

## Assignment 1 Solutions

**Problem 1.** Create a vector V with 8 elements (7,2,1,0,3,-1,-3,4).

- Transform that vector into a rectangular matrix A of dimensions 4X2 (4- rows, 2-columns).

Solution:

# First, I use the c command to create a vector from the given values.

```
> v <- c(7,2,1,0,3,-1,-3,4)
```

```
> v
```

```
[1] 7 2 1 0 3 -1 -3 4
```

# matrix() is used create a matrix from the values in the vector v that is 4x2

```
> A <- matrix(v, nrow=4, ncol=2)
```

```
> A
```

```
 [,1] [,2]
```

```
[1,]  7   3
```

```
[2,]  2  -1
```

```
[3,]  1  -3
```

```
[4,]  0   4
```

- Create a matrix transpose to the above matrix A. Call that matrix AT.

Solution:

# The transpose function is used on the matrix A.

```
> AT <- t(A)
```

```
> AT
```

```
 [,1] [,2] [,3] [,4]
```

```
[1,]  7   2   1   0
```

```
[2,]  3  -1  -3   4
```

- Calculate matrix products: A\*AT and AT\*A. Present the results. What are the dimensions of those two product matrices.

Solution:

# Multiplication is done using R's matrix multiplication command %\*%

```
> product <- A%*% AT
```

```
> product
```

```
 [,1] [,2] [,3] [,4]
```

```
[1,] 58 11 -2 12
```

```
[2,] 11  5  5 -4
```

```
[3,] -2  5 10 -12
```

```
[4,] 12 -4 -12 16
```

# The dimensions are 4x4 for A \* AT.

```
> product <- AT %*% A
```

```
> product
```

```
  [,1] [,2]
```

```
[1,]  54  16
```

```
[2,]  16  35
```

# The dimensions are 2x2 for AT \* A

- Square matrixes sometimes have an inverse matrix. Try calculating inverse matrices (or matrixes, if you prefer) of above matrices (matrixes)  $A*AT$  and  $AT*A$ .

# The solve function is used to find the inverse of the matrices.

```
> solve(A %*% AT)
```

#  $A*AT$  is a singular matrix, meaning its determinant is zero and it cannot be inverted.

```
> solve(AT %*% A)
```

```
  [,1] [,2]
```

```
[1,] 0.021419829 -0.009791922
```

```
[2,] -0.009791922 0.033047736
```

- Extend the above vector V with the ninth number of value -2. Do it elegantly by concatenating two vectors ().

```
> v <- c(v, c(-2))
```

```
> v
```

```
[1] 7 2 1 0 3 -1 -3 4 -2
```

- Transform that extended vector into a 3X3 matrix B.

Solution:

```
> B <- matrix(v, nrow=3, ncol=3)
```

```
> B
```

```
  [,1] [,2] [,3]
```

```
[1,]  7  0 -3
```

```
[2,]  2  3  4
```

```
[3,]  1 -1 -2
```

- Calculate the inverse matrix of matrix B. Call it Binv. Demonstrate that the product of B and Binv is the same as the product of Binv and B and is equal to what?

Solution:

# The inverse is calculated using the solve function on the matrix B.

```
> Binv = solve(B)
```

```
> Binv
```

```
  [,1] [,2] [,3]
```

```
[1,] -2  3  9
```

```
[2,]  8 -11 -34
```

```
[3,] -5  7 21
```

```
> B %*% Binv
```

```
      [,1]      [,2]      [,3]
[1,]  1 0.000000e+00 0.000000e+00
[2,]  0 1.000000e+00 -1.421085e-14
[3,]  0 1.776357e-15 1.000000e+00
```

```
> Binv %*% B
```

```
      [,1] [,2]      [,3]
[1,] 1.000000e+00  0 0.000000e+00
[2,] -7.105427e-15  1 -1.421085e-14
[3,] -3.552714e-15  0 1.000000e+00
```

# By rounding the matrices above to the 6th decimal place, both result in the matrix below  
# which is the identity matrix.

```
      [,1] [,2] [,3]
[1,] 1.0  0.0  0.0
[2,] 0.0  1.0  0.0
[3,] 0.0  0.0  1.0
```

- Determine the eigenvectors of matrices B.

