Design and Analysis
of Algorithms I

# Introduction

# Karatsuba Multiplication

# Example

$x = 5678$, $a = 56$, $b = 78$

$y = 1234$, $c = 12$, $d = 34$

Step 1: compute $a \cdot c = 672$

Step 2: Compute $b \cdot d = 2652$

Step 3: Compute $(a+b)(c+d) = 134 \cdot 46 = 6164$

Step 4: Compute ③ − ② − ① $= 2840$

Step 5:

$$
\begin{array}{r}
6720000 \\
2652 \\
284000 \\
\hline
7006652
\end{array}
= (1234)(5678)
$$

# A Recursive Algorithm

Write $x = 10^{\frac{n}{2}} a + b$ and $y = 10^{\frac{n}{2}} c + d$
where $a, b, c, d$ are $\frac{n}{2}$-digit numbers.

[example: $a = 56, b = 78, c = 12, d = 34$]

**Then:** $x \cdot y = (10^{\frac{n}{2}} a + b) \cdot (10^{\frac{n}{2}} c + d)$

$$= 10^n ac + 10^{\frac{n}{2}} (ad + bc) + bd \qquad (*)$$

(simple base case omitted)

**Idea:** recursively compute $ac, ad, bc, bd$, then compute $(*)$ in the straightforward way.

# Karatsuba Multiplication

Recall: $x \cdot y = 10^n ac + 10^{\frac{n}{2}}(ad+bc) + bd$ $(*)$

Step 1: recursively compute $ac$

Step 2: recursively compute $bd$

Step 3: recursively compute $(a+b)(c+d) = \cancel{ac} + ad + bc + \cancel{bd}$

Gauss's trick: $3 - 1 - 2 = ad + bc$

Upshot: only need 3 recursive multiplications!
(and some additions)