Spatially and Spectrally Consistent Deep Functional Maps

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Code available here

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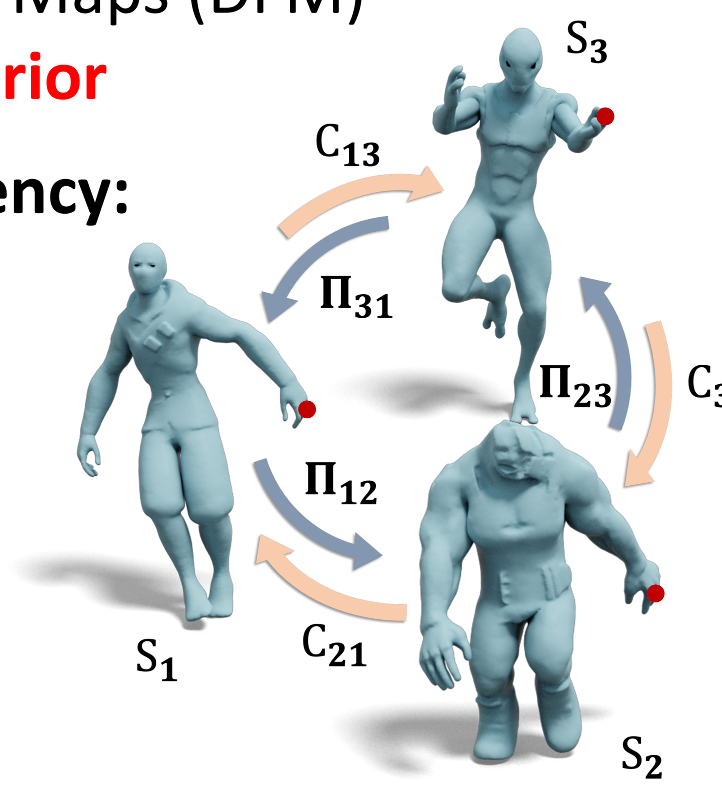
Introduction

- **Goal:** Enhance Deep Functional Maps (DFM) network via **cycle consistency prior**
- **There are two types of consistency:**
  - **Spectrally cycle consistency:**

    $$C_{21}C_{32}C_{13} = I_d$$

  - **Spatially cycle consistency:**

    $$\Pi_{31}\Pi_{23}\Pi_{12}(p) = p$$

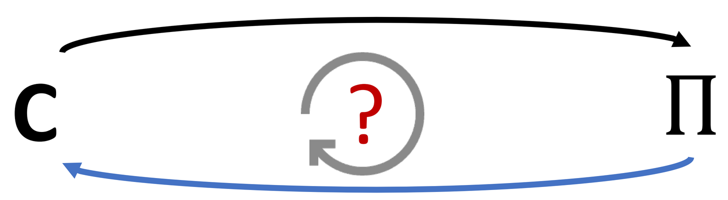


😊 **Good News:** DFM is *spectrally* cycle consistent -- we have proven it!

😢 **Bad News:** DFM is **not** always *spatially* cycle consistent.

- **Cause:** Functional maps are not necessarily **proper** [1]

$$\Pi^* = argmin_{\Pi}\|\Pi\Phi_2 C^T - \Phi_1\|_F^2$$



$$C^* = \Phi_2^{+}\Pi\Phi_1$$

- **Solution:** A simple yet effective **two-branch** DFM design
- **Features:**

✓ Induce light computational overhead.

✓ High-quality map estimation and strong generalization performance.

✓ Can be plugged in any DFM frameworks.

Reference:

[1] J. Ren, et al. Discrete optimization for shape matching. SGP 2021.

[2] L. Li, et al. Learning multi-resolution functional maps with spectral attention for robust shape matching. NeurIPS, 2022.

[3] D. Cao and F. Bernard. Unsupervised deep multi-shape matching. ECCV, 2022.

