

# Basic Data Structures

## *Stacks, Queues and Lists*

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- 1 Introduction
- 2 Primitive Data Types
- 3 Composite Data Types
- 4 Abstract Data Types
  - Stacks
  - Queues
  - Pointers
  - Lists

# When the world is built...

'Let there be data.'

'Okay, now we have the data. Let us store the data... wait, where is my list?'

'What? There's no list in this programming language?! I have to create one!'

# From the origin

In Python, when we are going to store some values (e.g. randomly generate some numbers), we usually use the codes below:

## Python: Generate Random Numbers

```
list_0=[]  
for i in range(10):  
    list_0.append(random.randint(1,100))
```

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In Python, when we are going to store some values (e.g. randomly generate some numbers), we usually use the codes below:

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for i in range(10):  
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```

Here we used an abstract data structure, namely a list ( $list_0$ ). How are the ten numbers stored?

# From the origin

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$i = 0$ :  $n_0$

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$i = 0$ : 

$n_0$
-------

$i = 1$ : 

$n_0$	$n_1$
-------	-------



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So the way that we place data into a list is appending a cell at the end each time.

# Different Data Types

We are going to talk about three main classes of data types:

- Primitive Data Types (PDT)
- Composite Data Types (CDT)
- Abstract Data Types (ADT)

The lower is the more advanced.

# Primitive Data Types

## Primitive Data Types (PDT)

Primitive Data Types usually indicate the most basic types that a programming language could provide. They are something like a building brick in Mine-craft.

On the next page, you are going to guess what names of those types are. But don't worry, you know them!

# List of Primitive Data Types

Type	Example



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	2, 3, 4, 5, 6, 7

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For example, an array is a composite data types, which is known as matrix in Mathematics. The difference between an array and other primitive types is that an array can get more than one value. (For an integer variable, it can only contain one value, namely the number. But for an array, it can have several cells of values, just as a matrix can have  $m \times n$  numbers. )

$n_{11}$	$n_{12}$	$n_{13}$
$n_{21}$	$n_{22}$	$n_{23}$
$n_{31}$	$n_{32}$	$n_{33}$

0.1	0.2	0.3
0.4	0.5	0.6
0.7	0.8	0.9

'A'	'B'	'C'
'D'	'E'	'F'
'G'	'H'	'I'

The array above contains 9 values, which are 9 integers (or floats, or anything else).

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In contrast to float type, a complex type ( $a + bi$ ) is a CDT. (Why?)



All class types are CDTs.

A class defines an object with two things: Attributes (data) and methods (functions). Since it can be constructed to comprise two or more attributes, it is a CDT.

(You may wish to revisit Niko's slides upon OOP)

# Abstract Data Types

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Using a metaphor, an ADT is like an algebraic structure (e.g. groups, rings) in Mathematics.

Don't be frightened by 'Mathematics'! You are only required to learn specific structures at this stage.

We are going to talk about three important ADTs: Stacks, queues and linked lists.

## Stack

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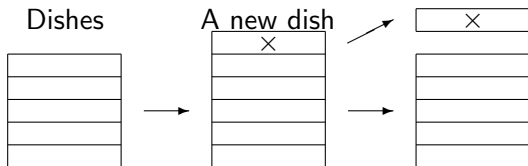
Imagine you have stacked dishes in your cupboard and you are going to take a dish for dinner. You can't get the one at the bottom of the dish stack, otherwise you will break all dishes it underpins. The only dish you can take is the one at the top, which is the last one you have stacked. Now obeying LIFO rule, the dish stack can be viewed as a stack.

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Only the last dish put in could be the first to take out.

# Stack Operations

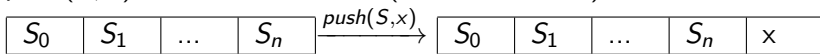
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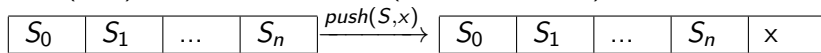
- $push(S, x)$ : INSERT  $x$  into  $S$ . (At the end of  $S$ )



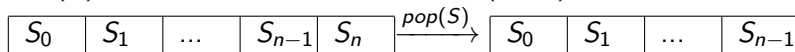
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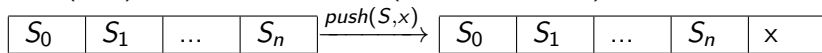
- $pop(S)$ : DELETE an element from  $S$ . (LIFO)



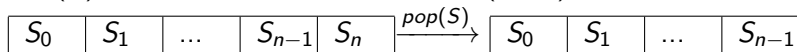
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- $isEmpty(S)$ : Return *True* if the stack is empty, otherwise return *False*.

