A Markov Chain Analysis of College Football Overtime

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1 Abstract

This study aims to model NCAA football's overtime format using Markov Chain techniques. The NCAA's new overtime structure was introduced in 2021 and its unique alternating-possession format is highly discrete, lending itself nicely to Markov Chain analysis. We constructed the model using possession, score, and game period characteristics to describe states and scoring rates to describe transition probabilities. Using historical play-by-play data, we find the conventional belief that it offers an advantage to defer possession in the first overtime does indeed have statistical basis though the benefit may not be as strong as widely believed. We also use our model to examine the risk/reward of attempting a two point conversion to win the game at the end of the first overtime, concluding that in most cases it is the optimal strategy. Our model can be applied to team specific data and be used to answer various other strategic questions that arise in college football overtime.

2 Introduction

2.1 The Rules

In 2021, the NCAA introduced a rule change for college football games going into overtime. The format has always involved alternating miniature drives from the 25 yard line, but, in response to a 5 hour, 74-72 Texas A&M marathon victory over LSU in 2018, the NCAA has made efforts to shorten theses games, with the most recent changes coming in 2021. Since that season, the format has consisted of 3 periods which look as follows. In the first frame both teams get one possession, starting at their opponents' 25 yard line. Teams can score in any standard way (field goal, touchdown, extra point or two point conversion, etc.) or not score at all. If after this period the score remains tied, they advance to the second overtime. This period functions the same as 1st with the slight distinction that, after any touchdowns scored, teams must attempt a two point conversion, as opposed to kicking for an extra point. Finally, if the score remains tied after both of these periods, the game moves to the third overtime which consists of alternating two point conversion attempts. The first team to lead after an equal amount of attempts wins the game. Through all periods, it is the case that the team that responded to the opponents' possession in the period prior leads off the following period. The only exception is the first period, for which a coin is flipped and the winner decides if they would like to go first or second.

2.2 Our Approach

The format of these overtimes, particularly post 2021, lends itself to the discretization of largely independent drive attempts, and a seemingly obvious tool for this kind of problem is the Markov Chain. There is a history of Markov Chains being used to model football and understand the probabilities that govern it. A paper that serves as one of the inspirations for our investigation is that of Goldner (2012), which aims to model football drives specifically and has 9 absorbing states on top of 340 other states based on down, distance, and yard-line for detailed drive analysis. Goldner uses the absorption probabilities to calculate expected points from any field position. Our paper will discretize drives as a whole, aiming to predict not drive outcomes but game outcomes.

Previous research by Wilson (2021) in the area of college overtime focused on the idea that going second in overtime produces a competitive advantage. Through machine learning models such as logistic regression and decision trees, Wilson's argues against the idea that going second has any meaningful effect on win percentage. This is one of many factors we hope to investigate further using our model, which is described in section 3. Further, our model can be used to study other questions relating to the NCAA overtime period. For example, we will analyze the question of, if down by 7 points in the first overtime, should the responding team attempt to end the game with a two point conversion. The model's exhaustive nature creates a tool that can estimate, given a stage in overtime and a range of team parameters, win probabilities and the implications of various strategic decisions.

2.3 The Data

In order to both provide the Markov Chain with real-world inputs for its variables, and confirm our assumptions align with true game events, we brought in recent NCAA game-play data. We used the R library "cfbfastr" for detailed play-by-play data, which is an R API wrapper for the website collegefootballdata.com. We subsetted the data from 2021 - 2023, representing complete seasons after the overtime rule change, and focus on just Power 5 teams in our analysis.

3 The Model

3.1 Assumptions

In order to discretize a football game, which is a continuous-state random process, one has to make a few assumptions to simplify and define concretely the state space. We confirmed each of our assumptions using the 2021-2023 play-by-play data. Our first assumption, that defensive touchdowns or safeties are negligible, is grounded in our data, which shows that, of plays within the 25-yard line, across 986 games, only 0.089% of plays resulted in defensive scores. No safeties occurred under these same conditions. This confirms the extreme rarity of these outcomes in overtime situations. By treating these events as negligible, we avoid the need to include additional states to account for such rare defensive scoring plays, which would complicate the analysis without adding much value.

Building on this, our next assumption is that teams avoid two-point conversions during the first overtime period, reflecting their preference for lower-risk strategies early on. This is supported by the fact that only 3.85% of touchdowns in the first overtime included two-point attempts. Teams tend to prioritize ensuring points on the board, favoring the safer extra point. Later in this paper, we will discuss a probabilistic look at this strategy, but by incorporating this assumption for now, the model avoids including human decision-making in what is a stochastic process model.

Similarly, we assume that extra points are nearly guaranteed, based on their success rate of 98.82% in the play-by-play data. Teams rely on extra points as a predictable scoring mechanism, reflecting their reliability in real-world scenarios. Treating PATs as an assured point simplifies state transitions after touchdowns, allowing the model to focus on more variable outcomes such as failed drives or successful field goals.

Finally, our model relies on the assumption that teams will attempt a field goal to win when any score guarantees victory (i.e., they are responding to a scoreless drive from their opponent). Data shows that, in these situations, second-possession teams will rush for a few yards to improve their field position before kicking on fourth down. Out of the 6 games that included this situation in our dataset, teams attempted to rush for a few yards, and then kick, in all of them. This behavior highlights the strategic adjustments teams make under the pressure of sudden death, where prioritizing a sure score is paramount.

