Influential Factors of the Number of Days an Animal Spends at the Shelter

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1 Introduction

Data on animals admitted to the Dallas animal shelter were collected over the course of a year, from October 2016 to September 2017. For each animal admitted to the shelter, the following information was recorded - the type of animal being admitted, the month and year it was admitted, the reason for the animal being admitted, the final outcome for the animal, whether the animal was micro-chipped, and the number of days the animal spent at the shelter.

This report will investigate which of these factors are influential in determining the number of days an animal spends at the shelter before its final outcome is decided.

2 Exploratory Data Analysis

The first five lines of the raw data:

Table 1: Raw data

animal_type	month	year	intake_type	outcome_type	chip_status	time_at_shelter
CAT	9	2017	STRAY	ADOPTION	UNABLE TO SCAN	9
DOG	6	2017	STRAY	EUTHANIZED	SCAN NO CHIP	4
DOG	12	2016	STRAY	ADOPTION	SCAN NO CHIP	21
DOG	9	2017	STRAY	ADOPTION	SCAN NO CHIP	4
CAT	11	2016	OWNER SURRENDER	ADOPTION	SCAN CHIP	7

Levels of each explanatory variable:

```
animal_type :
[1] "BIRD"     "CAT"      "DOG"      "WILDLIFE"

month :
 [1] "1"  "2"  "3"  "4"  "5"  "6"  "7"  "8"  "9"  "10" "11" "12"

year :
[1] "2016" "2017"

intake_type :
[1] "CONFISCATED"     "OWNER SURRENDER" "STRAY"

outcome_type :
[1] "ADOPTION"          "DIED"              "EUTHANIZED"
```

```
[4] "FOSTER"            "RETURNED TO OWNER"

chip_status :
[1] "SCAN CHIP"      "SCAN NO CHIP"   "UNABLE TO SCAN"
```

All the explanatory variables are categorical variables and each explanatory variable has multiple levels.

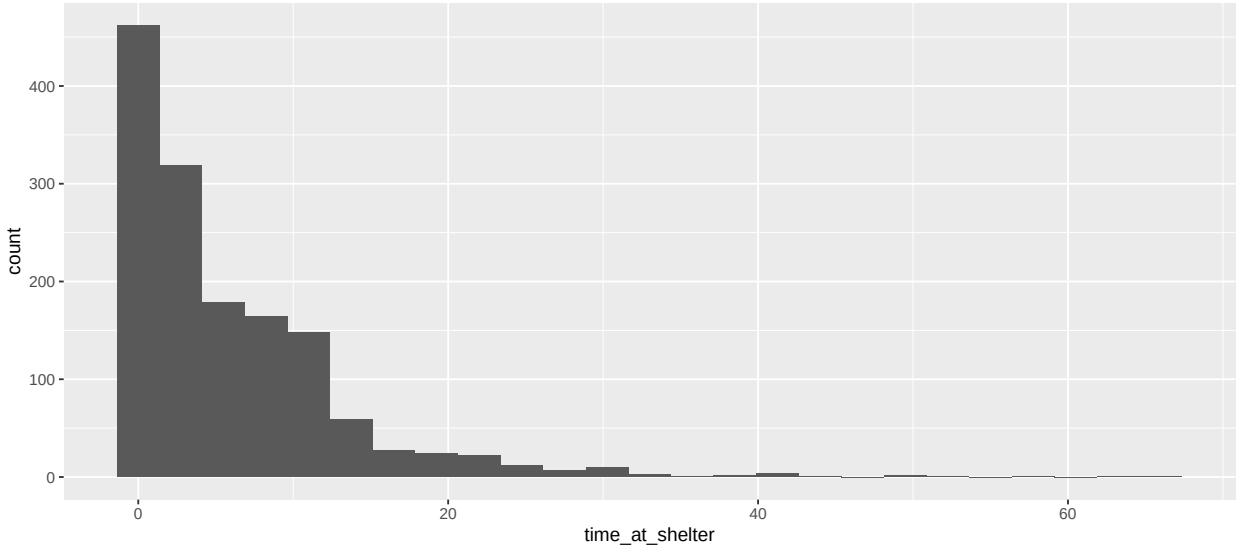

Figure 1: Histogram of number of days spent at the shelter

Figure 1 displays the histogram of the response variable, which is the number of days spent at the shelter. The histogram shows evidence of the response variable being right-skewed and following a Poisson distribution.

Figure 2: Pair plots of the variables

The explanatory variables are all categorical and their box plots are shown in Figure 2. The median time at

shelter appears to be low for all the explanatory variables, which is due to the median time at shelter being 4.

Since in Figure 1 the response variable is right-skewed, a median of the response variable is calculated. The figures below display the median of each category of the different explanatory variables.

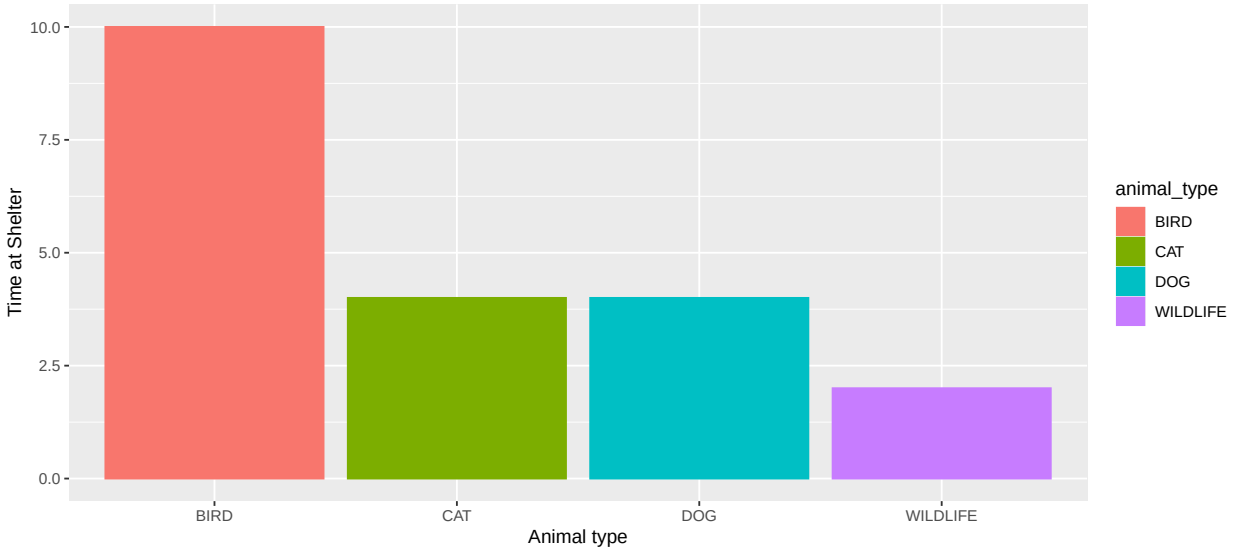

Figure 3: Bar plot of animal type vs median time at shelter

Table 2: Summary statistics on the time at shelter by animal type

animal_type	n	Mean	St.Dev	Min	Q1	Median	Q3	Max
BIRD	3	9.333333	8.020806	1	5.5	10	13.5	17
CAT	270	5.903704	7.366027	0	1.0	4	8.0	50
DOG	1163	6.110920	7.375513	0	1.0	4	9.0	66
WILDLIFE	14	4.500000	6.525099	0	0.0	2	6.5	23

From Figure 3, the median value of time at shelter seems different for each category except cat and dog. This could be because the sample sizes for bird and wildlife are much smaller than those of dog and cat, so this result could be skewed.

