Chapter 5 Risk and Return: Past and Prologue

Chapter Objectives

- Compute various measures of return on multi-year investments.
- Use either historical data the past performance of stocks and bonds or forward-looking scenario analysis to characterize the risk and return features of these investments.
- Determine the expected return and risk of portfolios that are constructed by combining risky assets with risk-free investments in Treasury bills.
- Use the Sharpe ratio to evaluate the performance of a portfolio and provide a guide for capital allocation.

5.1 RATES OF RETURN

- A key measure of investors' success is the rate at which their funds have grown during the investment period.
- The total **holding-period return (HPR)** of a share of stock depends on the increase (or decrease) in the price of the share over the investment period as well as on any dividend income the share has provided.
- The rate of return is defined as dollars earned over investment period, (price appreciation as well as dividends) per dollar invested:

$$\begin{aligned} \text{HPR} &= \frac{\text{Ending price - Beginning price + Cash dividend}}{\text{Beginning price}} \\ &= \text{Capital gains yield + Dividend yield} \end{aligned} \tag{5.1}$$

  - This definition of the HPR assumes that the dividend is paid at the end of the holding period.
  - When dividends are received earlier, the definition ignores reinvestment income between the receipt of the dividend and the end of the holding period.

- The percentage return from dividends, cash dividends/beginning price, is called the *dividend yield*, and so the dividend yield plus the capital gains yield equals the HPR.

- This definition of holding return is easy to modify for other types of investments.
  - For example, the HPR on a bond would be calculated using the same formula, with interest or coupon payments taking the place of dividends.
- Example 5.1: *Holding-Period Return*
  - The price of a share of an indexed stock is currently \$100, and your time horizon is one year. You expect the cash dividend during the year to be \$4, so your expected dividend yield is 4%.
    - ✓ Your HPR will depend on the price one year from now.
      - ➢ Suppose your best guess is that it will be \$110 per share. Then your expected *capital gain* will be \$10, so your capital gains yield is \$10/\$100 = .10, or 10%.
    - ✓ The HPR is the sum of the dividend yield plus the capital gain yield, 4% + 10% = 14%.
    - ✓ $\text{HPR} = \dfrac{\$110 - \$100 + \$4}{\$100} = .14, \text{ or } 14\%$

Measuring Investment Returns over Multiple Periods

- The HPR is a simple and unambiguous measure of investment return over a single period. But often investors are interested in average returns over longer periods of time.
  - For example, you might want to measure how well a mutual fund has performed over the preceding five-year period.
    - ✓ In this case, return measurement is ambiguous.
- Consider a fund that starts with \$1 million under management at the beginning of the year.
  - It receives additional funds to invest from new and existing shareholders and also redeems shares of existing shareholders so that cash inflow can be positive or negative.
  - The fund's quarterly results are given in Table 5.1 with negative numbers in parentheses.
  - The numbers indicate that when the firm does well (i.e., achieve a high HPR), it attracts new funds; otherwise it may suffer a net outflow.

TABLE 5.1 Quarterly cash flows and rates of return of a mutual fund

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
Assets under management at start of quarter ($ million)	1.0	1.2	2.0	0.8
Holding-period return (%)	10.0	25.0	(20.0)	20.0
Total assets before net inflows	1.1	1.5	1.6	0.96
Net inflow ($ million)*	0.1	0.5	(0.8)	0.6
Assets under management at end of quarter ($ million)	1.2	2.0	0.8	1.56

*New investment less redemptions and distributions, all assumed to occur at the end of each quarter.

- For example, the 10% return in the first quarter by itself increased assets under management by .10 × $1 million = $100,000; it also elicited new investments of $100,000, thus bringing assets under management to $1.2 million by the end of the quarter.
- An even better HPR in the second quarter elicited a larger net inflow, and the second quarter ended with $2 million under management.
- However, HPR in the third quarter was negative, and net inflows were negative.

- How would we characterize fund performance over the year, given that the fund experienced both cash inflows and outflows?
  - There are several candidate measures of performance, each with its own advantages and shortcomings. These measures may vary considerably, so it is important to understand their differences.
- Three methods are used to measure multi-period returns:
  - The *arithmetic average*
  - The *geometric average*
  - The *dollar-weighted return*
- ***ARITHMETIC AVERAGE***
  - The **arithmetic average** of the quarterly returns is just the sum of the quarterly returns divided by the number of quarters.
  - In the above example: (10% + 25% – 20% + 20%) ÷ 4 = 8.75%.
  - Because this statistic ignores compounding, it does not represent an equivalent, single quarterly rate for the year.

- However, without information beyond the historical sample, the arithmetic average is the best forecast of performance for the next quarter.

- ***GEOMETRIC AVERAGE***
  - The **geometric average** of the quarterly returns is equal to the single per-period return that would give the same cumulative performance as the sequence of actual returns.
  - We calculate the geometric average by compounding the actual period-by-period returns and then finding the single per-period rate that will compound to the same final value.
  - In our example, the geometric average quarterly return, r_G, is defined by:

    $$(1 + .10) \times (1 + .25) \times (1 - .20) \times (1 + .20) = (1 + r_G)^4$$

    - ✓ The left-hand side of this equation is the compounded value of a $1 investment earning r_G each quarter.
    - ✓ We solve for r_G:

    $$r_G = [(1 + .10) \times (1 + .25) \times (1 - .20) \times (1 + .25)]^{1/4} - 1 = .0719, \text{ or } 7.19\% \qquad (5.2)$$

- The geometric return is also called a *time-weighted average return* because it ignores the quarter-to-quarter variation in funds under management.
- In fact, an investor will obtain a larger cumulative return when high returns are earned in periods when additional sums have been invested and low returns are earned when less money is at risk.
- In Table 5.1:
  - ✓ The higher returns (25% and 20%) were achieved in quarters 2 and 4, when the fund managed $1,200,000 and $800,000, respectively.
  - ✓ The lower returns (-20% and 10%) occurred when the fund managed $2,000,000 and $l,000,000, respectively.
    - In this case, better returns were earned when *less* money was under management—an unfavorable combination.

- Published data on past returns earned by mutual funds actually are *required* to be time-weighted returns.
  - ✓ The rationale for this practice is that since the fund manager does not have full control over the amount of assets under management, we should not weight returns in one period more heavily than those in other periods when assessing "typical" past performance.

DOLLAR-WEIGHTED RETURN

- To account for the varying amounts under management, we treat the fund cash flows as we would a capital budgeting problem in corporate finance and compute the portfolio manager's internal rate of return (IRR).
- The initial value of $1 million and the net cash inflows are treated as the cash flows associated with an investment "project." The year-end "liquidation value" of the project is the final cash flow of the project.

- In our example, investor's net cash flows are as follows:

	Quarter				
	0	**1**	**2**	**3**	**4**
Net cash flow ($million)	-1.0	-.1	-.5	.8	-.6+1.56=.96

  - ✓ The entry for time 0 reflects the starting contribution of $1 million.
  - ✓ The negative entries for times 1 and 2 are additional net inflows in those quarters, whereas the positive value for quarter 3 signifies a withdrawal of funds.
  - ✓ Finally, the entry for time 4 represents the sum of the final (negative) cash flow plus the value of the portfolio at the end of the fourth quarter.
    - The latter is the value for which the portfolio could have been liquidated at year-end.
- The **dollar-weighted average return** is the internal rate of return of the project.

- The IRR is the interest rate that sets the present value of the cash flows realized on the portfolio (including the \$1.56 million for which the portfolio can be liquidated at the end of the year) equal to the initial cost of establishing the portfolio.
  - ✓ It therefore is the interest rate that satisfies the following equation:

$$0=-1.0+\frac{-.1}{1+IRR}+\frac{-.5}{(1+IRR)^2}+\frac{.8}{(1+IRR)^3}+\frac{.96}{(1+IRR)^4} \qquad (5.3)$$
$$\Rightarrow IRR=.0338=3.38\%$$

- The dollar-weighted return in this example is smaller than the time-weighted return of 7.19% because, as we noted, the portfolio returns were higher when less money was under management.
  - ✓ The difference between the dollar- and time-weighted average return in this case is quite large.

Conventions for Annualizing Rates of Return

- Returns on assets with regular cash flows, such as mortgages (with monthly payments) and bonds (with semiannual coupons), usually are quoted as annual percentage rates, or APRs, which annualize per-period rates using a simple interest approach, ignoring compound interest:

  APR = Per-period rate × Periods per year

  - However, because it ignores compounding, the APR does not equal the rate at which your invested funds actually grow.

- The latter is called the *effective annual rate* (EAR).
  - When there are n compounding periods in the year, we first recover the rate per period as APR/n and then compound that rate for the number of periods in a year. For example, n = 12 for mortgages and n = 2 for bonds making payments semiannually.

$$1+\text{EAR}=(1+\text{Rate per period})^n=\left(1+\frac{\text{APR}}{n}\right)^n \Rightarrow \text{EAR}=\left(1+\frac{\text{APR}}{n}\right)^n-1 \qquad (5.4)$$

- Because the rate earned each period is APR/n, after one year (when n periods have passed), your cumulative return would be $(1 + \text{APR}/n)^n$.
  - Note that one needs to know the holding period when given an APR in order to convert it to an effective rate.
- Rearranging Equation 5.4, we can also find APR given EAR:

$$\text{APR} = [(1 + \text{EAR})^{1/n} - 1] \times n$$

- EAR diverges from APR by greater amounts as n becomes larger (we compound cash flows more frequently). 預見
  - In the limit, we can envision continuous compounding when n becomes extremely large in Equation 5.4.
  - With continuous compounding, the APR is related to EAR by

$$1 + \text{EAR} = e^{\text{APR}} \Rightarrow \text{EAR} = e^{\text{APR}} - 1 \qquad (5.5\text{A})$$

or equivalently,

$$\text{APR} = \ln(1 + \text{EAR}) \qquad (5.5\text{B})$$

- Example 5.2: *Annualizing Treasury-Bill Returns*
  - Suppose you buy a $10,000 face value Treasury bill maturing in six months for $9,900. On the bill's maturity date, you collect the face value.
    - ✓ Because there are no other interest payments, the holding period return for this six-month investment is:

$$\text{HPR} = \frac{\text{Cash income} + \text{Price change}}{\text{Initial price}} = \frac{0 + \$100}{\$9{,}900} = 0.0101 = 1.01\%$$

    - ✓ The APR on this investment is therefore 1.01% × 2 = 2.02%.
    - ✓ The effective annual rate is higher:

$$1 + \text{EAR} = (1 + \frac{0.1212}{12})^2 = (1.0101)^2 = 1.0203$$

which implies that EAR = .0203 = 2.03%

5.2 INFLATION AND THE REAL RATE OF INTEREST

- A 10% annual rate of return means that your investment is worth l0% more at the end of the year than it was at the beginning.
  - This does not necessarily mean, however, that you can have buy 10% more goods and services with that money, for it is possible that in the course of the year prices of goods also have increased.
  - If prices have changed, the increase in your purchasing power will not match the increase in your dollar wealth.
- At any time, the prices of some goods may rise while the prices of other goods may fall; the *general* trend in prices is measured by examining changes in the consumer price index (CPI).
  - The CPI measures the cost of purchasing a bundle of goods that is considered representative of the “consumption basket” of a typical urban family of four.
    - ✓ Increases in the cost of this standardized consumption basket are indicative of a general trend toward higher prices.
  - The **inflation rate** is measured by the rate of increase of the CPI.

- Suppose the rate of inflation (the percentage change in the CPI, denoted by i) for the last year amounted to $i = 6\%$.
  - The purchasing power of money is thus reduced by 6% a year.
    - ✓ Therefore, part of your investment earnings are offset by the reduction in the purchasing power of the dollars you will receive at the end of the year.
    - ✓ With a 10% interest rate, after you net out the 6% reduction in the purchasing power of money, you are left with a net increase in purchasing power of about 4%.
- Thus, we need to distinguish between a **nominal interest rate**—the growth rate of your money—and a **real interest rate**—the growth rate of your purchasing power.
- If we call R the nominal rate, r the real rate, and i the inflation rate, then we conclude

$$r \approx R - i \tag{5.6}$$

  - In words, the real rate of interest is the nominal rate reduced by the rate of inflation.

