Letter to Director

Dear Sir/Madam,

Thank you for hiring us to find the best player for one individual sport. We have decided to investigate the G.O.A.T. of F1.

Formula 1 (F1) is the premier league of racing, watched by over 400 million fans across the globe and bringing in more than 2 million USD yearly. Races are held on tracks in more than 15 countries and attract fierce competition from well-funded teams like Ferrari and Mercedes. In 2020, Lewis Hamilton won his seventh F1 title, equalling the record set by Michael Schumacher, a previous champion of the sport. So, which of them is the G.O.A.T (Greatest of All Time).

To accurately determine the true G.O.A.T. of F1, we constructed a model that took into account the following factors:

- Scores of drivers adjusted for changes to scoring system, to account for inflated points of contemporary drivers so as to not disfavour drivers of the past.
- Scores of teams adjusted for improvements in engine and car design, to account for effect of cars on drivers' performances so as to not disfavour drivers from less well-funded teams with less access to advanced machinery.
- Competitive balance of players and teams in a given year, to account for level of playing field.

Player	Our Rank	Real Rank
Michael Schumacher	1	6
Lewis Hamilton	2	1
Sebastian Vettel	3	2
Fernando Alonso	4	3
Kimi Raikkonen	5	4

According to our results, the rankings of F1 drivers are largely similar to their rankings if only raw points were considered. Our model gives Michael Schumacher as the top rated driver and Hamilton second, even though both are tied in terms of F1 titles. Schumacher spent his career in many different teams, some of which are not as well funded, so his score might have been artificially lowered by poorer average team support.

In general, our model is a good gauge of a driver's ability. Even though we are trying to determine which driver is the absolute G.O.A.T., in reality there is little difference between the top drivers' abilities, as we believe each driver possesses unique qualities that make each of them special in their own way. As the acclaimed driver Sebastian Vettel once said, S̈imply racing in a Formula One car is an achievement." For all the focus we put on determining the G.O.A.T., sometimes we ought to remember the difficulties many drivers have to overcome just to be able to drive, and salute them for their relentless efforts.

Yours sincerely, Team IMMC

Contents

1 Introduction

In sports, the No. 1 position has always been the most coveted (Hochbaum, 2006), being the subject of countless athletes' efforts and aspirations. Rankings from established sports websites such as ESPN are widely held as a good gauge of athletes' abilities (Smith, 2009). In men's tennis for example, Roger Federer, Rafael Nadal and Novak Djokovic are commonly seen by tennis fans and observers alike as the top three players in contemporary times (Radicchi, 2011). But which player can claim to be the *greatest* is a topic that sparks heated debate among sport fans, commentators and athletes alike. Is Tom Brady or Jimmy Anderson the best National Football League (NFL) player? LeBron James or Michael Jordan for basketball? What about Lee Chong Wei and Lin Dan in badminton? These are questions that will not have clear answers for years to come.

From a technical perspective, the question of which player can be considered the Greatest of All Time (G.O.A.T.) seems deceptively simple. At first glance, it seems that comparing the score of different players from different eras is sufficient. However, one needs to consider that other factors such as change in scoring system, quality of equipment and game rules have varied over the years.

In Formula One (F1) for example, modern drivers have had access to more advanced racing cars than the pioneering drivers of the 1950s (Economist, 2018). Clearly, to accurately compare contemporary players of a sport with players past, such factors must be considered in order to avoid disadvantaging the later. In light of this, our team embarked on the assignment to determine what makes a player the G.O.A.T. and how to measure "Greatness".

We acknowledge that "Greatness" is not only based on personal accomplishment but also should include the athletes' impact on their sports or the contribution beyond the playing field (Kreider, 2021). However, in this paper, we define the "Greatness" largely in terms of athletes' performance in a particular sport during their career, as it is indeed of greater importance compared to other factors and athletes' performance data is more quantifiable and objective.

1.1 Problem Restatement

Task 1- warm-up: Based on the 2018 Grand Slam results, find ways to determine the best female player.
Task 1 (a): Identify key factors and develop models to determine the best female tennis player 2018.
Task 1 (b): Subsequently, use our model to choose the best player and analyse the result.

Task 2 (a): By choosing another sport other than women's tennis, identify significant factors and gather data to develop a model to determine the G.O.A.T of that sport.
Task 2 (b): Discuss necessary changes needed to make our G.O.A.T. model from Task 2a applicable to ANY individual sport. Such changes include accounting for factors present in other sports but not in the our chosen sport form Task 2a.

Task 3: Identify the differences between team sports and individual sports and discuss the changes needed to allow our model to be able to determine the G.O.A.T. of team sports accurately.

2 Overall Assumptions

Assumption 1: The determination of the athletes' rankings is largely based on their performances in the recorded data.
Justification: It is difficult to quantify an athlete's reputation and image or contribution to the sport or the society at large, even though that might be worthy of consideration if we were interested to find out who the public believes to be the greatest player ever.

Assumption 2: Published rankings of athletes by media outlets reflects their actual abilities relative to one another.
Justification: Rankings are published by trusted sources such as ESPN who have an established record in sports coverage and are corroborated by other outlets as well. As such, we can use the established ranking as a way to assess whether our model is reliable.

Assumption 3: Injured athletes would not perform better without injuries.
Justification: It is difficult to forecast how an athlete would have performed had one participated in the competition or played the game without the injury, as factors such as scale of injury have to be taken into account which are difficult to quantify.

Assumption 4: The talent pool of a particular sport does not change significantly across the years, which means it is not affected by the change in affluence of countries, popularity of the sport or population growth.
Justification: The most recent example will be e-sports for which there has been a rise in its popularity, especially during the pandemic. As a result, the competitiveness of the sport as a whole increases and thus it is harder for a player to win the championship (Economist, 2020). However, for most sports, it is unrealistic to account for their historical popularity and quantify it in reasonable terms.

3 Notations

Symbol	Meaning
RS	a score used to indicate the greatness of player
GSRS	Rank Score for Gram Slam Model
ATRS	All Time Rank Score
G	criterions for the Grand Slam Model
C	criterions for tournament ranking
F	criterions for G.O.A.T Model (Formula 1)

4 Task 1: Best Women's Tennis Player 2018

4.1 Background

The widely anticipated four Grand Slams (Australian Open, French Open, Wimbledon and US Open) that are held yearly represent the highest level of competition for women's tennis (Pollard, 2006). Each tournament requires the player to play with a draw of 128 competitors within a period of two weeks to win the finals' trophy or higher ranking points for prizes (Corral, 2009). Together, the Grand Slams serve as important indicators of individual players' skills as they are pitted against acclaimed opponents with differing playing styles (Hizan et al, 2015). However, Grand Slam playoffs cannot be the sole factor considered when evaluating each player's ability (Cui et al, 2018). In order to determine the greatest tennis player of 2018, other factors ought to be considered.

4.2 Assumptions and Justifications

Assumption 1: The 2018 tournament performances of players are largely reflective of their actual abilities, with the luck factor taken into account.
Justification: Although situations could arise in which an athlete randomly meets another player of significantly higher skill level in a tournament, leading to her early elimination and reducing her score, such situations are relatively unlikely. Furthermore, given the large number of matches in the Grand Slam, the impact of such anomalies would be minimised in the average.

