Para realizar o trabalho de classificação/regressão de dados, modelos de redes neurais são úteis para a realização da tarefa. O modelo Backpropagation é utilizado para encontrar pesos de uma rede neural que ajustam os dados de entrada e os de saída, minimizando o erro entre as saídas desejadas e as saídas calculadas pela rede neural. Considere a próxima base de dados binária:

	Α	В	C	
Fe	eat	tu	re	es
Cara	act	erí	sti	icas

As

Características

devem

possuir relação

com

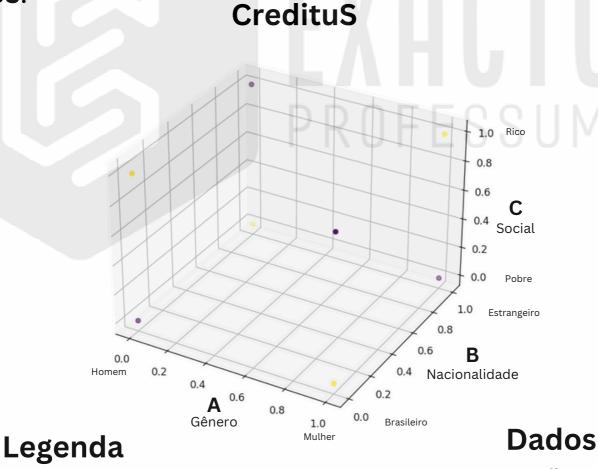
o Alvo

A	В	С	S
0	0	0	0
0	0	PROF	- E <sub>1</sub> S S
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

s Target Alvo

Acredita-se que haja uma função ou uma **estrutura de funções** aninhadas que possam gerar a coluna de target utilizando, como entradas, as colunas de **features**. Essa dinâmica é entendida como ajuste dos dados.

O ajuste de dados em classe (adimplência, julgamento, etc.) chama-se classificação de dados. O ajuste de dados do tipo números reais (temperatura, altura, etc.) chama-se regressão de dados.

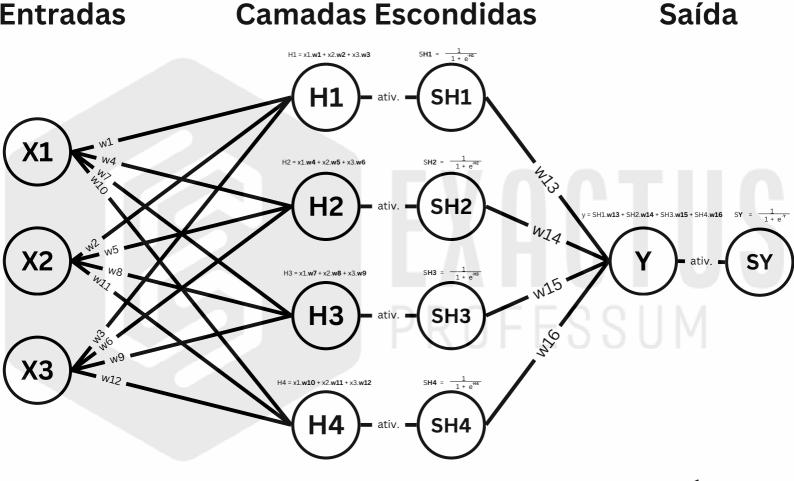


Adimplente s=0

Inadimplente s=1

Não Linearmente Separáveis!!!

Para dados **não-linearmente** separáveis, o modelo **backpropagation** proporciona uma estrutura funcional capaz de realizar classificação ou regressão não-lineares.



$$H1 = x1.w1 + x2.w2 + x3.w3$$

$$H2 = x1.w4 + x2.w5 + x3.w6$$

$$H3 = x1.w7 + x2.w8 + x3.w9$$

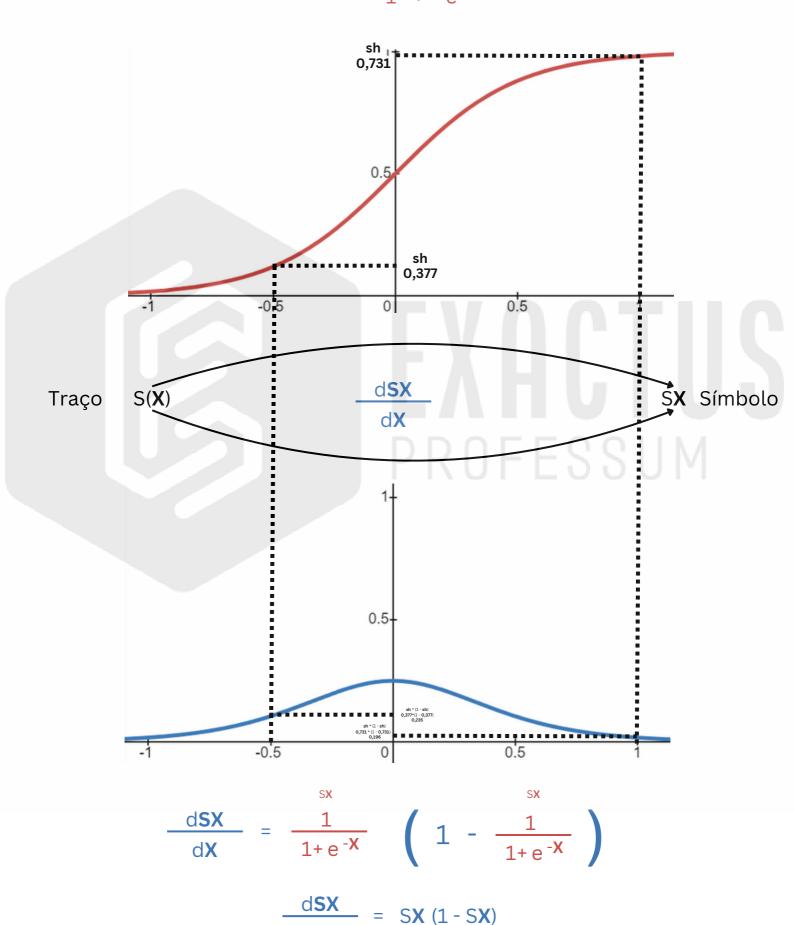
$$H4 = x1.w10 + x2.w11 + x3.w12$$

Erro = 
$$(S - SY)^2$$

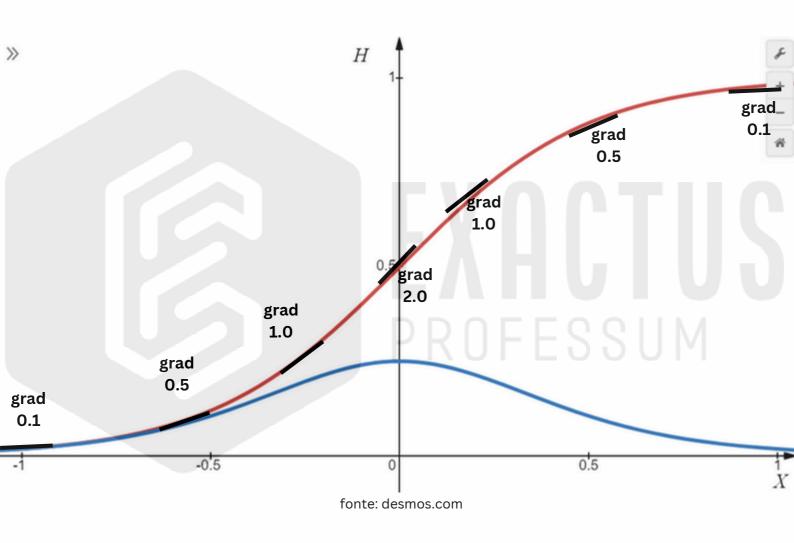
SH1 = 
$$\frac{1}{1 + e^{-H1}}$$
  
SH2 =  $\frac{1}{1 + e^{-H2}}$   
SH3 =  $\frac{1}{1 + e^{-H3}}$   
SH4 =  $\frac{1}{1 + e^{-H4}}$   
SY =  $\frac{1}{1 + e^{-Y}}$ 