- In fact, the exact relationship between the real and nominal interest rate is given by

$$
\begin{aligned}
&1+r=\frac{1+R}{1+i} \\
&\Rightarrow\ 1+R=(1+r)(1+i)=1+r+i+ri\approx 1+r+i \text{ if } ri \text{ is small} \qquad (5.7)\\
&\Rightarrow\ R\approx r+i \\
&\Rightarrow\ r\approx R-i
\end{aligned}
$$

  - In words, the growth factor of your purchasing power, $1 + r$, equals the growth factor of your money, $1 + R$, divided by the new price level that is $1 + i$ times its value in the previous period.

- The exact relationship can be rearranged to

$$
1+r=\frac{1+R}{1+i} \ \Rightarrow\ r=\frac{1+R}{1+i}-1 \ \Rightarrow\ r=\frac{R-i}{1+i} \qquad (5.8)
$$

which shows that the approximate rule overstates the real rate by the factor $1 + i$.

- Example 5.3: *Real versus Nominal Rates*
  - If the interest rate on a one-year CD is 8%, and you expect inflation to be 5% over the coming year, then using the approximation given in Equation 5.6, you expect the real rate to be $r = 8\% - 5\% = 3\%$.
  - Using the exact formula given in Equation 5.8, the real rate is $r = (.08 - .05)/(1 + .05) = .0286$, or 2.86%.
    - ✓ Therefore, the approximation rule overstates the expected real rate by only 0.14 percentage points.
    - ✓ The approximation rule of Equation 5.7 is more accurate for small inflation rates and is perfectly exact for continuously compounded rates.

The Equilibrium Nominal Rate of Interest

- Because investors should be concerned with their real returns—the increase in their purchasing power—they will demand higher nominal rates of return in the face of expected inflation.
  - This higher rate is necessary to maintain the expected real return offered by an investment.

- Irving Fisher (1930) argued that the nominal rate ought to increase one-for-one with increases in the expected inflation rate.
  - Using $E(i)$ to denote the current expected inflation over the coming period, the *Fisher equation* is

$$R = r + E(i) \tag{5.9}$$

    - ✓ Suppose the real rate of interest is 2%, and expected inflation rate is 4%, so that the nominal interest rate is about 6%.
    - ✓ If expected inflation rises to 5%, the nominal interest rate should climb to roughly 7%.
    - ✓ The increase in the nominal rate offsets the increase in expected inflation, giving investors an unchanged growth of purchasing power at a 2% rate.
- The Fed in normally concerned with price stability, that is, keeping inflation both moderate and predictable.
  - When it is successful, T-bill rates tend to follow anticipated inflation, thereby resulting in a stable real rate, just as Fisher hypothesized.

- The Fisher equation is more prone to fail in extreme and, more importantly, unanticipated economic conditions.
  - For example, in the face of a booming economy, when inflation substantially exceeds expectations, nominal rates can lag and real rates will fall to unusually low, even negative, levels.
  - At the other end, when the economy tanks and inflation falls rapidly, if interest rates lag behind, real rates will rise to abnormally high levels.
  - In even more extreme downturn such as the Great Depression, inflation can be negative, with the general level of prices actually falling; nominal rates, however, cannot sink below zero, meaning that the real rate can be extremely high.
  - Hence, the empirical validity of the Fisher equation may be quite sensitive to boom and bust episodes within any particular period.

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5.3 RISK AND RISK PREMIUMS

- Any investment involves some degree of uncertainty about future holding-period returns, and in many cases that uncertainty is considerable.
- Sources of investment risk range from macroeconomic fluctuations, to the changing fortunes of various industries, to asset-specific unexpected development.

● Scenario Analysis and Probability Distributions

- When we attempt to quantify risk, we begin with the question: What HPRs are possible, and how likely are they?
  - ◆ A good way to approach this question is to devise a list of possible economic outcomes, or *scenarios*, and specify both the likelihood (i.e., the probability) of each scenario and the HPR the asset will realize in that scenario.
    - ✓ Therefore, this approach is called **scenario analysis.**
  - ◆ The list of possible HPRs with associated probabilities is called the **probability distribution** of HPRs.

- Consider an investment in a broad portfolio of stocks, say, an index fund, which we will refer to as the "stock market." A very simple scenario analysis for the stock market (assuming only four possible scenarios) is illustrated in spreadsheet 5.1.

	A	B	C	D	E	F	G	H
1				**Column B x**	**Deviation from**	**Column B x**	**Spreadsheet 5.1**	
2	**Scenario**	**Probability**	**HPR (%)**	**Column C**	**Mean Return**	**Squared Deviation**	**Scenario analysis**	
3	1 Severe recession	0.05	-37	-1.85	-47.00	110.45	**for the stock market**	
4	2 Mild recession	0.25	-11	-2.75	-21.00	110.25		
5	3 Normal growth	0.40	14	5.60	4.00	6.40		
6	4 Boom	0.30	30	9.00	20.00	120.00		
7	Column Sum = Expected return =			10.00	Variance =	347.10		
8	Square Root of Variance = Standard deviation (%) =					18.63		

- The probability distribution lets us derive measurements for both the reward and the risk of the investment.
  - ◆ The reward from the investment is its **expected return,** which you can think of as the average HPR you would earn if you were to repeat an investment in the asset many times.
  - ◆ The expected return also is called the *mean of the distribution* of HPRs and often is referred to as the *mean return*.

- To compute the expected return from the data provided, we label scenarios by s and denote the HPR in each scenario as $r(s)$, with probability $p(s)$.
  - The expected return, denoted $E(r)$, is then the weighted average of returns in all possible scenarios, $s = 1,\ldots, S$, with weights equal to the probability of that particular scenario.

$$E(r)=\sum_{s=1}^{S} p(s)r(s) \tag{5.10}$$

- Each entry in column D of Spreadsheet 5.1 corresponds to one of the products in the summation in Equation 5.10.
  - The value in cell D7, which is the sum of these products, is therefore the expected return. Therefore, $E(r) = 10\%$.

- Because there is risk to the investment, and the actual return may be (a lot) more or less than 10%.
  - If a "boom" materializes, the return will be better, 30%, but in a severe recession, the return will be a disappointing -37%.
  - How can we quantify the uncertainty of the investment?
- The "surprise" return in any scenario is the difference between the actual return and the expected return.
  - For example, in a boom (scenario 4), the surprise is 20%: $r(4) – E(r) = 30\% – 10\% = 20\%$.
  - In a severe recession (scenario 1), the surprise is -47%: $r(1) – E(r) = -37\% – 10\% = -47\%$.
- Uncertainty surrounding the investment is a function of both the magnitudes and the probabilities of the possible surprises.
  - To summarize risk with a single number, we define the **variance** as the expected value of the *squared* deviation from the mean (the expected value of the squared "surprise" across scenarios).

$$\mathrm{Var}(r) \equiv \sigma^2 = \sum_{s=1}^{S} p(s)[r(s) - E(r)]^2 \quad (5.11)$$

- ✓ We square the deviations because otherwise, negative deviations would offset positive deviations with the result that the expected deviation from the mean return would necessarily be zero.
  - ➢ Squared deviations are necessarily positive.
- ✓ Squaring (which is a nonlinear transformation) exaggerates large (positive or negative) deviations and deemphasizes small deviations.

- ■ Another result of squaring deviations is that the variance has a dimension of percent squared.
  - ◆ To give the measure of risk the same dimension as expected return (%), we use the **standard deviation,** defined as the square root of the variance:

$$SD(r) \equiv \sigma = \sqrt{\mathrm{Var}(r)} \quad (5.12)$$

- Example 5.4: *Expected Return and Standard Deviation*
  - Applying Equation 5.10 to the data in Spreadsheet 5.1, we find that the expected rate of return on the stock index fund is

    $E(r) = .05 \times (-37\%) + .25 \times (-11\%) + .40 \times 14\% + .30 \times 30\% = 10\%$
  - We use Equation 5.11 to find the variance.
    - ✓ First we take the difference between the holding period return in each scenario and the mean return, then we square the difference, and finally we multiply the probability of each scenario.
    - ✓ The sum of the probability-weighted squared deviation is the variance.

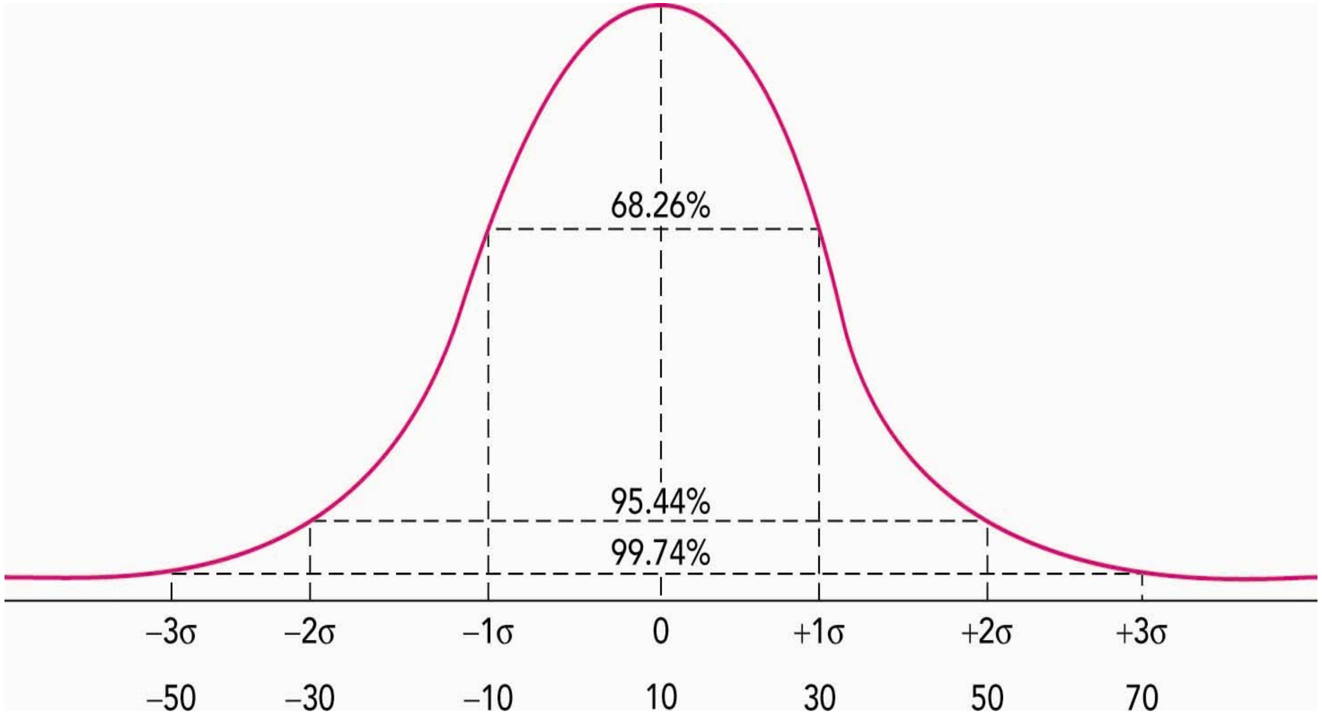
      $\sigma^2 = .05(-37 - 10)^2 + .25(-11 - 10)^2 + .40(14 - 10)^2 + .30(30 - 10)^2 = 347.10$
    - ✓ So the standard deviation is

      $\sigma = \sqrt{347.10} = 18.63\%$

- Column F of Spreadsheet 5.1 replicates these calculations.
  - ✓ Each entry in that column is the squared deviation from the mean multiplied by the probability of that scenario.
  - ✓ The sum of the probability-weighted squared deviations that appears in cell F7 is the variance, and the square root of that value is the standard deviation (in cell F8).

The Normal Distribution

- The normal distribution is central to the theory *and* practice of investments.
  - Its familiar bell-shaped plot is symmetric, with identical values for all three standard measures of "typical" results: the mean (the expected value discussed earlier), the median (the value above and below which we expect 50% of the observations), and the mode (the most likely value).
- Figure 5.1 illustrates a normal distribution with a mean of 10% and standard deviation (SD) of 20%.



- Notice that the probabilities are highest for outcomes near the mean and are significantly lower for outcomes far from the mean.
- But what do we mean by an outcome "far" from the mean?
  - ✓ A return 15% below the mean would hardly be noteworthy if typical volatility were high, for example, if the standard deviation of returns were 20%, but that same outcome would be highly unusual if the standard deviation were only 5%.

- ✓ For this reason, it is often useful to think about deviations from the mean in terms of how many standard deviations they represent.

