Option pricing using different techniques

Programming in Finance I

Rolands Repetto

Faculty of Economics
Università della Svizzera Italiana
Lugano, Switzerland

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1 Introduction

An option is a contract with a predetermined exercise date or period. Options are products that can yield a return much higher than simply holding and selling stocks, and this is why options, futures and derivatives took over the world of finance since their apparition in the 80s. (Hull, 2012) Two main type of option contracts appear: Call and Put. This is the case if our contract stipulates the 'option' to buy (call) or sell (put) the predetermined stock at a predetermined strike price. However, in this project I include exotic products such as Asian, Digital, Knock-in, and Knock-Out options.

Today, option prices are set using complex implied volatility sourced from current market conditions. As scholars, we will use historical volatility of returns to power our a simplified version of Monte Carlo simulations and binary trees. These results may differ from those on Bloomberg terminals, however main goal is to present a welcoming method for option pricing - easy to understand by anyone. In this paper, I will explain how these models work and will discuss the results' discrepancy between Binomial Methods and Monte Carlo Methods. Furthermore, introduction of Greeks will take place, which are an inevitable concept in the realm of options, evaluating the sensitivity of price change.

2 Economic Problem of pricing an option

The main economic problem of option pricing is the uncertainty associated with the future asset prices. Option pricing models aim to recreate the option price according to various factors, such as volatility, risk-free rate, current asset price, and expiration date. Nonetheless, accurately predicting the future movement of an assets price is still one of the main challenges in the financial industry. (Hull, 2012) Consequently, this leads to several discrepancies between the theoretical and actual value of the asset. Eventually, it allows traders to speculate or hedge in financial markets. (Hull, 2012)

However, the predictable models are not realistic. The model like Black-Scholes was created for a perfect capital market where no taxes, dividends, transactions costs existed. Thus, the model follows this float and demonstrates a perfect option's exercise. Yet the models' components, for instance, risk-free rate and volatility are assumed to remain constant throughout the period of option contract. (Black-Scholes, 1973) None of these will remain constant in a real world, for example, as seen in 2022, when the economy experienced high inflation leading to the increase of the interest rates. Consequently, stock market crashed and option prices were affected.

3 Description of mathematical and computational methods

In this research paper, we modeled the stock price of Tesla (TSLA) and Pfizer (PFE). We tried to calculate their current option price, using their historical daily returns. We used two main methods to calculate our underlying assets' option price - Monte Carlo Method and Binomial method.

3.1 Monte Carlo Simulation

Monte Carlo simulation is often used in financial industry and it mostly deals with derivative pricing and evaluation. It is a numerical method of solving mathematical problems by random sampling. (Pucci, 2019) This method is a great tool for option pricing to generate various random simulation of underlying asset's price. The simulation involves risk neutral environment, then we calculate the value of the option for the each path and, eventually, we take the average to calculate the present value of the option.

The method is used to calculate the expected value of the function of option price and these would be three main steps to estimate option price:

1. Calculate optimal N - number of simulations
2. Produce N independent random draws $\xi_1, \xi_2, ...\xi_n$
3. Use mean as estimator for the expectations: $\widehat{E[\xi]} = \frac{1}{N}\sum \xi_i$ (Gruber, 2023)

Since we have chosen a discrete time period of 252 trading days, in order to simulate Monte Carlo method, we used Geometric Brownian Motion:

$$\frac{\Delta x_t}{x_t} = \varepsilon_t \qquad \varepsilon_t \sim N(\mu, \sigma^2) \tag{1}$$

Geometric Brownian Motion helped us describe the stochastic movement of our financial assets and predict their prices in the next 252 day. The logarithmic value of our underlying assets' price follows Brownian motion with a drift, which represents average or expected return of underlying, and a diffusion, which captures random fluctuations or volatility in asset's price. (Bendob and Bentouir, 2019) To represent this random walk theory as T future stock price, we had to combine drift and diffusion into an equation:

$$S_{(t+1)} = S_t * e^{[(r-0.5\sigma^2)*\sqrt{\Delta}+\sigma*\sqrt{\Delta}*\varepsilon_t]} \tag{2}$$