Assumption 2: All top tennis athletes competed in all four majors in the Grand Slam and those who names did not appear were assumed to be eliminated before reaching the round of 16.
Justification: Considering the prestige and prize pools of majors in Grand Slams, it is reasonable to assume that all established players would compete in these competitions.

Assumption 3: Factors such as style, techniques, handedness, age or external factors like equipment and umpire bias are not considered.
Justification: We believe the performance of athletes is of the greatest importance, and should form the basis of our evaluation of their abilities.

4.3 Model Development

In order to quantitatively analyse a performance, we construct a model to find the best player. To begin with, we will find relevant factors relating to player performance and generate an $a \times b$ evaluation matrix E , where a is the number of the selected players, b is the number of the factors and x_{ij} represents the value of i player's jth factor.

$$E = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1b} \\ x_{21} & x_{22} & \cdots & x_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ x_{a1} & x_{a2} & \cdots & x_{ab} \end{bmatrix}$$

Then, we perform **normalization** to translate the evaluation matrix E to E' to ensure that all factors are on the same scale. We denote the normalized factor value as y_{ij}.

$$y_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{a} \tilde{x}_{ij}^{2}}}, \ i = 1, ..., a, \ j = 1, ..., b$$

Next, we chose to use the *Entropy Weight Method* to determine the relative importance of different factors based on the normalised evaluation matrix E'.
Firstly, the standardized value of the jth factor of the i player is given by:

$$s_{ij} = y_{ij} / \sum_{i=1}^{a} y_{ij}$$

Secondly, the entropy value of the jth factor is defined as:

$$t_j = -\sum_{i=1}^{a} t_{ij} \ln(t_{ij}) / \ln a$$

Thirdly, the weight of each factor will be:

$$w_j = (1 - t_j) / \sum_{j=1}^{b} (1 - t_j)$$

In order to evaluate the performance of individual player, the *Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)* is used to compare different players. **Firstly,** we define V_j^+ and V_j^- as the maximum and minimum value in the jth column of matrix E'.

$$V^+ = \left(V_1^+, ..., V_a^+\right) = \left(\max\{y_{11}, ..., y_{b1}\}, ..., \max\{y_{1a}, ..., y_{ab}\}\right)$$

$$V^- = \left(V_1^-, ..., V_a^-\right) = \left(\min\{y_{11}, ..., y_{b1}\}, ..., \min\{y_{1a}, ..., y_{ab}\}\right)$$

Secondly, we define S_i^+ and S_i^- as the *weighted Euclidean distance* from the ith player to the best possible factor and the worst possible factor respectively.

$$S_i^+ = \sqrt{\sum_{j=1}^{a} w_j \left(V_j^+ - y_{ij}\right)^2}, \ S_i^- = \sqrt{\sum_{j=1}^{a} w_j \left(V_j^- - y_{ij}\right)^2}$$

Finally, the rank score RS_i of each player can be computed as follows:

$$RS_i = \frac{T_i}{\sum_{i=1}^{b} T_i}$$

where

$$T_i = \frac{S_i^-}{S_i^+ + S_i^-}$$

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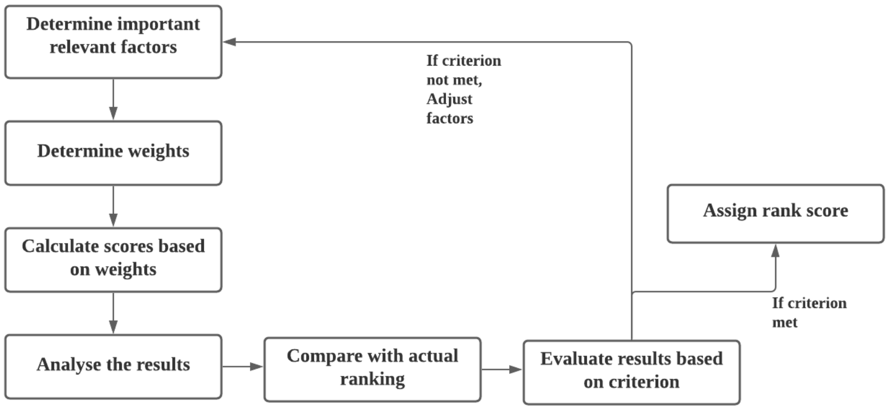

Figure 1: Flowchart of our development of the Model

4.4 Grand Slam Model

In order to determine who performed the best in the four grand slam competitions, we consider the following factors:

Meaning	Symbol	Weights
Points Earned in the Grand Slam	G_1	0.4575
Strength of Schedule	G_2	0.5425

Points Earned by Player in the Grand Slam: The Grand Slam constitutes 4 single-elimination tournaments. The further a player progresses in the tournament, the higher her score is, as her progression shows that she is able to win against increasingly skilled players.

Strength of Schedule: This is to account for the relative difficulty of matches. A victory against a seemingly stronger player will be much more significant than a victory against a much weaker player in the eyes of our model.

Grand Slam Rank Score (GS Rank Score): we calculate the GSRS as the numerical value that will determine the ranking of the players solely based on the 2018 Grand Slam results.

The results of our model is displayed as follow:

Player	G_1	G_2	GS Rank Score	Rank	Real Rank
S. Halep	2	2.5	1.00	1	1
S. Williams	1.75	1.75	0.75	2	16
A. Kerber	2	1.5	0.70	3	2
M. Keys	1.5	1.6875	0.69	4	17
N. Osaka	1.25	1.5625	0.61	5	5
S. Stephens	1.25	1.5	0.59	6	6
C. Wozniacki	1.25	1.125	0.49	7	3
E. Mertens	0.75	1.3125	0.44	8	12
D. Kasatkina	0.75	1.1875	0.41	9	10
K. Pliskova	0.75	1.1875	0.41	9	8

Comparing the top 10 players as generated by our model with the rankings provided by

Women's Tennis Association (WTA), we observed that the predicted ranking of most athletes like S. Halep, A. Kerber, N. Osaka, D. Kasatkina and K. Pliskova are similar to the actual rankings.

Some of the anomalies are explained:
(i) For players whose predicted ranking is higher than WTA ranking:
Since our model only took into account Grand Slam results, we gave players who performed well in the Grand Slam a more favourable ranking. However, their actual rankings may be lower as they might have participated in other competitions (which also contribute to their WTA ranking) in which they may not have performed as well in.

- (14 positions higher): S. Williams performed well in the 4 majors in Grand Slam, and was 1st runner up for both *Wimbledon* and *US Open*.
- (13 positions higher): M. Keys had a consistent performance during 4 majors with her qualifying for 2 *semifinals* and 1 *quarterfinal*, thus she is ranked 4th in our model.
- (4 positions higher): E. Mertens qualified for *Australian Open Semifinal* and lost to the eventual Champion C. Wozniacki. In addition, she qualified for two fourth rounds in the majors in the Grand Slam.

