Optimization with Python

- Linear Programming
- Mixed-Integer Linear Programming
- Nonlinear Programming
- Mixed-Integer Nonlinear Programming
- Heuristics (GA and Particle Swarm)
- Constraint Programming

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Outlines

- Installing Python and Packages
- Starting with Python
- Linear Programming (LP)
- Mixed-Integer Linear Programming (MILP)
- Nonlinear Programming (NLP)
- Mixed-Integer Nonlinear Programming (MINLP)
- Heuristics (GA and Particle Swarm)
- Constraint Programming
- Practical and good examples

What is optimization

- Search for the optimal decision
- Any problem of planning:
  long, medium, short term, operational
- Applied in the decision-making for investments, operations, route problems, cost reduction...

- Ex.: We want to maximize the revenue for the sale of 2 products (x and y), each product costs 1 dollar. What is the required daily production?

  Constraints
  2y <= x+8 (time of production)
  2x + y <= 14 (raw material)
  2x <= y+10 (historical sales)
  x,y <= 10 (maximum daily production)

$$\max x + y$$
$$-x + 2y \leq 8$$
$$2x + y \leq 14$$
$$2x - y \leq 10$$
$$0 \leq x \leq 10$$
$$0 \leq y \leq 10$$

PROBLEM UNDERSTANDING → PROBLEM MODELING → **RESOLUTION** → RESULTS

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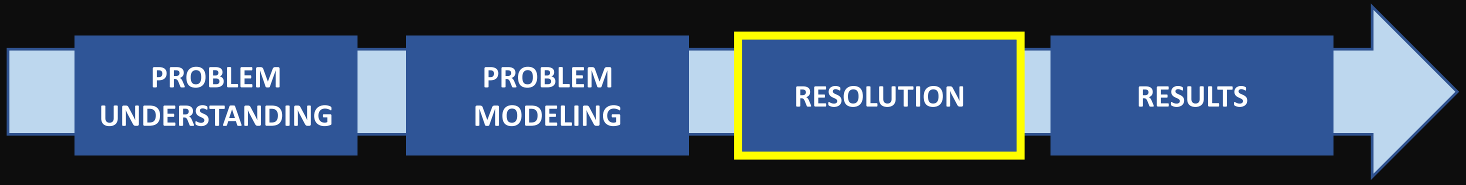

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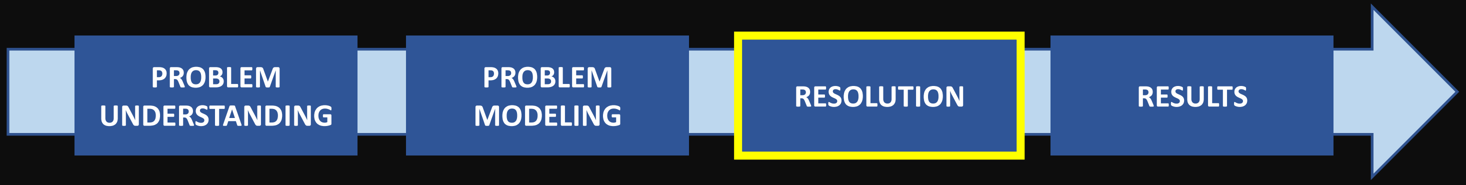
x and y integers



INTRODUCTION TO MATHEMATICAL MODELING

- What is mathematical modeling
- How do we solve mathematical problems?
- Type of variables
- Objective Function and Constraints
- How to model your problem?
- Some examples
- How to learn more?

WHAT IS MATHEMATICAL MODELING

Real-world problem → Mathematics

Very specific skill

I wish to minimize the total cost of logistics in a food delivery business

$$\min \sum_{i} Cost(i)$$

$$Cost(i) = \ldots..$$

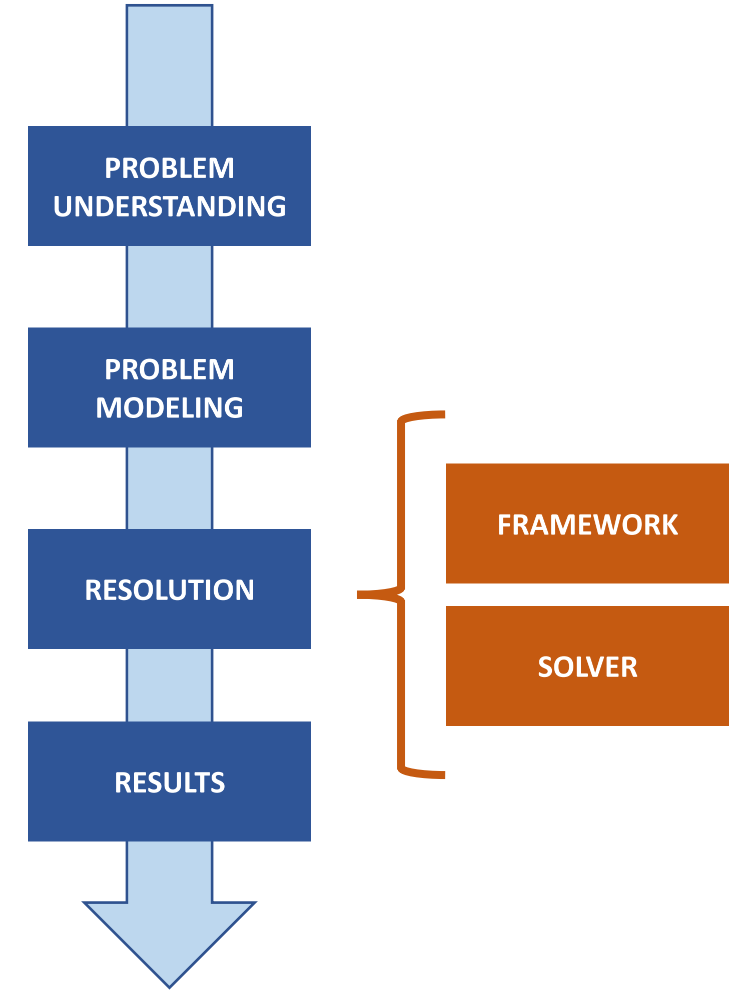
HOW DO WE SOLVE OPTIMIZATION PROBLEMS?

First, understand the problem!

Second, convert your problem to math

Third, solve your problem

Fourth, discuss the results



TYPE OF VARIABLES

What is variable?

What is a parameter?

What is an index?

What is a set?

$$\boldsymbol{min\ C_1 + C_2}$$

$$C_1 = 0.1P_1^2 + 0.5$$

$$C_2 = P_2 + 3$$

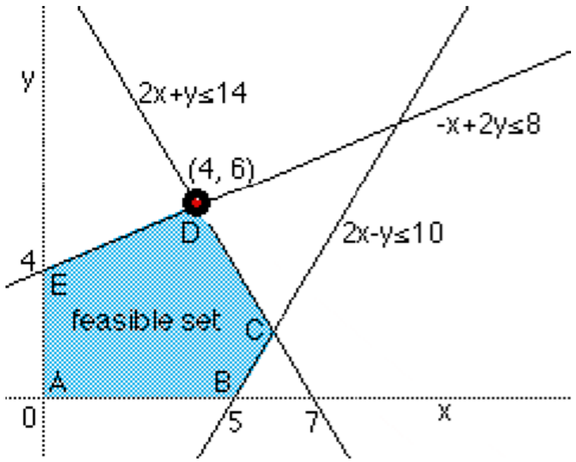
$$P_1 + P_2 = P_T$$

- Continuous
- Integer (discrete)
- Binary
- Others

OBJECTIVE FUNCTION AND CONSTRAINTS

How to define if a solution is better than other?

What defines if a solution is feasible?



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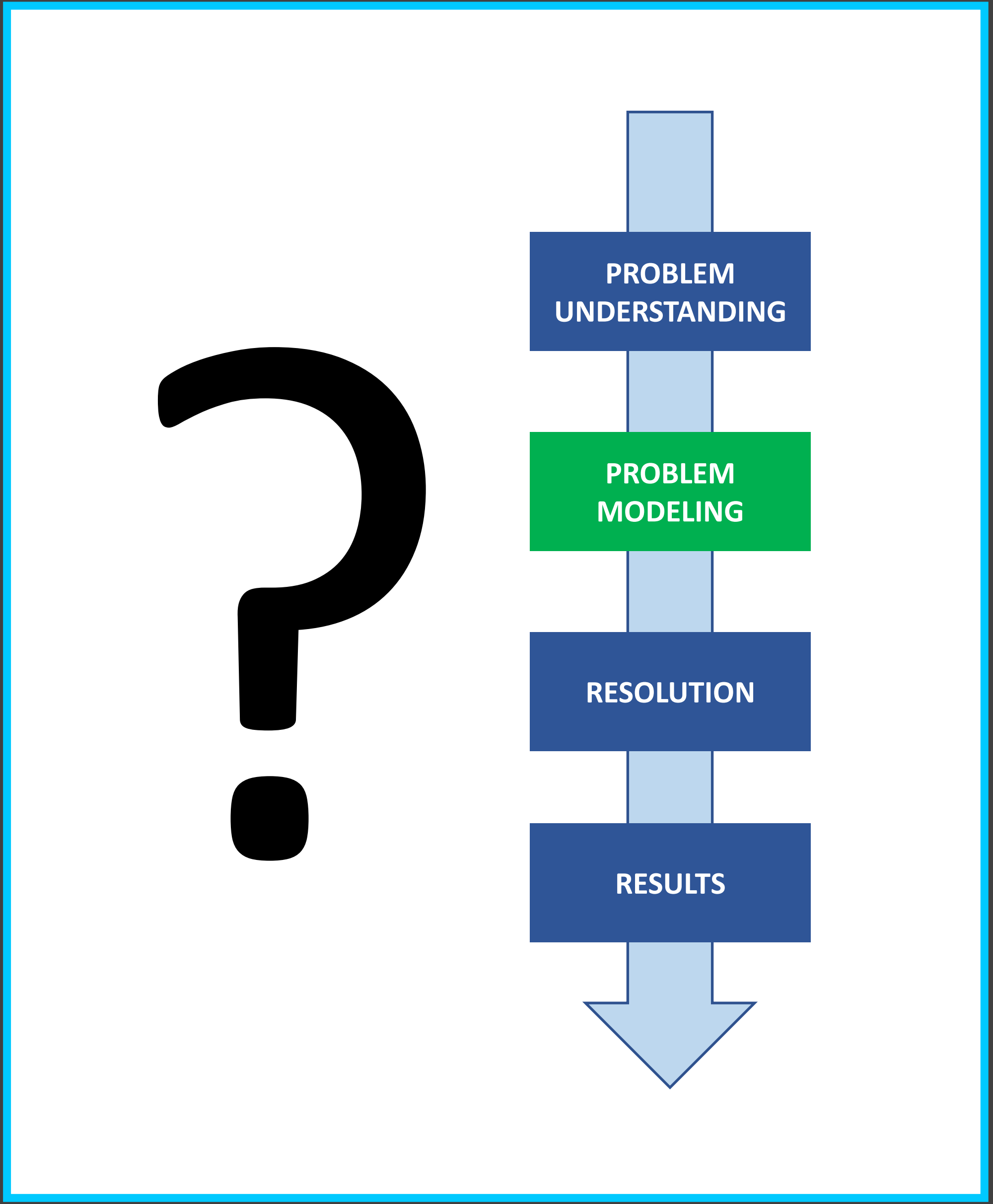
$$x, y \geq 0$$

HOW TO MODEL YOUR PROBLEM?