[4] N. Donati, et al. Deep orientation-aware functional maps: Tackling symmetry issues in shape matching. CVPR, 2022.

[5] M. Eisenberger, et al. Deep shells: Unsupervised shape correspondence with optimal transport. NeurIPS, 2020.

Acknowledgement:

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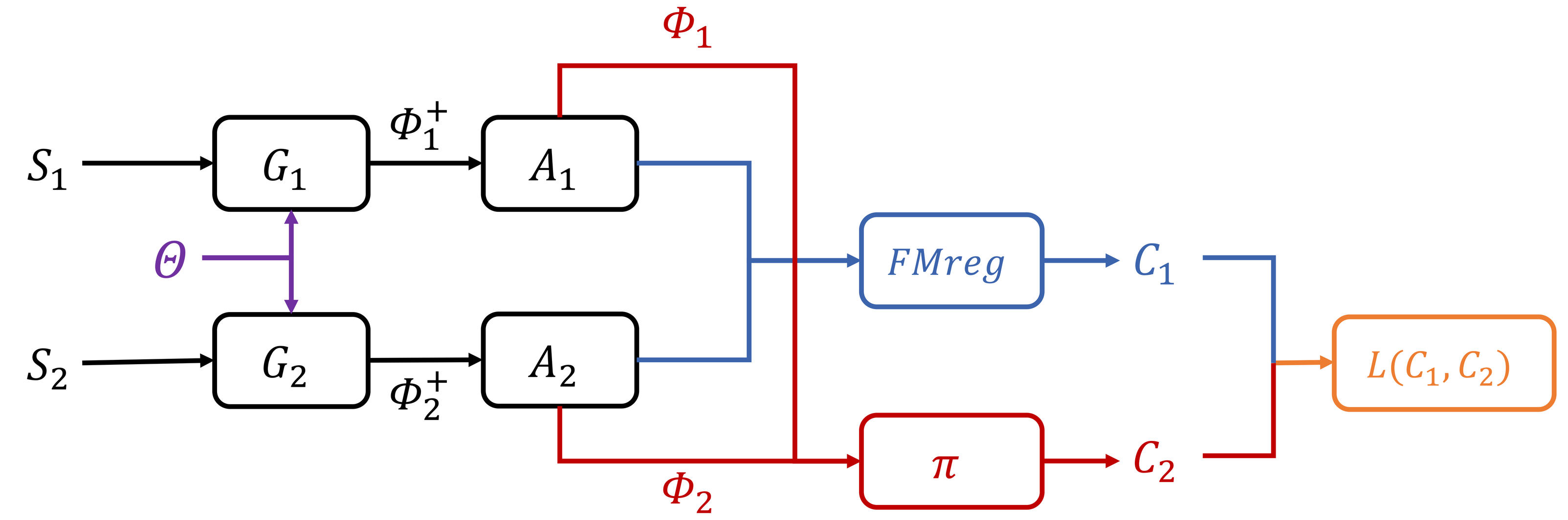
Method

Two-branch DFM network



Input: Learned features spectrally represented as A_1, A_2.

Branch 1: Estimate Fmap as $C_1 = argmin_C\|CA_1 - A_2\| + E_{Lap}(C)$.

Branch 2: Estimate point-wise map, for a pair of indices p, q, we have

$$\Pi(q,p) = \frac{\exp(-\alpha\delta_{qp})}{\sum_{p'}\exp(-\alpha\delta_{qp'})}, \text{ where } \delta_{qp} = \|\Phi_1[p]A_1 - \Phi_2[q]A_2\|_2.$$