(ii) For players whose predicted ranking is lower than WTA ranking:
Likewise, since our model only took into account Grand Slam results, we gave players who performed poorly in the majors in Grand Slam a less favourable ranking. Their actual rankings may be higher as they also participated in other competitions (which contribute to their WTA ranking) and may have fared better in those competitions.

- (No.7 by WTA ranking): P. Kvitova, who did not appear in any of the Grand Slam results at all (meaning she did not qualify for the 4th round), became the first three-time champion in *Madrid Open* history.
- (19 positions lower): E. Svitolina won her biggest title of her career then in *2018 BNP Paribas WTA Finals Singapore*, defeating S.Stephens. The competition is not among the four majors.
- (6 positions lower): K. Bertens defeated then world No.1 S. Halep and won *Cincinnati Final*.
- (4 positions lower): C. Wozniacki won the championship in *Australian Open* in the Grand Slam. However, she did not qualify at all for 2 other majors.

4.5 2018 Overall Model

In this model, we aim to investigate who is the best women player in 2018 by looking at all competitions in 2018. As such, the difficulty of each competition is an important factor to be considered. In tennis, the four majors are considered to be the most important games among all competitions, so it is more difficult to win a grand slam championship than a non-grand slam one.

To begin with, we want to determine the weight of competition based on some key factors about tennis competition:

Meaning	Symbol	Weight
Number of Participants	C_1	0.2773
Winner Average Points	C_2	0.2004
Winners' Average Rank	C_3	0.2332
Losers' Average Points	C_4	0.1259
Losers' Average Rank	C_5	0.1632

Weights of Competition w_i: As participants in each game varies, different games should have different weights to indicate its competitiveness.

Number of Participants: In general, when more players participate in a tournament, the tournament is taken to be more prestigious. However, there will be *exceptions* like the BNP Paribas WTA Finals which only has 8 players even though it is highly prestigious.

Winner Average Points /Losers' Average Points: Average sum of points earned by winners and losers in each match in a tournament respectively. This might reflect the competitiveness of the competition.

Winners' Average Rank /Losers' Average Rank: Average rank of winners and losers in each match in a tournament respectively. This is a good indicator of the prestige of the tournament.

Competition Points Earned p_i: Players who win a tournament are given 100 points, while those who lose in finals, semifinals, quarterfinals, round of 16 and round of 32 are awarded 60, 34, 18, 8 and 5 points respectively. These ratios reflect the points that are given to players in an actual tournament.

tournament	C_1	C_2	C_3	C_4	C_5	w_i	Rank
BNP Paribas WTA Finals	8	4039	4463	5	7	0.0304	1
French Open	128	2481	1303	85	51	0.0289	2
Australian Open	128	2245	1288	88	53	0.0284	3
US Open	128	2106	1339	91	49	0.0284	4
Wimbledon	128	1831	1355	78	56	0.0281	5
BNP Paribas Open	96	2165	1660	58	50	0.0272	6
Mutua Madrid Open	64	2974	1986	44	28	0.027	7
Sony Ericsson Open	96	2013	1582	69	57	0.0263	8
Internazionali BNL d'Italia	56	3034	2003	49	22	0.0263	9
China Open	61	2774	1963	48	27	0.0261	10

In this table, the four majors all receive high score as expected. One anomaly here is the BNP Paribas WTA Finals, and since it is a final with only 8 top players playing, this competition is assigned a very high score.
Similarly, we have used *Entropy Weight Method* to determine weights of factors and *TOPSIS* to derive W_i. Finally, we calculate the rank score of each player:

$$RS = \sum w_i * p_i$$

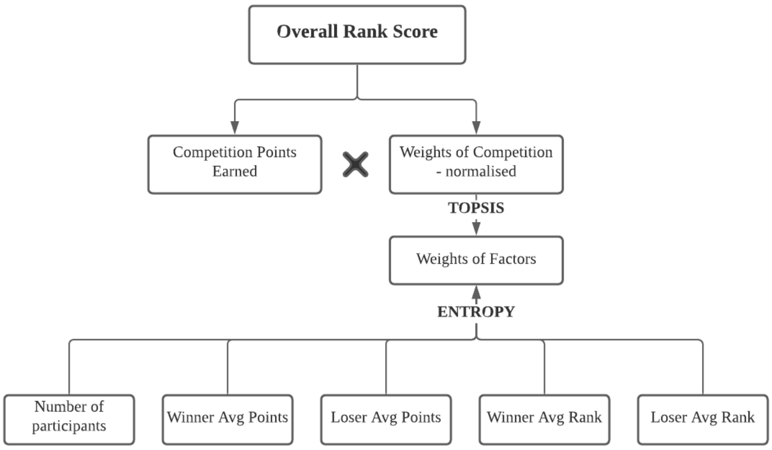

Figure 2: Flowchart of our 2018 Overall Model

4.6 Model Evaluation

We present the results of our model as Fig.3 in which the Win Rate is plotted against the **Overall Rank Score**. The Win Rate is the percentage of the competitions won by the players for which we believe it is an important indication of one's ability.
In our results, S.Halep is the overall best female tennis player in 2018. For the rest, E. Svitolina, K.Pliskova, C.Wozniacki, P.Kvitova, A.Kerber, K.Bertens, N.Osaka, S.Stephens, A.Sabalenka are the top 10 in the predicted ranking by our model which is the same Top. 10 as ranking given by Women Tennis Association. In this case, our model is comparatively accurate in determining the best player.

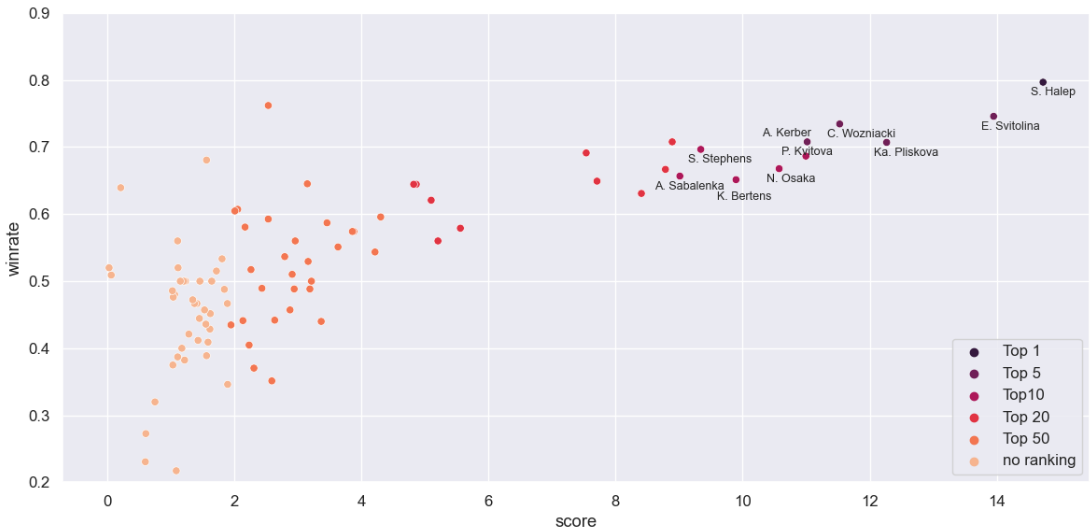

Figure 3: Predicted players ranking in 2018

5 Task 2(a): Determining the G.O.A.T of Formula One

5.1 Background

Formula One (F1) is the highest class of international auto racing for single-seat formula racing cars authorised by the Fédération Internationale de l'Automobile (FIA). It has been one of the premier forms of racing globally since 1950. Drivers compete on both street courses and specially designed racing courses, with race venues scattered across the world.

