Exploring the “Edge of Chaos” in 1D Cellular Automata

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Introduction

In 1984, Stephen Wolfram published his influential paper “University and Complexity in Cellular Automata”. In this paper he provides a classification of the behaviors of one-dimensional cellular automata. His four-part classification is as follows:

- **Class I:** Evolution to a fixed, homogeneous state
- **Class II**: Evolution to a simple, periodic structures
- **Class III:** Evolution into chaotic, aperiodic patterns
- **Class IV:** Evolution into complex, localized structures of varying length

Out of these four classifications, Class IV behavior is the most interesting and useful. Many things in the universe from life to physics closely follows a behavior similar to that of Class IV. Therefore, focusing on this level of behavior would prove most beneficial in the processes of understanding, simulating, and replicating biological systems. For this project, we will be examining the Wolfram Class IV behavior in hopes to better understand what conditions will lead to the spontaneous emergence of a complex dynamics of information that evolves to ultimately dominate the behavior of a one-dimensional cellular automata.

After executing 50 table walkthroughs – each with 12 overall steps – we have a total of 600 available CAs to classify and observe. Each step within a walkthrough has separate, calculated parameter values. In this instance, we will be investigating the lambda and entropy values of each CA. The parameter lambda is the measurement of randomness in a CA. The lower the value of lambda, the less random a given CA is – and vice versa. The parameter entropy, in relation to cellular automata, measures the overall uniformity in states of the cells within a CA. Higher entropy translates to more unpredictability or chaos. Lower entropy results in higher levels of predictability and uniformity. Lambda and entropy are both calculated in two ways – **complete (λ)/(H) and totalistic (λ_T)/(H_T).** The formulas utilized in calculating entropy and lambda are as follows:

$$H = -\sum_s p_s \lg p_s \qquad \lambda = \frac{T - n_0}{T} = 1 - {}^{n_0}\!/_{T} = 1 - p_0$$

$$H_T = -\sum_s p_s \lg p_s \qquad \lambda_T = \frac{S - m_0}{S} = 1 - {}^{m_0}\!/_{S}$$

After thorough analyzation and classification of the 600 cellular automata and their data, the follow statistics were calculated.

	Lambda (λ)	Lambda_t (λ_T)	Entropy (H)	Entropy_t (H_T)
Average	0.48300	0.46154	1.38697	1.46500
St-Dev	0.31947	0.28804	0.70039	0.70597
Min	0.00000	0.00000	0.00000	0.00000
Max	0.99200	0.92308	2.31052	2.29547

Table 1: General Statistics for All Classifications (Classes I, II, III, & IV)

	Percent Frequency (%)	Average Lambda (λ)
Class I	39.000	0.25708
Class II	5.500	0.62158
Class III	51.000	0.64923
Class IV	4.500	0.71156

Table 2: Behavioral Classification Frequency

	Lambda (λ)	Lambda_t (λ_T)	Entropy (H)	Entropy_t (H_T)
Average	0.71156	0.67094	1.95846	2.04783
St-Dev	0.12340	0.11133	0.21857	0.14755
Min	0.49600	0.46154	1.53934	1.77586
Max	0.92308	0.92308	2.24161	2.19969

