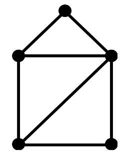
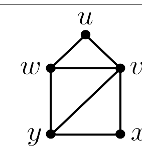
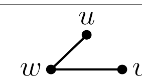
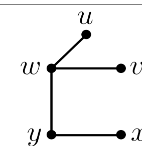
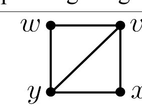
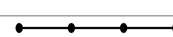
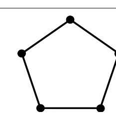
Common Graph Theory Terminology

Notation	**Name**	**Definition**	**Example**
G=(V,E)	Graph	A *graph* G consists of a finite non-empty set V of *vertices* and a set E of 2-element subsets of V called *edges.*	 Example: a graph
u~v	Adjacency	Given, u,v∈V. If {u,v}=e is an edge in G, we say u is *adjacent* to v.	 Example: u~v in G
N(v), or $N_G(v)$	Neighborhood	The set of vertices adjacent to u is called the *neighborhood* of v.	
$N_G[v]$	Closed Neighborhood	The *closed neighborhood* of v is N(v)∪{v}.	
H⊆G	subgraph	A graph H is said to be a *subgraph* of G if V(H)⊆V(G) and E(H)⊆E(G).	 Example: subgraph of G
	Spanning subgraph	A subgraph is said to be *spanning* if V(H)=V(G).	 Example: spanning subgraph of G
	Induced subgraph	A subgraph is said to be *induced* if ∀u,v∈V, if {u,v}∈E(G) then {u,v}∈E(H).	 Example: induced subgraph of G
K_n where |V|=n	Complete Graph	A graph G is said to be *complete* if ∀u,v∈V(G), {u,v}∈E(G).	
	clique	A *clique* is a subgraph that is isomorphic to a complete graph.	
P_n where |V|=n	path	A *path* is a graph P=(V,E) where V(P)={p_1, … p_n} and E(P)={{p_i , p_{i+1}}: 1 ≤ i ≤ n-1}.	 Example: P_4
C_n where |V|=n	cycle	A *cycle* is a path with the additional edge {p_n, p_1}.	 Example: C_5
	bipartite	A graph is *bipartite* if there exists S⊆V s.t. there are no edges between vertices in S and there are no edges between verties of V\S. Equivalently, a graph is bipartite if it does not contain any odd cycles.	

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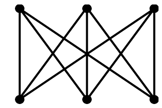
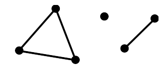
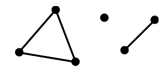
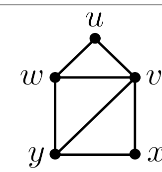
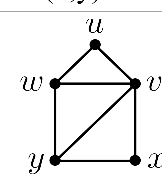
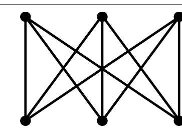
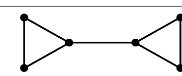
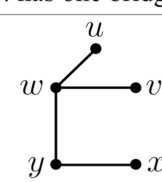
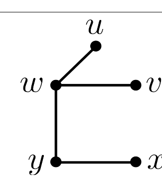
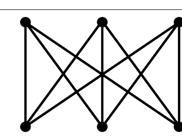
$K_{n,m}$ where $\|S\|=n$ and $\|V\setminus S\|=m$	Complete bipartite	A *complete bipartite* graph is a bipartite graph where $\forall u\in S$ and $\forall v\in V\setminus S$ $\{u,v\}\in E$.	 Example: $K_{3,3}$
	connected	A graph is said to be *connected* if $\forall u,v\in V$ there is a path $P\subseteq G$ such that $u,v\in P$. If this is not the case the graph is said to be *disconnected*.	 Example: disconnected
	component	A *component* is a maximal connected subgraph.	 Example: graph with 3 components
$d_G(u,v)$ or $d(u,v)$	distance	Let G be a connected graph. For $u,v\in V(G)$, the *distance* between u and v is the number of edges of the shortest path that contains u and v.	 Example: d(u,y)=2.
deg(v)	degree	The *degree* of a vertex v in G is the number of edges incident to v in G.	 Example: deg(v)=4, $\Delta(G)=4$, $\delta(G)=2$
$\Delta(G)$	Max degree	$\Delta(G)=\max\{\deg(v) : v\in V(G)\}$	
$\delta(G)$	Min degree	$\delta(G)=\min\{\deg(v) : v\in V(G)\}$	
	r-regular	A graph G is said to be *r-regular* if every vertex of G has degree r. That is, $\Delta(G)=\delta(G)$.	 Example: $\Delta(G)=\delta(G)=3$
	bridge	An edge e is said to be a *bridge* if G\e has more components than G.	 Example: has one bridge
	tree	A *tree* is a connected graph T that contains no cycles as subgraphs.	 Example: a tree
	claw-free	A graph is said to be *claw-free* if it does not contain an induced subgraph isomorphic to $K_{1,3}$.	 Example: not claw-free
	planar	A graph is said to be *planar* if it can be drawn on the plane without crossing edges.	 Example: not a planar graph

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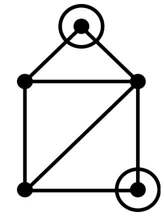
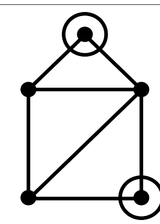
cr(G)	Crossing number	The crossing number of a graph is the lowest number of edge crossings in a plane drawing of the graph.	
G^C or $\overline{G}$	Graph Complement	The complement of a graph G=(V,E) is a graph G^C=(V,E') where $E \cup E' = E(K_n)$ and $E \cap E' = \varnothing$.	
	Independent set	An *independent set* of a graph G=(V,E) is a set $S \subseteq V$ where $\forall u,v \in S$, $\{u,v\} \notin E$.	 Example: an independent set
α(G)	Independence number	The *independence number* of a graph G is the size of the largest independent set of G.	
	Dominating set	A *dominating set* of a graph G=(V,E) is a set $S \subseteq V$ where $\forall v \in V \setminus S$, $\exists u \in S$ s.t. u is adjacent to v.	 Example: a dominating set
γ(G)	Domination number	The *domination number* of a graph G is the size of the smallest dominating set.	
	coloring	An *s-coloring* of a graph G, is a function c: $V \rightarrow \{1, \dots, s\}$.	
	Proper coloring	A coloring c of a graph G is *proper*, if vertices v, if u is adjacent to v, $c(u) \neq c(v)$.	
χ(G)	Chromatic number	$\chi(G) = \min\{s : G \text{ has a proper s-coloring}\}$	