- ■ We can transform any normally distributed return, r_i, into a "standard deviation score," by first subtracting its mean (to obtain distance from the mean or the return "surprise") and then dividing by the standard deviation (which enables us to measure distance from the mean in units of standard deviations).

$$sr_i = \frac{r_i - E(r_i)}{\sigma_i} \tag{5.13A}$$

- ◆ This standardized return, which we have denoted sr_i, is normally distributed with a mean of zero and a standard deviation of 1.
  - ✓ We therefore say that sr_i is a "standard normal" variable.

- ■ Conversely, we can start with a standard normal return, sr_i, and recover the original return by multiplying by the standard deviation and adding back the mean return:

$$r_i = E(r_i) + sr_i \times \sigma_i \tag{5.13B}$$

- In fact, this is how we drew Figure 5.1.
  - ✓ Start with a standard normal (mean = 0 and SD = 1).
  - ✓ Next, multiply the distance from the mean by the assumed standard deviation of 20%.
  - ✓ Finally, recenter the mean away from zero by adding 10%.
  - ✓ This gives us a normal variable with mean 10% and standard deviation 20%.

- Figure 5.1 shows that when returns are normally distributed, roughly two-thirds (more precisely, 68.26%) of the observations fall within one standard deviation of the mean.
  - ✓ That is, the probability that any observation in a sample of returns would be no more than one standard deviation away from the mean is 68.26%.
  - Deviations from the mean of more than two SDs are even rarer: 95.44% of the observations are expected to lie within this range.
  - Finally, only 2.6 out of 1,000 observations are expected to deviate from the mean by three or more SDs.

- Two special properties of the normal distribution lead to critical simplifications of investment management when returns are normally distributed:
  - 1. The return on a portfolio comprising two or more assets whose returns are normally distributed also will be normally distributed.
  - 2. The normal distribution is completely described by its mean and standard deviation.
    - ✓ No other statistic is needed to learn about the behavior of normally distributed returns.
  - 3. The above two properties in turn imply this far-reaching conclusion: The standard deviation is the appropriate measure of risk for a portfolio of assets with normally distributed returns.
    - ✓ In this case, no other statistic can improve the risk assessment conveyed by the standard deviation of a portfolio.

- Suppose you are worried about the magnitude of your possible investment losses. You may try to think about worst-case scenarios for your portfolio.
  - You might ask: “How much would I lose in a fairly extreme outcome, for example, if my return were in the fifth percentile of the distribution?”
    - ✓ You can expect your investment experience to be worse than this value only 5% of the time and better 95% of the time. 說法
    - ✓ In investments parlance, this cutoff is called the **value at risk** (denoted by **VaR,** to distinguish it from Var, the common notation for variance).
    - ✓ A loss-averse investor might desire to limit portfolio VaR, that is, limit the loss corresponding to a probability of 5%.
- For normally distributed returns, VaR can be derived from the mean and standard deviation of the distribution.
  - We calculate it using Excel’s standard normal function = NORMSINV(0.05).
    - ✓ This function computes the fifth percentile of a normal distribution with a mean of zero and a variance of 1, which turns out to be -1.64485.

- In other words, a value that is 1.64485 standard deviations below the mean would correspond to a VaR of 5%.

$$\text{VaR} = E(r) + (-1.64485)\sigma \qquad (5.14)$$

- We also can obtain this value directly from Excel's *nonstandard* normal distribution function = NORMINV(.05, $E(r)$, σ).

- When faced with a sample of actual returns that may not be normally distributed, we must estimate the VaR directly.
  - The 5% VaR is the fifth-percentile rate of return.
    - ✓ For a sample of 100 returns this is straightforward: If the rates are ordered from high to low, count the fifth observation from the bottom.
- Calculating the 5% VaR for samples where 5% of the observations don't make an integer requires interpolation.

- Suppose we have 72 monthly observations so that 5% of the sample is 3.6 observations.
  - ✓ We approximate the VaR by going .6 of the distance from the third to the fourth rate from the bottom. Suppose these rates are -42% and -37%. The interpolated value for VaR is then -42% + .6(42% – 37%) = -39%.

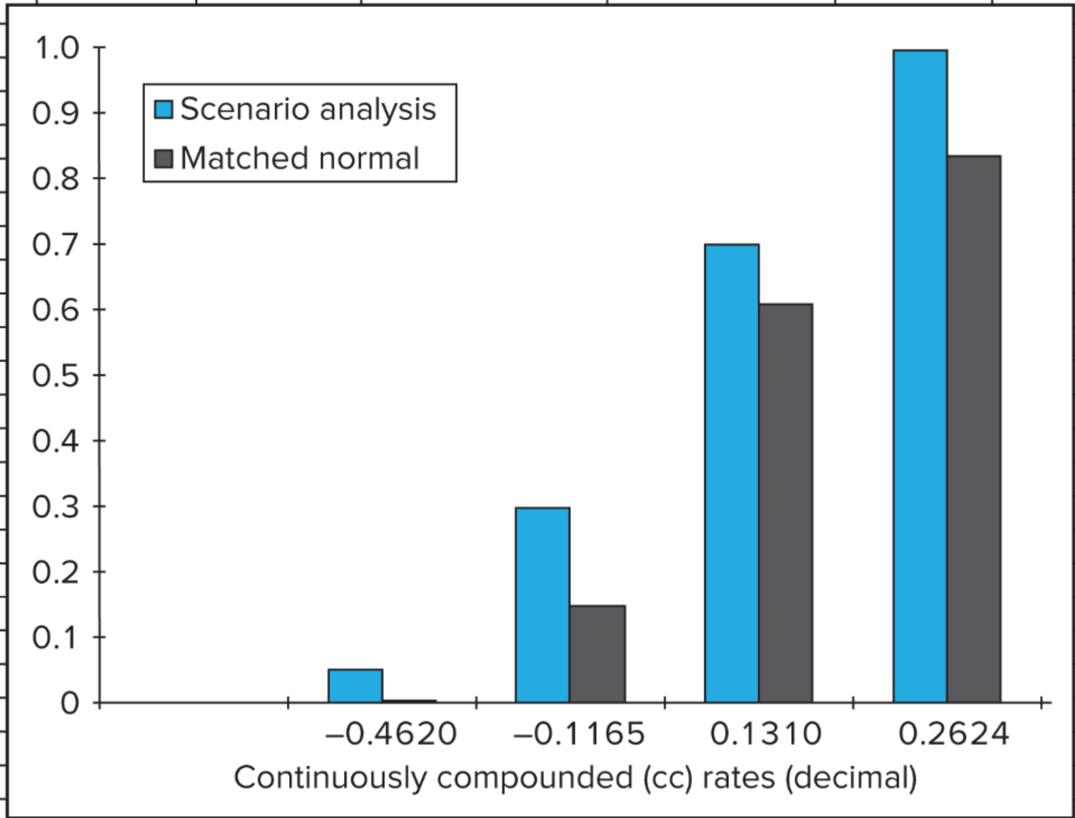
- In practice, analysts sometimes compare the historical sample VaR to the VaR implied by a normal distribution with the same mean and SD as the sample rates.
  - The difference between these VaR values indicates the deviation of the observed rates from normality.

Deviation from Normality and Value at Risk

- How can the returns specified in the scenario analysis in Spreadsheet 5.1 be judged against the normal distribution?
  - As prescribed above, we first convert the effective rates in each scenario to continuously compounded rates using Equation 5.5.
  - Obviously, it is naïve to believe that this simple analysis specifies the probabilities of all possible rates.

- But while we cannot explicitly pin down probabilities of rates other than those given in the table, we can get a good sense of the entire spectrum of potential outcomes by examining the distribution of the assumed scenario rates, as well as their mean and standard deviation.

- Figure 5.2 shows the known points from the cumulative distribution of the scenario analysis next to the corresponding points from a "matched normal distribution" (a normal distribution with the same mean and standard deviation).
  - Below the graph, we see a table of the actual distributions.
    - ✓ The mean in cell D34 is computed from the formula =SUMPRODUCT($B$30:$B$33, D30:D33), where the probability cells B30:B33 are fixed to allow copying to the right.
      - The Excel function SUMPRODUCT multiplies each term in the first column specified (in this case, the probabilities in column B) with the corresponding terms in the second column specified (in this case, the returns in column D), and then adds up those products. This gives us the expected rate of return across scenario.



	A	B	C	D	E	F	G
27							Cumulative
28				**Scenario Analysis**		**Squared**	**Corresponding**
29		**Probability**	**Cumulative**	**Effective Rate**	**cc Rate (decimal)**	**Deviations**	**Normal**
30		0.05	0.05	–0.37	–0.4620	0.2926	0.0020
31		0.25	0.3	–0.11	–0.1165	0.0382	0.1494
32		0.40	0.7	0.14	0.1310	0.0027	0.6092
33		0.30	1	0.30	0.2624	0.0337	0.8354
34			Mean	0.10	0.0789		
35					SD	0.1881	
36							
37		VaR Scenario Analysis		–0.37	–0.4620		
38		VaR corresponding Normal		–0.2058	–0.2304		
39							

- ✓ Similarly, the SD in cell F35 is computed from =SUMPRODUCT($B30:$B33, F30:F33)^0.5.
- ✓ The 5% VaR of the normal distribution in cell E38 is computed from =NORMINV(0.05, E34, F35).
- ✓ VaR values appear in cell D37 and D38. The VaR from the scenario analysis, -37%, is far worse than the VaR corresponding to the matched normal distribution, -20.58%.
  - ➢ This immediately suggests that the scenario analysis entails a higher probability of extreme losses than would be consistent with a normal distribution.
  - ➢ On the other hand, the normal distribution allows for the possibility of extremely large returns, beyond the maximum return of 30% envisioned in the scenario analysis.

    - ✓ We conclude that the scenario analysis has a distribution that is skewed to the left compared to the normal.
        - ➢ It has a longer left tail (larger losses) and a shorter right tail (smaller gains).
        - ➢ It makes up for this negative attribute with a larger probability of positive, but not extremely large, gains (14% and 30%).

- This example shows when and why the VaR is an important statistic.
    - ◆ When returns are normal, knowing just the mean and standard deviation allows us to fully describe the entire distribution.
        - ✓ In that case, we do not need to estimate VaR explicitly—we can calculate it exactly from the properties of the normal distribution.
    - ◆ But when returns are not normal, the VaR conveys important additional information beyond mean and standard deviation, specifically, the probability of severe losses arising from deviations from normality.
        - ✓ The financial crisis of 2008-2009 demonstrated that bank portfolio returns are far from normally distributed, with exposure to unlikely but catastrophic returns in extreme market meltdowns.

- ✓ The international Basel accord on bank regulation requires banks to monitor portfolio VaR to better control risk.

- Because risk is largely driven by the likelihood of extreme negative returns, two statistics are used to indicate whether a portfolio's probability distribution differs significantly from normality with respect to potential extreme values.
  - The first is **kurtosis,** which compares the frequency of extreme values to that of the normal distribution.
    - ✓ The kurtosis of the normal distribution is zero.
      - Positive values indicate higher frequency of extreme values than this benchmark.
      - A negative value suggests that extreme values are less frequent than with the normal distribution.
    - ✓ Kurtosis sometimes is called "fat tail risk," as plots of probability distributions with higher likelihood of extreme events will be higher than the normal distribution at both ends or "tails" of the distribution.
      - In other words, the distributions exhibit "fat tails."

- ✓ Similarly, exposure to extreme events is often called *tail risk*, because these are outcomes in the far reaches or "tail" of the probability distribution.
- ◆ The second statistic is the **skew,** which measures the asymmetry of the distribution.
  - ✓ Skew takes on a value of zero if, like the normal, the distribution is symmetric.
    - ➢ Negative (positive) skew suggests that extreme negative (positive) values are more frequent than extreme positive (negative) ones.
- ◆ Nonzero values for kurtosis and skew indicate that special attention should be paid to the VaR, in addition to the use of standard deviation as measure of portfolio risk.

● Using Time Series of Return

- ■ Scenario analysis postulates a probability distribution of future returns. But where do the probabilities and rates of return come from?
  - ◆ In large part, they come from observing a sample history of returns.
    - ✓ Suppose we observe a 10-year time series of monthly returns on a diversified portfolio of stocks.
    - ✓ We can interpret each of the 120 observations as one potential "scenario" offered to us by history.

- ✓ Adding judgment to this history, we can develop a scenario analysis of future returns.

- ◆ As a first step, we estimate the expected return, standard deviation, and VaR for the sample history.
  - ✓ We assume that each of the 120 returns represents one independent draw from the historical probability distribution.
    - ➢ Hence, each return is assigned an equal probability of 1/120 = .0083.
  - ✓ When you use a fixed probability in Equation 5.10, you obtain the simple average of the observations and this is often used to estimate the mean return.