However, since the 1950s, racing technology has greatly advanced (Economist, 2020). For example, the 1950 Alfa Romeo 158 had 700 brake horsepower, compare to the 2020 Alfa Romeo C39 which has over 1000 brake horsepower. This, along with improvements of track surface, reduction in time for refuelling, has greatly raised the playing field (Stuart, 2020).

5.2 Assumptions and Justifications

Assumption 1: The model assumes that the races that drivers "Do Not Finish" (DNF) are due to mechanical errors and hence they are excluded from our calculations.
Justification: It is extremely difficult to determine whether DNF results more from mechanical errors or from rough driving caused by drivers negligence.

Assumption 2: The average abilities of F1 racers have remained largely similar over the years.
Justification: The difference between racing records over time is caused by improvements in technology, and the innate abilities of racers have been largely similar. Therefore, the likelihood of players winning during a period can be calculated by observing if there are any dominant players in the field during that era.

Assumption 3: We do not take into account the driver's fastest lap when determining driver's ability. Consequently, we do not award players with an extra point for completing race with the fastest timing in any lap.
Justification: Due to technological advancements, the contemporary drivers will be favored because they can achieve a shorten duration with the aid of better car performance.

Assumption 4: Location of racetrack does not matter
Justification: It is highly unlikely that that drivers will fare worse on the Singapore track as compared to the Monaco track, for example

Assumption 5: The circuit on which the Formula one competition is held on does not matter.
Justification: It does not matter whether the competition is held on street circuit or road circuit.

5.3 G.O.A.T Model

In order to determine the G.O.A.T., it is not enough for us to focus on few competitions or player performance at particular year. Over the years, the rule of playing may have changed significantly, and the talent pool never remains the same. It seems that every era has its own "G.O.A.T", and it is impossible for those star players to compete against each other. As such, we need to find ways to compare players from different eras and evaluate their individual player performance.

Moreover, in our model, we need take into consideration other factors that might affect the performance of drivers i.e. the teams that players are in. Stronger teams may have more advanced cars, thus making it easier for players to win their races. We group these factors as **External Factors**.

As shown in Figure 4 (Left), the number of players and teams have been decreasing over the years. In the past, each team could send as many players as they wanted, but in recent years there has been changes in rules, thus allowing each team to send only 2 players. In light of this, the number of players participating in recent years have been decreasing. The number of F1 teams have been decreasing as well over the years as some teams are incurring huge financial losses.

As shown by Figure 4 (Right), the amount of points allocated to players and teams have been increasing throughout the year. Contemporary players are awarded more points than players from previous era even if they same rank.

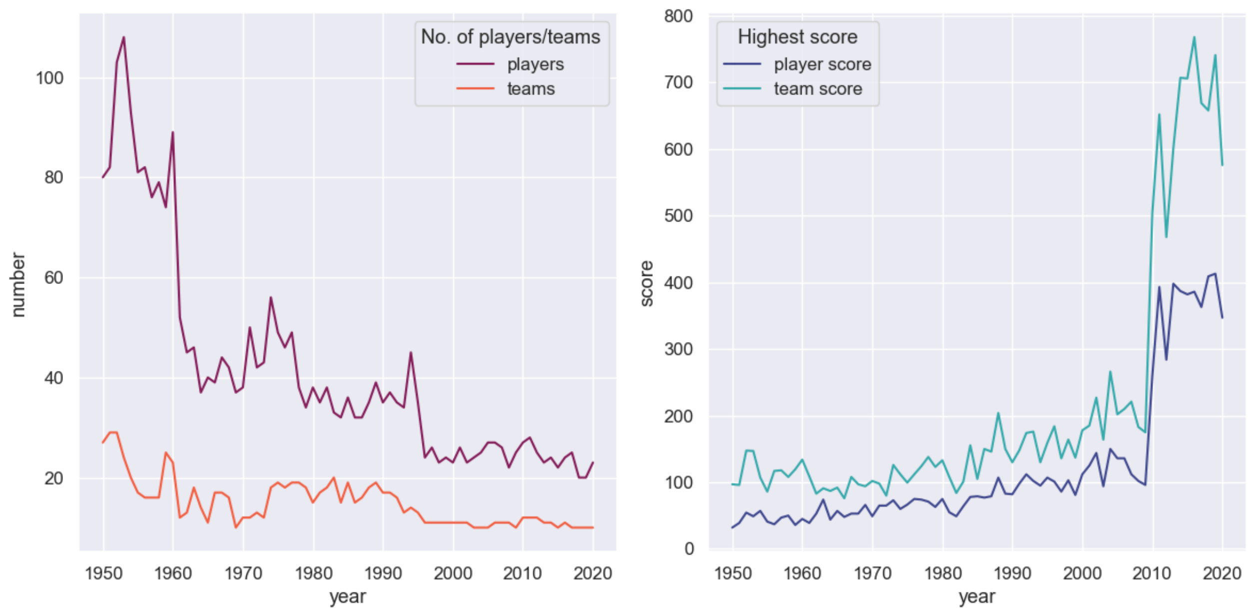

Figure 4: Information about Formula 1: (left) information about yearly No. of participants/teams and (right) information about the yearly highest score by player/team

Meaning	Symbol	Weight
Player Score	PS	na
External Factor	EF	na
Average Player Points in a year	F_1	na
Average Team Points in a year	F_2	na
Total Player Points in a year	F_3	na
Total Team Points in a year	F_4	na

Player Score : This numerical value is indicative of one's own performance and ability.

External Factor: Performance of F1 drivers may be influenced by the teams they are in. Some teams have more advanced cars, thus giving its players a greater competitive edge (Mitchell, 2020). Thus, we considered the total number of points that teams have amassed each year to determine relative strength of each team

Average Player Points: Average points players amassed in a year as a fraction of total points awarded to players in a year. This takes into account differences in point system of F1 over the year and helps to normalise our data

Average Team Points: Similarly, we need Average Team Points to help us in calculation of our external factor. This is given by the average points teams amassed in a year as a fraction of total points awarded to teams in a year.

5.3.1 Models: Determination of All Time Rank Score

Firstly, we calculated Average Player Points in year by

$$F_1 = \frac{F_3}{n}$$

where n is the average No. of team each year since 1950.

Secondly, we calculate the Points Earned by Player in a particular year by adding all the points he got. As the number of points allocated for players who perform well in the championships have increased over the years, we divide the Points Earned by Player by F_1, normalising the Points Earned by Player each year. In that way, we are no longer comparing the points earned by player but rather the **relative performance** that year.

