CHAPTER 05
RISK AND RETURN: PAST AND PROLOGUE

1. The 1% VaR will be less than –30%. As percentile or probability of a return declines so does the magnitude of that return. Thus, a 1 percentile probability will produce a smaller VaR than a 5 percentile probability.

2. The geometric return represents a compounding growth number and will artificially inflate the annual performance of the portfolio.

3. The excess return on the portfolio will be the same as long as you are consistent: you can use either real rates for the returns on both the portfolio and the risk-free asset, or nominal rate for each. Just don't mix and match! So the average excess return, the numerator of the Sharpe ratio, will be unaffected. Similarly, the standard deviation of the excess return also will be unaffected, again as long as you are consistent.

4. Decrease. Typically, standard deviation exceeds return. Thus, an underestimation of 4% in each will artificially decrease the return per unit of risk. To return to the proper risk return relationship the portfolio will need to decrease the amount of risk free investments.

5. Using Equation 5.10, we can calculate the mean of the HPR as:

$$E(r) = \sum_{s=1}^{S} p(s)\, r(s) = (0.3 \times 0.44) + (0.4 \times 0.14) + [0.3 \times (-0.16)] = 0.14 \text{ or } 14\%$$

Using Equation 5.11, we can calculate the variance as:

$$Var(r) = \sigma^2 = \sum_{s=1}^{S} p(s)\, [\, r(s) - E(r)]^2$$

$$= [0.3 \times (0.44 - 0.14)^2] + [0.4 \times (0.14 - 0.14)^2] + [0.3 \times (-0.16 - 0.14)^2]$$

$$= 0.054$$

Taking the square root of the variance, we get (using equation 5.12) $SD(r) = \sigma =$

$\sqrt{Var(r)} = \sqrt{0.054} = 0.2324$ or 23.24%

6. We use the below equation to calculate the holding period return of each scenario:

$$HPR = \frac{\text{Ending Price} - \text{Beginning Price} + \text{Cash Dividend}}{\text{Beginning Price}}$$

a. The holding period returns for the three scenarios are:

Boom: (50 – 40 + 2)/40 = 0.30 = 30%

Normal: (43 – 40 + 1)/40 = 0.10 = 10%

Recession: (34 – 40 + 0.50)/40 = –0.1375 = –13.75%

$$E(HPR) = \sum_{s=1}^{S} p(s)\, r(s)$$

$$= [(1/3) \times 0.30] + [(1/3) \times 0.10] + [(1/3) \times (-0.1375)]$$

$$= 0.0875 \text{ or } 8.75\%$$

$$Var(HPR) = \sum_{s=1}^{S} p(s)\, [\, r(s) - E(r)]^2$$

$$= [(1/3) \times (0.30 - 0.0875)^2] + [(1/3) \times (0.10 - 0.0875)^2]$$

$$+ [(1/3)\, (-0.1375 - 0.0875)^2]$$

$$= 0.031979$$

$$SD(r) = \sigma = \sqrt{Var(r)} = \sqrt{319.79} = 0.1788 \text{ or } 17.88\%$$

b. E(r) = (0.5 × 8.75%) + (0.5 × 4%) = 6.375%

σ = 0.5 × 17.88% = 8.94%

7.

a. Time-weighted average returns are based on year-by-year rates of return.

Year	Return = [(Capital gains + Dividend)/Price]
2010-2011	(110 – 100 + 4)/100 = 0.14 or 14.00%
2011-2012	(90 – 110 + 4)/110 = –0.1455 or –14.55%
2012-2013	(95 – 90 + 4)/90 = 0.10 or 10.00%

Arithmetic mean: [0.14 + (–0.1455) + 0.10]/3 = 0.0315 or 3.15%

Geometric mean: $\sqrt[3]{(1 + 0.14) \times [1 + (-0.1455)] \times (1 + 0.10)} - 1$
= 0.0233 or 2.33%

b.

