Fall 2021

Homework 13

Stochastic Processes, Reactive Reporting in R, Monte Carlo Simulations (MC Methods in Calculus Applications)

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Markov Stochastic Process

• A process with a finite number N of possible states (or outcomes, or event−results) in which the probability p of being in a particular state S at step/observation−moment n+1 depends only on the state occupied at step n.

– Example: Number of state options N=2 (binary state) is common

RW case.

Markov Process Memory

• A Markov process is a stochastic process that satisfies the Markov property of no memory of the past (or short memory)

– In simpler terms, it is a process for which predictions can be made regarding future outcomes based solely on its present state.

• Such predictions are just as good as the ones that could be made knowing the process's full history.

• Given the present state of the system, its future and past states are independent.

Discrete Time Markov Chain

• **If the time parameter is discrete:**

$$\{t_1, t_2, t_3, \ldots\ldots, t_{n-2}, t_{n-1}, t_n\}$$

it is called Discrete Time Markov Chain (DTMC).

What is a Markov Chain?

• Markov process or Markov chain, is a model of one special type of discrete−time stochastic process.

– Model is made in the LW with certain assumptions.

– Model must match observed stochastic system in the RW.

• LW Model−assumptions and RW system−attributes must

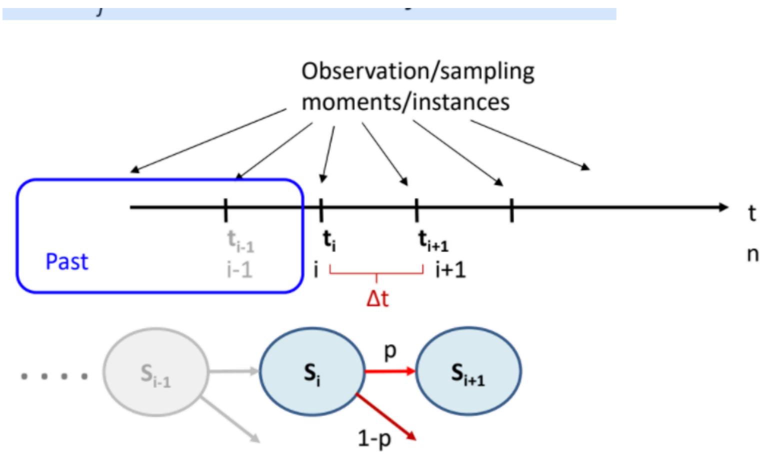
match.

S Sn−2 Sn−1 Sn n+1 . . . . p

1−p

t tn n+1 tn−1 tn−2

Discrete Time Markov Process State Change



• If the system moves from state Si over a period of time Δt, to state Sj a transition from i to j has occurred.

Assumption: Memoryless DTM Process

• It is assumed that probability of transition in any of the

possible states (of the state space) does not depend on the

past states (i−1, i−2, ...) or state changes.

Stationary Markov Process

• DTM Assumption I: The state of the system at time t+1

depends only on the state of the system at time t

Stationary Markov Process

• DTM Assumption I − Stationary Assumption: Transition probabilities are independent of time t.

– If probabilities do not change over time, process is stationary.

Example: Auto Insurance Risk

• Based on insurance company data, a motorist that is currently labeled as low risk has a 20% chance of moving to the high risk category and an 80% chance of remaining low risk.

– It is harder (less probable) to go from “Low Risk” to “High Risk”

than the opposite.

Example: Auto Insurance Risk

• A “state” of the observed entity or phenomena in this example is a motorist that can be in at any of the two states at a given time.

• A motorist is either in the “High Risk” or the “Low Risk” state.

• A motorist cannot be in both states, (Must be in one of the

N=2 states).

– States are exclusive.

– Over the period between insurance renewals, a motorist can transfer between states.

– Problem description

• Provides the probability of transitioning state to state.

• Does not provide anything about the probability of starting

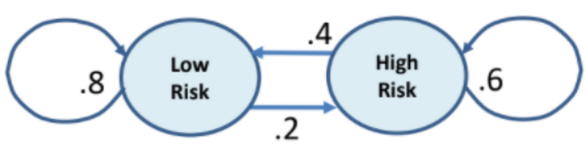
in either state.