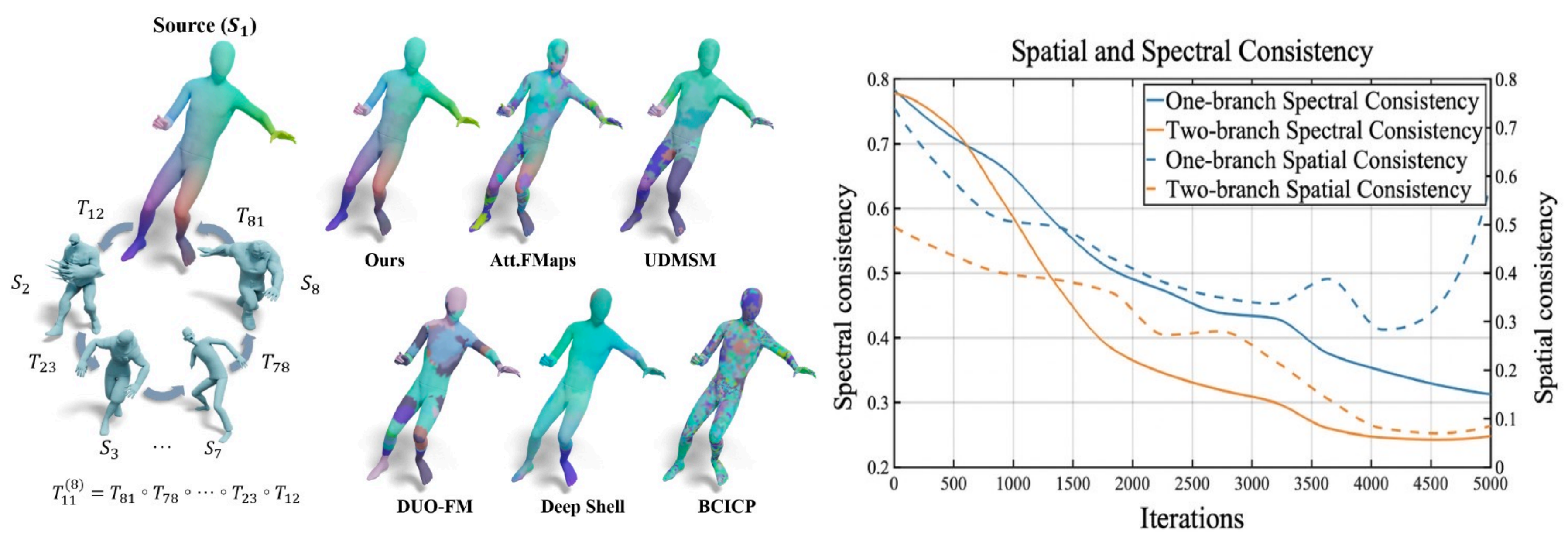
Enforcing consistency: We first convert Π to $C_2 = \Phi_2^{+}\Pi\Phi_1$, and then formulate the training loss as:

$$\mathcal{L}(\mathbf{C}_1, \mathbf{C}_2) = \|\mathbf{C}_1^T\mathbf{C}_1 - \mathbf{I}\|^2 + \|\mathbf{C}_1 - \mathbf{C}_2\|^2$$