# The eigenvectors are found by using the eigen function and then by extracting the vectors.

# The eigenvectors are the columns 1, 2, and 3 in the matrix C below.

```
> C <- eigen(B)$vectors
```

```
> C
```

```
      [,1]      [,2]      [,3]
[1,] 0.86822600 0.1825742 0.2159107
[2,] 0.49436902 -0.9128709 -0.8426423
[3,] 0.04222416 0.3651484 0.4932914
```

- Construct a new matrix C which is made by using each eigenvector of matrix B as a column. Calculate the product of matrix C and matrix B and the product of matrix B and C. Is there any significance to the elements of the product matrixes.

# C is found in the previous step and is the \$vectors portion of the result of eigen(B)

```
> B %*% C
```

```
      [,1]      [,2]      [,3]
[1,] 5.9509095 0.1825742 0.03150095
[2,] 3.3884557 -0.9128709 -0.12293986
[3,] 0.2894087 0.3651484 0.07197025
```

```
> C %*% B
```

```

      [,1] [,2] [,3]
[1,] 6.658641 0.3318118 -2.3062028
[2,] 0.792199 -1.8959704 -3.4493061
[3,] 1.519157 0.6021537 0.3473382

```

# The two matrices above don't have significant values. The matrix below of C-inverse \* B \* C produces a diagonal matrix with the eigenvalues of B in the diagonal.

```

> solve(C) %*% B %*% C
      [,1] [,2] [,3]
[1,] 6.854102e+00 -1.665335e-15 -1.776357e-15
[2,] 3.108624e-15 1.000000e+00 -1.776357e-15
[3,] -1.762479e-15 -1.221245e-15 1.458980e-01

```

- Transform matrix B into a matrix with names columns and named rows.

```

# the rows and columns are named by assigning the dimnames of B
> dimnames(B) <- list(c("row1", "row2", "row3"), c("col1", "col2", "col3"))
> B
      col1 col2 col3
row1    7    0   -3
row2    2    3    4
row3    1   -1   -2

```

- Transformed that fully "named" matrix into a data.frame.

```

# the transformation is done with the data.frame() function on the matrix B
> dFrame = data.frame(B)
> dFrame
      col1 col2 col3
row1    7    0   -3
row2    2    3    4
row3    1   -1   -2

```

- Ask the object you just created what is its class().

```

> class(dFrame)
[1] "data.frame"

```

**Problem 2.** Consider file 2006Data.csv upload to the class site in Assignment 01 folder. File represents actual measurement of power consumption in a country somewhere in a California. Import data contained in that file into a data frame. You are expected to Google and find a function that will let you perform that import.

Solution:

# The read.csv() function is used to import the data

```
> mydata = read.csv("2006Data.csv")
```

```
> head(mydata)
```

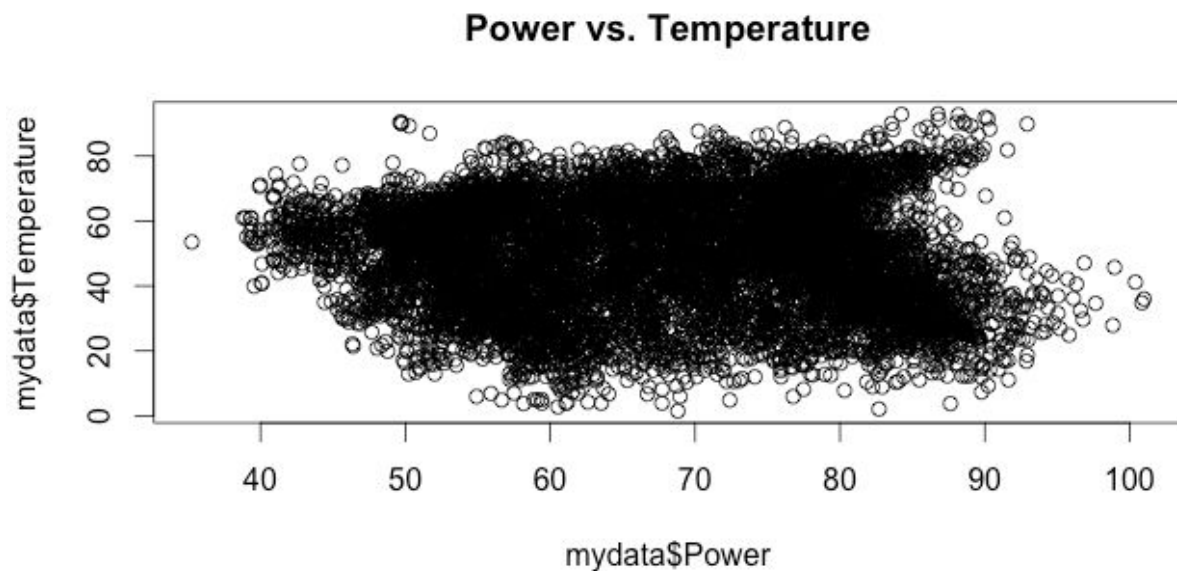
	Month	Day	Hour	DayOfWeek	Holiday	Power	Temperature	X
1	1	1	1	7	0	54.5448	19.0000	NA
2	1	1	2	7	0	52.3898	18.8500	NA
3	1	1	3	7	0	51.6344	17.8650	NA
4	1	1	4	7	0	51.5597	17.2800	NA
5	1	1	5	7	0	51.7148	15.9182	NA
6	1	1	6	7	0	52.6898	16.2400	NA

