Ravi Raju
MA 521
Homework #9
4/18/2019

Chapter 7: 2.16, 2.17, 3.6, 3.7

Exercise 2.16 For each of the following sequences $(a_n)_{n=1}^{\infty}$, prove whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges. (If it converges, you do not need to find the limit.)

1. $a_n = \sqrt{n+1} - \sqrt{n}$.
2. $a_n = \frac{\sqrt{n+1}-\sqrt{n}}{n}$.
3. $a_n = (\sqrt[n]{n} - 1)^n$.
4. $a_n = \frac{(-1)^n}{\log n}$ for $n \geq 2$ (and $a_1 = 0$).

1.
2.
3.
4.

$\blacksquare$

Exercise 2.17 Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{1+z^n}.$$

Determine which values of $z \in \mathbb{R} (z \neq -1)$ make the series convergent and which make it divergent. Prove your answers are correct.

$\blacksquare$

Exercise 3.6 Assume that $\sum_{n=1}^{\infty} a_n$ converges absolutely. Prove that $\sum_{n=1}^{\infty} \frac{\sqrt{|a_n|}}{n}$ converges. (Hint: Use the inequality $2AB \leq A^2 + B^2$, valid for any real numbers A, B).

$\blacksquare$

Exercise 3.7

1. Assume that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge absolutely. Prove that $\sum_{n=1}^{\infty}(a_n + b_n)$ absolutely as well.
2. Assume that $\sum_{n=1}^{\infty} a_n$ converges. Does it follow that $\sum_{n=1}^{\infty} a_2 n$ converges? Give a proof or counterexample.
3. Assume that $\sum_{n=1}^{\infty} a_n$ converges absolutely. Does it follow that $\sum_{n=1}^{\infty} a_2 n$ converges absolutely? Give a proof or counterexample.

1.

2.

3.

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