Stochastic Optimization in Asset Pricing and Portfolio Choice

Ising Model and Simulated Annealing

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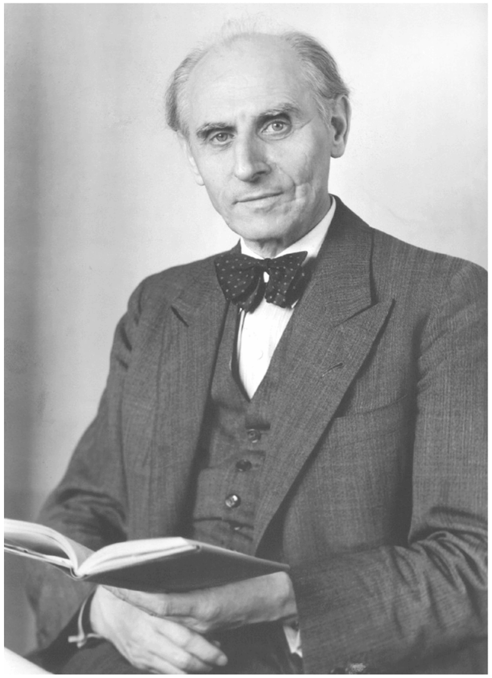
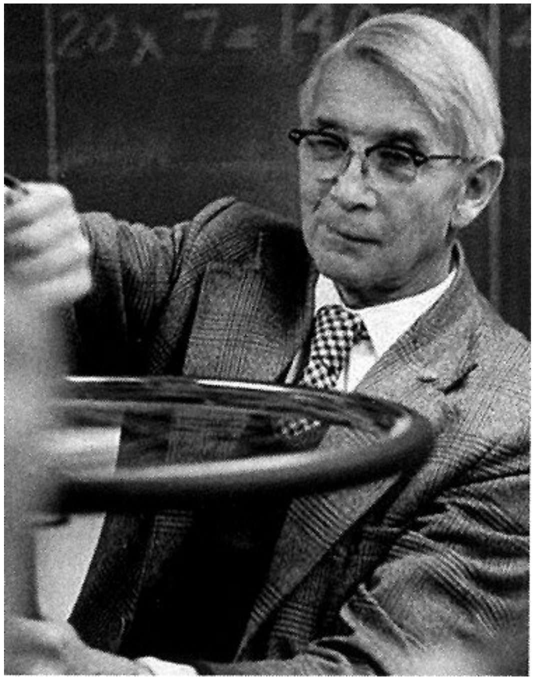
Temary

Fig. 1: Wilhelm Lenz and Ernst Ising

This model explains ferromagnetic phenomena. It was developed in 1924 and is one of the most studied models in statistical mechanics.

The configuration of a system determines its properties. Every magnetic moment is represented by a spin ($\sigma_i = \pm 1$). The energy of the system (or Hamiltonian) is defined as follows:

$$E = -J \underbrace{\sum_{\langle i,j \rangle} \sigma_i \sigma_j}_{\text{interaction between neighboring}} - h \overbrace{\sum_i \sigma_i}^{\text{effect of an applied magnetic field}} \quad (1)$$

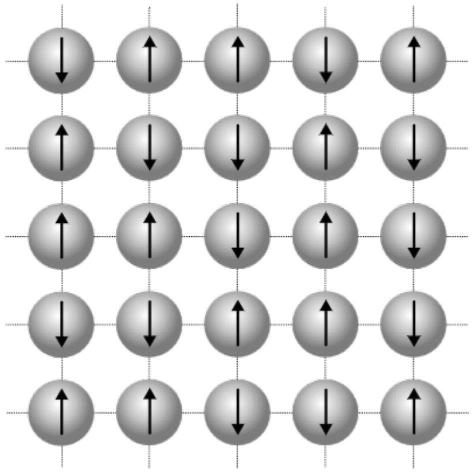
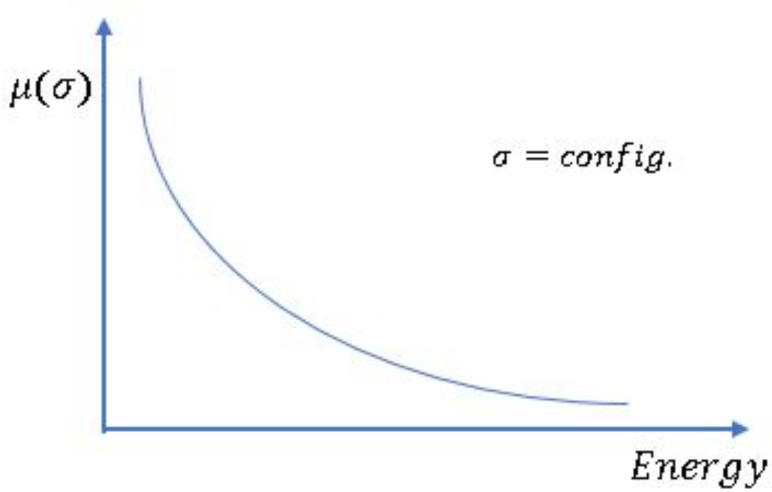