- ■ As mentioned earlier, the same principle applies to the VaR.
  - ◆ We sort the returns from high to low.
    - ✓ The bottom six observations comprise the lower 5% of the distribution.
    - ✓ The sixth observation from the bottom is just at the fifth percentile, and so would be the 5% VaR for the historical sample.

- Estimating variance from Equation 5.11 requires a minor modification.
  - Remember that variance is the expected value of squared deviations from the mean return.
    - ✓ But the true mean is not observable; we *estimate* it using the sample average.
  - If we compute variance as the average of squared deviations from the sample average, we will slightly underestimate it because this procedure ignores the fact that the average necessarily includes some estimation error.
    - ✓ The necessary correction turns out to be simple: with a sample of n observations, we divide the sum of the squared deviations from the sample average by $n-1$ instead of n.
    - ✓ Thus, the estimates of variance and standard deviation from a time series of returns, r_t, are as follows:

$$\bar{r}_t = \frac{1}{n}\sum r_t \qquad \mathrm{Var}(r_t) = \frac{1}{n-1}\sum (r_t - \bar{r}_t)^2 \qquad \mathrm{SD}(r_t) = \sqrt{\mathrm{Var}(r_t)} \tag{5.15}$$

- Example 5.5: *Historical Means and Standard Deviations*
  - To illustrate how to calculate average returns and standard deviations form historical data, let's compute these statistics for the returns on the S&P 500 portfolio using five years of data from the following table.

Year	(1) Rate of Return	(2) Deviation from Average Return	(3) Squared Deviation
1	16.9%	0.2%	0.0
2	31.3	14.6	213.2
3	−3.2	−19.9	396.0
4	30.7	14.0	196.0
5	7.7	−9.0	81.0
Total	83.4%		886.2

$$\text{Average rate of return} = 83.4/5 = 16.7$$

$$\text{Variance} = \frac{1}{5-1} \times 886.2 = 221.6$$

$$\text{Standard deviation} = \sqrt{221.6} = 14.9\%$$

- ✓ The average return over this period is 16.7%, computed by dividing the sum of column (1) by the number of observations.
- ✓ In column (2), we take the deviation of each year's return from the 16.7% average return.
- ✓ In column (3), we calculate the squared deviation. The variance is, from Equation 5.15, the sum of five squared deviations divided by (5 – 1). The standard deviation is the square root of the variance.
- ✓ If you input the column of rates into a spreadsheet, the AVERAGE and STDEV functions will give you the statistics directly.

● Risk Premiums and Risk Aversion

- ■ How much, if anything, should you invest in an index stock fund described in Spreadsheet 5.1?
  - ◆ First, you must ask how much of an expected reward is offered to compensate for the risk involved in investing money in stocks.

- We measure the “reward” as the difference between the expected HPR on the index stock fund and the **risk-free rate,** the rate you can earn on Treasury bills.
  - We call this difference the **risk premium** on common stocks.
    - ✓ If the risk-free rate in the example is 4% per year, and the expected index fund return is 10%, then the risk premium on stocks is 6% per year.
- The rate of return on Treasury bills also varies over time.
  - However, we know the rate of return we will earn on T-bills *at the beginning* of the holding period, while we can’t know the return we will earn on risky assets until the end of the holding period.
- Therefore, to study the risk premium on risky assets we compile a series of **excess returns,** that is, returns in excess of the T-bill rate in each period.
  - A reasonable forecast of an asset’s risk premium is the average of its historical excess returns.
- The degree to which investors are willing to commit funds to stocks depends on **risk aversion.**

- It seems obvious that investors are risk averse in the sense that, without a positive risk premium, they would not be willing to invest in stocks.
- In theory then, there must always be a positive risk premium on all risky assets in order to induce risk-averse investors to hold the existing supply of these assets.
- A positive risk premium distinguishes speculation from gambling.
  - Investors taking on risk to earn a risk premium are speculating.
    - ✓ Speculation is undertaken *despite* the risk because the speculator sees a favorable risk-return trade-off.
  - In contrast, gambling is the assumption of risk for no purpose beyond the enjoyment of the risk itself.
    - ✓ Gamblers take on risk even without the prospect of a risk premium.
- To determine an investor's optimal portfolio strategy, we need to quantify his degree of risk aversion.
  - To do so, we look at how he is willing to trade off risk against expected return.
  - An obvious benchmark is the risk-free asset, which has neither volatility nor risk premium: It pays a certain rate of return, r_f.

- Risk-averse investors will not hold risky assets without the prospect of earning some premium above the risk-free rate.
- An individual's degree of risk aversion can be inferred by contrasting the premium on the investor's entire wealth (the complete portfolio, C), $E(r_C) - r_f$, against the variance of the portfolio return, σ_C^2.
- We assume normality here, so variance adequately measure risk.
- The risk premium and volatility of the *individual* assets in the complete wealth portfolio are of no concern to the investor here.
- All that counts is the bottom line: *complete portfolio* risk premium versus *complete portfolio* risk.

- A natural way to proceed is to measure risk aversion by the risk premium necessary to compensate an investor for investing his entire wealth in a portfolio, say Q, with a variance, σ_Q^2.
  - This approach relies on the principle of *revealed preference:* We infer preferences from the choices individuals are willing to make.

- ◆ We will measure risk aversion by the risk premium offered by the complete portfolio per unit of variance.
  - ✓ This ratio measures the compensation that an investor has apparently required (per unit of variance) to be induced to hold this portfolio.
  - ✓ For example, if we were to observe that the entire wealth of an investor is held in a portfolio with annual risk premium of .10 (10%) and variance of .0256 (SD = 16%), we would infer this investor's degree of risk aversion as:

$$A = \frac{E(r_Q) - r_f}{\sigma_Q^2} = \frac{0.10}{0.0256} = 3.91 \tag{5.16}$$

- ◆ We call the ratio of a portfolio's risk premium to its variance the **price of risk.**

■ To get an idea of the level of the risk aversion exhibited by investors in U.S. capital markets, we can look at a representative portfolio held by these investors.

- ◆ Assume that all short-term borrowing offsets lending; that is, average net borrowing and lending are zero.

- In that case, the average investor holds a complete portfolio represented by a stock-market index; call it *M*.
  - ✓ A common proxy for the market index is the S&P 500.
- Using a long-term series of historical returns on the S&P 500 to estimate investors' expectations about mean and variance, we can recast Equation 5.16 with these stock market data to obtain an estimate of average risk aversion:

$$\bar{A} = \frac{\text{Average}(r_M) - r_f}{\text{Sample } \sigma_M^2} \approx \frac{0.08}{0.04} = 2 \tag{5.17}$$

- The price of risk of the market index portfolio, which reflects the risk aversion of the average investor, is sometimes called the *market price of risk*.
  - Conventional wisdom holds that plausible estimates for the value of *A* lie in the range of 1.5 – 4.

- **The Sharpe (Reward-to-volatility) Measure**
  - Risk aversion implies that investors will accept a lower reward (as measured by their portfolio risk premium) in exchange for a sufficient reduction in standard deviation.
  - A statistic commonly used to rank portfolios in terms of this risk-return trade-off is the **Sharpe (reward-to-volatility) ratio,** defined as:

$$S = \frac{\text{Portfolio risk premium}}{\text{Standard deviation of portfolio excess return}} = \frac{E(r_P) - r_f}{\sigma_P} \qquad (5.18)$$

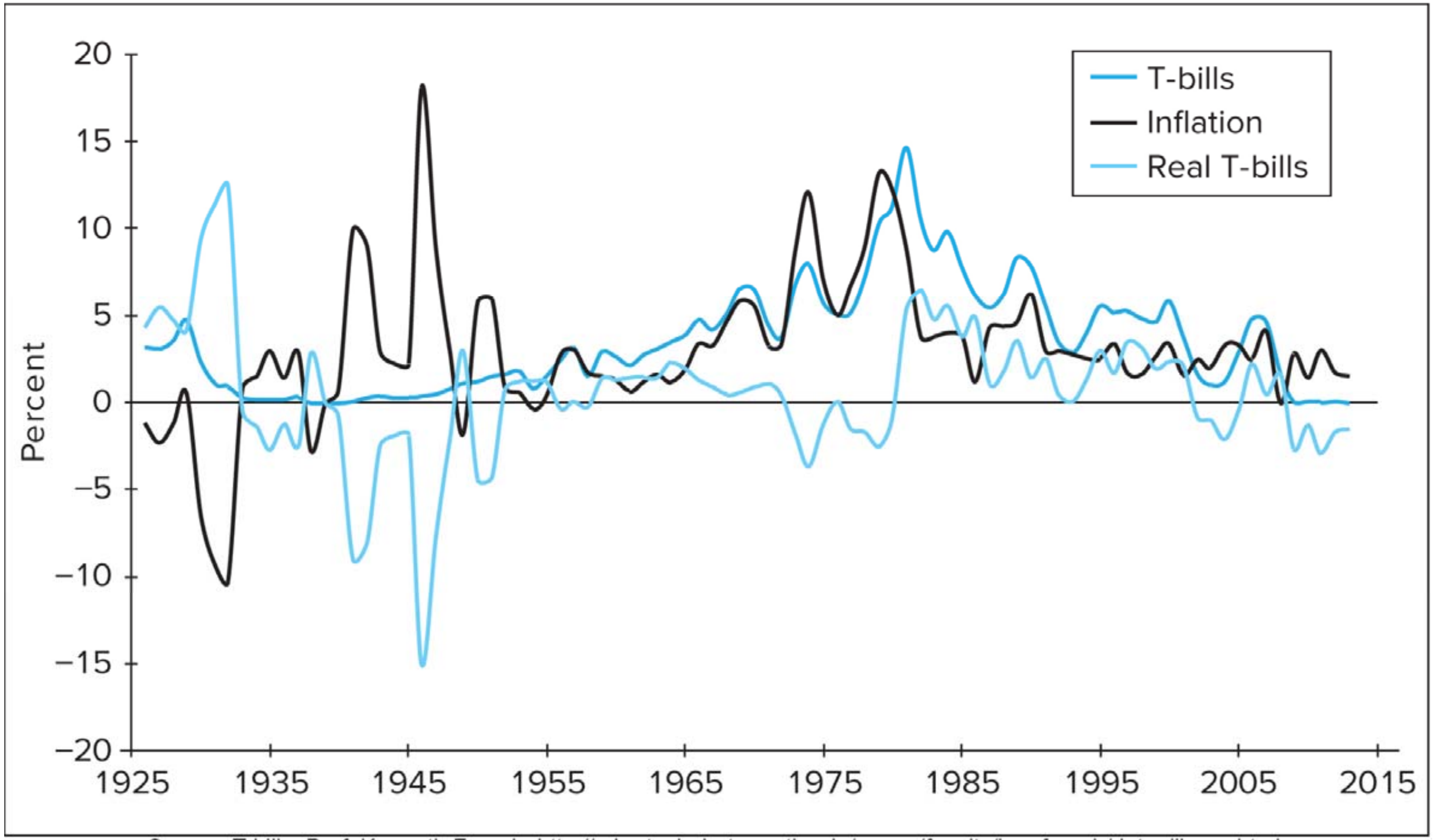
    - A risk-free asset would have a risk premium of zero and a standard deviation of zero.
    - Therefore, the Sharpe ratio of a risky portfolio quantifies the incremental reward (the increase in risk premium) for each increase of 1% in the portfolio standard deviation.
      - ✓ For example, the Sharpe measure of a portfolio with an annual risk premium of 8% and standard deviation of 20% is 8/20 = 0.4.
    - A higher Sharpe measure indicates a better reward per unit of SD, in other words, a more efficient portfolio.
    - Portfolio analysis in terms of mean and standard deviation (or variance) of excess returns is called **mean-variance analysis.**

- A warning: We will see in the next chapter that while standard deviation and VaR is a useful risk measure for diversified portfolios, these are not useful ways to think about the risk of individual securities.
  - Therefore, the Sharpe ratio is a valid statistic *only* for ranking portfolios; it is *not* useful for comparing individual assets.

5.4 THE HISTORICAL RRCORD

● History of U.S. Interest Rates, Inflation, and Real Interest Rates

- Figure 5.3 plots nominal interest rates, inflation rates, and real rates in the U.S. between 1926 and 2013.



