

2020 INFORMS Demo Formulation

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1 Data

1.1 Sets

- Let \mathcal{T} be the set of tasks that need to be completed.
- Let \mathcal{G} be the set of resource groups.
- Let \mathcal{R} be the set of resources that can complete the tasks.
- Let \mathcal{Q}_r be the set of tasks that resource $r \in \mathcal{R}$ can perform.
- Let \mathcal{P}_t be the set of resources that a task $t \in \mathcal{T}$ can be assigned to.
- Let \mathcal{S}_g be the set of resources that belong to resource group $g \in \mathcal{G}$.

1.2 Data Inputs

- Let $\eta_t \geq 0$ be the number of hours for a task $t \in \mathcal{T}$.
- Let $\lambda_r \geq 0$ be the cost per hour for a resource $r \in \mathcal{R}$.
- Let $\gamma_r \geq 0$ be the number of hours that a resource $r \in \mathcal{R}$ can work in one day.
- Let $\alpha_g \geq 0$ be the minimum number of tasks, $t \in \mathcal{T}$, that a resource group $g \in \mathcal{G}$ can work in one day.
- Let $\beta_g \geq 0$ be the maximum number of tasks, $t \in \mathcal{T}$, that a resource group $g \in \mathcal{G}$ can work in one day.

1.3 Data Transformations

- Let θ_{rt} be the cost for a resource $r \in \mathcal{R}$ performing a task $t \in \mathcal{T}$.

$$\theta_{rt} = \eta_t \cdot \lambda_r \quad \forall r \in \mathcal{R}, t \in \mathcal{T}$$

2 Decision Variables

- Let $x_{rt} \in \{0, 1\}$ be one if the resource $r \in \mathcal{R}$ is assigned to perform task $t \in \mathcal{T}$, 0 otherwise.
- Let $y_g \in \mathbb{Z} \geq 0$ be the number of tasks assigned to resources in $g \in \mathcal{G}$.

3 Constraints

3.1 Assign Each Task to only One Resource

Each task can only be assigned to be performed one task in the model.

$$\sum_{r \in \mathcal{R}} x_{rt} = 1 \quad \forall t \in \mathcal{T}$$

3.2 Ensure Each Resource Stays within their Allowed Hours

Each resource can only be assigned tasks whose total time must be no more than their maximum hours for a day.

$$\sum_{t \in \mathcal{T}} \eta_t \cdot x_{rt} \leq \gamma_r \quad \forall r \in \mathcal{R}$$

3.3 Count Tasks Assigned to Resource Group

Determine the number of tasks assigned to resources in each resource group

$$\sum_{r \in \mathcal{S}_g} \sum_{t \in \mathcal{Q}_r} x_{rt} = y_g \quad \forall g \in \mathcal{G}$$

3.4 Resource Groupss are Task Count Limited

3.4.1 Ensure Resource Group Meets Minimum Task Processing Ensure each resource group is only scheduled up to its maximum number of tasks

$$y_g \geq \alpha_g \quad \forall g \in \mathcal{G}$$

3.4.2 Ensure Resource Group Meets Maximum Task Processing Ensure each resource group is only scheduled up to its maximum number of tasks

$$y_g \leq \beta_g \quad \forall g \in \mathcal{G}$$

4 Objective

The objective function for the model is to minimize the cost of assigning the resources to tasks.

$$\text{Minimize } \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} \theta_{rt} \cdot x_{rt}$$