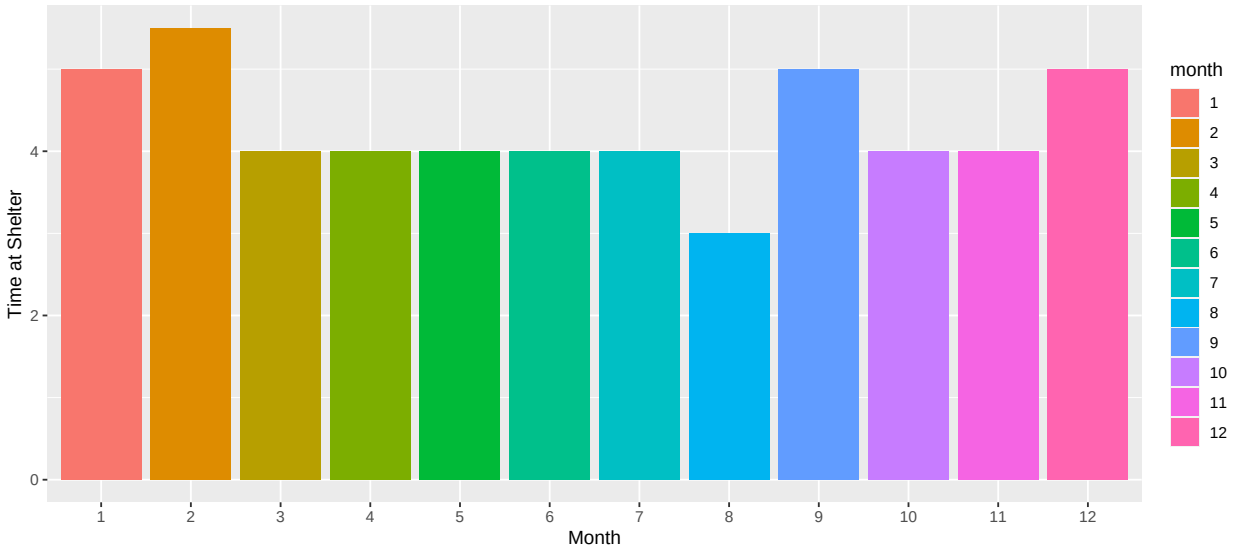

Figure 4: Bar plot of month vs median time at shelter

Table 3: Summary statistics on the time at shelter by month

month	n	Mean	St.Dev	Min	Q1	Median	Q3	Max
1	99	6.888889	7.618303	0	1	5.0	10	40
2	82	7.707317	9.646195	0	2	5.5	10	66
3	108	5.287037	7.163055	0	1	4.0	7	42
4	115	5.069565	5.549967	0	1	4.0	6	31
5	139	6.000000	8.062258	0	0	4.0	8	63
6	163	6.184049	6.325765	0	1	4.0	9	29
7	162	5.845679	6.315289	0	0	4.0	10	30
8	127	4.078740	4.922585	0	0	3.0	6	31
9	114	5.456140	4.954912	0	1	5.0	8	22
10	123	6.967480	9.716418	0	1	4.0	8	50
11	110	6.236364	7.911120	0	1	4.0	7	53
12	108	7.888889	9.075317	0	2	5.0	11	59

From Figure 4, the median value of time at shelter is similar for each month. All the summary statistics are similar.

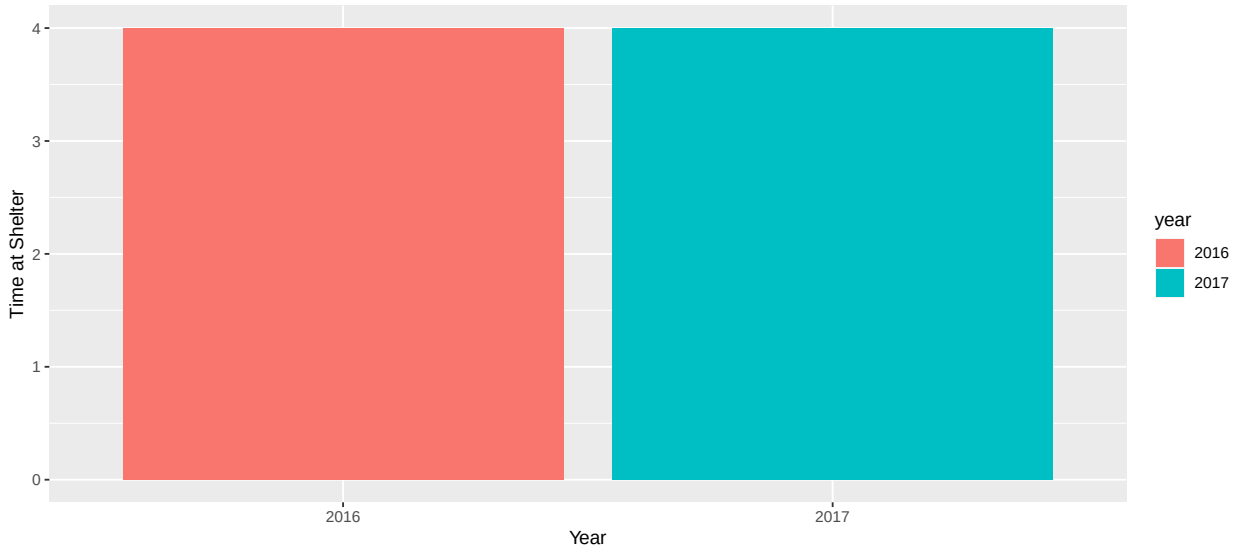

Figure 5: Bar plot of year vs median time at shelter

```
[1] FALSE
```

There is no overlap between the months and years, since the data was recorded over the period of a year. According to Figure 5, there is no obvious difference between the two years and the relationship between the response variable and month variable is similar to the relationship between the response variable and the year variable. In fact, both variables represent the same information, namely when the animal was admitted. Therefore, the variable year is removed.

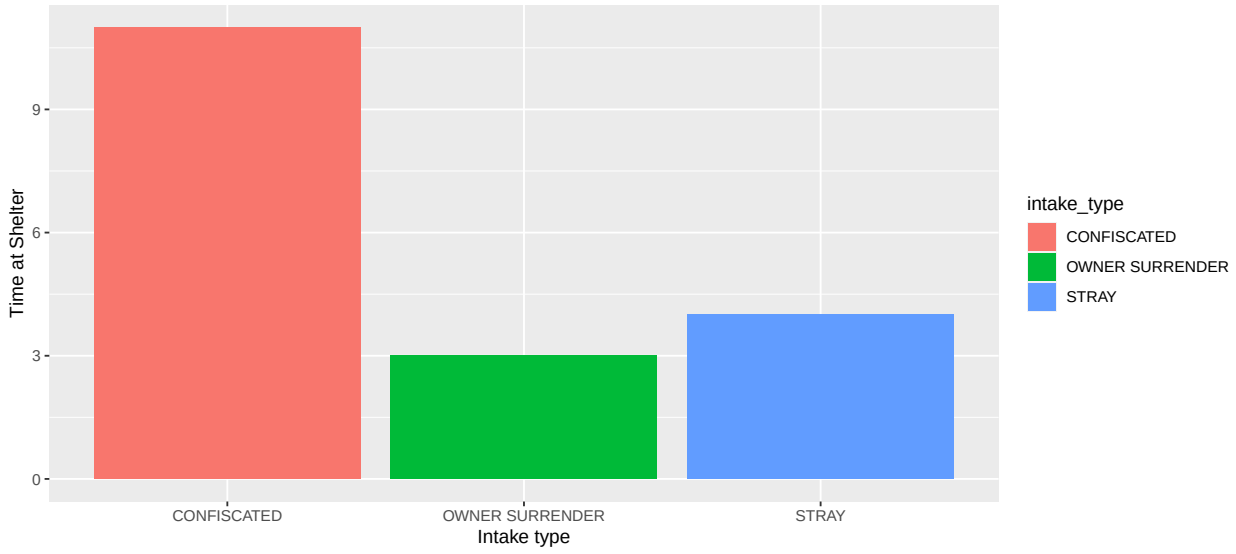

Figure 6: Bar plot of intake type vs median time at shelter

From Figure 6, an obvious difference is shown between each category.