What is the purpose of the model?

• Description of the entity, system or phenomena using one or more languages such as:

– Mathematics (Quantitative language), or

– Graphics/images/pictures (Visual language).



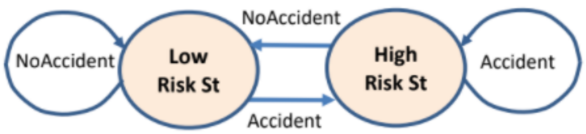
– Example: “State machine” like directed graph.

What is “State Machine” or “State Automata”?

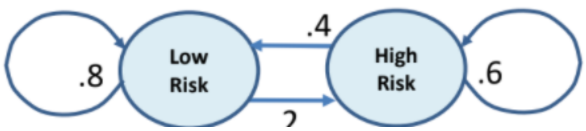
• A directed graph (Without time sequencing).

– Visual model made of vertices/dots/circles denoting states and labeled edges/arrows denoting state−transitions.

• In state machines edge−label denotes a reason/cause for state change/transition.



In Markov graph model, label edges denote probabilities of having a state change happen.



What is "Automata"?

• A "programmed" device that can take a state of operation depending upon an external event.

– Example: Washing machine is an automata.

– It takes different operational state determined by the selection of the washing option (Hot or Cold, Extra Rinse, Short or Long Wash, etc.)

• Washing machine automata appear as being programmable.

– Only one program is available (Loaded in the device").

– One program contains all wash−cycles that ma be selected.

» Machine is not programmable but configurable.

DTM Chains (DTMC) Model as a Tree

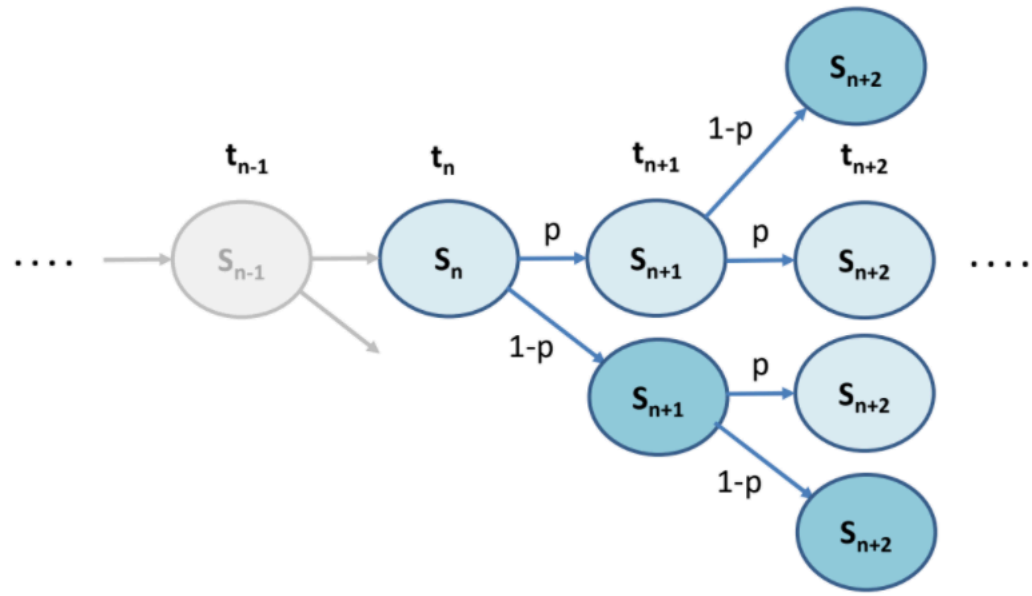
• DTMC ia a model of a process that proceeds in steps

– In discrete time, as a sequence of random trials.

