Descrizione del primo schema de Keller-Segel

December 14, 2017

1 Descrizione

- Time semi-discretization: we fix time steps $t_m = k \cdot m$ $(m = 0, 1, 2, ...)$ and given inital data $u^0, v^0 \in V = H^1(\Omega)$. For each find $m \geq 0$, we search u^{m+1} and v^{m+1} such that

$$(1/k)u^{m+1} - \Delta u^{m+1} = (1/k)u^m - k_1 \nabla \cdot (u^m \nabla v^m), \tag{1}$$

$$(1/k)v^{m+1} - \Delta v^{m+1} + k_2 v^{m+1} = (1/k)v^m + k_3 u^m. \tag{2}$$

As in other schema, we assume Neumann boundary conditions for each unknown: $\nabla u^{m+1} \cdot \mathbf{n} = 0$, $\nabla v^{m+1} \cdot \mathbf{n} = 0$, $m \geq 0$. Color highlights differences with previous Schema.

2 Validation

Here we propose some ideas to validate the software which we are developing (using `FreeFEM++`).

Test 1. Reproduction of results contained in Giuseppe's paper

Idea: if we use the data contained in this paper (inital data u^0 and v^0, time step, k_i parameters...), we must obtain the same results. For instance, blow-up in the time step which is reflected in graphics contained in that paper (*write here more details!*).

Test 2. Comparison with exact solution

Idea:

- To compute a exact solution, (u, v), to (a *modified version* of) Keller-Segel equations
- Use (a *modified version* of) scheme (1)–(2) and finite elements to aproximate the solution, (u_h^m, v_h^m).
- Compute errors $\|u - u_h^m\|_{L^2(\Omega)}$, $\|v - v_h^m\|_{L^2(\Omega)}$. When $k \to 0$ and $h \to 0$, errors must vanish.

Test 3. Discrete energy law

Idea: analyze the discrete energy law of scheme (1)–(2). Use *FreeFEM++* to compute this energy law. Plot the results and test if they are agree with the previous analysis.