3.2 States

Our state design captures the 4 characteristics that precisely describe the game scenario a team could face when beginning an overtime possession. These characteristics are

- Team in Possession (A or B)
- Score Differential (many possibilities)
- Overtime Period (1, 2, or 3)
- Overtime Frame (1 or 2)

Overtime frame describes whether the team with possession is leading that overtime period (frame 1) or responding to a drive from their opponent (frame 2). We will call our two teams team A and team B and assume, without loss of generality, that team A starts with the ball. This creates 17 states, per the format of NCAA overtime as described above. Not all score/possession/period combinations are possible due to the complexity of these rules. This amounts to 15 transient states and 2 absorbing states, the victories. See Figure 1 for detailed state notation.

Team with Possession	Score Differential(for A)	Overtime Period	Overtime Frame	Notation
A	+0	1	1	$\boldsymbol{A}(+0)B_{1,1}$
B	+0	1	2	$A(+0)\boldsymbol{B}_{1,2}$
B	+3	1	2	$A(+3)\boldsymbol{B}_{1,2}$
B	+7	1	2	$A(+7)\boldsymbol{B}_{1,2}$
B	+0	2	1	$A(+0)\boldsymbol{B}_{2,1}$
A	+0	2	2	$\boldsymbol{A}(+0)B_{2,2}$
A	−3	2	2	$\boldsymbol{A}(-3)B_{2,2}$
A	−6	2	2	$\boldsymbol{A}(-6)B_{2,2}$
A	−8	2	2	$\boldsymbol{A}(-8)B_{2,2}$
A	+0	3	1	$\boldsymbol{A}(+0)B_{2,2}$
B	+0	3	2	$A(+0)\boldsymbol{B}_{3,2}$
B	+2	3	2	$A(+2)\boldsymbol{B}_{3,2}$
B	+0	3	1	$A(+0)\boldsymbol{B}_{3,1}$
A	−2	3	2	$\boldsymbol{A}(-2)B_{3,2}$
B	+0	3	2	$\boldsymbol{A}(+0)B_{3,2}$
A Victory	N/A	N/A	N/A	W_A
B Victory	N/A	N/A	N/A	W_B

Figure 1: State Definitions

3.3 Transition Probabilities

To describe the probability of the game moving from one state to another, we must define some rate parameters for each team corresponding to their respective offensive efficiencies. We do so as follows:

t_A = A's touchdown rate from the 25 yard line	t_B = B's touchdown rate from the 25 yard line
f_A = A's FG rate from the 25 yard line	f_B = B's FG rate from the 25 yard line
p_A = A's TD rate from the 25, given no FGs possible	p_B = A's TD rate, given no FGs possible
c_A = A's 2pt conversion rate	c_B = B's 2pt conversion rate
k_A = A's FG rate between 20-25 yd line	k_B = B's FG rate between 20-25 yd line

These probabilities are carefully defined. t_A corresponds to the rate at which team A, given they have a first down at the 25 yard line, is able to score a touchdown. This rate also assumes that a FG is a viable, though less preferable, option on the drive. f_A, then, represents the percent of times that team A has a first down on the 25 yard line, attempts to score a touchdown, and is forced to settle for a field goal which is successful. This is not to be confused with FG percentage, though that value will be naturally factored into f_A since we do not include scenarios where a FG is forced and subsequently missed into our calculation of this value. These probabilities are designed to capture the dynamics at play in the first and second overtime periods. It is often the case that a game state will mean the team with the ball will try their hardest to score a touchdown and, if this effort is unsuccessful, settle for a field goal. The only scenarios where this will not be a team's basic strategy are those where they are responding to an opponent's touchdown (ie. $A(-7)\boldsymbol{B}_{1,2}$, $\boldsymbol{A}(-6)B_{2,2}$, and $\boldsymbol{A}(-8)_{2,2}$) or failed score altogether (ie. $A(+0)\boldsymbol{B}_{1,2}$ and $\boldsymbol{A}(+0)B_{2,2}$). These touchdown deficit states are what the parameters p_A and p_B aim to address. These rates represent teams' TD probability from the 25 yard line knowing they must score a touchdown (they would lose with a field goal). One would imagine this rate would be greater than t; teams will be more aggressive, going for it on every fourth down, and taking more shots at the endzone, etc. The probability c_A is a straight-forward parameter representing a teams 2 point conversion rate from the 3-yard line. Finally, k_A represents team A's field goal rate between the 20-25 yard line. This is used in the scenario where a team simply needs a FG to win. As discussed with our assumptions above, that team will normally attempt to rush for a couple of yards and attempt kick a FG, so we assumed these FGs are uniformally distributed between the 20-25 yard line.

3.4 The Matrix

Having established our model states and transition probabilities, we can now build our 17×17 transition matrix. It would be tedious to describe the rational behind every possible state transition but, to capture the general idea, we depict two in Figure 2.

Using this type of logic, we fill in our matrix. The generalized version is included in Appendix A, but in

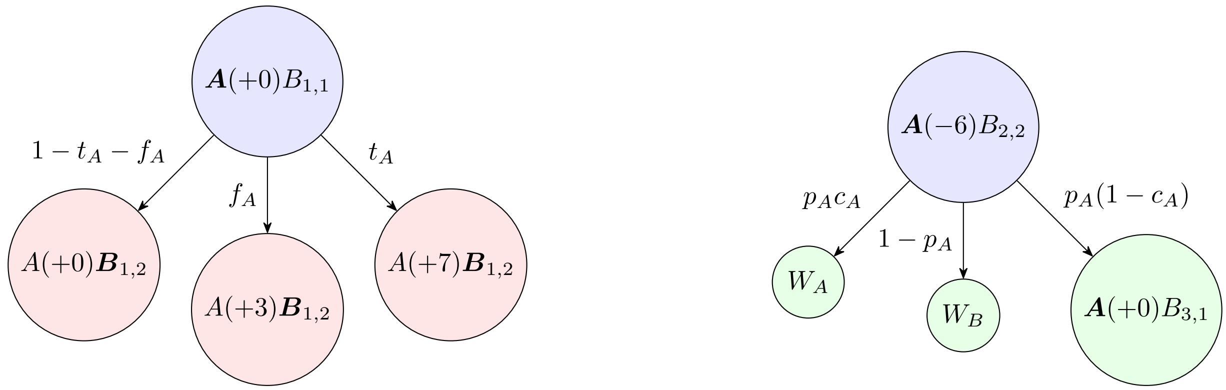


Figure 2: Sample transition probabilities between states.