Source: T-bills: Prof. Kenneth French, http://mba.tuck.dart mouth.edu/pages/faculty/ken.french/data_library.html; Inflation: Bureau of Labor Statistics, www.bls.gov; Real rate: authors' calculations

- The Fisher relation is clearly erratic.
  - ✓ In periods of economic stability, when inflation is around its long-term average of 3%, T-bill rates generally track inflation.
  - ✓ In periods of extreme high or low inflation, however, nominal (T-bill) rates seem to lag, and real rates therefore go to extremes.
  - ✓ Nevertheless, it appears that fluctuations in the real rates have moderated considerably in the last 30 years, presumably due to better monetary control by the Fed as well as more accurate investor anticipation of inflation.

■ Table 5.2 quantifies what we see in the figure.

- The geometric mean of the historical T-bill rate was 3.5%, implying that a $1 investment at the beginning of 1926 would have grown to $20.56 by the end of 2013 (88 years later).
- But adjusted for inflation, the geometric mean of the real T-bill rate was only 0.51%, implying that your purchasing power would have grown after 88 years by only 57%, from $1 to $1.57.

TABLE 5.2 Annual rate of return statistics for U.S. T-bills, inflation, and real T-bill returns, 1926–2013

	T-Bill Rate	Inflation Rate	Real T-Bill Rate
Geometric average (%)	3.50	2.97	0.51
58 years since 1956	4.76	3.81	0.92
Arithmetic average (%)	3.54	3.05	0.59
Lowest rate*	−0.02 (1938)	−10.27 (1932)	−15.05 (1946)
Highest rate	14.71 (1981)	18.13 (1946)	12.52 (1932)
Standard deviation	3.11	4.10	3.85
58 years since 1956	3.04	2.85	2.21
	Correlation with inflation rate		
1926–2013	0.41		−0.72
1926–1955	−0.30		−0.98
1956–1985	0.72		−0.48
1986–2013	0.53		0.01

*Negative yields on bills should be impossible, as one can always hold cash (with a zero yield) as an alternative. In the 1930s, however, Treasury securities were exempt from personal property taxes in some states and were required as collateral in some transactions. These advantages drove up their price above par value, resulting in a slightly negative yield in 1938.

Source: T-bills: Professor Kenneth French website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Inflation data: Bureau of Labor Statistics

高潮

- Despite numerous recessions in the latter part of the sample, culminating in the Great Recession of 2008-2009, the geometric means of all three rates for the 58 years since 1956 are higher.
  - ✓ Investing for 88 years at the higher real rate that characterized the more recent part of the sample, 0.92%, your purchasing power would have increased by the start of 2014 to $2.24 (instead of $1.57). 微不足道
  - ✓ Of course, this is still paltry compared to the nominal growth in wealth corresponding to the higher nominal rate (4.76%).
    陰險
    - ➢ Such is the insidious effect of even moderate inflation.
  - ✓ Notice that the lowest real rate (in 1946) occurs in the year of highest inflation, and the highest real rate (1932) occurs when inflation is at its lowest (deflation is highest).

- The standard deviation of the T-bill rate actually does not represent short-term investment risk.
  - ✓ After all, investors know the rate they will earn on their bills in advance.
  - ✓ Rather, the standard deviation reflects changes over time in nominal rates demanded by investors.
  - ✓ There is little evidence that this rate has become less volatile.
  - ✓ But there is an indication that the volatility of inflation and real rates have markedly declined, suggesting the Fed has become more effective in moderating economic business cycles.
- A related aspect of the data provides additional insight into inflation and the Fisher equation.
  - ✓ If investors can anticipate inflation well, then the nominal T-bill rate will track the rate of inflation closely, resulting in high correlation between the two.
    - Because nominal rates will rise with anticipated inflation rates, the real rate will by largely unaffected and therefore uncorrelated with either of them, instead fluctuation randomly with variation in demand and supply of capital.

- ✓ Conversely, if investors were wholly unable to predict future inflation, the nominal T-bill rate would be uncorrelated with inflation.
  - ➢ In this case, any surprise in inflation would reduce the real rate by an equal amount, and the real rate would be highly *negatively* correlated with inflation.
- ✓ Table 5.2 shows that the correlation between T-bill and inflation rates has gone from a negative value prior to 1955 to a positive since then.
  - ➢ Consistent with this pattern, the correlation between the real T-bill rate and the inflation rate was highly negative before 1955, but since then has been almost precisely zero.
  - ➢ These changes signal the greater predictability of inflation in recent decades, which is good news for financial markets.

◆ Inflation-indexed bonds called Treasury Inflation-Protected Securities (TIPS) were introduced in the U.S. in 1977.

- ✓ These are bonds of 5- to 30-year original maturities with coupons and principal that increase at the rate of inflation.

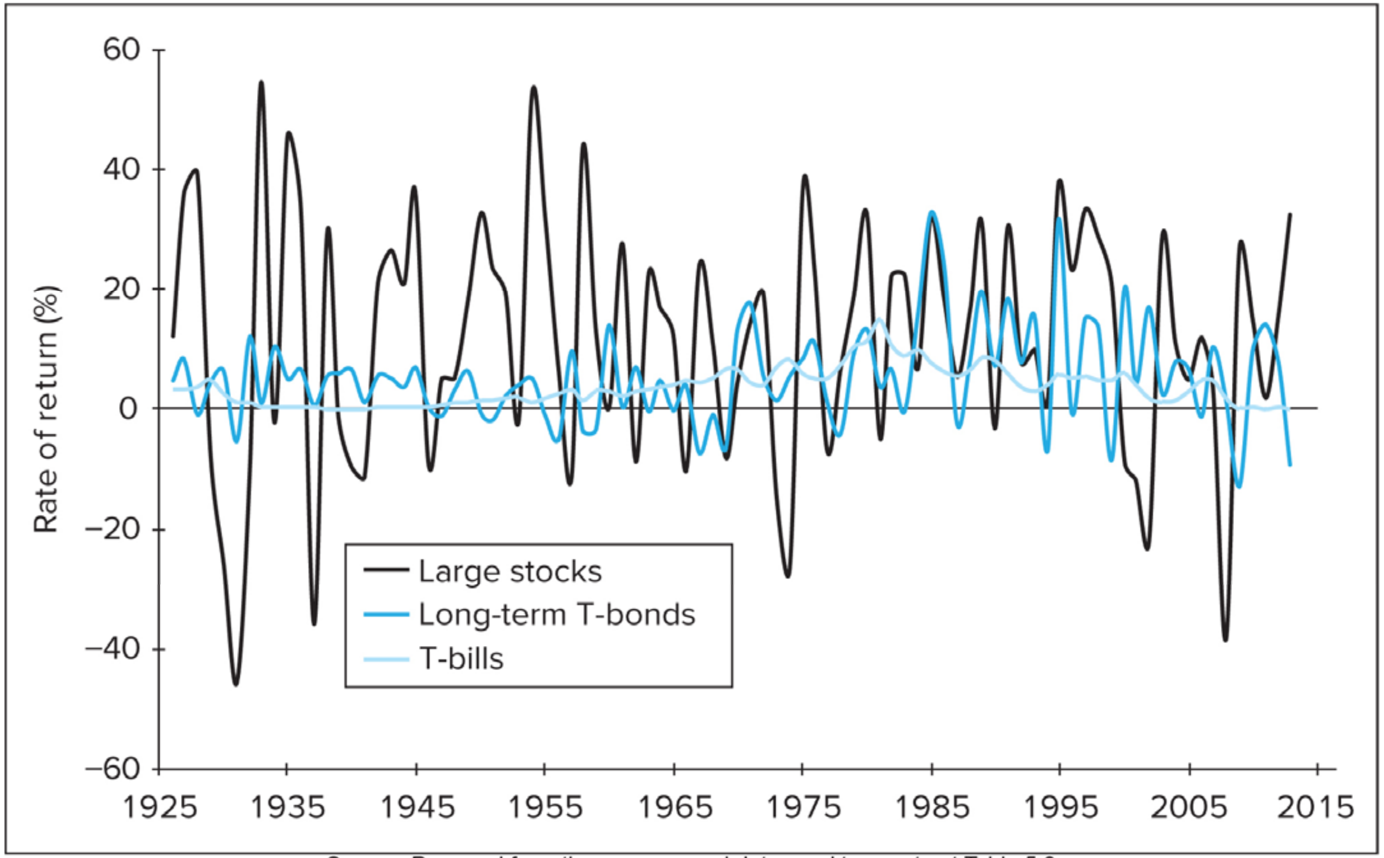
- ✓ The difference between nominal rates on conventional T-bonds and the rates on equal-maturity TIPS provides a measure of expected inflation (often called *break-even inflation*) over that maturity.

World and U.S. Risky Stock and Bond Portfolios

- We begin our examination of risk with the return history of five risky asset classes.
  - Three well-diversified stock portfolios:
    - ✓ World large stocks
    - ✓ U.S. large stocks
    - ✓ U.S. small stocks
  - Two long-term bond portfolios:
    - ✓ World bonds
    - ✓ U.S. Treasury bonds.
  - The 88 annual observations for each of the five time series of returns span the period 1926-2013.

- Until 1969, the "World Portfolio" of stocks is constructed from a diversified sample of large capitalization stocks of 16 developed countries weighted in proportion to gross domestic product.
  - Since 1970 this portfolio has been diversified across 24 developed countries (almost 6,000 stocks) with weighted determined by the relative capitalization of each market.
- "Large Stocks" is the Standard & Poor's market value-weighted portfolio of 500 U.S. common stocks selected from the largest market capitalization stocks.
- "Small U.S. Stocks" are the smallest 20% of the stocks trading on the NYSE, NASDAQ, and Amex (currently about 1,000 stocks).
- The World Portfolio of bonds was constructed from the same set of countries as the World Portfolio of stocks, using long-term government bonds from each country.
- Until 1996, "Long-Term T-Bonds" were represented by U.S. government bonds with at least a 20-year maturity and approximately current-level coupon rate.
  - Since 1996, this bond series has been measured by the Barcaly's Long-Term Treasury Bond Index.

■ Figure 5.4 provides one view of the hierarchy of risk.



Source: Prepared from the same annual data used to construct Table 5.3

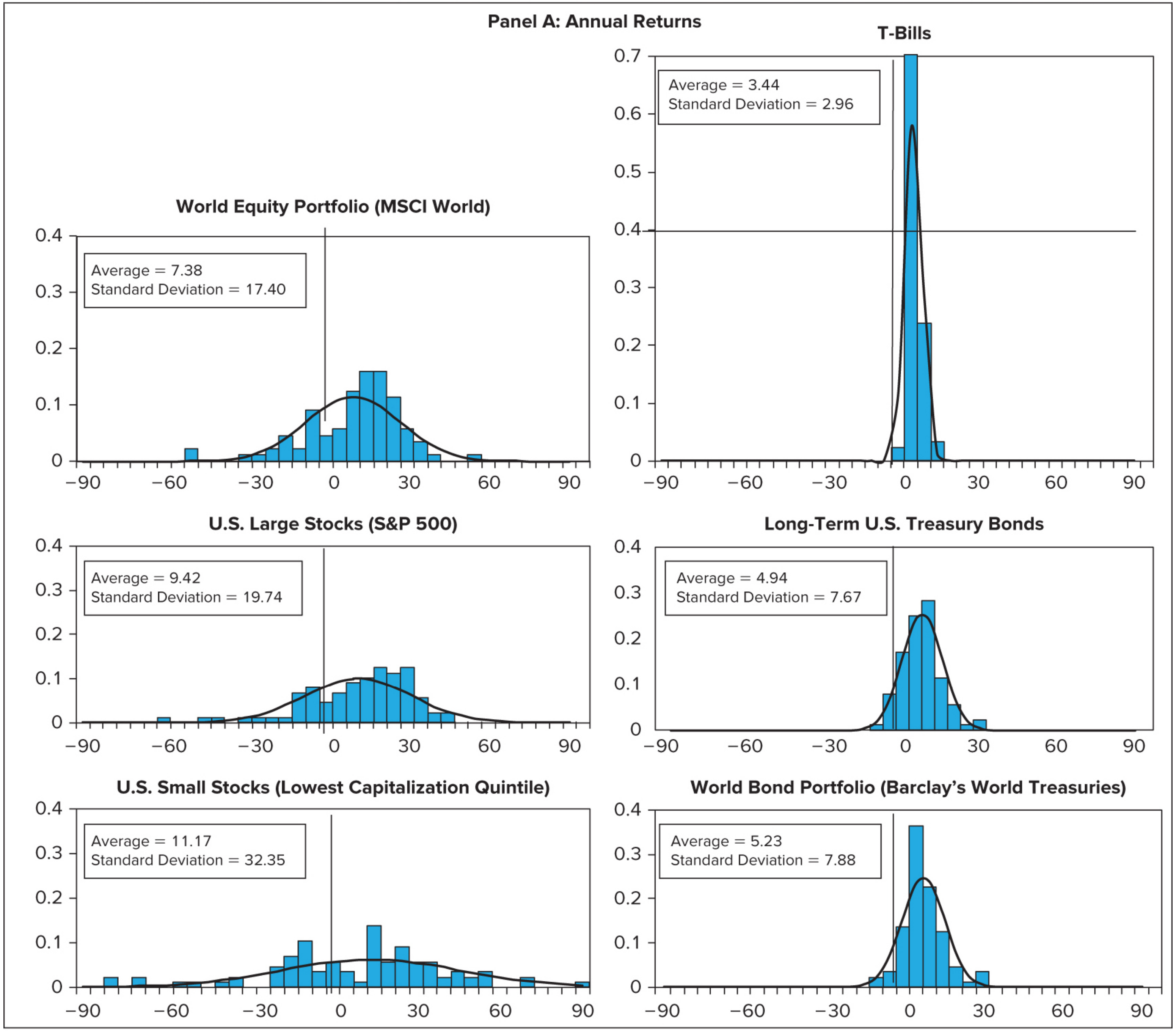
◆ Here we plot the year-by-year returns on U.S. large stocks, long-term Treasury bonds, and T-bills.

