Notation Guide - Wavelet Scattering for Tissue Classification

1. Basic Image and Signal Notation

- $f(x, y)$ or $f(\mathbf{x})$: Input image, where $\mathbf{x} = (x, y) \in \mathbb{R}^2$
- $\|\cdot\|$: L^2 norm, $\|f\| = \left(\int |f(\mathbf{x})|^2 d\mathbf{x}\right)^{1/2}$
- $*$: Convolution operator, $(f * g)(\mathbf{x}) = \int f(\mathbf{u}) g(\mathbf{x} - \mathbf{u}) d\mathbf{u}$

2. Wavelet Transform Components

- $\phi(\mathbf{x})$: Scaling function (low-pass filter)
- $\psi(\mathbf{x})$: Mother wavelet (band-pass filter)
- $\psi_{\lambda,\theta}(\mathbf{x})$: Wavelet at scale λ and orientation θ
- $\lambda \in 2^{\mathbb{Z}}$: Scale parameter (dyadic scales: $\lambda = 2^j, j \in \mathbb{Z}$)
- $\theta \in [0, \pi)$: Orientation parameter
- J : Number of scales
- L : Number of orientations per scale

3. Morlet Wavelet Family

The complex Morlet (Gabor) wavelet:

$$\psi(\mathbf{x}) = e^{i\boldsymbol{\xi}\cdot\mathbf{x}} \eta(\mathbf{x}) \tag{1}$$

where $\eta(\mathbf{x})$ is a Gaussian window and $\boldsymbol{\xi}$ is the central frequency.

Scaled and rotated version:

$$\psi_{\lambda,\theta}(\mathbf{x}) = \lambda^{-2} \psi\left(\lambda^{-1} R_\theta^{-1} \mathbf{x}\right) \tag{2}$$

where R_θ is the rotation matrix by angle θ.

4. Wavelet Transform Operations

- $Wf(\lambda, \theta)$: 2D continuous wavelet transform

$$Wf(\lambda, \theta) = \{f * \phi, (f * \psi_{\lambda,\theta})_{\lambda,\theta}\} \tag{3}$$

- $|Wf|(\lambda, \theta)$: Wavelet modulus transform

$$|Wf|(\lambda, \theta) = \{f * \phi, (|f * \psi_{\lambda,\theta}|)_{\lambda,\theta}\} \tag{4}$$

5. Scattering Transform Notation

Path Notation

- $p = (\lambda_1, \theta_1, \lambda_2, \theta_2, \ldots, \lambda_m, \theta_m)$: Frequency path
- m : Path length (order of scattering)
- $U[\lambda, \theta]$: Wavelet modulus operator, $U[\lambda, \theta]f = |f * \psi_{\lambda,\theta}|$

Scattering Coefficients by Order

Zeroth-order (low-pass filtered):

$$S^0 f = f * \phi \tag{5}$$

First-order (single wavelet convolution):

$$S^1 f(\lambda_1, \theta_1) = |f * \psi_{\lambda_1,\theta_1}| * \phi \tag{6}$$

Second-order (cascade of two wavelet convolutions):

$$S^2 f(\lambda_1, \theta_1, \lambda_2, \theta_2) = \big||f * \psi_{\lambda_1,\theta_1}| * \psi_{\lambda_2,\theta_2}\big| * \phi \tag{7}$$

m**-th order** (cascade of m wavelet convolutions):

$$S^m f(p) = \big|\cdots\big||f * \psi_{\lambda_1,\theta_1}| * \psi_{\lambda_2,\theta_2}\big|\cdots * \psi_{\lambda_m,\theta_m}\big| * \phi \tag{8}$$

Complete Scattering Transform

$$Sf = \begin{cases} S^0 f \\ S^1 f(\lambda_1, \theta_1) \\ S^2 f(\lambda_1, \theta_1, \lambda_2, \theta_2) \\ \vdots \\ S^m f(p) \end{cases} \tag{9}$$

6. Machine Learning Notation

- $\mathcal{D} = \{(f_i, y_i)\}_{i=1}^N$: Dataset with N samples
- f_i : i-th image
- $y_i \in \{1, 2, \ldots, C\}$: Class label for i-th image
- C : Number of classes (8 tissue types)
- $\mathbf{s}_i = Sf_i$: Scattering feature vector for image f_i
- d : Dimension of feature vector (61 in our implementation)
- $\mathbf{X} \in \mathbb{R}^{N \times d}$: Feature matrix
- $\mathbf{y} \in \{1, \ldots, C\}^N$: Label vector

7. SVM Classification

- $K(\mathbf{s}_i, \mathbf{s}_j)$: Kernel function
- For cubic polynomial kernel: $K(\mathbf{s}_i, \mathbf{s}_j) = (\mathbf{s}_i^T \mathbf{s}_j + c)^3$
- α_i : Lagrange multipliers
- $\hat{y}$: Predicted class label

8. Dataset Notation

- $N_{\text{train}} = 4000$: Number of training images
- $N_{\text{test}} = 1000$: Number of test images
- $N_c = 500$: Number of training images per class
- Image size: 150×150 pixels
- Tissue types: {TUMOR, STROMA, COMPLEX, LYMPHO, DEBRIS, MUCOSA, ADIPOSE, EMPTY}

9. Performance Metrics

- Accuracy: $\text{Acc} = \frac{1}{N_{\text{test}}} \sum_{i=1}^{N_{\text{test}}} \mathbb{I}(\hat{y}_i = y_i)$
- Per-class accuracy: $\text{Acc}_c = \frac{\text{TP}_c}{N_c}$
- Confusion matrix: $\mathbf{C} \in \mathbb{R}^{C \times C}$, where C_{ij} is the number of samples from class i classified as class j

10. Key Mathematical Properties

- **Translation invariance**: $S(f(\mathbf{x} - \mathbf{c})) = S(f(\mathbf{x}))$ for large support of ϕ
- **Energy preservation**: $\|f\|^2 = \sum_p \|S^m f(p)\|^2$
- **Lipschitz continuity**: $\|Sf_1 - Sf_2\| \leq C\|f_1 - f_2\|$ for deformations
- **Stability to deformations**: Provably stable to diffeomorphisms

Quick Reference Table

Symbol	Meaning
f	Input image
ϕ	Scaling function (low-pass)
$\psi_{\lambda,\theta}$	Wavelet at scale λ, orientation θ
$S^0 f$	Zeroth-order scattering
$S^1 f$	First-order scattering
$S^2 f$	Second-order scattering
J	Number of scales (3 in our work)
L	Number of orientations (4 in our work)
m	Scattering order/path length
d	Feature dimension (61)
C	Number of classes (8)