In-class Quiz #7 Name: KEY

Please circle your answers! (:

(1) a) Find $h'(x)$ if $h(x) = \ln\left(g\left(\frac{3^x}{e^{x-1}}\right)\right)$.

$$h'(x) = \frac{1}{g\left(\frac{3^x}{e^{x-1}}\right)} g'\left(\frac{3^x}{e^{x-1}}\right) \frac{(e^{x-1})(3^x)\ln 3 - (3^x)(e^{x-1})}{(e^{x-1})^2}$$

$$= \frac{1}{g\left(\frac{3^x}{e^{x-1}}\right)} g'\left(\frac{3^x}{e^{x-1}}\right) \frac{(e^{x-1})(3^x)[\ln 3 - 1]}{(e^{x-1})^2}$$

$$\boxed{= \frac{1}{g\left(\frac{3^x}{e^{x-1}}\right)} g'\left(\frac{3^x}{e^{x-1}}\right) \frac{(3^x)[\ln 3 - 1]}{e^{x-1}}}$$

b) Find $\frac{dy}{dx}$ if $y = x^{(x^3)}$. $\Leftrightarrow \ln y = \ln\left(x^{(x^3)}\right)$

$\Leftrightarrow \ln y = x^3 \ln x$

So,

$$\frac{1}{y}\frac{dy}{dx} = x^3\left(\frac{1}{x}\right) + (\ln x)(3x^2)$$

$$\frac{dy}{dx} = [x^2 + 3x^2 \ln x]\,y$$

$$\boxed{\frac{dy}{dx} = [x^2 + 3x^2 \ln x]\,x^{(x^3)}}$$

(2) You are adding a layer of special insulating material to the outer surface of the **hemispherical** dome of a large building. The radius of the dome is 50 feet and the insulating material will have a thickness of 0.10 feet.

Note: The volume V of a sphere of radius r is $V = \frac{4\pi}{3} r^3$.

a) (11 pts.) Use differentials to estimate how much of the insulating material (in cubic feet) you will have to purchase for the project.

$V = \frac{1}{2}\left(\frac{4\pi}{3} r^3\right)$ is the volume of a hemisphere

$= \frac{2\pi}{3} r^3$

$\frac{dV}{dr} = \frac{2\pi}{3}(3r^2)$

$\Rightarrow dV = 2\pi r^2 dr$, and when $r = 50$ and $dr = 0.10$,

$dV = 2\pi (50)^2 (0.10)$

$\Leftrightarrow dV = 2\pi (2500)(0.10)$

$\Leftrightarrow \boxed{dV = 500\pi \text{ ft}^3}$

b) (3 pts.) Structural engineers estimate the dome of the building can safely support an additional 90,000 pounds. The insulating material you are using weighs 60 pounds per cubic foot. Use your result from part (a) to determine if your insulating project puts the building in jeopardy.

Weight $\approx (500\pi)(60) = 30{,}000\pi$ pounds

$30{,}000\pi > 90{,}000$ so the project puts the building in jeopardy.

(3) A sample of Strontium-90 will decay to 25% of its original amount in 56 days.

a) ~~(6 pts.)~~ A sample has a mass of 60 mg initially. Find the amount remaining after 70 days. Give your final answer in terms of e and natural logarithms.

Let m = mass of Strontium-90

$m(t) = 60e^{kt}$

we know $m(56) = 0.25(60) = 15$, so $15 = 60e^{k(56)}$

$\Leftrightarrow \frac{1}{4} = e^{k(56)}$

$\Leftrightarrow \ln\frac{1}{4} = \ln e^{k(56)}$

$\Leftrightarrow \ln\frac{1}{4} = 56k$

Thus, $k = \frac{\ln(\frac{1}{4})}{56}$

So $m(t) = 60e^{\frac{\ln(\frac{1}{4})}{56}t}$.

At $t = 70$, we have $\boxed{m(70) = 60e^{\frac{\ln(\frac{1}{4})}{56}\cdot 70} \text{ mg}}$

b) ~~(6 pts.)~~ How long will it take the initial amount of 60 mg to decay to 6 mg? Give your final answer in terms of natural logarithms.

Recall $m(t) = 60e^{\ln(\frac{1}{4})t}$ from above

we want to solve $m(t) = 6$, so:

$6 = 60e^{\frac{\ln(\frac{1}{4})}{56}t}$

$\Leftrightarrow \frac{6}{60} = e^{\frac{\ln(\frac{1}{4})}{56}t}$

$\Leftrightarrow \ln(\frac{1}{10}) = \ln e^{\frac{\ln(\frac{1}{4})}{56}t}$

$\Leftrightarrow \ln(\frac{1}{10}) = \frac{\ln(\frac{1}{4})}{56}t$

Thus, $\boxed{t = \frac{56\ln(\frac{1}{10})}{\ln(\frac{1}{4})} \text{ days}}$

(4) (16 pts.) Let f and g be everywhere differentiable functions. Let $h(x) = \frac{1}{27}[f(g(x))]^2$. Use the values in the table below to answer questions a-c below.

	$x=0$	$x=1$	$x=2$	$x=3$
$f(x)$	0	1	8	27
$f'(x)$	0	3	12	27
$g(x)$	1	3	9	19
$g'(x)$	0	2	4	12

a) ~~(4 pts.)~~ Find $h(1)$.

$$h(x) = \frac{1}{27}[f(g(x))]^2$$

$$\Rightarrow h(1) = \frac{1}{27}[f(g(1))]^2$$

$$= \frac{1}{27}[f(3)]^2$$

$$= \frac{1}{27}[27]^2$$

$$\boxed{= 27}$$

b) ~~(7 pts.)~~ Find $h'(1)$.

$$h'(x) = \frac{2}{27}[f(g(x))]f'(g(x))g'(x)$$

$$\Rightarrow h'(1) = \frac{2}{27}[f(g(1))]f'(g(1))g'(1)$$

$$= \frac{2}{27}[f(g(1))]f'(3)g'(1)$$

$$= \frac{2}{27}[f(3)]f'(3)g'(1)$$

$$= \frac{2}{27}(27)(27)(2)$$

$$\boxed{= 108}$$

c) ~~(5 pts.)~~ Find the equation of the line tangent to the curve $y = h(x)$ at $x = 1$.

$$\boxed{y - 27 = 108(x-1)}$$