Coefficient of competitive balance for players
In addition, we considered the level of playing field and the competitive balance of each year, which takes into account relative strengths of players competing in the competition (Budzinski amp; Feddersen, 2020). A high coefficient for competitive balance would signal that the playing field is relatively balanced. On the contrary, a low coefficient for competitive balance would signal that playing field is tilted towards dominant players.

The result of competitive balance is quoted:

	1950 -2018	1950 -1959	1960 -1969	1970 -1979	1980 -1989	1990 -1999	2000 -2009	2010 -2018
F1 Drivers	0.350	0.360	0.171	0.171	0.267	0.171	0.360	0.222
F1 Teams	0.503	-	0.233	0.300	0.133	0.350	0.400	0.056

External Factor
We sorted the data to find out the teams that players have joined throughout their career and considered the relative strengths of teams that players have joined on yearly basis. Just as players, we calculate the normalised relative team performance by dividing the total points earned with the Average Team Points F_2.

Coefficient of competitive balance for teams

Likewise, we considered the coefficients of competitive balance for teams, as certain teams might dominate some eras due to improvements in their car design. This takes into account the effect of the team on driver's performance as cars which are more advanced can help to boost performance of drivers.

With all that, we are able to find out the Rank Score of a player for particular year using Entropy Weight Method and TOPSIS.
Therefore for the **All Time Rank Score**, we consider the accumulation of overall score in each year with both the effect of **PS** and **EF**. A player with higher **PS** performs better in races throughout their career. A higher **EF** would mean that effect of team on performance of players is more pronounced. Hence, we performed *Positivization* on **EF**.

Similar as Task 1, we use *Entropy Weight Method* with Normalization to determine the weights of PS, EF and coefficients of competitive balance for players and teams. The weight is as follows:

Meaning	Symbol	Weight
Relative Player Performance	F_5	0.6679
Relative Team Performance	F_6	0.0514
Player Competitive Balance	F_7	0.0588
Team Competitive Balance	F_8	0.2218

Then we use *TOPSIS* to calculate **All Time Rank Score**.

player	PS	EF	All Time Rank Score
Michael Schumacher	83.7504	50.5592	9.568
Lewis Hamilton	72.4086	47.7384	8.256
Sebastian Vettel	61.3202	38.8035	7.023
Fernando Alonso	55.1345	31.2946	6.388
Kimi RÃ¤ikkÃ¶nen	55.2268	39.1882	6.350
Alain Prost	52.8904	31.8977	6.065
Graham Hill	42.2005	34.5452	4.958
Rubens Barrichello	42.1872	35.1464	4.939
Ayrton Senna	40.1344	26.4935	4.621
Jenson Button	38.7187	25.8858	4.587

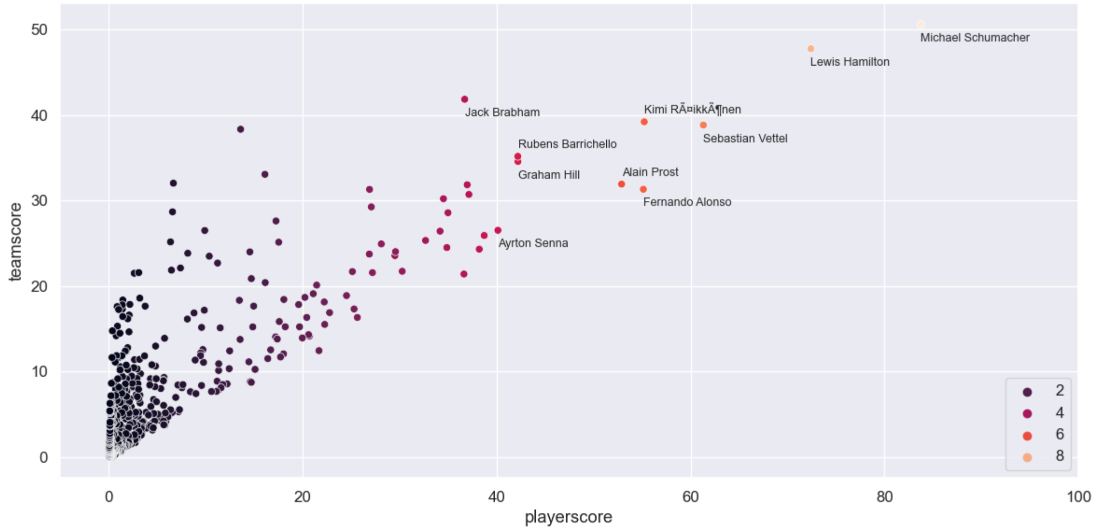

Figure 5: All Players with their PS (x axis) and ES (y axis) and All Time Rank Score (Color)

Comparing the top 10 formula 1 players generated by our model with the top 10 players who have amassed the most number of points throughout their career, we observe that the rankings are largely similar.

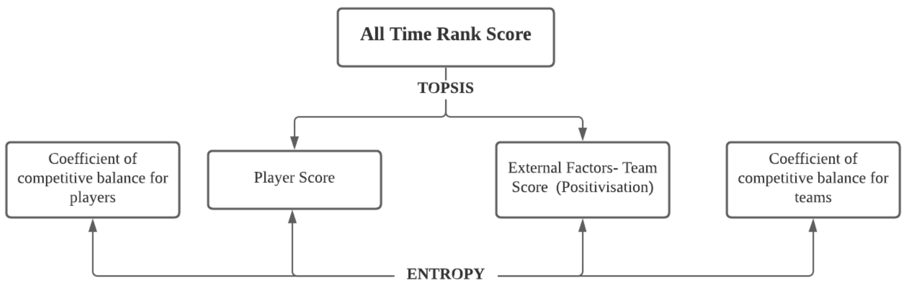

Figure 6: Flow Chart for G.O.A.T Model

5.4 Model Evaluation

Anomalies:

(i) For players whose predicted ranking is higher than actual ranking:

Reason 1: We normalised the points awarded to players in the tournament to ensure that players in the past and players who are competing in today's age are given similar points for finishing in same position. Over the years, the number of points who given to players who finished in top few positions have been increasing i.e. in 1950, 10 points were awarded to first place winner but in today's age the same player would be awarded with 25 points. The rankings provided by Formula 1 are a culmination of raw points obtained by players throughout their career, thus players who competed in past may be less favourably ranked. For example, players like Graham Hill dominated the 1960s but was not ranked highly in official rankings due to smaller number of points awarded during that period.

Reason 2: This could be due to the presence of dominant players in the playing field during that era, which can result in players not being able to pick up as many points. Our model reflects this by incorporating the coefficient of competitive balance, which accounts for strength of player's competitors. The 1980s saw strong competitiors like Alain Prost, Nelson Piquet and Ayrton Senna. Alain Prost won 4 titles, while Nelson Piquet and Ayrton Senna won 3 titles each. Thus each player was awarded lower points due to presence of other strong opponents who were of similar standard and stood equal chance of winning the races. The competitive balance during that era was higher, thus our model gave these 3 players a higher ranking.