Create a scatter plot of power consumption vs. temperature and power consumption vs. hour of the day.

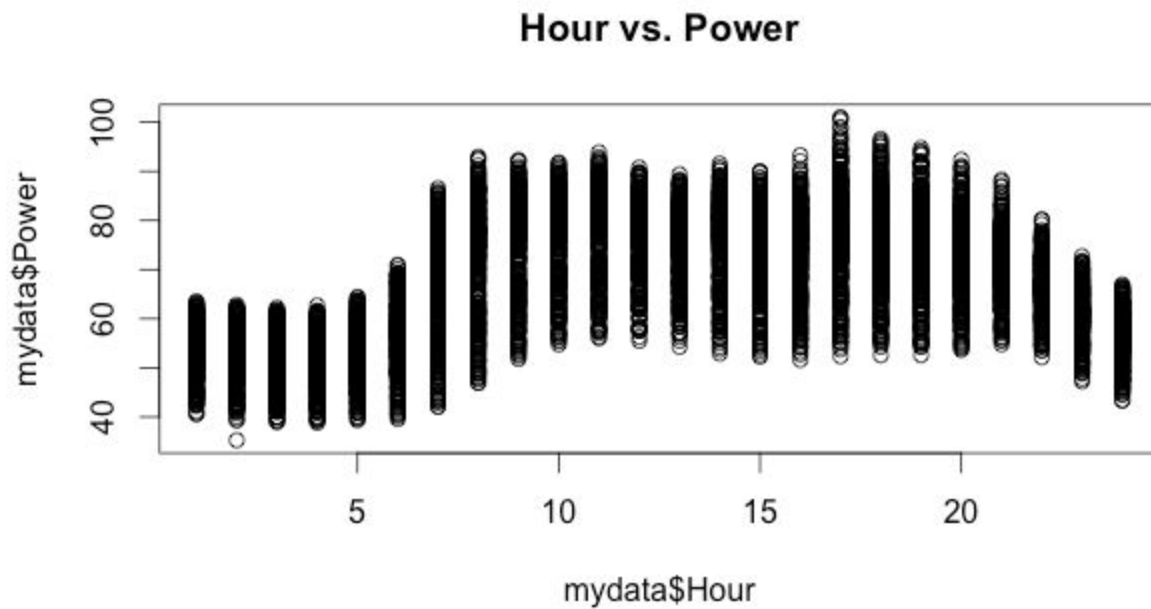
Solution:

# I used the plot function and selected the appropriate data from the whole data frame mydata.

```
> plot(mydata$Power, mydata$Temperature, main="Power vs. Temperature")
```



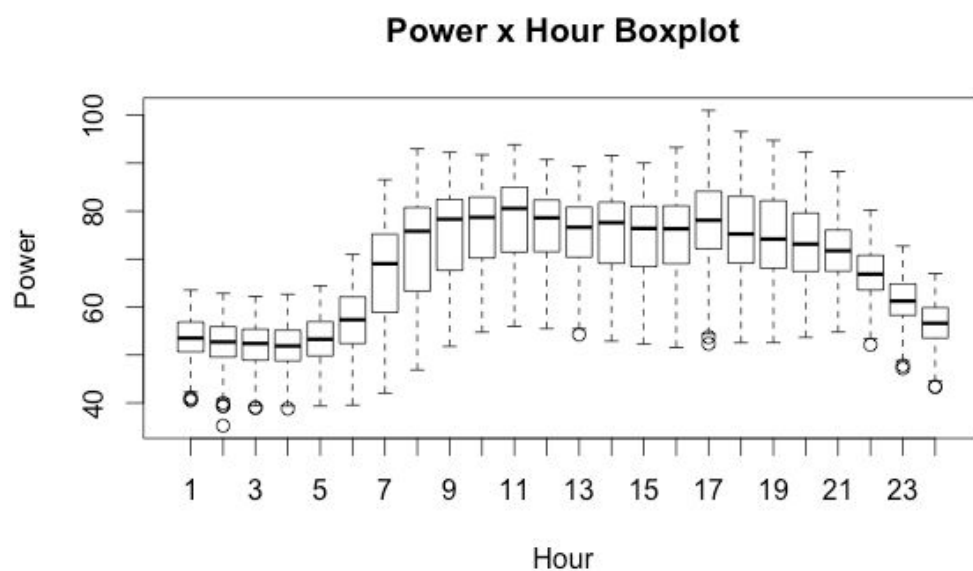
```
> plot(mydata$Hour, mydata$Power, main="Hour vs. Power")
```