10 Funções Para **ajustar** os dados **Y** + Função **Erro** 

$$SX = \frac{1}{1 + e^{-X}}$$



$$SH = \frac{1}{1 + e^{-H}}$$



$$\frac{dSH}{dH} = SH(1-SH)$$

# Ativação Sigmoide

Pesos

**Inicialmente** 

**Sorteados** 

#### **Primeira Camada**

$$w1 = -0.150$$

$$w2 = 0.608$$

$$w3 = 0.252$$

$$w4 = 1.572$$

$$w5 = 1.791$$

$$w6 = 0.507$$

#### w7 = 0.222

$$w8 = -0.382$$

$$w9 = -0.991$$

$$w10 = -0.127$$

$$w11 = -0.077$$

$$w12 = 0.364$$

#### Segunda Camada

$$w13 = 0.876$$

$$w14 = -0.087$$

$$w15 = 0.401$$

$$w16 = 1.463$$

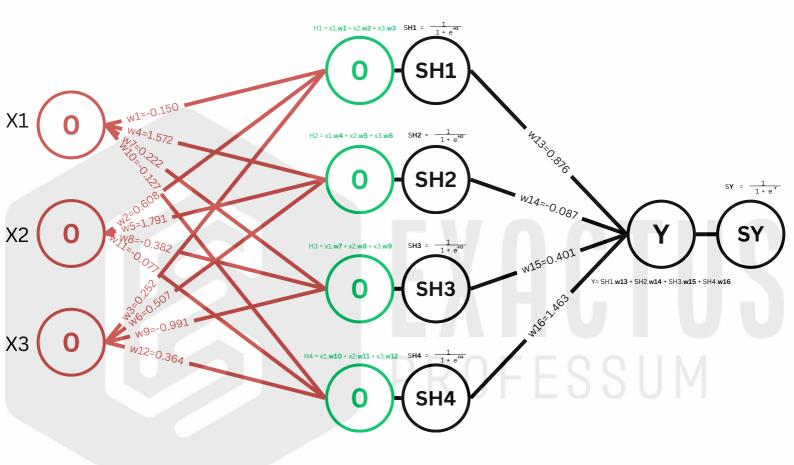
### \*Forward Propagation

Iteração 1

**Entradas** 

#### **Camadas Escondidas**

Saída



$$H1 = -0.0,150 + 0.0,608 + 0.0,252 = 0$$

$$H2 = + 0 . 1,572 + 0 . 1,791 + 0 . 0,507 = 0$$

$$H3 = -0.0,222 - 0.0,382 - 0.0,991 = 0$$

$$H4 = -0.0,127 - 0.0,077 + 0.0,364 = 0$$

# →Forward Propagation Iteração 1

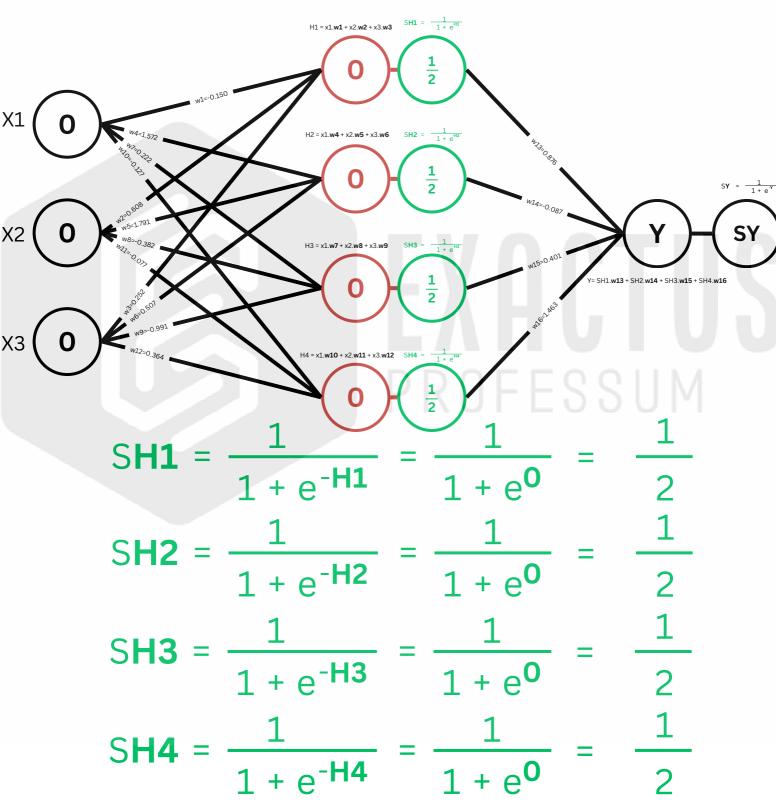
#### Forward Propagation

Iteração 1

#### **Entradas**

#### **Camadas Escondidas**

#### Saída

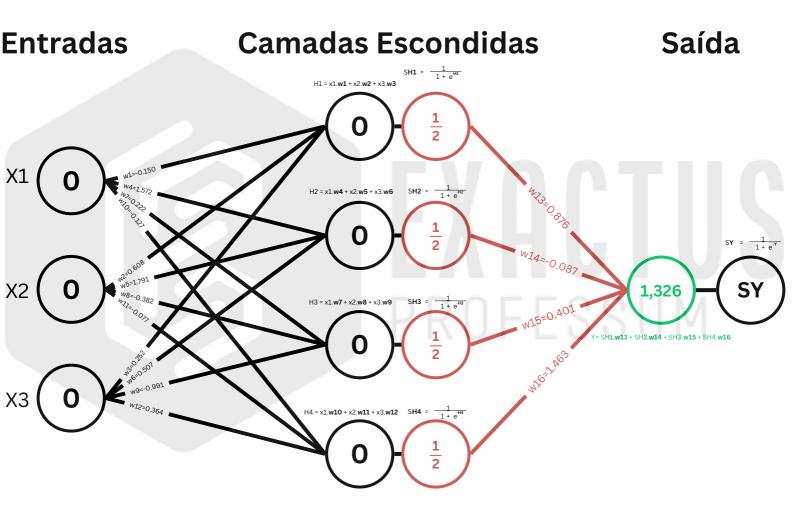


Forward Propagation

Iteração 1

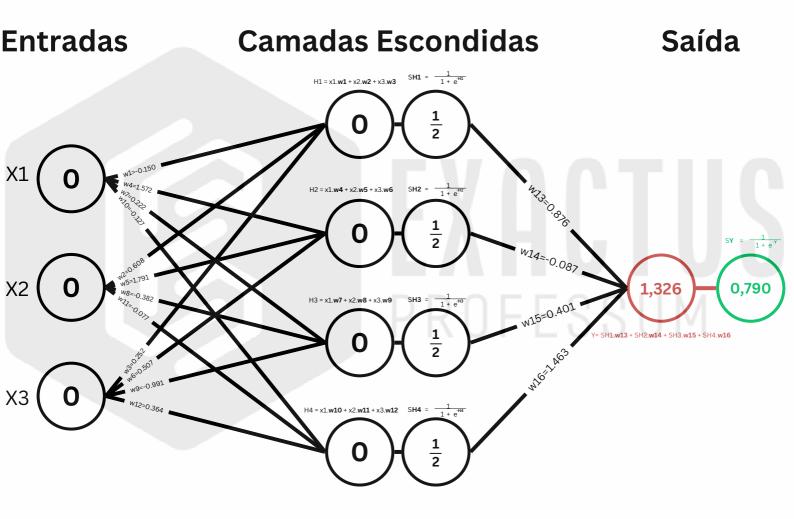
Forward Propagation

Iteração 1



Y = 0,5 . **0,876 -** 0,5 . **0,087** + 0,5 . **0,401** + 0,5 . **1,463** = **1,326** 

→Forward Propagation → Iteração 1



$$SY = \frac{1}{1 + e^{-Y}} = \frac{1}{1 + e^{-1,326}} = 0.790$$

→Forward Propagation ———— Iteração 1

#### Cálculo do Erro Iteração 1

Erro = 
$$0.5 * (S - SY)^2$$

S: Saída Esperada

SY: Saída Calculada

Lição	A	В	C	S
1	0	0	0	0



Erro = 
$$0.5 * (0 - 0.790)^2$$

$$Erro = 0.5 * (-0.790)^2$$

$$Erro = 0.5 * 0.043$$

$$Erro = 0,312$$

#### Cálculo do Erro Iteração 1

# ◆ Backward Propagation ← Iteração 1

E(w1, w2, w3, w4, w5, w6, w7, w8, w9, w10, w11, w12, w13, w14, w15 e w16)

 $\mathsf{O}$ 

**Backward** Propagation

consiste

em

realizar a **correção** dos

pesos (variáveis)

w1, w2, w3, w4, w5, w6, w7, w8, w9, w10, w11, w12, w13, w14, w15 e w16

para

minimizar

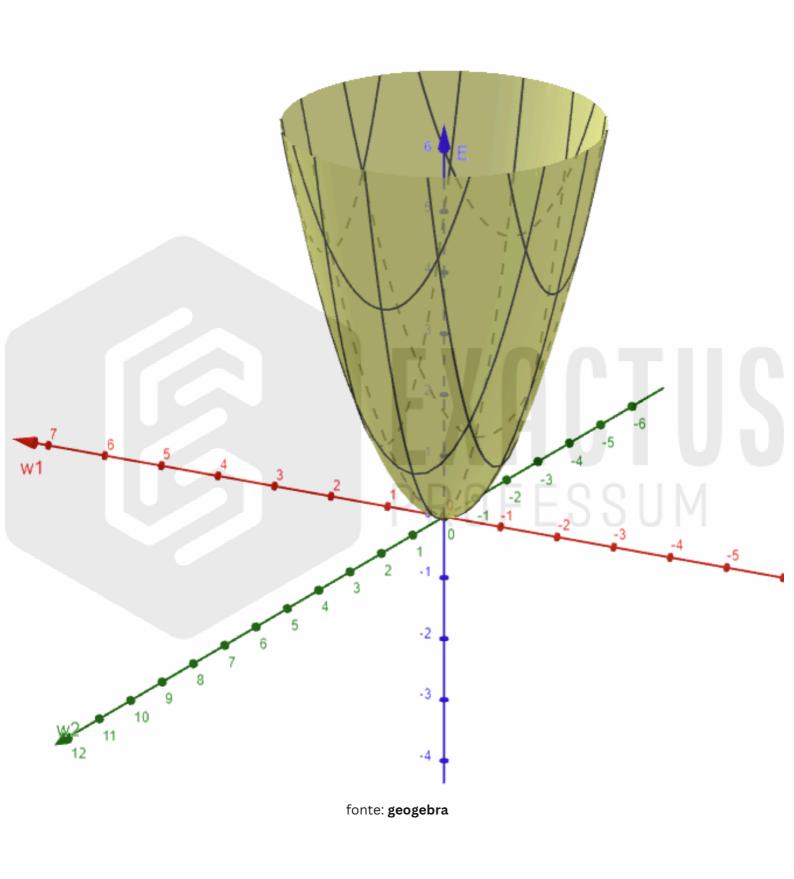
O

**Erro** 

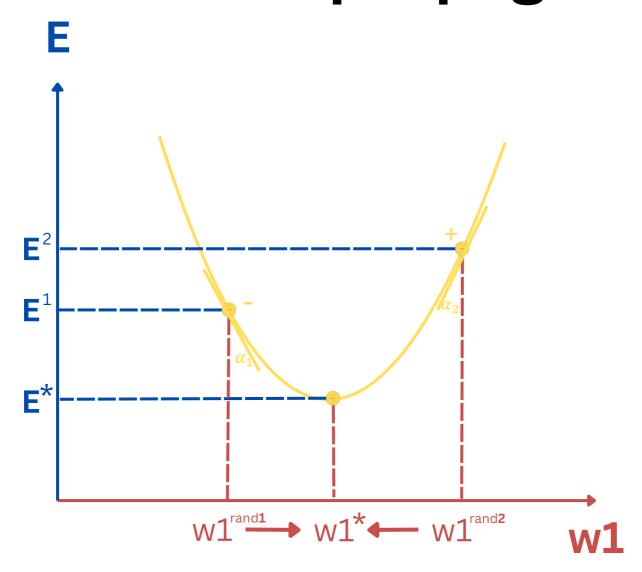
Erro = 0,5 \* 
$$(S - \frac{w_1}{w_1} \frac{w_2}{s} \frac{w_3}{v_1} \frac{w_4}{w_5})^2$$

Backward Propagation +

Iteração 1



E(w1, w2)



