overlap <- data.frame(
    "x" = df_control$x,
    "y" = pmin(df_control$y, df_tx$y)
)
# colors
overlap_fill <- "#2980b9"
u3_fill <- "#3498db"
control_fill <- "#7f8c8d"
treatment_fill <- "#2c3e50"
p0 <- ggplot(
    df_tx,
    aes(
        x,
        y,
        fill = "treatment",
    )
) +
    # fill treatment group
    geom_polygon(
        linewidth = 1,
    ) +
    # fill control group
    geom_polygon(
        data = df_control,
        aes(
            fill = "control"
        ),
        linewidth = 1
    ) +
```

```r
# overlap
geom_polygon(
    data = poly_overlap,
    color = NA,
    fill = overlap_fill,
) +
# line treatment
geom_polygon(
    linewidth = 1,
    color = "white",
    alpha = 0.5,
    fill = NA
) +
# line control
geom_polygon(
    data = df_control,
    linewidth = 1,
    color = "white",
    alpha = 0.5,
    fill = NA
) +
geom_vline(
    xintercept = 0,
    linetype = "dotted"
) +
geom_vline(
    xintercept = mean1,
    linetype = "dotted"
) +
annotate(
    geom = "text",
    label = "Control",
    x = 0,
    y = dnorm(0, 0, SD) * 1.1
) +
annotate(
    geom = "text",
    label = "Treatment",
    x = mean1,
    y = dnorm(mean1, mean1, SD) * 1.1
) +
```

```
    scale_color_manual(
        values = c(
            "control" = control_fill,
            "treatment" = treatment_fill
        )
    ) +
    scale_fill_manual(
        values = c(
            "control" = control_fill,
            "treatment" = treatment_fill
        )
    ) +
    labs(x = "Therapist effects", y = NULL) +
    theme_minimal() +
    theme(
        legend.position = "none",
        panel.grid.minor.y = element_blank(),
        panel.grid.major.y = element_blank(),
        axis.text.y = element_blank()
    )
p0
ggsave("figures/fig_overlap.svg", width = 8, height = 3)
```
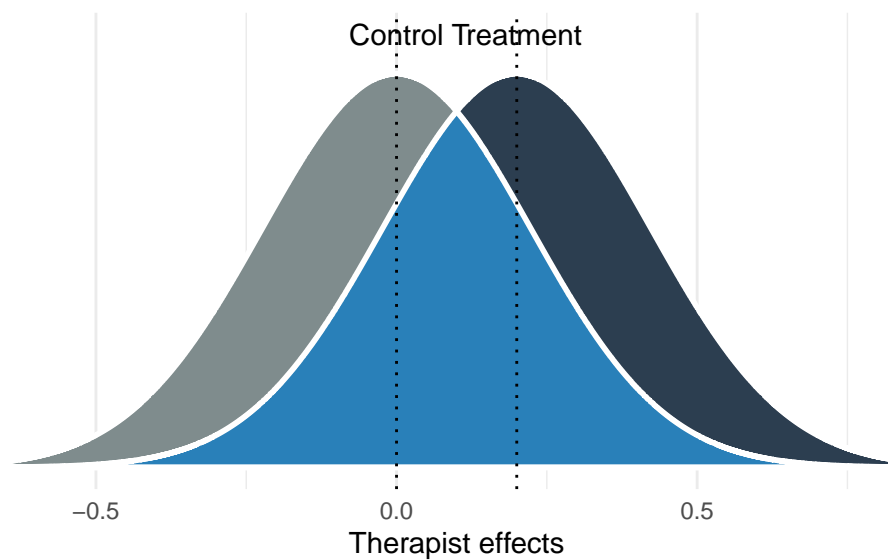

Figure 1: The proportion of therapists from each treatment group that have overlapping effects.

Cohen's U_3

Cohen's U_3 can be calculated using the examples below, and several equivalent parameterizations are shown, using both standardized and raw effect sizes.

```r
# standardize using therapist SD
cohensd <- 0.2
tot_sd <- sqrt(p$sd_therapist^2 + p$sd_error^2)
ATE <- cohensd * tot_sd
# standardize using therapist SD
z <- ATE / p$sd_therapist
pnorm(z)
```

```
[1] 0.8144533
```

```r
# unstandardized effect
pnorm(ATE, sd = p$sd_therapist)
```

```
[1] 0.8144533
```

```r
# cohen's d parameterization
pnorm(
    cohensd,
    sd = p$sd_therapist / tot_sd
)
```

```
[1] 0.8144533
```

```r
ICC <- p$sd_therapist^2 / (p$sd_therapist^2 + p$sd_error^2)
pnorm(
    cohensd,
    sd = sqrt(ICC)
)
```

```
[1] 0.8144533
```

Plot Cohen's U_3

Figure 2 visualizes Cohen's U_3.