Subsequently create a boxplot with power on the vertical axis and hour of the day on the horizontal axis. The objective is to present the distribution (variation) of power consumption for every hour of the day.

Solution:

```
# I used the boxplot method and selected the appropriate data from the data frame mydata
> boxplot(mydata$Power ~ mydata$Hour, main="Power x Hour Boxplot", xlab="Hour",
ylab="Power")
```



**Problem 3.** Separate temperature scale in a reasonable number of intervals: 50 or 100. Calculate average power consumption, minimum power consumption and maximum power consumptions for every interval.

Solution:

```
# 100 temperature intervals are made with the cut function. Then the power data is aggregated
# by which temperature interval it is in. The mean, min., and max. power values for each
# interval are found by setting the FUN parameter in aggregate() to the respective function.
```

```
> intervals= cut(mydata$Temperature, 100)
> storeMean = aggregate(mydata$Power, by=list(intervals), FUN=mean)
```

```
# First, the mean power value for each temperature interval is found.
```

```
> head(storeMean)
```

	Group.1	x
1	(1.38,2.39]	75.75205
2	(2.39,3.31]	60.53260
3	(3.31,4.22]	65.52837
4	(4.22,5.14]	61.51257
5	(5.14,6.05]	66.88447
6	(6.05,6.97]	62.21377

```
# Next the max. power value for each temperature interval is found.
```

```
> storeMax = aggregate(mydata$Power, by=list(intervals), FUN=max)
```

```
> head(storeMax)
```

	Group.1	x
1	(1.38,2.39]	82.6960
2	(2.39,3.31]	60.5326
3	(3.31,4.22]	87.6270
4	(4.22,5.14]	72.3954
5	(5.14,6.05]	76.7704
6	(6.05,6.97]	66.7038

```
# Lastly, the minimum power value for each temperature interval is found.
```

```
> storeMin = aggregate(mydata$Power, by=list(intervals), FUN=min)
```

```
> head(storeMin)
```

	Group.1	x
1	(1.38,2.39]	68.8081
2	(2.39,3.31]	60.5326
3	(3.31,4.22]	58.1601
4	(4.22,5.14]	56.6477
5	(5.14,6.05]	54.9205
6	(6.05,6.97]	55.9044

# Note: Complete listings of mean, max., and min. powers by interval are listed at the end.

Present those three sets of values on a single scatter graph (perhaps in different colours).

Solution:

# The plot function is used and additional data sets for the min. and mean power are added with  
# the points function. Also each data set is colored differently with the par function.