Fig. 2: Ex. of the Ising model on a 2D square lattice

Each configuration generates different energy.
We set a measure for each configuration as follows:

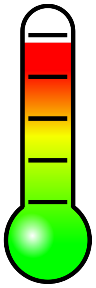
$$\mu(\sigma) \propto e^{\frac{-E}{T}} \tag{2}$$

Fig. 3: Nature seeks the minimum energy

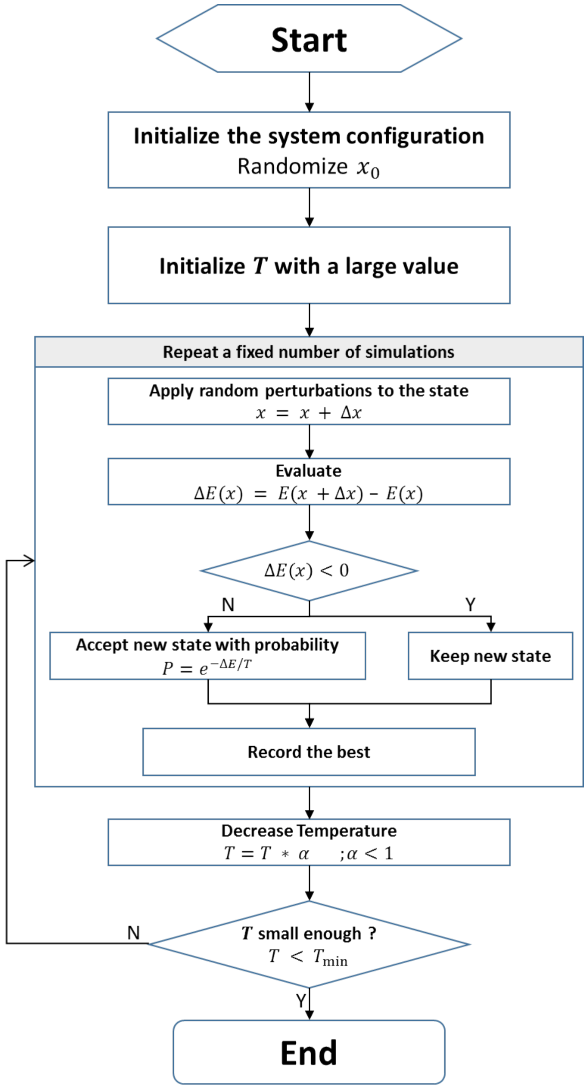


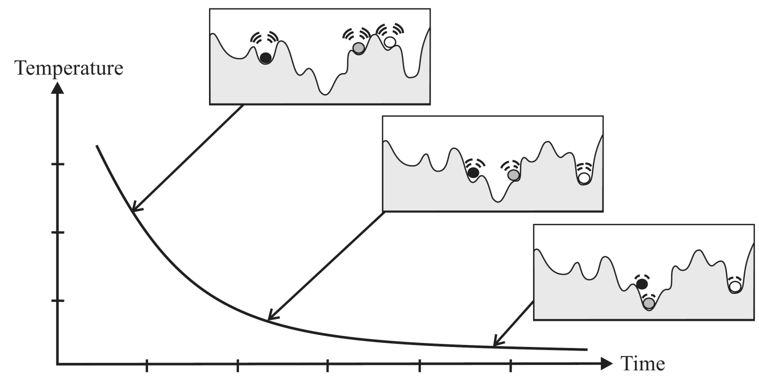
It's a metaheuristic algorithm to seek global minimun in a large search space with many local minima.

