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ABSTRACT

If direct verification of low incomes on a case by case basis is too costly, then to some extent the allocation of a poverty alleviation budget will have to be directed towards broadly defined groups in the population. There is then a leakage of expenditure to those above the poverty line. The object of this paper is to formulate precisely the problem of optimal direction of a poverty alleviation budget towards these broadly defined groups, taking into account the leakages involved. Using a recently suggested decomposable class of poverty indices the paper shows that the budgetary rules so derived have surprising and intricate features, and simple guidelines on budgetary stance towards a group, such as favoring disproportionately groups with greater poverty, turn out to be misleading.

1. Introduction

The problem that I wish to tackle in this paper, albeit in an abstract setting, is one which most policy oriented economists working on income distribution, social security and poverty will have thought about at one time or another. We start off with an income distribution in society and an agreed poverty line, and find that some income receiving units are below the poverty line. Suppose a "poverty alleviation" budget is made available to us. If the poor can be identified directly then the budget can be targeted towards low incomes efficiently. However, in many cases such direct identification and verification on a case by case basis is too costly (as in the case of a population scattered across a sparsely populated region in a developing country). We are then left with the option of using ancillary information and directing budgetary expenditure towards broadly defined groups - using regional expenditure, price support for particular crops or benefits for different categories of welfare claimants, for example.

The question posed in this paper is thus as follows: What budgetary rules should be used in directing expenditure towards broadly defined groups, taking into account the leakages involved, if our object is to minimize overall poverty subject to the budget constraint? For example, should most be spent per capita on the group which has highest poverty? As we shall see, the answer to the last question is "not necessarily," and budgetary rules for poverty alleviation turn out to have intricate and unexpected features.

Before discussing poverty alleviation we have to specify the poverty index we will be concerned with. Section 2 of this paper introduces and discusses two alternative formalizations of the absorption of budgetary expenditure by the groups under consideration. Sections 3 and 4 contain the main results while Section 5 concludes the paper with suggestions for further research.

2. The Poverty Index and the Basic Model

2.1 The Poverty Index

Let the distribution of income y be denoted by F(y), its density by f(y), and the poverty line income by z. Foster et. al. (1984) have recently suggested the following class of poverty measures:

$$P_\alpha = \int_{\underline{y}}^{z} \left(\frac{z-y}{z}\right)^\alpha f(y)dy \; ; \qquad \alpha \geq 0 \tag{1}$$

As α varies, this class of measures takes on different properties. For $\alpha = 0$ the index collapses to

$$P_0 = F(z), \tag{2}$$

the fraction of units below the poverty line, which is known as the head count ratio. The head count ratio has certain limitations as a poverty measure since it focuses on the number of the poor and not on how poor the poor are. For this reason, its use as a poverty measure has been criticized in the literature (see Sen, 1976)

For $\alpha = 1$, the index becomes

$$P_1 = \int_{\underline{y}}^{z} \left(\frac{z-y}{z}\right) f(y)dy \tag{3}$$

While sensitive to how far below the poverty line the average poor person falls, P_1 is not sensitive to the distribution of income among the poor. For example, a transfer of income from the poorest person to the next poorest person would leave P_1 unchanged. As Foster et. al. (1984) indicate, such "transfer insensitivity" is overcome by choosing values of α greater than 1. They suggest $\alpha = 2$ as a possibility.

The most interesting property of the P_α class of indices from our point of view is that they are additively decomposable across population subgroups, so that if the population is divided into a mutually exclusive and exhaustive groups with the i^{th} group having a population share x_i, then

$$P_\alpha = \sum_{i=1}^{n} x_i P_{i,\alpha} \; ; \qquad \sum_{i=1}^{n} x_i = 1 \tag{4}$$

where $P_{i,\alpha}$ is the P_α measure for the i^{th} group. The expression (4) is verified easily from (1) by noting that

$$f(y) = \sum_{i=1}^{n} x_i f_i(y) \; ; \quad \sum_{i=1}^{n} x_i = 1 \tag{5}$$

where $f_i(y)$ is the income density function of the i^{th} group. Since our focus in this paper is on population subgroups within the national population, decomposability turns out to be an attractive property.

