Homework:01 For CSCI6650 - P001 Intelligent Agents

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Problem Statement

Given two side vectors $\vec{AB}$ and $\vec{AD}$ defining a rectangle, along with a test point $\vec{AP}$, the objective is to determine whether the test point $\vec{P}$ lies inside, outside, or on the boundary of the rectangle using the determinant method.

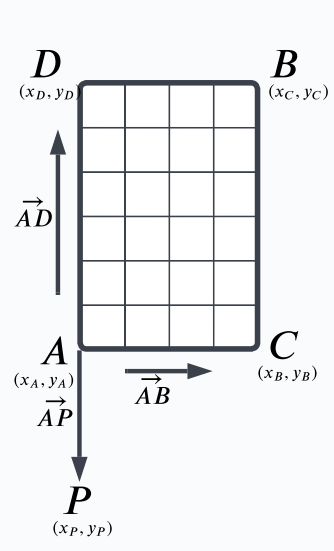

Figure 1: **Problem Illustration**

Objective

The primary goal is to ascertain the spatial relationship between the test point $\vec{P}$ and the rectangle defined by vectors $\vec{AB}$ and $\vec{AD}$. Specifically, the objective is to categorize $\vec{P}$ as inside, outside, or on the boundary of the rectangle.

Method

1. **Vector Definitions:**
   - Define vectors $\vec{AB} = \langle x_B - x_A, y_B - y_A \rangle$ and $\vec{AD} = \langle x_D - x_A, y_D - y_A \rangle$.
   - Define the vector $\vec{AP} = \langle x_P - x_A, y_P - y_A \rangle$ representing the displacement from point $\vec{A}$ to the test point $\vec{P}$.
2. **Matrix Formation:**
   - Create a 2×2 matrix M using $\vec{AB}$ and $\vec{AD}$:

$$M = \begin{bmatrix} x_B - x_A & x_D - x_A \\ y_B - y_A & y_D - y_A \end{bmatrix}$$

3. **Determinant Calculation:**
   - Calculate the determinant of matrix M, denoted as $\det(M)$:

$$\det(M) = (x_B - x_A) \cdot (y_D - y_A) - (y_B - y_A) \cdot (x_D - x_A)$$

4. **Matrix Modification:**
   - Create a matrix M_{AP} by replacing the second column of M with $\vec{AP}$:

$$M_{\text{AP}} = \begin{bmatrix} x_B - x_A & x_P - x_A \\ y_B - y_A & y_P - y_A \end{bmatrix}$$

5. **Determinant Calculation (Modified Matrix):**
   - Calculate the determinant of matrix M_{AP}, denoted as $\det(M_{\text{AP}})$:

$$\det(M_{\text{AP}}) = (x_B - x_A) \cdot (y_P - y_A) - (y_B - y_A) \cdot (x_P - x_A)$$

6. **Spatial Classification:**
   - Check the sign of $\det(M) \cdot \det(M_{\text{AP}})$:
     - If $\det(M) \cdot \det(M_{\text{AP}}) > 0$, then $\vec{P}$ is inside the rectangle.
     - If $\det(M) \cdot \det(M_{\text{AP}}) < 0$, then $\vec{P}$ is outside the rectangle.
     - If $\det(M) \cdot \det(M_{\text{AP}}) = 0$, then $\vec{P}$ is on the boundary of the rectangle.

This method leverages the determinant of the matrix formed by vectors to determine the signed area of parallelograms, enabling the classification of the test point's position with respect to the rectangle.

Code(Python):

Now, here's a Python function that checks if a point is inside a rectangle using the determinant method:

```
import numpy as np

def is_point_inside_rectangle(A, B, D, P):
    # Create matrices
    M = np.array([[B[0] - A[0], D[0] - A[0]], [B[1] - A[1], D[1] - A[1]]])
    M_AP = np.array([[B[0] - A[0], P[0] - A[0]], [B[1] - A[1], P[1] - A[1]]])

    # Calculate determinants
    det_M = np.linalg.det(M)
    det_M_AP = np.linalg.det(M_AP)

    # Check if the point is inside the rectangle
    return det_M * det_M_AP > 0

# Example usage
A = (0, 0)
B = (5, 0)
D = (0, 3)
P = (2, 1)

result = is_point_inside_rectangle(A, B, D, P)
print(f"The point P {P} is inside the rectangle: {result}")
```

The function `is_point_inside_rectangle` returns `True` if the point is inside the rectangle and `False` otherwise.