6 Sensitivity Analysis

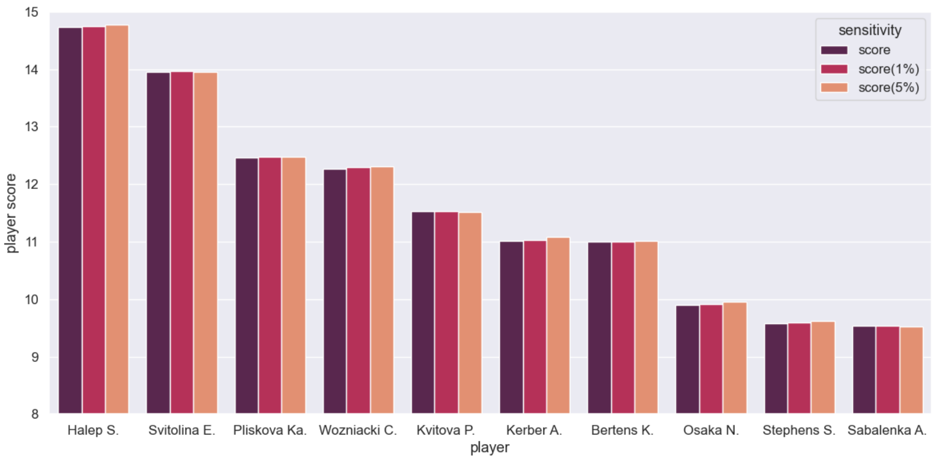

Figure 7: Sensitivity test for Rank Score of woman tennis players

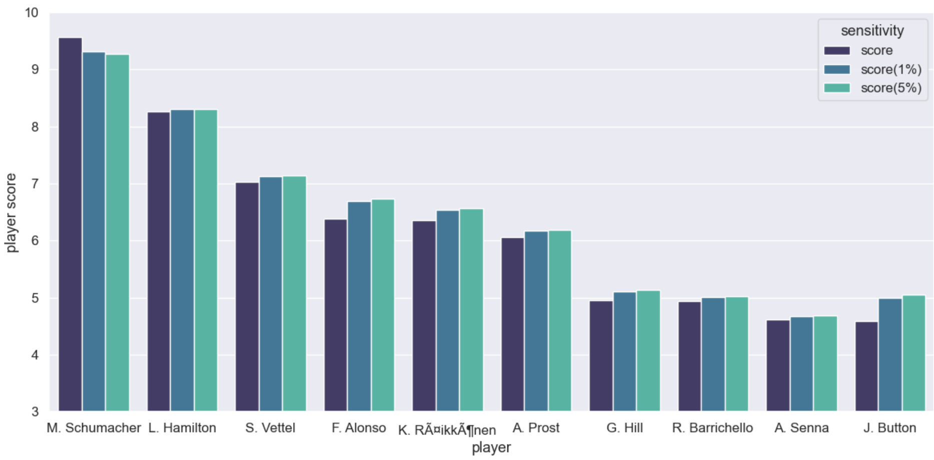

Figure 8: Sensitivity test for All Time Rank Score for Formula One players

To test the sensitivity of our model, we adjust the weights of different factors that contributes to Rank Score, to see if the Rank Score undergoes significant changes. We scale up the weights by 1% and 5%, generating Fig.7 and Fig.8 for 2018 Overall Model (woman tennis players) and G.O.A.T. Model (F1) respectively. As can be seen from the figures, the Rank Score of both woman tennis player and F1 players remain relatively constant. This shows that our model is relatively stable.

7 Task 2(b): Application of Task 2 Model to ANY individual sport

Factors to consider:
individual performance
1. Sports vs Non-interactive sports
For individual sports like Tennis, there is interaction between athletes when they serve shots to each other. However, for other sports such as Swimming, athletes are competing against an inanimate standard, with little direct interaction between athletes. In such cases, the **Strength of Schedule (SS)**, as used previously in our Grand Slam Model, would be irrelevant. Moreover, the breaking of records (if it is not due to technology, which can be assessed by comparing whether average performance of atheletes has improved as a whole) should be given additional credits as it shows that the athlete has overcome human limits to some extent.

external factor
2. Environmental condition
Sports such as Tennis and Formula 1 are played outdoors. For sports like Sailing, the environment might be of great importance. Thus, players' performance can be affected by environmental conditions (McCullough, 2018). For example, athletes' performance may be affected if the playing ground is wet from rain. In such cases, **External Factor** might be given different weights. However, for indoor sports such as Badminton and Table Tennis, environmental factors play a less significant role.

time factor
3. Technological advancement
Over the years, advances in technology has led to great improvements in sports equipment, such as the tyres and vehicles in F1. (Straw, 2020) As such, when comparing G.O.A.T of all time, it is reasonable to give older games **more weights**, or **scale up** the past performance by relevant percentage in **Players Earned Points** to account for the technological factor. Moreover, for different sports, the degree to which their equipment has advanced might be different. Thus **the extent of scaling** is different. For example, the degree to which sprinters' speeds have been boosted by better shoes is different from the degree to which cyclists' speeds have been boosted by lighter bicycles.

Additionally, technologies like Hawk-Eye (tennis), Microchip Balls (soccer) have been employed in various sports and enabled increased accuracy in judging, making past competition records relatively inaccurate. Thus, **data standardization** might be needed.

4. Change in competition rules or official equipment
For example, Badminton changed its scoring system in 2006 from scoring 15 points (Male) and 13 points (Female) respectively to scoring 21 points in 1 round. Other sports like table tennis have changed the size of the ball in 2000 (Baggaley, 2013). Such changes create inconsistency in the data collected (Andrews, 2013). As such, we need to avoid calculation based on absolute points gained and choose to use **percentage points gained** similar to F1 Model.

5. Longevity of players' careers:

Unlike determining the best tennis player in year 2018, when we decide the G.O.A.T, it is not enough to take only one or two performances of a particular player, for which there will be luck element involved. Moreover, it would be good to calculate one's **average performance** throughout one's career to determine whether if one stays in the top position for considerably time. For instance, Michael Phelps is considered as G.O.A.T. in competitive swimming as he has obtained gold medals in 4 Olympiad and 5 Summer Games straight (Chase, 2016).

6. Gender split
Most sports have separate categories for male and female players. For example, men's soccer and women's soccer are separate events at the World Cup. This is a reflection of biological differences between men and women and to level the playing field such that men's greater muscular strength do not cause them to dominate over female players (Miller, 1993). We ought to determine the G.O.A.T. according to gender as well.

8 Task 3: Differences between Team Sports and Individual Sports

Similar to the factors identified in Task 2, team sports will also be affected by Environmental conditions, Technological Advancements, Change in Competition Rules, Gender Split and Longevity of Players' Careers.

Some factors that we may need to consider in overall determination of G.O.A.T
1. Each role should have their own G.O.A.T.
A team consists of different roles. For example, in soccer, a team consists of strikers, midfielders and defenders. The role of players directly affects their abilities to score, as the striker would be much likely to score than a defender. Moreover, the skill sets of players in different role would differ as well. Hence, the points each player gained will not be a good gauge of their individual ability. Thus, some team sports should determine the G.O.A.T. by role.