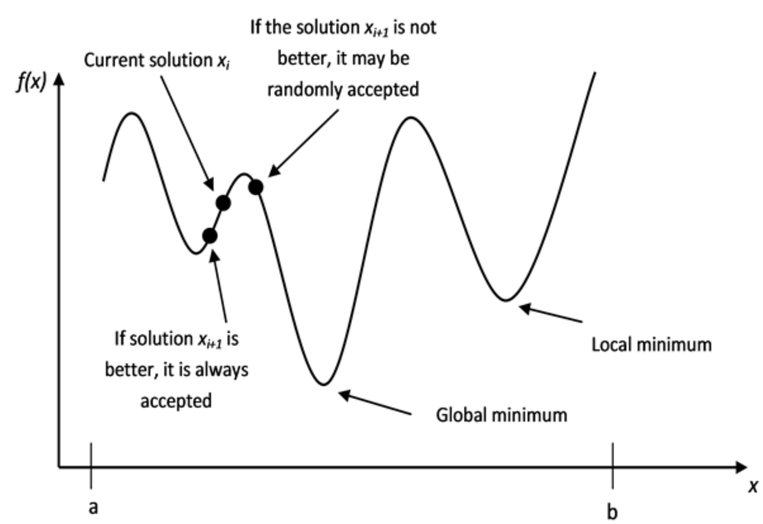
- Explore successors widly randomly : HIGH TEMP
- As time goes by, explore less widly : COOL DOWN
- Until there's a time where things settle : COLD



How it works?


Start
Initialize the system configuration
Randomize x_0
Initialize T with a large value
Repeat a fixed number of simulations
Apply random perturbations to the state
$x = x + \Delta x$
Evaluate
$\Delta E(x) = E(x + \Delta x) - E(x)$
$\Delta E(x) < 0$
N
Y
Accept new state with probability
$P = e^{-\Delta E/T}$
Keep new state
Record the best
Decrease Temperature
$T = T * \alpha \quad ; \alpha < 1$
N
T small enough ?
$T < T_{\min}$
Y
End


Temperature
Time


f(x)
Current solution x_i
If the solution x_{i+1} is not better, it may be randomly accepted
If solution x_{i+1} is better, it is always accepted
Local minimum
Global minimum
a
b
x

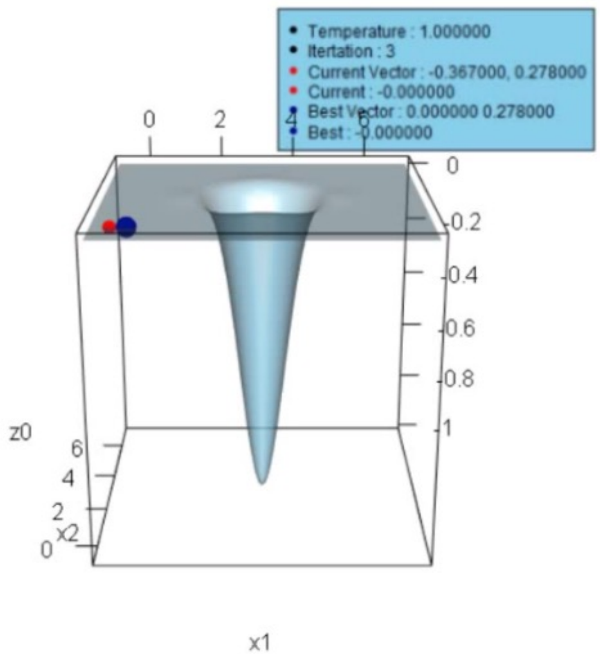
Testing some functions

Features added:

- Way of moving: $(0,0,...,0,\pm 1,0,...,0,0)$
- Dilatation of steps: Radius for searching neighborhoods.
- Boundary jumps.