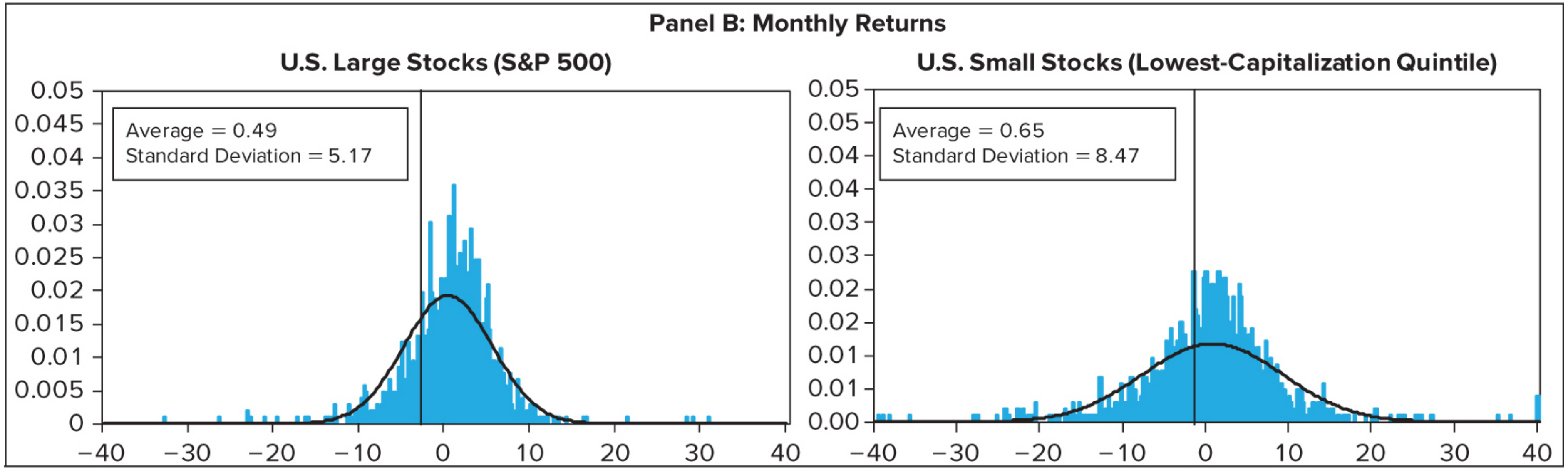
◆ Risk is reflected by wider swings of returns from year to year.

- Small stocks are the most risky, followed by large stocks and then long-term bonds.
- At the same time, the higher average return offered by riskier assets is evident, consistent with investor risk aversion.
- T-bill returns are by far the least volatile.
  - ✓ That is, changes in interest rates are of a smaller order than fluctuations in returns on risky assets.

- Another view of return history is presented in Figure 5.5, Panel A, which shows histograms of the continuously compounded annual returns of the five risky portfolios and of Treasury bills.
  - The higher risk of large stocks, and even more so, of small stocks is reflected in their wider dispersion of returns.
- To better assess the normality of these risky returns, Panel B presents histograms of the 1,058 *monthly* excess returns, expressed as continuously compounded rates, on U.S. small and large stocks.

- The bell-shaped curves in both Panels A and B are "matched" normal distributions, constructed with the same mean and variance as the historical sample.
  - Deviations from normality are suggested by the higher frequency of extreme (or tail) outcomes, the lower incidence of midrange positive or negative returns (compared to the "shoulders" of the normal distribution), and finally, the higher incidence of returns in the vicinity of the mean.


Panel A: Annual Returns
World Equity Portfolio (MSCI World)
Average = 7.38
Standard Deviation = 17.40
T-Bills
Average = 3.44
Standard Deviation = 2.96
U.S. Large Stocks (S&P 500)
Average = 9.42
Standard Deviation = 19.74
Long-Term U.S. Treasury Bonds
Average = 4.94
Standard Deviation = 7.67
U.S. Small Stocks (Lowest Capitalization Quintile)
Average = 11.17
Standard Deviation = 32.35
World Bond Portfolio (Barclay's World Treasuries)
Average = 5.23
Standard Deviation = 7.88
−90
−60
−30
0
30
60
90
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7



Source: Prepared from the same data used to construct Table 5.3

- Table 5.3 presents statistics on the return history of the five portfolios over the 88-year period.
  - We start with the geometric averages of total returns in the top panel of the table.
    - ✓ This is the equivalent, constant annual rate of return earned over the period.

TABLE 5.3 Statistics for asset-class index portfolios, 1926–2013 (annual rates in U.S. dollars, %)

	World Markets		U.S. Markets		
	Large Stocks	Government Bonds	Small Stocks	Large Stocks	U.S. Long-Term Treasuries
A. Total Returns (%)					
Geometric average	8.24	5.37	11.82	9.88	5.07
Lowest return	−39.94 (1931)	−13.50 (1946)	−54.27 (1937)	−45.56 (1931)	−13.82 (2009)
Highest return	70.81 (1933)	34.12 (1985)	159.05 (1933)	54.56 (1933)	32.68 (1985)
B. Risk (Measured Using Excess Returns)					
Standard deviation	18.89	8.44	37.29	20.52	8.01
Value at risk (VaR) 5%	−25.88	−10.67	−48.33	−31.96	−11.51
C. Deviation from Normality*					
VaR assuming normality	−22.54	−10.43	−36.96	−23.51	−10.23
Actual VaR minus normal	−3.34	−0.24	−11.37	−8.46	−1.28
Skew	−0.09	0.68	0.83	−0.31	0.39
Kurtosis	1.08	1.39	1.97	−0.05	0.53
D. Returns in Excess of One-Month T-Bill Rates					
Average excess return	6.32	2.16	13.94	8.34	1.83
Standard error	2.01	0.90	3.97	2.19	0.85
E. Sharpe Ratios for 1926–2013 and Three Subperiods					
Entire period	0.33	0.26	0.37	0.41	0.23
1926–1955	0.43	0.22	0.40	0.46	0.59
1956–1985	0.19	0.05	0.38	0.28	−0.11
1986–2013	0.35	0.52	0.37	0.48	0.41
F. Correlations of Excess Returns					
With inflation	−0.13	−0.27	−0.02	−0.06	−0.17
With T-bill rates	−0.25	−0.16	−0.22	−0.17	−0.12

Note: Colored entries indicate measures of tail risk that exceed values consistent with a normal distribution.

Sources: World portfolio of Equities: 1926–1969 Dimson, Marsh and Staunton (Equity Premia Around the World); 1970–2013 Bloomberg—MSCI World in US$; World bonds: 1926–1987 Dimson, Marsh, and Staunton (ibid); 1988–2013 Bloomberg: Barclay's Global Treasuries in U.S.$.; Small Stocks: Kenneth French Data Library, Lowest Quantile; Large stocks: Center for Research in Security Price (CRSP), S&P 500; Long-Term Treasury bonds: Bloomberg: Barclay's U.S. Long-Term Treasury index; T-bills: Kenneth French Data Library, monthly rollover of 30-day T-bills; Inflation data: Economagic—Bureau of Labor Statistics cpiu dec2dec.

- To appreciate the power of compounding, consider the growth of invested funds in each of these asset classes.
  - ✓ Investing $1 in each at the beginning of 1926 would have yielded the following growth of invested funds by the end of 2013:

	Government Bonds		Stocks		
	World	U.S.	World	U.S. Large	U.S. Small
Geometric average (%)	5.37	5.07	8.24	9.88	11.82
Growth of $1 (nominal value)	100	77	1,060	3,982	18,629

  - ✓ Obviously, even small differences in rates of return compound to large dollar values over time.

- The *geometric* average of total return is backward looking.
  - ✓ It measures the single compound rate of growth that was *actually* earned over a period.
  - ✓ Only part of that return is attributable to risk.
    - ➢ The component that is a reward for bearing risk is the *increment* to the safe rate, which we measure by the excess return over the one-month T-bill rate.
  - ✓ Therefore, portfolio risk should be measured from the volatility of *excess* returns.
- Observe the high standard deviations of excess returns in Table 5.3.
  - ✓ For large stock portfolios, the annual SD of excess returns over the 88-year period was nearly 20% (and a lot higher for small stocks).
    - ➢ This represents a high level of risk.
  - ✓ The average return in "bad years" (years with returns below the 88-year average) was -11.4%, around 20% (or one standard deviation) less than the 88-year average of 9.4%; the return in "good years" was about one standard deviation, or 21%, above the average return.

- ✓ The dispersion of returns for small stocks, with a standard deviation of 37.3%, is even higher.
- ✓ Investments in long-term bond portfolios also entail high risk.

■ The fat tails we observed in the histogram of Figure 5.5 tell us that we should devote some attention to risk measures that focus on extreme events, for example, VaR (value at risk).

- ◆ Table 5.3, Panel B, documents that the threat of extreme returns is real: in each of the 5% of the worst-return years, the excess return on small stocks was less than -48%.
- ◆ Even for Treasury bonds, excess return was less than -10% in each of the 5% worst years.
  - ✓ This degree of extreme poor performance is greater than the VaR of normal distributions with the same means and standard deviations.
- ◆ The colored entries in Panel C indicate measures of tail risk that exceed values consistent with a normal distribution.
  - ✓ All three stock portfolios show *actual* losses in the 5% worst years that are greater than would be expected from normal distributions.

- Deviation from normality is also evident in the skew and kurtosis statistics shown in Panel C.
  - These measures are derived by raising deviations from the mean to even higher powers than variance.
  - Just as variance is the average value of *squared* deviations from the average, skew is the average value of those deviations raised to the *third* power (expressed as a multiple of the third power of standard deviation).
  - Similarly, kurtosis is calculated from the average value of deviations raised to the *fourth* power (expressed as a multiple of the fourth power of standard deviation).
  - Because they raise deviations to higher powers than variance, they are highly sensitive to extreme outcomes and therefore are considered measures of “tail risk,” that is, outcomes in the tail of the probability distribution.
  - Because skew raises deviations to the third power (an odd exponent), negative deviations remain negative; therefore, negative values of skew indicate a risk of extreme *bad* outcomes.

- In contrast, kurtosis takes deviations to an even power, which makes all outcomes positive.
  - Therefore, it measures the tendency to observe extreme outcomes in *either* end of the probability distribution, positive or negative.
  - It is therefore a measure of "fat tails," that is, exposure to extreme good or bad results.
  - The expected value of this statistic for a normally distributed variable is 3.
    - Therefore, it is common to express kurtosis by first subtracting 3, making a value of zero consistent with normality and higher values indicative of fat tails.
- Because they are very sensitive to one or two extreme outcomes, skew and kurtosis estimates are statistically reliable only in large samples.
- Nevertheless, the history we have calls for caution.
  - For example, while the skew of U.S. small stocks is positive (and therefore, is not a worry for investors), the kurtosis of small stocks is 1.97, indicating the risk of fat tails beyond the risk implicit in the already high SD of 37.29%.

