In-Class Quiz 2

Name: KEY

Please **circle** answers (:

1. Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to \infty} \frac{4x - 7}{2x + 6} = \lim_{x \to \infty} \frac{\frac{4x}{x} - \frac{7}{x}}{\frac{2x}{x} + \frac{6}{x}} = \lim_{x \to \infty} \frac{4 - \frac{7}{x}}{2 + \frac{6}{x}} = \frac{4-0}{2+0} = \frac{4}{2} \boxed{= 2}$$

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to -\infty} \frac{x - 1}{x^2 + 7} = \lim_{x \to -\infty} \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{7}{x^2}} = \lim_{x \to -\infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 + \frac{7}{x^2}} = \frac{0-0}{1+0} = \frac{0}{1} \boxed{= 0}$$

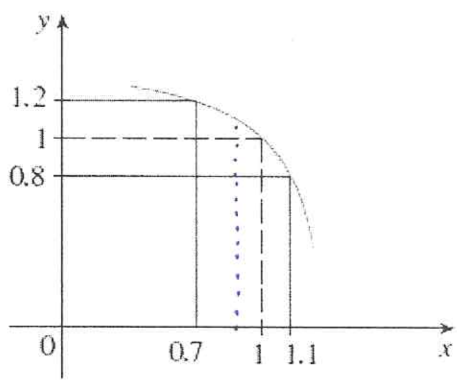
Find the limit.

$$\lim_{x \to -\infty} (x^6 + x^7) = \boxed{-\infty}$$

2. Use the given graph of f to find a number δ such that

if $|x - 1| < \delta$ then $|f(x) - 1| < 0.2$

$\delta = \boxed{0.1}$



.1 .1

3. How would you "remove the discontinuity" of f? In other words, how would you define $f(4)$ in order to make f continuous at 4?

$$f(x) = \frac{x^2 - 3x - 4}{x - 4} = \frac{(x-4)(x+1)}{(x-4)} = (x+1)$$

$$f(x) = x+1 \Rightarrow f(4) = 4+1 = 5$$

In order to make f continuous at $x=4$, $f(4)$ must be defined so that $f(4)=5$

4. Explain, using the theorems, why the function is continuous at every number in its domain.

$$F(x) = \frac{2x^2 - x - 9}{x^2 + 1}$$

$2x^2-x-9$ is a polynomial so it is a continuous function. x^2+1 is a polynomial so it is also a continuous function being divided by another continuous function; Therefore, $F(x)$ must also be continuous in its domain.

State the domain.

$(-\infty,\infty)$

5. Let $f(x) = \begin{cases} ax^2 + x + a, & \text{if } x \le 0 \\ \frac{x^2+a^2}{x^2+1}, & \text{if } x > 0. \end{cases}$

For what value(s) of a (if any) does $\lim_{x\to 0} f(x)$ exist?

$ax^2 + x + a \overset{?}{=} \frac{x^2+a^2}{x^2+1}$ when? look at where $x \to 0$

$a(0)^2 + (0) + a = \frac{(0)^2+a^2}{(0)^2+1} \Rightarrow a = a^2 \Rightarrow \boxed{a = 0 \text{ or } a = 1}$

6. Consider the function $g(x) = \begin{cases} x^3 - x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$

Show that $\lim_{x\to 0} g(x) = 0$.

Do not worry about this one.

You need to do an ε-δ proof for this that is harder than I thought it was.

My bad!

7. In the following problem we will use the formal definition of a limit to show that $\lim_{x\to 1}(x^2 - 1) = 0$.

a) State the formal definition of a limit.

Let $f(x)$ be a function that is defined on an interval containing $x = a$ (except possibly at $x = a$). We say that $\lim_{x\to a} f(x) = L$. For all numbers $\varepsilon > 0$ there is a number $\delta > 0$ so that $|f(x) - L| < \varepsilon$ and $0 < |x - a| < \delta$.

b) Use the formal definition of a limit to show that $\lim_{x\to 1}(x^2 - 1) = 0$.

Let's prove it for $\lim_{x\to a} x^2 - a^2$!

proof: Let $\varepsilon > 0$. We wish to find $\delta > 0$ s.t. for any $x \in \mathbb{R}$, $0 < |x-a| < \delta$ implies $|x^2 - a^2| < \varepsilon$. We claim that the choice $\delta = \min\left\{\frac{\varepsilon}{|2a|+1}, 1\right\}$ is an appropriate choice of δ. First, note that $|2a|$ is always nonnegative, so $|2a|+1$ is always positive. This means $\frac{\varepsilon}{|2a|+1} > 0$ for any $a \in \mathbb{R}$, so we have actually chosen $\delta > 0$. Next we observe that if $|x-a| < \delta = \min\left\{\frac{\varepsilon}{|2a|+1}, 1\right\}$, then

$$|x^2 - a^2| = |x+a||x-a|$$

$$< (|2a|+1)|x-a| \quad \text{since } |x+a| < |2a|+1 \text{ if } |x-a| < 1$$

$$< (|2a|+1)\frac{\varepsilon}{|2a|+1} \quad \left(|x-a| < \frac{\varepsilon}{|2a|+1}\right)$$

$$= \varepsilon$$

Thus, we see that this choice of δ forces $|x^2 - a^2| < \varepsilon$ whenever $0 < |x-a| < \delta$. It follows that $\lim x^2 - a^2 = 0$