Figure 3 we include the matrix that assumes two equal teams, ie. $t = t_A = t_B$, $f = f_A = f_B$, $c = c_A = c_B$, $p = p_A = p_B$, and $k = k_A = k_B$.

	$\boldsymbol{A}(+0)B_{1,1}$	$A(+0)\boldsymbol{B}_{1,2}$	$A(+3)\boldsymbol{B}_{1,2}$	$A(+7)\boldsymbol{B}_{1,2}$	$A(+0)\boldsymbol{B}_{2,1}$	$\boldsymbol{A}(+0)B_{2,2}$	$\boldsymbol{A}(-3)B_{2,2}$	$\boldsymbol{A}(-6)B_{2,2}$	$\boldsymbol{A}(-8)B_{2,2}$	$\boldsymbol{A}(+0)B_{3,1}$	$A(+2)\boldsymbol{B}_{3,2}$	$A(+0)\boldsymbol{B}_{3,2}$	$A(+0)\boldsymbol{B}_{3,1}$	$\boldsymbol{A}(-2)B_{3,2}$	$\boldsymbol{A}(+0)B_{3,2}$	$A_{\mathbf{W}}$	$B_{\mathbf{W}}$
$\boldsymbol{A}(+0)B_{1,1}$	0	$1-t-f$	f	t	0	0	0	0	0	0	0	0	0	0	0	0	0
$A(+0)\boldsymbol{B}_{1,2}$	0	0	0	0	$1-k$	0	0	0	0	0	0	0	0	0	0	0	k
$A(+3)\boldsymbol{B}_{1,2}$	0	0	0	0	f	0	0	0	0	0	0	0	0	0	0	$1-t-f$	t
$A(+7)\boldsymbol{B}_{1,2}$	0	0	0	0	p	0	0	0	0	0	0	0	0	0	0	$1-p$	0
$A(+0)\boldsymbol{B}_{2,1}$	0	0	0	0	0	$1-f-t$	f	$(1-c)t$	ct	0	0	0	0	0	0	0	0
$\boldsymbol{A}(+0)B_{2,2}$	0	0	0	0	0	0	0	0	0	$1-k$	0	0	0	0	0	k	0
$\boldsymbol{A}(-3)B_{2,2}$	0	0	0	0	0	0	0	0	0	f	0	0	0	0	0	t	$1-t-f$
$\boldsymbol{A}(-6)B_{2,2}$	0	0	0	0	0	0	0	0	0	$p(1-c)$	0	0	0	0	0	pc	$1-p$
$\boldsymbol{A}(-8)B_{2,2}$	0	0	0	0	0	0	0	0	0	pc	0	0	0	0	0	0	$1-pc$
$\boldsymbol{A}(+0)B_{3,1}$	0	0	0	0	0	0	0	0	0	0	c	$1-c$	0	0	0	0	0
$A(+2)\boldsymbol{B}_{3,2}$	0	0	0	0	0	0	0	0	0	0	0	0	c	0	0	$1-c$	0
$A(+0)\boldsymbol{B}_{3,2}$	0	0	0	0	0	0	0	0	0	0	0	0	$1-c$	0	0	0	c
$A(+0)\boldsymbol{B}_{3,1}$	0	0	0	0	0	0	0	0	0	0	0	0	0	c	$1-c$	0	0
$\boldsymbol{A}(-2)B_{3,2}$	0	0	0	0	0	0	0	0	0	c	0	0	0	0	0	0	$1-c$
$\boldsymbol{A}(+0)B_{3,2}$	0	0	0	0	0	0	0	0	0	$1-c$	0	0	0	0	0	c	0
$A_{\mathbf{W}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
$B_{\mathbf{W}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 3: Markov Matrix, assuming equal team rates.

4 Analysis

4.1 Estimating Rates

In R, with our cfb play-by-play data, we made a important few decisions in data handling to best estimate the transition probabilities. The goal was to find the Power 5 average rates from the 2021 to 2023 seasons to best represent likelihoods in the context of the new overtime structure.

First, to find the touchdown (t) and field goal (f) success rate, we define a sample space, of all drives, out of which to work: S : {touchdown, field goal, other}. The random process mapping onto this space is then all first downs around the 25 yard line. A range of the 23 to 27 yard lines were used to expand the qualifying plays in the dataset, under the assumption that these are negligible differences from 25 yards, and setting up on exactly the 25th yard line is as rare as setting up on any other specific yard line. All drives will end either in a touchdown, a field goal, or some other outcome in which the possession team does not score (punts, turnovers, etc). Touchdown success rate is then the probability of a first down around the 25 yard line resulting in a touchdown, and the same set up is used for field goals. We decided to consider touchdown and field goal success rates from just periods 1 through 3, in order to avoid potential confounds or outlier outcomes such as a team running out of time in the game during their drive, or driving harder to save a game.

Two-point conversion rate (c) was defined as number of two-point conversion successes divided by the sum of two-point conversion successes and failures (across all four periods).