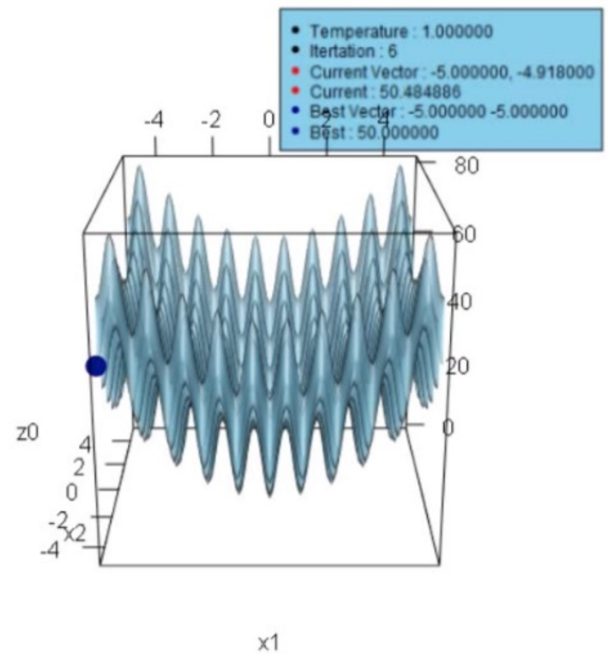
A convex function

$$-\cos x \;\; \cos y \; e^{-(x-pi)^2-(y-pi)^2}$$



A function with many local minima

$$20+(x^2-10\cos(2\pi x))+(y^2-10\cos(2\pi y))$$

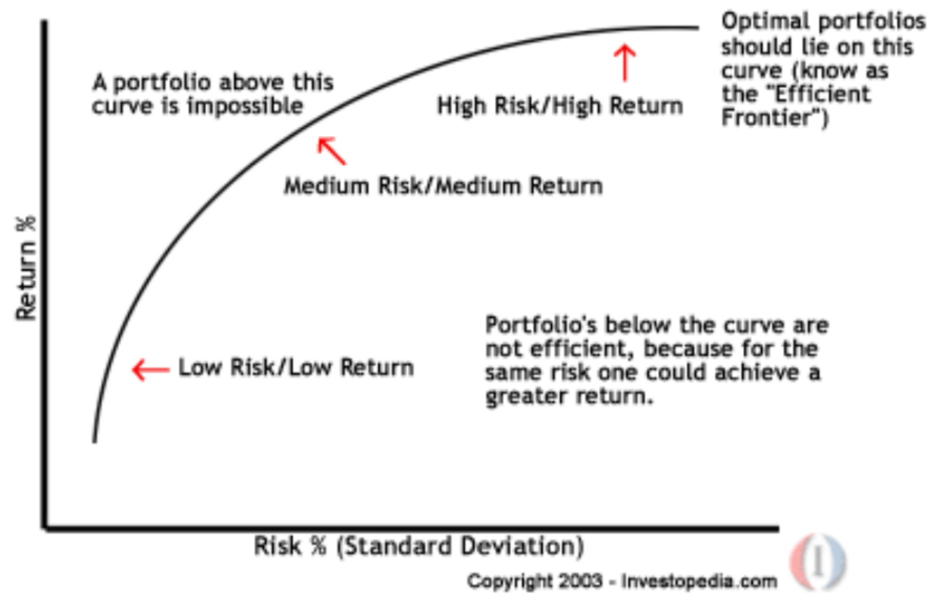


Efficient Market

- Assets price evolves as random walk.
- Everything is centered in mean and variance.
- Expected Return has normal distribution.
- It's impossible "to beat the market".
- Stock are accurately priced.
- Stock's price resume all the available information.

- In a efficient market, investors only have expected return and related risk.
- There are a lot of assets, different expected returns and risk.