– Appearing like a series of probability trees

S Sn−1 n . . . . Sn+1

Appearing like a series of probability trees



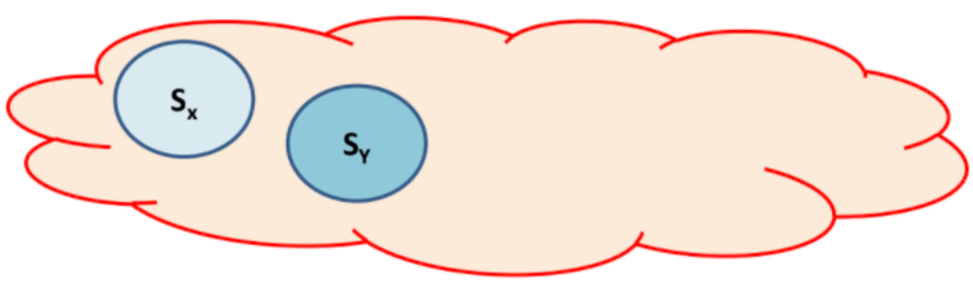
DTMC Model State Space

• DTMC model uses states from the finite countable set known as state space.

– Example: State space made of only N=2 states, (binary data receiver of 1’s or 0’s).

• Observed phenomena can be in only one “state” at each step

• When the next (n+1) step occurs, the process can be in the same state or move to another of the N−1 states of the state space, e.g., N=2:



Is state transition automata−like graph model, time aware?

• No,

– This model is not showing timing or time−sequencing of the events evolution.

• Transitions between states are defined by probabilities.

• – This model can be used to find the probability of the system being in any given state many steps into the future of a process.

Transition Matrix Model

• It is square, since all possible states must be used both as rows and as columns.

• All entries are between 0 and 1, because all entries represent probabilities.

• The sum of the entries in any row must be 1, since the numbers in the row give the probability of changing from the state at the left to any one of the states indicated across the top.

What is the transition matrix? What is it used for?

• Give 3 of its features.

• It is used as a model of the discrete Markov process.

• Main features are:

– It is square.

– Entries are between 0 and 1.

– The sum of entries in any row is equal to 1.

What would be the probabilities of the “Risk” states for a

given motorist after 1 year, 10 and 20 years?

- Create Markov state transition matrix:

Kth State Probabilities

• Assume that

– A number of steps from the current state (from now) is k.

– Initial/current state probability vector is vi.

– State transition matrix is P.

– kth state probabilities vector is vk.

vk = vi Pk

Example: Weather Forecast

• A meteorologist studying the weather in a region decides to classify each day as simply sunny or cloudy. After analyzing several years of weather records, he finds:

– The day after a sunny day is sunny 80% of the time, and cloudy 20% of the time; and

– The day after a cloudy day is sunny 60% of the time, and cloudy 40% of the time.

Example: Weather Forecast

• We can setup up a Markov chain to model this process.

- State transition diagram is

- State transition matrix is:

From/To	Sunny	Cloudy
Sunny	0.8	0.2
Cloudy	0.4	0.6

• There are just two states:

– Sunny, and

– Cloudy.

```
> install.packages("expm")
--- Please select a CRAN mirror for use in this session ---
trying URL 'https://ftp.osuosl.org/pub/cran/bin/macosx/contrib/4.1/expm_0.999-6.tgz'
Content type 'application/x-gzip' length 238006 bytes (232 KB)
==================================================
downloaded 232 KB


The downloaded binary packages are in
	/var/folders/6r/5cl8vzcn2_l3qxg48sb0llqw0000gn/T//RtmpZPOZNR/downloaded_packages
> library(expm)
Loading required package: Matrix

Attaching package: ‘expm’

The following object is masked from ‘package:Matrix’:

    expm
```

```
> m0 %^% 2
     [,1] [,2]
[1,] 0.44 0.56
[2,] 0.28 0.72
> m0 %^% 10
          [,1]      [,2]
[1,] 0.3334032 0.6665968
[2,] 0.3332984 0.6667016
> m0 %^% 20
          [,1]      [,2]
[1,] 0.3333333 0.6666667
[2,] 0.3333333 0.6666667
```

```
> vi<-c(0.5,0.5)
> vi
[1] 0.5 0.5
> m0.data<-c(0.6,0.4,0.2,0.8)
> m0<-matrix(m0.data,nrow=2,ncol=2,byrow=TRUE)
> m0
     [,1] [,2]
[1,]  0.6  0.4
[2,]  0.2  0.8
```