2.2 The Model of Expenditure Absorption

Suppose, then, that we have a poverty alleviation budget B and our object is to minimize poverty as measured by the P_α class of indices. If we could observe and verify each individual unit's income directly, 2

how would the budget be allocated? The answer depends on which poverty index we are using. If the object is to minimize the head count ratio (which is P_α with $\alpha = 0$) then clearly the first priority are people closest to the poverty line. The money would go to people just below the poverty line till they are just above it, then to the next poorest, and so on. When $\alpha = 1$, it does not matter which of the poor gets the money: the income gap is reduced by the same amount whether the expenditure is on those just below the poverty line or those well below it. With $\alpha > 1$, P_α becomes convex in the income gap so that the marginal gain is greater at the lowest level of income - the expenditure should thus be directed at the poorest of the poor. However the point is that such <u>specific</u> targeting of expenditure to different incomes below the poverty line may not be a feasible option, as discussed in the Introduction.

So how is the absorption of budgetary expenditure, directed at incomes generally, to be modeled? The specific model will vary from case to case, but we will consider here two prototypes: one where all incomes increase by an equal additive amount and the other where all incomes increase by the same multiplicative factor. The former can be thought of in terms of the raising of tax thresholds or general benefits to all, while the latter models a change in marginal tax rates or, in a developing country context, raising the price offered to farmers of a particular crop. If the total budget is B and we normalize population size at unity, then post expenditure income m in the two respective cases is given by

$$m = y + B \tag{6}$$

$$m = y\ (1 + \frac{B}{\mu}) \tag{7}$$

where μ is the mean of f(y). The post expenditure poverty indices in the additive and multiplicative cases, denoted P_α and $\hat{P}_\alpha$ respectively, are then given by

$$P_\alpha(B) = \int_{\underline{y}}^{z-B} (\frac{z-y-B}{z})^\alpha f(y)dy \tag{8}$$

$$\hat{P}_\alpha(B) = \int_{\underline{y}}^{z/(1+\frac{B}{\mu})} \{\frac{z-y(1+\frac{B}{\mu})}{z}\}^\alpha \quad f(y)dy \tag{9}$$

Our major concern is how the total budget B should be divided between broadly identifiable population subgroups. Before turning to that question in the next section, however, we will derive the shadow price of budgetary expenditure treating the whole of society as a single group. This is interesting in its own right but also lays the basis for the next section, since we will establish budgetary rules for group expenditure essentially by comparing the shadow prices of expenditure for each group.

Differentiating (8) and (9) with respect to B, and after some manipulation, we get

$$\frac{\partial P_\alpha(B)}{\partial B} = - \frac{\alpha}{z} P_{\alpha-1}(B) < 0 \tag{10}$$

$$\frac{\partial \hat{P}_\alpha(B)}{\partial B} = - \frac{\alpha\mu}{B+\mu} [\hat{P}_{\alpha-1}(B) - \hat{P}_\alpha(B)] < 0 \tag{11}$$

Thus in the additive case if the objective is to minimize P_α then the shadow price of budgetary expenditure is negatively proportional to $P_{\alpha-1}$. In the multiplicative case the situation is more complicated, with the marginal response of P_α being proportional to the difference between $\hat{P}_{\alpha-1}$ and $\hat{P}_\alpha$. Using the fact that $\hat{P}_\alpha(B) > P_\alpha(B)$ for all α and B, it can be checked that

$$\left| \frac{\partial P_\alpha(B)}{\partial B} \right| > \left| \frac{\partial \hat{P}_\alpha(B)}{\partial B} \right| \tag{12}$$

so that ceteris paribus additive programs reduce poverty more effectively than multiplicative ones.

Finally, by differentiating a second time with respect to B it can be shown that

$$\frac{\partial^2 P_\alpha(B)}{\partial B^2} = \frac{\alpha^2}{z^2} P_{\alpha-2}(B) > 0 \tag{13}$$

$$\frac{\partial^2 \hat{P}_\alpha(B)}{\partial B^2} = \frac{\alpha\mu^2}{(B+\mu)^2} [(1-\alpha)(\hat{P}_{\alpha-1}-\hat{P}_\alpha) + \alpha(\hat{P}_{\alpha-2}-\hat{P}_{\alpha-1})] > 0 \tag{14}$$

where the second inequality is derived using the convexity of $\hat{P}_\alpha$ as a function of α. Thus there are diminishing marginal returns to increasing budgetary expenditure in both the additive and the multiplicative cases.

3. Budgetary Rules with Identifiable Subgroups: Directional Indicators

Suppose now that we are given more information about the population, which allows us to classify it into mutually exclusive and exhaustive groups. These can be thought of as regional groups, racial groups, crop groups or welfare groups. In practice these groups may not be mutually exclusive (e.g. some farmers may grow more than one crop or one household may both have an unemployed head as well as a large number of children). However, in what follows we concentrate on the mutually exclusive case, leaving the generalization as an important area for further research.

