In-class Quiz #5 Name: Key

Please circle your answers! (:

1. (a) Differentiate.

$$y = \sec\theta\tan\theta$$

$y' = \sec\theta(\sec^2\theta) + \tan\theta(\sec\theta\tan\theta)$

$\boxed{= \sec^3\theta + \sec\theta\tan^2\theta}$

(b) Differentiate with respect to t.

$$y = d\cos t + t^2\sin t$$

$y' = d(-\sin t) + \cos t(0) + t^2(\cos t) + \sin t(2t)$

$\boxed{= t^2\cos t + 2t\sin t - d\sin t}$

(c) Differentiate.

$$y = \frac{2x}{5 - \cot x}$$

$y' = \dfrac{(5-\cot x)(2) - (2x(-\csc^2 x))}{(5-\cot x)^2}$

$\boxed{= \dfrac{10 - 2\cot x + 2x\csc^2 x}{(5-\cot x)^2}}$

(d) Differentiate.

$$f(\theta) = \frac{\sec\theta}{6 + \sec\theta}$$

$f'(\theta) = \dfrac{(6+\sec\theta)(\sec\theta\tan\theta) - (\sec\theta)(\sec\theta\tan\theta)}{(6+\sec\theta)^2}$

$\boxed{= \dfrac{6\sec\theta\tan\theta}{(6+\sec\theta)^2}}$

2. (a) Find the derivative of the function.

$$y = \sqrt{5 + 2e^{4x}}$$

$y' = \frac{1}{2}(5+2e^{4x})^{-1/2}(8e^{4x})$

$= \frac{8e^{4x}}{2\sqrt{5+2e^{4x}}}$

$\boxed{= \frac{4e^{4x}}{\sqrt{5+2e^{4x}}}}$

(b) Find the derivative of the function.

$$y = \frac{r}{\sqrt{r^2 + 8}}$$

$y' = \frac{(r^2+8)^{1/2}(1) - r\left(\frac{1}{2}(r^2+8)^{-1/2}\cdot 2r\right)}{\left(\sqrt{r^2+8}\right)^2}$

$\boxed{= \frac{\sqrt{r^2+8} - 2r^2}{2\sqrt{r^2+8}\,(r^2+8)}}$

(c) Find the derivative of the function.

$$F(t) = e^{9t\sin 2t}$$

Aside: $\frac{d}{dt} 9t\sin 2t = 9t\cos 2t \cdot 2 + \sin(2t) \cdot$

$F'(t) \boxed{= e^{9t\sin 2t}\left(18t\cos(2t) + 9\sin(2t)\right)}$

(d) Find the derivative of the function.

$$y = \sin(\tan 6x)$$

$y' = \cos(\tan 6x)\sec^2(6x)\cdot 6$

$\boxed{= 6\cos(\tan 6x)\sec^2(6x)}$

3. (a) Find dy/dx by implicit differentiation.

$$5 \cos x \sin y = 4$$

$y' =$

$$\cos x (\cos y) \frac{dy}{dx} + \sin y (-\sin x) = 0$$

$$\Leftrightarrow \cos x \cos y \left(\frac{dy}{dx}\right) - \sin x \sin y = 0$$

$$\Leftrightarrow \frac{dy}{dx} (\cos x \cos y) = \sin x \sin y$$

$$\Leftrightarrow \frac{dy}{dx} \boxed{= \frac{\sin x \sin y}{\cos x \cos y}}$$

(b) Find dy/dx by implicit differentiation.

$$e^{x/y} = 7x - y$$

$y' =$

$$7 - \frac{dy}{dx} = e^{x/y} \cdot \left[\frac{y - x \frac{dy}{dx}}{y^2} \right]$$

$$\Leftrightarrow -\frac{dy}{dx}(y^2 - x) = e^{x/y} y - 7y^2$$

$$\Leftrightarrow \boxed{\frac{-y e^{x/y} + 7y^2}{y^2 - x e^{x/y}} = \frac{dy}{dx}}$$

(c) Find dy/dx by implicit differentiation.

$$\tan^{-1}(2x^2y) = x + 5xy^2$$

$y' =$ My bad on this one! We aren't entirely ready for this hard yet.

$\boxed{\text{Oops}}$

(d) Find dy/dx by implicit differentiation.

$$e^y \cos x = 2 + \sin(xy)$$

$y' =$

$$e^y(-\sin x) + \cos x \, e^y \frac{dy}{dx} = 0 + \cos(xy)\left[x \frac{dy}{dx} + y\right]$$

$$\Leftrightarrow -\sin x e^y + \frac{dy}{dx} \cos x e^y = \frac{dy}{dx} x \cos(xy) + y \cos(xy)$$

$$\Leftrightarrow \frac{dy}{dx}(\cos x e^y - x \cos(xy)) = y \cos(xy) + \sin x e^y$$

$$\Leftrightarrow \frac{dy}{dx} \boxed{= \frac{y \cos(xy) + \sin x e^y}{\cos x e^y - x \cos(xy)}}$$

4. Find the following limits. Show all work. Do **NOT** use L'Hopital's Rule.

2 a) $\lim_{x \to 0} \frac{A\tan(3x)}{Bx}$ (A and B are real number constants)

$$\lim_{x \to 0} \frac{A\tan(3x)}{Bx} = \frac{0}{0}, \text{ Indeterminate Form}$$

$$= \frac{A}{B} \lim_{x \to 0} \frac{\tan(3x)}{x}$$

$$= \frac{A}{B} \lim_{x \to 0} \frac{\left(\frac{\sin(3x)}{\cos(3x)}\right)}{x}$$

$$= \frac{A}{B} \lim_{x \to 0} \frac{\sin(3x)}{x\cos(3x)}$$

$$= \frac{3A}{B} \lim_{x \to 0} \left[\frac{\sin(3x)}{3x} \cdot \frac{1}{\cos(3x)}\right]$$

$$= \frac{3A}{B} \lim_{x \to 0} \frac{\sin(3x)}{3x} \lim_{x \to 0} \frac{1}{\cos(3x)}$$

$$= \frac{3A}{B}(1)(1)$$

$$\boxed{= \frac{3A}{B}}$$

1 b) $\lim_{x \to 0} \frac{3\sec^5 x}{\sin^{-1} x + 4}$ $= \frac{3(\sec 0)^5}{\sin^{-1}(0) + 4} = \frac{3(1)^5}{0+4} \boxed{= \frac{3}{4}}$

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$$\boxed{\begin{array}{ll} \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}\csc^{-1}x = -\frac{1}{x\sqrt{x^2-1}} \\ \frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}} \\ \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} & \frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2} \end{array}}$$