EXAMPLE 1

Mayke wish to define the best investments that he should make with his money. He has a **total of 100,000 USD** and the following options for investment.

A) Low risk fund with historical gains of 5% per year
B) Medium risk fund with historical gains 10% per year
C) High risk fund with historical gains of 12% per year

Mayke wish to control the risk of his investments with maximum of 10% of his money in the investment with high risk, 20% in the investment with medium risk.

Which is the decision of investments in A, B, and C that **maximize the return** of investment for Mayke?

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Which is the decision of investments in A, B, and C that **maximize the return** of investment for Mayke?

Variables and indexes

R_A, R_B, R_C → Return of investment A,B, and C

C_A, C_B, C_C → Invested capital in fund A, B, and C

Constraints (rules)

$$C_A + C_B + C_C = 100{,}000$$

$$R_A = 0.05C_A$$

$$R_B = 0.10C_B$$

$$R_C = 0.12C_C$$

$$0 \leq C_B \leq 0.2 * 100{,}000$$

$$0 \leq C_C \leq 0.1 * 100{,}000$$

Objective Function

$$\max(R_A + R_B + R_C)$$

EXAMPLE 2

Mayke wish to define the best investments that he should make with his money. He has a **total of 100,000 USD** and the following options for investment.

A) Low risk fund with historical gains of 5% per year
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D) Especial fund: $10^{-6} * (C_D)^2$

Mayke wish to control the risk of his investments with maximum of 10% of his money in the investment with high risk, 20% in the investment with medium risk, and 30% in the especial fund.

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Index

A, B, C, D → funds

Variables

C_A, C_B, C_C, C_D → Invested capital in the fund

R_A, R_B, R_C, R_D → Return of investment from the fund

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Objective Function

$$\max R_A + R_B + R_C + R_D$$

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<u>Objective Function</u>

$$\max R_A + R_B + R_C + R_D$$

Same as

$$\max \sum_{f \in F} R_f$$

Set $F = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$

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$$R_D = 10^{-6}(C_D)^2$$

$$R_f = return(C_f) \ \forall f$$

$$0 \leq C_B \leq 0.2 * 100{,}000$$

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$$C_f^{min} \leq C_f \leq C_f^{max} \quad \forall f$$

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Objective Function

$$\max \sum_{f \in F} R_f$$

Constraints (rules)

$$\sum_{f \in F} C_f = 100{,}000$$

$$R_f = return(C_f) \quad \forall f$$

$$C_f^{min} \leq C_f \leq C_f^{max} \quad \forall f$$

EXAMPLE 3

You have a company of shoes with 3 very large machines, and you wish to minimize the total cost of production.

The total cost of production of each machine is:

$A)\ \ C_A = 0.1P_A^2 + 0.5P_A + 0.1$

$B)\ \ C_B = 0.3P_B + 0.5$

$C)\ \ C_C = 0.01P_C^3$

where C is the cost of production of P products for each machine

In the next month, you have a **demand of 10,000 shoes**. What is the number of products that should be assigned to each machine in order to **minimize the total cost**?

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Variables

C_A, C_B, C_C → Cost of production of machines A,B,C

P_A, P_B, P_C → Number of production produced by each machine (integer)

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$$P_A, P_B, P_C \geq 0$$

Objective Function

$$\min C_A + C_B + C_C$$

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Constraints (rules)

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Objective Function

$$\min \sum_m C_m$$

$m = \{A, B, C\}$

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Objective Function

$$\min \sum_m C_m$$

Constraints (rules)

$$\sum_m P_m = 10{,}000$$

$$C_A = 0.1P_A^2 + 0.5P_A + \beta_A 0.1$$

$$C_B = 0.3P_B + \beta_B 0.5$$

$$C_C = 0.01P_C^3$$

$$P_A \leq \beta_A M$$

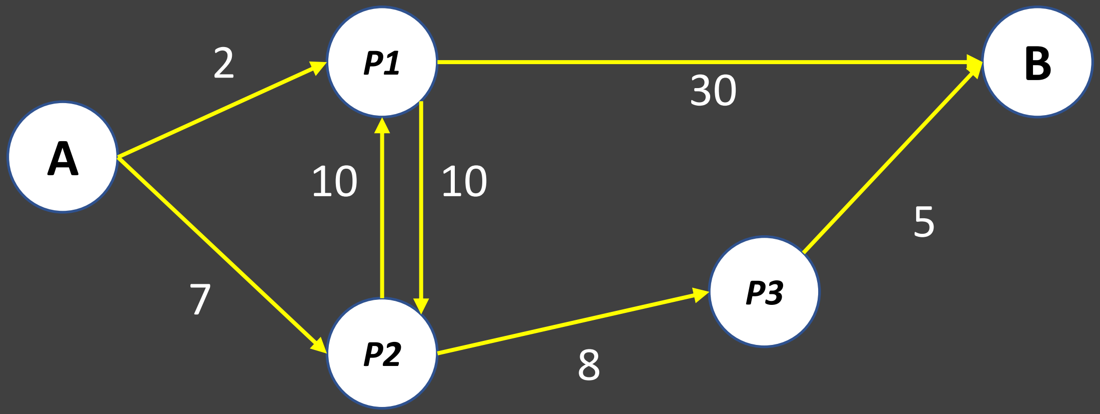
$$P_B \leq \beta_B M$$

$$P_A, P_B, P_C \geq 0$$

M: very large number $\quad$ $m = \{A, B, C\}$

EXAMPLE 4

Create a generic formulation to minimize the path from point A to B



The numbers are the distances from one point to another

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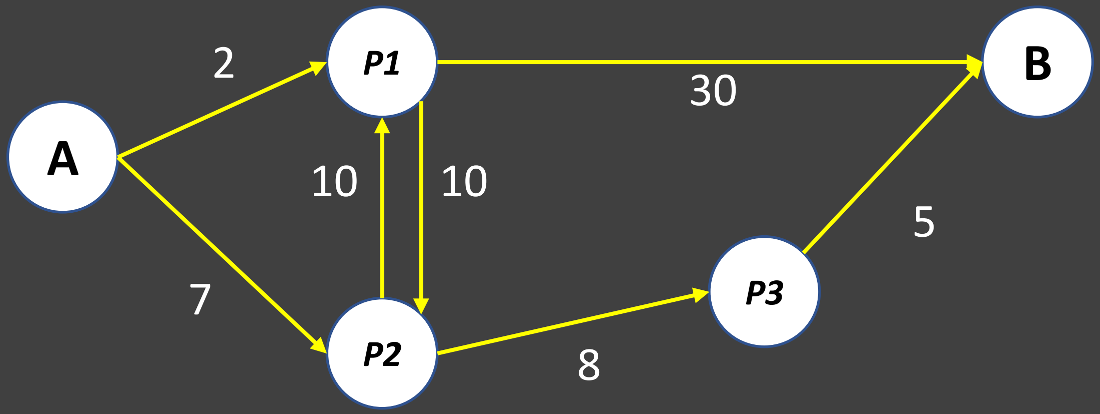

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Variables

$x_{i,j}$ → Binary decision on connection point i to j

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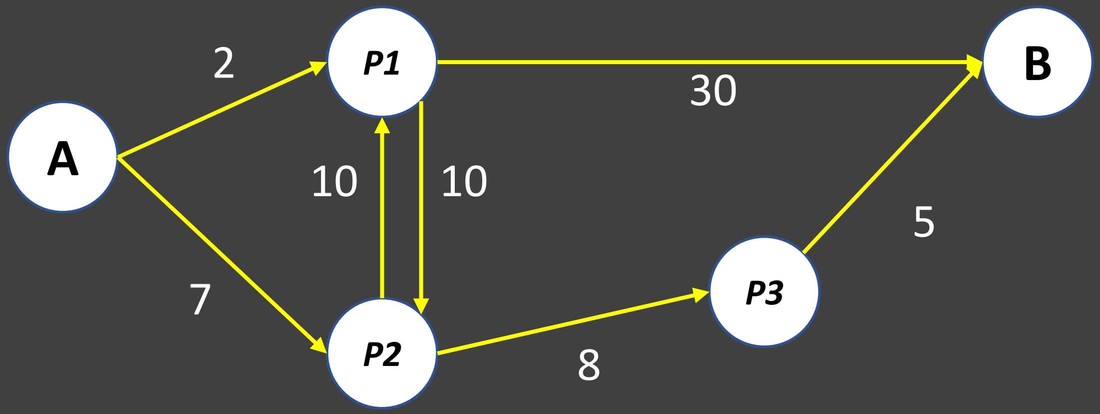

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$D_{i,j}$ → Distance from point i to j

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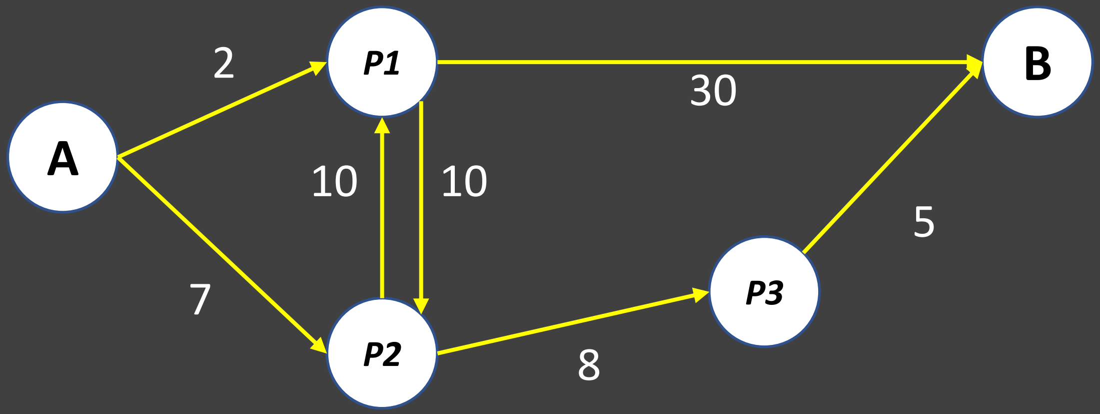

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$D_{i,j}$ → Distance from point i to j

Sets

Ω_i^{in} → set of nodes that connect to arcs entering node i

Ω_i^{out} → set of nodes that connect to arcs exiting node i

Example:

$\Omega_{P1}^{in} = \{A, P2\}$ $\Omega_B^{in} = \{P1, P3\}$

$\Omega_{P1}^{out} = \{P2, B\}$ $\Omega_A^{out} = \{P1, P2\}$

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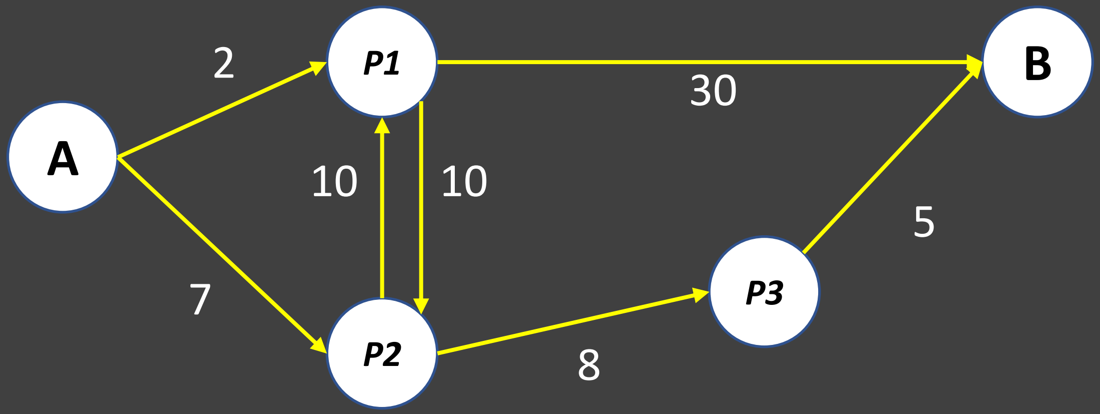

The numbers are the distances from one point to another

Variables, Parameters and sets

$x_{i,j}$ → Binary decision on connection point i to j

$D_{i,j}$ → Distance from point i to j

Ω_i^{in} → set of nodes that connect to arcs entering node i

Ω_i^{out} → set of nodes that connect to arcs exiting node i

Objective Function

$$\min \sum_{(i,j)} x_{i,j} D_{i,j}$$

Constraints

$$\sum_{j \in \Omega_A^{out}} x_{A,j} = 1$$

$$\sum_{i \in \Omega_B^{in}} x_{i,B} = 1$$

$$\sum_{j \in \Omega_i^{out}} x_{i,j} = \sum_{j \in \Omega_i^{in}} x_{j,i} \qquad \forall i \backslash \{A, B\}$$

EXAMPLE 5

Petter has a construction company. He needs to assign 5 of the company's teams to work in some of the constructions below:

A) Revenue of 500, requires 1 team
B) Revenue of 4,000, requires 3 teams
C) Revenue of 3,000, requires 2 teams
D) Revenue of 2,000, requires 1 team
E) Revenue of 2,000, requires 5 teams

Select the constructions that would maximize the revenue.
- Each construction can be made just once
- Not all constructions will be selected

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Variables

x_o → Binary decision on selecting (or not) construction o

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NT^{max} → Total number of teams (available) (5)

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Objective Function

$$\max \sum_o x_o R_o$$

Constraints

$$\sum_o x_o NT_o \leq NT^{max}$$

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Select the constructions that would maximize the revenue.
- Each construction can be made just once
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Solution: Constructions B and C

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R_o → Revenue of construction o

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Objective Function

$$\max \sum_o x_o R_o$$

Constraints

$$\sum_o x_o NT_o \leq NT^{max}$$

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C) Revenue of 3,000, requires 2 teams
D) Revenue of 2,000, requires 1 team
E) Revenue of 2,000, requires 5 teams

Select the constructions that would maximize the revenue.
- Each construction can be made just once
- Not all constructions will be selected
- Construction C can only be selected if A is selected
- Construction D can only be selected if A and C are selected

Variables

x_o → Binary decision on selecting (or not) construction o

Parameters

R_o → Revenue of construction o

NT_o → Number of teams required for the construction o

NT^{max} → Total number of teams (available) (5)

Objective Function

$$\max \sum_o x_o R_o$$

Constraints

$$\sum_o x_o NT_o \leq NT^{max}$$

EXAMPLE 6

Petter has a construction company. He needs to assign 5 of the company's teams to work in some of the constructions below:

A) Revenue of 500, requires 1 team
B) Revenue of 4,000, requires 3 teams
C) Revenue of 3,000, requires 2 teams
D) Revenue of 2,000, requires 1 team
E) Revenue of 2,000, requires 5 teams

Select the constructions that would maximize the revenue.
- Each construction can be made just once
- Not all constructions will be selected
- Construction C can only be selected if A is selected
- Construction D can only be selected if A and C are selected

Solution: Constructions A, C, and D

Variables

x_o → Binary decision on selecting (or not) construction o

Parameters

R_o → Revenue of construction o

NT_o → Number of teams required for the construction o

NT^{max} → Total number of teams (available) (5)

Objective Function

$$\max \sum_o x_o R_o$$

Constraint

$$\sum_o x_o NT_o \leq NT^{max}$$

$$x_C \leq x_A$$

$$x_D \leq x_A$$

$$x_D \leq x_C$$

EXAMPLE 6

Petter has a construction company. He needs to assign 5 of the company's teams to work in some of the constructions below:

A) Revenue of 500, requires 1 team
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- Each construction can be made just once
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- Construction C can only be selected if A is selected
- Construction D can only be selected if A and C are selected

Variables

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Parameters

R_o → Revenue of construction o

NT_o → Number of teams required for the construction o

NT^{max} → Total number of teams (available)

Objective Function

$$\max \sum_o x_o R_o$$

Constraints

$$\sum_o x_o NT_o \leq NT^{max}$$

$$x_C \leq x_A$$

$$x_D \leq x_A * x_C \quad (???)$$

EXAMPLE 7

Mark wishes to define the scheduling of costumers that he must attend in the next 3 days.

The list of jobs (demands) with the duration of job and its profit is defined below:
A) duration 2h, profit 200 USD
B) duration 3h, profit 500 USD
C) duration 5h, profit 300 USD
D) duration 2h, profit 100 USD
E) duration 6h, profit 1,000 USD
F) duration 4h, profit 300 USD

Mark wants to maximize the profit for the next 3 days working 6h per day. Which demands he should attend per day?
- Neglect the traveling time
- Each demand only can be attended once

EXAMPLE 7

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F) duration 4h, profit 300 USD

Mark wants to maximize the profit for the next 3 days working 6h per day. Which demands he should attend per day?
- Neglect the traveling time
- Each demand only can be attended once

Variables

$x_{j,d}$ → Binary decision on attending (or not) job j in day d

Parameters

P_j → Profit for the job

D_j → Duration of the job in hours

Th → Number of hours in a working day (6)

EXAMPLE 7

Mark wishes to define the scheduling of costumers that he must attend in the next 3 days.

The list of jobs (demands) with the duration of job and its profit is defined below:
A) duration 2h, profit 200 USD
B) duration 3h, profit 500 USD
C) duration 5h, profit 300 USD
D) duration 2h, profit 100 USD
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$x_{j,d}$ → Binary decision on attending (or not) job j in day d

Parameters

P_j → Profit for the job

D_j → Duration of the job in hours

Th → Number of hours in a working day (6)

Objective Function

$$\max \sum_j \sum_d x_{j,d} P_j$$

Constraints

$$\sum_j x_{j,d} Dj \leq Th \quad \forall d$$

$$\sum_d x_{j,d} \leq 1 \quad \forall j$$

EXAMPLE 7

Mark wishes to define the scheduling of costumers that he must attend in the next 3 days.

The list of jobs (demands) with the duration of job and its profit is defined below:
A) duration 2h, profit 200 USD
B) duration 3h, profit 500 USD
C) duration 5h, profit 300 USD
D) duration 2h, profit 100 USD
E) duration 6h, profit 1,000 USD
F) duration 4h, profit 300 USD

Solution: Profit Total = 2100.0
Job E in day 1 (duration 6, profit 1000)
Job B in day 2 (duration 3, profit 500)
Job D in day 2 (duration 2, profit 100)
Job A in day 3 (duration 2, profit 200)
Job F in day 3 (duration 4, profit 300)

Variables

$x_{j,d}$ → Binary decision on attending (or not) job j in day *d*

Parameters

P_j → Profit for the job

D_j → Duration of the job in hours

Th → Number of hours in a working day (6)

Objective Function

$$\max \sum_j \sum_d x_{j,d} P_j$$

Constraints

$$\sum_j x_{j,d} Dj \leq Th \ \ \forall d$$

$$\sum_d x_{j,d} \leq 1 \quad \forall j$$

EXAMPLE 8

Mark wishes to define the scheduling of costumers that he must attend in the next 3 days.

The list of jobs (demands) with the duration of job and its profit is defined below:
A) duration 2h, profit 200 USD
B) duration 3h, profit 500 USD
C) duration 5h, profit 300 USD
D) duration 2h, profit 100 USD
E) duration 6h, profit 1,000 USD
F) duration 4h, profit 300 USD

Mark wants to maximize the profit for the next 3 days working 6h per day. Which demands he should attend per day?
- Neglect the traveling time
- Each demand only can be attended once
- Mark wish to do a maximum of 1 job per day

Variables

$x_{j,d}$ → Binary decision on attending (or not) job j in day *d*

Parameters

P_j → Profit for the job

D_j → Duration of the job in hours

Th → Number of hours in a working day (6)

Objective Function

$$\max \sum_j \sum_d x_{j,d} P_j$$

Constraints

$$\sum_j x_{j,d} Dj \leq Th \quad \forall d$$

$$\sum_d x_{j,d} \leq 1 \quad \forall j$$

EXAMPLE 8

Mark wishes to define the scheduling of costumers that he must attend in the next 3 days.

The list of jobs (demands) with the duration of job and its profit is defined below:
A) duration 2h, profit 200 USD
B) duration 3h, profit 500 USD
C) duration 5h, profit 300 USD
D) duration 2h, profit 100 USD
E) duration 6h, profit 1,000 USD
F) duration 4h, profit 300 USD

Mark wants to maximize the profit for the next 3 days working 6h per day. Which demands he should attend per day?
- Neglect the traveling time
- Each demand only can be attended once
- Mark wish to do a maximum of 1 job per day

Variables

$x_{j,d}$ → Binary decision on attending (or not) job *j* in day *d*

Parameters

P_j → Profit for the job

D_j → Duration of the job in hours

Th → Number of hours in a working day (6)

Objective Function

$$\max \sum_j \sum_d x_{j,d} P_j$$

Constraints

$$\sum_j x_{j,d} Dj \leq Th \quad \forall d$$

$$\sum_d x_{j,d} \leq 1 \quad \forall j$$

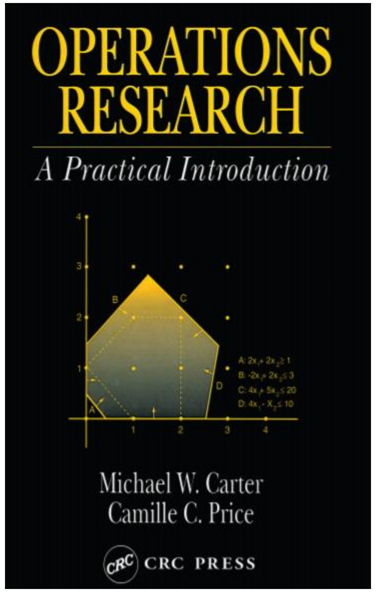
$$\sum_j x_{j,d} \leq 1 \quad \forall d$$

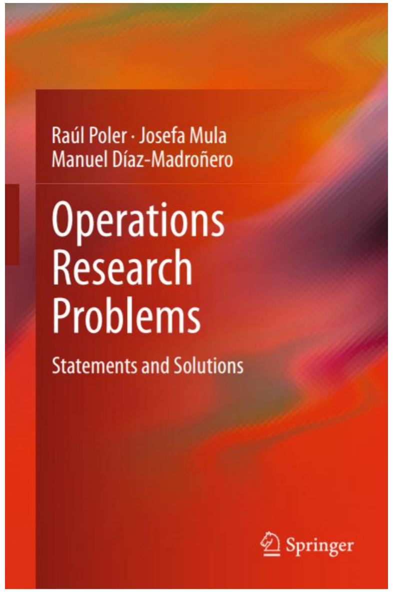
HOW TO LEARN MORE?