- As a whole, the tail risk measures in Table 5.3 suggest that SD alone can provide too narrow a view of stock risk.

- Identifying the reward one can expect from bearing the risk quantified in Table 5.3 begins by comparing the average excess returns provided by each asset classes. These are shown in Panel D.
  - The *arithmetic* average excess return is forward looking: It provides the best statistical estimate of the expected excess return in the next period.
  - Bear in mind that this forecast of future returns comes with some impression, which we measure using the standard error.
    - ✓ The standard error (which equals $\mathrm{SD}/\sqrt{n}$) measures the likely sampling variation in the estimate of mean excess return due to year-by-year variability in returns.
  - We find that even with a fairly long 88-year history, the standard error of the average excess return of the stock portfolio is large, between 2% and 4% for the different stock portfolios.

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- ◆ Nevertheless, these standard error are all less than half the average excess return, implying that we can discern a significant *risk premium* even though all the noise.
- ■ But are these risk premiums fair compensation for the degree of risk posed by the investments?
  - ◆ This central question has generated enormous volumes of theoretical and empirical research.
  - ◆ To answer it, we must determine what an appropriate reward-risk relationship should be.
  - ◆ We will defer that discussion until the next section; here we just point out that the differences between the historical average excess returns on the five portfolios are significant.
- ■ Example 5.6: *The Risk Premium and Growth of Wealth*
  - ◆ The potential import of the risk premium can be illustrated with a simple example.
  - ◆ Consider two investors with $1 million as of December 31, 2000.
    - ✓ One invests in the small-stock portfolio, and the other in T-bills.

- Suppose both investors reinvest all income from their portfolios and liquidate their investments eight years later, on December 31, 2010.
- We can find the annual rates of return for this period from the spreadsheet of returns available in Connect or through your course instructor.
- We compute a “wealth index” for each investment by compounding wealth at the end of each year by the return earned in the following year.

	Small Stocks		T-Bills	
Year	Return (%)	Wealth Index	Return (%)	Wealth Index
2000		1		1
2001	29.25	1.2925	3.86	1.0386
2002	−11.77	1.1404	1.63	1.0555
2003	74.75	1.9928	1.02	1.0663
2004	14.36	2.2790	1.19	1.0790
2005	3.26	2.3533	2.98	1.1111
2006	17.69	2.7696	4.81	1.1646
2007	−8.26	2.5408	4.67	1.2190
2008	−39.83	1.5288	1.64	1.2390
2009	36.33	2.0842	0.05	1.2396
2010	29.71	2.7034	0.08	1.2406

- ✓ For example, we calculate the value of the wealth index for small stocks as of 2003 by multiplying the value as of 2002 (1.1404) by 1 plus the rate of return earned in 2003 (measured in decimals), that is, by 1 + .7475, to obtain 1.9928.

◆ The final value of each portfolio as of December 31, 2010, equals its initial value ($1 million) multiplied by the wealth index at the end of the period:

Date	Small Stocks	T-Bills
December 31, 2000	$1,000,000	$1,000,000
December 31, 2010	$2,703,420	$1,240,572

- ✓ The difference in total return is dramatic.
  - ➢ Even with its devastating 2008 return, the value of the small stock portfolio after eight years is 118% more than that of the T-bill portfolio.

- We can also calculate the geometric average return of each portfolio over this period.
  - ✓ For T-bills, the geometric average over the eight-year period is computed from:

    $(1 + r_G)^{10} = 1.2406$

    $\Rightarrow 1 + r_G = 1.2406^{1/10} = 1.0218$

    $\Rightarrow r_G = 2.18\%$
  - ✓ Similarly, the geometric average for small stocks is 10.46%.
    - ➢ The difference in geometric average reflects the difference in cumulative wealth provided by the small-stock portfolio over this period.

- Turning to investment performance, recall that the Sharpe ratios shown in Panel E (the ratio of average excess return to SD) should be applied only to portfolios that are candidates for the entire risky investment.
  - As we shall see in the next chapter, none of the five portfolios in Table 5.3 is sufficiently diversified for this purpose, yet the stock portfolios come closest to meeting this criterion.

- Sharpe ratios for the entire 88-year period are larger for the stock than for the bond portfolios, indicating that bond-only portfolios are flatly inadequate candidates for the overall investment portfolio.
- The Sharpe ratio for large U.S. stocks implies that diversified stock portfolios have yielded an incremental average excess return of about 0.4% for every 1% of SD.
- The ratio is a bit smaller for non-U.S. and smaller stocks.

5.5 ASSET ALLOCATION ACROSS RISKY AND RISK-FREE PORTFOLIOS

- History shows that long-term bonds have been riskier than investments in Treasury bills and that stock investments have been riskier still. At the same time, the riskier investments have offered higher average returns.
- But investors do not make all-or-nothing choices from these investment classes.
  - They normally include securities from all asset classes in their portfolios.
- A simple strategy to control portfolio risk is to specify the fraction of the portfolio invested in broad asset classes such as stocks, bonds, and safe assets.
  - This aspect of portfolio management is called **asset allocation** and plays an important role in the determination of portfolio performance.
- The most basic form of asset allocation classifies all assets as either risky or risk free.
  - The fraction of the portfolio placed in risky assets is called the **capital allocation to risky assets** and speaks directly to investor risk aversion.

- To focus on the capital allocation decision, we think about an investor who allocates funds between T-bills and a portfolio of risky assets.
  - We can envision the risky portfolio, *P*, as a mutual fund or ETF (exchange-traded fund) that includes a bundle of risky assets in desired, fixed proportions.
  - Thus, when we shift wealth into and out of *P*, we do not change the relative proportion of the various securities within the risky portfolio.
  - We put off until the next chapter the question of how to best construct the risky portfolio.
  - We call the overall portfolio composed of the risk-free asset and the risky portfolio the **complete portfolio** that includes the investor's entire wealth.

The Risk-Free Asset

- The power to tax and to control the money supply lets the government, and only the government, issue default-free (Treasury) bonds.
  - The default-free guarantee by itself is not sufficient to make the bonds risk-free in real terms, since inflation affects the purchasing power of the proceeds from the bonds.

- The only risk-free asset in real terms would be a price-indexed government bond such as TIPS.
- Even then, a default-free, perfectly indexed bond offers a guaranteed real rate to an investor only if the maturity of the bond is identical to the investor’s desired holding period.
- These qualifications notwithstanding, it is common to view Treasury bills as *the* risk-free asset.
  - ✓ Any inflation uncertainty over the course of a few weeks, or even months, is negligible compared to the uncertainty of stock market returns.

■ In practice, most investors treat a broader range of money market instruments as effectively risk-free assets.

- All the money market instruments are virtually immune to interest rate risk (unexpected fluctuations in the price of a bond due to changes in market interest rates) because of their short maturities, and all are fairly safe in terms of default or credit risk.

- Money market mutual funds hold, for the most part, three types of securities: Treasury bills, bank certificates of deposit (CDs), and commercial paper.
  - The instruments differ slightly in their default risk.
  - The yields to maturity on CDs and commercial paper for identical maturities are always slightly higher than those of T-bills.
    - ✓ A history of this yield spread for 90-day CDs is shown in Figure 2.2 in Chapter 2.
- Money market funds have changed their relative holdings of these securities over time, but by and large, the risk of CDs and commercial paper is minuscule compared to that of long-term bonds.

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  - Hence, we treat money market funds, as well as T-bills, as representing the most easily accessible risk-free asset for most investors.

Portfolio Expected Return and Risk

- Consider the hierarchy of decisions an investor must make in a capital allocation framework.
  - The properties of the risky portfolio are summarized by expected return and risk, as measured by standard deviation.

- The risk-free asset has a standard deviation of zero: Its rate of return is known.
- The investor must decide on the fraction of the *complete* portfolio that will be allocated to the risky portfolio.
  - ✓ To make this determination, one must first determine the expected return and risk corresponding to any possible allocation.
    - ➢ This is the technical part of the allocation decision.
  - ✓ Given the trade-off between risk and return, which is common to all investors, each individual can choose his or her preferred allocation between the risky portfolio and the risk-free asset.
    - ➢ This choice depends on personal preferences, specifically risk aversion.
- We begin with the risk-return trade-off.

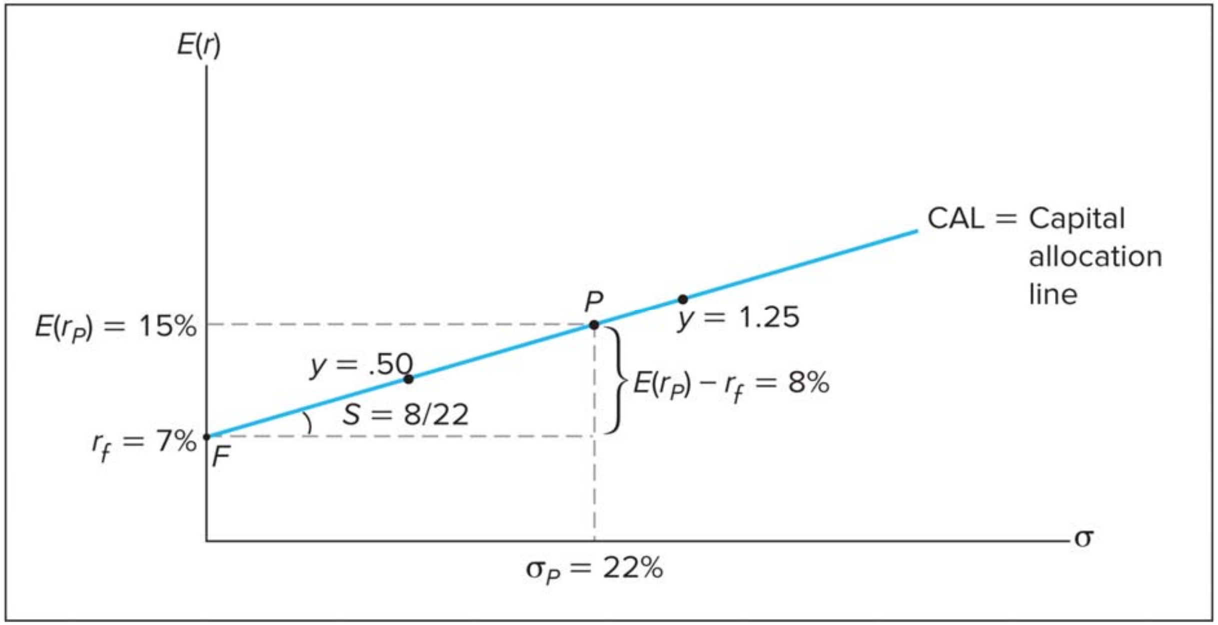
■ Because the composition of the optimal risky portfolio, *P*, already has been determined, the only concern here is with the proportion of the investment budget (y) to be allocated to it.

- The remaining proportion $(1 - y)$ is to be invested in the risk-free asset, which has a rate of return denoted r_f.

- We denote the *actual* risky rate of return by r_P, the *expected* rate of return on P by $E(r_P)$, and its standard deviation by σ_P.
- In the numerical example, we assume $E(r_P) = 15\%$, $\sigma_P = 22\%$, and $r_f = 7\%$.
  - ✓ Thus, the risk premium on the risky asset is $E(r_P) - r_f = 8\%$.

■ Let's start with two extreme cases.

- If you invest all of your funds in the risky asset, that is, $y = 1$, the expected return on your complete portfolio will be 15% and the standard deviation will be 22%.
  - ✓ This combination of risk and return is plotted as point P in Figure 5.6.



- At the other extreme, you might put all of your funds into the risk-free asset, that is, $y = 0$.
  - ✓ In this case, your portfolio would behave just as the risk-free asset, and you would earn a riskless return of 7%. (This choice is plotted as point *F* in Figure 5.6.)
- Now consider more moderate choices.
  - ✓ For example, if you allocate equal amounts of your *complete portfolio*, *C*, to the risky and risk-free assets, that is, if you choose $y = 0.5$, the expected return on the complete portfolio will be an average of $E(r_P)$ and r_f.
    - Therefore, $E(r_C) = .5 \times 7\% + .5 \times 15\% = 11\%$.
    - The risk premium of the complete portfolio is therefore 11% – 7% = 4%, which is half of the risk premium of *P*.
    - The standard deviation of the portfolio also is one-half of *P*'s, that is, 11%.
    - When you reduce the fraction of the complete portfolio allocated to the risky asset by half, you reduce both the risk and risk premium by half.

