Assignment 2

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1. CHAPTER III MISCELLANEOUS EXAMPLES VI Q6

Find the area of parallelogram formed by the lines

i. 12x - 5y = 7a,
ii. 5x - 12y = -7a,
iii. 5x - 12y = -126a,
iv. 12x - 5y = 126a

SOLUTION

Let's find the intersection of line (i) and (ii),

In matrix form, we have

$$\mathbf{AM} = \mathbf{B}$$
$$\mathbf{M} = \mathbf{A}^{-1}\mathbf{B}$$
$$\mathbf{M} = \begin{bmatrix} 12 & -5 \\ 5 & -12 \end{bmatrix}^{-1} \begin{bmatrix} 7a \\ -7a \end{bmatrix}$$
$$\mathbf{M} = \frac{-1}{119} \begin{bmatrix} -12 & 15 \\ -5 & 12 \end{bmatrix} \begin{bmatrix} 7a \\ -7a \end{bmatrix}$$
$$\mathbf{M} = \begin{bmatrix} a \\ a \end{bmatrix}$$

Similarly solving for lines (i) and (iii), we get

$$\mathbf{AN} = \mathbf{C}$$
$$\mathbf{N} = \mathbf{A}^{-1}\mathbf{C}$$
$$\mathbf{N} = \begin{bmatrix} 12 & -5 \\ 5 & -12 \end{bmatrix}^{-1} \begin{bmatrix} 7a \\ -126a \end{bmatrix}$$
$$\mathbf{N} = \frac{-1}{119} \begin{bmatrix} -12 & 5 \\ -5 & 12 \end{bmatrix} \begin{bmatrix} 7a \\ -126a \end{bmatrix}$$
$$\mathbf{N} = \begin{bmatrix} 6a \\ 13a \end{bmatrix}$$

Similarly solving for lines (ii) and (iv), we get

$$\mathbf{EO} = \mathbf{D}$$
$$\mathbf{O} = \mathbf{E}^{-1}\mathbf{D}$$
$$\mathbf{O} = \begin{bmatrix} 5 & -12 \\ 12 & -5 \end{bmatrix}^{-1} \begin{bmatrix} -7a \\ 126a \end{bmatrix}$$
$$\mathbf{O} = \frac{1}{119} \begin{bmatrix} -5 & 12 \\ -12 & 5 \end{bmatrix} \begin{bmatrix} -7a \\ 126a \end{bmatrix}$$
$$\mathbf{O} = \begin{bmatrix} 13a \\ 6a \end{bmatrix}$$

The area of the ||gm is given by the norm

$$||\mathbf{NM} \times \mathbf{OM}||$$

Since,

$$\mathrm{NM} = \begin{pmatrix} 5a \\ 12a \end{pmatrix}$$
$$\mathrm{OM} = \begin{pmatrix} 12a \\ 5a \end{pmatrix}$$

Therefore,

$$\mathbf{NM} \times \mathbf{OM} = \left(25a^2 - 144a^2\right)$$
$$= -119a^2$$

Eventually we get **Area** $= 119a^2$ units