```r
SD <- sqrt(0.05)
ES <- 0.2
mean1 <- ES
# create x axis
x_min <- 0 - 3 * SD
x_max <- mean1 + 3 * SD
x <- seq(x_min, x_max, length.out = 2e4)
df_control <- rbind(
    data.frame("x" = x_min, "y" = 0),
    data.frame("x" = x, "y" = dnorm(x, 0, SD)),
    data.frame("x" = x_max, "y" = 0)
)
df_tx <- rbind(
    data.frame("x" = x_min, "y" = 0),
    data.frame("x" = x, "y" = dnorm(x, mean1, SD)),
    data.frame("x" = x_max, "y" = 0)
)
poly_u3 <- rbind(
    data.frame("x" = x_min, "y" = 0),
    poly_overlap[poly_overlap$x <= 0, ],
    data.frame("x" = 0, "y" = 0)
)
# colors
overlap_fill <- "#2980b9"
u3_fill <- "#3498db"
control_fill <- "#7f8c8d"
treatment_fill <- "#2c3e50"
# plot
p0 <- ggplot(
    df_control,
    aes(
        x,
        y,
        fill = "control"
    )
) +
    # fill control group
    geom_polygon(
```

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```
        linewidth = 1,
    ) +
    # fill treatment group
    geom_polygon(
        data = df_tx,
        aes(
            fill = "treatment"
        ),
        linewidth = 1
    ) +
    # overlap
    geom_polygon(
        data = poly_u3,
        color = NA,
        fill = u3_fill
    ) +
    # line control
    geom_polygon(
        linewidth = 1,
        color = "white",
        alpha = 0.5,
        fill = NA
    ) +
    # line treatment
    geom_polygon(
        data = df_tx,
        linewidth = 1,
        color = "white",
        alpha = 0.5,
        fill = NA
    ) +
    geom_vline(
        xintercept = 0,
        linetype = "dotted"
    ) +
    geom_vline(
        xintercept = mean1,
        linetype = "dotted"
    ) +
    annotate(
        geom = "text",
```

```
        label = "Control",
        x = 0,
        y = dnorm(0, 0, SD) * 1.1
    ) +
    annotate(
        geom = "text",
        label = "Treatment",
        x = mean1,
        y = dnorm(mean1, mean1, SD) * 1.1
    ) +
    scale_color_manual(
        values = c(
            "control" = control_fill,
            "treatment" = treatment_fill
        )
    ) +
    scale_fill_manual(
        values = c(
            "control" = control_fill,
            "treatment" = treatment_fill
        )
    ) +
    labs(x = "Therapist effects", y = NULL) +
    theme_minimal() +
    theme(
        legend.position = "none",
        panel.grid.minor.y = element_blank(),
        panel.grid.major.y = element_blank(),
        axis.text.y = element_blank()
    )
p0
ggsave("figures/fig_u3.svg", width = 8, height = 3)
```
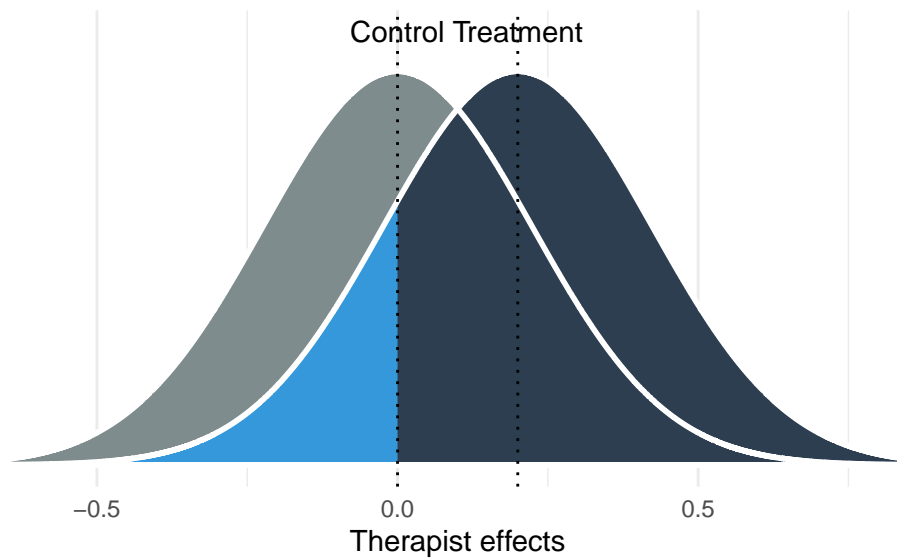
Figure 2: A visualization of Cohen's U3, the proportion of the therapists in the treatment group with causal effects above the average therapist in the control group

