# Tema 2. Modelización estadística.

1)

| Y\X | 0 | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|---|
| [0,4] | 3 | 3 | 1 | 0 | 0 | 0 | 7 |
| (4,6] | 3 | 4 | 2 | 0 | 0 | 0 | 9 |
| (6,8] | 1 | 3 | 2 | 1 | 0 | 0 | 7 |
| (8,12] | 0 | 1 | 1 | 2 | 3 | 2 | 9 |
| | 7 | 11 | 6 | 3 | 3 | 2 | 32 |

a) $Y/1 \leq x \leq 3$

| Y\X | 1 | 2 | 3 | $n_J^{1\leq x\leq 3}$ | $f_i$ | $x_i$ | $x_i^3$ | $x^4$ |
|---|---|---|---|---|---|---|---|---|
| [0,4] | 3 | 1 | 0 | 4 | 1/5 | 2 | 8 | 16 |
| (4,6] | 4 | 2 | 0 | 6 | 3/10 | 5 | 125 | 625 |
| (6,8] | 3 | 2 | 1 | 6 | 3/10 | 7 | 343 | 2401 |
| (8,12] | 1 | 1 | 2 | 4 | 1/5 | 10 | 1000 | 10000 |
| | 11 | 6 | 3 | 20 | | | | |

$\bar{x} = 6$. $\mu_3 = m_3 - 3m_2\bar{x} + 2\bar{x}^3 = 342 - 7.74 + 2\cdot 6^3 = 0$

$\sigma_x = 2{,}64575$

$m_3 = 342$ $\qquad g_1 = \frac{0}{\sigma^3} = 0$ Es simétrica.

$m_2 = 43$

$\mu_4 = m_4 - 4m_3\bar{x} + 6m_2\bar{x}^2 - 3\bar{x}^4 = 2911 - 8208 + 9288 - 3888$

$m_4 = 2911$ $\qquad \mu_4 = 103$.

$g_2 = \frac{103}{\sigma^4} - 3 = \frac{103}{49} - 3 = -0'897 < 0$

Es platicúrtica.

b) Rectas de regresión de X sobre Y y de Y sobre X.

Regresión X sobre Y

$x = a' + b'y$ $\qquad N = 32.$ $\quad \Sigma y_i = 198.$ $\quad \Sigma y_i^2 = 1496$

$\Sigma x_i = 54$ $\quad \Sigma x_i y_i = 430$

$$\begin{pmatrix} 32 & 198 \\ 198 & 1496 \end{pmatrix}\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} 54 \\ 430 \end{pmatrix}$$

$a' = 99/197 = -0'5025$

$b' = 767/2167 = 0'3539$

$$x = -0'5025 + 0'3539y$$

$$\left.\begin{array}{l} Na + b\sum x_i = \sum y_i \\ \sum x_i a + b\sum x_i^2 = \sum x_i y_i \end{array}\right\} \begin{array}{l} 32a + b\,54 = 198 \\ 54a + b\,160 = 430 \end{array}$$

$a = 3{,}8384 \qquad b = 1{,}3920$

$$\boxed{y = 3{,}8384 + 1{,}3920x}$$

c) $r = \sqrt{bb'} = \sqrt{1{,}3920 \cdot 0'3539} = \underline{\underline{0{,}70187}}$

$\sigma_{res}^2 = MSE \Rightarrow$ ¿?

$\sigma_y^2 = 8'464$

$\sigma_x^2 = 2'152$

$\sigma_{res}^2 = \sigma_y^2 - b^2\sigma_x^2$

$\sigma_{res}^2 = 8'464 - 1'39^2 \cdot 2'152 = 4{,}30612$

$\sigma_{res} = \sqrt{4{,}30612} = \underline{\underline{2{,}0751}}$

2) $\mathrm{cov}(x,y) = \sum_{i=1}^{k}\sum_{j=1}^{p}(x_i-\bar{x})(y_j-\bar{y})f_{ij} = \sum_{i=1}^{k}\sum_{j=1}^{p}x_i\, y_j\, f_{ij} - \bar{x}\bar{y}$

$$\sum_{i=1}^{k}\sum_{j=1}^{p}(x_iy_j - x_i\bar{y} - \bar{x}y_j + \bar{x}\bar{y})f_{ij} = \sum_{i=1}^{k}\sum_{j=1}^{p}x_iy_jf_{ij} \underbrace{- x_i\bar{y}f_{ij} - \bar{x}y_jf_{ij} + \bar{x}\bar{y}f_{ij}}^{1}$$