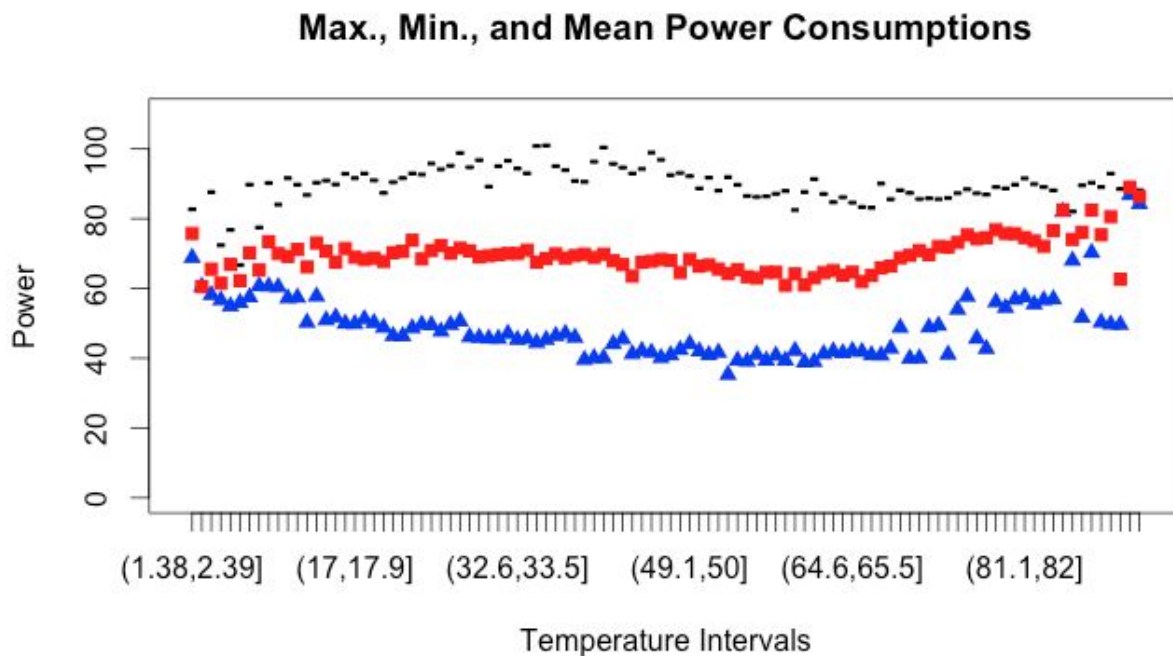
```
> plot(storeMax, ylim=c(0, 110), xlab="Temperature Intervals", ylab="Power", main="Max., Min.,  
and Mean Power Consumptions")
```

```
> par(pch=17, col="blue")
```

```
> points(storeMin)
```

```
> par(pch=15, col="red")
```

```
> points(storeMean)
```



Calculate three covariance matrixes between temperature and each of those power indicators (min, average, max).

Solution:

# The intervals are extracted and saved as mp. The intervals are converted to a string and split  
# until the beginning and ending value of each interval is reached. Those values are converted  
# to a numeric and lastly, the average of the start and end of each interval is calculated.

```
> mp = storeMean$Group.1
```

```
> midpoints = (aggregate(as.data.frame(mp), by=list(mp), FUN=function(x)
```

```
(as.numeric(substr(unlist(strsplit(as.character(x), ","))[1], 2,
```

```
10))+as.numeric(unlist(strsplit(unlist(strsplit(as.character(x), ","))[2], "J")[1])[1])/2 ))$mp
```

```
> head(midpoints)
```



```
[1] 1.885 2.850 3.765 4.680 5.595 6.510
```

### **Temperature and minimum power covariance**

```
# the interval midpoints and the min. power values are put into a matrix.  
# The resulting covariance of this 100x2 matrix is a 4x4 matrix.  
# 705.190794 is the covariance of the midpoints with themselves.  
# 7.906748 is the covariance of the midpoints with the min. power values, meaning there is a  
# positive correlation between these two variables.  
# 92.406717 is the covariance of min. power values with themselves.  
> cov(matrix(c(midpoints, storeMin$x), nrow=100, ncol=2))  
      [,1] [,2]  
[1,] 705.190794 7.906748  
[2,] 7.906748 92.406717
```

### **Temperature and mean power covariance**

```
# The midpoints and the mean power are put into a matrix for the covariance calculation.  
# 705.19079 again represents the same covariance of the midpoints with themselves.  
# 49.85253 is the covariance between the mean power values and the midpoints.  
# This suggest a strong positive correlation between these two variables that is stronger than  
# that between the min. power points and the midpoints.  
# 27.35601 is the covariance of the mean power values with themselves.  
> cov(matrix(c(midpoints, storeMean$x), nrow=100, ncol=2))  
      [,1] [,2]  
[1,] 705.19079 49.85253  
[2,] 49.85253 27.35601
```

### **Temperature and maximum power covariance**

```
# The midpoints and the max. power are put into a matrix for the covariance calculation.  
# 705.19079 again represents the same cov. of the midpoints with themselves.  
# 3.382451 is the covariance between the max. power values and the midpoints.  
# This suggest a weak positive correlation between these two variables that is weaker than  
# that between the min. power or mean power and the midpoints.  
# 37.496919 is the covariance of the max. power values with themselves.  
> cov(matrix(c(midpoints, storeMax$x), nrow=100, ncol=2))  
      [,1] [,2]  
[1,] 705.190794 3.382451  
[2,] 3.382451 37.496919
```