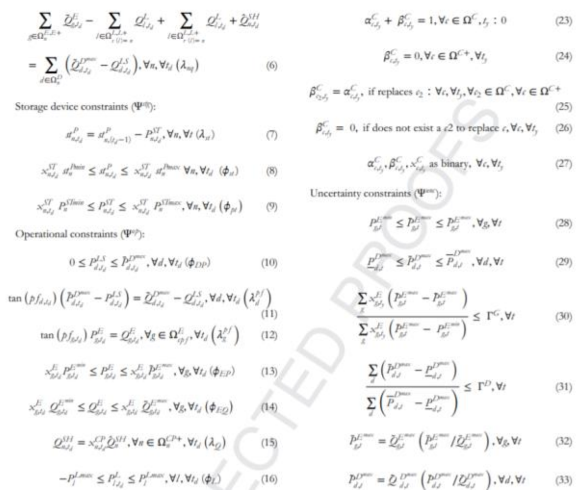
Try to solve exercises from books

Try to read articles and REALLY understand the proposed models

PRACTICE!!!







Installing Python and Packages

How to start with Python

WinPython

Portable

https://winpython.github.io/

Anaconda

Python distribution platform
https://www.anaconda.com/

Python Installation

https://www.python.org/

Installing Python

- Windows:
  - Go to https://www.python.org/
  - Download and install
  - Check version!
  - Update PIP

- Linux
  - Check your version:
    python --version

Packages

- Command Prompt:
  pip install PACKAGE_NAME

IDE Spyder

pip install spyder

Jupyter Notebook

pip install jupyterlab

Starting with Python

Starting with Python

Lists
Tuples
Dictionaries

Starting with Python

If
For
While

Inline commands

Starting with Python

Functions

Starting with Python

Numpy

Starting with Python

Pandas

Starting with Python

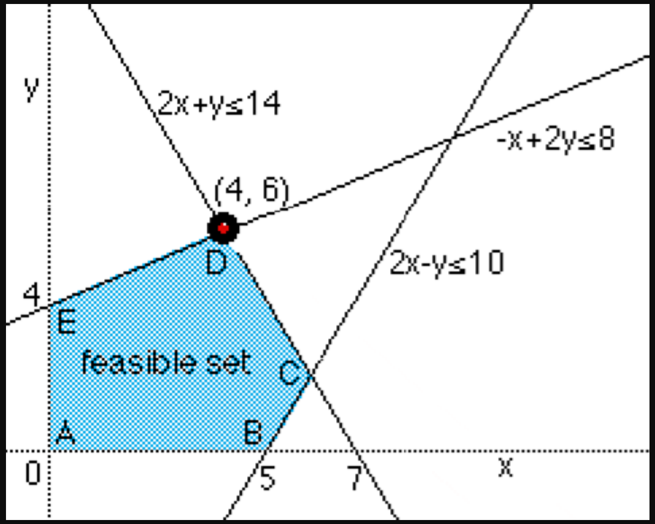
Pandas
Reading from Excel
and some functions

Starting with Python

Matplotlib

Linear Programming (LP)

Introduction



$$\max x + y$$

$$-x + 2y \leq 8$$

$$2x + y \leq 14$$

$$2x - y \leq 10$$

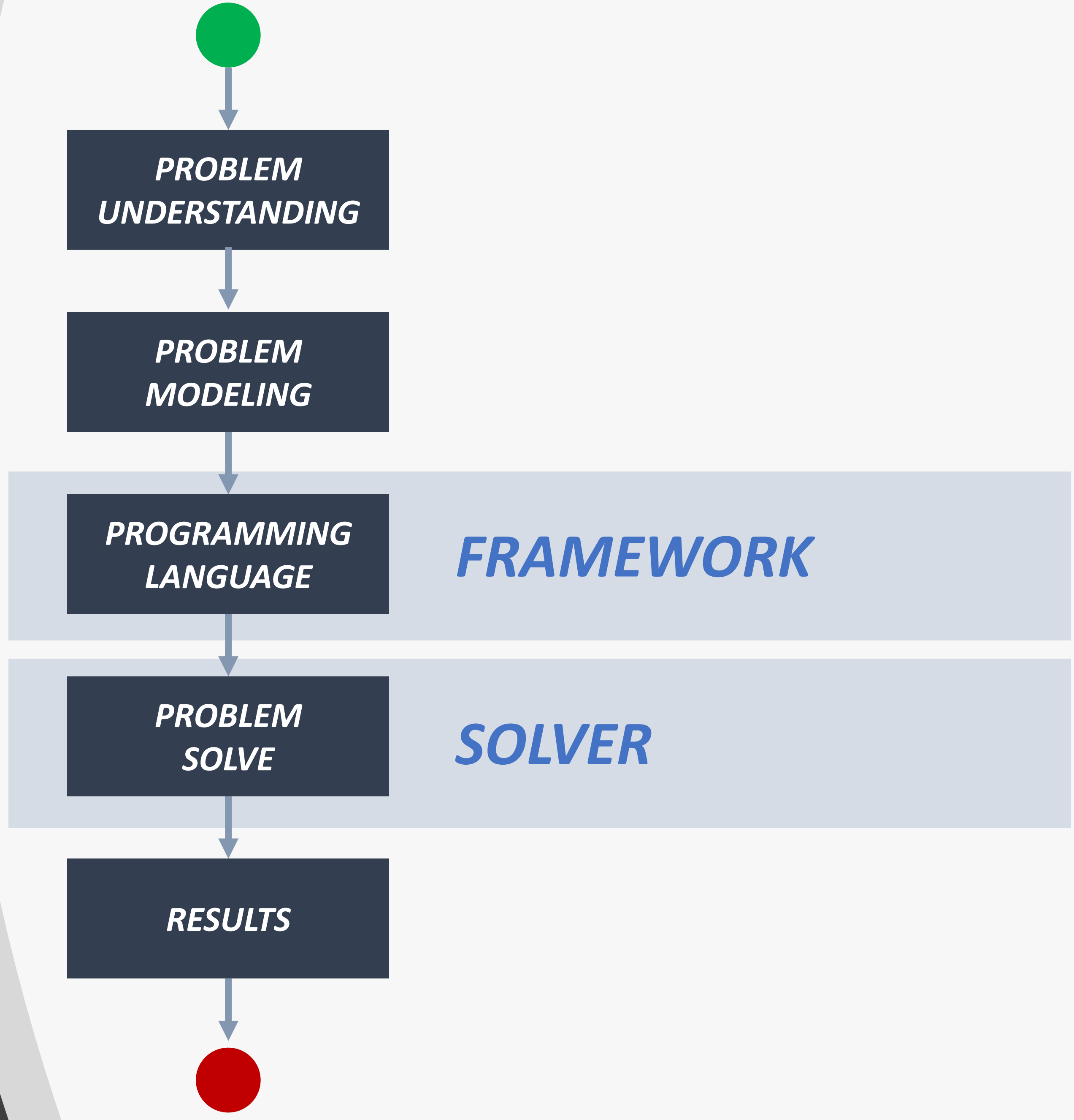
$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

Linear Programming

Solver
vs
Framework

Solver vs Framework



Linear Programming

Or-Tools

https://developers.google.com/optimization

Linear Programming

Scip

https://www.scipopt.org/

Download and instal SCIP

Install package PYSCIPOPT
Set environment variable SCIPOPTDIR

Package documentation
https://github.com/SCIP-Interfaces/PySCIPOpt

Linear Programming

Gurobi

https://www.gurobi.com/
Download and install Gurobi
Activate Gurobi

CPLEX

https://www.ibm.com/products/ilog-cplex-optimization-studio
Download and install

GLPK

http://sourceforge.net/projects/winglpk/
Download
Add GLPK/win64 to the environment variable PATH

Linear Programming

Pyomo

pip install pyomo

Documentation
https://pyomo.readthedocs.io/

Linear Programming

PuLP

pip install cython
pip install pulp

More information in
https://github.com/coin-or/pulp

Linear Programming

Which solver should I choose?

Which framework and solver

Framework (AML)	Linear Problems	Nonlinear Problems	How easy to start with	How easy to configure a new solver and about documentation
Pyomo	X	X	High	High
Ortools	X		Very High	Low
PuLP	X		High	High
SCIP	X	X	Very High	Not possible / Low
SciPy	X	X	Low	Medium

Solver	Linear Problemas	Nonlinear	Free / Commercial
Gurobi	X		COMMERCIAL
Cplex	X		COMMERCIAL
CBC	X		FREE
GLPK	X		FREE
IPOPT		X	FREE
SCIP	X	X	FREE
Baron		X	COMMERCIAL

Exercise

Show the optimal solution and processing time for the following problem:

$$\min -4x - 2y$$

$$x + y \leq 8$$

$$8x + 3y \geq -24$$

$$-6x + 8y \leq 48$$

$$3x + 5y \leq 15$$

$$x \leq 3$$

$$y \geq 0$$

Try to solve it by your self

Check the solution in the resource of this class

Estimated time: 30min

Observation: Use the package *time* to compute the processing time

Working with Pyomo

Working with Pyomo

Using different solvers

CBC:

https://projects.coin-or.org/Cbc

https://bintray.com/coin-or/download/Cbc/

Working with Pyomo

Arrays and Summations

Arrays and Summation

Power Generation (Pg)

ID	Cost	Power Generation
0	0,10	20 kW
1	0,05	10 kW
2	0,30	40 kW
3	0,40	50 kW
4	0,01	5 kW

Load Points (Pd)