$x_i\bar{y}f_{i1} + x_i\bar{y}f_{i2} + \dots + x_i f_{ij} =$
$= x_i\bar{y}f_{i\cdot} = \bar{x}\bar{y}$

$$= \sum_{i=1}^{k}\sum_{j=1}^{p}x_iy_jf_{ij} - \bar{x}\bar{y} - \cancel{\bar{x}\bar{y}} + \cancel{\bar{x}\bar{y}}$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{p}x_iy_jf_{ij} - \bar{x}\bar{y}$$

3)

| $X$ | $Y_1$ | $Y_2$ |
|---|---|---|
| 1 | 4 | 1 |
| 2 | 2 | 3 |
| 3 | 3 | 5 |
| 4 | 2 | 7 |
| 5 | 4 | 9 |

Con $Y_1$:

$$\begin{pmatrix} N & \Sigma y_i \\ \Sigma y_i & \Sigma y_i^2 \end{pmatrix}\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \Sigma x_i \\ \Sigma x_iy_i \end{pmatrix}$$

$N=5 \quad \Sigma y_i = 15 \quad \Sigma y^2 = 49 \quad \Sigma x_i = 15 \quad \Sigma xy = 45$

$$\begin{cases} 15 = 5a' + b'15 \to a' = \dfrac{15-15b'}{5} \Rightarrow a' = 3-3b' \to \boxed{a'=3} \\ 45 = a'15 + b'49 \end{cases}$$

$\to 45 = (3-3b')15 + b'49$

$45 = 45 - 45b' + 49b'$

$r=0$

$0 = 4b' \Rightarrow \boxed{b'=0}$

Recta $X$ sobre $Y_1$

$\boxed{x=3}$

Con $Y_2$:

$$\begin{pmatrix} N & \Sigma y_i \\ \Sigma y_i & \Sigma y^2 \end{pmatrix}\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \Sigma x_i \\ \Sigma x_iy_i \end{pmatrix}$$

$N=5 \quad \Sigma y_i = 25 \quad \Sigma y^2 = 165. \quad \Sigma x_i = 15.$
$\Sigma xy = 95$

$$\begin{cases} 15 = 5a' + b'25 \to a' = \dfrac{15-25b'}{5} \Rightarrow a' = 3-5b' \quad \boxed{a' = 1/2} \\ 95 = a'25 + b'165 \end{cases}$$