	Date			
	1/1/2010	1/1/2011	1/1/2012	1/1/2013
Net Cash Flow	–300	–208	110	396

Time	Net Cash flow	Explanation
0	–300	Purchase of three shares at $100 per share
1	–208	Purchase of two shares at $110, plus dividend income on three shares held
2	110	Dividends on five shares, plus sale of one share at $90
3	396	Dividends on four shares, plus sale of four shares at $95 per share

The dollar-weighted return is the internal rate of return that sets the sum of the present value of each net cash flow to zero:

$$0 = -\$300 + \frac{-\$208}{1+\text{IRR}} + \frac{\$110}{(1+\text{IRR})^2} + \frac{\$396}{(1+\text{IRR})^3}$$

Dollar-weighted return = Internal rate of return = –0.1661%

8.

a. Given that A = 4 and the projected standard deviation of the market return = 20%, we can use the below equation to solve for the expected market risk premium:

$$A = 4 = \frac{\text{Average}(r_M) - r_f}{\text{Sample } \sigma_M^2} = \frac{\text{Average}(r_M) - r_f}{(20\%)^2}$$

$E(r_M) - r_f = A\sigma_M^2 = 4 \times (0.20)^2 = 0.16$ or 16%

b. Solve $E(r_M) - r_f = 0.09 = A\sigma_M^2 = A \times (0.20)^2$, we can get

$A = 0.09/0.04 = 2.25$

c. Increased risk tolerance means decreased risk aversion (A), which results in a decline in risk premiums.

9. From Table 5.3, we find that for the period 1926 – 2013, the mean excess return for S&P 500 over 1-month T-bills is 8.34%.

E(r) = Risk-free rate + Risk premium = 5% + 8.34% = 13.34%

10. To answer this question with the data provided in the textbook, we look up the historical excess returns of the large stocks, small stocks, and Treasury Bonds for 1926-2013 from Table 5.3.

Excess Return – Arithmetic Average
Large Stocks: 8.34%
Small Stocks: 13.94%
Long-Term T-Bonds: 1.83%

To estimate the HPR expected by today's investors for each asset class, add the *current* T-bill rate to the historical excess return:

Large Stocks: 8.34% + 5% = 13.34%
Small Stocks: 13.94%+ 5% = 18.95%
Long-Term T-Bonds: 1.83%+ 5% = 6.83%

11.

a. The expected cash flow is: $(0.5 \times \$50{,}000) + (0.5 \times \$150{,}000) = \$100{,}000$
With a risk premium of 10%, the required rate of return is 15%. Therefore, if the value of the portfolio is X, then, in order to earn a 15% expected return:

Solving $X \times (1 + 0.15) = \$100{,}000$, we get $X = \$86{,}957$

b. If the portfolio is purchased at \$86,957, and the expected payoff is \$100,000, then the expected rate of return, E(r), is:

$$\frac{\$100{,}000 - \$86{,}957}{\$86{,}957} = 0.15 = 15\%$$

The portfolio price is set to equate the expected return with the required rate of return.

c. If the risk premium over T-bills is now 15%, then the required return is:

$5\% + 15\% = 20\%$

The value of the portfolio (X) must satisfy:

$X \times (1 + 0.20) = \$100{,}000 \Rightarrow X = \$83{,}333$

d. For a given expected cash flow, portfolios that command greater risk premiums must sell at lower prices. The extra discount in the purchase price from the expected value is to compensate the investor for bearing additional risk.

12.

a. Allocating 70% of the capital in the risky portfolio *P*, and 30% in risk-free asset, the client has an expected return on the complete portfolio calculated by adding up the expected return of the risky proportion (*y*) and the expected return of the proportion (1 - *y*) of the risk-free investment:

$$E(r_C) = y \times E(r_P) + (1 - y) \times r_f$$

$$= (0.7 \times 0.17) + (0.3 \times 0.07) = 0.14 \text{ or } 14\% \text{ per year}$$

The standard deviation of the portfolio equals the standard deviation of the risky fund times the fraction of the complete portfolio invested in the risky fund:

$\sigma_C = y \times \sigma_P = 0.7 \times 0.27 = 0.189$ or 18.9% per year

b. The investment proportions of the client's overall portfolio can be calculated by the proportion of risky portfolio in the complete portfolio times the proportion allocated in each stock.

Security		Investment Proportions
T-Bills		30.0%
Stock A	$0.7 \times 27\% =$	18.9%

Stock B	$0.7 \times 33\% =$	23.1%
Stock C	$0.7 \times 40\% =$	28.0%

c. We calculate the reward-to-variability ratio (Sharpe ratio) using Equation 5.14.

