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MA 521
Homework #4
2/28/2018

1.9, 1.10, 1.13, 1.14, 1.15, 1.21, 1.24

Exercise 1.9 Let E_1 and E_2 be subsets of a metric space (X, d). Prove that

$$\mathrm{Lim}_X(E_1 \cup E_2) = \mathrm{Lim}_X(E_1) \cup \mathrm{Lim}_X(E_2).$$

For $\subset$, assume $x \in \mathrm{Lim}_X(E_1 \cup E_2)$ and U is a neighborhood of x in X. So, $\emptyset \neq U \cap (E_1 \cup E_2) \setminus \{x\}$. $U \cap [(E_1 \setminus \{x\}) \cup (E_2 \setminus \{x\})] = [U \cap (E_1 \setminus \{x\}) \cup U \cap (E_2 \setminus \{x\})] = \mathrm{Lim}_X(E_1) \cup \mathrm{Lim}_X(E_2)$. For $\supset$, using Proposition 1.8, $E_1 \subset E_1 \cup E_2 \rightarrow \mathrm{Lim}_X(E_1) \subset \mathrm{Lim}_X(E_1 \cup E_2)$. Similarly, $E_2 \subset E_1 \cup E_2 \rightarrow \mathrm{Lim}_X(E_2) \subset \mathrm{Lim}_X(E_1 \cup E_2)$. So, $\mathrm{Lim}_X(E_1) \cup \mathrm{Lim}_X(E_2) \subset \mathrm{Lim}_X(E_1 \cup E_2)$. So, $\mathrm{Lim}_X(E_1 \cup E_2) = \mathrm{Lim}_X(E_1) \cup \mathrm{Lim}_X(E_2)$. ■

Part A: Problem C We defined addition of rational numbers in terms of representatives: $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$. Show that the addition of rational numbers is well-defined.

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