Table 3: General Statistics for Class IV Behavior

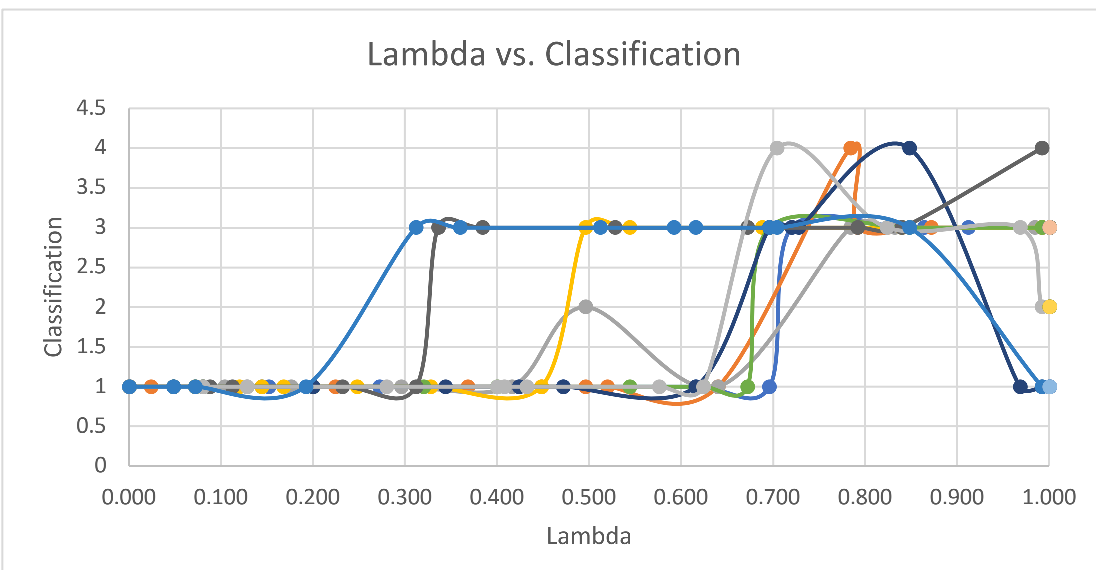
As shown by the second table, overall class III behavior (chaotic) is by far the most common classification. Furthermore, class IV behavior is the most rare or infrequent classification assigned to cellular automata. It is interesting to note that the second most infrequent classification is class II behavior (periodic). Standard deviation shows how much a group of values are spread out from the average. A low value for standard deviation means that most of the values are close to the overall average for that parameter. In regard to class IV behavior, the standard deviation values for lambda and lambda-t are the lowest. With this information, one could theorize that the parameter lambda is the most correlated with class IV behavior.

Theory & Methodology

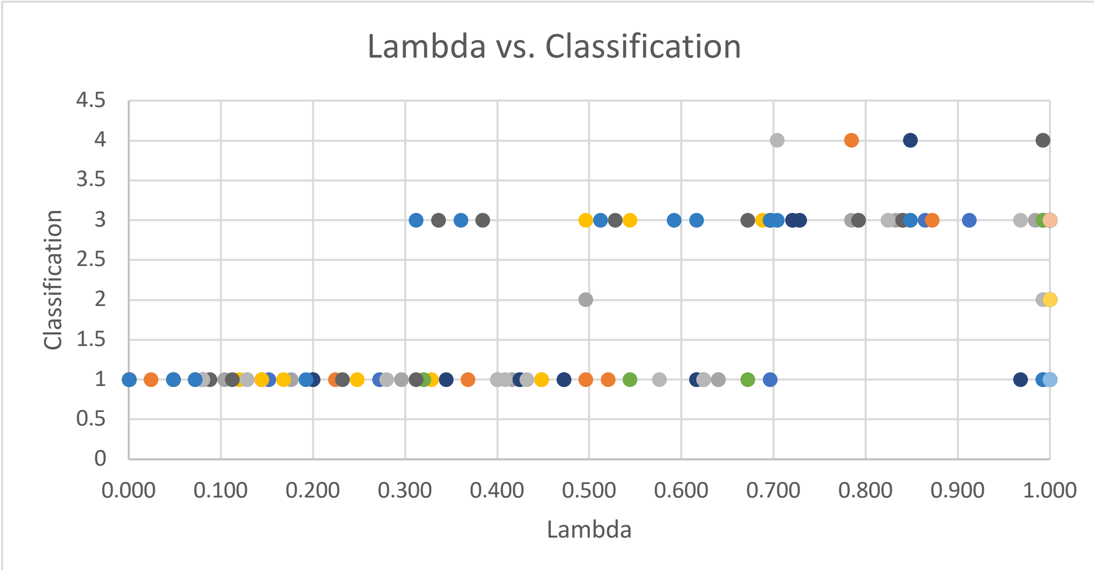
The theory behind this experiment is that there must exist some parameter or condition that is directly associated with the emergence of class IV behavior in cellular automata. To clearly and accurately test this theory, one has to be systematic in their approach to the problem. Firstly, a random initial state and rules must be maintained for the duration of the experiments. After the CA's initial state has be set up, for each state a simple parameter of the rule will be altered or decimated. Systematically following these guidelines will allow for affective observation and evaluation of the CA's qualitative behavior.

