



Design and Analysis
of Algorithms I

Introduction

Integer Multiplication

Integer Multiplication

Input: two n -digit numbers x and y .

Output: the product $x \cdot y$.

"Primitive Operation": add or multiply
2 single-digit numbers.

NOTE:

typed versions
of slides also
available!

The Grade-School Algorithm

Handwritten multiplication of 5678 by 1234:

$$\begin{array}{r}
 \begin{array}{cccc}
 2 & 2 & 3 & 3 \\
 5 & 6 & 7 & 8 \\
 \times & 1 & 2 & 3 & 4 \\
 \hline
 2 & 2 & 7 & 1 & 2 \\
 1 & 7 & 0 & 3 & 4 & - \\
 1 & 1 & 3 & 5 & 6 & - & - \\
 5 & 6 & 7 & 8 & - & - & - \\
 \hline
 7 & 0 & 0 & 6 & 6 & 5 & 2
 \end{array}
 \end{array}$$

n rows

$\leq 2n$ operations (per row)

Upshot: #operations overall $\leq \text{constant} \cdot n^2$
(like 4)

The Algorithm Designer's Mantra

“Perhaps the most important principle for the good algorithm designer is to refuse to be content.”

-Aho, Hopcroft, and Ullman, *The Design and Analysis of Computer Algorithms*, 1974

CAN WE DO BETTER?

(than the obvious /straightforward solution)