My Favorite Graph Theory Conjectures

Kirsti Wash

November 20, 2013

Abstract

The two most popular conjectures within the realm of graph products date back to the 60's. The first was posed in 1966 by Clemson's very own Stephen Hedetniemi. A k-coloring of a graph G is a mapping $f : V(G) \to \{1, \ldots, k\}$ such that if $uv \in E(G)$, then $f(u) \neq f(v)$. The chromatic number of G, denoted $\chi(G)$, is the smallest integer k such that G has a k-coloring. Hedetniemi conjectured that the chromatic number of the direct product $G \times H$ is actually equal to $\min\{\chi(G), \chi(H)\}$. Despite numerous attempts at a proof, we only know the conjecture holds when $\chi(G), \chi(H) \leq 4$.

The second conjecture was posed in 1968 by Vizing. Given a graph G, a set $D \subset V(G)$ is a dominating set of G if every vertex of $V(G) \backslash D$ is adjacent to at least one vertex in D. The domination number of G, denoted $\gamma(G)$, is the minimum cardinality of a dominating set of G. Vizing conjectured that the domination number of the Cartesian product $G \Box H$ is at least $\gamma(G)\gamma(H)$. To date, the best known lower bound is $\gamma(G \Box H) \geq \frac{1}{2}\gamma(G)\gamma(H) + \frac{1}{2}\min\{\gamma(G), \gamma(H)\}$.

In this talk, we will survey the previous attempts at proving these conjectures. In each case we shall see that although it takes only 90 seconds to explain the conjecture, we are no closer to solving these problems than we were 50 years ago.