Of course with more information we can do better, since at worst the information could simply be ignored. But how exactly are we to utilize this information? Typically we have survey results which provide us with estimates of poverty in each of the subgroups. Should, for example, more expenditure be directed per capita towards groups with higher poverty? In order to answer this question let us start with the case of two groups with population shares x_1 and x_2; $x_1 + x_2 = 1$. Let us also stick initially with the case of additive absorption. If an amount B_1 is allocated to group 1 and an amount $B_2 = B-B_1$ is allocated to group 2, every income in group 1 goes up by $\frac{B_1}{x_1}$ while every income in group 2 goes up by $\frac{B_2}{x_2} = \frac{B-B_1}{x_2}$. Then from (4) and (8),

$$P_\alpha(B_1,B) = x_1 P_{1,\alpha}\left(\frac{B_1}{x_1}\right) + x_2 P_{2,\alpha}\left(\frac{B-B_1}{x_2}\right) \tag{15}$$

Differentiating this with respect to B_1, we get

$$\frac{\partial P_\alpha}{\partial B_1} = -\frac{\alpha}{z}\left[P_{1,\alpha-1} - P_{2,\alpha-1}\right]$$

$$\gtreqless 0 \quad \Leftrightarrow \quad P_{1,\alpha-1} \lesseqgtr P_{2,\alpha-1} \tag{16}$$

The expression in (16) is of some interest. It highlights the fact that any index of poverty is a statement about the incomes of the poor on the <u>average</u>, while optimal allocation of budgetary expenditure requires <u>marginal</u> information. It so happens that in the additive absorption case, for fairly obvious reasons located in the structure of the P_α measure, the marginal response of the α^{th} order measure is negatively proportional to the $(\alpha-1)^{th}$ order measure. Thus if the objective is to minimize the α^{th} order measure P_α, the appropriate <u>indicator</u> to use is the $(\alpha-1)^{th}$ order index for each group - more should be directed towards the group for which $P_{\alpha-1}$ is greater. For $\alpha = 1$, this means that the head count ratio in a group is the appropriate directional indicator for budgetary stance. Thus even if P_o is rejected as a poverty measure in comparison with P_1, the former still has a role in guiding budgetary rules for poverty alleviation.

Many policy instruments work by changing individual incomes in a multiplicative manner. What are the directional rules for poverty alleviation in this case? Recall that in the multiplicative case we use hats on the poverty indices. Thus with two groups and an amount B_1 allocated to the first group we have

$$\hat{P}_\alpha(B_1,B) = x_1\hat{P}_{1,\alpha}(\frac{B_1}{x_1\mu_1}) + x_2\hat{P}_{2,\alpha}(\frac{B-B_1}{x_2\mu_2}) \quad (17)$$

Differentiating this and using (11) we get

$$\frac{\partial\hat{P}_\alpha(B_1,B)}{\partial B_1} = -\frac{\alpha x_1}{B_1+\mu_1x_1}[\hat{P}_{1,\alpha-1} - \hat{P}_{1,\alpha}] + \frac{\alpha x_2}{B-B_1+\mu_2x_2}[\hat{P}_{2,\alpha-1}-\hat{P}_{2,\alpha}]$$

$$\gtreqless 0 \iff \frac{\alpha x_1}{B_1+\mu_1x_1}[\hat{P}_{1,\alpha-1} - \hat{P}_{1,\alpha}] \lesseqgtr \frac{\alpha x_2}{B_2+\mu_2x_2}[\hat{P}_{2,\alpha-1} - \hat{P}_{2,\alpha}] \quad (18)$$

As in the additive case, (18) tells us that if the objective is to minimize α^{th} order poverty then the budgetary stance should be favorable to the group with the higher $(\alpha-1)^{th}$ order poverty. However, expression (18) has what might be termed an unconventional feature to it, namely that ceteris paribus the budgetary stance should favor the group with the lower α^{th} order poverty. Again, this should come as no surprise once it is realized that budgetary rules for poverty alleviation rely on marginal changes in the income distribution, while poverty is a statement about the poor on average.

4. Comparative Statics of the Optimum: The Additive Absorption Case With Many Groups

Quite often the question facing a policy maker is the following: Given an increase or a decrease in the total budget, which group should benefit or suffer the most? Of course if the allocation between the two groups is not at an optimum, then all of the increase or decrease should go to one group or the other. The interesting analytical question arises at the optimum, since now some of the increase (decrease) will go to one group and some to the other. In what follows we characterize the comparative statics of the optimal allocation with respect to changes in the total budget, for the case of additive absorption.