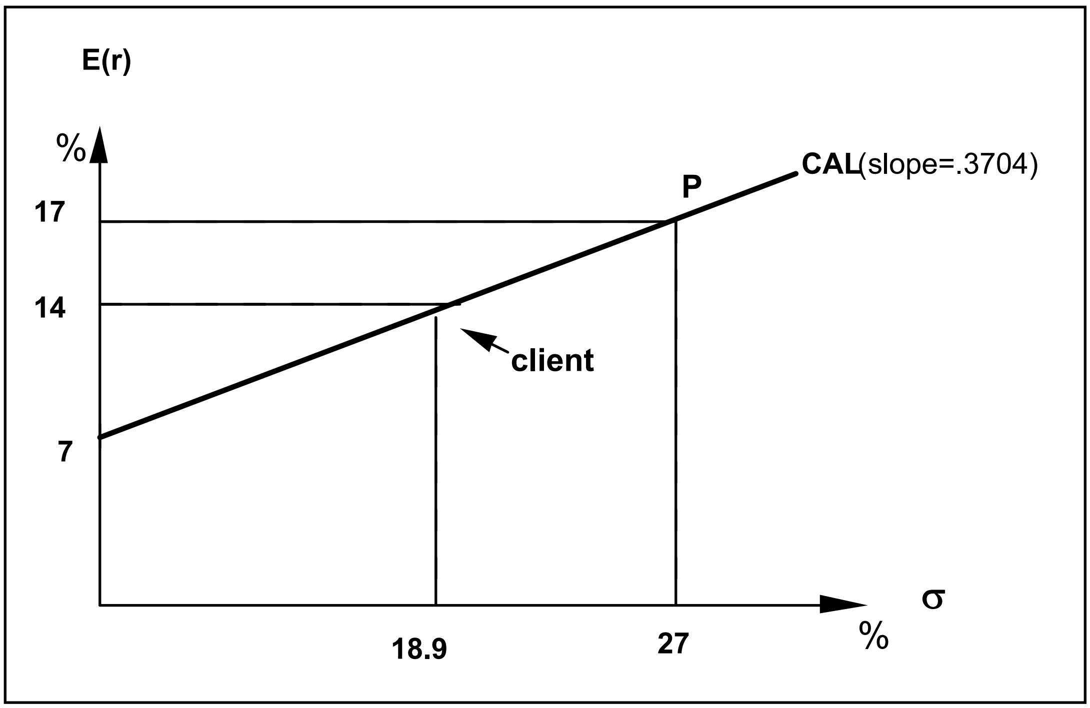
For the risky portfolio:

$$S = \frac{\text{Portfolio Risk Premium}}{\text{Standard Deviation of Portfolio Excess Return}}$$

$$= \frac{E(r_P) - r_f}{\sigma_P} = \frac{0.17 - 0.07}{0.27} = 0.3704$$

For the client's overall portfolio:

$$S = \frac{E(r_C) - r_f}{\sigma_C} = \frac{0.14 - 0.07}{0.189} = 0.3704$$



13.

a. $E(r_C) = y \times E(r_P) + (1 - y) \times r_f$

$$= y \times 0.17 + (1 - y) \times 0.07 = 0.15 \text{ or } 15\% \text{ per year}$$

Solving for y, we get $y = \frac{0.15 - 0.07}{0.10} = 0.8$

Therefore, in order to achieve an expected rate of return of 15%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b. The investment proportions of the client's overall portfolio can be calculated by the proportion of risky asset in the whole portfolio times the proportion allocated in each stock.

Security		Investment Proportions
T-Bills		20.0%
Stock A	$0.8 \times 27\% =$	21.6%
Stock B	$0.8 \times 33\% =$	26.4%
Stock C	$0.8 \times 40\% =$	32.0%

c. The standard deviation of the complete portfolio is the standard deviation of the risky portfolio times the fraction of the portfolio invested in the risky asset:

$$\sigma_C = y \times \sigma_P = 0.8 \times 0.27 = 0.216 \text{ or } 21.6\% \text{ per year}$$

14.

a. Standard deviation of the complete portfolio= $\sigma_C = y \times 0.27$
If the client wants the standard deviation to be equal or less than 20%, then:

$y = (0.20/0.27) = 0.7407 = 74.07\%$

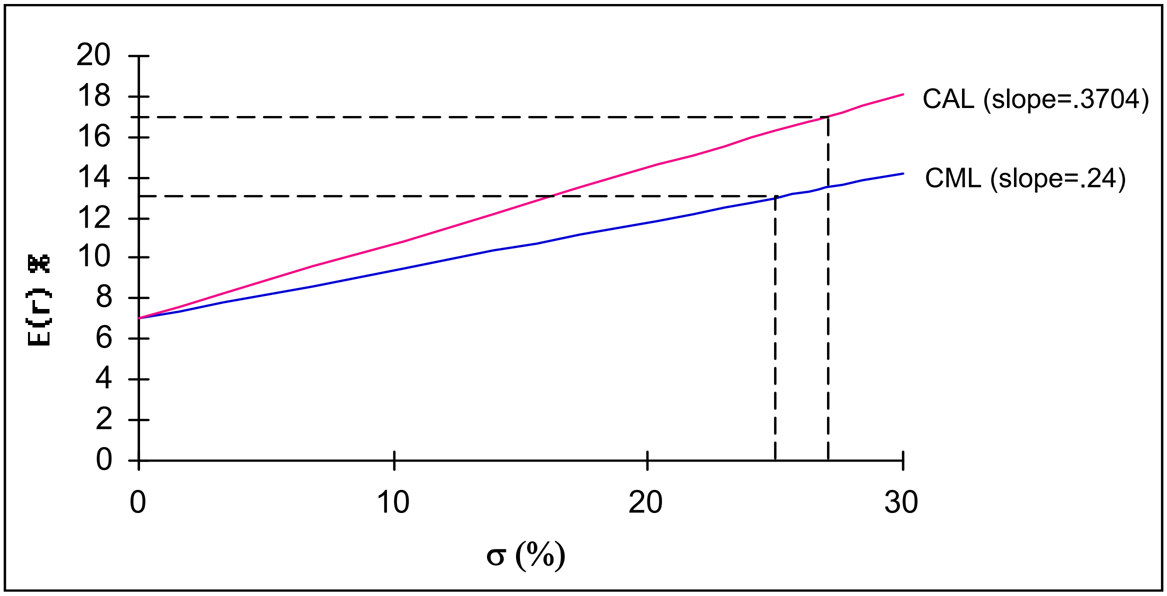
He should invest, at most, 74.07% in the risky fund.

b. $E(r_C) = r_f + y \times [E(r_P) - r_f] = 0.07 + 0.7407 \times 0.10 = 0.1441$ or 14.41%

15.

a. Slope of the CML $= \dfrac{E(r_M) - r_f}{\sigma_M} = \dfrac{0.13 - 0.07}{0.25} = 0.24$

See the diagram below:



b. Your fund allows an investor to achieve a higher expected rate of return for any given standard deviation than would a passive strategy, i.e., a higher expected return for any given level of risk.

16.

a. With 70% of his money in your fund's portfolio, the client has an expected rate of return of 14% per year and a standard deviation of 18.9% per year. If he shifts that money to the passive portfolio (which has an expected rate of return of 13% and standard deviation of 25%), his overall expected return and standard deviation would become:

$$E(r_C) = r_f + 0.7 \times [E(r_M) - r_f]$$

In this case, $r_f = 7\%$ and $E(r_M) = 13\%$. Therefore:

$$E(r_C) = 0.07 + (0.7 \times 0.06) = 0.112 \text{ or } 11.2\%$$

The standard deviation of the complete portfolio using the passive portfolio would be:

$$\sigma_C = 0.7 \times \sigma_M = 0.7 \times 0.25 = 0.175 \text{ or } 17.5\%$$

Therefore, the shift entails a decline in the mean from 14% to 11.2% and a decline in the standard deviation from 18.9% to 17.5%. Since both mean return *and* standard deviation fall, it is not yet clear whether the move is beneficial. The disadvantage of the shift is apparent from the fact that, if your client is willing to accept an expected return on his total portfolio of 11.2%, he can achieve that return with a lower standard deviation using your fund portfolio rather than the passive portfolio. To achieve a target mean of 11.2%, we first write the mean of the complete portfolio as a function of the proportions invested in your fund portfolio, *y*:

$$E(r_C) = 7\% + y \times (17\% - 7\%) = 7\% + 10\% \times y$$

Because our target is $E(r_C) = 11.2\%$, the proportion that must be invested in your fund is determined as follows:

$$11.2\% = 7\% + 10\% \times y \Rightarrow y = \frac{11.2\% - 7\%}{10\%} = 0.42$$