To estimate the transition probability p, we needed to answer the question: How do we define "under-pressure" touchdowns to find the probability that a team scores a touchdown when one is needed to have a chance at winning the game? We experimented with two different approaches to define such instances. Approach 1 was to use all available historical overtime data (from 2014 to 2023), where the team with second possession in the first or second overtime is down by a touchdown (6 to 8 points). We divided the number of touchdowns by the number of drives with those conditions. Approach 2 was to use fourth period data from 2021 to 2023, of drives with a first down between the 23rd and 27th yard line, and 1-3 minutes left on the clock. The team in possession is down by either 4 to 8 points, or more than 12 points, where the losing team should not kick a field goal because they need one or multiple touchdowns to tie the game. Again, we divided the number of touchdowns that resulted from such instances by the number of drives with these conditions. We discuss on what basis we picked the optimal of the two approaches in the results section.

Lastly, field goal success rate where the drive was set up between the 20 to 25 yard line (k) was calculated as the number of successful field goals divided by the sum of successful and failed field goals.

Listed below are the league average transition probabilities we calculated:

- $t = 0.4639$
- $f = 0.2239$
- Approach 1 $p = 0.5238$
- Approach 2 $p = 0.4727$
- $c = 0.5464$
- $k = 0.7214$

4.2 Markov Methods

The goal of our analysis is to understand the difference in win probabilities given team characteristics and initial possession. To this end, we calculate the absorption probabilities of the two absorbing states, W_A and W_B, using our Markov matrix. Plugging the matrix into the Python package SymPy, we get rather complex expressions for A and B's respective win probabilities. For space considerations, these expressions are included in Appendix B. It is hard to make much useful sense of these expressions, though they might provide a useful tool should a coach or analyst have these rates for both teams headed into an overtime. Of more interest to this study is the scenario where the two teams are assumed to be perfectly even, thus using the Equivalent Rates matrix above. First of all, this assumption may be a safe one in many circumstances; if two teams make it to overtime, it is likely the case they are evenly matched. Secondly, an analysis of this scenario should provide insight into any inherent advantages assumed by team A or team B as a result of the overtime structure alone, independent of football ability. Replacing the team specific terms in the expressions above, we get the simpler win probabilities

$$\begin{aligned}
P(\text{A Win}) = -\frac{1}{2}(&-f^4 - f^3 - f^2k^2 + f^2kt + f^2t^2 + f^2t + 2f^2 - fk^2t + fk^2 - 2fkpt + fkt^2 + fkt \\
&- 2fk + fpt - ft^2 + 2ft - f - kpt^2 + kpt + kt^3 - kt^2 - kt + k - p^2t^2 + pt^3 + pt^2 - t^3 + t^2 - t - 1) \\
P(\text{B Win}) = \frac{1}{2}(&-f^4 - 2f^3t + f^3 + f^2k^2 - 3f^2kt - f^2t^2 + 3f^2t - 2f^2 + fk^2t - fk^2 + 2fkpt - 3fkt^2 + fkt \\
&+ 2fk - 2fpt^2 - fpt + 3ft^2 - 4ft + f + kpt^2 - kpt - kt^3 + kt^2 + kt - k + p^2t^2 - pt^3 - pt^2 + t^3 - t^2 + t - 1)
\end{aligned}$$

These polynomials are more manageable and more relevant to our analysis. We also note immediately that the c parameter is not present in these equations. It appears that the structure of overtime is such that 2-point conversion attempts are symmetric for each team, that is either team gains a unique advantage in relation to their two point conversion chances.

4.3 Results

Our first step of analysis will be to check for an inherent advantage for either team given estimated offensive efficiency rate and our absorption probabilities. We calculate these absorption probabilities for both of our p parameter estimates (probability of touchdown given a touchdown is necessary).

$$\begin{aligned}
P(\text{A wins}|t = 0.4639;\ f = 0.2239;\ p = 0.5238;\ k = 0.7214) = 48.81\% \quad &\Longrightarrow P(\text{B wins}) &= 51.19\% \\
P(\text{A wins}|t = 0.4629\ ; f = 0.2229\ ; p = 0.4727;\ k = 0.7214) = 49.53\% \quad &\Longrightarrow P(\text{B wins}) &= 50.47\%
\end{aligned}$$

Our two different estimates for p have similar implications. Both an assumed increase in touchdown rate given a must score a touchdown from 46.4% to 52.4% and from 46.4% to 47.27% have the effect that the team responding in the first overtime, Team B, is at a probabilistic advantage when all team attributes are equal. We emphasize that Approach 1 to calculating p (ie. looking at historical overtime specific data) relies on fewer assumptions and produces results that align closer to intuition, that is a bigger jump in TD rate. A team going for it on every fourth down should be decently more likely to score. These factors, along with the fact that Approach 1 works directly with OT data, lead us to choose that result as our default value of p going forward. Either way, it is safe to conclude that the conventional strategy of deferring possession in overtime is indeed, the statistically correct one, providing (given our p assumption) a 2.38% probabilistic edge to Team B. The rules of overtime are essentially symmetric, except for the ordering of possessions, so it make sense that when teams are better at responding to the opponent than driving normally, the team who gets to do that first will be at an advantage.

