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MA 521
Homework #5
2/28/2018

1.25, 1.26, 1.28, 1.30, 1.31, 2.7, 2.9, 2.16, 2.18.

Exercise 1.25 Let (X, d) be a metric space. Let E and Y be subsets of X such that $E \subset Y$. Prove that

$$\mathrm{Cl}_Y(E) = \mathrm{Cl}_X(E) \cap Y.$$

$\mathrm{Cl}_Y(E) = E \cup \mathrm{Lim}_Y(E) = E \cup (\mathrm{Lim}_X(E) \cap Y)$ (by Exercise 1.10) $= (E \cup \mathrm{Lim}_X(E)) \cap Y = \mathrm{Cl}_X(E) \cap Y$. ■

Exercise 1.26 Let (X, d) be a metric space.

1. Prove that for any collection $\mathbb{E}$ of subsets of X, we have

$$\bigcup_{E \in \mathbb{E}} \overline{E} \subset \overline{\bigcup_{E \in \mathbb{E}} E}$$

   and equality holds if $\mathbb{E}$ is finite.

2. Prove that for any collection $\mathbb{E}$ of subsets of X, we have

$$\bigcap_{E \in \mathbb{E}} \overline{E} \supset \overline{\bigcap_{E \in \mathbb{E}} E}$$

   and equality holds if $\mathbb{E}$ is finite.

3. Give examples that demonstrate that equality might fail in part (1) is $\mathbb{E}$ is not finite, and equality might fail in part (2) even if $\mathbb{E}$ is finite.

1. Let $x \in \bigcup_{E \in \mathbb{E}} \overline{E}$. So, for some E, $x = E \cup \mathrm{Lim}_X(E)$. $\overline{\bigcup_{E \in \mathbb{E}}(E)} = [\bigcup_{E \in \mathbb{E}} E] \cup [\mathrm{Lim}_X(\bigcup_{E \in \mathbb{E}})]$. So, $E \subset \bigcup_{E \in \mathbb{E}} E$ and $\mathrm{Lim}_X(E) \subset \mathrm{Lim}_X(\bigcup_{E \in \mathbb{E}})$ by Exercise 1.9. So, $x \in \overline{\bigcup_{E \in \mathbb{E}} E}$. For $\supset$, let $K = \bigcup_{E \in \mathbb{E}} \overline{E}$. Since K is union of finite number of closed sets, K is a closed set. All $x \in \bigcap_{E \in \mathbb{E}}(E) \rightarrow x \in \mathrm{Cl}_X(E)$ and so $x \subset K$. Thus, $\bigcap_{E \in \mathbb{E}}(E) \subset K$.
2.
3.

■