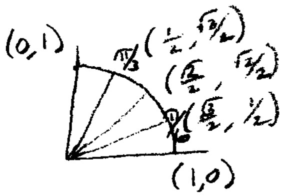
8/23/06

Review for the Basic Skills Test - Key

① $9^\circ \cdot \frac{\pi}{180} = \frac{\pi}{20}$ radians

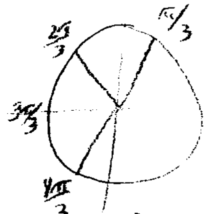
② $\frac{5\pi}{12} \cdot \frac{180}{\pi} = \frac{5 \cdot 30}{2} = 5 \cdot 15 = 75^\circ$

③



$(\cos\theta, \sin\theta)$

ⓐ



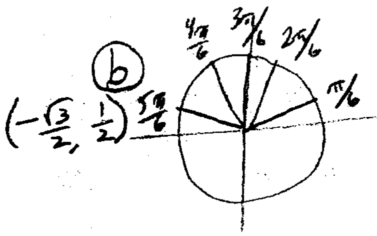
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\cos\frac{4\pi}{3} = -\frac{1}{2}$ $\qquad$ $\sec\frac{4\pi}{3} = -2$

$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ $\qquad$ $\csc\frac{4\pi}{3} = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

$\tan\frac{4\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$ $\qquad$ $\cot\frac{4\pi}{3} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

ⓑ



$\sin\frac{5\pi}{6} = \frac{1}{2}$

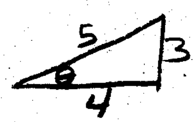
$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan\frac{5\pi}{6} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\cot\frac{5\pi}{6} = -\sqrt{3}$ $\qquad$ $\sec\frac{5\pi}{6} = \frac{-2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ $\qquad$ $\csc\frac{5\pi}{6} = 2$

④ $\sin\theta = \frac{3}{5} = \frac{opp}{hyp}$

$0 < \theta < \pi/2$



$\sin\theta = \frac{3}{5}$ $\qquad$ $\csc\theta = 5/3$

$\cos\theta = \frac{4}{5}$ $\qquad$ $\sec\theta = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

⑤ ⓐ $[0, 2\pi]$

$2\cos x - 1 = 0$

$\cos x = 1/2$

$x = \pi/3 \qquad x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

ⓑ $2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$|\sin x| = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sin x = \frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$

$x = \pi/4, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

ⓒ $\sin 2x = \cos x$

$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$2\sin x\cos x - 1) = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $2\sin x - 1 = 0$

$\sin x = 1/2$

$x = \pi/2, \frac{3\pi}{2} \qquad x = \pi/6 \quad x = \frac{5\pi}{6}$

ⓓ $\sin x = \tan x$

$\sin x - \tan x = 0$

$\sin x\left(1 - \frac{1}{\cos x}\right) = 0$

$\sin x = 0$ or $\frac{1}{\cos x} = 1$

$x = 0, \pi, 2\pi \qquad 1 = \cos x$

$x = 0, 2\pi$

(6) (a) $f(-4) = -2 \quad g(3) = 4$

(b) $f(x) = g(x)$ at $x = -2$ and $x = 2$

(c) $f(x) = -1$ when $x = -3$ and $x = 4$

(d) $f(x)$ is decreasing on $(0, 4)$

(e) f has domain $[-4, 4]$ and range $[-2, 3]$

g has domain $[-4, 3]$ and range $[\frac{1}{2}, 4]$

(7) $f(x) = 3x^2 - x + 2$

$$f(2) = 3(2^2) - 2 + 2 = 12$$

$$f(-2) = 3(-2)^2 - -2 + 2 = 12 + 2 + 2 = 16$$

$$f(a) = 3a^2 - a + 2$$

$$f(-a) = 3(-a)^2 - -a + 2 = 3a^2 + a + 2$$

$$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - (a+1) + 2$$

$$= 3a^2 + 6a + 3 - a - 1 + 2 = 3a^2 + 5a + 4$$

$$2f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4$$

$$f(2a) = 3(2a)^2 - (2a) + 2 = 12a^2 - 2a + 2$$

$$f(a^2) = 3(a^2)^2 - a^2 + 2 = 3a^4 - a^2 + 2$$

$$[f(a)]^2 = (3a^2 - a + 2)^2 = (3a^2 - a + 2)(3a^2 - a + 2)$$

$$= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4$$

$$= 9a^4 - 6a^3 + 13a^2 - 4a + 4$$

$$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - (a+h) + 2$$

$$= 3a^2 + 6ah + 3h^2 - a - h + 2$$

(8) $f(x) = x - x^2$

- $f(2+h) = 2 + h - (2+h)^2 = 2 + h - (4 + 4h + h^2)$
  $= 2 + h - 4 - 4h - h^2 = -h^2 - 3h - 2$

- $f(x+h) = (x+h) - (x+h)^2 = x + h - x^2 - 2xh - h^2$

- $\dfrac{f(x+h) - f(x)}{h} = \dfrac{x + h - x^2 - 2xh - h^2 - (x - x^2)}{h} = \dfrac{\cancel{x} + h - \cancel{x^2} - 2xh - h^2 - \cancel{x} + \cancel{x^2}}{h}$