ID	Load Demand
0	50 kW
1	20 kW
2	30 kW

***Only generators 0 and 3 can provide power to load point 0**

$$\min \sum_{i_g=0}^{4} C_g(i_g)P_g(i_g)$$

$$\sum_{i_g=0}^{4} P_g(i_g) = \sum_{i_c=0}^{2} P_d(i_d)$$

$$P_d(0) \leq P_g(0) + P_g(3)$$

$$P_g(i_g) \geq 0 \quad \forall i_g$$

$$P_g(i_g) \leq P_g(i_g)^{LIM} \quad \forall i_g$$

Working with Pyomo

Print your model, constraints, and summary

Working with Pyomo

Pyomo constraint rules

Working with Pyomo

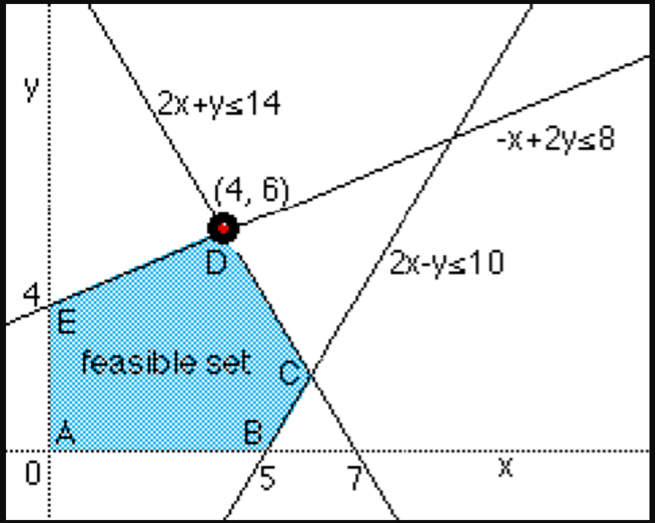
Some NonLinear equations

*- binary * continuos*
*b * x*

- continuos²
*x ** 2*

Mixed-Integer Linear Programming (MILP)

Introduction



$$\max x + y$$

$$-x + 2y \leq 8$$

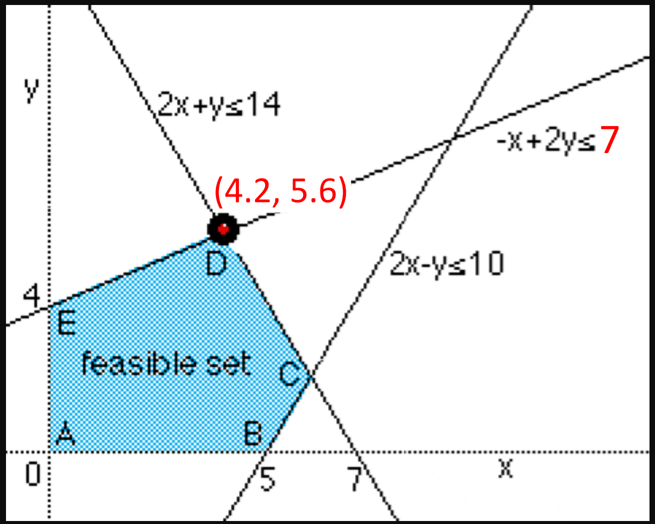
$$2x + y \leq 14$$

$$2x - y \leq 10$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

Introduction



$$\max x + y$$

$$-x + 2y \leq 7$$

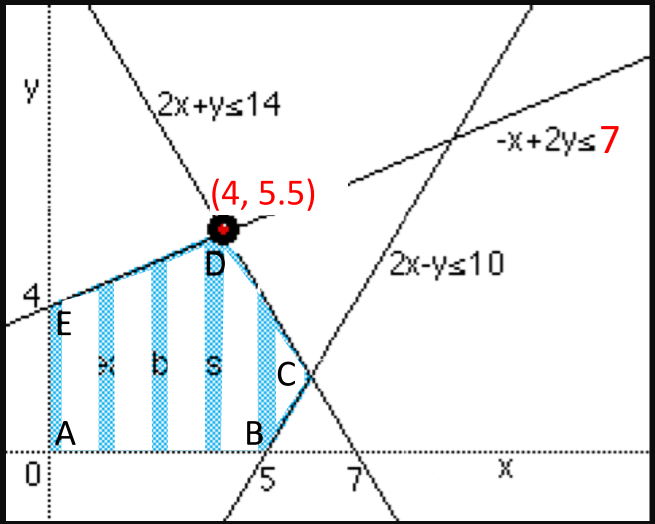
$$2x + y \leq 14$$

$$2x - y \leq 10$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

Introduction



$$\max x + y$$

$$-x + 2y \leq 7$$

$$2x + y \leq 14$$

$$2x - y \leq 10$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

x as integer

MILP

Pyomo

model.x = pyo.Var(within=Integers)

model.x = pyo.Var(within=Binary)

https://pyomo.readthedocs.io/en/stable/pyomo_modeling_components/Sets.html#predefined-virtual-sets

MILP

Ortools

Change the Solver (from GLOP to):
CBC
Gurobi
Cplex

*x = solver.**IntVar**(0,10,'x')*

MILP

SCIP

x = model.addVar('x', vtype='INTEGER')

Exercise

Find the optimal solution for the following problem

$$\min \sum_{i=1}^{5} x_i + y$$

Tip 1
From 1 to 5

$$\sum_{i=1}^{5} x_i + y \leq 20$$

$$x_i + y \geq 15\ , \forall i$$

Tip 2
For all i

$$\sum_{i=1}^{5} i \cdot x_i \geq 10$$

$$x_5 + 2y \geq 30$$

$$x_i, y \geq 0$$

x_i integer, $\forall i$

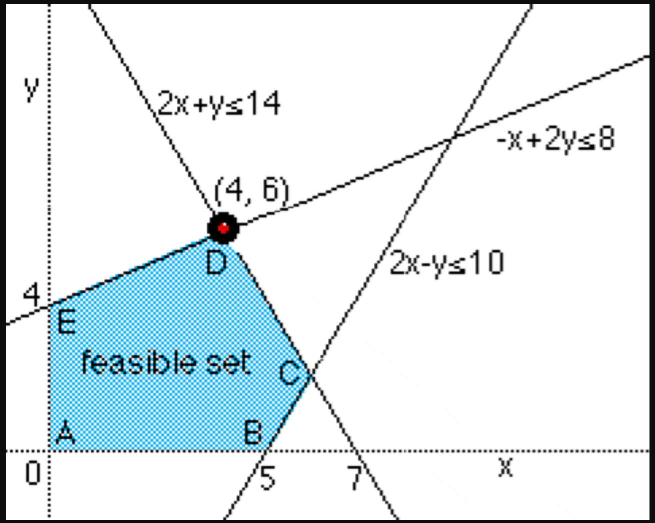
Try to solve it by your self

Check the solution in the next class

Estimated time: 1 hour

Nonlinear Programming (NLP)

Introduction



$$\max x + y$$

$$-x + 2y \leq 8$$

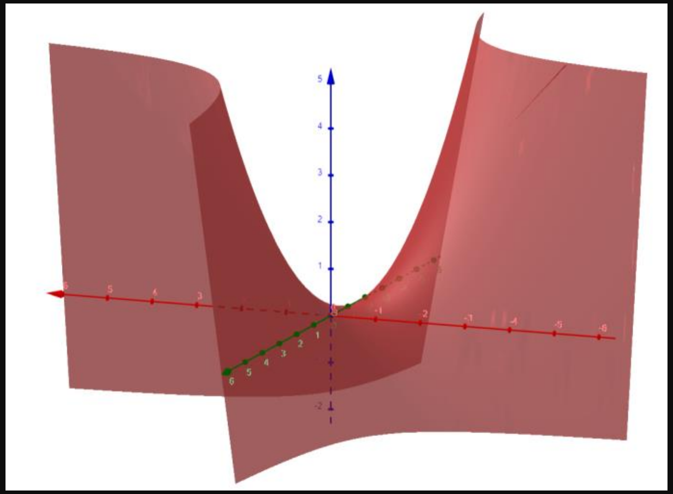
$$2x + y \leq 14$$

$$2x - y \leq 10$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

Introduction



$$\max x + xy$$

$$-x + 2y\boldsymbol{x} \leq 8$$

$$2x + y \leq 14$$

$$2x - y \leq 10$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

NLP

Pyomo: IPOPT

Search for ipopt binaries
https://www.coin-or.org/download/binary/Ipopt

Unzip in C:\

Pyomo
opt = SolverFactory(
‘ipopt',
executable='C:\\ipopt\\bin\\ipopt.exe')

NLP

SCIP

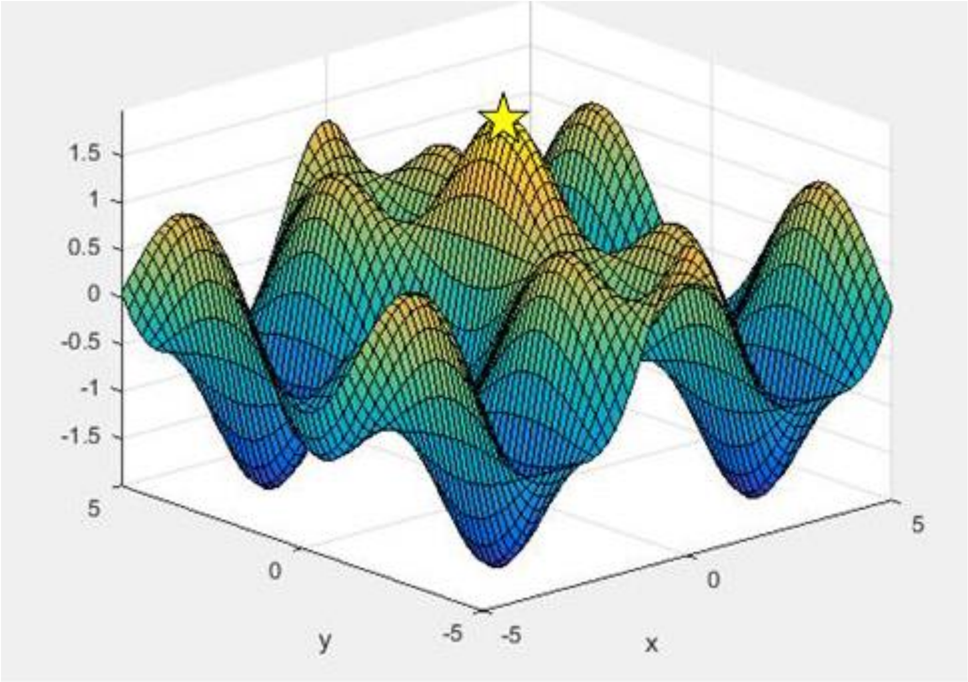
Exercise

Find the optimal solution for the following problem

$$\max \cos(x+1) + \cos(x)\, cos(y)$$

$$-5 \leq x \leq 5$$

$$-5 \leq y \leq 5$$



Explore the following options

model.x = pyo.Var(initialize=N)   N can be any number (0)

opt.options['tol'] = N   N can be any number (1e-6)

