Probability and Statistics Exam, 10

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1) The probability of a certain basketball player making a free throw is known to be 0.6. Find the probability of the following events:

   a) **(1 point)** A : the player makes his first free throw only on the 5^{th} shot;

   b) **(2 points)** B : the player makes the first at least 7 consecutive free throws.

~~2)~~ **(2 points)** Let $X \in \chi^2(2, 1/2)$. Find the pdf of $Y = \sqrt{X}$.

~~3)~~ Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = \theta x^{\theta-1}$, for $0 < x < 1$, with $\theta > 0$, unknown.

   ~~a)~~ **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

   ~~b)~~ **(2 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.

10. A food store owner receives 1-liter water bottles from two suppliers. After some complaints from customers, he wants to check the accuracy of the weights of 1-liter water bottles. He finds the following weights (the two populations are approximately normally distributed):

	Weights													
Supplier A	1021	980	1017	988	1005	998	1014	985	995	1004	1030	1015	995	1023
Supplier B	1070	970	993	1013	1006	1002	1014	997	1002	1010	975			

a. At the 5% significance level, do the population variances seem to differ?
b. At the same significance level, on average, does Supplier A seem to be more reliable?

. A study is conducted to compare the total printing time in seconds of two brands of laser printers on ious tasks. Data below are for the printing of charts (normality of the two populations is assumed):

	Printing times											
Brand A	29.8	30.6	29.0	27.7	29.9	29.6	30.5	31.1	30.2	28.1	29.4	28.5
Brand B	31.5	30.2	31.2	29.0	31.4	31.1	32.5	33.0	31.3	30.9	30.7	29.9

a. At the 5% significance level, do the population variances seem to differ?
b. At the same significance level, does the Brand A printer seem to be faster, on average?

07.02.2021

18 1) A contestant participates in a game show where two important prizes are offered. His chances of winning the two prizes are $\frac{2}{3}$ and $\frac{1}{2}$, respectively.

a) **(1 point)** Find the probability that the contestant loses at least one prize.

b) **(1 point)** Let X denote the number of prizes the contestant wins. Find the pdf of X.

c) **(1 point)** How many prizes can the contestant expect to win?

19 2) **(1.5 points)** Let X be a random variable with pdf $f(x) = \frac{1}{5}e^{-x/5}$, $x > 0$. Find the pdf of $Y = 2\sqrt{X}$. P19

20 3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a $Gamma(2, 2\theta)$ distribution, with $\theta > 0$ unknown. (for $X \in Gamma(a, b)$, the pdf is $f(x; a, b) = \frac{1}{b^a\Gamma(a)}x^{a-1}e^{-x/b}$, $x > 0$, $E(X) = ab$, $V(X) = ab^2$, $\Gamma(n+1) = n!, n \in \mathbb{N}$)

a) **(0.5 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

b) **(1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.

c) **(1 point)** Is $\overline{\theta}$ an absolutely correct estimator? Explain.

d) **(1.5 points)** Find the efficiency $e(\overline{\theta})$.

~~21~~ ~~1)~~ **(1.5 points)** Let X be a random variable with pdf $f(x) = 10e^{-10x}$, $x > 0$. Find the pdf of $Y = \frac{1}{3}X - 5$.

2) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = C_4^x\theta^x(1-\theta)^{4-x}$, $x = 0, 1, \ldots, 4$, $E(X) = 4\theta$, $V(X) = 4\theta(1-\theta)$, with $\theta \in (0,1)$ unknown.

a) **(0.5 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

b) **(1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.

c) **(1 point)** Is $\overline{\theta}$ an absolutely correct estimator? Explain.

d) **(1.5 points)** Find the efficiency $e(\overline{\theta})$.

~~22~~ ~~3)~~ Documents, as rare events, arrive at a printer, on average, every 10 seconds.

~~a)~~ **(1 point)** Find the probability that at least 8 printing jobs arrive in one minute.

~~b)~~ **(1 point)** In a minute, knowing that at least 5 documents arrived at the printer, what is the probability that exactly 3 did?