#### **Direcionar** W1 para w\*:

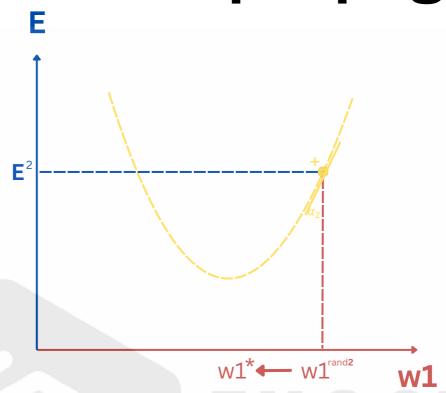
$$tg(\alpha_1) = \frac{\partial E}{\partial w_1} = k_1 (-) \qquad tg(\alpha_2) = \frac{\partial E}{\partial w_2} = k_2 (+)$$

$$new W_1^{rand1} = \frac{\partial E}{\partial w_1} = k_1 (-) \qquad new W_1^{rand2} = \frac{\partial E}{\partial w_2} = k_2 (+)$$

$$new W_1^{rand1} = \frac{\partial E}{\partial w_1} - (k1) \qquad new W_1^{rand2} = \frac{\partial E}{\partial w_1} - (k2)$$

$$new W_1^{rand1} = \frac{\partial E}{\partial w_1} - \left(\frac{\partial E}{\partial w_1}\right) \qquad new W_1^{rand2} = \frac{\partial E}{\partial w_1} - \left(\frac{\partial E}{\partial w_1}\right)$$

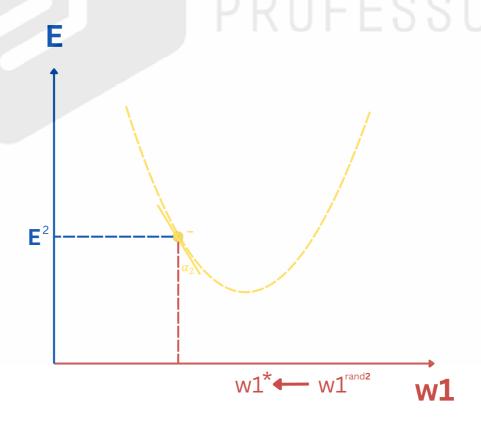
#### **Gradiente Descendente**



Corrigindo o peso w1 com k muito grande:

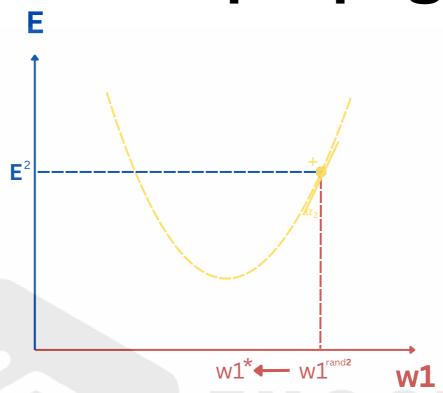
$$W_1^{new} = W_1^{old} - \left(\frac{\partial E}{\partial W_1}\right)$$

$$PRO - ESSUM$$



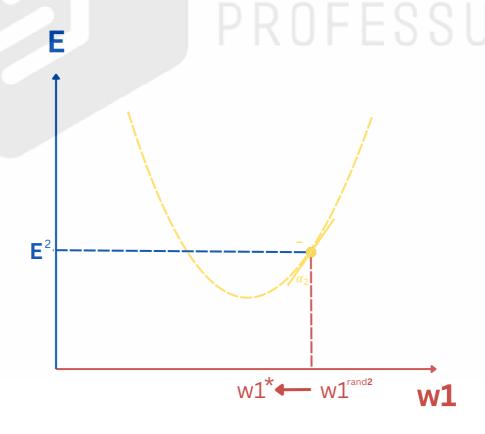
Passo Muito grande

**Gradiente Descendente** 



Corrigindo o peso w1 com com k reajustado (LR):

$$W_1^{\text{new}} = W_1^{\text{old}} - LR. \left( \frac{\partial E}{\partial W_1} \right)$$



Passo mais adequado pelo Learning Rate

**Gradiente Descendente** 

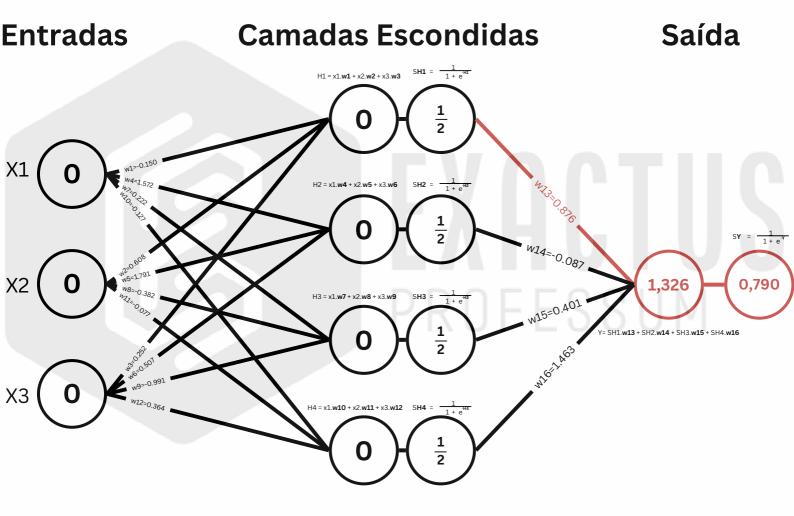
Correções de todos os pesos **Ws**:

$$\begin{split} W_1^{new} &= W_1^{old} - LR. \left(\frac{\partial E}{\partial W_1}\right) & W_9^{new} &= W_9^{old} - LR. \left(\frac{\partial E}{\partial W_9}\right) \\ W_2^{new} &= W_2^{old} - LR. \left(\frac{\partial E}{\partial W_2}\right) & W_{10}^{new} &= W_{10}^{old} - LR. \left(\frac{\partial E}{\partial W_{10}}\right) \\ W_3^{new} &= W_3^{old} - LR. \left(\frac{\partial E}{\partial W_3}\right) & W_{11}^{new} &= W_{11}^{old} - LR. \left(\frac{\partial E}{\partial W_{11}}\right) \\ W_4^{new} &= W_4^{old} - LR. \left(\frac{\partial E}{\partial W_4}\right) & W_{12}^{new} &= W_{12}^{old} - LR. \left(\frac{\partial E}{\partial W_{12}}\right) \\ W_5^{new} &= W_5^{old} - LR. \left(\frac{\partial E}{\partial W_5}\right) & W_{13}^{new} &= W_{13}^{old} - LR. \left(\frac{\partial E}{\partial W_{13}}\right) \\ W_6^{new} &= W_6^{old} - LR. \left(\frac{\partial E}{\partial W_6}\right) & W_{14}^{new} &= W_{14}^{old} - LR. \left(\frac{\partial E}{\partial W_{14}}\right) \\ W_7^{new} &= W_7^{old} - LR. \left(\frac{\partial E}{\partial W_7}\right) & W_{15}^{new} &= W_{15}^{old} - LR. \left(\frac{\partial E}{\partial W_{15}}\right) \\ W_8^{new} &= W_8^{old} - LR. \left(\frac{\partial E}{\partial W_8}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) & W_{16}^{new} &= W_{16}^{old} - LR. \left(\frac{\partial E}{\partial W_{16}}\right) \\ W_{16}^{new} &= W_{16}^{old} - LR.$$

#### LR = 0.5 Correção dos Pesos



Corrigindo Erro, em função de w13 **E(w13)** 



$$\frac{\partial E}{\partial w_{13}} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial w_{13}}$$

── Backward Propagation ← ── Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^1 * (-1) = (\mathbf{S} - \mathbf{SY})^1 * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{w_{13}}} = SH1.\,\mathbf{1} + 0 + 0 + 0 = SH1$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

A

В

(c)

**W13** 

**Gradiente** 

$$\frac{\partial E}{\partial \mathbf{w_{13}}} = 0,790 \ x \ 0,165 \ x \ 0,500 = 0,065$$

Backward Propagation ← Iteração 1

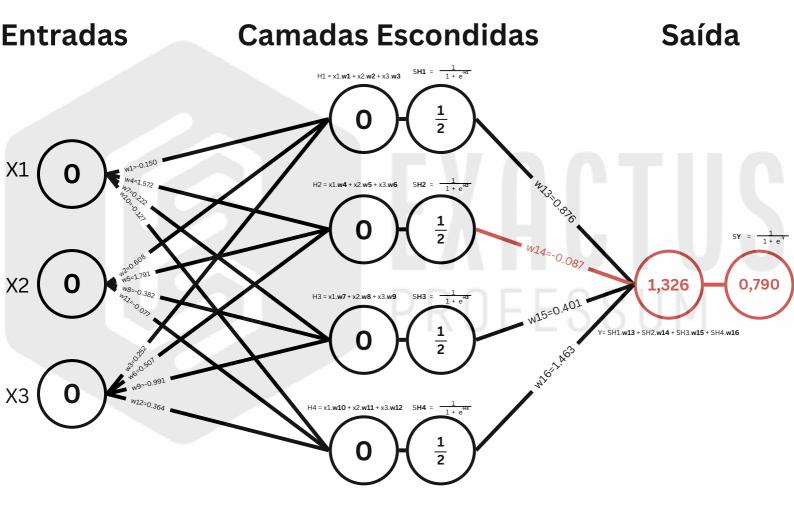
$$w_{13}^{new} = w_{13}^{old} - 0.50 x \left( \frac{\partial E}{\partial w_{13}} \right)$$

$$w_{13}^{new} = 0.876 - 0.50 * (0.065)$$

$$w_{13}^{new} = 0.843$$



Corrigindo Erro, em função de w14 **E(w14)** 



$$\frac{\partial E}{\partial \mathbf{w_{14}}} = \frac{\partial \mathbf{E}}{\partial \mathbf{SY}} x \frac{\partial \mathbf{SY}}{\partial \mathbf{Y}} x \frac{\partial \mathbf{Y}}{\partial \mathbf{w_{14}}}$$

← Backward Propagation ← Iteração 1

### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^1 * (-1) = (\mathbf{S} - \mathbf{SY})^1 * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\binom{\mathsf{c}}{\mathsf{c}}$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{w_{14}}} = 0 + SH2 \cdot \mathbf{1} + 0 + 0 = SH2$$

$$\frac{\partial E}{\partial SY} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