The standard deviation of the portfolio would be:

$$\sigma_C = y \times 27\% = 0.42 \times 27\% = 11.34\%$$

Thus, by using your portfolio, the same 11.2% expected rate of return can be achieved with a standard deviation of only 11.34% as opposed to the standard deviation of 17.5% using the passive portfolio.

b. The fee would reduce the reward-to-variability ratio, i.e., the slope of the CAL. Clients will be indifferent between your fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let *f* denote the fee:

$$\text{Slope of CAL with fee} = \frac{17\% - 7\% - f}{27\%} = \frac{10\% - f}{27\%}$$

$$\text{Slope of CML (which requires no fee)} = \frac{13\% - 7\%}{25\%} = 0.24$$

Setting these slopes equal and solving for *f*:

$$\frac{10\% - f}{27\%} = 0.24$$

$$10\% - f = 27\% \times 0.24 = 6.48\%$$

$$f = 10\% - 6.48\% = 3.52\% \text{ per year}$$

17. Assuming no change in tastes, that is, an unchanged risk aversion, investors perceiving higher risk will demand a higher risk premium to hold the same portfolio they held before. If we assume that the risk-free rate is unaffected, the increase in the risk premium would require a higher expected rate of return in the equity market.

18. Expected return for your fund = T-bill rate + risk premium = 6% + 10% = 16%
Expected return of client's overall portfolio = (0.6 × 16%) + (0.4 × 6%) = 12%
Standard deviation of client's overall portfolio = 0.6 × 14% = 8.4%

19. $$\text{Reward to volatility ratio} = \frac{\text{Portfolio Risk Premium}}{\text{Standard Deviation of Portfolio Excess Return}}$$

$$= \frac{10\%}{14\%} = 0.7143$$

20.

Excess Return (%)

	Average	Std Dev	Sharpe Ratio	5% VaR
1926-2013	13.94	37.29	0.37	-36.96
1926-1955	19.73	49.46	0.40	-46.25
1956-1985	12.22	32.35	0.38	-32.39
1986-2013	9.59	25.85	0.37	-27.94

a. In three out of four time frames presented, small stocks provide worse ratios than large stocks.

b. Small stocks show a markedly high standard deviation in the earliest subperiod, but risk has been on a declining trend since.

21. For geometric real returns, we take the geometric average return and the real geometric return data from Table 5.3 and then calculate the inflation in each time frame using the equation: Inflation rate = (1 + Nominal rate)/(1 + Real rate) – 1.

Geometric Real Returns (%) – Large Stocks

	Average	Inflation	Real Return
1926-2013	9.88	2.97	6.71
1926-1955	9.66	1.36	8.18
1956-1985	9.62	4.97	4.51
1986-2013	10.50	2.76	7.53

Risk Return Ratio – Large Stocks

	Arithmetic Real Return	Std Dev	Real Return to Risk
1926-2013	8.71	20.19	0.43
1926-1955	11.20	25.18	0.44
1956-1985	5.94	17.15	0.35
1986-2013	9.02	17.37	0.52

The VaR is not calculated.

Comparing with the excess return statistics in Table 5.4, in three out of four time frames the arithmetic real return is larger than the excess return, and the standard deviation of the real return in each time frame is lower than that of the excess return.

22.

Nominal Returns (%) – Small Stocks

	Nominal Return	Std Dev	Return to Risk

1926-2013	17.48	36.73	0.48
1926-1955	20.82	49.10	0.42
1956-1985	18.06	31.88	0.57
1986-2013	13.30	25.20	0.53

Real Return (%) – Small Stocks

	Arithmetic Real Return	Std Dev	Return to Risk
1926-2013	14.14	36.08	0.39
1926-1955	19.04	48.34	0.39
1956-1985	12.80	30.85	0.42
1986-2013	10.32	24.89	0.41

The VaR is not calculated.
Comparing the nominal rate with the real rate of return, the real rates in all time frames and their standard deviation are lower than those of the nominal returns.

