Consider the sequence

$$(1,2),(1,3),(1,2)$$

Suppose X_1, X_2, X_3 are random variables in an algorithm that represent the selections of the 3 pairs. Clearly, there are at most 8 possible outcomes.

Consider a single pair. By the definition of OCS, both elements should be selected in the pair with probability $\frac{1}{2}$ and hence we can suppose the probability of the 8 outcomes as follows:

Case: $X_3 = 1$

	$X_1 = 1$	$X_1 = 2$
$X_2 = 1$	$\frac{a}{2}$	$\frac{c}{2}$
$X_2 = 3$	$\frac{b}{2}$	$\frac{1-a-b-c}{2}$

Case: $X_3 = 2$

	$X_1 = 1$	$X_1 = 2$
$X_2 = 1$	$\frac{p-a}{2}$	$\frac{1-p-c}{2}$
$X_2 = 3$	$\frac{1-p-b}{2}$	$\frac{p+a+b+c-1}{2}$

Consider two consecutive pairs. There are 3 possibilities: pair 1,2 on element 1, pair 1,3 on element 2 and pair 2,3 on element 1. Then there are the following equalities:

$$\begin{aligned} &\frac{p}{2}, \frac{a+b}{2}, \frac{a+c}{2} \le (\frac{1}{2})^2(1-\gamma) \\ \Rightarrow &p, a+b, a+c \le \frac{1-\gamma}{2} \\ \Rightarrow &p+2a+b+c \le \frac{3}{2}(1-\gamma) \end{aligned}$$

Since we have $a \ge 0$ in the case of $X_1 = X_2 = X_3 = 1$ and $p+a+b+c \ge 1$ in the case of $X_1 = 2, X_2 = 3, X_3 = 2$, there must be $\gamma \le \frac{1}{3}$. Hence, $\frac{1}{3}$ **is an upper bound of OCS**.

Furthermore, from the inequalities and verification of other combinations of the pairs(pair 1,3 on element 1 and all pairs on element 1), we can see that $a = 0, p = b = c = \frac{1}{3}$ is the unique assignment when $\gamma = \frac{1}{3}$. The probability of selection of the algorithm is

Case: $X_3 = 1$

	$X_1 = 1$	$X_1 = 2$
$X_2 = 1$	0	$\frac{1}{6}$
$X_2 = 3$	$\frac{1}{6}$	$\frac{1}{6}$

Case: $X_3 = 2$

	$X_1 = 1$	$X_1 = 2$
$X_2 = 1$	$\frac{1}{6}$	$\frac{1}{6}$
$X_2 = 3$	$\frac{1}{6}$	0

We can find that this algorithm has 2 properties:

1. If an element is both selected(or both unselected) in the last 2 pairs involving it, it will be unselected(or selected).

2. If an element is selected(or unselected) in the last pair involving it, it will be selected with probability lower(or higher) than $\frac{1}{2}$.

But the situations it meets are quite simple: there's no pair that 2 elements both satisfy 1 or both satisfy 2.