3.1.1 American Option using Monte Carlo Method

In our pricing of an American option with Monte Carlo Method, we had to use the Least Squares Approach. This is because it was necessary to choose between exercising or continuing at each early exercise point. (Hull, 2012) Since most traders that buy the American option can do some computations, in order to exit at a preferable time we needed to implement their strategy in the pricing, if not American option prices would be too low. Thanks to AI we were able to implement a complex regression at the top of our code which implements the strategy. (Hull, 2012)

The trader has a tool for guessing the best exercise date (the regression). Every day, the regression looks at the stock price and helps the trader to decide if he should buy or wait for a better price. The regression does this by comparing: the current value of exercising the option and the predicted future value of the option if we don't exercise it today. Picking the Winning Paths: in-the-money is a filter that picks out only the winning paths, where exercising the option would be profitable. (Hull, 2012)

The regression helps us finding what payoffs the simulations that are doing better (in the money) might get in the future (will they keep increasing or not?). We predict the continuation values. This enables the trader to not sell to early.

3.2 Binomial Method

Binomial option pricing model allows us to see the evolution of the option price towards the periods. The model is based on certain assumptions:

1. The environment is considered to be a risk neutral.
2. There is no possibility for an arbitrage opportunity.
3. The asset is traded in a perfect market- no taxes, no transaction costs, consisting of one risk-free asset with risk-free rate r_f.
4. Eventually, the underlying asset can go either up or down by using risk-neutral probability.

The Binomial Model consists of three steps:

- The creation of binomial price tree - it helps to predict the possible future option's moves.

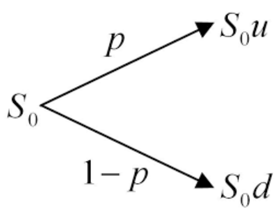

Figure 1: binomial tree

- The option valuation at each final period.
- Te calculation of option price in the previous period - backward induction.
- Final valuation at initial node (current time). (Pucci, 2019)

The binary tree method is much more effective for pricing American options. Options are evaluated by starting at the end of the tree (time T) and working backward. This is known as backward induction. Because a risk neutral world is being assumed, the value of each node at time T − delta t can be calculated as the expected value at time T discounted at rate r for a time period delta t. As the option is American it is necessary to check at each node to see whether early exercise is preferable this is included in our code. Eventually, by working back through all the nodes we are able to obtain the value of the option at time 0. (Bendob and Bentouir, 2019)

3.3 Main underlying assets - Tesla and Pfizer

We chose these two companies because of difference between them. Tesla is a growth stock and is more volatile, while Pfizer is more oriented in paying its earnings as dividends, thus its volatility is expected to be much lower.

Before conducting any simulations, we had to find the statistical components of our underlying assets, such as sample mean and sample standard deviation. These components were deviated from the historical returns between time frame of **11-28-2017** until **11-28-2023**. The following steps were conducted to find these significant components:

1. Historical six year Tesla's and Pfizer's stock price data was extracted from yahoo finance and cleaned.

2. To calculate daily returns we used Adjusted closing price.

3. We used python function $pctchange[:1]$ to calculate daily returns.

4. Lastly, we calculated sample mean and sample standard deviation.

4 Results - interpretation and discussion

Results can be tailored to our own desires by changing the parameters. Each function has various variables depending on the type of option being calculated. Simply adjust the desired variable in the cell where the result is printed.

Using Monte Carlo method and Binomial method, we simulated derivative prices for European, American, Digital, Knock-out and Knock-in, and Asian options. Additionally, we calculated Greeks for European and American options. Nevertheless, to conduct any simulations we had to set main option parameters for the simulations:

- The initial market price **S0** would be final Tesla's and Pfizer's stock price in our data, which are USD 236.08 and USD 30.13 respectively. Risk-free interest rate will be 5.37 %.

- Strike price **K** will be chosen independently based on model.

- Time of expiration - **1 year** and number of periods which are relevant for time steps in simulations - 252 days.
- The number of simulations: for Monte Carlo we will use **M=10000**, but for Binomial method we will use **N** between **10 and 36** as number of steps. The higher number of steps, the more accurate we price the option. However, in this case our computers are not able to process high number of steps quickly, therefore, we chose these numbers. (Pucci, 2019)

4.1 Sample statistics of underlying assets

After cleaning the data and calculating sample statistics parameters with **1502** as its number of samples, we found out that Tesla has an average daily return of **0.24 %** and average daily standard deviation of **4.01 %**. However, the average daily return of Pfizer was **0.019 %** and average daily volatility of **1.6 %**. These results display that Tesla stock in a period of six years has been a quite volatile stock and that has produced a significant gain, while Pfizer had a steady growth with decent volatility.

