In-class Quiz #6

Name: KEY

*Please **circle** your answers (:*

(1) a) Differentiate with respect to t.

$$y = d\cos t + t^2 \sin t$$

$$y' = d(-\sin t) + \cos t \cdot (0) + t^2(\cos t) + \sin(2t)$$

$$= t^2\cos t + 2t\sin t - d\sin t$$

b) Differentiate.

$$y = \frac{2x}{5 - \cot x}$$

$$y' = \frac{(5 - \cot x)(2) - (2x(\csc^2 x))}{(5 - \cot x)^2}$$

$$= \frac{10 - 2\cot x - 2x\csc^2 x}{(5 - \cot x)^2}$$

Note: Key for Quiz #5 is wrong!

(2) Find the derivative of the function.

a) $$y = \frac{r}{\sqrt{r^2 + 8}}$$

$$y' = \frac{(r^2+8)^{1/2}(1) - r\left(\frac{1}{2}(r^2+8)^{-1/2}\cdot 2r\right)}{(\sqrt{r^2+8})^2}$$

$$= \frac{(r^2+8)^{1/2} - r^2(r^2+8)^{-1/2}}{r^2+8}$$

Note: Key for Quiz #5 is wrong!

b) $F(t) = e^{9t\sin 2t}$

Aside: $\frac{d}{2t}\, 9t\sin 2t = 9t\cos 2t \cdot (2) + \sin(2t)$

$$F'(t) = e^{9t\sin 2t}\left[(18t\cdot\cos(2t)) + 9\sin(2t)\right]$$

(3) a) Find dy/dx by implicit differentiation.

$5 \cos x \sin y = 4$

$y' =$

$\cos x (\cos y)\frac{dy}{dx} + \sin y(-\sin x) = 0$

$\Leftrightarrow \cos x \cos y \left(\frac{dy}{dx}\right) - \sin x \sin y = 0$

$\Leftrightarrow \frac{dy}{dx}(\cos x \cos y) = \sin x \sin y$

$\Leftrightarrow \boxed{\frac{dy}{dx} = \frac{\sin x \sin y}{\cos x \cos y}}$

b) Find dy/dx by implicit differentiation.

$e^{x/y} = 7x - y$

$y' =$

$e^{x/y} \cdot \left[\frac{y - x\frac{dy}{dx}}{y^2}\right] = 7 - \frac{dy}{dx}$

$\Leftrightarrow -\frac{dy}{dx}(y^2 - xe^{x/y}) = e^{x/y}y - 7y^2$

$\Leftrightarrow \boxed{\frac{-ye^{x/y} + 7y^2}{y^2 - xe^{x/y}} = \frac{dy}{dx}}$

(4) a) Differentiate the function.

$f(x) = \sin(x)\ln(4x)$

$f'(x) = \sin x \left(\frac{1}{4x}\right)4 + \ln(4x)\cos x$

$\boxed{= \frac{\sin x}{x} + \ln 4x \cos x}$

b) Differentiate the function.

$G(y) = \ln \frac{(2y+1)^5}{\sqrt{y^2+1}} = \ln(2y+1)^5 - \ln(\sqrt{y^2+1}) = 5\ln(2y+1) - \frac{1}{2}\ln(y^2+1)$

$G'(y) = 5\frac{1}{2y+1}(2) - \frac{1}{2}\cdot\frac{1}{y^2+1}(2y)$

$\boxed{= \frac{10}{2y+1} - \frac{y}{y^2+1}}$

2 (5) *Leave onswers NOT simplified! (Only for this question)*

The height (in meters) of a projectile shot vertically upward from a point 2 m above ground level with an initial velocity of 25.5 m/s is $h = 2 + 25.5t - 4.9t^2$ after t seconds. ~~(Round your answers to two decimal places.)~~

(a) Find the velocity after 2 s and after 4 s.

$v(2) = 25.5 - 9.8(2)$ m/s

$v(4) = 25.5 - 9.8(4)$ m/s

(b) When does the projectile reach its maximum height?

Set $25.5 - 9.8t = 0$

$\Rightarrow t = \frac{25.5}{9.8}$ s

(c) What is the maximum height?

Plug the answer for (b) into the equation for h above (answer in m)

(d) When does it hit the ground?

Set $h = 0$ and use the positive t you acquire (answer in s)

(e) With what velocity does it hit the ground?

Plug the answer for (d) into the equation for v above (answer in m/s)

2 (6) a) Find $\frac{dy}{dx}$ if $y = \sqrt{[f(x)]^2 g(2x)}$.

$y = \left([f(x)]^2 g(2x)\right)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}\left([f(x)]^2 g(2x)\right)^{-1/2} \cdot \frac{d}{dx}\left[[f(x)]^2 g(2x)\right]$

$$\boxed{\frac{dy}{dx} = \frac{1}{2}\left([f(x)]^2 g(2x)\right)^{-1/2}\left[[f(x)]^2 g'(2x)(2) + g(2x)(2)f(x)f'(x)\right]}$$

b) Find $\frac{dy}{dx}$ if $\sin(x)\cos(y) = \sin(x) + \cos(y)$.

$\sin(x)\left[-\sin(y)\frac{dy}{dx}\right] + \cos(y)\cos(x) = \cos(x) - \sin(y)\frac{dy}{dx}$

$\frac{dy}{dx}\left[-\sin(x)\sin(y) + \sin(y)\right] = \cos(x) - \cos(y)\cos(x)$

$$\boxed{\frac{dy}{dx} = \frac{\cos(x) - \cos(y)\cos(x)}{-\sin(x)\sin(y) + \sin(y)}}$$

$\sin^{-1} = \frac{1}{\sqrt{1-x^2}}$

$\tan^{-1} = \frac{1}{1+x^2}$

$\sec^{-1} = \frac{1}{|x|\sqrt{x^2+1}}$

(7) Fill out the following table:

$$\frac{d}{dz}\arcsin z = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dz}\arccos z = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dz}\arctan z = \frac{1}{1+x^2}$$

$$\frac{d}{dz}\operatorname{arccot} z = -\frac{1}{1+x^2}$$

$$\frac{d}{dz}\operatorname{arcsec} z = \frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dz}\operatorname{arccsc} z = -\frac{1}{|x|\sqrt{x^2+1}}$$

(8) Draw the best picture you can of a squirrel intercepting a Frisbee from a mountain lion while trying to convince his girlfriend, a llama, why he should buy a pet armadillo.