# Stacks in Python

In Python, we can construct a stack class in Python using the following code:

## Python: Stack class

```
class Stack:
    def __init__(self):
        self.items = []

    def isEmpty(self):
        return self.items == []

    def push(self, item):
        self.items.append(item)

    def pop(self):
        return self.items.pop()
```

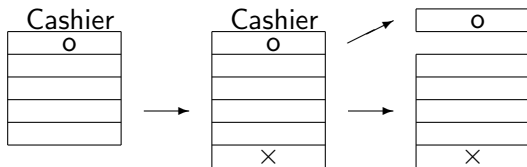
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Imagine you have a queue of consumers waiting to pay a cashier. First come first served, the consumer has come first can leave earlier than other consumers. Now obeying FIFO rule, the waiting queue can be viewed as a queue.



Even if a new consumer comes, the first consumer still leaves the first.

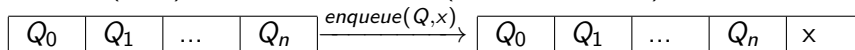
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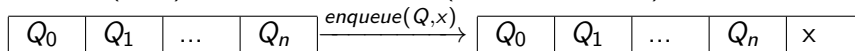




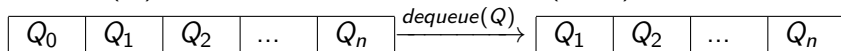
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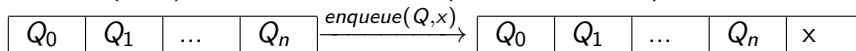
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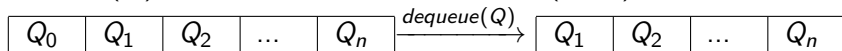
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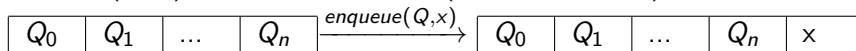


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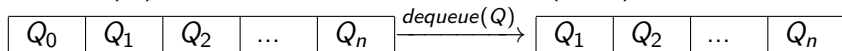
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How to construct a queue in Python is left as an exercise. (Pretty similar to the stack class. )

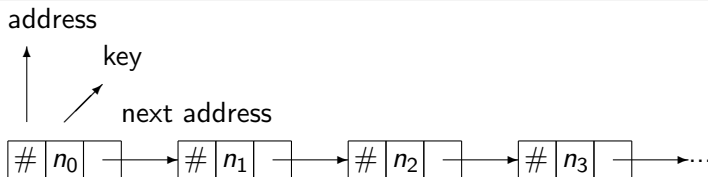
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A set of pointers is like a hint-hunt game, where in every step you can find the hint to the next treasure, and next treasure at hinted location (address) contains gold (key) and the next hint (next address) to another treasure. In this chain every treasure could be found.

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- In some languages, the number of elements in an array must be declared prior to using it. (And unable to expand.) In lists you can expand as long as memory permits!
- The idea of arrays and that of lists are different. In an array all data are stored in cells which are continuous neighbours, but in a list all data are only linked and are not necessarily next to each other (Recall the pointers!).



## Linked Lists

A linked list is an abstract data type that represents a sequence of values. Lists are like sequences in Mathematics.

In the context of this talk we define linked lists to be the linked chains of pointers.

## Linked Lists

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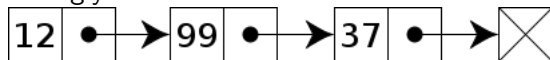
## Doubly Linked Lists

In a doubly linked list, each node contains, besides the next-node link, a second link field pointing to the previous node in the sequence, called **prev** (Previous).

# Graphic Illustration

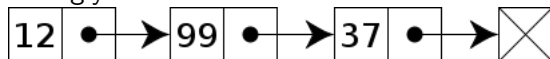
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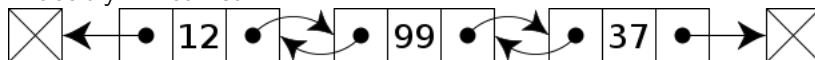


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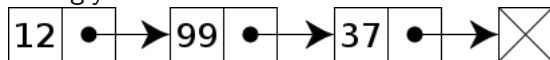


A doubly linked list:

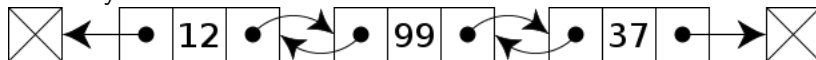


# Graphic Illustration

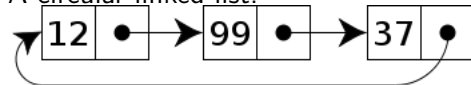
A singly linked list:



A doubly linked list:



A circular linked list:



# The End