Student's Printed Name: ________________________ **CUID:** __________

Instructor: ____________________________________ **Section:** __________

Instructions: You are **not** permitted to use a calculator on this test. You are **not** allowed to use a textbook, notes, cell phone, laptop, PDA, or any technology on this test. All devices must be turned off while you are in the testing room.

During this test, any communication with any person (other than the instructor or a designated proctor) in any form, including written, signed, verbal, or digital, is understood to be a violation of academic integrity.

No part of this test may be removed from the examination room.

Read each question very carefully. In order to receive full credit, you must:

1. Show legible, logical, and relevant justification which supports your answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.
4. Give exact numerical values whenever possible.

You have **90 minutes** to complete the entire test.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

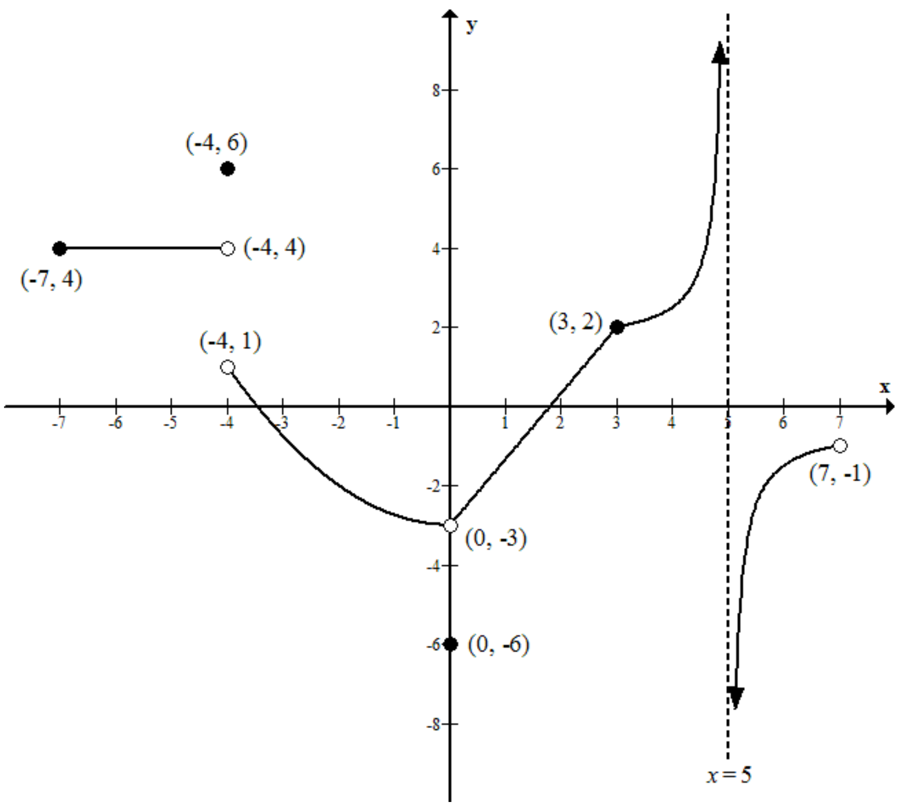
Student's Signature: ____________________________________

Do not write below this line.

Free Response Problem	Possible Points	Points Earned	Free Response Problem	Possible Points	Points Earned
1.	8		4.a.	7	
2.a.	8		4.b.	7	
2.b.	8		4.c.	7	
2.c.	8		5.	7	
2.d.	8		6.	9	
3.	8		7.	8	
			8.	7	
			Test Total	100	

Read each question carefully. In order to receive full credit you must show legible, logical, and relevant justification which supports your final answer. Give answers as exact values. You are NOT permitted to use a calculator on any portion of this test.