Appendix:

> storeMean

	Group.1	x
1	(1.38,2.39]	75.75205
2	(2.39,3.31]	60.53260
3	(3.31,4.22]	65.52837
4	(4.22,5.14]	61.51257
5	(5.14,6.05]	66.88447
6	(6.05,6.97]	62.21377
7	(6.97,7.88]	70.20718
8	(7.88,8.8]	65.26590
9	(8.8,9.71]	73.39637
10	(9.71,10.6]	69.99766
11	(10.6,11.5]	69.16680
12	(11.5,12.5]	71.18797
13	(12.5,13.4]	66.18224
14	(13.4,14.3]	73.04802
15	(14.3,15.2]	70.67881
16	(15.2,16.1]	67.54408
17	(16.1,17]	71.44375
18	(17,17.9]	68.94623
19	(17.9,18.9]	68.31896
20	(18.9,19.8]	68.71435
21	(19.8,20.7]	67.73369
22	(20.7,21.6]	70.25384
23	(21.6,22.5]	70.61595
24	(22.5,23.4]	73.80868
25	(23.4,24.4]	68.44159
26	(24.4,25.3]	70.88065
27	(25.3,26.2]	72.30138
28	(26.2,27.1]	70.10909
29	(27.1,28]	71.55183
30	(28,28.9]	70.75086
31	(28.9,29.8]	69.03743
32	(29.8,30.8]	69.37282
33	(30.8,31.7]	69.71369
34	(31.7,32.6]	70.01482
35	(32.6,33.5]	70.00666
36	(33.5,34.4]	70.95129
37	(34.4,35.3]	67.51793
38	(35.3,36.3]	68.55177
39	(36.3,37.2]	69.91988

40 (37.2,38.1] 68.72625  
41 (38.1,39] 69.43315  
42 (39,39.9] 69.81562  
43 (39.9,40.8] 68.87853  
44 (40.8,41.7] 69.81403  
45 (41.7,42.7] 67.93662  
46 (42.7,43.6] 66.83172  
47 (43.6,44.5] 63.54442  
48 (44.5,45.4] 67.49919  
49 (45.4,46.3] 67.80343  
50 (46.3,47.2] 68.30028  
51 (47.2,48.2] 68.03177  
52 (48.2,49.1] 64.55237  
53 (49.1,50] 68.31634  
54 (50,50.9] 66.38534  
55 (50.9,51.8] 66.74457  
56 (51.8,52.7] 65.55811  
57 (52.7,53.6] 64.22405  
58 (53.6,54.6] 65.38439  
59 (54.6,55.5] 63.35608  
60 (55.5,56.4] 62.99458  
61 (56.4,57.3] 64.62322  
62 (57.3,58.2] 64.63714  
63 (58.2,59.1] 60.87621  
64 (59.1,60.1] 64.16369  
65 (60.1,61] 60.99559  
66 (61,61.9] 63.04476  
67 (61.9,62.8] 64.55285  
68 (62.8,63.7] 65.11167  
69 (63.7,64.6] 63.79745  
70 (64.6,65.5] 64.67096  
71 (65.5,66.5] 61.96465  
72 (66.5,67.4] 63.73172  
73 (67.4,68.3] 65.90883  
74 (68.3,69.2] 66.40768  
75 (69.2,70.1] 68.76676  
76 (70.1,71] 69.48880  
77 (71,71.9] 70.85030  
78 (71.9,72.9] 69.67759  
79 (72.9,73.8] 72.02108  
80 (73.8,74.7] 71.83831  
81 (74.7,75.6] 73.19687  
82 (75.6,76.5] 75.34647

83	(76.5,77.4]	74.23531
84	(77.4,78.4]	74.56746
85	(78.4,79.3]	76.81973
86	(79.3,80.2]	75.74894
87	(80.2,81.1]	75.70026
88	(81.1,82]	74.57160
89	(82,82.9]	73.64694
90	(82.9,83.8]	72.06534
91	(83.8,84.8]	76.58159
92	(84.8,85.7]	82.48933
93	(85.7,86.6]	74.05720
94	(86.6,87.5]	76.09365
95	(87.5,88.4]	82.44285
96	(88.4,89.3]	75.38355
97	(89.3,90.3]	80.57612
98	(90.3,91.2]	62.62437
99	(91.2,92.1]	89.02307
100	(92.1,93.1]	86.39373