23.

Results	T Bill	S&P 500	Market
Average	3.56	5.57	5.63
SD	2.96	20.33	20.41
Skew	0.90	-0.87	-0.88
Kurtosis	0.70	1.06	0.92
Percentile (5%)	0.05	-31.51	-35.04
Normal (5%)	-1.31	-27.86	-27.93
Min	-0.04	-61.89	-59.69
Max	13.73	43.24	45.13
Serial corr	0.91	0.05	0.06
Corr(SP500,Mkt)		0.99	
Corr(pf, risk-free)		-0.12	-0.15

Comparison
The combined market index represents the Fama-French market factor (Mkt). It is better diversified than the S&P 500 index since it contains approximately ten times as many stocks. The total market capitalization of the additional stocks, however, is relatively small compared to the S&P 500. As a result, the performance of the value-weighted portfolios is expected to be quite similar, and the correlation of the excess returns very high. Even though the sample contains 84 observations, the standard deviation of the annual returns is relatively high, but the difference between the two indices is very small. When comparing the continuously compounded excess returns, we see that the difference between the two portfolios is indeed quite small, and the correlation coefficient between their returns is 0.99. Both deviate from the normal distribution as seen from the negative skew and positive kurtosis. Accordingly, the VaR (5% percentile) of the two is smaller than what is expected from a normal distribution

with the same mean and standard deviation. This is also indicated by the lower minimum excess return for the period. The serial correlation is also small and indistinguishable across the portfolios.

As a result of all this, we expect the risk premium of the two portfolios to be similar, as we find from the sample. It is worth noting that the excess return of both portfolios has a small negative correlation with the risk-free rate. Since we expect the risk-free rate to be highly correlated with the rate of inflation, this suggests that equities are not a perfect hedge against inflation. More rigorous analysis of this point is important, but beyond the scope of this question.

CFA 1

Answer: $V(12/31/2011) = V(1/1/2005) \times (1 + g)^7 = \$100{,}000 \times (1.05)^7 = \$140{,}710.04$

CF 2

Answer: *a*. and *b*. are true. The standard deviation is non-negative.

CFA 3

Answer: c. Determines most of the portfolio's return and volatility over time.

CFA 4

Answer: Investment 3.

For each portfolio: Utility = $E(r) - (0.5 \times 4 \times \sigma^2)$

Investment	E(r)	σ	Utility
1	0.12	0.30	-0.0600
2	0.15	0.50	-0.3500
3	0.21	0.16	0.1588
4	0.24	0.21	0.1518

We choose the portfolio with the highest utility value.

CFA 5

Answer: Investment 4.
When an investor is risk neutral, A = 0 so that the portfolio with the highest utility is the portfolio with the highest expected return.

CFA 6

Answer: b. Investor's aversion to risk.

CFA 7

Answer:

$E(r_X) = [0.2 \times (-0.20)] + (0.5 \times 0.18) + (0.3 \times 0.50) = 0.20$ or 20%

$E(r_Y) = [0.2 \times (-0.15)] + (0.5 \times 0.20) + (0.3 \times 0.10) = 0.10$ or 10%

CFA 8

Answer:

$\sigma_X^2 = [0.2 \times (-0.20 - 0.20)^2] + [0.5 \times (0.18 - 0.20)^2] + [0.3 \times (0.50 - 0.20)^2] = 0.0592$

$\sigma_X = 0.2433 = 24.33\%$

$\sigma_Y^2 = [0.2 \times (-0.15 - 0.10)^2] + [0.5 \times (0.20 - 0.10)^2] + [0.3 \times (0.10 - 0.10)^2] = 0.0175$

$\sigma_Y = 0.1323 = 13.23\%$

CFA 9

Answer:

$E(r) = (0.9 \times 0.20) + (0.1 \times 0.10) = 0.19$ or 19%

CFA 10

Answer:

The probability is 0.5 that the state of the economy is neutral. Given a neutral economy, the probability that the performance of the stock will be poor is 0.3, and the probability of both a neutral economy and poor stock performance is:

$0.3 \times 0.5 = 0.15$

CFA 11

Answer:

$E(r) = (0.1 \times 0.15) + (0.6 \times 0.13) + (0.3 \times 0.07) = 0.114$ or 11.4%