With n groups the overall poverty index is as in (4), and the first order conditions for optimal group allocations $B_1, B_2, B_3, ---, B_n$

$(\sum_{i=1}^{n} B_i = B)$ are given by

$$
\begin{aligned}
P_{1,\alpha-1}\left(\frac{B_1}{x_1}\right) &= P_{n,\alpha-1}\left(\frac{B_n}{x_n}\right) \\
P_{2,\alpha-1}\left(\frac{B_2}{x_2}\right) &= P_{n,\alpha-1}\left(\frac{B_n}{x_n}\right) \\
&\vdots \\
P_{n-1,\alpha-1}\left(\frac{B_{n-1}}{x_{n-1}}\right) &= P_{n,\alpha-1}\left(\frac{B_n}{x_n}\right)
\end{aligned}
\tag{19}
$$

Differentiating this system with respect to B and defining

$$a_i = \frac{P_{i,\alpha-2}}{x_i} \quad ; \qquad i = 1,2,\ldots, n \tag{20}$$

we get:

$$[a_n E_{n-1} + \hat{\underset{\sim}{a}}_{n-1}]\, \underset{\sim}{b}_{n-1} = a_n \underset{\sim}{e}_{n-1} \tag{21}$$

where $\underset{\sim}{a}_{n-1}$ is the column vector $(a_1, a_2, \ldots, a_{n-1})'$, $\hat{a}_{\sim n-1}$ is the diagonal matrix with $\underset{\sim}{a}_{n-1}$ along the diagonal, $\underset{\sim}{e}_{n-1}$ is the column vector of (n-1) ones, E_{n-1} is the (n-1) x (n-1) matrix of ones, and $\underset{\sim}{b}_{n-1}$ is the column vector $\left(\frac{dB_1}{dB}, \frac{dB_2}{dB}, \ldots\ldots, \frac{dB_{n-1}}{dB}\right)'$. Thus

$$\underset{\sim}{b}_{n-1} = [a_n E_{n-1} + \hat{\underset{\sim}{a}}_{n-1}]^{-1} [a_n \underset{\sim}{e}_{n-1}] \tag{22}$$

Now using standard results in matrix algebra (see, for example, Maddala, 1977)

$$[a_n E_{n-1} + \hat{\underset{\sim}{a}}_{n-1}]^{-1}$$

$$= [\hat{\underset{\sim}{a}}_{n-1}]^{-1} - \left(\frac{a_n}{1+a_n \underset{\sim}{e}'_{n-1}[\hat{a}_{n-1}]^{-1}\underset{\sim}{e}_{n-1}}\right)[\hat{\underset{\sim}{a}}_{n-1}]^{-1}\underset{\sim}{e}_{n-1}\underset{\sim}{e}'_{n-1}[\hat{\underset{\sim}{a}}_{n-1}]^{-1} \tag{23}$$

Using this, (22) gives us

$$\underset{\sim}{b}_{n-1} = a_n[\hat{\underset{\sim}{a}}_{n-1}]^{-1}\underset{\sim}{e}_{n-1} - \frac{a_n\underset{\sim}{e}'_{n-1}[\hat{\underset{\sim}{a}}_{n-1}]^{-1}\underset{\sim}{e}_{n-1}}{1+a_n\underset{\sim}{e}'_{n-1}[\hat{\underset{\sim}{a}}_{n-1}]^{-1}\underset{\sim}{e}_{n-1}}\{a_n[\hat{\underset{\sim}{a}}_{n-1}]^{-1}\underset{\sim}{e}_{n-1}\} \tag{24}$$

Simplifying and picking out the i^{th} element in $\underset{\sim}{b}_{n-1}$ gives us

$$b_i = \frac{dB_i}{dB} = \frac{\prod_{k\neq i} a_k}{\sum_{i=1}^{n} \left(\prod_{k\neq i} a_k\right)} \quad ; \quad i = 1,2,\ldots, n \tag{25}$$

It can then be shown that

$$\frac{dB_i}{dB} - x_i \gtrless 0. \iff P_{i,\alpha-2} \lessgtr \sum_{j\neq i}\beta_j P_{j,\alpha-2}$$

where

$$\beta_j = \frac{\prod_{k\neq i,j} a_k}{\sum_{j\neq i}\left(\prod_{k\neq i,j} a_k\right)} \quad ; \quad \sum_{j\neq i}\beta_j = 1 \tag{26}$$

Thus the optimal rule for increasing (decreasing) the per capita allocation to a group depends on whether the $(\alpha-2)^{th}$ order poverty for the group is less than (greater than) a weighted average of all the $(\alpha-2)^{th}$ order poverty indices in the other groups.