Table 4: Summary statistics on the time at shelter by intake type

intake_type	n	Mean	St.Dev	Min	Q1	Median	Q3	Max
CONFISCATED	77	10.896104	9.564992	0	5	11	13	63
OWNER SURRENDER	467	5.141328	7.215962	0	1	3	7	53
STRAY	906	6.128035	7.063027	0	1	4	8	66

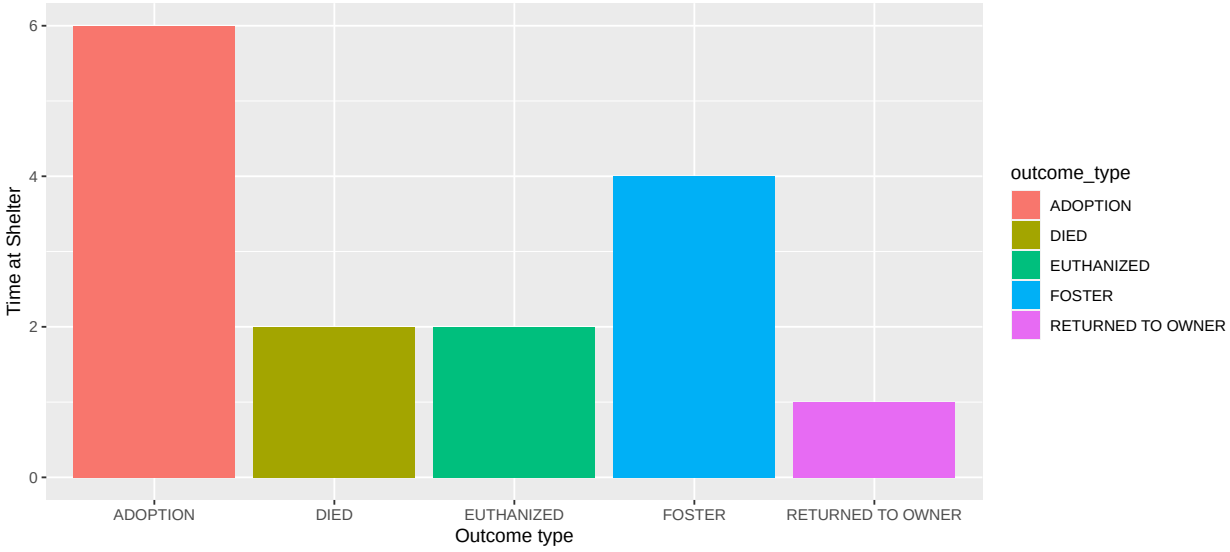

Figure 7: Bar plot of outcome type vs median time at shelter

Table 5: Summary statistics on the time at shelter by outcome type

outcome_type	n	Mean	St.Dev	Min	Q1	Median	Q3	Max
ADOPTION	636	8.523585	7.618321	0	4	6	10.25	66
DIED	25	4.360000	6.531207	0	1	2	5.00	33
EUTHANIZED	489	4.777096	7.380844	0	0	2	6.00	63
FOSTER	29	6.482759	8.708045	0	1	4	7.00	37
RETURNED TO OWNER	271	2.723247	3.952610	0	0	1	4.00	22

Figure 7 shows there is an obvious difference between each category. The sample size of DIED and FOSTER are small compared with the other categories.

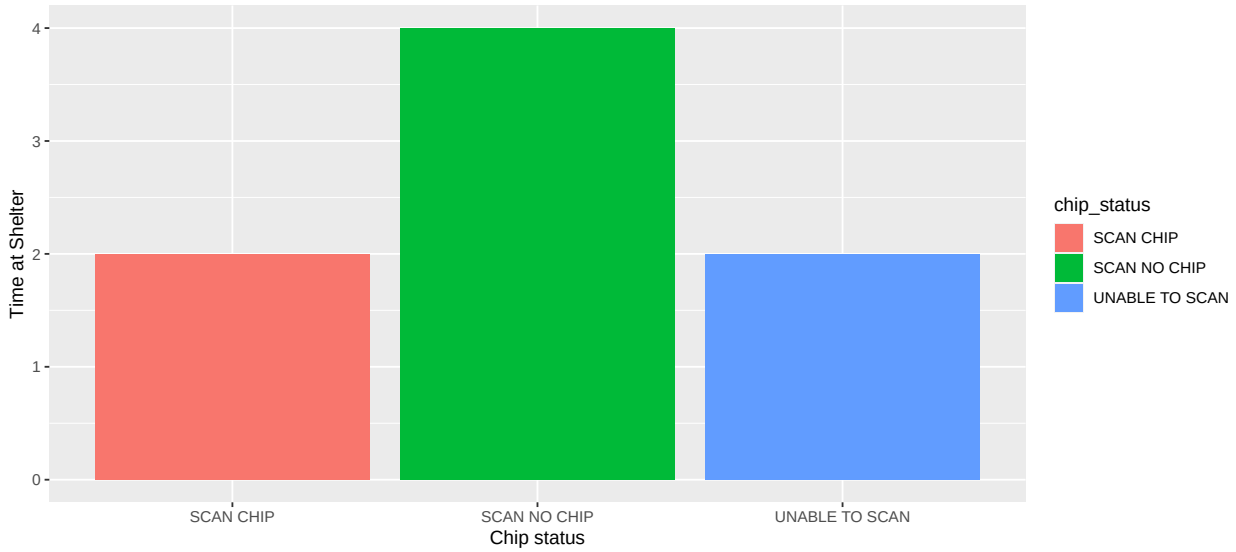

Figure 8: Bar plot of chip status vs median time at shelter

Table 6: Summary statistics on the time at shelter by chips status

chip_status	n	Mean	St.Dev	Min	Q1	Median	Q3	Max
SCAN CHIP	285	6.000000	8.582655	0	1	2	10	66
SCAN NO CHIP	1110	6.141441	7.038910	0	1	4	8	63
UNABLE TO SCAN	55	4.818182	6.944465	0	0	2	6	31

From Figure 7, some differences exist. The sample size of UNABLE TO SCAN is small compared with others.

3 Formal Data Analysis——Fitting a Poisson model

Since the response variable is count data, a Poisson model is fit to the data. The response variables, Y_i, are assumed to be independently, identically distributed, following a Poisson distribution:

$$Y_i \sim Poi(\mu_i)$$

The following formula is then used to estimate the number of days spent at the shelter:

$$log(\hat{Y}_i) = log(\hat{\mu}_i) = log(n_i) + \sum_{j=1}^{k} x_{ij}\hat{\beta}_j$$

where $\hat{Y}_i$ is the expected number of days spent at the shelter from exposure n_i. $\hat{\mu}_i$ is the expected mean, x_{ij} is the j^{th} covariate, where $j = 1, ..., k$, and $\hat{\beta}_j$ is the coefficient of covariate j. k equals the number of all the covariates used to fit the model.