2. Team cooperation
Since the final results hinge on the entire team's collaborative efforts, team cooperation is essential for teams to perform well and for G.O.A.T to perform at their peak. For example, in football, for a striker to be able to score, their teammates must cooperate to pass the ball to them. Without cooperation, strikers, regardless of how good they are, would find it very difficult to score. Hence, we can introduce another factor in the G.O.A.T. model such as **Team Score** to incorporate one's contribution to the team.
Moreover, **different weight** will be assigned to team factors and individual performance in different sports.

3. Taking one for the team
In certain games, the coach may decide to sacrifice the scoring opportunities of certain players in order to enhance the likelihood of team victory. In such cases, the players might be disadvantaged (and the information usually will not reflect in the data) when we consider his/her own performance during the game, especially in the realm of e-sports (Tang, 2018).**Team Corporation Score** or **Undermine** one's own performance score by giving it less weight.

4. Team Spirit
Not only the team corporation matters, team spirit also matters. In this case, the role that

one plays in raising the team's morale or uniting the team is important. While we acknowledge that this factor is hard to quantify, one possible way is to give it a value based on the review by one's coaches and teammates.

5. Relative weight of Individual Performance and Team Performance
It is possible that due to the low competency of one's teammates, an athlete will not achieve high results for team sports. Such a scenario is common in e-sports where it is known as "ELO Hell" (Wilde, 2018), where in Massively Mutliplayer Role Playing Games (MMORG), players are unable to improve their ranking due to being assigned teammates of poor calibre. In such cases, the individual should be of greater importance and teammates ability could be an **inversely related factor** or we could **compare** the Team Performance before and after that one player joins.

9 Strength and Weakness of our models

9.1 Strengths

1. Consideration of the difficulty of each match
A victory against a much more skilled player carries much more weight than a victory against a similarly skilled player. We accounted for the divergent difficulties of each match by calculating the **Strength of Schedule** and incorporating it into our model.

2. Large dataset
Because the question requires for the Greatest player over a time span, we collected as much data as possible to ensure that the player's performance is consistently well in various situations and facing different opponents. For example, in Model 1b, we included the expanded dataset from all 2018 tournaments to more accurately determine the best women's tennis player of 2018.

3. Accounted for External Factors in Individual Sports
Our model is realistic as even in individual sports like F1, there are team factors. For Model 2, we recognised that team support for refuelling is crucial to achieving shorter lap times. As such, the quality of support given by the refuelling team is important. We accounted for this by considering the track record of each team and penalising drivers of teams with great track records such as Mercedes while boosting drivers of teams with sub-par track records to a small extent (Though this factor is not essential in this model, when applying to other sports, it might be significant).

4. Account for change in External factor over the years
Our Model is highly thorough as we considered the detailed change of EF across the years. For Model 2, we recognised that drivers may be part of many different teams throughout their careers by manually calculating the number of points accumulated by each driver during their stay with each team. This allows to factor in more accurately any performance bonus or penalty caused by team support.

5. Possible broad application
By categorising the factors into two broad terms, our models can be adopted to ANY other sports more easily with change of different sets of sub factors.

6. Specific strength of models we used
When determining the **Rank Score**, we use relatively more impartial and objective method.

Moreover, The Entropy Weight Method is objective in assigning the weights of different factors (Qiyue, 2010) whereas TOPSIS is indicative of variances in different factors (Wang, 2006).

9.2 Weakness

1.Certain factors pertaining to individual performance are not taken into consideration

We simplified our models by excluding factors that are less relevant, more complicated or implicit such as Nationality. Nationality should be taken into consideration because, similar to College Admissions, athletes' backgrounds determine the amount of investment, resources and experience they can receive from their countries. Sports like skating is also unfriendly for tropical countries like Singapore.

2.External Factors are not fully accounted for

Our models did not account for differences in competing environments for which might be influencing the performance of players significantly. For example, the surfaces are different in the Grand Slam, which will give advantage to players of certain style (Girard et al, 2009). Due to unavailability of data on F1 competing environment, we did not include it as part of our EF.

10 Conclusion

In conclusion, we managed to address all the tasks by identifying the relevant individual, external, time factors which influence the determination of Best Player in a year or G.O.A.T of a particular sport and developing appropriate models. Overall, we use TOPSIS and Entropy Weight Method to in our model development. By evaluating our model results with the established ranking and using sensitivity tests, we confirm the reliability of our models.
Specifically, our model deduces that S.Halep. is the best 2018 woman tennis player both based on just the Grand Slam results and 2018 all year performances. For F1, we believe Michael Schumacher is G.O.A.T.

Despite that our model has taken consideration of Time factor by calculating the Relative Performance, the modeling domain presented by this problem is vast and there is a large room for improvement. In future studies, given more time and resources, we can factor in more time-related factors which might be significant in certain sports such as general improvement in nutrition over the years (which may lead to performance boost of contemporary players). Similarly, External Factors (EF) can be expanded and more detailed if more data are available. For example, the tyres and the cars in F1 can be taken into consideration because the techniques along possessed by Players should be of the greatest importance. EF can also be more thorough if we are given the availability of data such as environment condition for outdoor games or even the audience support, for which might influence the players' morale and thus performance.

Moreover, if time permits, we could incorporate others aspects of 'greatness' and come up with an assessing criteria for each aspect, such as how to quantifying one's contribution to the sports or society. Furthermore, we could even attempt to consider the subjective elements when people determine the 'greatness'. For instance, some might consider the techniques is more important than the physical build for an athlete to be considered as 'great'. Other factors such as charisma, personal traits can also be of consideration for some sport fans. In this case, we can possibly assigning differing weight to different factors in the lens of varying groups and come out with multiple G.O.A.T. in various aspects within a single sport, for which it is a celebration of diversity, too.

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12 Appendix

Table 1 Results of the four Grand Slam Majors

players name	Average Score	Opponents	Entropy Weighted Scores	Rank
S Halep	2.000	2.500	1.000	1
S Williams	1.750	1.750	0.747	2
A Kerber	2.000	1.500	0.703	3
M Keys	1.500	1.688	0.688	4
N Osaka	1.250	1.563	0.606	5
S Stephens	1.250	1.500	0.590	6
C Wozniacki	1.250	1.125	0.492	7
E Mertens	0.750	1.313	0.445	8
D Kasatkina	0.750	1.188	0.412	9
Ka Pliskova	0.750	1.188	0.412	9
A Sevastova	0.750	0.875	0.328	11
C Suarez Navarro	0.750	0.813	0.312	12
G Muguruza	0.750	0.750	0.295	13
J Ostapenko	0.750	0.688	0.279	14
K Bertens	0.500	0.813	0.270	15
M Sharapova	0.500	0.813	0.270	15
C Garcia	0.250	0.875	0.264	17
J Gorges	0.750	0.625	0.263	18
S-w Hsieh	0.250	0.750	0.229	19
D Cibulkova	0.500	0.625	0.217	20
L Tsurenko	0.500	0.563	0.199	21
C Giorgi	0.500	0.500	0.182	22
E Svitolina	0.500	0.438	0.164	23
Y Putintseva	0.500	0.438	0.164	23
A Kontaveit	0.250	0.500	0.156	25
B Bencic	0.250	0.500	0.156	25
K Kanepi	0.250	0.438	0.137	27
A Sabalenka	0.500	0.313	0.131	28
M Buzarnescu	0.250	0.375	0.118	29
B Strycova	0.250	0.313	0.099	30
A Barty	0.250	0.188	0.060	31
A Sasnovich	0.250	0.188	0.060	31
A Van Uytvanck	0.250	0.188	0.060	31
D Vekic	0.250	0.188	0.060	31
P Martic	0.250	0.188	0.060	31
D Allertova	0.250	0.125	0.040	36
E Makarova	0.250	0.125	0.040	36
M Vondrousova	0.250	0.125	0.040	36
E Rodina	0.250	0.000	0.000	39
M Rybarikova	0.250	0.000	0.000	39