1. **(8 pts.)** Use the graph of $f(x)$ to answer the following questions. **(1 pt. each)**
Infinite limits should be answered with "$=\infty$" or "$=-\infty$", whichever is appropriate.
If the limit does not exist (and cannot be answered as ∞ or $-\infty$), state "DNE."



a) $\lim\limits_{x\to -5} f(x) = 4$

b) $\lim\limits_{x\to 5^+} f(x) = -\infty$

c) $\lim\limits_{x\to 0} f(x) = -3$

d) $\lim\limits_{x\to -4} f(x)$ DNE

e) $\lim\limits_{x\to 3} f(x) = 2$

f) $\lim\limits_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$ DNE

g) $\lim\limits_{x\to 7^-} f(x) = -1$

h) $\lim\limits_{x\to -6^+} \dfrac{df}{dx} = 0$

2. **(8 pts. each)** Find the following limits. Show all work. Do **NOT** use L'Hopital's Rule.

a) $$\lim_{x\to-\infty}\left(e^{\ln\left(7+\frac{1}{x}+\frac{2}{x^2}\right)}\sec\left(\frac{1}{x^3}\right)\right)$$

$$=\lim_{x\to-\infty}\left(\left(7+\frac{1}{x}+\frac{2}{x^2}\right)\sec\left(\frac{1}{x^3}\right)\right)=\lim_{x\to-\infty}\left(7+\frac{1}{x}+\frac{2}{x^2}\right)\lim_{x\to-\infty}\sec\left(\frac{1}{x^3}\right)=(7+0+0)\sec(0)=7(1)=7$$

Work on Problem:	**Points Awarded:**
Finds limit for argument of natural log function.	3 points
Finds limit for argument of secant function	3 points
Simplifies exponential and natural log as inverse functions	1 point
Final answer	1 point

Notes:
- Subtract ½ point for missing notation such as:, $x \to a$ (w/o "limit") , omitting =, including $\lim_{x\to a}$ after substitution
- Maximum of 1 point deduction for all notation errors

b)

$$\lim_{t\to5}\frac{\sqrt{4t+16}-6}{25t-t^3}$$

$$=\lim_{t\to5}\frac{\sqrt{4t+16}-6}{25t-t^3}\cdot\frac{\sqrt{4t+16}+6}{\sqrt{4t+16}+6}=\lim_{t\to5}\frac{(4t+16)-36}{(25t-t^3)(\sqrt{4t+16}+6)}=\lim_{t\to5}\frac{4t-20}{t(25-t^2)(\sqrt{4t+16}+6)}$$

$$=\lim_{t\to5}\frac{4(t-5)}{t(5-t)(5+t)(\sqrt{4t+16}+6)}=\lim_{t\to5}\frac{-4}{t(5+t)(\sqrt{4t+16}+6)}$$

$$=\frac{-4}{5(5+5)(\sqrt{4(5)+16}+6)}=-\frac{-4}{5(10)(12)}=-\frac{1}{150}$$

Work on Problem:	**Points Awarded:**
Recognizes implicitly or explicitly indeterminate form 0/0.	1 point
Uses conjugate to rewrite	3 points
Algebra to get a reduced form for which substitution works	3 points
Correctly evaluates limit.	1 point

Notes:
- Subtract ½ point for the untrue statement: $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$.
- Subtract 1 point for not presenting the correct reduced form before substitution
- Maximum of 1 point deduction for all notation errors

c)

$$\lim_{x\to 4} \sin\left(\cos^{-1}\left(\frac{\ln e^{(x^2-1)}}{30}\right)\right)$$

$$= \sin\left(\cos^{-1}\left(\frac{\ln e^{(4^2-1)}}{30}\right)\right) = \sin\left(\cos^{-1}\left(\frac{\ln e^{(15)}}{30}\right)\right) = \sin\left(\cos^{-1}\left(\frac{15}{30}\right)\right) = \sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Work on Problem:	**Points Awarded:**
Substitutes.	1 point
Simplifies exponential and natural log as inverse functions	2 points
Finds $\cos^{-1}(1/2)$	3 points
Finds $\sin(\pi/3)$	2 points

Notes:

- Subtract ½ point for missing notation such as:, $x \to a$ (w/o "limit") , omitting =, including $\lim_{x\to a}$ after substitution
- Maximum of 1 point deduction for all notation errors

d)

$$\lim_{x\to-\infty}\left(\frac{\sqrt{16x^4+64x^2}+x^2}{2x^2-\pi^5}\right)$$

$$=\lim_{x\to-\infty}\frac{\sqrt{x^4\left(16+{}^{64}\!/_{x^2}\right)}+x^2}{2x^2-\pi^5}=\lim_{x\to-\infty}\frac{x^2\sqrt{\left(16+{}^{64}\!/_{x^2}\right)}+x^2}{2x^2-\pi^5}=\lim_{x\to-\infty}\frac{\frac{x^2}{x^2}\sqrt{\left(16+{}^{64}\!/_{x^2}\right)}+\frac{x^2}{x^2}}{\frac{2x^2}{x^2}-\frac{\pi^5}{x^2}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\left(16+{}^{64}\!/_{x^2}\right)}+1}{2-\frac{\pi^5}{x^2}}=\frac{\sqrt{(16+0)}+1}{2-0}=\frac{4+1}{2}=\frac{5}{2}$$

Work on Problem:	**Points**
Recognizes implicitly or explicitly indeterminate form ∞/∞.	1 point
Rewrites to get a form for which substitution works	5 points
Correctly evaluates limit.	2 points

Notes:
- Award 3 points total for recognizing ∞/∞ (I.F.) and the need to rewrite, but incorrect algebra leads to incorrect answer or leads to incorrect work to arrive at answer.
- Subtract ½ point for the untrue statement: $\lim\limits_{x\to-\infty}\dfrac{f(x)}{g(x)}=\dfrac{\infty}{\infty}$
- Subtract ½ point for missing notation such as:, $x \rightarrow a$ (w/o "limit") , omitting =, including $\lim\limits_{x\to a}$ after substitution
- Maximum of 1 point deduction for all notation errors

3. **(8 pts.)** Let $f(x)=\dfrac{2x^2-9x-5}{x-5}$**.** Use the epsilon-delta definition of a limit to prove

$\lim\limits_{x\to 5} f(x)=11$**.**

we want

$|f(x)-11|<\varepsilon$

$\left|\dfrac{2x^2-9x-5}{x-5}-11\right|<\varepsilon$

$\left|\dfrac{(2x+1)(x-5)}{x-5}-11\right|<\varepsilon$

$|(2x+1)-11|<\varepsilon$

$|2(x-5)|<\varepsilon$

$|x-5|<\dfrac{\varepsilon}{2}\Rightarrow$ choose $\delta=\dfrac{\varepsilon}{2}$

Proof:

Let $\varepsilon>0$ be given.

Choose $\delta=\dfrac{\varepsilon}{2}\Rightarrow 0<|x-5|<\delta=\dfrac{\varepsilon}{2}$. Then

$$|f(x)-11|=\left|\frac{2x^2-9x-5}{x-5}-11\right|=\left|\frac{(2x+1)(x-5)}{x-5}-11\right|=|(2x+1)-11|=|2x-10|=2|x-5|<2\delta=2\left(\frac{\varepsilon}{2}\right)=\varepsilon.$$

Work on Problem:	**Points**
Determines a value for δ	3 points
Proof	5 points

Notes:

- Subtract 1pt. if epsilon and delta were switched but the student was consistent throughout. At least 2 points were taken off if they were switched halfway through the problem
- Subtract 1pt. if '=' was used instead of '<' in finding a value for delta
- Subtract 1pt. if limit notation was used
- Subtract ½ point if the absolute values were missing more than once
- Subtract ½ if "let epsilon > 0 be given" (or something similar) was missing from the proof

4. **(7 pts. each)** Find the derivatives of the following functions. Assume $g(x)$ is a differentiable function wherever it appears. Do NOT simplify your answers.