3.1 Variable selection using AIC

```
Start:  AIC=12146.91
```

```
time_at_shelter ~ animal_type + month + intake_type + outcome_type +
    chip_status

               Df Deviance   AIC
<none>              8079.3 12147
- animal_type   3   8092.7 12154
- chip_status   2   8116.0 12180
- month        11   8225.1 12271
- intake_type   2   9018.1 13082
- outcome_type  4   9957.4 14017

Call:  glm(formula = time_at_shelter ~ animal_type + month + intake_type +
    outcome_type + chip_status, family = "poisson", data = data10)

Coefficients:
                  (Intercept)                 animal_typeCAT
                     2.997158                       0.441668
                animal_typeDOG             animal_typeWILDLIFE
                     0.485824                       0.225305
                       month2                         month3
                     0.075718                      -0.132108
                       month4                         month5
                    -0.193819                      -0.005919
                       month6                         month7
                    -0.035721                      -0.057427
                       month8                         month9
                    -0.413755                      -0.082308
                      month10                        month11
                     0.101852                      -0.055580
                      month12      intake_typeOWNER SURRENDER
                     0.114138                      -1.451530
              intake_typeSTRAY                outcome_typeDIED
                    -1.031365                      -0.649881
       outcome_typeEUTHANIZED              outcome_typeFOSTER
                    -0.592552                      -0.279520
outcome_typeRETURNED TO OWNER         chip_statusSCAN NO CHIP
                    -1.531722                      -0.171716
    chip_statusUNABLE TO SCAN
                    -0.247414

Degrees of Freedom: 1449 Total (i.e. Null);  1427 Residual
Null Deviance:      10550
Residual Deviance: 8079     AIC: 12150
```

Using AIC as a selection criteria, the model with the minimum AIC is selected and hence the best fit for the data is the saturated model.

3.2 P-value and confidence intervals for the Poisson model

```
Call:
glm(formula = time_at_shelter ~ ., family = "poisson", data = data10)

Deviance Residuals:
```

```
    Min       1Q   Median       3Q      Max
-6.9146  -1.9976  -0.8903   0.6306  12.7550

Coefficients:
                              Estimate Std. Error z value Pr(>|z|)
(Intercept)                   2.997158   0.197263  15.194  < 2e-16 ***
animal_typeCAT                0.441668   0.195885   2.255 0.024150 *
animal_typeDOG                0.485824   0.194425   2.499 0.012462 *
animal_typeWILDLIFE           0.225305   0.231453   0.973 0.330336
month2                        0.075718   0.055370   1.367 0.171470
month3                       -0.132108   0.057115  -2.313 0.020721 *
month4                       -0.193819   0.056691  -3.419 0.000629 ***
month5                       -0.005919   0.052007  -0.114 0.909386
month6                       -0.035721   0.050097  -0.713 0.475818
month7                       -0.057427   0.050613  -1.135 0.256526
month8                       -0.413755   0.058842  -7.032 2.04e-12 ***
month9                       -0.082308   0.056140  -1.466 0.142617
month10                       0.101852   0.051801   1.966 0.049273 *
month11                      -0.055580   0.054389  -1.022 0.306833
month12                       0.114138   0.051633   2.211 0.027065 *
intake_typeOWNER SURRENDER   -1.451530   0.043649 -33.254  < 2e-16 ***
intake_typeSTRAY             -1.031365   0.039395 -26.180  < 2e-16 ***
outcome_typeDIED             -0.649881   0.097578  -6.660 2.74e-11 ***
outcome_typeEUTHANIZED       -0.592552   0.025262 -23.456  < 2e-16 ***
outcome_typeFOSTER           -0.279520   0.076201  -3.668 0.000244 ***
outcome_typeRETURNED TO OWNER -1.531722   0.042358 -36.161  < 2e-16 ***
chip_statusSCAN NO CHIP      -0.171716   0.028935  -5.934 2.95e-09 ***
chip_statusUNABLE TO SCAN    -0.247414   0.068726  -3.600 0.000318 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 10551.2  on 1449  degrees of freedom
Residual deviance:  8079.3  on 1427  degrees of freedom
AIC: 12147

Number of Fisher Scoring iterations: 6
```