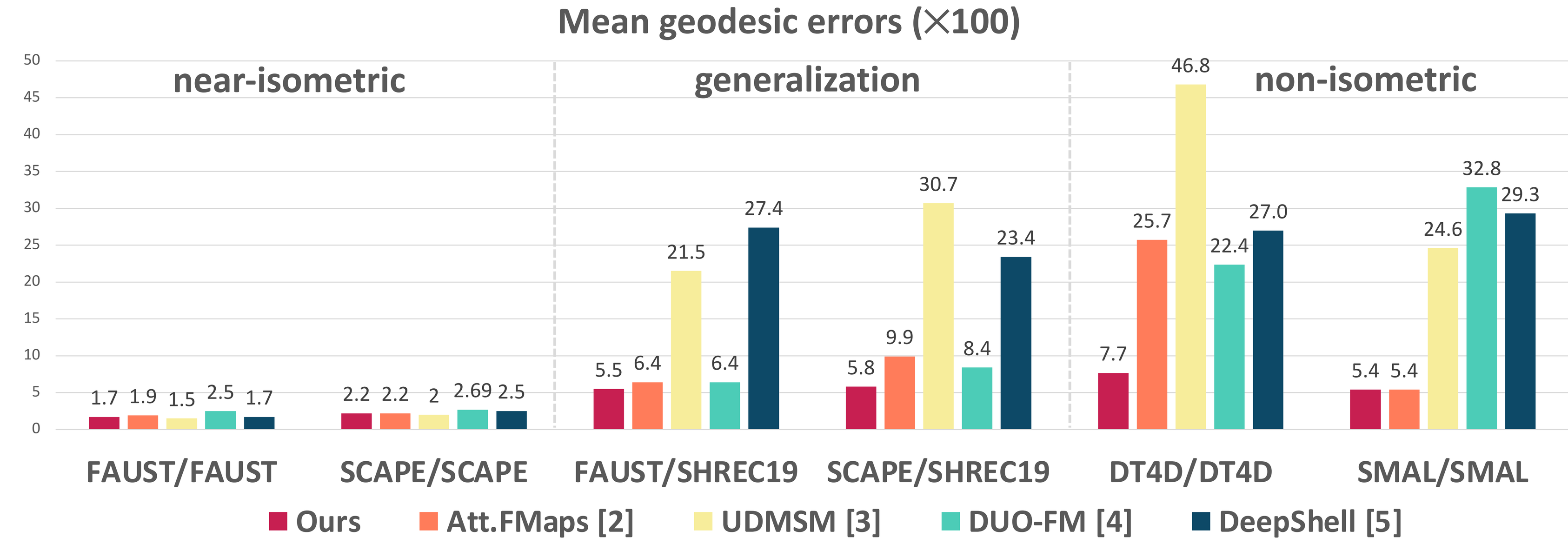
Key remark: the parameter α plays an important role, which is gradually increased during training – starting from fuzzy maps, converged at sharp ones in the end.

Results

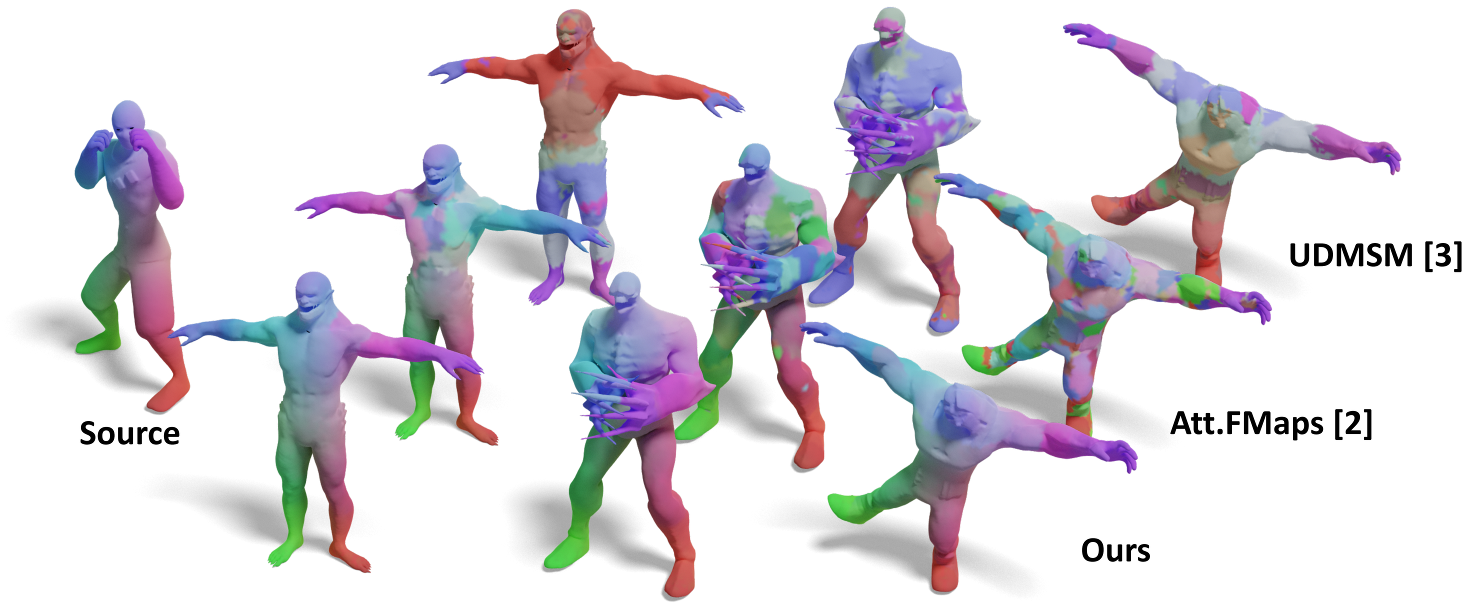
Validation of spatially cycle consistency