To interpret the general result in (26), consider the special case of two groups. With n = 2, this becomes

$$\frac{dB_1}{dB} - x_1 \gtrless 0 \iff P_{1,\alpha-2} \lessgtr P_{2,\alpha-2} \tag{27}$$

Thus (27) tells us that the group with the higher $(\alpha-2)^{th}$ order index should be favored disproportionately. For α-2, this means that the stance should be more favorable to the group with the higher head count ratio; but this should not be surprising since the head count ratio is here playing the role of an indicator for minimization of the poverty index P_2.

With n = 3, (26) becomes

$$\frac{dB_1}{dB} - x_1 \gtrless 0 \iff P_{1,\alpha-2} \lessgtr \frac{a_3}{a_2+a_3} P_{2,\alpha-2} + \frac{a_2}{a_2+a_3} P_{3,\alpha-2}$$

$$\frac{dB_2}{dB} - x_2 \gtrless 0 \iff P_{2,\alpha-2} \lessgtr \frac{a_3}{a_1+a_3} P_{1,\alpha-2} + \frac{a_1}{a_1+a_3} P_{3,\alpha-2}$$

$$\frac{dB_3}{dB} - x_3 \gtrless 0 \iff P_{3,\alpha-2} \lessgtr \frac{a_2}{a_1+a_2} P_{1,\alpha-2} + \frac{a_1}{a_1+a_2} P_{2,\alpha-2} \tag{28}$$

Thus whether or not the optimal per capita allocation to a group increases with the overall budget depends on how the $(\alpha-2)^{th}$ order measure for this group compares with the $(\alpha-2)^{th}$ order measure for the other two groups. In particular, if $P_{\alpha-2}$ values for the other two groups is less than a weighted sum of the $P_{\alpha-2}$ values for the other two groups (the weights being given in (28), then its per capita allocation increases with an increase in the total budget.

5. Conclusions and Further Research

If poverty alleviation cannot proceed entirely on a case by case basis, because the "means testing" of each applicant to the poverty alleviation budget is too costly, then to some extent policy has to be directed towards broadly defined groups. This has in fact been the general framework for policy analysis both of social security provisions in developed countries (e.g. Atkinson, 1969) and of poverty target groups in developing countries (e.g. Anand, 1983). This paper is a first attempt at formalizing and making precise leakages and tradeoffs involved in directing expenditure at different groups. Using a stylized model of how this expenditure is absorbed, we have derived budget allocation rules which depend on group characteristics. We have shown that budgetary rules for poverty alleviation involve an intricate combination of these characteristics - a simple rule such as directing more expenditure towards a group with higher poverty is not necessarily optimal.

A number of topics remain for further research and we end with a discussion of these. The analysis in this paper has been restricted to the P_α class of measures. An extension to other measures (see, for example, the collection in Donaldson and Weymark, 1985) presents intractabilities because of non-decomposability across population sub-groups, but simulation using specific distributions would perhaps be an appropriate strategy. In any case, an important area of further research must be the empirical application of the analysis to actual poverty alleviation programs.

Our assumption that the groups are mutually exclusive will limit empirical applicability since in practice income receiving units in poverty may satisfy more than one defining criterion (e.g. a household in poverty may have an unemployed head as well as a large number of children, or a poor farm household may grow more than one crop). The formalization and analysis of this case is an area for further research. In fact one could go further than simply assuming a given partition of the population into broad groups according to easily verifiable characteristics, and pose the question of the most effective division of the population into the target groups. The administrative costs of having a large number of target groups will have to be balanced against greater efficiency in reaching the poor. The formal analysis of the optimal partitioning of a population by observable characteristics, in order to optimize an objective function defined on unobservables which are correlated with the observables, is an open area for research.

In this paper we have restricted attention to additive and multiplicative absorption. One extension would be to a mixed linear scheme with additive and multiplicative components, or to more general non-linear schemes. Moreover, while this paper has been motivated by poverty alleviation, there is no reason why the analysis cannot be straightforwardly extended to the reduction of inequality, or the optimization of more general welfare functions. Finally, the basic model of expenditure absorption can be reinterpreted to allow the formulation of a methodology for analyzing the impact of macroeconomic adjustment, via its effect on the sectoral composition of national output, on poverty and inequality. A start is made in this direction in Kanbur (1985).

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