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MA 521
Homework #4
2/28/2018

1.9, 1.10, 1.13, 1.14, 1.15, 1.21, 1.24

Exercise 1.9 Let E_1 and E_2 be subsets of a metric space (X, d). Prove that

$$\text{Lim}_X(E_1 \cup E_2) = \text{Lim}_X(E_1) \cup \text{Lim}_X(E_2).$$

For $\subset$, assume $x \in \text{Lim}_X(E_1 \cup E_2)$ and U is a neighborhood of x in X. So, $\emptyset \neq U \cap (E_1 \cup E_2) \setminus \{x\}$. $U \cap [(E_1 \setminus \{x\}) \cup (E_2 \setminus \{x\})] = [U \cap (E_1 \setminus \{x\}) \cup U \cap (E_2 \setminus \{x\})] = \text{Lim}_X(E_1) \cup \text{Lim}_X(E_2)$. For $\supset$, using Proposition 1.8, $E_1 \subset E_1 \cup E_2 \rightarrow \text{Lim}_X(E_1) \subset \text{Lim}_X(E_1 \cup E_2)$. Similarly, $E_2 \subset E_1 \cup E_2 \rightarrow \text{Lim}_X(E_2) \subset \text{Lim}_X(E_1 \cup E_2)$. So, $\text{Lim}_X(E_1) \cup \text{Lim}_X(E_2) \subset \text{Lim}_X(E_1 \cup E_2)$. So, $\text{Lim}_X(E_1 \cup E_2) = \text{Lim}_X(E_1) \cup \text{Lim}_X(E_2)$. ■

Exercise 1.10 Let (X, d) be a metric space, and assume $E \subset Y \subset X$. Prove that

$$\text{Lim}_Y(E) = \text{Lim}_X(E) \cap Y.$$

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Exercise 1.13 Let (X, d) be a metric space, and assume $Y \subset X$. Let $(x_n)_{n=1}^{\infty}$ be a sequence in Y and let x be a point of X. Prove that the following two statements are equivalent:

1. $x_n \rightarrow x$ in X, and $x \in Y$.
2. $x_n \rightarrow x$ in Y.

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Exercise 1.14 Let (X, d) be a metric space, and let $(x_n)_{n=1}^{\infty}$ be a sequence in Y and let x be a point of X. Prove that the following two statements are equivalent:

1. $x_n \rightarrow x$ in X
2. For every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $n \geq N$ implies $x_n \in B_X(x, \epsilon)$(i.e.$d(x, x_n), \epsilon$).
3. $d(x, x_n) \rightarrow 0$ as $n \rightarrow \infty$.

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Exercise 1.15 Let $(s_n)_{n=1}^{\infty}$ and $(t_n)_{n=1}^{\infty}$ be sequences of real numbers, with $t_n > 0$ for each $n \in \mathbb{N}$. Assume that $t_n \rightarrow 0$ as $n \rightarrow \infty$.

- Prove that if $|s_n - s| < t_n$ for all $n \in \mathbb{N}$, then $s_n \rightarrow s$ as $n \rightarrow \infty$.
- Prove that if $\frac{1}{n} \rightarrow 0$ as $n \rightarrow \infty$.

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Exercise 1.21 Let (X, d) be a metric space, and let E be a subset of X. Prove that $\mathrm{Lim}_X(E)$ is a closed set of X.

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Exercise 1.24 Let (X, d) be a metric space, and let E be a subset of X. Prove that

$$X \setminus \mathrm{Cl}_X(E) = \mathrm{Int}_X(X \setminus E)$$

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