

Ans to the Q No 1

We know, Entropy $H(Y)$ of a random variable Y

$$H(Y) = - \sum_{i=1}^k P(Y=y_i) \log_2 P(Y=y_i)$$

$$\text{Here, } P(\text{Taste} = \text{Meh}) = \frac{5}{10} = \frac{1}{2}$$

$$P(\text{Taste} = \text{Yummy}) = \frac{5}{10} = \frac{1}{2}$$

So, initial entropy of target variable variable taste

$$\begin{aligned} H(\text{Taste}) &= -P(\text{Taste} = \text{Meh}) \log_2 P(\text{Taste} = \text{Meh}) \\ &\quad - P(\text{Taste} = \text{Yummy}) \log_2 P(\text{Taste} = \text{Yummy}) \\ &= -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \\ &= -\log_2\left(\frac{1}{2}\right) = -(-1) = 1 \end{aligned}$$

Ans to the Q No-2

$$IG(\text{Visual defects}) = H(\text{Taste}) - H(\text{Taste} | \text{Visual defects})$$

Now, conditional entropy of $H(Y|X)$ of a random variable Y conditioned on a random variable X

$$H(Y|X) = - \sum_{j=1}^v P(X=x_j) \sum_{i=1}^k P(Y=y_i | X=x_j) \log_2 P(Y=y_i | X=x_j)$$

$$\text{Here } P(\text{Visual defects} = \text{Some}) = \frac{3}{10}$$

$$P(\text{Visual defects} = \text{None}) = \frac{4}{10} = \frac{2}{5}$$

$$P(\text{Visual defects} = \text{Many}) = \frac{3}{10}$$

$$H(Taste | Visual defects) \\ = -P(Visual defects = some) \left[P(Taste = Meh | Visual defects = some) \log_2 P(Taste = Meh) \right]$$

For convenience, we denote

Taste by T, visual defects by VD, Meh by Me, Yummy by Y, Some by S, None by N, Many by M

$$H(T | VD) = -P(VD = S) \left[P(T = Me | VD = S) \log_2 P(T = Me | VD = S) + P(T = Y | VD = S) \log_2 P(T = Y | VD = S) \right] - P(VD = N) \left[P(T = Me | VD = N) \log_2 P(T = Me | VD = N) + P(T = Y | VD = N) \log_2 P(T = Y | VD = N) \right] - P(VD = M) \left[P(T = Me | VD = M) \log_2 P(T = Me | VD = M) + P(T = Y | VD = M) \log_2 P(T = Y | VD = M) \right]$$

$$= -\frac{3}{10} (1 \log_2 1 + 0 \log_2 0) - \frac{2}{5} \left(\frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2} \right) - \frac{3}{10} (0 \log_2 0 + 1 \log_2 1)$$

$$= 0 + \frac{2}{5} + 0 = 0.4$$

Visual Defects		
Some	None	Many
Meh: 3 Yummy: 0	Meh: 2 Yummy: 2	Yummy: 3 Meh: 0

$$\text{Therefore, } I(G) I_G(\text{Visual defects}) = 1 - 0.4 = 0.6$$

Ans to the Q No-3

$H(Taste | Visual Defects == Some)$

$$= -P(T=Me | VD=S) \log_2 P(T=Me | VD=S)$$

$$- P(T=Y | VD=S) \log_2 P(T=Y | VD=S)$$

$$= -1 \log_2 1 - 0 \log_2 0 = 0$$

$H(Taste | Visual Defects == None)$

$$= -P(T=Me | VD=N) \log_2 P(T=Me | VD=N)$$

$$- P(T=Y | VD=N) \log_2 P(T=Y | VD=N)$$

$$= -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = -\log_2 \frac{1}{2}$$

$$= -(-1)$$

$$= 1$$