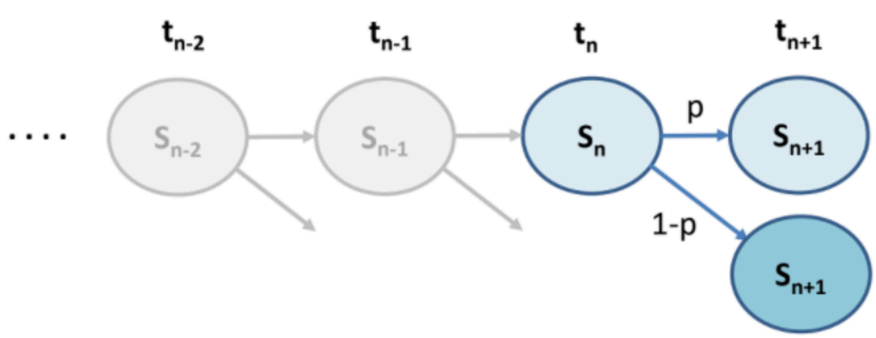
```
Error: unexpected input in "v0 <-"
> v0<-c(0.6,0.4,0.2,0.8)
> v0
[1] 0.6 0.4 0.2 0.8
> m0<-matrix(v0,nrow=2,ncol=2,byrow=TRUE)
> m0
     [,1] [,2]
[1,]  0.6  0.4
[2,]  0.2  0.8
> m0 %*% m0
     [,1] [,2]
[1,] 0.44 0.56
[2,] 0.28 0.72
> m0 %^% 2
     [,1] [,2]
[1,] 0.44 0.56
[2,] 0.28 0.72
>
```

```
Error: unexpected input in "v0 <-"
> v0<-c(0.6,0.4,0.2,0.8)
> v0
[1] 0.6 0.4 0.2 0.8
```

Random/Stochastic Process as
Mathematical Model

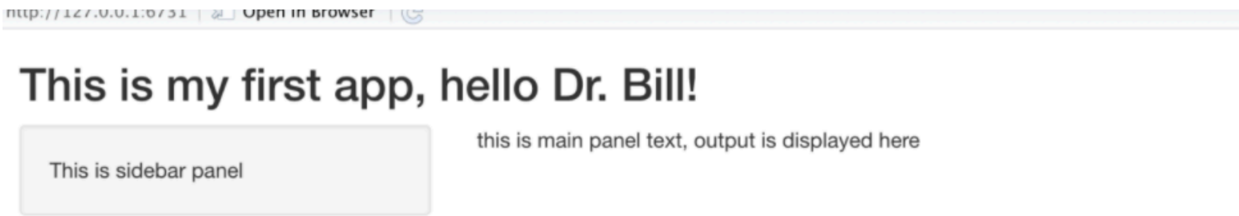
• Stochastic processes are mostly mathematical models of statistical variation in time of observed or engineered system's parameters.
– Definition: A model is a descriptive story of some phenomena, process, or system, that generates events.
• Mathematical model is a story presented in mathematical language (One or more of the Mathematical dialects).
• Processes is a time discrete or continuous sequence of events.
• Stochastic process models are used for better systems understanding, design, or redesign.