- To generalize, the risk premium of the complete portfolio, C, will equal the risk premium of the risky asset times the fraction of the portfolio invested in the risky asset.

$$E(r_C) - r_f = y[E(r_P) - r_f] \quad \Rightarrow \quad E(r_C) = r_f + y[E(r_P) - r_f] \tag{5.19}$$

- The standard deviation of the complete portfolio will equal the standard deviation of the risky asset times the fraction of the portfolio invested in the risky asset.

$$\sigma_C = y\sigma_P \tag{5.20}$$

- In sum, both the risk premium and the standard deviation of the complete portfolio increase in proportion to the investment in the risky portfolio.
- Therefore, the points that describe the risk and return of the complete portfolio for various asset allocations, that is, for various choices of y, all plot on the straight line connecting F and P as shown in Figure 5.6, with an intercept of r_f and slope (rise/run) equal to the familiar Sharpe ratio of P:

$$S = \frac{E(r_P) - r_f}{\sigma_P} = \frac{15 - 7}{22} = .36 \tag{5.21}$$

The Capital Allocation Line

- The line plotted in Figure 5.6 depicts the risk-return combinations available by varying asset allocation, that is, by choosing different values of *y*.
  - For this reason, it is called the **capital allocation line (CAL).**
    - ✓ The slope, *S*, of the CAL equals the increase in expected return that an investor can obtain per unit of additional standard deviation, or equivalently, extra return per extra risk.
    - ✓ This is the Sharpe ratio, after William Sharpe who first suggested its use.
    - ✓ It is obvious why Sharpe ratio is sometimes called the *reward-to-volatility ratio*.
- Notice that the Sharpe ratio is the same for risky portfolio *P* and the complete portfolio that mixes *P* and the risk-free asset.

	Expected Return	Risk Premium	Standard Deviation	Reward-to-Volatility Ratio
Portfolio *P*:	15%	8%	22%	$\frac{8}{22} = 0.36$
Portfolio *C*:	11%	4%	11%	$\frac{4}{11} = 0.36$

- In fact, the Sharpe ratio is the same for all complete portfolios that plot on the capital allocation line.
  - While the risk-return combinations differ according to the investor's choice of y, the *ratio* of reward to risk is constant.
- What about points on the line to the right of portfolio P in the investment opportunity set?
  - You can construct complete portfolios to the right of point P by borrowing, that is, by choosing $y > 1$.
    - ✓ This means that you borrow a proportion of $y - 1$ and invest both the borrowed funds and your own wealth in the risky portfolio P.
    - ✓ If you can borrow at the risk-free rate, $r_f = 7\%$, then your rate of return will be $r_C = -(y - 1)r_f + yr_P = r_f + y(r_P - r_f)$.
    - ✓ This complete portfolio has risk premium of $y[E(r_P) - r_f]$ and SD $= y\sigma_P$.
    - ✓ Verify that your Sharpe ratio equals that of any other portfolio on the same CAL.

- Example 5.7: *Levered Complete Portfolios*
  - Suppose the investment budget is \$300,000, and an investor borrows an additional \$120,000, investing the \$420,000 in the risky asset.
    - ✓ This is a levered position in the risky asset, which is financed in part by borrowing.
    - ✓ In that case

      $$y = \frac{420{,}000}{300{,}000} = 1.4$$

      and $1 - y = 1 - 1.4 = -0.4$, reflecting a short position in the risk-free asset, or a borrowing position.
    - ✓ Rather than lending at a 7% interest rate, the investor borrows at 7%. With weights of -0.4 in the risk-free asset and 1.4 in the risky portfolio, the portfolio rate of return is

      $$E(r_C) = (-0.4) \times 7\% + 1.4 \times 15\% = 18.2\%$$

- Another way to find this portfolio rate of return is as follows.
  - ✓ Your expect to earn \$63,000 (15% of \$420,000) and pay \$8,400 (7% of \$120,000) in interest on the loan.
  - ✓ Simple subtraction yields an expected profit of \$54,600 (= \$63,000 – \$8,400), which is 18.2% (= \$54,600/\$300,000) of your investment budget of \$300,000. Therefore, $E(r_C) = 18.2\%$.
  - ✓ Your portfolio still exhibits the same reward-to-volatility or Sharpe ratio:

$$\sigma_C = 1.4 \times 22\% = 30.8\%$$

$$S = \frac{E(r_C) - r_f}{\sigma_C} = \frac{18.2 - 7}{30.8} = \frac{11.2}{30.8} = .36$$

  - ✓ As you might have expected, the levered portfolio has both a higher expected return and a higher standard deviation than an unlevered position in the risky asset.

● Risk Aversion and Capital Allocation

- We have developed the CAL, the graph of all feasible risk-return combinations available from allocating the complete portfolio between a risky portfolio and a risk-free asset.
- The investor confronting the CAL now must choose one optimal combination from the set of feasible choices.
  - This choice entails a trade-off between risk and return.
  - Individual investors with different levels of risk aversion, given an identical capital allocation line, will choose different positions in the risky asset.
  - Specifically, the more risk-averse investors will choose to hold *less* of the risky asset and *more* of the risk-free asset.
- How can we find the best allocation between the risky portfolio and risk-free asset?
  - Recall from Equation 5.16 that a particular investor's degree of risk aversion (A) measures the price of risk she demands from the complete portfolio in which her entire wealth is invested.
  - The compensation for risk demanded by the investor must be compared to the price of risk offered by the risky portfolio, P.

- We can find the investor's preferred capital allocation, *y*, by dividing the risky portfolio's price of risk by the investor's risk aversion, her *required* price of risk:

$$y = \frac{\text{Available risk premium to variance ratio}}{\text{Required risk premium to variance ratio}}$$

$$= \frac{[E(r_P) - r_f]/\sigma_P^2}{A} = \frac{[E(r_P) - r_f]}{A\sigma_P^2} \tag{5.22}$$

- Notice that when the price of risk of the available risky portfolio exactly matches the investor's degree of risk aversion, her entire wealth will be invested in it ($y = 1$).

■ What would the investor of Equation 5.16 (with $A = 3.91$) do when faced with the market index portfolio of Equation 5.17 (with price of risk = 2)?

- Equation 5.22 tells us that this investor would invest $y = 2/3.91 = 0.51$ (51%) in the market index portfolio and a proportion $1 - y = 0.49$ in the risk-free asset.

- Graphically, more risk-averse investors will choose portfolios near point *F* on the capital allocation line plotted in Figure 5.6.
- More risk-tolerant investors will choose points closer to *P*, with higher expected return and higher risk.
- The most risk-tolerant investors will choose portfolios to the right of point *P*.
  - These levered portfolios provide even higher expected returns, but even greater risk.
- The investor's asset allocation choice also will depend on the trade-off between risk and return.
  - When the Sharpe ratio is higher, investors will take on riskier positions.
    - ✓ Suppose an investor reevaluates the probability distribution of the risky portfolio and now perceives a greater expected return without an accompanying increase in the standard deviation.
    - ✓ This amounts to an increase in Sharpe ratio or, equivalently, an increase in the slope of the CAL.
    - ✓ As a result, this investor will choose a higher *y*, that is, a greater position in the risky portfolio.

- One role of a professional financial adviser is to present investment opportunity alternatives to clients, obtain an assessment of the client’s risk tolerance, and help determine the appropriate complete portfolio.

5.6 PASSIVE STRATEGIES AND THE CAPITAL MARKET LINE

- A **passive strategy** is based on the premise that securities are fairly priced, and it avoids the costs involved in undertaking security analysis.
  - Such a strategy might at first blush appear to be naïve.
  - However, we will see in Chapter 8 that intense competition among professional money managers might indeed force security prices to levels at which further security analysis in unlikely to turn up significant profit opportunities.
    - ✓ Passive investment strategies may make sense for many investors.
- To avoid the costs of acquiring information on any individual stock or group of stocks, we may follow a "neutral" diversification approach.
  - Select a diversified portfolio of common stocks that mirrors the corporate sector of the broad economy.
    - ✓ This results in a value-weighted portfolio, which, for example, invests a proportion in GM stock that equals the ratio of GM's market value to the market value of all listed stocks.

- Such strategies are called *indexing*.
  - The investor chooses a portfolio of all the stocks in a broad market index such as the S&P 500.
    - ✓ The rate of return on the portfolio then replicates the return on the index.
  - Indexing has become a popular strategy for passive investors.
- We call the capital allocation line provided by one-month T-bills and a broad index of common stocks the **capital market line (CML).**
  - That is, a passive strategy using the broad stock market index as the risky portfolio generates an investment opportunity set that is represented by the CML.

Historical Evidence on the Capital Market Line

- Table 5.4 is a small cut-and-paste from Table 5.3, which concentrates on S&P 500 data, a popular choice for a broad stock-market index.
  - As we discussed earlier, the large standard deviation of its rate of return implies that we cannot reject the hypothesis that the entire 88-year period is characterized by the same Sharpe ratio.

TABLE 5.4 Excess return statistics for the S&P 500

	Excess Return (%)			
	Average	Std Dev	Sharpe Ratio	5% VaR
1926–2013	8.34	20.23	0.41	−25.88
1926–1955	11.67	25.40	0.46	NA*
1956–1985	5.01	17.58	0.28	NA*
1986–2013	8.33	17.73	0.47	NA*

*Too few observations for meaningful calculation.

- ◆ Using this history as a guide, investors might reasonably forecast a risk premium of around 8% coupled with a standard deviation of approximately 20%, resulting in a Sharpe ratio of .4.
- ◆ To hold a complete portfolio with these risk-return characteristics, the "average" investor (who choose $y = 1$) must have a coefficient of risk aversion of $.08/.20^2 = 2$.
  - ✓ But that average investor would need some courage.

- ◆ As the VaR figures in Table 5.4 indicate, history suggests a probability of 5% of an annual loss on a market index portfolio exceeding 25%.
  - ✓ This substantial risk, together with differences in risk aversion across individuals, might explain the large differences we observe in portfolio positions across investors.
- ■ Finally, notice the instability of the excess returns on the S&P 500 across the 30-year subperiods in Table 5.4.
  - ◆ The great variability in excess returns raises the question of whether the 8% historical average really is a reasonable estimate of the risk premium looking into the future.
  - ◆ It also suggests that different investors may come to different conclusions about future excess returns, another reason for capital allocations to vary.

Costs and Benefits of Passive Investing

- The fact that an individual's capital allocation decision is hard does not imply that its implementation needs to be complex.
  - A passive strategy is simple and inexpensive to implement: Choose a broad index fund or ETF and divide your savings between it and a money market fund.
- To justify spending your own time and effort or paying a professional to pursue an active strategy requires some evidence that those activities are likely to be profitable.
  - As we shall see later in the text, this is much harder to come by than you might expect!

    Acquire, obtain
- To choose an active strategy, an investor must be convinced that the benefits outweigh the cost, and the cost can be quite large.
  - As a benchmark, annual expense ratios for index funds are around 20 and 50 basis points for U.S. and international stocks, respectively.
  - The cost of utilizing a money market fund is smaller still, and T-bills can be purchased at no cost.

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- Here is a very cursory idea of the cost of active strategies.
  - The annual expense ratio of an active stock mutual fund averages around 1% of invested assets.
  - Mutual funds that invest in more exotic assets such as real estate or precious metals can be more expensive still.
  - A hedge fund will cost you 1% to 2% of invested assets plus 10% or more of any returns above the risk-free rate.
  - If you are wealthy and seek more dedicated portfolio management, costs will be even higher.
- Because of the power of compounding, an extra 1% of annual costs can have large consequences for the future value of your portfolio.
  - With a risk-free rate of 2% and a risk premium of 8%, you might expect your wealth to grow by a factor of $1.10^{30} = 17.45$ over a 30-year investment horizon.
  - If fees are 1%, then your net return is reduced to 9%, and your wealth grows by a factor of only $1.09^{30} = 13.26$ over the same horizon.

- That seemingly small management fee reduces your final wealth by about one-quarter.

- The potential benefits of active strategies are discussed in detail in Chapter 8.
  - The news is generally not that good for active investors.
- However, the factors that keep the active management industry going are as follows:
  - The large potential of enrichment from successful investments—the same power of compounding works in your favor if you can add even a few basis points to total return
  - The difficulty in assessing performance
  - Uninformed investors who are willing to pay for professional money management
- There is no question that some money managers can outperform passive strategies.
  - The problem is (1) how do you identify them and (2) do their fees outstrip their potential.
- Whatever the choice one makes, one thing is clear: The CML using the passive market index is not an obviously inferior choice.