Probability of Superiority

The probability of superiority can be calculated using the examples below, and several equivalent parameterizations are shown, using both standardized and raw effect sizes.

```
cohensd <- 0.2
tot_sd <- sqrt(p$sd_therapist^2 + p$sd_error^2)
ATE <- cohensd * tot_sd
z <- ATE / p$sd_therapist
# standardize using therapist SD
pnorm(z / sqrt(2))
```

[1] 0.7364554

```
# raw ES
pnorm(ATE / sqrt(2), sd = p$sd_therapist)
```

[1] 0.7364554

```r
# cohen's d
ICC <- p$sd_therapist^2/(p$sd_therapist^2 + p$sd_error^2)
pnorm(
    cohensd / sqrt(2),
    sd = p$sd_therapist / tot_sd
)
```

```
[1] 0.7364554
```

```r
pnorm(
    cohensd / sqrt(2),
    sd = sqrt(ICC)
)
```

```
[1] 0.7364554
```

```r
# using simulation
n <- 1e5
cc <- rnorm(n, 0, p$sd_therapist)
tx <- rnorm(n, ATE, p$sd_therapist)
mean(tx > cc)
```

```
[1] 0.73714
```

Table

Table 4 show the overlap measures for a range of treatment effects and ICCs. σ_e^2 is constant, so only the treatment and therapist effects are varied.

```r
icc <- c(0.01, 0.05, 0.1, 0.2)
d <- c(0.2, 0.5, 0.8)
sigma_error2 <- 1
sigma_u2 <- get_therapist_sd_from_icc(icc, sigma_error2)^2
grid <- expand_grid(d, sigma_u2)
grid <- grid %>%
    mutate(
        sigma_error2,
```

```r
        icc = sigma_u2 / (sigma_u2 + sigma_error2),
        ate = d * sqrt((sigma_u2 + sigma_error2))
    )
map2_dfr(
    grid$d,
    grid$icc,
    function(d, icc) {
        data.frame(
            d = d,
            icc = icc,
            overlap = 2 * pnorm(-abs(d) / 2, sd = sqrt(icc)),
            u3 = pnorm(d, sd = sqrt(icc)),
            prob_superiority = pnorm(d / sqrt(2), sd = sqrt(icc))
        )

    }
) %>%
mutate(
    overlap = round(overlap * 100, 0),
    u3 = round(u3 * 100, 0),
    prob_superiority = round(prob_superiority, 2),
) %>%
left_join(
    select(grid, ate, d, icc, sigma_error2, sigma_u2),
    by = c("d", "icc")
) %>%
arrange(d, icc) %>%
rename(
    "Overlap (%)" = overlap,
    "$U_3$ (%)" = u3,
    "Pr. superiority" = prob_superiority,
    "Cohen's *d*" = d,
    "ICC" = icc,
    "ATE" = ate,
    "$\\sigma_e^2$" = sigma_error2,
    "$\\sigma_u^2$" = sigma_u2
) %>%
kable(digits = 2)
```

Table 4: Interpreting Therapist Effects Using Overlap Measures

Cohen's d	ICC	Overlap (%)	U_3 (%)	Pr. superiority	ATE	σ_e^2	σ_u^2
0.2	0.01	32	98	0.92	0.20	1	0.01
0.2	0.05	65	81	0.74	0.21	1	0.05
0.2	0.10	75	74	0.67	0.21	1	0.11
0.2	0.20	82	67	0.62	0.22	1	0.25
0.5	0.01	1	100	1.00	0.50	1	0.01
0.5	0.05	26	99	0.94	0.51	1	0.05
0.5	0.10	43	94	0.87	0.53	1	0.11
0.5	0.20	58	87	0.79	0.56	1	0.25
0.8	0.01	0	100	1.00	0.80	1	0.01
0.8	0.05	7	100	0.99	0.82	1	0.05
0.8	0.10	21	99	0.96	0.84	1	0.11
0.8	0.20	37	96	0.90	0.89	1	0.25