  $= \dfrac{h - 2xh - h^2}{h} = 1 - 2x - h$ when $h \neq 0$

(9) (a) $f(x) = \frac{x}{3x-1}$ $\quad 3x-1 \neq 0$

$3x \neq 1$

$x \neq \frac{1}{3}$

domain of $f(x)$ is $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

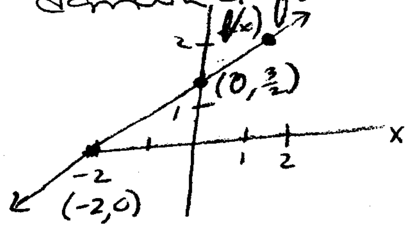
(b) $f(t) = \sqrt{t} + \sqrt[3]{t}$ $\quad t \geq 0$ for $\sqrt{t}$

domain of $f(t)$ is $[0, \infty)$

(10) (a) $f(x) = \frac{1}{2}(x+3)$

$= \frac{1}{2}x + \frac{3}{2}$

domain of $f(x)$ is $\mathbb{R}$



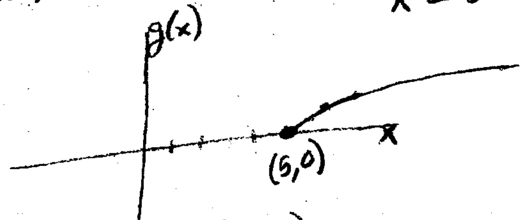
y-intercept $\frac{3}{2}$

$m = \frac{1}{2}$

(b) $g(x) = \sqrt{x-5}$ $\quad x-5 \geq 0$

$x \geq 5$

domain of $g(x)$ is $[5, \infty)$



(11) $(-2, 1)$ and $(4, -6)$

$m = \frac{-6-1}{4--2} = \frac{-7}{6}$

point slope form

$y - 1 = -\frac{7}{6}(x - -2)$

$y - 1 = -\frac{7}{6}x - \frac{7}{3}$

$y = -\frac{7}{6}x - \frac{4}{3}$ slope-intercept form

(12) P: perimeter

A: area

l: length

w: width

$P = 20 = 2l + 2w$

$10 = l + w$

$10 - l = w$

$A = l \cdot w$

$A = l(10 - l) = 10l - l^2$ m^2

(13) (a) $x(x+3) = 18$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6 \quad x = 3$

(b) $2x^2 + 2x = 5$

$2x^2 + 2x - 5 = 0$

$(2x \quad)(x \quad) = 0$ can not be factored

quadratic formula $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{4 - 4(2)(-5)}}{4}$

$= \frac{-2 \pm \sqrt{4 + 40}}{4} = \frac{-2 \pm \sqrt{44}}{4} = \frac{-2 \pm 2\sqrt{11}}{4}$

$= \frac{-1 \pm \sqrt{11}}{2}$

(14) 2:00 time $x = 0$ hr constant speed $\Rightarrow$ linear model

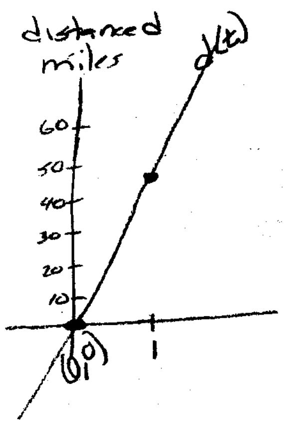
2:50 time $x = \frac{5}{6}$ hr

make ordered pair (time, distance)

have two points $(0,0)$ and $(\frac{5}{6}, 40)$

y-intercept

$$m = \frac{40-0}{\frac{5}{6}-0} = \frac{40}{\frac{5}{6}} = 40 \cdot \frac{6}{5} = 48 \text{ mph}$$



$d = 48t$ miles where t is hours travelled since 2:00 pm

note: makes sense with distance = rate × time

slope of line represents velocity

(15) (a) $(0, 15)$
$(10, 19.34)$

"increases by # for every 10 ft" $\Rightarrow$ linear model

ordered pairs are (depth below ocean surface, pressure)
(d, p)

$$m = \frac{19.34-15}{10-0} = \frac{4.34}{10} = .434$$

$p - 15 = .434(d-0)$

$p = .434d + 15$ lb/in^2 where d is depth in ft below ocean surface

(b) $p = 100 = .434d + 15$

$85 = .434d$

$d = \frac{85}{.434} = 195.853$ ft

(16) (a) $y = f(x) + 3$ (b) $y = f(x) - 3$ (c) $y = f(x-3)$

(d) $y = f(x - -3) = f(x+3)$ (e) $y = -f(x)$ (f) $y = f(-x)$

17. a) $y = f(x-4)$ shifts $f(x)$ right 4 units graph 3

b) $y = f(x)+3$ shifts $f(x)$ up 3 units graph 1

c) $y = \frac{1}{2}f(x)$ compresses $f(x)$ by a factor of 2 graph 4

d) $y = -f(x+4)$ shifts $f(x)$ left by 4 units and reflects across horizontal axis graph 5

e) $y = 2f(x+6)$ shifts $f(x)$ left by 6 units and stretches the graph by a factor of 2 graph 2

18. $g(x) = x^2+1$ $f(x) = \sqrt{2x+3}$

$(f\circ g)(x) = f(g(x)) = \sqrt{2(x^2+1)+3} = \sqrt{2x^2+5}$

$(g\circ f)(x) = g(f(x)) = g(\sqrt{2x+3}) = (\sqrt{2x+3})^2+1 = 2x+3+1$
$= 2x+4$

$(f\circ f)(x) = f(f(x)) = \sqrt{2\sqrt{2x+3}+3}$

$(g\circ g)(x) = g(g(x)) = (x^2+1)^2+1 = x^4+2x^2+1+1 = x^4+2x^2+2$

19. a) $F(x) = (x^2+1)^{10}$ could be decomposed to $f(g(x)) = (f\circ g)(x)$ where $f(x) = x^{10}$ and $g(x) = x^2+1$

b) $F(x) = \dfrac{x^2}{x^2+4}$ could be decomposed to $f(g(x)) = (f\circ g)(x)$ where $f(x) = \dfrac{x}{x+4}$ and $g(x) = x^2$