For these observations, I opted to use the simulator supplied through Dr. Van Hornweder's help page. The code was originally written by a student who took this course in Fall 2007. Later, in 2009, it was edited to streamline the processing and gathering of data and images. This program creates and assigns a random rule then runs a designated amount of experiments. For each experiment, it steps through each table 12 times. For each step, the program calculates the values for each parameter and stores them in a .csv file named *MasterExperiment.csv*. Furthermore, the program creates .html files containing illustrations of each CA at each of its 12 steps. To calculate each parameter, the program used the aforementioned functions provided earlier. To calculate the statistics used in my analysis, I relied solely on the functions built into excel and numbers. The functions are as follows:

Results and Findings

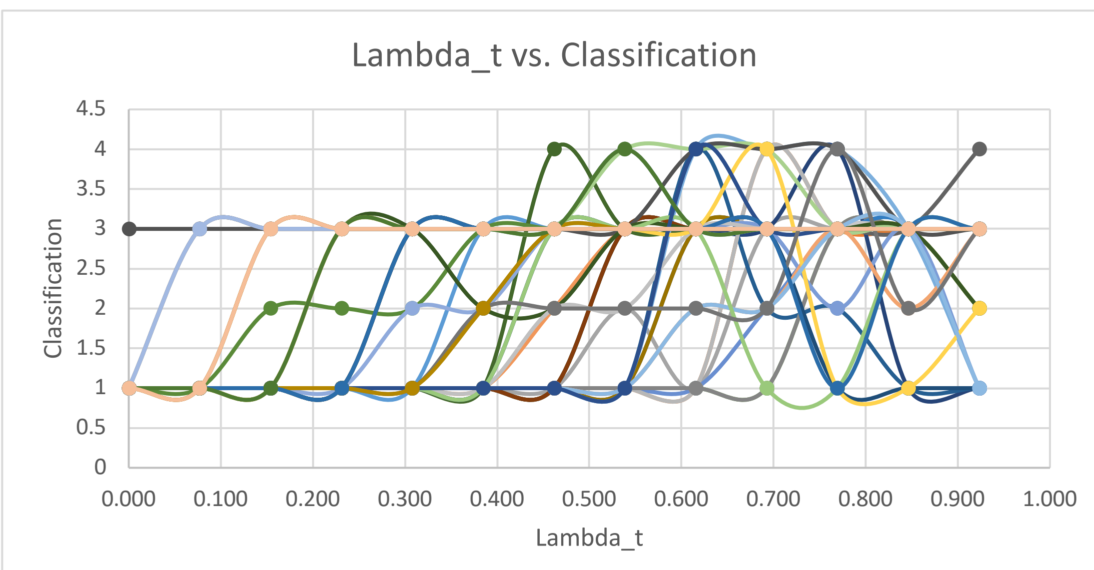


Graph 1.1: X-Y Scatter Plot of Lambda Values Against Behavior Classification

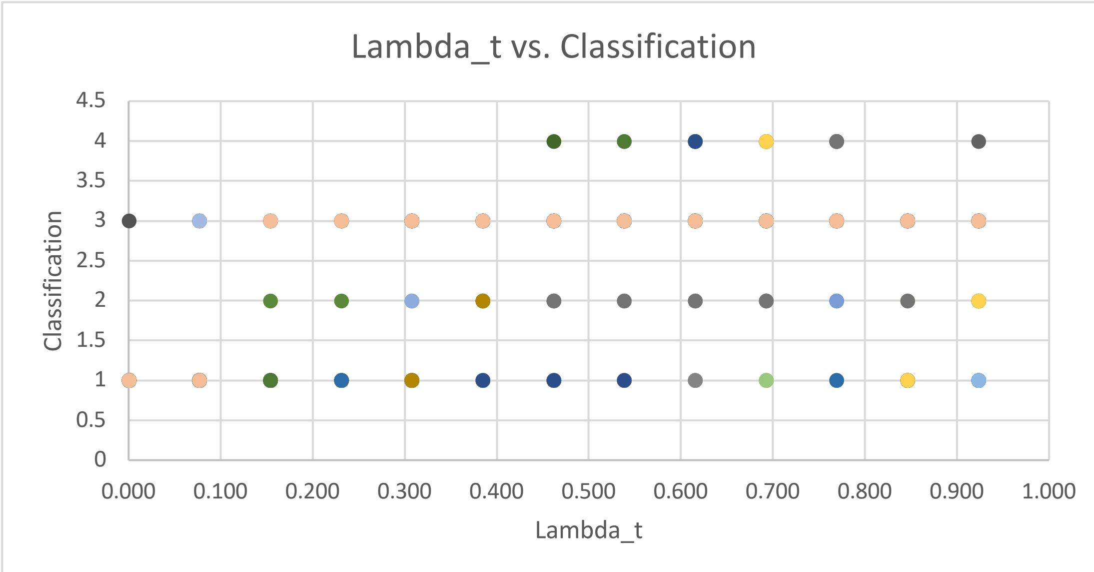


Graph 1.2: X-Y Scatter Plot of Lambda Values Against Behavior Classification

The two graphs above illustrate the relationship between lambda and the classification of CA behavior. From the graphs, it appears that as lambda approaches the range of 0.70 to 0.85, class IV behavior begins to emerge. This is reasonable considering the calculated average lambda value for class IV behavior is 0.712.

