Darion Question

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1 Introduction

Much to the chagrin of the group, Darion asked the question:

> You have two buckets of wine, one contains 0.5 liter red wine and the other contains 0.5 liters white wine (equal volumes). A man takes a full cup from the red wine and pours it into the bucket white wine. He mixes the solution and proceeds to take a cup from the mixed solution and pours it back into the red wine bucket.
>
> Is there more red wine in the white wine or is there more white wine in the red wine?

In this document, I will attempt to find a general solution to this problem.

2 Solution

Solution, hey get it?!

Let r and w be the amounts of red and white wine, and let R and W indicate the red and white buckets:

$$R \equiv r \quad (1) \qquad\qquad W \equiv w \quad (2)$$

Let p be the percentage of buckets that fits in our cup. We start by removing pr from the red wine and adding it to the white wine:

$$R \equiv (1-p)r \quad (3) \qquad\qquad W \equiv w + pr \quad (4)$$

Now we need to move wine from the white bucket back to the red bucket; the percentage that needs to be moved is:

$$p_1 = \frac{p}{1+p} \tag{5}$$

So our buckets now contain:

$$R \equiv (1-p)r + p_1(w+pr) \tag{6}$$

$$W \equiv w + pr - p_1(w+pr) \tag{7}$$

$$R \equiv r - pr + p_1(w+pr) \tag{8}$$

$$W \equiv w + pr - p_1(w+pr) \tag{9}$$

Solving for red:

$$R \equiv r - pr + p_1(w+pr) \tag{10}$$

Substituting for p_1:

$$R \equiv r - pr + \frac{p}{1+p}(w+pr) \tag{11}$$

$$R \equiv r - pr + \frac{pw + p^2 r}{1+p} \tag{12}$$

Multiplying the two left terms by $(1+p)/(1+p)$

$$R \equiv \frac{(1+p)(r-pr) + pw + p^2 r}{1+p} \tag{13}$$

$$R \equiv \frac{r + pr - pr - p^2 r + pw + p^2 r}{1+p} \tag{14}$$

$$R \equiv \frac{r + pw}{1+p} \tag{15}$$

Solving for white:

$$W \equiv w + pr - p_1(w+pr) \tag{16}$$

Substituting for p_1:

$$W \equiv w + pr - \frac{p}{1+p}(w+pr) \tag{17}$$

Multiplying the two left terms by $(1+p)/(1+p)$

$$W \equiv \frac{(1+p)(w+pr) - p(w+pr)}{1+p} \tag{18}$$

$$W \equiv \frac{w + pr + pw + prw - pw - prw}{1+p} \tag{19}$$

$$W \equiv \frac{w + pr}{1+p} \tag{20}$$

Yielding:

$$R \equiv \frac{r + pw}{1 + p} \tag{21}$$

$$W \equiv \frac{w + pr}{1 + p} \tag{22}$$

Since we started with the same amounts for r and w, we can see that both buckets now contain the opposite amounts of red and white wine. If this initial amount is I, the amount of original wine, O, and foreign wine, F in each bucket after this is over is given by:

$$O = \frac{I}{1 + p} \tag{23}$$

$$F = \frac{Ip}{1 + p} \tag{24}$$

We started with $I = 500ml$, with a cup size of 1 cup, or $236.588ml$, so $p = 0.473176$ therefore each bucket has:

$$O = \frac{500ml}{1 + 0.473176} = 339.402759752ml \tag{25}$$

$$F = \frac{500 \cdot 0.473176}{1 + 0.473176} = 160.597240248ml \tag{26}$$

So the original red bucket now contains $339ml$ of red wine and $161ml$ of white wine, making it a very fine blend indeed.