Table 2 Full Tournaments Ranking

tournament	no_participant	winscore	losescore	wrank	lrank	Score	Rank
BNP Paribas WTA Finals	8	4039	4463	5	7	0.0304	1
French Open	128	2481	1303	85	51	0.0289	2
Australian Open	128	2245	1288	88	53	0.0284	3
US Open	128	2106	1339	91	49	0.0284	3
Wimbledon	128	1831	1355	78	56	0.0281	5
BNP Paribas Open	96	2165	1660	58	50	0.0272	6
Mutua Madrid Open	64	2974	1986	44	28	0.027	7
Sony Ericsson Open	96	2013	1582	69	57	0.0263	8
Internazionali BNL d'Italia	56	3034	2003	49	22	0.0263	8
China Open	61	2774	1963	48	27	0.0261	10
Qatar Total Open	56	2968	2016	51	30	0.026	11
Rogers Cup	56	3101	1904	54	24	0.0259	12
Western Southern Financial Group Women's Open	56	2641	2101	48	34	0.0257	13
Porsche Tennis Grand Prix	28	3070	2599	41	33	0.0254	14
Dubai Duty Free Tennis Championships	28	3158	2260	34	24	0.0245	15
Wuhan Open	56	1980	2086	41	36	0.0244	16
WTA Elite Trophy	12	2701	2579	19	17	0.0238	17
AEGON Classic	32	2648	1934	43	27	0.0225	18
Eastbourne International	48	2445	1566	65	32	0.0218	19
Brisbane International	30	2459	1836	46	41	0.0211	20
Sydney International	30	1829	2037	49	44	0.0206	21
Toray Pan Pacific Open	28	2301	1736	70	30	0.0201	22
Connecticut Open	30	1976	1631	69	35	0.0191	23
Family Circle Cup	56	1525	1082	86	57	0.018	24
Ladies Trophy	28	1982	1738	75	74	0.0179	25
Kremlin Cup	28	1749	1544	61	49	0.0178	26
Mallorca Open	32	1250	1151	70	66	0.015	27
BGL BNP Paribas Luxembourg Open	32	1495	947	76	58	0.0149	28
Shenzhen Open	32	1791	900	102	57	0.0148	29
ASB Classic	32	1813	982	112	67	0.0144	30
Generali Ladies Linz	32	1089	1084	76	69	0.0143	31
JT Banka Prague Open	32	1297	875	91	67	0.0136	32
Tianjin Open	32	1879	987	114	91	0.0136	32
Mubadala Silicon Valley Classic	28	1265	942	92	62	0.0135	34
Istanbul Cup	32	1249	1035	100	76	0.0134	35
Aegon Open	32	1408	753	114	62	0.0131	36
Ricoh Open	32	1079	912	95	75	0.0129	37
Korea Open	32	1427	807	128	69	0.0126	38
Abierto Mexicano	32	1147	885	96	84	0.0125	39
Monterrey Open	32	1723	604	135	72	0.0125	39
Samsung Open	32	1010	859	92	80	0.0125	39
Hobart International	32	994	887	97	82	0.0123	42
Hungarian Ladies Open	32	945	812	97	76	0.0123	42
Prudential Hong Kong Tennis Open	32	1388	935	127	83	0.0123	42
Internationaux de Strasbourg	32	1239	783	135	65	0.0122	45
Japan Women's Tennis Open	32	894	651	109	77	0.0115	46
Citi Open	32	959	842	118	90	0.0112	47
NÃ$\frac{1}{4}$rnberger Versicherungscup	32	831	923	124	101	0.0106	48
Bucharest Open	32	973	601	134	88	0.0102	49
Moscow River Cup	32	971	884	114	127	0.0101	50
Grand Prix SAR Lalla Meryem	32	915	675	121	103	0.01	51
Guangzhou Open	32	821	569	146	85	0.0098	52
Taiwan Open	32	709	626	134	109	0.009	53
Tashkent Open	32	525	564	125	141	0.008	54
Ladies Championship Gstaad	32	643	495	166	119	0.0074	55
Coupe Banque Nationale	32	612	576	169	150	0.0065	56
Copa Claro Colsanitas	32	534	478	182	155	0.0059	57
Jiangxi Women's Tennis Open	32	554	367	209	159	0.0055	58

Table 3 Top 20 Best Women Tennis Player 2018

	Score	Rank	Real rank
Halep S.	14.7292	1	1
Svitolina E.	13.951	2	4
Pliskova Ka.	12.4678	3	8
Wozniacki C.	12.2668	4	3
Kvitova P.	11.528	5	7
Kerber A.	11.0164	6	2
Bertens K.	10.9961	7	9
Osaka N.	9.896	8	5
Stephens S.	9.5749	9	6
Sabalenka A.	9.5414	10	13
Goerges J.	9.0123	11	14
Barty A.	8.8917	12	15
Sevastova A.	8.7849	13	11
Kasatkina D.	8.4068	14	10
Muguruza G.	7.7081	15	18
Garcia C.	7.6529	16	19
Mertens E.	7.538	17	12
Wang Q.	6.8324	18	21
Kontaveit A.	5.5573	19	20

Table 4 Top 20 Formula 1 players

player	score	rank	real rank
Michael Schumacher	9.568365214	1	6
Lewis Hamilton	8.255703275	2	1
Sebastian Vettel	7.023388485	3	2
Fernando Alonso	6.387767611	4	3
Kimi RÃ¤ikkÃ¶nen	6.349710248	5	4
Alain Prost	6.065093172	6	13
Graham Hill	4.957707811	7	32
Rubens Barrichello	4.939163172	8	15
Ayrton Senna	4.621280062	9	16
Jenson Button	4.587326802	10	8
Nelson Piquet	4.456706908	11	19
David Coulthard	4.294032549	12	17
Jack Brabham	4.284787929	13	39
Juan Manuel Fangio	4.229508408	14	31
Jackie Stewart	4.196133605	15	27
Nigel Mansell	4.118696054	16	20
Nico Rosberg	4.019233344	17	18
Felipe Massa	4.005532147	18	9
Jim Clark	3.909271044	19	37
Niki Lauda	3.811153963	20	21