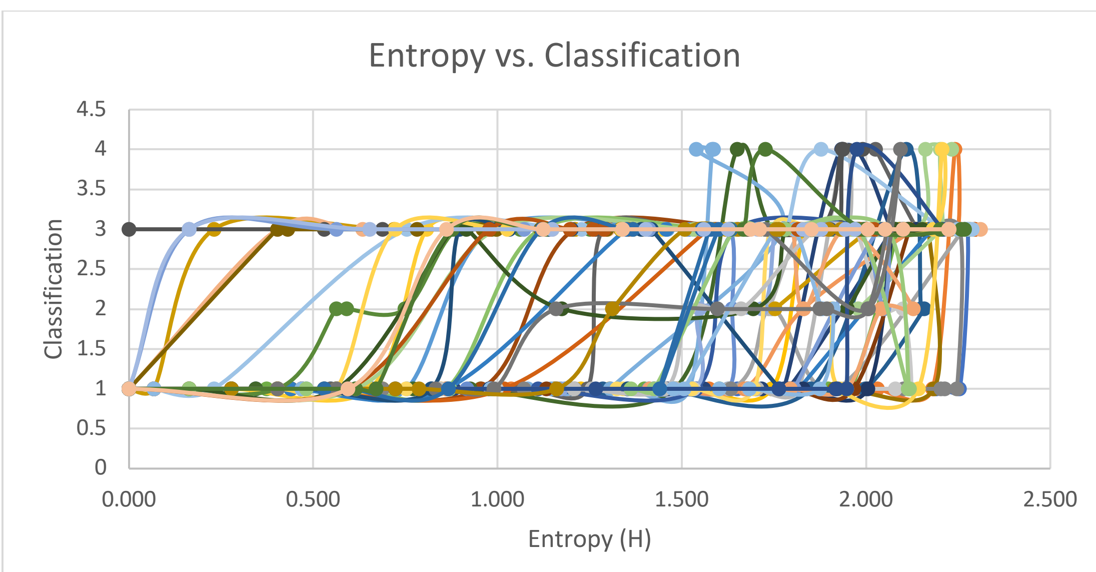


Graph 2.1: X-Y Scatter Plot of Lambda_t Values Against Behavior Classification

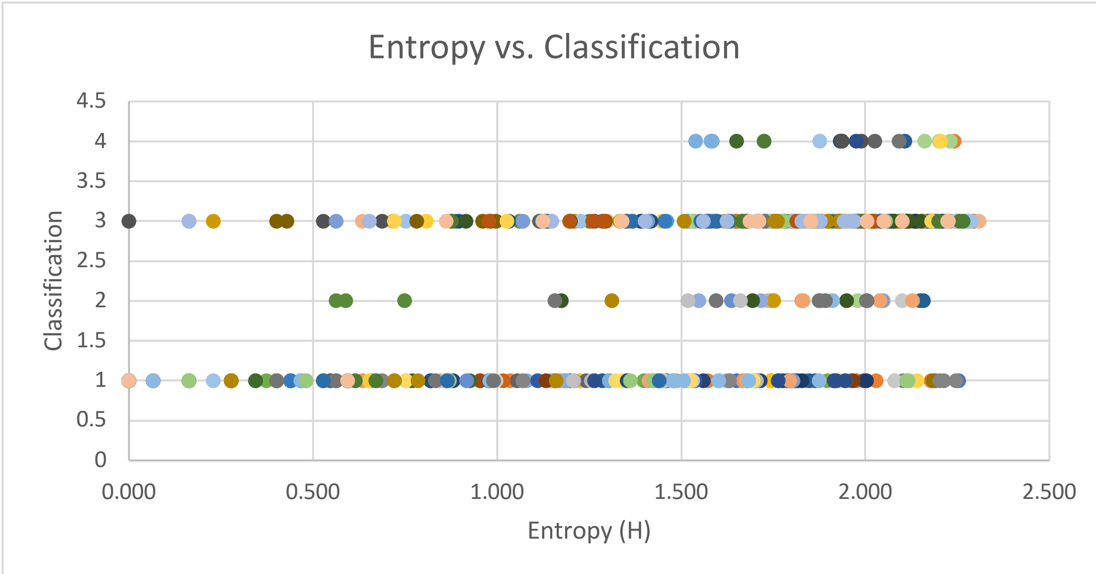


Graph 2.2: X-Y Scatter Plot of Lambda_t Values Against Behavior Classification

Lambda_t appears to be correlated to class IV behavior in a similar fashion to lambda. However, the striking difference between these parameter's correlation is the range at where class IV behavior occurs. For lambda_t, class IV behavior begins to emerge most frequently within the range of 0.46 to 0.77. As stated in the statistical charts above, the average lambda_t value for class IV behavior is 0.671.