~~c)~~ **(1 point)** How many documents can be expected to arrive at the printer in a minute and a half?

1) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a $Gamma(2, \theta)$ distribution, with $\theta > 0$ unknown. (for $X \in Gamma(a, b)$, the pdf is $f(x; a, b) = \frac{1}{b^a\Gamma(a)}x^{a-1}e^{-x/b}$, $x > 0$, $E(X) = ab$, $V(X) = ab^2, \Gamma(n+1) = n!, n \in \mathbb{N}$)

a) **(0.5 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

b) **(1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.

c) **(1 point)** Is $\overline{\theta}$ an absolutely correct estimator? Explain.

d) **(1.5 points)** At the significance level $\alpha \in (0, 1)$, find a most powerful test for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$.

2) A student has to take two exams. His chances of passing the two tests are $\frac{4}{5}$ and $\frac{3}{4}$, respectively.

a) **(1 point)** Find the probability that the student fails at least one exam.

b) **(1 point)** Let X denote the number of exams the student passes. Find the pdf of X.

c) **(1 point)** How many tests can the student expect to pass?

3) **(1.5 points)** The pdf of the random variable X is given by $f_X(x) = 2e^{-2x}$, $x > 0$. Find the pdf of $Y = \frac{1}{3}X - 2$.

1) **(1.5 points)** Let X be a random variable with pdf $f(x) = 5e^{-5x}$, $x > 0$. Find the pdf of $Y = \sqrt{X}$.

2) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x; \lambda) = \lambda^x(1-\lambda)^{1-x}$, $x = 0, 1$, $E(X) = \lambda$, $V(X) = \lambda(1 - \lambda)$, where $\lambda \in (0, 1)$ is unknown.

a) **(0.5 points)** Find the method of moments estimator, $\hat{\lambda}$, for λ.

b) **(1.5 points)** Find the maximum likelihood estimator, $\overline{\lambda}$, for λ.

c) **(1 point)** Is $\overline{\lambda}$ an absolutely correct estimator? Explain.

d) **(1.5 points)** Find the efficiency $e(\overline{\lambda})$.

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3) There are 10 plates of hot food left at a cafeteria, 6 of which are vegetarian. Four students come to lunch late and randomly pick up a plate.

a) **(1 point)** Find the probability that at least half of the students got a vegetarian lunch.

b) **(1 point)** How many students can expect to get a non-vegetarian lunch?

c) **(1 point)** Let X be the number of students getting a vegetarian lunch. Prove that $5P(-3 < X < 3) \geq 1$.

1) The probability that the battery of a particular car brand does not start is 0.03.

- **a) (1 point)** Find the probability that the battery only starts on the 3^{rd} attempt.
- **b) (1 point)** Find the probability that the battery starts only on the first 25 attempts.
- **c) (1 point)** How many times in a row is the battery expected to start before it dies?

~~25~~ **2)** **(1.5 points)** Let X be a random variable with pdf $\begin{pmatrix} -1 & 0 & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$. Find the pdf of $Y = X^2 + 1$.

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = (1+\theta)x^\theta$, for $0 < x < 1$, with $\theta > -1$, unknown.

- **a) (1 point)** Find the population mean and variance.
- **b) (1 point)** Find the method of moments estimator, $\hat{\theta}$, for θ.
- **c) (1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.
- ! 27 **d) (1 point)** Compute the Fisher information of the sample.

1) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}$, $x > 0$, $E(X) = \theta$, $V(X) = \theta^2$, where $\theta > 0$ is unknown.

- **a) (0.5 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.
- **b) (1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.
- **c) (1 point)** Is $\overline{\theta}$ an absolutely correct estimator? Explain.
- **d) (1.5 points)** At the significance level $\alpha \in (0,1)$, find a most powerful test for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

2) There are 30 computers in a classroom, 4 of which are very slow. Twenty students come to class and are seated randomly each in front of a computer.