> storeMax

	Group.1	x
1	(1.38,2.39]	82.6960
2	(2.39,3.31]	60.5326
3	(3.31,4.22]	87.6270
4	(4.22,5.14]	72.3954
5	(5.14,6.05]	76.7704
6	(6.05,6.97]	66.7038
7	(6.97,7.88]	89.7771
8	(7.88,8.8]	77.4909
9	(8.8,9.71]	90.2336
10	(9.71,10.6]	84.0427
11	(10.6,11.5]	91.6283
12	(11.5,12.5]	89.7539
13	(12.5,13.4]	86.8044
14	(13.4,14.3]	90.3355
15	(14.3,15.2]	90.9417
16	(15.2,16.1]	89.8396
17	(16.1,17]	92.8717
18	(17,17.9]	91.6216
19	(17.9,18.9]	92.9389
20	(18.9,19.8]	91.0105

21 (19.8,20.7] 87.3819  
22 (20.7,21.6] 90.4748  
23 (21.6,22.5] 91.6367  
24 (22.5,23.4] 92.9723  
25 (23.4,24.4] 92.6620  
26 (24.4,25.3] 95.8191  
27 (25.3,26.2] 94.1587  
28 (26.2,27.1] 95.1873  
29 (27.1,28] 98.8383  
30 (28,28.9] 94.7358  
31 (28.9,29.8] 96.7866  
32 (29.8,30.8] 89.1891  
33 (30.8,31.7] 95.0473  
34 (31.7,32.6] 96.6031  
35 (32.6,33.5] 94.3725  
36 (33.5,34.4] 93.0051  
37 (34.4,35.3] 100.8437  
38 (35.3,36.3] 100.9896  
39 (36.3,37.2] 95.0553  
40 (37.2,38.1] 93.9437  
41 (38.1,39] 90.8336  
42 (39,39.9] 90.5679  
43 (39.9,40.8] 96.3287  
44 (40.8,41.7] 100.3930  
45 (41.7,42.7] 95.7247  
46 (42.7,43.6] 94.6591  
47 (43.6,44.5] 92.9579  
48 (44.5,45.4] 94.2719  
49 (45.4,46.3] 98.9550  
50 (46.3,47.2] 96.8664  
51 (47.2,48.2] 92.4966  
52 (48.2,49.1] 93.0825  
53 (49.1,50] 92.1957  
54 (50,50.9] 88.6777  
55 (50.9,51.8] 91.7737  
56 (51.8,52.7] 88.0476  
57 (52.7,53.6] 91.9065  
58 (53.6,54.6] 89.7125  
59 (54.6,55.5] 86.5059  
60 (55.5,56.4] 86.2429  
61 (56.4,57.3] 86.3998  
62 (57.3,58.2] 87.1483  
63 (58.2,59.1] 87.9378

64	(59.1,60.1]	82.5046
65	(60.1,61]	87.6648
66	(61,61.9]	91.3635
67	(61.9,62.8]	87.1151
68	(62.8,63.7]	84.7115
69	(63.7,64.6]	86.1453
70	(64.6,65.5]	84.5971
71	(65.5,66.5]	83.3536
72	(66.5,67.4]	83.1391
73	(67.4,68.3]	90.0549
74	(68.3,69.2]	85.5745
75	(69.2,70.1]	88.1541
76	(70.1,71]	87.3791
77	(71,71.9]	85.6309
78	(71.9,72.9]	85.8301
79	(72.9,73.8]	85.6128
80	(73.8,74.7]	85.9253
81	(74.7,75.6]	87.3303
82	(75.6,76.5]	88.4328
83	(76.5,77.4]	87.2949
84	(77.4,78.4]	86.9132
85	(78.4,79.3]	89.0667
86	(79.3,80.2]	88.6840
87	(80.2,81.1]	89.7736
88	(81.1,82]	91.5709
89	(82,82.9]	89.9356
90	(82.9,83.8]	89.0597
91	(83.8,84.8]	88.1065
92	(84.8,85.7]	82.7765
93	(85.7,86.6]	82.0439
94	(86.6,87.5]	89.5421
95	(87.5,88.4]	90.3311
96	(88.4,89.3]	89.0548
97	(89.3,90.3]	92.9252
98	(90.3,91.2]	88.5200
99	(91.2,92.1]	90.2133
100	(92.1,93.1]	88.1772