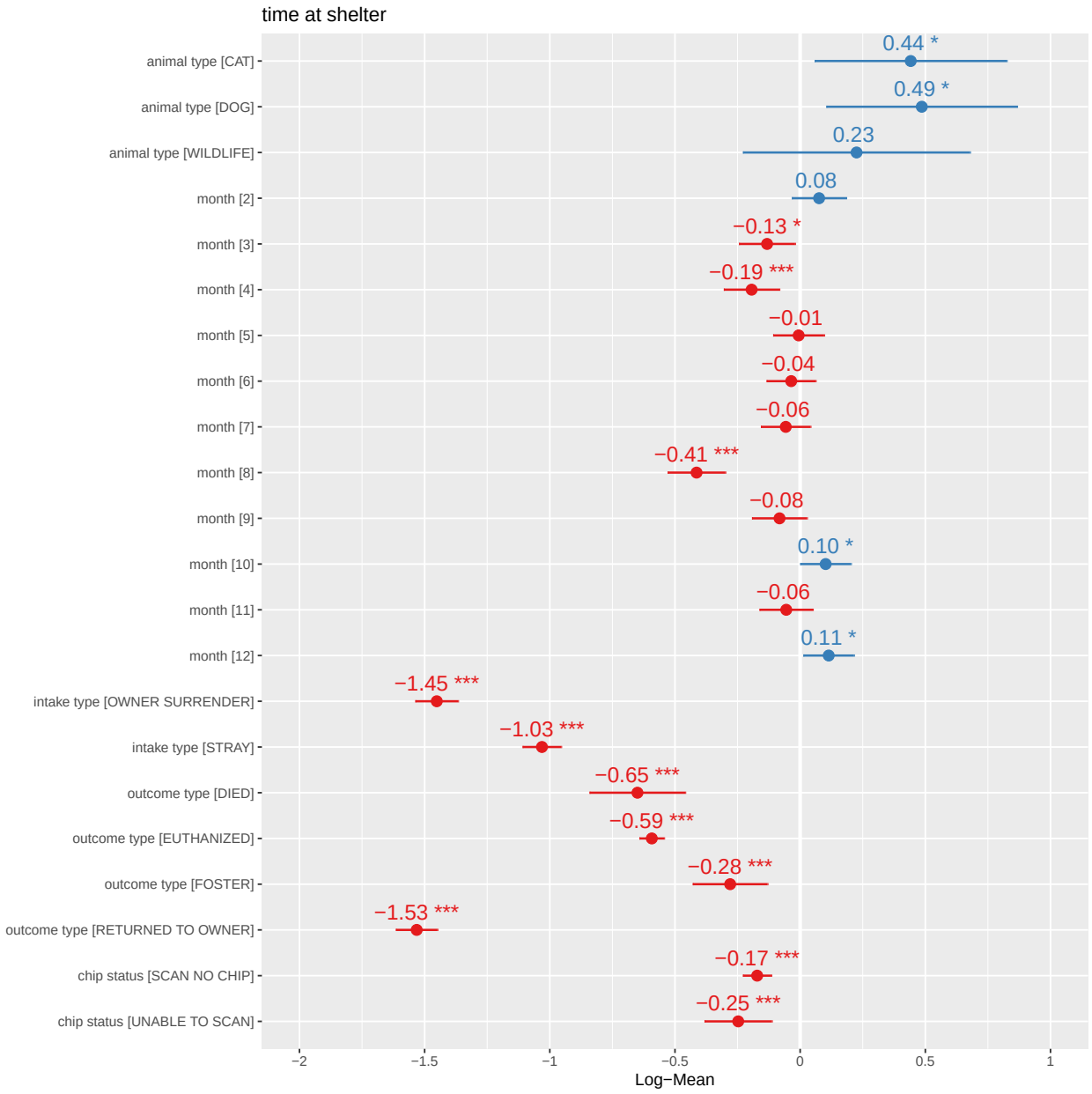

Figure 9: Confidence Intervals of the Poisson Model

Figure 9 displays the confidence intervals for each level of each categorical variable in comparison to the respective baseline category. All the levels of the categorical variables intake type, outcome type and chip status are significant. Two levels are significant in the factor animal type and one is insignificant. Five out of eleven categories of month are significant and the others are not.

3.3 Goodness of fit and overdispersion for the Poisson model

```
$results
[1] "Goodness-of-fit test for Poisson assumption"

$chisq
```

```
[1] 8079.325

$df
[1] 1427

$p.value
[1] 0
```

Since the p-value is smaller than 0.05, the null hypothesis is rejected and the over-dispersion is significant.

A rootogram can be used to check the over-dispersion. It is easy to visualize whether the model is over-fitting or under-fitting the values using the zero line. If the bar is below the zero line then that value has been under-fitted. And if there is a space between the zero line and the bar then it has been over-fitted. For the model to be fitted correctly, the bar should sit as close to the zero line as possible.

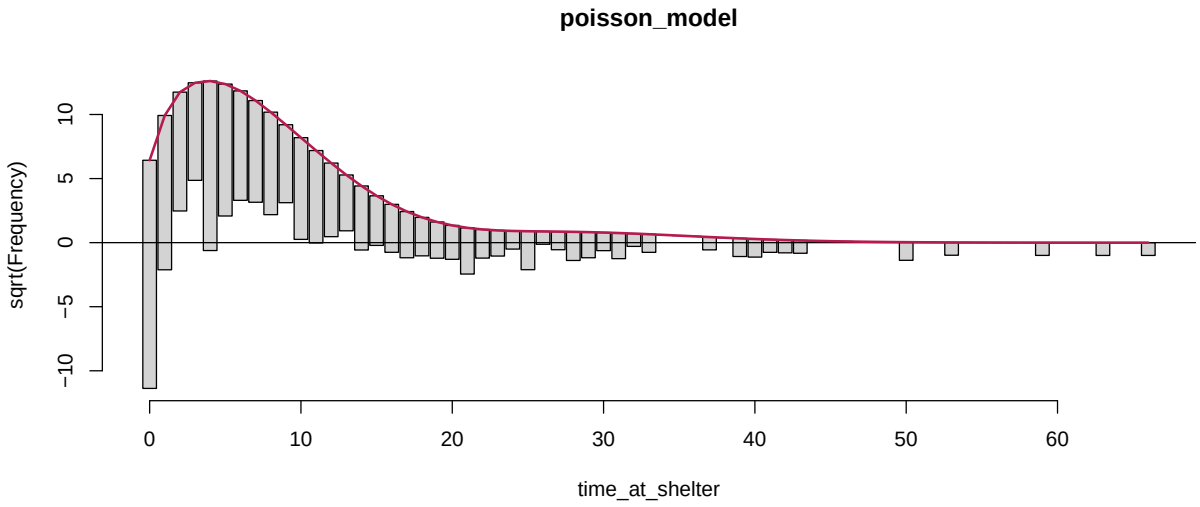

Figure 10: Rootogram of the Poisson Model

In Figure 10, the Poisson model is severely under-fitting zero counts. There were 317 zero counts observed in the data set but the model only fitted 41. It is also over-fitting the lower positive counts and under-fitting the higher counts, suggesting there is over-dispersion due to excess zeroes in the model. Hence a hurdle model will be fitted to provide a better fit.

4 Formal Data Analysis——Fitting a Hurdle model

4.1 Fitting a Binomial-Poisson hurdle model

Now a Binomial-Poisson hurdle model is fit to the data. A Binomial model is first used to determine whether an animal will be admitted to the shelter or not. Assuming the response variables, Y_i are independently, identically distributed:

$$Y_i \sim Bin(n_i, p_i)$$

where p_i is the probability that an animal will be admitted to the shelter.

The log-odds are then:

$$log(\frac{p_i}{1-p_i}) = \sum_{j=1}^{k} \beta_j x_{ij}$$

where β_j is the coefficient of the j^{th} covariate, x_{ij}, with $j = 1, ...k$. k equals the number of all the covariates used to fit the model.

Once an animal is admitted to the shelter, a truncated Poisson model is fit to the data. The number of days spent at the shelter will be positive and can be estimated using the following formula:

$$\hat{Y_i} = (1-p_i)\frac{\hat{\mu_i}}{1-exp(-\hat{\mu_i})}$$

where $\hat{Y_i}$ is the number of days spent at the shelter, $\hat{\mu_i}$ is the expected mean and p_i is the probability that an animal will be admitted to the shelter.

The value of $log(\hat{\mu_i})$ can be found using:

$$log(\hat{\mu_i}) = \sum_{j=1}^{k} \hat{\beta_j} x_{ij}$$

where $j = 1, ..., k$ represents the j^{th} covariate, $\hat{\beta_0}$ denotes the intercept term and $\hat{\beta_j}$ is the coefficient of the j^{th} explanatory variable, x_{ij}. k equals the number of all the covariates used to fit the model.

```
Call:
hurdle(formula = time_at_shelter ~ ., data = data10, dist = "poisson",
    zero.dist = "binomial")

Pearson residuals:
    Min      1Q  Median      3Q     Max
-4.3608 -1.0287 -0.5823  0.4795 14.9926

Count model coefficients (truncated poisson with log link):
                             Estimate Std. Error z value Pr(>|z|)
(Intercept)                 2.9579923  0.1983275  14.915  < 2e-16 ***
animal_typeCAT              0.3743137  0.1965591   1.904 0.056867 .
animal_typeDOG              0.3213099  0.1951832   1.646 0.099723 .
animal_typeWILDLIFE         0.4412799  0.2325810   1.897 0.057786 .
month2                     -0.0007866  0.0555725  -0.014 0.988706
month3                     -0.1913094  0.0574189  -3.332 0.000863 ***
month4                     -0.2968745  0.0570389  -5.205 1.94e-07 ***
month5                     -0.0358694  0.0522504  -0.686 0.492405
month6                     -0.1290100  0.0505296  -2.553 0.010675 *
month7                     -0.0908291  0.0508464  -1.786 0.074043 .
month8                     -0.3531232  0.0594007  -5.945 2.77e-09 ***
month9                     -0.1700644  0.0563869  -3.016 0.002561 **
month10                     0.0425144  0.0518410   0.820 0.412164
month11                    -0.0777278  0.0545280  -1.425 0.154023
month12                     0.0460268  0.0517740   0.889 0.374006
intake_typeOWNER SURRENDER -1.1067328  0.0453104 -24.426  < 2e-16 ***
intake_typeSTRAY           -0.7609702  0.0407405 -18.678  < 2e-16 ***
outcome_typeDIED           -0.6233442  0.0998502  -6.243 4.30e-10 ***
outcome_typeEUTHANIZED     -0.2197569  0.0254704  -8.628  < 2e-16 ***
```

```
outcome_typeFOSTER            -0.1110361  0.0769153  -1.444 0.148847
outcome_typeRETURNED TO OWNER -0.9857031  0.0450846 -21.863  < 2e-16 ***
chip_statusSCAN NO CHIP       -0.2019465  0.0290236  -6.958 3.45e-12 ***
chip_statusUNABLE TO SCAN     -0.2152199  0.0686741  -3.134 0.001725 **
Zero hurdle model coefficients (binomial with logit link):
                               Estimate Std. Error z value Pr(>|z|)
(Intercept)                    1.905e+01  6.099e+02   0.031    0.975
animal_typeCAT                -1.328e+01  6.099e+02  -0.022    0.983
animal_typeDOG                -1.266e+01  6.099e+02  -0.021    0.983
animal_typeWILDLIFE           -1.454e+01  6.099e+02  -0.024    0.981
month2                         7.990e-01  4.898e-01   1.631    0.103
month3                         3.817e-01  4.040e-01   0.945    0.345
month4                         3.724e-01  4.020e-01   0.926    0.354
month5                        -9.406e-04  3.735e-01  -0.003    0.998
month6                         4.541e-01  3.702e-01   1.227    0.220
month7                         1.809e-01  3.643e-01   0.497    0.620
month8                        -2.548e-01  3.782e-01  -0.674    0.500
month9                         3.331e-01  3.984e-01   0.836    0.403
month10                        3.409e-01  3.981e-01   0.856    0.392
month11                        5.129e-02  4.062e-01   0.126    0.900
month12                        4.482e-01  4.345e-01   1.032    0.302
intake_typeOWNER SURRENDER    -3.171e+00  5.161e-01  -6.143 8.07e-10 ***
intake_typeSTRAY              -2.406e+00  4.857e-01  -4.955 7.25e-07 ***
outcome_typeDIED              -8.929e-01  8.223e-01  -1.086    0.278
outcome_typeEUTHANIZED        -2.999e+00  2.661e-01 -11.273  < 2e-16 ***
outcome_typeFOSTER            -2.137e+00  5.383e-01  -3.969 7.21e-05 ***
outcome_typeRETURNED TO OWNER -4.203e+00  3.115e-01 -13.491  < 2e-16 ***
chip_statusSCAN NO CHIP       -1.024e-01  1.978e-01  -0.518    0.605
chip_statusUNABLE TO SCAN     -6.084e-01  3.793e-01  -1.604    0.109
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Number of iterations in BFGS optimization: 30
Log-likelihood: -5193 on 46 Df
```

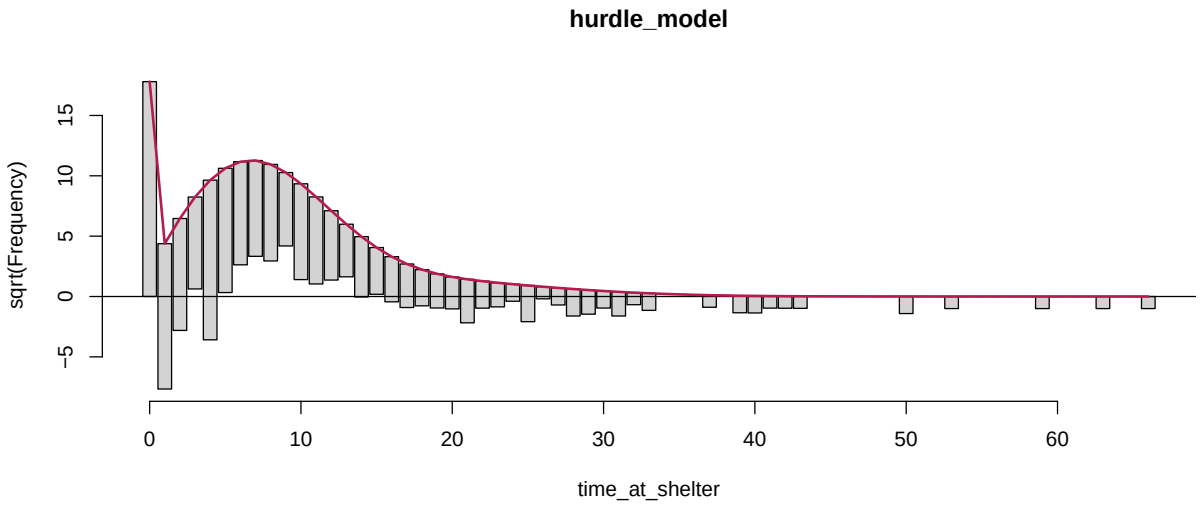

Figure 11: Rootogram of the Poisson Hurdle Model

In Figure 11 counts 1,2 and 4 are being severely under-fitted, while 6-9 are being over-fitted. There is also under-fitting at the higher counts which suggests over-dispersion. Therefore, a negative binomial hurdle model shall be fitted to address this.

4.2 Fitting a Binomial-Negative Binomial hurdle model

The Binomial-Negative Binomial hurdle model works in a similar way to the Binomial-Poisson hurdle model. However, once an animal is admitted to the shelter, a truncated Negative Binomial model is fit instead.The number of days spent at the shelter can then be estimated using the following formula:

$$\hat{Y}_i = (1 - p_i)\frac{\hat{\mu}_i}{1 - (1 + \alpha\hat{\mu}_i)^{-1/\alpha}}$$

where $\hat{Y}_i$ is the number of days spent at the shelter, $\hat{\mu}_i$ is the expected mean, p_i is the probability that an animal will be admitted to the shelter and $\alpha(\geq 0)$ is the dispersion parameter that is assumed not to depend on covariates.

The value of $log(\hat{\mu}_i)$ can be found using:

$$log(\hat{\mu}_i) = \sum_{j=1}^{k} \hat{\beta}_j x_{ij}$$

where $j = 1, ..., k$ represents the j^{th} covariate, $\hat{\beta}_0$ denotes the intercept term and $\hat{\beta}_j$ is the coefficient of the j^{th} explanatory variable, x_{ij}. k equals the number of all the covariates used to fit the model.