- **a) (1 point)** Find the probability that at most half of the slow computers are used.
- **b) (1 point)** Knowing that at least one slow computer was used, what is the probability that 2 students use a slow computer?
- **c) (1 point)** Let X be the number of students using a slow computer. Show that $9P(-3 < X < 3) \geq 1$.

3) **(1.5 points)** Let $X \in N(0,1)$. Find the pdf of $Y = 2X + 1$.

24

1) Ten people donate blood at a clinic, 3 of which have blood type 0 ("universal donors"). Five people get in first.

a) (1 point) Find the probability that at least 2 of them do not have blood type 0.

b) (1 point) What is the expected number of universal donors going in?

c) (1 point) Let X be the number of universal donors that go in. Prove that $4P((X \leq -2) \cup (X \geq 2)) \leq 3$.

2) (1.5 points) Let X be a random variable with pdf $f(x) = \frac{1}{2}e^{-x/2}$, $x > 0$. Find the pdf of $Y = \sqrt{X}$.

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = C_5^x\ \theta^x(1-\theta)^{5-x}$, $x = \overline{0,5}$, $E(X) = 5\theta$, $V(X) = 5\theta(1-\theta)$, where $\theta \in (0,1)$ is unknown.

a) (0.5 points) Find the method of moments estimator, $\hat{\theta}$, for θ.

b) (1.5 points) Find the maximum likelihood estimator, $\overline{\theta}$, for θ.

c) (1 point) Is $\overline{\theta}$ an absolutely correct estimator? Explain.

d) (1.5 points) Find the efficiency $e(\overline{\theta})$.

Examen - 11.02.2018 - Probabilităţi şi Statistică - 90 minute - I

1. (2p) Un program nou este testat cu ajutorul a 3 teste independente. Fiecare dintre cele trei teste găseşte o posibilă eroare în program cu probabilităţile 0.6, 0.5, respectiv 0.7. S-a constatat că programul nou conţine o eroare. Care este probabilitatea ca

a) toate cele trei teste să găsească eroarea? **b)** cel puţin un test să găsească eroarea?
c) cel mult două teste să găsească eroarea? **d)** exact două teste să găsească eroarea?

2. (2p) Funcţia de densitate a unei variabile aleatoare continue X este $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} ax^2, & |x| \leq 1, \\ 0, & |x| > 1, \end{cases}$$

unde $a > 0$ este necunoscut. Determinaţi:
a) valoarea lui a;
b) funcţia de repartiţie a lui X;
) varianţa lui X;
) dacă evenimentele $X > -\frac{1}{2}$ şi $X < \frac{1}{2}$ sunt independente sau nu.

3. (2p) Un producător de cipuri de calculatoare afirmă că nu mai mult de 1% din cipurile, pe care le produce, ar fi defecte. O companie de electronice, este impresionată de această afirmaţie şi a cumpărat o cantitate mare de astfel de cipuri. Pentru a verifica dacă afirmaţia producătorului este adevărată, compania a decis să testeze un eşantion de 9900 de cipuri şi constată că 110 dintre aceste cipuri sunt defecte. Poate fi afirmaţia producătorului acceptată sau sunt suficiente dovezi pentru a fi respinsă? (α=0.01)

a	0.005	0.01	0.03	0.06	0.94	0.97	0.99	0.995
$norminv(a,0,1)$	-2.58	-2.33	-1.88	-1.55	1.55	1.88	2.33	2.58
$tinv(a,9899)$	-2.58	-2.3	-1.9	-1.6	1.6	1.9	2.3	2.58
$chi2inv(a,9899)$	9540	9575	9636	9681	10119	10165	10229	10265

Justificaţi toate răspunsurile.

Probability and Statistics Exam, 2

4 A software firm produces accounting programs. The probability that one of their programs is defective is 0.2. A chain store purchases 20 accounting programs from that firm.

a) **(1 point)** Find the probability that exactly 3 programs are defective.

b) **(1.5 point)** Find the probability that at least 80% of the programs are working properly.

c) **(1.5 points)** Let X denote the number of programs that are working properly. Find the probability distribution function of X. What type of distribution is it?

d) **(0.5 points)** What is the expected number of programs that are working properly?