> storeMin

	Group.1	x
1	(1.38,2.39]	68.8081
2	(2.39,3.31]	60.5326

3 (3.31,4.22] 58.1601  
4 (4.22,5.14] 56.6477  
5 (5.14,6.05] 54.9205  
6 (6.05,6.97] 55.9044  
7 (6.97,7.88] 57.4954  
8 (7.88,8.8] 60.7024  
9 (8.8,9.71] 60.6043  
10 (9.71,10.6] 60.4221  
11 (10.6,11.5] 57.2195  
12 (11.5,12.5] 57.4083  
13 (12.5,13.4] 50.2237  
14 (13.4,14.3] 57.7248  
15 (14.3,15.2] 50.9279  
16 (15.2,16.1] 51.7148  
17 (16.1,17] 49.9430  
18 (17,17.9] 49.8641  
19 (17.9,18.9] 51.2449  
20 (18.9,19.8] 50.1434  
21 (19.8,20.7] 48.7796  
22 (20.7,21.6] 46.3978  
23 (21.6,22.5] 46.3474  
24 (22.5,23.4] 48.6415  
25 (23.4,24.4] 49.6580  
26 (24.4,25.3] 49.5164  
27 (25.3,26.2] 47.6825  
28 (26.2,27.1] 49.6092  
29 (27.1,28] 50.6035  
30 (28,28.9] 46.0882  
31 (28.9,29.8] 45.9109  
32 (29.8,30.8] 45.6591  
33 (30.8,31.7] 45.6151  
34 (31.7,32.6] 47.0494  
35 (32.6,33.5] 45.3238  
36 (33.5,34.4] 45.6437  
37 (34.4,35.3] 44.4238  
38 (35.3,36.3] 45.3269  
39 (36.3,37.2] 46.4085  
40 (37.2,38.1] 47.0528  
41 (38.1,39] 45.9313  
42 (39,39.9] 39.5643  
43 (39.9,40.8] 40.0857  
44 (40.8,41.7] 40.0671  
45 (41.7,42.7] 44.1056

46 (42.7,43.6] 45.4897  
47 (43.6,44.5] 41.2311  
48 (44.5,45.4] 42.1209  
49 (45.4,46.3] 41.6288  
50 (46.3,47.2] 40.1007  
51 (47.2,48.2] 40.8894  
52 (48.2,49.1] 42.4239  
53 (49.1,50] 44.1051  
54 (50,50.9] 41.9975  
55 (50.9,51.8] 40.9541  
56 (51.8,52.7] 41.6193  
57 (52.7,53.6] 35.2605  
58 (53.6,54.6] 39.3752  
59 (54.6,55.5] 39.0662  
60 (55.5,56.4] 41.0658  
61 (56.4,57.3] 39.3408  
62 (57.3,58.2] 40.7643  
63 (58.2,59.1] 39.3739  
64 (59.1,60.1] 42.1571  
65 (60.1,61] 38.8142  
66 (61,61.9] 38.9765  
67 (61.9,62.8] 41.2297  
68 (62.8,63.7] 41.9802  
69 (63.7,64.6] 41.4718  
70 (64.6,65.5] 42.1162  
71 (65.5,66.5] 41.8363  
72 (66.5,67.4] 40.9116  
73 (67.4,68.3] 40.8431  
74 (68.3,69.2] 42.7762  
75 (69.2,70.1] 48.7543  
76 (70.1,71] 39.9362  
77 (71,71.9] 39.9917  
78 (71.9,72.9] 48.8566  
79 (72.9,73.8] 49.3744  
80 (73.8,74.7] 41.0651  
81 (74.7,75.6] 53.9464  
82 (75.6,76.5] 57.6834  
83 (76.5,77.4] 45.6383  
84 (77.4,78.4] 42.7008  
85 (78.4,79.3] 56.0804  
86 (79.3,80.2] 54.4724  
87 (80.2,81.1] 56.8079  
88 (81.1,82] 57.5287



89 (82,82.9] 55.4768  
90 (82.9,83.8] 56.6712  
91 (83.8,84.8] 56.9069  
92 (84.8,85.7] 81.9917  
93 (85.7,86.6] 68.0002  
94 (86.6,87.5] 51.6770  
95 (87.5,88.4] 70.2406  
96 (88.4,89.3] 50.2647  
97 (89.3,90.3] 49.7290  
98 (90.3,91.2] 49.6333  
99 (91.2,92.1] 86.8271  
100 (92.1,93.1] 84.2437