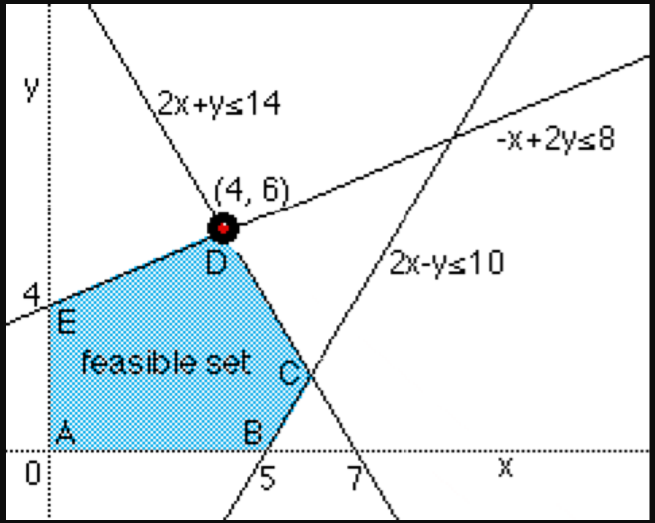
Try to solve it by yourself

Check the solution in the next class

Estimated time: 20 min

Mixed-Integer Nonlinear Programming (MINLP)

Introduction



$$\max x + y$$

$$-x + 2y \leq 8$$

$$2x + y \leq 14$$

$$2x - y \leq 10$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

Introduction

$$\max x + xy$$

$$-x + 2yx \leq 8$$

$$2x + y \leq 14$$

$$2x - y \leq 10$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 10$$

x **integer**

MINLP

Pyomo: Couenne

https://projects.coin-or.org/Couenne

https://www.coin-or.org/download/binary/Couenne/

Unzip in C:

opt = SolverFactory('couenne', executable='C:\\couenne\\bin\\couenne.exe')

MINLP

Decomposition Pyomo + MindtPy

opt = SolverFactory('mindtpy')

opt.solve(model, mip_solver='gurobi', nlp_solver='ipopt')

MINLP

SCIP

MINLP

Genetic Algorithm

pip install geneticalgorithm

https://pypi.org/project/geneticalgorithm/

MINLP

Particle Swarm

pip install pyswarm

https://pythonhosted.org/pyswarm/

CP

Ortools

https://developers.google.com/optimization/cp/integer_opt_cp

Maximize $2x + 2y + 3z$
subject to

$x + {}^{7}/_{2}\, y + {}^{3}/_{2}\, z$	$\leq$	25
$3x - 5y + 7z$	$\leq$	45
$5x + 2y - 6z$	$\leq$	37
x, y, z	$\geq$	0

x, y, z integers

Constraint Programming (CP)

CP

Ortools

https://developers.google.com/optimization/cp/integer_opt_cp

Maximize $2x + 2y + 3z$
subject to

$x + {}^7/_2\, y + {}^3/_2\, z$	$\leq$	25
$3x - 5y + 7z$	$\leq$	45
$5x + 2y - 6z$	$\leq$	37
x, y, z	$\geq$	0

x, y, z integers

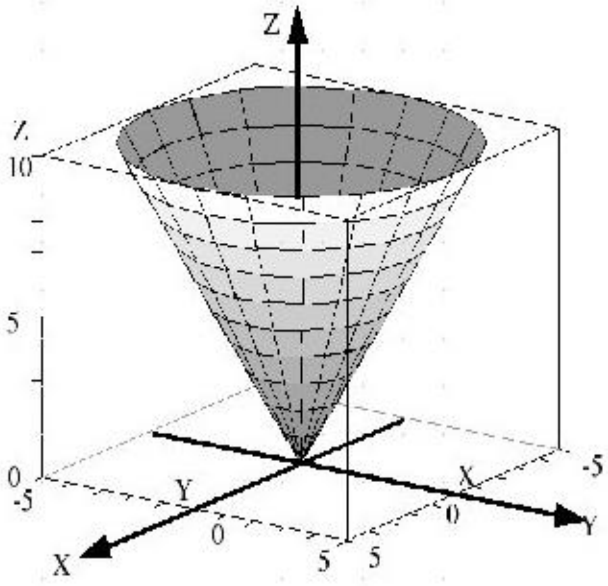
Special Cases

Introduction

- Linear or NonLinear Models
- Special NonLinear Models
- Linearizations

SCOP

Second-Order Cone Programming



Pyomo + Gurobi

SCOP Example

Suppose that you have 3 machines to manufacture shoes, and the cost of each machine is:

$$C_1 = 0.01n_1^2 + 2n_1$$
$$C_2 = 6n_2$$
$$C_3 = 7n_3$$

where C_i is cost for production of machine *i*, n_i is the number of shoes manufactured in machine *i*

Each machine has a limit of production of 1.000 shoes.

For a total production of 2.100 shoes, how many shoes should each machine made in order to minimize the total cost?

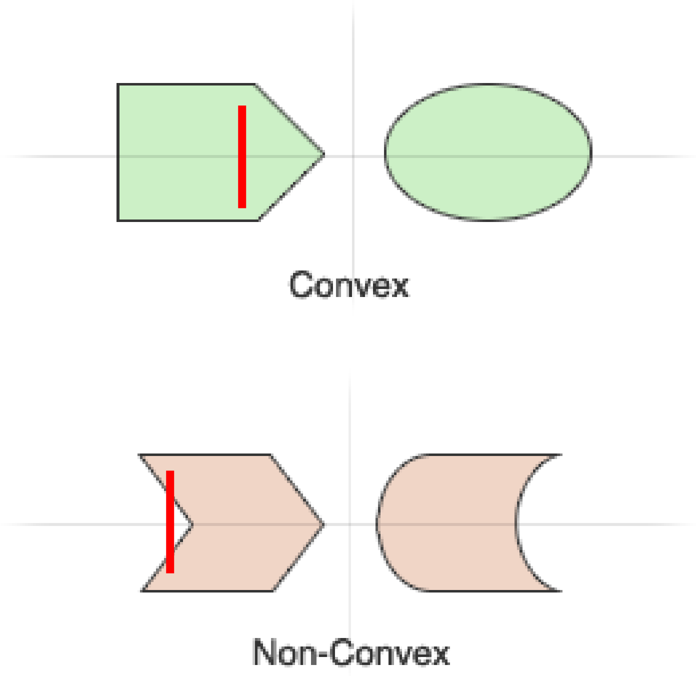
$$\min C_1 + C_2 + C_3$$

$$n_1 + n_2 + n_3 = 2100$$
$$C_1 = 0.01n_1^2 + 2n_1$$
$$C_2 = 6n_2$$
$$C_3 = 7n_3$$
$$0 \leq n_1, n_2, n_3 \leq 1000$$
$$n_1, n_2, n_3 \; as \; integers$$

NonConvex QP

NonConvex Quadratic Programming

https://www.gurobi.com/resource/non-convex-quadratic-optimization/



NonConvex QP Example

Suppose that you have 3 machines to manufacture shoes, and the cost of each machine is:

$$C_1 = 0.01n_1^2 + 2n_1 \qquad C_2 = 6n_2\boldsymbol{n_1} \qquad C_3 = 7n_3$$

where C_i is cost for production of machine *i*, n_i is the number of shoes manufactured in machine *i*

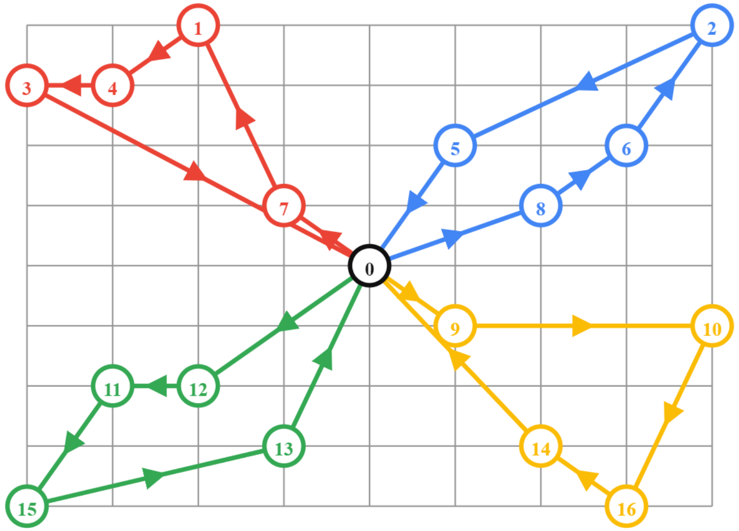
Each machine has a limit of production of 1.000 shoes.
For a total production of 2.100 shoes, how many shoes should each machine made in order to minimize the total cost?

$$\min C_1 + C_2 + C_3$$

$$n_1 + n_2 + n_3 = 2100$$
$$C_1 = 0.01n_1^2 + 2n_1$$
$$\boldsymbol{C_2 = 6n_2n_1}$$
$$C_3 = 7n_3$$
$$0 \le n_1, n_2, n_3 \le 1000$$
$$n_1, n_2, n_3\ as\ integers$$

Routing Problems

https://developers.google.com/optimization/routing/vrp



OR-TOOLS VRP

Linearization BigM

Binary * Continuos

$$C = b * x$$
$$b \text{ is binary}$$

Similar to

$$-b * M \leq C \leq b * M$$
$$-(1 - b) * M \leq C - x \leq (1 - b) * M$$
$$b \text{ is binary}$$

Linearization
BigM
Binary * Continuos

$$C = b * x$$
$$b \text{ is binary}$$

Similar to

$$-0 * M \leq C \leq 0 * M$$
$$-(1-0) * M \leq C - x \leq (1-0) * M$$

Linearization BigM
Binary * Continuos

$$C = b * x$$
$$b \text{ is binary}$$

Similar to

$$0 \leq C \leq 0$$
$$-M \leq C - x \leq M$$

Linearization BigM

Binary * Continuos

$$C = b * x$$

$$b \text{ is binary}$$

Similar to

$$C = 0$$

x can be any value

Linearization BigM

Binary * Continuos

$$C = b * x$$

$$b \text{ is binary}$$

Similar to

$$-1 * M \leq C \leq 1 * M$$

$$-(1-1) * M \leq C - x \leq (1-1) * M$$

Linearization BigM

Binary * Continuos

$$C = b * x$$
$$b \text{ is binary}$$

Similar to

$$-M \leq C \leq M$$
$$0 \leq C - x \leq 0$$

Linearization
BigM
Binary * Continuos

$$C = b * x$$
$$b \text{ is binary}$$

Similar to

$$-M \leq C \leq M$$
$$C - x = 0$$

Linearization BigM

Binary * Continuos

$$C = b * x$$

$$b \textit{ is integer}$$

Similar to

C can be any value

$$C = x$$

*Binary*Continuous Example*

Suppose that you have 3 machines to manufacture shoes, and the cost of each machine is:

$$\boldsymbol{C_1 = 2n_1} \qquad C_2 = 6n_2 \qquad C_3 = 7n_3$$

where C_i is cost for production of machine *i*, n_i is the number of shoes manufactured in machine *i*

Each machine has a limit of production of 1.000 shoes.
Machine 2 has a start cost of 1000.
For a total production of 2.100 shoes, how many shoes should each machine made in order to minimize the total cost?

$$\min C_1 + C_2 + C_3$$
$$n_1 + n_2 + n_3 = 2100$$
$$\boldsymbol{C_1 = 2n_1}$$
$$\boldsymbol{C_2 = b * (6n_2 + 1000)}$$
$$\boldsymbol{n_2 \le b * 1000}$$
$$C_3 = 7n_3$$
$$0 \le n_1, n_2, n_3 \le 1000$$
$$n_1, n_2, n_3 \ as\ integers$$

$b\ as\ binary$
b=1 represents that machine 2 is ON
b=0 represents that machine 2 is OFF

$$\boldsymbol{-b * M \le C_2 \le b * M}$$
$$\boldsymbol{-(1-b) * M \le C_2 - (6n_2 + 1000) \le (1-b) * M}$$

Linearization

Binary * Binary

$$C = b_1 * b_2$$

$$b_1 \text{ and } b_2 \text{ are binaries}$$

Similar to

$$C = z$$

$$z \le b_1$$

$$z \le b_2$$

$$z \ge b_1 + b_2 - 1$$

$$z, b_1, b_2 \text{ are binaries}$$

*Binary*Binary Example*

Suppose that you have 3 machines to manufacture shoes, and the cost of each machine is:

$$\boldsymbol{C_1 = 2n_1} \qquad C_2 = 6n_2 \qquad C_3 = 7n_3$$

where C_i is cost for production of machine *i*, n_i is the number of shoes manufactured in machine *i*

Each machine has a limit of production of 1.000 shoes.