$\to 95 = (3-5b')25 + b'165$

$19 = (3-5b')5 + 33b'$

$19 = 15 - 25b' + 33b'$

$4 = 8b' \Rightarrow \boxed{b' = 1/2}$

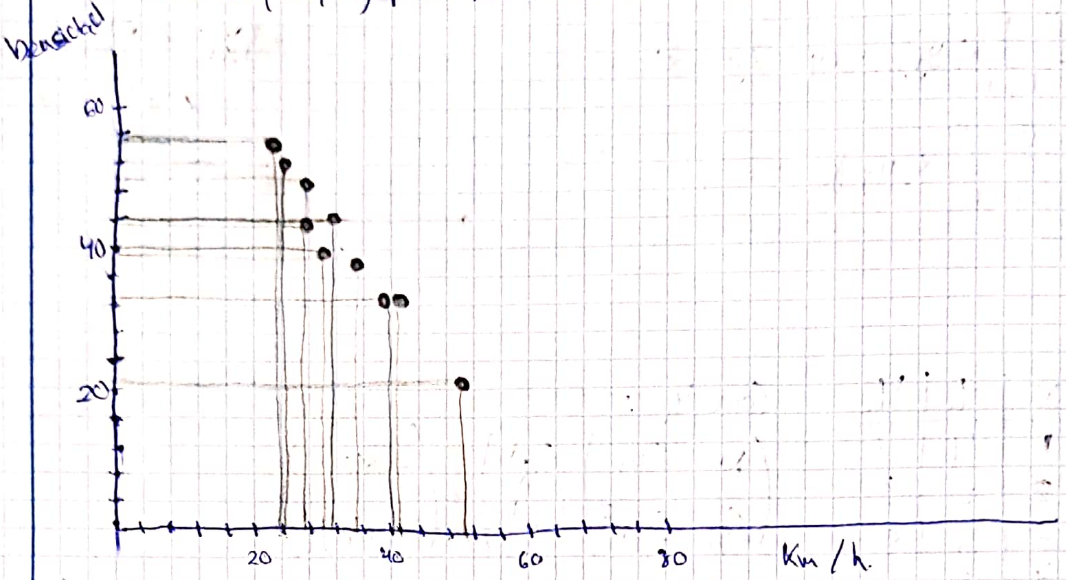
$x = \frac{1}{2} + \frac{1}{2}y$

$b' = \dfrac{\mathrm{Cov}(x,y)}{\sigma_y^2} \Rightarrow \mathrm{Cov}(x,y) = b'\sigma_y^2 = \frac{1}{2}\cdot 8 = 4$

$r = \dfrac{\mathrm{Cov}(x,y)}{\sigma_x\sigma_y} = \dfrac{4}{4} = 1$

4)

| Densidad (veh/Km) | 43 | 55 | 40 | 52 | 39 | 33 | 50 | 33 | 44 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| Velocidad (Km/h) | 27'0 | 23'8 | 30'7 | 21'0 | 34'8 | 41'4 | 27'0 | 40'4 | 31'7 | 51'2 |




b) -0'968, sentido descendente (negativo), relación entre las variables (se acerca a -1)

c) $Cov(x,y) = \frac{\Sigma xy}{N} - \bar{x}\bar{y} = 1282,26 - 1361,2 = -78,94.$

$r = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{-78,94}{81,52} = -0'968319$

d)

5) $r = \frac{Cov(x,y)}{\sigma_x \sigma_y}$ si $Cov(x,y) = 0 \Rightarrow r = \frac{0}{\sigma_x \sigma_y} = 0$

por lo tanto $r = 0$ son linealmente incorreladas

7) $\begin{cases} x - 2y = 4 \\ 2x - 9y = 8 \end{cases}$ $N = 10$ $\Rightarrow y = \frac{1x}{2} - \frac{4}{2} \Rightarrow y = \frac{1}{2}x - 2.$

$\sigma_x^2 = 9.$ $b = \frac{1}{2}$

$Cov(x,y) = b \cdot \sigma_x^2 = \frac{1}{2} 9 = \frac{9}{2} = \boxed{4'5 = Cov(x,y)}$

$\Rightarrow x = \frac{8}{2} + \frac{9}{2}y \Rightarrow 4 + \frac{9}{2}y = x$ $b' = \frac{9}{2}$ $\Rightarrow \frac{Cov(x,y)}{b'} = \sigma_y^2 = \frac{4'5}{4'5} = \boxed{1 = \sigma_y^2}$

$r = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{4'5}{\sqrt{9}} = 1,5$

MP2

7) $x - 2y = 4$ $N = 10$ $\sigma_x^2 = 9$

$2x - 9y = 8$

(1) $\begin{cases} y = \frac{1}{2}x - 2 \\ y = \frac{2}{9}x - \frac{8}{9} \end{cases}$

Identificamos rectas Y sobre X y X sobre Y.

$Y \backslash X$ : $y = a + bx$

$X/Y$ : $x = a' + b'y \rightarrow y = -\frac{a'}{b'} + \frac{x}{b'} \Rightarrow$ pendiente $\frac{1}{b'}$

Sabiendo que $|b| \leq \frac{1}{|b'|} \Rightarrow$ si $\frac{1}{2} \leq \frac{2}{9}$ NO SE CUMPLE

si $\frac{2}{9} \leq \frac{1}{2} \rightarrow$ SE CUMPLE entonces $\boxed{b = \frac{2}{9}}$ y $\frac{1}{b'} = \frac{1}{2} \Rightarrow \boxed{b' = 2}$

$Cov(x,y) = \sigma_x^2 \cdot b = 9 \cdot \frac{2}{9} = 2$

$r = \sqrt{bb'} = \sqrt{\frac{2}{9} \cdot 2} = 0,\overline{6}$

$\frac{Cov(x,y)}{b'} = \sigma_y^2 = 2/2 = 1$

b) Para conseguir la media, sabemos que el punto donde cortan ambas rectas los valores son las medias $(\bar{x}, \bar{y})$

En (1) sustituyo y.