4.2 European Options

To calculate European options, we used USD 260 as strike price for Tesla to calculate call option, and USD 40 as strike price for Pfizer to calculate put option. In Binomial method, we had 36 steps. The results were:

Stock	Monte Carlo	Binomial Tree
Tesla	USD 0.70	USD 0.62
Pfizer	USD 7.81	USD 7.77

Table 1: European options' simulation results

The results in Table 1 demonstrate that simulations produced slightly identical results. Both methods were able to produce almost identical results for Pfizer's put option. Likewise, Tesla's prices were different between methods. Option prices are lower with Binomial method. It can be explained that this method breaks down time into discrete intervals and calculates the possible future price of underlying assets' at each interval. Meanwhile Monte Carlo uses more stochastic approach like Geometric Brownian motion and follows random walk. This method is more flexible.

4.3 American Options

For the American options, we had the same numbers of Strike price and number of steps for Binomial Tree. The results in Table 2 indicate that simulations produced close outputs comparing

Stock	Monte Carlo	Binomial Tree
Tesla	USD 0.65	USD 0.62
Pfizer	USD 9.30	USD 9.86

Table 2: American options' simulation results

with European options. While calls were priced quite close with respect to European options, simulations created more expensive puts for Pfizer. It can be explained with additional premium that is paid on purchasing American options. American options can be exercised any time and therefore dealers require additional premium for this privilege. (Hull, 2012)

4.4 Option Greeks

In this project, we also measured option Greeks for the European and American Options. Briefly, option Greeks help traders to measure various risk factors that influence option pricing. There are four main Greeks:

1. Delta (Δ) - measures impact of a change in the price of underlying. Measures are between -1 and 1.
2. Gamma (γ) - measures the rate of change of delta.
3. Theta (θ) - measures impact of a change in time remaining.
4. Vega (ν) - measures impact of a change in volatility.

Stock	Delta	Gamma	Theta	Vega
Tesla (MC) - EU	0.177	0.012	-1.45	54.6
Tesla (MC) - US	0.161	0.011	-2.62	51.5
Tesla (BM) - EU	0.181	0.013	-2.46	40.3
Tesla (BM) - US	0.181	0.013	-2.46	40.3

Table 3: Greeks for Tesla's American and European options

In tables 3 and 4 we demonstrate Greeks for Tesla's and Pfizer's options. For the Monte Carlo Method and Binomial Method we use abbreviations **MC** and **BM** respectively, and for European and American option - **EU** and **US**. We immediately find out that Greeks for Tesla stock were produced identical by Binomial method and quite similar by Monte Carlo method. However,

Stock	Delta	Gamma	Theta	Vega
Pfizer (MC) - EU	-0.999	4.79e-15	2.04	-0.235
Pfizer (MC) - US	-0.949	1.95e-14	0.541	0.110
Pfizer (BM) - EU	-1.00	-1.27e-13	2.04	-8.88e-11
Pfizer (BM) - US	-1.00	0.00	0.00	0.00

Table 4: Greeks for Pfizer's American and European options

Greeks for Pfizer were slightly different or extremely small like gamma.

Delta for call options were positive, meaning, that when Tesla stock rises by USD 0.17, the call option increases by this amount respectively. (Summa, 2021) Delta is negative for puts as it is demonstrated in Pfizer's results. Puts have negative relationship (correlation) with an underlying. Gammas for both underlying assets are mostly positive and small digit parameters. Tesla possesses greater gammas than Pfizer. This indicates that Tesla options have higher risks during unstable and unfavorable underlying movements, while Pfizer is more stable.(Summa, 2021)

Thetas are all negative for Tesla and positive for Pfizer. European options have higher Thetas than American because European options can be exercised at the maturity. There is less time to make profits. Eventually, Vegas are high for Tesla options and extremely low or negative for Pfizer stock. Therefore, Tesla stock is volatile and its options will respond more to changes in implied volatility. (Summa, 2021)

There is another Greek called Rho which we did not include in our analysis. The Rho of an option shows the rate of change in option price with respect to the interest rate. Given our simplified model we believed that changes in interest rates would add another degree of complexity and we decided to focus on volatility, time and price. (Hull, 2012)

As we can see the Greeks are influenced by the strike price and the fact their position (in or out of the market). Experts in structured products will use these values for complex strategies such as delta hedging or making a portfolio gamma neutral (Hull, 2012).