Machine 2 can only be ON if Machine 1 is ON

For a total production of 2.100 shoes, how many shoes should each machine made in order to minimize the total cost?

$$\min C_1 + C_2 + C_3$$

$$n_1 + n_2 + n_3 = 2100$$

$$\boldsymbol{C_1 = 2n_1}$$

$$\boldsymbol{n_1 \leq b_1 * 1000}$$

$$C_2 = 6n_2$$

$$\boldsymbol{n_2 \leq b_1 * b_2 * 1000}$$

$$C_3 = 7n_3$$

$$0 \leq n_1, n_2, n_3 \leq 1000$$

$$n_1, n_2, n_3 \ as \ integers$$

$$b_1, b_2 \ as \ binary$$

b_i=1 represents that machine *i* is ON

b_i=0 represents that machine *i* is OFF

$$\boldsymbol{n_2 \leq z * 1000}$$

$$\boldsymbol{z \leq b_1}$$

$$\boldsymbol{z \leq b_2}$$

$$\boldsymbol{z \geq b_1 + b_2 - 1}$$

$$\boldsymbol{z \ as \ binary}$$

*Binary*Binary Example*

A good alternative would be using b2<b1, as the following example.
Using this alternative, you do not need to use b2*b1; however, for this class, I will continue with b2*b1 so we can practice how to work with the multiplication of two binaries variables

$$\min C_1 + C_2 + C_3$$

$$n_1 + n_2 + n_3 = 2100$$

$$\boldsymbol{C_1 = 2n_1}$$
$$\boldsymbol{n_1 \leq b_1 * 1000}$$
$$C_2 = 6n_2$$
$$\boldsymbol{n_2 \leq b_2 * 1000}$$
$$\boldsymbol{b_2 \leq b_1}$$
$$C_3 = 7n_3$$
$$0 \leq n_1, n_2, n_3 \leq 1000$$
$$n_1, n_2, n_3 \ as\ integers$$

$$b_1, b_2 \ as\ binary$$

Advanced Features for Pyomo

Case Study

Case Study

Suppose you have **4 machines** on your computer's factory

You have to define the number of computers that each machine must produce during the next **10 hours** in order to maximize the total production. The objective function of our problem is given by

$$max \sum_m \sum_t x_{m,t}$$

where $x_{m,t}$ represents the number of computers produced by a machine *m* at hour *t*.

Some machines have dependencies on others and the constraints are (for each t)

$$2x_{2,t} - 8x_{3,t} \leq 0 \qquad (1)$$

$$x_{2,t} - 2x_{3,t-2} + x_{4,t} \geq 1 \qquad (2)$$

And the capacity production is given by (for all t)

$$\sum_m x_{m,t} \leq 50 \qquad (3)$$

$$x_{1,t} + x_{2,t-1} + x_{3,t} + x_{4,t} \leq 10 \qquad (4)$$

$$0 \leq x_{m,t} \leq 10 \qquad (5)$$

Solve the problem and find the optimal production of computers for each machine for each hour of the next 10 hours.

Note the $x_{2,t-1}$, in the last constraint, represents the second machine at hour *t-1*
Obs.: Constraints (2) and (4) does not exist for t<3 and t<2, respectively

Case Study

$$max \sum_{m} \sum_{t} x_{m,t}$$

$$2x_{2,t} - 8x_{3,t} \leq 0 \qquad \forall t$$

$$x_{2,t} - 2x_{3,t-2} + x_{4,t} \geq 1 \qquad \forall t > 2$$

$$\sum_{m} x_{m,t} \leq 50 \qquad \forall t$$

$$x_{1,t} + x_{2,t-1} + x_{3,t} + x_{4,t} \leq 10 \qquad \forall t > 1$$

$$0 \leq x_{m,t} \leq 10 \qquad \forall m, \forall t$$

Solver progress

*opt.solve(model, **tee=True**)*

Gap Limit

```
opt.options['MIPgap'] = 0.0001
```

https://www.gurobi.com/documentation/9.1/refman/mipgap2.html

Time Limit

```
opt.options[‘TimeLimit'] = 60
```

Inequallity

$$A \leq x \leq B$$

model.C = pyo.Constraint(**pyo.inequality(A,x,B)**)

summation

$$\sum_{m}\sum_{t} x_{m,t}$$

pyo.summation(x)

Parameters and Sets

```
pyo.Param(initialize=VALUE)

pyo.Set(initialize=LIST)

pyo.RangeSet(BEGIN,END)
```

Constraint's Rules

```python
model.C1 = pyo.Constraint(expr = 2*x + 2*y == 0)
model.C2 = pyo.Constraint(expr = x - 3*y >= 5)
```

SAME AS

```python
model.C1 = pyo.Constraint(rule=myrule1)
model.C2 = pyo.Constraint(rule=myrule2)

def myrule1(model):
        return 2*model.x + 2*model.y == 0
def myrule2(model):
        return model.x - 3*model.y >= 5
```

Rules in the Objective Function

```
pyo.Objective(expr = pyo.summation(x), sense=pyo.maximize)
```

SAME AS

pyo.Objective(**rule=myobj**, sense=pyo.maximize)

```
def myobj(model):
    return pyo.summation(model.x)
```

Constraint’s Rules with indexes

```
model.C1 = pyo.ConstraintList()
for t in model.setT:
   model.C1.add(expr = 2*x[2,t] - 8*x[3,t] <= 0)
```

SAME AS

```
model.C1 = pyo.Constraint(model.setT, rule=myrule)

def myrule(model, t):
   return 2*model.x[2,t] - 8*model.x[3,t] <= 0
```

*pyo.Constraint(RangeIndex1, RangeIndex2, ... , **rule=myrule**)*

Warmstart

https://pyomo.readthedocs.io/en/stable/working_models.html#warm-starts

```
instance.y[0] = 1
instance.y[1] = 0

opt = pyo.SolverFactory("cplex")

results = opt.solve(instance, warmstart=True)
```

Differential Algebraic Equations (DAE)

https://pyomo.readthedocs.io/en/stable/modeling_extensions/dae.html

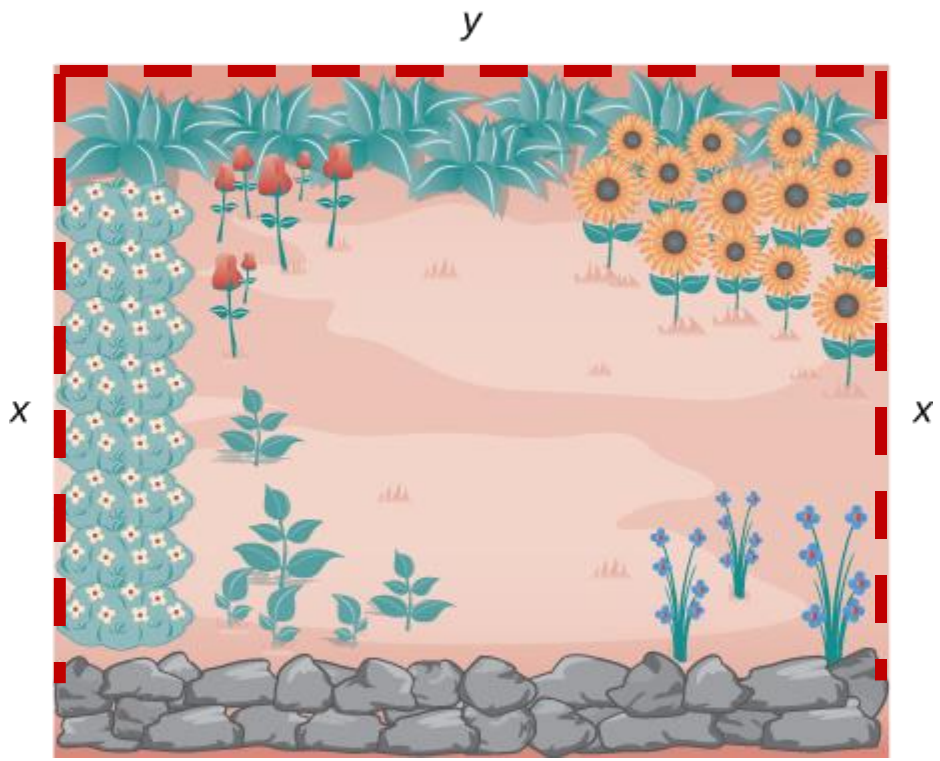
```
Required imports
>>> from pyomo.environ import *
>>> from pyomo.dae import *

>>> model = ConcreteModel()
>>> model.s = Set(initialize=['a','b'])
>>> model.t = ContinuousSet(bounds=(0,5))
>>> model.l = ContinuousSet(bounds=(-10,10))

>>> model.x = Var(model.t)
>>> model.y = Var(model.s,model.t)
>>> model.z = Var(model.t,model.l)

Declare the first derivative of model.x with respect to model.t
>>> model.dxdt = DerivativeVar(model.x, withrespectto=model.t)

Declare the second derivative of model.y with respect to model.t
Note that this DerivativeVar will be indexed by both model.s and model.t
>>> model.dydt2 = DerivativeVar(model.y, wrt=(model.t,model.t))

Declare the partial derivative of model.z with respect to model.l
Note that this DerivativeVar will be indexed by both model.t and model.l
>>> model.dzdl = DerivativeVar(model.z, wrt=(model.l), initialize=0)

Declare the mixed second order partial derivative of model.z with respect
to model.t and model.l and set bounds
>>> model.dz2 = DerivativeVar(model.z, wrt=(model.t, model.l), bounds=(-10, 10))
```

Practical Examples

Fence in the Garden

What is the largest area that we can fence in a garden using 100 meters of fence? Define the dimensions of this garden as well.