$\frac{2}{9}x - \frac{8}{9} = \frac{1}{2}x - 2 \Rightarrow \frac{10}{9} = \frac{5}{18}x \Rightarrow \boxed{x = 4}$ $\boxed{y = 0}$

8) $y = a \cdot e^{bx}$ $\quad Y = A + bx.$

$\ln y = \ln a + bx \ln e$

$\boxed{\ln y = \ln a + bx}$ $\quad Y = \ln y. \quad A = \ln a.$

$y = a \cdot x^{b}$ $\quad Y = A + bX$

$\boxed{\ln y = \ln a + b \ln x}$ $\quad Y = \ln y;\ A = \ln a\ ,\ X = \ln x.$

$y = \dfrac{1}{a + bx}$. $\quad Y = a + bx$

$\boxed{a + bx = \dfrac{1}{y}}$ $\quad Y = \dfrac{1}{y}$

9) $Y = A + Bx.\ ;\ y = a \cdot b^{x} \Rightarrow \ln y = \ln a + x \ln b\ ,\ Y = \ln y\ ,\ A = \ln a, B = \ln b$

$NA + B\Sigma x_i = \Sigma Y$ $\quad N = 5 \quad \Sigma x_i = 15 \quad \Sigma x_i^2 = 55$

$A \Sigma x_i + B \Sigma x_i^2 = \Sigma x_i Y_i$ $\quad \Sigma Y = 9{,}5592 \quad \Sigma x_i Y_i = 32{,}695$

$5A + B15 = 9{,}5592 \to A = 1{,}911 - 3B.$

$15A + 55B = 32{,}695 \to 15(1{,}911 - 3B) + 55B = 32{,}695.$

$28{,}677 - 45B + 55B = 32{,}695.$

$10B = 4{,}017$

$e^{B} = b \leftarrow B = \ln b \leftarrow \boxed{B = 0{,}4017}$ $\quad b = 1{,}4943$

$\boxed{A = 0{,}7066}$ $\quad a = 2{,}027.$

$\boxed{y = 2{,}027 \cdot 1{,}4943^{x}}$

10) $y = a x^{b}$ $\quad Y = \ln y \quad A = \ln a \quad X = \ln x. \quad Y = A + bX.$

$N = 5 \quad \Sigma X = 4{,}7875 \quad \Sigma X^2 = 6{,}1995 \quad \Sigma Y = 6{,}1092 \quad \Sigma XY = 9{,}0805$

$NA + b\Sigma X = \Sigma Y \to 5A + b\Sigma X = \Sigma Y \to A = \dfrac{\Sigma Y}{5} - \dfrac{\Sigma X}{5} b.$

$A\Sigma X + b\Sigma X^2 = \Sigma XY \to \left(\dfrac{\Sigma Y}{5} - \dfrac{\Sigma X}{5} b\right) \cdot \Sigma X + b \Sigma X^2 = \Sigma XY$

$\dfrac{\Sigma Y \cdot \Sigma X}{5} - \dfrac{(\Sigma X)^2}{5} b + b \Sigma X^2 = \Sigma XY$

$1{,}6154\, b = 3{,}2309$

$\boxed{b \simeq 2}$

$A = -0'69314 \to \boxed{a = 0'5}$

$\boxed{Y = 0'5 \cdot x^{2}}$

11) $y = \frac{1}{a+bx}$  $Y = \frac{1}{y}$

$N = 5$  $\sum x_i = 15$  $\sum x_i^2 = 55$  $\sum Y = 15,\overline{03}$  $\sum xY = 55,\overline{09}$

$Na + b\sum x_i = \sum Y$

$a\sum x_i + b\sum x^2 = \sum xY$

$-3(5a + b15 = 15,\overline{03}) \rightarrow a = \frac{15,\overline{03} - 15}{5} = 6,06 \cdot 10^{-3}$