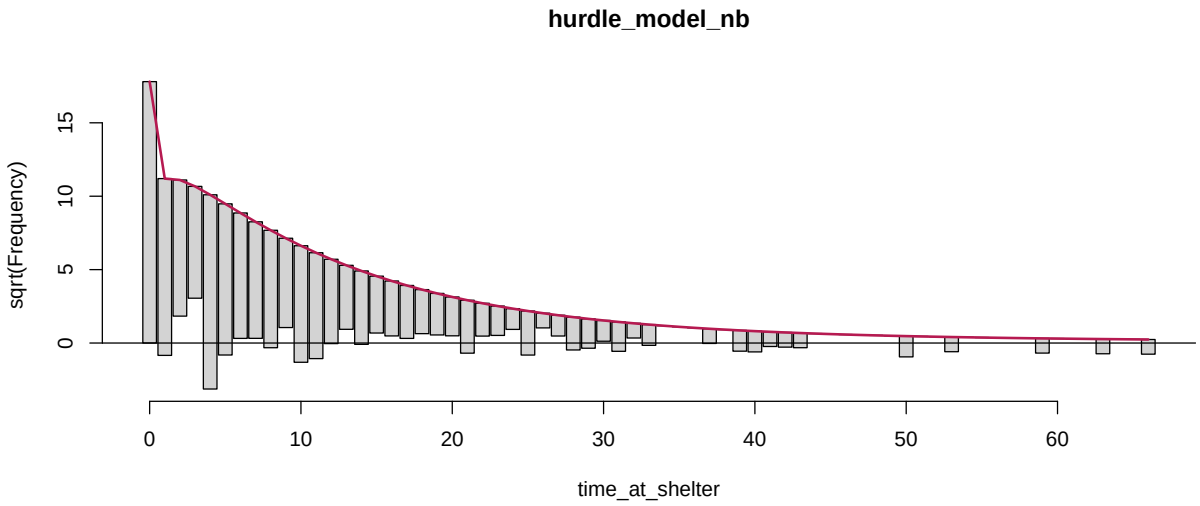

Figure 12: Rootogram of the Negative Binomial Hurdle Model

The AIC of the Poisson hurdle model is 10478 and the AIC of the Negative Binomial hurdle model is 7781. From this, the Negative Binomial model shows a much better fit to the data. However, in Figure 12 some values are still being under-fitted.

4.3 Variable selection using AIC for negative binomial hurdle model

```
Start:  AIC=7780.7
time_at_shelter ~ animal_type + month + intake_type + outcome_type +
    chip_status

               Df    AIC
- month        22 7767.3
<none>            7780.7
- chip_status   4 7782.2
- animal_type   6 7787.7
- intake_type   4 7942.5
- outcome_type  8 8245.8

Step:  AIC=7767.26
time_at_shelter ~ animal_type + intake_type + outcome_type +
    chip_status

               Df    AIC
<none>            7767.3
- chip_status   4 7767.7
- animal_type   6 7776.1
+ month        22 7780.7
- intake_type   4 7931.5
- outcome_type  8 8248.1

Call:
hurdle(formula = time_at_shelter ~ animal_type + intake_type + outcome_type +
```

```
    chip_status, data = data10, dist = "negbin", zero.dist = "binomial")

Count model coefficients (truncated negbin with log link):
                  (Intercept)                 animal_typeCAT
                       2.4956                         0.9004
               animal_typeDOG            animal_typeWILDLIFE
                       0.8454                         0.9344
   intake_typeOWNER SURRENDER               intake_typeSTRAY
                      -1.3568                        -0.9797
            outcome_typeDIED         outcome_typeEUTHANIZED
                      -0.7449                        -0.2824
          outcome_typeFOSTER  outcome_typeRETURNED TO OWNER
                      -0.1796                        -1.2008
      chip_statusSCAN NO CHIP      chip_statusUNABLE TO SCAN
                      -0.1833                        -0.1427
Theta = 1.5067

Zero hurdle model coefficients (binomial with logit link):
                  (Intercept)                 animal_typeCAT
                      19.1526                       -13.1510
               animal_typeDOG            animal_typeWILDLIFE
                     -12.4842                       -14.4181
   intake_typeOWNER SURRENDER               intake_typeSTRAY
                      -3.2086                        -2.4313
            outcome_typeDIED         outcome_typeEUTHANIZED
                      -0.9783                        -2.9986
          outcome_typeFOSTER  outcome_typeRETURNED TO OWNER
                      -2.0942                        -4.2473
      chip_statusSCAN NO CHIP      chip_statusUNABLE TO SCAN
                      -0.1077                        -0.5265
```

Using AIC as a selection criteria, the model with the minimum AIC is selected and hence the best fit for the data is the model with animal type, chip status, intake type and outcome type as the explanatory variables.

4.4 P-value and confidence intervals for negative binomial hurdle model

```
Call:
hurdle(formula = time_at_shelter ~ animal_type + intake_type + outcome_type +
    chip_status, data = data10, dist = "negbin", zero.dist = "binomial")

Pearson residuals:
    Min      1Q  Median      3Q     Max
-1.1815 -0.6457 -0.3219  0.2380  8.9096

Count model coefficients (truncated negbin with log link):
                            Estimate Std. Error z value Pr(>|z|)
(Intercept)                  2.49559    0.53328   4.680 2.87e-06 ***
animal_typeCAT               0.90035    0.54405   1.655 0.097943 .
animal_typeDOG               0.84537    0.54038   1.564 0.117726
animal_typeWILDLIFE          0.93442    0.63104   1.481 0.138667
intake_typeOWNER SURRENDER  -1.35684    0.13723  -9.887  < 2e-16 ***
intake_typeSTRAY            -0.97973    0.12565  -7.797 6.33e-15 ***
outcome_typeDIED            -0.74487    0.20889  -3.566 0.000363 ***
```

```
outcome_typeEUTHANIZED        -0.28239    0.06371  -4.432 9.32e-06 ***
outcome_typeFOSTER            -0.17956    0.19697  -0.912 0.361973
outcome_typeRETURNED TO OWNER -1.20077    0.10457 -11.483  < 2e-16 ***
chip_statusSCAN NO CHIP       -0.18330    0.07284  -2.517 0.011851 *
chip_statusUNABLE TO SCAN     -0.14273    0.17540  -0.814 0.415789
Log(theta)                     0.40994    0.07215   5.682 1.33e-08 ***
Zero hurdle model coefficients (binomial with logit link):
                            Estimate Std. Error z value Pr(>|z|)
(Intercept)                  19.1526   612.0501   0.031    0.975
animal_typeCAT              -13.1510   612.0498  -0.021    0.983
animal_typeDOG              -12.4842   612.0498  -0.020    0.984
animal_typeWILDLIFE         -14.4181   612.0502  -0.024    0.981
intake_typeOWNER SURRENDER   -3.2086     0.5150  -6.231 4.64e-10 ***
intake_typeSTRAY             -2.4313     0.4848  -5.016 5.29e-07 ***
outcome_typeDIED             -0.9783     0.8054  -1.215    0.225
outcome_typeEUTHANIZED       -2.9986     0.2648 -11.322  < 2e-16 ***
outcome_typeFOSTER           -2.0942     0.5372  -3.898 9.69e-05 ***
outcome_typeRETURNED TO OWNER -4.2473    0.3101 -13.697  < 2e-16 ***
chip_statusSCAN NO CHIP      -0.1077     0.1944  -0.554    0.579
chip_statusUNABLE TO SCAN    -0.5265     0.3724  -1.414    0.157
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Theta: count = 1.5067
Number of iterations in BFGS optimization: 20
Log-likelihood: -3859 on 25 Df
```

Figure 13: Confidence Intervals of the Negative Binomial Hurdle Model

Figure 13 displays the confidence intervals for each level of each categorical variable in comparison to the respective baseline category. In the Binomial model, all the levels of the categorical variables intake type and outcome type are significant, while all the levels of the categorical variables animal type and chip status are insignificant. In the Truncated Poisson model, all the levels of the categorical variable intake type are significant and all the levels of animal type are insignificant.

Since the variable animal type is not significant for the model, animal type is removed to fit a new model.

The AIC of the new model only increases by 8.83, so the factor animal type is removed to make the model

simpler.