A

B

C

Gradiente

$$\frac{\partial E}{\partial \mathbf{w_{14}}} = 0,790 \ x \ 0,165 \ x \ 0,500 = 0,065$$

**W14** 

**←** Backward Propagation **←** 

Iteração 1



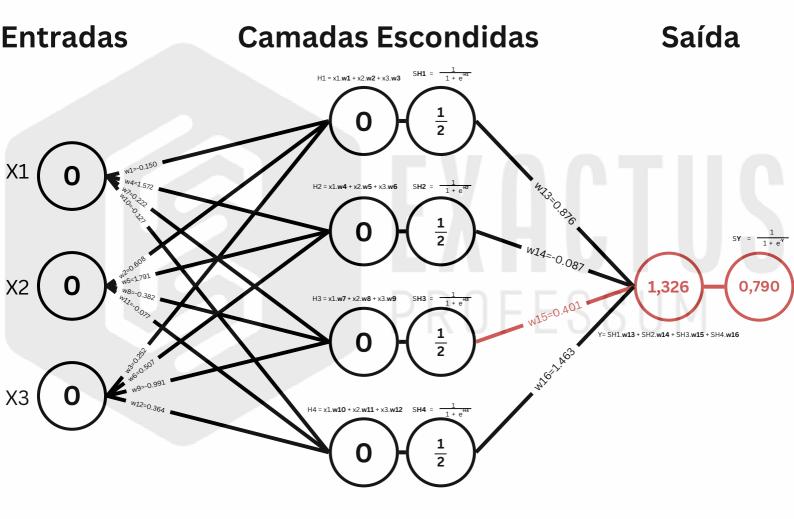
$$w_{14}^{new} = w_{14}^{old} - 0.50 x \left( \frac{\partial E}{\partial w_{14}} \right)$$

$$w_{14}^{new} = -0.087 - 0.50 * (0.065)$$

$$w_{14}^{\text{new}} = -0.119$$



Corrigindo Erro, em função de w15 **E(w15)** 



$$\frac{\partial E}{\partial \mathbf{w_{15}}} = \frac{\partial \mathbf{E}}{\partial \mathbf{SY}} x \frac{\partial \mathbf{SY}}{\partial \mathbf{Y}} x \frac{\partial \mathbf{Y}}{\partial \mathbf{w_{15}}}$$

——— Backward Propagation ←———

Iteração 1



**Iteração 1** 

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^1 * (-1) = (\mathbf{S} - \mathbf{SY})^1 * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\bigcirc$$

$$\frac{\partial Y}{\partial w_{15}} = 0 + 0 + SH3 \cdot 1 + 0 = SH3$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial w_{15}} = SH3 = 0,500$$

Gradiente

$$\frac{\partial E}{\partial w_{15}} = 0,790 \ x \ 0,165 \ x \ 0,500 = 0,065$$
 W15

Backward Propagation ←
 Iteração 1

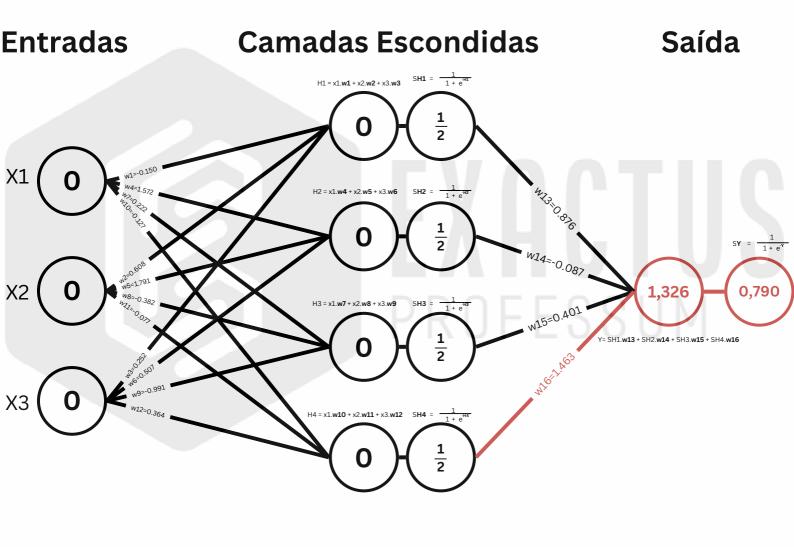
$$w_{15}^{new} = w_{15}^{old} - 0.50 x \left( \frac{\partial E}{\partial w_{15}} \right)$$

$$w_{15}^{\text{new}} = 0.401 - 0.50 * (0.065)$$

$$w_{15}^{new} = 0.368$$



Corrigindo Erro, em função de w16 **E(w16)** 



$$\frac{\partial E}{\partial \mathbf{w_{16}}} = \frac{\partial \mathbf{E}}{\partial \mathbf{SY}} x \frac{\partial \mathbf{SY}}{\partial \mathbf{Y}} x \frac{\partial \mathbf{Y}}{\partial \mathbf{w_{16}}}$$

—— Backward Propagation ←———

Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^1 * (-1) = (\mathbf{S} - \mathbf{SY})^1 * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\left( c\right)$$

$$\frac{\partial Y}{\partial w_{16}} = 0 + 0 + 0 + SH4 \cdot \mathbf{1} = SH4$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial w_{16}}$$
 = SH4 ============= 0,500

A

B

(c)

**Gradiente** 

$$\frac{\partial E}{\partial w_{16}} = 0,790 \ x \ 0,165 \ x \ 0,500 = 0,065$$
 W16

Backward Propagation +
 Iteração 1

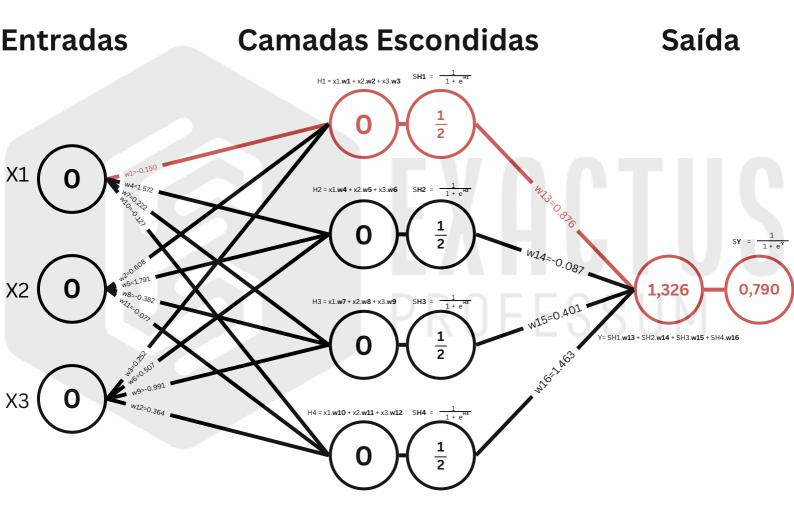
$$w_{16}^{new} = w_{16}^{old} - 0.50 x \left( \frac{\partial E}{\partial w_{16}} \right)$$

$$w_{16}^{new} = 1,463 - 0,50 * (0,065)$$

$$w_{16}^{new} = 1,430$$



Corrigindo Erro, em função de w1 **E(w1)** 



$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH1} x \frac{\partial SH1}{\partial H1} x \frac{\partial H1}{\partial w_1}$$

- Backward Propagation ← Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial E}{\partial SY} = \frac{2}{2} * (S - SY)^{1} * (-1) = (S - SY)^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH1}} = \mathbf{1}.w13 + 0 + 0 + 0$$

$$\frac{\partial SH1}{\partial H1} = SH1 * (1 - SH1)$$

$$\frac{\partial \mathbf{H1}}{\partial \mathbf{w_1}} = X1.\,\mathbf{1} + 0 + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH1}$$
 = w13 ========== 0,876

$$\frac{\partial SH1}{\partial H1}$$
 = SH1 \* (1 - SH1) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H1}{\partial w_1} = X1 = 200000$$

**Gradiente** 
$$\frac{\partial E}{\partial w_1} = 0.790 \ x \ 0.165 \ x \ 0.876 \ x \ 0.250 \ x \ 0.000 = 0.000$$

**W1** 

**Backward Propagation** 



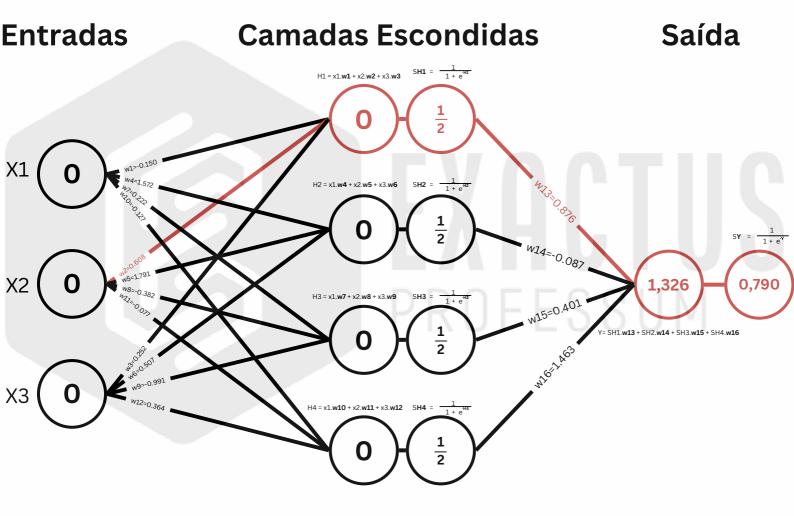
$$W_1^{new} = W_1^{old} - 0.50 \left( \frac{\partial E}{\partial W_1} \right)$$

$$W_1^{\text{new}} = -0.150 - 0.50 * (0.000)$$

$$W_1^{new} = -0.150$$



Corrigindo Erro, em função de w2 **E(w2)** 



$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH1} x \frac{\partial SH1}{\partial H1} x \frac{\partial H1}{\partial w_2}$$

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH1}} = \mathbf{1}.w13 + 0 + 0 + 0$$

$$\frac{\partial SH1}{\partial H1} = SH1 * (1 - SH1)$$

$$\frac{\partial \mathbf{H1}}{\partial \mathbf{w_2}} = 0 + X2.\,\mathbf{1} + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - 0,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH1}$$
 = w13 =========== 0,876

$$\frac{\partial SH1}{\partial H1}$$
 = SH1 \* (1 - SH1) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H1}{\partial w_2}$$
 = X2 =========== 0,000