$15a + b55 = 55,\overline{09}$

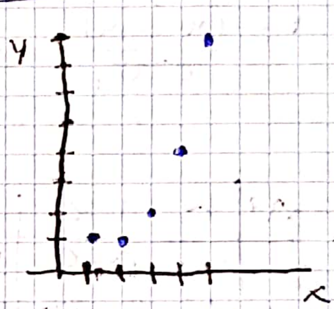
$0 + 10b = 10$

$\boxed{b = 1}$  $\boxed{a = 6,\overline{06} \cdot 10^{-3}}$

$$\boxed{Y = \frac{1}{6 \cdot 10^3 + x}}$$

12)

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| y | 1 | 1 | 2 | 4 | 8 |




a) No sería conveniente, mejor sería usar una exponencial.

b) $y = a \cdot b^x$, $Y = \ln y$, $A = \ln a$, $B = \ln b$.

$N = 5$  $\sum x_i = 15$  $\sum x_i^2 = 55$  $\sum Y = 4,1588$  $\sum xY = 18,02182$

$3(A5 + B15 = \sum Y) \rightarrow A = \frac{\sum Y}{5} - \frac{15}{5} \cdot 0'5545 = -0,83172$

$15A + 55B = \sum xY$  $a = 0,4352$

$0 + 10B = 5,5451$

$B = 0'5545$  $y = 0,4352 \cdot 1,7411^x$

$b = 1,7411$

Escaneado con CamScanner

13)

| $x \backslash y$ | 0 | 1 | 2 | |
|---|---|---|---|---|
| 20 | 2 | 0 | 0 | 2 |
| 30 | 1 | 3 | 2 | 6 |
| 40 | 1 | 3 | 2 | 6 |
| 50 | 2 | 0 | 0 | 2 |
| | 6 | 6 | 4 | 16 |

$\to$

| x | y | $n_i$ |
|---|---|---|
| 20 | 0 | 2 |
| 30 | 0 | 1 |
| 40 | 0 | 1 |
| 50 | 0 | 2 |
| 20 | 1 | 0 |
| 30 | 1 | 3 |
| 40 | 1 | 3 |
| 50 | 1 | 0 |
| 20 | 2 | 0 |
| 30 | 2 | 2 |
| 40 | 2 | 2 |
| 50 | 2 | 0 |
| | | 16 |

$N = 16 \quad \Sigma x_i = 560 \quad \Sigma x_i^2 = 20800$

$\Sigma y_i = 14 \quad \Sigma xy = 490.$

$$\begin{cases} 16a + b\,560 = 14 \\ 560\,a + b\,20800 = 490 \end{cases}$$

$0 + b \cdot 1200 = 0$

$b = 0$

$\to a = \frac{14}{16} - \frac{560}{16} \cdot b$

$a = \frac{7}{8} - 35 \cdot 0 \Rightarrow a = \frac{7}{8} = 0'875$

b) $r = \frac{Cov(x,y)}{\sigma_x \sigma_y}$ . y $Cov(x,y) = b \cdot \sigma_x^2 = 0$ . ya que $b = 0$.

entonces $r = 0$.

c)

| $x \backslash y$ | 0 | 1 | 2 | |
|---|---|---|---|---|
| 20 | 2/16 | 0 | 0 | 1/8 |
| 30 | 1/16 | 3/16 | 2/16 | 6/16 |
| 40 | 1/16 | 3/16 | 2/16 | 6/16 |
| 50 | 2/16 | 0 | 0 | 1/8 |
| | 6/16 | 6/16 | 1/4 | 1 |

No son independientes

Por ejemplo:

$f_{3\cdot} \cdot f_{\cdot 3} \neq f_{33} \Rightarrow \frac{6}{16} \cdot \frac{1}{14} \neq \frac{2}{16}$

d)

$$\begin{pmatrix} N & \Sigma x_i & \Sigma x_i^2 \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \Sigma y_i \\ \Sigma x y_i \\ \Sigma y x^2 \end{pmatrix}$$

