Problem Statement

For the processes and controllers below, analyze the stability of cascade control system

Primary Process: $G_{p1} = \frac{1.0\exp(-4s)}{(10s+1)}$

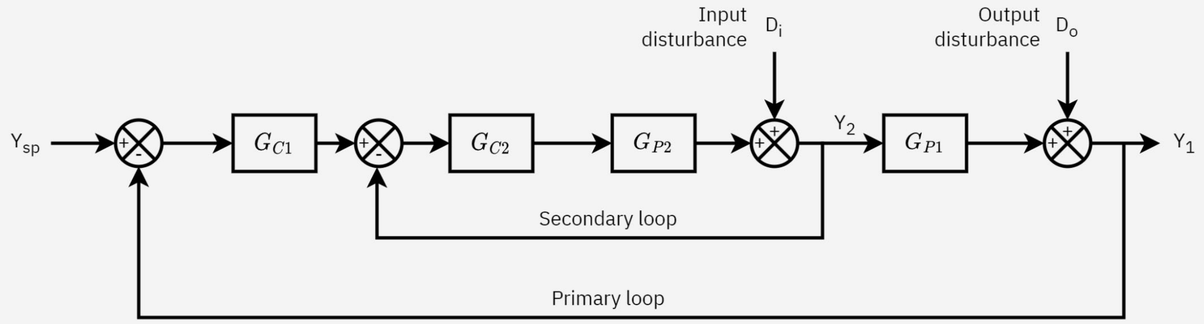
Secondary Process: $G_{p2} = \frac{5\exp(-s)}{2s+1}$

Primary Controller: $G_{c1} = 2.5\left(1 + \frac{1.0}{5s}\right)$

Secondary Controller: $G_{c2} = 0.1$

Solution

We analyze the primary and secondary loops separately. First we close the secondary loop and assess its behavior. Then we analyze the primary loop.



- Primary process

$$G_{p1} = \frac{K_{p1}e^{-\theta_1 s}}{\tau_1 s + 1}$$

- Secondary process

$$G_{p2} = \frac{K_{p2}e^{-\theta_2 s}}{\tau_2 s + 1}$$

- Primary controller

$$G_c = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

- Secondary controller

$$G_c = K_c$$

```
% G_p1
kp1 = 1.0;
theta1 = 4;
tau1 = 10;

num = [kp1];
den = [tau1 1];

gp1 = tf(num, den, 'InputDelay', theta1)
```

```
gp1 =

                  1
  exp(-4*s) * --------
              10 s + 1

Continuous-time transfer function.
Model Properties
```

```
% G_p2
kp2 = 5.0;
theta2 = 1;
tau2 = 2;
gp2 = tf([kp2], [tau2 1], 'InputDelay', theta2)
```

```
gp2 =

                  5
  exp(-1*s) * -------
              2 s + 1

Continuous-time transfer function.
Model Properties
```

```
% G_c1
s = tf('s');

kc1 = 2.5;
tauI1 = 5.0;
gc1 = 2.5 * (1 + 1/(tauI1*s))
```

```
gc1 =

  12.5 s + 2.5
  ------------
      5 s

Continuous-time transfer function.
Model Properties
```

```
% G_c2

kc2 = 0.1;
gc2 = tf([kc2])
```

```
gc2 =
```

```
  0.1

Static gain.
Model Properties
```

```
% Calculate KL2 = K_c2*K_p2
KL2 = dcgain(gc2) * dcgain(gp2)
```

```
KL2 = 0.5000
```

Secondary loop analysis

```
% Inner loop characteristic equation
T_inner = 1 + gc2*gp2
```

```
T_inner =

  A =
         x1
   x1  -0.5

  B =
        u1
   x1  0.5

  C =
        x1
   y1  0.5

  D =
       u1
   y1   1

  (values computed with all internal delays set to zero)

  Internal delays (seconds): 1
```

```

% Returns a state space. We need to approximate delay time

% Simplified approximation e^(-θs) = 1-θs
delay = 1 - theta2* s;
gp2_approx = (kp2*delay)/(tau2*s + 1)
```

```
gp2_approx =

  -5 s + 5
  --------
  2 s + 1

Continuous-time transfer function.
Model Properties
```

```

Gcl2_approx = feedback(gc2*gp2_approx,1)
```

```
Gcl2_approx =

  -0.5 s + 0.5
  ------------
  1.5 s + 1.5

Continuous-time transfer function.
Model Properties
```

```
den = Gcl2_approx.Denominator{1};  % Extract denominator coefficients
poles = roots(den)
```

```
poles = -1
```

```
isstable(Gcl2_approx)
```

```
ans = logical
   1
```

```
% First order Pade approximation e^(-θs) = [1-(θ/2)s]/[1+(θ/2)s]
gp2_pade = pade(gp2,1)
```

```
gp2_pade =

      -5 s + 10
  ---------------
  2 s^2 + 5 s + 2

Continuous-time transfer function.
Model Properties
```

```
Gcl2_pade = feedback(gc2*gp2_pade,1)
```

```
Gcl2_pade =

      -0.5 s + 1
  -----------------
  2 s^2 + 4.5 s + 3

Continuous-time transfer function.
Model Properties
```

```
den = Gcl2_pade.Denominator{1}  % Extract denominator coefficients
```

```
den = 1×3
    2.0000    4.5000    3.0000
```

```
poles = roots(den)
```

```
poles = 2×1 complex
  -1.1250 + 0.4841i
  -1.1250 - 0.4841i
```

```
isstable(Gcl2_pade)
```

```
ans = logical
   1
```

Primary loop analysis

```
% Augmented primary process

gp1_pade = pade(gp1,1)
```

```
gp1_pade =

       -s + 0.5
  ------------------
  10 s^2 + 6 s + 0.5

Continuous-time transfer function.
Model Properties
```

```
Gpa = Gcl2_pade*gp1_pade
```

```
Gpa =

           0.5 s^2 - 1.25 s + 0.5
  ----------------------------------------
  20 s^4 + 57 s^3 + 58 s^2 + 20.25 s + 1.5

Continuous-time transfer function.
Model Properties
```

```
Gcl1_pade = feedback(gc1*Gpa,1)
```

```
Gcl1_pade =

            6.25 s^3 - 14.38 s^2 + 3.125 s + 1.25
  ----------------------------------------------------------
  100 s^5 + 285 s^4 + 296.2 s^3 + 86.88 s^2 + 10.62 s + 1.25

Continuous-time transfer function.
Model Properties
```

```
den = Gcl1_pade.Denominator{1}  % Extract denominator coefficients
```

```
den = 1×6
  100.0000  285.0000  296.2500   86.8750   10.6250    1.2500
```

```


poles = roots(den)
```

```
poles = 5×1 complex
  -1.2360 + 0.6742i
  -1.2360 - 0.6742i
  -0.2955 + 0.0000i
  -0.0413 + 0.1401i
  -0.0413 - 0.1401i
```

```
isstable(Gcl1_pade)
```

```
ans = logical
   1
```