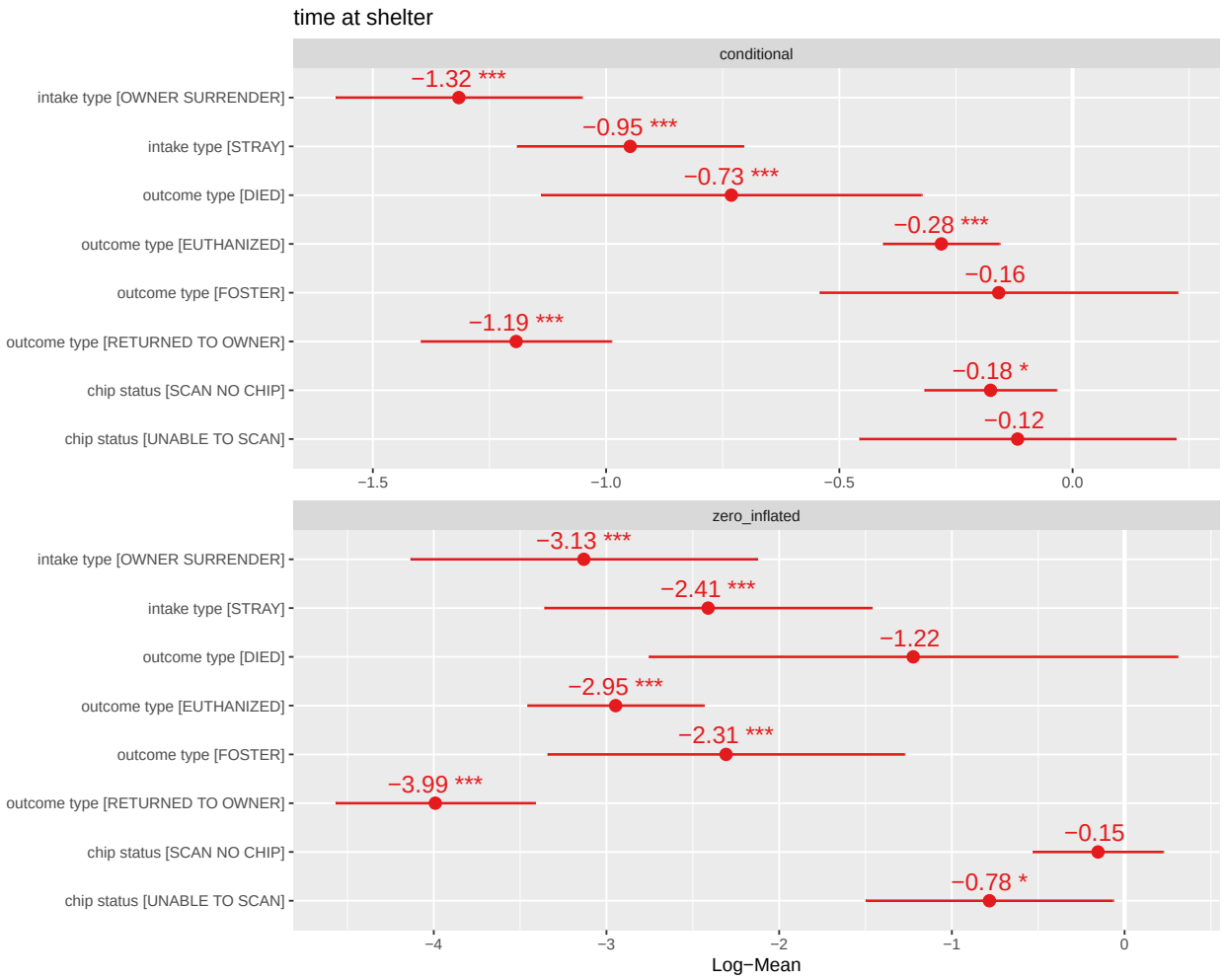

Figure 14: Confidence Intervals of the Negative Binomial Hurdle Model

From Figure 14, according to the p-value of each categorical variable, all the factors are influential.

4.5 Goodness of fit for the negative binomial hurdle model

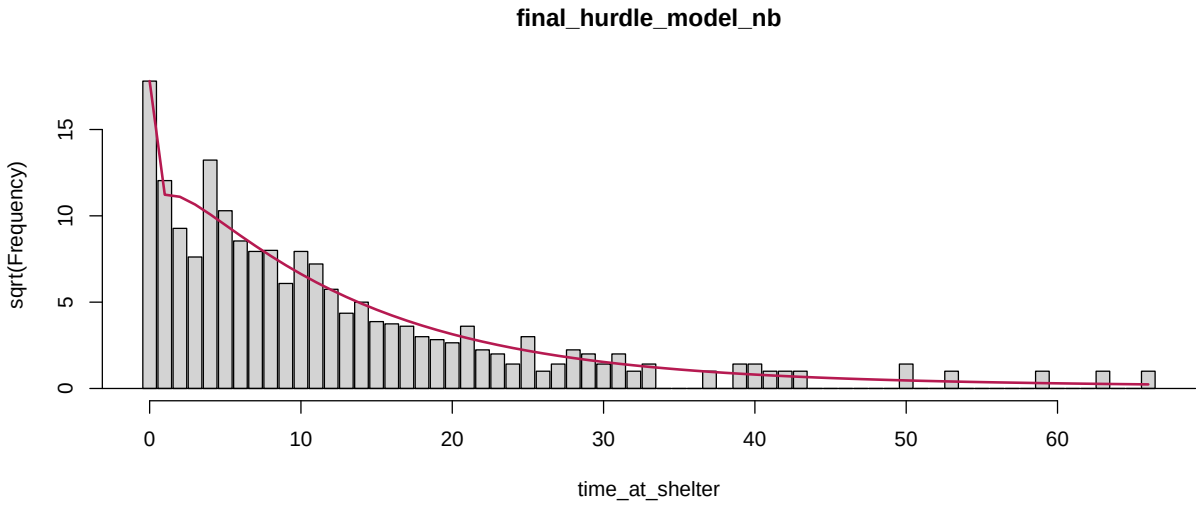

Figure 15: Rootogram of Negative Binomial Hurdle Model with reduced variables

The final model provides an adequate fit to the data. It has the lowest AIC of 7776.09 and as seen from Figure 15, the model, represented by the red line, fits most of the values of the count data well.

If a animal is admitted to the shelter, the time spent at the shelter can then be estimated using:

$$\begin{aligned} log(\hat{Y}_i) = log(\hat{\mu}_i) = 3.31 &+ -1.32 \cdot I(owner\ surrender) + -0.95 \cdot I(stray) \\ &+ -0.73 \cdot I(died) + -0.28 \cdot I(euthanized) + -0.16 \cdot I(foster) \\ &+ -1.19 \cdot I(returned\ to\ owner) + -0.18 \cdot I(scan\ no\ chip) + -0.12 \cdot I(unable\ to\ scan) \end{aligned}$$

where Y_i is the time spent at the shelter, 3.31 is the sum of the coefficients for the baseline categories of every factor. $I()$ is the indicator function:

$$I(A) = \begin{cases} 1 & if\ a \in A \\ 0 & if\ a \notin A \end{cases}$$

where a is the animal.

The coefficients stand for the multiplicative effect. For example, keeping other variables unchanged and comparing with the baseline category, if an animal belongs to *owner surrender*, the time spent at the shelter will be multiplied by $exp^{-1.32}$.

5 Conclusions

Due to the excess zeroes present in the data, the Poisson model is not a suitable fit to the data. The model which provides the best fit to the data is the negative binomial Hurdle model which includes intake type, outcome type and chip status as explanatory variables. Hence these factors are the most influential in determining the number of days an animal spends at the shelter before its final outcome is decided.