$N = 16 \quad \Sigma x_i = 560 \quad \Sigma x_i^2 = 20800 \quad \Sigma y_i = 14 \quad \Sigma xy = 490.$

$\Sigma x_i^3 = 812000 \quad \Sigma x_i^4 = 33040000 \quad \Sigma yx^2 = 17500$

$$\left(\begin{array}{ccc|c} 16a & 560b & 20800c & 14 \\ 560a & 20800b & 812000c & 490 \\ 20800a & 812000b & 33040000c & 17500 \end{array}\right)$$

$\to a = -35/6 = -5{,}8\overline{3}$

$b = 49/120 = 0{,}408\overline{3}$

$c = -7/1200 = -5{,}8\overline{3} \cdot 10^{-3}$

$\sigma_x^2 = 75 \quad \sigma_x = 8{,}66025$

$\sigma_y = 0{,}7806$

$\sigma_{res}^2 = \sigma_y^2 - b^2\sigma_x^2 = 0{,}609375 - \left(\frac{49}{120}\right)^2 \cdot 75 =$

$\sigma_{res}^2 = -11{,}8958\overline{3}$

$S_{xy} = \left(\left(\sum_i x_i y_i\right) \frac{1}{n}\right) - \bar{x}\bar{y} = 0.$

$r = \frac{S_{xy}}{S_x S_y} = 0.$

14)

| Carga (Newtons) | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| Extensión (mm) | 10 | 19 | 29 | 40 | 48 | 56 |

$E = aC$ $N=6$ $\Sigma C_i = 42$ $\Sigma E_i = 202$ $\Sigma C_i E_i = 1742$ $\Sigma C_i^2 = 364$

$M = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{pmatrix}$ $A = (a_0)$ $a_0 = a$, $E = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix}$

$M^t M A = \Sigma C^2 \cdot a$
$M^t y = \Sigma C_i E_i$ $\rightarrow \Sigma C_i^2 \cdot a = \Sigma C_i E_i$

$\boxed{a = \frac{1742}{364} = 4'7857}$

$\boxed{E = 4'7857 \cdot C}$

15)

| x | 5 | 5 | 5 | 5 | 10 | 10 | 10 | 10 | 15 | 15 | 15 | 15 | 20 | 20 | 20 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| z | 28 | 30 | 48 | 74 | 29 | 50 | 57 | 42 | 20 | 24 | 31 | 47 | 9 | 18 | 22 | 31 |

a) $z = a + bx + cy$

$M^t M A = M^t z$

$$M^t M A = \begin{pmatrix} N & \Sigma x_i & \Sigma y_i \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i y_i \\ \Sigma y_i & \Sigma x_i y_i & \Sigma y_i^2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \Sigma z_i \\ \Sigma x_i z_i \\ \Sigma y_i z_i \end{pmatrix} = M^t z$$

b) $N = 16$ $\Sigma x_i = 200$ $\Sigma x_i^2 = 3000$ $\Sigma y_i = 40$ $\Sigma y_i^2 = 120$ $\Sigma x_i y_i = 500$
$\Sigma z_i = 560$ $\Sigma x_i z_i = 6110$ $\Sigma y_i \cdot z_i = 1580$

$$\left(\begin{array}{ccc|c} 16 & 200 & 40 & 560 \\ 200 & 3000 & 80 & 6110 \\ 40 & 500 & 120 & 1580 \end{array}\right)$$

$a = 139/4 = 34'75$
$b = -89/50 = -1'78$
$c = 9$

16) $z = ax + by$ $\Sigma x_i z_i = 41$ $\Sigma y_i z_i = 76$.

$\Sigma x = 4$ $\Sigma x_i^2 = 19$
$\Sigma y = 22$ $\Sigma y_i^2 = 76$ $\rightarrow \begin{pmatrix} \Sigma x^2 & \Sigma xy \\ \Sigma xy & \Sigma y^2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \Sigma zx \\ \Sigma zy \end{pmatrix}$
$\Sigma z = 24$ $\Sigma x_i y_i = 34$

$$\left(\begin{array}{cc|c} 19 & 34 & 41 \\ 34 & 76 & 76 \end{array}\right) \rightarrow \begin{array}{l} a = 1'8472 \\ b = 0'17361 \end{array}$$