The two 'response' parameters in our model in this respect are p and k. Recall that p represents a team's ability to respond to a TD with a TD of their own, while k represents a team's ability to respond to a scoreless drive with a game winning field goal. We can analyze the effects of these parameters on win rates using a graphical approach. Assuming all other parameters to be NCAA averages, we graph how varying rates affect the teams' win probabilities in Figure 3. We see that an increase in either p or k in turn increases B's win probability. The parameter p's positive correlation to B's win probability is a straightforward confirmation of the fact that when teams are more efficient responding to TDs than scoring TDs first, the team who responds first is less likely to lose. Parameter k's effect is subtler. Team B has the chance to kick a field goal for a win in the first overtime period, (if team A, for whatever reason, fails to score on the opening drive) while Team A is not afforded a chance at this opportunity until the second OT period. The more reliably teams can make field goals from inside 25 yards, the more valuable this opportunity is, so it figures that an increase in k corresponds to an increase in B's likelihood of winning. We have also marked the equilibrium points on each graph. These represent the theoretical rates at which neither team would have an inherent advantage all other rates being average. These exact values are $p_{eq} = 0.441$ and $k_{eq} = 0.559$. We note both of these points are below our estimates for p and k respectively. This implies that even with slightly lower 'response' parameters Team B will still be at an advantage. Moreover, p_{eq} is less than $t = 0.4639$, meaning Team B could be $\sim 1\%$ worse at responding to TD's than scoring TD's regularly, but still be probabilistically advantaged given the effects described above of parameter k.

While f and t are not directly related to this 'response effect', a graphical analysis of the two still provides useful insights into the underlying structures that govern win probability. Such study into the effects of c would be trivial, since, as we noted above, c behaves perfectly symmetrically in the Markov Chain and its value has no effect on generalized win probability. To analyze t and f however requires a bit more care, since the two are linearly bounded and dependent on each other. For example, if the rate at which teams are scoring touchdowns t is decreasing, it is safe to assume the rate at which teams are settling for FG's f is increasing. To handle this dependence, we assume the rate at which teams score neither, ie. $1 - f - t$,

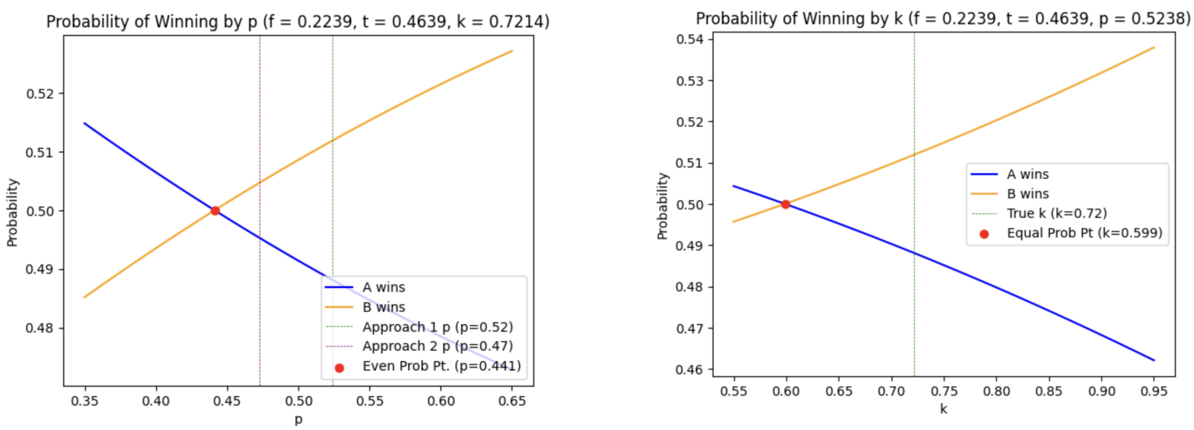
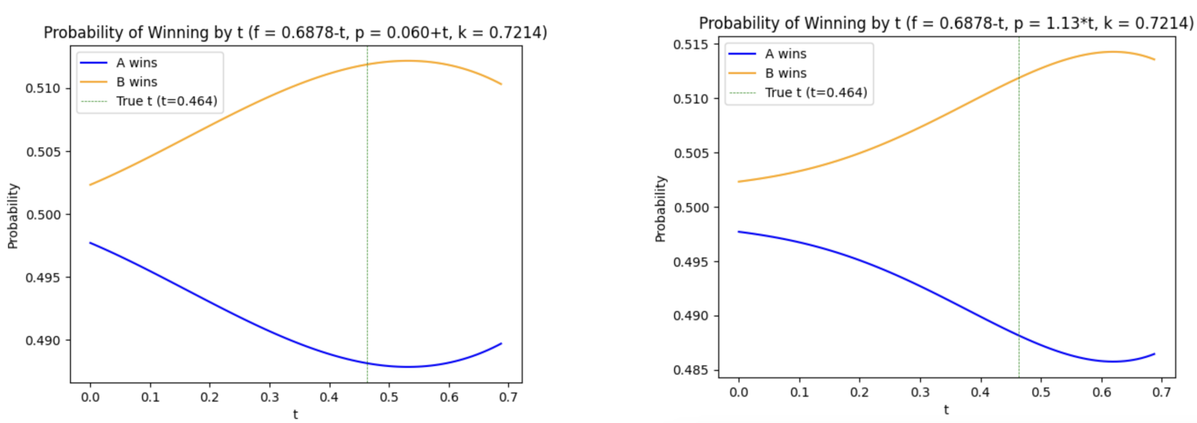


Figure 4: Win Probabilities by p and k

is constant. Equivalently, the rate at which teams score one or the other is constant. In our analysis, that means as we vary t, we will also vary f such that $t + f = 0.5036 + 0.2256 = 0.6878$ is always true. This is mathematically equivalent to varying f and adjusting t accordingly. We also have to consider a varying t's effect on p. Both approaches' values of p aligned with our intuition that teams are more likely to score when they need to score. But how can we reasonably adjust p based on a teams t? Two approaches present themselves. First, we could hold the difference between $p + t$ constant, so in the case of our data as we vary t we let $p = t + (p_{\text{observed}} - t_{\text{observed}}) = t + 0.0599$. Alternatively, we could keep their ratio constant, that is $p = t \cdot \frac{p_{\text{observed}}}{t_{\text{observed}}} = 1.129 \cdot t$. It is not clear which is a more accurate approach, so we try both in Figure 4.



