Suppose we have the distribution

$$P(x) = \lambda e^{-\lambda x} \quad \text{Exponential Distribution}$$

$$x \in [0, \infty]$$

then $$H = -\int_0^\infty \lambda e^{-\lambda x} \ln(\lambda e^{-\lambda x})\, dx$$

$$H = \int_0^\infty \lambda^2 x e^{-\lambda x}\, dx - \lambda \ln\lambda \int_0^\infty e^{-\lambda x}\, dx$$

$$= \underbrace{\lambda \int_0^\infty x P(x)\, dx}_{\text{Expectation of } P(x)} - \ln\lambda \underbrace{\int_0^\infty P(x)\, dx}_{1}$$

$$= \lambda\left(\frac{1}{\lambda}\right) - \ln\lambda \Rightarrow H = 1 - \ln(\lambda)$$