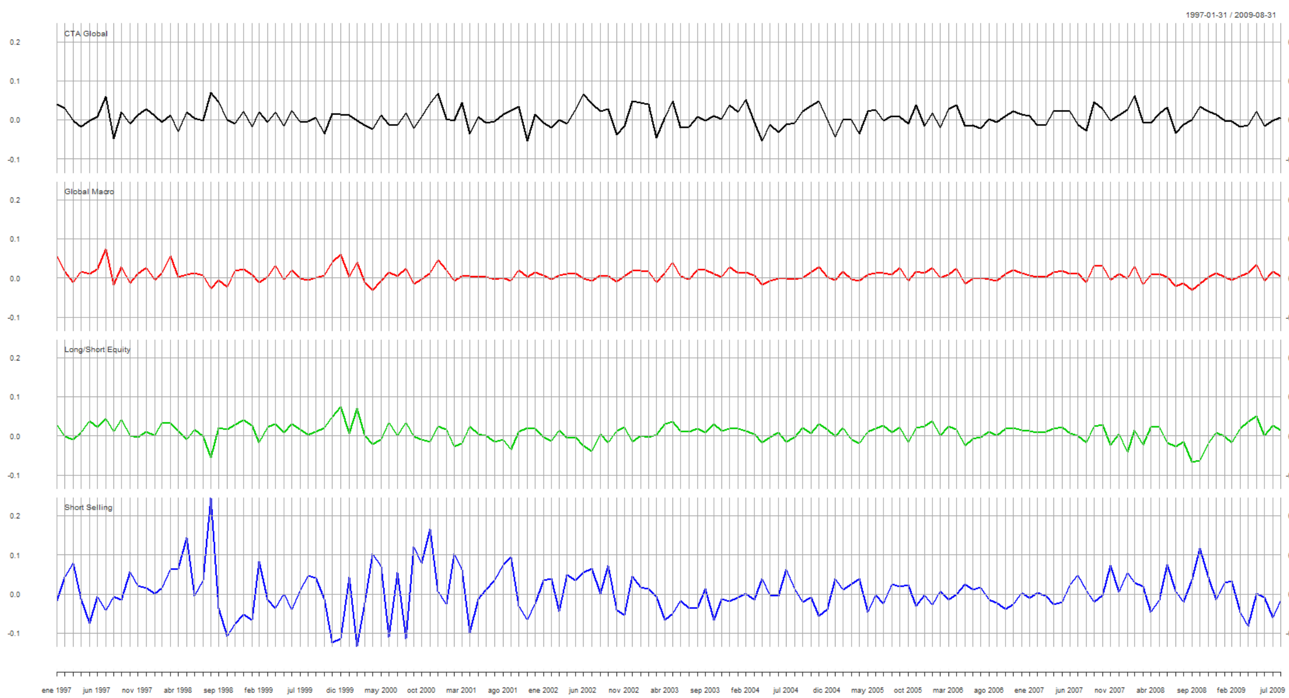
Solution:



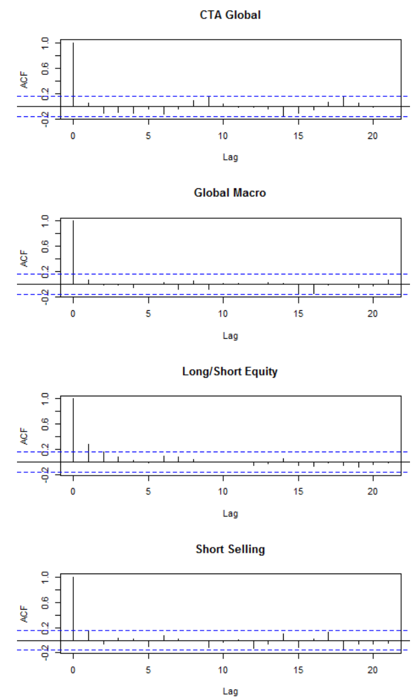
Application: Choosing the Portfolio

We used the PortfolioAnalytics R-package for building an efficient market for testing the SA algorithm: (CTA Global, Global Macro, Long/Short Equity, Short Selling)