(a) Constant Difference $p - t$ Approach

(b) Constant Ratio p/t Approach

Figure 5: Win Probabilities by (t, f) pairs

While our two approaches for approximating p produced graphs of different exact shapes, the general trends remain the same. Examining the graphs, we see that relative TD and FG rate (t, f) pairs indeed have a large effect on win probabilities. In the extreme, when the vast majority of scores are touchdowns, Team B sees its largest advantage, while when touchdowns are very rare this advantage diminishes greatly. Practically, this means that in games where defenses are playing very well, and thus the offenses are more likely to settle for a field goal, the generalized advantage of possessing the ball second is decreased. It is also worth noting the scale of the edge on both of these graphs. Any edge is still worth capitalizing on, but if there are other factors that might guide a teams decision to defer (fatigue, momentum, etc), this mathematical advantage can likely be ignored.

4.4 A Strategic Analysis

Our model also serves to provide insight into some potential strategies in overtime. One such strategy that could be considered is the question if, down 7 points, team B should attempt a 2-point conversion after a touchdown. Obviously, they risk failing the attempt and losing the game right there, but the question remains if their 2 point conversion rate is higher then their chance of winning in the rest of overtime. This is an easy question to answer given our model. We assume all rates (except 2 point conversion rate c) align with NCAA averages as calculated above. The absorption probability in question is the probability of ending in state W_B given starting in state $A(+0)\boldsymbol{B}_{2,1}$, that is the state that would result if team B played it safe and kicked a typical extra point to tie the game. This probability, taken from the 2nd column of the 5th row in our absorption matrix, is

$$\begin{aligned}
P\left(W_B|A(+0)\boldsymbol{B}_{2,1}\right) &= \frac{1-f^2+fk-2ft+f+kt-k-pt+t}{2} \\
&= \frac{1}{2}(1-(.2239)^2+(.2239)(.7214)-2(.2239)(0.4639)+0.2239) \\
&+ (0.7214)(.4639)-(.7214)-(.5238)(.4639)+(.4639)) \qquad \text{(NCAA average rates)} \\
&= 48.09\%
\end{aligned}$$

Note that we used here the approach 1 value for p, given the reasons stated above that support it as a stronger estimate. This result implies that if team B has a 2 point conversion rating higher than 48.09% and assumes their opponent team A has about the same conversion rate, it is indeed the strategically correct choice to attempt a 2-point conversion to decide the game then and there. Given NCAA football's average 2-pt conversion rate of 54.64%, we can say that the vast majority of teams should be going for two in this situation, but it still will come down to a team by team basis. While this threshold is an interesting observation for general teams, we again emphasize that this probability was calculated using NCAA averages. More relevant to real-game application is the team-specific win rate for team B against an average team A. We grab the expression from our generalized Markov transition matrix, specifically

$$P_B = P\left(W_B | A(+0)\boldsymbol{B}_{2,1} \textbf{ and } t_A = 0.4639;\ f_A = 0.2239;\ p_A = 0.5238;\ k_A = 0.7214\right)$$

Team B, knowing their specific rates (t_B, f_B, c_B, p_B, and k_B), would calculate their edge gained (or lost) by going for 2 as

$$\begin{aligned}\textbf{Team B Edge} &= c_B - P_B \\ &= c_B - .236c_Bt_B \\ &- \frac{c_B\left(-.0220c_Bt_B + .0248f_B + .0186t_B - .126\right)}{.009c_B^2 + .0842c_B - .794} - .312f_B - .476t_B \\ &- \frac{.454\left(-.006c_B^3t_B + .005c_B^2f_B - .018c_B^2t_B - .026c_B^2 + .025c_Bf_B + .019c_Bt_B - .126c_B\right)}{.009c_B^2 + .084c_B - .794}\end{aligned}$$

If we assume that f_B, t_B are at the NCAA average, we can plot this expression as a function of c_B, and determine precisely for which 2 point conversion rates team B gains a positive edge by going for 2 in the situation we are studying. See Figure 5 for this graph.

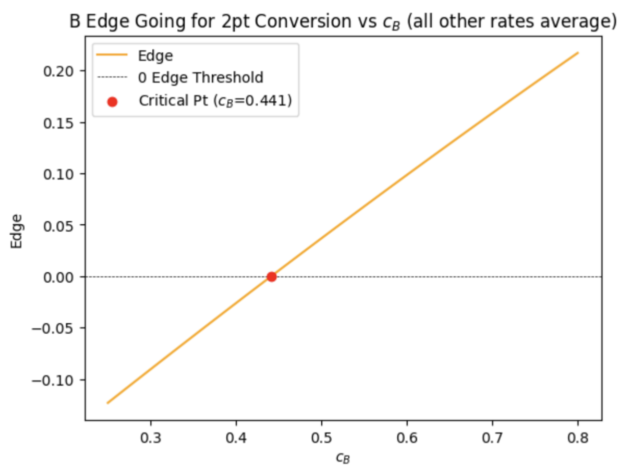


Figure 6

We see from the graph a critical point at $c_B = 0.441$. Assuming all other rates align with NCAA averages, this implies team B, when down 7 in the first overtime, should go for 2 after a touchdown if their two point conversion rate is greater than 44.1% and kick an extra point otherwise. This type of analysis could obviously be specified to particular teams if we know their respective scoring rates.