**Gradiente** 
$$\frac{\partial E}{\partial w_2} = 0.790 \ x \ 0.165 \ x \ 0.876 \ x \ 0.250 \ x \ 0.000 = 0.000$$

W2

**Backward Propagation** 

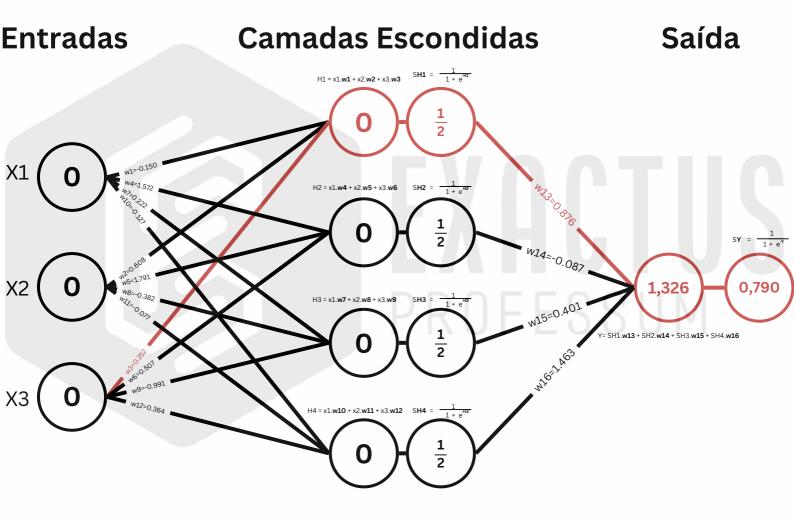
$$W_2^{\text{new}} = W_2^{\text{old}} - 0.50 \left( \frac{\partial E}{\partial W_2} \right)$$

$$W_2^{\text{new}} = 0.608 - 0.50 * (0.000)$$

$$W_2^{new} = 0,608$$



Corrigindo Erro, em função de w3 **E(w3)** 



$$\frac{\partial E}{\partial \mathbf{w_3}} = \frac{\partial \mathbf{E}}{\partial \mathbf{SY}} x \frac{\partial \mathbf{SY}}{\partial \mathbf{Y}} x \frac{\partial \mathbf{Y}}{\partial \mathbf{SH1}} x \frac{\partial \mathbf{SH1}}{\partial \mathbf{H1}} x \frac{\partial \mathbf{H1}}{\partial \mathbf{w_3}}$$

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH1}} = \mathbf{1}.w13 + 0 + 0 + 0$$

$$\frac{\partial SH1}{\partial H1} = SH1 * (1 - SH1)$$

$$\frac{\partial H\mathbf{1}}{\partial w_3} = 0 + 0 + X3.\mathbf{1}$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH1}$$
 = w13 =========== 0,876

$$\frac{\partial SH1}{\partial H1}$$
 = SH1 \* (1 - SH1) = 0,5 \* (1 - 0,5) === 0,250

**Gradiente** 
$$\frac{\partial E}{\partial w_3} = 0.790 \ x \ 0.165 \ x \ 0.876 \ x \ 0.250 \ x \ 0.000 = 0.000$$

W3



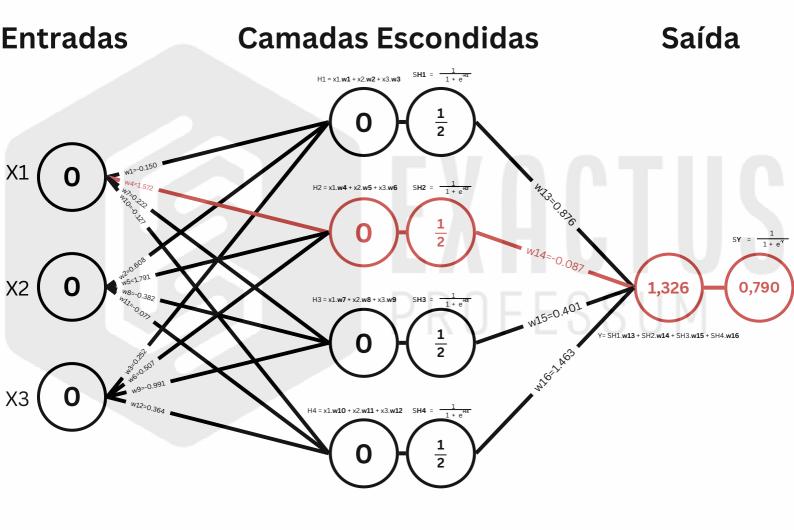
$$W_3^{new} = W_3^{old} - 0.50 \left( \frac{\partial E}{\partial W_3} \right)$$

$$W_3^{new} = 0.252 - 0.50 * (0.000)$$

$$W_3^{new}=0.252$$



Corrigindo Erro, em função de w4 **E(w4)** 



$$\frac{\partial E}{\partial w_4} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH2} x \frac{\partial SH2}{\partial H2} x \frac{\partial H2}{\partial w_4}$$

Backward Propagation ← \_\_\_\_\_\_
 Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH2}} = 0 + w14 \cdot \mathbf{1} + 0 + 0$$

$$\frac{\partial SH2}{\partial H2} = SH2 * (1 - SH2)$$

$$\frac{\partial \mathbf{H2}}{\partial \mathbf{w_4}} = X1 \cdot \mathbf{1} + 0 + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH2}$$
 = w14 =========-0,087

$$\frac{\partial SH2}{\partial H2}$$
 = SH2 \* (1 - SH2) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H2}{\partial w_4}$$
 = X1 ========== 0,000

Gradiente

$$\frac{\partial E}{\partial \mathbf{w_4}} = 0,790 \ x \ 0,165 \ x \ (-0,087) \ x \ 0,250 \ x \ 0,000 = 0,000$$

W4

**Backward Propagation** 

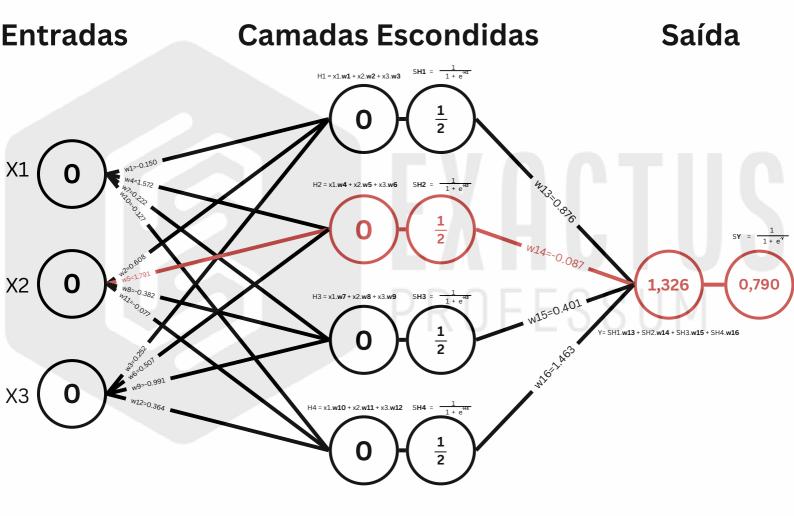
$$W_4^{new} = W_4^{old} - 0.50 \left( \frac{\partial E}{\partial W_4} \right)$$

$$W_4^{new} = 1,572 - 0.50 * (0,000)$$

$$W_4^{new} = 1,572$$



Corrigindo Erro, em função de w5 **E(w5)** 



$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH2} x \frac{\partial SH2}{\partial H2} x \frac{\partial H2}{\partial w_5}$$

- Backward Propagation ← Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH2}} = 0 + w14 \cdot \mathbf{1} + 0 + 0$$

$$\frac{\partial SH2}{\partial H2} = SH2 * (1 - SH2)$$

$$\frac{\partial H2}{\partial w_5} = 0 + X2 \cdot \mathbf{1} + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH2}$$
 = w14 ============ 0,087

$$\frac{\partial SH2}{\partial H2}$$
 = SH2 \* (1 - SH2) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H2}{\partial w_5}$$
 = X2 ========== 0,000

Gradiente

$$\frac{\partial E}{\partial \mathbf{w_5}} = 0,790 \ x \ 0,165 \ x \ 0,087 \ x \ 0,250 \ x \ 0,000 = 0,000$$

W5



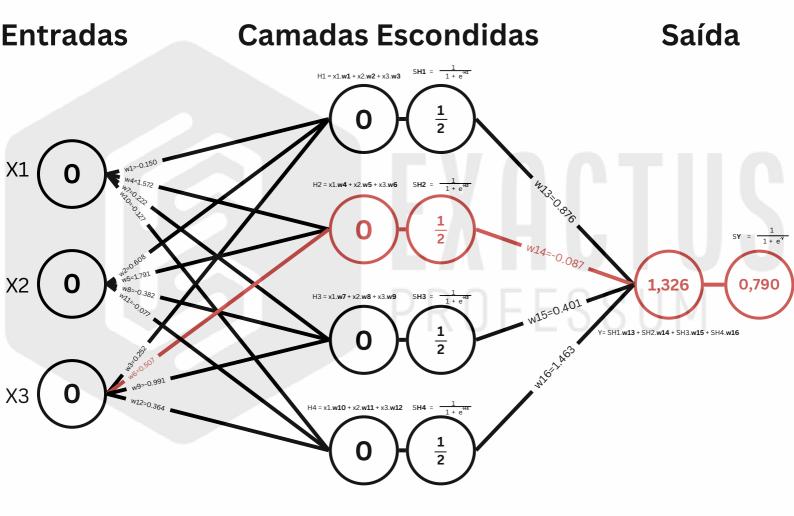
$$W_5^{\text{new}} = W_5^{\text{old}} - 0.50 \left( \frac{\partial E}{\partial W_5} \right)$$

$$W_5^{new} = 1,791 - 0.50 * (0,000)$$

$$W_5^{new} = 1,791$$



Corrigindo Erro, em função de w6 **E(w6)** 



$$\frac{\partial E}{\partial w_6} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH2} x \frac{\partial SH2}{\partial H2} x \frac{\partial H2}{\partial w_6}$$