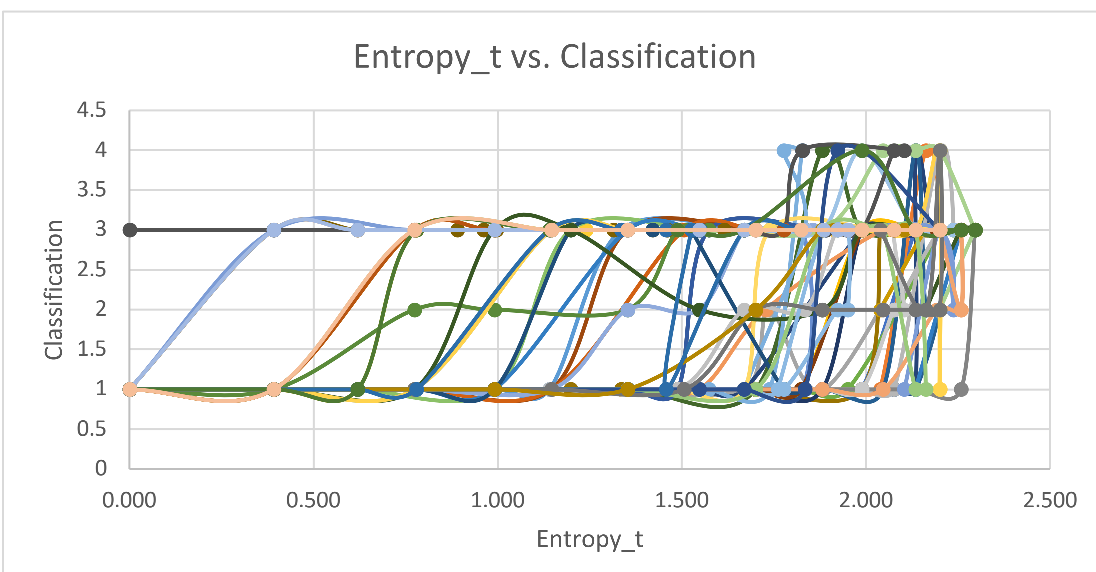


Graph 3.1: X-Y Scatter Plot of Entropy Values Against Behavior Classification

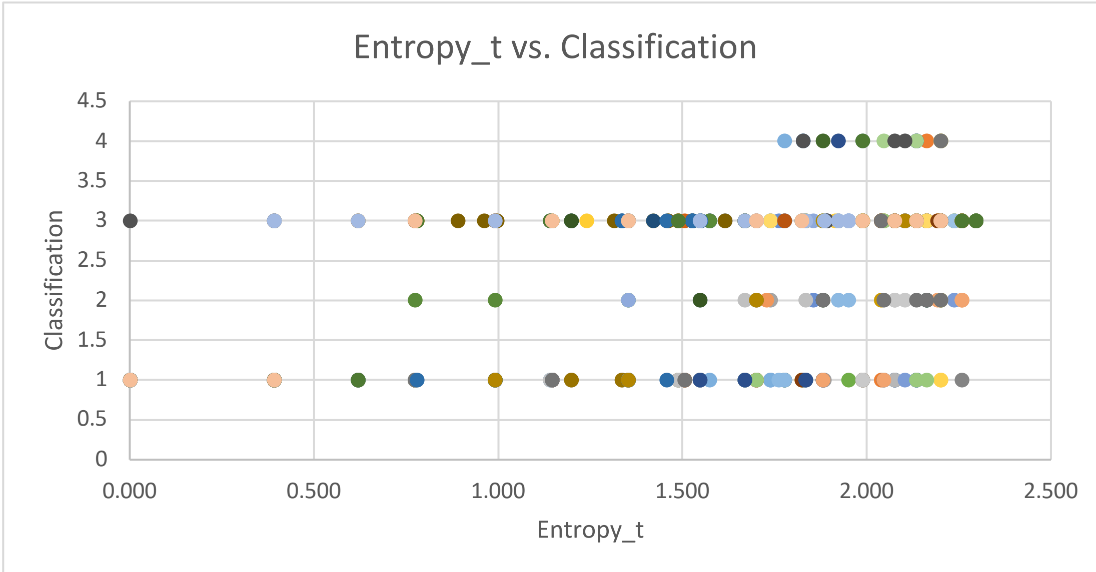


Graph 3.2: X-Y Scatter Plot of Entropy Values Against Behavior Classification

Judging from the two entropy graphs above, it appears that this parameter is more strongly correlated with the occurrence of class IV behavior. However, interestingly enough, this parameter had the highest calculated standard deviation. As the entropy values approach the range of 1.5 to 2.2, observed instances of class IV become more frequent. The calculated average of entropy for class IV behavior is 1.96.