Note: The garden is already fenced by a wall of rocks in one of its sides.



$$\max xy$$

$$2x + y \leq 100$$

$$x, y > 0$$

Maximize Revenue

A car rental company wish to to maximize its revenue.

From the historical sales data, it is known that practicing a rent price (p) between 50 and 200 dollar, the number of cars rented per day is N(p) = 1001-5p.

What is the rent price that maximizes the daily revenue?
And what is the expected number of cars to be rented?

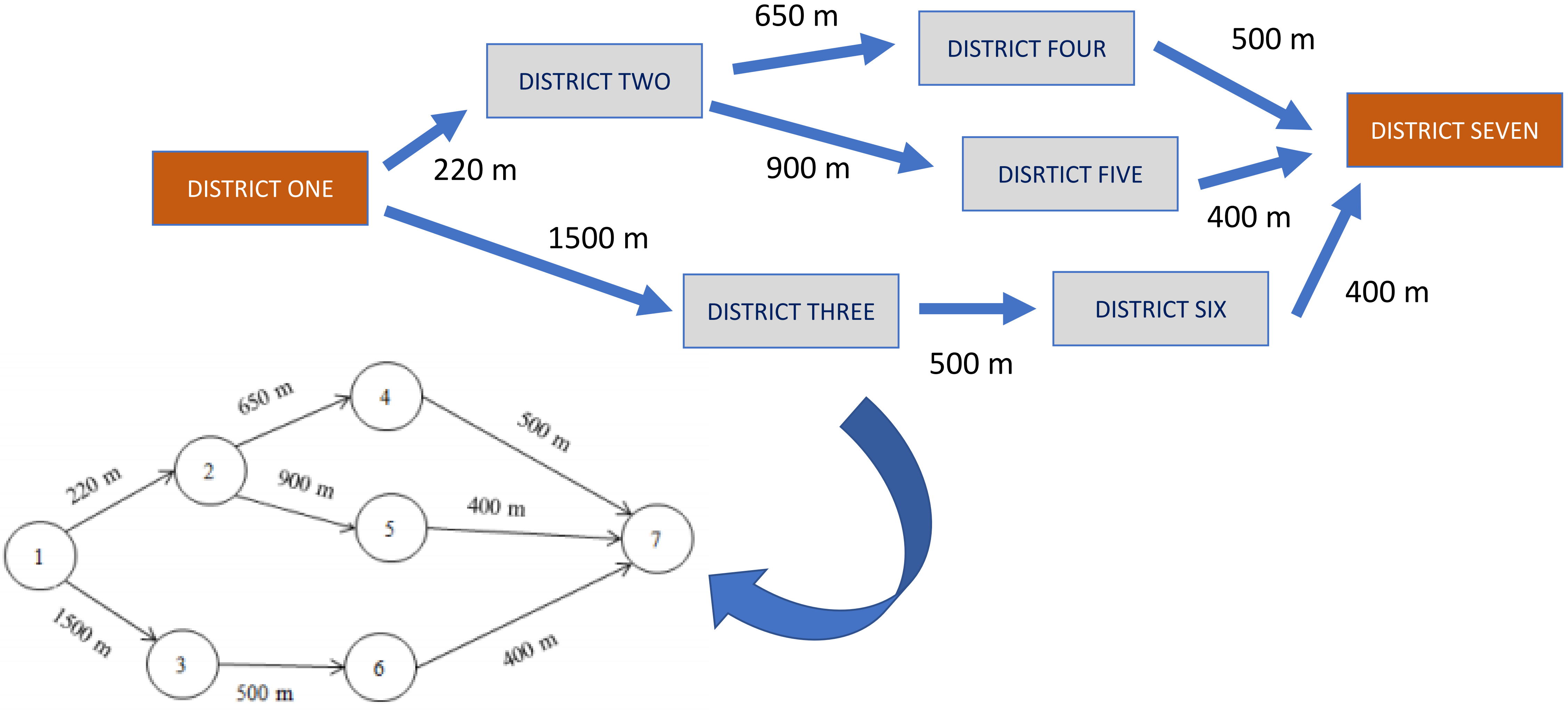
$$\max p\, N$$

$$N = 1001 - 5p$$

$$50 \leq p \leq 200$$

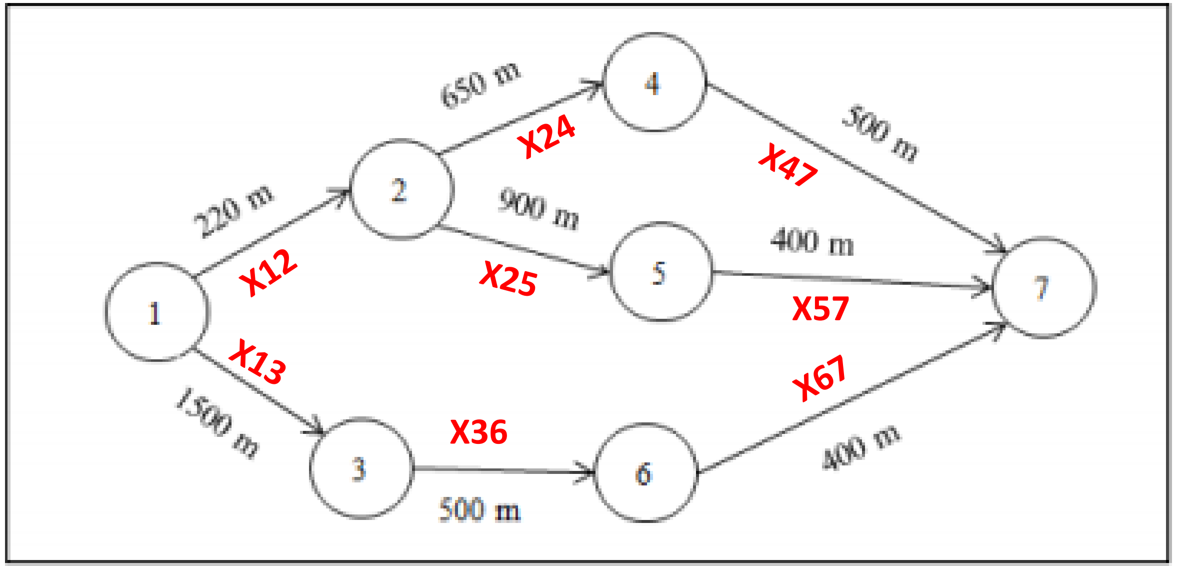
$$N \textit{ integer}$$

Route Optimization



Route Optimization

Which is the best route from point 1 to point 7?



$$\min \sum x_{ij} D_{x_{ij}}$$

$$\sum_{out} x_{ij} = 1 \quad \textit{origin node}$$

$$\sum_{in} x_{ij} = 1 \quad \textit{destination node}$$

$$\sum_{out} x_{ij} \leq 1 \quad \forall node/(origin, destination)$$

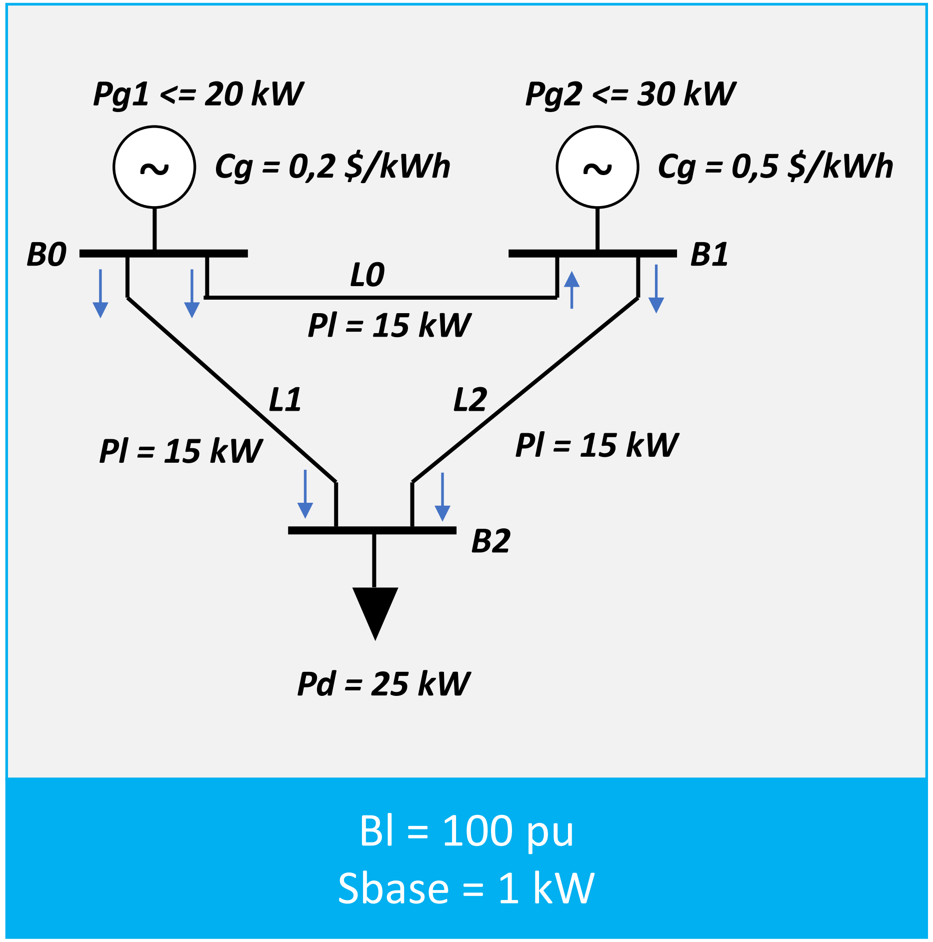
$$\sum_{in} x_{ij} \leq 1 \quad \forall node/(origin, destination)$$

$$\sum_{in} x_{ij} = \sum_{out} x_{ij} \quad \forall node/(origin, destination)$$

$$all\ \ x_{ij}\ \ binary$$

Linear Optimal Power Flow: Power Systems

What are the optimal values for the generation power units in the following system? Consider the objective function as the minimization of the power generation cost.



$$\min \sum_{g} C_g P_g$$

$$\sum_{g \in \Omega_n^G} P_g - \sum_{l \in \Omega_{n=l(s)}^L} P_l + \sum_{l \in \Omega_{n=l(r)}^L} P_l = \sum_{d \in \Omega_n^D} P_d \quad \forall n$$

$$P_l = B_l(\theta_{l(n=s)} - \theta_{l(n=r)}) \quad \forall l$$

$$0 \le P_g \le P_g^{max} \quad \forall g$$

$$-P_l^{max} \le P_l \le P_l^{max} \quad \forall l$$

$$-\pi \le \theta_n \le \pi \quad \forall n$$

$$\theta_n = 0 \quad n{:}\, ref(0)$$

Congratulations!!

Challenges:
https://math.libretexts.org/Courses/Mount_Royal_University/MATH_1200%3A_Calculus_for_Scientists_I/3%3A_Applications_of_Derivatives/3.6%3A_Applied_Optimization_Problems