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH2}} = 0 + w14 \cdot \mathbf{1} + 0 + 0$$

$$\frac{\partial SH2}{\partial H2} = SH2 * (1 - SH2)$$

$$\frac{\partial H2}{\partial w_6} = 0 + 0 + X3.1$$

$$\partial$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH2}$$
 = w14 ========= 0,087

$$\frac{\partial SH2}{\partial H2}$$
 = SH2 \* (1 - SH2) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H2}{\partial w_6}$$
 = X3 =========== 0,000

Gradiente

$$\frac{\partial E}{\partial \mathbf{w_6}} = 0,790 \ x \ 0,165 \ x \ 0,087 \ x \ 0,250 \ x \ 0,000 = 0,000$$

**W6** 

#### Backward Propagation •

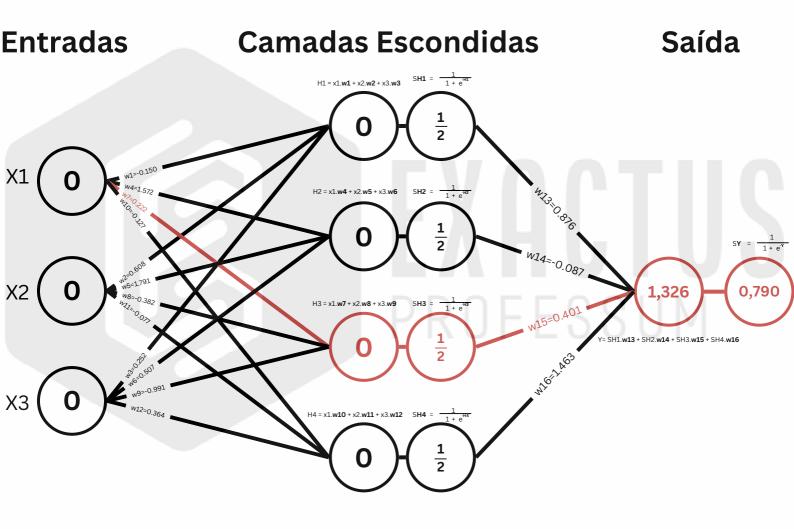
$$W_6^{\text{new}} = W_6^{\text{old}} - 0.50 \left( \frac{\partial E}{\partial W_6} \right)$$

$$W_6^{\text{new}} = 0.507 - 0.50 * (0.000)$$

$$W_6^{new} = 0.507$$



Corrigindo Erro, em função de w7 **E(w7)** 



$$\frac{\partial E}{\partial w_7} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH3} x \frac{\partial SH3}{\partial H3} x \frac{\partial H3}{\partial w_7}$$

Backward Propagation ← \_\_\_\_\_\_
 Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial Y}{\partial SH3} = 0 + 0 + w15 \cdot \mathbf{1} + 0$$

$$\frac{\partial SH3}{\partial H3} = SH3 * (1 - SH3)$$

$$\frac{\partial \mathbf{H3}}{\partial \mathbf{w_7}} = X1 \cdot \mathbf{1} + 0 + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH3}$$
 = w15 ============= 0,401

$$\frac{\partial SH3}{\partial H3}$$
 = SH3 \* (1 - SH3) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H3}{\partial w_7}$$
 = X1 =========== 0,000

Gradiente

$$\frac{\partial E}{\partial w_7} = 0,790 \ x \ 0,165 \ x \ 0,401 \ x \ 0,250 \ x \ 0,000 = 0,000$$

W7

#### **Backward Propagation**

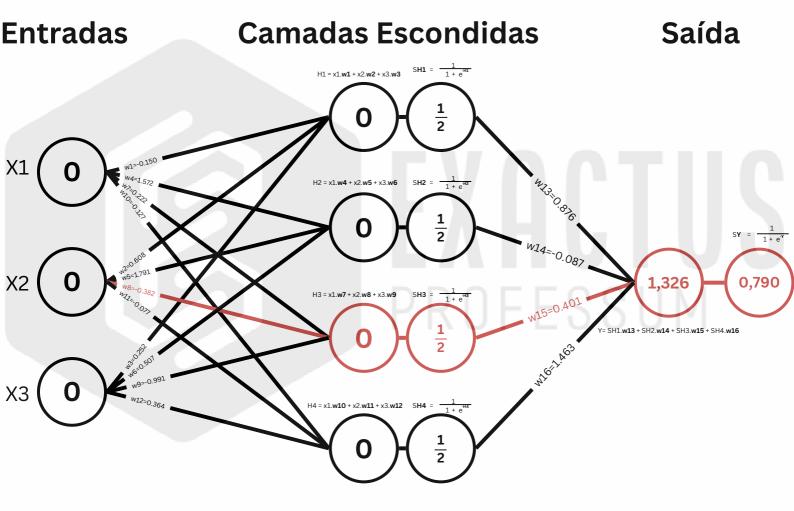
$$W_7^{\text{new}} = W_7^{\text{old}} - 0.50 \left( \frac{\partial E}{\partial W_7} \right)$$

$$W_7^{\text{new}} = 0.222 - 0.50 * (0.000)$$

$$W_7^{new} = 0.222$$



Corrigindo Erro, em função de w8 **E(w8)** 



$$\frac{\partial E}{\partial w_8} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH3} x \frac{\partial SH3}{\partial H3} x \frac{\partial H3}{\partial w_8}$$

#### **Backward Propagation**

Iteração 1

$$\frac{\partial E}{\partial SY} = \frac{2}{2} * (S - SY)^{1} * (-1) = (S - SY)^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial Y}{\partial SH3} = 0 + 0 + w15 \cdot \mathbf{1} + 0$$

$$\frac{\partial SH3}{\partial H3} = SH3 * (1 - SH3)$$

$$\frac{\partial H3}{\partial w_8} = 0 + X2 \cdot \mathbf{1} + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial SH3}{\partial H3}$$
 = SH3 \* (1 - SH3) = 0,5 \* (1 - 0,5) === 0,250

Gradiente

$$\frac{\partial E}{\partial \mathbf{w_8}} = 0.790 \ x \ 0.165 \ x \ 0.401 \ x \ 0.250 \ x \ 0.000 = 0.000$$

W8

**Backward Propagation** 

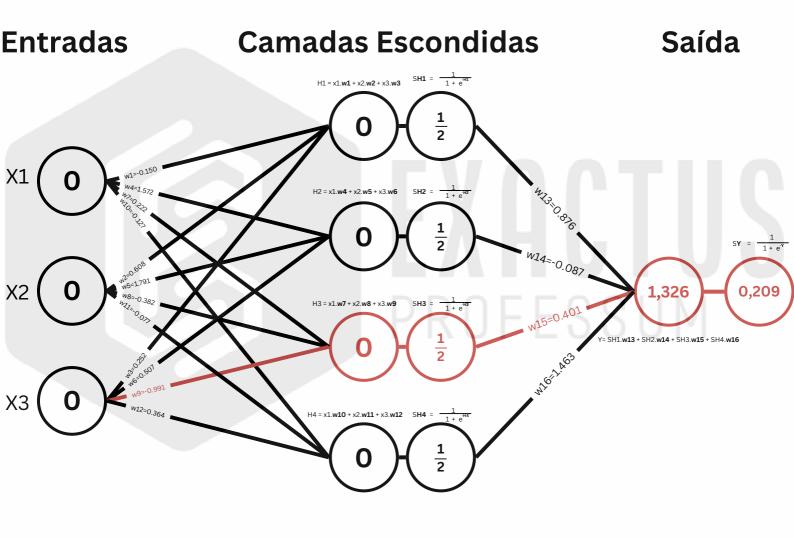
$$W_8^{new} = W_8^{old} - 0.50 \left( \frac{\partial E}{\partial W_8} \right)$$

$$W_8^{\text{new}} = -0.382 - 0.50 * (0.000)$$

$$W_8^{new} = -0.382$$



Corrigindo Erro, em função de w9 **E(w9)** 



$$\frac{\partial E}{\partial w_9} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH3} x \frac{\partial SH3}{\partial H3} x \frac{\partial H3}{\partial w_9}$$

Backward Propagation ← \_\_\_\_\_
 Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^1 * (-1) = (\mathbf{S} - \mathbf{SY})^1 * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial Y}{\partial SH3} = 0 + 0 + w15 \cdot \mathbf{1} + 0$$

$$\frac{\partial SH3}{\partial H3} = SH3 * (1 - SH3)$$

$$\frac{\partial H3}{\partial w_9} = 0 + 0 + X3.1$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial Y}{\partial SH3}$$
 = w15 =========== 0,401

$$\frac{\partial SH3}{\partial H3}$$
 = SH3 \* (1 - SH3) = 0,5 \* (1 - 0,5) === 0,250

Gradiente

$$\frac{\partial E}{\partial w_9} = 0,790 \ x \ 0,165 \ x \ 0,401 \ x \ 0,250 \ x \ 0,000 = 0,000$$

W9

**Backward Propagation** 



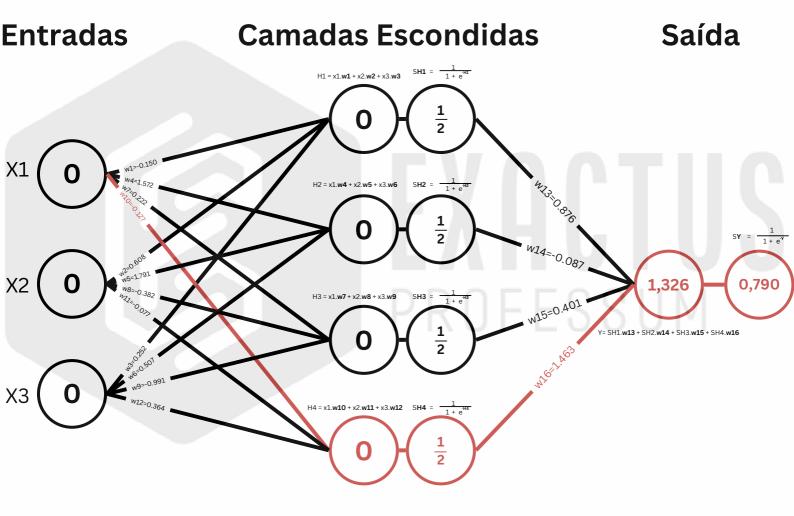
$$W_9^{\text{new}} = W_9^{\text{old}} - 0.50 \left( \frac{\partial E}{\partial W_9} \right)$$

$$W_9^{\text{new}} = -0.991 - 0.50 * (0.000)$$

$$W_9^{new} = -0.991$$



Corrigindo Erro, em função de w10 **E(w10)** 



$$\frac{\partial E}{\partial w_{10}} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH4} x \frac{\partial SH4}{\partial H4} x \frac{\partial H4}{\partial w_{10}}$$