5 Discussion/Conclusion

Our model proved successful in analyzing the structure of NCAA football overtime and its effects on team win probabilities. The conventional strategy of opting to defer the ball in first overtime does indeed align with this probabilistic analysis. We distilled this fact primarily to the effect of the 'response' parameters in our models. Namely, the p and k parameters were shown to be positively correlated to the deferring team's win probability and current estimates for the rates in the NCAA showed did indeed give the edge to the team who responds in the first overtime. There are two scenarios where the responding team gains probabilistic advantage: if the leading team score a touchdown or if the leading team does not score at all. In the former case, the responding team benefits from an increased chance of touchdown (given $p > t$ in our model), and in the later case the responding team is able to kick a high likelihood field goal for the win. These response effects advantage the team who gets to take advantage of them first, that is the team deferring in the first OT. That said, this edge is small; based on the current NCAA averages we estimated, the deferring team has a 51.19% chance of winning, all else being equal. So for a coach deciding whether to receive or defer the ball, other factors such as momentum, player sentiment, or fatigue of the offensive or defensive unit could all reasonably be factored into this decision.

What we have built can also be used as a tool for specific team analysis. While we set rates equal to study the fundamental advantage of possessing the ball second, the more specified matrix could be populated with a team and their opponent's exact rates to determine the optimal decision in a narrower context. We also can use it to analyze specific potential strategies, as we did with the second period of 1st overtime 2 point conversion question. That argument concluded that in a game with two otherwise average teams, the team that responds in the first overtime should attempt a two point conversion (assuming their opponent is up 7) if their success rate is greater than 44.1%. One could imagine applying similar analysis to tight 4th down decisions, but that would require detailed situational factors not of wide enough scope for this paper.

Obviously, our model is not perfect. We distill a very large sample space of possible football outcomes into 2-4 possibilities per drive, and we did not account for effects such as momentum, home/away relative offensive or defensive advantage, player fatigue, or the many other factors that inevitably shape human athletic interaction. That said, given our understanding of the game and that our results are consistent with football dynamics, we are confident that our conclusions provide valuable insight into the inner workings of the new college football overtime rules. Should we continue our research into overtime dynamics, we will discretize the game further, instead of dividing by possession breaking it down by yard increments and advancement probabilities.

6 References

- https://www.espn.com/college-football/story/_/id/39111711/what-ncaa-college-football-rules
- Goldner, Keith. 2012. "A Markov Model of Football: Using Stochastic Processes to Model a Football Drive." *Journal of Quantitative Analysis in Sports* 8: n. pag. https://doi.org/10.1515/1559-0410.1400
- Wilson, Rick L. 2020. "College Football Overtime Outcomes: Implications for In-Game Decision-Making." *Frontiers in Artificial Intelligence* 3. https://doi.org/10.3389/frai.2020.00061.

- Gilani, Saiem, Akshay Easwaran, Jared Lee, and Eric Hess. 2021. "cfbfastR: The SportsDataverse's R Package for College Football Data." https://cfbfastR.sportsdataverse.org/.

Appendices

A Markov Matrix (Team Specific Rates)

	$\boldsymbol{A}(+0)B_{1,1}$	$A(+0)\boldsymbol{B}_{1,2}$	$A(+3)\boldsymbol{B}_{1,2}$	$A(+7)\boldsymbol{B}_{1,2}$	$A(+0)\boldsymbol{B}_{2,1}$	$\boldsymbol{A}(+0)B_{2,2}$	$\boldsymbol{A}(-3)B_{2,2}$	$\boldsymbol{A}(-6)B_{2,2}$	$\boldsymbol{A}(-8)B_{2,2}$	$\boldsymbol{A}(+0)B_{3,1}$	$A(+2)\boldsymbol{B}_{3,2}$	$A(+0)\boldsymbol{B}_{3,2}$	$A(+0)\boldsymbol{B}_{3,1}$	$\boldsymbol{A}(-2)B_{3,2}$	$\boldsymbol{A}(+0)B_{3,2}$	$A_{\mathbf{W}}$	$B_{\mathbf{W}}$
$\boldsymbol{A}(+0)B_{1,1}$	0	$1-t_A-f_A$	f_A	t_A	0	0	0	0	0	0	0	0	0	0	0	0	0
$A(+0)\boldsymbol{B}_{1,2}$	0	0	0	0	$1-k_B$	0	0	0	0	0	0	0	0	0	0	0	k_B
$A(+3)\boldsymbol{B}_{1,2}$	0	0	0	0	f_B	0	0	0	0	0	0	0	0	0	0	$1-t_B-f_B$	t_B
$A(+7)\boldsymbol{B}_{1,2}$	0	0	0	0	p_B	0	0	0	0	0	0	0	0	0	0	$1-p_B$	0
$A(+0)\boldsymbol{B}_{2,1}$	0	0	0	0	0	$1-f_B-t_B$	f_B	$(1-c_B)t_B$	c_Bt_B	0	0	0	0	0	0	0	0
$\boldsymbol{A}(+0)B_{2,2}$	0	0	0	0	0	0	0	0	0	$1-k_A$	0	0	0	0	0	k_A	0
$\boldsymbol{A}(-3)B_{2,2}$	0	0	0	0	0	0	0	0	0	f_A	0	0	0	0	0	t_A	$1-t_A-f_A$
$\boldsymbol{A}(-6)B_{2,2}$	0	0	0	0	0	0	0	0	0	$p_A(1-c_A)$	0	0	0	0	0	p_Ac_A	$1-p_A$
$\boldsymbol{A}(-8)B_{2,2}$	0	0	0	0	0	0	0	0	0	p_Ac_A	0	0	0	0	0	0	$1-p_Ac_A$
$\boldsymbol{A}(+0)B_{3,1}$	0	0	0	0	0	0	0	0	0	0	c_A	$1-c_A$	0	0	0	0	0
$A(+2)\boldsymbol{B}_{3,2}$	0	0	0	0	0	0	0	0	0	0	0	0	c_B	0	0	$1-c_B$	0
$A(+0)\boldsymbol{B}_{3,2}$	0	0	0	0	0	0	0	0	0	0	0	0	$1-c_B$	0	0	0	c_B
$A(+0)\boldsymbol{B}_{3,1}$	0	0	0	0	0	0	0	0	0	0	0	0	0	c_B	$1-c_B$	0	0
$\boldsymbol{A}(-2)B_{3,2}$	0	0	0	0	0	0	0	0	0	c_A	0	0	0	0	0	0	$1-c_A$
$\boldsymbol{A}(+0)B_{3,2}$	0	0	0	0	0	0	0	0	0	$1-c_A$	0	0	0	0	0	c_A	0
$A_{\mathbf{W}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
$B_{\mathbf{W}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