5 Let $X_1, X_2, ..., X_n$ be a random sample drawn from a *Poisson* distribution with parameter λ, unknown. (for $X \in \mathcal{P}(\lambda)$, the pdf is $p(x; \lambda) = \frac{\lambda^x}{x!} e^{-\lambda}$, $x = 0, 1, ...,$ $E(X) = V(X) = \lambda$).

a) **(1.5 points)** Find the maximum likelihood estimator, $\overline{\lambda}$, for λ.

b) **(0.5 points)** Is it an absolutely correct estimator? Explain.

c) **(1.5 points)** Find the efficiency of $\overline{\lambda}$, $e(\overline{\lambda})$.

d) **(1 point)** At the significance level $\alpha \in (0, 1)$, find a most powerful test for testing $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$.

Probability and Statistics Exam, 9

6 1) The battery of a particular car brand starts with probability 0.95. Find the probability of the following events:

a) **(1 point)** A : the battery only starts on the 5^{th} attempt;

b) **(2 points)** B : the battery starts on the first at least 20 consecutive attempts.

7 2) **(2 points)** Let $X \in Exp(\mu)$. Find the pdf of $Y = \sqrt{X}$.

8 3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x; \theta) = \frac{2}{\theta^2} x$, for $0 < x < \theta$, with $\theta > 0$ unknown.

a) **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

b) **(2 points)** Is $\hat{\theta}$ an absolutely correct estimator? Explain.

Probability and Statistics Exam, 13

1) A contestant participates in a game show where three important prizes are offered. His chances of winning the three prizes are $\frac{1}{6}$, $\frac{1}{3}$ and $\frac{1}{2}$, respectively.

a) **(1 point)** Find the probability that the contestant wins exactly one prize.

b) **(1.5 points)** Find the probability that the contestant loses at least two prizes.

c) **(1.5 points)** Let X denote the number of prizes won by the contestant. Find the probability distribution function of X.

d) **(1 point)** How many prizes can the contestant expect to win?

2) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a $Gamma(2, 3\theta)$ distribution, with $\theta > 0$ unknown. (for $X \in Gamma(a,b)$, the pdf is $f(x;a,b) = \frac{1}{b^a\Gamma(a)}x^{a-1}e^{-x/b}$, $x > 0$, $E(X) = ab$, $V(X) = ab^2$)

a) **(1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.

b) **(0.5 points)** Is it an absolutely correct estimator? Explain.

c) **(2 points)** Find the efficiency of $\overline{\theta}$, $e(\overline{\theta})$.

Probability and Statistics Exam, 8

9 **1)** A basketball player makes a free throw with probability 0.7. Find the probability of the following events:

a) **(1 point)** A : the player makes his first free throw only on the 4^{th} shot;

b) **(2 points)** B : the player makes the first at least 10 consecutive free throws.

10 **2)** **(2 points)** Let $X \in N(0,1)$. Find the pdf of $Y = X^2$. What type of distribution is it? ~~R~~10

11 **3)** Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = \frac{1}{\theta}$, for $0 < x < \theta$, with $\theta > 0$ unknown.

a) **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

b) **(2 points)** Is $\hat{\theta}$ an absolutely correct estimator? Explain.

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Probability and Statistics Exam, 7

1) The probability that the battery of a particular car brand does not start is 0.03. Find the probability of the following events:

   a) **(1 point)** A : the battery only starts on the 3^{rd} attempt;

   b) **(2 points)** B : the battery starts on the first at least 25 consecutive attempts.

2) **(2 points)** Let $X \in \chi^2(2,1)$. Find the pdf of $Y = \sqrt{X}$.

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = (1+\theta)x^{\theta}$, for $0 < x < 1$, with $\theta > -1$, unknown.

   a) **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

   b) **(2 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.