a)

$$f(x) = \frac{1+xe^x}{1+e^x}$$

$$f'(x) = \frac{(1+e^x)(xe^x+e^x)-(1+xe^x)e^x}{(1+e^x)^2}$$

Work on Problem:	**Points**
Applies Quotient Rule correctly	4 points
Applies Product Rule correctly	3 point
Notes: •	

b)

$$f(x) = \frac{(x^2+1)g(x)}{x^3}$$

$$f'(x) = \frac{(x^3)\left[(x^2+1)g'(x)+g(x)(2x)\right]-(x^2+1)g(x)(3x^2)}{(x^3)^2}$$

Work on Problem:	**Points**
Applies Quotient Rule correctly	4 points
Applies Product Rule correctly	3 point
Notes: •	

c)

$$h(t) = \pi^5\left(\sqrt{t}-e^3-\sqrt{7}\right)$$

$$h'(t) = \pi^5\left(\frac{1}{2}t^{-1/2}-0-0\right)$$

$$h'(t) = \frac{\pi^5}{2}t^{-1/2}$$

Work on Problem:	**Points**
Derivative of $\sqrt{t}$	3 points
Derivative of two constants (1 point each)	2 points
Constant multiple	2 point
Notes: •	

5. **(7 pts)** Let $h(x) = \dfrac{-2f(x)}{g(x)}$.

Find $h'(1)$ if $f(1) = -3$, $g(1) = 4$, $f'(1) = -2$, and $g'(1) = 7$.

$$h(x) = \frac{-2f(x)}{g(x)}$$

$$h'(x) = (-2)\left[\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}\right]$$

$$h'(1) = (-2)\left[\frac{g(1)f'(1) - f(1)g'(1)}{[g(1)]^2}\right]$$

$$h'(1) = (-2)\left[\frac{(4)(-2) - (-3)(7)}{[4]^2}\right]$$

$$h'(1) = (-2)\left[\frac{(-8) - (-21)}{[4]^2}\right]$$

$$h'(1) = (-2)\left[\frac{13}{16}\right]$$

$$h'(1) = -\frac{13}{8}$$

Work on Problem:	**Points**
Applies Quotient Rule correctly	4 points
Substitutes given values	2 point
Final answer	1 point

Notes:

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6. **(9 pts.)** Let $f(x) = 3x^3 - 13x$.

a) **(3 pts.)** Find all values of x such that the line tangent to $f(x)$ is parallel to the line $y = 23x + 1$.

$$f(x) = 3x^3 - 13x$$
$$f'(x) = 9x^2 - 13$$

solve

$$f'(x) = 23$$
$$9x^2 - 13 = 23$$
$$9x^2 = 36$$
$$x^2 = 4$$
$$x = \pm 2$$

Work on Problem:	**Points**
Derivative of *f*	1 point
Sets *f '* equal to 23	1 point
Two solutions (1/2 each)	1 point

Notes:
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b) **(3 pts.)** Find the equation of the line **normal** (perpendicular) to $f(x)$ at $x = 1$. Put your final answer in slope-intercept form ($y = mx + b$).

$$f(x) = 3x^3 - 13x$$
$$f'(1) = 9(1)^2 - 13 = -4$$
$$\Rightarrow m_{normal} = \frac{1}{4}$$

$$f(1) = -10 \Rightarrow \text{ point}(1, -10)$$
$$y - (-10) = \frac{1}{4}(x - 1)$$
$$y = \frac{1}{4}x - \frac{1}{4} - 10$$
$$y = \frac{1}{4}x - \frac{41}{4}$$

Work on Problem:	**Points**
Calculates *f '*(1)	1/2 point
Calculates *f*(1)	1/2 point
Determines slope of normal	1 point
Equation of normal line	1 point

Notes:
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c) **(3 pts.)** Let $g(x) = -10x + 7$. Show that there exists a solution to the equation $f(x) = g(x)$.

