What does computation have to offer?

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What do computational scientists do

- Invent: Create algorithms to solve for numerical solutions where analytic solutions are not possible
- Improve: Create new/Augment existing algorithms to make intractable problems tracktable (or just speed them up)
- Analyze: Use functional analysis to prove the uniqueness and existense of solutions as well as the stability of algorithms
- Code: Freefem, MATLAB, Fortran (People still use this), C++, Deal.II, etc... There are many paths to choose from

What would I work on

Algorithms to simulate:

- Fluid flow
  - Turbulence models
- Porous media flow
  - Help Driscol's determine where to plant berry bushes
- Fluid-structure interactions
  - Improve blood flow simulations through arteries
- Discontinuous Galerkin (DG) simulations
  - Earth's mantle simulations
  - Containment of nuclear waste

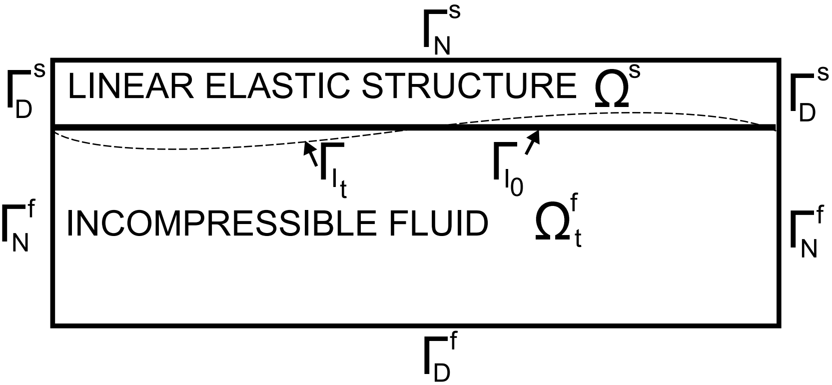
Problems look something like this:

$$
\begin{aligned}
\rho_f \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) - 2\nu_f \, \nabla \cdot D(\mathbf{u}) + \nabla p &= \mathbf{f}_f \quad \text{in } \Omega_t^f \,, \\
\nabla \cdot \mathbf{u} &= 0 \quad \text{in } \Omega_t^f \,.
\end{aligned}
$$

What is in it for you?

- Mixture of math and programming (if you are on the fence between the two)
- Large variety of problems to work on (weather, plate tectonics, fluid flow, etc...)
- Can always fall back on a programming job (also pays well)
- More money for research than most other fields of mathematics

Domain


Γ_N^s
Γ_D^s
LINEAR ELASTIC STRUCTURE Ω^s
Γ_D^s
Γ_{I_t}
Γ_{I_0}
Γ_N^f
INCOMPRESSIBLE FLUID Ω_t^f
Γ_N^f
Γ_D^f

FSI Equations

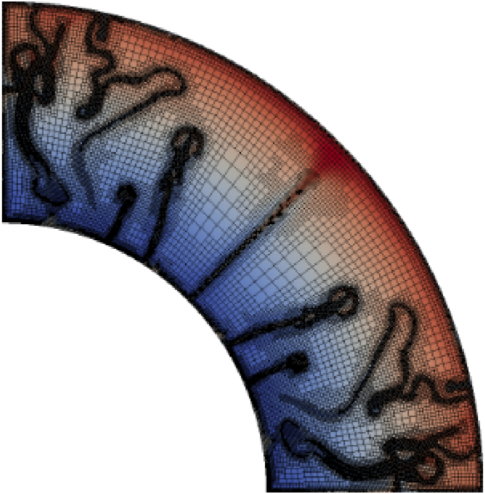
System of Fluid-Structure Equations

$$
\begin{aligned}
\rho_f \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) - 2\nu_f \, \nabla \cdot D(\mathbf{u}) + \nabla p &= \mathbf{f}_f \quad \text{in } \Omega_t^f \,, \\
\nabla \cdot \mathbf{u} &= 0 \quad \text{in } \Omega_t^f \,, \\
\rho_s \frac{\partial^2 \boldsymbol{\eta}}{\partial t^2} - 2\nu_s \, \nabla \cdot D(\boldsymbol{\eta}) - \lambda \nabla \cdot (\nabla \cdot \boldsymbol{\eta}) &= \mathbf{f}_s \quad \text{in } \Omega^s \,,
\end{aligned}
$$

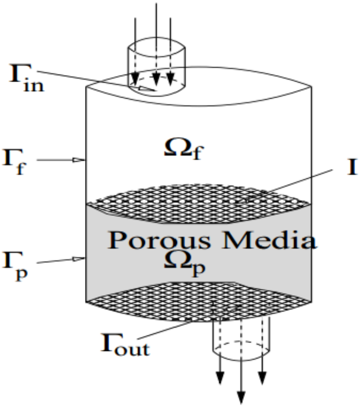
where $\mathbf{u}$ denotes the velocity vector of fluid, p the pressure of fluid, ρ_f the density of the fluid, ν_f the fluid viscosity, $\boldsymbol{\eta}$ the displacement of the structure, ρ_s the structure density; $D(\mathbf{v}) := (\nabla \mathbf{v} + \nabla \mathbf{v}^T)/2$

Fluid-structure Interaction Problems

Earth's Mantle Simulation



Darcy Flow



Γ_{in}
Γ_f
Ω_f
I
Γ_p
Porous Media
Ω_p
Γ_{out}

Turbulent Flow