4.5 Some other options

We analyzed and simulated Knock-out, Knock-in, Digital and Asian Options. Number of binomial steps were 20. The main difference between knock-out and knock-in options is that when the underlying reaches the barrier, the option becomes worthless in knock-out contract. However, in knock-in contract, the underlying has to go over the predetermined barrier to activate option. (Chen, 2022)

In this case, we simulated call option for Tesla with an exercise price of USD 230 and barrier of

Stock	Tesla MC	Tesla BM	Pfizer MC	Pfizer BM
Knock-out	USD 0.21	USD 0.16	USD 0.29	USD 0.24
Knock-in	USD 3.12	USD 3.08	USD 1.57	USD 1.16

Table 5: Knock-out and Knock-in options

USD 240, and put option for Pfizer with strike price and barrier of USD 32 and USD 27 respectively. Although we mostly used call for Tesla and put for Pfizer, we swapped them for Knock-in options to get better results, avoiding low digit numbers.

The price estimates of the options with the two methods do not match perfectly. We can provide two reasons for this: first, being a barrier option, we could no longer use backward induction in the binomial tree because it does not consider the paths taken by prices. To address this, we used permutations to calculate all possible paths, considering all combinations of u and d, that the price could take. So, the first reason is the shift away from using backward induction, which had worked well for the first two options. Another reason could be that the Monte Carlo method performs better for barrier options due to its daily price simulations, allowing it to capture micro-fluctuations in prices. (Hull, 2012)

Method	Monte Carlo	Binomial
TSLA Digital	USD 0.17	USD 0.15
Pfizer Digital	USD 0.11	USD 0.50
TSLA Asian	USD 16.56	USD 22.57
Pfizer Asian	USD 0.84	USD 0.11

Table 6: American Digital and Asian option prices

Our research and simulations involved pricing American Digital and Asian options. In a few words, Digital option is a contract with a predetermined payoff if stock reaches certain price during the length of the contract. In our simulations, we had USD 260 for Tesla and USD 30 for Pfizer. Our payoff was USD 1. We observed that results were almost the same with both methods. While with Monte Carlo it is fairly easy to compute the percentage of paths that touched the barrier, with Binary Tree method we need to go back to every path. We achieved this thanks to permutations. (Hull, 2012)

Asian options are option where the payoff depends on the arithmetic average of the price of the underlying asset during a life of the option. (Hull, 2012) However, in our project we also added the geometrical average as we saw that option of this type are also popular. Another type of Asian option is an average strike option. An average strike option call pays off max (0, St – Save) and average put pays off max (0,Save – St). This is indicated by Price = True in our code: "payoff = max(0, avgvalue - K)" if the Price = 'else' then the option will be of the original type.

For Tesla, we used the average strike price method, while for Pfizer we used the Vanilla Asian Option method. We used arithmetic mean for Tesla and geometric mean for Pfizer. As results demonstrate us, Tesla is more expensive because it is in-the-money, therefore, it has intrinsic value. Pfizer is call and close to the money. In the calculation of Asian option prices for the two stocks, we observe larger differences. This can be attributed to the fact that in the binomial tree, the price changes only N times, whereas in the Monte Carlo method, we have 252 observations in a year. This makes both the average more accurate and the price fluctuations more realistic. Additionally, as in the case of the digital option, knock-out option, and knock-in option, we had to use permutations to calculate the path within the tree and then take the average.

5 Conclusion

The goal of our project was to provide and understandable approach to option pricing. Our project delved into classic options, as well as exotic options. The results from our research indicated the small discrepancy between the two pricing methods. This proves that both methods are relatively reliable to price an option under current conditions.

However, Binomial Method would have produced a closer match with Monte Carlo Method if we had used higher number of steps. The constraint of computer processing power did not allow us to produce closer results. In the future it would be interesting to understand how to build a portfolio of options using hedging techniques with Greeks and taking advantage of eventual mis-pricing.

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