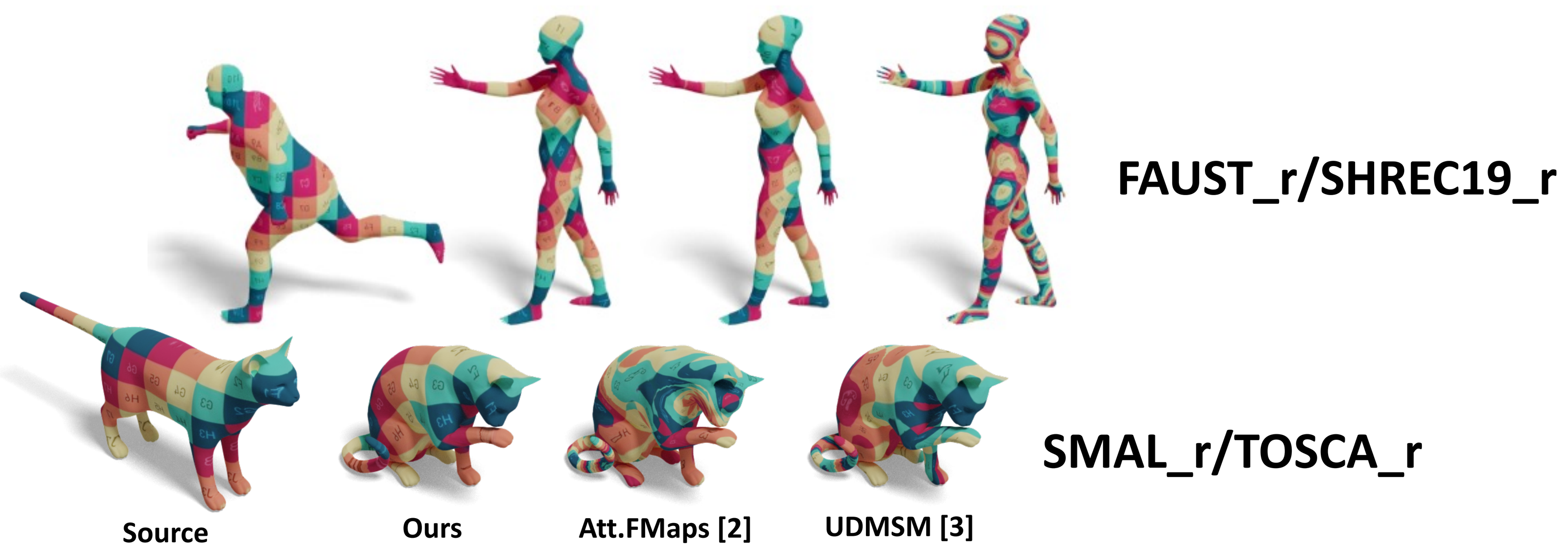
Quantitative results on standard benchmarks



Highly non-isometric shape matching



Robust generalization performance



One pair fine-tune with model trained on human shapes