B Complete Absorption Probabilities

$$
\begin{aligned}
P(W_A|\boldsymbol{A}(+0)B_{1,1}) =& \frac{1}{2c_Ac_B - c_A - c_B} \cdot c_Ac_Bf_A^2f_B^2 + c_Ac_Bf_A^2f_Bk_B - c_Ac_Bf_A^2f_B - c_Ac_Bf_Af_B^2k_A \\
&+ 2c_Ac_Bf_Af_B^2t_A - c_Ac_Bf_Af_B^2 - c_Ac_Bf_Af_Bk_Ak_B - c_Ac_Bf_Af_Bk_At_B + 2c_Ac_Bf_Af_Bk_A \\
&+ 3c_Ac_Bf_Af_Bk_Bt_A - 2c_Ac_Bf_Af_Bk_B + c_Ac_Bf_Af_Bp_At_B + c_Ac_Bf_Af_Bp_Bt_A \\
&- 3c_Ac_Bf_Af_Bt_A - c_Ac_Bf_Af_Bt_B + c_Ac_Bf_Af_B - c_Ac_Bf_Ak_Ak_Bt_B \\
&+ c_Ac_Bf_Ak_Ak_B + c_Ac_Bf_Ak_At_B - c_Ac_Bf_Ak_A + c_Ac_Bf_Ak_Bp_At_B \\
&- c_Ac_Bf_Ak_Bt_B + c_Ac_Bf_Ak_B - c_Ac_Bf_Ap_At_B - c_Ac_Bf_At_B + c_Ac_Bf_A \\
&- c_Ac_Bf_Bk_Ak_Bt_A + c_Ac_Bf_Bk_Ak_B - c_Ac_Bf_Bk_Ap_Bt_A + c_Ac_Bf_Bk_At_A \\
&- c_Ac_Bf_Bk_A + 2c_Ac_Bf_Bk_Bt_A^2 - 3c_Ac_Bf_Bk_Bt_A + c_Ac_Bf_Bk_B \\
&+ 2c_Ac_Bf_Bp_Bt_A^2 - c_Ac_Bf_Bp_Bt_A - 2c_Ac_Bf_Bt_A^2 + 3c_Ac_Bf_Bt_A - c_Ac_Bf_B \\
&- c_Ac_Bk_Ak_Bt_At_B + c_Ac_Bk_Ak_Bt_A + c_Ac_Bk_Ak_Bt_B - c_Ac_Bk_Ak_B \\
&- c_Ac_Bk_Ap_Bt_At_B + c_Ac_Bk_Ap_Bt_A + c_Ac_Bk_At_At_B - c_Ac_Bk_At_A \\
&- c_Ac_Bk_At_B + c_Ac_Bk_A + c_Ac_Bk_Bp_At_At_B - c_Ac_Bk_Bp_At_B \\
&- c_Ac_Bk_Bt_At_B + c_Ac_Bk_Bt_A + c_Ac_Bk_Bt_B - c_Ac_Bk_B \\
&+ c_Ac_Bp_Ap_Bt_At_B - c_Ac_Bp_At_At_B + c_Ac_Bp_At_B - c_Ac_Bp_Bt_At_B \\
&- c_Ac_Bp_Bt_A + c_Ac_Bt_At_B + c_Ac_Bt_A - c_Ac_Bt_B + c_Ac_B \\
&- c_Af_A^2f_B^2 - c_Af_A^2f_Bk_B + c_Af_A^2f_B - c_Af_Af_B^2t_A + c_Af_Af_B^2 \\
&- 2c_Af_Af_Bk_Bt_A + 2c_Af_Af_Bk_B - c_Af_Af_Bp_At_B - c_Af_Af_Bp_Bt_A \\
&+ 2c_Af_Af_Bt_A + c_Af_Af_Bt_B - 2c_Af_Af_B - c_Af_Ak_Bp_At_B + c_Af_Ak_Bt_B \\
&- c_Af_Ak_B + c_Af_Ap_At_B - c_Af_Bk_Bt_A^2 + 2c_Af_Bk_Bt_A - c_Af_Bk_B \\
&- c_Af_Bp_Bt_A^2 + c_Af_Bp_Bt_A + c_Af_Bt_A^2 - 2c_Af_Bt_A + c_Af_B \\
&- c_Ak_Bp_At_At_B + c_Ak_Bp_At_B + c_Ak_Bt_At_B - c_Ak_Bt_A \\
&- c_Ak_Bt_B + c_Ak_B + c_Ap_Ap_Bt_At_B - c_Ap_At_At_B \\
&+ c_Ap_At_B - c_Ap_Bt_At_B + c_Ap_Bt_A - c_At_At_B - c_At_A + c_At_B - c_A \\
P(\text{B win}) =& 1 - P(\text{A win})
\end{aligned}
$$