Return of Assets



ACF of Returns



Application: Results

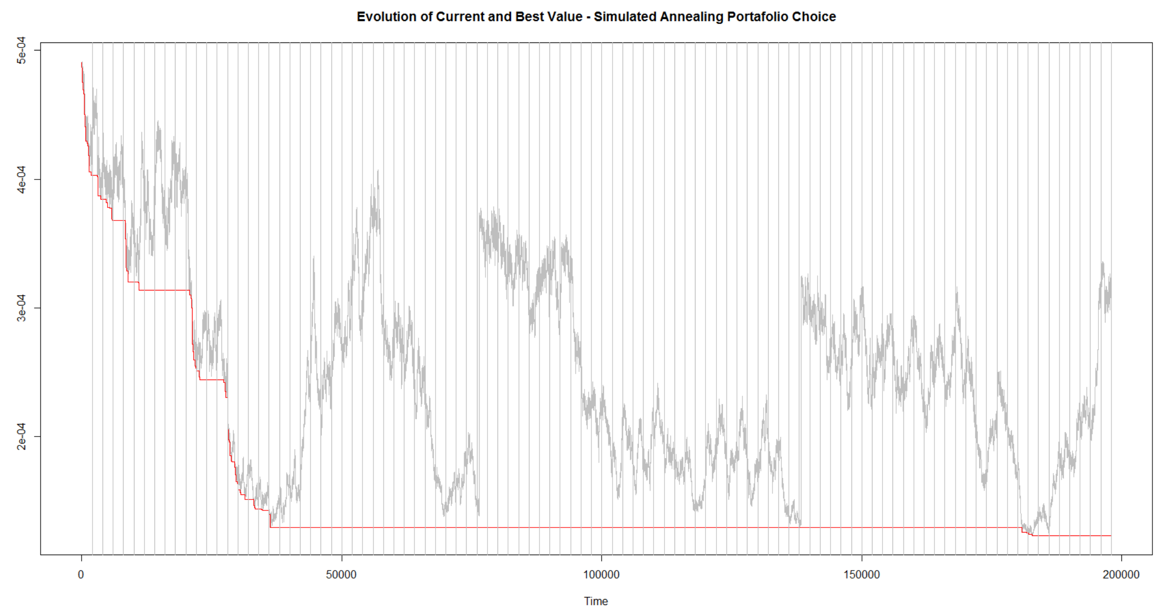
Parameters:
N. Simul = 2000
alpha = 0.97
x_0 = All 3rd asset.
Minimun return = 0.7%

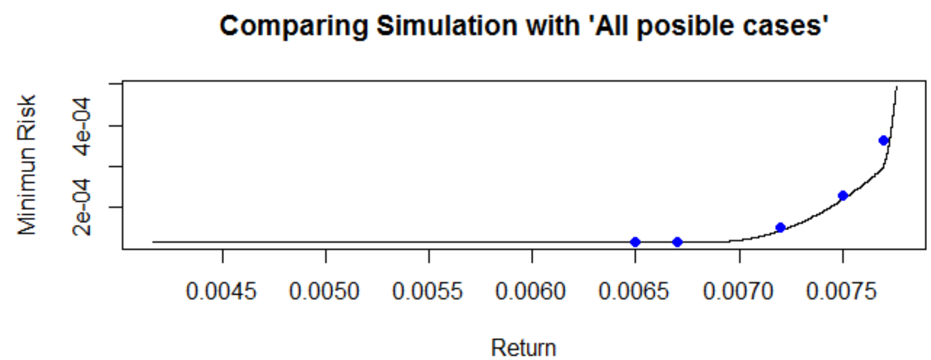
Results:
Asset Allocation:

Portfolio	%
CTA Global	3.9
Global Macro	27.2
Long/Short Equity	50.3
Short Selling	18.6

Return expected:
0.7017168%

Risk Asociated:
0.000122902





If we want to work with real scenarios, we will need to add more constraints. We start with two of them:

- Limiting the number of assets in an efficient portfolio *(cardinality constraint)*
- Prescribing lower and upper bounds on the fraction of the capital invested in each asset *(quantity constraint)*

This model is known as *Limited Asset Markowitz* model (LAM)

$$
\begin{aligned}
&\text{Min} \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \\
&st \\
&\qquad \sum_{i=1}^{n}\mu_i x_i = \rho \\
&\qquad \sum_{i=1}^{n} x_i = 1 \\
&\qquad x_i = 0 \text{ or } \ell_i \le x_i \le u_i, \quad i = 1, \ldots, n \\
&\qquad |supp(x)| \le K, \\
&\qquad \text{where } supp(x) = \{i : x_i > 0\}.
\end{aligned}
$$

Some aditional restriccions.