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH4}} = 0 + 0 + 0 + w16.\mathbf{1}$$

$$\frac{\partial SH4}{\partial H4} = SH4 * (1 - SH4)$$



$$\frac{\partial \mathbf{H4}}{\partial \mathbf{w_{10}}} = X1 \cdot \mathbf{1} + 0 + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial SH4}{\partial H4}$$
 = SH4 \* (1 - SH4) = 0,5 \* (1 - 0,5) === 0,250

**Gradiente** 

$$\frac{\partial E}{\partial w_{10}} = 0,790 \ x \ 0,165 \ x \ 1,463 \ x \ 0,250 \ x \ 0,000 = 0,000$$
 **W10**

**Backward Propagation** 



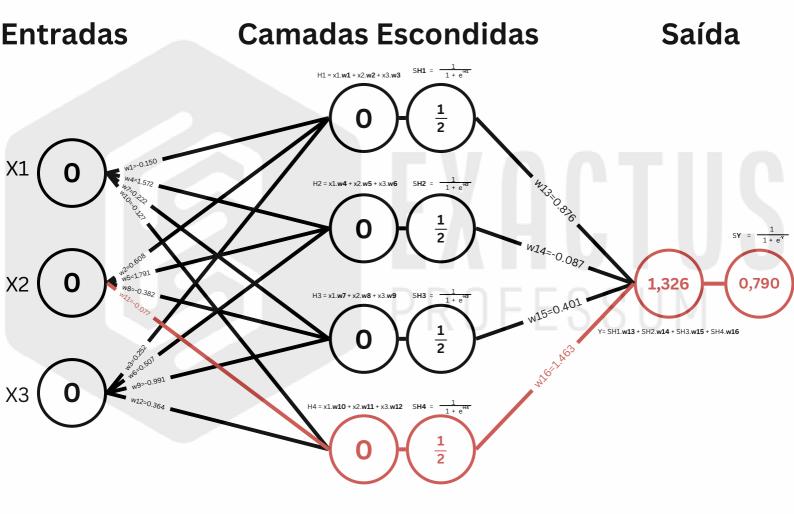
$$W_{10}^{new} = W_{10}^{old} - 0.50 \left( \frac{\partial E}{\partial W_{10}} \right)$$

$$W_{10}^{new} = -0.127 - 0.50 * (0.000)$$

$$W_{10}^{new} = -0.127$$



Corrigindo Erro, em função de w11 **E(w11)** 



$$\frac{\partial E}{\partial w_{11}} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH4} x \frac{\partial SH4}{\partial H4} x \frac{\partial H4}{\partial w_{11}}$$

- Backward Propagation ← Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial E}{\partial SY} = \frac{2}{2} * (S - SY)^{1} * (-1) = (S - SY)^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH4}} = 0 + 0 + 0 + w16.\mathbf{1}$$

$$\frac{\partial SH4}{\partial H4} = SH4 * (1 - SH4)$$

$$\frac{\partial \mathbf{H4}}{\partial \mathbf{w_{11}}} = 0 + X2 \cdot \mathbf{1} + 0$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - 0,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial SH4}{\partial H4}$$
 = SH4 \* (1 - SH4) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H4}{\partial w_{11}} = X2 = 2 = 0,000$$

**Gradiente** 

$$\frac{\partial E}{\partial w_{11}} = 0,790 \ x \ 0,165 \ x \ 1,463 \ x \ 0,250 \ x \ 0,000 = 0,000$$
 **W11**

**Backward Propagation** 



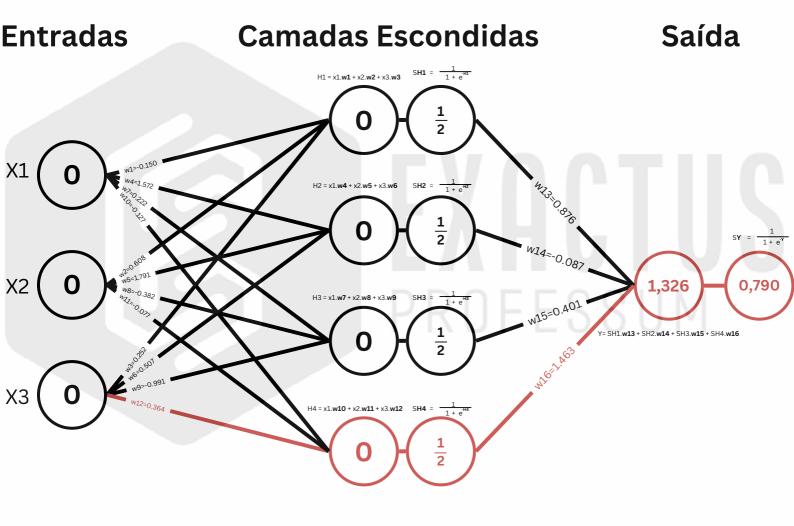
$$W_{11}^{new} = W_{11}^{old} - 0.50 \left( \frac{\partial E}{\partial W_{11}} \right)$$

$$W_{11}^{new} = -0.077 - 0.50 * (0.000)$$

$$W_{11}^{new} = -0.077$$



Corrigindo Erro, em função de w12 **E(w12)** 



$$\frac{\partial E}{\partial w_{12}} = \frac{\partial E}{\partial SY} x \frac{\partial SY}{\partial Y} x \frac{\partial Y}{\partial SH4} x \frac{\partial SH4}{\partial H4} x \frac{\partial H4}{\partial w_{12}}$$

- Backward Propagation ← Iteração 1

#### **Backward Propagation**

Iteração 1

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = \frac{2}{2} * (\mathbf{S} - \mathbf{SY})^{1} * (-1) = (\mathbf{S} - \mathbf{SY})^{1} * (-1)$$

$$\frac{\partial SY}{\partial Y} = SY * (1 - SY)$$

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{SH4}} = 0 + 0 + 0 + w16.\mathbf{1}$$

$$\frac{\partial SH4}{\partial H4} = SH4 * (1 - SH4)$$

$$\frac{\partial \mathbf{H4}}{\partial \mathbf{w_{12}}} = 0 + 0 + X3.\mathbf{1}$$

$$\frac{\partial \mathbf{E}}{\partial \mathbf{SY}} = (S - SY)^{1} * (-1) = (O - O,790) * (-1) = 0,790$$

$$\frac{\partial SY}{\partial Y}$$
 = SY \* (1 - SY) = 0,790 \* (1 - 0,790) = 0,165

$$\frac{\partial SH4}{\partial H4}$$
 = SH4 \* (1 - SH4) = 0,5 \* (1 - 0,5) === 0,250

$$\frac{\partial H4}{\partial w_{12}}$$
 = X3 ============ 0,000

Gradiente

$$\frac{\partial E}{\partial \mathbf{w_{12}}} = 0,790 \ x \ 0,165 \ x \ 1,463 \ x \ 0,250 \ x \ 0,000 = 0,000$$

**W12** 

**Backward Propagation** 

$$W_{12}^{new} = W_{12}^{old} - 0.50 \left( \frac{\partial E}{\partial W_{12}} \right)$$

$$W_{12}^{new} = 0.364 - 0.50 * (0.000)$$

$$W_{12}^{new} = 0.364$$

$$w_1 = -0.150$$
 $w_1 = -0.150$ 
 $w_2 = 0.608$ 
 $w_2 = 0.608$ 
 $w_3 = 0.252$ 
 $w_3 = 0.252$ 
 $w_4 = 1.572$ 
 $w_4 = 1.572$ 
 $w_5 = 1.791$ 
 $w_5 = 1.791$ 
 $w_6 = 0.507$ 
 $w_6 = 0.507$ 
 $w_7 = 0.222$ 
 $w_8 = -0.382$ 
 $w_9 = -0.991$ 
 $w_9 = -0.991$ 
 $w_{10} = -0.127$ 
 $w_{10} = -0.127$ 
 $w_{11} = -0.077$ 
 $w_{12} = -0.150$ 
 $w_{13} = 0.843$ 
 $w_{14} = -0.119$ 
 $w_{15} = 0.368$ 
 $w_{15} = 0.368$ 
 $w_{16} = 1.463$ 
 $w_{16} = 1.430$ 