Graph 4.1: X-Y Scatter Plot of Entropy_t Values Against Behavior Classification



Graph 4.2: X-Y Scatter Plot of Entropy_t Values Against Behavior Classification

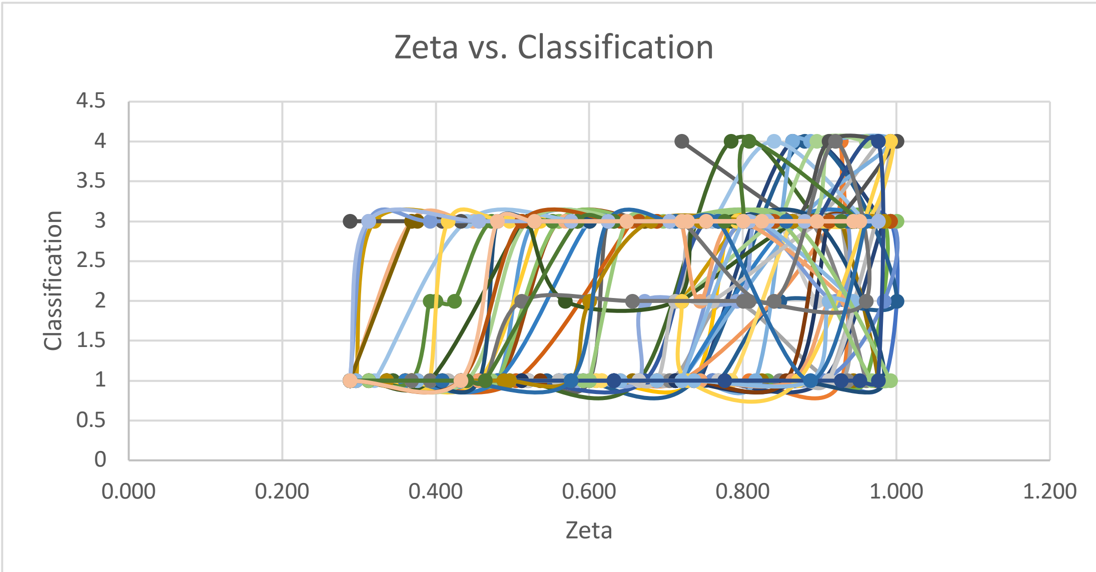
The parameter entropy_t seems to just as correlated with class IV behavior as its counterpart, entropy. However, as seen in the graph above, the range at which class IV behavior emerges is smaller and more compact. The range at which class IV behavior occurs is from 1.7 to 2.2. The calculated average for entropy_t in class IV CAs is found to be 2.05.