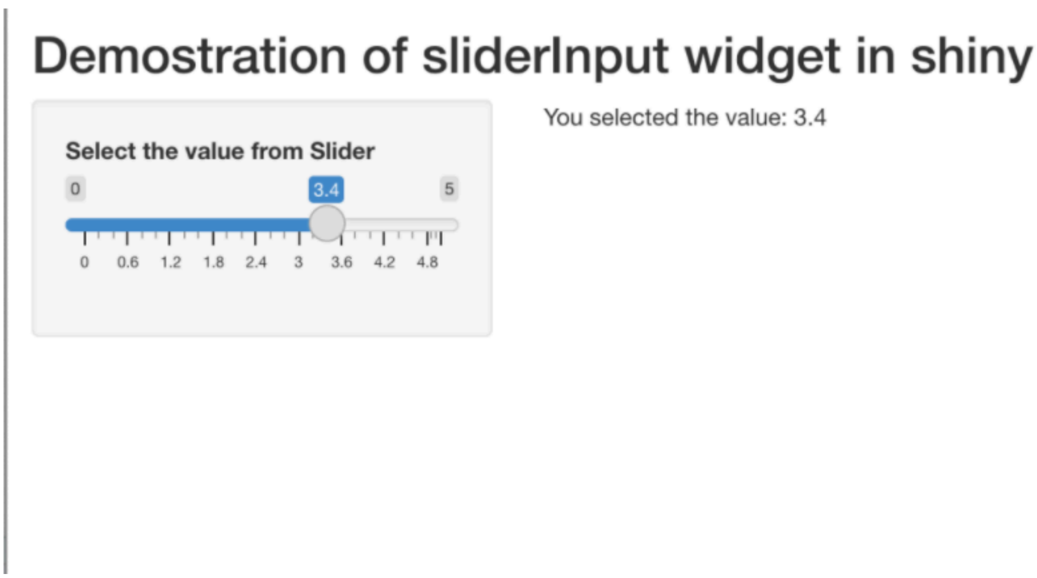
Answer: What is a Markov Chain?
• What is Markov chain?
• A model of a RW random process made in LW.
• Where RW random process has certain characteristics:
– It is memoryless (Minimal memory).
– It is stationary.

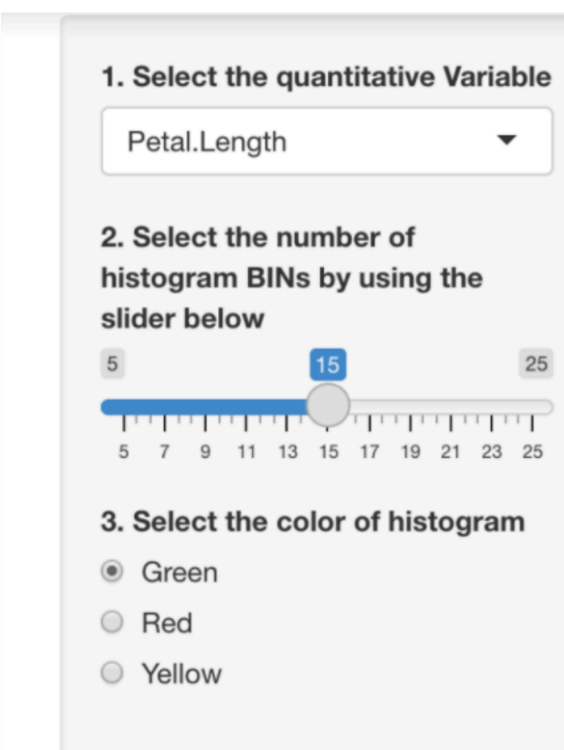


```
> library("shiny")
> uiX<-fluidPage()
> serverX<-function(input,output){}
> shinyApp)ui=uX,server=serverX
Error: unexpected ')' in "shinyApp)"
> shinyApp(ui=uX,server=serverX)
Error in force(ui) : object 'uX' not founc
> shinyApp(ui=uiX,server=serverX)

Listening on http://127.0.0.1:7745
```


```
Error: object 'n' not found
> ui <- fluidPage(
+ titlePanel(title="First app . . ."),
+ sidebarLayout(position = "right",
+ sidebarPanel("Sidebar pannel ..."),
+ mainPanel("Main panel, ..."))
+ )
> server<-function(input,output){}
> shinyApp(ui=ui,server=server)

Listening on http://127.0.0.1:7745
```


http://127.0.0.1:6731
Open in Browser
This is my first app, hello Dr. Bill!
This is sidebar panel
this is main panel text, output is displayed here


Demostration of sliderInput widget in shiny
Select the value from Slider
0
3.4
5
0 0.6 1.2 1.8 2.4 3 3.6 4.2 4.8
You selected the value: 3.4


1. Select the quantitative Variable
Petal.Length
2. Select the number of histogram BINs by using the slider below
5
15
25
5 7 9 11 13 15 17 19 21 23 25
3. Select the color of histogram
Green
Red
Yellow