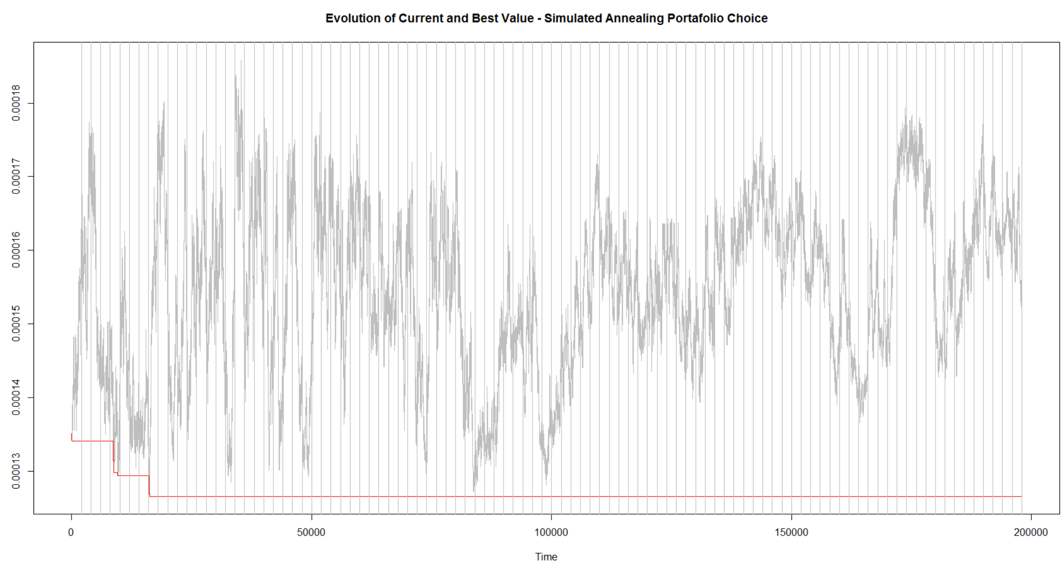
Parameters:
N. Simul = 2000
alpha = 0.97
$x_0 = (0.15, 0.15, 0.55, 0.15)$
Minimun return = 0.7%
Minimun % = 10% all assets
N. Portfolios chosen $\geqslant 2$

Results:
Asset Allocation:

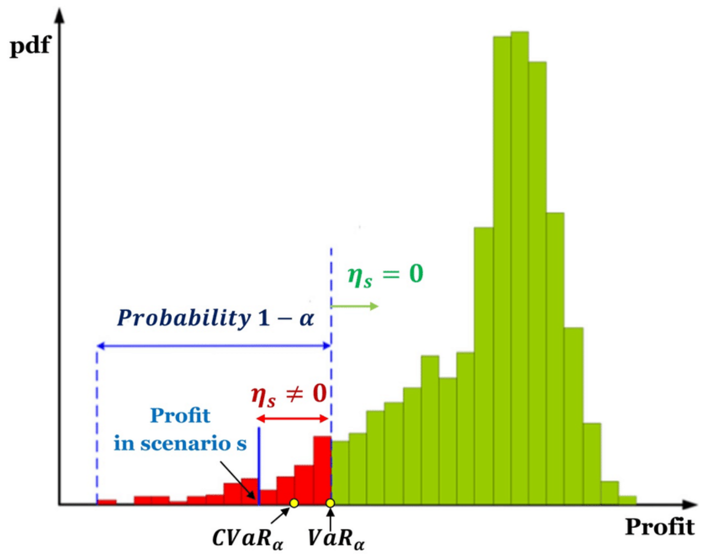
Portfolio	%
CTA Global	10.3
Global Macro	18.5
Long/Short Equity	54.2
Short Selling	17.0

Return expected:
0.7001054%

Risk Asociated:
0.0001265596



Now, in scenarios with assets heavy-tailed, the constraints about expected return and volatility are not enough. A more convenient risk-return model would be q using CVaR as a risk measure. This model can be written as follows:



Conditional value-at-risk (CVaR) calculation.

$$
\begin{aligned}
\min \quad & CVaR(x,\epsilon) \\
st \quad & \\
& \sum_{i=1}^{n} \mu_i x_i = \rho \\
& \sum_{i=1}^{n} x_i = 1 \\
& x_i = 0 \text{ or } \ell_i \leq x_i \leq u_i, \quad i = 1, \dots, n \\
& |supp(x)| \leq K,
\end{aligned}
$$

This new kind of objective functions (like *VaR*, *CVaR*,...) are not convex, they have many local minima and non linear constraints. Simulated Annealing algorithm is a good alternative to strugle with this situations.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SELL!
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BYE?
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Buy!
Buy!
BUY!!
Buy!
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