The Zeta Parameter

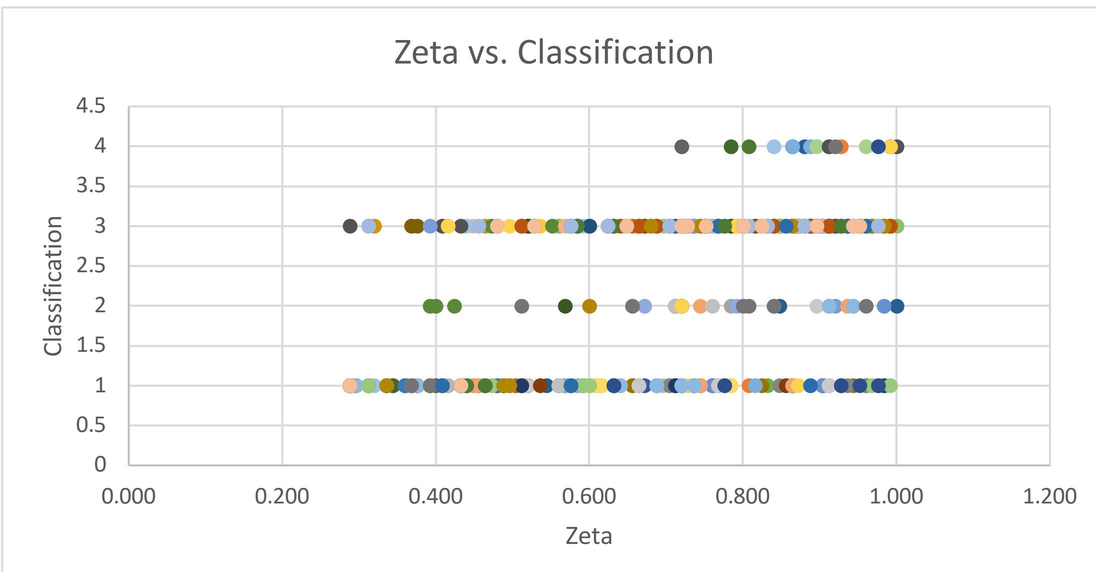
In addition to the four parameters that we have observed up to this point, I created a formula to calculate a fifth. The purpose of this was to find another parameter that could be more or as correlated to the emergence of class IV behavior as the others. The formula I created to calculate Zeta is as follows:

$$Z = \frac{1}{1-\text{abs}(\text{lambda} - 0.712)}$$

I reached this formula from the observation that the parameter lambda appeared to be the most correlated with class IV behavior. However, there is only a specific range of lambda in which class IV behavior begins to emerge. To center the parameter Zeta on this range, I took the absolute value of the difference of the calculated lambda and the average value for lambda in class IV CAs. The graphs below show the correlation of zeta to the classification of CA behavior. As you can see the parameter zeta is very much correlated with the emergence of class IV behavior. In fact, its calculated standard deviation of 0.081 is lower than all other parameters.



Graph 5.1: X-Y Scatter Plot of Zeta Values Against Behavior Classification



Graph 5.2: X-Y Scatter Plot of Zeta Values Against Behavior Classification

Conclusion

More commonly than not, cellular automata tend to evolve into either a static, unchanging state (class I), or a chaotic, unpredictable state (class III). However, both of these states are practically useless because they have no correlation or connection to anything. Through both prediction and observation, we have found that class IV behavior is certainly the most infrequent of the classifications.

Class IV behavior is such an interesting and important occurrence because it is found echoing throughout the universe. The emergence and cycle of life, the spreading of particles, and many other natural phenomena share some form of class IV behavior. This was the driving reason for observing which parameters and circumstances most frequently lead to the emergence of class IV behavior. Through this experiment, the calculations, and the illustrations provided by the graphs, all the parameters discussed have a high level of influence in the emergence of class IV behavior. However, from both a statistical and observational standpoint, the lambda parameter seems to be the *most* correlated with class IV behavior. It appears that once it rises to a certain threshold, the occurrence of class IV behavior becomes much more frequent.