$f(x) = g(x)$
$3x^3 - 13x = -10x + 7$
$3x^3 - 3x - 7 = 0$
Let $h(x) = 3x^3 - 3x - 7$
h is continuous on its domain $(-\infty, \infty)$.

$h(0) = -7 < 0$
$h(2) = 11 > 0$

By the Intermediate Value Theorem there exists some $c \in (0, 2)$ such that $h(c) = 0$
$\Rightarrow h$ has an x-intercept at $x = c$
$\Rightarrow f(x) = g(x)$ has a solution at $x = c$.

Work on Problem:	**Points**
Establishes continuity	1/2 point
Finds an *x* such that $f(x) < 0$	1/2 point
Finds an *x* such that $f(x) > 0$	1/2 point
Mentions IVT	1/2 point
Conclusion that f(x) = g(x) has a solution	1/2 point

Notes:
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7. **(8 pts.)** Use the **limit definition** of the derivative to find $f'(x)$ if $f(x)=\dfrac{x}{2x+1}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{x+h}{2(x+h)+1}-\dfrac{x}{2x+1}}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{x+h}{2x+2h+1}-\dfrac{x}{2x+1}}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{(x+h)(2x+1)-x(2x+2h+1)}{(2x+2h+1)(2x+1)}}{h}$$

$$=\lim_{h\to 0}\frac{2x^2+x+2xh+h-2x^2-2xh-x}{h(2x+2h+1)(2x+1)}$$

$$=\lim_{h\to 0}\frac{h}{h(2x+2h+1)(2x+1)}$$

$$=\lim_{h\to 0}\frac{1}{(2x+2h+1)(2x+1)}$$

$$=\frac{1}{(2x+1)^2}$$

Work on Problem	**Points**
States the formula for the limit definition of the derivative. Okay if implied	2 Points
Substitutes $(x+h)$ correctly into the definition	2 Points
Simplifies to a form where direct substitution works • 1 point for getting a common denominator for $f(x+h)$ and $f(x)$ • 1 point for expanding the polynomials in the numerator after combining $f(x+h)$ and $f(x)$ into a single function • 1 point for simplifying terms to the point where the limit can be evaluated using direct substitution	3 Points
Correctly evaluates limit (no credit if this doesn't follow from work)	1 Point
Notes: • Subtract 8 points for not using the limit definition of the derivative • Subtract 2 points if no work is shown between simplifying from getting a common denominator and taking the limit • Max of 2 points total for all notation errors • Subtract 1 point if $\lim\limits_{h\to 0}$ is missing at any step where it should be present • Subtract ½ point if $\lim\limits_{h\to 0}$ is written after direct substitution (after the limit has been taken) • Work is only followed after a mistake if the mistake does not reduce the difficulty of the problem	

8. **(7 pts.)** Let $f(x)$ be a function such that the following inequality is true for all real numbers a and b. Find $\lim_{x\to a} f(x)$.

$$b-|x-a| \le 2f(x) \le b+|x-a|.$$

$$\lim_{x\to a}\left(b-|x-a|\right)$$
$$= b-|a-a|$$
$$= b-0$$
$$= b$$

$$\lim_{x\to a}\left(b+|x-a|\right)$$
$$= b+|a-a|$$
$$= b+0$$
$$= b$$

By the Squeeze Theorem

$$\lim_{x\to a} 2f(x) = b$$
$$\Rightarrow 2\lim_{x\to a} f(x) = b$$
$$\Rightarrow \lim_{x\to a} f(x) = \frac{b}{2}$$

Work on Problem:	**Points**
Set up inequality of limits or manipulate the given inequality (can be omitted)	1 point
Limits of two outside functions (2 each)	4 points
Mentions Squeeze (Sandwich) Theorem	1 point
Final answer	1 points

Notes:

- -1/2 point for limit notation errors
- Maximum of 1 point deduction for all notation errors
- Max of 5 points awarded for appropriate work without a correct answer via Squeeze Theorem