(part-1)
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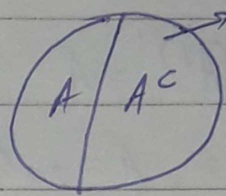
Events & Their Complements

Complements

$A + A^C$ = Sample Space.



complement: everything the event is NOT

Eg:- All possible outcomes → A, B, C

$P(A) + P(B)$ = Sum of prob. of A & B

$P(A) + P(B) + P(C) = 1$

$P(A) + P(B) > 1$?

- doesn't make sense
- double counting some events
- occuring simultaneously

$P(A) + P(B) < 1$?

- some events are not counted for.

$P < 1$?

- not guarenteed to occur

Complement of $A = A^c = A'$

$\rightarrow (A')' = A$

Eg: A = rolling an even number

A' = rolling an odd number

Eg: A = getting 1, 2, 4, 5, 6 $= \left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right) = \frac{5}{6}$ (difficult to calculate)

A' = not getting 3 $= 1 - \frac{1}{6} = \frac{5}{6}$ (easy to calculate)

If $P(A) + P(B) + P(C) = 1$, then $A' = B + C$

$P(A') = 1 - P(A)$

Combinatorics

- deals with combination of objects from a specific finite set with certain restrictions of repetition, order or other.

Permutation Variation Combination

$$\underbrace{\text{Permutation} \quad \text{Variation} \quad \text{Combination}}$$

Number of favourable outcomes

OR

Number of all elements in sample space

Permutation

- Number of different possible ways we can arrange a set of elements.

Eg: winners of a race : A, B, C

$P(3)$ = total number of different ways these drivers could split the medals.

(A,B,C) (A,C,B) (B,A,C) (B,C,A) (C,A,B) (C,B,A) = Permutation

$$P_n = n!$$

$$\cancel{{}^nP_r = \frac{n!}{(n-r)!}}$$

~~(r objects are chosen from n objects)~~
~~(& arranging into r slots)~~

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Factorial

$n! = n \times (n-1) \times (n-2) \ldots 1$ (n: natural no.)

Eg: $3! = 3 \times 2 \times 1 = 6$

properties:-

- negative numbers don't have factorials
- $0! = 1$
- $n! = n \times (n-1)!$, $(n+1)! = (n+1)\, n!$
- $(n+k)! = n! \times (n+1) \times (n+2) \ldots (n+k)$
- $(n-k)! = \dfrac{n!}{(n-k+1) \times (n-k+2) \ldots n}$
- if $n > k$, $\dfrac{n!}{k!} = (k+1)(k+2) \ldots n$

Variations (similar to permutation)

- total number of ways we can pick & arrange some elements of a given set

- with repetition: $\bar{V}^n_p = n^p$

n: total available elements

p: number of positions we need to fill

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- Variation without Repetition
  - cannot use same element twice

$$V_p^n = \frac{n!}{(n-p)!}$$

Combinations:

- number of ways we can pick certain elements of a set

⇒ All different permutation of a single combination are different variations.

$$^nC_r = \frac{n!}{(n-r)!\,r!} = \frac{V_r^n}{P_r}$$

Symmetry of Combination

- picking more elements leads to having fewer combinations

$$\frac{n!}{r!(n-r)!} = \boxed{^nC_r = {}^nC_{n-r}} = \frac{n!}{(n-r)!\,(n-(n-r))!} = \frac{n!}{(n-r)!\,r!}$$

$r > \frac{n}{2} > n-r$, apply symmetry.

Combinations with separate sample spaces

- mixture of different smaller individual events

Ex: $\overset{1}{\text{Sandwich}} + \overset{1}{\text{Side}} + \overset{1}{\text{drinks}}$

3 types 2 types 2 types

$$= {}^{3}C_{1} \times {}^{2}C_{1} \times {}^{2}C_{1} = 12$$

Summary:

pick : combination

arrange : permutation

pick & arrange : variation

No Repetition	Repetition
$C = \frac{V}{P}$	$\bar{V}_p^n = n^p$
$P = P_n = n!$	$\bar{C}_p^n = \frac{(n+p-1)!}{p!\,(n-1)!}$
$V = V_p^n = \frac{n!}{(n-p)!}$	
$C = C_p^n = \frac{n!}{(n-p)!\,p!}$	