Pesos Anteriores → Pesos Atualizados

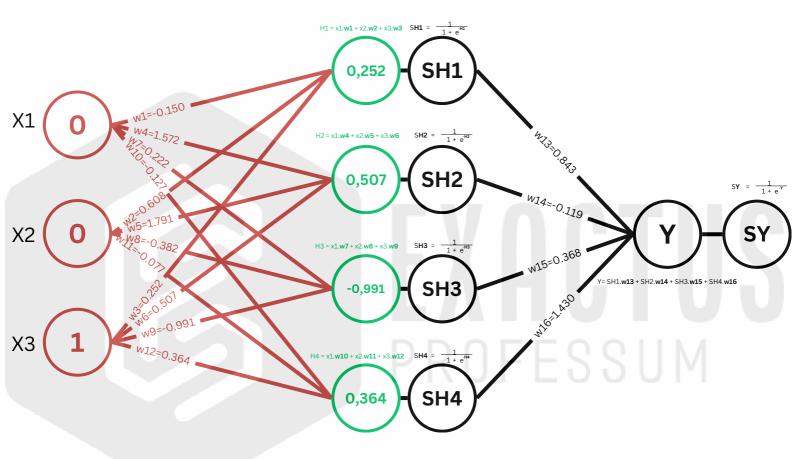
#### **→Forward Propagation**

Iteração 2

**Entradas** 

#### **Camadas Escondidas**

Saída



$$H1 = -0.0,150 + 0.0,608 + 1.0,252 = 0,252$$

$$H2 = + 0 . 1,572 + 0 . 1,791 + 1 . 0,507 = 0,507$$

$$H4 = -0.0,127 - 0.0,077 + 1.0,364 = 0,364$$

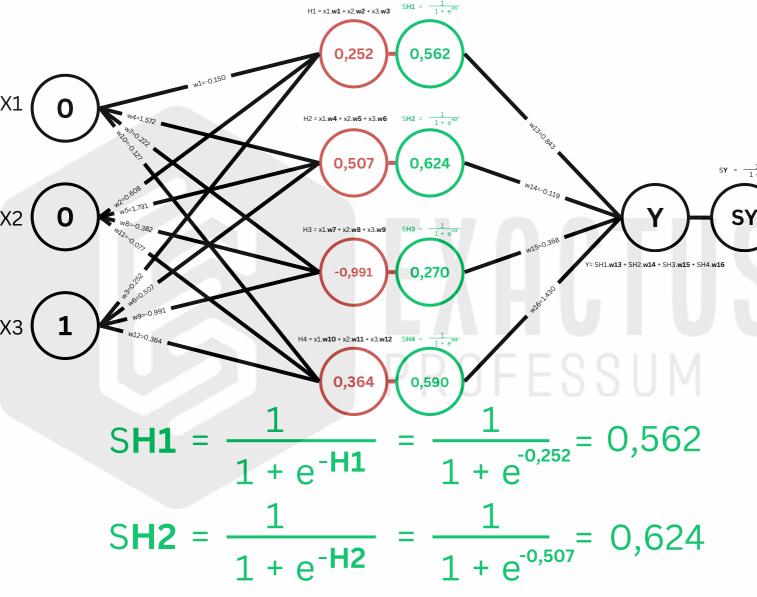
#### →Forward Propagation

Iteração 2

#### Forward Propagation

Iteração 2

#### **Entradas Camadas Escondidas** Saída 0,562



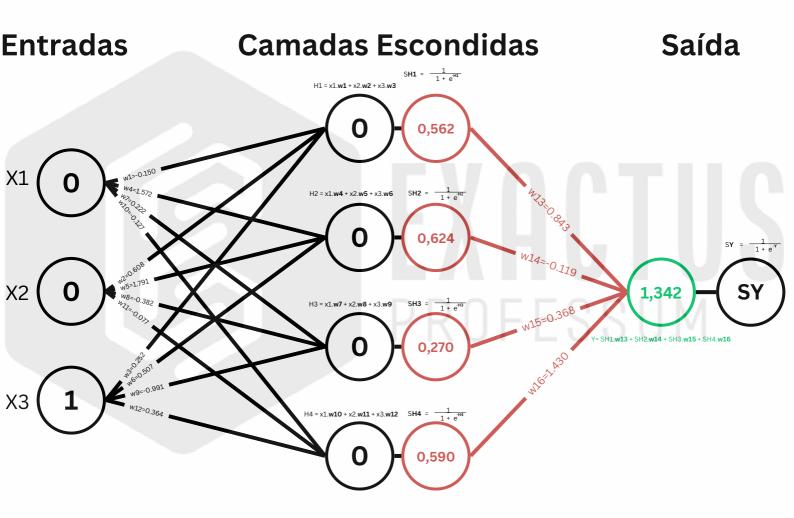
SH3 = 
$$\frac{1}{1 + e^{-H3}} = \frac{1}{1 + e^{0.991}} = 0,270$$

SH4 = 
$$\frac{1}{1 + e^{-H4}}$$
 =  $\frac{1}{1 + e^{-0.364}}$  = 0,590

Forward Propagation

Iteração 2

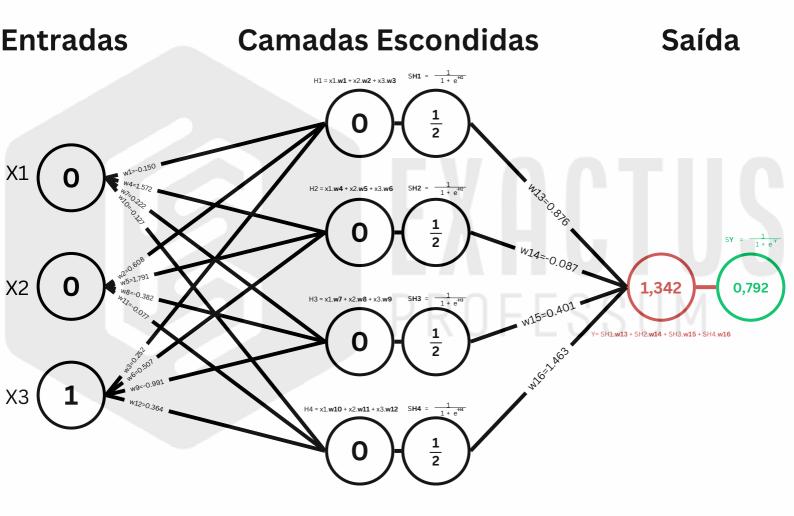
→ Forward Propagation → Iteração 2



 $Y = 0.562 \cdot 0.843 - 0.624 \cdot 0.119 + 0.270 \cdot 0.368 + 0.590 \cdot 1.430 = 1.342$ 

Forward Propagation

Iteração 2



$$SY = \frac{1}{1 + e^{-Y}} = \frac{1}{1 + e^{-1,342}} = 0.792$$

→Forward Propagation ——— Iteração 2

#### Cálculo do Erro Iteração 2

Erro = 
$$0.5 * (S - SY)^2$$

S: Saída Esperada

SY: Saída Calculada

	X1	X2	ХЗ	S
Lição 2	0	0	1	1



Erro = 
$$0.5 * (1 - 0.792)^2$$

Erro = 
$$0.5 * (0.208)^2$$

$$Erro = 0.5 * 0.043$$

$$Erro = 0,021$$

#### Cálculo do Erro Iteração 2

## ◆ Backward Propagation ◆ Iteração 2

E(w1, w2, w3, w4, w5, w6, w7, w8, w9, w10, w11, w12, w13, w14, w15 e w16)

O

**Backward** Propagation

consiste

em

realizar a **correção** dos

pesos (variáveis)

w1, w2, w3, w4, w5, w6, w7, w8, w9, w10, w11, w12, w13, w14, w15 e w16

para

minimizar

0

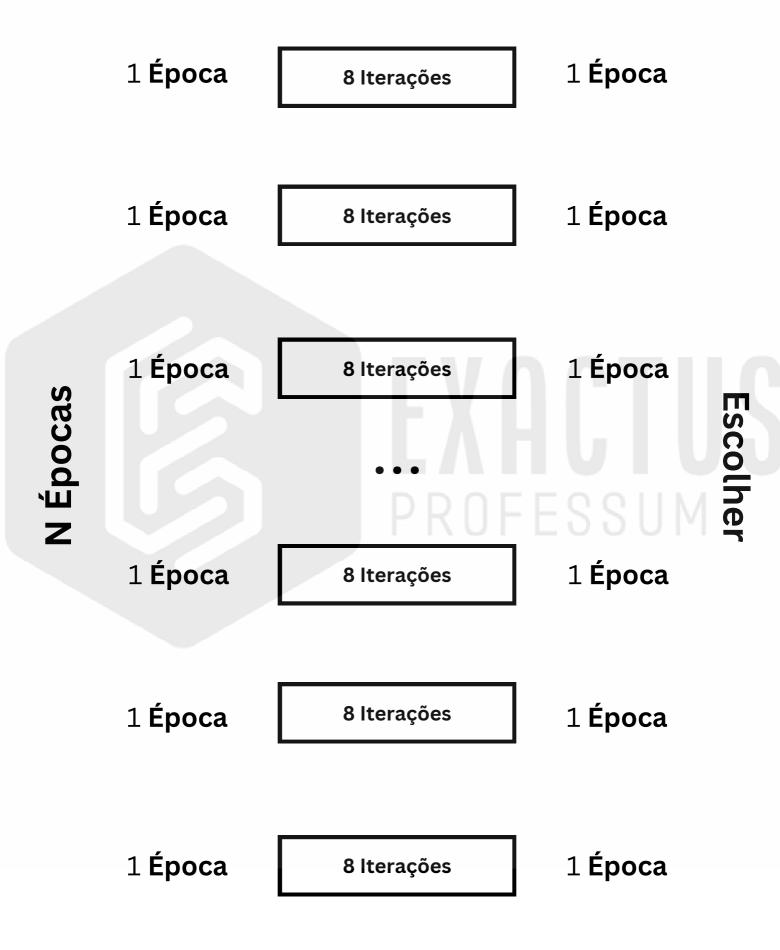
**Erro** 

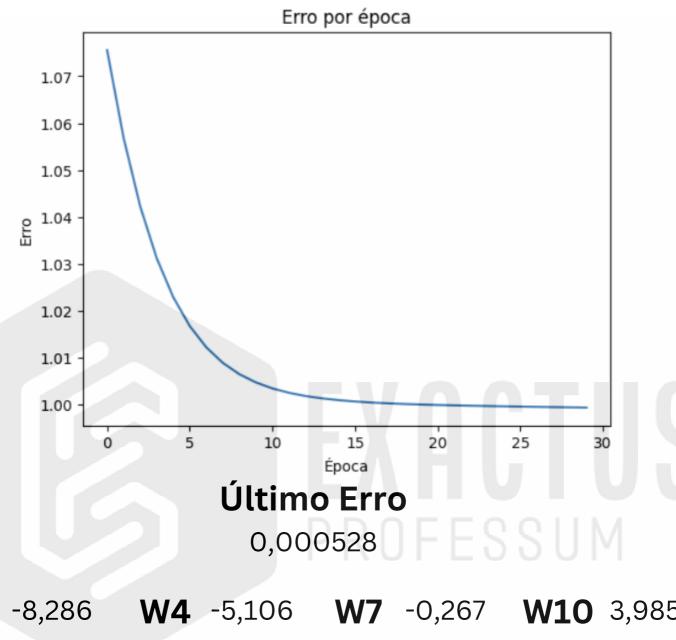
Erro = 0,5 \* (S 
$$-\frac{w_1}{w_{15}}$$
  $\frac{w_2}{S}$   $\frac{w_3}{Y}$   $\frac{w_4}{w_5}$   $\frac{w_5}{w_6}$  )<sup>2</sup>

# ◆ Backward Propagation ◆ Iteração 2

Iteração 8

8 Iterações





W1	-8,286	W4	-5,106	W7	-0,267	W10	3,985
W2	4,502	W5	9,349	W8	-4,102	W11	4,683
W3	3,904	W6	-5,191	W9	-0,294	W12	-8,395
			W13	10,087			

W14 -15,451W15 -12,697W16 10,165